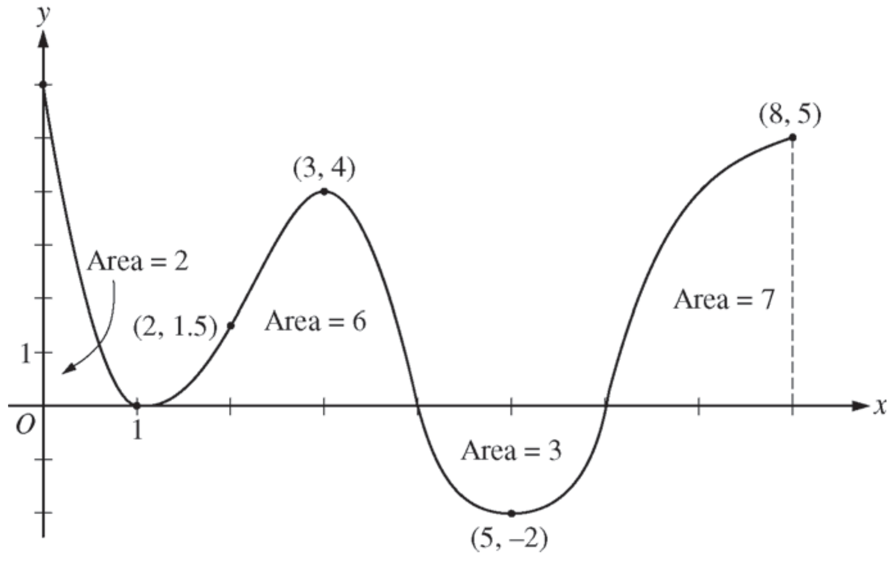

Graph of f'

4. The figure above shows the graph of f', the derivative of a twice-differentiable function f, on the closed interval $0 \le x \le 8$. The graph of f' has horizontal tangent lines at $x = 1$, $x = 3$, and $x = 5$. The areas of the regions between the graph of f' and the x-axis are labeled in the figure. The function f is defined for all real numbers and satisfies $f(8) = 4$.

(a) Find all values of x on the open interval $0 < x < 8$ for which the function f has a local minimum. Justify your answer.

(b) Determine the absolute minimum value of f on the closed interval $0 \le x \le 8$. Justify your answer.

(c) On what open intervals contained in $0 < x < 8$ is the graph of f both concave down and increasing? Explain your reasoning.

(d) The function g is defined by $g(x) = (f(x))^3$. If $f(3) = -\frac{5}{2}$, find the slope of the line tangent to the graph of g at $x = 3$.

5. Consider the differential equation $\frac{dy}{dx} = y^2(2x + 2)$. Let $y = f(x)$ be the particular solution to the differential equation with initial condition $f(0) = -1$.

   (a) Find $\lim_{x \to 0} \frac{f(x) + 1}{\sin x}$. Show the work that leads to your answer.

   (b) Use Euler's method, starting at $x = 0$ with two steps of equal size, to approximate $f\left(\frac{1}{2}\right)$.

   (c) Find $y = f(x)$, the particular solution to the differential equation with initial condition $f(0) = -1$.

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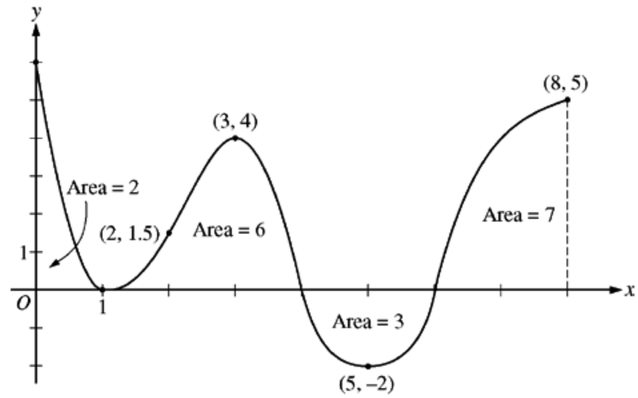
Question 4



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(a) $x = 6$ is the only critical point at which f' changes sign from negative to positive. Therefore, f has a local minimum at $x = 6$.

1 : answer with justification

(b) From part (a), the absolute minimum occurs either at $x = 6$ or at an endpoint.

$$f(0) = f(8) + \int_8^0 f'(x)\,dx = f(8) - \int_0^8 f'(x)\,dx = 4 - 12 = -8$$

$$f(6) = f(8) + \int_8^6 f'(x)\,dx = f(8) - \int_6^8 f'(x)\,dx = 4 - 7 = -3$$

$$f(8) = 4$$

The absolute minimum value of f on the closed interval $[0, 8]$ is -8.

$3 : \begin{cases} 1 : \text{considers } x = 0 \text{ and } x = 6 \\ 1 : \text{answer} \\ 1 : \text{justification} \end{cases}$

(c) The graph of f is concave down and increasing on $0 < x < 1$ and $3 < x < 4$, because f' is decreasing and positive on these intervals.

$2 : \begin{cases} 1 : \text{answer} \\ 1 : \text{explanation} \end{cases}$

(d) $g'(x) = 3[f(x)]^2 \cdot f'(x)$

$$g'(3) = 3[f(3)]^2 \cdot f'(3) = 3\left(-\frac{5}{2}\right)^2 \cdot 4 = 75$$

$3 : \begin{cases} 2 : g'(x) \\ 1 : \text{answer} \end{cases}$