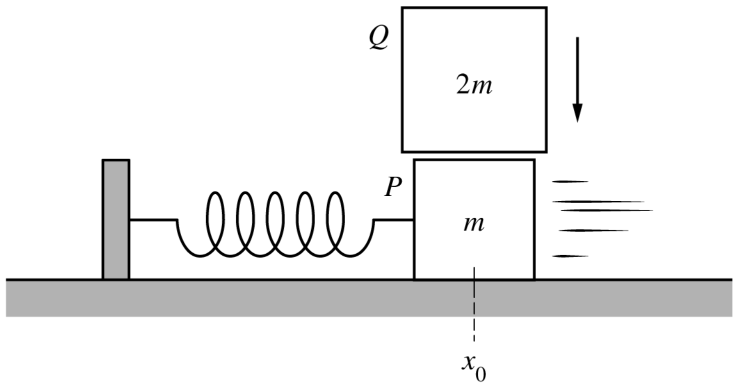
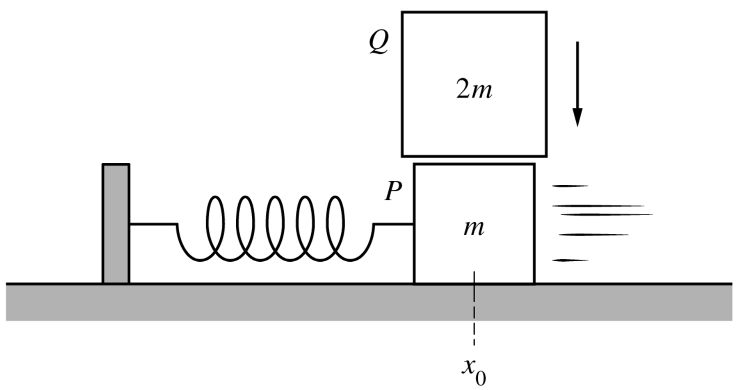



5. (7 points, suggested time 13 minutes)

Block $P$ of mass $m$ is on a horizontal, frictionless surface and is attached to a spring with spring constant $k$. The block is oscillating with period $T_P$ and amplitude $A_P$ about the spring's equilibrium position $x_0$. A second block $Q$ of mass $2m$ is then dropped from rest and lands on block $P$ at the instant it passes through the equilibrium position, as shown above. Block $Q$ immediately sticks to the top of block $P$, and the two-block system oscillates with period $T_{PQ}$ and amplitude $A_{PQ}$.

(a) Determine the numerical value of the ratio $T_{PQ}/T_P$.




(b) The figure is reproduced above. How does the amplitude of oscillation $A_{PQ}$ of the two-block system compare with the original amplitude $A_P$ of block $P$ alone?

____ $A_{PQ} < A_P$ ____ $A_{PQ} = A_P$ ____ $A_{PQ} > A_P$

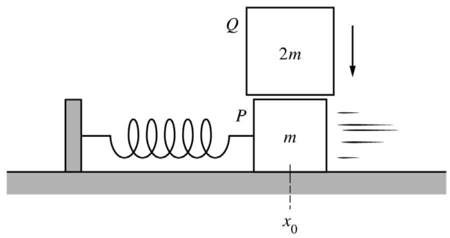
In a clear, coherent paragraph-length response that may also contain diagrams and/or equations, explain your reasoning.

**STOP**

**END OF EXAM**

## Question 5

**7 points total** | **Distribution of points**




Block $P$ of mass $m$ is on a horizontal, frictionless surface and is attached to a spring with spring constant $k$. The block is oscillating with period $T_P$ and amplitude $A_P$ about the spring's equilibrium position $x_0$. A second block $Q$ of mass $2m$ is then dropped from rest and lands on block $P$ at the instant it passes through the equilibrium position, as shown above. Block $Q$ immediately sticks to the top of block $P$, and the two-block system oscillates with period $T_{PQ}$ and amplitude $A_{PQ}$.

(a) LO / SP: 3.B.3.1 / 6.4
1 point

Determine the numerical value of the ratio $T_{PQ}/T_P$.

| | | |
|---|---|---|
| For a correct answer: $\sqrt{3}$ | | 1 point |

**Question 5 (continued)**

**Distribution of points**

(b) LO / SP: 3.B.3.1 / 6.4, 7.2; 3.B.3.4 / 2.2, 6.2; 4.C.1.1 / 1.4, 2.2; 4.C.1.2 / 6.4; 5.B.3.1/ 2.2, 6.4; 5.B.3.3 / 1.4, 2.2; 5.B.4.1 / 6.4, 7.2; 5.B.4.2 / 1.4, 2.1, 2.2; 5.D.2.1 / 6.4, 7.2
6 points

How does the amplitude of oscillation $A_{PQ}$ of the two-block system compare with the original amplitude $A_P$ of block $P$ alone?

