

## 1999 CALCULUS AB

$t$ (hours)	$R(t)$ (gallons per hour)
0	9.6
3	10.4
6	10.8
9	11.2
12	11.4
15	11.3
18	10.7
21	10.2
24	9.6

3. The rate at which water flows out of a pipe, in gallons per hour, is given by a differentiable function  $R$  of time  $t$ . The table above shows the rate as measured every 3 hours for a 24-hour period.
- (a) Use a midpoint Riemann sum with 4 subdivisions of equal length to approximate  $\int_0^{24} R(t)dt$ . Using correct units, explain the meaning of your answer in terms of water flow.
- (b) Is there some time  $t$ ,  $0 < t < 24$ , such that  $R'(t) = 0$ ? Justify your answer.
- (c) The rate of water flow  $R(t)$  can be approximated by  $Q(t) = \frac{1}{79}(768 + 23t - t^2)$ .  
Use  $Q(t)$  to approximate the average rate of water flow during the 24-hour time period.  
Indicate units of measure.
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4. Suppose that the function  $f$  has a continuous second derivative for all  $x$ , and that  $f(0) = 2$ ,  $f'(0) = -3$ , and  $f''(0) = 0$ . Let  $g$  be a function whose derivative is given by  $g'(x) = e^{-2x}(3f(x) + 2f'(x))$  for all  $x$ .
- (a) Write an equation of the line tangent to the graph of  $f$  at the point where  $x = 0$ .
  - (b) Is there sufficient information to determine whether or not the graph of  $f$  has a point of inflection when  $x = 0$ ? Explain your answer.
  - (c) Given that  $g(0) = 4$ , write an equation of the line tangent to the graph of  $g$  at the point where  $x = 0$ .
  - (d) Show that  $g''(x) = e^{-2x}(-6f(x) - f'(x) + 2f''(x))$ . Does  $g$  have a local maximum at  $x = 0$ ? Justify your answer.
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