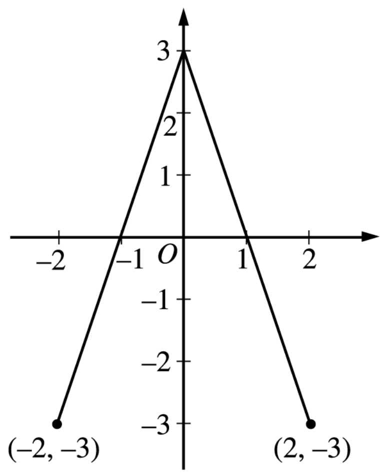
CALCULUS BC

SECTION II, Part B

Time—45 minutes

Number of problems—3

No calculator is allowed for these problems.



Graph of f

4. The graph of the function f shown above consists of two line segments. Let g be the function given by $g(x) = \int_0^x f(t)\,dt$.

   (a) Find $g(-1)$, $g'(-1)$, and $g''(-1)$.

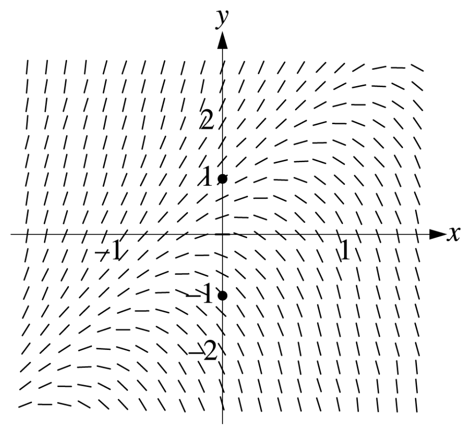
   (b) For what values of x in the open interval $(-2, 2)$ is g increasing? Explain your reasoning.

   (c) For what values of x in the open interval $(-2, 2)$ is the graph of g concave down? Explain your reasoning.

   (d) On the axes provided, sketch the graph of g on the closed interval $[-2, 2]$.
   (Note: The axes are provided in the pink test booklet only.)

5. Consider the differential equation $\frac{dy}{dx} = 2y - 4x$.

   (a) The slope field for the given differential equation is provided. Sketch the solution curve that passes through the point $(0, 1)$ and sketch the solution curve that passes through the point $(0, -1)$.
   (Note: Use the slope field provided in the pink test booklet.)



   (b) Let f be the function that satisfies the given differential equation with the initial condition $f(0) = 1$. Use Euler's method, starting at $x = 0$ with a step size of 0.1, to approximate $f(0.2)$. Show the work that leads to your answer.

   (c) Find the value of b for which $y = 2x + b$ is a solution to the given differential equation. Justify your answer.

   (d) Let g be the function that satisfies the given differential equation with the initial condition $g(0) = 0$. Does the graph of g have a local extremum at the point $(0, 0)$? If so, is the point a local maximum or a local minimum? Justify your answer.

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6. The Maclaurin series for the function f is given by

$$f(x) = \sum_{n=0}^{\infty} \frac{(2x)^{n+1}}{n+1} = 2x + \frac{4x^2}{2} + \frac{8x^3}{3} + \frac{16x^4}{4} + \cdots + \frac{(2x)^{n+1}}{n+1} + \cdots$$

   on its interval of convergence.

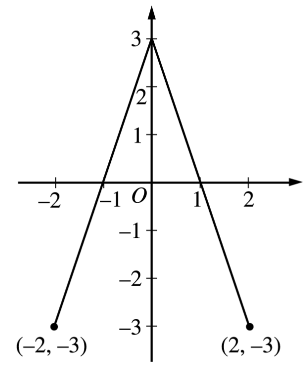
   (a) Find the interval of convergence of the Maclaurin series for f. Justify your answer.

   (b) Find the first four terms and the general term for the Maclaurin series for $f'(x)$.

   (c) Use the Maclaurin series you found in part (b) to find the value of $f'\left(-\frac{1}{3}\right)$.

END OF EXAMINATION

Question 4



Graph of f

The graph of the function f shown above consists of two line segments. Let g be the function given by $g(x) = \int_0^x f(t)\,dt$.

(a) Find $g(-1)$, $g'(-1)$, and $g''(-1)$.

(b) For what values of x in the open interval $(-2,2)$ is g increasing? Explain your reasoning.

(c) For what values of x in the open interval $(-2,2)$ is the graph of g concave down? Explain your reasoning.

(d) On the axes provided, sketch the graph of g on the closed interval $[-2,2]$.

(a) $g(-1) = \int_0^{-1} f(t)\,dt = -\int_{-1}^{0} f(t)\,dt = -\dfrac{3}{2}$

$g'(-1) = f(-1) = 0$

$g''(-1) = f'(-1) = 3$

$$3\begin{cases} 1: \ g(-1) \\ 1: \ g'(-1) \\ 1: \ g''(-1) \end{cases}$$

(b) g is increasing on $-1 < x < 1$ because $g'(x) = f(x) > 0$ on this interval.

$$2\begin{cases} 1: \ \text{interval} \\ 1: \ \text{reason} \end{cases}$$

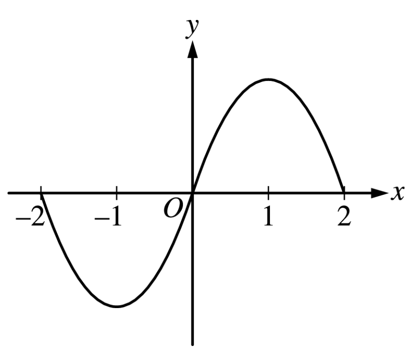
(c) The graph of g is concave down on $0 < x < 2$ because $g''(x) = f'(x) < 0$ on this interval.

or

because $g'(x) = f(x)$ is decreasing on this interval.

$$2\begin{cases} 1: \ \text{interval} \\ 1: \ \text{reason} \end{cases}$$

(d)



$$2\begin{cases} 1: \ g(-2) = g(0) = g(2) = 0 \\ 1: \ \text{appropriate increasing/decreasing} \\ \quad\ \ \text{and concavity behavior} \\ \quad\ \ < -1 > \text{vertical asymptote} \end{cases}$$