EEN452 - Control and Operation of Electric Power Systems

Part 2: Synchronous machine model (simplified)

https://sps.cut.ac.cy/courses/een452/

Dr Petros Aristidou
Department of Electrical Engineering, Computer Engineering & Informatics
Last updated: January 27, 2021

Today's learning objectives

After this part of the lecture and additional reading, you should be able to . . .

1. . . . describe and analyse the behaviour of a synchronous machine under different operating conditions;
2. . . . use simple computational models to show the behaviour of synchronous machines;
3. . . . link the synchronous machine with the system dynamics.

Much of the material was adapted from the courses delivered by Prof. Thierry Van Cutsem at the University of Liege.

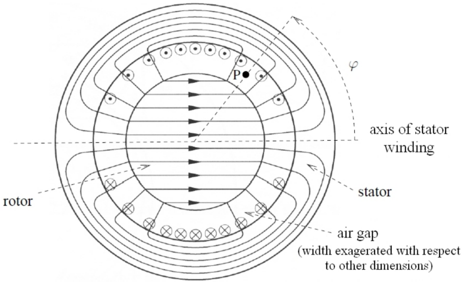
Why synchronous machines?

- produce the major part of the electric energy
  - range from a few kVA to a few hundred MVA
  - the biggest are rated 1500 MVA
- play an important role:
  - they impose the frequency of sinusoidal voltages and currents
  - they provide an ”energy buffer” (through the kinetic energy stored in their rotating masses)
  - they can produce or consume reactive power (needed to regulate voltage).

1 Magnetic field created by the stator

- stator (or armature) = motionless, separated from the rotor by a small air gap
- subjected to varying magnetic flux $\rightarrow$ built up of thin laminations to decrease eddy (or Foucault) currents
- equipped with three windings, distributed 120 degrees apart in space.

Magnetic field created by a direct current flowing in one of the stator windings:

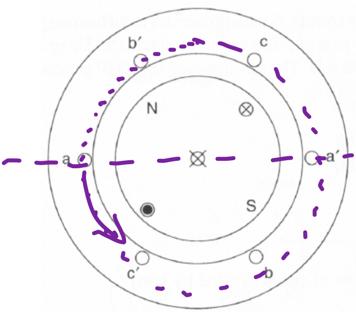


1 Magnetic field created by the stator

The magnetic field lines cross the air gap radially. The amplitude $B(\varphi)$ of the magnetic flux density at point P:

- is a periodic function of φ with period 2π
- this function has a "staircase" shape
- is made as close as possible to a sinusoid, by properly distributing the conductors along the air gap.

Layout of the three phases (each winding is represented by a single turn for clarity):



φ

Total flux density created by the three phases at point P corresponding to angle φ:

$$\boxed{B_{3\varphi}(\varphi) = ki_a \cos(\varphi) + ki_b \cos(\varphi - \frac{2\pi}{3}) + ki_c \cos(\varphi - \frac{4\pi}{3})}$$

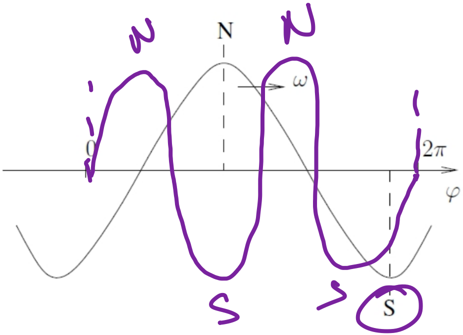
If three-phase alternating currents are flowing in the windings:

$$\begin{aligned}
B_{3\varphi}(\varphi) =& \sqrt{2}kI \Big[\cos(\omega t + \psi)\cos(\varphi) + \cos(\omega t + \psi - \frac{2\pi}{3})\cos(\varphi - \frac{2\pi}{3}) \\
&+ \cos(\omega t + \psi - \frac{4\pi}{3})\cos(\varphi - \frac{4\pi}{3})\Big] \\
=& \frac{\sqrt{2}kI}{2}\Big[\cos(\omega t + \psi + \varphi) + \cos(\omega t + \psi - \varphi) + \cos(\omega t + \psi + \varphi - \frac{4\pi}{3}) \\
&+ \cos(\omega t + \psi - \varphi) + \cos(\omega t + \psi + \varphi - \frac{2\pi}{3}) + \cos(\omega t + \psi - \varphi)\Big] \\
=& \frac{3\sqrt{2}kI}{2}\cos(\omega t + \psi - \varphi)
\end{aligned}$$

