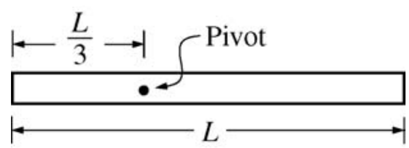
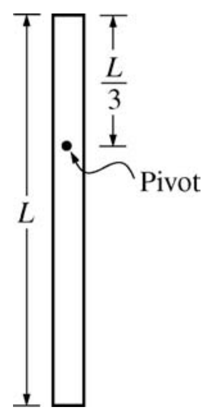


Mech. 3.

A uniform rod of mass M and length L is attached to a pivot of negligible friction as shown above. The pivot is located at a distance $\frac{L}{3}$ from the left end of the rod. Express all answers in terms of the given quantities and fundamental constants.

(a) Calculate the rotational inertia of the rod about the pivot.

(b) The rod is then released from rest from the horizontal position shown above. Calculate the linear speed of the bottom end of the rod when the rod passes through the vertical.



(c) The rod is brought to rest in the vertical position shown above and hangs freely. It is then displaced slightly from this position. Calculate the period of oscillation as it swings.

END OF SECTION II, MECHANICS

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Question 3

15 points total | **Distribution of points**

(a) 4 points

Using the integral expression for the moment of inertia:

$I = \int r^2\, dm$

For a correct change of variables — 1 point

$dm = (M/L)\,dr$

For correct limits of integration — 1 point

$I = \int_{-L/3}^{2L/3} \frac{M}{L} r^2\, dr$ $\left(\begin{array}{l}\text{or other appropriate combination of limits and distance expression,} \\ \text{such as replacing } r \text{ by } x-(L/3) \text{ and integrating from zero to } L\end{array}\right)$

For correctly integrating — 1 point

$I = \frac{M}{L} \left.\frac{r^3}{3}\right|_{-L/3}^{2L/3}$

$I = \frac{M}{3L}\left[\left(\frac{2L}{3}\right)^3 - \left(-\frac{L}{3}\right)^3\right]$

For the correct answer — 1 point

$I = \frac{ML^2}{9}$

Alternate Solution — *Alternate points*

For any statement of the parallel axis theorem — *1 point*

$I = I_{cm} + mr^2$, where r is the distance from the center of mass to the pivot point

For a correct value of the center of mass inertia (calculated or remembered) — *1 point*

$I_{cm} = \frac{1}{12} ML^2$

For indicating that $r = L/6$ — *1 point*

$I = \frac{1}{12} ML^2 + M\left(\frac{L}{6}\right)^2$

For the correct answer — *1 point*

$I = \frac{ML^2}{9}$

Appropriate credit was also awarded for adding inertias for the parts of the rod on either side of the pivot. Credit was given for either calculating the inertias or remembering an appropriate expression for inertia.

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Question 3 (continued)

		Distribution of points

(b) 7 points

For any indication of conservation of energy — 2 points

For correctly calculating the change in potential energy of the rod (or the work done on it) — 2 points

For example: $\Delta U = Mgh_{cm} = Mg\left(\frac{L}{6}\right)$

For writing a conservation equation that includes a rotational kinetic energy (regardless of whether the potential energy is correct) — 1 point

$$\frac{1}{2}I\omega^2 = \frac{MgL}{6}$$

For any indication that ω is linear speed divided by a distance (regardless of whether the correct distance is used) — 1 point

Substituting and solving for υ :

$$\frac{1}{2}\left(\frac{ML^2}{9}\right)\left(\frac{\upsilon}{r}\right)^2 = \frac{MgL}{6}$$

$$\frac{\upsilon^2}{r^2} = \frac{MgL}{6}\,\frac{18}{ML^2} = \frac{3g}{L}$$

$$\upsilon^2 = \frac{3g}{L}r^2 = \frac{3g}{L}\left(\frac{2L}{3}\right)^2 = \frac{4}{3}gL$$

For the correct answer — 1 point

$$\upsilon = 2\sqrt{\frac{gL}{3}}$$

(c) 4 points

For an equation for the period of a physical pendulum — 1 point

$$T = 2\pi\sqrt{\frac{I}{mgd}}$$

For substitution of the inertia from part (a) — 1 point

For indicating that the distance d is the distance from the pivot to the center of mass, i.e. $d = \frac{L}{2} - \frac{L}{3} = \frac{L}{6}$ — 1 point

$$T = 2\pi\sqrt{\frac{ML^2/9}{MgL/6}}$$

For the correct answer — 1 point

$$T = 2\pi\sqrt{\frac{2L}{3g}}$$

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Question 3 (continued)

Distribution of points

(c) (continued)

Alternate solution — *Alternate points*

For an equation relating the angular acceleration to the torque and inertia — *1 point*

$$\alpha = \frac{d^2\theta}{dt^2} = \frac{\tau}{I}$$

For substituting the inertia from part (a) and the torque as a function of θ — *1point*

$$\frac{d^2\theta}{dt^2} = \frac{-Mg\,(L/6)\sin\theta}{ML^2/9} = -\frac{3}{2}\frac{g}{L}\sin\theta$$

For using the approximation $\sin\theta \approx \theta$ — *1 point*

$$\frac{d^2\theta}{dt^2} = -\frac{3}{2}\frac{g}{L}\theta$$

Taking $\theta = k\sin\omega t$, the second derivative is $\dfrac{d^2\theta}{dt^2} = -\omega^2 k\sin\omega t$

Substituting into the differential equation and solving for ω:

$$-\omega^2 k\sin\omega t = -\frac{3}{2}\frac{g}{L}k\sin\omega t$$

$$\omega = \sqrt{\frac{3}{2}\frac{g}{L}}$$

Using the relationship between T and ω:

$T = 2\pi/\omega$

For the correct answer — *1 point*

$$T = 2\pi\sqrt{\frac{2L}{3g}}$$

No credit was awarded for the incorrect approach of using the period of a simple pendulum, $T = 2\pi\sqrt{\ell/g}$, and the length $2L/3$, which happens to give the same result as the correct method

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