The magnitude of the acceleration a was measured for different values of M_1 and M_2, and the data are shown below.

M_1 (kg)	1.0	2.0	5.0	6.0	10.0
M_2 (kg)	2.0	3.0	12.0	8.0	14.0
a (m/s^2)	3.02	1.82	4.21	1.15	1.71

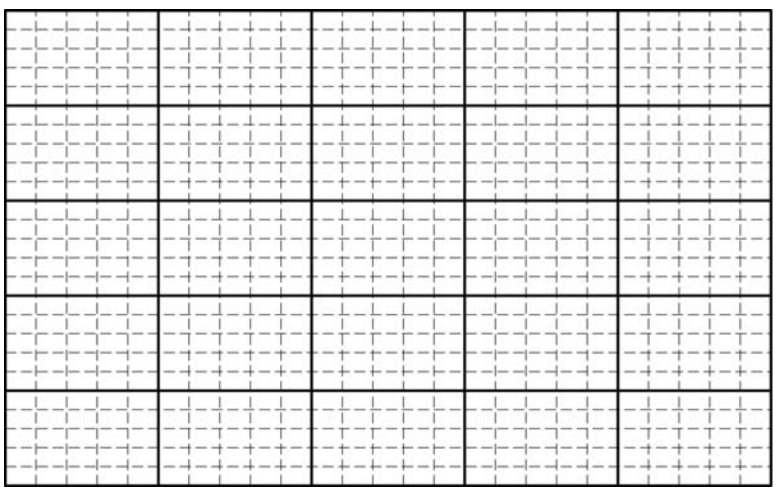
(c) Indicate below which quantities should be graphed to yield a straight line whose slope could be used to calculate a numerical value for the acceleration due to gravity g.

Vertical axis: ____________________

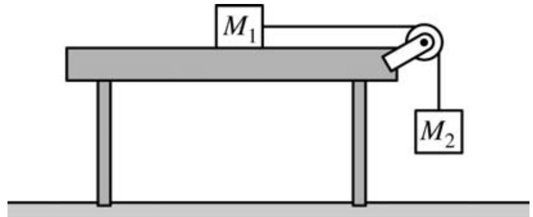
Horizontal axis: ____________________

Use the remaining rows in the table above, as needed, to record any quantities that you indicated that are not given.

(d) Plot the data points for the quantities indicated in part (c) on the graph below. Clearly scale and label all axes including units, if appropriate. Draw a straight line that best represents the data.



(e) Using your straight line, determine an experimental value for g.



The experiment is now repeated with a modification. The Atwood's machine is now set up so that the block of mass M_1 is on a smooth, horizontal table and the block of mass M_2 is hanging over the side of the table, as shown in the figure above.

(f) For the same values of M_1 and M_2, is the magnitude of the tension in the string when the blocks are moving higher, lower, or equal to the magnitude of the tension in the string when the blocks are moving in the first experiment?

____ Higher ____ Lower ____ Equal to

Justify your answer.

(g) The value determined for the acceleration due to gravity g is lower than in the first experiment. Give one physical factor that could account for this lower value and explain how this factor affected the experiment.

Note: Figure not drawn to scale.

2. A block of mass m starts at rest at the top of an inclined plane of height h, as shown in the figure above. The block travels down the inclined plane and makes a smooth transition onto a horizontal surface. While traveling on the horizontal surface, the block collides with and attaches to an ideal spring of spring constant k. There is negligible friction between the block and both the inclined plane and the horizontal surface, and the spring has negligible mass. Express all algebraic answers for parts (a), (b), and (c) in terms of m, h, k, and physical constants, as appropriate.

(a)

i. Derive an expression for the speed of the block just before it collides with the spring.

ii. Is the speed halfway down the incline greater than, less than, or equal to one-half the speed at the bottom of the inclined plane?

____ Greater than ______ Less than ____ Equal to

Justify your answer.

(b) Derive an expression for the maximum compression of the spring.

(c) Determine an expression for the time from when the block collides with the spring to when the spring reaches its maximum compression.

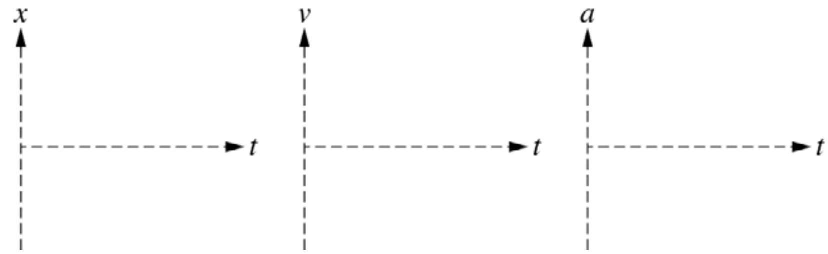
The block is again released from rest at the top of the incline, and when it reaches the horizontal surface it is moving with speed v_0. Now suppose the block experiences a resistive force as it slides on the horizontal surface. The magnitude of the resistive force F is given as a function of speed v by $F = \beta v^2$, where β is a positive constant with units of $\mathrm{kg/m}$.

