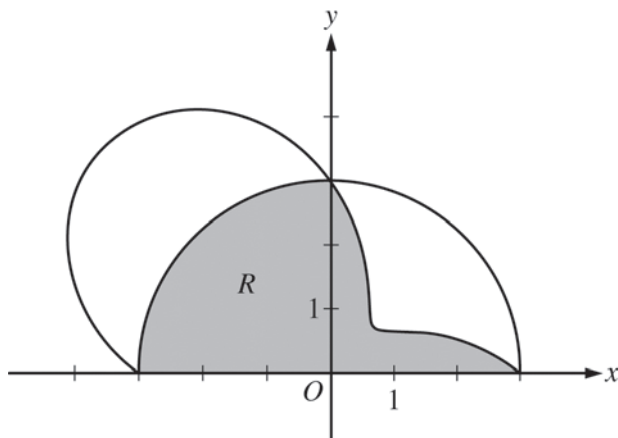
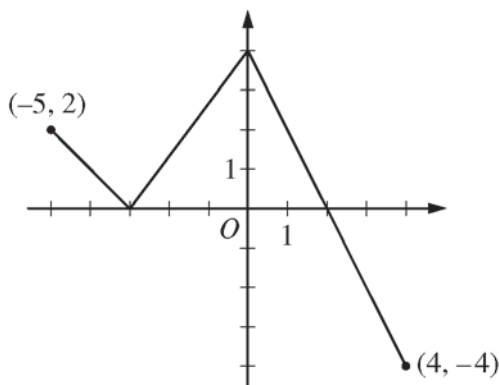


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2. The graphs of the polar curves $r = 3$ and $r = 3 - 2\sin(2\theta)$ are shown in the figure above for $0 \leq \theta \leq \pi$.
- (a) Let R be the shaded region that is inside the graph of $r = 3$ and inside the graph of $r = 3 - 2\sin(2\theta)$. Find the area of R .
- (b) For the curve $r = 3 - 2\sin(2\theta)$, find the value of $\frac{dx}{d\theta}$ at $\theta = \frac{\pi}{6}$.
- (c) The distance between the two curves changes for $0 < \theta < \frac{\pi}{2}$. Find the rate at which the distance between the two curves is changing with respect to θ when $\theta = \frac{\pi}{3}$.
- (d) A particle is moving along the curve $r = 3 - 2\sin(2\theta)$ so that $\frac{d\theta}{dt} = 3$ for all times $t \geq 0$. Find the value of $\frac{dr}{dt}$ at $\theta = \frac{\pi}{6}$.
-

END OF PART A OF SECTION II

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SECTION II, Part B****Time—60 minutes****Number of problems—4****No calculator is allowed for these problems.**Graph of f

3. The function f is defined on the closed interval $[-5, 4]$. The graph of f consists of three line segments and is shown in the figure above. Let g be the function defined by $g(x) = \int_{-3}^x f(t) dt$.
- (a) Find $g(3)$.
- (b) On what open intervals contained in $-5 < x < 4$ is the graph of g both increasing and concave down? Give a reason for your answer.
- (c) The function h is defined by $h(x) = \frac{g(x)}{5x}$. Find $h'(3)$.
- (d) The function p is defined by $p(x) = f(x^2 - x)$. Find the slope of the line tangent to the graph of p at the point where $x = -1$.
-

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Question 2

The graphs of the polar curves $r = 3$ and $r = 3 - 2\sin(2\theta)$ are shown in the figure above for $0 \leq \theta \leq \pi$.

(a) Let R be the shaded region that is inside the graph of $r = 3$ and inside the graph of $r = 3 - 2\sin(2\theta)$. Find the area of R .

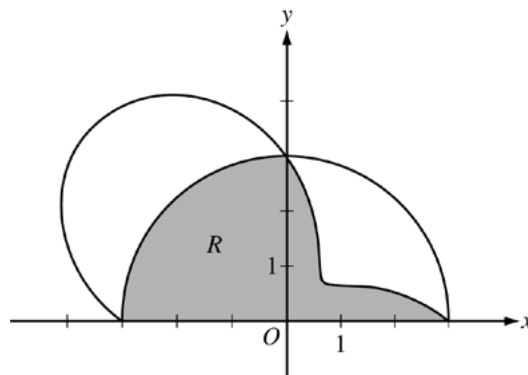
(b) For the curve $r = 3 - 2\sin(2\theta)$, find the value of $\frac{dx}{d\theta}$ at

$$\theta = \frac{\pi}{6}.$$

(c) The distance between the two curves changes for $0 < \theta < \frac{\pi}{2}$.

Find the rate at which the distance between the two curves is changing with respect to θ when $\theta = \frac{\pi}{3}$.

(d) A particle is moving along the curve $r = 3 - 2\sin(2\theta)$ so that $\frac{d\theta}{dt} = 3$ for all times $t \geq 0$. Find the value of $\frac{dr}{dt}$ at $\theta = \frac{\pi}{6}$.



$$\begin{aligned} \text{(a) Area} &= \frac{9\pi}{4} + \frac{1}{2} \int_0^{\pi/2} (3 - 2\sin(2\theta))^2 d\theta \\ &= 9.708 \text{ (or } 9.707) \end{aligned}$$

$$3 : \begin{cases} 1 : \text{integrand} \\ 1 : \text{limits} \\ 1 : \text{answer} \end{cases}$$

$$\begin{aligned} \text{(b) } x &= (3 - 2\sin(2\theta))\cos\theta \\ \left. \frac{dx}{d\theta} \right|_{\theta=\pi/6} &= -2.366 \end{aligned}$$

$$2 : \begin{cases} 1 : \text{expression for } x \\ 1 : \text{answer} \end{cases}$$

(c) The distance between the two curves is $D = 3 - (3 - 2\sin(2\theta)) = 2\sin(2\theta)$.

$$\left. \frac{dD}{d\theta} \right|_{\theta=\pi/3} = -2$$

$$2 : \begin{cases} 1 : \text{expression for distance} \\ 1 : \text{answer} \end{cases}$$

$$\begin{aligned} \text{(d) } \frac{dr}{dt} &= \frac{dr}{d\theta} \cdot \frac{d\theta}{dt} = \frac{dr}{d\theta} \cdot 3 \\ \left. \frac{dr}{dt} \right|_{\theta=\pi/6} &= (-2)(3) = -6 \end{aligned}$$

$$2 : \begin{cases} 1 : \text{chain rule with respect to } t \\ 1 : \text{answer} \end{cases}$$