This is the equation of a wave rotating in the air gap at the angular speed ω

1 Magnetic field created by the stator

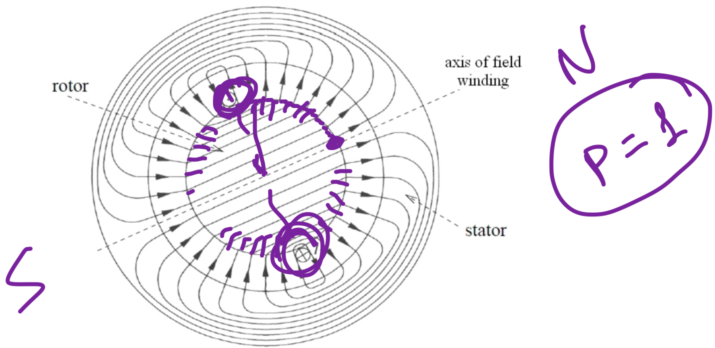
If we "unroll" the air-gap:



- The three-phase alternating currents all together produce the same magnetic field as a magnet (or a coil carrying a direct current) rotating at the angular speed ω
- North pole of magnet $\rightarrow$ maximum of $B(\varphi)$
- South pole of magnet $\rightarrow$ minimum of $B(\varphi)$

1 Magnetic field created by the rotor

Magnetic field created by this direct current (field winding represented by a single turn for clarity):

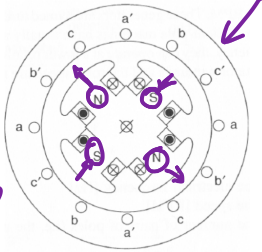


- rotor = rotating part, separated from the rotor by the air gap
- carries a winding in which a direct current flows, in steady-state operation
- referred to as field winding

1 Machines with multiple pairs of poles

Some turbines operate at a lower speed but AC voltages and currents at the stator must keep the same period $T = \frac{1}{f}$ → 50HZ → 20ms

- the rotor carries p **pairs of poles**
- during a period T, the rotor makes only $\frac{1}{p}$ of a whole revolution
- the stator carries p sets of (a, b, c) windings
- one winding spans an angle of π/p radians
- during a period T, each stator winding is still swept by one North and one South pole of rotor



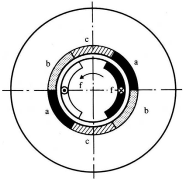
example for $p = 2$

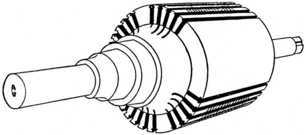
- $\boxed{\text{speed: } \frac{60 \cdot f}{p} rpm}$

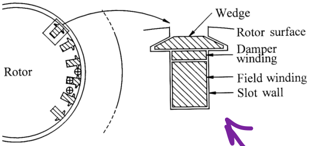
50HZ AC DC AC 100HZ

- The various windings relative to a given phase are connected (in series or parallel) to end up with a three-phase machine.

Cyprus University of Technology







- Driven by steam or gas turbines, which rotate at high speed
- $p = 1$ (conventional thermal units) or $p = 2$ (nuclear units)

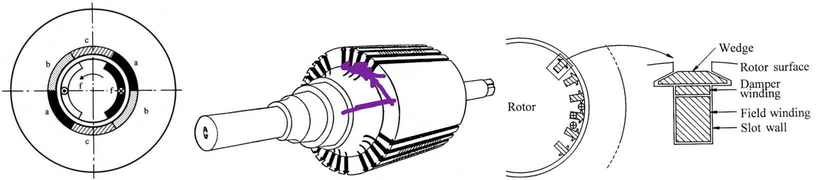
3000 rpm

↳ 1500 rpm

- cylindrical rotor made up of solid steel forging
- diameter $<<$ length (centrifugal force)

2 Round-rotor generators (or turbo-alternators)

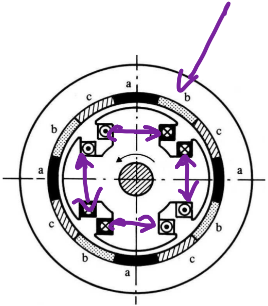
Cyprus University of Technology

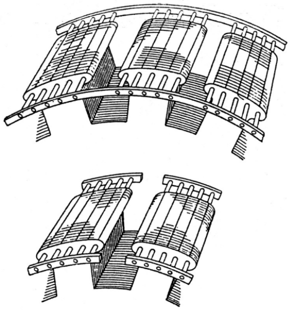


- field winding made up of conductors distributed on the rotor, in milled slots $\rightsquigarrow \sin(a)$ $N_c \rightsquigarrow \Phi_{max}$ I_f
- even if the generator efficiency is around 99%, the heat produced by Joule losses has to be evacuated.
- Large generators are cooled by hydrogen (heat evacuation 7 times better than air) or water (12 times better) flowing in the hollow stator conductors.

