5. Die A has four 9's and two 0's on its faces. Die B has four 3's and two 11's on its faces. When either of these dice is rolled, each face has an equal chance of landing on top. Two players are going to play a game. The first player selects a die and rolls it. The second player rolls the remaining die. The winner is the player whose die has the higher number on top.

   a. Suppose you are the first player and you want to win the game. Which die would you select? Justify your answer.

   b. Suppose the player using die A receives 45 tokens each time he or she wins the game. How many tokens must the player using die B receive each time he or she wins in order for this to be a fair game? Explain how you found your answer.

      (A fair game is one in which the player using die A and the player using die B both end up with the same number of tokens in the long run.)

GO ON TO THE NEXT PAGE

STATISTICS

SECTION II

Part B

Question 6

Spend about 25 minutes on this part of the exam.

Percent of Section II grade—25

6. Researchers want to see whether training increases the capability of people to correctly predict outcomes of coin tosses. Each of twenty people is asked to predict the outcome (heads or tails) of 100 independent tosses of a fair coin. After training, they are retested with a new set of 100 tosses. (All 40 sets of 100 tosses are independently generated.) Since the coin is fair, the probability of a correct guess by chance is 0.5 on each toss. The numbers correct for each of the 20 people were as follows.

Score Before Training (number correct)	Score After Training (number correct)
46	61
48	62
50	53
54	46
54	50
54	52
54	53
54	59
54	60
54	61
55	55
56	59
57	55
58	50
58	56
61	58
61	64
63	57
64	61
65	54
Sum 1,120	Sum 1,126

To answer the following questions, you may want to enter these data into your calculator. As a check that you have entered the data correctly, the sum of the first column is 1,120 and the sum of the second column is 1,126.

a. Do the data suggest that after training people can correctly predict coin toss outcomes better than the 50 percent expected by chance guessing alone?

   Give appropriate statistical evidence to support your conclusion.

GO ON TO THE NEXT PAGE

AP® STATISTICS
1999 SCORING GUIDELINES

Question 5

Solution

Possible Outcomes

Die A	Die B	Winner	Prob
9	3	A	(2/3)(2/3) = 4/9
9	11	B	(2/3)(1/3) = 2/9
0	3	B	(1/3)(2/3) = 2/9
0	11	B	(1/3)(1/3) = 1/9

OR

	DIE A						
DIE B		0	0	9	9	9	9
	3	B	B	A	A	A	A
	3	B	B	A	A	A	A
	3	B	B	A	A	A	A
	3	B	B	A	A	A	A
	11	B	B	B	B	B	B
	11	B	B	B	B	B	B

Winner	Prob
A	16/36 = 4/9
B	20/36 = 5/9

a. Choose die B, because the probability of winning is higher (5/9 compared to 4/9 for die A)

b. Let X be the number of tokens the player using die B should receive. For the game to be fair, we need

   $45(4/9) = X(5/9)$

   Solving this equation for X gives X = 36. Player B should receive 36 tokens.

Question 5 (cont.)

Points

4 Complete Response
Part (a) and Part (b) must both be correct.

3 Substantial Response
Part (a) or Part (b) is correct and the other part is partially correct.

2 Developing Response
Part (a) or Part (b) is correct and the other part is incorrect.
OR
Part (a) and Part (b) are both partially correct.

1 Minimal Response
Part (a) or Part (b) is partially correct.

Correct Responses
Part (a)

- Correctly reasons*, using a probability argument, that die B should be chosen.

Part (b)

- Gives a correct answer* to part (b) and gives a complete explanation of how the answer was determined.

Partially Correct Responses
Part (a)

- Chooses the correct die, has a correct approach, but the explanation is incomplete.
- Gives an incorrect answer to part (a) based on a correct approach.
- Chooses the incorrect die, A, based on an expected value argument, but correctly computes expected values of 6 for die A and 5.667 for die B.

Part (b)

- Arrives at an answer, has a correct approach, but with an incomplete explanation. (A common example of this is not demonstrating the use of probabilities.)

Question 5 (cont.)

Incorrect Responses
Part (a)

- Uses an expected value argument but gets the wrong expected values.
- Choosing die A because the probability of rolling a 9 on Die A = 2/3 and the probability of rolling an 11 on die B = 1/3.
- An answer without an explanation receives no credit.

Part (b)

- An answer without an explanation receives no credit.

Notes:
- Papers with minor errors, such as transposition errors or arithmetic errors, that give reasonable answers, should not be penalized.
- Another possibility is that the student takes a simulation approach to the problem. It is possible for this approach to earn up to a score of 4. Papers taking this approach should be referred to a table leader.