

Graph of  $f$ 

4. Let  $f$  be a continuous function defined on the closed interval  $-4 \leq x \leq 6$ . The graph of  $f$ , consisting of four line segments, is shown above. Let  $G$  be the function defined by  $G(x) = \int_0^x f(t) dt$ .
- On what open intervals is the graph of  $G$  concave up? Give a reason for your answer.
  - Let  $P$  be the function defined by  $P(x) = G(x) \cdot f(x)$ . Find  $P'(3)$ .
  - Find  $\lim_{x \rightarrow 2} \frac{G(x)}{x^2 - 2x}$ .
  - Find the average rate of change of  $G$  on the interval  $[-4, 2]$ . Does the Mean Value Theorem guarantee a value  $c$ ,  $-4 < c < 2$ , for which  $G'(c)$  is equal to this average rate of change? Justify your answer.

**Write your responses to this question only on the designated pages in the separate Free Response booklet. Write your solution to each part in the space provided for that part.**

5. Let  $y = f(x)$  be the particular solution to the differential equation  $\frac{dy}{dx} = y \cdot (x \ln x)$  with initial condition  $f(1) = 4$ . It can be shown that  $f''(1) = 4$ .
- (a) Write the second-degree Taylor polynomial for  $f$  about  $x = 1$ . Use the Taylor polynomial to approximate  $f(2)$ .
- (b) Use Euler's method, starting at  $x = 1$  with two steps of equal size, to approximate  $f(2)$ . Show the work that leads to your answer.
- (c) Find the particular solution  $y = f(x)$  to the differential equation  $\frac{dy}{dx} = y \cdot (x \ln x)$  with initial condition  $f(1) = 4$ .

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