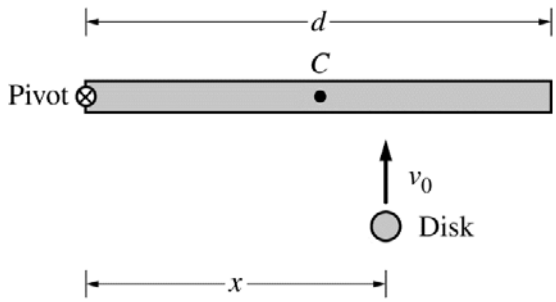

Top View

3. (12 points, suggested time 25 minutes)

The left end of a rod of length d and rotational inertia I is attached to a frictionless horizontal surface by a frictionless pivot, as shown above. Point C marks the center (midpoint) of the rod. The rod is initially motionless but is free to rotate around the pivot. A student will slide a disk of mass m_{disk} toward the rod with velocity v_0 perpendicular to the rod, and the disk will stick to the rod a distance x from the pivot. The student wants the rod-disk system to end up with as much angular speed as possible.

(a) Suppose the rod is much more massive than the disk. To give the rod as much angular speed as possible, should the student make the disk hit the rod to the left of point C, at point C, or to the right of point C ?

____ To the left of C ____ At C ____ To the right of C

Briefly explain your reasoning without manipulating equations.

(b) On the Internet, a student finds the following equation for the postcollision angular speed ω of the rod in this situation: $\omega = \frac{m_{disk} x v_0}{I}$. Regardless of whether this equation for angular speed is correct, does it agree with your qualitative reasoning in part (a) ? In other words, does this equation for ω have the expected dependence as reasoned in part (a) ?

____ Yes ____ No

Briefly explain your reasoning without deriving an equation for ω.

(c) Another student deriving an equation for the postcollision angular speed ω of the rod makes a mistake and comes up with $\omega = \frac{I x v_0}{m_{disk} d^4}$. Without deriving the correct equation, how can you tell that this equation is not plausible—in other words, that it does not make physical sense? Briefly explain your reasoning.

For parts (d) and (e), do NOT assume that the rod is much more massive than the disk.

(d) Immediately before colliding with the rod, the disk's rotational inertia about the pivot is $m_{disk}x^2$ and its angular momentum with respect to the pivot is $m_{disk}v_0x$. Derive an equation for the postcollision angular speed ω of the rod. Express your answer in terms of d, m_{disk}, I, x, v_0, and physical constants, as appropriate.

(e) Consider the collision for which your equation in part (d) was derived, except now suppose the disk bounces backward off the rod instead of sticking to the rod. Is the postcollision angular speed of the rod when the disk bounces off it greater than, less than, or equal to the postcollision angular speed of the rod when the disk sticks to it?

____ Greater than   ____ Less than   ____ Equal to

Briefly explain your reasoning.

Question 3

12 points total | **Distribution of points**

(a) 1 point

Correct answer: "To the right of *C*"
Reasoning cannot earn credit if the incorrect selection is made.

For an explanation that the torque exerted by the disk or the angular momentum of the disk is greater when farther from the pivot — 1 point

Example 1: The disk exerts a greater torque on the rod when it pushes the rod farther from the pivot.
Example 2: The disk has greater angular momentum when it's farther from the pivot. The disk loses almost all its speed during the collision and hence gives the rod almost all its angular momentum. So the rod ends up with more angular momentum when the disk hits it farther from the pivot.

(b) 2 points

Correct answer: "Yes"
If "No" is selected, the explanation may still earn full credit if an incorrect selection was made in part (a).

For a selection consistent with the selection from part (a) — 1 point
For indicating that the equation shows that ω increases with increasing x — 1 point

Example: According to the equation, ω increases with x; a bigger x produces a bigger angular speed. This agrees with my reasoning from part (a), where I said a bigger x creates a bigger angular speed after the collision.

(c) 3 points

For focusing on functional dependence (instead of, for example, considering units/dimensions) — 1 point
For addressing m_{disk}, I, or both — 1 point
For correctly concluding that the equation is wrong because of the dependence on m_{disk}, I, or both — 1 point

Example: If m_{disk} is large, then more angular momentum will be transferred during the collision. But the equation shows the angular speed decreasing with increasing m_{disk}, because it is in the denominator.

Question 3 (continued)

		Distribution of points
(d)	4 points	
	For using an expression of conservation of angular momentum for the disk and rod Note: This point is not awarded for equating angular and linear momentum.	1 point
	For indicating that the initial angular momentum of the system is equal to $m_{disk} v_0 x$	1 point
	For a dimensionally correct expression for the post-collision angular momentum that includes $I\omega$	1 point
	For indicating the correct rotational inertia of the system after the collision: $I + m_{disk} x^2$	1 point
(e)	2 points	
	Correct answer: "Greater than"	
	For indicating, either directly or by analogy to the linear case, that the disk's angular momentum with respect to the pivot changes more in the bouncy scenario than in the original scenario OR for using a similar argument in terms of impulse Note: This point is for describing what happens to the disk.	1 point
	For using conservation of angular momentum or momentum-impulse reasoning to conclude that the rod gains more angular momentum, and hence more angular speed, in the bouncy scenario Note: This point is for describing what happens to the rod.	1 point
	Example: After the bouncy collision, the disk has angular momentum in the clockwise direction. To keep the system angular momentum constant, the magnitude of the rod's counterclockwise angular momentum must be greater than before.	