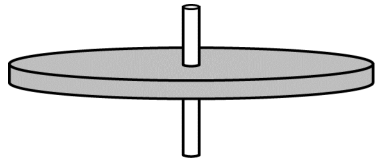


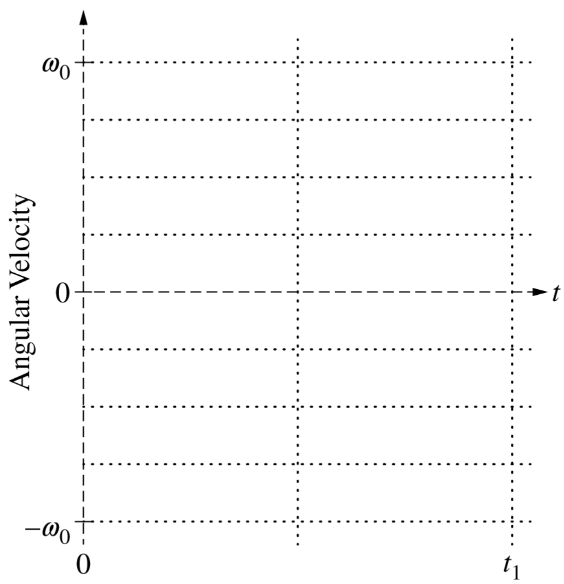
3. (12 points, suggested time 25 minutes)

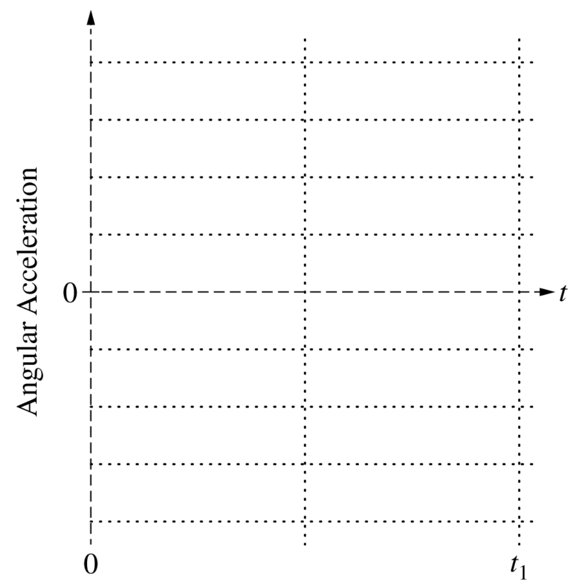
The disk shown above spins about the axle at its center. A student's experiments reveal that, while the disk is spinning, friction between the axle and the disk exerts a constant torque on the disk.

(a) At time $t = 0$ the disk has an initial counterclockwise (positive) angular velocity ω_0. The disk later comes to rest at time $t = t_1$.

i. On the grid at left below, sketch a graph that could represent the disk's angular velocity as a function of time t from $t = 0$ until the disk comes to rest at time $t = t_1$.

ii. On the grid at right below, sketch the disk's angular acceleration as a function of time t from $t = 0$ until the disk comes to rest at time $t = t_1$.



