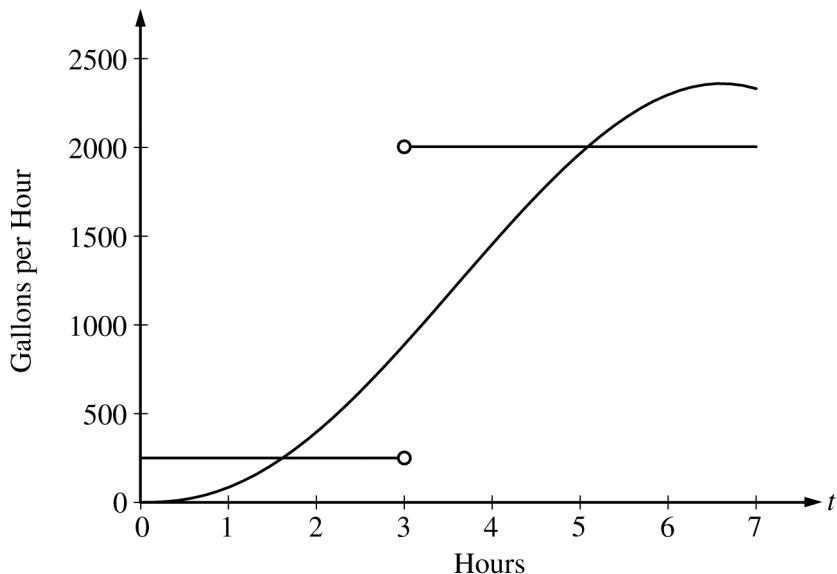


2007 AP® CALCULUS AB FREE-RESPONSE QUESTIONS



2. The amount of water in a storage tank, in gallons, is modeled by a continuous function on the time interval $0 \leq t \leq 7$, where t is measured in hours. In this model, rates are given as follows:

- (i) The rate at which water enters the tank is $f(t) = 100t^2 \sin(\sqrt{t})$ gallons per hour for $0 \leq t \leq 7$.
- (ii) The rate at which water leaves the tank is

$$g(t) = \begin{cases} 250 & \text{for } 0 \leq t < 3 \\ 2000 & \text{for } 3 < t \leq 7 \end{cases} \text{ gallons per hour.}$$

The graphs of f and g , which intersect at $t = 1.617$ and $t = 5.076$, are shown in the figure above. At time $t = 0$, the amount of water in the tank is 5000 gallons.

- (a) How many gallons of water enter the tank during the time interval $0 \leq t \leq 7$? Round your answer to the nearest gallon.
 - (b) For $0 \leq t \leq 7$, find the time intervals during which the amount of water in the tank is decreasing. Give a reason for each answer.
 - (c) For $0 \leq t \leq 7$, at what time t is the amount of water in the tank greatest? To the nearest gallon, compute the amount of water at this time. Justify your answer.
-

WRITE ALL WORK IN THE PINK EXAM BOOKLET.

2007 AP® CALCULUS AB FREE-RESPONSE QUESTIONS

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
1	6	4	2	5
2	9	2	3	1
3	10	-4	4	2
4	-1	3	6	7

3. The functions f and g are differentiable for all real numbers, and g is strictly increasing. The table above gives values of the functions and their first derivatives at selected values of x . The function h is given by $h(x) = f(g(x)) - 6$.
- Explain why there must be a value r for $1 < r < 3$ such that $h(r) = -5$.
 - Explain why there must be a value c for $1 < c < 3$ such that $h'(c) = -5$.
 - Let w be the function given by $w(x) = \int_1^{g(x)} f(t) dt$. Find the value of $w'(3)$.
 - If g^{-1} is the inverse function of g , write an equation for the line tangent to the graph of $y = g^{-1}(x)$ at $x = 2$.
-

WRITE ALL WORK IN THE PINK EXAM BOOKLET.

END OF PART A OF SECTION II

**AP[®] CALCULUS AB
2007 SCORING GUIDELINES**

Question 2

The amount of water in a storage tank, in gallons, is modeled by a continuous function on the time interval $0 \leq t \leq 7$, where t is measured in hours. In this model, rates are given as follows:

- (i) The rate at which water enters the tank is

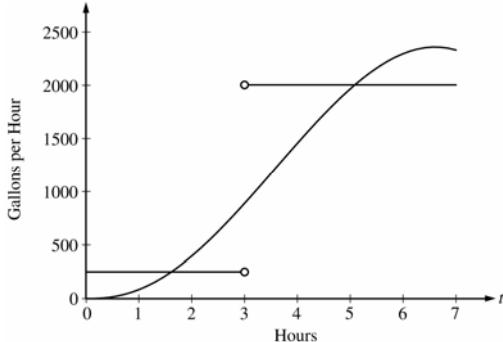
$$f(t) = 100t^2 \sin(\sqrt{t}) \text{ gallons per hour for } 0 \leq t \leq 7.$$

- (ii) The rate at which water leaves the tank is

$$g(t) = \begin{cases} 250 & \text{for } 0 \leq t < 3 \\ 2000 & \text{for } 3 < t \leq 7 \end{cases} \text{ gallons per hour.}$$

The graphs of f and g , which intersect at $t = 1.617$ and $t = 5.076$, are shown in the figure above. At time $t = 0$, the amount of water in the tank is 5000 gallons.

- (a) How many gallons of water enter the tank during the time interval $0 \leq t \leq 7$? Round your answer to the nearest gallon.
- (b) For $0 \leq t \leq 7$, find the time intervals during which the amount of water in the tank is decreasing. Give a reason for each answer.
- (c) For $0 \leq t \leq 7$, at what time t is the amount of water in the tank greatest? To the nearest gallon, compute the amount of water at this time. Justify your answer.



(a) $\int_0^7 f(t) dt \approx 8264$ gallons

2 : $\left\{ \begin{array}{l} 1 : \text{integral} \\ 1 : \text{answer} \end{array} \right.$

- (b) The amount of water in the tank is decreasing on the intervals $0 \leq t \leq 1.617$ and $3 \leq t \leq 5.076$ because $f(t) < g(t)$ for $0 \leq t < 1.617$ and $3 < t < 5.076$.

2 : $\left\{ \begin{array}{l} 1 : \text{intervals} \\ 1 : \text{reason} \end{array} \right.$

- (c) Since $f(t) - g(t)$ changes sign from positive to negative only at $t = 3$, the candidates for the absolute maximum are at $t = 0, 3$, and 7 .

t (hours)	gallons of water
0	5000
3	$5000 + \int_0^3 f(t) dt - 250(3) = 5126.591$
7	$5126.591 + \int_3^7 f(t) dt - 2000(4) = 4513.807$

5 : $\left\{ \begin{array}{l} 1 : \text{identifies } t = 3 \text{ as a candidate} \\ 1 : \text{integrand} \\ 1 : \text{amount of water at } t = 3 \\ 1 : \text{amount of water at } t = 7 \\ 1 : \text{conclusion} \end{array} \right.$

The amount of water in the tank is greatest at 3 hours. At that time, the amount of water in the tank, rounded to the nearest gallon, is 5127 gallons.