

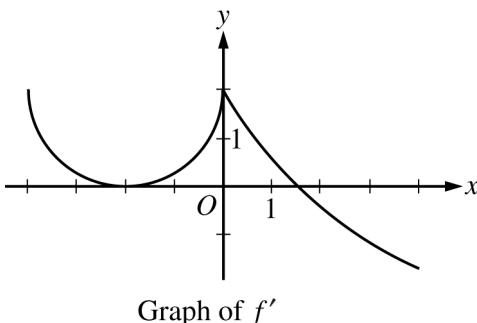
2009 AP® CALCULUS AB FREE-RESPONSE QUESTIONS

x	2	3	5	8	13
$f(x)$	1	4	-2	3	6

5. Let f be a function that is twice differentiable for all real numbers. The table above gives values of f for selected points in the closed interval $2 \leq x \leq 13$.
- (a) Estimate $f'(4)$. Show the work that leads to your answer.
- (b) Evaluate $\int_2^{13} (3 - 5f'(x)) dx$. Show the work that leads to your answer.
- (c) Use a left Riemann sum with subintervals indicated by the data in the table to approximate $\int_2^{13} f(x) dx$. Show the work that leads to your answer.
- (d) Suppose $f'(5) = 3$ and $f''(x) < 0$ for all x in the closed interval $5 \leq x \leq 8$. Use the line tangent to the graph of f at $x = 5$ to show that $f(7) \leq 4$. Use the secant line for the graph of f on $5 \leq x \leq 8$ to show that $f(7) \geq \frac{4}{3}$.
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WRITE ALL WORK IN THE PINK EXAM BOOKLET.

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Graph of f'

6. The derivative of a function f is defined by $f'(x) = \begin{cases} g(x) & \text{for } -4 \leq x \leq 0 \\ 5e^{-x/3} - 3 & \text{for } 0 < x \leq 4 \end{cases}$.

The graph of the continuous function f' , shown in the figure above, has x -intercepts at $x = -2$ and $x = 3 \ln\left(\frac{5}{3}\right)$. The graph of g on $-4 \leq x \leq 0$ is a semicircle, and $f(0) = 5$.

- For $-4 < x < 4$, find all values of x at which the graph of f has a point of inflection. Justify your answer.
- Find $f(-4)$ and $f(4)$.
- For $-4 \leq x \leq 4$, find the value of x at which f has an absolute maximum. Justify your answer.

WRITE ALL WORK IN THE PINK EXAM BOOKLET.

END OF EXAM

**AP[®] CALCULUS AB
2009 SCORING GUIDELINES**

Question 5

x	2	3	5	8	13
$f(x)$	1	4	-2	3	6

Let f be a function that is twice differentiable for all real numbers. The table above gives values of f for selected points in the closed interval $2 \leq x \leq 13$.

- (a) Estimate $f'(4)$. Show the work that leads to your answer.
- (b) Evaluate $\int_2^{13} (3 - 5f'(x)) dx$. Show the work that leads to your answer.
- (c) Use a left Riemann sum with subintervals indicated by the data in the table to approximate $\int_2^{13} f(x) dx$. Show the work that leads to your answer.
- (d) Suppose $f'(5) = 3$ and $f''(x) < 0$ for all x in the closed interval $5 \leq x \leq 8$. Use the line tangent to the graph of f at $x = 5$ to show that $f(7) \leq 4$. Use the secant line for the graph of f on $5 \leq x \leq 8$ to show that $f(7) \geq \frac{4}{3}$.

(a) $f'(4) \approx \frac{f(5) - f(3)}{5 - 3} = -3$

(b)
$$\begin{aligned} \int_2^{13} (3 - 5f'(x)) dx &= \int_2^{13} 3 dx - 5 \int_2^{13} f'(x) dx \\ &= 3(13 - 2) - 5(f(13) - f(2)) = 8 \end{aligned}$$

(c)
$$\begin{aligned} \int_2^{13} f(x) dx &\approx f(2)(3 - 2) + f(3)(5 - 3) \\ &\quad + f(5)(8 - 5) + f(8)(13 - 8) = 18 \end{aligned}$$

- (d) An equation for the tangent line is $y = -2 + 3(x - 5)$. Since $f''(x) < 0$ for all x in the interval $5 \leq x \leq 8$, the line tangent to the graph of $y = f(x)$ at $x = 5$ lies above the graph for all x in the interval $5 < x \leq 8$.

Therefore, $f(7) \leq -2 + 3 \cdot 2 = 4$.

An equation for the secant line is $y = -2 + \frac{5}{3}(x - 5)$.

Since $f''(x) < 0$ for all x in the interval $5 \leq x \leq 8$, the secant line connecting $(5, f(5))$ and $(8, f(8))$ lies below the graph of $y = f(x)$ for all x in the interval $5 < x < 8$.

Therefore, $f(7) \geq -2 + \frac{5}{3} \cdot 2 = \frac{4}{3}$.

1 : answer

2 :
$$\begin{cases} 1 : \text{uses Fundamental Theorem} \\ \text{of Calculus} \\ 1 : \text{answer} \end{cases}$$

2 :
$$\begin{cases} 1 : \text{left Riemann sum} \\ 1 : \text{answer} \end{cases}$$

4 :
$$\begin{cases} 1 : \text{tangent line} \\ 1 : \text{shows } f(7) \leq 4 \\ 1 : \text{secant line} \\ 1 : \text{shows } f(7) \geq \frac{4}{3} \end{cases}$$