2. The table below shows the political party registration by gender of all 500 registered voters in Franklin Township.

PARTY REGISTRATION–FRANKLIN TOWNSHIP

	Party W	Party X	Party Y	Total
Female	60	120	120	300
Male	28	124	48	200
Total	88	244	168	500

(a) Given that a randomly selected registered voter is a male, what is the probability that he is registered for Party Y?

(b) Among the registered voters of Franklin Township, are the events "is a male" and "is registered for Party Y" independent? Justify your answer based on probabilities calculated from the table above.

(c) One way to display the data in the table is to use a segmented bar graph. The following segmented bar graph, constructed from the data in the party registration–Franklin Township table, shows party-registration distributions for males and females in Franklin Township.
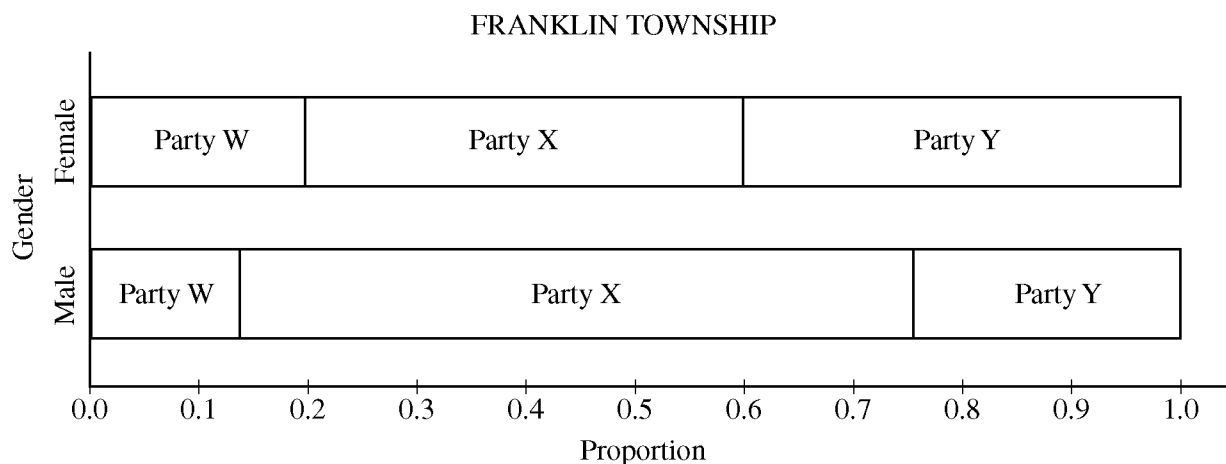
FRANKLIN TOWNSHIP

In Lawrence Township, the proportions of all registered voters for Parties W, X, and Y are the same as for Franklin Township, and party registration is independent of gender. Complete the graph below to show the distributions of party registration by gender in Lawrence Township.

LAWRENCE TOWNSHIP

GO ON TO THE NEXT PAGE.
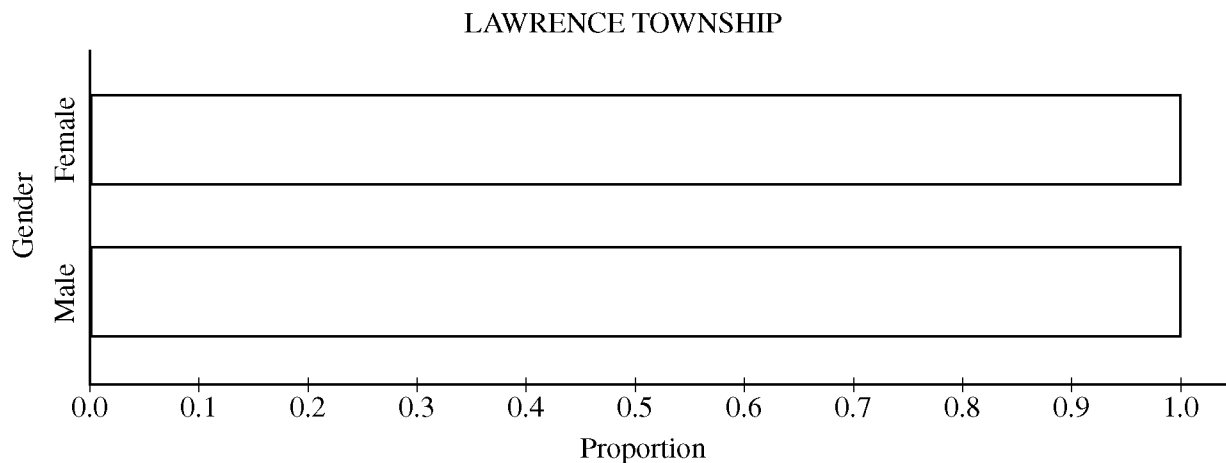
3. An apartment building has nine floors and each floor has four apartments. The building owner wants to install new carpeting in eight apartments to see how well it wears before she decides whether to replace the carpet in the entire building.

The figure below shows the floors of apartments in the building with their apartment numbers. Only the nine apartments indicated with an asterisk (*) have children in the apartment.

