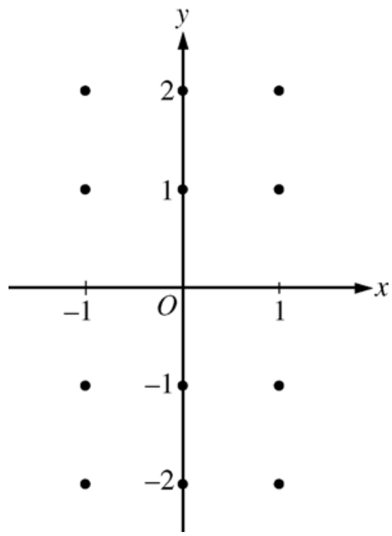
6. Consider the differential equation $\dfrac{dy}{dx} = -\dfrac{2x}{y}$.

   (a) On the axes provided, sketch a slope field for the given differential equation at the twelve points indicated.
   (**Note: Use the axes provided in the pink test booklet.**)



   (b) Let $y = f(x)$ be the particular solution to the differential equation with the initial condition $f(1) = -1$. Write an equation for the line tangent to the graph of f at $(1, -1)$ and use it to approximate $f(1.1)$.

   (c) Find the particular solution $y = f(x)$ to the given differential equation with the initial condition $f(1) = -1$.

WRITE ALL WORK IN THE TEST BOOKLET.

END OF EXAM

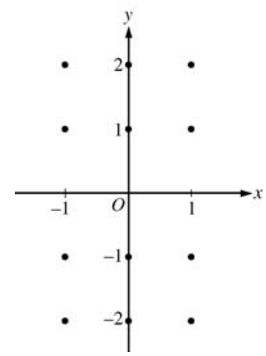
Question 6

Consider the differential equation $\frac{dy}{dx} = -\frac{2x}{y}$.

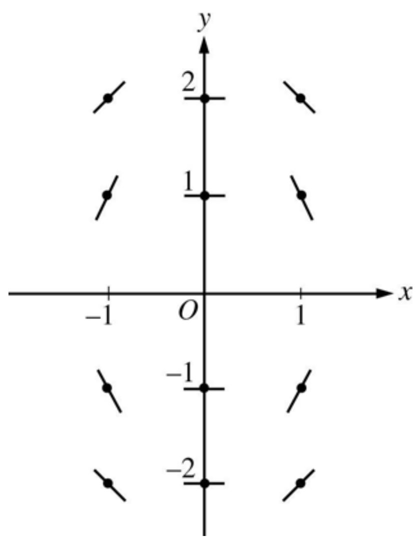
(a) On the axes provided, sketch a slope field for the given differential equation at the twelve points indicated.
(**Note: Use the axes provided in the pink test booklet.**)

(b) Let $y = f(x)$ be the particular solution to the differential equation with the initial condition $f(1) = -1$. Write an equation for the line tangent to the graph of f at $(1, -1)$ and use it to approximate $f(1.1)$.

(c) Find the particular solution $y = f(x)$ to the given differential equation with the initial condition $f(1) = -1$.



(a)



$2 : \begin{cases} 1 : \text{zero slopes} \\ 1 : \text{nonzero slopes} \end{cases}$

(b) The line tangent to f at $(1, -1)$ is $y + 1 = 2(x - 1)$.
Thus, $f(1.1)$ is approximately -0.8.

$2 : \begin{cases} 1 : \text{equation of the tangent line} \\ 1 : \text{approximation for } f(1.1) \end{cases}$

(c) $\frac{dy}{dx} = -\frac{2x}{y}$

$y\,dy = -2x\,dx$

$\frac{y^2}{2} = -x^2 + C$

$\frac{1}{2} = -1 + C;\ C = \frac{3}{2}$

$y^2 = -2x^2 + 3$

Since the particular solution goes through $(1, -1)$, y must be negative.

Thus the particular solution is $y = -\sqrt{3 - 2x^2}$.

$5 : \begin{cases} 1 : \text{separates variables} \\ 1 : \text{antiderivatives} \\ 1 : \text{constant of integration} \\ 1 : \text{uses initial condition} \\ 1 : \text{solves for } y \end{cases}$

Note: max $2/5$ [1-1-0-0-0] if no constant of integration

Note: $0/5$ if no separation of variables