

## 2006 AP<sup>®</sup> CALCULUS BC FREE-RESPONSE QUESTIONS

5. Consider the differential equation  $\frac{dy}{dx} = 5x^2 - \frac{6}{y-2}$  for  $y \neq 2$ . Let  $y = f(x)$  be the particular solution to this differential equation with the initial condition  $f(-1) = -4$ .
- (a) Evaluate  $\frac{dy}{dx}$  and  $\frac{d^2y}{dx^2}$  at  $(-1, -4)$ .
- (b) Is it possible for the  $x$ -axis to be tangent to the graph of  $f$  at some point? Explain why or why not.
- (c) Find the second-degree Taylor polynomial for  $f$  about  $x = -1$ .
- (d) Use Euler's method, starting at  $x = -1$  with two steps of equal size, to approximate  $f(0)$ . Show the work that leads to your answer.
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6. The function  $f$  is defined by the power series

$$f(x) = -\frac{x}{2} + \frac{2x^2}{3} - \frac{3x^3}{4} + \cdots + \frac{(-1)^n nx^n}{n+1} + \cdots$$

for all real numbers  $x$  for which the series converges. The function  $g$  is defined by the power series

$$g(x) = 1 - \frac{x}{2!} + \frac{x^2}{4!} - \frac{x^3}{6!} + \cdots + \frac{(-1)^n x^n}{(2n)!} + \cdots$$

for all real numbers  $x$  for which the series converges.

- (a) Find the interval of convergence of the power series for  $f$ . Justify your answer.
- (b) The graph of  $y = f(x) - g(x)$  passes through the point  $(0, -1)$ . Find  $y'(0)$  and  $y''(0)$ . Determine whether  $y$  has a relative minimum, a relative maximum, or neither at  $x = 0$ . Give a reason for your answer.
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**WRITE ALL WORK IN THE PINK EXAM BOOKLET.**

**END OF EXAM**