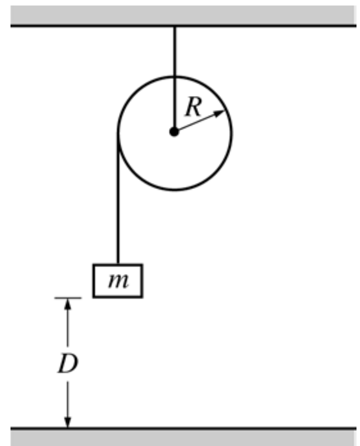


Mech. 2.

A solid disk of unknown mass and known radius R is used as a pulley in a lab experiment, as shown above. A small block of mass m is attached to a string, the other end of which is attached to the pulley and wrapped around it several times. The block of mass m is released from rest and takes a time t to fall the distance D to the floor.

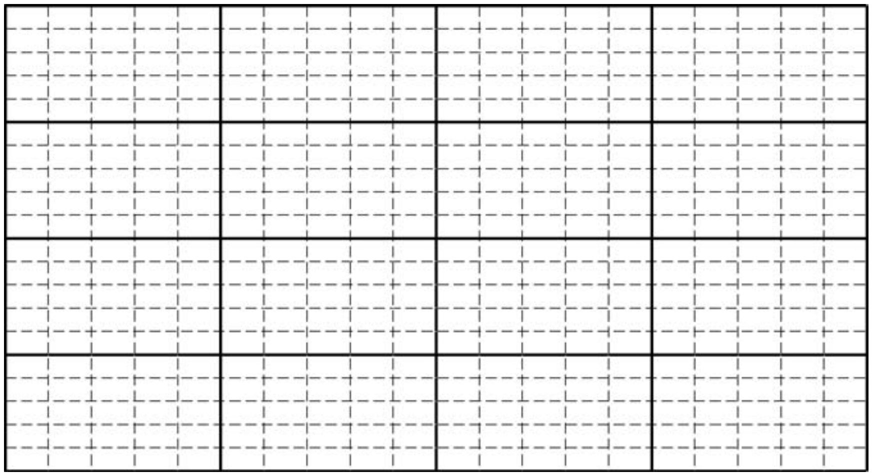
(a) Calculate the linear acceleration a of the falling block in terms of the given quantities.

(b) The time t is measured for various heights D and the data are recorded in the following table.

D (m)	t (s)
0.5	0.68
1	1.02
1.5	1.19
2	1.38

i. What quantities should be graphed in order to best determine the acceleration of the block? Explain your reasoning.

ii. On the grid below, plot the quantities determined in (b)i., label the axes, and draw the best-fit line to the data.



iii. Use your graph to calculate the magnitude of the acceleration.

(c) Calculate the rotational inertia of the pulley in terms of m, R, a, and fundamental constants.

(d) The value of acceleration found in (b)iii, along with numerical values for the given quantities and your answer to (c), can be used to determine the rotational inertia of the pulley. The pulley is removed from its support and its rotational inertia is found to be greater than this value. Give one explanation for this discrepancy.

Question 2

15 points total — **Distribution of points**

(a) 2 points

Using the kinematic equation:

$x = \upsilon_0 t + \frac{1}{2}at^2$

For applying this equation with $x = D$ and $\upsilon_0 = 0$ — 1 point

$D = \frac{1}{2}at^2$

For the correct answer — 1 point

$a = \frac{2D}{t^2}$

(b)

i. 2 points

For indicating a correct pair of quantities to graph — 1 point

For a correct explanation — 1 point

Method 1: Graph quantities such as D versus t^2 or υ versus t. Such graphs are linear and the slope will yield the acceleration.

Method 2: Graph quantities such as a versus t or a versus x, using the equation from part (a) to determine the accelerations. Such graphs are linear and the a intercept will yield the acceleration.

Method 3: Graph D versus t, which is parabolic. The equation that describes the best fit curve is quadratic, and the second derivative is the acceleration.

