x	$h(x)$	$h'(x)$	$h''(x)$	$h'''(x)$	$h^{(4)}(x)$
1	11	30	42	99	18
2	80	128	$\dfrac{488}{3}$	$\dfrac{448}{3}$	$\dfrac{584}{9}$
3	317	$\dfrac{753}{2}$	$\dfrac{1383}{4}$	$\dfrac{3483}{16}$	$\dfrac{1125}{16}$

3. Let h be a function having derivatives of all orders for $x > 0.$ Selected values of h and its first four derivatives are indicated in the table above. The function h and these four derivatives are increasing on the interval $1 \le x \le 3.$

   (a) Write the first-degree Taylor polynomial for h about $x = 2$ and use it to approximate $h(1.9).$ Is this approximation greater than or less than $h(1.9)$? Explain your reasoning.

   (b) Write the third-degree Taylor polynomial for h about $x = 2$ and use it to approximate $h(1.9).$

   (c) Use the Lagrange error bound to show that the third-degree Taylor polynomial for h about $x = 2$ approximates $h(1.9)$ with error less than $3 \times 10^{-4}.$

WRITE ALL WORK IN THE PINK EXAM BOOKLET.
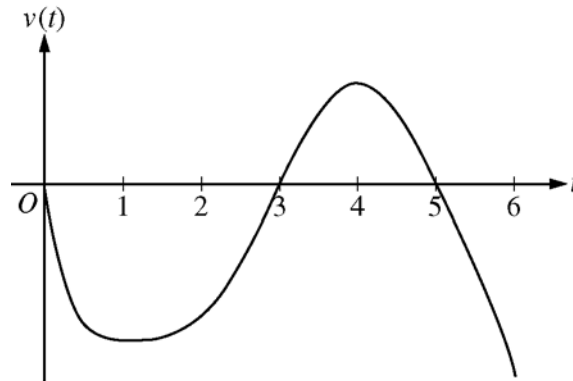
END OF PART A OF SECTION II

**CALCULUS BC
SECTION II, Part B
Time—45 minutes
Number of problems—3**

No calculator is allowed for these problems.

Graph of v

4. A particle moves along the x-axis so that its velocity at time t, for $0 \le t \le 6$, is given by a differentiable function v whose graph is shown above. The velocity is 0 at $t = 0$, $t = 3$, and $t = 5$, and the graph has horizontal tangents at $t = 1$ and $t = 4$. The areas of the regions bounded by the t-axis and the graph of v on the intervals $[0, 3]$, $[3, 5]$, and $[5, 6]$ are 8, 3, and 2, respectively. At time $t = 0$, the particle is at $x = -2$.

(a) For $0 \le t \le 6$, find both the time and the position of the particle when the particle is farthest to the left. Justify your answer.

(b) For how many values of t, where $0 \le t \le 6$, is the particle at $x = -8$? Explain your reasoning.

(c) On the interval $2 < t < 3$, is the speed of the particle increasing or decreasing? Give a reason for your answer.

(d) During what time intervals, if any, is the acceleration of the particle negative? Justify your answer.

WRITE ALL WORK IN THE PINK EXAM BOOKLET.

GO ON TO THE NEXT PAGE.

AP® CALCULUS BC
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Question 3

x	$h(x)$	$h'(x)$	$h''(x)$	$h'''(x)$	$h^{(4)}(x)$
1	11	30	42	99	18
2	80	128	$\dfrac{488}{3}$	$\dfrac{448}{3}$	$\dfrac{584}{9}$
3	317	$\dfrac{753}{2}$	$\dfrac{1383}{4}$	$\dfrac{3483}{16}$	$\dfrac{1125}{16}$

Let h be a function having derivatives of all orders for $x > 0$. Selected values of h and its first four derivatives are indicated in the table above. The function h and these four derivatives are increasing on the interval $1 \leq x \leq 3$.

(a) Write the first-degree Taylor polynomial for h about $x = 2$ and use it to approximate $h(1.9)$. Is this approximation greater than or less than $h(1.9)$? Explain your reasoning.

(b) Write the third-degree Taylor polynomial for h about $x = 2$ and use it to approximate $h(1.9)$.

(c) Use the Lagrange error bound to show that the third-degree Taylor polynomial for h about $x = 2$ approximates $h(1.9)$ with error less than 3×10^{-4}.

(a) $P_1(x) = 80 + 128(x - 2)$, so $h(1.9) \approx P_1(1.9) = 67.2$

$P_1(1.9) < h(1.9)$ since h' is increasing on the interval $1 \leq x \leq 3$.

$4 : \begin{cases} 2 : P_1(x) \\ 1 : P_1(1.9) \\ 1 : P_1(1.9) < h(1.9) \text{ with reason} \end{cases}$

(b) $P_3(x) = 80 + 128(x - 2) + \dfrac{488}{6}(x - 2)^2 + \dfrac{448}{18}(x - 2)^3$

$h(1.9) \approx P_3(1.9) = 67.988$

$3 : \begin{cases} 2 : P_3(x) \\ 1 : P_3(1.9) \end{cases}$

(c) The fourth derivative of h is increasing on the interval $1 \leq x \leq 3$, so $\max\limits_{1.9 \leq x \leq 2} \left| h^{(4)}(x) \right| = \dfrac{584}{9}$.

Therefore, $\left| h(1.9) - P_3(1.9) \right| \leq \dfrac{584}{9} \dfrac{|1.9 - 2|^4}{4!}$

$\qquad\qquad\qquad\qquad = 2.7037 \times 10^{-4}$

$\qquad\qquad\qquad\qquad < 3 \times 10^{-4}$

$2 : \begin{cases} 1 : \text{form of Lagrange error estimate} \\ 1 : \text{reasoning} \end{cases}$