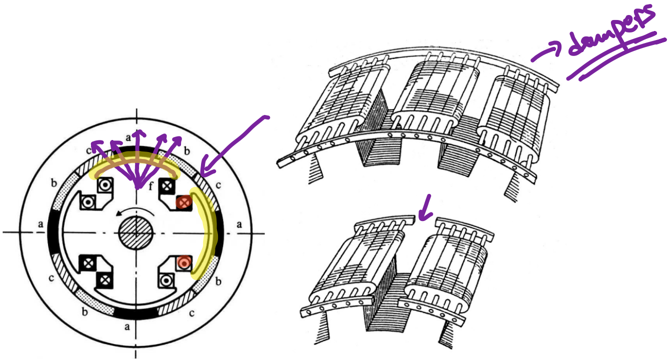
2 Round-rotor generators (or turbo-alternators)

2 Salient-pole generators



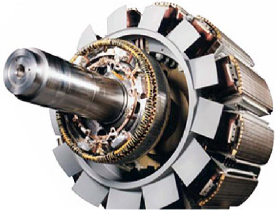


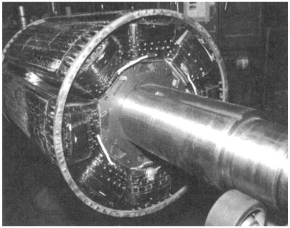
- Driven by hydraulic turbines (or diesel engines), which rotate at low speed
- p is much higher (at least 4) $\rightarrow$ it is more convenient to have field windings concentrated and placed on the poles
- air gap is not constant: min. in front of a pole, max. in between two poles

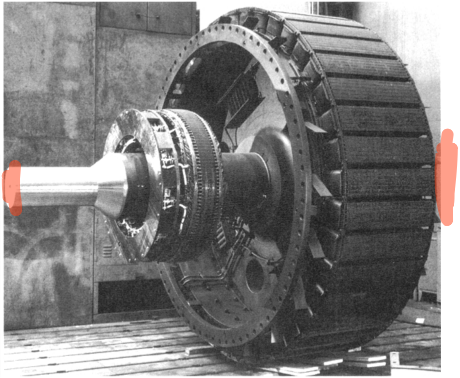


- poles are shaped to also minimize space harmonics (see slide 6)
- diameter $>>$ length (to have space for the many poles)
- rotor is laminated (poles easier to construct)
- generators usually cooled by the flow of air around the rotor

2 Salient-pole generators







$\frac{3000}{4}$

3 Number of windings

Cyprus University of Technology

There are typically 5 physical windings on a synchronous machine:

- 3 stator windings (a-phase, b-phase, and c-phase)
- 1 main field winding
- Damper or Amortisseur windings on the pole-faces
  - **round-rotor machines**: copper/brass bars placed in the same slots at the field winding, and interconnected to form a damper cage (similar to the squirrel cage of an induction motor)
  - **salient-pole machines**: copper/brass rods embedded in the poles and connected at their ends to rings or segments
  - They can be continuous or noncontinuous (see fig. in slide 15)

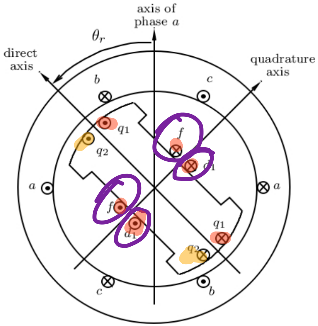
3 Damper windings

- **in perfect steady state**: the magnetic fields produced by both the stator and the rotor are fixed relative to the rotor $\rightarrow$ no current induced in dampers[1]
- **after a disturbance**: the rotor moves with respect to stator magnetic field $\rightarrow$ currents are induced in the dampers. . .
  . . . which, according to Lenz's law, create a damping torque helping the rotor to align on the stator magnetic field
- **Round-rotor generators**: the solid rotor offers a path for eddy currents, which produce an effect similar to those of amortisseurs.

[1] Amortisseur means "dead"

3 Modeled windings

Number of rotor windings = degree of sophistication of model. But more detailed model $\rightarrow$ more data are needed while measurement devices can be connected only to the field winding.

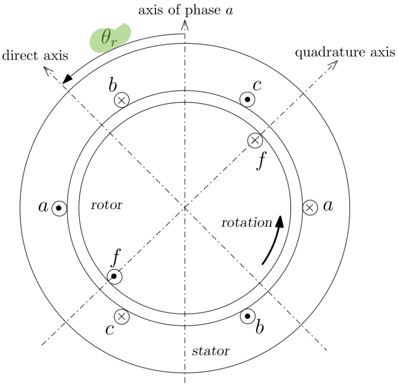


- 3 stator windings
- Most widely used model: 3 or 4 rotor windings:
  - f: field winding, $\cancel{d_1}$, $\cancel{q_1}$: damper windings
  - $\cancel{q_2}$: accounts for eddy currents in rotor; not used in (laminated) salient-pole generators

