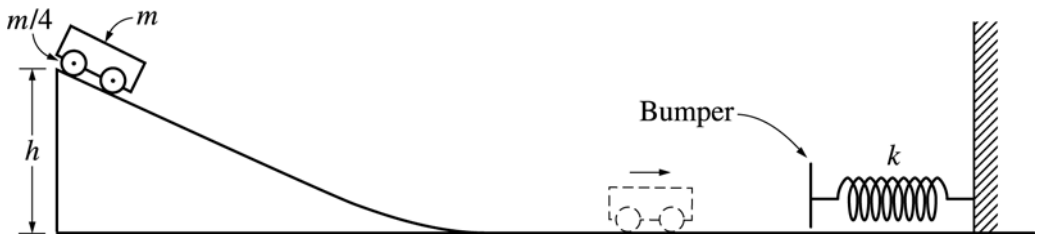

Mech 2.

The cart shown above is made of a block of mass m and four solid rubber tires each of mass $m/4$ and radius r. Each tire may be considered to be a disk. (A disk has rotational inertia $\frac{1}{2}ML^2$, where M is the mass and L is the radius of the disk.) The cart is released from rest and rolls without slipping from the top of an inclined plane of height h. Express all algebraic answers in terms of the given quantities and fundamental constants.

(a) Determine the total rotational inertia of all four tires.

(b) Determine the speed of the cart when it reaches the bottom of the incline.

(c) After rolling down the incline and across the horizontal surface, the cart collides with a bumper of negligible mass attached to an ideal spring, which has a spring constant k. Determine the distance x_m the spring is compressed before the cart and bumper come to rest.

(d) Now assume that the bumper has a non-neglible mass. After the collision with the bumper, the spring is compressed to a maximum distance of about 90% of the value of x_m in part (c). Give a reasonable explanation for this decrease.

Question 2

15 points total **Distribution of points**

(a) 2 points

For determining the rotational inertia of each tire 1 point

$I = \frac{1}{2}ML^2 = \frac{1}{2}\frac{m}{4}r^2 = \frac{1}{8}mr^2$

For the correct total rotational inertia for all 4 tires 1 point

$I_{tot} = 4I = \frac{1}{2}mr^2$

(b) 7 points

For an indication of the conservation of mechanical energy 1 point

$E_{top} = E_{bottom}$; $\Delta U = -\Delta K$; or equivalent

For correct expressions for energies at the top 1 point

$K_{top} = 0;\ U_{top} = mgh + 4\left(\frac{1}{4}mgh\right) = 2mgh$

For a correct expression for potential energy at the bottom and for recognizing that kinetic energy at the bottom is the sum of translational and rotational kinetic energies 1 point

$U_{bottom} = 0;\ K_{bottom} = K_{trans} + K_{rot}$

For a correct expression for translational kinetic energy at the bottom 1 point

$K_{trans} = \frac{1}{2}(2m)v^2 = mv^2$

For a correct expression for rotational kinetic energy at the bottom 1 point

$K_{rot} = \frac{1}{2}I\omega^2$

For recognition of the relationship between translational and rotational velocity 1 point

$\omega = \frac{v}{r}$

Substituting these expressions to determine total kinetic energy at the bottom

$K_{bottom} = mv^2 + \frac{1}{2}\left(\frac{1}{2}mr^2\right)\frac{v^2}{r^2} = \frac{5}{4}mv^2$

Setting potential energy at the top equal to the kinetic energy at the bottom

$\frac{5}{4}mv^2 = 2mgh$

For the correct solution for v 1 point

$v = \sqrt{\frac{8}{5}gh}$

Question 2 (cont'd.)

Distribution of points

(c) 4 points

For recognition that energy is conserved (Although it is an inelastic collision, the mass of the bumper is negligibly small, thus its kinetic energy is negligible and there is no loss of energy.) 1 point

For a correct expression for potential energy of the spring at maximum compression 1 point

$U_K = \frac{1}{2}kx_m^2$

For applying conservation of energy by equating the potential energy of the spring at maximum compression EITHER to the gravitational potential energy of the cart and wheels at the top of the inclined plane OR to the kinetic energy of the cart and wheels at the bottom of the inclined plane 1 point

$\frac{1}{2}kx_m^2 = 2mgh$ OR $\frac{1}{2}kx_m^2 = \frac{5}{4}Mv^2$

For a correct solution of either of these equations for x_m (including a correct substitution for v from part (b) for the second equation) or an answer consistent with work done in (b) 1 point

$x_m = 2\sqrt{\frac{mgh}{k}}$

Notes:

If there was no attempt to use energy then 1 point only was given for $F = -kx = ma = \frac{dp}{dt}$ or a similar expression.

Further, if there was written recognition of the necessity to slow down the wheel rotation, torque, angular acceleration, etc, then 1 point was given.

(d) 2 points

For an explanation that discusses the inelastic collision with a loss of mechanical energy or a reduced velocity resulting in a smaller compression. The discussion should have been correctly stated. If there were any incorrect statements, then 1 point was subtracted. Points were neither added or subtracted for irrelevant statements, such as references to friction. 2 points