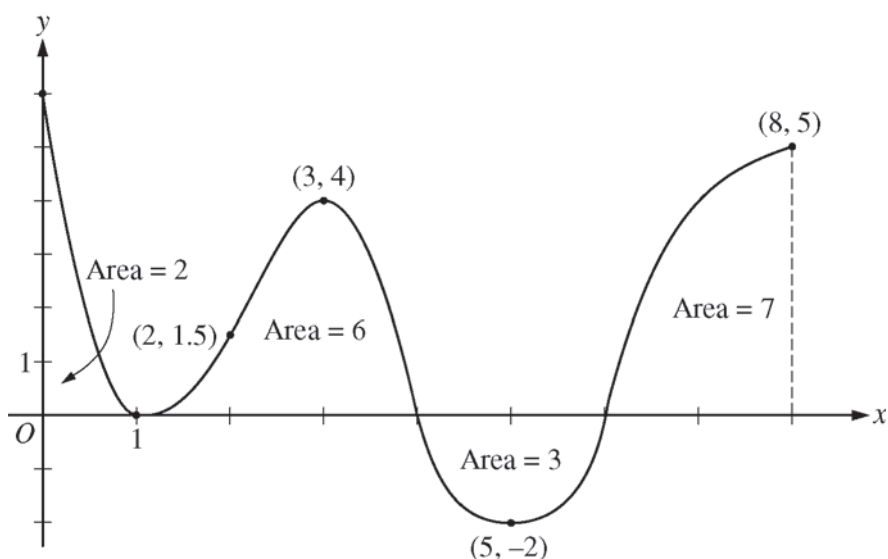


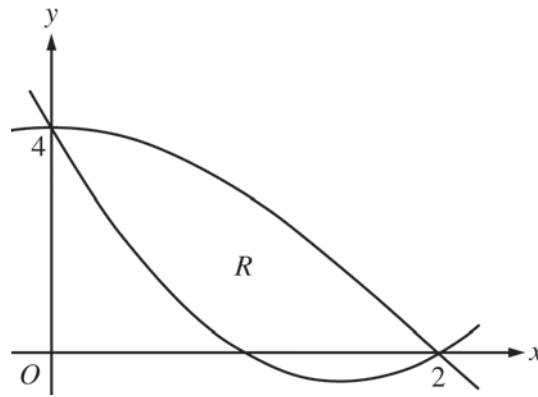
2013 AP[®] CALCULUS AB FREE-RESPONSE QUESTIONS



Graph of f'

4. The figure above shows the graph of f' , the derivative of a twice-differentiable function f , on the closed interval $0 \leq x \leq 8$. The graph of f' has horizontal tangent lines at $x = 1$, $x = 3$, and $x = 5$. The areas of the regions between the graph of f' and the x -axis are labeled in the figure. The function f is defined for all real numbers and satisfies $f(8) = 4$.
- Find all values of x on the open interval $0 < x < 8$ for which the function f has a local minimum. Justify your answer.
 - Determine the absolute minimum value of f on the closed interval $0 \leq x \leq 8$. Justify your answer.
 - On what open intervals contained in $0 < x < 8$ is the graph of f both concave down and increasing? Explain your reasoning.
 - The function g is defined by $g(x) = (f(x))^3$. If $f(3) = -\frac{5}{2}$, find the slope of the line tangent to the graph of g at $x = 3$.

2013 AP[®] CALCULUS AB FREE-RESPONSE QUESTIONS

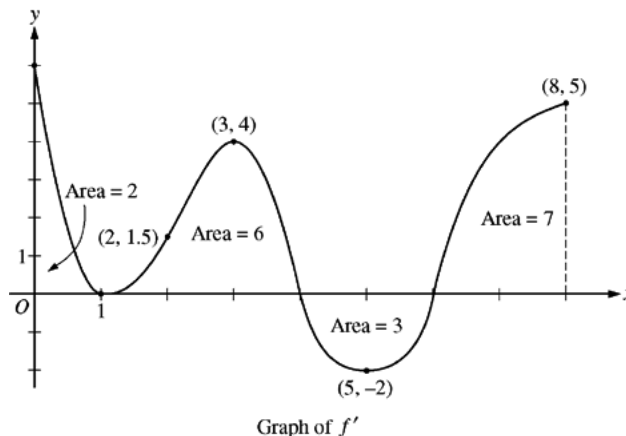


5. Let $f(x) = 2x^2 - 6x + 4$ and $g(x) = 4\cos\left(\frac{1}{4}\pi x\right)$. Let R be the region bounded by the graphs of f and g , as shown in the figure above.
- Find the area of R .
 - Write, but do not evaluate, an integral expression that gives the volume of the solid generated when R is rotated about the horizontal line $y = 4$.
 - The region R is the base of a solid. For this solid, each cross section perpendicular to the x -axis is a square. Write, but do not evaluate, an integral expression that gives the volume of the solid.
-

AP[®] CALCULUS AB
2013 SCORING GUIDELINES

Question 4

The figure above shows the graph of f' , the derivative of a twice-differentiable function f , on the closed interval $0 \leq x \leq 8$. The graph of f' has horizontal tangent lines at $x = 1$, $x = 3$, and $x = 5$. The areas of the regions between the graph of f' and the x -axis are labeled in the figure. The function f is defined for all real numbers and satisfies $f(8) = 4$.



- (a) Find all values of x on the open interval $0 < x < 8$ for which the function f has a local minimum. Justify your answer.
- (b) Determine the absolute minimum value of f on the closed interval $0 \leq x \leq 8$. Justify your answer.
- (c) On what open intervals contained in $0 < x < 8$ is the graph of f both concave down and increasing? Explain your reasoning.
- (d) The function g is defined by $g(x) = (f(x))^3$. If $f(3) = -\frac{5}{2}$, find the slope of the line tangent to the graph of g at $x = 3$.

- (a) $x = 6$ is the only critical point at which f' changes sign from negative to positive. Therefore, f has a local minimum at $x = 6$.

- (b) From part (a), the absolute minimum occurs either at $x = 6$ or at an endpoint.

$$\begin{aligned} f(0) &= f(8) + \int_8^0 f'(x) \, dx \\ &= f(8) - \int_0^8 f'(x) \, dx = 4 - 12 = -8 \end{aligned}$$

$$\begin{aligned} f(6) &= f(8) + \int_8^6 f'(x) \, dx \\ &= f(8) - \int_6^8 f'(x) \, dx = 4 - 7 = -3 \end{aligned}$$

$$f(8) = 4$$

The absolute minimum value of f on the closed interval $[0, 8]$ is -8 .

- (c) The graph of f is concave down and increasing on $0 < x < 1$ and $3 < x < 4$, because f' is decreasing and positive on these intervals.

- (d) $g'(x) = 3[f(x)]^2 \cdot f'(x)$

$$g'(3) = 3[f(3)]^2 \cdot f'(3) = 3\left(-\frac{5}{2}\right)^2 \cdot 4 = 75$$

1 : answer with justification

3 : $\begin{cases} 1 : \text{considers } x = 0 \text{ and } x = 6 \\ 1 : \text{answer} \\ 1 : \text{justification} \end{cases}$

2 : $\begin{cases} 1 : \text{answer} \\ 1 : \text{explanation} \end{cases}$

3 : $\begin{cases} 2 : g'(x) \\ 1 : \text{answer} \end{cases}$