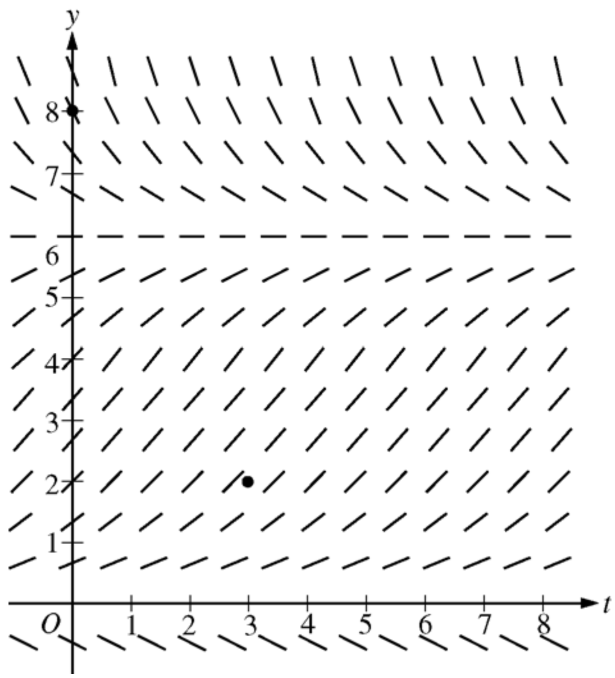
6. Consider the logistic differential equation $\frac{dy}{dt} = \frac{y}{8}(6 - y)$. Let $y = f(t)$ be the particular solution to the differential equation with $f(0) = 8$.

   (a) A slope field for this differential equation is given below. Sketch possible solution curves through the points $(3, 2)$ and $(0, 8)$.

   (Note: Use the axes provided in the exam booklet.)



   (b) Use Euler's method, starting at $t = 0$ with two steps of equal size, to approximate $f(1)$.

   (c) Write the second-degree Taylor polynomial for f about $t = 0$, and use it to approximate $f(1)$.

   (d) What is the range of f for $t \geq 0$?

WRITE ALL WORK IN THE PINK EXAM BOOKLET.

END OF EXAM

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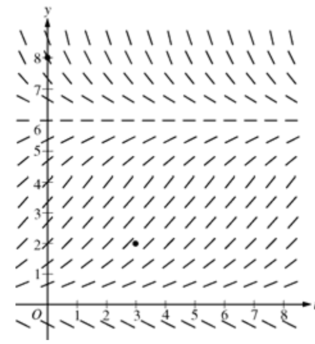
Question 6

Consider the logistic differential equation $\frac{dy}{dt} = \frac{y}{8}(6 - y)$. Let $y = f(t)$ be the particular solution to the differential equation with $f(0) = 8$.

(a) A slope field for this differential equation is given below. Sketch possible solution curves through the points $(3, 2)$ and $(0, 8)$.
(Note: Use the axes provided in the exam booklet.)

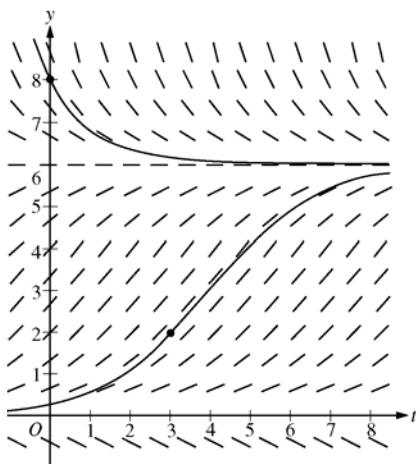


(b) Use Euler's method, starting at $t = 0$ with two steps of equal size, to approximate $f(1)$.

(c) Write the second-degree Taylor polynomial for f about $t = 0$, and use it to approximate $f(1)$.

(d) What is the range of f for $t \geq 0$?

(a)



$2 : \begin{cases} 1\text{: solution curve through } (0,8) \\ 1\text{: solution curve through } (3,2) \end{cases}$

(b) $f\left(\frac{1}{2}\right) \approx 8 + (-2)\left(\frac{1}{2}\right) = 7$

$f(1) \approx 7 + \left(-\frac{7}{8}\right)\left(\frac{1}{2}\right) = \frac{105}{16}$

$2 : \begin{cases} 1 : \text{Euler's method with two steps} \\ 1 : \text{approximation of } f(1) \end{cases}$

(c) $\frac{d^2y}{dt^2} = \frac{1}{8}\frac{dy}{dt}(6 - y) + \frac{y}{8}\left(-\frac{dy}{dt}\right)$

$f(0) = 8$; $f'(0) = \left.\frac{dy}{dt}\right|_{t=0} = \frac{8}{8}(6 - 8) = -2$; and

$f''(0) = \left.\frac{d^2y}{dt^2}\right|_{t=0} = \frac{1}{8}(-2)(-2) + \frac{8}{8}(2) = \frac{5}{2}$

The second-degree Taylor polynomial for f about $t = 0$ is $P_2(t) = 8 - 2t + \frac{5}{4}t^2$.

$f(1) \approx P_2(1) = \frac{29}{4}$

$4 : \begin{cases} 2 : \frac{d^2y}{dt^2} \\ 1 : \text{second-degree Taylor polynomial} \\ 1 : \text{approximation of } f(1) \end{cases}$

(d) The range of f for $t \geq 0$ is $6 < y \leq 8$.

1 : answer