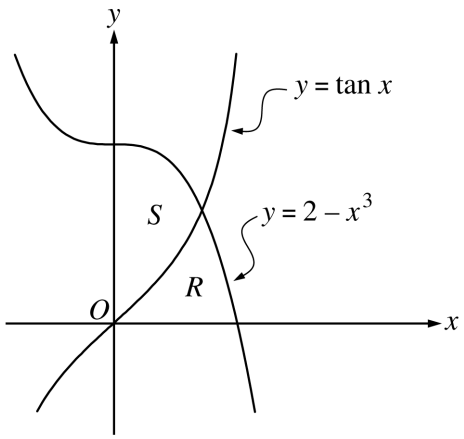
CALCULUS AB
SECTION II, Part A
Time—45 minutes
Number of problems—3

A graphing calculator is required for some problems or parts of problems.



1. Let R and S be the regions in the first quadrant shown in the figure above. The region R is bounded by the x-axis and the graphs of $y = 2 - x^3$ and $y = \tan x$. The region S is bounded by the y-axis and the graphs of $y = 2 - x^3$ and $y = \tan x$.

   (a) Find the area of R.

   (b) Find the area of S.

   (c) Find the volume of the solid generated when S is revolved about the x-axis.

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t (days)	$W(t)$ (°C)
0	20
3	31
6	28
9	24
12	22
15	21

2. The temperature, in degrees Celsius (°C), of the water in a pond is a differentiable function W of time t. The table above shows the water temperature as recorded every 3 days over a 15-day period.

   (a) Use data from the table to find an approximation for $W'(12)$. Show the computations that lead to your answer. Indicate units of measure.

   (b) Approximate the average temperature, in degrees Celsius, of the water over the time interval $0 \le t \le 15$ days by using a trapezoidal approximation with subintervals of length $\Delta t = 3$ days.

   (c) A student proposes the function P, given by $P(t) = 20 + 10te^{(-t/3)}$, as a model for the temperature of the water in the pond at time t, where t is measured in days and $P(t)$ is measured in degrees Celsius. Find $P'(12)$. Using appropriate units, explain the meaning of your answer in terms of water temperature.

   (d) Use the function P defined in part (c) to find the average value, in degrees Celsius, of $P(t)$ over the time interval $0 \le t \le 15$ days.

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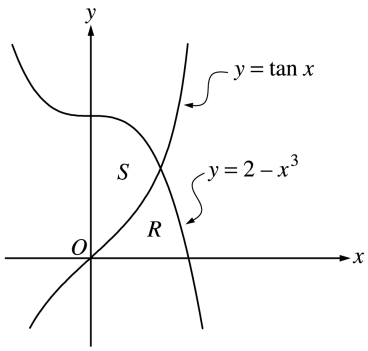
Question 1

Let R and S be the regions in the first quadrant shown in the figure above. The region R is bounded by the x-axis and the graphs of $y = 2 - x^3$ and $y = \tan x$. The region S is bounded by the y-axis and the graphs of $y = 2 - x^3$ and $y = \tan x$.



(a) Find the area of R.

(b) Find the area of S.

(c) Find the volume of the solid generated when S is revolved about the x-axis.

Point of intersection

$2 - x^3 = \tan x$ at $(A, B) = (0.902155, 1.265751)$

(a) Area $R = \displaystyle\int_0^A \tan x\,dx + \int_A^{\sqrt[3]{2}} \left(2 - x^3\right) dx = 0.729$

or

Area $R = \displaystyle\int_0^B \left((2 - y)^{1/3} - \tan^{-1} y\right) dy = 0.729$

or

Area $R = \displaystyle\int_0^{\sqrt[3]{2}} \left(2 - x^3\right) dx - \int_0^A \left(2 - x^3 - \tan x\right) dx = 0.729$

3 : { 1 : limits; 1 : integrand; 1 : answer }

(b) Area $S = \displaystyle\int_0^A \left(2 - x^3 - \tan x\right) dx = 1.160$ or 1.161

or

Area $S = \displaystyle\int_0^B \tan^{-1} y\,dy + \int_B^2 (2 - y)^{1/3}\,dy = 1.160$ or 1.161

or

Area S

$= \displaystyle\int_0^2 (2 - y)^{1/3}\,dy - \int_0^B \left((2 - y)^{1/3} - \tan^{-1} y\right) dy$

$= 1.160$ or 1.161

3 : { 1 : limits; 1 : integrand; 1 : answer }

(c) Volume $= \pi \displaystyle\int_0^A \left(\left(2 - x^3\right)^2 - \tan^2 x\right) dx$

$= 2.652\pi$ or 8.331 or 8.332

3 : { 1 : limits and constant; 1 : integrand; 1 : answer }

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