



Cyprus  
University of  
Technology

## EEN320 - Power Systems I (Συστήματα Ισχύος Ι)

Part 6: Fundamentals of power system operation

Dr Petros Aristidou

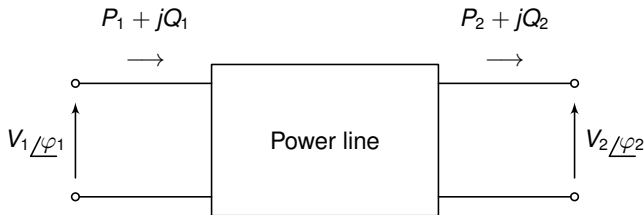
Department of Electrical Engineering, Computer Engineering & Informatics

Last updated: April 10, 2020

After this part of the lecture and additional reading, you should be able to . . .

- ① . . . describe and analyse the behaviour of a transmission line under different operating conditions;
- ② . . . explain the Ferranti effect.

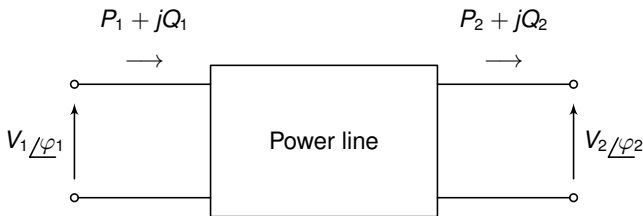
- In this part of the lecture, we investigate the stationary current and voltage relations as well as the resulting active and reactive power flows on an AC power line
- For this purpose, we use the wave equation discussed in the previous part of the lecture (Part 5)
- Thereby, we focus on a series of practically relevant scenarios
- The analysis is performed under two assumptions:
  - 1) The operating conditions are balanced → analysis is performed via single-phase equivalent circuits
  - 2) The network is in steady-state (for assessment of dynamic phenomena other models are required)
- Furthermore, we consider all powers *per phase*. The corresponding three-phase power can be calculated using the conventions introduced in Part 2.



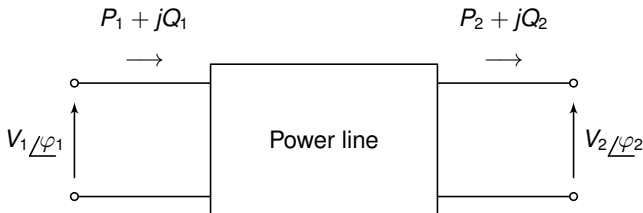
- Several ways to mathematically describe power flow over a power line
- Usually, we use complex voltage together with active and reactive powers at each end of line
- This yields 8 real quantities

$$V_1, \varphi_1, P_1, Q_1, V_2, \varphi_2, P_2, Q_2$$

- Which of the above quantities are decoupled (i.e. independent) of each other and which are not?

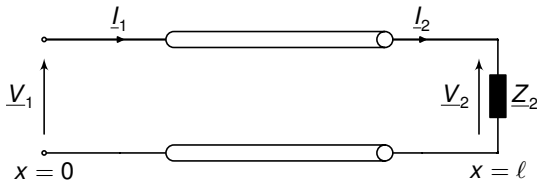


- Not all quantities in above graphic are independent of each other
- Examples:
  - $\underline{V}_1$  and  $\underline{V}_2$  are coupled via line characteristics (see previous lectures)
- Therefore it is customary to take one angle, e.g.  $\varphi_2$ , as reference; hence, one "loses" one quantity in the formulas
- Power flows are also coupled; if  $P_1$  and  $Q_1$  are fixed, then  $P_2$  and  $Q_2$  can be computed if  $\underline{V}_1$  or  $\underline{V}_2$  is fixed, too
- If  $\underline{V}_1$  and  $\underline{V}_2$  are fixed,  $P_1$ ,  $P_2$ ,  $Q_1$  and  $Q_2$  are also fixed and can *not* be adjusted independently



- $V_1, \varphi_1, V_2$ : powers result from line characteristics and given quantities; practical example: power line connects two bulk "stiff" power networks
- $V_1, P_2, Q_2$  (or  $P_1, Q_1, V_2$ ): By fixing voltage on one end of line and power on other end, remaining quantities follow; practical example: consumer with fixed power demand connected via power line to network
- $V_1, P_1, Q_1$ : By fixing quantities at sending end of line, voltage and powers at receiving end follow; practical example: power plant that feeds network over power line

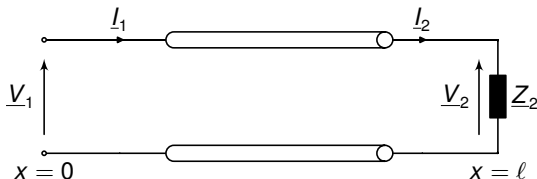
## 2 Surge impedance loading - Meaning



- Surge impedance loading (SIL) = power delivered when line is loaded with its surge impedance, i.e.

