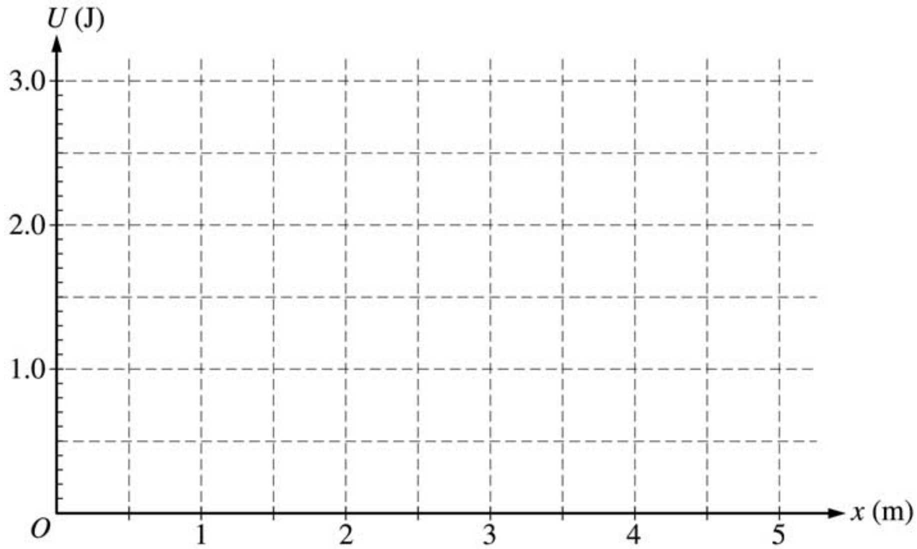
Mech 3.

An object of mass 0.5 kg experiences a force that is associated with the potential energy function $U(x) = \dfrac{4.0}{2.0 + x}$, where $U$ is in joules and $x$ is in meters.

(a) On the axes below, sketch the graph of $U(x)$ versus $x$.




(b) Determine the force associated with the potential energy function given above.

(c) Suppose that the object is released from rest at the origin. Determine the speed of the particle at $x = 2$ m.

In the laboratory, you are given a glider of mass 0.5 kg on an air track. The glider is acted on by the force determined in part (b). Your goal is to determine experimentally the validity of your theoretical calculation in part (c).

(d) From the list below, select the additional equipment you will need from the laboratory to do your experiment by checking the line next to each item. If you need more than one of an item, place the number you need on the line.

___ Meterstick ___ Stopwatch ___ Photogate timer ___ String ___ Spring

___ Balance ___ Wood block ___ Set of objects of different masses

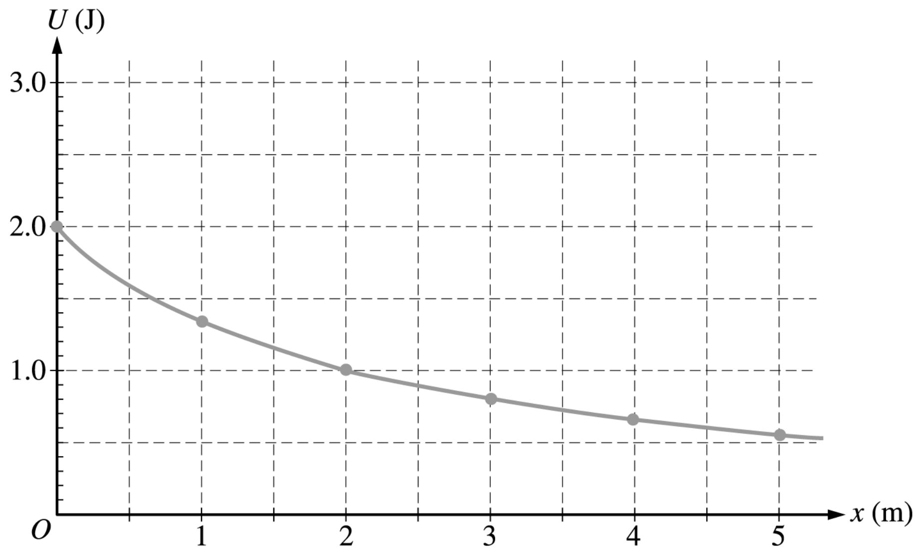
(e) Briefly outline the procedure you will use, being explicit about what measurements you need to make in order to determine the speed. You may include a labeled diagram of your setup if it will clarify your procedure.

**END OF SECTION II, MECHANICS**

## Question 3

**15 points total** | **Distribution of points**

(a) 3 points




| | |
|---|---|
| For correct shape: concave upward and monotonically decreasing | 1 point |
| For correct $y$ intercept: $U(0) = 2.0$ J | 1 point |
| For value of $U$ at $x = 5$ within the range $0.50 \text{ J} \le U(5) \le 0.75 \text{ J}$ | 1 point |

(b) 4 points

For the expression for force as a derivative of potential relative to position — 2 points

$$F = -\frac{dU}{dx}$$

Only one point was awarded if the negative sign was missing. A non-calculus statement such as $F = -\frac{\Delta U}{\Delta x}$ earned no credit.

For taking a derivative of the potential function — 1 point

$$F = -\frac{d}{dx}\left(\frac{4.0}{2.0 + x}\right)$$

For the correct answer or answer consistent with non-inclusion of minus sign in above equation — 1 point

$$F = \frac{4.0}{(x + 2.0)^2}$$

**Question 3 (cont'd.)**

| | **Distribution of points** |
|---|---|
| (c) 3 points | |
| For explicitly indicating the use of conservation of energy or the net work-energy theorem in words or by an equation (but not just $K = U$) | 1 point |
| $\Delta U = U(0) - U(2) = \frac{1}{2}mv^2$ | |
| $2.0\text{ J} - 1.0\text{ J} = \frac{1}{2}(0.5\text{ kg})v^2$ | |
| For the correct answer | 2 points |
| $v = 2.0$ m/s | |
| (d) 2 points | |
| For indicating items of equipment consistent with the procedure described in part (e) (at least two of the items if more than two were used) | 2 points |
| Note: If part (e) was not attempted, only 1 point maximum was awarded. Unreasonable indications, such as all the items being checked, were not awarded any points. | |
| (e) 3 points | |
| For a complete description of any correct procedure. | 3 points |

Partial credit was awarded for less complete descriptions. The following were common examples. Other examples, though rarely cited, could receive partial or full credit.

1. Using photogates

   Place the photogates near $x = 2$ m and a small distance apart (such as a glider length). Measure the distance between the photogates. Measure the time the glider takes to travel between the photogates. Obtain the speed from distance/time.

   Note: No points were given if the distance measured was from 0 to 2 m and the time to travel 2 m was used.

2. Using a spring

   The spring constant $k$ of the spring must be known, or if not, then measured. Set up the spring at $x = 2$ m so that it is compressed when struck by the glider. Measure the distance of maximum compression $x_m$. The velocity can then be determined from the equation $\frac{1}{2}kx_m = \frac{1}{2}mv^2$.

3. Treating the glider as a projectile

   Adjust the starting point so that $x = 2$ m is at end of the track. Thus the glider leaves the track at this point and becomes a projectile. The height of the track determines the time interval $t$ that the glider is in the air. The horizontal distance $x$ from the end of the track to the point where the glider hits the ground is measured and then the velocity is computed from $x/t$.