EEN320 - Power Systems I (Συστήματα Ισχύος Ι)

Part 7: Introduction to rotating machines

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Today's learning objectives

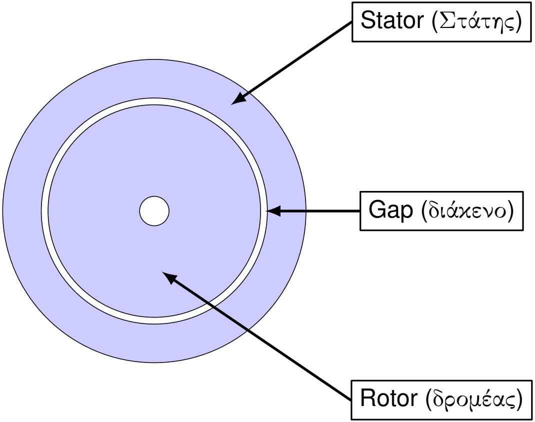
After this part of the lecture and additional reading, you should be able to ...

1. ...explain the basic principles of electromechanical energy conversion;
2. ...explain the fundamental principles of rotating machines.

Outline

1. Basic rotating machines principles
2. Machine stator and rotor
3. Power flows, efficiency and losses
4. Synchronous machine characteristics
5. Synchronous motor

1 Basic rotating machine components



1 Lorentz Force Law review

From electromagnetics, we know that Lorentz Force Law:

$$\underline{F} = q(\underline{E} + \underline{v} \times \underline{B})$$

where:

- $\underline{F}$ is the force (newtons) on a particle of charge q (coulombs) in the presence of electric and magnetic fields
- $\underline{E}$ is the electric field in volts per meter
- $\underline{B}$ is the magnetic field in teslas
- $\underline{v}$ is the velocity of the particle q relative to the magnetic field, in meters per second.

Ignoring the electric field:

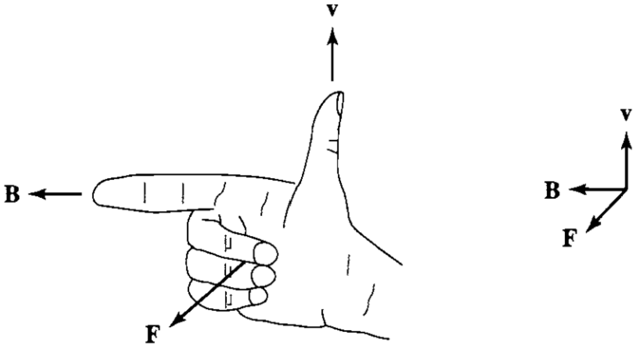


Figure 3.1 Right-hand rule for determining the direction magnetic-field component of the Lorentz force $\mathbf{F} = q(\mathbf{v} \times \mathbf{B})$.

Fitzgerald, A. E., Kingsley, C., & Umans, S. D. (2003). Electric machinery. McGraw-Hill.

1 Application of Lorentz Force Law on a rotor

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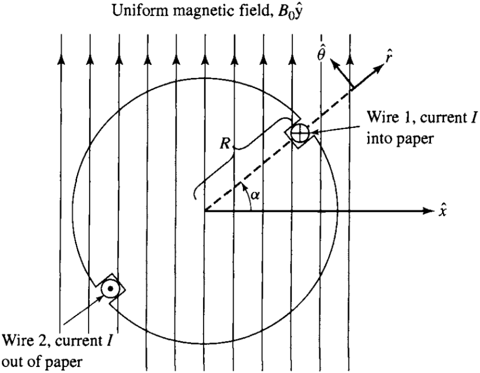
A non-magnetic rotor (δρομέας) containing a single-turn coil is placed in a uniform magnetic field of magnitude B_0 generated by the stator (see later), as shown below. The coil sides are at radius R and the wire carries current I. Find the θ-directed torque as a function of rotor position. Assume that the rotor is of length ℓ.



Adapted from "Fitzgerald, A. E., Kingsley, C., & Umans, S. D. (2003). Electric machinery. McGraw-Hill".

1 Application of Lorentz Force Law on a rotor

The force per unit length (in N) acting on the wire is given by

$$\underline{F} = \underline{I} \times \underline{B} \text{ N}$$

For wire of length ℓ and current I is given as:

$$F = -IB_0\ell \sin(\alpha)$$

For two wires:

$$F = -2IB_0\ell \sin(\alpha)$$

The total torque (in Nm) is then:

$$T = -2IRB_0\ell \sin(\alpha)$$

And, if we assume a rotation $\alpha = \omega t$:

$$T = -2IRB_0\ell \sin(\omega t)$$

1 Application of Lorentz Force Law on a rotor

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Since there is a current flowing, the rotor also produces a magnetic field. The magnetic flux density B_R generated by the rotor due to the current I is:

$$B_R = \mu H_R = \frac{\mu I}{G}$$

where G depends on the geometry of the rotor loop. For a circular one then $G = 2R$. For a rectangular one, G depends on the length-to-width ratio.
So, we get:

$$T = \frac{AG}{\mu} B_R B_0 \sin(\alpha)$$

where $A = 2R\ell$ is the area of the wire on the rotor if assumed rectangular.
We can rewrite as:

$$\boxed{T = kB_R B_0 \sin(\alpha) = k\underline{B}_R \times \underline{B}_0}$$

where $k = AG/\mu$ is a factor depending on the machine construction.

