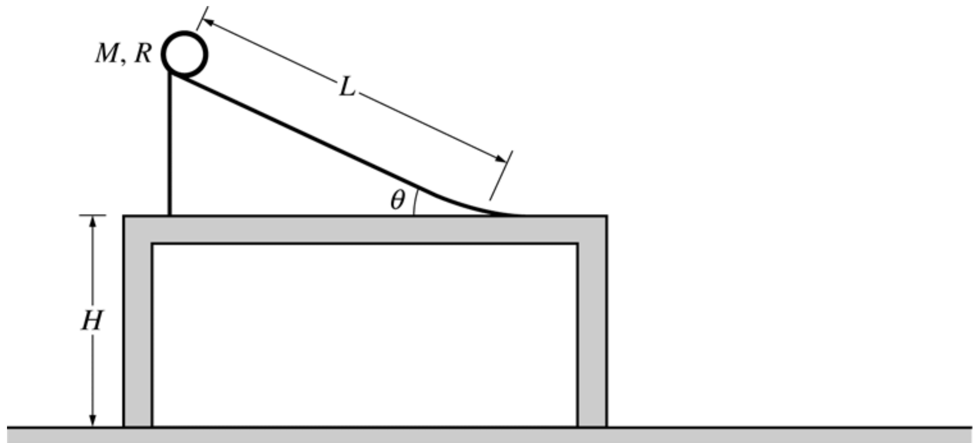

Mech 3.

A thin hoop of mass M, radius R, and rotational inertia MR^2 is released from rest from the top of the ramp of length L above. The ramp makes an angle θ with respect to a horizontal tabletop to which the ramp is fixed. The table is a height H above the floor. Assume that the hoop rolls without slipping down the ramp and across the table. Express all algebraic answers in terms of given quantities and fundamental constants.

(a) Derive an expression for the acceleration of the center of mass of the hoop as it rolls down the ramp.

(b) Derive an expression for the speed of the center of mass of the hoop when it reaches the bottom of the ramp.

(c) Derive an expression for the horizontal distance from the edge of the table to where the hoop lands on the floor.

(d) Suppose that the hoop is now replaced by a disk having the same mass M and radius R. How will the distance from the edge of the table to where the disk lands on the floor compare with the distance determined in part (c) for the hoop?

____ Less than ____ The same as ____ Greater than

Briefly justify your response.

END OF EXAM

Question 3

15 points total | **Distribution of points**

(a) and (b)
These two parts were scored together because of the different approaches that could be used to answer them. The parts could be answered in either order.

<u>Approach using translational and rotational dynamics</u>

(a) 5 points

For use of Newton's 2nd law in both translational <u>and</u> rotational forms — 1 point
$\sum F = ma_{cm}$ and $\sum \tau = I\alpha_{cm}$

For a correct equation applying Newton's second law in translational form — 1 point
$Mg\sin\theta - f = Ma_{cm}$

For a correct equation applying Newton's second law in rotational form — 1 point
$fR = I\alpha_{cm}$

For a correct relationship between linear and angular acceleration for rolling without slipping — 1 point
$\alpha_{cm} = \dfrac{a_{cm}}{R}$

Substituting for I and α_{cm} into the rotational equation above
$fR = MR^2 \dfrac{a_{cm}}{R}$
$f = Ma_{cm}$

Substituting this expression for f into the equation for translational motion above
$Mg\sin\theta - Ma_{cm} = Ma_{cm}$

For the correct answer — 1 point
$a_{cm} = \dfrac{g}{2}\sin\theta$

(b) 3 points

For a correct kinematic equation containing a and υ — 1 point
$\upsilon^2 = \upsilon_0^2 + 2a\Delta x$, $\upsilon_0 = 0$

For correct substitution of the expression for acceleration from part (a) — 1 point
For correct substitution of the distance traveled — 1 point
$\upsilon^2 = 2\left(\dfrac{g}{2}\sin\theta\right)L$
$\upsilon = \sqrt{gL\sin\theta}$

Question 3 (continued)

