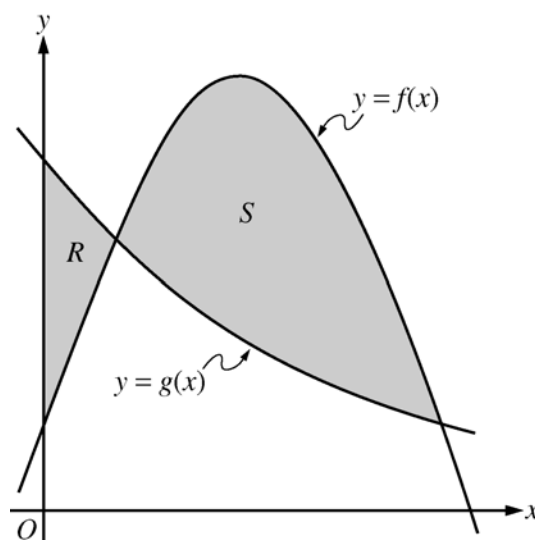


2005 AP[®] CALCULUS BC FREE-RESPONSE QUESTIONS

CALCULUS BC
SECTION II, Part A
Time—45 minutes
Number of problems—3

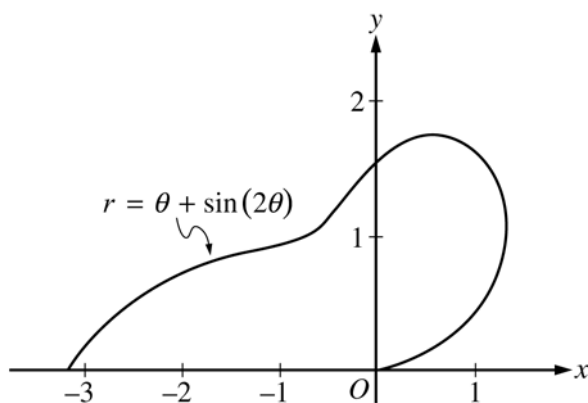
A graphing calculator is required for some problems or parts of problems.



1. Let f and g be the functions given by $f(x) = \frac{1}{4} + \sin(\pi x)$ and $g(x) = 4^{-x}$. Let R be the shaded region in the first quadrant enclosed by the y -axis and the graphs of f and g , and let S be the shaded region in the first quadrant enclosed by the graphs of f and g , as shown in the figure above.
- (a) Find the area of R .
 - (b) Find the area of S .
 - (c) Find the volume of the solid generated when S is revolved about the horizontal line $y = -1$.

WRITE ALL WORK IN THE TEST BOOKLET.

2005 AP[®] CALCULUS BC FREE-RESPONSE QUESTIONS



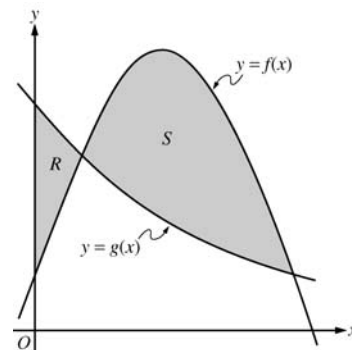
2. The curve above is drawn in the xy -plane and is described by the equation in polar coordinates $r = \theta + \sin(2\theta)$ for $0 \leq \theta \leq \pi$, where r is measured in meters and θ is measured in radians. The derivative of r with respect to θ is given by $\frac{dr}{d\theta} = 1 + 2\cos(2\theta)$.
- Find the area bounded by the curve and the x -axis.
 - Find the angle θ that corresponds to the point on the curve with x -coordinate -2 .
 - For $\frac{\pi}{3} < \theta < \frac{2\pi}{3}$, $\frac{dr}{d\theta}$ is negative. What does this fact say about r ? What does this fact say about the curve?
 - Find the value of θ in the interval $0 \leq \theta \leq \frac{\pi}{2}$ that corresponds to the point on the curve in the first quadrant with greatest distance from the origin. Justify your answer.
-

WRITE ALL WORK IN THE TEST BOOKLET.

AP[®] CALCULUS BC
2005 SCORING GUIDELINES

Question 1

Let f and g be the functions given by $f(x) = \frac{1}{4} + \sin(\pi x)$ and $g(x) = 4^{-x}$. Let R be the shaded region in the first quadrant enclosed by the y -axis and the graphs of f and g , and let S be the shaded region in the first quadrant enclosed by the graphs of f and g , as shown in the figure above.



- (a) Find the area of R .
- (b) Find the area of S .
- (c) Find the volume of the solid generated when S is revolved about the horizontal line $y = -1$.

$$f(x) = g(x) \text{ when } \frac{1}{4} + \sin(\pi x) = 4^{-x}.$$

f and g intersect when $x = 0.178218$ and when $x = 1$.
 Let $a = 0.178218$.

$$(a) \quad \int_0^a (g(x) - f(x)) \, dx = 0.064 \text{ or } 0.065$$

3 : $\begin{cases} 1 : \text{limits} \\ 1 : \text{integrand} \\ 1 : \text{answer} \end{cases}$

$$(b) \quad \int_a^1 (f(x) - g(x)) \, dx = 0.410$$

3 : $\begin{cases} 1 : \text{limits} \\ 1 : \text{integrand} \\ 1 : \text{answer} \end{cases}$

$$(c) \quad \pi \int_a^1 ((f(x) + 1)^2 - (g(x) + 1)^2) \, dx = 4.558 \text{ or } 4.559$$

3 : $\begin{cases} 2 : \text{integrand} \\ 1 : \text{limits, constant, and answer} \end{cases}$