Begin your response to **QUESTION 3** on this page.

3. A car maker produces four different models of cars: A, B, C, and D. A group of researchers is investigating which model of car has the longest distance traveled per gallon of gas (mileage). Higher mileage is considered better than lower mileage. The researchers will conduct a study in which they contact several owners of each model of car and ask them to estimate their mileage.

(a) Is this an observational study or an experiment? Justify your answer in context.

Model D has an autopilot feature, in which the car controls its own motion with human supervision. James owns a Model D car and will investigate whether using the autopilot feature results in higher mileage than not using the autopilot. James will drive his car on 70 different days to and from work, using the same route at the same time each day. James will record the mileage each day.

(b) James will use a completely randomized design to conduct his investigation. Describe an appropriate method James could use to randomly assign the two treatments, driving using the autopilot feature and driving without using the autopilot feature, to 35 days each.

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 3** on this page.

(c) After the investigation was completed, James verified that the conditions for inference were met and conducted a hypothesis test. He discovered the mean mileage when using the autopilot feature was significantly higher than the mean mileage when not using the autopilot feature.

James is a member of a Model D club with thousands of members who all drive Model D cars. He will give a presentation at a Model D club members' meeting later this year and would like to state that the results of his hypothesis test apply to all Model D cars in his club. Another member of the club who is a statistician tells James his findings do not apply to all Model D cars in the club. What change would James need to make to his original study to be able to generalize to all Model D cars in the club?

GO ON TO THE NEXT PAGE.

Begin your response to **QUESTION 4** on this page.

4. In an online game, players move through a virtual world collecting geodes, a type of hollow rock. When broken open, these geodes contain crystals of different colors that are useful in the game. A red crystal is the most useful crystal in the game. The color of the crystal in each geode is independent and the probability that a geode contains a red crystal is 0.08.

(a) Sarah, a player, will collect and open geodes until a red crystal is found.

(i) Calculate the mean of the distribution of the number of geodes Sarah will open until a red crystal is found. Show your work.

(ii) Calculate the standard deviation of the distribution of the number of geodes Sarah will open until a red crystal is found. Show your work.

(b) Another player, Conrad, decides to play the game and will stop opening geodes after finding a red crystal or when 4 geodes have been opened, whichever comes first. Let Y = the number of geodes Conrad will open. The table shows the partially completed probability distribution for the random variable Y.

Number of geodes Conrad will open, y	1	2	3	4
Probability, $P(Y = y)$	0.08	0.0736		

GO ON TO THE NEXT PAGE.

Question 3: Focus on Sampling and Experimental Design   **4 points**

General Scoring Notes

- Each part of the question (indicated by a letter) is initially scored by determining if it meets the criteria for essentially correct (E), partially correct (P), or incorrect (I). The response is then categorized based on the scores assigned to each letter part and awarded an integer score between 0 and 4 (see the table at the end of the question).

- The model solution represents an ideal response to each part of the question, and the scoring criteria identify the specific components of the model solution that are used to determine the score.

Model Solution	Scoring
(a) This is an observational study. The researchers had the car owners estimate their mileage. The car owners were not randomly assigned a car model, so no treatment was imposed.	**Essentially correct (E)** if the response satisfies the following three components: 1. Identifies an observational study 2. Provides a justification based on no treatment being imposed 3. Includes context **Partially correct (P)** if the response satisfies only two of the three components required for E. **Incorrect (I)** if the response does not meet the criteria for E or P.

Additional Notes:

- A response that states the study is an experiment receives a score of I for part (a).
- A response that states "No random assignment of treatment" may satisfy component 2.
- A response may satisfy component 3 with car models, drivers, cars, owners, or members. Mileage alone does not satisfy component 3.

