5. Consider the differential equation $\dfrac{dy}{dx} = 5x^2 - \dfrac{6}{y-2}$ for $y \neq 2$. Let $y = f(x)$ be the particular solution to this differential equation with the initial condition $f(-1) = -4$.

   (a) Evaluate $\dfrac{dy}{dx}$ and $\dfrac{d^2 y}{dx^2}$ at $(-1, -4)$.

   (b) Is it possible for the x-axis to be tangent to the graph of f at some point? Explain why or why not.

   (c) Find the second-degree Taylor polynomial for f about $x = -1$.

   (d) Use Euler's method, starting at $x = -1$ with two steps of equal size, to approximate $f(0)$. Show the work that leads to your answer.

6. The function f is defined by the power series

$$f(x) = -\frac{x}{2} + \frac{2x^2}{3} - \frac{3x^3}{4} + \cdots + \frac{(-1)^n n x^n}{n+1} + \cdots$$

   for all real numbers x for which the series converges. The function g is defined by the power series

$$g(x) = 1 - \frac{x}{2!} + \frac{x^2}{4!} - \frac{x^3}{6!} + \cdots + \frac{(-1)^n x^n}{(2n)!} + \cdots$$

   for all real numbers x for which the series converges.

   (a) Find the interval of convergence of the power series for f. Justify your answer.

   (b) The graph of $y = f(x) - g(x)$ passes through the point $(0, -1)$. Find $y'(0)$ and $y''(0)$. Determine whether y has a relative minimum, a relative maximum, or neither at $x = 0$. Give a reason for your answer.

WRITE ALL WORK IN THE PINK EXAM BOOKLET.

END OF EXAM