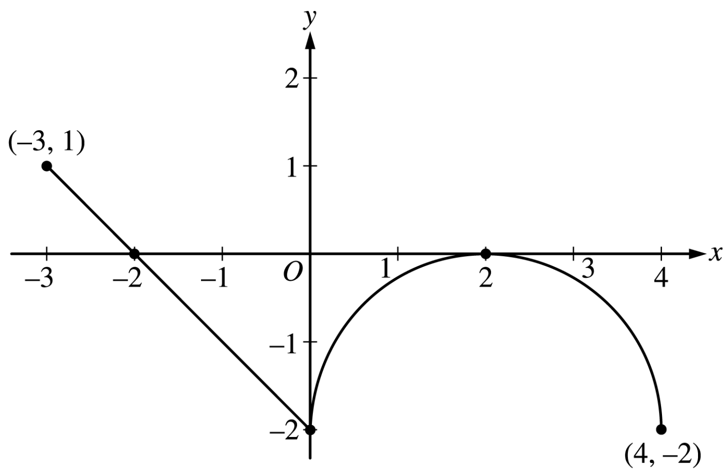
CALCULUS BC
SECTION II, Part B
Time—45 minutes
Number of problems—3

No calculator is allowed for these problems.



Graph of f'

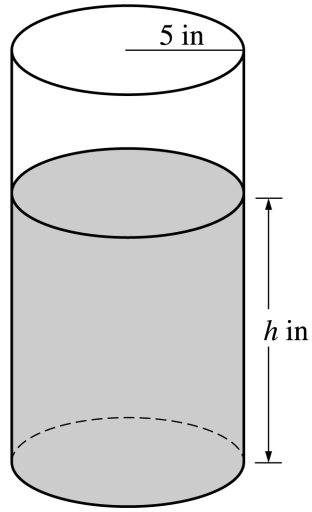
4. Let f be a function defined on the closed interval $-3 \le x \le 4$ with $f(0) = 3$. The graph of f', the derivative of f, consists of one line segment and a semicircle, as shown above.

   (a) On what intervals, if any, is f increasing? Justify your answer.

   (b) Find the x-coordinate of each point of inflection of the graph of f on the open interval $-3 < x < 4$. Justify your answer.

   (c) Find an equation for the line tangent to the graph of f at the point $(0, 3)$.

   (d) Find $f(-3)$ and $f(4)$. Show the work that leads to your answers.



5. A coffeepot has the shape of a cylinder with radius 5 inches, as shown in the figure above. Let h be the depth of the coffee in the pot, measured in inches, where h is a function of time t, measured in seconds. The volume V of coffee in the pot is changing at the rate of $-5\pi\sqrt{h}$ cubic inches per second. (The volume V of a cylinder with radius r and height h is $V = \pi r^2 h$.)

   (a) Show that $\dfrac{dh}{dt} = -\dfrac{\sqrt{h}}{5}$.

   (b) Given that $h = 17$ at time $t = 0$, solve the differential equation $\dfrac{dh}{dt} = -\dfrac{\sqrt{h}}{5}$ for h as a function of t.

   (c) At what time t is the coffeepot empty?

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6. The function f is defined by the power series

$$f(x) = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n+1)!} = 1 - \frac{x^2}{3!} + \frac{x^4}{5!} - \frac{x^6}{7!} + \cdots + \frac{(-1)^n x^{2n}}{(2n+1)!} + \cdots$$

   for all real numbers x.

   (a) Find $f'(0)$ and $f''(0)$. Determine whether f has a local maximum, a local minimum, or neither at $x = 0$. Give a reason for your answer.

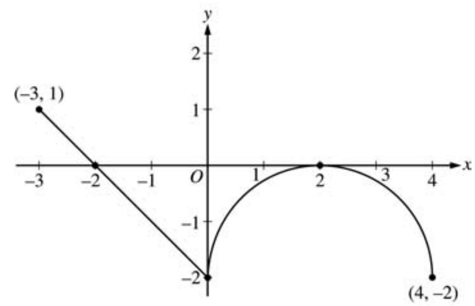
   (b) Show that $1 - \dfrac{1}{3!}$ approximates $f(1)$ with error less than $\dfrac{1}{100}$.

   (c) Show that $y = f(x)$ is a solution to the differential equation $xy' + y = \cos x$.

END OF EXAMINATION

Question 4

Let f be a function defined on the closed interval $-3 \le x \le 4$ with $f(0) = 3$. The graph of f', the derivative of f, consists of one line segment and a semicircle, as shown above.



Graph of f'

(a) On what intervals, if any, is f increasing? Justify your answer.

(b) Find the x-coordinate of each point of inflection of the graph of f on the open interval $-3 < x < 4$. Justify your answer.

(c) Find an equation for the line tangent to the graph of f at the point $(0, 3)$.

(d) Find $f(-3)$ and $f(4)$. Show the work that leads to your answers.

(a) The function f is increasing on $[-3, -2]$ since $f' > 0$ for $-3 \le x < -2$.

$2 : \begin{cases} 1 : \text{interval} \\ 1 : \text{reason} \end{cases}$

(b) $x = 0$ and $x = 2$

f' changes from decreasing to increasing at $x = 0$ and from increasing to decreasing at $x = 2$

$2 : \begin{cases} 1 : x = 0 \text{ and } x = 2 \text{ only} \\ 1 : \text{justification} \end{cases}$

(c) $f'(0) = -2$

Tangent line is $y = -2x + 3$.

$1 : \text{equation}$

(d)
$$f(0) - f(-3) = \int_{-3}^{0} f'(t)\,dt = \frac{1}{2}(1)(1) - \frac{1}{2}(2)(2) = -\frac{3}{2}$$

$$f(-3) = f(0) + \frac{3}{2} = \frac{9}{2}$$

$$f(4) - f(0) = \int_{0}^{4} f'(t)\,dt = -\left(8 - \frac{1}{2}(2)^2\pi\right) = -8 + 2\pi$$

$$f(4) = f(0) - 8 + 2\pi = -5 + 2\pi$$

$4 : \begin{cases} 1 : \pm\left(\frac{1}{2} - 2\right) \\ \quad \text{(difference of areas of triangles)} \\ 1 : \text{answer for } f(-3) \text{ using FTC} \\ 1 : \pm\left(8 - \frac{1}{2}(2)^2\pi\right) \\ \quad \text{(area of rectangle} \\ \quad - \text{area of semicircle)} \\ 1 : \text{answer for } f(4) \text{ using FTC} \end{cases}$