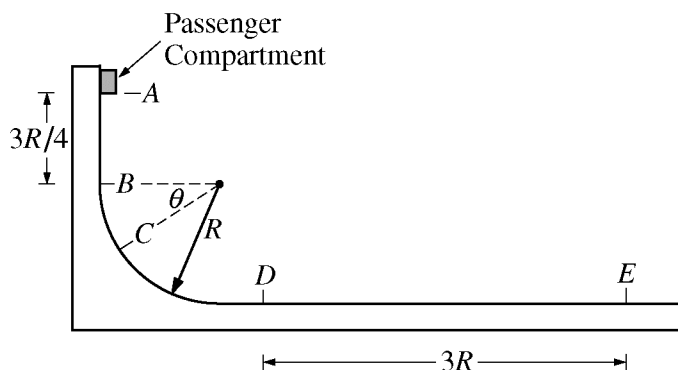


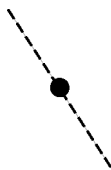
2011 AP<sup>®</sup> PHYSICS C: MECHANICS FREE-RESPONSE QUESTIONS



Mech. 2.

An amusement park ride features a passenger compartment of mass  $M$  that is released from rest at point  $A$ , as shown in the figure above, and moves along a track to point  $E$ . The compartment is in free fall between points  $A$  and  $B$ , which are a distance of  $3R/4$  apart, then moves along the circular arc of radius  $R$  between points  $B$  and  $D$ . Assume the track is frictionless from point  $A$  to point  $D$  and the dimensions of the passenger compartment are negligible compared to  $R$ .

- (a) On the dot below that represents the passenger compartment, draw and label the forces (not components) that act on the passenger compartment when it is at point  $C$ , which is at an angle  $\theta$  from point  $B$ .



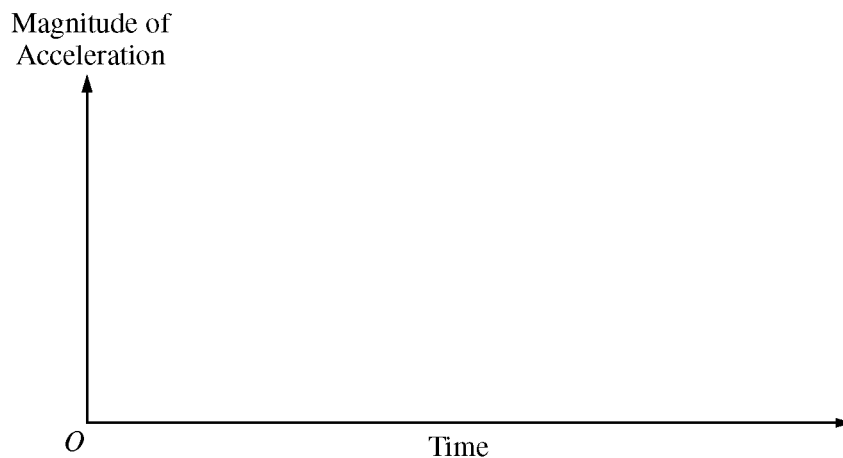
- (b) In terms of  $\theta$  and the magnitudes of the forces drawn in part (a), determine an expression for the magnitude of the centripetal force acting on the compartment at point  $C$ . If you need to draw anything besides what you have shown in part (a) to assist in your solution, use the space below. Do NOT add anything to the figure in part (a).
- (c) Derive an expression for the speed  $v_D$  of the passenger compartment as it reaches point  $D$  in terms of  $M$ ,  $R$ , and fundamental constants, as appropriate.

A force acts on the compartment between points  $D$  and  $E$  and brings it to rest at point  $E$ .

- (d) If the compartment is brought to rest by friction, calculate the numerical value of the coefficient of friction  $\mu$  between the compartment and the track.

**2011 AP<sup>®</sup> PHYSICS C: MECHANICS FREE-RESPONSE QUESTIONS**

- (e) Now consider the case in which there is no friction between the compartment and the track, but instead the compartment is brought to rest by a braking force  $-k\mathbf{v}$ , where  $k$  is a constant and  $\mathbf{v}$  is the velocity of the compartment. Express all algebraic answers to the following in terms of  $M$ ,  $R$ ,  $k$ ,  $v_D$ , and fundamental constants, as appropriate.
- Write, but do NOT solve, the differential equation for  $v(t)$ .
  - Solve the differential equation you wrote in part i.
  - On the axes below, sketch a graph of the magnitude of the acceleration of the compartment as a function of time. On the axes, explicitly label any intercepts, asymptotes, maxima, or minima with numerical values or algebraic expressions, as appropriate.



