

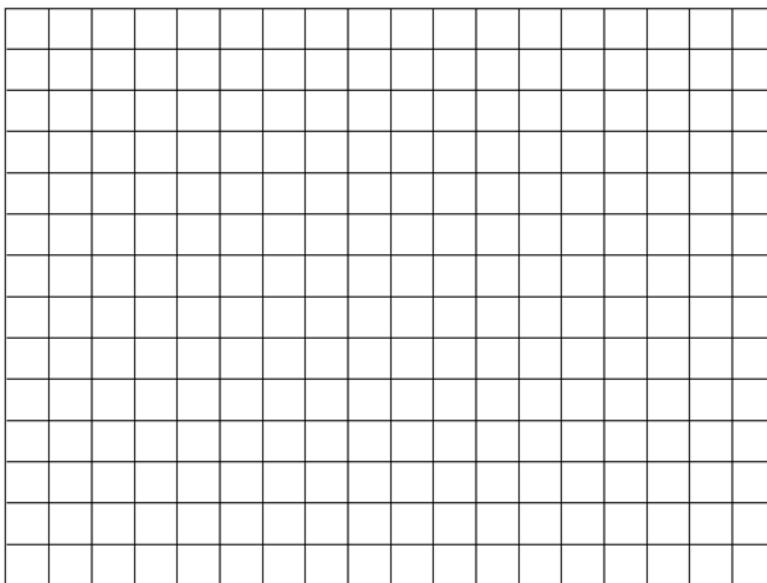
2008 AP® STATISTICS FREE-RESPONSE QUESTIONS

4. An experiment was conducted to study the effect of temperature on the reliability of an electronic device used in an undersea communications system. The experiment was done in a laboratory where tanks of seawater were maintained at either 10°C, 30°C, 50°C, or 70°C. After the electronic devices were submerged in the tanks for 5,000 hours, each device was inspected to determine if it was still working. The following table provides information on the number of devices tested at each temperature and the number of working devices at the end of the 5,000-hour test.

Seawater temperature	10°C	30°C	50°C	70°C
Number of working devices	29	42	21	12
Number of devices tested	30	50	30	20

You may assume that the result for any single device is not influenced by the result for any other device.

- (a) Using the information in the table, construct a scatterplot that would be useful for showing the effect of water temperature on the ability of the devices to work for at least 5,000 hours.



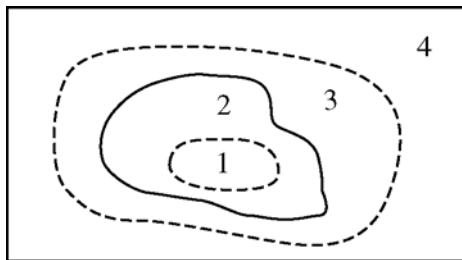
- (b) Comment on any trend or pattern that is revealed by the scatterplot you constructed.
(c) An estimate of the proportion of devices that would work after 5,000 hours of submersion in 40°C seawater can be obtained by averaging the estimates at 30°C and 50°C. Compute this estimate and the associated standard error.

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5. A study was conducted to determine where moose are found in a region containing a large burned area. A map of the study area was partitioned into the following four habitat types.

- (1) Inside the burned area, not near the edge of the burned area,
- (2) Inside the burned area, near the edge,
- (3) Outside the burned area, near the edge, and
- (4) Outside the burned area, not near the edge.

The figure below shows these four habitat types.



Note: Figure not drawn to scale.

The proportion of total acreage in each of the habitat types was determined for the study area. Using an aerial survey, moose locations were observed and classified into one of the four habitat types. The results are given in the table below.

Habitat Type	Proportion of Total Acreage	Number of Moose Observed
1	0.340	25
2	0.101	22
3	0.104	30
4	0.455	40
Total	1.000	117

- (a) The researchers who are conducting the study expect the number of moose observed in a habitat type to be proportional to the amount of acreage of that type of habitat. Are the data consistent with this expectation? Conduct an appropriate statistical test to support your conclusion. Assume the conditions for inference are met.
- (b) Relative to the proportion of total acreage, which habitat types did the moose seem to prefer? Explain.

