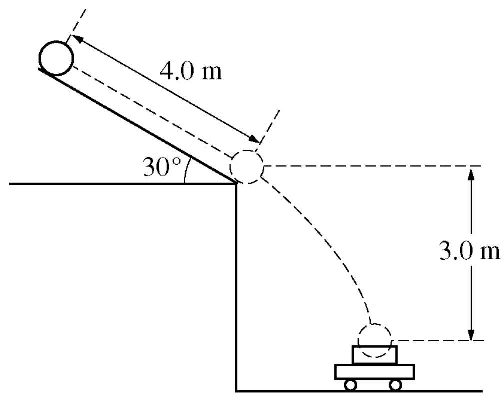
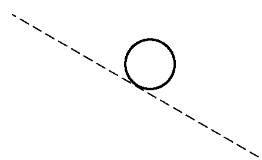


Note: Figure not drawn to scale.

Mech. 2.

A bowling ball of mass 6.0 kg is released from rest from the top of a slanted roof that is 4.0 m long and angled at $30°$, as shown above. The ball rolls along the roof without slipping. The rotational inertia of a sphere of mass M and radius R about its center of mass is $\frac{2}{5}MR^2$.

(a) On the figure below, draw and label the forces (not components) acting on the ball at their points of application as it rolls along the roof.



(b) Calculate the force due to friction acting on the ball as it rolls along the roof. If you need to draw anything other than what you have shown in part (a) to assist in your solution, use the space below. Do NOT add anything to the figure in part (a).

(c) Calculate the linear speed of the center of mass of the ball when it reaches the bottom edge of the roof.

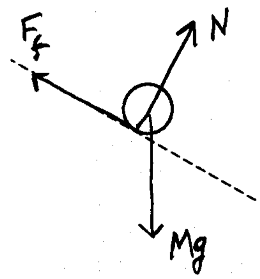
(d) A wagon containing a box is at rest on the ground below the roof so that the ball falls a vertical distance of 3.0 m and lands and sticks in the center of the box. The total mass of the wagon and the box is 12 kg. Calculate the horizontal speed of the wagon immediately after the ball lands in it.

GO ON TO THE NEXT PAGE.

Question 2

15 points total **Distribution of points**

(a) 3 points



For each correct force for which the vector is drawn with the correct direction, label and point of application, 1 point was awarded. — 3 points
One earned point was deducted if any components or extraneous forces are present.

(b) 5 points

Starting with Newton's second law (linear form):
$F_{net} = Ma$
For expressing F_{net} in terms of gravitational and frictional forces — 1 point
For including the correct component of the weight — 1 point
$Mg\sin\theta - F_f = Ma$
$\tau = I\alpha$
For correct substitution of torque into Newton's second law (angular form) — 1 point
$RF_f = I\alpha$
$RF_f = (2/5)MR^2\alpha$
For the correct relationship between angular and linear acceleration (either explicitly stated or used in the calculation) — 1 point
$\alpha = a/R$
$RF_f = (2/5)MR^2(a/R)$
Solving for Ma
$Ma = (5/2)F_f$
Substituting into the linear equation above
$Mg\sin\theta - F_f = (5/2)F_f$
$F_f = (2/7)Mg\sin\theta = (2/7)(6.0\text{ kg})(9.8\text{ m/s}^2)(\sin 30°)$
For the correct value of F_f — 1 point
$F_f = 8.4\text{ N}$

Notes: Credit is awarded for solutions that use the value of v calculated in (c) to calculate acceleration and, from there, the value of the frictional force.
If $Mg\cos\theta$ is used, the point was awarded for a value of 14.5 N.

Question 2 (continued)

Distribution of points

(c) 3 points

For an expression of conservation of energy — 1 point

For including gravitational potential energy, translational kinetic energy and rotational kinetic energy in a correct energy equation or statement — 1 point

$$Mg\Delta h = \frac{1}{2}M\upsilon^2 + \frac{1}{2}I\omega^2$$

$$Mgd\sin\theta = \frac{1}{2}M\upsilon^2 + \frac{1}{2}\left(\frac{2}{5}MR^2\right)\omega^2$$

$$\omega = \upsilon/R$$

$$Mgd\sin\theta = \frac{1}{2}M\upsilon^2 + \frac{1}{5}M\upsilon^2 = \frac{7}{10}M\upsilon^2$$

$$\upsilon = \sqrt{(10/7)gd\sin\theta} = \sqrt{(10/7)\left(9.8\ \text{m/s}^2\right)(4.0\ \text{m})(\sin 30^\circ)}$$

For a correct numerical answer — 1 point

$\upsilon = 5.3\ \text{m/s}$

Alternate solution (kinematics method) — *Alternate points*

For determination of the linear acceleration in terms of mass and frictional force — *1 point*

$a = 5F_f/2M$ from work shown in part (b)

$$a = 5(8.4\ \text{N})/2(6.0\ \text{kg}) = 3.5\ \text{m/s}^2$$

For a correct substitution into an appropriate kinematics equation using $\upsilon_0 = 0$ — *1 point*

$$\upsilon^2 = \upsilon_0^2 + 2ad = 2ad$$

$$\upsilon = \sqrt{2ad} = \sqrt{(2)\left(3.5\ \text{m/s}^2\right)(4.0\ \text{m})}$$

For a correct numerical answer — *1 point*

$\upsilon = 5.3\ \text{m/s}$

(d) 3 points

For a correct statement of conservation of momentum — 1 point

$$M_i\upsilon_i = M_f\upsilon_f$$

$$\upsilon_f = \left(M_i/M_f\right)\upsilon_i$$

For correctly equating υ_i with the horizontal component of the ball as it leaves the roof — 1 point

For setting M_f equal to the total mass of the ball and the wagon/box — 1 point

$$\upsilon_f = \left(M_i/M_f\right)\upsilon\cos\theta = [(6.0\ \text{kg})/(18.0\ \text{kg})]\left(5.3\ \text{m/s}^2\right)\cos 30^\circ$$

$$\upsilon_f = 1.5\ \text{m/s}$$

Units 1 point

For correct units in at least two of the parts (b), (c) and (d) — 1 point