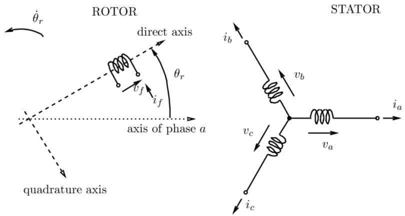
4 Outline

4 Simplifying assumptions

- round rotor
- saturation of magnetic material neglected
- on the rotor: field winding only (acceptable since focus is on steady-state operation)
- single pair of poles (does not affect the electrical behaviour)
- **ignore the damper windings**



4 Relations between voltages, fluxes and currents



$v_a = -R_a i_a - L\frac{di_a}{dt}$
$= -R_a i_a - \frac{d\psi_a}{dt}$

Stator:

$$v_a = -R_a i_a - \frac{d\psi_a}{dt} \quad v_b = -R_b i_b - \frac{d\psi_b}{dt} \quad v_c = -R_c i_c - \frac{d\psi_c}{dt}$$

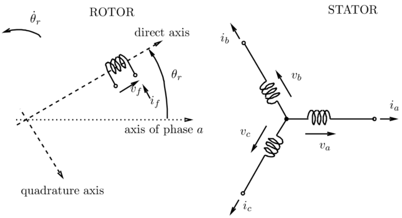
R_a : Resistance of each phase $\quad \psi_a$, ψ_b and ψ_c : flux linkages in phases

Field winding:

$$\boxed{v_f = -R_f i_f - \frac{d\psi_f}{dt}}$$

R_f : Resistance of winding $\quad \psi_f$: flux linkages in winding

4 Relations between voltages, fluxes and currents



- The magnetic circuit and all rotor winding circuits are symmetrical with respect to the polar and inter-polar (between-poles) axes
- We give these axes special names:
  - Polar axis: Direct, or d-axis
  - Interpolar axis: Quadrature, or q-axis
- q-axis is 90^o from the d-axis but can be modeled both as *leading* or *lagging*. Both assumptions are correct and used by textbooks. $\rightarrow$ in this course, we assume **lagging**.

4 Inductance matrix

$$\begin{bmatrix} \psi_a \\ \psi_b \\ \psi_c \\ \psi_f \end{bmatrix} = \underbrace{\begin{bmatrix} L_o & -L_m & -L_m & L_{af}\cos\theta_r \\ -L_m & L_o & -L_m & L_{af}\cos(\theta_r - \frac{2\pi}{3}) \\ -L_m & -L_m & L_o & L_{af}\cos(\theta_r - \frac{4\pi}{3}) \\ L_{af}\cos\theta_r & L_{af}\cos(\theta_r - \frac{2\pi}{3}) & L_{af}\cos(\theta_r - \frac{4\pi}{3}) & L_{ff} \end{bmatrix}}_{\mathbf{L}(\theta_r)} \begin{bmatrix} i_a \\ i_b \\ i_c \\ i_f \end{bmatrix}$$

where $L_o, L_m > 0$.

- Self-inductance of any stator winding is constant (due to round rotor)
- mutual inductance between any two phases is constant (due to round rotor)
- . . . and negative since a positive current i_x in phase x creates a negative flux ψ_y in phase y ($x \neq y$)

4 Inductance matrix

$$\begin{bmatrix} \psi_a \\ \psi_b \\ \psi_c \\ \psi_f \end{bmatrix} = \underbrace{\begin{bmatrix} L_o & -L_m & -L_m & L_{af}\cos\theta_r \\ -L_m & L_o & -L_m & L_{af}\cos(\theta_r - \frac{2\pi}{3}) \\ -L_m & -L_m & L_o & L_{af}\cos(\theta_r - \frac{4\pi}{3}) \\ L_{af}\cos\theta_r & L_{af}\cos(\theta_r - \frac{2\pi}{3}) & L_{af}\cos(\theta_r - \frac{4\pi}{3}) & L_{ff} \end{bmatrix}}_{\mathbf{L}(\theta_r)} \begin{bmatrix} i_a \\ i_b \\ i_c \\ i_f \end{bmatrix}$$

where $L_o, L_m > 0$.

- self-inductance of field winding is constant (path of magnetic field identical whatever the position of the rotor)
- mutual inductance between one phase and the field winding is maximum and positive when $\theta_r = 0$, zero when $\theta_r = \frac{\pi}{2}$, minimum and negative when $\theta_r = \pi$

