STATISTICS

Section II

Part B

Question 6

Spend about 25 minutes on this part of the exam.

Percent of Section II grade—25

Directions: Show all your work. Indicate clearly the methods you use, because you will be graded on the correctness of your methods as well as on the accuracy of your results and explanation.

6. A pharmaceutical company has developed a new drug to reduce cholesterol. A regulatory agency will recommend the new drug for use if there is convincing evidence that the mean reduction in cholesterol level after one month of use is more than 20 milligrams/deciliter (mg/dl), because a mean reduction of this magnitude would be greater than the mean reduction for the current most widely used drug.

   The pharmaceutical company collected data by giving the new drug to a random sample of 50 people from the population of people with high cholesterol. The reduction in cholesterol level after one month of use was recorded for each individual in the sample, resulting in a sample mean reduction and standard deviation of 24 mg/dl and 15 mg/dl, respectively.

   (a) The regulatory agency decides to use an interval estimate for the population mean reduction in cholesterol level for the new drug. Provide this 95 percent confidence interval. Be sure to interpret this interval.

   (b) Because the 95 percent confidence interval includes 20, the regulatory agency is not convinced that the new drug is better than the current best-seller. The pharmaceutical company tested the following hypotheses.

   $$H_0\colon\ \mu = 20 \text{ versus } H_a\colon\ \mu > 20,$$

   where μ represents the population mean reduction in cholesterol level for the new drug.

   The test procedure resulted in a t-value of 1.89 and a p-value of 0.033. Because the p-value was less than 0.05, the company believes that there is convincing evidence that the mean reduction in cholesterol level for the new drug is more than 20. Explain why the confidence interval and the hypothesis test led to different conclusions.

   (c) The company would like to determine a value L that would allow them to make the following statement.

   We are 95 percent confident that the true mean reduction in cholesterol level is greater than L.

   A statement of this form is called a one-sided confidence interval. The value of L can be found using the following formula.

   $$L = \bar{x} - t^{*}\frac{s}{\sqrt{n}}$$

   This has the same form as the lower endpoint of the confidence interval in part (a), but requires a different critical value, t^{*}. What value should be used for t^{*}?

   Recall that the sample mean reduction in cholesterol level and standard deviation are 24 mg/dl and 15 mg/dl, respectively. Compute the value of L.

   (d) If the regulatory agency had used the one-sided confidence interval in part (c) rather than the interval constructed in part (a), would it have reached a different conclusion? Explain.

END OF EXAMINATION

Table entry for z is the probability lying below z.

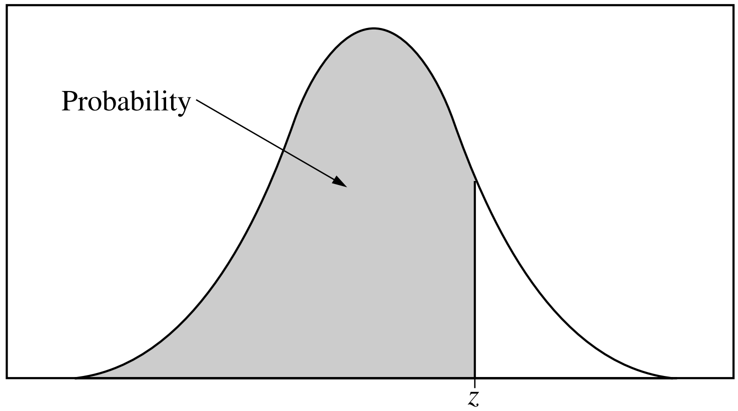

Table A (***Continued***)

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998

Question 6

Solution

Part (a):

Step 1: States and checks appropriate conditions.

We are told that the sample was randomly selected. Since the sample size is large (e.g., n =50 > 30), the one sample t interval should be valid. Alternatively, we could assume that the reduction in cholesterol level after one month is (at least approximately) normally distributed, but we have no way to check this assumption with the information provided.

Step 2: Identifies the appropriate confidence interval by name or formula.

One sample t interval for μ, the mean reduction in cholesterol for the new drug or $\bar{x} \pm t^*_{n-1} \dfrac{s}{\sqrt{n}}$.

Step 3: Correct mechanics.

Degrees of freedom = n-1 = 49.

$$\bar{x} \pm t^*_{n-1} \frac{s}{\sqrt{n}} = 24 \pm 2.0096 \frac{15}{\sqrt{50}} = 24 \pm 4.2631 = (19.7369, 28.2631).$$

Step 4: Interprets the confidence interval in context.

