STATISTICS
SECTION II
Part B
Question 6

Spend about 25 minutes on this part of the exam.

Percent of Section II grade—25

Directions: Show all your work. Indicate clearly the methods you use, because you will be graded on the correctness of your methods as well as on the accuracy of your results and explanation.

6. A survey given to a random sample of students at a university included a question about which of two well-known comedy shows, S or F, students preferred. The students were asked the question, “Do you prefer S or F ?” The responses are shown below.

Preference		
S	F	Total
185	139	324

(a) Based on the results of this survey, construct and interpret a 95% confidence interval for the proportion of students in the population who would respond S to the question, “Do you prefer S or F ?”

(b) What is the meaning of “95% confidence” in part (a) ?

(c) In a follow-up survey, a separate group of randomly selected students was asked “Do you prefer F or S ?” The responses are shown below.

Preference		
S	F	Total
68	88	156

Based on these two surveys, is there evidence that the stated preference depends on the order in which the comedy shows were listed in the survey question? Justify your answer.

(d) Suppose the test in part (c) indicates that the order in which the shows were listed does make a difference.

Is the pooled value $\frac{185 + 68}{324 + 156} = 0.527$ a reasonable estimate for the proportion of students at the university who would respond S ? If so, justify your answer. If not, what would be a more reasonable estimate? Explain why.

END OF EXAMINATION

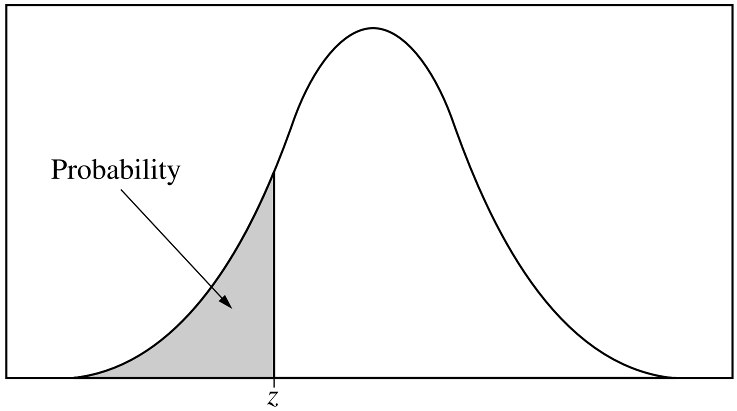

Table entry for z is the probability lying below z.

