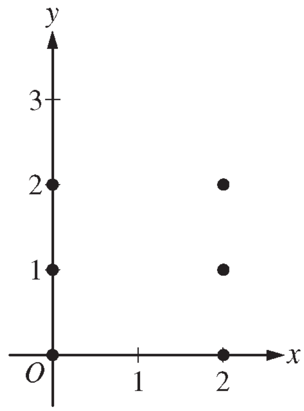
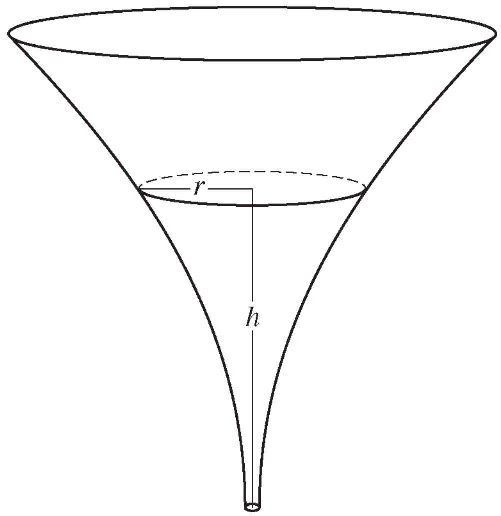
4. Consider the differential equation $\frac{dy}{dx} = \frac{y^2}{x - 1}$.

(a) On the axes provided, sketch a slope field for the given differential equation at the six points indicated.



(b) Let $y = f(x)$ be the particular solution to the given differential equation with the initial condition $f(2) = 3$. Write an equation for the line tangent to the graph of $y = f(x)$ at $x = 2$. Use your equation to approximate $f(2.1)$.

(c) Find the particular solution $y = f(x)$ to the given differential equation with the initial condition $f(2) = 3$.



5. The inside of a funnel of height 10 inches has circular cross sections, as shown in the figure above. At height h, the radius of the funnel is given by $r = \frac{1}{20}\left(3 + h^2\right)$, where $0 \le h \le 10$. The units of r and h are inches.

(a) Find the average value of the radius of the funnel.

(b) Find the volume of the funnel.

(c) The funnel contains liquid that is draining from the bottom. At the instant when the height of the liquid is $h = 3$ inches, the radius of the surface of the liquid is decreasing at a rate of $\frac{1}{5}$ inch per second. At this instant, what is the rate of change of the height of the liquid with respect to time?

Question 4

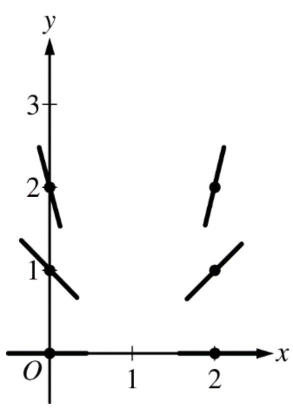
Consider the differential equation $\dfrac{dy}{dx} = \dfrac{y^2}{x-1}$.

(a) On the axes provided, sketch a slope field for the given differential equation at the six points indicated.

(b) Let $y = f(x)$ be the particular solution to the given differential equation with the initial condition $f(2) = 3$. Write an equation for the line tangent to the graph of $y = f(x)$ at $x = 2$. Use your equation to approximate $f(2.1)$.

(c) Find the particular solution $y = f(x)$ to the given differential equation with the initial condition $f(2) = 3$.

(a)



$2 : \begin{cases} 1 : \text{zero slopes} \\ 1 : \text{nonzero slopes} \end{cases}$

(b) $\left.\dfrac{dy}{dx}\right|_{(x,\,y)=(2,\,3)} = \dfrac{3^2}{2-1} = 9$

An equation for the tangent line is $y = 9(x-2) + 3$.

$f(2.1) \approx 9(2.1-2) + 3 = 3.9$

$2 : \begin{cases} 1 : \text{tangent line equation} \\ 1 : \text{approximation} \end{cases}$

(c) $\dfrac{1}{y^2}\,dy = \dfrac{1}{x-1}\,dx$

$\displaystyle\int \frac{1}{y^2}\,dy = \int \frac{1}{x-1}\,dx$

$-\dfrac{1}{y} = \ln|x-1| + C$

$-\dfrac{1}{3} = \ln|2-1| + C \Rightarrow C = -\dfrac{1}{3}$

$-\dfrac{1}{y} = \ln|x-1| - \dfrac{1}{3}$

$y = \dfrac{1}{\frac{1}{3} - \ln(x-1)}$

Note: This solution is valid for $1 < x < 1 + e^{1/3}$.

$5 : \begin{cases} 1 : \text{separation of variables} \\ 2 : \text{antiderivatives} \\ 1 : \text{constant of integration and uses initial condition} \\ 1 : \text{solves for } y \end{cases}$

Note: max $3/5$ [1-2-0-0] if no constant of integration

Note: $0/5$ if no separation of variables