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Question 4

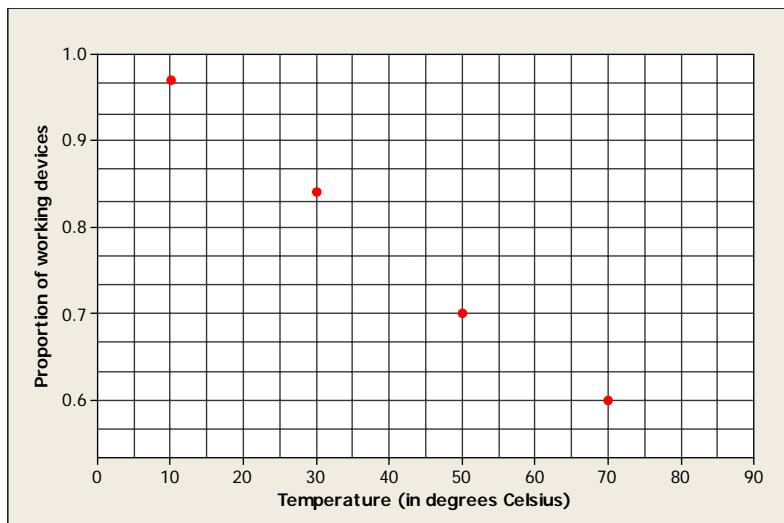
Intent of Question

The primary goals for this question were to assess a student's ability to (1) create and interpret a scatterplot and (2) estimate a proportion and the associated standard error.

Solution

Part (a):

Create a new variable p that indicates the proportion (number working / number tested) of working devices. A scatterplot of the proportion of working devices and temperature in degrees Celsius is shown below.



Part (b):

The scatterplot clearly shows a very strong, negative, linear association between the temperature and proportion of working devices at the end of the 5,000 hours. As the temperature increases, the proportion of working devices decreases.

Part (c):

The estimated proportion is the linear combination $\hat{p}_{40^\circ} = \frac{1}{2}\left(\frac{X_{30^\circ}}{n_{30^\circ}}\right) + \frac{1}{2}\left(\frac{X_{50^\circ}}{n_{50^\circ}}\right) = \frac{1}{2}\left(\frac{42}{50}\right) + \frac{1}{2}\left(\frac{21}{30}\right) = 0.77$.

Because the results for the two devices are independent, the variance of the estimated proportion

$$\text{is } \text{Var}(\hat{p}_{40^\circ}) = \left(\frac{1}{2}\right)^2 \left(\frac{\left(\frac{42}{50}\right)\left(\frac{8}{50}\right)}{50} \right) + \left(\frac{1}{2}\right)^2 \left(\frac{\left(\frac{21}{30}\right)\left(\frac{9}{30}\right)}{30} \right).$$

Thus, the standard error is given by $\sqrt{\text{Var}(\hat{p}_{40^\circ})} = \left(\frac{1}{2}\right)\sqrt{0.0027 + 0.007} = 0.0492$.

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Question 4 (continued)

Notes:

- Students can give two possible approaches to “averaging the estimates at 30°C and 50°C.” The first is the arithmetic mean $\hat{p}_{40^\circ} = \frac{\frac{42}{50} + \frac{21}{30}}{2} = 0.77$ with its standard error as given in the model solution. The second is the pooled estimate $\hat{p}_{40^\circ} = \frac{42 + 21}{50 + 30} = 0.7875$. The standard error of this pooled estimate is calculated as follows:

$$\text{Var}\left(\frac{X_{30^\circ} + X_{50^\circ}}{n_{30^\circ} + n_{50^\circ}}\right) = \frac{1}{80^2} \left(n_{30^\circ} p_{30^\circ} q_{30^\circ} + n_{50^\circ} p_{50^\circ} q_{50^\circ} \right) = \frac{1}{80^2} \left(50 \left(\frac{42}{50} \right) \left(\frac{8}{50} \right) + 30 \left(\frac{21}{30} \right) \left(\frac{9}{30} \right) \right) = 0.00203.$$
$$\text{SE} = \sqrt{\text{Var}\left(\frac{X_{30^\circ} + X_{50^\circ}}{n_{30^\circ} + n_{50^\circ}}\right)} = \sqrt{0.00203} = 0.0451.$$

- Any standard error of a single proportion is not acceptable.

4 Complete Response

All three parts essentially correct

3 Substantial Response

Two parts essentially correct and one part partially correct

2 Developing Response

Two parts essentially correct and no parts partially correct

OR

One part essentially correct and one or two parts partially correct

OR

Three parts partially correct

1 Minimal Response

One part essentially correct and no parts partially correct

OR

No parts essentially correct and two parts partially correct