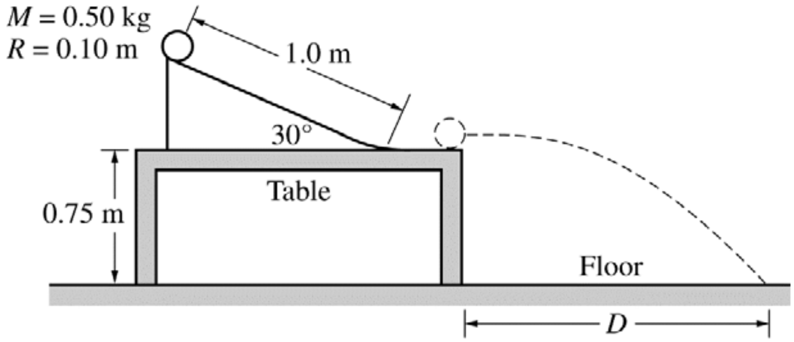

3. A uniform solid cylinder of mass $M = 0.50$ kg and radius $R = 0.10$ m is released from rest, rolls without slipping down a 1.0 m long inclined plane, and is launched horizontally from a horizontal table of height 0.75 m. The inclined plane makes an angle of $30°$ with the horizontal. The cylinder lands on the floor a distance D away from the edge of the table, as shown in the figure above. There is a smooth transition from the inclined plane to the horizontal table, and the motion occurs with no frictional energy losses. The rotational inertia of a cylinder around its center is $MR^2/2$.

(a) Calculate the total kinetic energy of the cylinder as it reaches the horizontal table.

(b) Calculate the angular velocity of the cylinder around its axis at the moment it reaches the floor.

(c) Calculate the ratio of the rotational kinetic energy to the total kinetic energy for the cylinder at the moment it reaches the floor.

(d) Calculate the horizontal distance D.

A sphere of the same mass and radius is now rolled down the same inclined plane. The rotational inertia of a sphere around its center is $\frac{2}{5}MR^2$.

(e)

i. Is the total kinetic energy of the sphere at the moment it reaches the floor greater than, less than, or equal to the total kinetic energy of the cylinder at the moment it reaches the floor?

____ Greater than ____ Less than ____ Equal to

Justify your answer.

ii. Is the rotational kinetic energy of the sphere at the moment it reaches the floor greater than, less than, or equal to the rotational kinetic energy of the cylinder at the moment it reaches the floor?

____ Greater than ____ Less than ____ Equal to

Justify your answer.

iii. Is the horizontal distance the sphere travels from the table to where it hits the floor greater than, less than, or equal to the horizontal distance the cylinder travels from the table to where it hits the floor?

____ Greater than ____ Less than ____ Equal to

Justify your answer.

STOP

END OF EXAM

Question 3

15 points total — **Distribution of points**

(a) 2 points

For correctly applying conservation of energy to the cylinder rolling down the incline — 1 point

$U_{g\_top} = K_{table}$

$K_{table} = mgh = (0.50\ \text{kg})\left(9.8\ \text{m/s}^2\right)(1.0\ \text{m})(\sin 30)$

For a correct answer with units — 1 point

$K_{table} = 2.45\ \text{J}$ (or 2.5 J using $g = 10\ \text{m/s}^2$)

(b) 3 points

For correctly setting the kinetic energy of the cylinder equal to the sum of both the linear and rotational kinetic energy — 1 point

$$K = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$$

For correctly substituting into the above equation for the linear velocity and moment of inertia of the cylinder — 1 point

$$K = \frac{1}{2}MR^2\omega^2 + \frac{1}{2}\left(\frac{1}{2}MR^2\right)\omega^2 = \frac{1}{2}M(R\omega)^2 + \frac{1}{4}M(R\omega)^2 = \frac{3}{4}M(R\omega)^2$$

$$\omega = \sqrt{\frac{4K}{3MR^2}} = \sqrt{\frac{(4)(2.45\ \text{J})}{(3)(0.50\ \text{kg})(0.10\ \text{m})^2}}$$

For correct substitution into the equation above — 1 point

$\omega = 25.6\ \text{rad/s}$ (or $26.0\ \text{rad/s}$ using $g = 10\ \text{m/s}^2$)

