

2009 AP<sup>®</sup> STATISTICS FREE-RESPONSE QUESTIONS

STATISTICS

SECTION II

Part B

Question 6

Spend about 25 minutes on this part of the exam.

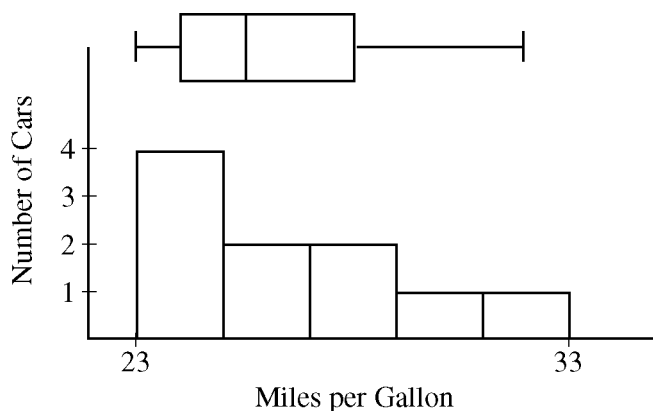
Percent of Section II score—25

**Directions:** Show all your work. Indicate clearly the methods you use, because you will be graded on the correctness of your methods as well as on the accuracy and completeness of your results and explanations.

6. A consumer organization was concerned that an automobile manufacturer was misleading customers by overstating the average fuel efficiency (measured in miles per gallon, or mpg) of a particular car model. The model was advertised to get 27 mpg. To investigate, researchers selected a random sample of 10 cars of that model. Each car was then randomly assigned a different driver. Each car was driven for 5,000 miles, and the total fuel consumption was used to compute mpg for that car.

- (a) Define the parameter of interest and state the null and alternative hypotheses the consumer organization is interested in testing.

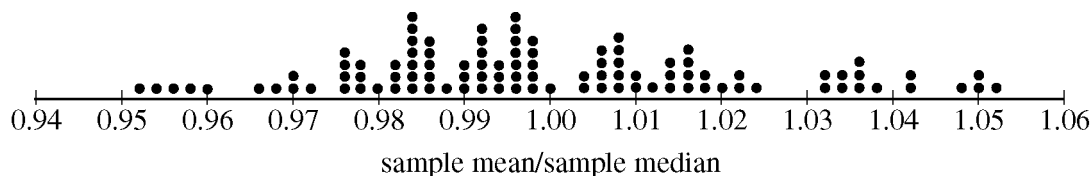
One condition for conducting a one-sample  $t$ -test in this situation is that the mpg measurements for the population of cars of this model should be normally distributed. However, the boxplot and histogram shown below indicate that the distribution of the 10 sample values is skewed to the right.



- (b) One possible statistic that measures skewness is the ratio  $\frac{\text{sample mean}}{\text{sample median}}$ . What values of that statistic (small, large, close to one) might indicate that the population distribution of mpg values is skewed to the right? Explain.

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- (c) Even though the mpg values in the sample were skewed to the right, it is still possible that the population distribution of mpg values is normally distributed and that the skewness was due to sampling variability. To investigate, 100 samples, each of size 10, were taken from a normal distribution with the same mean and standard deviation as the original sample. For each of those 100 samples, the statistic  $\frac{\text{sample mean}}{\text{sample median}}$  was calculated. A dotplot of the 100 simulated statistics is shown below.



In the original sample, the value of the statistic  $\frac{\text{sample mean}}{\text{sample median}}$  was 1.03. Based on the value of 1.03 and the dotplot above, is it plausible that the original sample of 10 cars came from a normal population, or do the simulated results suggest the original population is really skewed to the right? Explain.

- (d) The table below shows summary statistics for mpg measurements for the original sample of 10 cars.

| Minimum | Q1 | Median | Q3 | Maximum |
|---------|----|--------|----|---------|
| 23      | 24 | 25.5   | 28 | 32      |

Choosing only from the summary statistics in the table, define a formula for a different statistic that measures skewness.

What values of that statistic might indicate that the distribution is skewed to the right? Explain.