11*	12
1st Floor	
14	13

21	22*
2nd Floor	
24	23*

31	32
3rd Floor	
34	33

41	42
4th Floor	
44	43

51*	52
5th Floor	
54	53

61	62
6th Floor	
64	63

* = Children in the apartment

71	72
7th Floor	
74*	73*

81	82
8th Floor	
84*	83

91	92*
9th Floor	
94	93*

(a) For convenience, the apartment building owner wants to use a cluster sampling method, in which the floors are clusters, to select the eight apartments. Describe a process for randomly selecting eight different apartments using this method.

(b) An alternative sampling method would be to select a stratified random sample of eight apartments, where the strata are apartments with children and apartments with no children. A stratified random sample of size eight might include two randomly selected apartments with children and six randomly selected apartments with no children. In the context of this situation, give one statistical advantage of selecting such a stratified sample as opposed to a cluster sample of eight apartments using the floors as clusters.

GO ON TO THE NEXT PAGE.

Question 2

Intent of Question

The primary goals of this question were to assess students' ability to (1) determine a conditional probability from a table of data; (2) use a table of data to determine whether or not two events are independent; (3) demonstrate an understanding of the concept of independence by constructing a graph that displays independence between two variables.

Solution

Part (a):

Of the 200 male registered voters in Franklin Township, 48 are registered for Party Y. Therefore the conditional probability that a randomly selected voter is registered for Party Y, given that the voter is a male, is $\frac{48}{200} = 0.24.$

Part (b):

No, the events "is a male" and "is registered for Party Y" are not independent. One justification of this conclusion is to note that the conditional probability of the event "is registered for Party Y" given the event "is a male" — which was computed in part (a) — is not equal to the probability of the event "is registered for Party Y," as shown below.

$$P\left(\text{is registered for Party Y} \mid \text{is a male}\right) = 0.24$$

$$P\left(\text{is registered for Party Y}\right) = \frac{168}{500} = 0.336$$

Because $0.24 \neq 0.336,$ the two events are not independent.

Part (c):

The marginal proportions of voters registered for each of the three political parties (without regard to gender) are given below.

$$\text{Party W: } \frac{88}{500} = 0.176$$

$$\text{Party X: } \frac{244}{500} = 0.488$$

$$\text{Party Y: } \frac{168}{500} = 0.336$$

Because party registration is independent of gender in Lawrence Township, the proportions of males and females registered for each party must be identical to each other and also identical to the marginal proportion of voters registered for that party. Using the order Party W, Party X, and Party Y, the graph for Lawrence Township is displayed below.

LAWRENCE TOWNSHIP
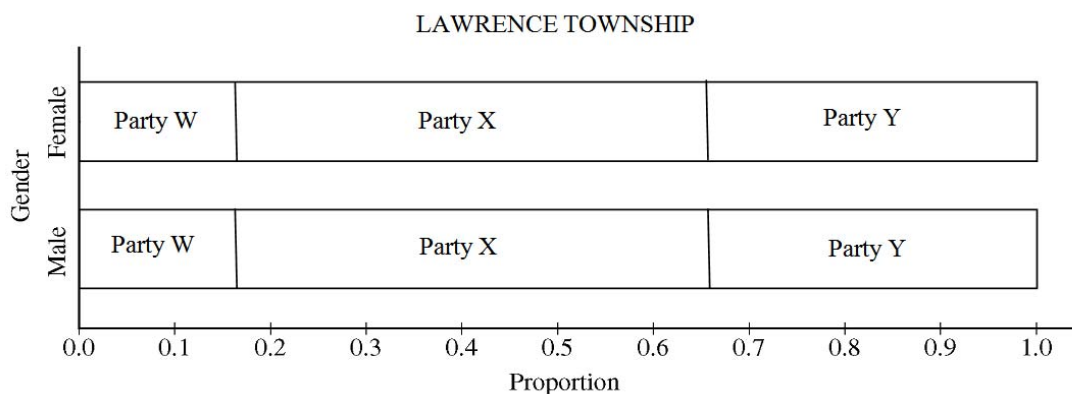
Scoring

Parts (a), (b) and (c) are scored as essentially correct (E), partially correct (P) or incorrect (I).

Part (a) is scored as follows:

Essentially correct (E) if the response has the correct conditional probability *AND* shows the work.

Partially correct (P) if the response has the correct reverse conditional probability (of being a male given that he is registered for Party Y),
    OR
if the response has the correct conditional probability *BUT* does not show work.

Incorrect (I) if the response fails to meet the criteria for E or P.

Part (b) is scored as follows:

Essentially correct (E) if the response identifies two values whose inequality implies a lack of independence between the events *AND* includes the following three components:
  1. Correct computations of the two values.
  2. An explicit statement of whether the two values are equal or unequal.
  3. An appropriate conclusion about the independence of the events.

Partially correct (P) if the response identifies two values whose inequality implies a lack of independence between the events but includes only two of the three components listed above.

Incorrect (I) if the response fails to meet the criteria for E or P.

Part (c) is scored as follows:

Essentially correct (E) if the response shows the same conditional distribution of party registration for both males and females *AND* includes the following two components:
  1. Correct proportions for each party.
  2. Correct labels (Party W, Party X, Party Y).