

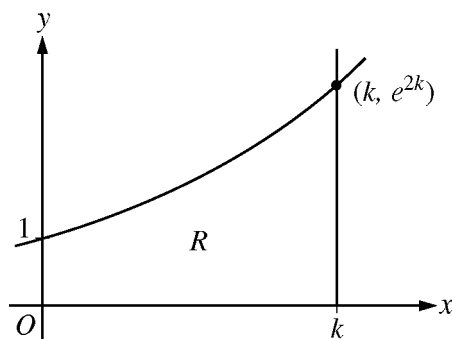
2011 AP<sup>®</sup> CALCULUS BC FREE-RESPONSE QUESTIONS

CALCULUS BC  
SECTION II, Part B

Time—60 minutes

Number of problems—4

No calculator is allowed for these problems.

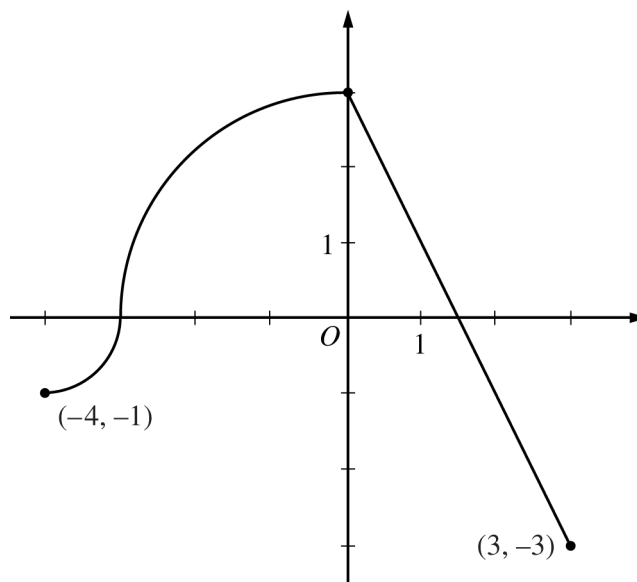


3. Let  $f(x) = e^{2x}$ . Let  $R$  be the region in the first quadrant bounded by the graph of  $f$ , the coordinate axes, and the vertical line  $x = k$ , where  $k > 0$ . The region  $R$  is shown in the figure above.
- (a) Write, but do not evaluate, an expression involving an integral that gives the perimeter of  $R$  in terms of  $k$ .
- (b) The region  $R$  is rotated about the  $x$ -axis to form a solid. Find the volume,  $V$ , of the solid in terms of  $k$ .
- (c) The volume  $V$ , found in part (b), changes as  $k$  changes. If  $\frac{dk}{dt} = \frac{1}{3}$ , determine  $\frac{dV}{dt}$  when  $k = \frac{1}{2}$ .

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WRITE ALL WORK IN THE EXAM BOOKLET.

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Graph of  $f$

4. The continuous function  $f$  is defined on the interval  $-4 \leq x \leq 3$ . The graph of  $f$  consists of two quarter circles and one line segment, as shown in the figure above. Let  $g(x) = 2x + \int_0^x f(t) \, dt$ .
- Find  $g(-3)$ . Find  $g'(x)$  and evaluate  $g'(-3)$ .
  - Determine the  $x$ -coordinate of the point at which  $g$  has an absolute maximum on the interval  $-4 \leq x \leq 3$ . Justify your answer.
  - Find all values of  $x$  on the interval  $-4 < x < 3$  for which the graph of  $g$  has a point of inflection. Give a reason for your answer.
  - Find the average rate of change of  $f$  on the interval  $-4 \leq x \leq 3$ . There is no point  $c$ ,  $-4 < c < 3$ , for which  $f'(c)$  is equal to that average rate of change. Explain why this statement does not contradict the Mean Value Theorem.

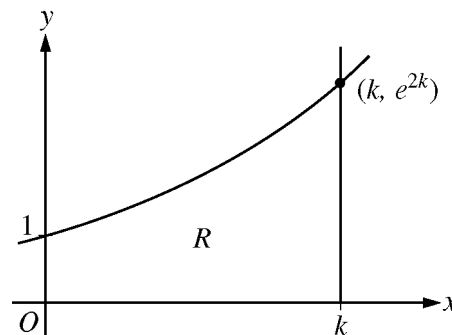
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**WRITE ALL WORK IN THE EXAM BOOKLET.**

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**2011 SCORING GUIDELINES**

**Question 3**

Let  $f(x) = e^{2x}$ . Let  $R$  be the region in the first quadrant bounded by the graph of  $f$ , the coordinate axes, and the vertical line  $x = k$ , where  $k > 0$ . The region  $R$  is shown in the figure above.



- (a) Write, but do not evaluate, an expression involving an integral that gives the perimeter of  $R$  in terms of  $k$ .
- (b) The region  $R$  is rotated about the  $x$ -axis to form a solid. Find the volume,  $V$ , of the solid in terms of  $k$ .
- (c) The volume  $V$ , found in part (b), changes as  $k$  changes. If  $\frac{dk}{dt} = \frac{1}{3}$ , determine  $\frac{dV}{dt}$  when  $k = \frac{1}{2}$ .

(a)  $f'(x) = 2e^{2x}$

$$\text{Perimeter} = 1 + k + e^{2k} + \int_0^k \sqrt{1 + (2e^{2x})^2} dx$$

$$3 : \begin{cases} 1 : f'(x) \\ 1 : \text{integral} \\ 1 : \text{answer} \end{cases}$$

(b)  $\text{Volume} = \pi \int_0^k (e^{2x})^2 dx = \pi \int_0^k e^{4x} dx = \frac{\pi}{4} e^{4x} \Big|_{x=0}^{x=k} = \frac{\pi}{4} e^{4k} - \frac{\pi}{4}$

$$4 : \begin{cases} 1 : \text{integrand} \\ 1 : \text{limits} \\ 1 : \text{antiderivative} \\ 1 : \text{answer} \end{cases}$$

(c)  $\frac{dV}{dt} = \pi e^{4k} \frac{dk}{dt}$

When  $k = \frac{1}{2}$ ,  $\frac{dV}{dt} = \pi e^2 \cdot \frac{1}{3}$ .

$$2 : \begin{cases} 1 : \text{applies chain rule} \\ 1 : \text{answer} \end{cases}$$