Power Systems I & II

Exercises

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Contents

Preface

This selection of exercises has been selected, created, or curated to support the courses Power Systems I and II taught at the Cyprus University of Technology. If you find any mistakes or issues with the material, please contact me.

Chapter 1

Basics of electrical engineering

In this introductory chapter we want to refresh some basics of electrical engineering. We repeat above all important terms and tools from AC engineering: The representation of alternating quantities through complex numbers, definitions of impedances in AC circuits and the concepts of power and energy.

1.1 Complex Numbers

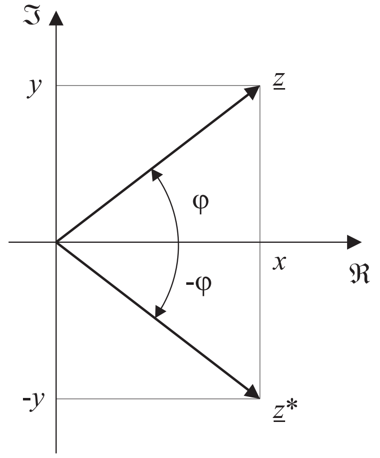


Figure 1.1: Geometric representation of a complex number $\underline{z}$ and its conjugate $\underline{z}^*$.

We look at the interpretation of a complex number represented in figure 1.1. With the real part x on the x-coordinate and the imaginary part y on the y-coordinate we obtain a vector

$$\underline{z} = x + jy = |\underline{z}|\angle\varphi \tag{1.1}$$

where j is the imaginary unit:[1]

$$j^2 = -1 \tag{1.2}$$

The operators $\Re$ and $\Im$ form the real and imaginary part of their argument:

$$\Re(\underline{z}) = x \tag{1.3a}$$

$$\Im(\underline{z}) = y \tag{1.3b}$$

A complex number can be represented either by orthogonal coordinates (real part x and imaginary part y) or by polar coordinates (absolute value $z = |\underline{z}|$ and phase φ). A complex number

[1] The definition $j = \sqrt{-1}$ can lead to a mathematical paradox when using the square root function "incautiously".

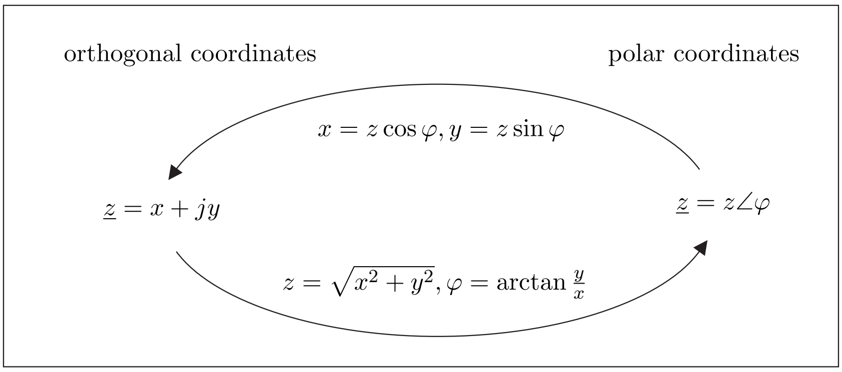


Figure 1.2: Conversion between orthogonal and polar coordinates.

$\underline{z} = z\angle\varphi$ given in polar coordinates can be converted to orthogonal coordinates by means of the following relationships:

$$x = z\cos\varphi \tag{1.4a}$$
$$y = z\sin\varphi \tag{1.4b}$$

The inverse operation, i.e. the conversion of a number $\underline{z} = x+jy$ given in orthogonal coordinates to polar coordinates is done as follows:

$$z = |\underline{z}| = \sqrt{x^2 + y^2} \tag{1.5}$$

$$\varphi = \arctan\frac{\Im(\underline{z})}{\Re(\underline{z})} = \arctan\frac{y}{x} \tag{1.6}$$

The conversions between the two coordinate planes are summarized in figure 1.2.

By means of the Eulerian relationship

$$e^{j\varphi} = \cos\varphi + j\sin\varphi \tag{1.7}$$

complex numbers can be represented in exponential notation:

$$\underline{z} = x + jy = ze^{j\varphi} \tag{1.8}$$

This notation can be very beneficial for certain applications. Please note that the product of two exponential numbers is $e^a e^b = e^{a+b}$. Furthermore the following relationship holds true for the reciprocal value of an exponential number: $1/e^a = e^{-a}$.

If one replaces the imaginary part of a complex number $\underline{z}$ by its negative value, one obtains the conjugate complex number for $\underline{z}$:

$$\underline{z}^* = x - jy = z\angle - \varphi = ze^{-j\varphi} \tag{1.9}$$

The conjugation of a complex number can be geometrically interpreted as reflection in the real axis (see figure 1.1).

In the following, we want to repeat the four basic arithmetic operations for two complex numbers $\underline{z}_1 = x_1 + jy_1$ and $\underline{z}_2 = x_2 + jy_2$.

- Addition/subtraction: To sum two complex numbers, we add up their real and imaginary part, respectively:

$$\underline{z}_1 \pm \underline{z}_2 = (x_1 + jy_1) \pm (x_2 + jy_2) = x_1 \pm x_2 + j(y_1 \pm y_2) \tag{1.10}$$

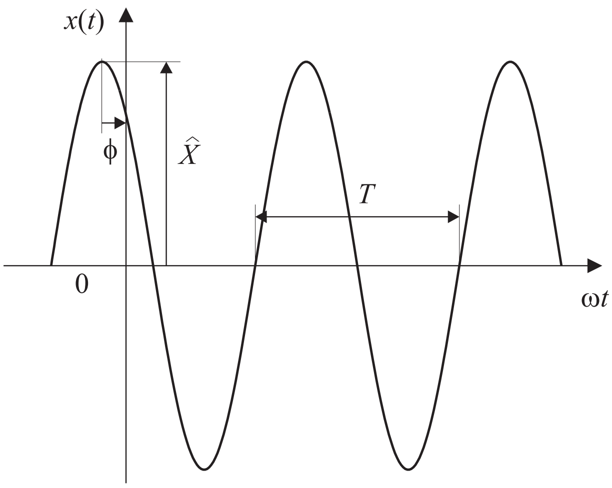


Figure 1.3: Sinusoidal oscillation with amplitude $\widehat{X}$, angular phase shift ϕ and cycle duration $T = 2\pi/\omega$.

- Multiplication: To calculate the product of two complex numbers, we multiply their absolute values and add up their phase angles:

$$\underline{z}_1 \cdot \underline{z}_2 = |\underline{z}_1| \cdot |\underline{z}_2| \cdot e^{j(\varphi_1+\varphi_2)} = z_1 z_2 \angle (\varphi_1 + \varphi_2) \tag{1.11}$$

- Division: The quotient of two complex numbers results from the quotient of the absolute values and the difference of the phase angles:

$$\frac{\underline{z}_1}{\underline{z}_2} = \frac{|\underline{z}_1|}{|\underline{z}_2|} \cdot e^{j(\varphi_1-\varphi_2)} = \frac{z_1}{z_2} \angle (\varphi_1 - \varphi_2) \tag{1.12}$$

One can, of course, carry out each of these operations with a different representation (orthogonal/polar coordinates), but the calculations are in general simpler in the described forms.

1.2 Sinusoidal quantities

Sinusoidal AC quantities can be defined as sine or cosine quantities. In principle it does not matter which definition one chooses. Depending on the calculations to be carried out, one or the other form can lead to more calculation efforts. All properties that will be discussed in the following hold true for both sine and cosine functions.

1.2.1 Definitions

Figure 1.3 shows a symmetrical, sinusoidal oscillation

$$x(t) = \widehat{X} \cos(\omega t + \phi) \tag{1.13}$$

The maximum value of the oscillation $\widehat{X}$ is called amplitude. The phase angle ϕ defines the shift of the phase with respect to the reference angle (in this case 0Â°); T corresponds to the duration of a cycle, also called the period of the oscillation. We can now calculate the following quantities:

- Frequency f, angular frequency ω:

$$f = \frac{1}{T}, \quad \omega = 2\pi f = \frac{2\pi}{T} \tag{1.14}$$

The unit of frequency is Hertz (Hz), where 1 Hz = 1 s^{-1}; the unit of the angular frequency is strictly speaking s^{-1}, too. However, it is often given in radians per second (rad/s).