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Conceptual explanation:

1. A magnetic north and south poles can be associated with the stator and rotor of a machine due to the current flows;

2. Similar to a compass needle trying to align with the earth's magnetic field, these two sets of fields attempt to align;

3. If one of the fields (stator or rotor) rotates, the other ones tries to ”catch up”. Torque is associated with their displacement from alignment:
   - In a motor, the stator magnetic field rotates ahead of that of the rotor, ”pulling” on it and performing work
   - In a generator, the rotor magnetic field rotates ahead of that of the stator, ”pulling” on it and performing work

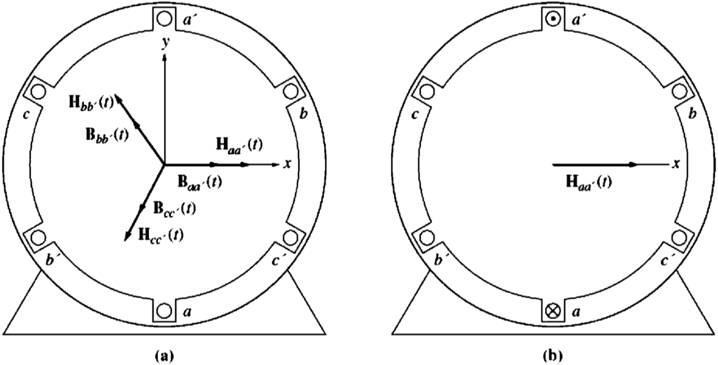
The torque in a real machine, depends on:

1. The strength of the rotor magnetic field;
2. The strength of the external (stator) magnetic field;
3. The $\sin$ of the angle between them; and,
4. A constant depending on the construction of the machine.

2 Outline

2 Three-phase machine stator

Assume now a three-phase stator (στάτης) with windings aa', bb', cc' as shown in the figure below:



Chapman, S.J. (2005). Electric machinery fundamentals (4e). McGraw-Hill.

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We feed the three coils with (in Ampere):

$$I_{aa'}(t) = I_M \sin(\omega t)$$
$$I_{bb'}(t) = I_M \sin(\omega t - 120^\circ)$$
$$I_{cc'}(t) = I_M \sin(\omega t - 240^\circ)$$

Which generates magnetic field intensity (in Ampere-turns/m):

$$\underline{H}_{aa'}(t) = H_M \sin(\omega t) \underline{/0^\circ}$$
$$\underline{H}_{bb'}(t) = H_M \sin(\omega t - 120^\circ) \underline{/120^\circ}$$
$$\underline{H}_{cc'}(t) = H_M \sin(\omega t - 240^\circ) \underline{/240^\circ}$$

- The direction of the field is shown on the figure and given by the ”right-hand rule”. The phase shown at the end is the spacial degrees.
- The magnitude changes sinusoidally but direction same.

2 Three-phase machine stator: magnetic field

The flux density is given by $\underline{B} = \mu \underline{H}$ (in Tesla):

$$
\begin{aligned}
\underline{B}_{aa'}(t) &= B_M \sin(\omega t) \underline{/0^\circ} \\
\underline{B}_{bb'}(t) &= B_M \sin(\omega t - 120^\circ) \underline{/120^\circ} \\
\underline{B}_{cc'}(t) &= B_M \sin(\omega t - 240^\circ) \underline{/240^\circ}
\end{aligned}
$$

where $B_M = \mu H_M$.

Examples:

$\omega t = 0^\circ$

$$\begin{aligned}\underline{B}_{net} &= \underline{B}_{aa'} + \underline{B}_{bb'} + \underline{B}_{cc'} \\ &= 0 + \left(-\frac{\sqrt{3}}{2}B_M\right)\underline{/120^\circ} + \left(\frac{\sqrt{3}}{2}B_M\right)\underline{/240^\circ} \\ &= 1.5B_M\underline{/-90^\circ}\end{aligned}$$

$\omega t = 90^\circ$

$$\begin{aligned}\underline{B}_{net} &= \underline{B}_{aa'} + \underline{B}_{bb'} + \underline{B}_{cc'} \\ &= B_M\underline{/0^\circ} + \left(-\frac{1}{2}B_M\right)\underline{/120^\circ} + \left(-\frac{1}{2}B_M\right)\underline{/240^\circ} \\ &= 1.5B_M\underline{/0^\circ}\end{aligned}$$

In the general case[1]:

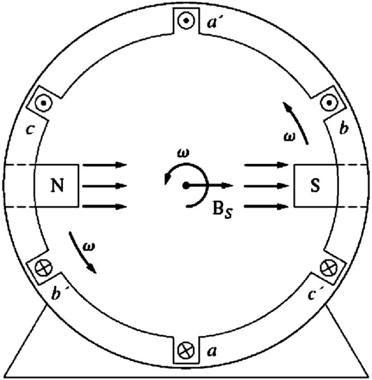
$$\underline{B}_{net} = 1.5B_M \left(\sin(\omega t)\underline{\hat{x}} - \cos(\omega t)\underline{\hat{y}}\right)$$

How about changing the rotation of the field?
$\rightarrow$ We swap the current in two of the phases:

$$\underline{B}_{net} = 1.5B_M \left(\sin(\omega t)\underline{\hat{x}} + \cos(\omega t)\underline{\hat{y}}\right)$$

[1] Try to prove this using the trigonometric relations used in part 2.