We are 95% confident that the mean reduction in cholesterol for the new drug in the population of people with high cholesterol is between 19.74 and 28.26 mg/dl.

Part (b):

The decision based on a 95% confidence interval only corresponds to the two-sided test of significance at the 5% level, not necessarily the one-sided test. The confidence interval in (a) is equivalent to testing H_0: $\mu = 20$ against H_a: $\mu \neq 20$. In this test, the tail probability would be doubled, and this two-sided p-value, .066, is larger than .05, failing to reject the null hypothesis. However, in testing H_0: $\mu = 20$ against H_a: $\mu > 20$, the one-sided p-value of .033 is small enough to reject H_0 at the 5% level.

Alternatively, if we had compared the p-value of .033 to an alpha level of .025, the conclusions would match.

Question 6 (cont'd.)

Part (c):

The critical value for the lower confidence bound is the 95th percentile (instead of the 97.5th percentile) of the *t* distribution with 49 degrees of freedom.

$$t^* = 1.676$$

and

$$L = 24 - 1.676\frac{15}{\sqrt{50}} = 24 - 3.5553 = 20.4447$$

Part (d):

Yes, the decision would change. Since the lower bound *L* is more than 20, the agency would now be convinced that μ is greater than 20 and the new drug is statistically significantly better than the current drug.

Scoring

Part (a) is scored as

Essentially correct if all four steps (check conditions, identify procedure, calculate interval, interpret interval) are correct. Each step is scored as correct or incorrect, no partial credit is given for the steps.

- Ok if only state "*n* is large" or if assume population distribution is normal with some justification/recognition that this is only an assumption.
- Student can use either *t* interval or *z* interval but needs to give name of procedure (state *t* or *z* somewhere) or provide critical value.
- A correct interpretation of the confidence level does not count for step 4. An incorrect interpretation of level prevents credit for correct interpretation of interval.

Partially correct if two or three steps are correct.

Incorrect if at most one step is correct.

Part (b) is scored as

Essentially correct if the student discussion includes:

1. the confidence interval is two-sided and the test is one-sided, and
2. a quantitative linkage between the procedures (e.g., doubling the p-value, halving the level of significance, using 90% as the confidence level [with clear connection to the $\alpha = .05$ significance level]).

Question 6 (cont'd.)

Partially correct if

- Student states only that the confidence interval is two-sided and the test of significance is one-sided.
- Student has the right concept but discussion is very poor.

Incorrect if

- Student solution is only a restatement of the conclusions of each procedure.
- Student solution is only a restatement that the results differ.
- Student solution states that a 90% confidence interval is shorter and would not include 20 (but gives no indication of how 90% relates to the level of significance of the test) or that they might not differ if a different level of α was used (with no value stated).
- Student says confidence intervals and test are different procedures (e.g., vary in specificity) and lead to different conclusions.
- Student says one uses the t value and the other uses the z value.

Note: Students may refer to the duality between confidence intervals and hypothesis tests in their solutions, but they must describe this duality in order to receive credit.

Part (c) is scored as

Essentially correct if

1. a reasonable one-sided value of t^* is given ($\pm$ 1.676 or $\pm$ 1.684), and
2. L is calculated correctly using the value of t^* provided (20.445 or 20.428).

Partially correct if the student identifies an incorrect reasonable critical value (z^*, two-sided t^* but see note below, or wrong df) and then uses this value to calculate L.

Incorrect if the student identifies a nonsensical critical value (e.g., uses the test statistic, the p-value, α, $t^*/2$, 20) and/or obtains a lower bound larger than the sample mean (unless it is from an incorrectly substituted one-sided t^* value).

Part (d) is scored as

Essentially correct if

1. the student correctly justifies whether or not the conclusion has changed, and
2. the student makes a correct conclusion from the one-sided confidence interval and supports their conclusion (e.g., compares L to the value of 20, believes $\mu \geq L$, is 20 in the interval).

Partially correct if student gives only component 2 or has component 1 with weak justification.

Incorrect if an answer (yes or no) is provided with no reasonable explanation.

NOTE: If the student's work in part (c) is merely a reworking of the lower endpoint in (a) and they obtain the same lower bound and comment that the calculation is the same, they will receive at most one point among parts (a), (c) and (d). "ACD Rule"