(d)

i. Write, but do NOT solve, a differential equation for the speed of the block on the horizontal surface as a function of time t before it reaches the spring. Express your answer in terms of m, h, k, β, v, and physical constants, as appropriate.

ii. Using the differential equation from part (d)i, show that the speed of the block $v(t)$ as a function of time t can be written in the form $\dfrac{1}{v(t)} = \dfrac{1}{v_0} + \dfrac{\beta t}{m}$, where v_0 is the speed at $t = 0$.

(e) Sketch graphs of position x as a function of time t, velocity v as a function of time t, and acceleration a as a function of time t for the block as it is moving on the horizontal surface before it reaches the spring.



Question 2

15 points total | **Distribution of points**

(a)

i. 2 points

For correctly using conservation of energy for the block moving down the incline — 1 point

$U_g = K$

$mgh = \frac{1}{2}mv^2$

For a correct answer — 1 point

$v = \sqrt{2gh}$

ii. 1 point

Correct answer: "Greater than"

For a correct justification — 1 point

Example: The speed is proportional to the square root of the change in height. So if the height is reduced by a factor of 2, the speed is reduced by a factor of $\sqrt{2} \approx 1.41$. Therefore, the speed halfway down the ramp is more than half the speed at the bottom of the ramp.

Note: If the incorrect selection is made, the justification cannot earn credit.

(b) 1 point

For correctly using conservation of energy, consistent with part (a), for the block compressing the spring — 1 point

$U_g = U_S$

$mgh = \frac{1}{2}kx_{\max}^2$

$x_{\max} = \sqrt{\frac{2mgh}{k}}$

(c) 2 points

For indicating a simple harmonic motion approach — 1 point

For a correct answer — 1 point

$t = \frac{1}{4}T = \left(\frac{1}{4}\right)\left(2\pi\sqrt{\frac{m}{k}}\right) = \frac{\pi}{2}\sqrt{\frac{m}{k}}$

Question 2 (continued)

Distribution of points

(d)

i. 3 points

For correctly applying Newton's second law for the horizontally sliding block — 1 point

For correctly indicating that the direction of F_{net} is opposite the direction of motion — 1 point

$F_{net} = ma$

$-\beta v^2 = ma$

For expressing the equation as a differential equation — 1 point

$$-\beta v^2 = m\frac{dv}{dt}$$

ii. 3 points

For correctly separating variables — 1 point

$$-\frac{\beta}{m}dt = \frac{1}{v^2}dv$$

For correctly integrating the equation above — 1 point

$$\int -\frac{\beta}{m}dt = \int \frac{1}{v^2}dv$$

$$-\frac{\beta}{m}[t] = \left[-\frac{1}{v}\right]$$

For using the correct limits or constant of integration — 1 point

$$-\frac{\beta}{m}[t]_0^t = \left[-\frac{1}{v}\right]_{v_0}^{v(t)}$$

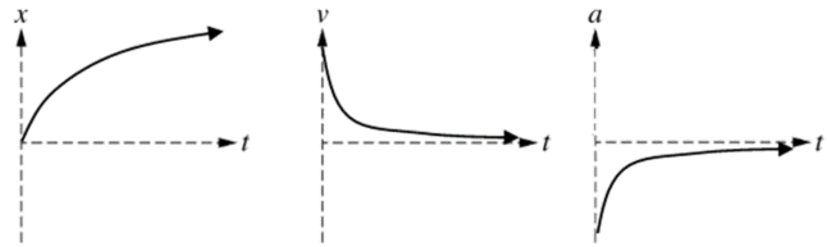
$$-\frac{\beta t}{m} = -\frac{1}{v(t)} - \left(-\frac{1}{v_0}\right)$$

$$\frac{1}{v(t)} = \frac{1}{v_0} + \frac{\beta t}{m}$$

Question 2 (continued)

Distribution of points

(e) 3 points



For a displacement curve that is concave down and approaching a horizontal asymptote	1 point
For a velocity curve that is concave up and has the horizontal axis as an asymptote	1 point
For an acceleration curve that is concave down and has the horizontal axis as an asymptote	1 point

Note: If an incorrect nonlinear velocity graph is generated, 1 point is earned if the position and acceleration graphs are consistent with the velocity graph.

Note: Full credit is earned if all three graphs are flipped vertically.