This is equivalent to a two-pole (north-south) field rotating:



The magnetic poles complete **one** full *mechanical* rotation for every **one** *electrical* cycle:

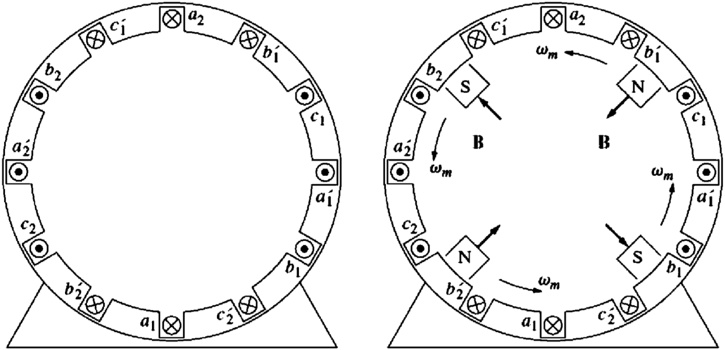
$$f_e = f_m$$

$$\omega_e = \omega_m$$

Chapman, S.J. (2005). Electric machinery fundamentals (4e). McGraw-Hill.

2 Three-phase machine stator: four-pole

This is equivalent to **two** two-pole (north-south) field rotating:



The magnetic poles complete **one** full *mechanical* rotation for every **two** *electrical* cycle:

$$\theta_e = 2\theta_m, \quad f_e = 2f_m, \quad \omega_e = 2\omega_m$$

Chapman, S.J. (2005). Electric machinery fundamentals (4e). McGraw-Hill.

In general for a *P*-poles machine:

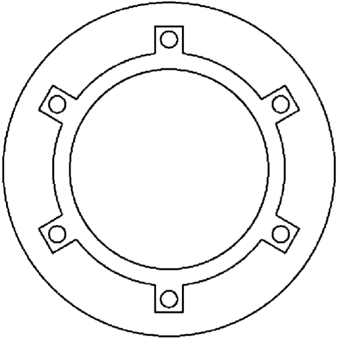
$$\boxed{\theta_e = \frac{P}{2}\theta_m} \text{ (rad)}$$

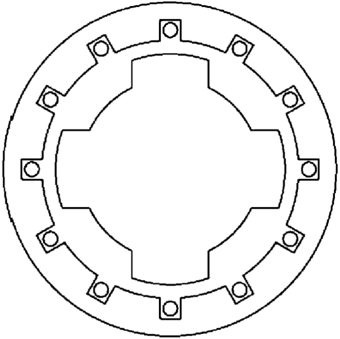
$$\boxed{f_e = \frac{P}{2}f_m} \text{ (Hz)}$$

$$\boxed{\omega_e = \frac{P}{2}\omega_m} \text{ (rad/s)}$$

$$\boxed{n = \frac{120 f_e}{P}} \text{ (rounds per minute)}$$

In general, there are two type of rotors: (a) cylindrical or nonsalient-pole (κυλινδρικός δρομέας) (b) salient-pole (έκτυπους πόλους).


(a)


(b)

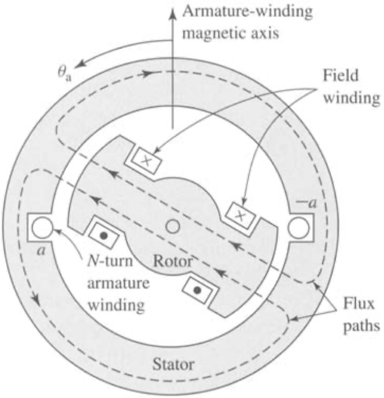
Chapman, S.J. (2005). Electric machinery fundamentals (4e). McGraw-Hill.

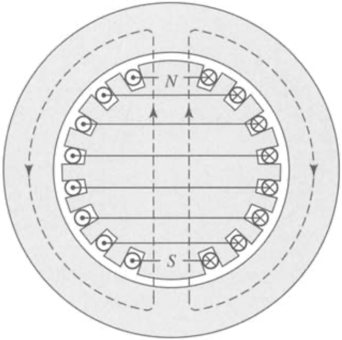
2 Three-phase machine rotor type

- Salient-pole is characteristic of hydroelectric generators because hydraulic turbines operate at lower speeds
  $\rightarrow$ lower speeds requires higher number of poles
  $\rightarrow$ salient poles are better mechanically for large number of poles.

- Steam and gas turbines operate better at high speeds and are commonly two- or four-pole cylindrical-rotor.

2 Three-phase machine rotor type





Salient pole field windings create the magnetic field. The construction of the poles generates a sinusoidal field.

Cylindrical rotor needs an uneven distribution of the conductors to generate a sinusoidal magnetic field.

