Application of the HAA Algorithm to the Fine-Structure Constant α^{-1} (CODATA 2022). As a test case, we apply the HAA algorithm to the reciprocal value of the fine-structure constant, $\alpha^{-1} = 137.035999177$.

Algorithm Parameters.

- Target value: $\alpha^{-1} = 137.035999177$
- Initial approximation: $S_0 = \lfloor \alpha^{-1} \rfloor = 137$
- Required relative accuracy: $\varepsilon = 9.99 \times 10^{-294}$
- Maximum number of restarts: 30
- Maximum base index: $q_{\max} = 650$

TABLE 1. Results of the HAA algorithm applied to α^{-1}

Parameter	**Value**
Number of restarts	1
Total number of corrections	384
Absolute error	$\Delta = 1.366872529 \times 10^{-291}$
Relative error	$\delta = 9.974550756 \times 10^{-294}$
Number of accurate decimals	~ 290
Dominant term	$2\kappa_4$
Number of corrections with sign +	190
Number of corrections with sign −	194

Analytic Representation. $\alpha^{-1} \approx 137 + 2\kappa_{4} + \kappa_{5} + \kappa_{6} + \kappa_{7} - \kappa_{8} - \kappa_{9} + \kappa_{10} - \kappa_{11} - \kappa_{13} + \kappa_{14} + \kappa_{17} + \kappa_{18} + \kappa_{22} - \kappa_{23} + \kappa_{25} + \kappa_{28} - \kappa_{29} + \kappa_{31} - \kappa_{32} - \kappa_{33} - \kappa_{35} - \kappa_{36} + \kappa_{37} - \kappa_{39} - \kappa_{40} + \kappa_{43} - \kappa_{45} - \kappa_{46} - \kappa_{47} + \kappa_{48} - \kappa_{50} - \kappa_{53} - \kappa_{54} + \kappa_{55} + \kappa_{56} - \kappa_{57} + \kappa_{59} - \kappa_{64} - \kappa_{65} + \kappa_{68} - \kappa_{70} - \kappa_{71} + \kappa_{76} - \kappa_{78} - \kappa_{81} + \kappa_{82} - \kappa_{83} - \kappa_{85} - \kappa_{87} - \kappa_{88} - \kappa_{90} + \kappa_{92} + \kappa_{93} + \kappa_{94} + \kappa_{98} - \kappa_{99} - \kappa_{100} - \kappa_{101} - \kappa_{102} + \kappa_{103} + \kappa_{105} - \kappa_{108} + \kappa_{110} + \kappa_{114} + \kappa_{115} - \kappa_{117} + \kappa_{119} - \kappa_{120} - \kappa_{124} - \kappa_{125} + \kappa_{126} - \kappa_{127} + \kappa_{128} - \kappa_{129} - \kappa_{130} + \kappa_{131} - \kappa_{132} + \kappa_{134} + \kappa_{135} - \kappa_{137} + \kappa_{140} - \kappa_{141} + \kappa_{142} - \kappa_{143} - \kappa_{144} + \kappa_{145} - \kappa_{149} - \kappa_{150} + \kappa_{151} - \kappa_{152} - \kappa_{153} + \kappa_{154} - \kappa_{155} + \kappa_{156} + \kappa_{158} + \kappa_{159} + \kappa_{160} - \kappa_{161} - \kappa_{162} + \kappa_{164} - \kappa_{165} - \kappa_{166} - \kappa_{171} + \kappa_{172} + \kappa_{174} + \kappa_{175} - \kappa_{176} + \kappa_{177} - \kappa_{178} - \kappa_{179} - \kappa_{180} - \kappa_{181} - \kappa_{183} - \kappa_{185} - \kappa_{186} - \kappa_{187} - \kappa_{191} + \kappa_{193} + \kappa_{194} + \kappa_{197} + \kappa_{198} + \kappa_{199} - \kappa_{202} + \kappa_{203} - \kappa_{207} + \kappa_{208} + \kappa_{209} + \kappa_{210} - \kappa_{211} - \kappa_{215} - \kappa_{216} - \kappa_{217} - \kappa_{218} - \kappa_{220} - \kappa_{221} - \kappa_{222} + \kappa_{223} - \kappa_{224} + \kappa_{225} + \kappa_{226} - \kappa_{227} - \kappa_{229} - \kappa_{231} - \kappa_{232} + \kappa_{233} - \kappa_{234} - \kappa_{236} - \kappa_{238} + \kappa_{239} + \kappa_{240} - \kappa_{243} + \kappa_{244} + \kappa_{245} - \kappa_{246} - \kappa_{247} + \kappa_{248} + \kappa_{249} - \kappa_{250} - \kappa_{251} - \kappa_{253} + \kappa_{254} - \kappa_{255} - \kappa_{256} + \kappa_{257} + \kappa_{258} + \kappa_{261} - \kappa_{262} + \kappa_{263} + \kappa_{265} + \kappa_{266} - \kappa_{267} - \kappa_{268} - \kappa_{269} + \kappa_{271} + \kappa_{273} - \kappa_{277} - \kappa_{279} - \kappa_{281} - \kappa_{282} - \kappa_{283} - \kappa_{284} + \kappa_{285} + \kappa_{286} - \kappa_{287} + \kappa_{289} + \kappa_{294} + \kappa_{295} + \kappa_{296} + \kappa_{298} - \kappa_{300} - \kappa_{308} + \kappa_{310} + \kappa_{312} - \kappa_{315} + \kappa_{316} - \kappa_{317} + \kappa_{318} - \kappa_{319} + \kappa_{320} - \kappa_{322} - \kappa_{323} + \kappa_{326} - \kappa_{327} + \kappa_{330} - \kappa_{331} + \kappa_{333} + \kappa_{335} + \kappa_{336} - \kappa_{338} + \kappa_{339} + \kappa_{340} + \kappa_{341} + \kappa_{343} + \kappa_{344} - \kappa_{346} - \kappa_{347} - \kappa_{349} - \kappa_{352} + \kappa_{353} - \kappa_{355} - \kappa_{356} - \kappa_{357} + \kappa_{358} - \kappa_{363} - \kappa_{366} - \kappa_{368} - \kappa_{370} + \kappa_{371} - \kappa_{372} - \kappa_{373} + \kappa_{375} + \kappa_{378} - \kappa_{379} + \kappa_{381} + \kappa_{382} + \kappa_{383} + \kappa_{384} + \kappa_{386} + \kappa_{392} + \kappa_{393} + \kappa_{394} + \kappa_{396} - \kappa_{397} + \kappa_{402} + \kappa_{403} + \kappa_{404} - \kappa_{405} + \kappa_{407} + \kappa_{408} + \kappa_{410} - \kappa_{411} - \kappa_{412} + \kappa_{413} + \kappa_{414} + \kappa_{415} + \kappa_{416} + \kappa_{418} + \kappa_{420} + \kappa_{423} + \kappa_{424} - \kappa_{426} + \kappa_{427} - \kappa_{429} + \kappa_{431} + \kappa_{432} + \kappa_{433} + \kappa_{434} + \kappa_{435} + \kappa_{437} + \kappa_{438} + \kappa_{439} - \kappa_{442} - \kappa_{444} + \kappa_{446} - \kappa_{447} - \kappa_{448} + \kappa_{449} - \kappa_{450} - \kappa_{452} - \kappa_{453} - \kappa_{455} - \kappa_{458} - \kappa_{462} + \kappa_{464} + \kappa_{465} + \kappa_{467} + \kappa_{468} +$

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