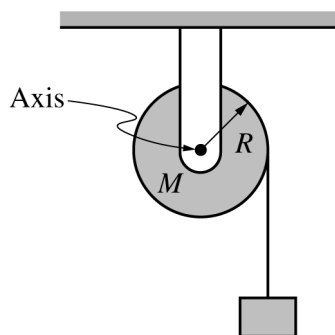


Begin your response to **QUESTION 4** on this page.

4. (7 points, suggested time 13 minutes)

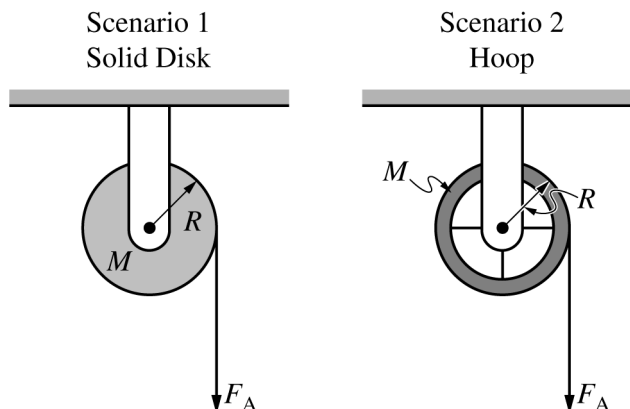
A block of unknown mass is attached to a long, lightweight string that is wrapped several turns around a pulley mounted on a horizontal axis through its center, as shown. The pulley is a uniform solid disk of mass  $M$  and radius  $R$ . The rotational inertia of the pulley is described by the equation  $I = \frac{1}{2}MR^2$ . The pulley can rotate about its center with negligible friction. The string does not slip on the pulley as the block falls.

When the block is released from rest and as the block travels toward the ground, the magnitude of the tension exerted on the block by the string is  $F_T$ .

- (a) Determine an expression for the magnitude of the angular acceleration  $\alpha_D$  of the disk as the block travels downward. Express your answer in terms of  $M$ ,  $R$ ,  $F_T$ , and physical constants as appropriate.

**GO ON TO THE NEXT PAGE.**

Continue your response to **QUESTION 4** on this page.



Scenarios 1 and 2 show two different pulleys. In Scenario 1, the pulley is the same solid disk referenced in part (a). In Scenario 2, the pulley is a hoop that has the same mass  $M$  and radius  $R$  as the disk. Each pulley has a lightweight string wrapped around it several turns and is mounted on a horizontal axle, as shown. Each pulley is free to rotate about its center with negligible friction.

In both scenarios, the pulleys begin at rest. Then both strings are pulled with the same constant force  $F_A$  for the same time interval  $\Delta t$ , causing the pulleys to rotate without the string slipping. After time interval  $\Delta t$ , the change in angular momentum of the disk is equal to the change in angular momentum of the hoop, but the change in rotational kinetic energy for the disk is greater than that of the hoop.

- (b) Consider scenarios 1 and 2 at the end of time interval  $\Delta t$ . In a clear, coherent paragraph-length response that may also contain equations and drawings, explain why the change in angular momentum of both pulleys is the same but the change in rotational kinetic energy is greater for the disk.

**GO ON TO THE NEXT PAGE.**

**Question 4: Short Answer Paragraph Argument****7 points**

- 
- (a) For a correct expression for the angular acceleration of the pulley in terms of the appropriate **1 point**

quantities:  $\alpha_D = \frac{2F_T}{MR}$

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**Example Response**

$$\alpha_D = \frac{RF_T}{\frac{1}{2}MR^2} \quad \text{OR} \quad \alpha_D = \frac{2F_T}{MR}$$

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**Total for part (a) 1 point**

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<b>(b)</b>	For indicating that the torque, $\tau$ , is the same for both pulleys	<b>1 point</b>
	For indicating that the impulse, $\tau\Delta t$ , (or change in momentum $\Delta L$ ) is the same for both pulleys because $\tau$ and $\Delta t$ are the same	<b>1 point</b>
	For indicating that the rotational inertia, $I$ , of the disk and hoop are different	<b>1 point</b>
	For providing reasoning that because the rotational inertia, $I$ , are different for the disk and hoop, the kinematic quantities ( $\Delta\theta$ , $\omega$ , $\alpha$ ) are also different for the disk and hoop	<b>1 point</b>
	For <b>one</b> of the following: <ul style="list-style-type: none"> <li>• Relating <math>I</math> and <math>\omega</math> to reason that <math>\Delta K</math> is greater for the disk</li> <li>• Indicating that because <math>\Delta\theta</math> is greater for the disk the work done on the disk is greater</li> </ul>	<b>1 point</b>
	For a logical, relevant, and internally consistent argument that follows the guidelines described in the published requirements for the paragraph-length response	<b>1 point</b>

**Example Response**

*The rotational inertia,  $I$ , of the hoop is larger than the rotational inertia of the disk because the hoop's mass is all on the outside instead of distributed throughout like the disk. Equal forces are applied to both pulleys at the same distance, which means that the torques exerted on the pulleys will also be equal. Since the same torque is applied to both pulleys for the same time period, the change in angular momentum will be the same for the disk and hoop. The magnitude of the angular velocity for the hoop will be smaller than that of the disk since angular velocity is inversely proportional to the rotational inertia ( $\omega = \frac{L}{I}$ ).*

*Since kinetic energy is proportional to rotational inertia and the square of angular velocity ( $K_R = \frac{1}{2}I\omega^2$ ), the difference in angular velocity more greatly affects the rotational kinetic energy. That means the disk will have a greater rotational kinetic energy than the hoop.*

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**Total for part (b) 6 points**

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**Total for question 4 7 points**