

$t$ (seconds)	0	60	90	120	135	150
$f(t)$ (gallons per second)	0	0.1	0.15	0.1	0.05	0

1. A customer at a gas station is pumping gasoline into a gas tank. The rate of flow of gasoline is modeled by a differentiable function  $f$ , where  $f(t)$  is measured in gallons per second and  $t$  is measured in seconds since pumping began. Selected values of  $f(t)$  are given in the table.

- (a) Using correct units, interpret the meaning of  $\int_{60}^{135} f(t) dt$  in the context of the problem. Use a right Riemann sum with the three subintervals  $[60, 90]$ ,  $[90, 120]$ , and  $[120, 135]$  to approximate the value of

$$\int_{60}^{135} f(t) dt.$$

- (b) Must there exist a value of  $c$ , for  $60 < c < 120$ , such that  $f'(c) = 0$ ? Justify your answer.

- (c) The rate of flow of gasoline, in gallons per second, can also be modeled by  $g(t) = \left(\frac{t}{500}\right) \cos\left(\left(\frac{t}{120}\right)^2\right)$  for

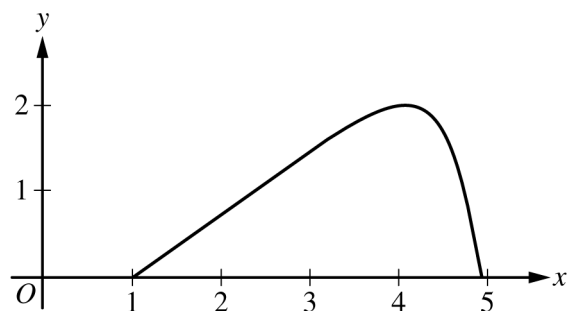
$0 \leq t \leq 150$ . Using this model, find the average rate of flow of gasoline over the time interval  $0 \leq t \leq 150$ .

Show the setup for your calculations.

- (d) Using the model  $g$  defined in part (c), find the value of  $g'(140)$ . Interpret the meaning of your answer in the context of the problem.

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**Write your responses to this question only on the designated pages in the separate Free Response booklet. Write your solution to each part in the space provided for that part.**



2. For  $0 \leq t \leq \pi$ , a particle is moving along the curve shown so that its position at time  $t$  is  $(x(t), y(t))$ , where  $x(t)$  is not explicitly given and  $y(t) = 2 \sin t$ . It is known that  $\frac{dx}{dt} = e^{\cos t}$ . At time  $t = 0$ , the particle is at position  $(1, 0)$ .
- Find the acceleration vector of the particle at time  $t = 1$ . Show the setup for your calculations.
  - For  $0 \leq t \leq \pi$ , find the first time  $t$  at which the speed of the particle is 1.5. Show the work that leads to your answer.
  - Find the slope of the line tangent to the path of the particle at time  $t = 1$ . Find the  $x$ -coordinate of the position of the particle at time  $t = 1$ . Show the work that leads to your answers.
  - Find the total distance traveled by the particle over the time interval  $0 \leq t \leq \pi$ . Show the setup for your calculations.

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**Part A (AB or BC): Graphing calculator required****Question 1****9 points****General Scoring Notes**

The model solution is presented using standard mathematical notation.

Answers (numeric or algebraic) need not be simplified. Answers given as a decimal approximation should be correct to three places after the decimal point. Within each individual free-response question, at most one point is not earned for inappropriate rounding.

$t$ (seconds)	0	60	90	120	135	150
$f(t)$ (gallons per second)	0	0.1	0.15	0.1	0.05	0

A customer at a gas station is pumping gasoline into a gas tank. The rate of flow of gasoline is modeled by a differentiable function  $f$ , where  $f(t)$  is measured in gallons per second and  $t$  is measured in seconds since pumping began. Selected values of  $f(t)$  are given in the table.

Model Solution	Scoring
(a) Using correct units, interpret the meaning of $\int_{60}^{135} f(t) dt$ in the context of the problem. Use a right Riemann sum with the three subintervals $[60, 90]$ , $[90, 120]$ , and $[120, 135]$ to approximate the value of $\int_{60}^{135} f(t) dt$ .	
$\int_{60}^{135} f(t) dt$ represents the total number of gallons of gasoline pumped into the gas tank from time $t = 60$ seconds to time $t = 135$ seconds.	Interpretation with units <b>1 point</b>
$\int_{60}^{135} f(t) dt$ $\approx f(90)(90 - 60) + f(120)(120 - 90) + f(135)(135 - 120)$ $= (0.15)(30) + (0.1)(30) + (0.05)(15) = 8.25$	Form of Riemann sum <b>1 point</b>
	Answer <b>1 point</b>

**Scoring notes:**

- To earn the first point the response must reference gallons of gasoline added/pumped and the time interval  $t = 60$  to  $t = 135$ .
- To earn the second point at least five of the six factors in the Riemann sum must be correct.
- If there is any error in the Riemann sum, the response does not earn the third point.
- A response of  $(0.15)(30) + (0.1)(30) + (0.05)(15)$  earns both the second and third points, unless there is a subsequent error in simplification, in which case the response would earn only the second point.

The average rate of flow of gasoline, in gallons per second, is 0.096 (or 0.095).

**Scoring notes:**

- The exact value of  $\frac{1}{150} \int_0^{150} g(t) dt$  is  $\frac{12}{125} \sin\left(\frac{25}{16}\right)$ .
- A response may present the average value formula in single or multiple steps. For example, the following response earns both points:  $\int_0^{150} g(t) dt = 14.399504$  so the average rate is 0.0959967.
- A response that presents the average value formula in multiple steps but provides incorrect or incomplete communication (e.g.,  $\int_0^{150} g(t) dt = \frac{14.399504}{150} = 0.0959967$ ) earns 1 out of 2 points.
- A response of  $\int_0^{150} g(t) dt = 0.0959967$  does not earn either point.
- Degree mode: A response that presents answers obtained by using a calculator in degree mode does not earn the first point it would have otherwise earned. The response is generally eligible for all subsequent points (unless no answer is possible in degree mode or the question is made simpler by using degree mode). In degree mode,  $\frac{1}{150} \int_0^{150} g(t) dt = 0.149981$  or 0.002618.

**Total for part (c) 2 points**

- (d) Using the model  $g$  defined in part (c), find the value of  $g'(140)$ . Interpret the meaning of your answer in the context of the problem.

$g'(140) \approx -0.004908$ $g'(140) = -0.005$ (or $-0.004$ )	$g'(140)$	<b>1 point</b>
The rate at which gasoline is flowing into the tank is decreasing at a rate of 0.005 (or 0.004) gallon per second per second at time $t = 140$ seconds.	Interpretation	<b>1 point</b>

**Scoring notes:**

- The exact value of  $g'(140)$  is  $\frac{1}{500} \cos\left(\frac{49}{36}\right) - \frac{49}{9000} \sin\left(\frac{49}{36}\right)$ .
- The value of  $g'(140)$  may appear only in the interpretation.
- To be eligible for the second point a response must present some numerical value for  $g'(140)$ .
- To earn the second point the interpretation must include “the rate of flow of gasoline is changing at a rate of [the declared value of  $g'(140)$ ]” and “at  $t = 140$ ” (or equivalent).
- An interpretation of “decreasing at a rate of  $-0.005$ ” or “increasing at a rate of 0.005” does not earn the second point.
- Degree mode: In degree mode,  $g'(140) = 0.001997$  or 0.00187.

**Total for part (d) 2 points****Total for question 1 9 points**