- Average over time $\overline{X}$:
$$\overline{X} = \frac{1}{T}\int_0^T x(t)dt \tag{1.15}$$
For symmetrical sinusoidal oscillations, we have $\overline{X} = 0$.

- Effective value (root mean square value) X: The root mean square (RMS) value of an AC current is defined as that value of a DC current which would generate the same amount of heat in an ohmic resistance. In general, the RMS value can be determined for any periodic signal $x(t)$:
$$X = \sqrt{\frac{1}{T}\int_0^T x^2(t)dt} \tag{1.16}$$
For a sinusoidal oscillation as illustrated in figure 1.3, the relationship between the RMS value and the amplitude is the following:
$$X = \frac{\widehat{X}}{\sqrt{2}} \tag{1.17}$$

1.2.2 Representation of sinusoidal quantities as phasors

We begin with the consideration of an exponential function $e^{j\omega t}$. At any time t we obtain a complex number as value of the function. At time $t = 0$ this number is purely real and equals 1; for $\omega t = \pi/2$ we obtain a purely imaginary number which equals j, etc. The absolute value is always $|e^{j\omega t}| = 1$. We can interpret this function as unit vector rotating around the origin with the angular frequency ω. Depending on time t, we obtain different projections on the real and imaginary axis.

At first we define a complex number $\underline{X}$, which contains the position of the phasor at time $t = 0$:
$$x(t) = \widehat{X}\cos(\omega t + \phi) \quad \rightarrow \quad \underline{X} = \frac{\widehat{X}}{\sqrt{2}}(\cos\phi + j\sin\phi) = Xe^{j\phi} \tag{1.18}$$
The rotation of this phasor with ω can be expressed by multiplying with $e^{j\omega t}$. We can therefore represent the sinusoidal signal as a complex phasor $\underline{X}$ rotating with ω. The instantaneous value of the function $x(t)$ at time t becomes then
$$x(t) = \sqrt{2}\cdot\Re\left(\underline{X}e^{j\omega t}\right) \tag{1.19}$$

1.3 Impedance

1.3.1 Definitions

In an AC circuit, different quantities for ,,impedances” are defined:

- Resistance (active resistance) R: Resistance of ohmic elements
- Reactance (reactive impedance) X: Impedance of inductances and capacitances
- Impedance $\underline{Z}$: complex quantity with real part R and imaginary part X.
$$\underline{Z} = R + jX \tag{1.20}$$
The unit of R, X and $\underline{Z}$ is Ohm (Ω). Figure 1.5 shows an impedance $\underline{Z}$ in the complex plane.

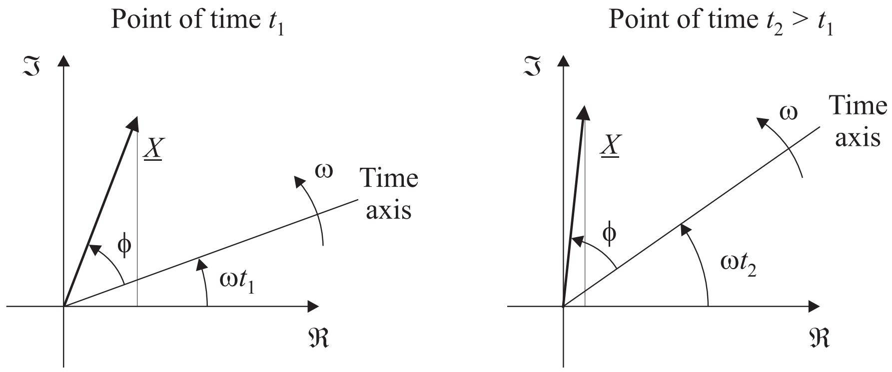


Figure 1.4: Representation of a sinusoidal AC quantity as rotating phasor at two different points in time. $\underline{X}$ encloses a fixed angle ϕ with the time axis. The time axis rotates anti-clockwise with the angular frequency ω. The length of the phasor $\underline{X}$ corresponds to the RMS value X; the projection of $\underline{X}$ on the real axis thus corresponds to $\frac{x(t)}{\sqrt{2}}$.

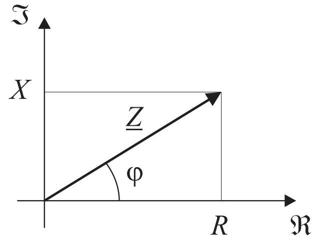


Figure 1.5: Representation of an impedance $\underline{Z} = R + jX$ in the complex plane.

- Conductance G: Conductance of ohmic elements
- Susceptance B: AC conductance of inductances and capacitances
- Admittance $\underline{Y}$: complex quantity with real part G and imaginary part B.

$$\underline{Y} = G + jB \tag{1.21}$$

The unit of G, B and $\underline{Y}$ is Siemens (S); 1 S = 1 Ω^{-1}.

The admittance of a passive bipole corresponds to its inverse impedance:

$$\underline{Y} = \underline{Z}^{-1} \tag{1.22}$$

For serial and parallel connections of impedances, the same relationships as for ohmic resistances hold true. The total impedance $\underline{Z}$ of two impedances $\underline{Z}_1$ and $\underline{Z}_2$ connected in series is

$$\underline{Z} = \underline{Z}_1 + \underline{Z}_2 \tag{1.23}$$

For the parallel connection of the two impedances, we obtain

$$\underline{Z} = \underline{Z}_1 \parallel \underline{Z}_2 = \frac{\underline{Z}_1 \cdot \underline{Z}_2}{\underline{Z}_1 + \underline{Z}_2} \tag{1.24}$$

1.3.2 Impedance of R, L and C

For an ohmic resistance, the equation $u = iR$ applies; voltage and current are always in phase, i.e. the impedance becomes purely real:

$$\underline{Z}_R = R \tag{1.25}$$

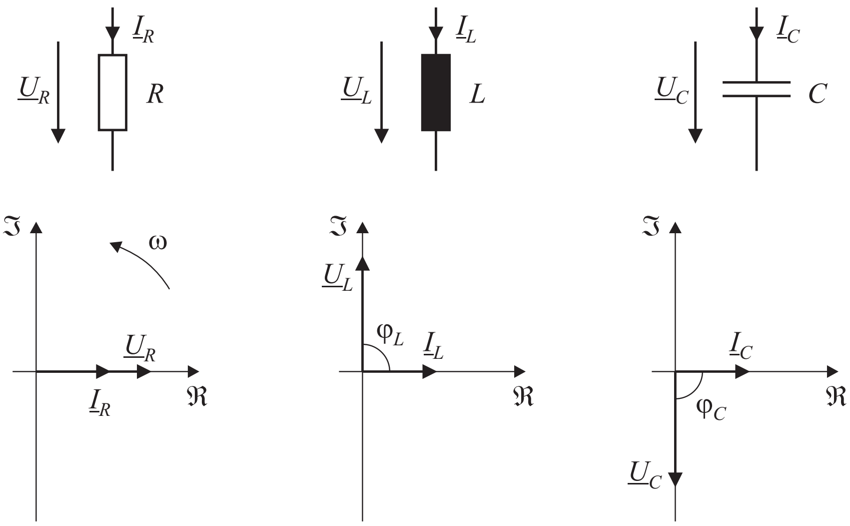


Figure 1.6: Voltage at R, L and C as consequence of an imposed real current.

The relationship between current and voltage for an inductance is given by $u_L = L\frac{di_L}{dt}$. For sinusoidal quantities $\underline{U}_L$ and $\underline{I}_L$, a phase shift between current and voltage results from the differential term. From $\underline{U}_L = j\omega L\underline{I}_L$, we obtain the impedance of an inductance

$$\underline{Z}_L = j\omega L = jX_L \tag{1.26}$$

The multiplication with j corresponds to a rotation of the voltage by 90^o with respect to the current (in sense of rotation of ω):

$$\varphi_L = \angle\left(\underline{U}_L, \underline{I}_L\right) = \frac{\pi}{2} = 90^o \tag{1.27}$$

At an inductance, the voltage leads the current by 90Â°. This fact is represented as phasor diagram in figure 1.6. Voltage and current at a capacitance evolve according to $i_C = C\frac{du_c}{dt}$; the impedance becomes then[2]

$$\underline{Z}_C = \frac{1}{j\omega C} = -j\frac{1}{\omega C} = jX_C \tag{1.28}$$

In this case, we obtain a negative phase shift between voltage and current:

$$\varphi_C = \angle\left(\underline{U}_C, \underline{I}_C\right) = -\frac{\pi}{2} = -90^o \tag{1.29}$$

At a capacitance, the voltage lags the current by 90Â°.