____ $A_{PQ} < A_P$ ____ $A_{PQ} = A_P$ ____ $A_{PQ} > A_P$

In a clear, coherent paragraph-length response that may also contain diagrams and/or equations, explain your reasoning.

| | | |
|---|---|---|
| Correct answer: $A_{PQ} < A_P$ .<br>Note: The response is graded even if an incorrect selection is made. | | |
| For applying conservation of momentum to the collision | | 1 point |
| For correctly finding that the post-collision speed has decreased (or, for determining that $v_f = \frac{1}{3}v_i$) | | 1 point |
| Note: The first 2 points can be earned for stating that the collision is inelastic | | |
| For stating or implying that the system's kinetic energy has decreased (or, for calculating a lower final kinetic energy) | | 1 point |
| For stating or implying kinetic energy of blocks right after collision equals maximum potential energy of spring<br>OR<br>For stating or implying that the maximum potential energy equals the total mechanical energy just after the collision (Simply stating that $E_{tot} = \frac{1}{2}kA^2$ is sufficient.) | | 1 point |
| For stating or implying that maximum potential energy is reached when the displacement from equilibrium equals the amplitude of oscillation | | 1 point |
| Note: The previous 2 points can be earned in a single sentence in which one or both of the points is implicit. | | |
| For a logical, relevant, and internally consistent argument that addresses the required argument or question asked and follows the guidelines described in the published requirements for the paragraph-length response | | 1 point |

| | | |
|---|---|---|
| *Alternate solution for first 2 points in part (b):* | | |
| *A frictional force is exerted on block P by Q, so it slows down.*<br>*Note: The third point can be earned if energy loss is stated.* | | *2 points* |

## Question 5 (continued)

| | |
|---|---|
| Example:<br>The second block (*Q*) adds mass without changing the horizontal momentum of the two-block system. In effect, block *P* (mass *m*) becomes block *PQ* (mass 3*m*). This reduces the speed at equilibrium from $v_{max}$ to $v_{max}/3$ according to conservation of momentum. To see how this affects amplitude, we must analyze what happens to the maximum kinetic energy (*K*) of the oscillating mass:<br>$K_P = \frac{1}{2}mv_{max}^2 \qquad K_{PQ} = \frac{1}{2}(3m)\left(\frac{v_{max}}{3}\right)^2 = \frac{1}{6}mv_{max}^2 = \frac{1}{3}K_P$<br>Because the maximum *K* is reduced, this means the maximum potential energy in the spring is also reduced (to $1/3$ of its former value). Because amplitude is related to maximum potential energy $\left(U_{max} = \frac{1}{2}kA^2\right)$, the amplitude of block *PQ* is less than that of block *P*. | |

## Question 5 (continued)

Learning Objectives (LO)

**LO 3.B.3.1**: The student is able to predict which properties determine the motion of a simple harmonic oscillator and what the dependence of the motion is on those properties. [See Science Practices 6.4, 7.2]

**LO 3.B.3.4**: The student is able to construct a qualitative and/or a quantitative explanation of oscillatory behavior given evidence of a restoring force. [See Science Practices 2.2, 6.2]

**LO 4.C.1.1**: The student is able to calculate the total energy of a system and justify the mathematical routines used in the calculation of component types of energy within the system whose sum is the total energy. [See Science Practices 1.4, 2.1, 2.2]

**LO 4.C.1.2**: The student is able to predict changes in the total energy of a system due to changes in position and speed of objects or frictional interactions within the system. [See Science Practice 6.4]

**LO 5.B.3.1**: The student is able to describe and make qualitative and/or quantitative predictions about everyday examples of systems with internal potential energy. [See Science Practices 2.2, 6.4, 7.2]

**LO 5.B.3.3**: The student is able to apply mathematical reasoning to create a description of the internal potential energy of a system from a description or diagram of the objects and interactions in that system. [See Science Practices 1.4, 2.2]

**LO 5.B.4.1**: The student is able to describe and make predictions about the internal energy of systems. [See Science Practices 6.4, 7.2]

**LO 5.B.4.2**: The student is able to calculate changes in kinetic energy and potential energy of a system, using information from representations of that system. [See Science Practices 1.4, 2.1, 2.2]

**LO 5.D.2.1**: The student is able to qualitatively predict, in terms of linear momentum and kinetic energy, how the outcome of a collision between two objects changes depending on whether the collision is elastic or inelastic. [See Science Practices 6.4, 7.2]