

2018 AP® CALCULUS AB FREE-RESPONSE QUESTIONS

**CALCULUS AB
SECTION II, Part A
Time—30 minutes
Number of questions—2**

A GRAPHING CALCULATOR IS REQUIRED FOR THESE QUESTIONS.

1. People enter a line for an escalator at a rate modeled by the function r given by

$$r(t) = \begin{cases} 44\left(\frac{t}{100}\right)^3\left(1 - \frac{t}{300}\right)^7 & \text{for } 0 \leq t \leq 300 \\ 0 & \text{for } t > 300, \end{cases}$$

where $r(t)$ is measured in people per second and t is measured in seconds. As people get on the escalator, they exit the line at a constant rate of 0.7 person per second. There are 20 people in line at time $t = 0$.

- How many people enter the line for the escalator during the time interval $0 \leq t \leq 300$?
- During the time interval $0 \leq t \leq 300$, there are always people in line for the escalator. How many people are in line at time $t = 300$?
- For $t > 300$, what is the first time t that there are no people in line for the escalator?
- For $0 \leq t \leq 300$, at what time t is the number of people in line a minimum? To the nearest whole number, find the number of people in line at this time. Justify your answer.

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2. A particle moves along the x -axis with velocity given by $v(t) = \frac{10 \sin(0.4t^2)}{t^2 - t + 3}$ for time $0 \leq t \leq 3.5$.

The particle is at position $x = -5$ at time $t = 0$.

(a) Find the acceleration of the particle at time $t = 3$.

(b) Find the position of the particle at time $t = 3$.

(c) Evaluate $\int_0^{3.5} v(t) \, dt$, and evaluate $\int_0^{3.5} |v(t)| \, dt$. Interpret the meaning of each integral in the context of the problem.

(d) A second particle moves along the x -axis with position given by $x_2(t) = t^2 - t$ for $0 \leq t \leq 3.5$. At what time t are the two particles moving with the same velocity?

END OF PART A OF SECTION II

**AP[®] CALCULUS AB/CALCULUS BC
2018 SCORING GUIDELINES**

Question 1

(a) $\int_0^{300} r(t) dt = 270$

2 : $\begin{cases} 1 : \text{integral} \\ 1 : \text{answer} \end{cases}$

According to the model, 270 people enter the line for the escalator during the time interval $0 \leq t \leq 300$.

(b) $20 + \int_0^{300} (r(t) - 0.7) dt = 20 + \int_0^{300} r(t) dt - 0.7 \cdot 300 = 80$

2 : $\begin{cases} 1 : \text{considers rate out} \\ 1 : \text{answer} \end{cases}$

According to the model, 80 people are in line at time $t = 300$.

- (c) Based on part (b), the number of people in line at time $t = 300$ is 80.

1 : answer

The first time t that there are no people in line is

$$300 + \frac{80}{0.7} = 414.286 \text{ (or } 414.285\text{)} \text{ seconds.}$$

- (d) The total number of people in line at time t , $0 \leq t \leq 300$, is modeled by

$$20 + \int_0^t r(x) dx - 0.7t.$$

4 : $\begin{cases} 1 : \text{considers } r(t) - 0.7 = 0 \\ 1 : \text{identifies } t = 33.013 \\ 1 : \text{answers} \\ 1 : \text{justification} \end{cases}$

$$r(t) - 0.7 = 0 \Rightarrow t_1 = 33.013298, t_2 = 166.574719$$

t	People in line for escalator
0	20
t_1	3.803
t_2	158.070
300	80

The number of people in line is a minimum at time $t = 33.013$ seconds, when there are 4 people in line.