Cyprus University of Technology

- rotation speed equal to nominal angular frequency:

$$\dot{\theta}_r = \omega_N \quad \theta_r = \theta_r^o + \omega_N t$$

θ_r^0: rotor position at $t = 0$

- constant direct current in field winding: $i_f = I_f$

- balanced three-phase voltages and currents in stator:

$$v_a(t) = \sqrt{2} V \cos(\omega_N t + \theta) \qquad i_a(t) = \sqrt{2} I \cos(\omega_N t + \psi)$$
$$v_b(t) = \sqrt{2} V \cos(\omega_N t + \theta - \frac{2\pi}{3}) \quad i_b(t) = \sqrt{2} I \cos(\omega_N t + \psi - \frac{2\pi}{3})$$
$$v_c(t) = \sqrt{2} V \cos(\omega_N t + \theta - \frac{4\pi}{3}) \quad i_c(t) = \sqrt{2} I \cos(\omega_N t + \psi - \frac{4\pi}{3})$$

with the corresponding phasors:

$$\underline{V} = V e^{j\theta} \qquad \underline{I} = I e^{j\theta}$$

Cyprus University of Technology

$$\psi_a = L_o\sqrt{2}I\cos(\omega_N t + \psi) - L_m\underbrace{\sqrt{2}I\cos(\omega_N t + \psi - \frac{2\pi}{3})}_{i_b} - L_m\underbrace{\sqrt{2}I\cos(\omega_N t + \psi - \frac{4\pi}{3})}_{i_c} + L_{af}\cos(\omega_N t + \theta_r^o)I_f$$

Adding and subtracting $L_m\sqrt{2}I\cos(\omega_N t + \psi)$ yields:

$$\begin{aligned}\psi_a =& L_o\sqrt{2}I\cos(\omega_N t + \psi) + L_m\sqrt{2}I\cos(\omega_N t + \psi) \\ & - L_m\sqrt{2}I\underbrace{\left(\cos(\omega_N t + \psi) + \cos(\omega_N t + \psi - \frac{2\pi}{3}) + \cos(\omega_N t + \psi - \frac{4\pi}{3})\right)}_{0} \\ & + L_{af}I_f\cos(\omega_N t + \theta_r^o) \\ =& \boxed{\underbrace{\sqrt{2}(L_o + L_m)I\cos(\omega_N t + \psi)}_{\psi_a^s} + \underbrace{L_{af}I_f\cos(\omega_N t + \theta_r^o)}_{\psi_a^r}}\end{aligned}$$

ψ_a^s: flux of the rotating field produced by the three stator currents

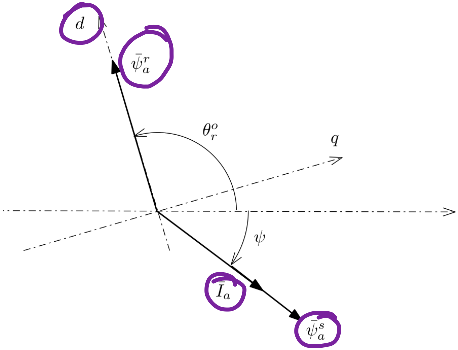
ψ_a^r: flux of the field created by the current i_f

5 Flux linkage in one stator winding (phase a)

Both flux components being sinusoidal functions of time (with angular frequency ω_N), they can be characterized by phasors:

$$\underline{\psi}_a^s = (L_o + L_m) I e^{j\psi} \quad \text{and} \quad \underline{\psi}_a^r = \frac{L_{af}}{\sqrt{2}} I_f e^{j\theta_r^o}$$

Phasor diagram:



Horizontal axis
= axis on which rotating vectors are projected
= axis to which the rotor position is referred, i.e. axis of phase a

5 Flux linkage in field winding

ia

ib

ic

$$
\begin{aligned}
\psi_f = {} & L_{ff} I_f + L_{af}\cos(\omega_N t + \theta_r^o)\sqrt{2} I \cos(\omega_N t + \psi) \\
& + L_{af}\cos(\omega_N t + \theta_r^o - \frac{2\pi}{3})\sqrt{2} I \cos(\omega_N t + \psi - \frac{2\pi}{3}) \\
& + L_{af}\cos(\omega_N t + \theta_r^o - \frac{4\pi}{3})\sqrt{2} I \cos(\omega_N t + \psi - \frac{4\pi}{3}) \\
= {} & L_{ff} I_f + \frac{\sqrt{2} L_{af}}{2} I \left[\cos(\theta_r^o - \psi) + \cos(2\omega_N t + \theta_r^o + \psi)\right] \\
& + \frac{\sqrt{2} L_{af}}{2} I \left[\cos(\theta_r^o - \psi) + \cos(2\omega_N t + \theta_r^o + \psi - \frac{4\pi}{3})\right] \\
& + \frac{\sqrt{2} L_{af}}{2} I \left[\cos(\theta_r^o - \psi) + \cos(2\omega_N t + \theta_r^o + \psi + \frac{4\pi}{3})\right] \\
= {} & \underbrace{L_{ff} I_f}_{\psi_f^r} + \underbrace{\frac{3\sqrt{2} L_{af}}{2} I \cos(\theta_r^o - \psi)}_{\psi_f^s}
\end{aligned}
$$

ψ_f^s: flux of the rotating field produced by the three stator currents; constant magnitude; at an angle $\theta_r^o - \psi$ wrt to field winding

