3. The depth from the surface of Earth to a refracting layer beneath the surface can be estimated using methods developed by seismologists. One method is based on the time required for vibrations to travel from a distant explosion to a receiving point. The depth measurement (M) is the sum of the true depth (D) and the random measurement error (E). That is, $M = D + E$. The measurement error (E) is assumed to be normally distributed with mean 0 feet and standard deviation 1.5 feet.

   (a) If the true depth at a certain point is 2 feet, what is the probability that the depth measurement will be negative?

   (b) Suppose three independent depth measurements are taken at the point where the true depth is 2 feet. What is the probability that <u>at least one</u> of these measurements will be negative?

   (c) What is the probability that the <u>mean</u> of the three independent depth measurements taken at the point where the true depth is 2 feet will be negative?

4. Patients with heart-attack symptoms arrive at an emergency room either by ambulance or self-transportation provided by themselves, family, or friends. When a patient arrives at the emergency room, the time of arrival is recorded. The time when the patient's diagnostic treatment begins is also recorded.

   An administrator of a large hospital wanted to determine whether the mean wait time (time between arrival and diagnostic treatment) for patients with heart-attack symptoms differs according to the mode of transportation. A random sample of 150 patients with heart-attack symptoms who had reported to the emergency room was selected. For each patient, the mode of transportation and wait time were recorded. Summary statistics for each mode of transportation are shown in the table below.

Mode of Transportation	Sample Size	Mean Wait Time (in minutes)	Standard Deviation of Wait Times (in minutes)
Ambulance	77	6.04	4.30
Self	73	8.30	5.16

   (a) Use a 99 percent confidence interval to estimate the difference between the mean wait times for ambulance-transported patients and self-transported patients at this emergency room.

   (b) Based only on this confidence interval, do you think the difference in the mean wait times is statistically significant? Justify your answer.

5. A biologist is interested in studying the effect of growth-enhancing nutrients and different salinity (salt) levels in water on the growth of shrimps. The biologist has ordered a large shipment of young tiger shrimps from a supply house for use in the study. The experiment is to be conducted in a laboratory where 10 tiger shrimps are placed randomly into each of 12 similar tanks in a controlled environment. The biologist is planning to use 3 different growth-enhancing nutrients (A, B, and C) and two different salinity levels (low and high).

   (a) List the treatments that the biologist plans to use in this experiment.

   (b) Using the treatments listed in part (a), describe a completely randomized design that will allow the biologist to compare the shrimps' growth after 3 weeks.

   (c) Give one statistical advantage to having only tiger shrimps in the experiment. Explain why this is an advantage.

   (d) Give one statistical disadvantage to having only tiger shrimps in the experiment. Explain why this is a disadvantage.

Question 4

Intent of Question

The primary goals of this question are to evaluate a student's ability to: (1) identify and compute an appropriate confidence interval, after checking the necessary conditions; (2) interpret the interval in the context of the question; and (3) use the confidence interval to conduct an appropriate test of significance.

Solution

Part (a):

Step 1: Identifies the appropriate confidence interval by name or formula and checks appropriate conditions.

Two sample *t* interval for $\mu_A - \mu_S$, the difference in mean waiting times, or

$(\bar{x}_A - \bar{x}_S) \pm t^*_{df}\sqrt{\frac{s_A^2}{n_A} + \frac{s_S^2}{n_S}}$ [See the next page for possible values of *df*.]

Conditions: 1. Independent random samples
2. Large samples or normal population distributions

One sample of 150 patients divided into two groups after sampling does not meet the condition of two independent random samples with fixed sample sizes. Nevertheless, it is reasonable to assume that mode of transportation splits the patients into two independent groups. Secondly, use of the two sample *t* interval is reasonable because each sample size is large (e.g., $n_A = 77 > 30$ and $n_S = 73 > 30$).

Alternatively, we could assume that the waiting times are (at least approximately) normally distributed, but we have no way to check this assumption with the information provided.

