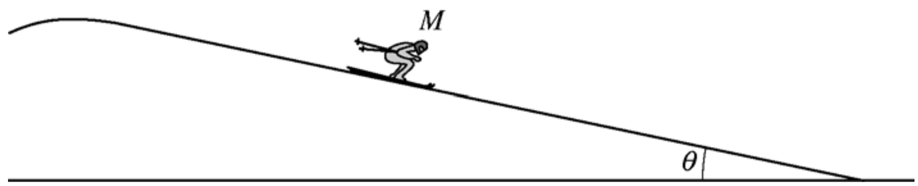
PHYSICS C: MECHANICS

SECTION II

Time—45 minutes

3 Questions

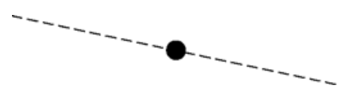
Directions: Answer all three questions. The suggested time is about 15 minutes for answering each of the questions, which are worth 15 points each. The parts within a question may not have equal weight. Show all your work in the pink booklet in the spaces provided after each part, NOT in this green insert.



Mech. 1.

A skier of mass M is skiing down a frictionless hill that makes an angle θ with the horizontal, as shown in the diagram. The skier starts from rest at time $t = 0$ and is subject to a velocity-dependent drag force due to air resistance of the form $F = -bv$, where v is the velocity of the skier and b is a positive constant. Express all algebraic answers in terms of M, b, θ, and fundamental constants.

(a) On the dot below that represents the skier, draw a free-body diagram indicating and labeling all of the forces that act on the skier while the skier descends the hill.

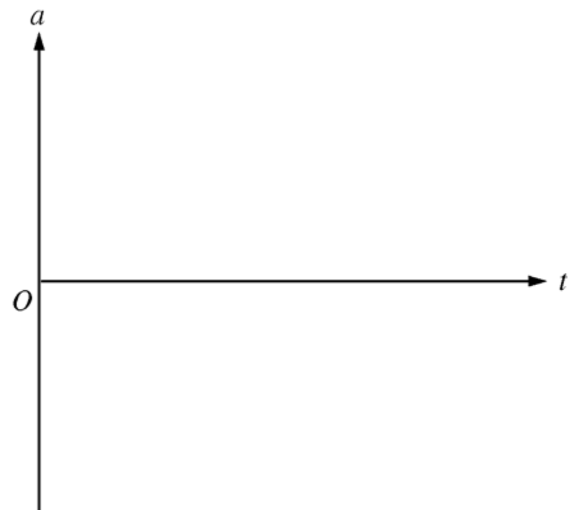


(b) Write a differential equation that can be used to solve for the velocity of the skier as a function of time.

(c) Determine an expression for the terminal velocity v_T of the skier.

(d) Solve the differential equation in part (b) to determine the velocity of the skier as a function of time, <u>showing all your steps</u>.

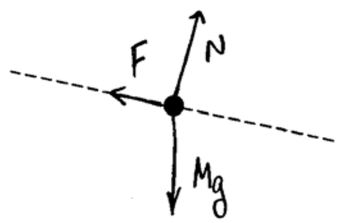
(e) On the axes below, sketch a graph of the acceleration a of the skier as a function of time t, and indicate the initial value of a. Take downhill as positive.



Question 1

15 points total **Distribution of points**

(a) 3 points



For a correctly drawn and labeled weight vector, originating on the dot and with an arrowhead (Alternatively, correctly drawn and labeled components instead of the total weight vector was acceptable.) — 1 point

For a correctly drawn and labeled normal force vector, originating on the dot and with an arrowhead — 1 point

For a correctly drawn and labeled drag-force vector, originating on the dot and with an arrowhead — 1 point

One point was deducted if there were any extra vectors on the point, including components drawn with arrowheads.

(b) 4 points

For any expression of $F = Ma$ or any dimensionally correct application of $F = Ma$ — 1 point

For correctly expressing the component of the weight parallel to the plane as $Mg\sin\theta$ — 1 point

For correctly expressing the drag force as $-kx$ — 1 point

$Ma = Mg\sin\theta - bv$

For a dimensionally correct differential equation, including dv/dt and expressions for the drag force and the component of the weight parallel to the plane — 1 point

$$M\frac{dv}{dt} = Mg\sin\theta - bv$$

One point was deducted if the algebraic signs of the weight component and the drag force were not opposite somewhere in the solution, OR if only one of these two terms was included.

(c) 2 points

For an indication that $F_{\text{net}} = 0$, $a = 0$, or the parallel component of the weight $= bv_T$ — 1 point

$0 = Mg\sin\theta - bv_T$

$bv_T = Mg\sin\theta$

For the correct expression for the terminal velocity (or one consistent with part (b)) — 1 point

$v_T = Mg\sin\theta/b$

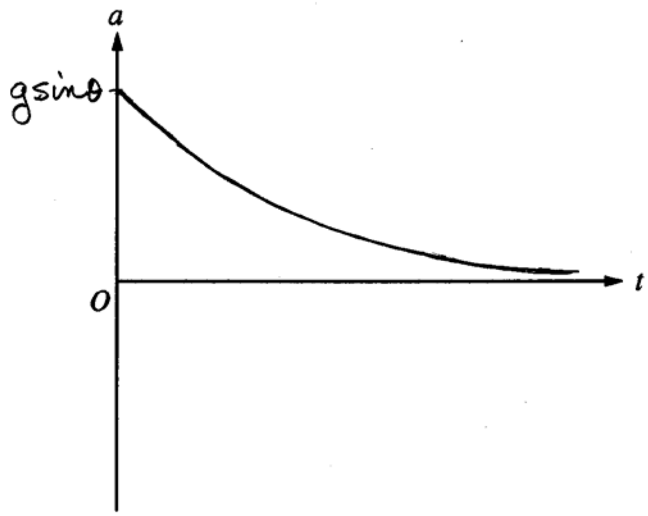
Question 1 (continued)

	Distribution of points
(d) 3 points	
For taking the differential equation from part (b) and correctly separating the variables in preparation for integration (definite or indefinite integral)	1 point
$M\dfrac{dv}{dt} = Mg\sin\theta - bv$	
$\dfrac{dv}{Mg\sin\theta - bv} = \dfrac{dt}{M}$	
For correct integration of both sides of equation	1 point
For example, using a method involving an indefinite integral	
Letting $u = Mg\sin\theta - bv$, so $du = -b\,dv$	
$-\dfrac{1}{b}\dfrac{du}{u} = \dfrac{dt}{M}$	
$\displaystyle\int \frac{du}{u} = -\frac{b}{M}\int dt$	
$\ln u = -\dfrac{b}{M}t + \ln C$	
$u = Ce^{-bt/M}$	
$Mg\sin\theta - bv = Ce^{-bt/M}$	
Using $v = 0$ at $t = 0$	
$Mg\sin\theta = C$	
$Mg\sin\theta - bv = Mg\sin\theta\; e^{-bt/M}$	
$-bv = Mg\sin\theta\; e^{-bt/M} - Mg\sin\theta$	
For a correct final expression for $v(t)$	1 point
$v = \left(Mg\sin\theta/b\right)\left(1 - e^{-bt/M}\right)$	

Question 1 (continued)

Distribution of points

(e) 3 points



For the correct initial value of *a* (or a value consistent with part (b))	1 point
For a negatively sloped curve, concave up	1 point
For a curve asymptotic to the *t* axis (This point was awarded even if the curve was not otherwise correct.)	1 point