PHYSICS C: ELECTRICITY AND MAGNETISM
SECTION II
Time—45 minutes
3 Questions

Directions: Answer all three questions. The suggested time is about 15 minutes for answering each of the questions, which are worth 15 points each. The parts within a question may not have equal weight. Show all your work in this booklet in the spaces provided after each part, NOT in the green insert.

E&M. 1.

A spherically symmetric charge distribution has net positive charge Q_0 distributed within a radius of R.
Its electric potential V as a function of the distance r from the center of the sphere is given by the following.

$$V(r) = \frac{Q_0}{4\pi\epsilon_0 R}\left[-2 + 3\left(\frac{r}{R}\right)^2\right] \text{ for } r < R$$

$$V(r) = \frac{Q_0}{4\pi\epsilon_0 r} \text{ for } r > R$$

Express all algebraic answers in terms of the given quantities and fundamental constants.

(a) For the following regions, indicate the direction of the electric field $E(r)$ and derive an expression for its magnitude.

i. $r < R$

____ Radially inward ____ Radially outward

ii. $r > R$

____ Radially inward ____ Radially outward

(b) For the following regions, derive an expression for the enclosed charge that generates the electric field in that region, expressed as a function of r.

i. $r < R$

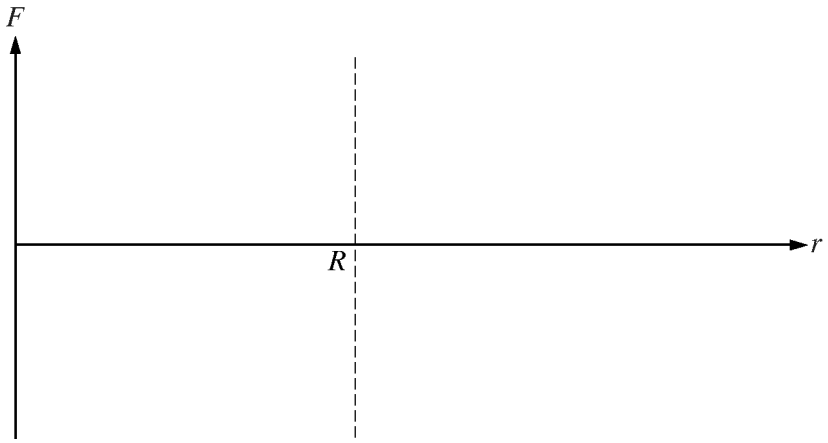
ii. $r > R$

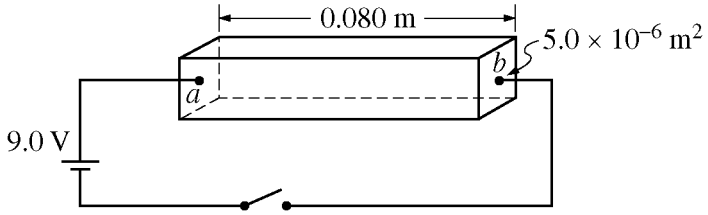
(c) Is there any charge on the surface of the sphere ($r = R$) ?

____ Yes ____ No

If there is, determine the charge. In either case, explain your reasoning.

(d) On the axes below, sketch a graph of the force that would act on a positive test charge in the regions $r < R$ and $r > R$. Assume that a force directed radially outward is positive.



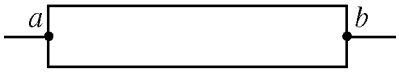


E&M. 2.

A 9.0 V battery is connected to a rectangular bar of length 0.080 m, uniform cross-sectional area 5.0×10^{-6} m^2, and resistivity 4.5×10^{-4} Ω•m , as shown above. Electrons are the sole charge carriers in the bar. The wires have negligible resistance. The switch in the circuit is closed at time $t = 0$.

(a) Calculate the power delivered to the circuit by the battery.

(b) On the diagram below, indicate the direction of the electric field in the bar.

