x	2	3	5	8	13
$f(x)$	1	4	–2	3	6

5. Let f be a function that is twice differentiable for all real numbers. The table above gives values of f for selected points in the closed interval $2 \le x \le 13$.

(a) Estimate $f'(4)$. Show the work that leads to your answer.

(b) Evaluate $\int_2^{13} (3 - 5f'(x))\, dx$. Show the work that leads to your answer.

(c) Use a left Riemann sum with subintervals indicated by the data in the table to approximate $\int_2^{13} f(x)\, dx$. Show the work that leads to your answer.

(d) Suppose $f'(5) = 3$ and $f''(x) < 0$ for all x in the closed interval $5 \le x \le 8$. Use the line tangent to the graph of f at $x = 5$ to show that $f(7) \le 4$. Use the secant line for the graph of f on $5 \le x \le 8$ to show that $f(7) \ge \frac{4}{3}$.

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6. The Maclaurin series for e^x is $e^x = 1 + x + \frac{x^2}{2} + \frac{x^3}{6} + \cdots + \frac{x^n}{n!} + \cdots$. The continuous function f is defined by $f(x) = \frac{e^{(x-1)^2} - 1}{(x-1)^2}$ for $x \ne 1$ and $f(1) = 1$. The function f has derivatives of all orders at $x = 1$.

(a) Write the first four nonzero terms and the general term of the Taylor series for $e^{(x-1)^2}$ about $x = 1$.

(b) Use the Taylor series found in part (a) to write the first four nonzero terms and the general term of the Taylor series for f about $x = 1$.

(c) Use the ratio test to find the interval of convergence for the Taylor series found in part (b).

(d) Use the Taylor series for f about $x = 1$ to determine whether the graph of f has any points of inflection.

WRITE ALL WORK IN THE PINK EXAM BOOKLET.

END OF EXAM

AP® CALCULUS BC
2009 SCORING GUIDELINES

Question 5

x	2	3	5	8	13
$f(x)$	1	4	–2	3	6

Let f be a function that is twice differentiable for all real numbers. The table above gives values of f for selected points in the closed interval $2 \le x \le 13$.

(a) Estimate $f'(4)$. Show the work that leads to your answer.

(b) Evaluate $\int_2^{13} (3 - 5f'(x))\,dx$. Show the work that leads to your answer.

(c) Use a left Riemann sum with subintervals indicated by the data in the table to approximate $\int_2^{13} f(x)\,dx$. Show the work that leads to your answer.

(d) Suppose $f'(5) = 3$ and $f''(x) < 0$ for all x in the closed interval $5 \le x \le 8$. Use the line tangent to the graph of f at $x = 5$ to show that $f(7) \le 4$. Use the secant line for the graph of f on $5 \le x \le 8$ to show that $f(7) \ge \frac{4}{3}$.

(a) $f'(4) \approx \dfrac{f(5) - f(3)}{5 - 3} = -3$

1 : answer

(b) $\int_2^{13} (3 - 5f'(x))\,dx = \int_2^{13} 3\,dx - 5\int_2^{13} f'(x)\,dx$
$= 3(13 - 2) - 5(f(13) - f(2)) = 8$

2 : $\begin{cases} 1 : \text{uses Fundamental Theorem of Calculus} \\ 1 : \text{answer} \end{cases}$

(c) $\int_2^{13} f(x)\,dx \approx f(2)(3-2) + f(3)(5-3)$
$+ f(5)(8-5) + f(8)(13-8) = 18$

2 : $\begin{cases} 1 : \text{left Riemann sum} \\ 1 : \text{answer} \end{cases}$

(d) An equation for the tangent line is $y = -2 + 3(x - 5)$. Since $f''(x) < 0$ for all x in the interval $5 \le x \le 8$, the line tangent to the graph of $y = f(x)$ at $x = 5$ lies above the graph for all x in the interval $5 < x \le 8$.

Therefore, $f(7) \le -2 + 3 \cdot 2 = 4$.

An equation for the secant line is $y = -2 + \frac{5}{3}(x - 5)$.

Since $f''(x) < 0$ for all x in the interval $5 \le x \le 8$, the secant line connecting $(5, f(5))$ and $(8, f(8))$ lies below the graph of $y = f(x)$ for all x in the interval $5 < x < 8$.

Therefore, $f(7) \ge -2 + \frac{5}{3} \cdot 2 = \frac{4}{3}$.

4 : $\begin{cases} 1 : \text{tangent line} \\ 1 : \text{shows } f(7) \le 4 \\ 1 : \text{secant line} \\ 1 : \text{shows } f(7) \ge \frac{4}{3} \end{cases}$