

## 2012 AP<sup>®</sup> CALCULUS BC FREE-RESPONSE QUESTIONS

2. For  $t \geq 0$ , a particle is moving along a curve so that its position at time  $t$  is  $(x(t), y(t))$ . At time  $t = 2$ , the particle is at position  $(1, 5)$ . It is known that  $\frac{dx}{dt} = \frac{\sqrt{t+2}}{e^t}$  and  $\frac{dy}{dt} = \sin^2 t$ .
- (a) Is the horizontal movement of the particle to the left or to the right at time  $t = 2$ ? Explain your answer. Find the slope of the path of the particle at time  $t = 2$ .
- (b) Find the  $x$ -coordinate of the particle's position at time  $t = 4$ .
- (c) Find the speed of the particle at time  $t = 4$ . Find the acceleration vector of the particle at time  $t = 4$ .
- (d) Find the distance traveled by the particle from time  $t = 2$  to  $t = 4$ .
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END OF PART A OF SECTION II

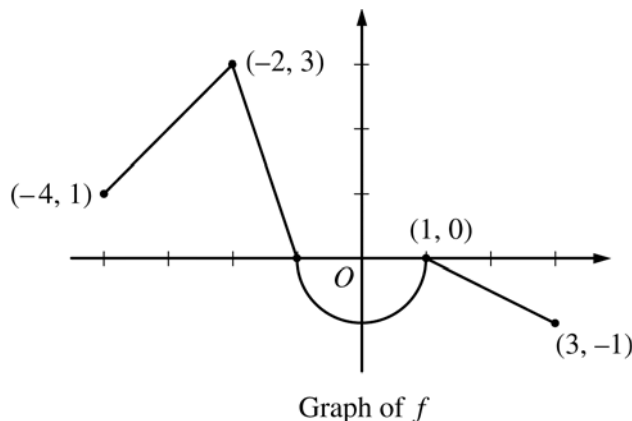
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CALCULUS BC  
SECTION II, Part B

Time—60 minutes

Number of problems—4

No calculator is allowed for these problems.



3. Let  $f$  be the continuous function defined on  $[-4, 3]$  whose graph, consisting of three line segments and a semicircle centered at the origin, is given above. Let  $g$  be the function given by  $g(x) = \int_1^x f(t) dt$ .
- Find the values of  $g(2)$  and  $g(-2)$ .
  - For each of  $g'(-3)$  and  $g''(-3)$ , find the value or state that it does not exist.
  - Find the  $x$ -coordinate of each point at which the graph of  $g$  has a horizontal tangent line. For each of these points, determine whether  $g$  has a relative minimum, relative maximum, or neither a minimum nor a maximum at the point. Justify your answers.
  - For  $-4 < x < 3$ , find all values of  $x$  for which the graph of  $g$  has a point of inflection. Explain your reasoning.

**AP<sup>®</sup> CALCULUS BC**  
**2012 SCORING GUIDELINES**

**Question 2**

For  $t \geq 0$ , a particle is moving along a curve so that its position at time  $t$  is  $(x(t), y(t))$ . At time  $t = 2$ , the particle is at position  $(1, 5)$ . It is known that  $\frac{dx}{dt} = \frac{\sqrt{t+2}}{e^t}$  and  $\frac{dy}{dt} = \sin^2 t$ .

- (a) Is the horizontal movement of the particle to the left or to the right at time  $t = 2$ ? Explain your answer. Find the slope of the path of the particle at time  $t = 2$ .
- (b) Find the  $x$ -coordinate of the particle's position at time  $t = 4$ .
- (c) Find the speed of the particle at time  $t = 4$ . Find the acceleration vector of the particle at time  $t = 4$ .
- (d) Find the distance traveled by the particle from time  $t = 2$  to  $t = 4$ .

(a)  $\left. \frac{dx}{dt} \right|_{t=2} = \frac{2}{e^2}$

Because  $\left. \frac{dx}{dt} \right|_{t=2} > 0$ , the particle is moving to the right at time  $t = 2$ .

$$\left. \frac{dy}{dx} \right|_{t=2} = \frac{\left. dy/dt \right|_{t=2}}{\left. dx/dt \right|_{t=2}} = 3.055 \text{ (or 3.054)}$$

3 :  $\begin{cases} 1 : \text{moving to the right with reason} \\ 1 : \text{considers } \frac{dy/dt}{dx/dt} \\ 1 : \text{slope at } t = 2 \end{cases}$

(b)  $x(4) = 1 + \int_2^4 \frac{\sqrt{t+2}}{e^t} dt = 1.253 \text{ (or 1.252)}$

2 :  $\begin{cases} 1 : \text{integral} \\ 1 : \text{answer} \end{cases}$

(c) Speed =  $\sqrt{(x'(4))^2 + (y'(4))^2} = 0.575 \text{ (or 0.574)}$

$$\begin{aligned} \text{Acceleration} &= \langle x''(4), y''(4) \rangle \\ &= \langle -0.041, 0.989 \rangle \end{aligned}$$

2 :  $\begin{cases} 1 : \text{speed} \\ 1 : \text{acceleration} \end{cases}$

(d) Distance =  $\int_2^4 \sqrt{(x'(t))^2 + (y'(t))^2} dt$   
 $= 0.651 \text{ (or 0.650)}$

2 :  $\begin{cases} 1 : \text{integral} \\ 1 : \text{answer} \end{cases}$