3. At an archaeological site that was an ancient swamp, the bones from 20 brontosaur skeletons have been unearthed. The bones do not show any sign of disease or malformation. It is thought that these animals wandered into a deep area of the swamp and became trapped in the swamp bottom. The 20 left femur bones (thigh bones) were located and 4 of these left femurs are to be randomly selected without replacement for DNA testing to determine gender.

   (a) Let X be the number out of the 4 selected left femurs that are from males. Based on how these bones were sampled, explain why the probability distribution of X is not binomial.

   (b) Suppose that the group of 20 brontosaurs whose remains were found in the swamp had been made up of 10 males and 10 females. What is the probability that all 4 in the sample to be tested are male?

   (c) The DNA testing revealed that all 4 femurs tested were from males. Based on this result and your answer from part (b), do you think that males and females were equally represented in the group of 20 brontosaurs stuck in the swamp? Explain.

   (d) Is it reasonable to generalize your conclusion in part (c) pertaining to the group of 20 brontosaurs to the population of all brontosaurs? Explain why or why not.

4. Two antibiotics are available as treatment for a common ear infection in children.

   - Antibiotic A is known to effectively cure the infection 60 percent of the time. Treatment with antibiotic A costs $50.
   - Antibiotic B is known to effectively cure the infection 90 percent of the time. Treatment with antibiotic B costs $80.

   The antibiotics work independently of one another. Both antibiotics can be safely administered to children. A health insurance company intends to recommend one of the following two plans of treatment for children with this ear infection.

   - Plan I: Treat with antibiotic A first. If it is not effective, then treat with antibiotic B.
   - Plan II: Treat with antibiotic B first. If it is not effective, then treat with antibiotic A.

   (a) If a doctor treats a child with an ear infection using plan I, what is the probability that the child will be cured?

   If a doctor treats a child with an ear infection using plan II, what is the probability that the child will be cured?

   (b) Compute the expected cost per child when plan I is used for treatment.

   Compute the expected cost per child when plan II is used for treatment.

   (c) Based on the results in parts (a) and (b), which plan would you recommend?

   Explain your recommendation.

5. A rural county hospital offers several health services. The hospital administrators conducted a poll to determine whether the residents' satisfaction with the available services depends on their gender. A random sample of 1,000 adult county residents was selected. The gender of each respondent was recorded and each was asked whether he or she was satisfied with the services offered by the hospital. The resulting data are shown in the table below.

	Male	Female	Total
Satisfied	384	416	800
Not Satisfied	80	120	200
Total	464	536	1,000

(a) Using a significance level of 0.05, conduct an appropriate test to determine if, for adult residents of this county, there is an association between gender and whether or not they were satisfied with services offered by the hospital.

(b) Is $\frac{800}{1,000}$ a reasonable estimate for the proportion of all adult county residents who are satisfied with the services offered by this hospital? Explain why or why not.

GO ON TO THE NEXT PAGE.

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2004 SCORING GUIDELINES

Question 4

Solution

Part (a):

Let A be the event "antibiotic A works."
Let B be the event "antibiotic B works."

The probability that a child will be cured with Plan I is:

$$\begin{aligned} P(\text{Cure}_{\text{I}}) &= P(\text{A}) + P(\text{not A})P(\text{B}) \\ &= 0.6 + (0.4 \times 0.9) \\ &= 0.96 \end{aligned}$$

The probability that a child will be cured with Plan II is:

$$\begin{aligned} P(\text{Cure}_{\text{II}}) &= P(\text{B}) + P(\text{not B})P(\text{A}) \\ &= 0.9 + (0.1 \times 0.6) \\ &= 0.96 \end{aligned}$$

Part (b):

Treatment with antibiotic A costs \$50, and treatment with antibiotic B costs \$80.

The expected cost per child when Plan I is used for treatment is:

$$\begin{aligned} E(\text{Cost}_{\text{I}}) &= \$50 \times 0.6 + \$130 \times 0.4 \\ &= \$30 + \$52 \\ &= \$82 \end{aligned}$$

The expected cost per child when Plan II is used for treatment is:

$$\begin{aligned} E(\text{Cost}_{\text{II}}) &= \$80 \times 0.9 + \$130 \times 0.1 \\ &= \$72 + \$13 \\ &= \$85 \end{aligned}$$

Part (c):

Since the probability that a child will be cured is the same under either plan, some other criterion must be used to make a recommendation. From a financial point of view, Plan I should be recommended because the expected cost per child is less than Plan II.

