5. A large university provides housing for 10 percent of its graduate students to live on campus. The university's housing office thinks that the percentage of graduate students looking for housing on campus may be more than 10 percent. The housing office decides to survey a random sample of graduate students, and 62 of the 481 respondents say that they are looking for housing on campus.

   (a) On the basis of the survey data, would you recommend that the housing office consider increasing the amount of housing on campus available to graduate students? Give appropriate evidence to support your recommendation.

   (b) In addition to the 481 graduate students who responded to the survey, there were 19 who did not respond. If these 19 had responded, is it possible that your recommendation would have changed? Explain.

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Section II

Part B

Question 6

Spend about 25 minutes on this part of the exam.

Percent of Section II grade—25

6. The manager of a cultured pearl farm has received a special order for two pearls between 7 millimeters and 9 millimeters in diameter. From past experience, the manager knows that the pearls found in his oyster bed have diameters that are normally distributed with a mean of 8 millimeters and a standard deviation of 2 millimeters. Assume that every oyster contains one pearl.

   The manager wants to know how many oysters he should expect to open to find two pearls of the appropriate size for this special order. Complete the following parts to design a simulation to answer the manager's question.

   (a) Determine the probability of finding a pearl of the appropriate size in an oyster selected at random. (Express this probability as a number between 0 and 1. Round this probability to the nearest tenth.)

   (b) Describe how you would use a table of random digits to carry out a simulation to determine the number of oysters needed to find two pearls of the appropriate size. Include a description of what each of the digits 0, 1, 2, 3, 4, 5, 6, 7, 8, and 9 will represent in your simulation.

   (c) Perform your simulation 3 times. (That is, run 3 trials of you simulation.) Start at the upper left most digit in the first row of the table and move across. Make your procedure clear so that someone can follow what you did. You must do this by marking directly on or above the table.

48747	76595	32588	38392	84422	80016	37890
71950	22494	00369	51269	87073	73694	97751
17857	52352	21392	22930	43776	10503	58249
80993	52010	88856	23882	73613	57648	47051
63016	73572	22684	02409	37565	52457	01257
40615	63910	09596	10241	03413	77576	74872
57431	29251	77848	98037	81230	38561	69580
06181	97842	48327	37976	81333	10264	77769

Free-Response Scoring Guidelines: Question 5

4 Complete Response

The response is substantially complete for all four sections.

3 Substantial Response

The response is substantially complete for three of the four sections.

2 Developing Response

The response is substantially complete for two of the four sections.

1 Minimal Response

The response is substantially complete for one of the four sections, or the response must show some evidence that they understand the role of variation in making inference from data.

Solutions and Scoring

Section 1: Checking the assumptions in Part (a)

Students must show (1), or they may show both (2) AND (3) in the following list:

1. Shows $np > 10$ and $n(1-p) > 10$ with numbers substituted for n and p; must use $p = 0.1$.
2. States only that $np > 10$ and $n(1-p) > 10$ [$np > 5$ and $n(1-p) > 5$ are OK].
3. States that the observations must be independent or points out SRS conducted.

Section 2: Doing the mechanics in Part (a)

Students should correctly state null and alternative hypotheses.

Because the 481 respondents appear to constitute a random sample, it is appropriate to test the null hypothesis: $p = .10$ against the alternative hypothesis: $p > .10$ where p denotes the true proportion of graduate students who prefer campus housing.

Students should show that the test statistic is:

$$z = \frac{\hat{p} - p_0}{\sqrt{\dfrac{p_0(1-p_0)}{n}}} = \frac{0.129 - 0.100}{\sqrt{\dfrac{0.1(0.9)}{481}}} = 2.12 \quad \text{where } \hat{p} = \frac{62}{481} = 0.129,$$

the observed sample proportion of those who desire on-campus housing.

Students should show the associated p-value, .017 (one-sided), or they may use an appropriate rejection region approach.