$$\underline{Z}_2 = \underline{Z}_w = \sqrt{\frac{R' + j\omega L'}{G' + j\omega C'}}$$

- SIL also called natural loading
- In the following, we consider two cases
  - Lossless line ( $R' = G' = 0$ )
  - Lossy line ( $R' \neq 0, G' \neq 0$ )



- Lossless power line:  $R' = G' = 0 \rightarrow$  surge impedance  $Z_w = \sqrt{\frac{L'}{C'}}$

- Active power delivered at end of line

$$P_2 = \frac{|V_2|^2}{Z_2} = \frac{|V_2|^2}{Z_w}$$

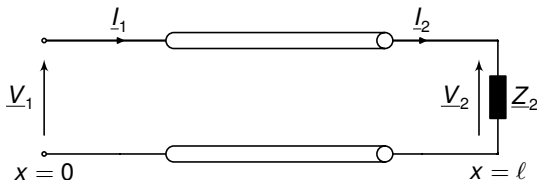
- Reactive power delivered at end of line ( $Z_2 = Z_w$  is real in lossless case)

$$Q_2 = 0$$

- Current at end of line

$$I_2 = \frac{V_2}{Z_2} = \frac{V_2}{Z_w}$$





- From solution of wave equation with  $x = 0$  and  $\underline{\gamma} = j\omega\sqrt{L'C'} = j\beta$  (see Part 5, Sect. 4.2)

$$\underline{V}_1 = \cosh(j\beta\ell)\underline{V}_2 + \underline{Z}_W \sinh(j\beta\ell)\underline{I}_2$$

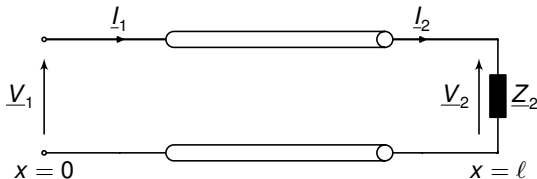
$$\underline{I}_1 = \frac{\underline{V}_2}{\underline{Z}_W} \sinh(j\beta\ell) + \cosh(j\beta\ell)\underline{I}_2$$

- With  $\cosh(j\beta) = \cos(\beta)$  and  $\sinh(j\beta) = j\sin(\beta)$  we obtain

$$\underline{V}_1 = \cos(\beta\ell)\underline{V}_2 + j\underline{Z}_W \sin(\beta\ell)\underline{I}_2$$

$$\underline{I}_1 = j\frac{\underline{V}_2}{\underline{Z}_W} \sin(\beta\ell) + \cos(\beta\ell)\underline{I}_2$$

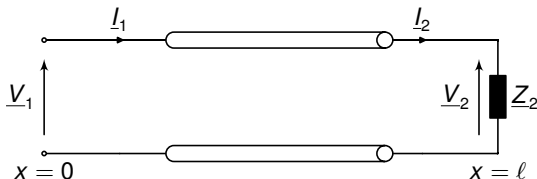
## 2.1 SIL of lossless power line - Sending end (2)



- Using  $\underline{I}_2 = \frac{\underline{V}_2}{\underline{Z}_2}$  yields

$$\begin{aligned}\underline{V}_1 &= \cos(\beta\ell)\underline{V}_2 + jZ_W \sin(\beta\ell)\frac{\underline{V}_2}{Z_W} \\ &= \underline{V}_2(\cos(\beta\ell) + j\sin(\beta\ell)) = \underline{V}_2 e^{j\beta\ell} \\ \underline{I}_1 &= j\frac{\underline{V}_2}{Z_W} \sin(\beta\ell) + \cos(\beta\ell)\frac{\underline{V}_2}{Z_W} \\ &= \underline{I}_2(\cos(\beta\ell) + j\sin(\beta\ell)) = \underline{I}_2 e^{j\beta\ell}\end{aligned}$$

→ Voltage and current are shifted by angle  $\beta\ell$  at end of line  
Thereby, their amplitudes remain unchanged



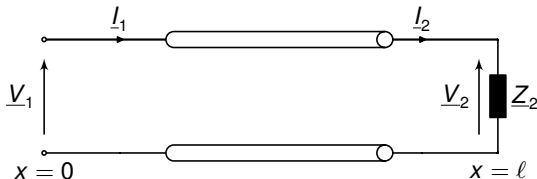
- For active power at both end of lines, we have that (as line is lossless)

$$P_1 = \underline{V}_1 \underline{I}_1^* = \underline{V}_2 \underline{I}_2^* = P_2 = \frac{|\underline{V}_1|^2}{Z_w}$$

- This particular loading of line is called *surge impedance loading (SIL)*

$$P_{SIL} = \frac{|\underline{V}|^2}{Z_w}$$

- For this loading we achieve optimal transmission conditions (amplitudes of voltage and current remain constant along whole line)
- In practice, loading usually differs from SIL