Step 2: Correct Mechanics

Degrees of freedom = $\min\{(77-1),(73-1)\} = 72$.

$(6.04 - 8.30) \pm 2.6459\sqrt{\frac{4.30^2}{77} + \frac{5.16^2}{73}}$

$-2.26 \pm 2.6459 \cdot (0.7777)$

-2.26 ± 2.0577

$(-4.3177, -0.2023)$

Step 3: Interpretation

Based on this sample, we are 99 percent confident that the true difference in the populations' mean waiting times (ambulance − self) is between −4.3177 minutes and −0.2023 minutes.

Equivalently,

With 99 percent confidence, the true mean wait time for those who arrive by ambulance is shorter than those who are self transported by somewhere between 0.2 and 4.3 minutes.

Part (b):

Since zero is not in the 99 percent confidence interval of plausible values for the true difference in means, we can reject $H_0 : \mu_A - \mu_S = 0$ in favor of the alternative $H_a : \mu_A - \mu_S \neq 0$ at the $\alpha = .01$ significance level.

Question 4 (continued)

Thus, we have statistically significant evidence that there is a difference in the mean wait times for the two groups.

Scoring

Part (a) is scored according to the number of correct steps. Each step is scored as essentially correct (E) or incorrect (I). Part (b) is scored as essentially correct (E) or incorrect (I).

Step 1: Identification of method and check of conditions.
A score of essentially correct (E) requires each of the following; otherwise, the score is I.

- A correct confidence interval procedure should be named or a correct formula given.
- Normality must be assessed by either checking that *EACH* sample size is sufficiently large (e.g., >30) OR by stating that each population needs to be normally distributed but that the data are not given to check that.
- It must be stated that the two groups are assumed to be independent random samples.

Step 2: Mechanics
An identifiable minor error in Step 2 will not necessarily change a score from essentially correct to incorrect. The following confidence intervals are all scored E.

Solutions to Step 2			
Procedure	**d.f.**	**t***	**Confidence Interval**
Unequal Variances	140.37	2.61140	(–4.2910, –0.2291)
Large samples	∞	2.576	(–4.26, –0.26)
Pooled variance	148	2.6095	(–4.2797, –0.2404)
Conservative Approach	72	2.6459	(–4.3177, –0.2023)
Unequal Variances	100 (table)	2.626	(–4.302, –0.218)
Unequal Variances	1000 (table)	2.581	(–4.267, –0.253)
Conservative Approach	60 (table)	2.660	(–4.329, –0.191)
Conservative Approach	80 (table)	2.639	(–4.312, –0.208)

Step 3: Interpretation of Confidence Interval
For a score of essentially correct (E), the interpretation must be about a difference of population means AND be in context AND mention the 99 percent level of confidence.

Note: If the interpretation is correct and also explains the meaning of "confidence level," then that extra explanation must be correct for a score of E. If it is not, the score is I.
Note: The correct interpretation of the confidence interval for Step 3 of part (a) may be found in part (b).

Part (b) is essentially correct (E) if the student concludes that the mean times differ because zero is not contained in the 99 percent confidence interval. The confidence level or the significance level must be stated and the conclusion must be stated in context.

Part (b) is incorrect (I) if the student only says the mean wait time for patients transported via ambulance is significantly lower without indicating the significance level adjustment needed for a one-sided test.

Question 4 (continued)

4 Complete Response

All three steps of the confidence interval in part (a) are essentially correct and part (b) is essentially correct.

3 Substantial Response

All three steps of the confidence interval in part (a) are essentially correct and part (b) is incorrect.
OR
Two steps of the confidence interval in part (a) are essentially correct and part (b) is essentially correct.

2 Developing Response

Two steps of the confidence interval in part (a) are essentially correct and part (b) is incorrect.
OR
One step of the confidence interval in part (a) is essentially correct and part (b) is essentially correct.

1 Minimal Response

One step of the confidence interval in part (a) is essentially correct and part (b) is incorrect.
OR
Part (b) is essentially correct.