Mech 2.

A rubber ball of mass m is dropped from a cliff. As the ball falls, it is subject to air drag (a resistive force caused by the air). The drag force on the ball has magnitude bv^2, where b is a constant drag coefficient and v is the instantaneous speed of the ball. The drag coefficient b is directly proportional to the cross-sectional area of the ball and the density of the air and does not depend on the mass of the ball. As the ball falls, its speed approaches a constant value called the terminal speed.

(a) On the figure below, draw and label all the forces on the ball at some instant before it reaches terminal speed.

(b) State whether the magnitude of the acceleration of the ball of mass m increases, decreases, or remains the same as the ball approaches terminal speed. Explain.

(c) Write, but do NOT solve, a differential equation for the instantaneous speed v of the ball in terms of time t, the given quantities, and fundamental constants.

(d) Determine the terminal speed v_t in terms of the given quantities and fundamental constants.

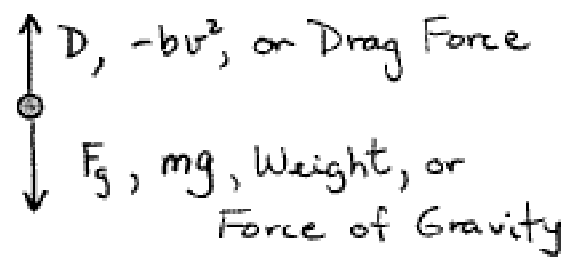
(e) Determine the energy dissipated by the drag force during the fall if the ball is released at height h and reaches its terminal speed before hitting the ground, in terms of the given quantities and fundamental constants.

GO ON TO THE NEXT PAGE.

2000 Physics C Solutions **Distribution of points**

Mech. 2 (15 points)

(a) 3 points



For a vector arrow pointing downward	1 point
For a vector arrow pointing upward	1 point
For correct force labels on both vectors	1 point
For any extra vectors drawn, deduct 1 point	

(b) 3 points

For indicating that the acceleration decreases	1 point
For a correct explanation that includes a correct mention of forces.	2 points

For example, as the ball approaches terminal speed, the velocity increases, so the drag force increases and gets closer in magnitude to the gravitational force. The resultant force, which is the difference between the gravitational and drag forces, gets smaller, and since it is proportional to the acceleration, the acceleration decreases.

Partial credit of 1 point given for only a statement including a basic definition of terminal velocity (e.g., at terminal velocity v = constant, so a must decrease from 9.8 m/s^2 to zero)

(c) 2 points

For an expression for the resultant force on the ball — 1 point

$F = mg - bv^2$

Since $F = ma = m\dfrac{dv}{dt}$, then $m\dfrac{dv}{dt} = mg - bv^2$

For a correct differential equation — 1 point

$\dfrac{dv}{dt} = g - \dfrac{b}{m}v^2$

Students did not need to use the convention + and – for up and down, respectively, but they did have to be consistent in their sign notation for credit. The integral form of the differential equation was also acceptable.

Distribution of points

Mech. 2 (continued)

(d) 3 points

For recognition that acceleration is zero at terminal speed — 1 point
For setting the drag force equal to the gravitational force — 1 point
$mg = bv_t^2$
For a correct solution for v_t — 1 point
$v_t = \sqrt{\frac{mg}{b}}$

Full credit also given for writing answer only with no other work shown

(e) 4 points

For a correct statement of work-energy, recognizing that the energy dissipated by the drag force is equal to the initial energy minus the final energy — 1 point
For correct recognition of both initial potential energy mgh and final kinetic energy $\frac{1}{2}mv_t^2$ — 1 point
$\Delta E = mgh - \frac{1}{2}mv_t^2$
For correct substitution of v_t from part (d) — 1 point
$\Delta E = mgh - \frac{1}{2}m\left(\frac{mg}{b}\right)$
For correct answer — 1 point
$\Delta E = mg\left(h - \frac{m}{2b}\right)$

Alternate partial solution (for maximum credit of 2 points) — *(Alternate points)*
For a correct integral for work — *(1 point)*
$W = \int P\,dt$ OR $W = \int F\,dx$
For correct substitutions for P or F — *(1 point)*
$W = \int bv^3\,dt$ OR $W = \int bv^2\,dx$ OR $W = \int kv^2\,dx$