

2017 AP® CALCULUS AB FREE-RESPONSE QUESTIONS

5. Two particles move along the x -axis. For $0 \leq t \leq 8$, the position of particle P at time t is given by

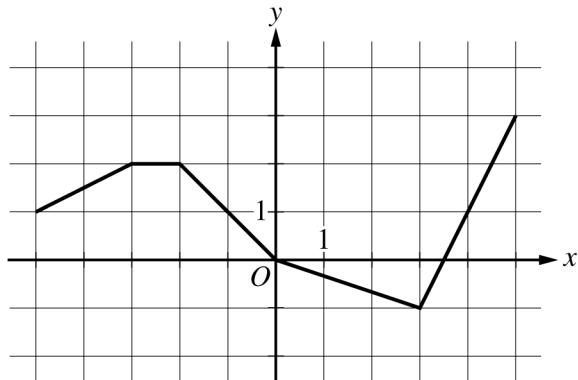
$$x_P(t) = \ln(t^2 - 2t + 10),$$
 while the velocity of particle Q at time t is given by $v_Q(t) = t^2 - 8t + 15.$

Particle Q is at position $x = 5$ at time $t = 0$.

- (a) For $0 \leq t \leq 8$, when is particle P moving to the left?
 - (b) For $0 \leq t \leq 8$, find all times t during which the two particles travel in the same direction.
 - (c) Find the acceleration of particle Q at time $t = 2$. Is the speed of particle Q increasing, decreasing, or neither at time $t = 2$? Explain your reasoning.
 - (d) Find the position of particle Q the first time it changes direction.
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x	$g(x)$	$g'(x)$
-5	10	-3
-4	5	-1
-3	2	4
-2	3	1
-1	1	-2
0	0	-3



Graph of h

6. Let f be the function defined by $f(x) = \cos(2x) + e^{\sin x}$.

Let g be a differentiable function. The table above gives values of g and its derivative g' at selected values of x .

Let h be the function whose graph, consisting of five line segments, is shown in the figure above.

- (a) Find the slope of the line tangent to the graph of f at $x = \pi$.
- (b) Let k be the function defined by $k(x) = h(f(x))$. Find $k'(\pi)$.
- (c) Let m be the function defined by $m(x) = g(-2x) \cdot h(x)$. Find $m'(2)$.
- (d) Is there a number c in the closed interval $[-5, -3]$ such that $g'(c) = -4$? Justify your answer.

STOP

END OF EXAM

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2017 SCORING GUIDELINES

Question 5

(a) $x'_P(t) = \frac{2t - 2}{t^2 - 2t + 10} = \frac{2(t - 1)}{t^2 - 2t + 10}$

$t^2 - 2t + 10 > 0$ for all t .

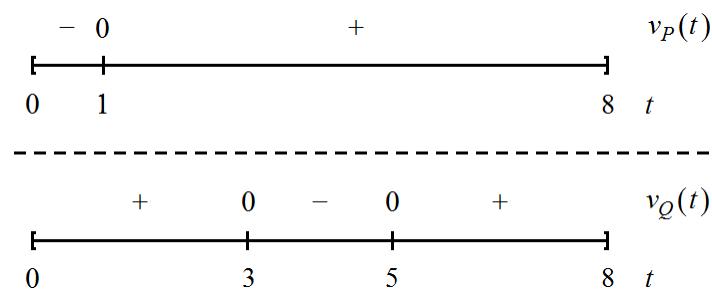
$$x'_P(t) = 0 \Rightarrow t = 1$$

$x'_P(t) < 0$ for $0 \leq t < 1$.

Therefore, the particle is moving to the left for $0 \leq t < 1$.

(b) $v_Q(t) = (t - 5)(t - 3)$

$$v_Q(t) = 0 \Rightarrow t = 3, t = 5$$



Both particles move in the same direction for $1 < t < 3$ and $5 < t \leq 8$ since $v_P(t) = x'_P(t)$ and $v_Q(t)$ have the same sign on these intervals.

(c) $a_Q(t) = v'_Q(t) = 2t - 8$

$$a_Q(2) = 2 \cdot 2 - 8 = -4$$

$$a_Q(2) < 0 \text{ and } v_Q(2) = 3 > 0$$

At time $t = 2$, the speed of the particle is decreasing because velocity and acceleration have opposite signs.

(d) Particle Q first changes direction at time $t = 3$.

$$\begin{aligned} x_Q(3) &= x_Q(0) + \int_0^3 v_Q(t) dt = 5 + \int_0^3 (t^2 - 8t + 15) dt \\ &= 5 + \left[\frac{1}{3}t^3 - 4t^2 + 15t \right]_{t=0}^{t=3} = 5 + (9 - 36 + 45) = 23 \end{aligned}$$

2 : $\begin{cases} 1 : x'_P(t) \\ 1 : \text{interval} \end{cases}$

2 : $\begin{cases} 1 : \text{intervals} \\ 1 : \text{analysis using } v_P(t) \text{ and } v_Q(t) \end{cases}$

Note: 1/2 if only one interval with analysis

Note: 0/2 if no analysis

2 : $\begin{cases} 1 : a_Q(2) \\ 1 : \text{speed decreasing with reason} \end{cases}$

3 : $\begin{cases} 1 : \text{antiderivative} \\ 1 : \text{uses initial condition} \\ 1 : \text{answer} \end{cases}$