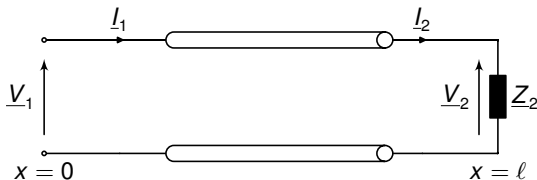


- For SIL, reactive power flow on line is zero
- At each point on line, reactive power "absorption" of line inductance *equals* reactive power "production" of line capacitance

$$Q'_C = Q'_L \Rightarrow V^2 \omega C' = I^2 \omega L' \Rightarrow \frac{V^2}{I^2} = \frac{L'}{C'} = Z_w^2$$

- Surge impedance of overhead lines (OHLs) between 200 – 400  $\Omega$
- OHL inductance significantly larger than OHL capacitance
- Reactive power "absorbed" by OHL inductance exceeds reactive power "produced" by OHL capacitance even for small currents
- OHLs often operated above their SIL; then they "absorb" reactive power
- Compared to OHLs, cables have very low surge impedance ( $\approx 30 - 50 \Omega$ )
- SIL usually above thermal limit of cable
- Cables usually "produce" reactive power

## 2.2 SIL of lossy power line - Sending end (1)

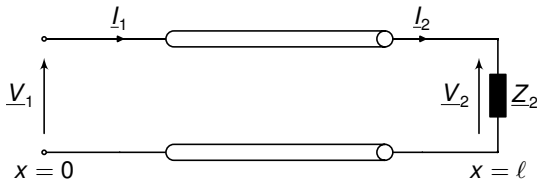


- Lossy line  $\rightarrow \underline{Z}_w$  is complex
- As before, we consider the case  $\underline{Z}_2 = \underline{Z}_w$
- Current at receiving end of line

$$I_2 = \frac{V_2}{\underline{Z}_2} = \frac{V_2}{\underline{Z}_w}$$

- Apparent power at receiving end of line

$$\underline{S}_2 = P_2 + jQ_2 = \underline{V}_2 I_2^* = \frac{|\underline{V}_2|^2}{\underline{Z}_w^*}$$

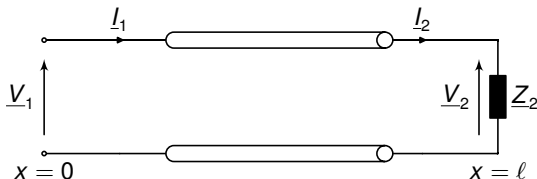


- From solution of wave equation with  $x = 0$  and  $\underline{\gamma} = \alpha + j\beta$  (see Part 5)

$$\begin{aligned}
 \underline{V}_1 &= \cosh(\underline{\gamma}\ell) \underline{V}_2 + \underline{Z}_w \sinh(\underline{\gamma}\ell) \underline{I}_2 = \cosh(\underline{\gamma}\ell) \underline{V}_2 + \underline{Z}_w \sinh(\underline{\gamma}\ell) \frac{\underline{V}_2}{\underline{Z}_w} \\
 &= \underline{V}_2 (\cosh(\underline{\gamma}\ell) + \sinh(\underline{\gamma}\ell)) = \underline{V}_2 e^{\underline{\gamma}\ell} \\
 \underline{I}_1 &= \frac{\underline{V}_2}{\underline{Z}_w} \sinh(\underline{\gamma}\ell) + \cosh(\underline{\gamma}\ell) \underline{I}_2 \\
 &= \underline{I}_2 (\cosh(\underline{\gamma}\ell) + \sinh(\underline{\gamma}\ell)) = \underline{I}_2 e^{\underline{\gamma}\ell}
 \end{aligned}$$

Note: To obtain the last equality, we have used  $\cosh(x) + \sinh(x) = e^x$

## 2.2 SIL of lossy power line - Sending end (3)



- Apparent power at sending end

$$\underline{S}_1 = P_1 + jQ_1 = \underline{V}_1 \underline{I}_1^* = \underline{V}_2 \frac{\underline{V}_2^*}{\underline{Z}_w^*} e^{2\alpha\ell} = \underline{S}_2 e^{2\alpha\ell}$$

- As in lossless case, phase angle between voltage and current remains constant along line; phase shift is proportional to  $\beta x$
- But now, active and reactive power decrease with line length; same applies to voltage and current



- Typical values for OHLs

Rated voltage in kV	132	275	400
$\underline{Z}_w$ [ $\Omega$ ]	373	302	296
$P_{SIL}$ [MW]	47	250	540

Source: B. M. Weedy et al., "Electric Power Systems", John Wiley & Sons, 2012

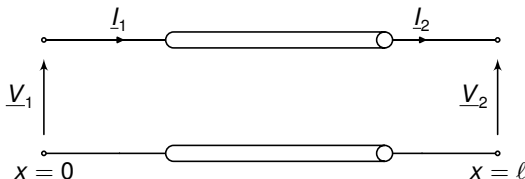
- Typical values for cables

Rated voltage in kV	115	230	500
$\underline{Z}_w$ [ $\Omega$ ]	36.2	37.1	50.4
$P_{SIL}$ [MW]	365	1426	4960

