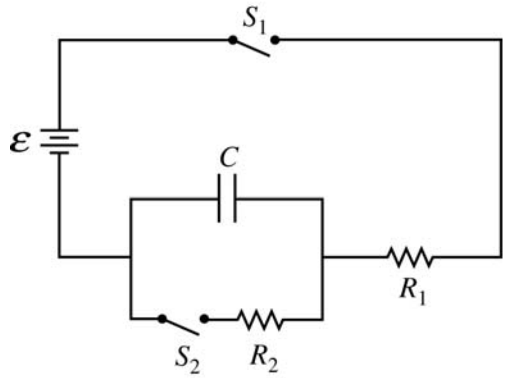

E&M 2.

The circuit above contains a capacitor of capacitance C, a power supply of emf $\mathcal{E}$, two resistors of resistances R_1 and R_2, and two switches, S_1 and S_2. Initially, the capacitor is uncharged and both switches are open. Switch S_1 then gets closed at time $t = 0$.

(a) Write a differential equation that can be solved to obtain the charge on the capacitor as a function of time t.

(b) Solve the differential equation in part (a) to determine the charge on the capacitor as a function of time t.

Numerical values for the components are given as follows:

$\mathcal{E} = 12$ V
$C = 0.060$ F
$R_1 = R_2 = 4700\ \Omega$

(c) Determine the time at which the capacitor has a voltage 4.0 V across it.

After switch S_1 has been closed for a long time, switch S_2 gets closed at a new time $t = 0$.

(d) On the axes below, sketch graphs of the current I_1 in R_1 versus time and of the current I_2 in R_2 versus time, beginning when switch S_2 is closed at new time $t = 0$. Clearly label which graph is I_1 and which is I_2.

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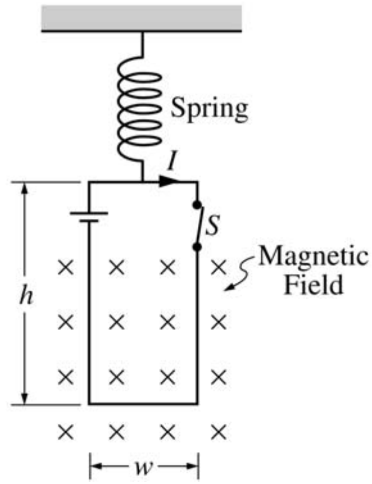
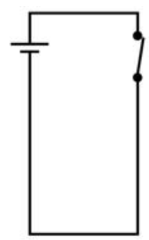


E&M 3.

A loop of wire of width w and height h contains a switch and a battery and is connected to a spring of force constant k, as shown above. The loop carries a current I in a clockwise direction, and its bottom is in a constant, uniform magnetic field directed into the plane of the page.

(a) On the diagram of the loop below, indicate the directions of the magnetic forces, if any, that act on each side of the loop.



(b) The switch S is opened, and the loop eventually comes to rest at a new equilibrium position that is a distance x from its former position. Derive an expression for the magnitude B_0 of the uniform magnetic field in terms of the given quantities and fundamental constants.

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Question 2

15 points total | **Distribution of points**

(a) 4 points

From Kirchhoff's loop rule, the sum of the potential differences around the circuit is zero.

For any statement of the loop rule — 1 point

$$\mathcal{E} - V_R - V_C = 0$$

For correct substitution of the potential differences across both resistor and capacitor — 1 point

$$\mathcal{E} - iR_1 - \frac{q}{C} = 0$$

For substituting the differential definition of current — 1 point

$$i = \frac{dq}{dt}$$

For a correct answer including correct signs (a correct answer with no supporting work was awarded full credit) — 1 point

$$\mathcal{E} - R_1\frac{dq}{dt} - \frac{q}{C} = 0$$

Alternate solution — *Alternate points*

For a correct exponential expression for current as a function of time — *1 point*

$$I = I_0 e^{-t/R_1C}$$

For applying Ohm's law to determine the initial current — *1 point*

$$I_0 = \frac{\mathcal{E}}{R_1}$$

For substituting the differential definition of current — *1 point*

$$I = \frac{dq}{dt}$$

For a correct answer (a correct answer with no supporting work was awarded full credit) — *1 point*

$$\frac{dq}{dt} = \frac{\mathcal{E}}{R_1}e^{-t/R_1C}$$

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Question 2 (continued)

Distribution of points

(b) 3 points

Using the differential equation from part (a)

$$\mathcal{E} - \frac{dq}{dt}R_1 - \frac{q}{C} = 0$$

For separating the variables of the differential equation — 1 point

$$\frac{dq}{\mathcal{E}C - q} = \frac{dt}{R_1 C}$$

$$\int_0^q \frac{dq}{q - \mathcal{E}C} = \int_0^t -\frac{dt}{R_1 C}$$

For integrating the expression — 1 point

$$\ln(q - \mathcal{E}C)\Big|_0^q = -\frac{t}{R_1 C}\Bigg|_0^t$$

$$\ln(q - \mathcal{E}C) - \ln(-\mathcal{E}C) = \ln\frac{q - \mathcal{E}C}{-\mathcal{E}C} = -\frac{1}{R_1 C}(t - 0)$$

$$\frac{q - \mathcal{E}C}{-\mathcal{E}C} = e^{-t/R_1 C}$$

$$q - \mathcal{E}C = -\mathcal{E}Ce^{-t/R_1 C}$$

For a correct answer (a correct answer without supporting work in parts (a) or (b) was awarded 1 point) — 1 point

$$q = \mathcal{E}C\left(1 - e^{-t/R_1 C}\right)$$

Alternate solution — *Alternate points*

Using the differential equation from part (a) (alternate)

$$\frac{dq}{dt} = \frac{\mathcal{E}}{R_1}e^{-t/R_1 C}$$

For separating the variables of the differential equation — *1 point*

$$dq = \frac{\mathcal{E}}{R_1}e^{-t/R_1 C}dt$$

$$\int_0^q dq = \int_0^t \frac{\mathcal{E}}{R_1}e^{-t/R_1 C}dt$$

For integrating the expression — *1 point*

$$q\Big|_0^q = \frac{\mathcal{E}}{R_1}\left(-R_1 Ce^{-t/R_1 C}\right)\Big|_0^t$$

$$q = -R_1 C\left(\frac{\mathcal{E}}{R_1}e^{-t/R_1 C} - \frac{\mathcal{E}}{R_1}\right)$$

For a correct answer (a correct answer without supporting work in parts (a) or (b) was awarded 1 point) — *1 point*

$$q = \mathcal{E}C\left(1 - e^{-t/R_1 C}\right)$$

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