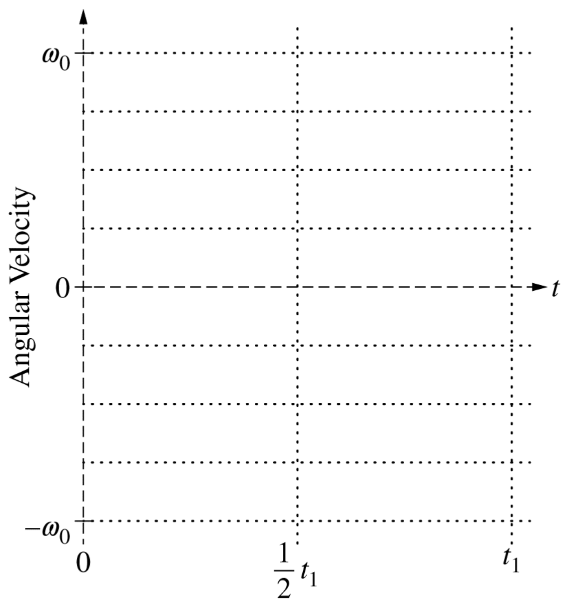


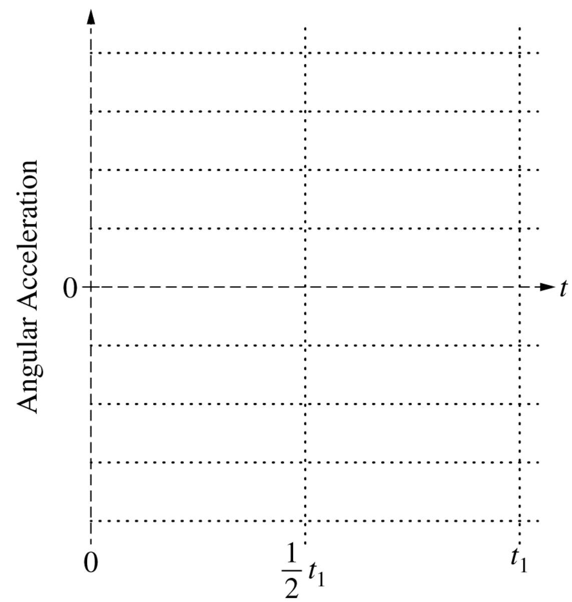
(b) The magnitude of the frictional torque exerted on the disk is τ_0. Derive an equation for the rotational inertia I of the disk in terms of τ_0, ω_0, t_1, and physical constants, as appropriate.

(c) In another experiment, the disk again has an initial positive angular velocity ω_0 at time $t = 0$. At time $t = \frac{1}{2}t_1$, the student starts dripping oil on the contact surface between the axle and the disk to reduce the friction. As time passes, more and more oil reaches that contact surface, reducing the friction even further.

i. On the grid at left below, sketch a graph that could represent the disk's angular velocity as a function of time from $t = 0$ to $t = t_1$, which is the time at which the disk came to rest in part (a).

ii. On the grid at right below, sketch the disk's angular acceleration as a function of time from $t = 0$ to $t = t_1$.





(d) The student is trying to mathematically model the magnitude τ of the torque exerted by the axle on the disk when the oil is present at times $t > \frac{1}{2}t_1$. The student writes down the following two equations, each of which includes a positive constant (C_1 or C_2) with appropriate units.

(1) $\tau = C_1\left(t - \frac{1}{2}t_1\right)$ (for $t > \frac{1}{2}t_1$)

(2) $\tau = \dfrac{C_2}{\left(t + \frac{1}{2}t_1\right)}$ (for $t > \frac{1}{2}t_1$)

Which equation better mathematically models this experiment?

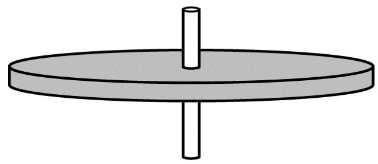
____ Equation (1) ____ Equation (2)

Briefly explain why the equation you selected is plausible and why the other equation is not plausible.

GO ON TO THE NEXT PAGE.

Question 3

12 points total **Distribution of points**



The disk shown above spins about the axle at its center. A student's experiments reveal that, while the disk is spinning, friction between the axle and the disk exerts a constant torque on the disk.

(a) LO / SP: 3.A.1.1 / 1.5, 2.2; 3.F.1.1 / 1.4; 3.F.2.1 / 6.4; 4.D.2.1 / 1.2, 1.4
4 points

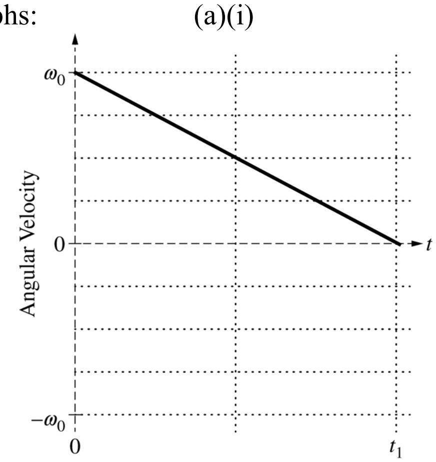
At time $t = 0$ the disk has an initial counterclockwise (positive) angular velocity ω_0. The disk later comes to rest at time $t = t_1$.

i. On the grid at left below, sketch a graph that could represent the disk's angular velocity as a function of time t from $t = 0$ until the disk comes to rest at time $t = t_1$.