Fitzgerald, A. E., Kingsley, C., & Umans, S. D. (2003). Electric machinery. McGraw-Hill.

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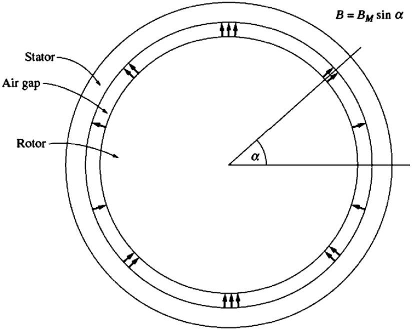
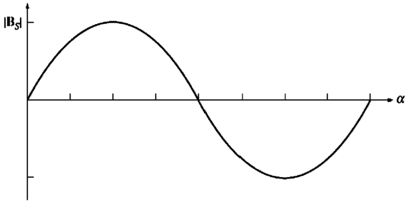




The conductors on the surface of the cylindrical rotor should be distributed as $n_C = N_C \cos(\alpha)$ with N_C the number of conductors at 0° (maximum).

If we freeze the rotor and we observe the magnetic field generated by it at an angle α, it will be sinusoidal:

$$\boxed{B = B_M \sin(\alpha)}$$

Chapman, S.J. (2005). Electric machinery fundamentals (4e). McGraw-Hill.

2 Three-phase machine rotor supply

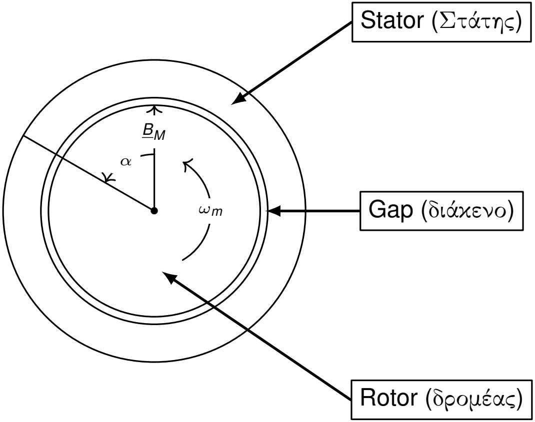
- In **all** three phase AC machines, the stator is fed with AC voltages, leading to a rotating magnetic field.
- In **synchronous (σύγχρονες)** machines, the rotor is fed with a DC current leading to a constant magnetic field.
  $\rightarrow$ As the constant magnetic field of the rotor tries to align to the rotating magnetic field of the stator, the rotor will rotate at constant *synchronous* speed (defined by the electrical frequency and the number of poles).
- In **induction (επαγωγής)** machines, the rotor is short-circuited (with a resistance) and alternating currents are induced by the stator field.
  $\rightarrow$ Think of it like a three-phase transformer: the AC currents in the stator (primary) generate a magnetic field that induces AC currents in the rotor (secondary).
  $\rightarrow$ The rotor does not rotate *synchronously* but it 'slips', meaning it operates at a different frequency than the stator.

2 Three-phase machine induced voltage

If we start rotating a rotor at a speed ω_m in a P-pole machine, the magnetic field observed at a constant location with angle α on the stator is now:

$$\boxed{B = B_M \cos(\omega t - \alpha)}$$

if we measure α from the direction of the peak flux density B_M and $\omega = \frac{P}{2}\omega_m$.

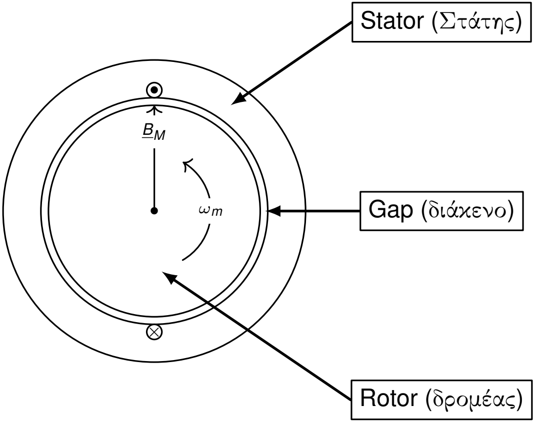


2 Three-phase machine induced voltage

✍ Beware that the field in the rotor is generated by a DC current, thus it has a **constant** direction. The field fluctuation is caused by the mechanical rotation of rotor. This is not the same in the stator field we studied previously where the field was fluctuating due to the sinusoidal currents in the three-phase windings.

2 Three-phase machine induced voltage

We place one stator winding as shown below at the point of peak flux density ($\alpha = 0$):



The magnetic field generated by the rotor B_M is seen by the stator winding as a varying field given by $B = B_M \cos(\omega t)$.

