

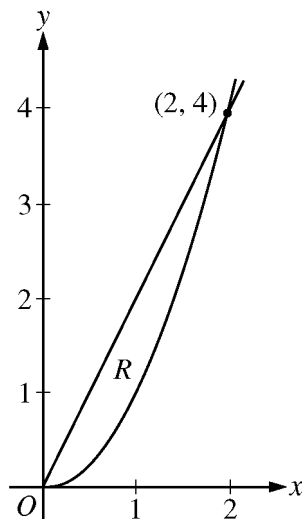
2009 AP<sup>®</sup> CALCULUS AB FREE-RESPONSE QUESTIONS

CALCULUS AB  
SECTION II, Part B

Time—45 minutes

Number of problems—3

No calculator is allowed for these problems.



4. Let  $R$  be the region in the first quadrant enclosed by the graphs of  $y = 2x$  and  $y = x^2$ , as shown in the figure above.
- (a) Find the area of  $R$ .
  - (b) The region  $R$  is the base of a solid. For this solid, at each  $x$  the cross section perpendicular to the  $x$ -axis has area  $A(x) = \sin\left(\frac{\pi}{2}x\right)$ . Find the volume of the solid.
  - (c) Another solid has the same base  $R$ . For this solid, the cross sections perpendicular to the  $y$ -axis are squares. Write, but do not evaluate, an integral expression for the volume of the solid.
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WRITE ALL WORK IN THE PINK EXAM BOOKLET.

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$x$	2	3	5	8	13
$f(x)$	1	4	-2	3	6

5. Let  $f$  be a function that is twice differentiable for all real numbers. The table above gives values of  $f$  for selected points in the closed interval  $2 \leq x \leq 13$ .
- (a) Estimate  $f'(4)$ . Show the work that leads to your answer.
- (b) Evaluate  $\int_2^{13} (3 - 5f'(x)) \, dx$ . Show the work that leads to your answer.
- (c) Use a left Riemann sum with subintervals indicated by the data in the table to approximate  $\int_2^{13} f(x) \, dx$ .  
Show the work that leads to your answer.
- (d) Suppose  $f'(5) = 3$  and  $f''(x) < 0$  for all  $x$  in the closed interval  $5 \leq x \leq 8$ . Use the line tangent to the graph of  $f$  at  $x = 5$  to show that  $f(7) \leq 4$ . Use the secant line for the graph of  $f$  on  $5 \leq x \leq 8$  to show that  $f(7) \geq \frac{4}{3}$ .
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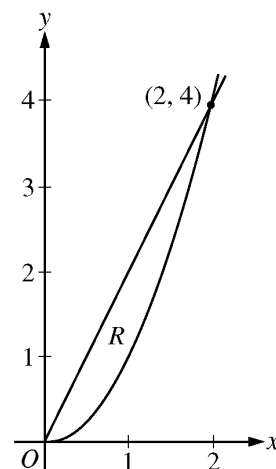
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**AP<sup>®</sup> CALCULUS AB**  
**2009 SCORING GUIDELINES**

**Question 4**

Let  $R$  be the region in the first quadrant enclosed by the graphs of  $y = 2x$  and  $y = x^2$ , as shown in the figure above.

- (a) Find the area of  $R$ .
- (b) The region  $R$  is the base of a solid. For this solid, at each  $x$  the cross section perpendicular to the  $x$ -axis has area  $A(x) = \sin\left(\frac{\pi}{2}x\right)$ . Find the volume of the solid.
- (c) Another solid has the same base  $R$ . For this solid, the cross sections perpendicular to the  $y$ -axis are squares. Write, but do not evaluate, an integral expression for the volume of the solid.



$$\begin{aligned} \text{(a) Area} &= \int_0^2 (2x - x^2) dx \\ &= x^2 - \frac{1}{3}x^3 \Big|_{x=0}^{x=2} \\ &= \frac{4}{3} \end{aligned}$$

3 :  $\begin{cases} 1 : \text{integrand} \\ 1 : \text{antiderivative} \\ 1 : \text{answer} \end{cases}$

$$\begin{aligned} \text{(b) Volume} &= \int_0^2 \sin\left(\frac{\pi}{2}x\right) dx \\ &= -\frac{2}{\pi} \cos\left(\frac{\pi}{2}x\right) \Big|_{x=0}^{x=2} \\ &= \frac{4}{\pi} \end{aligned}$$

3 :  $\begin{cases} 1 : \text{integrand} \\ 1 : \text{antiderivative} \\ 1 : \text{answer} \end{cases}$

$$\text{(c) Volume} = \int_0^4 \left(\sqrt{y} - \frac{y}{2}\right)^2 dy$$

3 :  $\begin{cases} 2 : \text{integrand} \\ 1 : \text{limits} \end{cases}$