

2007 AP® CALCULUS BC FREE-RESPONSE QUESTIONS

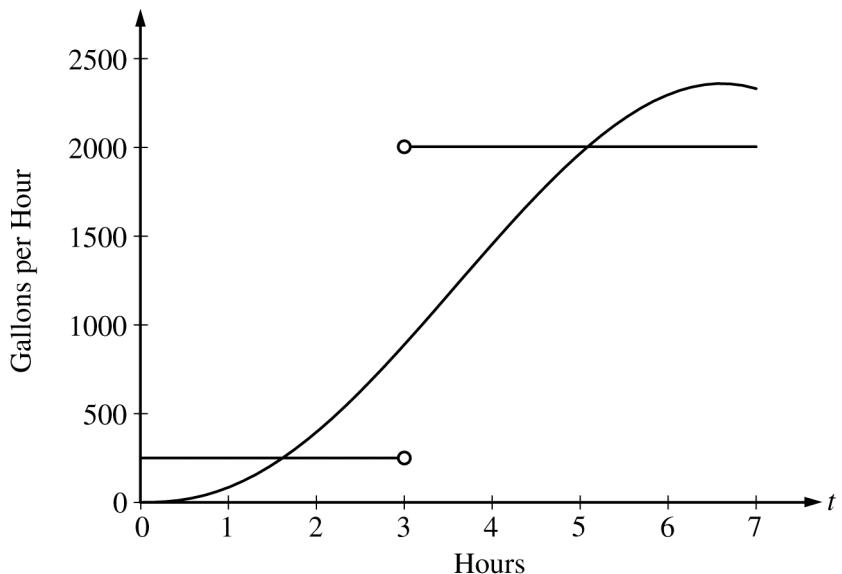
**CALCULUS BC
SECTION II, Part A
Time—45 minutes
Number of problems—3**

A graphing calculator is required for some problems or parts of problems.

1. Let R be the region in the first and second quadrants bounded above by the graph of $y = \frac{20}{1+x^2}$ and below by the horizontal line $y = 2$.
 - (a) Find the area of R .
 - (b) Find the volume of the solid generated when R is rotated about the x -axis.
 - (c) The region R is the base of a solid. For this solid, the cross sections perpendicular to the x -axis are semicircles. Find the volume of this solid.
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WRITE ALL WORK IN THE PINK EXAM BOOKLET.

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2. The amount of water in a storage tank, in gallons, is modeled by a continuous function on the time interval $0 \leq t \leq 7$, where t is measured in hours. In this model, rates are given as follows:

- (i) The rate at which water enters the tank is $f(t) = 100t^2 \sin(\sqrt{t})$ gallons per hour for $0 \leq t \leq 7$.
- (ii) The rate at which water leaves the tank is

$$g(t) = \begin{cases} 250 & \text{for } 0 \leq t < 3 \\ 2000 & \text{for } 3 < t \leq 7 \end{cases} \text{ gallons per hour.}$$

The graphs of f and g , which intersect at $t = 1.617$ and $t = 5.076$, are shown in the figure above. At time $t = 0$, the amount of water in the tank is 5000 gallons.

- (a) How many gallons of water enter the tank during the time interval $0 \leq t \leq 7$? Round your answer to the nearest gallon.
 - (b) For $0 \leq t \leq 7$, find the time intervals during which the amount of water in the tank is decreasing. Give a reason for each answer.
 - (c) For $0 \leq t \leq 7$, at what time t is the amount of water in the tank greatest? To the nearest gallon, compute the amount of water at this time. Justify your answer.
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**AP[®] CALCULUS BC
2007 SCORING GUIDELINES**

Question 1

Let R be the region in the first and second quadrants bounded above by the graph of $y = \frac{20}{1+x^2}$ and below by the horizontal line $y = 2$.

- (a) Find the area of R .
- (b) Find the volume of the solid generated when R is rotated about the x -axis.
- (c) The region R is the base of a solid. For this solid, the cross sections perpendicular to the x -axis are semicircles. Find the volume of this solid.

$$\frac{20}{1+x^2} = 2 \text{ when } x = \pm 3$$

(a) Area = $\int_{-3}^3 \left(\frac{20}{1+x^2} - 2 \right) dx = 37.961 \text{ or } 37.962$

(b) Volume = $\pi \int_{-3}^3 \left(\left(\frac{20}{1+x^2} \right)^2 - 2^2 \right) dx = 1871.190$

(c) Volume = $\frac{\pi}{2} \int_{-3}^3 \left(\frac{1}{2} \left(\frac{20}{1+x^2} - 2 \right) \right)^2 dx$
= $\frac{\pi}{8} \int_{-3}^3 \left(\frac{20}{1+x^2} - 2 \right)^2 dx = 174.268$

1 : correct limits in an integral in
(a), (b), or (c)

2 : $\begin{cases} 1 : \text{integrand} \\ 1 : \text{answer} \end{cases}$

3 : $\begin{cases} 2 : \text{integrand} \\ 1 : \text{answer} \end{cases}$

3 : $\begin{cases} 2 : \text{integrand} \\ 1 : \text{answer} \end{cases}$