2 Three-phase machine induced voltage

Due to the rotating field, there is an induced voltage on the stator winding given by Faraday law:

$$e = -\frac{d\lambda}{dt}$$

with λ the flux linkage given by $\lambda = N_c\phi = N_c\Phi_M\cos(\omega t)$ (N_c the number of winding turns on the stator). Thus, the induced voltage is given as:

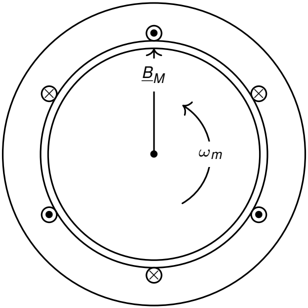
$$\boxed{e = -N_c\Phi_M\frac{d(\cos(\omega t))}{dt} = N_c\Phi_M\omega\sin(\omega t)}$$

In the particular case where the stator winding is rectangular with radius r and length ℓ, then the area is $A = 2r\ell$ and $\Phi_M = AB = 2r\ell B_M\cos(\omega t)$. Thus:

$$e = -2r\ell N_c B_M\frac{d(\cos(\omega t))}{dt} = 2r\ell N_c B_M\omega\sin(\omega t)$$

2 Three-phase machine induced voltage

Following the same analysis for three windings spaced 120° apart:



Gives (in Volt):

$$
\begin{aligned}
e_{aa'} &= N_c \Phi_M \omega \sin(\omega t) \\
e_{bb'} &= N_c \Phi_M \omega \sin(\omega t - 120^\circ) \\
e_{cc'} &= N_c \Phi_M \omega \sin(\omega t - 240^\circ)
\end{aligned}
$$

2 Three-phase machine induced voltage

The peak voltage at each phase is:

$$E_{max} = N_c \Phi_M \omega = N_c \Phi_M 2\pi f$$

with the RMS voltage:

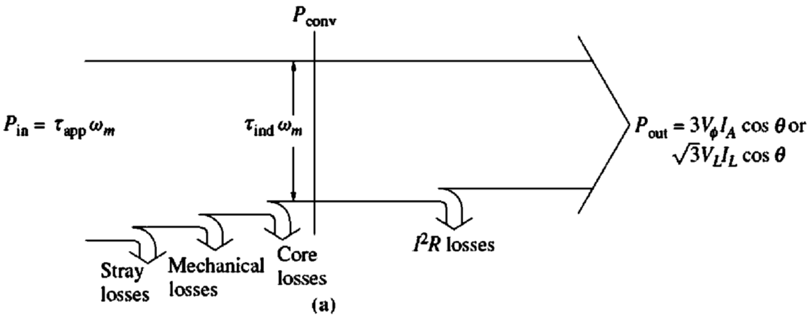
$$\boxed{E_{RMS} = \frac{N_c \Phi_M 2\pi f}{\sqrt{2}} = \sqrt{2} N_c \Phi_M \pi f = 4.44 N_c \Phi_M f}$$

- If the generator is connected in Y, then it's voltage is $\sqrt{3} E_{RMS}$.
- If the generator is connected in Delta, then it's voltage is E_{RMS}.

In phasor representation:

$$\underline{E}_A = E_{RMS} \underline{/0^\circ}$$
$$\underline{E}_B = E_{RMS} \underline{/-120^\circ}$$
$$\underline{E}_C = E_{RMS} \underline{/-240^\circ}$$

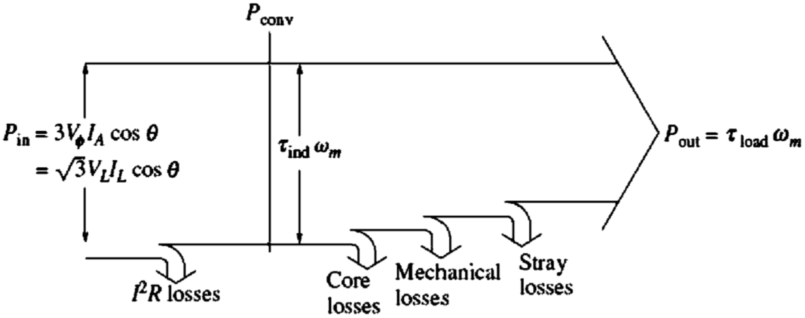
3 Power flows: Generator operation



- **Electrical losses:** $P = 3I^2R$
- **Core losses:** Losses in magnetic core (hysterisis, eddy currents, etc.)
- **Mechanical losses:** Friction and windage
- **Stray losses:** Everything not included above ($\approx 1\%$)

Chapman, S.J. (2005). Electric machinery fundamentals (4e). McGraw-Hill.

3 Power flows: Motor operation



(b)

Chapman, S.J. (2005). Electric machinery fundamentals (4e). McGraw-Hill.

3 Swing equation

The equation governing the rotor motion is called the *swing equation*:

$$J\frac{d^2\theta_m}{dt^2} = J\frac{d\omega_m}{dt} = T_a = T_m - T_e \quad \text{N-m}$$

where:

- J is the total moment of inertia of the rotor mass in $kg - m^2$
- θ_m is the angular position of the rotor with respect to a stationary axis in (rad)
- $\omega_m = \frac{d\theta_m}{dt}$ is the angular speed of the rotor with respect to a stationary axis in (rad/s)
- t is time in seconds (s)
- T_m is the mechanical torque supplied by the prime mover in N-m
- T_e is the electrical torque output of the alternator in N-m
- T_a is the net accelerating torque, in N-m

Multiplying both sides by ω_m, the can rewrite the equations as:

$$J\omega_m \frac{d^2\theta_m}{dt^2} = J\omega_m \frac{d\omega_m}{dt} = P_a = P_m - P_e \quad \text{W}$$

where P_a, P_m and P_e are the net, mechanical and electrical powers, respectively.

