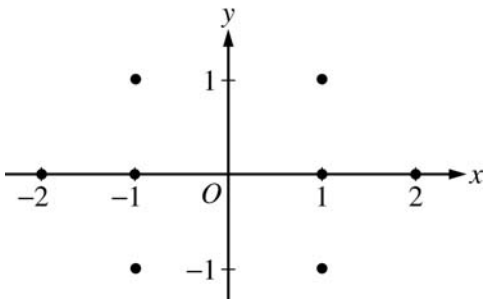
5. Consider the differential equation $\frac{dy}{dx} = \frac{1+y}{x}$, where $x \neq 0$.

   (a) On the axes provided, sketch a slope field for the given differential equation at the eight points indicated.
   (Note: Use the axes provided in the pink exam booklet.)



   (b) Find the particular solution $y = f(x)$ to the differential equation with the initial condition $f(-1) = 1$ and state its domain.

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6. The twice-differentiable function f is defined for all real numbers and satisfies the following conditions:

$$f(0) = 2, \quad f'(0) = -4, \text{ and } f''(0) = 3.$$

   (a) The function g is given by $g(x) = e^{ax} + f(x)$ for all real numbers, where a is a constant. Find $g'(0)$ and $g''(0)$ in terms of a. Show the work that leads to your answers.

   (b) The function h is given by $h(x) = \cos(kx) f(x)$ for all real numbers, where k is a constant. Find $h'(x)$ and write an equation for the line tangent to the graph of h at $x = 0$.

WRITE ALL WORK IN THE PINK EXAM BOOKLET.

END OF EXAM

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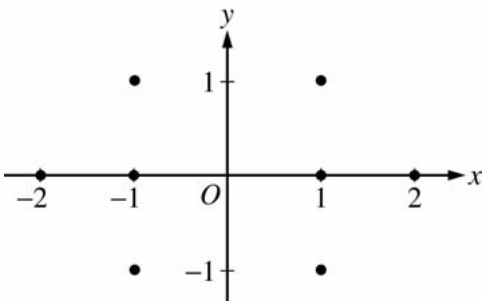
Question 5

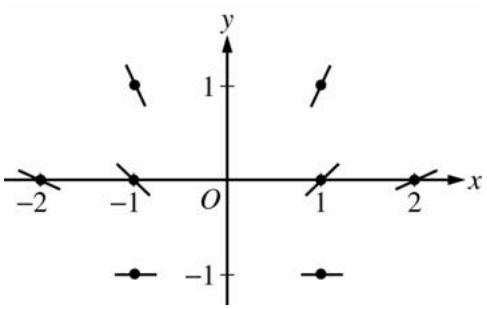
Consider the differential equation $\frac{dy}{dx} = \frac{1+y}{x}$, where $x \neq 0$.

(a) On the axes provided, sketch a slope field for the given differential equation at the eight points indicated.
(Note: Use the axes provided in the pink exam booklet.)



(b) Find the particular solution $y = f(x)$ to the differential equation with the initial condition $f(-1) = 1$ and state its domain.

(a)



2 : sign of slope at each point and relative steepness of slope lines in rows and columns

(b) $\frac{1}{1+y}\,dy = \frac{1}{x}\,dx$

$\ln|1+y| = \ln|x| + K$

$|1+y| = e^{\ln|x|+K}$

$1+y = C|x|$

$2 = C$

$1+y = 2|x|$

$y = 2|x| - 1$ and $x < 0$

or

$y = -2x - 1$ and $x < 0$

7 :
- 6 :
  - 1 : separates variables
  - 2 : antiderivatives
  - 1 : constant of integration
  - 1 : uses initial condition
  - 1 : solves for y

  Note: max $3/6$ [1-2-0-0-0] if no constant of integration

  Note: $0/6$ if no separation of variables
- 1 : domain