1.4 Power and Energy

We consider a passive bipole as shown in figure 1.7 with voltage $u = u(t)$ and current $i = i(t)$. The instantaneously (at time t) consumed power is the product of voltage and current:

$$p(t) = u(t)i(t) \tag{1.30}$$

The unit of p is Watt (W). The energy exchanged between two points of time t_1 and t_2 is the time integral of the instantaneous power:

$$w = \int_{t_1}^{t_2} p(t)dt \tag{1.31}$$

The unit of energy is thus Watt second (Ws); a Watt second corresponds to a Joule, which equals a Newton meter: (1 Ws = 1 J = 1 Nm).

[2]Please note that $\frac{1}{j} = \frac{j}{j^2} = \frac{j}{-1} = -j$.

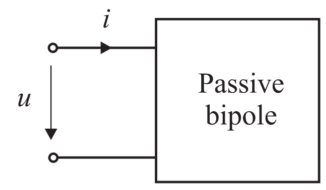


Figure 1.7: Passive bipole with voltage u and current i.

1.4.1 Instantaneous power at R, L and C

Now we want to examine the instantaneous power $p(t)$ at passive elements for arbitrary current and voltage forms $i = i(t)$ and $u = u(t)$.

- Ohmic resistance R:
$$p_R = ui = Ri^2 = \frac{u^2}{R} \tag{1.32}$$

- Inductance L: If a current i flows through a coil, the voltage $L\frac{di}{dt}$ occurs. The instantaneous power then becomes
$$p_L = ui = L\frac{di}{dt}i = \frac{1}{2}L\frac{di^2}{dt} \tag{1.33}$$
It can be shown that the power instantaneously exchanged by an inductance corresponds to the periodic change of the stored (magnetic) energy w_L:
$$w_L = \frac{1}{2}Li^2 \quad \Rightarrow \quad p_L = \frac{dw_L}{dt} \tag{1.34}$$

- Capacitance C: If a voltage u is applied to a capacitance, a current $C\frac{du}{dt}$ occurs. The power is then calculated as follows:
$$p_C = ui = uC\frac{du}{dt} = \frac{1}{2}C\frac{du^2}{dt} \tag{1.35}$$
Also in the case of the capacitance, the instantaneous power is the change of the energy w_C stored in the electric field:
$$w_C = \frac{1}{2}Cu^2 \quad \Rightarrow \quad p_C = \frac{dw_C}{dt} \tag{1.36}$$

1.4.2 AC power at R, L and C

Now we want to calculate the instantaneous power at passive bipoles for sinusoidal currents and voltages.

- Ohmic resistance flown through by current $i = \widehat{I}\cos(\omega t + \phi)$: The instantaneous power results in
$$\begin{aligned} p_R &= Ri^2 = R\widehat{I}^2\cos^2(\omega t + \phi) \\ &= \frac{R\widehat{I}^2}{2}\left\{1 + \cos\left[2(\omega t + \phi)\right]\right\} \end{aligned} \tag{1.37}$$
We see that the instantaneous power is composed of a constant part and a part oscillating with twice the voltage frequency. The average over time of the instantaneous power is a time-independent term:
$$\overline{P}_R = \frac{1}{2}R\widehat{I}^2 = RI^2 \tag{1.38}$$

- Inductance flown through by current $i = \widehat{I}\cos(\omega t + \phi)$: Also in this case, the instantaneous power oscillates with twice the voltage frequency, however around the zero position:

$$\begin{aligned} p_L &= ui = \underbrace{-\omega L\widehat{I}\sin(\omega t + \phi)}_{u=L\frac{di}{dt}}\widehat{I}\cos(\omega t + \phi) \\ &= -\frac{\omega L\widehat{I}^2}{2}\sin[2(\omega t + \phi)] \end{aligned} \quad (1.39)$$

We note that the power balance of an inductance is zero in the time average:

$$\overline{P}_L = 0 \quad (1.40)$$

The amplitude of the oscillating instantaneous power is defined as reactive power Q_L. The (inductive) reactive power consumed by an inductance results from equation 1.39:

$$\widehat{P}_L = Q_L = \frac{\omega L\widehat{I}^2}{2} = X_L I^2 \quad (1.41)$$

- Capacitance with $u = \widehat{U}\cos(\omega t + \phi)$: The product of voltage and current leads to

$$\begin{aligned} p &= ui = \widehat{U}\cos(\omega t + \phi)\underbrace{\left(-\omega C\widehat{U}\sin(\omega t + \phi)\right)}_{i=C\frac{du}{dt}} \\ &= -\frac{\omega C\widehat{U}^2}{2}\sin[2(\omega t + \phi)] \end{aligned} \quad (1.42)$$

We note that the capacitance does not exchange any power in the time average:

$$\overline{P}_C = 0 \quad (1.43)$$

The (capacitive) reactive power results again from the amplitude of the instantaneous power:

$$\widehat{P}_C = Q_C = \frac{U^2}{X_C} \quad (1.44)$$

We summarize that inductances and capacitances do exchange energy instantaneously when being operated with alternating current. They do not exchange power, however, if one considers the time average.

1.5 Exercises

Exercise 1.5.1 *Basic complex number calculations*

a) Three complex numbers are given:

$$\underline{z}_1 = 3 + j7; \quad \underline{z}_2 = -1 + j; \quad \underline{z}_3 = 5\angle 60°$$

Calculate

- $\underline{z}_1^2 + \underline{z}_2 \cdot \underline{z}_3$
- $\underline{z}_1 \cdot \underline{z}_2^* \cdot \underline{z}_3$
- $\underline{z}_1^{-1} \cdot \Im(\underline{z}_2)$

and give all results in orthogonal as well as in polar coordinates.

b) We know the absolute value and the power factor of an ohmic-inductive load $\underline{Z}$ (see figure 1.5): $|\underline{Z}| = 500\ \Omega$, $\cos\varphi = 0.85$. Calculate R and L for 50 Hz.

Exercise 1.5.2 *Basic phasor calculations*

We consider the signal shown in figure 1.8. Determine

a) the constants of the function $x(t) = \widehat{X}\sin(\omega t + \phi)$ so that this function describes the given signal;

b) the time average $\overline{X}$ as well as the RMS value X of the signal;

c) the complex number $\underline{X}$, so that $x(t) = \sqrt{2} \cdot \Re\left(\underline{X}e^{j\omega t}\right)$.

Answer the following questions:

d) How would the RMS value change if one doubled the frequency?

e) How would the RMS value change if one superimposed a positive offset to the signal?

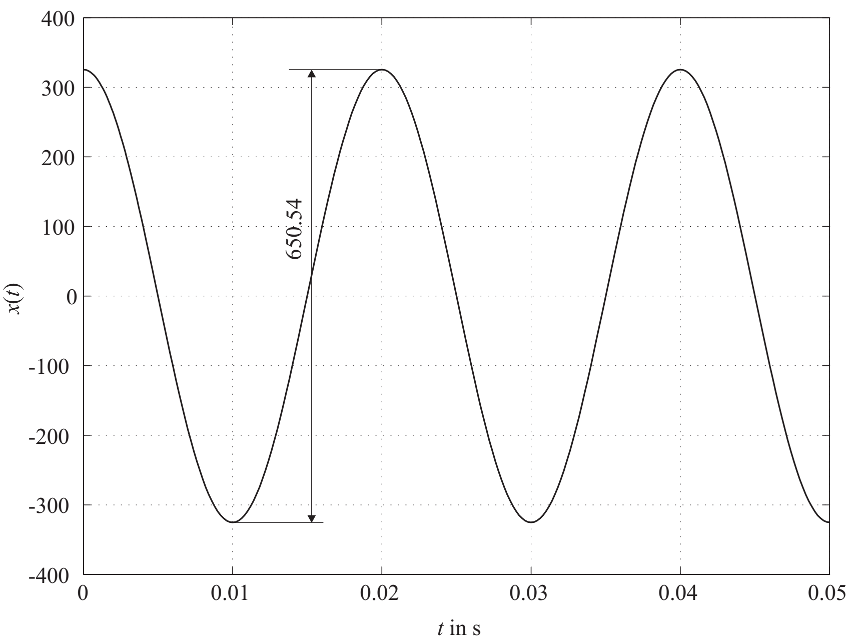


Figure 1.8: Sinusoidal AC signal.

