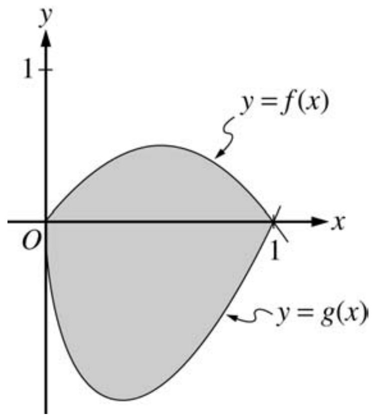


2. Let f and g be the functions given by $f(x) = 2x(1 - x)$ and $g(x) = 3(x - 1)\sqrt{x}$ for $0 \le x \le 1$. The graphs of f and g are shown in the figure above.

(a) Find the area of the shaded region enclosed by the graphs of f and g.

(b) Find the volume of the solid generated when the shaded region enclosed by the graphs of f and g is revolved about the horizontal line $y = 2$.

(c) Let h be the function given by $h(x) = kx(1 - x)$ for $0 \le x \le 1$. For each $k > 0$, the region (not shown) enclosed by the graphs of h and g is the base of a solid with square cross sections perpendicular to the x-axis. There is a value of k for which the volume of this solid is equal to 15. Write, but do not solve, an equation involving an integral expression that could be used to find the value of k.

---

3. An object moving along a curve in the xy-plane has position $(x(t), y(t))$ at time $t \geq 0$ with $\frac{dx}{dt} = 3 + \cos(t^2)$. The derivative $\frac{dy}{dt}$ is not explicitly given. At time $t = 2$, the object is at position $(1, 8)$.

   (a) Find the x-coordinate of the position of the object at time $t = 4$.

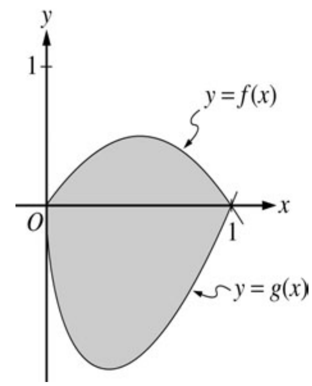
   (b) At time $t = 2$, the value of $\frac{dy}{dt}$ is -7. Write an equation for the line tangent to the curve at the point $(x(2), y(2))$.

   (c) Find the speed of the object at time $t = 2$.

   (d) For $t \geq 3$, the line tangent to the curve at $(x(t), y(t))$ has a slope of $2t + 1$. Find the acceleration vector of the object at time $t = 4$.

END OF PART A OF SECTION II

Question 2



Let f and g be the functions given by $f(x) = 2x(1-x)$ and $g(x) = 3(x-1)\sqrt{x}$ for $0 \le x \le 1$. The graphs of f and g are shown in the figure above.

(a) Find the area of the shaded region enclosed by the graphs of f and g.

(b) Find the volume of the solid generated when the shaded region enclosed by the graphs of f and g is revolved about the horizontal line $y = 2$.

(c) Let h be the function given by $h(x) = kx(1-x)$ for $0 \le x \le 1$. For each $k > 0$, the region (not shown) enclosed by the graphs of h and g is the base of a solid with square cross sections perpendicular to the x-axis. There is a value of k for which the volume of this solid is equal to 15. Write, but do not solve, an equation involving an integral expression that could be used to find the value of k.

(a) Area $= \int_0^1 (f(x) - g(x))\,dx$

$$= \int_0^1 \left(2x(1-x) - 3(x-1)\sqrt{x}\right) dx = 1.133$$

$2 : \begin{cases} 1 : \text{integral} \\ 1 : \text{answer} \end{cases}$

(b) Volume $= \pi\int_0^1 \left((2 - g(x))^2 - (2 - f(x))^2\right) dx$

$$= \pi\int_0^1 \left(\left(2 - 3(x-1)\sqrt{x}\right)^2 - (2 - 2x(1-x))^2\right) dx$$

$= 16.179$

$4 : \begin{cases} 1 : \text{limits and constant} \\ 2 : \text{integrand} \\ \quad \langle -1 \rangle \text{ each error} \\ \quad \text{Note: } 0/2 \text{ if integral not of form} \\ \qquad\qquad c\int_a^b \left(R^2(x) - r^2(x)\right) dx \\ 1 : \text{answer} \end{cases}$

(c) Volume $= \int_0^1 (h(x) - g(x))^2\,dx$

$$\int_0^1 \left(kx(1-x) - 3(x-1)\sqrt{x}\right)^2 dx = 15$$

$3 : \begin{cases} 2 : \text{integrand} \\ 1 : \text{answer} \end{cases}$