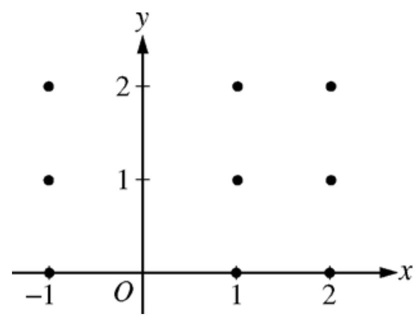
5. Consider the differential equation $\frac{dy}{dx} = \frac{y-1}{x^2}$, where $x \neq 0$.

   (a) On the axes provided, sketch a slope field for the given differential equation at the nine points indicated.
   (Note: Use the axes provided in the exam booklet.)



   (b) Find the particular solution $y = f(x)$ to the differential equation with the initial condition $f(2) = 0$.

   (c) For the particular solution $y = f(x)$ described in part (b), find $\lim_{x \to \infty} f(x)$.

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6. Let f be the function given by $f(x) = \frac{\ln x}{x}$ for all $x > 0$. The derivative of f is given by $f'(x) = \frac{1 - \ln x}{x^2}$.

   (a) Write an equation for the line tangent to the graph of f at $x = e^2$.

   (b) Find the x-coordinate of the critical point of f. Determine whether this point is a relative minimum, a relative maximum, or neither for the function f. Justify your answer.

   (c) The graph of the function f has exactly one point of inflection. Find the x-coordinate of this point.

   (d) Find $\lim_{x \to 0^+} f(x)$.

WRITE ALL WORK IN THE PINK EXAM BOOKLET.

END OF EXAM

AP® CALCULUS AB
2008 SCORING GUIDELINES

Question 6

Let f be the function given by $f(x) = \dfrac{\ln x}{x}$ for all $x > 0$. The derivative of f is given by $f'(x) = \dfrac{1 - \ln x}{x^2}$.

(a) Write an equation for the line tangent to the graph of f at $x = e^2$.

(b) Find the x-coordinate of the critical point of f. Determine whether this point is a relative minimum, a relative maximum, or neither for the function f. Justify your answer.

(c) The graph of the function f has exactly one point of inflection. Find the x-coordinate of this point.

(d) Find $\lim\limits_{x \to 0^+} f(x)$.

Solution	Scoring
(a) $f\left(e^2\right) = \dfrac{\ln e^2}{e^2} = \dfrac{2}{e^2}$, $f'\left(e^2\right) = \dfrac{1 - \ln e^2}{\left(e^2\right)^2} = -\dfrac{1}{e^4}$ An equation for the tangent line is $y = \dfrac{2}{e^2} - \dfrac{1}{e^4}\left(x - e^2\right)$.	$2 : \begin{cases} 1 : f\left(e^2\right) \text{ and } f'\left(e^2\right) \\ 1 : \text{answer} \end{cases}$
(b) $f'(x) = 0$ when $x = e$. The function f has a relative maximum at $x = e$ because $f'(x)$ changes from positive to negative at $x = e$.	$3 : \begin{cases} 1 : x = e \\ 1 : \text{relative maximum} \\ 1 : \text{justification} \end{cases}$
(c) $f''(x) = \dfrac{-\frac{1}{x}x^2 - (1 - \ln x)2x}{x^4} = \dfrac{-3 + 2\ln x}{x^3}$ for all $x > 0$ $f''(x) = 0$ when $-3 + 2\ln x = 0$ $x = e^{3/2}$ The graph of f has a point of inflection at $x = e^{3/2}$ because $f''(x)$ changes sign at $x = e^{3/2}$.	$3 : \begin{cases} 2 : f''(x) \\ 1 : \text{answer} \end{cases}$
(d) $\lim\limits_{x \to 0^+} \dfrac{\ln x}{x} = -\infty$ or Does Not Exist	1 : answer