Table A Standard normal probabilities

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
−3.4	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0002
−3.3	.0005	.0005	.0005	.0004	.0004	.0004	.0004	.0004	.0004	.0003
−3.2	.0007	.0007	.0006	.0006	.0006	.0006	.0006	.0005	.0005	.0005
−3.1	.0010	.0009	.0009	.0009	.0008	.0008	.0008	.0008	.0007	.0007
−3.0	.0013	.0013	.0013	.0012	.0012	.0011	.0011	.0011	.0010	.0010
−2.9	.0019	.0018	.0018	.0017	.0016	.0016	.0015	.0015	.0014	.0014
−2.8	.0026	.0025	.0024	.0023	.0023	.0022	.0021	.0021	.0020	.0019
−2.7	.0035	.0034	.0033	.0032	.0031	.0030	.0029	.0028	.0027	.0026
−2.6	.0047	.0045	.0044	.0043	.0041	.0040	.0039	.0038	.0037	.0036
−2.5	.0062	.0060	.0059	.0057	.0055	.0054	.0052	.0051	.0049	.0048
−2.4	.0082	.0080	.0078	.0075	.0073	.0071	.0069	.0068	.0066	.0064
−2.3	.0107	.0104	.0102	.0099	.0096	.0094	.0091	.0089	.0087	.0084
−2.2	.0139	.0136	.0132	.0129	.0125	.0122	.0119	.0116	.0113	.0110
−2.1	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0146	.0143
−2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183
−1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233
−1.8	.0359	.0351	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
−1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367
−1.6	.0548	.0537	.0526	.0516	.0505	.0495	.0485	.0475	.0465	.0455
−1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0571	.0559
−1.4	.0808	.0793	.0778	.0764	.0749	.0735	.0721	.0708	.0694	.0681
−1.3	.0968	.0951	.0934	.0918	.0901	.0885	.0869	.0853	.0838	.0823
−1.2	.1151	.1131	.1112	.1093	.1075	.1056	.1038	.1020	.1003	.0985
−1.1	.1357	.1335	.1314	.1292	.1271	.1251	.1230	.1210	.1190	.1170
−1.0	.1587	.1562	.1539	.1515	.1492	.1469	.1446	.1423	.1401	.1379
−0.9	.1841	.1814	.1788	.1762	.1736	.1711	.1685	.1660	.1635	.1611
−0.8	.2119	.2090	.2061	.2033	.2005	.1977	.1949	.1922	.1894	.1867
−0.7	.2420	.2389	.2358	.2327	.2296	.2266	.2236	.2206	.2177	.2148
−0.6	.2743	.2709	.2676	.2643	.2611	.2578	.2546	.2514	.2483	.2451
−0.5	.3085	.3050	.3015	.2981	.2946	.2912	.2877	.2843	.2810	.2776
−0.4	.3446	.3409	.3372	.3336	.3300	.3264	.3228	.3192	.3156	.3121
−0.3	.3821	.3783	.3745	.3707	.3669	.3632	.3594	.3557	.3520	.3483
−0.2	.4207	.4168	.4129	.4090	.4052	.4013	.3974	.3936	.3897	.3859
−0.1	.4602	.4562	.4522	.4483	.4443	.4404	.4364	.4325	.4286	.4247
−0.0	.5000	.4960	.4920	.4880	.4840	.4801	.4761	.4721	.4681	.4641

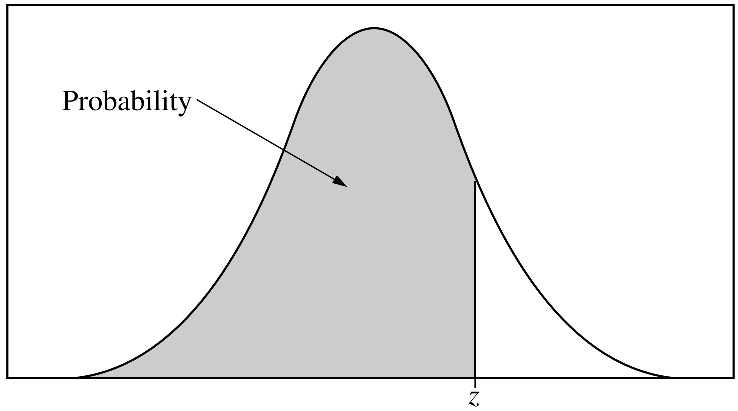

Table entry for z is the probability lying below z.

Table A ***(Continued)***

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998

Table entry for p is the point (χ^2) with probability p lying above it.

