

**2007 AP<sup>®</sup> CALCULUS BC FREE-RESPONSE QUESTIONS****CALCULUS BC  
SECTION II, Part B****Time—45 minutes****Number of problems—3****No calculator is allowed for these problems.**

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4. Let  $f$  be the function defined for  $x > 0$ , with  $f(e) = 2$  and  $f'$ , the first derivative of  $f$ , given by  $f'(x) = x^2 \ln x$ .
- (a) Write an equation for the line tangent to the graph of  $f$  at the point  $(e, 2)$ .
- (b) Is the graph of  $f$  concave up or concave down on the interval  $1 < x < 3$ ? Give a reason for your answer.
- (c) Use antidifferentiation to find  $f(x)$ .
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$t$ (minutes)	0	2	5	7	11	12
$r'(t)$ (feet per minute)	5.7	4.0	2.0	1.2	0.6	0.5

5. The volume of a spherical hot air balloon expands as the air inside the balloon is heated. The radius of the balloon, in feet, is modeled by a twice-differentiable function  $r$  of time  $t$ , where  $t$  is measured in minutes. For  $0 < t < 12$ , the graph of  $r$  is concave down. The table above gives selected values of the rate of change,  $r'(t)$ , of the radius of the balloon over the time interval  $0 \leq t \leq 12$ . The radius of the balloon is 30 feet when  $t = 5$ . (Note: The volume of a sphere of radius  $r$  is given by  $V = \frac{4}{3}\pi r^3$ .)
- (a) Estimate the radius of the balloon when  $t = 5.4$  using the tangent line approximation at  $t = 5$ . Is your estimate greater than or less than the true value? Give a reason for your answer.
- (b) Find the rate of change of the volume of the balloon with respect to time when  $t = 5$ . Indicate units of measure.
- (c) Use a right Riemann sum with the five subintervals indicated by the data in the table to approximate  $\int_0^{12} r'(t) dt$ . Using correct units, explain the meaning of  $\int_0^{12} r'(t) dt$  in terms of the radius of the balloon.
- (d) Is your approximation in part (c) greater than or less than  $\int_0^{12} r'(t) dt$ ? Give a reason for your answer.
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**WRITE ALL WORK IN THE PINK EXAM BOOKLET.**

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**2007 SCORING GUIDELINES**

**Question 4**

Let  $f$  be the function defined for  $x > 0$ , with  $f(e) = 2$  and  $f'$ , the first derivative of  $f$ , given by  $f'(x) = x^2 \ln x$ .

- (a) Write an equation for the line tangent to the graph of  $f$  at the point  $(e, 2)$ .  
 (b) Is the graph of  $f$  concave up or concave down on the interval  $1 < x < 3$ ? Give a reason for your answer.  
 (c) Use antidifferentiation to find  $f(x)$ .

(a)  $f'(e) = e^2$

An equation for the line tangent to the graph of  $f$  at the point  $(e, 2)$  is  $y - 2 = e^2(x - e)$ .

2 :  $\begin{cases} 1 : f'(e) \\ 1 : \text{equation of tangent line} \end{cases}$

(b)  $f''(x) = x + 2x \ln x$ .

For  $1 < x < 3$ ,  $x > 0$  and  $\ln x > 0$ , so  $f''(x) > 0$ . Thus, the graph of  $f$  is concave up on  $(1, 3)$ .

3 :  $\begin{cases} 2 : f''(x) \\ 1 : \text{answer with reason} \end{cases}$

- (c) Since  $f(x) = \int (x^2 \ln x) dx$ , we consider integration by parts.

$$\begin{aligned} u &= \ln x & dv &= x^2 dx \\ du &= \frac{1}{x} dx & v &= \int (x^2) dx = \frac{1}{3}x^3 \end{aligned}$$

Therefore,

$$\begin{aligned} f(x) &= \int (x^2 \ln x) dx \\ &= \frac{1}{3}x^3 \ln x - \int \left( \frac{1}{3}x^3 \cdot \frac{1}{x} \right) dx \\ &= \frac{1}{3}x^3 \ln x - \frac{1}{9}x^3 + C. \end{aligned}$$

Since  $f(e) = 2$ ,  $2 = \frac{e^3}{3} - \frac{e^3}{9} + C$  and  $C = 2 - \frac{2}{9}e^3$ .

Thus,  $f(x) = \frac{x^3}{3} \ln x - \frac{1}{9}x^3 + 2 - \frac{2}{9}e^3$ .

4 :  $\begin{cases} 2 : \text{antiderivative} \\ 1 : \text{uses } f(e) = 2 \\ 1 : \text{answer} \end{cases}$