Mech 2.

An explorer plans a mission to place a satellite into a circular orbit around the planet Jupiter, which has mass $M_J = 1.90 \times 10^{27}$ kg and radius $R_J = 7.14 \times 10^7$ m.

(a) If the radius of the planned orbit is R, use Newton's laws to show each of the following.

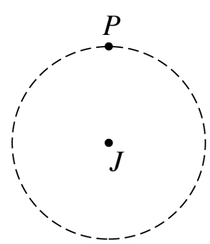
i. The orbital speed of the planned satellite is given by $v = \sqrt{\dfrac{GM_J}{R}}$.

ii. The period of the orbit is given by $T = \sqrt{\dfrac{4\pi^2 R^3}{GM_J}}$.

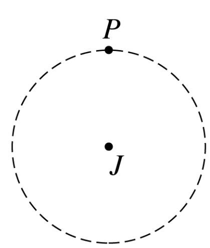
(b) The explorer wants the satellite's orbit to be synchronized with Jupiter's rotation. This requires an equatorial orbit whose period equals Jupiter's rotation period of 9 hr 51 min = 3.55×10^4 s. Determine the required orbital radius in meters.

(c) Suppose that the injection of the satellite into orbit is less than perfect. For an injection velocity that differs from the desired value in each of the following ways, sketch the resulting orbit on the figure. (J is the center of Jupiter, the dashed circle is the desired orbit, and P is the injection point.) Also, describe the resulting orbit qualitatively but specifically.

i. When the satellite is at the desired altitude over the equator, its velocity vector has the correct direction, but the speed is slightly <u>faster</u> than the correct speed for a circular orbit of that radius.



ii. When the satellite is at the desired altitude over the equator, its velocity vector has the correct direction, but the speed is slightly <u>slower</u> than the correct speed for a circular orbit of that radius.



Question 2

15 points total

2. (a) i. **3 points** **Distribution of Points**

There were two methods generally used to solve this part.

Method 1.

$F = ma_c$

$F = \dfrac{GM_J m}{R^2}$

For a statement of at least one of Newton's laws above **1 point**

Equating the two equations above and substituting expression for centripetal force:

$ma_c = \dfrac{GM_J m}{R^2}$

For substituting the expression for centripetal force **1 point**

$\dfrac{mv^2}{R} = \dfrac{GM_J m}{R^2}$

For a solution for v that follows algebraically from previous work **1 point**

$v = \sqrt{\dfrac{GM_J}{R}}$

Method 2.

$a = \dfrac{GM_J}{R^2}$ or $g = \dfrac{GM_J}{R^2}$

For statement of either of the above, which are derived from Newton's laws **1 point**

For a correct statement of centripetal acceleration **1 point**

$a_c = \dfrac{v^2}{R}$

Equating the two expressions above for a_c and solving for v:

$v = \sqrt{\dfrac{GM_J}{R}}$

For a solution for v that follows algebraically from previous work. **1 point**

Two points were awarded for an approach that started with $K = \dfrac{1}{2}mv^2 = \dfrac{GM_J m}{2R}$ and solved for v as long as there was no sign error in the equation and there were no incorrect statements regarding energy prior to the equation.

Question 2 (cont.)

2. (a) ii. **3 points** — **Distribution of Points**

There were three methods generally used to solve this part.

Method 1.

$$\upsilon = \frac{d}{t} = \frac{2\pi R}{T}$$

For an expression for υ in terms of the period T — **1 point**

For substitution of $2\pi R$ for the length d of the orbital path — **1 point**

Solving for T gives $T = \dfrac{2\pi R}{\upsilon}$

For correct substitution for υ from (a) i. — **1 point**

$$T = \frac{2\pi R}{\sqrt{\dfrac{GM_J}{R}}} = \sqrt{\frac{4\pi^2 R^3}{GM_J}}$$

Method 2.

$$T = \frac{2\pi}{\omega} = \frac{2\pi R}{\upsilon}$$

For the equation for T in terms of ω — **1 point**

For substitution of υ/R for ω in the equation — **1 point**

For correct substitution for υ from (a) i. — **1 point**

$$T = \frac{2\pi R}{\sqrt{\dfrac{GM_J}{R}}} = \sqrt{\frac{4\pi^2 R^3}{GM_J}}$$

Method 3.

$$F = \frac{m\upsilon^2}{R} = m\omega^2 R = m\left(\frac{2\pi}{T}\right)^2 R = \frac{GmM_J}{R^2}$$ — **2 points**

$$\frac{4\pi^2 mR}{T^2} = \frac{GmM_J}{R^2}$$

For $T^2 = \dfrac{4\pi^2 R^3}{GM_J}$ — **1 point**

Note: Direct use of $T^2 = \dfrac{4\pi^2 R^3}{GM_J}$ was awarded **1 point** only, if it was defined as Kepler's law or Law of Orbits.

Question 2 (cont.)

2. (b) **3 points** — **Distribution of Points**

For use of correct equation for T from (a) ii. or derivation of this relationship — **1 point**

$$T = \sqrt{\frac{4\pi^2 R^3}{GM_J}}$$

For correct solution for R or R^3, numerical or symbolic, from above equation — **1 point**

$$R^3 = \frac{GM_J T^2}{4\pi^2} \quad \underline{\text{or}} \quad R = \left(\frac{GM_J T^2}{4\pi^2}\right)^{1/3} \quad \underline{\text{or}}$$

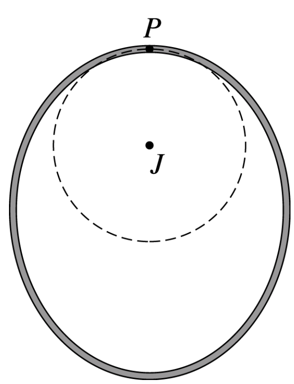
$$R = \left[\left(6.67\times10^{-11}\ \text{N}\bullet\text{m}^2/\text{kg}^2\right)\left(1.9\times10^{27}\ \text{kg}\right)\left(3.55\times10^{4}\ \text{s}\right)^2 \big/ 4\pi^2\right]^{1/3}$$

For a correct answer — **1 point**

$R = 1.59\times10^{8}\ \text{m}$

Note: If R_J was subtracted from R the answer point was only awarded if the difference was clearly indicated to be the height of the orbit above the surface.

2. (c) i. **3 points**



For stating that the orbit is an ellipse — **1 point**

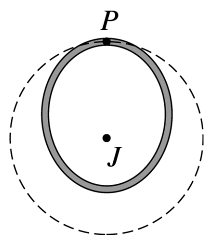
For diagram with orbit drawn completely outside the circle with point of contact only at point P and major axis along PJ. — **2 points**

Partial credit of **1 point** awarded for any path or orbit completely outside the circle.

No points were awarded in any part of path or orbit was inside the circle.

Question 2 (cont.)

2. (c) ii. **3 points**



Distribution of Points

For stating that the orbit is an ellipse	**1 point**
For diagram with orbit drawn completely inside the circle with point of contact only at point *P* and major axis along *PJ*. Partial credit of **1 point** awarded for any path or orbit completely inside the circle. **No points** were awarded if any part of path or orbit was outside the circle.	**2 points**

Note: **Three points** may also be awarded in this part for a path in which the satellite "crashes" into Jupiter only if there is specific reference to the scale of the orbit from part (b) and the given radius of Jupiter.