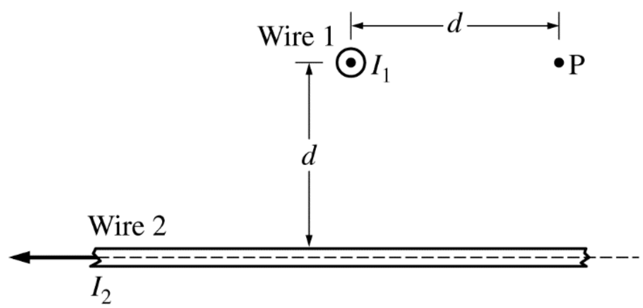

Figure 1. Side view

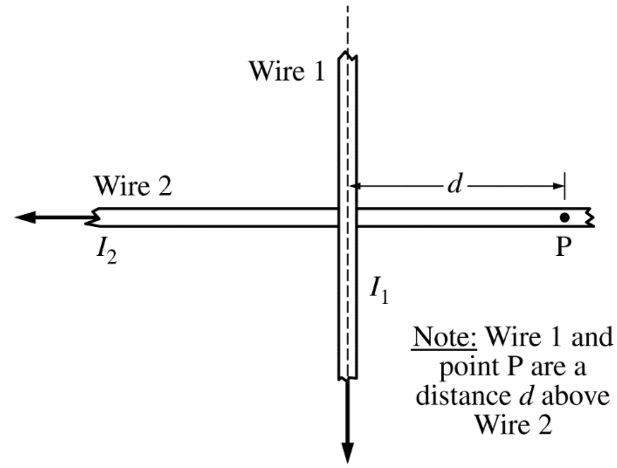

Figure 2. Top view

3. The figures above represent different views of two long, straight, horizontal wires, 1 and 2, carrying currents $I_1 = I$ and $I_2 = 2I$, respectively, in the directions shown. The wires are held in place. In Figure 1, the current in wire 1 is directed out of the page, and wire 1 is a distance d above wire 2. Point P is a horizontal distance d from wire 1 and a distance d directly above wire 2. Express your answers to parts (a) and (b) in terms of I, d, and physical constants, as appropriate.

   (a) Use Ampere's law to derive an expression for the magnitude of the magnetic field at point P due to wire 1.

   (b) Derive an expression for the magnitude of the net magnetic field at point P.

   (c) Calculate the numerical value of the angle to the horizontal for the direction of the net magnetic field at point P.

   (d) Wire 1 is now released. Which of the following best describes the initial motion of wire 1 due to the magnetic field of wire 2 ? Assume gravitational effects are negligible.

   ____ Wire 1 will not move.

   ____ Wire 1 will move upward as viewed in Figure 1.

   ____ Wire 1 will move downward as viewed in Figure 1.

   ____ Wire 1 will rotate clockwise as viewed in Figure 2.

   ____ Wire 1 will rotate counterclockwise as viewed in Figure 2.

   Justify your answer.

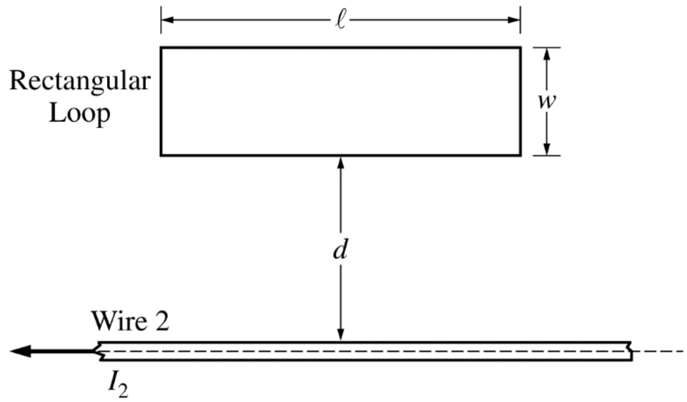

Figure 3. Side view

Wire 1 is now replaced by a conducting rectangular loop of length ℓ, width w, and resistance R. The loop is placed a distance d from wire 2, as shown. The loop, wire, and distance d are all in the plane of the page. The long side of the loop is parallel to the wire. The current I_2 for wire 2 is decreasing linearly as a function of time t according to the equation $I_2 = 2I_0(1 - kt)$, where k is a positive constant with units of s^{-1}.

(e) Of the following, select the integration that will give an expression for the flux Φ as a function of time t.

