CALCULUS AB
SECTION II, Part B
Time—45 minutes
Number of problems—3

No calculator is allowed for these problems.

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4. Water is pumped into an underground tank at a constant rate of 8 gallons per minute. Water leaks out of the tank at the rate of $\sqrt{t+1}$ gallons per minute, for $0 \le t \le 120$ minutes. At time $t = 0$, the tank contains 30 gallons of water.

   (a) How many gallons of water leak out of the tank from time $t = 0$ to $t = 3$ minutes?

   (b) How many gallons of water are in the tank at time $t = 3$ minutes?

   (c) Write an expression for $A(t)$, the total number of gallons of water in the tank at time t.

   (d) At what time t, for $0 \le t \le 120$, is the amount of water in the tank a maximum? Justify your answer.

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5. Consider the curve given by $xy^2 - x^3y = 6$.

   (a) Show that $\frac{dy}{dx} = \frac{3x^2y - y^2}{2xy - x^3}$.

   (b) Find all points on the curve whose x-coordinate is 1, and write an equation for the tangent line at each of these points.

   (c) Find the x-coordinate of each point on the curve where the tangent line is vertical.

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6. Consider the differential equation $\frac{dy}{dx} = \frac{3x^2}{e^{2y}}$.

   (a) Find a solution $y = f(x)$ to the differential equation satisfying $f(0) = \frac{1}{2}$.

   (b) Find the domain and range of the function f found in part (a).

END OF EXAMINATION

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Consider the differential equation $\dfrac{dy}{dx} = \dfrac{3x^2}{e^{2y}}$.

(a) Find a solution $y = f(x)$ to the differential equation satisfying $f(0) = \dfrac{1}{2}$.

(b) Find the domain and range of the function f found in part (a).

(a) $e^{2y}\,dy = 3x^2\,dx$

$\dfrac{1}{2}e^{2y} = x^3 + C_1$

$e^{2y} = 2x^3 + C$

$y = \dfrac{1}{2}\ln\left(2x^3 + C\right)$

$\dfrac{1}{2} = \dfrac{1}{2}\ln(0 + C); \quad C = e$

$y = \dfrac{1}{2}\ln(2x^3 + e)$

6
- 1 : separates variables
- 1 : antiderivative of dy term
- 1 : antiderivative of dx term
- 1 : constant of integration
- 1 : uses initial condition $f(0) = \dfrac{1}{2}$
- 1 : solves for y
- Note: 0/1 if y is not a logarithmic function of x

Note: max 3/6 [1-1-1-0-0-0] if no constant of integration

Note: 0/6 if no separation of variables

(b) Domain: $2x^3 + e > 0$

$x^3 > -\dfrac{1}{2}e$

$x > \left(-\dfrac{1}{2}e\right)^{1/3} = -\left(\dfrac{1}{2}e\right)^{1/3}$

Range: $-\infty < y < \infty$

3
- 1 : $2x^3 + e > 0$
- 1 : domain
  - Note: 0/1 if 0 is not in the domain
- 1 : range

Note: 0/3 if y is not a logarithmic function of x

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