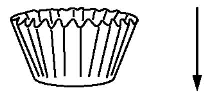
PHYSICS C: MECHANICS
SECTION II
Time—45 minutes
3 Questions

Directions: Answer all three questions. The suggested time is about 15 minutes for answering each of the questions, which are worth 15 points each. The parts within a question may not have equal weight. Show all your work in the pink booklet in the spaces provided after each part, NOT in this green insert.



Mech. 1.

Students are to conduct an experiment to investigate the relationship between the terminal speed of a stack of falling paper coffee filters and its mass. Their procedure involves stacking a number of coffee filters, like the one shown in the figure above, and dropping the stack from rest. The students change the number of filters in the stack to vary the mass m while keeping the shape of the stack the same. As a stack of coffee filters falls, there is an air resistance (drag) force acting on the filters.

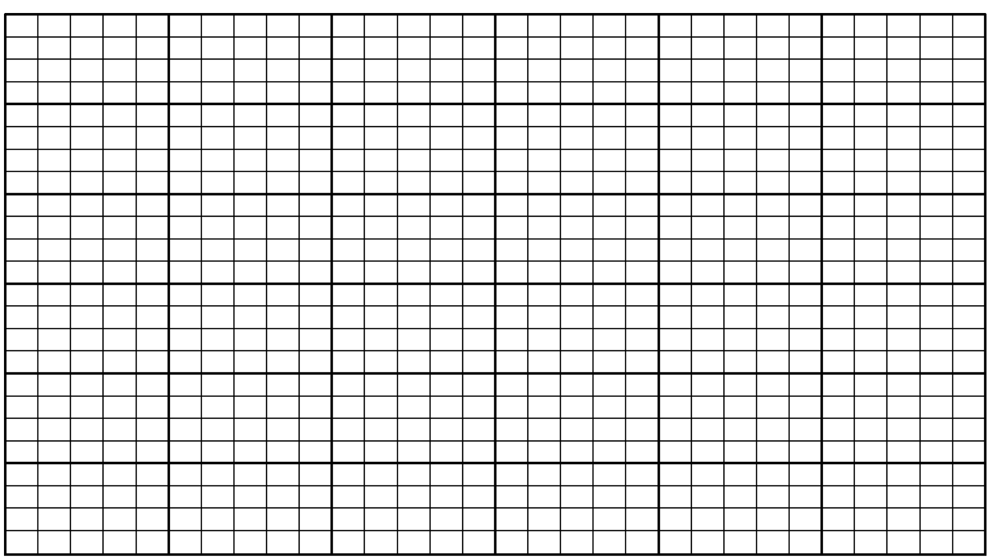
(a) The students suspect that the drag force F_D is proportional to the square of the speed v : $F_D = Cv^2$, where C is a constant. Using this relationship, derive an expression relating the terminal speed v_T to the mass m.

The students conduct the experiment and obtain the following data.

Mass of the stack of filters, m (kg)	1.12×10^{-3}	2.04×10^{-3}	2.96×10^{-3}	4.18×10^{-3}	5.10×10^{-3}
Terminal speed, v_T (m/s)	0.51	0.62	0.82	0.92	1.06

(b)

(i) Assuming the functional relationship for the drag force above, use the grid below to plot a linear graph as a function of m to verify the relationship. Use the empty boxes in the data table, as appropriate, to record any calculated values you are graphing. Label the vertical axis as appropriate, and place numbers on both axes.



m (kg)

(ii) Use your graph to calculate C.

A particular stack of filters with mass m is dropped from rest and reaches a speed very close to terminal speed by the time it has fallen a vertical distance Y.

(c)

(i) Sketch an approximate graph of speed versus time from the time the filters are released up to the time $t = T$ that the filters have fallen the distance Y. Indicate time $t = T$ and terminal speed $v = v_T$ on the graph.

(ii) Suppose you had a graph like the one sketched in (c)(i) that had a numerical scale on each axis. Describe how you could use the graph to approximate the distance Y.

