

Begin your response to **QUESTION 4** on this page.

4. In an online game, players move through a virtual world collecting geodes, a type of hollow rock. When broken open, these geodes contain crystals of different colors that are useful in the game. A red crystal is the most useful crystal in the game. The color of the crystal in each geode is independent and the probability that a geode contains a red crystal is 0.08.

(a) Sarah, a player, will collect and open geodes until a red crystal is found.

(i) Calculate the mean of the distribution of the number of geodes Sarah will open until a red crystal is found.
Show your work.

(ii) Calculate the standard deviation of the distribution of the number of geodes Sarah will open until a red crystal is found. Show your work.

(b) Another player, Conrad, decides to play the game and will stop opening geodes after finding a red crystal or when 4 geodes have been opened, whichever comes first. Let Y = the number of geodes Conrad will open. The table shows the partially completed probability distribution for the random variable Y .

Number of geodes Conrad will open, y	1	2	3	4
Probability, $P(Y = y)$	0.08	0.0736		

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Continue your response to **QUESTION 4** on this page.

(i) Calculate $P(Y = 3)$. Show your work.

(ii) Calculate $P(Y = 4)$. Show your work.

(c) Consider the table and your results from part (b).

(i) Calculate the mean of the distribution of the number of geodes Conrad will open. Show your work.

(ii) Interpret the mean of the distribution of the number of geodes Conrad will open, which was calculated in part (c-i).

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Begin your response to **QUESTION 5** on this page.

5. Baseball cards are trading cards that feature data on a player's performance in baseball games. Michelle is at a national baseball card collector's convention with approximately 20,000 attendees. She notices that some collectors have both regular cards, which are easily obtained, and rare cards, which are harder to obtain. Michelle believes that there is a relationship between the number of months a collector has been collecting baseball cards and whether the majority of the cards (cards appearing more often) in their collection are regular or rare. She obtains information from a random sample of 500 baseball card collectors at the convention and records how many full months they have been collecting baseball cards and whether the majority of the cards in their card collection are regular or rare. Her results are displayed in a two-way table.

Majority Type of Baseball Cards and Months of Collecting Baseball Cards

	Fewer Than 6 Months	6 - 10 Months	11 - 15 Months	16 - 20 Months	21 or More Months	Total
Has a Majority of Regular Baseball Cards	80	84	71	76	112	423
Has a Majority of Rare Baseball Cards	11	16	9	6	35	77
Total	91	100	80	82	147	500

- (a) If one collector from the sample is selected at random, what is the probability that the collector has been collecting baseball cards for 11 or more months and has a majority of regular baseball cards? Show your work.
- (b) Given that a randomly selected collector from the sample has been collecting baseball cards for fewer than 6 months, what is the probability the collector has a majority of regular baseball cards? Show your work.

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Question 4: Focus on Probability and Sampling Distributions**4 points****General Scoring Notes**

- Each part of the question (indicated by a letter) is initially scored by determining if it meets the criteria for essentially correct (E), partially correct (P), or incorrect (I). The response is then categorized based on the scores assigned to each letter part and awarded an integer score between 0 and 4 (see the table at the end of the question).
- The model solution represents an ideal response to each part of the question, and the scoring criteria identify the specific components of the model solution that are used to determine the score.

Model Solution	Scoring
(a) Let G represent the number of geodes a player opens until finding a red crystal. G follows a geometric distribution with $p = 0.08$.	Essentially correct (E) if the response satisfies at least three of the following four components: 1. In part (a-i) the response correctly calculates the mean 2. In part (a-i) the response provides supporting work for the calculation of the correct mean 3. In part (a-ii) the response correctly calculates the standard deviation 4. In part (a-ii) the response provides supporting work for the calculation of the correct standard deviation
(i) $\mu = E(G) = \frac{1}{0.08} \approx 12.5$ geodes	
(ii) $\sigma_G = \frac{\sqrt{1 - 0.08}}{0.08} \approx 11.99$ geodes	

Partially correct (P) if the response satisfies only two of the four components required for E.

Incorrect (I) if the response does not meet the criteria for E or P.

