

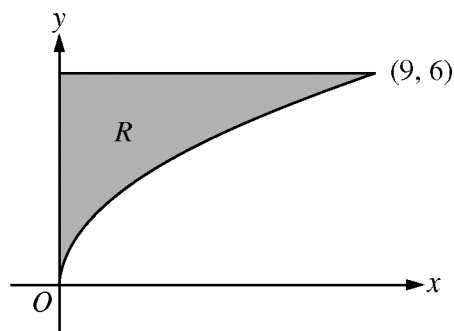
2010 AP[®] CALCULUS BC FREE-RESPONSE QUESTIONS

CALCULUS BC
SECTION II, Part B

Time—45 minutes

Number of problems—3

No calculator is allowed for these problems.



4. Let R be the region in the first quadrant bounded by the graph of $y = 2\sqrt{x}$, the horizontal line $y = 6$, and the y -axis, as shown in the figure above.
- (a) Find the area of R .
 - (b) Write, but do not evaluate, an integral expression that gives the volume of the solid generated when R is rotated about the horizontal line $y = 7$.
 - (c) Region R is the base of a solid. For each y , where $0 \leq y \leq 6$, the cross section of the solid taken perpendicular to the y -axis is a rectangle whose height is 3 times the length of its base in region R . Write, but do not evaluate, an integral expression that gives the volume of the solid.
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5. Consider the differential equation $\frac{dy}{dx} = 1 - y$. Let $y = f(x)$ be the particular solution to this differential equation with the initial condition $f(1) = 0$. For this particular solution, $f(x) < 1$ for all values of x .
- (a) Use Euler's method, starting at $x = 1$ with two steps of equal size, to approximate $f(0)$. Show the work that leads to your answer.
- (b) Find $\lim_{x \rightarrow 1} \frac{f(x)}{x^3 - 1}$. Show the work that leads to your answer.
- (c) Find the particular solution $y = f(x)$ to the differential equation $\frac{dy}{dx} = 1 - y$ with the initial condition $f(1) = 0$.
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$$f(x) = \begin{cases} \frac{\cos x - 1}{x^2} & \text{for } x \neq 0 \\ -\frac{1}{2} & \text{for } x = 0 \end{cases}$$

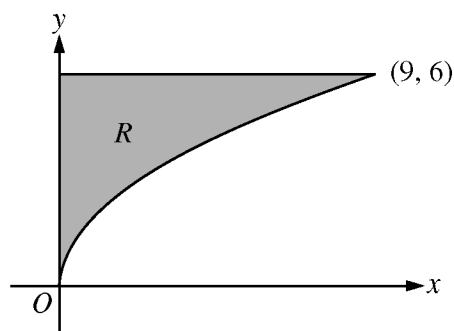
6. The function f , defined above, has derivatives of all orders. Let g be the function defined by $g(x) = 1 + \int_0^x f(t) dt$.
- (a) Write the first three nonzero terms and the general term of the Taylor series for $\cos x$ about $x = 0$. Use this series to write the first three nonzero terms and the general term of the Taylor series for f about $x = 0$.
- (b) Use the Taylor series for f about $x = 0$ found in part (a) to determine whether f has a relative maximum, relative minimum, or neither at $x = 0$. Give a reason for your answer.
- (c) Write the fifth-degree Taylor polynomial for g about $x = 0$.
- (d) The Taylor series for g about $x = 0$, evaluated at $x = 1$, is an alternating series with individual terms that decrease in absolute value to 0. Use the third-degree Taylor polynomial for g about $x = 0$ to estimate the value of $g(1)$. Explain why this estimate differs from the actual value of $g(1)$ by less than $\frac{1}{6!}$.
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END OF EXAM

AP[®] CALCULUS BC
2010 SCORING GUIDELINES

Question 4



Let R be the region in the first quadrant bounded by the graph of $y = 2\sqrt{x}$, the horizontal line $y = 6$, and the y -axis, as shown in the figure above.

- Find the area of R .
- Write, but do not evaluate, an integral expression that gives the volume of the solid generated when R is rotated about the horizontal line $y = 7$.
- Region R is the base of a solid. For each y , where $0 \leq y \leq 6$, the cross section of the solid taken perpendicular to the y -axis is a rectangle whose height is 3 times the length of its base in region R . Write, but do not evaluate, an integral expression that gives the volume of the solid.

(a) $\text{Area} = \int_0^9 (6 - 2\sqrt{x}) \, dx = \left(6x - \frac{4}{3}x^{3/2} \right) \Big|_{x=0}^{x=9} = 18$

3 : $\begin{cases} 1 : \text{integrand} \\ 1 : \text{antiderivative} \\ 1 : \text{answer} \end{cases}$

(b) $\text{Volume} = \pi \int_0^9 \left((7 - 2\sqrt{x})^2 - (7 - 6)^2 \right) dx$

3 : $\begin{cases} 2 : \text{integrand} \\ 1 : \text{limits and constant} \end{cases}$

(c) Solving $y = 2\sqrt{x}$ for x yields $x = \frac{y^2}{4}$.

Each rectangular cross section has area $\left(3 \frac{y^2}{4} \right) \left(\frac{y^2}{4} \right) = \frac{3}{16} y^4$.

$\text{Volume} = \int_0^6 \frac{3}{16} y^4 \, dy$

3 : $\begin{cases} 2 : \text{integrand} \\ 1 : \text{answer} \end{cases}$