1999

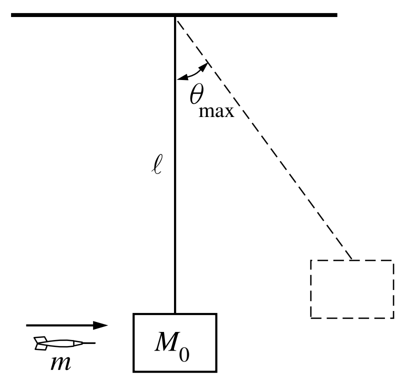
PHYSICS C

SECTION II, MECHANICS

Time—45 minutes

3 Questions

Directions: Answer all three questions. The suggested time is about 15 minutes for answering each of the questions, which are worth 15 points each. The parts within a question may not have equal weight. Show all your work in the pink booklet in the spaces provided after each part, NOT in this green insert.



Mech 1. In a laboratory experiment, you wish to determine the initial speed of a dart just after it leaves a dart gun. The dart, of mass m, is fired with the gun very close to a wooden block of mass M_0, which hangs from a cord of length ℓ and negligible mass, as shown above. Assume the size of the block is negligible compared to ℓ, and the dart is moving horizontally when it hits the left side of the block at its center and becomes embedded in it. The block swings up to a maximum angle θ_{max} from the vertical. Express your answers to the following in terms of m, M_0, ℓ, θ_{max}, and g.

(a) Determine the speed v_0 of the dart immediately before it strikes the block.

(b) The dart and block subsequently swing as a pendulum. Determine the tension in the cord when it returns to the lowest point of the swing.

(c) At your lab table you have only the following additional equipment.

Meter stick	Stopwatch	Set of known masses
Protractor	5 m of string	Five more blocks of mass M_0
Spring		

Without destroying or disassembling any of this equipment, design another practical method for determining the speed of the dart just after it leaves the gun. Indicate the measurements you would take, and how the speed could be determined from these measurements.

1999 Physics C Solutions | **Distribution of points**

Mech. 1 (15 points)

(a) 5 points

For conservation of momentum — 1 point

$mv_0 = (m+M_0)v$

$v = \dfrac{mv_0}{m+M_0}$

For conservation of mechanical energy — 1 point

$K + U = K' + U'$ or $\frac{1}{2} M_{\text{total}} v^2 = M_{\text{total}} gh$

For calculating h in terms of ℓ — 1 point

$h = \ell(1 - \cos\theta)$

For substituting for M_{total} and v in the energy equation — 1 point

$\frac{1}{2}(m + M_0)\left(\dfrac{mv_0}{m + M_0}\right)^2 = (m + M_0)g\ell(1 - \cos\theta)$

For the correct answer — 1 point

$v_0 = \dfrac{m+M_0}{m}\sqrt{2g\ell(1-\cos\theta)}$

(b) 4 points

For Newton's second law (not awarded if net force was set equal to zero) — 1 point

$F_{\text{net}} = ma$

For any equation that indicated that the tension minus the weight is <u>not</u> zero — 1 point

$T - M_{\text{total}}g = M_{\text{total}}a$

For an expression for the centripetal force — 1 point

$a = \dfrac{v^2}{r}$ or $\dfrac{v^2}{\ell}$

$T - M_{\text{total}}g = M_{\text{total}}\dfrac{v^2}{\ell}$

For correctly substituting for M_{total} and v — 1 point

$(m + M_0)\dfrac{1}{\ell}\left(\dfrac{mv_0}{m + M_0}\right)^2 = T - (m + M_0)g$

Solving for T

$T = (m+M_0)g(3-2\cos\theta)$

Distribution of points

Mech. 1 (continued)

(c) 4 points

For all of the following: — 4 points

1) A practical procedure that uses some or all of the apparatus listed and would work
2) Recognition of any assumptions that must be made
3) Indication of the proper mathematical computation using the variables measured

Two points were awarded if the description of the procedure was not complete but it would work, or if the mathematical work did not clearly specify the variables used, or any combination of the above.

No points were awarded if the procedure would not be feasible in a laboratory situation with the apparatus listed, or if the procedure was merely a repeat of that outlined in part (a).

(d) 2 points

$F_{net} = ma = -bv$

For expressing the acceleration as the time derivative of the speed, $a = \frac{dv}{dt}$ — 1 point

$$\int_{v_0}^{v} \frac{dv}{v} = \int_0^t -\frac{b}{m}\,dt$$

$$\ln\left(\frac{v}{v_0}\right) = -\frac{bt}{m}$$

$$v = v_0 e^{-bt/m}$$

$$\int_0^{\ell} ds = \int_0^t v_0 e^{-bt/m}\,dt$$

For a general expression for the length of the dart in the block as a function of time or for the expression for the total distance L — 1 point

$$\ell = \frac{mv_0}{b}\left(1 - e^{-bt/m}\right)$$

$$L = \frac{mv_0}{b}$$

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Mech. 1 (continued)

(d) (continued)

Alternate Solution 1 — *Alternate points*

For indicating that work equals the change in kinetic energy — *1 point*

$$\int F\,dx = \frac{1}{2}mv_0^2$$

$$\left|F_{average}\right| = \frac{bv_0}{2}$$

$$\int_0^L \frac{bv_0}{2}\,dx = \frac{1}{2}mv_0^2$$

$$\frac{bv_0}{2}L = \frac{1}{2}mv_0^2$$

For the correct expression for the total distance L — *1 point*

$$L = \frac{mv_0}{b}$$

Alternate Solution 2 — *Alternate points*

$$\mathbf{F}_{net}\,\Delta t = \Delta\mathbf{p}$$

For the above expression — *1 point*

$$\int_0^\infty -bv\,dt = -mv_0$$

Using $v = \frac{ds}{dt}$ and the fact that s goes from zero to L as time goes from zero to infinity

$$\int_0^L -b\,ds = -mv_0$$

$$-bL = -mv_0$$

For the correct expression for the total distance L — *1 point*

$$L = \frac{mv_0}{b}$$