4. (10 points, suggested time 20 minutes)

Some students are investigating the behavior of a circuit with four components in series: a resistor of resistance R, a capacitor of capacitance C, a battery with potential difference $\mathcal{E}$, and a switch. Initially, the capacitor is uncharged and the switch is open.

(a)

i. Determine the current in the resistor and the potential difference across the capacitor immediately after the switch is closed.

ii. Determine the current in the resistor and the potential difference across the capacitor a long time after the switch is closed.

(b) The switch is opened, the capacitor is discharged, and a second, identical capacitor is added to the circuit in series with the other components. The switch is then closed again.

i. A long time after the switch is closed, the energy stored in the single capacitor in the original circuit is U_1, and the total energy stored in the two capacitors in the new circuit is U_2. Calculate the ratio U_1/U_2.

ii. The two capacitors in series are to be replaced with a single capacitor that will have the same energy U_2. Indicate a plate area and a distance between the plates for the new capacitor, compared with one of the original capacitors, that will accomplish this. Support your reasoning using appropriate physics principles and/or mathematical models.

The students are then asked to design two circuits each containing a switch, a battery with a small internal resistance, a lightbulb, and a capacitor. In arrangement 1, the bulb should gradually light up after the switch is closed, becoming brightest after the switch has been closed a long time. In arrangement 2, the bulb should be brightest when the switch is first closed, getting dimmer with time, and going out completely when the switch has been closed for a long time.

(c) Using standard symbols, draw two circuit diagrams, one showing a possible circuit for arrangement 1 and the other showing a possible circuit for arrangement 2. Justify your circuit diagrams with a paragraph-length explanation referring to the properties of lightbulbs and capacitors in circuits and the conservation of energy and/or the conservation of charge.

STOP

END OF EXAM

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Question 4

10 points total **Distribution of points**

(a)

i. 1 point

For indicating that $I = \mathcal{E}/R$ and $V_C = 0$ 1 point

Because there is no charge on the capacitor, there is no potential difference across it. Therefore, entire battery potential is across the resistor, so the current is that potential divided by the resistance.

ii. 1 point

For indicating that $I = 0$ and $V_C = \mathcal{E}$ 1 point

Once the capacitor is fully charged, it allows no current to pass. Because all the components are in series, there is no current at all in the circuit. With no current, there is no potential difference across the resistor, so the entire battery potential is across the capacitor.

(b)

i. 2 points

For a calculation that indicates one of the following: 1 point

- The potential difference across each capacitor in the new circuit is half that across the single capacitor in the original circuit
- The equivalent capacitance of the new circuit is one-half the capacitance of the original circuit

$U_1 = (1/2)C\mathcal{E}^2$

$U_2 = 2\left[(1/2)C(\mathcal{E}/2)^2\right]$ or $(1/2)(C/2)\mathcal{E}^2$, which both equal $C\mathcal{E}^2/4$

For correctly calculating the ratio 1 point

$U_1/U_2 = \left(C\mathcal{E}^2/2\right)/\left(C\mathcal{E}^2/4\right) = 2$

ii. 1 point

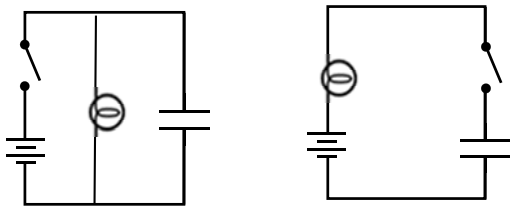
For any combination of area and spacing that is consistent with the student's answer for the ratio in part (b)(i), with a proper principle or model as support 1 point

Example: $U = (1/2)CV^2$. The potential difference across each of the single capacitors is the same. For the energy stored in the single new capacitor to be half that of the original single capacitor, the new capacitor must have half the capacitance. $C = \varepsilon_0 A/d$, so half the plate area with the same distance between the plates will accomplish this.

Question 4 (continued)

Distribution of points

(c) 5 points



Arrangement 1 Arrangement 2

For two correct circuit diagrams, each matched with the correct situation — arrangement 1 has lightbulb and capacitor in parallel, and arrangement 2 has them in series — 1 point

For indicating that the lightbulb is brightest when the current through it is maximum and that the capacitor eventually stops current from flowing in its branch when the potential difference across its plates is equal in magnitude to the emf of the battery (or something similar) — 1 point

Response using current

For indicating that in the series circuit (where the same current flows through both components) the most current flows right after the switch is closed and decreases as the capacitor charges

Response using potential difference

For indicating that in the series circuit (where the potential is shared) the resistor has its maximum potential difference right after the switch is closed, because the capacitor starts out uncharged (no potential difference) and then charges until it has the same potential as the battery — 1 point

For indicating that in the parallel circuit (where the current is shared between the components) the most current flows through the lightbulb a long time after the switch is closed, because the full current initially goes through the capacitor branch because it acts like a wire (very low potential difference), then ends up all through the lightbulb once the fully charged capacitor acts like an open circuit (same potential difference as battery)

For indicating that in a parallel circuit (where both components have the same potential difference) the bulb starts out with the same zero potential difference as the capacitor and ends up with the total battery potential — 1 point

For a response that has sufficient paragraph structure, as described in the published requirements for the paragraph-length response — 1 point