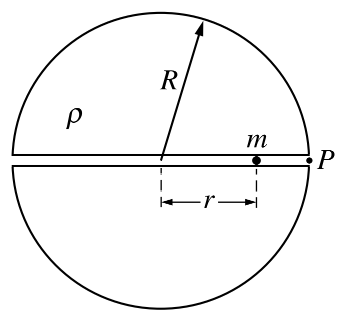
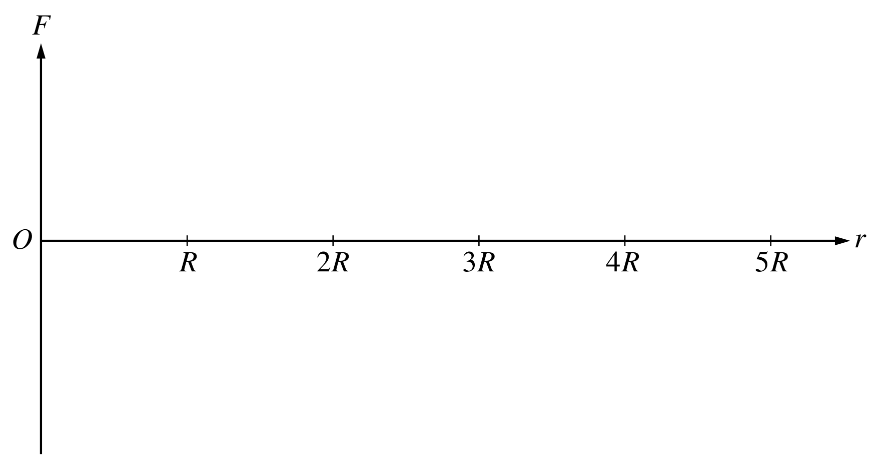
(d) The dart is now shot into a block of wood that is fixed in place. The block exerts a force $\mathbf{F}$ on the dart that is proportional to the dart's velocity $\mathbf{v}$ and in the opposite direction, that is $\mathbf{F} = -b\mathbf{v}$, where b is a constant. Derive an expression for the distance L that the dart penetrates into the block, in terms of m, v_0, and b.



Mech 2. A spherical, nonrotating planet has a radius R and a uniform density ρ throughout its volume. Suppose a narrow tunnel were drilled through the planet along one of its diameters, as shown in the figure above, in which a small ball of mass m could move freely under the influence of gravity. Let r be the distance of the ball from the center of the planet.

(a) Show that the magnitude of the force on the ball at a distance $r < R$ from the center of the planet is given by $F = -Cr$, where $C = \frac{4}{3}\pi G\rho m$.

(b) On the axes below, sketch the force F on the ball as a function of distance r from the center of the planet.

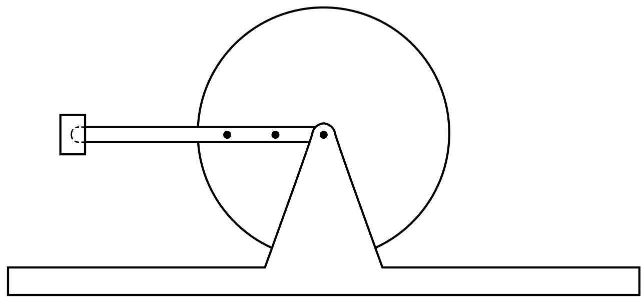


The string is now cut, and the disk-rod-block system is free to rotate.

(b) Determine the following for the instant immediately after the string is cut.

i. The magnitude of the angular acceleration of the disk

ii. The magnitude of the linear acceleration of the mass at the end of the rod



As the disk rotates, the rod passes the horizontal position shown above.

(c) Determine the linear speed of the mass at the end of the rod for the instant the rod is in the horizontal position.

S T O P

END OF SECTION II, MECHANICS

1999 Physics C Solutions | **Distribution of points**

Mech. 2 (15 points)

(a) 3 points

For indicating that the equation for gravitational force is applicable — 1 point

$$F = -\frac{Gm_1m_2}{r^2}$$

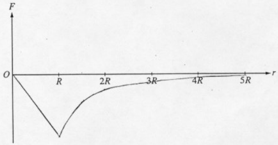
For using the proper expression for the mass of the planet enclosed by the radius — 1 point

$$F = -\frac{Gm\left(\rho\frac{4}{3}\pi r^3\right)}{r^2}$$

For proper cancellation of terms to show the final result — 1 point

$$F = -\left(\frac{4}{3}\pi G\rho m\right)r$$

(b) 4 points



For drawing a straight line from the origin to a distance R, and not going past R — 1 point

For having the maximum magnitude occur at R — 1 point

For having the curve from R to $5R$ decreasing in magnitude with proper curvature and appearing to reach an asymptote — 1 point

For recognizing that the force is always negative, i.e. the graph is always below the x-axis — 1 point

Mech. 2 (continued)

(c) 3 points

In this and all subsequent parts, either C or $\frac{4}{3}\pi G\rho m$ could be used.

For indicating the integral that needs to be calculated to determine the work — 1 point

$W = \int F\,dr = \int -Cr\,dr$

For using the proper limits on the integral (R to zero, not r) — 1 point

$W = \int_R^0 -Cr\,dr$

$W = -C\frac{r^2}{2}\Big|_R^0$

For the correct answer — 1 point

$W = \frac{CR^2}{2}$

Alternate Solution — *Alternate points*

For recognition that the work is the area under the curve, which is triangular — *1 point*

For using the corrects limits (zero to R) — *1 point*

For the correct answer — *1 point*

$W = \frac{CR^2}{2}$

(d) 2 points

For using conservation of energy or work-energy relationship — 1 point

$\Delta K = \Delta U = W$

$\frac{1}{2}mv^2 = \frac{CR^2}{2}$

For the correct answer — 1 point

$v = \sqrt{\frac{CR^2}{m}}$

An alternate solution indicating the potential energy as that of a harmonic oscillator also received full credit.

(e) 2 points

For indicating that the ball will move from the center to the surface of the planet — 1 point

For indicating that the ball will stop at the surface, return to the center, and continue oscillating in this manner, with no damping — 1 point

Describing the motion as simple harmonic oscillation with no damping earned full credit

Mech. 2 (continued)

(f) 1 point

For showing a proper application of Newton's first law 1 point

$F = ma$

$Cr = m\dfrac{d^2r}{dt^2}$

Alternately, one could relate the time to the period of oscillation, $T = 2\pi\sqrt{\dfrac{3}{4\pi G\rho}}$,

i.e. the time is one-fourth this period. The above equation was required; a more general form was not acceptable.