A useful representation is by introducing the inertia constant of the machine:

$$H = \frac{\text{stored kinetic energy in mega joules at synchronous speed}}{\text{machine rating in MVA}} = \frac{J\omega_s^2}{2S_{rated}} \quad \text{MJ/MVA}$$

where S_{rated} is the three-phase power rating of the machine in MVA.

3 Efficiency

The power efficiency of a machine is:

$$\boxed{\eta = \frac{P_{out}}{P_{in}} \times 100\%}$$

The voltage regulation is a measure of the ability of a generator to keep constant voltage at its terminals as load varies:

$$\boxed{VR = \frac{V_{no-load} - V_{full-load}}{V_{full-load}} \times 100\%}$$

A small *VR* is "better" as the voltage is more constant.

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The speed regulation is a measure of the ability of a motor to keep constant speed as load varies:

$$\boxed{SR = \frac{n_{no-load} - n_{full-load}}{n_{full-load}} \times 100\%}$$

$$\boxed{SR = \frac{\omega_{no-load} - \omega_{full-load}}{\omega_{full-load}} \times 100\%}$$

A small *SR* is ”better” as the speed is more constant.

4 Outline

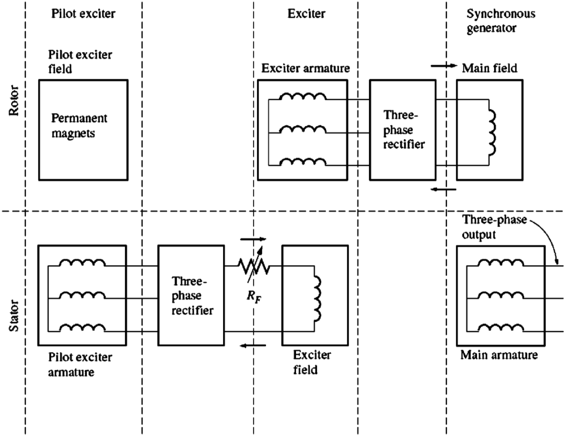
- To supply the rotor with DC current we:
  1. Supply from external DC source by means of slip rings and brushes.
  2. Supply from a special DC power source mounted on the shaft.
- We can make generator independent of external sources by including a small pilot exciter (usually, a small permanent magnet generator) mounted on the rotor shaft.
- It is reminded that:

$$\boxed{E_{RMS} = \sqrt{2} N_c \Phi_M \pi f}$$

with Φ_M depending on B_M which depends on the field current I_F. Thus, the generator output voltage E_{RMS} is proportional to I_F.

Chapman, S.J. (2005). Electric machinery fundamentals (4e). McGraw-Hill.

4 Field current example



A brushless excitation scheme that includes a pilot exciter. The permanent magnets of the pilot exciter produce the field current of the exciter. Which in ~~turn produces the field current of the~~ main machine.

Chapman, S.J. (2005). Electric machinery fundamentals (4e). McGraw-Hill.

4 (Simplified) equivalent circuit of synchronous generator

The induced voltage $\underline{E}_A$ is usually **different** from the real synchronous machine output $\underline{V}_A$:

1. There is a distortion in the magnetic field of the stator due to the current flowing in the windings, called *armature reaction*.
2. Self-inductance of the armature coils.
3. Resistance of the armature coils.
4. Effect of salient-pole rotor shape.

1. Armature reaction

- If the generator is feeding a load (inductive or capacitive), then the currents in the windings will generate their own field $\underline{B}_S$, distorting the rotor one. The net magnetic field will now be:

$$\underline{B}_{net} = \underline{B}_R + \underline{B}_S$$

- The rotor field $\underline{B}_R$ induces the voltage $\underline{E}_A$ as shown in Slide 31. The stator field $\underline{B}_S$ induces a voltage $\underline{E}_S$ which lies 90° behind the current $\underline{I}_A$ in the stator[2].

- The stator induced voltage can then be modelled as $\underline{E}_S = -jX\underline{I}_A$, with X a proportional constant.

[2] Why? Use Faraday's law to explain.

2+3. Self-inductance and resistance of armature coils

As expected, the coils in the stator have a self-inductance L_A and a resistance R_A that create a voltage drop $-jX_A\underline{I}_A$ and $-R_A\underline{I}_A$, respectively.