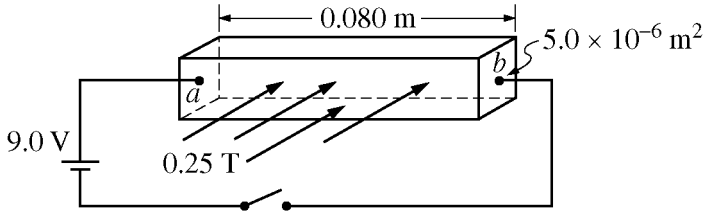


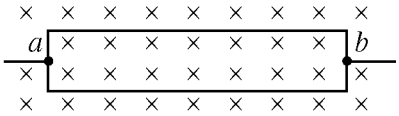
Side View

Explain your answer.

(c) Calculate the strength of the electric field in the bar.

A uniform magnetic field of magnitude 0.25 T perpendicular to the bar is added to the region around the bar, as shown below.

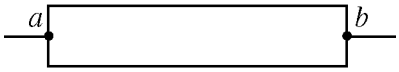




Side View

(d) Calculate the magnetic force on the bar.

(e) The electrons moving through the bar are initially deflected by the external magnetic field. On the diagram below, indicate the direction of the additional electric field that is created in the bar by the deflected electrons.



Side View

(f) The electrons eventually experience no deflection and move through the bar at an average speed of 3.5×10^{-3} m/s . Calculate the strength of the additional electric field indicated in part (e).

Question 1 (continued)

Distribution of points

(c) 2 points

There is charge residing on the surface of the sphere.

For indicating that the total enclosed charge equals the charge at the surface plus all the charge inside the sphere — 1 point

$Q_{enclosed,\ r>R} = Q_{surface} + Q_{enclosed,\ r<R \text{ when } r=R}$

$Q_{surface} = Q_{enclosed,\ r>R} - Q_{enclosed,\ r<R \text{ when } r=R}$

For substituting the correct value of $Q_{enclosed,\ r<R \text{ when } r=R}$ — 1 point

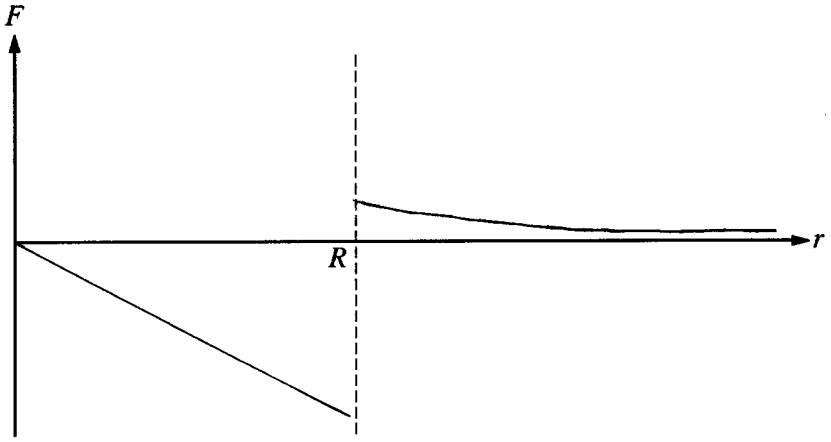
$$Q_{surface} = Q_0 - \left(-\frac{6Q_0R^3}{R^3}\right)$$

$$Q_{surface} = Q_0 - (-6Q_0)$$

$$Q_{surface} = 7Q_0$$

Note: The student could earn 1 point for a correct qualitative description of the charge configuration, even if the surface charge was not calculated.

(d) 3 points



For having the graph for $r < R$ consistent with the answer to part a (i) — 1 point

For having the graph for $r > R$ consistent with the answer to part a (ii), with a finite value at $r = R$ — 1 point

For having a step discontinuity at $r = R$ indicating the presence of charge at the surface (Having both graphs asymptotic to $r = R$ does not constitute the correct discontinuity.) — 1 point