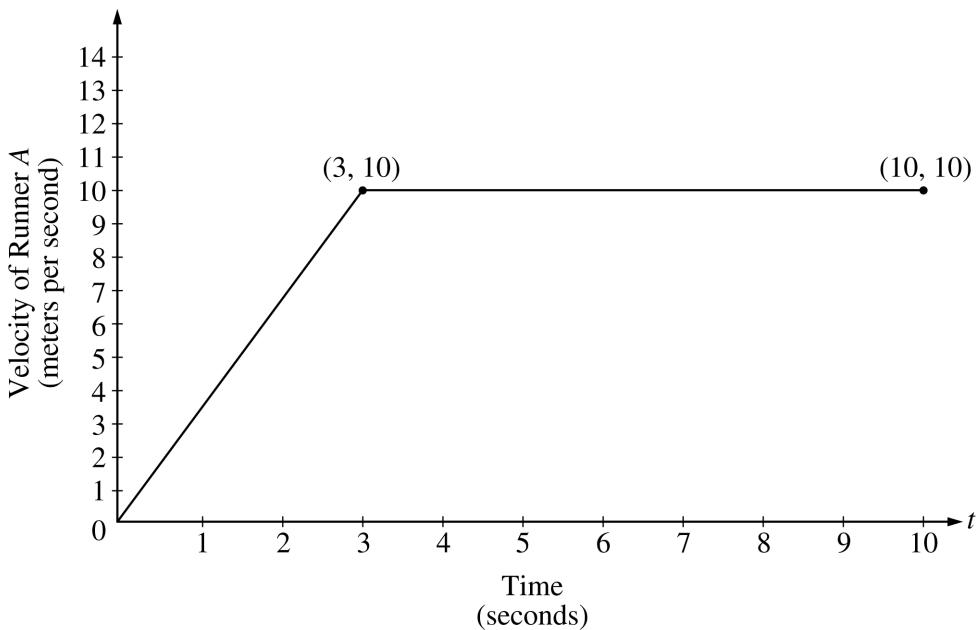


2000 AP® CALCULUS BC FREE-RESPONSE QUESTIONS

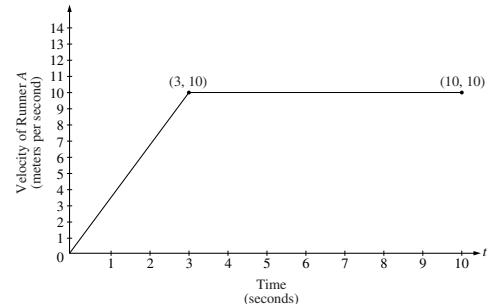


2. Two runners, *A* and *B*, run on a straight racetrack for $0 \leq t \leq 10$ seconds. The graph above, which consists of two line segments, shows the velocity, in meters per second, of Runner *A*. The velocity, in meters per second, of Runner *B* is given by the function v defined by $v(t) = \frac{24t}{2t + 3}$.
- Find the velocity of Runner *A* and the velocity of Runner *B* at time $t = 2$ seconds. Indicate units of measure.
 - Find the acceleration of Runner *A* and the acceleration of Runner *B* at time $t = 2$ seconds. Indicate units of measure.
 - Find the total distance run by Runner *A* and the total distance run by Runner *B* over the time interval $0 \leq t \leq 10$ seconds. Indicate units of measure.
-
3. The Taylor series about $x = 5$ for a certain function f converges to $f(x)$ for all x in the interval of convergence. The n th derivative of f at $x = 5$ is given by $f^{(n)}(5) = \frac{(-1)^n n!}{2^n (n+2)}$, and $f(5) = \frac{1}{2}$.
- Write the third-degree Taylor polynomial for f about $x = 5$.
 - Find the radius of convergence of the Taylor series for f about $x = 5$.
 - Show that the sixth-degree Taylor polynomial for f about $x = 5$ approximates $f(6)$ with error less than $\frac{1}{1000}$.

END OF PART A OF SECTION II

Two runners, A and B , run on a straight racetrack for $0 \leq t \leq 10$ seconds. The graph above, which consists of two line segments, shows the velocity, in meters per second, of Runner A . The velocity, in meters per second, of Runner B is given by the function v defined by $v(t) = \frac{24t}{2t+3}$.

- Find the velocity of Runner A and the velocity of Runner B at time $t = 2$ seconds. Indicate units of measure.
- Find the acceleration of Runner A and the acceleration of Runner B at time $t = 2$ seconds. Indicate units of measure.
- Find the total distance run by Runner A and the total distance run by Runner B over the time interval $0 \leq t \leq 10$ seconds. Indicate units of measure.



$$\begin{aligned} \text{(a) Runner } A: \text{velocity} &= \frac{10}{3} \cdot 2 = \frac{20}{3} \\ &= 6.666 \text{ or } 6.667 \text{ meters/sec} \end{aligned}$$

$$\text{Runner } B: v(2) = \frac{48}{7} = 6.857 \text{ meters/sec}$$

$$\begin{aligned} \text{(b) Runner } A: \text{acceleration} &= \frac{10}{3} = 3.333 \text{ meters/sec}^2 \\ \text{Runner } B: a(2) &= v'(2) = \left. \frac{72}{(2t+3)^2} \right|_{t=2} \\ &= \frac{72}{49} = 1.469 \text{ meters/sec}^2 \end{aligned}$$

$$\text{(c) Runner } A: \text{distance} = \frac{1}{2}(3)(10) + 7(10) = 85 \text{ meters}$$

$$\text{Runner } B: \text{distance} = \int_0^{10} \frac{24t}{2t+3} dt = 83.336 \text{ meters}$$

(units) meters/sec in part (a), meters/sec² in part (b), and meters in part (c), or equivalent.

$$2 \left\{ \begin{array}{l} 1: \text{velocity for Runner } A \\ 1: \text{velocity for Runner } B \end{array} \right.$$

$$2 \left\{ \begin{array}{l} 1: \text{acceleration for Runner } A \\ 1: \text{acceleration for Runner } B \end{array} \right.$$

$$4 \left\{ \begin{array}{l} 2: \text{distance for Runner } A \\ 1: \text{method} \\ 1: \text{answer} \\ 2: \text{distance for Runner } B \\ 1: \text{integral} \\ 1: \text{answer} \end{array} \right.$$

1: units