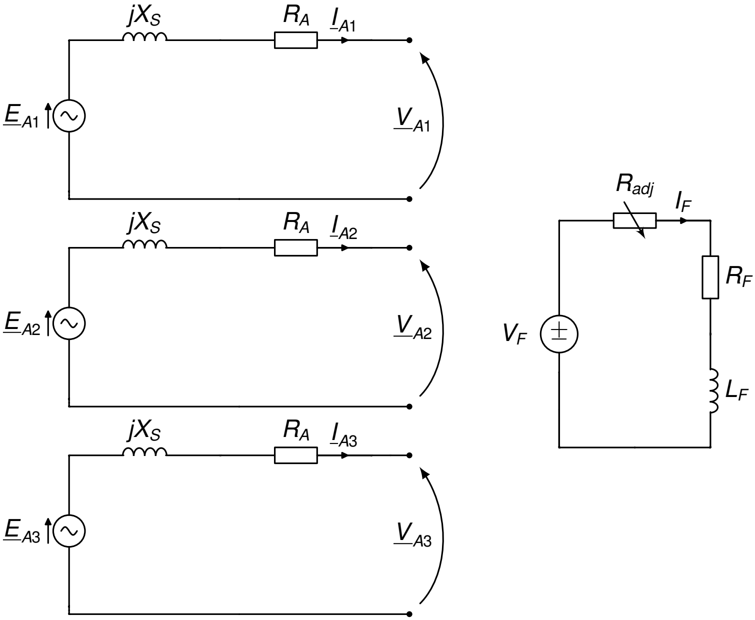
Equivalent model

Combining the above, the (simplified) equivalent model is given:

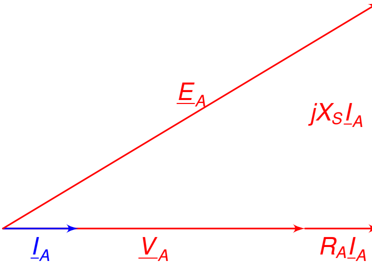
$$\boxed{\underline{V}_A = \underline{E}_A - jX\underline{I}_A - jX_A\underline{I}_A - R_A\underline{I}_A = \underline{E}_A - jX_S\underline{I}_A - R_A\underline{I}_A}$$

with $X_S = X + X_A$ the synchronous reactance of the generator.

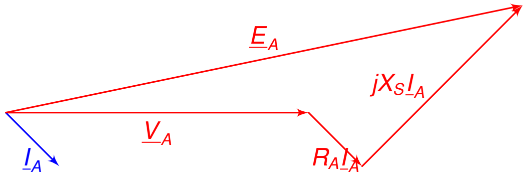
4 (Simplified) equivalent circuit of synchronous generator



For a load current with unit power factor:

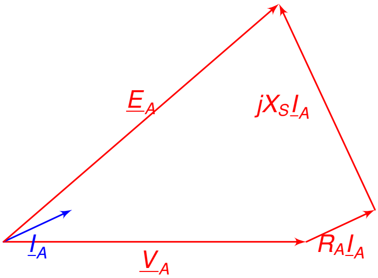


For a load current with lagging power factor:



To keep V_A constant for a lagging load, we need to increase E_A. How?

For a load current with leading power factor:



To keep V_A constant for a leading load, we need to decrease E_A. How?

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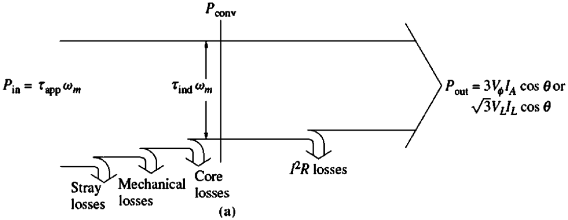
Let's get back the power flow diagram of a generator:



The power converted from mechanical to electrical is:

$$P_{conv} = \tau_{ind}\omega_m = 3E_A I_A \cos(\gamma)$$

where γ is the angle between $\underline{E}_A$ and $\underline{I}_A$.

Chapman, S.J. (2005). Electric machinery fundamentals (4e). McGraw-Hill.

4 Synchronous generator power and torque

The output power is:

$$P_{out} = 3V_A I_A \cos(\theta) \quad Q_{out} = 3V_A I_A \sin(\theta)$$

where θ is the angle between $\underline{V}_A$ and $\underline{I}_A$ and the power angle at the generator terminal.

If we ignore the resistance R_A (since $R_A << X_S$), we can use the power flow equations over a reactance, derived in Part 6, to get the generator power output and torque:

$$\boxed{P = \frac{3V_A E_A}{X_S}\sin(\delta)} \quad \boxed{\tau = \frac{3V_A E_A}{\omega_m X_S}\sin(\delta)}$$

with δ the angle between $\underline{E}_A$ and $\underline{V}_A$, also called *torque angle*.

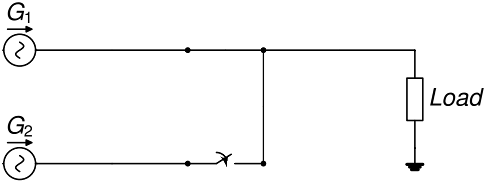
Q: What is the maximum power of the generator, if we keep E_A and V_A constant?

4 Parallel operation of synchronous generator or connection to grid

- Parallel operation of synchronous generators is necessary to increase security and reliability, minimise cost, and increase flexibility for dispatching and maintenance.

- When connecting a generator to the grid, if the switch is closed arbitrarily, the generators might be severely damaged, and the load may lose power.

- If the voltages are not **exactly** the same in each conductor being tied together, there will be a very large current flow when the switch is closed.



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- The rms line voltages of the two generators must be equal.
- The two generators must have the same phase sequence.
- The phase angles of the two α phases must be equal.
- The frequency of the new generator, called the oncoming generator, must be slightly higher than the frequency of the running system.