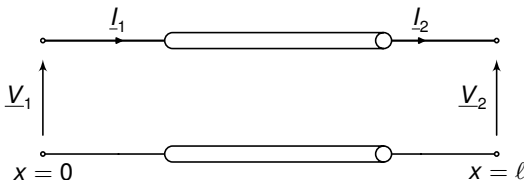
Source: P. Kundur, "Power System Stability", McGraw-Hill, 1994

- Next, we analyse the behaviour of a power line in two special cases
  - No load
  - Short circuit
- To simplify our calculations, we restrict ourselves to the lossless case

## 3.1 No load conditions - Setup



- No load condition can occur if
  - Voltage is applied to unloaded line
  - Load at end of line is disconnected
- Main characteristic:  $\underline{I}_2 = 0$



- Solution of wave equation with  $x = 0$  and  $\underline{\gamma} = j\omega\sqrt{L'C'} = j\beta$  yields (see Part 5, Sect. 4.2)

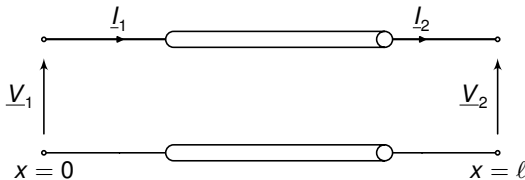
$$\underline{V}_1 = \cos(\beta\ell)\underline{V}_2$$

$$\underline{I}_1 = j\frac{\underline{V}_2}{Z_W} \sin(\beta\ell)$$

- Recall that we may fix one of two voltage angles. Setting  $\varphi_1 = 0$ , we have

$$V_1 = \cos(\beta\ell)V_2$$

$$I_1 = j\frac{V_2}{Z_W} \sin(\beta\ell)$$

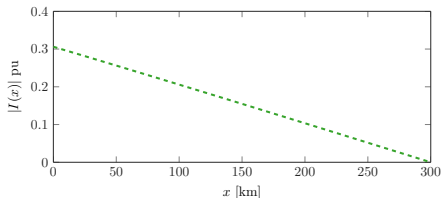
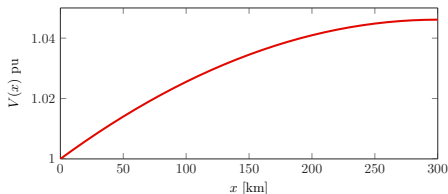
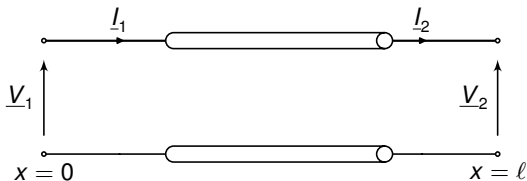


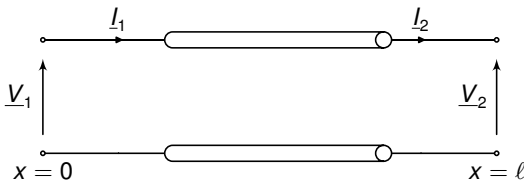
- Keeping  $V_1$  constant, we get

$$V_2 = \frac{V_1}{\cos(\beta\ell)}$$
$$I_1 = \frac{jV_2}{Z_w} \sin(\beta\ell) = \frac{jV_1 \tan(\beta\ell)}{Z_w}$$

- Voltage amplitude increases along line, while that of current decreases ( $I_2 = 0$ )
- This phenomenon is called *Ferranti effect* (because it was first observed by the British engineer Sebastian Ziani de Ferranti in 1887)

## 3.1 No load conditions - Ferranti effect illustration





- It holds that ( $\epsilon_0$  is electric constant and  $\mu_0$  magnetic constant)

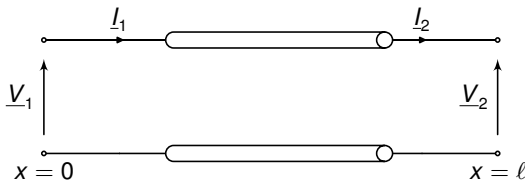
$$\beta = \omega \sqrt{L' C'} \approx \omega \sqrt{\epsilon \epsilon_0 \mu_0}$$

- Permittivity of air  $\epsilon = 1$
- For  $\omega = 2\pi 50$  [rad/s], we have that  $\beta \approx \frac{6^\circ}{100 \text{ km}}$

→ Extreme scenario: resonance; achieved for 50 Hz at  $\ell = 1500 \text{ km}$

$$\beta \ell = \frac{6^\circ \times 1500 \text{ km}}{100 \text{ km}} = 90^\circ = \frac{\pi}{2}$$