ψ_f^r: flux created by field current

---

https://en.wikipedia.org/wiki/List_of_trigonometric_identities

5 Voltage-current relation at stator

Replacing v_a, i_a, and ψ_a by their expressions:

$$\sqrt{2}V\cos(\omega_N t + \theta) = -R_a\sqrt{2}I\cos(\omega_N t + \psi) + \sqrt{2}\omega_N(L_o + L_m)I\sin(\omega_N t + \psi) + \sqrt{2}\frac{\omega_N L_{af}}{\sqrt{2}}I_f\sin(\omega_N t + \theta_r^o)$$

$-\frac{\pi}{2}$

Let's define:

- $\boxed{X = \omega_N(L_o + L_m)}$: the synchronous reactance of the machine
- $\boxed{E_q = \frac{\omega_N L_{af}}{\sqrt{2}} I_f}$ RMS value of an e.m.f. proportional to field current I_f

The above equation becomes:

$$\boxed{\sqrt{2}V\cos(\omega_N t + \theta) = -R_a\sqrt{2}I\cos(\omega_N t + \psi) + \sqrt{2}XI\cos(\omega_N t + \psi - \frac{\pi}{2}) + \sqrt{2}E_q\cos(\omega_N t + \theta_r^o - \frac{\pi}{2})}$$

5 Voltage-current relation at stator

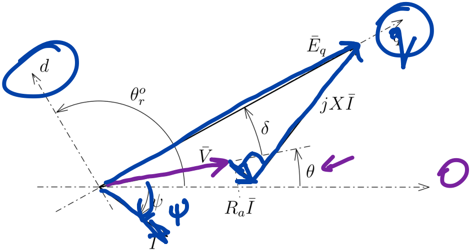
The corresponding phasor equation is:

$$Ve^{j\theta} = -R_a I e^{j\psi} + XIe^{j\psi} e^{-j\frac{\pi}{2}} + E_q e^{j(\theta_r^o - \frac{\pi}{2})}$$

or, simply:

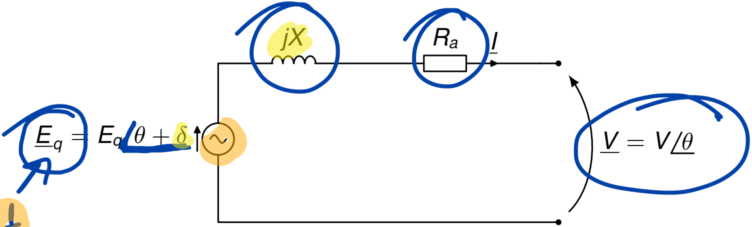
$$\boxed{\underline{V} = -R_a \underline{I} - jX\underline{I} + \underline{E}_q}$$

where $\underline{E}_q = E_q e^{j(\theta_r^o - \frac{\pi}{2})}$ is the phasor of the e.m.f. E_q, lying on the q axis



5 Per-phase equivalent circuit

Cyprus University of Technology



- The synchronous reactance X characterizes the steady-state operation of the machine
- δ is the phase shift between the internal e.m.f. $\underline{E}_q$ and the terminal voltage $\underline{V}$
- δ is called the internal angle, load angle, or power angle of the machine

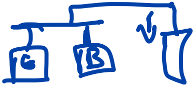
5 Nominal values and orders of magnitude

- Nominal voltage V_N: voltage for which the machine has been designed (in particular its insulation). If Na
  The real voltage may deviate from this value by a few %
- nominal current I_N: current for which machine has been designed (in particular the cross-section of its conductors).
  Maximum current that can be accepted without limit in time
- nominal apparent power $S_N = \sqrt{3} V_N I_N$

The machine parameters in per-unit on the base ($S_B = S_N$, $V_B = V_N/\sqrt{3}$):

- $R_a \approx 0.005$ pu
- $X \approx 1.5 - 2.5$ pu (for a round-rotor machine)

6 Power balance of the stator



$$p_{r\to s} = p_T + p_{Js} + \frac{dW_{ms}}{dt} \to 0$$

where

- $p_{r\to s}$: power transfer from rotor to stator
- p_T: three-phase instantaneous power leaving the stator
- p_{Js}: Joule losses in stator windings
- W_{ms}: magnetic energy stored in the stator windings

The nature of $p_{r\to s}$

- mechanical power for sure (torque applied to rotating masses)
- is there some electromagnetic transfer of power (like in a transformer)?

