6. Consider the curve defined by $2y^3 + 6x^2y - 12x^2 + 6y = 1$.

   (a) Show that $\dfrac{dy}{dx} = \dfrac{4x - 2xy}{x^2 + y^2 + 1}$.

   (b) Write an equation of each horizontal tangent line to the curve.

   (c) The line through the origin with slope -1 is tangent to the curve at point P. Find the x– and y–coordinates of point P.

END OF EXAMINATION

1998 AP Calculus AB Scoring Guidelines

6. Consider the curve defined by $2y^3 + 6x^2y - 12x^2 + 6y = 1$.
   (a) Show that $\dfrac{dy}{dx} = \dfrac{4x - 2xy}{x^2 + y^2 + 1}$.
   (b) Write an equation of each horizontal tangent line to the curve.
   (c) The line through the origin with slope -1 is tangent to the curve at point P. Find the x– and y–coordinates of point P.

(a) $6y^2\dfrac{dy}{dx} + 6x^2\dfrac{dy}{dx} + 12xy - 24x + 6\dfrac{dy}{dx} = 0$

$\dfrac{dy}{dx}(6y^2 + 6x^2 + 6) = 24x - 12xy$

$\dfrac{dy}{dx} = \dfrac{24x - 12xy}{6x^2 + 6y^2 + 6} = \dfrac{4x - 2xy}{x^2 + y^2 + 1}$

2 $\begin{cases} 1: & \text{implicit differentiation} \\ 1: & \text{verifies expression for } \dfrac{dy}{dx} \end{cases}$

(b) $\dfrac{dy}{dx} = 0$

$4x - 2xy = 2x(2 - y) = 0$

$x = 0$ or $y = 2$

When $x = 0$, $2y^3 + 6y = 1$; $y = 0.165$

There is no point on the curve with y coordinate of 2.

$y = 0.165$ is the equation of the only horizontal tangent line.

4 $\begin{cases} 1: & \text{sets } \dfrac{dy}{dx} = 0 \\ 1: & \text{solves } \dfrac{dy}{dx} = 0 \\ 1: & \text{uses solutions for } x \text{ to find equations of horizontal tangent lines} \\ 1: & \text{verifies which solutions for } y \text{ yield equations of horizontal tangent lines} \end{cases}$

Note: max 1/4 [1-0-0-0] if $dy/dx = 0$ is not of the form $g(x, y)/h(x, y) = 0$ with solutions for both x and y

(c) $y = -x$ is equation of the line.

$2(-x)^3 + 6x^2(-x) - 12x^2 + 6(-x) = 1$

$-8x^3 - 12x^2 - 6x - 1 = 0$

$x = -1/2, \quad y = 1/2$

–or–

$\dfrac{dy}{dx} = -1$

$4x - 2xy = -x^2 - y^2 - 1$

$4x + 2x^2 = -x^2 - x^2 - 1$

$4x^2 + 4x + 1 = 0$

$x = -1/2, \quad y = 1/2$

3 $\begin{cases} 1: & y = -x \\ 1: & \text{substitutes } y = -x \text{ into equation of curve} \\ 1: & \text{solves for } x \text{ and } y \end{cases}$

–or–

3 $\begin{cases} 1: & \text{sets } \dfrac{dy}{dx} = -1 \\ 1: & \text{substitutes } y = -x \text{ into } \dfrac{dy}{dx} \\ 1: & \text{solves for } x \text{ and } y \end{cases}$

Note: max 2/3 [1-1-0] if importing incorrect derivative from part (a)