	Distribution of points
Approach using torque about point of contact between hoop and ramp and parallel axis theorem	
(a) 5 points	
For use of Newton's 2nd law in rotational form and the parallel axis theorem $\sum \tau = I\alpha_{cm}$ and $I = I_{cm} + Mh^2$	1 point
For a correct rotational inertia about the point of contact using the parallel axis theorem $I = MR^2 + MR^2 = 2MR^2$	1 point
For a correct torque about the point of contact $\sum \tau = RMg\sin\theta$	1 point
For a correct relationship between linear and angular acceleration for rolling without slipping $\alpha_{cm} = \dfrac{a_{cm}}{R}$	1 point
Substituting for $\sum \tau$, I, and α_{cm} into the rotational equation above $RMg\sin\theta = 2MR^2 \dfrac{a_{cm}}{R}$	
For the correct answer $a_{cm} = \dfrac{g}{2}\sin\theta$	1 point
(b) 3 points	
For a solution to part (b) as in the previous approach with points allotted similarly	3 points

Question 3 (continued)

Distribution of points

Approach using conservation of energy and kinematics, working part (b) first

(b) 5 points

For a statement of conservation of energy containing potential and kinetic energy terms — 1 point

$\Delta U = K_{rot} + K_{trans}$

For a correct expression for the potential energy change — 1 point

For correct translational and rotational kinetic energies — 1 point

$MgL\sin\theta = \frac{1}{2}Mv^2 + \frac{1}{2}I\omega^2$

For a correct relationship between linear and angular velocity for rolling without slipping — 1 point

$\omega = \frac{v}{R}$

Substituting expressions for I and ω into the energy equation above

$MgL\sin\theta = \frac{1}{2}Mv^2 + \frac{1}{2}\left(MR^2\right)\left(\frac{v}{R}\right)^2$

$gL\sin\theta = \frac{1}{2}v^2 + \frac{1}{2}v^2 = v^2$

For the correct answer — 1 point

$v = \sqrt{gL\sin\theta}$

(a) 3 points

For a correct kinematic relationship — 1 point

$v^2 = v_0^2 + 2a\Delta x\ ,\ v_0 = 0$

For correct substitution of the expression for velocity — 1 point

For correct substitution of the distance traveled — 1 point

$gL\sin\theta = 2a_{cm}L$

$a_{cm} = \frac{g}{2}\sin\theta$

Question 3 (continued)

	Distribution of points
(c) 4 points	
Applying the kinematic equation for distance as a function of time to the vertical motion $H = gt^2/2$	
For a correct expression for the time between leaving the table and landing on the floor $t = \sqrt{2H/g}$	1 point
For use of zero acceleration in calculation of the horizontal distance traveled $x = \upsilon_x t$	1 point
For correct substitution of υ_x from part (b)	1 point
For correct substitution of *t* from previous calculation $d = \sqrt{gL\sin\theta}\sqrt{2H/g}$ $d = \sqrt{2LH\sin\theta}$	1 point
(d) 3 points	
For checking the space next to "Greater than"	1 point
For a sufficiently detailed justification containing no incorrect statements. Such an answer logically concludes, at a minimum, that the linear speed or velocity at the bottom of the ramp is greater for the disk because the rotational inertia of the disk is less. It is not necessary to state that the time of fall is the same.	2 points

One point was awarded for a minimal or partially correct answer.
No justification points were awarded if the space next to "Greater than" was not checked.

Examples of 2-point answers:

A disk will have smaller rotational inertia and will therefore have a greater rotational velocity. This will lead to a greater translational velocity, and a greater distance *x*.

The rotational inertia is less than the hoop, causing greater acceleration and more final speed at the end of the table.

The acceleration when $I = MR^2/2$ is $(2/3)g\sin\theta$, so the disk will be moving faster at the bottom of the ramp and will travel farther.

Examples of 1-point answers:

A disk has a larger rotational inertia, so it will have a greater kinetic energy and will therefore land farther from the ramp.

The moment of inertia for the disk is smaller, thus its rotational velocity is bigger, causing it to go further.

Less energy will be used to spin the disk than the hoop, and *I* of the disk is less than *I* of the hoop.