(d) Determine an expression for the approximate amount of mechanical energy dissipated, ΔE, due to air resistance during the time the stack falls a distance y, where $y > Y$. Express your answer in terms of y, m, v_T, and fundamental constants.

GO ON TO THE NEXT PAGE.

Question 1

15 points total **Distribution of points**

(a) 2 points

Starting with Newton's second law:

$F_{net} = mg - Cv^2 = ma$

For correctly indicating that at terminal velocity ($v = v_T$), the net force and acceleration are zero — 1 point

$mg - Cv_T^2 = 0$

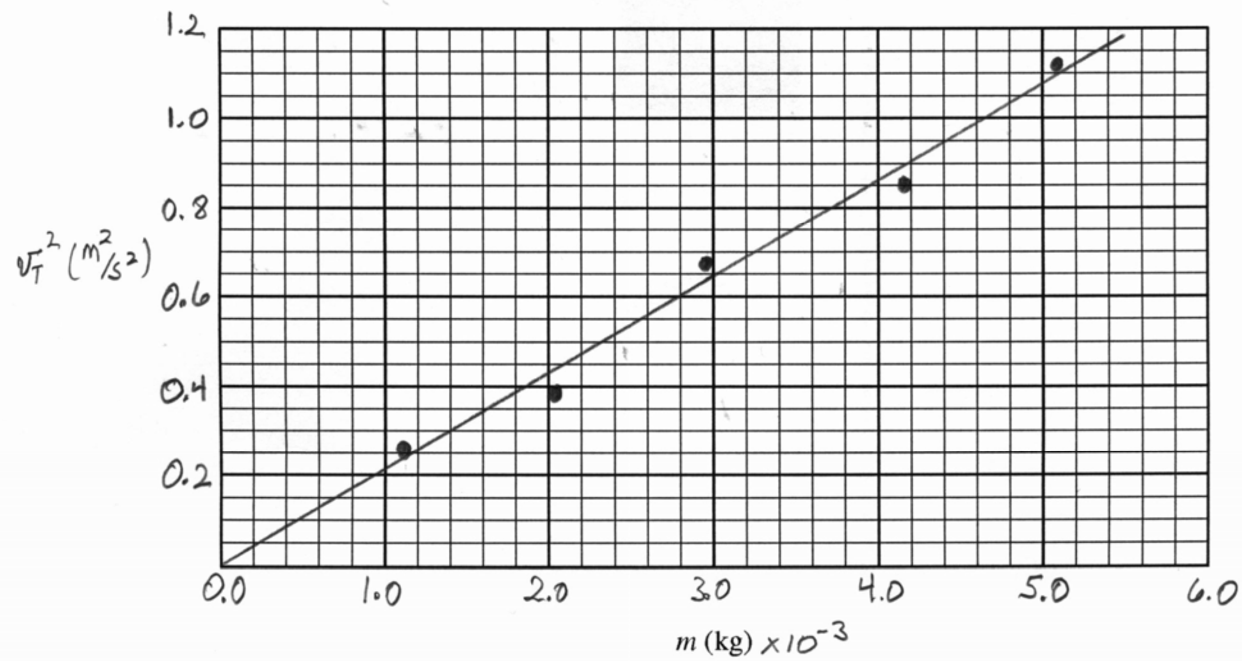
For a correct relationship between v_T and m — 1 point

$v_T^2 = \frac{g}{C}m$

(b)

(i) 4 points

Mass of the stack of filters, m (kg)	1.12×10^{-3}	2.04×10^{-3}	2.96×10^{-3}	4.18×10^{-3}	5.10×10^{-3}
Terminal speed, v_T (m/s)	0.51	0.62	0.82	0.92	1.06
v_T^2 (m^2/s^2)	0.26	0.38	0.67	0.85	1.12



Question 1 (continued)

Distribution of points

(b) (i) (continued)

For recording in the table a row of calculated data points that involve only v_T or a combination of v_T and g — 1 point

For graphing v_T^2 (or the equivalent) on the vertical axis to obtain a linear graph — 1 point

For including an appropriate linear scale on both axes — 1 point

For drawing a reasonable best-fit straight line — 1 point

Note: Correct graphs of v_T versus $\sqrt{m}$ are given credit provided the quantities being plotted are clearly indicated.

