

**2007 AP® CALCULUS AB FREE-RESPONSE QUESTIONS****CALCULUS AB  
SECTION II, Part B****Time—45 minutes****Number of problems—3****No calculator is allowed for these problems.**

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4. A particle moves along the  $x$ -axis with position at time  $t$  given by  $x(t) = e^{-t} \sin t$  for  $0 \leq t \leq 2\pi$ .
- (a) Find the time  $t$  at which the particle is farthest to the left. Justify your answer.
- (b) Find the value of the constant  $A$  for which  $x(t)$  satisfies the equation  $Ax''(t) + x'(t) + x(t) = 0$  for  $0 < t < 2\pi$ .
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$t$ (minutes)	0	2	5	7	11	12
$r'(t)$ (feet per minute)	5.7	4.0	2.0	1.2	0.6	0.5

5. The volume of a spherical hot air balloon expands as the air inside the balloon is heated. The radius of the balloon, in feet, is modeled by a twice-differentiable function  $r$  of time  $t$ , where  $t$  is measured in minutes. For  $0 < t < 12$ , the graph of  $r$  is concave down. The table above gives selected values of the rate of change,  $r'(t)$ , of the radius of the balloon over the time interval  $0 \leq t \leq 12$ . The radius of the balloon is 30 feet when  $t = 5$ .  
(Note: The volume of a sphere of radius  $r$  is given by  $V = \frac{4}{3}\pi r^3$ .)
- (a) Estimate the radius of the balloon when  $t = 5.4$  using the tangent line approximation at  $t = 5$ . Is your estimate greater than or less than the true value? Give a reason for your answer.
- (b) Find the rate of change of the volume of the balloon with respect to time when  $t = 5$ . Indicate units of measure.
- (c) Use a right Riemann sum with the five subintervals indicated by the data in the table to approximate  $\int_0^{12} r'(t) dt$ . Using correct units, explain the meaning of  $\int_0^{12} r'(t) dt$  in terms of the radius of the balloon.
- (d) Is your approximation in part (c) greater than or less than  $\int_0^{12} r'(t) dt$ ? Give a reason for your answer.
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**WRITE ALL WORK IN THE PINK EXAM BOOKLET.**

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6. Let  $f$  be the function defined by  $f(x) = k\sqrt{x} - \ln x$  for  $x > 0$ , where  $k$  is a positive constant.
- (a) Find  $f'(x)$  and  $f''(x)$ .
  - (b) For what value of the constant  $k$  does  $f$  have a critical point at  $x = 1$ ? For this value of  $k$ , determine whether  $f$  has a relative minimum, relative maximum, or neither at  $x = 1$ . Justify your answer.
  - (c) For a certain value of the constant  $k$ , the graph of  $f$  has a point of inflection on the  $x$ -axis. Find this value of  $k$ .
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**END OF EXAM**

**AP<sup>®</sup> CALCULUS AB**  
**2007 SCORING GUIDELINES**

**Question 5**

$t$ (minutes)	0	2	5	7	11	12
$r'(t)$ (feet per minute)	5.7	4.0	2.0	1.2	0.6	0.5

The volume of a spherical hot air balloon expands as the air inside the balloon is heated. The radius of the balloon, in feet, is modeled by a twice-differentiable function  $r$  of time  $t$ , where  $t$  is measured in minutes. For  $0 < t < 12$ , the graph of  $r$  is concave down. The table above gives selected values of the rate of change,  $r'(t)$ , of the radius of the balloon over the time interval  $0 \leq t \leq 12$ . The radius of the balloon is 30 feet when  $t = 5$ . (Note: The volume of a sphere of radius  $r$  is given by  $V = \frac{4}{3}\pi r^3$ .)

- (a) Estimate the radius of the balloon when  $t = 5.4$  using the tangent line approximation at  $t = 5$ . Is your estimate greater than or less than the true value? Give a reason for your answer.
- (b) Find the rate of change of the volume of the balloon with respect to time when  $t = 5$ . Indicate units of measure.
- (c) Use a right Riemann sum with the five subintervals indicated by the data in the table to approximate  $\int_0^{12} r'(t) dt$ . Using correct units, explain the meaning of  $\int_0^{12} r'(t) dt$  in terms of the radius of the balloon.
- (d) Is your approximation in part (c) greater than or less than  $\int_0^{12} r'(t) dt$ ? Give a reason for your answer.

- (a)  $r(5.4) \approx r(5) + r'(5)\Delta t = 30 + 2(0.4) = 30.8$  ft  
 Since the graph of  $r$  is concave down on the interval  $5 < t < 5.4$ , this estimate is greater than  $r(5.4)$ .

2 :  $\begin{cases} 1 : \text{estimate} \\ 1 : \text{conclusion with reason} \end{cases}$

- (b)  $\frac{dV}{dt} = 3\left(\frac{4}{3}\right)\pi r^2 \frac{dr}{dt}$   
 $\left.\frac{dV}{dt}\right|_{t=5} = 4\pi(30)^2 2 = 7200\pi \text{ ft}^3/\text{min}$

3 :  $\begin{cases} 2 : \frac{dV}{dt} \\ 1 : \text{answer} \end{cases}$

- (c)  $\int_0^{12} r'(t) dt \approx 2(4.0) + 3(2.0) + 2(1.2) + 4(0.6) + 1(0.5)$   
 $= 19.3$  ft  
 $\int_0^{12} r'(t) dt$  is the change in the radius, in feet, from  $t = 0$  to  $t = 12$  minutes.

2 :  $\begin{cases} 1 : \text{approximation} \\ 1 : \text{explanation} \end{cases}$

- (d) Since  $r$  is concave down,  $r'$  is decreasing on  $0 < t < 12$ . Therefore, this approximation, 19.3 ft, is less than  $\int_0^{12} r'(t) dt$ .

1 : conclusion with reason

Units of  $\text{ft}^3/\text{min}$  in part (b) and ft in part (c)

1 : units in (b) and (c)