Model Solution	Scoring
(b) Number the days in the experiment from 1 to 70. Using a random number generator, generate 35 unique integers from 1 to 70, inclusive. Assign the days with those 35 unique integers for James to drive the car with autopilot and assign the remaining 35 days for James to drive the car without autopilot. (Alternative solution) Using 70 equally sized slips of paper, label 35 "with autopilot" and 35 "without autopilot." Mix the slips of paper in a bag. Each day for the 70 days, select a slip of paper (without replacement) to determine the driving method for that day.	**Essentially correct (E)** if the response satisfies the following three components: 1. Creates appropriate labels for the units/treatments 2. Describes how to correctly implement the random process so that every possible random assignment is equally likely 3. The response indicates a random process that results in 35 days assigned to using autopilot and 35 days assigned to not using autopilot **Partially correct (P)** if the response satisfies only two of the three components required for E. **Incorrect (I)** if the response does not meet the criteria for E or P.

Additional Notes:

- For responses that use slips of paper (or marbles or equivalent) to represent treatments
  - To satisfy component 1 some slips must be labeled or assigned to represent autopilot and some labeled or assigned to represent no autopilot (e.g., blue marbles represent autopilot and yellow marbles represent no autopilot).
  - To satisfy component 2 the slips of paper must be mixed/shuffled, and the response must clearly link the treatment selected to a day (e.g., each day James selects a slip to determine the driving method).
  - To satisfy component 3 the response must indicate that there are 35 slips (or marbles of a specific color) for each treatment and the response must indicate that slips of paper (marbles) are selected without replacement.
- For responses that use slips of paper labeled from 1 to 70 (or an equivalent interval)
  - To satisfy component 1 the days must be labeled 1 to 70.
  - To satisfy component 2 the slips of paper must be mixed/shuffled, and the response must clearly link the day number selected to autopilot or no autopilot.
  - To satisfy component 3 the response must indicate that slips of paper are selected without replacement and that 35 days are assigned to each treatment.
- For responses that use a random number generator with days labeled from 1 to 70 (or an equivalent interval)
  - To satisfy component 1 the days must be labeled 1 to 70.
  - To satisfy component 2 the response indicates that the random number generator selects numbers 1 to 70 inclusive, and the response clearly links the day number selected to autopilot or no autopilot.
  - To satisfy component 3 the response must indicate that numbers are selected without repeats and that 35 days are assigned to each treatment.

- For responses that flip a coin for each day (or roll a die and note odd/even, generate a random number 1 or 2, or equivalent)
  - To satisfy component 1 each outcome must be linked to a treatment (e.g., heads equals autopilot, tails equals no autopilot).
  - To satisfy component 2 the response must clearly link the outcome of the coin flip to a day (e.g., each day James flips a coin to determine the driving method). Note: If the response includes a stopping rule (e.g., when 35 days are assigned one treatment, the remaining days are assigned the other treatment), component 2 is not satisfied because this plan increases the probability that the last days will have the same treatment, which does not meet the equally likely random assignment requirement.
  - To satisfy component 3 the response must indicate that 35 days are assigned to each treatment using a stopping rule. If there is no stopping rule, component 3 is not satisfied.
- If a response uses a random number generator or slips of paper with the numbers 1 to 70 and does not initially number the days from 1 to 70, component 1 may be satisfied if the response indicates a link between the number selected and the day (e.g., if 3 is selected, James uses autopilot on the third day).
- Responses that do not use any random process should be scored I. For example, "number the days from 1 to 70 and use autopilot on odd-numbered days."
- Responses that use blocking do not satisfy component 2 because all possible random assignments are not equally likely.
- If the response describes two ways to perform the random assignment, assign the score for the weaker assignment process.

Model Solution	Scoring
(c) In order to generalize his findings to all Model D cars in his club, he would need to randomly select Model D cars from the club. He would then need to carry out a similar study using the Model D cars that were randomly sampled from the club.	**Essentially correct (E)** if the response satisfies the following three components: 1. The response indicates that more cars must be sampled 2. The response indicates that random sampling is required for generalization 3. The response is in context, which would include sampling from the population of interest (Model D cars from his club) **Partially correct (P)** if the response satisfies only two of the three components required for E. **Incorrect (I)** if the response does not meet the criteria for E or P.

Additional Notes:

- Component 3 may be satisfied by using "members" of the club or "owners" in the club rather than cars.
- Any discussion of experimental design beyond the sample selection (such as conditions or two-sample design) should be ignored.