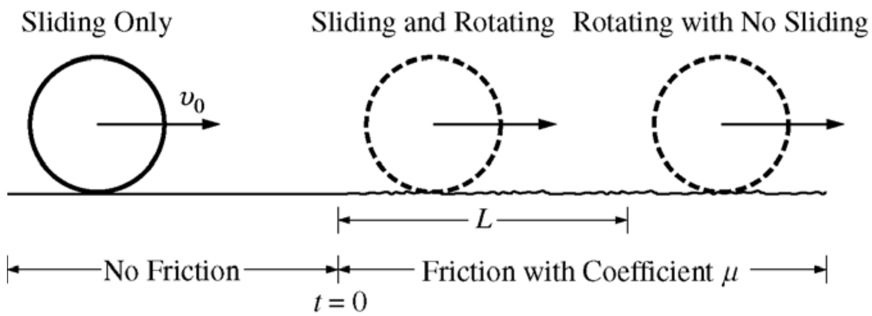

Mech. 3.

A ring of mass M, radius R, and rotational inertia MR^2 is initially sliding on a frictionless surface at constant velocity v_0 to the right, as shown above. At time $t = 0$ it encounters a surface with coefficient of friction μ and begins sliding and rotating. After traveling a distance L, the ring begins rolling without sliding. Express all answers to the following in terms of M, R, v_0, μ, and fundamental constants, as appropriate.

(a) Starting from Newton's second law in either translational or rotational form, as appropriate, derive a differential equation that can be used to solve for the magnitude of the following as the ring is sliding and rotating.

i. The linear velocity v of the ring as a function of time t

ii. The angular velocity ω of the ring as a function of time t

(b) Derive an expression for the magnitude of the following as the ring is sliding and rotating.

i. The linear velocity v of the ring as a function of time t

ii. The angular velocity ω of the ring as a function of time t

(c) Derive an expression for the time it takes the ring to travel the distance L.

(d) Derive an expression for the magnitude of the velocity of the ring immediately after it has traveled the distance L.

(e) Derive an expression for the distance L.

STOP

END OF EXAM

Question 3

15 points total | **Distribution of points**

(a)

i. 3 points

For starting with Newton's second law for translation, with friction as the net force — 1 point
$\Sigma F = -f = Ma$

For a correct expression for the frictional force — 1 point
$f = \mu Mg$

For indicating that linear acceleration is the time derivative of velocity — 1 point

$a = \dfrac{d\upsilon}{dt}$

$\dfrac{d\upsilon}{dt} = -\mu g$

ii. 3 points

For starting with Newton's second law for rotation, with a correct substitution for the rotational inertia — 1 point

$\tau = MR^2\alpha$

For a correct expression for the torque, using the frictional force — 1 point
$\tau = \mu MgR$

For indicating that the angular acceleration is the time derivative of the angular velocity — 1 point

$\alpha = \dfrac{d\omega}{dt}$

$\dfrac{d\omega}{dt} = \dfrac{\mu g}{R}$

(b)

i. 2 points

For setting up the integral of the function determined in part (a)-i — 1 point

$\int_{\upsilon_0}^{\upsilon} d\upsilon = -\int_0^t \mu g dt$

For the correct answer — 1 point
$\upsilon = \upsilon_0 - \mu gt$

Alternate solution — *Alternate points*

For a clear substitution of the acceleration from part (a)-i into the kinematics equation — *1 point*
$a = -\mu g$

$\upsilon = \upsilon_0 + at$

For the correct answer — *1 point*
$\upsilon = \upsilon_0 - \mu gt$

Question 3 (continued)

Distribution of points

(b) continued

ii. 2 points

For setting up the integral of the function determined in part (a)-ii — 1 point

$$\int_0^{\omega} d\omega = \int_0^{t} (\mu g/R)\,dt$$

For the correct answer — 1 point

$$\omega = \mu g t/R$$

Alternate solution — *Alternate points*

For a clear substitution of the angular acceleration from part (a)-ii into the correct rotational kinematics equation — *1 point*

$$\alpha = \frac{\mu g}{R}$$

$$\omega = \omega_0 + \alpha t$$

For the correct answer — *1 point*

$$\omega = \mu g t/R$$

(c) 2 points

For indicating that the linear speed is equal to $R\omega$ when the slipping stops — 1 point

$$\upsilon = R\omega$$

$$\upsilon_0 - \mu g t = R\left(\frac{\mu g t}{R}\right)$$

For the correct answer — 1 point

$$t = \frac{\upsilon_0}{2\mu g}$$

(d) 1 point

For substituting the time found in part (c) into a correct kinematics equation — 1 point

$$\upsilon = \upsilon_0 - \mu g\left(\frac{\upsilon_0}{2\mu g}\right)$$

$$\upsilon = \upsilon_0/2$$

Question 3 (continued)

Distribution of points

(e) 2 points

For setting up the integral of the velocity function determined in part (b)-i — 1 point

$$L = \int_0^t (\upsilon_0 - \upsilon g t)\,dt$$

For the correct answer, with correct supporting work — 1 point

$$L = \left[\upsilon_0 t - \frac{1}{2}\mu g t^2\right]_0^{\frac{\upsilon_0}{2\mu g}}$$

$$L = \frac{3\upsilon_0^2}{8\mu g}$$

Alternate solution #1 — *Alternate points*

For substituting the velocity from part (d) and the acceleration from part (a)-i into a correct equation that solves for L — *1 point*

$$\upsilon^2 = \upsilon_0^2 + 2a\Delta x$$

$$\left(\frac{\upsilon_0}{2}\right)^2 = \upsilon_0^2 + 2(-\mu g)L$$

For the correct answer, with correct supporting work — *1 point*

$$L = \frac{3\upsilon_0^2}{8\mu g}$$

Alternate solution #2 — *Alternate points*

For substituting the velocity from part (d) and the acceleration from part (a)-i into a correct equation that solves for L — *1 point*

<u>Note</u>: The time determined in part (c) must also be substituted.

$$\Delta x = \upsilon_0 t + \frac{1}{2}at^2$$

$$L = \upsilon_0\left(\frac{\upsilon_0}{2\mu g}\right) + \frac{1}{2}(-\mu g)\left(\frac{\upsilon_0}{2\mu g}\right)^2$$

For the correct answer, with correct supporting work — *1 point*

$$L = \frac{3\upsilon_0^2}{8\mu g}$$