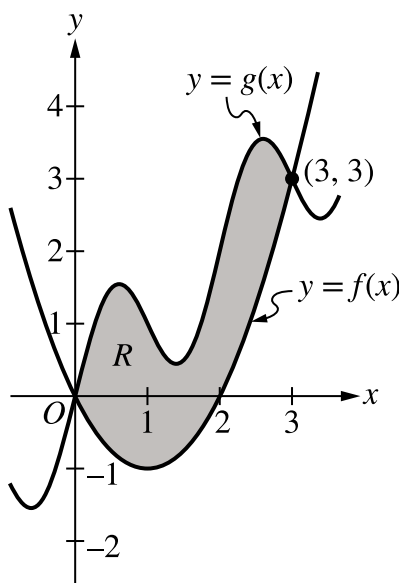**2.** The shaded region R is bounded by the graphs of the functions f and g, where $f(x) = x^2 - 2x$ and $g(x) = x + \sin(\pi x)$, as shown in the figure.

(Note: Your calculator should be in radian mode.)

A. Find the area of R. Show the setup for your calculations.

B. Region R is the base of a solid. For this solid, at each x the cross section perpendicular to the x-axis is a rectangle with height x and base in region R. Find the volume of the solid. Show the setup for your calculations.

C. Write, but do not evaluate, an integral expression for the volume of the solid generated when the region R is rotated about the horizontal line $y = -2$.

D. It can be shown that $g'(x) = 1 + \pi \cos(\pi x)$. Find the value of x, for $0 < x < 1$, at which the line tangent to the graph of f is parallel to the line tangent to the graph of g.

END OF PART A

3. A student starts reading a book at time $t = 0$ minutes and continues reading for the next 10 minutes. The rate at which the student reads is modeled by the differentiable function R, where $R(t)$ is measured in words per minute. Selected values of $R(t)$ are given in the table shown.
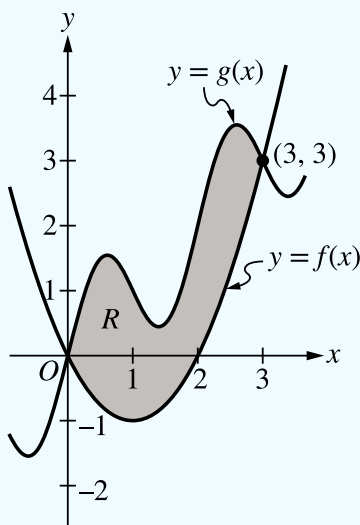
t (minutes)	0	2	8	10
$R(t)$ (words per minute)	90	100	150	162

A. Approximate $R'(1)$ using the average rate of change of R over the interval $0 \leq t \leq 2$. Show the work that leads to your answer. Indicate units of measure.

B. Must there be a value c, for $0 < c < 10$, such that $R(c) = 155$? Justify your answer.

C. Use a trapezoidal sum with the three subintervals indicated by the data in the table to approximate the value of $\int_0^{10} R(t)\,dt$. Show the work that leads to your answer.

D. A teacher also starts reading at time $t = 0$ minutes and continues reading for the next 10 minutes. The rate at which the teacher reads is modeled by the function W defined by $W(t) = -\frac{3}{10}t^2 + 8t + 100$, where $W(t)$ is measured in words per minute. Based on the model, how many words has the teacher read by the end of the 10 minutes? Show the work that leads to your answer.

Part A (AB): Graphing calculator required
Question 2   **9 points**

General Scoring Notes

- The model solution is presented using standard mathematical notation.
- Answers (numeric or algebraic) need not be simplified. Answers given as a decimal approximation should be accurate to three places after the decimal point. Within each individual free-response question, at most one point is not earned for inappropriate rounding.

The shaded region R is bounded by the graphs of the functions f and g, where $f(x) = x^2 - 2x$ and $g(x) = x + \sin(\pi x)$, as shown in the figure.

(Note: Your calculator should be in radian mode.)

Model Solution	**Scoring**

A   Find the area of R. Show the setup for your calculations.

$\int_0^3 (g(x) - f(x))\, dx$	Form of integrand	**Point 1 (P1)**
$= 5.136620$ The area is 5.137 (or 5.136).	Answer	**Point 2 (P2)**

Scoring Notes for Part A

- **P1** is earned for a response that presents an integrand of $g(x) - f(x)$, $|g(x) - f(x)|$, $f(x) - g(x)$, or $|f(x) - g(x)|$ in a definite integral, with or without the differential dx.

- **P1** could also be earned for a difference of definite integrals with integrands $g(x)$ and $f(x)$.

- **P2** is earned for the correct answer, with or without supporting work. A reported answer should be accurate to three places after the decimal point, rounded or truncated. An inappropriately rounded answer does not earn the point.

- Incorrect communication between the integral and the correct answer is treated as scratch work and is not considered in scoring.

  - $\int_0^3 (f(x) - g(x))\, dx = -5.137$ so the area is 5.137.