6 Power balance of the rotor

IF VF RF

$$\cancel{p_f} + \cancel{P_m} = \cancel{p_{Jf}} + \frac{dW_{mf}}{dt} + \frac{dW_c}{dt} + p_{r\to s}$$

$rpm = \frac{60 f}{p}$

where

- P_m: mechanical power provided by the turbine
- p_f: electrical power provided to the field winding by the excitation system
- p_{Jf}: Joule losses in the field winding
- W_{mf}: magnetic energy stored in the field winding
- W_c: kinetic energy of all rotating masses (generator and turbine)

Total electromagnetic energy stored in the machine:

$\frac{1}{2} L i^2$?

$$W_{m,tot} = \frac{1}{2}\begin{bmatrix} i_a & i_b & i_c & i_f \end{bmatrix} \boldsymbol{L}(\theta_r) \begin{bmatrix} i_a \\ i_b \\ i_c \\ i_f \end{bmatrix} = \frac{1}{2}\begin{bmatrix} i_a & i_b & i_c & i_f \end{bmatrix} \begin{bmatrix} \psi_a \\ \psi_b \\ \psi_c \\ \psi_f \end{bmatrix}$$

$$= \underbrace{\frac{1}{2}(i_a\psi_a + i_b\psi_b + i_c\psi_c)}_{W_{ms}} + \underbrace{\frac{1}{2} i_f \psi_f}_{W_{mf}}$$

$$J\frac{d^2\theta_r}{dt^2} = T_m - T_e$$

where
J: moment of inertia of all rotating masses
T_m: mechanical torque applied to the rotor by the turbine
T_e: electromagnetic torque applied to the rotor by the generator
Multiplying by the rotor speed $d\theta_r/dt$:

$$J\frac{d\theta_r}{dt}\frac{d^2\theta_r}{dt^2} = \frac{d\theta_r}{dt}T_m - \frac{d\theta_r}{dt}T_e$$

$$\Leftrightarrow \frac{dW_c}{dt} = P_m - \frac{d\theta_r}{dt}T_e$$

$\omega_m \cdot T_m = P_m$

$\parallel$

$\omega_e = 2\pi f$

P_e

and the power balance of the rotor becomes:

$$\boxed{p_f + \frac{d\theta_r}{dt}T_e = p_{Jf} + \frac{dW_{mf}}{dt} + p_{r\to s}}$$

6 Power balance of stator

$$\frac{1}{2} i_a \psi_a = (L_o + L_m) I^2 \cos^2(\omega_N t + \psi) + \frac{\sqrt{2}}{2} L_{af} I_f I \cos(\omega_N t + \theta_r^o) \cos(\omega_N t + \psi)$$
$$= \frac{1}{2}(L_o + L_m) I^2 + \frac{1}{2}(L_o + L_m) I^2 \cos(2\omega_N t + 2\psi) +$$
$$\frac{\sqrt{2}}{4} L_{af} I_f I \left[\cos(\theta_r^o - \psi) + \cos(2\omega_N t + \theta_r^o + \psi)\right]$$

By doing the same derivation for phases b and c, and adding all three results:

$$W_{ms} = \frac{1}{2}(i_a\psi_a + i_b\psi_b + i_c\psi_c) = \boxed{\frac{3}{2}(L_o + L_m) I^2 + \frac{3\sqrt{2}}{4} L_{af} I_f I \cos(\theta_r^o - \psi)}$$

W_{ms} is constant, i.e. $\frac{dW_{ms}}{dt} = 0$

https://en.wikipedia.org/wiki/List_of_trigonometric_identities

6 Power balance of stator

In three-phase balanced operation:

$$p_T = 3P$$

where P is the active power produced by one phase.

Hence, the power balance of the stator simply becomes :

$$p_{r \to s} = 3P + p_{Js}$$

Cyprus University of Technology

$$W_{mf} = \frac{1}{2} i_f \psi_f = \frac{1}{2} L_{ff} I_f^2 + \frac{3\sqrt{2}}{4} L_{af} I\, I_f \cos(\theta_r^o - \psi)$$

$$W_{mf} \text{ is constant, i.e. } \frac{dW_{mf}}{dt} = 0$$

$$\frac{d\psi_f}{dt} = 0 \quad \Rightarrow \quad V_f = R_f I_f \quad \Rightarrow \quad p_f = R_f I_f^2 = p_{Jf}$$

In steady state, the power entering the field winding is dissipated in Joule losses!

The field current aims at "magnetizing" the rotor, allowing the torque T_e to be created, but the field winding does not exchange power with the other windings.

$\omega_N T_e$

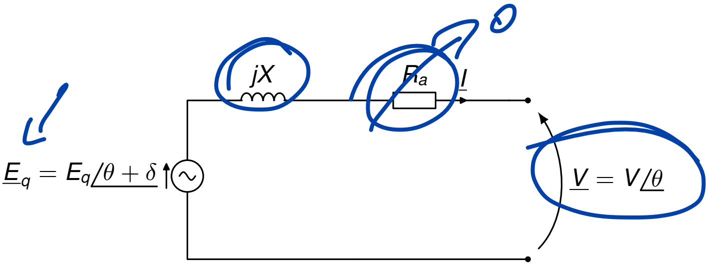
$$\frac{d\theta_r}{dt} = \omega_N \qquad \frac{dW_c}{dt} = 0 \qquad \boxed{T_m = T_e} \quad P_m = \omega_N T_e = \omega_N T_m$$

Hence, the power balance of the rotor simply becomes:

$$\boxed{p_{r\to s} = \omega_N T_e = \omega_N T_m = P_m}$$

where power $p_{r\to s}$ transfered from rotor to stator is purely mechanical!