Exercise 1.5.3 *Passive elements*

Three passive elements are given:

$$R = 140\ \Omega; \quad L = 1.2\ \text{mH}; \quad C = 33\ \mu\text{F}$$

Determine for a 50 Hz system the

a) impedance of the series connection of R, L and C;

b) admittance of the parallel connection of R, L and C.

Give all results in orthogonal as well as in polar coordinates.

Exercise 1.5.4 *Passive elements and frequency dependency*

We consider the circuit of figure 1.9. The values of the resistance and of the inductance are given: $R = 100\ \Omega$, $L = 50$ mH.

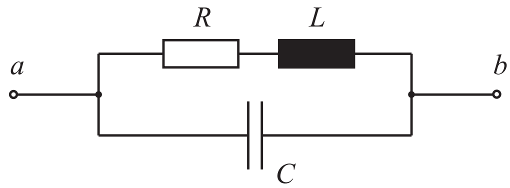


Figure 1.9: RLC-circuit.

a) For which value of the capacitance C does the impedance between the terminals a and b become purely ohmic at 50 Hz?

b) We assume that the circuit is now operated with the capacitance calculated in a), but at 60 Hz. Does the total impedance between a and b become inductive or capacitive?

Exercise 1.5.5 *Passive elements, voltage dependency, and energy profile*

a) How high is the amplitude of the instantaneous power at an inductance $L = 0.9$ mH if a current $i(t) = 247 \cdot \sin(314.159 \cdot t)$ A flows through it?

b) How much does the reactive power generated by a capacitance increase if one increases the frequency of the supply voltage from 50 to 60 Hz?

c) Figure 1.10 shows the demand for electric energy on a winter's day in a country with 7.5 million inhabitants. Estimate the energy consumption per capita on this day with the help of this diagram.

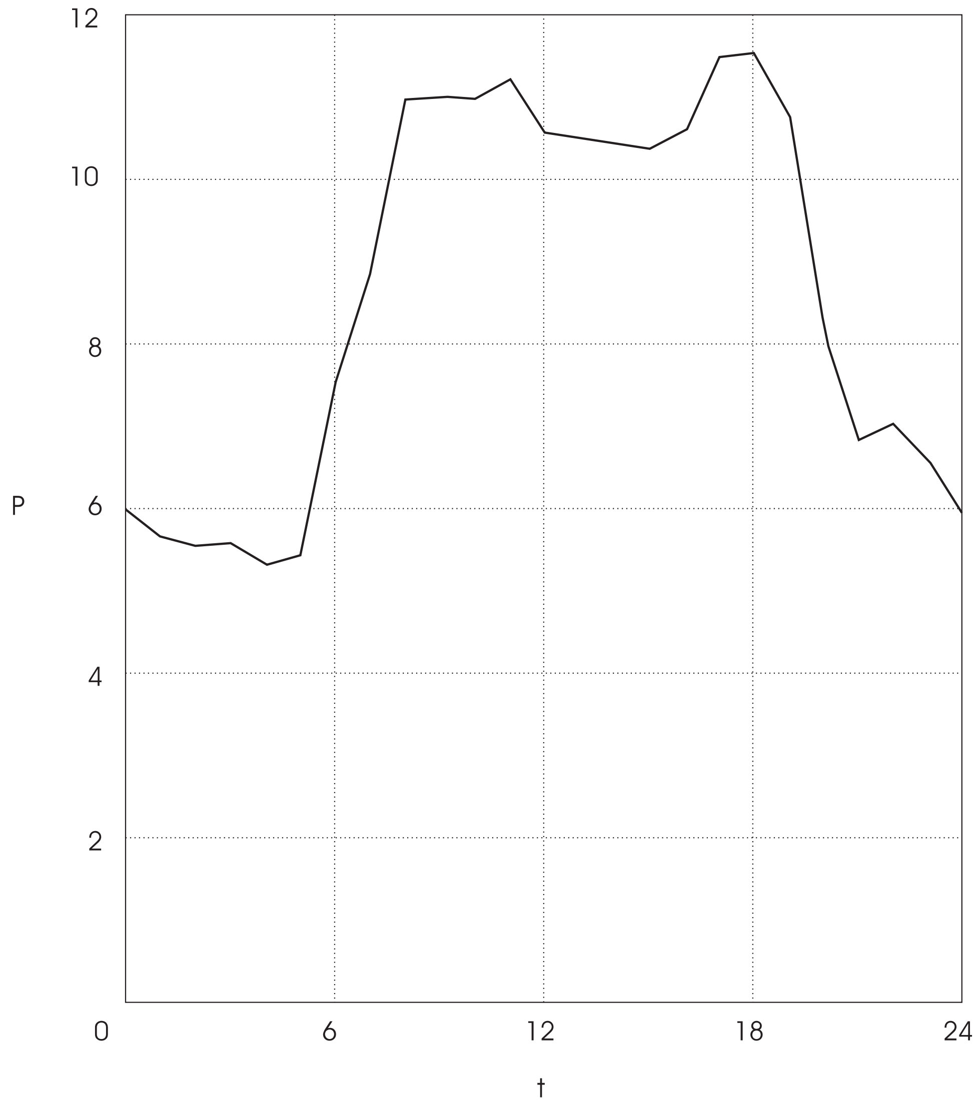


Figure 1.10: Consumption of electric energy on a winter's day.

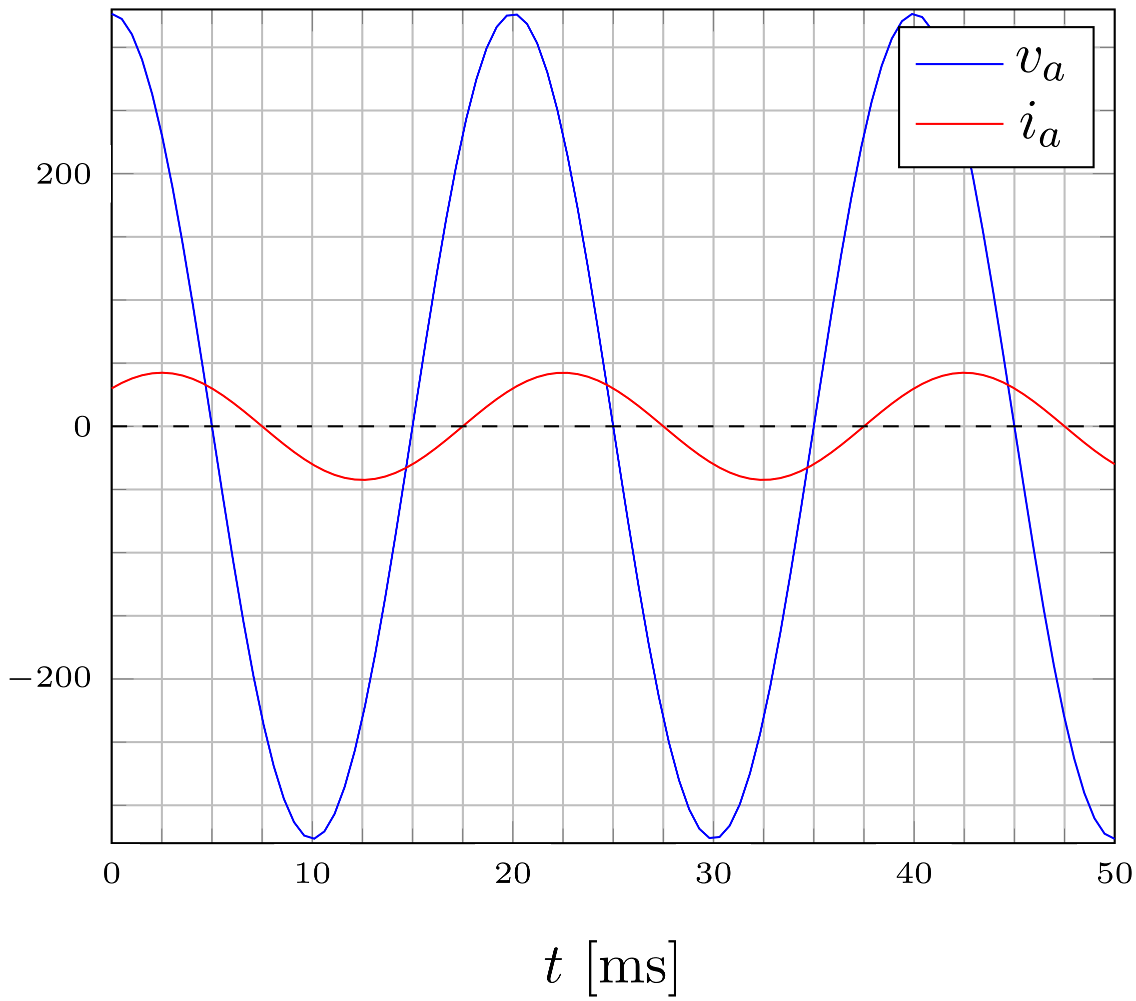
Exercise 1.5.6 *Three-phase electronic inverter*

