Mech. 2.

A student is given the set of orbital data for some of the moons of Saturn shown below and is asked to use the data to determine the mass M_S of Saturn. Assume the orbits of these moons are circular.

Orbital Period, T (seconds)	Orbital Radius, R (meters)		
8.14×10^4	1.85×10^8		
1.18×10^5	2.38×10^8		
1.63×10^5	2.95×10^8		
2.37×10^5	3.77×10^8		

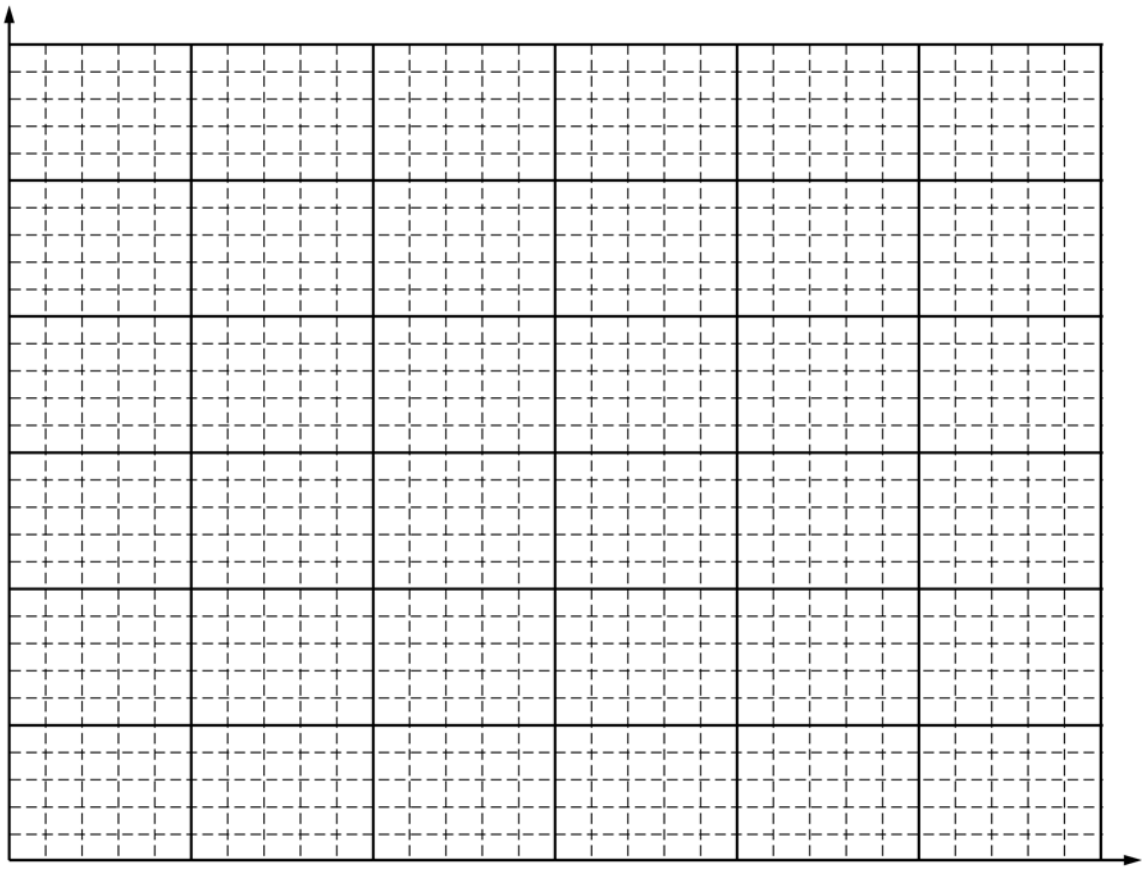
(a) Write an algebraic expression for the gravitational force between Saturn and one of its moons.

(b) Use your expression from part (a) and the assumption of circular orbits to derive an equation for the orbital period T of a moon as a function of its orbital radius R.

(c) Which quantities should be graphed to yield a straight line whose slope could be used to determine Saturn's mass?

(d) Complete the data table by calculating the two quantities to be graphed. Label the top of each column, including units.

(e) Plot the graph on the axes below. Label the axes with the variables used and appropriate numbers to indicate the scale.



(f) Using the graph, calculate a value for the mass of Saturn.

Question 2

15 points total **Distribution of points**

(a) 1 point

For a correct expression of Newton's law of gravity containing the mass of Saturn, with or without the minus sign 1 point

$F = \dfrac{GM_S m}{R^2}$, where m is the mass of a moon

(b) 3 points

Circular orbit means the force is centripetal.

For equating Newton's law of gravity to the centripetal force 1 point

$\dfrac{GM_S m}{R^2} = \dfrac{mv^2}{R}$ OR $F_g = F_c$

For correctly relating the orbital speed and the period 1 point

$v = 2\pi R/T$ OR $v = \omega r$ and $\omega = 2\pi/T$

$\dfrac{GM_S m}{R^2} = \dfrac{m}{R}\left(\dfrac{2\pi R}{T}\right)^2$

For a correct relationship between T and R 1 point

$GM_S T^2 = 4\pi^2 R^3$ OR $T = \sqrt{\dfrac{4\pi^2}{GM_S}}R^{3/2}$

(c) 1 point

For any correct pair of quantities that would yield a straight line when graphed 1 point

For example, T^2 versus R^3 OR T versus $R^{3/2}$

(d) 3 points

For appropriate and correct table headings, i.e., powers of T and R consistent with answer to part (b) 1 point

For correct units in both columns 1 point

For correct numerical values in at least 75 percent of the table entries, provided that the table will yield a linear graph 1 point

Full credit was awarded for completing only one column containing $R^{3/2}$.

(Example data on next page.)