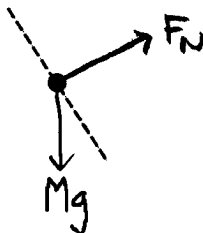
**AP<sup>®</sup> PHYSICS C: MECHANICS  
2011 SCORING GUIDELINES**

**Question 2**

**15 points total**

**Distribution  
of points**

(a) 2 points



For either a weight force or a normal force, correctly drawn and labeled  
For the second correct force and no additional forces, arrows or components

1 point  
1 point

(b) 1 point

For a correct expression for the centripetal force in terms of the forces drawn in part (a)  
For the example above:

1 point

$$F_c = F_N - Mg \sin \theta$$

*Alternate Solution*

*Alternate points*

*Applying conservation of energy, with the loss of potential energy equal to the kinetic energy at point C*

$$Mg \Delta h = Mv_C^2/2$$

$$v_C^2 = 2g \Delta h$$

$$\Delta h = 3R/4 + R \sin \theta$$

$$v_C^2 = 2g(3R/4 + R \sin \theta)$$

$$F_c = Mv_C^2/R$$

$$F_c = M(2g(3R/4 + R \sin \theta))/R$$

*For a correct answer*

$$F_c = 2Mg(3/4 + \sin \theta)$$

*1 point*

(c) 2 points

For applying conservation of energy, with the loss of potential energy equal to the kinetic energy at point D

1 point

$$Mg \Delta h = Mv_D^2/2$$

$$v_D^2 = 2g \Delta h$$

$$\Delta h = 3R/4 + R = 7R/4$$

$$v_D^2 = 2g(7R/4)$$

For a correct answer

$$v_D = \sqrt{(7/2)gR}$$

1 point

**AP<sup>®</sup> PHYSICS C: MECHANICS**  
**2011 SCORING GUIDELINES**

**Question 2 (continued)**

	<b>Distribution of points</b>
(d) 3 points	
Work-energy approach For equating the work done by the friction force to the kinetic energy of the compartment at point $D$	1 point
$W = \Delta K = 0 - \frac{1}{2} M v_D^2$	
For a correct expression for the frictional force	1 point
$f = \mu N = \mu Mg$ $W = \mathbf{F} \cdot \Delta \mathbf{r} = fd \cos 180 = -(\mu Mg)d$	
$(\mu Mg)d = \frac{1}{2} M v_D^2$	
For substituting the expression for $v_D$ from part (c), and $d = 3R$	1 point
$(\mu Mg)3R = \frac{1}{2} M \left( \frac{7}{2} gR \right)$ $3\mu = \frac{1}{2} \left( \frac{7}{2} \right)$ $\mu = 7/12$	
Note: Full credit is also earned for setting the initial potential energy at point $A$ , $U_A = mg \left( \frac{7R}{4} \right)$ , equal to the work done by the frictional force, and solving for $\mu$ .	
<i>Alternate solution</i>	<i>Alternate points</i>
For using both Newton's second law and a correct kinematics equation	1 point
$\mathbf{F}_{net} = m\mathbf{a}$ $v_f^2 - v_i^2 = 2ad$	
For a correct expression for the frictional force	1 point
$f = \mu N = \mu Mg$ $-\mu Mg = Ma$ $a = -\mu g$	
Substituting for $a$ , and the final and initial speeds in the kinematic equation	
$-v_D^2 = 2(-\mu g)d$	
For substituting the expression for $v_D$ from part (c), and $d = 3R$	1 point
$\frac{7}{2} gR = 2(\mu g)3R$ $\mu = 7/12$	