(ii) 3 points

For a correct calculation of the slope, using points on the student's best-fit line (not points from the data table) — 1 point

Using the example graph shown:

$$\text{slope} = \frac{(0.95\ - 0.30\ )\text{m}^2/\text{s}^2}{\left[(4.4 - 1.4)\times 10^{-3}\right]\text{kg}} = 217\ \text{m}^2/\text{kg}\bullet\text{s}^2$$

For a correct expression relating the slope to C — 1 point

$$\text{slope} = 217\text{m}^2/\text{kg}\bullet\text{s}^2 = \frac{g}{C}$$

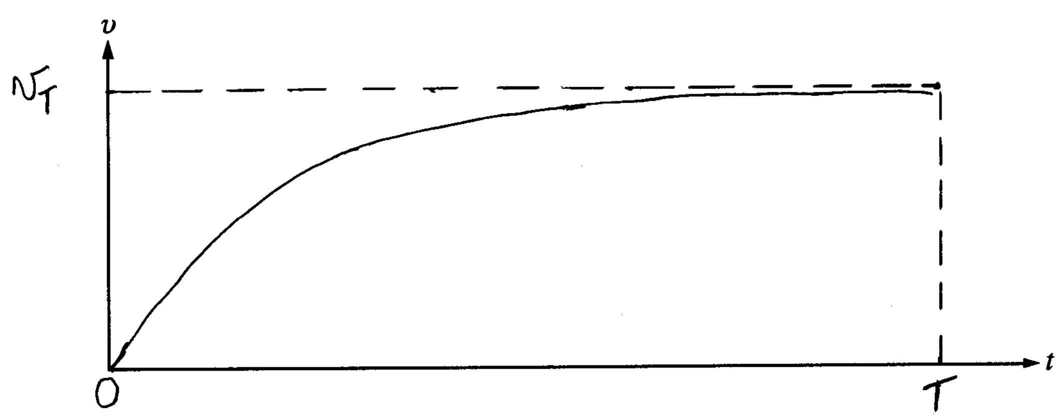
$$C = \frac{g}{\text{Slope}} = \frac{9.8\ \text{m}/\text{s}^2}{217\ \text{m}^2/\text{kg}\bullet\text{s}^2}$$

For correct units on C — 1 point

$C = 0.045\ \text{kg/m}$ $\quad (0.046\ \text{kg/m}$ using $g = 10\ \text{m}/\text{s}^2)$

(c)

(i) 3 points



For a graph that starts at the origin with an initial positive slope — 1 point

For a graph that is concave down throughout — 1 point

Question 1 (continued)

	Distribution of points
For labeling v_T and T at a point where the slope of the graph approaches zero	1 point

(c) (continued)

(ii) 1 point

For a verbal statement that clearly indicates the distance Y is the area under the curve between times $t = 0$ and $t = T$	1 point

Or an equivalent statement referring to the area under the curve when the v versus t graph ends at T

Or an equivalent mathematical expression with limits: $Y = \int_0^T v(t)\, dt$

(d) 2 points

For a correct indication that the mechanical energy dissipated is the change in mechanical energy during the time the stack falls a distance y	1 point

$\Delta E = \left(U_{final} + K_{final}\right) - \left(U_{initial} + K_{initial}\right) \quad \left(\text{or } \Delta E = \int_0^y Cv^2 dy\right)$

$\Delta E = \Delta U + \Delta K$

For correct substitutions of y, m, and v_T	1 point

$\Delta U = -mgy$

$\Delta K = \frac{1}{2}mv_T^2$

$\Delta E = \frac{1}{2}mv_T^2 - mgy$ (or any equation with an additional negative sign that has the correct relative signs for potential and kinetic energies)