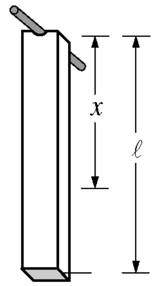

Mech. 2.

You are given a long, thin, rectangular bar of known mass M and length ℓ with a pivot attached to one end. The bar has a nonuniform mass density, and the center of mass is located a known distance x from the end with the pivot. You are to determine the rotational inertia I_b of the bar about the pivot by suspending the bar from the pivot, as shown above, and allowing it to swing. Express all algebraic answers in terms of I_b, the given quantities, and fundamental constants.

(a)

i. By applying the appropriate equation of motion to the bar, write the differential equation for the angle θ the bar makes with the vertical.

ii. By applying the small-angle approximation to your differential equation, calculate the period of the bar's motion.

(b) Describe the experimental procedure you would use to make the additional measurements needed to determine I_b. Include how you would use your measurements to obtain I_b and how you would minimize experimental error.

(c) Now suppose that you were not given the location of the center of mass of the bar. Describe an experimental procedure that you could use to determine it, including the equipment that you would need.

GO ON TO THE NEXT PAGE.

Question 2

15 points total **Distribution of points**

(a)

(i) 4 points

For the rotational form of Newton's second law — 1 point

$\tau = I\alpha$

For a correct expression of the magnitude of torque — 1 point

For correctly labeling the torque as negative — 1 point

$-Mgx\sin\theta = I_b\alpha$

For expressing α as the second time derivative of θ — 1 point

$$-Mgx\sin\theta = I_b\left(\frac{d^2\theta}{dt^2}\right)$$

(ii) 4 points

For the appropriate small angle approximation — 1 point

For small angles, $\sin\theta \approx \theta$

$$-Mgx\theta = I_b\left(\frac{d^2\theta}{dt^2}\right)$$

$$\left(\frac{d^2\theta}{dt^2}\right) + \left(\frac{Mgx}{I_b}\right)\theta = 0$$

For recognizing that the coefficient of θ is ω^2 — 1 point

$$\omega^2 = \frac{Mgx}{I_b}$$

For the relationship between T and ω (this point was awarded for the equation alone or with relevant work, but NOT as part of multiple random equations) — 1 point

$$T = \frac{2\pi}{\omega}$$

For the final expression for T (this point was awarded for the final correct answer with no supporting work) — 1 point

$$T = 2\pi\sqrt{\frac{I_b}{Mgx}}$$

Question 2 (continued)

Distribution of points

(b) 5 points

For an experimental procedure that includes:

A valid approach	1 point
How the variables will be measured or calculated, including equipment to be used	2 points
How these variables will be used to determine I_B	1 point
How to minimize error	1 point

Example 1: Displace the bar by a small angle and release it to oscillate. To reduce errors, time 10 complete oscillations with a stopwatch. Calculate the average value of the time for 10 oscillations and then divide by 10 to determine the period T. Calculate I_B from $T = 2\pi\sqrt{I_b/Mgx}$, using known values of M and x.

Example 2: Locate a photogate at the bottom of the bar's swing; set it to measure the amount of time the photogate is blocked. While the bar is hanging from its pivot point, displace the bar to a horizontal position and measure the height of the center of mass above the position of the photogate with a meter stick. Allow the bar to swing through the photogate and obtain the time the gate is blocked. To reduce errors, repeat this procedure five times and obtain an average time. Measure the width of the bar and use this and the time to determine the speed of the bar at the bottom of the swing, $\upsilon = \text{width}/\text{time}$. Calculate the angular speed of the bar from $\omega = \upsilon/\ell$. Apply conservation of energy to the bar: $Mgh = I_B\omega^2/2$. Calculate I_B from $I_B = 2Mgh/\omega^2$, using known values of M, measured value of h, and calculated value of ω.

(c) 2 points

For a valid procedure to locate the center of mass	1 point
For specifying the equipment to be used	1 point

Example 1: Place the bar on top of a fulcrum, e.g., the top of a prism. Adjust the position of the bar until it is balanced horizontally. The point at which this occurs is the center of mass.

Example 2: Place the bar near the edge of a desk or table. Slowly push the bar so it hangs off the table until it is just ready to tip. The point at which this occurs is the center of mass.