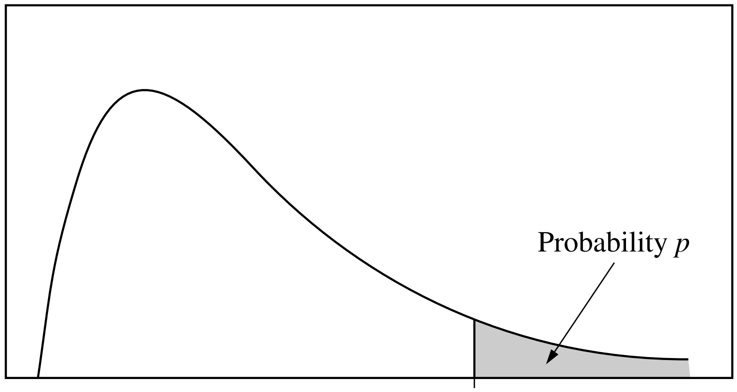

Table C χ^2 **critical values**

	Tail probability p											
df	.25	.20	.15	.10	.05	.025	.02	.01	.005	.0025	.001	.0005
1	1.32	1.64	2.07	2.71	3.84	5.02	5.41	6.63	7.88	9.14	10.83	12.12
2	2.77	3.22	3.79	4.61	5.99	7.38	7.82	9.21	10.60	11.98	13.82	15.20
3	4.11	4.64	5.32	6.25	7.81	9.35	9.84	11.34	12.84	14.32	16.27	17.73
4	5.39	5.99	6.74	7.78	9.49	11.14	11.67	13.28	14.86	16.42	18.47	20.00
5	6.63	7.29	8.12	9.24	11.07	12.83	13.39	15.09	16.75	18.39	20.51	22.11
6	7.84	8.56	9.45	10.64	12.59	14.45	15.03	16.81	18.55	20.25	22.46	24.10
7	9.04	9.80	10.75	12.02	14.07	16.01	16.62	18.48	20.28	22.04	24.3	26.02
8	10.22	11.03	12.03	13.36	15.51	17.53	18.17	20.09	21.95	23.77	26.1	27.87
9	11.39	12.24	13.29	14.68	16.92	19.02	19.68	21.67	23.59	25.46	27.8	29.67
10	12.55	13.44	14.53	15.99	18.31	20.48	21.16	23.21	25.19	27.11	29.59	31.42
11	13.70	14.63	15.77	17.28	19.68	21.92	22.62	24.72	26.76	28.73	31.26	33.14
12	14.85	15.81	16.99	18.55	21.03	23.34	24.05	26.22	28.30	30.32	32.91	34.82
13	15.98	16.98	18.20	19.81	22.36	24.74	25.47	27.69	29.82	31.88	34.53	36.48
14	17.12	18.15	19.41	21.06	23.68	26.12	26.87	29.14	31.32	33.43	36.12	38.11
15	18.25	19.31	20.60	22.31	25.00	27.49	28.26	30.58	32.80	34.95	37.70	39.72
16	19.37	20.47	21.79	23.54	26.30	28.85	29.63	32.00	34.27	36.46	39.2	41.31
17	20.49	21.61	22.98	24.77	27.59	30.19	31.00	33.41	35.72	37.95	40.79	42.88
18	21.60	22.76	24.16	25.99	28.87	31.53	32.35	34.81	37.16	39.42	42.3	44.43
19	22.72	23.90	25.33	27.20	30.14	32.85	33.69	36.19	38.58	40.88	43.8	45.97
20	23.83	25.04	26.50	28.41	31.41	34.17	35.02	37.57	40.00	42.34	45.3	47.50
21	24.93	26.17	27.66	29.62	32.67	35.48	36.34	38.93	41.40	43.78	46.80	49.01
22	26.04	27.30	28.82	30.81	33.92	36.78	37.66	40.29	42.80	45.20	48.27	50.51
23	27.14	28.43	29.98	32.01	35.17	38.08	38.97	41.64	44.18	46.62	49.73	52.00
24	28.24	29.55	31.13	33.20	36.42	39.36	40.27	42.98	45.56	48.03	51.18	53.48
25	29.34	30.68	32.28	34.38	37.65	40.65	41.57	44.31	46.93	49.44	52.62	54.95
26	30.43	31.79	33.43	35.56	38.89	41.92	42.86	45.64	48.29	50.83	54.0	56.41
27	31.53	32.91	34.57	36.74	40.11	43.19	44.14	46.96	49.64	52.22	55.4	57.86
28	32.62	34.03	35.71	37.92	41.34	44.46	45.42	48.28	50.99	53.59	56.89	59.30
29	33.71	35.14	36.85	39.09	42.56	45.72	46.69	49.59	52.34	54.97	58.30	60.73
30	34.80	36.25	37.99	40.26	43.77	46.98	47.96	50.89	53.67	56.33	59.70	62.16
40	45.62	47.27	49.24	51.81	55.76	59.34	60.44	63.69	66.77	69.70	73.40	76.09
50	56.33	58.16	60.35	63.17	67.50	71.42	72.61	76.15	79.49	82.66	86.66	89.56
60	66.98	68.97	71.34	74.40	79.08	83.30	84.58	88.38	91.95	95.34	99.61	102.7
80	88.13	90.41	93.11	96.58	101.9	106.6	108.1	112.3	116.3	120.1	124.8	128.3
100	109.1	111.7	114.7	118.5	124.3	129.6	131.1	135.8	140.2	144.3	149.4	153.2

