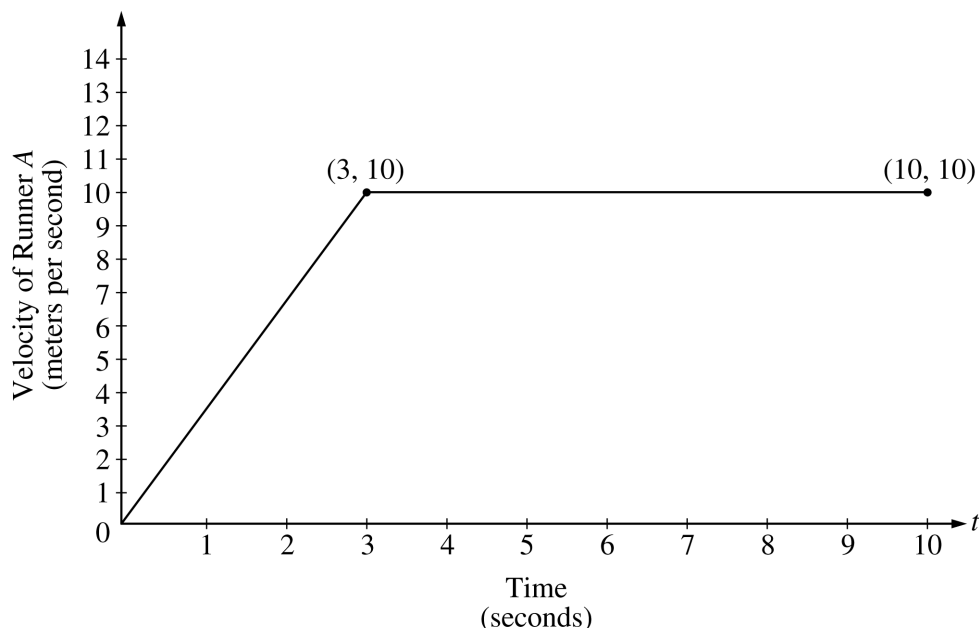


# 2000 AP® CALCULUS BC FREE-RESPONSE QUESTIONS

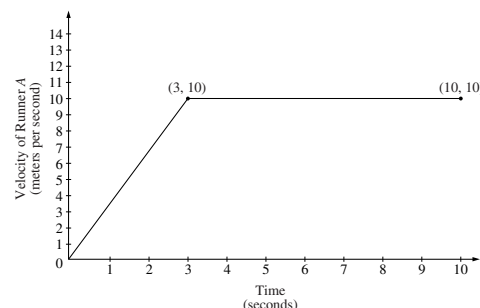


2. Two runners,  $A$  and  $B$ , run on a straight racetrack for  $0 \leq t \leq 10$  seconds. The graph above, which consists of two line segments, shows the velocity, in meters per second, of Runner  $A$ . The velocity, in meters per second, of Runner  $B$  is given by the function  $v$  defined by  $v(t) = \frac{24t}{2t + 3}$ .
- Find the velocity of Runner  $A$  and the velocity of Runner  $B$  at time  $t = 2$  seconds. Indicate units of measure.
  - Find the acceleration of Runner  $A$  and the acceleration of Runner  $B$  at time  $t = 2$  seconds. Indicate units of measure.
  - Find the total distance run by Runner  $A$  and the total distance run by Runner  $B$  over the time interval  $0 \leq t \leq 10$  seconds. Indicate units of measure.

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3. The Taylor series about  $x = 5$  for a certain function  $f$  converges to  $f(x)$  for all  $x$  in the interval of convergence. The  $n$ th derivative of  $f$  at  $x = 5$  is given by  $f^{(n)}(5) = \frac{(-1)^n n!}{2^n (n + 2)}$ , and  $f(5) = \frac{1}{2}$ .
- Write the third-degree Taylor polynomial for  $f$  about  $x = 5$ .
  - Find the radius of convergence of the Taylor series for  $f$  about  $x = 5$ .
  - Show that the sixth-degree Taylor polynomial for  $f$  about  $x = 5$  approximates  $f(6)$  with error less than  $\frac{1}{1000}$ .

**END OF PART A OF SECTION II**

Two runners,  $A$  and  $B$ , run on a straight racetrack for  $0 \leq t \leq 10$  seconds. The graph above, which consists of two line segments, shows the velocity, in meters per second, of Runner  $A$ . The velocity, in meters per second, of Runner  $B$  is given by the function  $v$  defined by  $v(t) = \frac{24t}{2t+3}$ .



- (a) Find the velocity of Runner  $A$  and the velocity of Runner  $B$  at time  $t = 2$  seconds. Indicate units of measure.
- (b) Find the acceleration of Runner  $A$  and the acceleration of Runner  $B$  at time  $t = 2$  seconds. Indicate units of measure.
- (c) Find the total distance run by Runner  $A$  and the total distance run by Runner  $B$  over the time interval  $0 \leq t \leq 10$  seconds. Indicate units of measure.

$$\begin{aligned} \text{(a) Runner } A: \text{ velocity} &= \frac{10}{3} \cdot 2 = \frac{20}{3} \\ &= 6.666 \text{ or } 6.667 \text{ meters/sec} \end{aligned}$$

$$\text{Runner } B: v(2) = \frac{48}{7} = 6.857 \text{ meters/sec}$$

$$\text{(b) Runner } A: \text{ acceleration} = \frac{10}{3} = 3.333 \text{ meters/sec}^2$$

$$\begin{aligned} \text{Runner } B: a(2) = v'(2) &= \left. \frac{72}{(2t+3)^2} \right|_{t=2} \\ &= \frac{72}{49} = 1.469 \text{ meters/sec}^2 \end{aligned}$$

$$\text{(c) Runner } A: \text{ distance} = \frac{1}{2}(3)(10) + 7(10) = 85 \text{ meters}$$

$$\text{Runner } B: \text{ distance} = \int_0^{10} \frac{24t}{2t+3} dt = 83.336 \text{ meters}$$

(units) meters/sec in part (a), meters/sec<sup>2</sup> in part (b), and meters in part (c), or equivalent.

$$2 \left\{ \begin{array}{l} 1: \text{ velocity for Runner } A \\ 1: \text{ velocity for Runner } B \end{array} \right.$$

$$2 \left\{ \begin{array}{l} 1: \text{ acceleration for Runner } A \\ 1: \text{ acceleration for Runner } B \end{array} \right.$$

$$4 \left\{ \begin{array}{l} 2: \text{ distance for Runner } A \\ 1: \text{ method} \\ 1: \text{ answer} \\ 2: \text{ distance for Runner } B \\ 1: \text{ integral} \\ 1: \text{ answer} \end{array} \right.$$

1: units