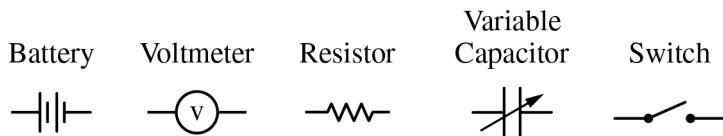


Begin your response to **QUESTION 2** on this page.

2. The plates of a certain variable capacitor have an adjustable area. An experiment is performed to study the potential difference across the capacitor as it discharges through a resistor. A circuit is to be constructed with the following available equipment: a single ideal battery of potential difference ΔV_0 , a single voltmeter, a single resistor of resistance R , a single uncharged variable capacitor set to capacitance C , and one or more switches as needed.



- (a) Using the symbols shown, draw a schematic diagram of a circuit that can charge the capacitor and may also be used to study the potential difference across the capacitor as it discharges through the resistor.

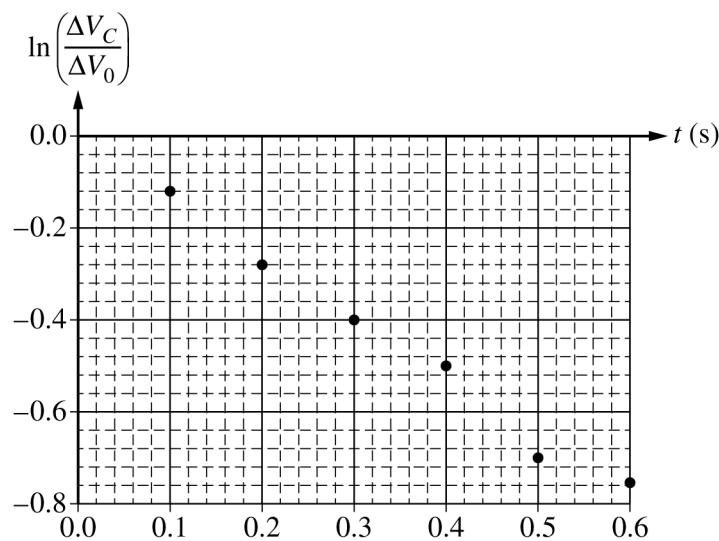
The capacitor is fully charged by the battery. At time $t = 0$, the capacitor starts discharging through the resistor.

- (b) Show that the potential difference ΔV_C across the capacitor as a function of time t is $\Delta V_C(t) = \Delta V_0 e^{-\frac{t}{RC}}$ as the capacitor discharges.

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Continue your response to **QUESTION 2** on this page.

- (c) The experiment is performed using a resistor of $R = 150 \text{ k}\Omega$. Data for the potential difference ΔV_C across the capacitor as a function of t are recorded and a plot of $\ln\left(\frac{\Delta V_C}{\Delta V_0}\right)$ as a function of t is created on the graph below.



- Draw the best-fit line for the data.
- Using the best-fit line, calculate a value for the unknown capacitance C .

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 2** on this page.

- (d) The capacitor is adjusted so that the surface area of the plates is increased, and the experiment is repeated. Would the slope of the best-fit line in the second experiment be more steep, less steep, or unchanged compared to the slope of the best-fit line in part (c)?

More steep Less steep Unchanged

Briefly justify your answer.

- (e) The ideal battery is then replaced with a non-ideal battery with internal resistance r , and the experiment is repeated.

- i. Would the slope of the graph in this final experiment change compared to the graph in part (c)?

Yes No

Briefly justify your answer.