Cyprus University of Technology



Assuming $R_a \approxeq 0$, active and reactive power in per-unit can be given as:

$$P = -\frac{VE_q}{X}\sin(\theta - (\theta + \delta)) = \frac{VE_q}{X}\sin(\delta)$$

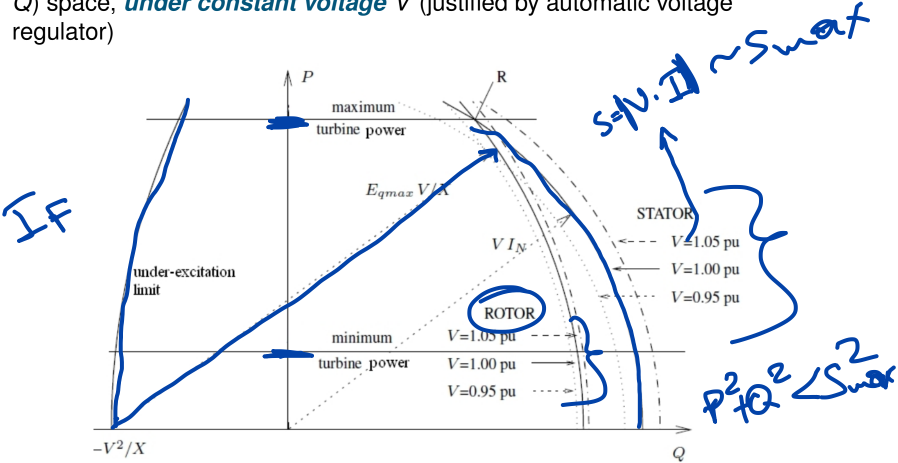
$$Q = -\frac{V^2}{X} + \frac{VE_q}{X}\cos(\theta - (\theta + \delta)) = -\frac{V^2}{X} + \frac{VE_q}{X}\cos(\delta)$$

7 Outline

7 Capability curves

Seen from the network, a generator is characterized by three variables: V, P and Q

The capability curves define the set of admissible operating points in the (P, Q) space, ***under constant voltage*** V (justified by automatic voltage regulator)



Cyprus University of Technology

Stator (heating) limit

stator current $I = I_N$ in per-unit: $S^2 = P^2 + Q^2 = V^2 I_N^2$

Rotor (heating) limit

field current $I_f = I_{fmax} \quad \Rightarrow E_q = E_{qmax} = \dfrac{\omega_N L_{af}}{\sqrt{2}} I_{fmax}$

With the same simplifying assumptions as before, and with $R_a = 0$:

$$P = \frac{E_{qmax} V}{X} \sin(\delta) \quad Q = \frac{E_{qmax} V}{X} \cos(\delta) - \frac{V^2}{X}$$

after eliminating δ:

$$\left(\frac{VE_{qmax}}{X}\right)^2 = \left(Q + \frac{V^2}{X}\right)^2 + P^2$$

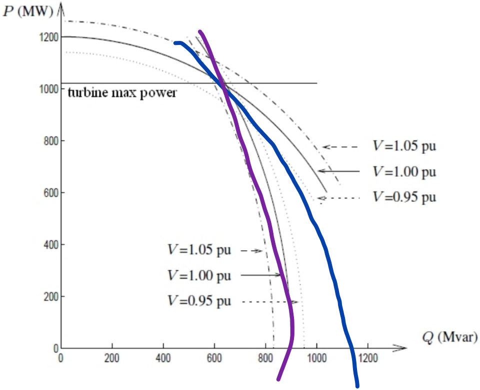
- Lower limit on active power caused by stability of combustion in thermal power plants
- maximum reactive power *in*creases when the active power *de*creases
  - to relieve an overloaded machine, P can be decreased but this power has to be produced by some other generators
- for a given value of P, the maximum reactive power increases with V
  - this holds true under the simplifying assumption of a non saturated machine; see next slide for a case with saturation
- in practice, under $V = 1$ pu, the two-by-two intersection points of respectively the turbine, the rotor and the stator limits are close to each other (”coherent” design of stator and rotor)
- the stator limits can be increased by a stronger cooling (e.g., higher hydrogen pressure in stator windings)

Under-excitation limit

Corresponds to a stability, not a thermal limit: absorbing more $Q \Rightarrow$ decreasing $E_q \Rightarrow$ decreasing $i_f \rightarrow$ maximum torque T_e decreases $\Rightarrow$ risk of losing synchronism.

7 Capability curves

Capability curves ($Q > 0$ part only) of a real machine with saturation taken into account



- the overall shape of the curves is the same
- but the rotor limit becomes more constraining when V increases