____ $\Phi = \int_{r=d}^{r=d+w} \frac{\mu_0(2I_0)(1-kt)\ell w}{2\pi} dr$ ____ $\Phi = \int_{r=d}^{r=w} \frac{\mu_0(2I_0)(1-kt)\ell w}{2\pi} dr$

____ $\Phi = \int_{r=d}^{r=d+w} \frac{\mu_0(2I_0)(1-kt)}{2\pi r} \ell dr$ ____ $\Phi = \int_{r=d}^{r=w} \frac{\mu_0(2I_0)(1-kt)}{2\pi r} \ell dr$

(f) Given that the flux through the rectangular loop as a function of time t is given by the equation $\Phi = \frac{\mu_0 I_0 \ell (1-kt)}{\pi} \ln\left(\frac{d+w}{d}\right)$, derive an expression for the magnitude of the current, if any, induced in the loop. Express your answers in terms of I_0, d, r, R, w, k, ℓ, and physical constants, as appropriate.

(g) What is the direction of the current, if any, induced in the loop as seen in Figure 3 ?

____ Clockwise ____ Counterclockwise

____ Undefined, because there is no current induced in the loop

Justify your answer.

STOP

END OF EXAM

Question 3 (continued)

Distribution of points

(c) 2 points

Calculate the numerical value of the angle to the horizontal for the direction of the net magnetic field at point P.

For correctly relating the angle to the individual magnetic fields		1 point
$\theta = \tan^{-1}\left(\dfrac{B_1}{B_2}\right)$		
For correctly substituting B_1 and B_2 into the equation		1 point
$\theta = \tan^{-1}\left(\dfrac{B_1}{B_2}\right) = \tan^{-1}\left(\dfrac{\left(\dfrac{\mu_0 I}{2\pi d}\right)}{\left(\dfrac{\mu_0 (2I)}{2\pi d}\right)}\right) == \tan^{-1}\left(\dfrac{1}{2}\right) = 26.6°$		

(d) 2 points

Wire 1 is now released. Which of the following best describes the initial motion of wire 1 due to the magnetic field of wire 2? Assume gravitational effects are negligible.

____ Wire 1 will not move.

____ Wire 1 will move upward as viewed in Figure 1.

____ Wire 1 will move downward as viewed in Figure 1.

____ Wire 1 will rotate clockwise as viewed in Figure 2.

____ Wire 1 will rotate counterclockwise as viewed in Figure 2.

Justify your answer.

For stating there is no translational motion since the magnetic forces on the wire cancel		1 point
For stating there is a net torque which causes rotation		1 point
Example: In Figure 2, the top portion of wire 1 will be in a magnetic field into the page from wire 2 and, thus, will experience a force to the right. The bottom portion of wire 1 will be in a magnetic field out of the page from wire 2 and, thus, will experience a force to the left. So the net force will be zero, but there will be a net clockwise torque, so the wire will rotate clockwise.		

Question 3 (continued)

Distribution of points

(f) 3 points

Given that the flux through the rectangular loop as a function of time t is given by the equation $\Phi = \frac{\mu_0 I_0 \ell (1 - kt)}{\pi} \ln\left(\frac{d + w}{d}\right)$, derive an expression for the magnitude of the current, if any, induced in the loop. Express your answers in terms of I_0, d, r, R, w, k, ℓ, and physical constants, as appropriate.

For attempting to take the time derivative of the magnetic flux to calculate the emf		1 point
$\varepsilon = \left\lvert -\frac{d\Phi}{dt} \right\rvert = \frac{\mu_0 I_0 \ell}{\pi} \ln\left(\frac{d + w}{d}\right) \left\lvert \frac{d}{dt}[(1 - kt)] \right\rvert$		
$\varepsilon = \frac{\mu_0 I_0 k \ell}{\pi} \ln\left(\frac{d + w}{d}\right)$		
For dividing the emf by the resistance to calculate the current		1 point
$I = \frac{\varepsilon}{R} = \frac{\frac{\mu_0 I_0 k \ell}{\pi} \ln\left(\frac{d + w}{d}\right)}{R}$		
For a correct answer		1 point
$I = \frac{\mu_0 I_0 k \ell}{\pi R} \ln\left(\frac{d + w}{d}\right)$		

(g) 3 points

What is the direction of the current, if any, induced in the loop as seen in Figure 3?

____ Clockwise ____ Counterclockwise

____ Undefined because there is no current induced in the loop

Justify your answer.

For selecting "Clockwise" with an attempt at a relevant justification		1 point
For indicating that the flux inside the loop will decrease		1 point
For using Lenz's law to relate the decrease in the flux to the clockwise current		1 point
Example: Because the current in the wire is decreasing, the flux in the loop will decrease. According to Lenz's law, the induced current should create a magnetic field to oppose this decrease. Thus the induced magnetic field must be into the page, and the current in the loop must be clockwise.		