

Begin your response to **QUESTION 2** on this page.

2. A developer wants to know whether adding fibers to concrete used in paving driveways will reduce the severity of cracking, because any driveway with severe cracks will have to be repaired by the developer. The developer conducts a completely randomized experiment with 60 new homes that need driveways. Thirty of the driveways will be randomly assigned to receive concrete that contains fibers, and the other 30 driveways will receive concrete that does not contain fibers. After one year, the developer will record the severity of cracks in each driveway on a scale of 0 to 10, with 0 representing not cracked at all and 10 representing severely cracked.

(a) Based on the information provided about the developer's experiment, identify each of the following.

- Experimental units
- Treatments
- Response variable

(b) Describe an appropriate method the developer could use to randomly assign concrete that contains fibers and concrete that does not contain fibers to the 60 driveways.

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Continue your response to **QUESTION 2** on this page.

Suppose the developer finds that there is a statistically significant reduction in the mean severity of cracks in driveways using the concrete that contains fibers compared to the driveways using concrete that does not contain fibers.

- (c) In terms of the developer's conclusion, what is the benefit of randomly assigning the driveways to either the concrete that contains fibers or the concrete that does not contain fibers?

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Begin your response to **QUESTION 3** on this page.

3. Bath fizzies are mineral tablets that dissolve and create bubbles when added to bathwater. In order to increase sales, the Fizzy Bath Company has produced a new line of bath fizzies that have a cash prize in every bath fizzy. Let the random variable,  $X$ , represent the dollar value of the cash prize in a bath fizzy. The probability distribution of  $X$  is shown in the table.

Cash prize, $x$	\$1	\$5	\$10	\$20	\$50	\$100
Probability of cash prize, $P(X = x)$	$P(X = \$1)$	0.2	0.05	0.05	0.01	0.01

(a) Based on the probability distribution of  $X$ , answer the following. Show your work.

(i) Calculate the proportion of bath fizzies that contain \$1.

(ii) Calculate the proportion of bath fizzies that contain at least \$10.

(b) Based on the probability distribution of  $X$ , calculate the probability that a randomly selected bath fizzy contains \$100, given that it contains at least \$10. Show your work.

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**Question 2: Focus on Sampling and Experimental Design****4 points****General Scoring Notes**

- Each part of the question (indicated by a letter) is initially scored by determining if it meets the criteria for essentially correct (E), partially correct (P), or incorrect (I). The response is then categorized based on the scores assigned to each letter part and awarded an integer score between 0 and 4 (see the table at the end of the question).
- The model solution represents an ideal response to each part of the question, and the scoring criteria identify the specific components of the model solution that are used to determine the score.

	Model Solution	Scoring
(a)	<p>Experimental units: 60 driveways.</p> <p>Treatments: Concrete with fibers and concrete without fibers.</p> <p>Response variable: Rating of the severity of the cracks after one year, on a scale of 0 to 10.</p>	<p><b>Essentially correct (E)</b> if the response satisfies the following three components:</p> <ol style="list-style-type: none"> <li>Identifies the experimental units as the driveways</li> <li>Identifies the treatments as concrete with fibers and concrete without fibers</li> <li>Identifies the response variable as the rating of the severity of the cracks</li> </ol> <p><b>Partially correct (P)</b> if the response satisfies only two of the three components.</p> <p><b>Incorrect (I)</b> if the response does not meet the criteria for E or P.</p>

**Additional Notes:**

- The number of experimental units is not necessary to satisfy component 1.
- The experimental units can be identified as “new homes needing driveways” to satisfy component 1.
- The phrases “on a scale of 0 to 10” and “after one year” are not required for component 3.
- Reasonable synonyms for “rating of the severity” such as “rating,” “severity,” or “on a scale of 0 to 10” are acceptable towards satisfying component 3.
- Identification of the response variable as a mean, or average, does not satisfy component 3.

Model Solution	Scoring
<p>(b) Number the 60 driveways from 01 to 60. Using a random number generator, generate two-digit integers between 01 and 60. Ignore 00 and any number greater than 60 until 30 unique numbers are obtained. Assign the driveways with those 30 unique numbers to receive concrete with fibers and the remaining 30 driveways to receive concrete without fibers.</p>	<p><b>Essentially correct (E)</b> if the response satisfies the following two components:</p> <ol style="list-style-type: none"> <li>1. Describes how to correctly use a random number generator, or some other appropriate random process, to assign driveways that have been labeled 1 to 60 to concrete with fibers and concrete without fibers so that every possible random assignment is equally likely</li> <li>2. The random process results in an equal number of driveways assigned to the concrete with fibers and the concrete without fibers</li> </ol> <p><b>Partially correct (P)</b> if the response describes how to implement a random process that satisfies only one of the two components.</p> <p><b>Incorrect (I)</b> if the response does not meet the criteria for E or P.</p>