    Note: This response earns **P1** for the integral. It also earns **P2** for the correct answer.

  - $\int_0^3 (f(x) - g(x))\, dx = 5.137$

    Note: This response earns **P1** for the integral. It also earns **P2** for the correct answer. (In this instance, incorrect linkage is not considered in scoring.)

- The exact answer is $\dfrac{4 + 9\pi}{2\pi}$.

B	Region R is the base of a solid. For this solid, at each x the cross section perpendicular to the x-axis is a rectangle with height x and base in region R. Find the volume of the solid. Show the setup for your calculations.		
	$\int_0^3 x(g(x) - f(x))\, dx$	Form of integrand	**Point 3 (P3)**
	$= 7.704930$ The volume of the solid is 7.705 (or 7.704).	Answer	**Point 4 (P4)**

Scoring Notes for Part B

- **P3** is earned for a definite integral with an integrand presented as a product of two nonconstant factors, with one of the factors equal to x, $g(x) - f(x)$, or $f(x) - g(x)$.

- The presence or absence of the differential dx will not be considered in scoring **P3** or **P4**.

- **P4** is earned for the correct answer, with or without supporting work. A reported answer should be accurate to three places after the decimal point, rounded or truncated. An inappropriately rounded answer does not earn the point, unless an earlier point was not earned due to inappropriate rounding.

- Incorrect or unclear communication between the integral and the correct answer is treated as scratch work and is not considered in scoring. For example:

  o $\int_0^3 x(f(x) - g(x))\, dx = -7.705$ so the volume is 7.705.

    Note: This response earns **P3** for the integral. It also earns **P4** for the correct answer.

  o $\int_0^3 x(f(x) - g(x))\, dx = 7.705$

    Note: This response earns **P3** for the integral. It also earns **P4** for the correct answer. (In this instance, incorrect linkage is not considered in scoring.)

- The exact answer is $\dfrac{12 + 27\pi}{4\pi}$.

C Write, but do not evaluate, an integral expression for the volume of the solid generated when the region R is rotated about the horizontal line $y = -2$.

$$\text{Volume} = \pi \int_0^3 \left((g(x) - (-2))^2 - (f(x) - (-2))^2 \right) dx$$

Form of integrand	**Point 5 (P5)**
Integrand	**Point 6 (P6)**
Limits, constant, and differential	**Point 7 (P7)**

Scoring Notes for Part C

- **P5** is earned for a definite integral with an integrand of $R^2 - r^2$ or $\left| R^2 - r^2 \right|$, where one of $\{R, r\}$ is correct or a difference between g and a nonzero constant, and the other is correct or a difference between f and a nonzero constant.

- **P6** is earned for the integral $\int_0^3 \left((g(x) + 2)^2 - (f(x) + 2)^2 \right) dx$, $\int_0^3 \left((f(x) + 2)^2 - (g(x) + 2)^2 \right) dx$, or a mathematically equivalent expression.

- Note **P5** and **P6** could be earned for a difference of definite integrals.

- A response that presents an integral expression that does not include the constant π is eligible for **P5** and **P6** but does not earn **P7**.

- To be eligible for **P7**, a response must have earned **P5**.

- A response that reverses the difference of squares must resolve the reversal with either the constant or the limits of integration AND include the differential to earn **P7**. For example:

  o A response of $-\pi \int_0^3 \left((f(x) + 2)^2 - (g(x) + 2)^2 \right) dx$ or $\pi \int_3^0 \left((f(x) + 2)^2 - (g(x) + 2)^2 \right) dx$ earns **P5**, **P6**, and **P7**.

  o A response of $\pi \int_0^3 \left((f(x) + 2)^2 - (g(x) + 2)^2 \right) dx$ earns **P5** and **P6** but does not earn **P7**.

- A response of only $\pi \int_0^3 \left((g(x) - (-2))^2 - (f(x) - (-2))^2 \right) dx$ earns **P5**, **P6**, and **P7**.

D	It can be shown that $g'(x) = 1 + \pi \cos(\pi x)$. Find the value of x, for $0 < x < 1$, at which the line tangent to the graph of f is parallel to the line tangent to the graph of g.		
	$f'(x) = g'(x) \implies 2x - 2 = 1 + \pi \cos(\pi x)$	$f'(x) = g'(x)$	**Point 8 (P8)**
	$\implies x = 0.675819$	Answer	**Point 9 (P9)**
	The lines tangent to the graphs of f and g are parallel at $x = 0.676$ (or 0.675).		

Scoring Notes for Part D

- **P8** is earned for a general statement, such as $f'(x) = g'(x)$, or for any correct equation formed by substituting $2x - 2$ for $f'(x)$, $1 + \pi \cos(\pi x)$ for $g'(x)$, or both.

- **P9** is earned for the correct answer, with or without supporting work. A reported answer should be accurate to three places after the decimal point, rounded or truncated. An inappropriately rounded answer does not earn the point, unless an earlier point was not earned due to inappropriate rounding.