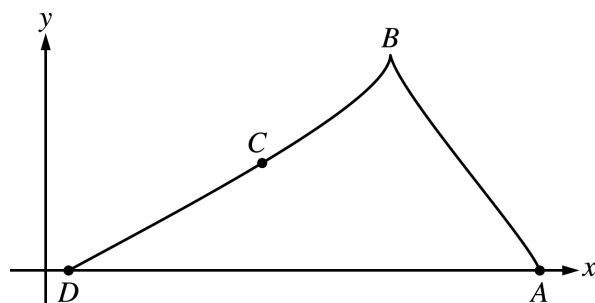
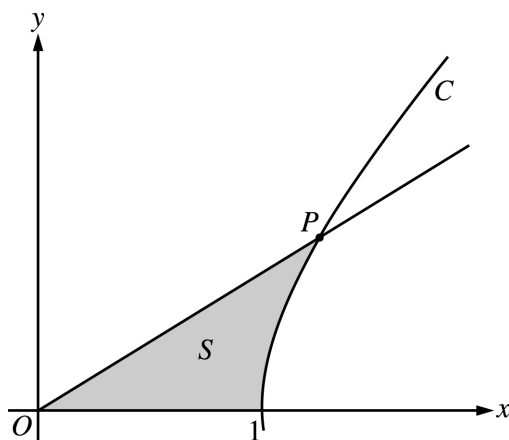


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2. A particle starts at point  $A$  on the positive  $x$ -axis at time  $t = 0$  and travels along the curve from  $A$  to  $B$  to  $C$  to  $D$ , as shown above. The coordinates of the particle's position  $(x(t), y(t))$  are differentiable functions of  $t$ , where  $x'(t) = \frac{dx}{dt} = -9\cos\left(\frac{\pi t}{6}\right)\sin\left(\frac{\pi\sqrt{t+1}}{2}\right)$  and  $y'(t) = \frac{dy}{dt}$  is not explicitly given. At time  $t = 9$ , the particle reaches its final position at point  $D$  on the positive  $x$ -axis.
- At point  $C$ , is  $\frac{dy}{dt}$  positive? At point  $C$ , is  $\frac{dx}{dt}$  positive? Give a reason for each answer.
  - The slope of the curve is undefined at point  $B$ . At what time  $t$  is the particle at point  $B$ ?
  - The line tangent to the curve at the point  $(x(8), y(8))$  has equation  $y = \frac{5}{9}x - 2$ . Find the velocity vector and the speed of the particle at this point.
  - How far apart are points  $A$  and  $D$ , the initial and final positions, respectively, of the particle?
-

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3. The figure above shows the graphs of the line  $x = \frac{5}{3}y$  and the curve  $C$  given by  $x = \sqrt{1 + y^2}$ . Let  $S$  be the shaded region bounded by the two graphs and the  $x$ -axis. The line and the curve intersect at point  $P$ .
- Find the coordinates of point  $P$  and the value of  $\frac{dx}{dy}$  for curve  $C$  at point  $P$ .
  - Set up and evaluate an integral expression with respect to  $y$  that gives the area of  $S$ .
  - Curve  $C$  is a part of the curve  $x^2 - y^2 = 1$ . Show that  $x^2 - y^2 = 1$  can be written as the polar equation  $r^2 = \frac{1}{\cos^2 \theta - \sin^2 \theta}$ .
  - Use the polar equation given in part (c) to set up an integral expression with respect to the polar angle  $\theta$  that represents the area of  $S$ .
- 

**END OF PART A OF SECTION II**