**STOP**

**END OF EXAM**

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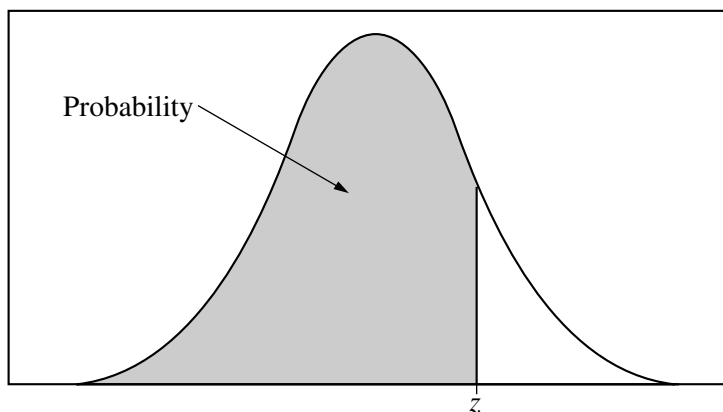


Table entry for  $z$  is the probability lying below  $z$ .

**Table A** (Continued)

| $z$ | .00   | .01   | .02   | .03   | .04   | .05   | .06   | .07   | .08   | .09   |
|-----|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| 0.0 | .5000 | .5040 | .5080 | .5120 | .5160 | .5199 | .5239 | .5279 | .5319 | .5359 |
| 0.1 | .5398 | .5438 | .5478 | .5517 | .5557 | .5596 | .5636 | .5675 | .5714 | .5753 |
| 0.2 | .5793 | .5832 | .5871 | .5910 | .5948 | .5987 | .6026 | .6064 | .6103 | .6141 |
| 0.3 | .6179 | .6217 | .6255 | .6293 | .6331 | .6368 | .6406 | .6443 | .6480 | .6517 |
| 0.4 | .6554 | .6591 | .6628 | .6664 | .6700 | .6736 | .6772 | .6808 | .6844 | .6879 |
| 0.5 | .6915 | .6950 | .6985 | .7019 | .7054 | .7088 | .7123 | .7157 | .7190 | .7224 |
| 0.6 | .7257 | .7291 | .7324 | .7357 | .7389 | .7422 | .7454 | .7486 | .7517 | .7549 |
| 0.7 | .7580 | .7611 | .7642 | .7673 | .7704 | .7734 | .7764 | .7794 | .7823 | .7852 |
| 0.8 | .7881 | .7910 | .7939 | .7967 | .7995 | .8023 | .8051 | .8078 | .8106 | .8133 |
| 0.9 | .8159 | .8186 | .8212 | .8238 | .8264 | .8289 | .8315 | .8340 | .8365 | .8389 |
| 1.0 | .8413 | .8438 | .8461 | .8485 | .8508 | .8531 | .8554 | .8577 | .8599 | .8621 |
| 1.1 | .8643 | .8665 | .8686 | .8708 | .8729 | .8749 | .8770 | .8790 | .8810 | .8830 |
| 1.2 | .8849 | .8869 | .8888 | .8907 | .8925 | .8944 | .8962 | .8980 | .8997 | .9015 |
| 1.3 | .9032 | .9049 | .9066 | .9082 | .9099 | .9115 | .9131 | .9147 | .9162 | .9177 |
| 1.4 | .9192 | .9207 | .9222 | .9236 | .9251 | .9265 | .9279 | .9292 | .9306 | .9319 |
| 1.5 | .9332 | .9345 | .9357 | .9370 | .9382 | .9394 | .9406 | .9418 | .9429 | .9441 |
| 1.6 | .9452 | .9463 | .9474 | .9484 | .9495 | .9505 | .9515 | .9525 | .9535 | .9545 |
| 1.7 | .9554 | .9564 | .9573 | .9582 | .9591 | .9599 | .9608 | .9616 | .9625 | .9633 |
| 1.8 | .9641 | .9649 | .9656 | .9664 | .9671 | .9678 | .9686 | .9693 | .9699 | .9706 |
| 1.9 | .9713 | .9719 | .9726 | .9732 | .9738 | .9744 | .9750 | .9756 | .9761 | .9767 |
| 2.0 | .9772 | .9778 | .9783 | .9788 | .9793 | .9798 | .9803 | .9808 | .9812 | .9817 |
| 2.1 | .9821 | .9826 | .9830 | .9834 | .9838 | .9842 | .9846 | .9850 | .9854 | .9857 |
| 2.2 | .9861 | .9864 | .9868 | .9871 | .9875 | .9878 | .9881 | .9884 | .9887 | .9890 |
| 2.3 | .9893 | .9896 | .9898 | .9901 | .9904 | .9906 | .9909 | .9911 | .9913 | .9916 |
| 2.4 | .9918 | .9920 | .9922 | .9925 | .9927 | .9929 | .9931 | .9932 | .9934 | .9936 |
| 2.5 | .9938 | .9940 | .9941 | .9943 | .9945 | .9946 | .9948 | .9949 | .9951 | .9952 |
| 2.6 | .9953 | .9955 | .9956 | .9957 | .9959 | .9960 | .9961 | .9962 | .9963 | .9964 |
| 2.7 | .9965 | .9966 | .9967 | .9968 | .9969 | .9970 | .9971 | .9972 | .9973 | .9974 |
| 2.8 | .9974 | .9975 | .9976 | .9977 | .9977 | .9978 | .9979 | .9979 | .9980 | .9981 |
| 2.9 | .9981 | .9982 | .9982 | .9983 | .9984 | .9984 | .9985 | .9985 | .9986 | .9986 |
| 3.0 | .9987 | .9987 | .9987 | .9988 | .9988 | .9989 | .9989 | .9989 | .9990 | .9990 |
| 3.1 | .9990 | .9991 | .9991 | .9991 | .9992 | .9992 | .9992 | .9992 | .9993 | .9993 |
| 3.2 | .9993 | .9993 | .9994 | .9994 | .9994 | .9994 | .9994 | .9995 | .9995 | .9995 |
| 3.3 | .9995 | .9995 | .9995 | .9996 | .9996 | .9996 | .9996 | .9996 | .9996 | .9997 |
| 3.4 | .9997 | .9997 | .9997 | .9997 | .9997 | .9997 | .9997 | .9997 | .9997 | .9998 |