- Then  $\cos(\beta \ell) = 0$  and  $V_2 \rightarrow \infty$



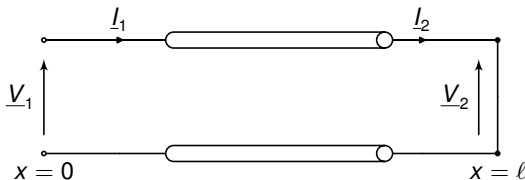
- Impedance at sending end

$$\underline{Z}_1 = \frac{\underline{V}_1}{\underline{I}_1} = -j \frac{Z_w}{\tan(\beta \ell)}$$

- We can see that impedance has *capacitive* character
- High loading currents required!
- In practice: amplitude of  $\underline{V}_1$  not stiff, but additionally increased by loading currents → need to be careful with voltage rise already for line lengths of 300km



## 3.2 Short circuit conditions - Voltage and current



- Short circuit  $\rightarrow \underline{V}_2 = 0$
- Solution of wave equation with  $x = 0$  and  $\underline{\gamma} = j\omega\sqrt{L'C'} = j\beta$  yields (see Part 5, Sect. 4.2)

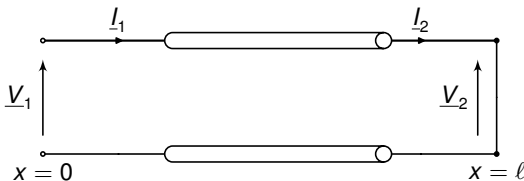
$$\underline{V}_1 = j\underline{I}_2 Z_w \sin(\beta\ell)$$

$$\underline{I}_1 = \underline{I}_2 \cos(\beta\ell)$$

- In analogy to voltage in no load condition, now current increases along line

$$\underline{I}_2 = \frac{\underline{I}_1}{\cos(\beta\ell)}$$

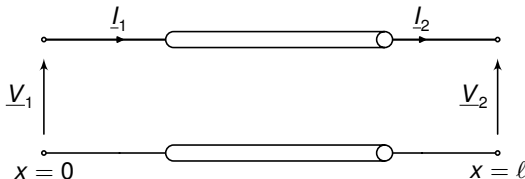
## 3.2 Short circuit conditions - Impedance



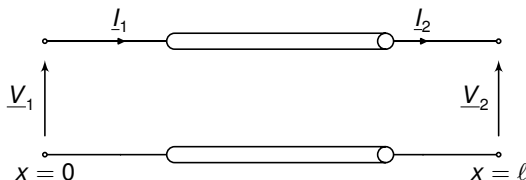
- Short circuit impedance

$$\frac{V_1}{I_1} = \underline{Z}_1 = jZ_w \tan(\beta\ell)$$

- For  $\omega = 2\pi 50$  [rad/s], short circuit impedance is *inductive* for line lengths  $< 1500$  km
- As before, resonance  $|I_2| \rightarrow \infty$  for  $\beta\ell = \pi/2$



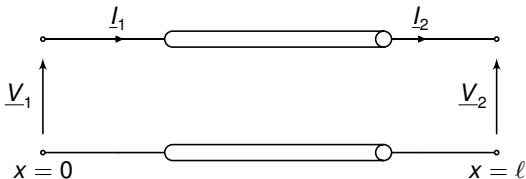
- Power transmission over a power line causes losses:
  - Ohmic components of line (resistance; conductance) cause active power losses
  - Reactive components of line (inductance; capacitance) influence reactive power flow
- Apparent power at receiving end of line differs from apparent power at sending end!
- For voltage relation along line, reactive power is most important as discussed hereafter for *lossless* line



- Wave equation for *lossless* line

$$\begin{bmatrix} \underline{V}_1 \\ \underline{I}_1 \end{bmatrix} = \begin{bmatrix} \cos(\beta\ell) & jZ_w \sin(\beta\ell) \\ j\frac{\sin(\beta\ell)}{Z_w} & \cos(\beta\ell) \end{bmatrix} \begin{bmatrix} \underline{V}_2 \\ \underline{I}_2 \end{bmatrix}$$

→ Apparent power  $\underline{S}_1 = \underline{V}_1 \underline{I}_1^* = P_1 + jQ_1$  at sending end is dependent on  
apparent power  $\underline{S}_2 = \underline{V}_2 \underline{I}_2^* = P_2 + jQ_2$  at receiving end of line



- Hence, we have

$$\underline{S}_1 = \underline{V}_1 I_1^* = P_1 + jQ_1 = j \cos(\beta \ell) \sin(\beta \ell) \left( |I_2|^2 - |\underline{V}_2|^2 \frac{1}{Z_w} \right) + \sin^2(\beta \ell) I_2 \underline{V}_2^* + \cos^2(\beta \ell) \underline{V}_2 I_2^*$$

