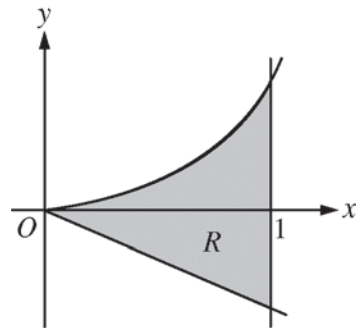

5. Let R be the shaded region bounded by the graph of $y = xe^{x^2}$, the line $y = -2x$, and the vertical line $x = 1$, as shown in the figure above.

   (a) Find the area of R.

   (b) Write, but do not evaluate, an integral expression that gives the volume of the solid generated when R is rotated about the horizontal line $y = -2$.

   (c) Write, but do not evaluate, an expression involving one or more integrals that gives the perimeter of R.

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6. The Taylor series for a function f about $x = 1$ is given by $\sum_{n=1}^{\infty}(-1)^{n+1}\frac{2^n}{n}(x-1)^n$ and converges to $f(x)$ for $|x-1| < R$, where R is the radius of convergence of the Taylor series.

   (a) Find the value of R.

   (b) Find the first three nonzero terms and the general term of the Taylor series for f', the derivative of f, about $x = 1$.

   (c) The Taylor series for f' about $x = 1$, found in part (b), is a geometric series. Find the function f' to which the series converges for $|x-1| < R$. Use this function to determine f for $|x-1| < R$.

STOP

END OF EXAM

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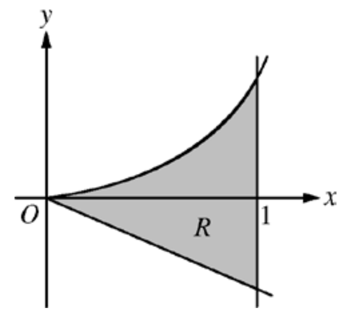
Question 5



Let R be the shaded region bounded by the graph of $y = xe^{x^2}$, the line $y = -2x$, and the vertical line $x = 1$, as shown in the figure above.

(a) Find the area of R.

(b) Write, but do not evaluate, an integral expression that gives the volume of the solid generated when R is rotated about the horizontal line $y = -2$.

(c) Write, but do not evaluate, an expression involving one or more integrals that gives the perimeter of R.

(a) $$\text{Area} = \int_0^1 \left(xe^{x^2} - (-2x) \right) dx$$
$$= \left[\frac{1}{2}e^{x^2} + x^2 \right]_{x=0}^{x=1}$$
$$= \left(\frac{1}{2}e + 1 \right) - \frac{1}{2} = \frac{e+1}{2}$$

$3 : \begin{cases} 1 : \text{integrand} \\ 1 : \text{antiderivative} \\ 1 : \text{answer} \end{cases}$

(b) $$\text{Volume} = \pi \int_0^1 \left[\left(xe^{x^2} + 2 \right)^2 - (-2x+2)^2 \right] dx$$

$3 : \begin{cases} 2 : \text{integrand} \\ 1 : \text{limits and constant} \end{cases}$

(c) $$y' = \frac{d}{dx}\left(xe^{x^2} \right) = e^{x^2} + 2x^2e^{x^2} = e^{x^2}\left(1 + 2x^2\right)$$

$$\text{Perimeter} = \sqrt{5} + 2 + e + \int_0^1 \sqrt{1 + \left[e^{x^2}\left(1 + 2x^2\right) \right]^2}\, dx$$

$3 : \begin{cases} 1 : y' = e^{x^2}\left(1 + 2x^2\right) \\ 1 : \text{integral} \\ 1 : \text{answer} \end{cases}$