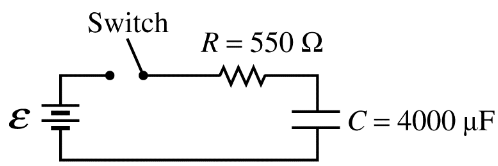
PHYSICS C: ELECTRICITY AND MAGNETISM

SECTION II

Time—45 minutes

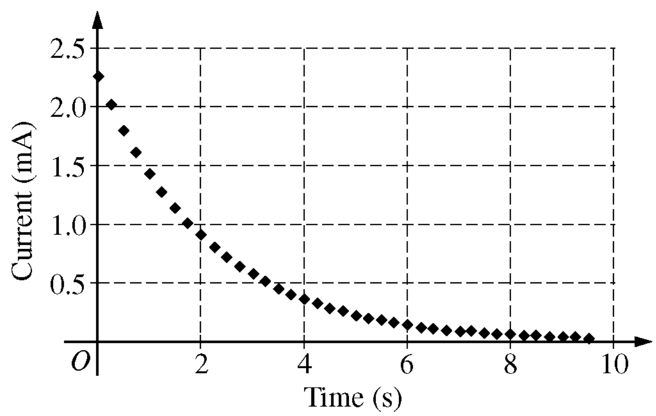
3 Questions

Directions: Answer all three questions. The suggested time is about 15 minutes for answering each of the questions, which are worth 15 points each. The parts within a question may not have equal weight. Show all your work in the pink booklet in the spaces provided after each part, NOT in this green insert.



E&M 1.

A student sets up the circuit above in the lab. The values of the resistance and capacitance are as shown, but the constant voltage $\mathcal{E}$ delivered by the ideal battery is unknown. At time $t = 0$, the capacitor is uncharged and the student closes the switch. The current as a function of time is measured using a computer system, and the following graph is obtained.

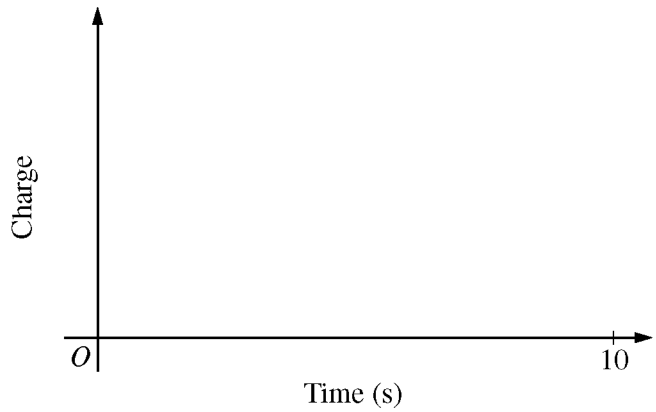


(a) Using the data above, calculate the battery voltage $\mathcal{E}$.

(b) Calculate the voltage across the capacitor at time $t = 4.0$ s.

(c) Calculate the charge on the capacitor at $t = 4.0$ s.

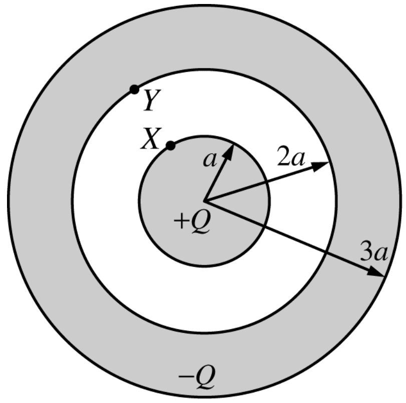
(d) On the axes below, sketch a graph of the charge on the capacitor as a function of time.



(e) Calculate the power being dissipated as heat in the resistor at $t = 4.0$ s.

(f) The capacitor is now discharged, its dielectric of constant $\kappa = 1$ is replaced by a dielectric of constant $\kappa = 3$, and the procedure is repeated. Is the amount of charge on one plate of the capacitor at $t = 4.0$ s now greater than, less than, or the same as before? Justify your answer.

_____Greater than   _____Less than   _____The same



E&M 2.

In the figure above, a nonconducting solid sphere of radius a with charge $+Q$ uniformly distributed throughout its volume is concentric with a nonconducting spherical shell of inner radius $2a$ and outer radius $3a$ that has a charge $-Q$ uniformly distributed throughout its volume. Express all answers in terms of the given quantities and fundamental constants.

(a) Using Gauss's law, derive expressions for the magnitude of the electric field as a function of radius r in the following regions.

i. Within the solid sphere ($r < a$)

ii. Between the solid sphere and the spherical shell ($a < r < 2a$)

iii. Within the spherical shell ($2a < r < 3a$)

iv. Outside the spherical shell ($r > 3a$)

(b) What is the electric potential at the outer surface of the spherical shell ($r = 3a$)? Explain your reasoning.

(c) Derive an expression for the electric potential difference $V_X - V_Y$ between points X and Y shown in the figure.

