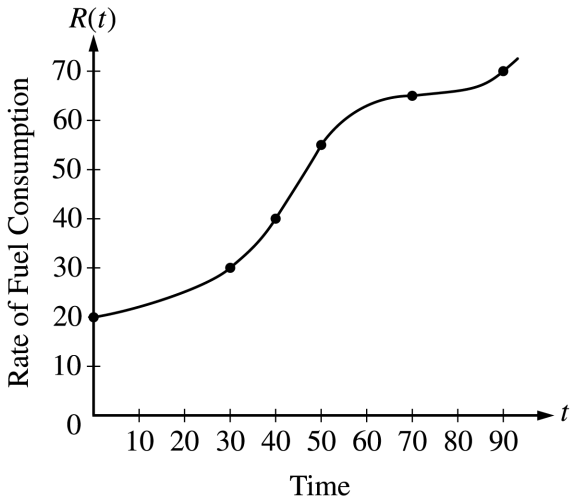


| $t$ (minutes) | $R(t)$ (gallons per minute) |
|---|---|
| 0 | 20 |
| 30 | 30 |
| 40 | 40 |
| 50 | 55 |
| 70 | 65 |
| 90 | 70 |

3. The rate of fuel consumption, in gallons per minute, recorded during an airplane flight is given by a twice-differentiable and strictly increasing function $R$ of time $t$. The graph of $R$ and a table of selected values of $R(t)$, for the time interval $0 \le t \le 90$ minutes, are shown above.

   (a) Use data from the table to find an approximation for $R'(45)$. Show the computations that lead to your answer. Indicate units of measure.

   (b) The rate of fuel consumption is increasing fastest at time $t = 45$ minutes. What is the value of $R''(45)$? Explain your reasoning.

   (c) Approximate the value of $\int_0^{90} R(t)\,dt$ using a left Riemann sum with the five subintervals indicated by the data in the table. Is this numerical approximation less than the value of $\int_0^{90} R(t)\,dt$? Explain your reasoning.

   (d) For $0 < b \le 90$ minutes, explain the meaning of $\int_0^{b} R(t)\,dt$ in terms of fuel consumption for the plane. Explain the meaning of $\frac{1}{b}\int_0^{b} R(t)\,dt$ in terms of fuel consumption for the plane. Indicate units of measure in both answers.

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**END OF PART A OF SECTION II**

**CALCULUS AB**
**SECTION II, Part B**
**Time—45 minutes**
**Number of problems—3**

**No calculator is allowed for these problems.**

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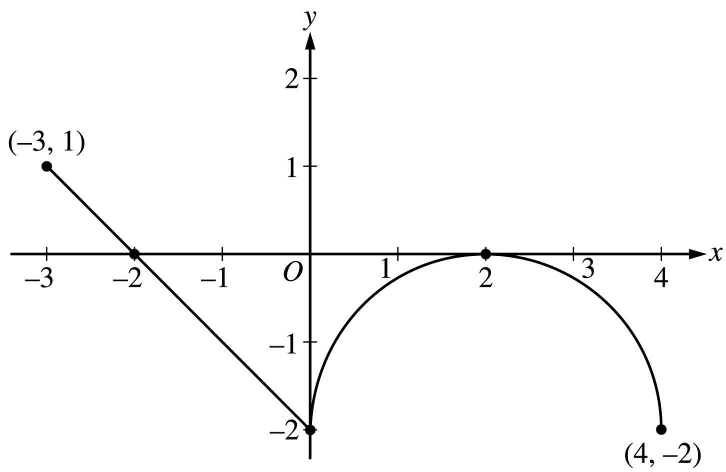


Graph of $f'$

4. Let $f$ be a function defined on the closed interval $-3 \le x \le 4$ with $f(0) = 3$. The graph of $f'$, the derivative of $f$, consists of one line segment and a semicircle, as shown above.

   (a) On what intervals, if any, is $f$ increasing? Justify your answer.

   (b) Find the $x$-coordinate of each point of inflection of the graph of $f$ on the open interval $-3 < x < 4$. Justify your answer.

   (c) Find an equation for the line tangent to the graph of $f$ at the point $(0, 3)$.

   (d) Find $f(-3)$ and $f(4)$. Show the work that leads to your answers.

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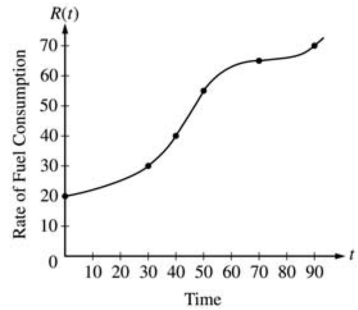
### Question 3

The rate of fuel consumption, in gallons per minute, recorded during an airplane flight is given by a twice-differentiable and strictly increasing function $R$ of time $t$. The graph of $R$ and a table of selected values of $R(t)$, for the time interval $0 \le t \le 90$ minutes, are shown above.




| $t$ (minutes) | $R(t)$ (gallons per minute) |
|---|---|
| 0 | 20 |
| 30 | 30 |
| 40 | 40 |
| 50 | 55 |
| 70 | 65 |
| 90 | 70 |

(a) Use data from the table to find an approximation for $R'(45)$. Show the computations that lead to your answer. Indicate units of measure.

(b) The rate of fuel consumption is increasing fastest at time $t = 45$ minutes. What is the value of $R''(45)$? Explain your reasoning.

(c) Approximate the value of $\int_0^{90} R(t)\,dt$ using a left Riemann sum with the five subintervals indicated by the data in the table. Is this numerical approximation less than the value of $\int_0^{90} R(t)\,dt$ ? Explain your reasoning.

(d) For $0 < b \le 90$ minutes, explain the meaning of $\int_0^b R(t)\,dt$ in terms of fuel consumption for the plane. Explain the meaning of $\frac{1}{b}\int_0^b R(t)\,dt$ in terms of fuel consumption for the plane. Indicate units of measure in both answers.

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(a) $R'(45) \approx \dfrac{R(50) - R(40)}{50 - 40} = \dfrac{55 - 40}{10}$
$= 1.5 \text{ gal/min}^2$

$2 : \begin{cases} 1 : \text{a difference quotient using numbers from table and interval that contains 45} \\ 1 : 1.5 \text{ gal/min}^2 \end{cases}$

(b) $R''(45) = 0$ since $R'(t)$ has a maximum at $t = 45$.

$2 : \begin{cases} 1 : R''(45) = 0 \\ 1 : \text{reason} \end{cases}$

(c) $\int_0^{90} R(t)\,dt \approx (30)(20) + (10)(30) + (10)(40)$
$+(20)(55) + (20)(65) = 3700$

Yes, this approximation is less because the graph of $R$ is increasing on the interval.

$2 : \begin{cases} 1 : \text{value of left Riemann sum} \\ 1 : \text{"less" with reason} \end{cases}$

(d) $\int_0^b R(t)\,dt$ is the total amount of fuel in gallons consumed for the first $b$ minutes.

$\frac{1}{b}\int_0^b R(t)\,dt$ is the average value of the rate of fuel consumption in gallons/min during the first $b$ minutes.

$3 : \begin{cases} 2 : \text{meanings} \\ \quad 1 : \text{meaning of } \int_0^b R(t)\,dt \\ \quad 1 : \text{meaning of } \frac{1}{b}\int_0^b R(t)\,dt \\ \quad < -1 > \text{ if no reference to time } b \\ 1 : \text{units in both answers} \end{cases}$