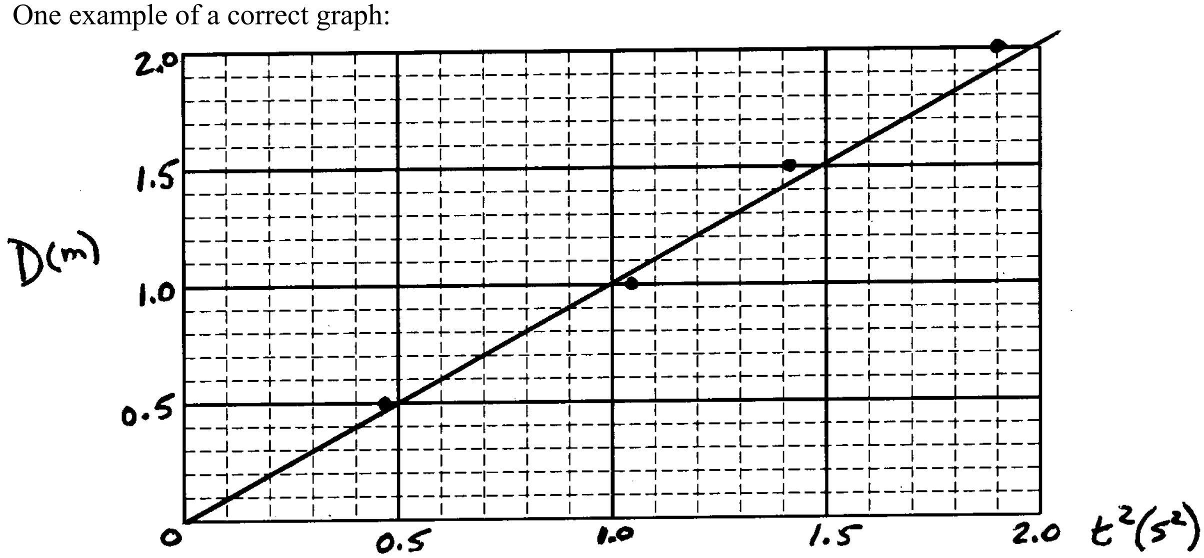
Question 2 (continued)

Distribution of points

(b) (continued)

ii. 3 points

One example of a correct graph:



For using data consistent with the answer to (b) i. — 1 point

For plotting the data correctly, including labeling the axes and their scales — 1 point

For constructing a best fit line or curve, depending on the data graphed — 1 point

iii. 2 points

For using points from the best fit line, not data points — 1 point

For correctly determining the acceleration, including correct units and explicitly showing how the points yield the answer — 1 point

For example, using the graph shown:

$$a = 2(\text{slope}) = 2\left(\frac{2.0\text{ m} - 0.5\text{ m}}{1.97\text{ s}^2 - 0.5\text{ s}^2}\right) = 2\left(\frac{1.5\text{ m}}{1.47\text{ s}^2}\right) = 2.04\text{ m/s}^2$$

Question 2 (continued)

Distribution of points

(c) 4 points

For a correct torque equation that includes the tension T in the string — 1 point

$TR = I\alpha$

For the correct relationship between α and a — 1 point

$\alpha = a/R$

Substituting for α and solving for I:

$$TR = I\frac{a}{R}$$

$$I = TR^2/a$$

For correctly applying Newton's second law for the block — 1 point

$ma = mg - T$

Solving for T:

$T = m(g - a)$

For the correct answer, obtained for substituting T into the equation for I — 1 point

$$I = \frac{m(g-a)R^2}{a} \quad \text{or} \quad I = mR^2\left(\frac{g}{a} - 1\right)$$

Alternate solution — *Alternate points*

For a correct application of conservation of energy for the system — *1 point*

The change in potential energy of the block equals the change in linear kinetic energy of the block plus the change in rotational kinetic energy of the pulley

$$mgD = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$$

For the correct relationship between the rotational and linear speeds — *1 point*

$$\omega = \frac{v}{R}$$

For a correct relationship between the linear speed, the acceleration, and the distance D — *1 point*

$v^2 = 2aD$

Substituting the last two expressions into the energy equation:

$$mgD = \frac{1}{2}mv^2 + \frac{1}{2}I\frac{v^2}{R^2} = \frac{1}{2}v^2\left(m + \frac{I}{R^2}\right)$$

$$mgD = \frac{1}{2}(2aD)\left(m + \frac{I}{R^2}\right)$$

For the correct answer — *1 point*

$$I = \frac{m(g-a)R^2}{a} \quad \text{or} \quad I = mR^2\left(\frac{g}{a} - 1\right)$$

Question 2 (continued)

	Distribution of points
(d) 2 points	
For a reasonable analysis of the answer to part (c) that explicitly indicates either the effective mass or radius is greater than given in the initial paragraph of the question, or the experimental acceleration is obtained in (b) iii is greater than it would be without experimental error.	1 point
For a possible physical reason for the discrepant value	1 point

Examples: The string was wrapped around the pulley several times, causing the effective radius at which the torque acted to be larger than the radius of the pulley used in the calculation.

The string slipped on the pulley, allowing the block to accelerate faster than it would have otherwise, resulting in a smaller experimental moment of inertia.

Note: Friction is not a correct answer, since the presence of friction would make the experimental value of the moment of inertia too large.