Question 2 (continued)

Distribution of points

(d) (continued)

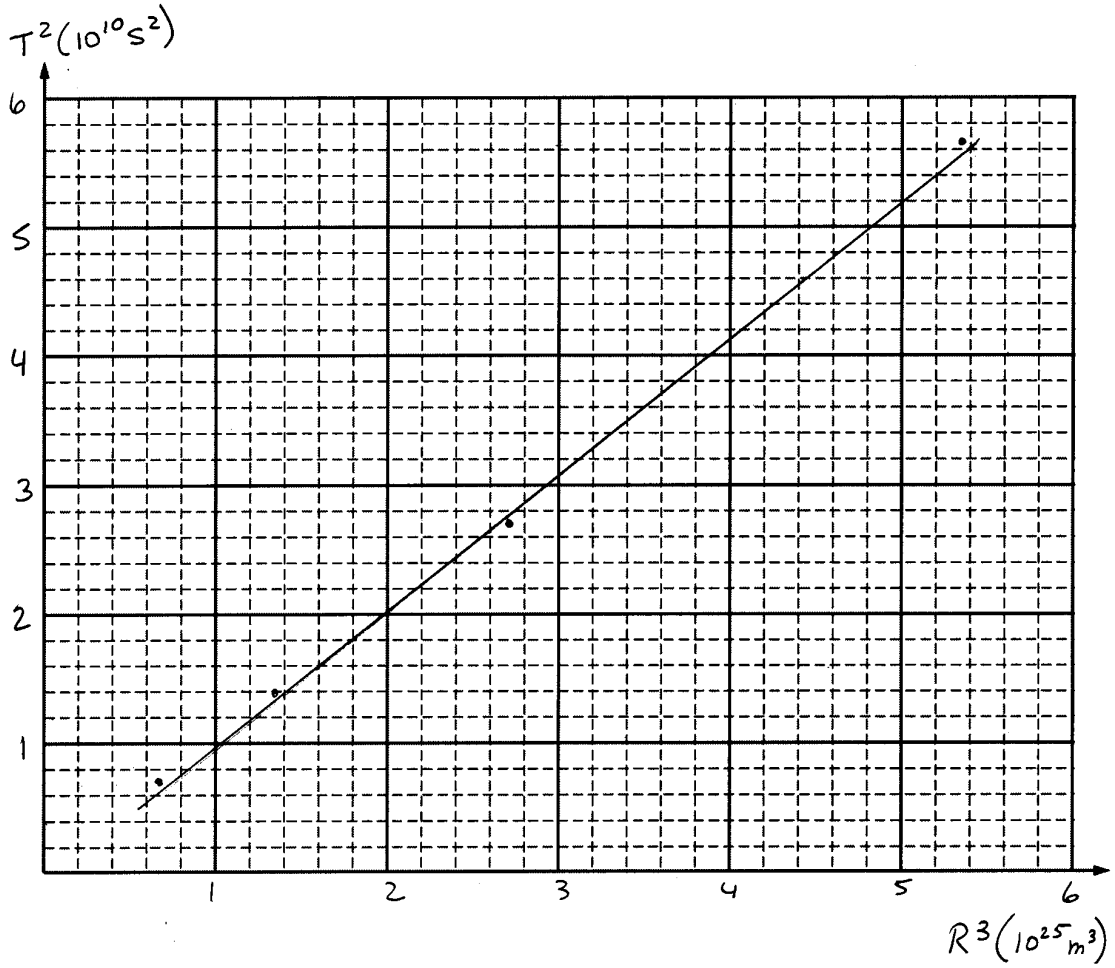
Example data

Orbital Period, T (seconds)	Orbital Radius, R (meters)	T^2 $\left(s^2\right)$	R^3 $\left(m^3\right)$
8.14×10^4	1.85×10^8	0.663×10^{10}	0.633×10^{25}
1.18×10^5	2.38×10^8	1.39×10^{10}	1.35×10^{25}
1.63×10^5	2.95×10^8	2.66×10^{10}	2.57×10^{25}
2.37×10^5	3.77×10^8	5.62×10^{10}	5.36×10^{25}

(e) 4 points

For axis labels consistent with the table and having correct units	1 point
For appropriate numbering of axes	1 point
For correctly plotting data from table (only awarded if numbering is correct)	1 point
For drawing a best-fit linear graph (only awarded if data define a linear graph)	1 point

Example response



Question 2 (continued)

Distribution of points

(f) 3 points

For correctly indicating a calculation of the slope of the graph (only awarded if graph is drawn) 1 point

From the graph shown here

$$\text{slope} = \frac{\left(5.6 \times 10^{10} - 1.4 \times 10^{10}\right)\text{m}^3}{\left(5.4 \times 10^{25} - 1.4 \times 10^{25}\right)\text{s}^2} = \frac{4.2 \times 10^{10}\ \text{m}^3}{4.0 \times 10^{25}\ \text{s}^2}$$

For relating the slope to the equation obtained in part (b) 1 point

In the example shown here, $T^2 = \frac{4\pi^2}{GM_S}R^3$.

$$\text{slope of line} = \frac{4\pi^2}{GM_S}$$

$$M_S = \frac{4\pi^2}{G\ (\text{slope})}$$

$$M_S = \frac{4\pi^2}{6.67 \times 10^{-11}\ \text{m}^3/\text{kg}\cdot\text{s}^2}\ \frac{4.0 \times 10^{25}\ \text{s}^2}{4.2 \times 10^{10}\ \text{m}^3}$$

For the correct numerical answer 1 point

$$M_S = 5.64 \times 10^{26}\ \text{kg}$$

Solutions for which the slope yields Saturn's mass directly (e.g., table columns of $4\pi^2R^3$ versus GT^2) received full credit.