To receive credit for Section 2, the student MUST state the null hypothesis and a 1-sided alternative and give a p-value (or appropriate rejection region). Overall, a student cannot receive a score of 4 on this question if they do not identify, by name or by formula, the statistical procedure they use. For example, they may say "1-sample test of a proportion" or give the formula for the test statistic (using symbols or numbers). If the score is reduced to 3 by some other error, failure to identify the statistical procedure used will not further reduce the score by another point.

Do not give credit for Section 2 if more than one or two of the following errors occur:

- Uses non-standard notation that is not defined
- Uses incorrect notation in either hypothesis
- Gives incorrect Z value because $\hat{p}$ was used in calculation of standard error
- Gives incorrect Z value because of minor arithmetic errors
- Does not show the calculation of the test statistic
- Makes inference based on 1 minus the correct p-value
- Gives a 2-sided p-value for a 1-sided test

Section 3: Writing the conclusion in Part (a)

Students must connect the work done in Section 2 with the conclusion given in Section 3 by noting that the observed test result is a rare event. They might say:

> Because the p-value is less than an alpha-level the student selects, OR because the p-value is so small, OR because there are only 17 chances in 1,000 of getting a z-value this large or larger when null hypothesis is true therefore we reject the null,

The student must ALSO state the conclusion in the context of the problem. For example,

> There is evidence to believe that more than 10% of the graduate students desire on-campus housing.

If a significance level is stated at .01, then the conclusion should be that there is insufficient evidence to say that more than 10% of the graduate students desire on-campus housing. The conclusion must be consistent with the alpha level the student gives.

A student should not lose credit for this section if they make one error such as stating that "the hypothesis has been proven" if this section is otherwise correct.

Section 4: Part (b)

Students must recognize that the situation most detrimental to the conclusion reached in part (a) would be the case in which all 19 said that they were not looking for housing on campus, and they must evaluate their effect on the p-value of the test statistic and on their conclusion in part (a).

In this situation, $\hat{p} = \dfrac{62}{500} = 0.124$
and the test statistic becomes $z = 1.79$ and the p-value is .0368. Although this p-value is larger than the one in part (a), there is still evidence to conclude that more than 10% of the graduate students desire on-campus housing (an any fixed significance level greater than .0368).

If the decision in Part (a) is not to recommend increasing the amount of housing, the student should evaluate
$\hat{p} = \dfrac{81}{500} = 0.162$
and the test statistic becomes $z = 4.62$ and the p-value is less than .000002. In this case the decision in Part (a) would change.

Other Possible Solutions Include:

(1) **Exact Binomial**

Part (a) $X \sim \text{BINOMIAL}(n = 481, p = .1)$ $\text{P}(X \geq 62) = .0238$

Part (b) $X \sim \text{BINOMIAL}(n = 500, p = .1)$ $\text{P}(X \geq 62) = .0465$

(2) **Normal Approximation to Binomial**

Part (a): $X \sim \text{BINOMIAL}(n = 481, p = .1) \Rightarrow X$ approximately $\text{N}(48.10, 6.58)$

$Z = \dfrac{x - \mu}{\sigma} = \dfrac{62 - 48.1}{6.58} = 2.113$ and $\text{P}(Z \geq 2.113) = 0.0173$

OR

with the correction for continuity: $Z = 2.037$ and $\text{P}(Z \geq 2.037) = 0.0208$

Part (b): $X \sim \text{BINOMIAL}(n = 500, p = .1) \Rightarrow X$ approximately $\text{N}(50.00, 6.71)$

$Z = \dfrac{x - \mu}{\sigma} = \dfrac{62 - 50}{6.71} = 1.789$ and $\text{P}(Z \geq 1.789) = 0.0368$

OR

with the correction for continuity: $Z = 1.714$ and $\text{P}(Z \geq 1.714) = 0.0432$

(3) **Chi-Square Test**

(4) **Appropriate One-Sided Confidence Interval**