(c) 2 points

For using a correct expression for the ratio of the rotational kinetic energy to the total kinetic energy of the cylinder — 1 point

$$\frac{K_{rot}}{K_{tot}} = \frac{\frac{1}{2}I\omega^2}{\frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 + mgh} = \frac{\frac{1}{4}M(R\omega)^2}{\frac{3}{4}M(R\omega)^2 + Mgh_{\text{table}}} = \frac{(R\omega)^2}{3(R\omega)^2 + 4gh_{\text{table}}}$$

For substituting into the above equation — 1 point

$$\frac{K_{rot}}{K_{tot}} = \frac{[(0.10\ \text{m})(25.6\ \text{rad/s})]^2}{(3)(0.10\ \text{m})^2(25.6\ \text{rad/s})^2 + (4)\left(9.81\ \text{m/s}^2\right)(0.75\ \text{m})}$$

$\frac{K_{rot}}{K_{tot}} = 0.133$ (or 0.135 using $g = 10\ \text{m/s}^2$)

Question 3 (continued)

Distribution of points

(c) (continued)

Alternate Solution — *Alternate Points*

For using a correct expression for the ratio of the rotational kinetic energy to the total potential energy of the cylinder — *1 point*

$$\frac{K_{rot}}{K_{tot}} = \frac{\frac{1}{2}I\omega^2}{U_{total}} = \frac{\frac{1}{4}M(R\omega)^2}{Mg(h+y)}$$

For substituting into the above equation — *1 point*

$$\frac{K_{rot}}{U_{total}} = \frac{\frac{1}{4}(R\omega)^2}{g(h+y)} = \frac{\frac{1}{4}[(0.10\ \text{m})(25.6\ \text{rad/s})]^2}{(9.81\ \text{m/s}^2)\left((1.0\ \text{m})\sin(30^\circ) + (0.75\ \text{m})\right)}$$

$$\frac{K_{rot}}{U_{total}} = 0.13 \text{ (or 0.135 using } g = 10\ \text{m/s}^2\text{)}$$

(d) 2 points

For correctly using motion in the vertical direction to calculate the time for the cylinder to reach the floor — 1 point

$$y = y_0 + v_{0y}t + \frac{1}{2}a_y t^2$$

$$y - y_0 = 0 - \frac{1}{2}gt^2$$

Determine the time for the cylinder to reach the floor

$$y - y_0 = -\frac{1}{2}gt^2$$

$$0 - y = -\frac{1}{2}gt^2$$

$$t = \sqrt{\frac{2y}{g}} = \sqrt{\frac{(2)(0.75\ \text{m})}{(9.81\ \text{m/s}^2)}} = 0.39\ \text{s}$$

For correctly using the equation for constant speed in the horizontal direction — 1 point

$$x - x_0 = v_{0x}t$$

$$x - x_0 = R\omega t$$

$$x - x_0 = R\omega\sqrt{\frac{2y}{g}}$$

$$D = R\omega t = (0.10\ \text{m})(25.6\ \text{rad/s})(0.39\ \text{s})$$

$$D = 1.0\ \text{m}$$

Question 3 (continued)

Distribution of points

(e)

i. 2 points

For selecting "Equal to" and attempting a relevant justification	1 point
For a correct justification	1 point

Example: Because the sphere falls the same height as the cylinder and because they have the same mass, the sphere-Earth system has the same initial potential energy and, therefore, the same total kinetic energy when it reaches the floor.

ii. 2 points

For selecting "Less than" and attempting a relevant justification	1 point
For a correct justification	1 point

Example: Because the rotational inertia of the sphere is less than the rotational inertia of the cylinder, the sphere will rotate faster and, because $v = r\omega$, will move with a greater linear speed. Because the mass is the same and the linear speed is greater, the sphere will have a greater linear kinetic energy. Because the total kinetic energies of the sphere and cylinder are the same, the sphere must have less rotational kinetic energy.

iii. 2 points

For selecting "Greater than" and attempting a relevant justification	1 point
For a correct justification	1 point

Example: Because the sphere has a greater linear speed as it leaves the table, it will travel a greater horizontal distance before it reaches the floor.