Question 6

Solution

Part (a):

p = proportion of students at this university who would respond S to the question, "Do you prefer S or F?"

Large sample confidence interval for a population proportion.

$$\hat{p} \pm 1.96\sqrt{\frac{\hat{p}\,(1-\hat{p})}{n}}$$

State and Check Assumptions and Conditions:

Simple random sample (given in the problem stem—need not be mentioned in solution). Also need large sample with $n\hat{p} \geq 5$ and $n\left(1-\hat{p}\right) \geq 5$. Here,

$n\hat{p} = 185$ $\quad\quad n(1-\hat{p}) = 139$ are both greater than 5 (or 10)

or $\hat{p} \pm 3\sqrt{\frac{\hat{p}\,(1-\hat{p})}{n}}$ is entirely in the interval (0,1).

We assume the university has at least 10(324) = 3240 students ($N \geq 10n$).

Calculations:

$$\hat{p} \pm 1.96\sqrt{\frac{\hat{p}\,(1-\hat{p})}{n}} = 0.571 \pm 1.96\sqrt{\frac{(.571)(.429)}{324}} = 0.571 \pm 0.054 = (0.517,\ 0.625)$$

Calculator solution: (0.5171, 0.62488). The procedure is specified in the stem, but students still need to check assumptions and conditions.

Interpretation:

Based on this sample, we can be 95 percent confident that the proportion of students at this university who would respond S to the question, "Do you prefer S or F?" is between 0.517 and 0.625.
OR
We have 95 percent confidence that the interval (0.5171, 0.62488) captures the proportion of students who would respond "S" to the question, "Do you prefer S or F?"

Question 6 (cont'd.)

Part (b):

Meaning of Confidence Level:

In repeated sampling, 95 percent of the intervals produced using this method will contain the proportion of students at this university who would respond S to the question "Do you prefer S or F?"

Part (c):

Approach 1: Hypothesis Test – Two Proportion *Z*- test

States a Correct Pair of Hypotheses:

H_0: $p_1 - p_2 = 0$
H_a: $p_1 - p_2 \neq 0$

where
p_1 = proportion of students at this university who would respond S with the original question wording
p_2 = proportion of students at this university who would respond S with the revised question wording

Note: A one-sided test is incorrect.

Name Test and State and Check Assumptions and Conditions:

Identifies a correct test (by name or by formula), and checks appropriate assumptions.

Two sample z-test for proportions

Note: Problem states that samples are random samples, so this does not need to be addressed in the assumptions.

Large samples: $n_1\hat{p}_1 = 185$; $n_1(1-\hat{p}_1) = 139$; $n_2\hat{p}_2 = 68$; $n_2(1-\hat{p}_2) = 88$
All are greater than 5 (or 10), so the sample sizes are large enough.

Question 6 (cont'd.)

