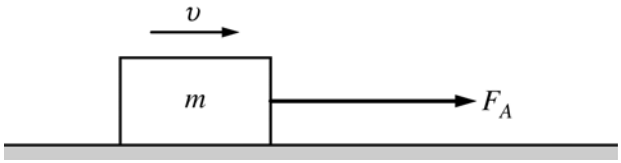


Mech 2.

A box of mass m initially at rest is acted upon by a constant applied force of magnitude F_A, as shown in the figure above. The friction between the box and the horizontal surface can be assumed to be negligible, but the box is subject to a drag force of magnitude kv where v is the speed of the box and k is a positive constant. Express all your answers in terms of the given quantities and fundamental constants, as appropriate.

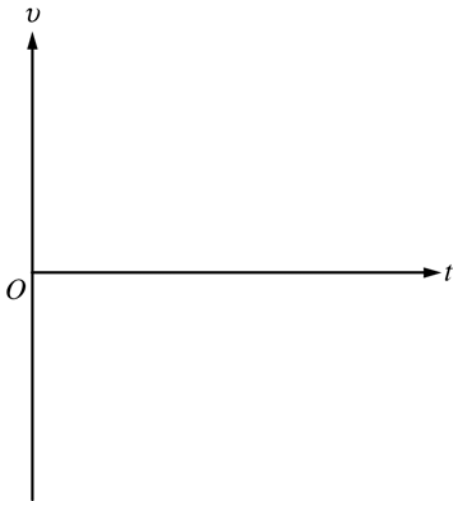
(a) The dot below represents the box. Draw and label the forces (not components) that act on the box.

(b) Write, but do not solve, a differential equation that could be used to determine the speed v of the box as a function of time t. If you need to draw anything other than what you have shown in part (a) to assist in your solution, use the space below. Do NOT add anything to the figure in part (a).

(c) Determine the magnitude of the terminal velocity of the box.

(d) Use the differential equation from part (b) to derive the equation for the speed v of the box as a function of time t. Assume that $v = 0$ at time $t = 0$.

(e) On the axes below, sketch a graph of the speed v of the box as a function of time t. Explicitly label any intercepts, asymptotes, maxima, or minima with numerical values or algebraic expressions, as appropriate.

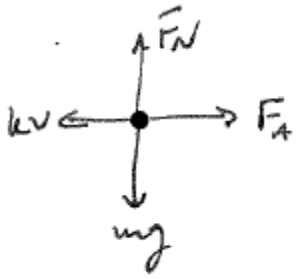


AP® PHYSICS C: MECHANICS
2013 SCORING GUIDELINES

Question 2

15 points total **Distribution of points**

(a) 4 points



For correctly showing and labeling the applied force directed to the right — 1 point
For correctly showing and labeling the downward gravitational force — 1 point
For correctly showing and labeling the upward normal force — 1 point
For correctly showing and labeling the drag force directed to the left — 1 point
One earned point was deducted for having any extraneous vectors

(b) 2 points

$F_{net} = ma$

For the correct substitution into Newton's second law — 1 point

$F_A - kv = ma$

For a correct differential equation — 1 point

$F_A - kv = m\frac{dv}{dt}$

(c) 1 point

Set $\frac{dv}{dt} = 0$ in the equation from part (b)

$F_A - kv = 0$

For the correct expression for the terminal velocity — 1 point

$v_T = \frac{F_A}{k}$

Question 2 (continued)

Distribution of points

(d) 5 points

Use the differential equation from part (b)

$F_A - k\upsilon = m\frac{d\upsilon}{dt}$

For demonstrating separation of variables — 1 point

$\frac{1}{m}dt = \frac{1}{F_A - k\upsilon}d\upsilon$

For demonstrating that the equation must be integrated — 1 point

$\int \frac{1}{m}dt = \int \frac{1}{F_A - k\upsilon}d\upsilon$

For demonstrating substitution using initial and final values (or evaluating the constant of integration using the boundary conditions) — 1 point

$\int_0^t \frac{1}{m}dt = \int_0^{\upsilon(t)} \frac{1}{F_A - k\upsilon}d\upsilon$

$\left[\frac{t}{m}\right]_0^t = -\frac{1}{k}\left[\ln\left(F_A - k\upsilon\right)\right]_0^{\upsilon(t)}$

For attempting to solve for $\upsilon(t)$ — 1 point

$-\frac{kt}{m} = \ln\left(\frac{F_A - k\upsilon(t)}{F_A}\right)$

$e^{-kt/m} = \frac{F_A - k\upsilon(t)}{F_A} = 1 - \frac{k\upsilon(t)}{F_A}$

$\frac{k\upsilon(t)}{F_A} = 1 - e^{-kt/m}$

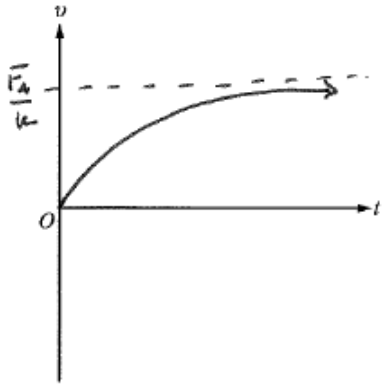
For a correct answer — 1 point

$\upsilon(t) = \frac{F_A}{k}\left(1 - e^{-kt/m}\right)$

Question 2 (continued)

Distribution of points

(e) 3 points



For a graph that begins at the origin, with a non-negative slope everywhere, and is concave downward — 1 point

For a graph with a horizontal asymptote — 1 point

For the correct label of the expression for the asymptote or maximum on the vertical axis — 1 point