A three-phase electronic inverter is *producing* active power. The time evolutions of the phase-to-neutral voltage $v_a(t)$ and the corresponding current $i_a(t)$ are shown in Fig. 1.11. The current orientation is not known. The RMS value of the phase voltage (resp. line current) are $400/\sqrt{3}$ V (resp. 30 A). The three phases are balanced.

a) Is the inverter producing or consuming reactive power?

b) Determine the three-phase active, reactive and apparent powers.

c) Assume that the inverter is lossless and the DC side can be represented by a Thevenin equivalent as shown in the figure. Compute the value of the DC current I_{DC}.



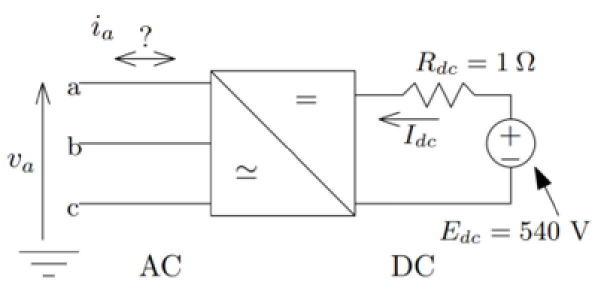


Figure 1.11: Three-phase electronic inverter.

Chapter 2

AC power circuits

2.1 Exercises

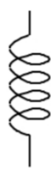
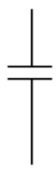
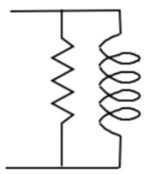
Exercise 2.1.1 *Y-D loads*

An electric radiator consisting of three Ohmic resistances is connected to a three-phase power output, which supplies a constant, symmetric voltage. Will a wye or a delta connection of the resistances lead to a faster warming of your house and how much faster? Show some brief calculations.

Exercise 2.1.2 *Power factor - load*

Fill the cells of the table below with the most appropriate answer among:

$$= 0 \qquad < 0 \qquad > 0 \qquad = 1 \qquad < 1$$

one-port :					
active power consumed					
reactive power produced					
$\cos\phi$					
$\tan\phi$ [1]					

[1]load convention is assumed

For the $cos\phi$ also state if it is capacitive (cap.) or inductive (ind.) where appropriate.

Exercise 2.1.3 *Power factor - line*

A transmission line is serving a load. How does the power factor of the load $cos\phi$ influence the maximum transferable active power of the line?

Exercise 2.1.4 *Symmetrical three-phase system analysis*

In this example, we will examine the power occurring in a symmetrical three-phase system. We consider the single phase diagram of a three-phase transmission system in figure 2.1.

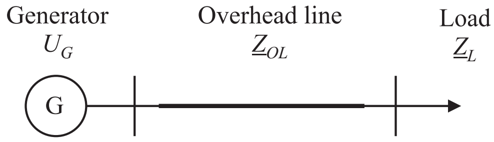


Figure 2.1: Generator-Line-Load.

A generator supplies power to a load via an overhead transmission line. The overhead line is modelled as a concentrated series impedance $\underline{Z}_{OL}$; the load is given as constant impedance $\underline{Z}_L$. The following quantities are known:

- Generator voltage $U_G = 20$ kV (phase to phase), $f = 50$ Hz
- Overhead line impedance $\underline{Z}_{OL} = R_{OL} + jX_{OL} = 2.5 + j3.0\ \Omega$
- Load impedance $\underline{Z}_L = R_L + jX_L = 110 + j50\ \Omega$ (by phase, in Y-connection)

The following problems are to be solved:

a) Draw the complete three-phase as well as the single phase equivalent circuit diagram for a balanced state.

b) Calculate the phase currents and the active, reactive and apparent power consumed by the load as well as the power factor of the load.

c) The power factor of the load shall be increased through a connection of capacitors (in parallel to the load). Thereby, the capacitors can be Δ- or Y-connected. Which capacitances C_Δ, C_Y would be necessary to compensate the power factor to a value of 0.98 (inductive)?

d) Compare the line losses (active and reactive power) with and without compensation.

e) Compare the voltages (absolute value and phase angle) of the load with and without compensation.

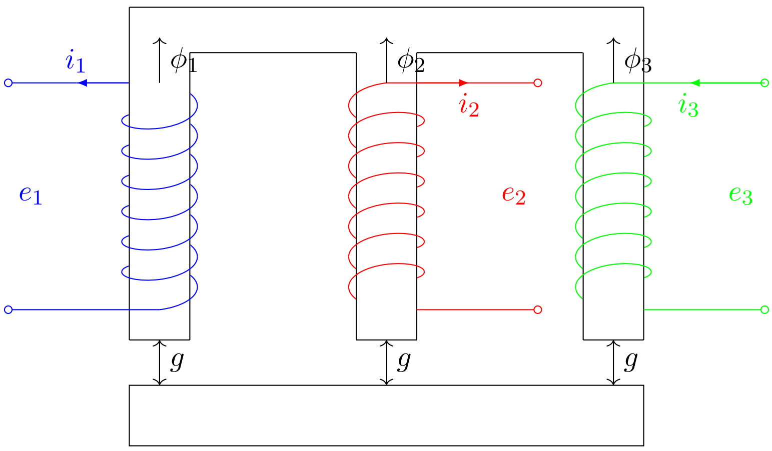
f) Reconsider the results from d) and e) and summarize the effects of reactive power compensation.

Chapter 3

Magnetic circuits and power transformers

3.1 Exercises

Exercise 3.1.1 *Three-coil magnetic core*



For the magnetic circuit of the image above, it is given that the three gaps have the same length g and cross-section A. The magnetic resistance of the core can be neglected.

(a) Compute the magnetic fluxes ϕ_1, ϕ_2, and ϕ_3 as a function of the currents i_1, i_2, and i_3.

(b) Give the necessary condition to have **no** magnetic flux in the middle leg. Compute the magnetic flux in the other two legs in this scenario.

(c) Compute the self-inductance of each coil and the mutual inductance between the three coils.

Exercise 3.1.2 *Single-phase transformer*

A single-phase transformer is rated at 30 kVA, 3.45/0.23 kV, 50 Hz. An ideal voltage source, connected to the high voltage side, supplies an impedance load. At a voltage of $V_L = 0.22$ kV the load has a power consumption of 15 kVA, power factor 0.85 lagging. Suppose the transformer is ideal.

(a) Draw the equivalent circuit of the system.

(b) Determine the voltage across the primary winding.

(c) Calculate the load impedance $\underline{Z}_L$.

(d) Determine the load impedance referred to the 3.45 kV side.

(e) What are the real and reactive power supplied to the 3.45 kV winding?