5 Outline

5 Synchronous motor equivalent model

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The operation is inverted:

- Three-phase voltage is applied on the stator, creating a **rotating** magnetic field $\underline{B}_S$.
- The DC field current generates a magnetic field $\underline{B}_R$.
- Since the stator magnetic field is rotating, the rotor magnetic field (and hence the rotor itself) will constantly try to catch up. The larger the angle between th e two magnetic fields (up to a certain maximum), the greater the **torque** on the rotor of the machine.
- The basic principle of synchronous motor operation is that the rotor ”chases” the rotating stator magnetic field around in a circle, never quite catching up with it.
- Most of the characteristics of the motor are the same as the generator seen before.

5 Synchronous motor equivalent model

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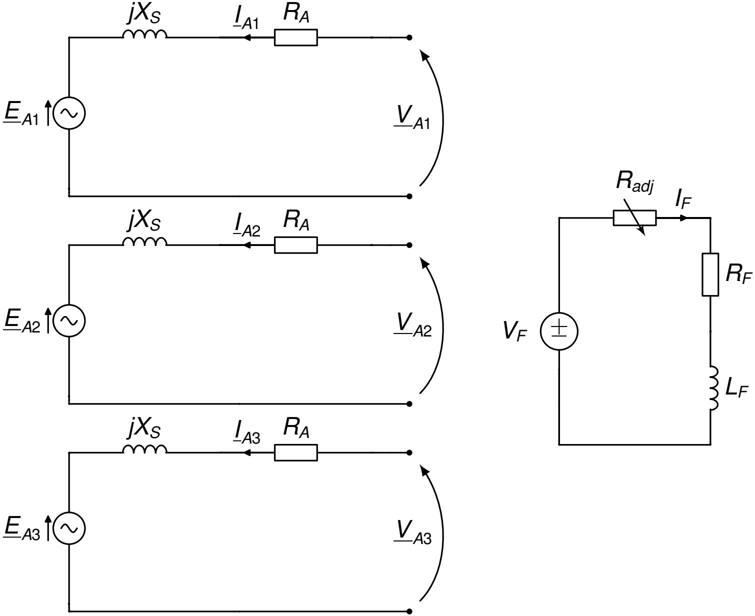


5 Synchronous motor equivalent model

With the equivalent model equation inverted to:

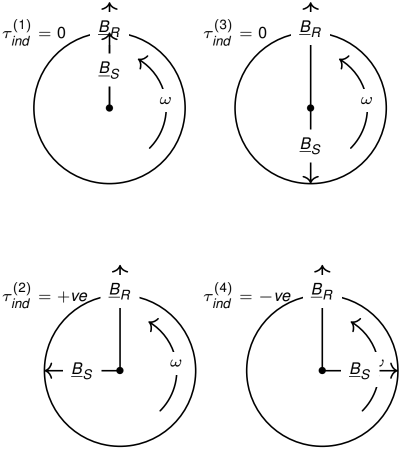
$$\boxed{\underline{E}_A = \underline{V}_A - jX_S\underline{I}_A - R_A\underline{I}_A}$$

- In a generator, $\underline{E}_A$, lies ahead of $\underline{V}_A$
- In a motor, $\underline{E}_A$ lies behind $\underline{V}_A$
- The angle between them is δ, also called torque angle
- The torque and power, similar to the generator case, is given by:

$$\boxed{P = \frac{3V_AE_A}{X_S}\sin(\delta)} \quad \boxed{\tau = \frac{3V_AE_A}{\omega_m X_S}\sin(\delta)}$$

Q: At which angle do we get the maximum or ”pull-out” torque?

If we try to start a motor by simply supplying voltages to the stator, it just vibrates and fails as the average torque is zero. Assume a 50 Hz two-pole generator:



1. At $t = 0$, the two magnetic fields are in the same direction, so the torque is $\tau_{ind} = k\underline{B}_R \times \underline{B}_S = 0$.
2. A quarter of a period later ($t = 1/200\ s$), the two fields are in 90° angle and the torque is clockwise.
3. Half a period later ($t = 2/200\ s$), the two fields are in 180° angle and the torque is again zero.
4. 3/4 of a period later ($t = 3/200\ s$), the two fields are in 270° angle and the torque is anti-clockwise.

5 Starting a synchronous motor

To start a motor, the most popular methods are:

1. Reduce the speed of the stator magnetic field to a low enough value that the rotor can accelerate and lock in with it during one half-cycle of the magnetic field 's rotation. This can be done by reducing the frequency of the applied electric power. Power electronics are used through AC-DC-AC conversion that can vary the frequency at the motor side.

2. Use an external prime mover to accelerate the synchronous motor up to synchronous speed, go through the paralleling procedure, and bring the machine on the line as a generator. Then, turning off or disconnecting the prime mover will make the synchronous machine a motor.

3. Use damper windings or amortisseur windings.

5 Summary

- Electromechanical energy conversion is achieved through the interaction of the magnetic fields in the stator and the rotor.
- Forces and torques develop to align the magnetic fields, dictated by the Lorentz law.
- In a generator, the magnetic field of the rotor induces voltages to the stator windings.
- In a motor, the rotating magnetic field of the stator induces a torque on the rotor and on the load.