

2001 AP[®] CALCULUS AB FREE-RESPONSE QUESTIONS

CALCULUS AB SECTION II, Part B

Time—45 minutes

Number of problems—3

No calculator is allowed for these problems.

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4. Let h be a function defined for all $x \neq 0$ such that $h(4) = -3$ and the derivative of h is given by

$$h'(x) = \frac{x^2 - 2}{x} \text{ for all } x \neq 0.$$

- (a) Find all values of x for which the graph of h has a horizontal tangent, and determine whether h has a local maximum, a local minimum, or neither at each of these values. Justify your answers.
- (b) On what intervals, if any, is the graph of h concave up? Justify your answer.
- (c) Write an equation for the line tangent to the graph of h at $x = 4$.
- (d) Does the line tangent to the graph of h at $x = 4$ lie above or below the graph of h for $x > 4$? Why?

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5. A cubic polynomial function f is defined by

$$f(x) = 4x^3 + ax^2 + bx + k$$

where a , b , and k are constants. The function f has a local minimum at $x = -1$, and the graph of f has a point of inflection at $x = -2$.

- (a) Find the values of a and b .
- (b) If $\int_0^1 f(x) dx = 32$, what is the value of k ?

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6. The function f is differentiable for all real numbers. The point $\left(3, \frac{1}{4}\right)$ is on the graph of $y = f(x)$, and the slope at each point (x, y) on the graph is given by $\frac{dy}{dx} = y^2(6 - 2x)$.

- (a) Find $\frac{d^2y}{dx^2}$ and evaluate it at the point $\left(3, \frac{1}{4}\right)$.
- (b) Find $y = f(x)$ by solving the differential equation $\frac{dy}{dx} = y^2(6 - 2x)$ with the initial condition $f(3) = \frac{1}{4}$.

END OF EXAMINATION

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Question 5

A cubic polynomial function f is defined by

$$f(x) = 4x^3 + ax^2 + bx + k$$

where a , b , and k are constants. The function f has a local minimum at $x = -1$, and the graph of f has a point of inflection at $x = -2$.

(a) Find the values of a and b .

(b) If $\int_0^1 f(x) dx = 32$, what is the value of k ?

(a) $f'(x) = 12x^2 + 2ax + b$

$$f''(x) = 24x + 2a$$

$$f'(-1) = 12 - 2a + b = 0$$

$$f''(-2) = -48 + 2a = 0$$

$$a = 24$$

$$b = -12 + 2a = 36$$

$$5 : \left\{ \begin{array}{l} 1 : f'(x) \\ 1 : f''(x) \\ 1 : f'(-1) = 0 \\ 1 : f''(-2) = 0 \\ 1 : a, b \end{array} \right.$$

(b) $\int_0^1 (4x^3 + 24x^2 + 36x + k) dx$

$$= x^4 + 8x^3 + 18x^2 + kx \Big|_{x=0}^{x=1} = 27 + k$$

$$27 + k = 32$$

$$k = 5$$

$$4 : \left\{ \begin{array}{l} 2 : \text{antidifferentiation} \\ \quad < -1 > \text{ each error} \\ 1 : \text{expression in } k \\ 1 : k \end{array} \right.$$