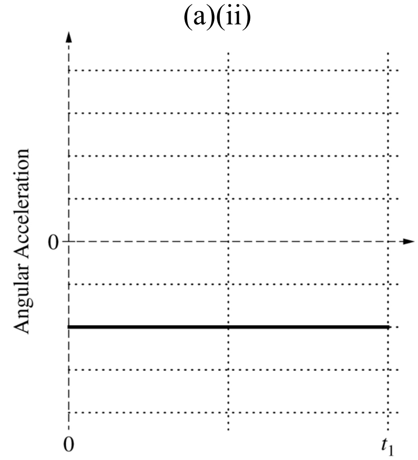
ii. On the grid at right below, sketch the disk's angular acceleration as a function of time t from $t = 0$ until the disk comes to rest at time $t = t_1$.

Example graphs:

(a)(i)



(a)(ii)



i. 2 points

For a curve that has an angular velocity of $+\omega_0$ at time $t = 0$ and decreases to zero at time $t = t_1$		1 point
For a curve that is a straight line with a negative slope showing the angular velocity approaching zero (can be a positive slope, if the initial angular velocity on the graph is negative)		1 point

Question 3 (continued)

Distribution of points

(a) (continued)

ii. 2 points

For a curve that is negative for the entire time		1 point
For a curve that is a constant nonzero function		1 point

(b) LO / SP: 3.A.1.1 / 1.5, 2.2; 3.F.2.1 / 6.4; 4.D.1.1 / 1.2, 1.4; 4.D.2.1 / 1.2, 1.4; 4.D.3.1 / 2.2
3 points

The magnitude of the frictional torque exerted on the disk is τ_0. Derive an equation for the rotational inertia I of the disk in terms of τ_0, ω_0, t_1, and physical constants, as appropriate.

For using an equation for the rotational version of Newton's second law		1 point
For using an appropriate rotational kinematics equation $\alpha = \Delta\omega/\Delta t$		1 point
For a correct answer in terms of the listed quantities, derived from first principles $I = \dfrac{\tau_0 t_1}{\omega_0}$ <u>Note</u>: This point is still earned if there is a minus sign, e.g., from using $-\tau_0$ or $-\omega_0$.		1 point

Alternate solution using angular momentum and rotational impulse:		
For defining and using angular momentum $L = I\omega$		1 point
For using rotational impulse $\Delta L = \tau\Delta t$		1 point
For a correct answer in terms of the listed quantities, derived from first principles $I = \dfrac{\tau_0 t_1}{\omega_0}$ *<u>Note</u>: This point is still earned if there is a minus sign, e.g., from using* $-\tau_0$ *or* $-\omega_0$.		1 point

Question 3 (continued)

Distribution of points

(c) LO / SP: 3.A.1.1 / 1.5, 2.2; 3.A.3.1 / 6.4, 7.2; 3.F.1.1 / 1.4; 3.F.2.1 / 6.4; 4.D.1.1 / 1.2, 1.4; 4.D.2.1 / 1.2,1.4
4 points

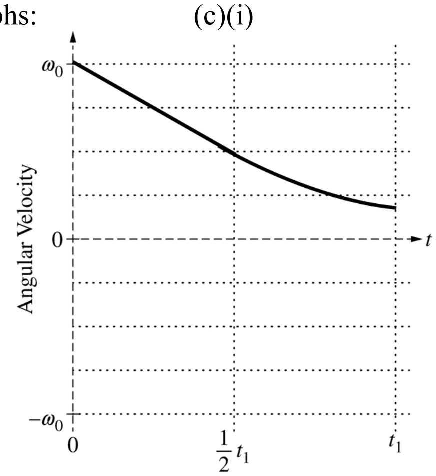
In another experiment, the disk again has an initial positive angular velocity ω_0 at time $t = 0$. At time $t = \frac{1}{2}t_1$, the student starts dripping oil on the contact surface between the axle and the disk to reduce the friction. As time passes, more and more oil reaches that contact surface, reducing the friction even further.

i. On the grid at left below, sketch a graph that could represent the disk's angular velocity as a function of time from $t = 0$ to $t = t_1$, which is the time at which the disk came to rest in part (a).