Additional Notes:

- An arithmetic or transcription error in a response can be ignored if correct work is shown.
- A response with the mean rounded to 12 or 13 does not satisfy component 1.
- A response with a standard deviation of 12 geodes satisfies component 3.

Model Solution

(b) (i) $P(Y = 3) = (0.92)^2(0.08) \approx 0.067712$

(ii) $P(Y = 4) = 1 - P(Y = 1 \text{ or } 2 \text{ or } 3)$
 $\approx 1 - (0.08 + 0.0736 + 0.067712)$
 ≈ 0.778688

OR

If Conrad opens 4 geodes, then he either finds no red geodes or he finds a red geode on the fourth one he opens; therefore,

$$P(Y = 4) = (0.92)^4 + (0.92)^3(0.08) \\ \approx 0.778688.$$

Scoring

Essentially correct (E) if the response satisfies at least three of the following four components:

1. In part (b-i) the response correctly calculates the probability of opening 3 geodes
2. In part (b-i) the response provides supporting work for the calculation of the correct probability
3. In part (b-ii) the response correctly calculates the probability of opening 4 geodes, consistent with the response to part (b-i)
4. In part (b-ii) the response provides supporting work for the calculation of the correct probability

Partially correct (P) if the response satisfies only two of the four components required for E.

Incorrect (I) if the response does not meet the criteria for E or P.

Additional Notes:

- A response may satisfy component 2 or component 4 by the following or a combination of the following:
 - Probability formula: Displaying a correct formula for computing the geometric probability, such as:
 - $(1 - 0.08)^2(0.08)$ or $(0.92)^2(0.08)$ for part (b-i)
 - Calculator function syntax: Using calculator function notation with the correct value of the parameter identified, such as:
 - $\text{geompdf}(p = 0.08, x = 3)$ for part (b-i)
 - $1 - \text{geomcdf}(p = 0.08, x = 3)$ for part (b-ii)
 - $\text{binompdf}(n = 4, p = 0.08, x = 0) + \text{geompdf}(p = 0.08, x = 4)$ for part (b-ii)
 - $\text{binompdf}(n = 4, p = 0.92, x = 4) + \text{geompdf}(p = 0.08, x = 4)$ for part (b-ii)
- An arithmetic or transcription error in a response can be ignored if correct work is shown.

Model Solution	Scoring
<p>(c) (i) $\mu = E(Y)$</p> $\approx (1)(0.08) + (2)(0.0736)$ $+ (3)(0.0677) + (4)(0.778688)$ $\approx 0.08 + 0.1472 + 0.2031 + 3.1148$ $\approx 3.545 \text{ geodes.}$ <p>(ii) The mean of 3.545 geodes is the average number of geodes that result from a long run of many, many trials of opening randomly selected geodes and counting the number opened until either a red geode is found or the fourth geode is opened.</p>	<p>Essentially correct (E) if the response satisfies both components 1 and 2 and at least two of components 3–5:</p> <ol style="list-style-type: none"> In part (c-i) the response states the correct mean of the distribution consistent with values calculated in part (b) In part (c-i) the response shows appropriate work to calculate the mean using the values calculated in part (b) In part (c-ii) the interpretation includes the concept of repeating the selection process over a long period of time In part (c-ii) the interpretation includes the concept of an average or mean In part (c-ii) the interpretation includes the context of number of geodes opened <p>Partially correct (P) if the response does not meet the criteria for E but satisfies two or three of components 1–4.</p> <p>Incorrect (I) if the response does not meet the criteria for E or P.</p>

Additional Notes:

- Supporting work for finding the expected value must include at least two of the terms in the equation to show the pattern, such as $1(0.08) + 2(0.0736) + \dots$.
- Calculator notation does not satisfy component 2, such as 1-VAR STATS(L1, L2).
- An arithmetic or transcription error in a response can be ignored if correct work is shown.
- The response $\frac{1}{(.08 + .0736 + .0677 + .0623)} \approx 3.526$ does not satisfy component 1 or 2.
- In part (c-i) if the response has incorrect values in part (b), but uses the values from the correct probability distribution, the response may satisfy components 1 and 2.
- The numerical value of the mean is not required to satisfy components 3–5.