- For our analysis, it is convenient to fix  $\underline{V}_2$  and express  $\underline{S}_1$  in terms of SIL

$$P_{SIL} = \frac{|\underline{V}_2|^2}{Z_w}$$

- Using the relations

$$|\underline{V}_2|^2 = P_{SIL} Z_w$$

$$\underline{I}_2^* = \frac{\underline{S}_2}{\underline{V}_2} \rightarrow |\underline{I}_2|^2 = \frac{|\underline{S}_2|^2}{|\underline{V}_2|^2} = \frac{|\underline{S}_2|^2}{P_{SIL} Z_w}$$

$$\underline{I}_2 \underline{V}_2^* = (\underline{V}_2 \underline{I}_2^*)^* = \underline{S}_2^* = P_2 - jQ_2$$

$$\cos^2(\beta\ell) - \sin^2(\beta\ell) = \cos(2\beta\ell)$$

$$\cos(\beta\ell) \sin(\beta\ell) = \frac{1}{2} \sin(2\beta\ell)$$

we can rewrite the equation for  $\underline{S}_1$  as follows

$$\underline{S}_1 = P_1 + jQ_1 = P_2 + j \left( Q_2 \cos(2\beta\ell) + \frac{1}{2} \sin(2\beta\ell) \left( \frac{|\underline{S}_2|^2}{P_{SIL}} - P_{SIL} \right) \right)$$

- For lossless line

$$P_1 = P_2 \rightarrow \text{can focus further analysis on reactive power flows}$$

- Relation of reactive power flows

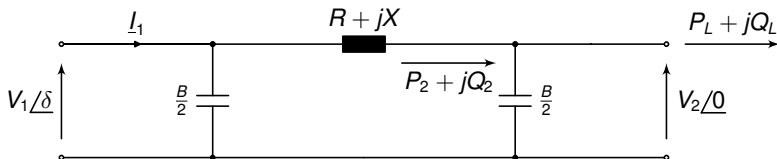
$$Q_1 = Q_2 \cos(2\beta\ell) + \frac{1}{2} \sin(2\beta\ell) \left( \frac{|\underline{S}_2|^2}{P_{SIL}} - P_{SIL} \right)$$

- $Q_1$  dependent on  $Q_2$  ("load demand") and reactive power demand of line
- With simplifying approximation  $\cos(2\beta\ell) \approx 1$ , reactive power demand of line given by

$$\Delta Q = Q_1 - Q_2 \approx \underbrace{\frac{1}{2} \sin(2\beta\ell) \frac{|\underline{S}_2|^2}{P_{SIL}}}_{\text{inductive component } Q_L} - \underbrace{\frac{1}{2} \sin(2\beta\ell) P_{SIL}}_{\text{capacitive component } Q_C}$$

- $|\underline{S}_2| = P_{SIL} \rightarrow \Delta Q = 0$
- $|\underline{S}_2| = 0 \rightarrow \Delta Q < 0$  line produces reactive power ( $Q_L = 0$ ,  $Q_C > 0$ )
- $|\underline{S}_2| > P_{SIL} \rightarrow \Delta Q > 0$  line absorbs reactive power ( $Q_L > Q_C$ )
- $|\underline{S}_2| < P_{SIL} \rightarrow \Delta Q < 0$  line produces reactive power ( $Q_L < Q_C$ )

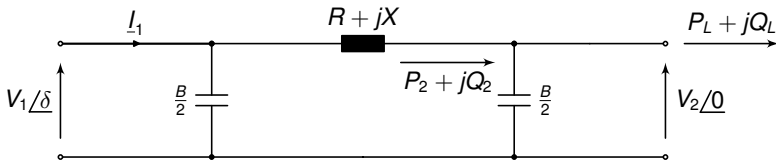
## 5 Voltage drop across a power line - Setup



- $\pi$ -model of power line of length  $\ell$  and  $G' = 0$ ,  $R = R'\ell$ ,  $X = \omega L'\ell$  and  $B = \omega C'\ell$
- Load at end of line:  $P_L + jQ_L$
- We want to derive a formula for voltage drop across line
- For this purpose it is convenient to define  $V_2$  on real line and denote angle between  $V_2$  and  $\underline{V}_1$  by  $\delta$



## 5 Voltage drop across a power line - A simplification (1)

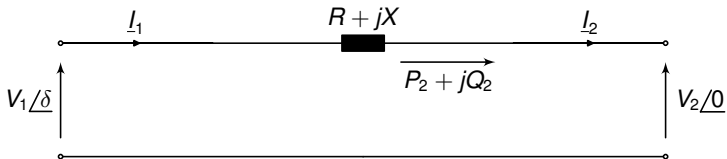


- Shunt elements  $B$  produce reactive power
- We can obtain "net" reactive power flow  $Q_2$  on line by subtracting reactive power  $Q_C$  produced by  $B$  from  $Q_L$ , i.e.,

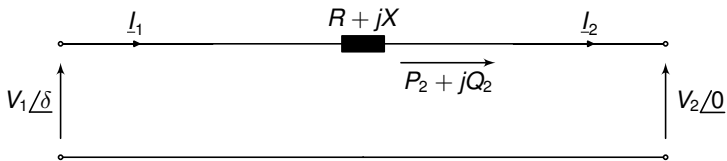
$$P_2 = P_L$$

$$Q_2 = Q_L - Q_C$$

## 5 Voltage drop across a power line - A simplification (2)



- By using  $Q_2 = Q_L - Q_C$  we can simplify considered circuit as shown above



- Current  $I_2$  as function of apparent power  $\underline{S}_2 = P_2 + jQ_2$  and  $\underline{V}_2 = V_2$

