CALCULUS AB
SECTION II, Part B
Time—45 minutes
Number of problems—3

No calculator is allowed for these problems.

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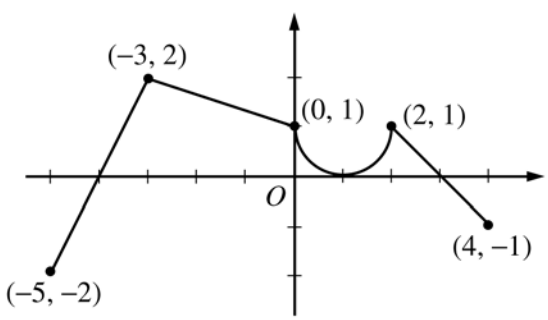
4. Consider the curve given by $x^2 + 4y^2 = 7 + 3xy$.

   (a) Show that $\dfrac{dy}{dx} = \dfrac{3y - 2x}{8y - 3x}$.

   (b) Show that there is a point P with x-coordinate 3 at which the line tangent to the curve at P is horizontal. Find the y-coordinate of P.

   (c) Find the value of $\dfrac{d^2y}{dx^2}$ at the point P found in part (b). Does the curve have a local maximum, a local minimum, or neither at the point P? Justify your answer.



Graph of f

5. The graph of the function f shown above consists of a semicircle and three line segments. Let g be the function given by $g(x) = \int_{-3}^{x} f(t)\,dt$.

   (a) Find $g(0)$ and $g'(0)$.

   (b) Find all values of x in the open interval $(-5, 4)$ at which g attains a relative maximum. Justify your answer.

   (c) Find the absolute minimum value of g on the closed interval $[-5, 4]$. Justify your answer.

   (d) Find all values of x in the open interval $(-5, 4)$ at which the graph of g has a point of inflection.

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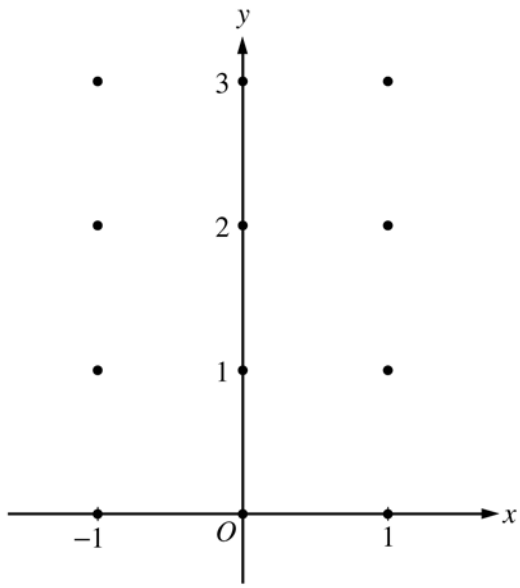
6. Consider the differential equation $\frac{dy}{dx} = x^2(y - 1)$.

   (a) On the axes provided, sketch a slope field for the given differential equation at the twelve points indicated. (**Note: Use the axes provided in the pink test booklet.**)



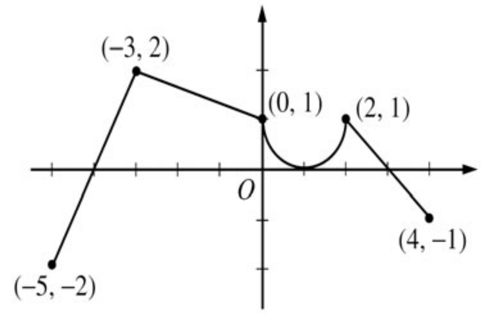
   (b) While the slope field in part (a) is drawn at only twelve points, it is defined at every point in the xy-plane. Describe all points in the xy-plane for which the slopes are positive.

   (c) Find the particular solution $y = f(x)$ to the given differential equation with the initial condition $f(0) = 3$.

END OF EXAMINATION

Question 5

The graph of the function f shown above consists of a semicircle and three line segments. Let g be the function given by $g(x) = \int_{-3}^{x} f(t)\,dt$.



Graph of f

(a) Find $g(0)$ and $g'(0)$.

(b) Find all values of x in the open interval $(-5, 4)$ at which g attains a relative maximum. Justify your answer.

(c) Find the absolute minimum value of g on the closed interval $[-5, 4]$. Justify your answer.

(d) Find all values of x in the open interval $(-5, 4)$ at which the graph of g has a point of inflection.

(a) $g(0) = \int_{-3}^{0} f(t)\,dt = \frac{1}{2}(3)(2+1) = \frac{9}{2}$ $g'(0) = f(0) = 1$	$2 : \begin{cases} 1 : g(0) \\ 1 : g'(0) \end{cases}$
(b) g has a relative maximum at $x = 3$. This is the only x-value where $g' = f$ changes from positive to negative.	$2 : \begin{cases} 1 : x = 3 \\ 1 : \text{justification} \end{cases}$
(c) The only x-value where f changes from negative to positive is $x = -4$. The other candidates for the location of the absolute minimum value are the endpoints. $g(-5) = 0$ $g(-4) = \int_{-3}^{-4} f(t)\,dt = -1$ $g(4) = \frac{9}{2} + \left(2 - \frac{\pi}{2}\right) = \frac{13 - \pi}{2}$ So the absolute minimum value of g is -1.	$3 : \begin{cases} 1 : \text{identifies } x = -4 \text{ as a candidate} \\ 1 : g(-4) = -1 \\ 1 : \text{justification and answer} \end{cases}$
(d) $x = -3, 1, 2$	2 : correct values $\langle -1 \rangle$ each missing or extra value