Exercise 3.1.3 *Per-unit calculations*

The reactance of a transformer is 0.15 pu with a power base value of 150 MVA. What is the transformer's reactance in pu, if we assume a power base value of 100 MVA? Clearly show your calculation.

Exercise 3.1.4 *Per-unit single-phase transformer*

A single-phase transformer is rated at 20 kVA, 600/230 V, 50 Hz. The equivalent leakage impedance of the transformer referred to the 600 V winding is $\underline{Z}_1 = 0.07\underline{/65^\circ}\ \Omega$.

(a) Determine the per unit leakage impedance referred to winding 1 using the transformer ratings as base values.

(b) Determine the per unit leakage impedance referred to winding 2 using the transformer ratings as base values.

Chapter 4

Per-unit system

4.1 Exercises

Exercise 4.1.1 *Benefits of per-uni system*

What is the advantage of selecting the base voltages of the p.u. system at the two sides of a transformer according to its voltage transformation ratio?

Exercise 4.1.2 *Per-unit analysis with transformers*

This example is taken from: A. R. Bergen, V. Vittal: Power Systems Analysis, 2nd Edition, Prentice Hall, 2000, ISBN 0-13-691990-1, pp. 164–166.

We consider a three-phase system with the single phase diagram shown in figure 4.1. The nominal transformer quantities are:

- Trafo 1: $S_{B,T1} = 5MVA$ (3-ph.), Prim.: 13.2 kV – Sec.: 132 kV, $x_{T1} = 10\%$=0.1 p.u.
- Trafo 2: $S_{B,T2} = 10MVA$ (3-ph.), Prim.: 138 kV – Sec.: 69 kV, $x_{T2} = 8\%$= 0.08 p.u.

The transformer voltages are given in phase-to-phase values. The impedance of the overhead line is $\underline{Z}_{OL} = 10 + j100\ \Omega$, the ohmic load impedance is $\underline{Z}_L = 300\ \Omega$. The RMS value of the phase-to-phase generator voltage is $U_G = 13.2$ kV.

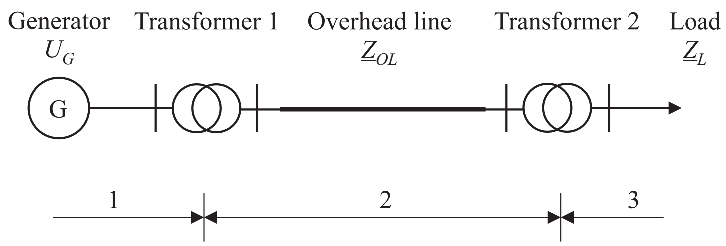


Figure 4.1: Power transmission with transformer-line-transformer.

The following problems are to be solved:

a) Pick a base value for the power (one for the entire system) and three base values for the voltage (for sections 1, 2 and 3) and express all impedances and the generator voltage in p.u. values.

b) Draw the single phase equivalent circuit.

Name	Type	Rated power [MVA]	Rated voltage [kV]	Reactance [pu] (on device base)
G	Generator	90	22	0.18
T1	Transformer	50	22/220	0.10
T2	Transformer	40	220/11	0.06
T3	Transformer	40	22/110	0.064
T4	Transformer	40	110/11	0.08
M	Motor	66.5	10.45	0.185
L1	Power line	135	220	0.4
L2	Power line	65	110	0.35

Table 4.1: Parameters of AC power system

c) Calculate the absolute value of the phase currents in the sections 1, 2 and 3, both in p.u. and in SI.

d) Calculate the three-phase apparent power of the load, both in p.u. and in SI.

Exercise 4.1.3 *Per-unit three-phase power system*

Consider the three-phase power system with one-line diagram given in Fig. 4.2. The three-phase load "L" at bus 4 consumes 57 MVA, 0.6 power factor lagging at a line voltage of 10.45 kV. The remaining system data is given in Table 4.1. For the transformers, the per unit values are expressed with respect to the voltage on the high voltage side. Select a common base of $S_{B3\phi} = 100$ MVA and $V_B = 22$ kV on the generator side.

(a) Draw the single-phase equivalent circuit of the system showing all impedances (including the load impedance).

(b) How many different voltage zones are there in the system? Determine the base voltage, the base power and the base impedance for each voltage zone.

(c) Determine the values of all impedances in per unit.

(d) Is the system (N-1) secure with respect to the failure of a power line? Justify your answer.

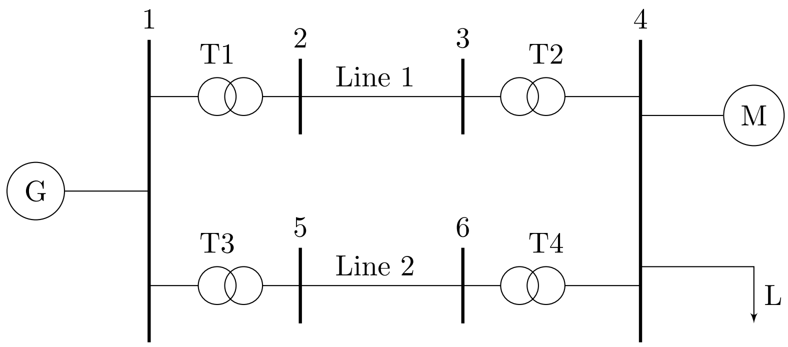


Figure 4.2: AC power system one-line diagram

Chapter 5

Transmission lines

5.1 Exercises

Exercise 5.1.1 *Safe magnetic field distance*

A balanced 132 kV three-phase circuit transfers 300 MVA. It is given that the following organisations have set these maximum safety limits for alternating magnetic fields of low frequency:

(a) ICNIRP: 0.1mT

(b) CENELEC: 0.64mT

(c) NRPB: 1.6mT

What is the minimum safe distance concerning the magnetic field?

Exercise 5.1.2 *Transmission line noise*

Losses and acoustic noise are generated by partial discharges along the overhead transmission conductors (so-called corona discharges). These unintended noise emissions can be mitigated by reducing the electric field on the conductor surface. Think about measures leading to a reduction of the electric field strength and thus to a noise reduction. While doing so, consider the following aspects:

(a) Conductor voltage

(b) Geometry of the line tower, arrangement of the conductors

(c) Cross section and form of the conductors

Exercise 5.1.3 *Line parameter calculation*

In this example, we compare the surge impedances and the surge impedance loadings of two different three-phase conductors. The following data are known:

	Overhead line	Cable
Per unit length series resistance R' in Ω/km	0.12	0.10
Per unit length shunt conductance G' in μS/km	0.05	1.00
Per unit length series inductance L' in mH/km	1.00	0.30
Per unit length shunt capacitance C' in nF/km	10	200
Nominal voltage U_N in kV (phase to phase)	110	110
Nominal current I_N in A (per phase)	330	290
Frequency f in Hz	50	50
Length l in km	85	45

Calculate the surge impedances of the two conductors.

The following exercises are to be solved:

(a) Calculate the surge impedances of the two conductors.

(b) Calculate the surge impedance loadings of the two conductors (for nominal voltage at the end of the line) and compare them to the corresponding nominal powers.

(c) Which data from the table didn't you need? Why not?

Exercise 5.1.4 *PI equivalent*

In this example, we want to calculate the concentrated elements of a Π equivalent circuit from the distributed line parameters. We know the following quantities:

- $R' = 0.12\ \Omega/\text{km}$, $L' = 1\ \text{mH/km}$, $G' = 0\ \mu\text{S/km}$, $C' = 10\ \text{nF/km}$
- Line length $l = 300$ km
- Frequency $f = 50Hz$

The following exercises are to be solved:

(a) Calculate the series impedance $\underline{Z}_l$ and the shunt admittance $\underline{Y}_q$ of the Π equivalent circuit with the exact relationships from the wave equation.

(b) Calculate the series impedance $\underline{Z}_l$ and the shunt admittance $\underline{Y}_q$ of the Π equivalent circuit with the simplified equations for $|\underline{\gamma} l| \ll 1$.

(c) How can we explain the presence of an ohmic component in the shunt element ($\Re\left\{\underline{Y}_q\right\} \neq 0$) in case a) although we assumed $G' = 0$?