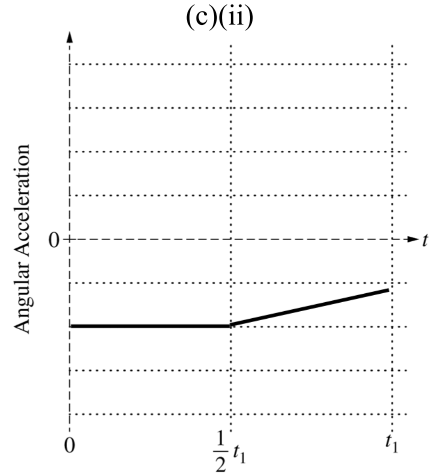
ii. On the grid at right below, sketch the disk's angular acceleration as a function of time from $t = 0$ to $t = t_1$.

Example graphs:

(c)(i)



(c)(ii)



i. 3 points

For curve with a clear change of slope or curvature at $\frac{1}{2}t_1$ and showing a decrease in speed thereafter		1 point
For a curve that indicates slowing at a decreasing rate between times $\frac{1}{2}t_1$ and t_1		1 point
For a curve that does not reach zero at or before time t_1		1 point

Question 3 (continued)

Distribution of points

(c) (continued)

ii. 1 point

For a curve with decreasing magnitude between times $\frac{1}{2}t_1$ and t_1 Note: The acceleration may reach zero at or before time t_1. If so, it must remain zero for the remaining time.		1 point

(d) LO / SP: 3.F.2.1 / 6.4
1 point

The student is trying to mathematically model the magnitude τ of the torque exerted by the axle on the disk when the oil is present at times $t > \frac{1}{2}t_1$. The student writes down the following two equations, each of which includes a positive constant $(C_1$ or $C_2)$ with appropriate units.

(1) $\tau = C_1\left(t - \frac{1}{2}t_1\right)$ (for $t > \frac{1}{2}t_1$)

(2) $\tau = \dfrac{C_2}{\left(t + \frac{1}{2}t_1\right)}$ (for $t > \frac{1}{2}t_1$)

Which equation better mathematically models this experiment?

____ Equation (1) ____ Equation (2)

Briefly explain why the equation you selected is plausible and why the other equation is not plausible.

Correct answer: "Equation (2)" Note: If the wrong selection is made, the explanation is not graded.		
For stating that Equation (2) is plausible because the frictional torque decreases with increasing time, whereas in Equation (1) torque increases with increasing time		1 point
Examples: Equation (2) because τ decreases. In Equation (1), it doesn't. Equation (2) is plausible because the frictional torque decreases as more oil reaches the contact surface over time. Equation (1) is not plausible because it shows friction increasing as more oil reaches the surface over time.		

Question 3 (continued)

Learning Objectives (LO)

LO 3.A.1.1: The student is able to express the motion of an object using narrative, mathematical, and graphical representations. [See Science Practices 1.5, 2.1, and 2.2]

LO 3.A.3.1: The student is able to analyze a scenario and make claims (develop arguments, justify assertions) about the forces exerted on an object by other objects for different types of forces or components of forces. [See Science Practices 6.4 and 7.2]

LO 3.F.1.1: The student is able to use representations of the relationship between force and torque. [See Science Practice 1.4]

LO 3.F.2.1: The student is able to make predictions about the change in the angular velocity about an axis for an object when forces exerted on the object cause a torque about that axis. [See Science Practice 6.4]

LO 4.D.1.1: The student is able to describe a representation and use it to analyze a situation in which several forces exerted on a rotating system of rigidly connected objects change the angular velocity and angular momentum of the system. [See Science Practices 1.2 and 1.4]

LO 4.D.2.1: The student is able to describe a model of a rotational system and use that model to analyze a situation in which angular momentum changes due to interaction with other objects or systems. [See Science Practices 1.2 and 1.4]

LO 4.D.3.1: The student is able to use appropriate mathematical routines to calculate values for initial or final angular momentum, or change in angular momentum of a system, or average torque or time during which the torque is exerted in analyzing a situation involving torque and angular momentum. [See Science Practice 2.2]