Calculations:

Correct mechanics, including P-value or rejection region (except for minor arithmetic errors)

For two sample proportion z-test:

$\hat{p}_1 = \dfrac{185}{324} = 0.571$ $\qquad$ $\hat{p}_2 = \dfrac{68}{156} = 0.436$

$$\hat{p} = \frac{185 + 68}{324 + 156} = \frac{253}{480} = 0.527$$

$$z = \frac{\hat{p}_1 - \hat{p}_2}{\sqrt{\dfrac{\hat{p}(1-\hat{p})}{n_1} + \dfrac{\hat{p}(1-\hat{p})}{n_2}}} = \frac{0.571 - 0.436}{\sqrt{\dfrac{(0.527)(0.473)}{324} + \dfrac{(0.527)(0.473)}{156}}} = \frac{0.135}{0.0487} = 2.77$$

P-value = 2(.0028) = .0056 from tables

(Calculator: $z = 2.776554085$, P-value = .0054939656)

If the proportions are not pooled, then $z = 2.795$ and $p = 0.00518$.

Conclusion:

Since the P-value (0.0055) is so small, we reject the null hypothesis that the proportions of this university's students who would respond S to the two survey questions are equal. We believe the order in which the choices are given affects the students' response.

Stating a correct conclusion in the context of the problem, using the result of the statistical test (i.e., linking the conclusion to the result of the hypothesis test).

If both an α and a P-value are given, the linkage is implied. If no α is given, the solution must be explicit about the linkage by giving a correct interpretation of the P-value or explaining how the conclusion follows from the P-value.

If the P-value in part 3 is incorrect but the conclusion is consistent with the computed P-value, part 4 should be considered as correct.

Question 6 (cont'd.)

Approach 2: Hypothesis Test – Chi-square Test for Homogeneity

States a correct pair of hypotheses:

H_0: population response proportions are the same for the two question wordings
H_a: population response proportions are not the same for the two question wordings

NOTE: Although the computations are the same, stating the hypotheses in terms of independence is not correct.

Name Test and State and Check Assumptions and Conditions:

Identifies a correct test (by name or by formula), and checks appropriate assumptions.

Chi-square test for homogeneity

Observed counts

	S	F	row total
Original wording	185	139	324
Revised wording	68	88	156
Column total	253	227	480

Expected counts

	S	F
Original wording	170.775	153.225
Revised wording	82.225	73.775

NOTE: Problem states that samples are random samples, so this does not need to be addressed in the assumptions. All expected counts are greater than five, so the sample sizes are large enough.

Calculations:

Correct mechanics, including P-value or rejection region (except for minor arithmetic errors)

For chi-square test:

$$\chi^2 = \sum \frac{(O-E)^2}{E} = \frac{(185\ -\ 170.775)^2}{170.775} + \ldots + \frac{(88\ -\ 73.775)^2}{73.775} = 7.709253$$

df = 1 from tables $0.005 < \text{P-value} < 0.01$

(Calculator: P-value = 0.0054938481)

Question 6 (cont'd.)

Conclusion:

Since the P-value is so small, we reject the null hypothesis that the proportions of students at this university who would respond S are the same for the two survey questions. We believe the order in which the choices are given affects the students' response.

Stating a correct conclusion in the context of the problem, using the result of the statistical test (i.e., linking the conclusion to the result of the hypothesis test).

If both an α and a P-value are given, the linkage is implied. If no α is given, the solution must be explicit about the linkage by giving a correct interpretation of the P-value or explaining how the conclusion follows from the P-value.

If the P-value in part 3 is incorrect but the conclusion is consistent with the computed P-value, part 4 should be considered as correct.

Approach 3: Two sample confidence interval

Name Test and State and Check Assumptions and Conditions:

Two-sample confidence interval.

Problem states that samples are random samples, so this does not need to be addressed in the assumptions.

Large samples:

$n_1\hat{p}_1 = 185;\quad n_1(1-\hat{p}_1) = 139;\quad n_2\hat{p}_2 = 68;\quad n_2(1-\hat{p}_2) = 88$

All are larger than 5 (or 10), so the sample sizes are large enough.

Calculations:

$$(\hat{p}_1 - \hat{p}_2) \pm z^* \sqrt{\frac{\hat{p}_1(1-\hat{p}_1)}{n_1} + \frac{\hat{p}_2(1-\hat{p}_2)}{n_2}}$$

90 percent CI: (0.05565, 0.21453)
95 percent CI: (0.04044, 0.22974)
99 percent CI: (0.01069, 0.25949)

Question 6 (cont'd.)