Question 1

15 points total **Distribution of points**

(a) 3 points

Writing the general loop rule for the circuit

$\mathcal{E} = IR + V_C$

$V_C = 0$ at time $= 0$, so $\mathcal{E} = I_0 R$

For correct substitution of a value for I_0 and a value for R into Ohm's law or the loop rule with the recognition that $V_C = 0$ at time $= 0$ — 1 point

For correctly reading the magnitude of I_0 from the graph and using it in a valid equation — 1 point

$2.2 \le I_0 \le 2.3$

For correctly using the current in mA — 1 point

$2.2 \text{ mA} \le I_0 \le 2.3 \text{ mA}$

$\mathcal{E} = \left(2.25 \times 10^{-3} \text{ A}\right)(550\ \Omega) = 1.24 \text{ V}$

(b) 3 points

For a correct loop rule equation — 1 point

$\mathcal{E} = IR + V_C$

For correctly reading $I(t = 4 \text{ s})$ from the graph — 1 point

$0.3 \text{ mA} \le I(t = 4 \text{ s}) \le 0.4 \text{ mA}$

For correct substitution of $\mathcal{E}$ from part (a) into a correct equation — 1 point

$V_C = 1.24 \text{ V} - \left(0.35 \times 10^{-3} \text{ A}\right)(550\ \Omega)$, using the middle of the range of acceptable values for I

$V_C = 1.05 \text{ V}$ (or value consistent with the value of I chosen within the acceptable range)

<u>Note</u>: *Use of a value for current that was not correctly expressed in mA was acceptable if the mA point was not awarded in part (a) (either for using an incorrectly expressed value or no value at all).*

Alternate solution — *Alternate points*

For correctly using an expression for V_C — *1 point*

$V_C = \mathcal{E}\left(1 - e^{-t/\tau}\right)$ *or equivalent*

For correct substitution for the time and the time constant — *1 point*

$$\frac{t}{\tau} = \frac{t}{RC} = \frac{4.0 \text{ s}}{(550\ \Omega)\left(4000 \times 10^{-6} \text{ F}\right)} = \frac{4.0 \text{ s}}{2.2 \text{ s}} = 1.82$$

For correct substitution of $\mathcal{E}$ *from part (a) into a correct equation* — *1 point*

$V_C = (1.24 \text{ V})\left(1 - e^{-1.82}\right)$

$V_C = 1.04 \text{ V}$

Question 1 (continued)

Distribution of points

(c) 2 points

For using V_C from part (b) in a correct equation — 1 point

$Q = CV = C(1.05\text{ V})$

For correct substitution of C — 1 point

$Q = \left(4000 \times 10^{-6}\text{ F}\right)(1.05\text{ V})$

$Q = 4.20 \times 10^{-3}\text{ C}$ or $4200 \times 10^{-6}\text{ C}$ (or value consistent with V_C from part (b))

Alternate solution — *Alternate points*

For a correct substitution of $\mathcal{E}$ or I_0 into a correct equation — *1 point*

$Q = C\mathcal{E}\left(1 - e^{-t/\tau}\right) = C(1.24\text{ V})\left(1 - e^{-t/\tau}\right)$

OR $Q = \int_0^4 \left(2.25 \times 10^{-3}\text{A}\right)\left(e^{-t/\tau}\right)dt$

For a correct substitution of C or τ into a correct equation — *1 point*

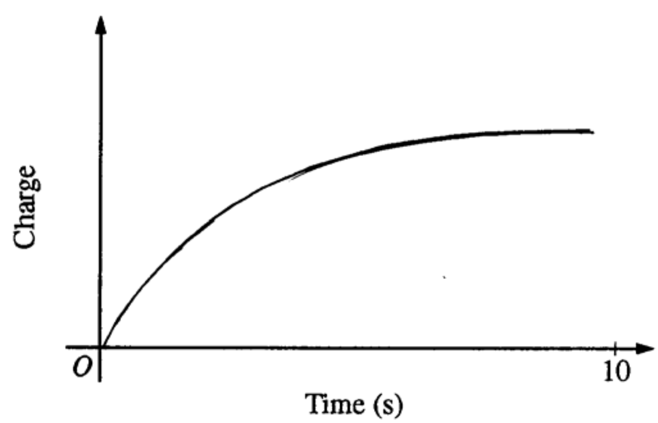
$Q = \left(4000 \times 10^{-6}\text{ F}\right)(1.24\text{ V})\left(1 - e^{-1.82}\right)$

OR $Q = \int_0^4 \left(2.25 \times 10^{-3}\text{A}\right)\left(e^{-t/(2.2\text{s})}\right)dt$

$Q = 4.16 \times 10^{-3}\text{ C}$ *or* $4160\ \mu\text{C}$

<u>Note</u>: *If the answer to (c) was correct using one of the exponential equations, it could be substituted into $V = Q/C$ in part (b) for full credit.*

(d) 2 points



For starting the graph at the origin — 1 point

For a sketch that is approximately exponential and approaching an asymptote — 1 point