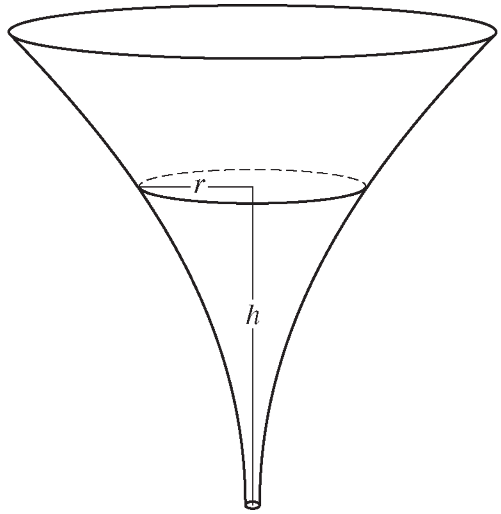
4. Consider the differential equation $\frac{dy}{dx} = x^2 - \frac{1}{2}y.$

   (a) Find $\frac{d^2y}{dx^2}$ in terms of x and $y.$

   (b) Let $y = f(x)$ be the particular solution to the given differential equation whose graph passes through the point $(-2, 8)$. Does the graph of f have a relative minimum, a relative maximum, or neither at the point $(-2, 8)$? Justify your answer.

   (c) Let $y = g(x)$ be the particular solution to the given differential equation with $g(-1) = 2$. Find $\lim_{x \to -1}\left(\frac{g(x) - 2}{3(x+1)^2}\right)$. Show the work that leads to your answer.

   (d) Let $y = h(x)$ be the particular solution to the given differential equation with $h(0) = 2$. Use Euler's method, starting at $x = 0$ with two steps of equal size, to approximate $h(1)$.



5. The inside of a funnel of height 10 inches has circular cross sections, as shown in the figure above. At height h, the radius of the funnel is given by $r = \frac{1}{20}\left(3 + h^2\right)$, where $0 \le h \le 10$. The units of r and h are inches.

(a) Find the average value of the radius of the funnel.

(b) Find the volume of the funnel.

(c) The funnel contains liquid that is draining from the bottom. At the instant when the height of the liquid is $h = 3$ inches, the radius of the surface of the liquid is decreasing at a rate of $\frac{1}{5}$ inch per second. At this instant, what is the rate of change of the height of the liquid with respect to time?

GO ON TO THE NEXT PAGE.

Question 4

Consider the differential equation $\frac{dy}{dx} = x^2 - \frac{1}{2}y.$

(a) Find $\frac{d^2y}{dx^2}$ in terms of x and $y.$

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(c) Let $y = g(x)$ be the particular solution to the given differential equation with $g(-1) = 2.$ Find $\lim_{x \to -1}\left(\frac{g(x) - 2}{3(x + 1)^2}\right).$ Show the work that leads to your answer.

(d) Let $y = h(x)$ be the particular solution to the given differential equation with $h(0) = 2.$ Use Euler's method, starting at $x = 0$ with two steps of equal size, to approximate $h(1).$

(a) $\frac{d^2y}{dx^2} = 2x - \frac{1}{2}\frac{dy}{dx} = 2x - \frac{1}{2}\left(x^2 - \frac{1}{2}y\right)$	2 : $\frac{d^2y}{dx^2}$ in terms of x and y
(b) $\left.\frac{dy}{dx}\right\rvert_{(x,\,y)=(-2,\,8)} = (-2)^2 - \frac{1}{2}\cdot 8 = 0$ $\left.\frac{d^2y}{dx^2}\right\rvert_{(x,\,y)=(-2,\,8)} = 2(-2) - \frac{1}{2}\left((-2)^2 - \frac{1}{2}\cdot 8\right) = -4 < 0$ Thus, the graph of f has a relative maximum at the point $(-2, 8)$.	2 : conclusion with justification
(c) $\lim_{x \to -1}(g(x) - 2) = 0$ and $\lim_{x \to -1} 3(x+1)^2 = 0$ Using L'Hospital's Rule, $\lim_{x \to -1}\left(\frac{g(x) - 2}{3(x+1)^2}\right) = \lim_{x \to -1}\left(\frac{g'(x)}{6(x+1)}\right)$ $\lim_{x \to -1} g'(x) = 0$ and $\lim_{x \to -1} 6(x+1) = 0$ Using L'Hospital's Rule, $\lim_{x \to -1}\left(\frac{g'(x)}{6(x+1)}\right) = \lim_{x \to -1}\left(\frac{g''(x)}{6}\right) = \frac{-2}{6} = -\frac{1}{3}$	3 : $\begin{cases} 2 : \text{L'Hospital's Rule} \\ 1 : \text{answer} \end{cases}$
(d) $h\left(\frac{1}{2}\right) \approx h(0) + h'(0)\cdot\frac{1}{2} = 2 + (-1)\cdot\frac{1}{2} = \frac{3}{2}$ $h(1) \approx h\left(\frac{1}{2}\right) + h'\left(\frac{1}{2}\right)\cdot\frac{1}{2} \approx \frac{3}{2} + \left(-\frac{1}{2}\right)\cdot\frac{1}{2} = \frac{5}{4}$	2 : $\begin{cases} 1 : \text{Euler's method} \\ 1 : \text{approximation} \end{cases}$