Conclusion:

Since the confidence interval does not include 0, there is evidence that the proportions of students at this university who respond S is not the same for the two question wordings.

Approach 4: (This approach can score at most partially correct for part (c))

One sample confidence interval for p_2.

All checks and assumptions must be included (same as in section (a)).

95 percent CI: (0.35808, 0.51371)

Since this interval does not overlap with the interval computed in part (a), $(0.517, 0.625)$, conclude that the proportions of this university's students who would respond S is not the same for the two question wordings.

Part (d):

If the sample sizes had been equal, it would be reasonable to combine the data from the two samples by pooling, which would be equivalent to averaging the two proportions in this case. But, since the wording of the question makes a difference, and more people were asked the original version than were asked the revised version, we cannot just pool.

OR

It is only reasonable to pool estimates if they are estimating the same population parameter. Here the proportion who would respond S differs with the survey question so the estimates should not be pooled.

Approach 1:

One reasonable approach would be to scale sample 2 up to a sample size of 324 while maintaining the same sample proportion. To do this, the number of S's in sample 2 would be multiplied by a factor of 2.076923 (It is OK if the student uses a factor of 2 for simplicity). This would result in two samples of sizes 324 with 185 S's in sample 1 and 141 S's in sample 2. This would result in an estimate of those who prefer S of

$$\hat{p} = \frac{185 + 141}{648} = \frac{326}{648} = 0.503$$

Question 6 (cont'd.)

Note: A comparable approach would be to scale sample 1 down to a sample size of 156 by using a multiplier of 0.481481 to obtain

$$\hat{p} = \frac{89 + 68}{312} = \frac{157}{312} = 0.503$$ This is very close to 0.5.

Approach 2:

The approach described above is equivalent to just averaging the two proportions, and so averaging the two given proportions is also an acceptable approach.

$$\hat{p} = \frac{\frac{185}{324} + \frac{68}{156}}{2} = \frac{0.571 + 0.436}{2} = 0.503$$

Note: A weighted average of the two proportions (with weights proportional to sample size) is equivalent to the given pooled value. If the student rejects the pooled value and proposes a weighted average of the two sample proportions as an alternative, part (d) is incorrect.

Scoring for Question 6

Parts (a) and (b) should be read together.

Part (a) is scored as essentially correct (E) if the assumptions are checked, the interval is computed correctly (except for minor arithmetic errors), and a correct interpretation of the interval is given.

It is partially correct (P) if the interval is computed correctly (except for minor arithmetic errors) but either the assumptions or the interpretation is not correct.

Otherwise, part (a) is scored as incorrect (I).

Part (b) is scored as either essentially correct (E) or incorrect (I). It is not possible to score partially correct on this part.

Part (c) is scored as essentially correct (E) if all four parts of the hypothesis test are correct. It is scored as partially correct (P) if two or three of the components of the test are correct. Otherwise, it is scored as incorrect (I).

Part (d) is scored as essentially correct (E) if the student produces a reasonable estimate that takes the different sample sizes into account, the explanation is correct and communication is good.

Question 6 (cont'd.)

It is partially correct (P) if
- a reasonable estimate is produced but the explanation is incorrect or weak
  OR
- a good explanation of why not to use the pooled estimate but no reasonable alternative is given

For parts (a), (b), and (c), essentially correct responses count as 1 part and partially correct responses count as ½ part.

4 **Complete Response**
Four parts correct.

3 **Substantial Response**
Three parts correct.

2 **Developing Response**
Two parts correct.

1 **Minimal Response**
One part correct.

If a paper is between two scores (for example, 2 ½ parts) use a holistic approach to determine whether to score up or down depending on the strength of the response and communication.

Note: If the paper is between two scores and (a) or (c) has the interpretation correct, then round up. If neither is correct, round down.