Chapter 6

Power system operation

6.1 Exercises

Exercise 6.1.1 *Maximum transferable power in an AC system*

In this example, we deal with the maximum transferable power in an AC system. We consider the transmission system of figure 6.1. A generator supplies power to a grid via a transformer and an overhead line. The generator is modelled as voltage source with an internal reactance x_S.[1] The amplitude of the internal generator voltage $\underline{u}_{Gi}$ is controlled to 1.0 p.u. The transformer and the overhead line are both represented with their reactance. The grid is considered to be an *infinite bus*, i.e. both the amplitude and the phase angle of the voltage $\underline{u}_N$ are constant and independent from the power infeed.

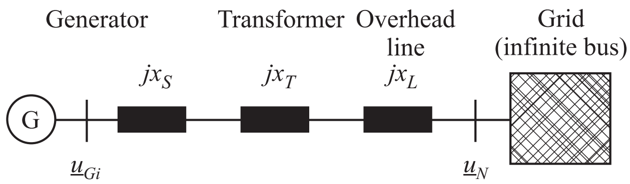


Figure 6.1: A generator supplies power to a grid.

The following data are known:

- Voltages: $\underline{u}_N = 1.0\angle 0^o$ p.u., $|\underline{u}_{Gi}| = 1.0$ p.u.
- Reactances: $x_S = 1.00$ p.u., $x_T = 0.10$ p.u., $x_L = 2.00$ p.u.

The following exercises are to be solved:

a) Which active power p_{max} (in p.u.) can be maximally transferred from the generator to the grid?

b) To which value x'_L should the reactance of the overhead line be reduced in order to keep a steady state stability margin of 30% when transmitting p_{max}?

c) Which reactive power would the generator and the grid exchange with full and reduced line reactance (cases a) and b)) when transmitting p_{max}?

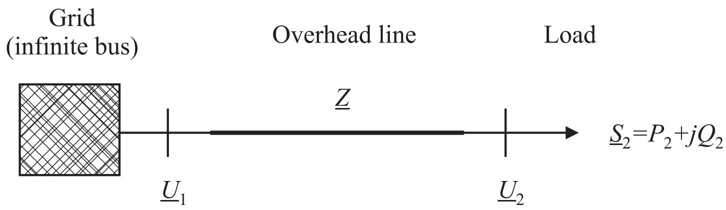


Figure 6.2: A load is connected to an infinite bus via an overhead line.

Exercise 6.1.2 *Voltage drop across an AC line*

In this example, we deal with the voltage drop along a line. We know the *voltage* $\underline{U}_1$ *at the beginning* of the line, the complex *power* $\underline{S}_2$ *at the end* of the line as well as the series impedance of the line $\underline{Z}$ (see figure 6.2):

- $\underline{U}_1 = 400\angle 0^o$ kV
- $\underline{S}_2 = P_2 + jQ_2 = 200$ MW $+j70$ MVar
- $\underline{Z} = R + jX = 10 + j100\ \Omega$

Calculate the magnitude (in kV) and the phase angle of the voltage $\underline{U}_1$ at the end of the line

a) with exact equations

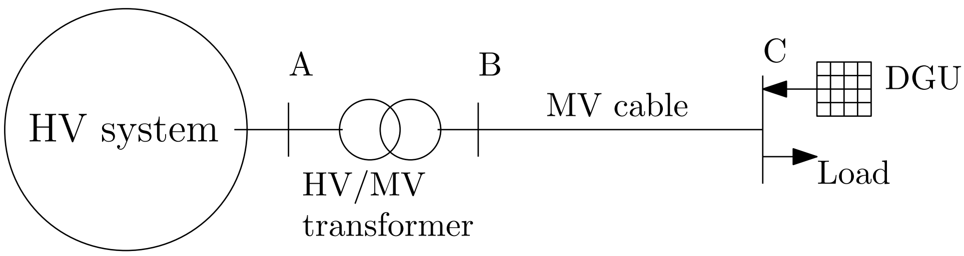
b) with the simplification $R = 0$

c) with the simplifications $R = 0$ and $P_2 = 0$

Exercise 6.1.3 *Distributed generator integration*

The following exercise illustrates, in a simplified manner, a concern due to the connection of Distributed Generators (DGs) to distribution grids. In situations of low load and high DG production, the active power flow reverts from distribution to transmission, and the voltage rises in the medium-voltage and low-voltage distribution grids. In case of overvoltage, customer appliances may be damaged and DGs may be disconnected by the protections of their power electronics.

Consider the following system:



System data:

- HV network: nominal voltage: 150 kV; magnitude of Thevenin e.m.f. $E_{th} = 1$ pu, the Thevenin impedance is $X_{th} = 5\,\text{Ohm}$
- transformer: nominal apparent power = 10 MVA; reactance $X_{trfo} = 0.14$ pu on the 10-MVA base; ratio $r = 138/11$ kV/kV

[1]For static considerations, it is common to approximate the internal reactance of synchronous generators with their *synchronous reactance* x_S. The theoretical background of this model is the two-axis theory (Park & Robertson, 1928).

- cable: nominal voltage = 11 kV; $R = 0.0\ \Omega$; $X = 1.2\ \Omega$
- load power factor: 0.95 inductive
- DG: nominal voltage = 11 kV; nominal apparent power = 5 MVA, nominal active power $P_N = 4.5$ MW

Answer the following questions:

a) Draw the per-unit single phase equivalent circuit, assuming a constant impedance load in Y connection and that the DG is not connected. Label clearly the nodes of buses A, B, and C, the load voltage and the system sources, currents and impedances.

b) Choose the appropriate base voltages, and express all impedances and the generator voltage in pu values. Use base power $S_{base} = 10$ MVA and base voltage on bus A $V_{b1} = 138$ kV.

c) Consider a "high" load $P_L = 5$ MW with the DG not in operation. Compute the load voltage V_C. Is it within the security margin of $\pm 5\%$?

d) Consider a "low" load $P_L = 1$ MW and a "high" production $P_{DG} = 4.5$ MW with $Q_{DG} = 0$ (unity power factor). Show that the voltage V_C takes a high value.

e) Now, consider the same P_L and P_{DG} of the previous question and the DG follows the VDE standard shown in Fig. 6.3. What is the PF the unit will work at? Will it consume or produce reactive power? How much? Show that the voltage V_C is brought back to a normal value.

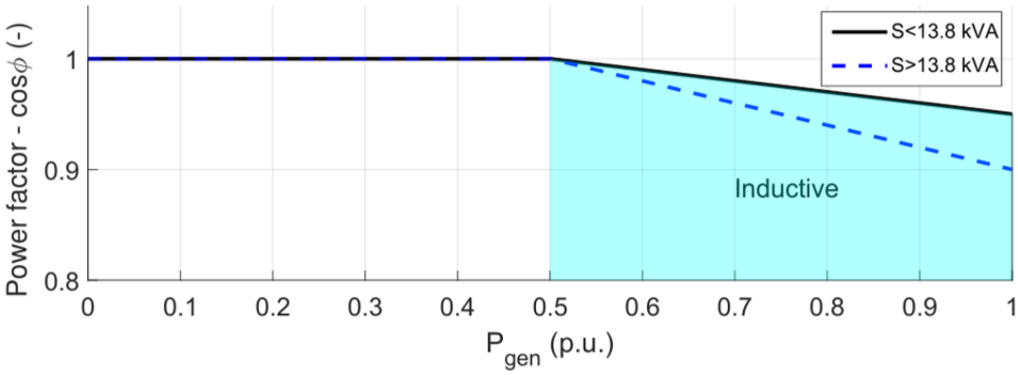


Figure 6.3: Reactive power control according to the VDE standard (P_{gen} is in per-unit of the nominal active power P_N of the DG)

f) Is the current in the DG acceptable if it operates as in the previous question?

g) Which reactive power must be consumed by the DG to bring the voltage V to 11.44 kV?

Chapter 7

Rotating machines

7.1 Exercises

Exercise 7.1.1

What is the principal difference between a synchronous machine and an induction machine?