**AP<sup>®</sup> PHYSICS C: MECHANICS  
2011 SCORING GUIDELINES**

**Question 2 (continued)**

**Distribution  
of points**

(e)

i. 2 points

$$\Sigma F = ma$$

For substituting the braking force into Newton's second law as the net force

1 point

For substituting the time derivative of velocity for the acceleration

1 point

$$-kv = M(dv/dt)$$

ii. 2 points

For separating the variables and integrating

1 point

$$dv/v = -(k/M)dt$$

$$\int_{v_D}^v dv/v = -(k/M) \int_0^t dt$$

$$\ln v|_{v_D}^v = -(k/M)t$$

$$\ln v - \ln v_D = \ln(v/v_D) = -(k/M)t$$

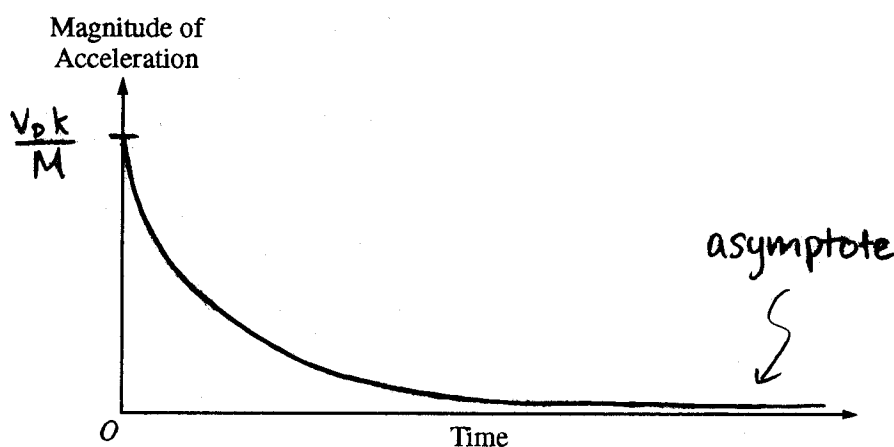
$$v/v_D = e^{-kt/M}$$

For a correct expression for the velocity as a function of time

1 point

$$v = v_D e^{-kt/M}$$

iii. 3 points



Taking the derivative of the equation for  $v$  from part (e) ii

$$a = dv/dt = d(v_D e^{-kt/M})/dt = -(k/M)v_D e^{-kt/M}$$

$$\text{At } t = 0, a = -kv_D/M$$

For a graph with a finite intercept on the vertical axis

1 point

For a graph that is concave upward and asymptotic to zero

1 point

For labeling the initial acceleration with the correct value

1 point

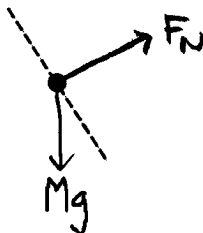
**AP<sup>®</sup> PHYSICS C: MECHANICS  
2011 SCORING GUIDELINES**

**Question 2**

**15 points total**

**Distribution  
of points**

(a) 2 points



For either a weight force or a normal force, correctly drawn and labeled  
For the second correct force and no additional forces, arrows or components

1 point  
1 point

(b) 1 point

For a correct expression for the centripetal force in terms of the forces drawn in part (a)  
For the example above:

1 point

$$F_c = F_N - Mg \sin \theta$$

*Alternate Solution*

*Alternate points*

*Applying conservation of energy, with the loss of potential energy equal to the kinetic energy at point C*

$$Mg \Delta h = Mv_C^2/2$$

$$v_C^2 = 2g \Delta h$$

$$\Delta h = 3R/4 + R \sin \theta$$

$$v_C^2 = 2g(3R/4 + R \sin \theta)$$

$$F_c = Mv_C^2/R$$

$$F_c = M(2g(3R/4 + R \sin \theta))/R$$

*For a correct answer*

$$F_c = 2Mg(3/4 + \sin \theta)$$

*1 point*

(c) 2 points

For applying conservation of energy, with the loss of potential energy equal to the kinetic energy at point D

