CALCULUS AB
SECTION II, Part B
Time—45 minutes
Number of problems—3

No calculator is allowed for these problems.

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4. Water is pumped into an underground tank at a constant rate of 8 gallons per minute. Water leaks out of the tank at the rate of $\sqrt{t+1}$ gallons per minute, for $0 \le t \le 120$ minutes. At time $t = 0$, the tank contains 30 gallons of water.

   (a) How many gallons of water leak out of the tank from time $t = 0$ to $t = 3$ minutes?

   (b) How many gallons of water are in the tank at time $t = 3$ minutes?

   (c) Write an expression for $A(t)$, the total number of gallons of water in the tank at time t.

   (d) At what time t, for $0 \le t \le 120$, is the amount of water in the tank a maximum? Justify your answer.

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5. Consider the curve given by $xy^2 - x^3y = 6$.

   (a) Show that $\dfrac{dy}{dx} = \dfrac{3x^2y - y^2}{2xy - x^3}$.

   (b) Find all points on the curve whose x-coordinate is 1, and write an equation for the tangent line at each of these points.

   (c) Find the x-coordinate of each point on the curve where the tangent line is vertical.

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6. Consider the differential equation $\dfrac{dy}{dx} = \dfrac{3x^2}{e^{2y}}$.

   (a) Find a solution $y = f(x)$ to the differential equation satisfying $f(0) = \dfrac{1}{2}$.

   (b) Find the domain and range of the function f found in part (a).

END OF EXAMINATION

Water is pumped into an underground tank at a constant rate of 8 gallons per minute. Water leaks out of the tank at the rate of $\sqrt{t+1}$ gallons per minute, for $0 \le t \le 120$ minutes. At time $t = 0$, the tank contains 30 gallons of water.

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(b) How many gallons of water are in the tank at time $t = 3$ minutes?

(c) Write an expression for $A(t)$, the total number of gallons of water in the tank at time t.

(d) At what time t, for $0 \le t \le 120$, is the amount of water in the tank a maximum? Justify your answer.

(a) Method 1: $\int_0^3 \sqrt{t+1}\,dt = \frac{2}{3}(t+1)^{3/2}\Big|_0^3 = \frac{14}{3}$

– or –

Method 2: $L(t)$ = gallons leaked in first t minutes

$$\frac{dL}{dt} = \sqrt{t+1}; \quad L(t) = \frac{2}{3}(t+1)^{3/2} + C$$

$$L(0) = 0; \quad C = -\frac{2}{3}$$

$$L(t) = \frac{2}{3}(t+1)^{3/2} - \frac{2}{3}; \quad L(3) = \frac{14}{3}$$

Method 1:

$$3\begin{cases} 2 : \text{definite integral} \\ \quad 1 : \text{limits} \\ \quad 1 : \text{integrand} \\ 1 : \text{answer} \end{cases}$$

– or –

Method 2:

$$3\begin{cases} 1 : \text{antiderivative with } C \\ 1 : \text{solves for } C \text{ using } L(0) = 0 \\ 1 : \text{answer} \end{cases}$$

(b) $30 + 8 \cdot 3 - \frac{14}{3} = \frac{148}{3}$

1 : answer

(c) Method 1:

$$A(t) = 30 + \int_0^t \left(8 - \sqrt{x+1}\right)dx$$
$$= 30 + 8t - \int_0^t \sqrt{x+1}\,dx$$

– or –

Method 2:

$$\frac{dA}{dt} = 8 - \sqrt{t+1}$$

$$A(t) = 8t - \frac{2}{3}(t+1)^{3/2} + C$$

$$30 = 8(0) - \frac{2}{3}(0+1)^{3/2} + C; \quad C = \frac{92}{3}$$

$$A(t) = 8t - \frac{2}{3}(t+1)^{3/2} + \frac{92}{3}$$

Method 1:

$$2\begin{cases} 1 : 30 + 8t \\ 1 : -\int_0^t \sqrt{x+1}\,dx \end{cases}$$

– or –

Method 2:

$$2\begin{cases} 1 : \text{antiderivative with } C \\ 1 : \text{answer} \end{cases}$$

(d) $A'(t) = 8 - \sqrt{t+1} = 0$ when $t = 63$

$A'(t)$ is positive for $0 < t < 63$ and negative for $63 < t < 120$. Therefore there is a maximum at $t = 63$.

$$3\begin{cases} 1 : \text{ sets } A'(t) = 0 \\ 1 : \text{ solves for } t \\ 1 : \text{ justification} \end{cases}$$