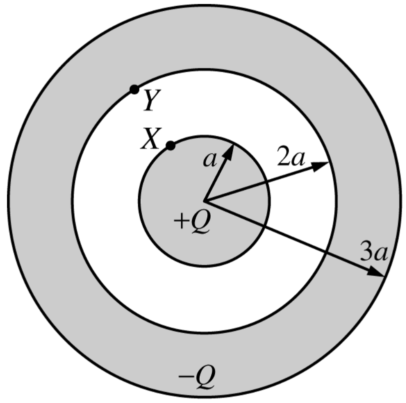


E&M 2.

In the figure above, a nonconducting solid sphere of radius a with charge $+Q$ uniformly distributed throughout its volume is concentric with a nonconducting spherical shell of inner radius $2a$ and outer radius $3a$ that has a charge $-Q$ uniformly distributed throughout its volume. Express all answers in terms of the given quantities and fundamental constants.

(a) Using Gauss's law, derive expressions for the magnitude of the electric field as a function of radius r in the following regions.

i. Within the solid sphere ($r < a$)

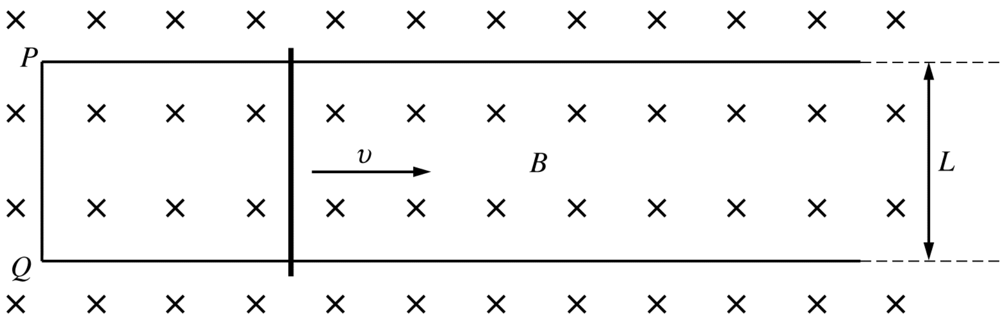
ii. Between the solid sphere and the spherical shell ($a < r < 2a$)

iii. Within the spherical shell ($2a < r < 3a$)

iv. Outside the spherical shell ($r > 3a$)

(b) What is the electric potential at the outer surface of the spherical shell ($r = 3a$)? Explain your reasoning.

(c) Derive an expression for the electric potential difference $V_X - V_Y$ between points X and Y shown in the figure.



Top View

E&M 3.

In the diagram above, a nichrome wire of resistance per unit length λ is bent at points P and Q to form horizontal conducting rails that are a distance L apart. The wire is placed within a uniform magnetic field of magnitude B pointing into the page. A conducting rod of negligible resistance, which was aligned with end PQ at time $t = 0$, slides to the right with constant speed v and negligible friction. Express all algebraic answers in terms of the given quantities and fundamental constants.

(a) Indicate the direction of the current induced in the circuit.

______Clockwise ______Counterclockwise

Justify your answer.

(b) Derive an expression for the magnitude of the induced current as a function of time t.

(c) Derive an expression for the magnitude of the magnetic force on the rod as a function of time.

(d) On the axes below, sketch a graph of the external force F_{ext} as a function of time that must be applied to the rod to keep it moving at constant speed while in the field. Label the values of any intercepts.

(e) The force pulling the rod is now removed. Indicate whether the speed of the rod increases, decreases, or remains the same.

______Increases _____Decreases ______Remains the same

Justify your answer.

END OF EXAM

Question 2

15 points total | **Distribution of points**

(a)

(i) 2 points

$\oint \mathbf{E} \cdot d\mathbf{A} = \dfrac{Q_{enc}}{\epsilon_0}$, where Q_{enc} is the charge enclosed by the Gaussian surface

Use a concentric sphere of radius $r < a$ as the Gaussian surface.

For correctly calculating Q_{enc} — 1 point

$$Q_{enc} = \rho V = \left(\frac{Q}{\frac{4}{3}\pi a^3}\right)\left(\frac{4}{3}\pi r^3\right) = \frac{Qr^3}{a^3}$$

E is normal to the surface everywhere, so applying Gauss's law,

$$E\left(4\pi r^2\right) = \frac{1}{\epsilon_0}\left(\frac{Qr^3}{a^3}\right)$$

For the correct answer (This point was not awarded if no supporting work was shown.) — 1 point

$$E = \frac{1}{4\pi\epsilon_0}\frac{Qr}{a^3} \quad \text{or} \quad E = \frac{kQr}{a^3}$$

(ii) 2 points

Use a concentric sphere of radius $a < r < 2a$ as the Gaussian surface.

For correctly identifying Q_{enc} — 1 point

$$Q_{enc} = Q$$

E is normal to the surface everywhere, so applying Gauss's law,

$$E\left(4\pi r^2\right) = \frac{Q}{\epsilon_0}$$

For the correct answer (This point was not awarded if no supporting work was shown.) — 1 point

$$E = \frac{1}{4\pi\epsilon_0}\frac{Q}{r^2} \quad \text{or} \quad E = \frac{kQ}{r^2}$$