1998 Calculus BC Free-Response Questions

CALCULUS BC

Section II

Time—1 hour and 30 minutes

Number of problems—6

Percent of total grade—50

A GRAPHING CALCULATOR IS REQUIRED FOR SOME PROBLEMS OR PARTS OF PROBLEMS ON THIS SECTION OF THE EXAMINATION.

REMEMBER TO SHOW YOUR SETUPS AS DESCRIBED IN THE GENERAL INSTRUCTIONS.

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1. Let R be the region in the first quadrant bounded by the graph of $y = 8 - x^{\frac{3}{2}}$, the x–axis, and the y–axis.
   (a) Find the area of the region R.
   (b) Find the volume of the solid generated when R is revolved about the x–axis.
   (c) The vertical line $x = k$ divides the region R into two regions such that when these two regions are revolved about the x–axis, they generate solids with equal volumes. Find the value of k.

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2. Let f be the function given by $f(x) = 2xe^{2x}$.
   (a) Find $\lim_{x \to -\infty} f(x)$ and $\lim_{x \to \infty} f(x)$.
   (b) Find the absolute minimum value of f. Justify that your answer is an absolute minimum.
   (c) What is the range of f?
   (d) Consider the family of functions defined by $y = bxe^{bx}$, where b is a nonzero constant. Show that the absolute minimum value of bxe^{bx} is the same for all nonzero values of b.

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1998 Calculus BC Scoring Guidelines

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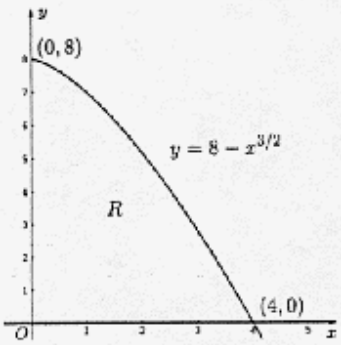
   (a) Find the area of the region R.
   (b) Find the volume of the solid generated when R is revolved about the x-axis.
   (c) The vertical line $x = k$ divides the region R into two regions such that when these two regions are revolved about the x-axis, they generate solids with equal volumes. Find the value of k.

(a)



$$A = \int_0^4 \left(8 - x^{3/2}\right) dx$$

$$= 8x - \frac{2}{5}x^{5/2}\Big|_0^4 = 32 - \frac{64}{5} = \frac{96}{5} = 19.2$$

3 { 2: integral; 1: integrand; 1: limits; 1: answer }

(b) $$V = \pi \int_0^4 \left(8 - x^{3/2}\right)^2 dx$$

$$= \frac{576\pi}{5} = 115.2\pi \approx 361.911$$

3 { 2: integral; 1: integrand; 1: limits and constant; 1: answer }

(c) $$\pi \int_0^k \left(8 - x^{3/2}\right)^2 dx = \frac{115.2\pi}{2}$$

[or $$\pi \int_0^k \left(8 - x^{3/2}\right)^2 dx = \pi \int_k^4 \left(8 - x^{3/2}\right)^2 dx$$]

$$\int_0^k \left(8 - x^{3/2}\right)^2 dx = 57.6$$

$$\int_0^k (64 - 16x^{3/2} + x^3)\, dx = 57.6$$

$$64k - \frac{32}{5}k^{5/2} + \frac{k^4}{4} = 57.6$$

$k \approx 0.995$ or 0.994

3 { 1: integral with k in limits; 1: equates volumes; 1: answer }

Note: 0/1 for answer in each part if no setup points earned

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