4. A company is considering implementing one of two quality control plans for monitoring the weights of automobile batteries that it manufactures. If the manufacturing process is working properly, the battery weights are approximately normally distributed with a specified mean and standard deviation.

   Quality control plan A calls for rejecting a battery as defective if its weight falls more than 2 standard deviations below the specified mean.

   Quality control plan B calls for rejecting a battery as defective if its weight falls more than 1.5 interquartile ranges below the lower quartile of the specified population.

   Assume the manufacturing process is under control.

   a. What proportion of batteries will be rejected by plan A ?

   b. What is the probability that at least 1 of 2 randomly selected batteries will be rejected by plan A ?

   c. What proportion of batteries will be rejected by plan B ?

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5. Die A has four 9's and two 0's on its faces. Die B has four 3's and two 11's on its faces. When either of these dice is rolled, each face has an equal chance of landing on top. Two players are going to play a game. The first player selects a die and rolls it. The second player rolls the remaining die. The winner is the player whose die has the higher number on top.

   a. Suppose you are the first player and you want to win the game. Which die would you select? Justify your answer.

   b. Suppose the player using die A receives 45 tokens each time he or she wins the game. How many tokens must the player using die B receive each time he or she wins in order for this to be a fair game? Explain how you found your answer.

      (A fair game is one in which the player using die A and the player using die B both end up with the same number of tokens in the long run.)

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Question 4

Solution

1. Plan A: defective if more than two standard deviations below the mean

For Plan A
P(defective) = P(z < -2) = .0228 (Tables) (.02275 Calculator)

So, plan A rejects .0228 or 2.28% as defective.

OR

Students use the Empirical Rule: about 95% of the area under the normal curve lies between z = -2 and z = 2, so 5% lies outside these limits, and about 2.5% lies below z = -2.

2. For Plan A:

P(at least 1 of 2 rejected)
= 1 - P(none rejected)
= 1 - 2 nCr 0 $(0.0228)^0$ $(0.9772)^2$
= 1 - $(.9772)^2$
= 1 - .9549
= .0451 (.04498 Calculator)

OR

P(at least 1 of 2 rejected)
= P(exactly 1 rejected) + P(exactly 2 rejected)
= 2 nCr 1 (0.0228) (0.9772) + 2 nCr 2 $(0.0228)^2$ $(0.9772)^0$
= 2 (.0228) (.9772) + $(0.0228)^2$
= .0446 + .0005
= .0451

OR

P(at least 1 of 2 rejected)
= P(1st rejected) + P(2nd rejected) - P(both rejected)
= .0228 + .0228 - $(.0228)^2$
= .0451

Note: If P(defective) = .025 then P(at least 1 of 2 rejected) = .04938

Question 4 (cont.)

3. Plan B: defective if more than 1.5 IQR below the lower quartile

Quartiles are at .67 std. dev. (.67449 Calculator)

IQR = 1.34 std. dev. (1.34898 Calculator)

1.5 IQR = 2.01 std. dev. (2.02347 Calculator)

Boundary for defective region is:

Q1 - 1.5 IQR = - .67 - 2.01 = - 2.68

For Plan B

P(defective) = P(z < -2.68) = .0037 (.00349 Calculator)

So, Plan B rejects .0037 or .37% as defective. (Tables) (.349% Calculator)

Points:

4 Complete Response
Except for minor arithmetic errors

- Correctly computes the defective proportion in (a) and shows appropriate work
- Correctly computes the probability in (b) and shows appropriate work
- Correctly determines boundary for defective region in part (c) and the corresponding defective proportion

3 Substantial Response
Except for minor errors

- Correctly computes the defective proportion in (a) and shows appropriate work
- Correctly computes the probability in (b) and shows appropriate work
- Makes an error in determining the IQR or boundary for the defective region in (c) but computes the proportion defective correctly for the stated boundaries

A paper can also get a 3 if the defective region is seen as two sided (i.e. more than two std. dev. from the mean rather than more than two std. dev. below the mean) and answers to all three parts are correct for this interpretation.
(a: .0456, b: .0891, c: .0074 ; or on Calculator: a: .0455, b: .0889, c: .006976)

OR

Part (c) is essentially correct and either (a) or (b) is correct