1 point

$$Mg \Delta h = Mv_D^2/2$$

$$v_D^2 = 2g \Delta h$$

$$\Delta h = 3R/4 + R = 7R/4$$

$$v_D^2 = 2g(7R/4)$$

For a correct answer

$$v_D = \sqrt{(7/2)gR}$$

1 point

**AP<sup>®</sup> PHYSICS C: MECHANICS**  
**2011 SCORING GUIDELINES**

**Question 2 (continued)**

	<b>Distribution of points</b>
(d) 3 points	
Work-energy approach For equating the work done by the friction force to the kinetic energy of the compartment at point $D$	1 point
$W = \Delta K = 0 - \frac{1}{2} M v_D^2$	
For a correct expression for the frictional force	1 point
$f = \mu N = \mu Mg$ $W = \mathbf{F} \cdot \Delta \mathbf{r} = fd \cos 180 = -(\mu Mg)d$	
$(\mu Mg)d = \frac{1}{2} M v_D^2$	
For substituting the expression for $v_D$ from part (c), and $d = 3R$	1 point
$(\mu Mg)3R = \frac{1}{2} M \left( \frac{7}{2} gR \right)$ $3\mu = \frac{1}{2} \left( \frac{7}{2} \right)$ $\mu = 7/12$	
Note: Full credit is also earned for setting the initial potential energy at point $A$ , $U_A = mg \left( \frac{7R}{4} \right)$ , equal to the work done by the frictional force, and solving for $\mu$ .	
<i>Alternate solution</i>	<i>Alternate points</i>
For using both Newton's second law and a correct kinematics equation	1 point
$\mathbf{F}_{net} = m\mathbf{a}$ $v_f^2 - v_i^2 = 2ad$	
For a correct expression for the frictional force	1 point
$f = \mu N = \mu Mg$ $-\mu Mg = Ma$ $a = -\mu g$	
Substituting for $a$ , and the final and initial speeds in the kinematic equation	
$-v_D^2 = 2(-\mu g)d$	
For substituting the expression for $v_D$ from part (c), and $d = 3R$	1 point
$\frac{7}{2} gR = 2(\mu g)3R$ $\mu = 7/12$	

**AP<sup>®</sup> PHYSICS C: MECHANICS  
2011 SCORING GUIDELINES**

**Question 2 (continued)**

**Distribution  
of points**

(e)

i. 2 points

$$\Sigma F = ma$$

For substituting the braking force into Newton's second law as the net force

1 point

For substituting the time derivative of velocity for the acceleration

1 point

$$-kv = M(dv/dt)$$

ii. 2 points

For separating the variables and integrating

1 point

$$dv/v = -(k/M)dt$$

$$\int_{v_D}^v dv/v = -(k/M) \int_0^t dt$$

$$\ln v|_{v_D}^v = -(k/M)t$$

$$\ln v - \ln v_D = \ln(v/v_D) = -(k/M)t$$

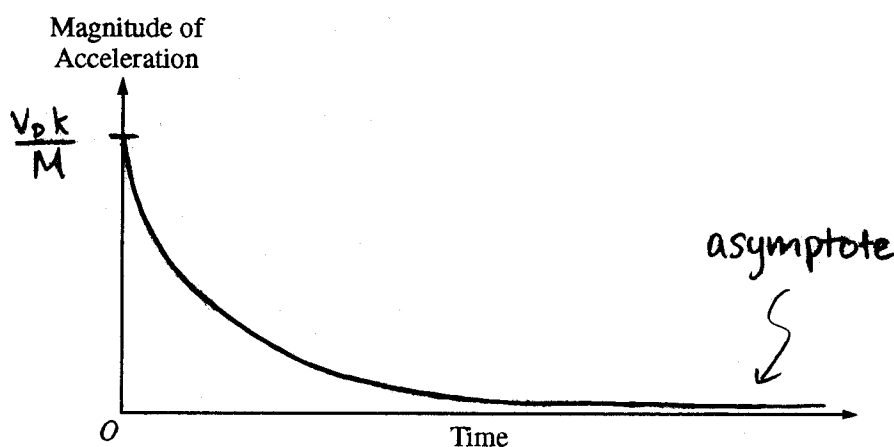
$$v/v_D = e^{-kt/M}$$

For a correct expression for the velocity as a function of time

1 point

$$v = v_D e^{-kt/M}$$

iii. 3 points



Taking the derivative of the equation for  $v$  from part (e) ii

$$a = dv/dt = d(v_D e^{-kt/M})/dt = -(k/M)v_D e^{-kt/M}$$

$$\text{At } t = 0, a = -kv_D/M$$

For a graph with a finite intercept on the vertical axis

1 point

For a graph that is concave upward and asymptotic to zero

1 point

For labeling the initial acceleration with the correct value

1 point