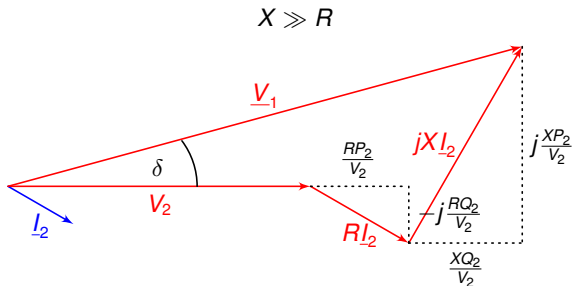
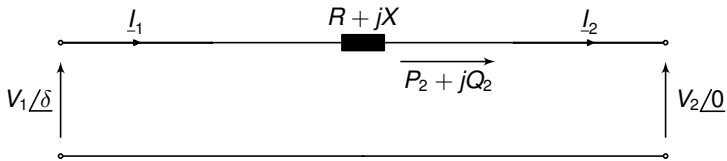
$$\underline{I}_1 = \underline{I}_2 = \frac{\underline{S}_2^*}{\underline{V}_2} = \frac{P_2 - jQ_2}{V_2}$$

- Voltage  $\underline{V}_1$

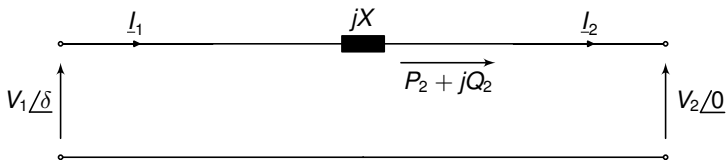
$$\begin{aligned}\underline{V}_1 &= V_2 + (R + jX)\underline{I}_2 = V_2 + (R + jX)\frac{P_2 - jQ_2}{V_2} \\ &= \left(V_2 + \frac{RP_2 + XQ_2}{V_2}\right) + j\left(\frac{XP_2 - RQ_2}{V_2}\right)\end{aligned}$$

$$|\underline{V}_1| = V_1 = \sqrt{\left(V_2 + \frac{RP_2 + XQ_2}{V_2}\right)^2 + \left(\frac{XP_2 - RQ_2}{V_2}\right)^2}$$

## 5 Voltage drop across a power line - Phasor diagram



## 5 Voltage drop across a power line - Lossless line



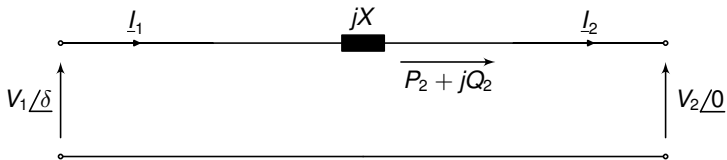
- Lossless line  $\rightarrow R = 0$
- Expression for  $V_1$  simplifies to

$$\underline{V}_1 = V_1 \cos(\delta) + jV_1 \sin(\delta) = \left( V_2 + \frac{XQ_2}{V_2} \right) + j \left( \frac{XP_2}{V_2} \right)$$

- Separating the real with the imaginary parts, gives

$$P_2 = \frac{V_1 V_2 \sin(\delta)}{X}$$

$$Q_2 = \frac{V_1 V_2 \cos(\delta) - V_2^2}{X}$$



- The magnitude of  $V_1$  is given by

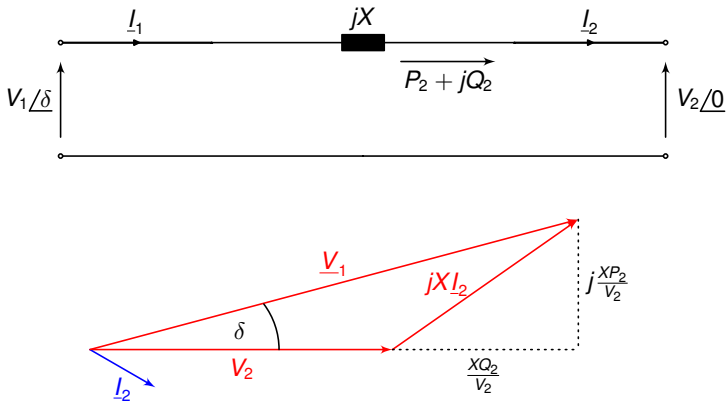
$$|\underline{V}_1| = V_1 = \sqrt{\left(V_2 + \frac{XQ_2}{V_2}\right)^2 + \left(\frac{XP_2}{V_2}\right)^2}$$

