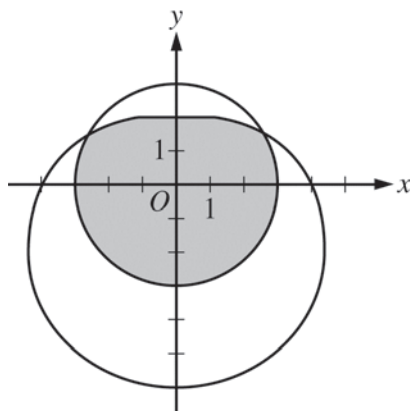


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2. The graphs of the polar curves $r = 3$ and $r = 4 - 2\sin \theta$ are shown in the figure above. The curves intersect when $\theta = \frac{\pi}{6}$ and $\theta = \frac{5\pi}{6}$.
- (a) Let S be the shaded region that is inside the graph of $r = 3$ and also inside the graph of $r = 4 - 2\sin \theta$. Find the area of S .
- (b) A particle moves along the polar curve $r = 4 - 2\sin \theta$ so that at time t seconds, $\theta = t^2$. Find the time t in the interval $1 \leq t \leq 2$ for which the x -coordinate of the particle's position is -1 .
- (c) For the particle described in part (b), find the position vector in terms of t . Find the velocity vector at time $t = 1.5$.
-

END OF PART A OF SECTION II

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SECTION II, Part B****Time—60 minutes****Number of problems—4****No calculator is allowed for these problems.**

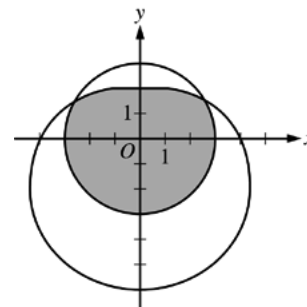
t (minutes)	0	1	2	3	4	5	6
$C(t)$ (ounces)	0	5.3	8.8	11.2	12.8	13.8	14.5

3. Hot water is dripping through a coffeemaker, filling a large cup with coffee. The amount of coffee in the cup at time t , $0 \leq t \leq 6$, is given by a differentiable function C , where t is measured in minutes. Selected values of $C(t)$, measured in ounces, are given in the table above.
- (a) Use the data in the table to approximate $C'(3.5)$. Show the computations that lead to your answer, and indicate units of measure.
- (b) Is there a time t , $2 \leq t \leq 4$, at which $C'(t) = 2$? Justify your answer.
- (c) Use a midpoint sum with three subintervals of equal length indicated by the data in the table to approximate the value of $\frac{1}{6} \int_0^6 C(t) dt$. Using correct units, explain the meaning of $\frac{1}{6} \int_0^6 C(t) dt$ in the context of the problem.
- (d) The amount of coffee in the cup, in ounces, is modeled by $B(t) = 16 - 16e^{-0.4t}$. Using this model, find the rate at which the amount of coffee in the cup is changing when $t = 5$.
-

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Question 2

The graphs of the polar curves $r = 3$ and $r = 4 - 2\sin \theta$ are shown in the figure above. The curves intersect when $\theta = \frac{\pi}{6}$ and $\theta = \frac{5\pi}{6}$.



- (a) Let S be the shaded region that is inside the graph of $r = 3$ and also inside the graph of $r = 4 - 2\sin \theta$. Find the area of S .
- (b) A particle moves along the polar curve $r = 4 - 2\sin \theta$ so that at time t seconds, $\theta = t^2$. Find the time t in the interval $1 \leq t \leq 2$ for which the x -coordinate of the particle's position is -1 .
- (c) For the particle described in part (b), find the position vector in terms of t . Find the velocity vector at time $t = 1.5$.

(a) $\text{Area} = 6\pi + \frac{1}{2} \int_{\pi/6}^{5\pi/6} (4 - 2\sin \theta)^2 d\theta = 24.709$ (or 24.708)

3 : $\begin{cases} 1 : \text{integrand} \\ 1 : \text{limits and constant} \\ 1 : \text{answer} \end{cases}$

(b) $x = r \cos \theta \Rightarrow x(\theta) = (4 - 2\sin \theta) \cos \theta$
 $x(t) = (4 - 2\sin(t^2)) \cos(t^2)$
 $x(t) = -1$ when $t = 1.428$ (or 1.427)

3 : $\begin{cases} 1 : x(\theta) \text{ or } x(t) \\ 1 : x(\theta) = -1 \text{ or } x(t) = -1 \\ 1 : \text{answer} \end{cases}$

(c) $y = r \sin \theta \Rightarrow y(\theta) = (4 - 2\sin \theta) \sin \theta$
 $y(t) = (4 - 2\sin(t^2)) \sin(t^2)$

3 : $\begin{cases} 2 : \text{position vector} \\ 1 : \text{velocity vector} \end{cases}$

Position vector $= \langle x(t), y(t) \rangle$
 $= \langle (4 - 2\sin(t^2)) \cos(t^2), (4 - 2\sin(t^2)) \sin(t^2) \rangle$

$v(1.5) = \langle x'(1.5), y'(1.5) \rangle$
 $= \langle -8.072, -1.673 \rangle$ (or $\langle -8.072, -1.672 \rangle$)