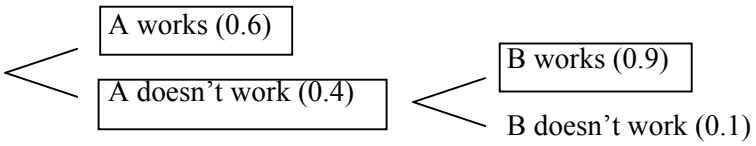
Question 4 (cont'd.)

Scoring

Each part is scored as essentially correct, partially correct, or incorrect.

Part (a) is essentially correct if the probabilities of cure are calculated correctly with justification for both plans.

Plan I:



$P(\text{Cure}_{\text{I}}) = 0.6 + (0.4 \times 0.9) = 0.96$

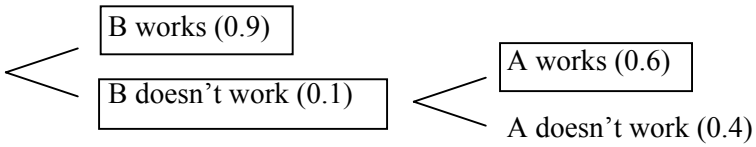
OR

$P(\text{Cure}_{\text{I}}) = P(\text{A} \cup \text{B}) = 0.6 + 0.9 - (0.6 \times 0.9) = 0.96$

OR

$P(\text{Cure}_{\text{I}}) = 1 - P(\text{not A})P(\text{not B}) = 1 - (0.4) \times (0.1) = 0.96$

Plan II:



$P(\text{Cure}_{\text{II}}) = 0.9 + (0.1 \times 0.6) = 0.96$

OR

$P(\text{Cure}_{\text{II}}) = P(\text{B} \cup \text{A}) = 0.9 + 0.6 - (0.9 \times 0.6) = 0.96$

OR

$P(\text{Cure}_{\text{II}}) = 1 - P(\text{not B})P(\text{not A}) = 1 - (0.1) \times (0.4) = 0.96$

Question 4 (cont'd.)

Part (a) is partially correct if

one of the two probabilities is calculated correctly with justification,

OR

both probabilities are correct with incomplete justifications.

Part (b) is essentially correct if the expected costs per child are calculated correctly with justification for both plans.

The expected cost per child when Plan I is used for treatment is:

$$E(\text{Cost}_{\text{I}}) = \$50 \times 0.6 + \$130 \times 0.4 = \$30 + \$52 = \$82$$

OR

$$E(\text{Cost}_{\text{I}}) = \$50 + 0.4 \times \$80 = \$50 + \$32 = \$82$$

The expected cost per child when Plan II is used for treatment is:

$$E(\text{Cost}_{\text{II}}) = \$80 \times 0.9 + \$130 \times 0.1 = \$72 + \$13 = \$85$$

OR

$$E(\text{Cost}_{\text{II}}) = \$80 + 0.1 \times \$50 = \$80 + \$5 = \$85$$

Part (b) is partially correct if
the expected cost per child is calculated correctly with justification for one of the two plans,

OR

both expected costs are correct with incomplete justifications,

OR

the expected costs are incorrectly calculated but the probabilities involved add up to 1. For example the following computations would receive a partial.

The expected cost per child when Plan I is used for treatment is:

$$= \$50 \times 0.6 + \$80 \times 0.4 = \$30 + \$32 = \$62$$

The expected cost per child when Plan II is used for treatment is:

$$= \$80 \times 0.9 + \$50 \times 0.1 = \$72 + \$5 = \$77$$

Question 4 (cont'd.)

In contrast, the following computations would receive an incorrect because the probabilities involved do not add up to 1.

The expected cost per child when Plan I is used for treatment is:

$$= \$50 \times 0.6 + \$130 \times 0.36 = \$30 + \$46.80 = \$76.80$$

The expected cost per child when Plan II is used for treatment is:

$$= \$80 \times 0.9 + \$130 \times 0.06 = \$72 + \$7.80 = \$79.80$$

Part (c) is essentially correct if the recommendation contains a statistical argument based on parts (a) and (b). That is, the student must base the recommendation on probabilities from part (a) and expected values from part (b). The following two examples are essentially correct:

Since the probability that a child will be cured is the same under either plan, some other criterion must be used to make a recommendation. From a financial point of view, Plan I should be recommended because the expected cost per child is less than Plan II.

Since the probability that a child will be cured is the same under either plan, some other criterion must be used to make a recommendation. Parents might prefer Plan II, regardless of its higher cost, because their child is more likely to need only the first drug.

Part (c) is partially correct if the recommendation contains a statistical argument based only on part (a) or (b) but not both.

Part (c) is incorrect if no recommendation is made.