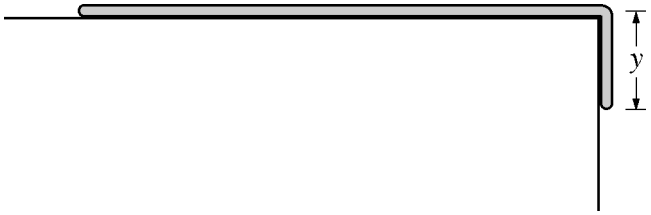
Mech. 3.

A block of mass $M/2$ rests on a frictionless horizontal table, as shown above. It is connected to one end of a string that passes over a massless pulley and has another block of mass $M/2$ hanging from its other end. The apparatus is released from rest.

(a) Derive an expression for the speed v_h of the hanging block as a function of the distance d it descends.

Now the block and pulley system is replaced by a uniform rope of length L and mass M, with one end of the rope hanging slightly over the edge of the frictionless table. The rope is released from rest, and at some time later there is a length y of rope hanging over the edge, as shown below. Express your answers to parts (b), (c), and (d) in terms of y, L, M, and fundamental constants.



(b) Determine an expression for the force of gravity on the hanging part of the rope as a function of y.

(c) Derive an expression for the work done by gravity on the rope as a function of y, assuming y is initially zero.

(d) Derive an expression for the speed v_r of the rope as a function of y.

(e) The hanging block and the right end of the rope are each allowed to fall a distance L (the length of the rope). The string is long enough that the sliding block does not hit the pulley. Indicate whether v_h from part (a) or v_r from part (d) is greater after the block and the end of the rope have traveled this distance.

____ v_h is greater. ____ v_r is greater. ____ The speeds are equal.

Justify your answer.

END OF EXAM

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Question 3

15 points total **Distribution of points**

(a) 4 points

For an indication of conservation of energy 1 point

$|\Delta U| = |\Delta K|$

$mgh = \frac{1}{2}mv^2$

The speed of both blocks is v_h.

For substituting $M/2$ into the expression for U 1 point

For substituting d for h in the expression for U 1 point

For substituting the sum of the masses, $\frac{M}{2} + \frac{M}{2}$, into the expression for K 1 point

$\frac{M}{2}gd = \frac{1}{2}\left(\frac{M}{2} + \frac{M}{2}\right)v_h^2$

$\frac{M}{2}gd = \frac{1}{2}Mv_h^2$

$v_h = \sqrt{gd}$

Alternate Solution *Alternate Points*

For an indication that Newton's second law applies *1 point*

$F_{net} = ma$

$\frac{M}{2}g = 2\left(\frac{M}{2}\right)a$

For solving for acceleration *1 point*

$a = \frac{g}{2}$

For selecting correct kinematics equation(s) *1 point*

$v^2 = v_0^2 + 2a(x - x_0)$ OR $x = \frac{1}{2}at^2$ and $v = at$

For substituting d for the vertical + distance *1 point*

$v_h^2 = 2a(d) = 2\frac{g}{2}(d)$ OR $d = \frac{1}{2}\frac{g}{2}t^2$ and $v_h = \frac{g}{2}t$ (*and combining by eliminating t*)

$v_h = \sqrt{gd}$

(b) 2 points

$F_g = mg$

For a correct expression for the force 2 points

$F_g = \frac{Mg}{L}y$

Note: Since the stem states "determine," no work was necessary to earn these points.
No partial credit was awarded for this part.

Question 3 (continued)

Distribution of points

(c) 3 points

For a correct integral expression for work. (If the nonintegral form of work was presented, no further work on this part was scored.) 1 point

$W = \int F\, dy$

For substituting F from part (b) into the integral 1 point

$W = \int \frac{Mg}{L} y\, dy$

$W = \frac{Mg}{L} \int y\, dy$

For correct integration 1 point

$W = \frac{Mg}{L} \frac{1}{2} y^2$

$W = \frac{Mg}{2L} y^2$

Alternate Solution *Alternate Points*

For a correct relationship between work and potential energy *1 point*

$W = -\Delta U$

$W = mg\, \Delta h_{cm}$

For substituting the expression for force of gravity from (b) *1 point*

For substituting $y/2$ *for* Δh_{cm} *1 point*

$W = \left(\frac{M}{L} y\right) g \frac{y}{2}$

$W = \frac{Mg}{2L} y^2$

(d) 3 points

For an indication of the work-energy relationship 1 point

$W = \Delta K = \frac{1}{2} m v^2$

For substituting the expression for W from part (c) 1 point

For substituting M into expression for ΔK 1 point

$\frac{Mg}{2L} y^2 = \frac{1}{2} M v_r^2$

$v_r = \sqrt{g/L}\, y$

Note: An alternate solution using Newton's second law and kinematics was also possible.

Question 3 (continued)

Distribution of points

(e) 3 points

For indicating that the speeds are equal 1 point
For a complete and correct justification, conceptual or symbolic 2 points

Example 1: Substituting L for d and y in the equations $v_h = \sqrt{gd}$ and $v_r = \sqrt{gy^2/L}$, respectively, yields $\sqrt{gL}$ in both cases.

Example 2: For the blocks, a mass of $M/2$ falls a distance L. For the rope, the center of mass of a mass of M falls a distance of $L/2$. The same amount of potential energy becomes kinetic energy. Equal total masses gaining equal kinetic energies means they acquire equal speeds.

Notes:

- Since this part could be answered without making reference to the rest of the problem, it was scored independently.
- A correct but incomplete justification was awarded 1 point.