**Additional Notes:**

- Examples of alternative random assignment processes that satisfy both component 1 and component 2 include:
  - Put 60 equally sized slips of paper labeled from 1 to 60 into a hat, mix well, and randomly select 30 slips of paper out of the hat, sampling without replacement. Assign the driveways with the numbers on the 30 selected slips of paper to concrete with fibers and assign the remaining 30 driveways to concrete without fibers.
  - Put 30 white marbles and 30 black marbles in an urn and mix well. Randomly select one marble from the urn. If the marble is white, assign the driveway with label 1 to concrete with fibers, otherwise assign the driveway with label 1 to concrete without fibers. Randomly select a marble from the remaining 59 marbles in the urn. If the marble is white, assign the driveway with label 2 to concrete with fibers, otherwise assign the driveway with label 2 to concrete without fibers. Continue this process of randomly selecting marbles from the urn without replacement until 30 driveways are assigned to concrete with fibers. The other 30 driveways are assigned to concrete without fibers.
  - Spin a spinner with 60 equally sized sections numbered from 1 to 60, ignoring repeats, until 30 unique numbers are generated. Assign the driveways with those 30 numbers to concrete with fibers and assign the remaining 30 driveways to concrete without fibers.
- The following random assignment processes are examples that would satisfy component 2 but would not satisfy component 1.
  - An example that does not specify without replacement: Put 60 equally sized slips of paper labeled from 1 to 60 into a bowl, mix well, and randomly select one slip out of the bowl and record the number. Continue this process of randomly selecting slips until 30 numbers are recorded. Assign the driveways with the 30 recorded numbers to concrete with fibers and assign the remaining 30 driveways to concrete without fibers.

- An example of a stopping rule with equal probabilities: Put 30 white marbles and 30 black marbles into an urn and mix well. Randomly select one marble from the urn. If the marble is white, assign the driveway with label 1 to concrete with fibers, otherwise assign the driveway with label 1 to concrete without fibers. Place the selected marble back into the urn, mix well, and randomly select a marble from the urn. If the marble is white, assign the driveway with label 2 to concrete with fibers, otherwise assign the driveway with label 2 to concrete without fibers. Continue this process of randomly selecting marbles from the urn with replacement until 30 driveways are assigned to concrete with fibers. The other 30 driveways are assigned to concrete without fibers.
  - An example where not all outcomes are equally likely: Toss 60 fair coins and record the number of heads. Continue tossing the 60 coins, and ignoring zero, until 30 unique numbers are recorded. Assign the driveways with the 30 recorded numbers to concrete with fibers and assign the remaining 30 driveways to concrete without fibers.
  - If there is not some type of labeling system (numbering the driveways), then component 1 is not satisfied.
  - If it is not clear that the random assignment process allows every possible random assignment of driveways to type of concrete to be equally likely, then component 1 is not satisfied.
  - If the response does not clearly indicate that random numbers are selected without using repeats, then component 1 is not satisfied.
  - A response that only assigns driveways to groups and does not indicate how the groups correspond to concrete with fibers and concrete without fibers does not satisfy component 1.
  - If a response describes two separate random assignment processes in detail (e.g., how to randomly assign number labels to driveways and how to randomly assign driveways to concrete type), score the combined random assignment process according to the two components.
  - If the response describes two ways to make the same random assignment (e.g., how to randomly assign driveways to treatments), assign the score for the weaker random assignment process.
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Model Solution	Scoring
<p>(c) The results were statistically significant, and because the driveways were randomly assigned to either the concrete with the fibers or the concrete without the fibers, there is evidence the treatment (type of concrete) caused the response (rating of severity of cracks).</p>	<p><b>Essentially correct (E)</b> if the response satisfies the following two components:</p> <ol style="list-style-type: none"> <li>1. The response indicates that random assignment enables the conclusion that type of concrete caused the rating of severity of the cracks</li> <li>2. The explanation is in context of the problem</li> </ol> <p><b>Partially correct (P)</b> if the response satisfies only component 1,  <i>OR</i>  the response indicates random assignment limits the effect of confounding variables in context.</p> <p><b>Incorrect (I)</b> if the response does not meet the criteria for E or P.</p>

**Additional Notes:**

- If the response also indicates an incorrect conclusion based on random assignment, such as generalization, representation of the population, reduction of bias, or reduction of variability, reduce the score from an E to a P or from a P to an I.
- Context includes either the treatment (concrete with fibers or concrete without fibers or “fibers”) or the severity rating.