- In most scenarios  $|XP_2/V_2| \ll V_2$  and expression for  $V_1$  can be further simplified to

$$|\underline{V}_1| = V_1 \approx V_2 + \frac{XQ_2}{V_2}$$

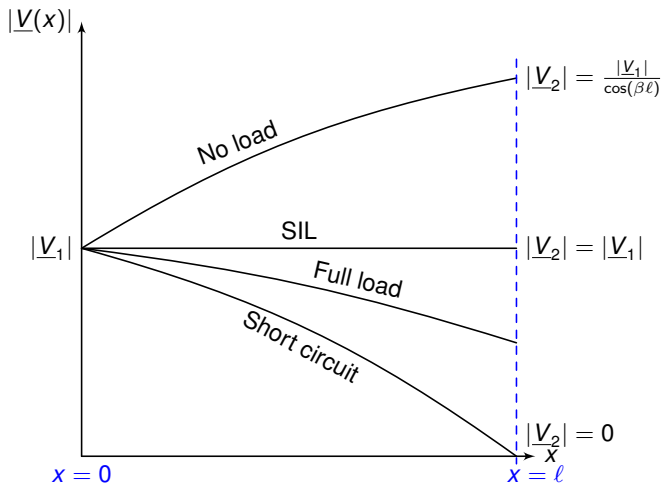
→  $\Delta V = V_1 - V_2$  mainly influenced by reactive power  $Q_2$ !

## 5 Voltage drop across a power line - Lossless line phasor diagram



→ Phase angle  $\delta$  mainly influenced by active power  $P_2$ !

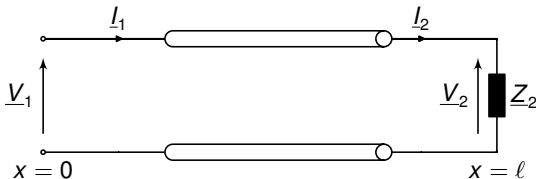
- We have assumed  $V_2$  and  $\underline{S}_2$  are known and we want to calculate  $\underline{V}_1$ ; often also  $V_1$  and  $\underline{S}_2$  given and we seek to compute  $\underline{V}_2$ ; this can be done in an equivalent manner



- The discussed scenarios mainly apply to OHLs; cables typically have different properties



## 6 Efficiency of a high-voltage power line - An example (1)



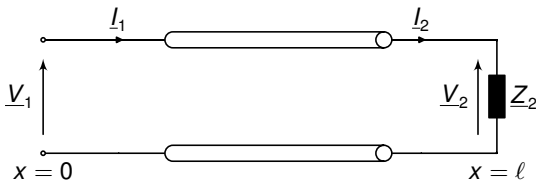
- Consider exemplary 200 km/420 kV ( $= V_{LL} = \sqrt{3}V_2$ ) power line with following characteristics

$$R' = 0.031 \, \Omega/\text{km}, \quad L' = 1.06 \, \text{mH}/\text{km}, \quad C' = 11.9 \, \text{nF}/\text{km}, \quad G' = 0, \quad f = 50 \, \text{Hz}$$

- Assume line is loaded with surge impedance  $\underline{Z}_2 = \underline{Z}_w$
- Propagation constant

$$\underline{\gamma} = \sqrt{(0.031 + j0.333)j3.74 \cdot 10^{-6}} = (0.052 + j1.117)10^{-3}$$
$$\underline{\gamma}\ell = \alpha\ell + j\beta\ell = 0.0104 + j0.2234$$

## 6 Efficiency of a high-voltage power line - An example (2)



- Characteristic impedance (neglecting imaginary part)

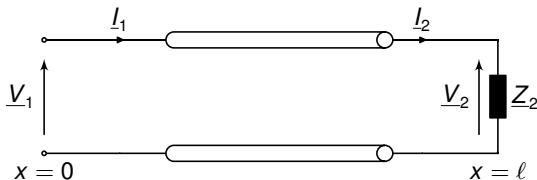
$$Z_w = 298.5 \, \Omega$$

- Active power drawn by load at receiving end of line

$$P_2 = \frac{V_{LL}^2}{Z_w} \approx 591 \, \text{MW}$$

- Current RMS magnitude (per phase)

$$I_2 = \frac{\frac{V_{LL}}{\sqrt{3}}}{Z_w} = \frac{420 \, \text{kV}}{\sqrt{3} \cdot 298.5 \, \Omega} = 812.4 \, \text{A}$$



- Line losses can be approximated by

$$\Delta P = P_1 - P_2 \approx 3R'\ell I_2^2 = 3 \cdot 0.031 \cdot 200 \cdot 812.4^2 = 12.3 \text{ MW}$$

- $P_1 = P_2 + \Delta P \approx 591 + 12.3 = 603.3 \text{ MW}$

- Alternative:** We can calculate exact value for  $P_1$  from wave equation (see Part 6 Section 2.2)

$$P_1 = P_2 e^{2\alpha\ell} = 603.6 \text{ MW}$$

→ Our approximation is fairly accurate

→ Very high efficiency for power transmission!

$$e^{-2\alpha\ell} = 0.979 \quad \leftrightarrow \quad 97.9\%$$