Exercise 7.1.2 *Synchronous generator voltages*

The following information is known about the simple two-pole synchronous generator in Fig. 7.1. The peak flux density of the rotor magnetic field is 0.2 T, and the mechanical rate of rotation of the shaft is 3000 r/min. The stator radius of the machine is 0.25 m, its coil length is 0.3 m, and there are 15 turns per coil. The machine is Y-connected.

a) What is the frequency of the generator?

b) What are the three phase voltages of the generator as a function of time?

c) What is the rms phase voltage of this generator?

d) What is the rms terminal voltage of this generator?

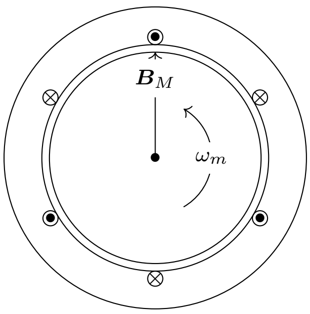


Figure 7.1: Three-phase machine.

Exercise 7.1.3

A three-phase, Y-connected, 50 Hz, two-pole synchronous machine has a stator with 2000 turns of wire per phase. What rotor flux would be required to produce terminal (line-to-line) voltage of 6 kV?

Exercise 7.1.4 *Synchronous generator field current*

(*Adapted from "Chapman, S.J. (2005). Electric machinery fundamentals (4e). McGraw-Hill."*)

A 480-V, 50 Hz, Delta-connected, four-pole synchronous generator has the Open-Circuit Characteristic curve shown in Fig 7.2 (this curve shows how the voltage V_A changes against the field current I_F if the generator is open circuited). This generator has a synchronous reactance of 0.1 Ω and an armature resistance of 0.015 Ω. At full load, the machine supplies 1200 A at 0.8 PF lagging. Under full-load conditions, the friction and windage losses are 40 kW and the core losses are 30 kW. Ignore any field circuit losses.

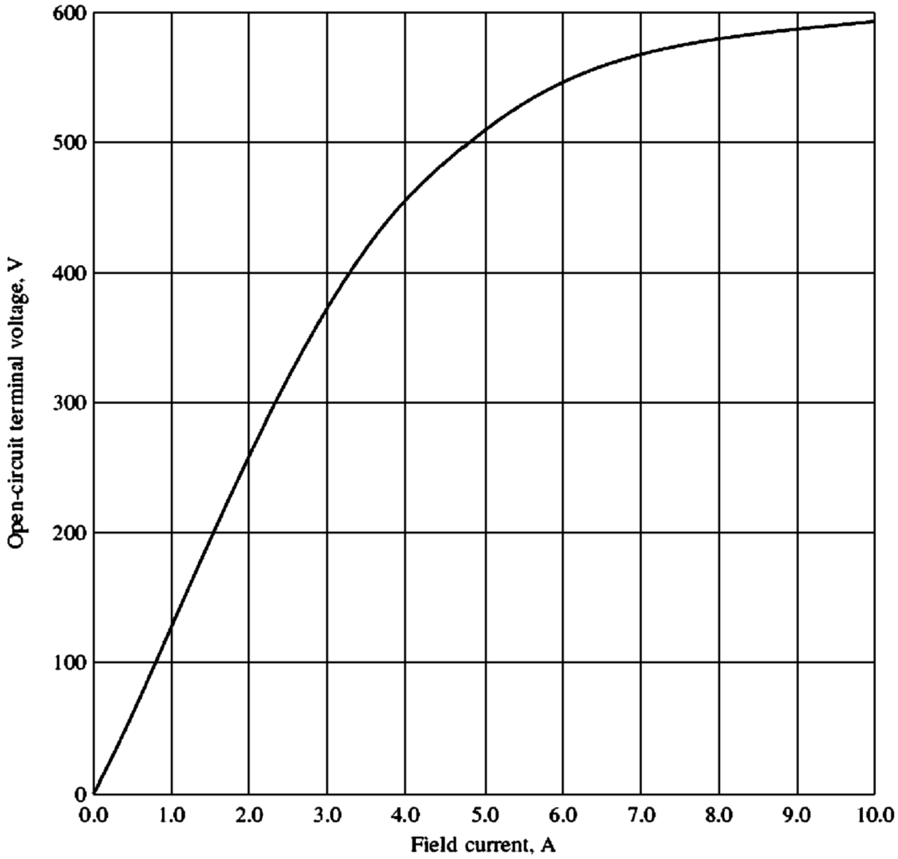


Figure 7.2: Open-Circuit Characteristic curve of synchronous generator.

a) What is the speed of rotation of this generator?

b) How much field current must be supplied to the generator to make the terminal voltage 480 V at no load?

c) If the generator is now connected to a load and the load draws 1200 A at 0.8 PF lagging, how much field current will be required to keep the terminal voltage equal to 480 V?

d) How much power is the generator now supplying? How much power is supplied to the generator by the prime mover? What is this machine's overall efficiency?

e) If the generator's load were suddenly disconnected from the line, what would happen to its terminal voltage?

f) Finally, suppose that the generator is connected to a load drawing 1200 A at 0.8 PF leading. How much field current would be required to keep V_A 480 V?

Exercise 7.1.5 *Synchronous generator analysis under varying load*

(*Adapted from "Chapman, S.J. (2005). Electric machinery fundamentals (4e). McGraw-Hill."*)

A 480-V, 50-Hz, Y-connected, six-pole synchronous generator has a per-phase synchronous reactance of 1.0Ω. Its full-load armature current is 60 A at a PF of 0.8 lagging. This generator has friction and windage losses of 1.5 kW and core losses of 1.0 kW at 50 Hz at full load. Since the armature resistance is being ignored, assume that the I^2R losses are negligible. The field current has been adjusted so that the terminal voltage is 480 V at no load.

a) What is the speed of rotation of this generator?

b) What is the terminal voltage of this generator if the following are true?

    i. It is loaded with the rated current at 0.8 PF lagging.
    ii. It is loaded with the rated current at 1.0 PF.
    iii. It is loaded with the rated current at 0.8 PF leading.

c) What is the efficiency of this generator (ignoring the unknown electrical losses) when it is operating at the rated current and 0.8 PF lagging?

d) How much shaft torque must be applied by the prime mover at full load? How large is the induced counter-torque?

e) What is the voltage regulation of this generator at 0.8 PF lagging? At 1.0 PF? At 0.8 PF leading?

Exercise 7.1.6 *Synchronous motor analysis under varying load*

(*Adapted from "Chapman, S.J. (2005). Electric machinery fundamentals (4e). McGraw-Hill."*)

A 208-V, 45-kVA, Delta-connected, 50-Hz synchronous machine has a synchronous reactance of 2.5 Ω and a negligible armature resistance. Its friction and windage losses are 1.5 kW, and its core losses are 1.0 kW. Initially, the shaft is supplying a 15-hp load, and the motor's PF is 0.8 leading.

a) Sketch the phasor diagram of this motor, and find the values of $\underline{I}_A$, $\underline{I}_L$ (line current) and $\underline{E}_A$.

b) Assume that the shaft load is now increased to 30 hp. Find $\underline{I}_A$, $\underline{I}_L$ and $\underline{E}_A$ after the load change. What is the new motor power factor?

c) Sketch the phasor diagram in response to this change.

Chapter 8

Power flow analysis

Chapter 9

Short-circuit analysis

Chapter 10

Unbalanced operation

Chapter 11

FACTS devices

Chapter 12

Answers to exercises

Exercise 1.5.1 a) $61.7\angle 139.4°$ b) $53.85\angle -8.2°$ c) $0.1313\angle -66.8°$

Exercise 1.5.3 a) $169.8\angle -34.5°\,\Omega$ b) $2.64\angle -89.8°\,S$

Exercise 1.5.4 a) $C = 4.88\,\mu$F

Exercise 1.5.5 a) $\widehat{p} = 8.62$ kW c) 27.2 kWh

Exercise 1.5.6 a) $\frac{\pi}{4}$ b) $P_{ac} = 14.7\ kW$; $Q_{ac} = 14.7\ kVAr$ c) $I_{dc} = 28.75$ A

Exercise 4.1.3 **Exercise 7.1.2** a) $f_e = 50\ Hz$ b) $E_{max} = 169.7\ V$ c) $E_{RMS} = 120\ V$ d) $V_T = 208\ V$