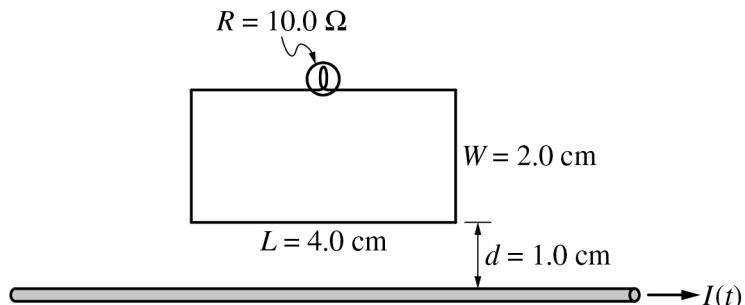
- ii. Would the vertical intercept of the graph in this final experiment change compared to the graph in part (c)?

Yes No

Briefly justify your answer.

GO ON TO THE NEXT PAGE.

Begin your response to **QUESTION 3** on this page.



3. A lightbulb of resistance $R = 10.0 \Omega$ is connected to a rectangular loop of wire of negligible resistance near a very long current-carrying wire. The rectangular loop has a length $L = 4.0 \text{ cm}$ and a width $W = 2.0 \text{ cm}$ and is positioned so one of the longer sides of the loop is a distance $d = 1.0 \text{ cm}$ above and parallel to the long wire, as shown. The current in the long wire is initially flowing to the right and is given by $I(t) = C - Dt$, where $C = 10.0 \text{ A}$ and $D = 2.0 \text{ A/s}$. At time $t = 5.0 \text{ s}$, the current in the long wire is instantaneously zero as the current changes direction.

- (a) What is the direction, if any, of the magnetic field produced by the induced current in the rectangular loop as the current in the long wire changes direction?

Into the page Out of the page No direction, because the field is zero

Justify your answer.

- (b) Calculate the magnetic flux through the loop due to only the long wire at time $t = 3.0 \text{ s}$.

GO ON TO THE NEXT PAGE.

-
- (c) For recognizing the electric field varies as an inverse square of the separation distance from the center of the nonconducting sphere 1 point

Example Response

$$E \propto \frac{1}{r^2}$$

For the correct magnitude, including units, of the electric field at $2R$ 1 point

Example Response

$$\begin{aligned} E_{\text{new}} &= \frac{E_{\text{old}}}{4} = \frac{8 \text{ N/C}}{4} \\ E_{\text{new}} &= 2 \text{ N/C} \end{aligned}$$

Total for part (c) 2 points

-
- (d) For using the equation relating potential difference to the electric field with an attempt at either integration limits or evaluating the integral 1 point

Example Response

$$\begin{aligned} V_f - V_i &= - \int_{s_i}^{s_f} Edr \\ \Delta V &= - \int_{r=R}^{r=4R} Edr \end{aligned}$$

For substituting the correct expression for the electric field or an expression for E that is consistent with the explicit functional dependence of $E(r)$ from part (c) 1 point

Example Response

$$\Delta V = - \int_{r=R}^{r=4R} -\frac{Q}{4\pi\epsilon_0 r^2} dr$$

$$\Delta V = \frac{Q}{4\pi\epsilon_0} \int_{r=R}^{r=4R} \frac{1}{r^2} dr$$

For using a differential expression consistent with the loop rule in the first point that includes a substitution of $I = -\frac{dq}{dt}$ for the current 1 point

Example Response

$$\begin{aligned}\frac{q}{C} &= -\frac{dq}{dt} R \\ -\frac{1}{RC} dt &= \frac{1}{q} dq \\ -\frac{1}{RC} \int_{t=0}^{t=t} dt &= \int_{q=q_0}^{q=q} \frac{1}{q} dq \\ \frac{-t}{RC} &= \ln\left(\frac{q(t)}{q_0}\right) \therefore \frac{q(t)}{q_0} = e^{-t/RC} \therefore \frac{q(t)}{C} = \frac{q_0}{C} e^{-t/RC} \therefore V(t) = V_0 e^{-t/RC}\end{aligned}$$

Scoring Note: The point can be earned regardless of the sign used in the substitution of $\frac{dq}{dt}$ for the current.

Total for part (b) 3 points

(c)(i) For drawing an appropriate best-fit line 1 point

Example Response