# AP<sup>®</sup> STATISTICS

## 2009 SCORING GUIDELINES

### Question 6 (continued)

**Part (a)** is scored as follows:

Essentially correct (E) if the student correctly states the hypotheses with a lower-tailed alternative hypothesis *AND* correctly defines the parameter of interest by referring to the mean of the population in context (e.g., population mean mpg, true mean fuel efficiency, mean mpg of all cars of this model, mean mpg of this car model).

Partially correct (P) if only one component is correct (hypotheses or definition of parameter).

Incorrect (I) otherwise.

*Notes:*

- If a student attempts to define the parameter more than once (e.g., saying “the parameter is . . .” and then later saying  $\mu = \dots$ ”), then these are treated as parallel solutions, and the worst attempt is scored.
- A symbol other than  $\mu$  in the hypotheses must be explicitly and correctly defined to get credit for the parameter component.
- If words are used in the hypotheses to describe the parameter,
  - the words do not count as the definition of the parameter, and
  - the words must be consistent with the definition of the parameter,or no credit is given for the parameter component.

**Part (b)** is scored as follows:

Essentially correct (E) if the student states that large values (or values greater than 1) of the statistic indicate right-skewness *AND* justifies the answer with a correct statement of how right-skewness affects the relationship between the mean and median.

Partially correct (P) if the student:

- states that large values of the statistic indicate right-skewness *BUT* only argues that in a normal (or symmetric) distribution the ratio should be close to 1 (i.e., does not discuss the right-skewness)  
*OR*
- makes a correct statement of how right-skewness affects the relationship between the mean and median *BUT* does not state that large values of the statistic indicate right-skewness  
*OR*
- has reversed the relationship between mean and median in right-skewed distributions or reversed left- and right-skewness *AND* states that small values of the statistic indicate right-skewness.

Incorrect (I) if the student says large values indicate right-skewness but gives no explanation or an incorrect explanation.

**Part (c)** is scored as follows:

Essentially correct (E) if the student states that it is plausible that the sample came from a normal population *AND* justifies the choice with specific numerical evidence from the dotplot describing the relative location of the value 1.03 (e.g., 14 percent of the values are above 1.03).