CALCULUS BC

SECTION II, Part B

Time—45 minutes

Number of problems—3

No calculator is allowed for these problems.

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4. A moving particle has position $(x(t),\ y(t))$ at time t. The position of the particle at time $t = 1$ is $(2,6)$, and the velocity vector at any time $t > 0$ is given by $\left(1 - \frac{1}{t^2}, 2 + \frac{1}{t^2}\right)$.

   (a) Find the acceleration vector at time $t = 3$.

   (b) Find the position of the particle at time $t = 3$.

   (c) For what time $t > 0$ does the line tangent to the path of the particle at $(x(t),\ y(t))$ have a slope of 8 ?

   (d) The particle approaches a line as $t \to \infty$. Find the slope of this line. Show the work that leads to your conclusion.

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5. Consider the curve given by $xy^2 - x^3y = 6$.

   (a) Show that $\dfrac{dy}{dx} = \dfrac{3x^2y - y^2}{2xy - x^3}$.

   (b) Find all points on the curve whose x-coordinate is 1, and write an equation for the tangent line at each of these points.

   (c) Find the x-coordinate of each point on the curve where the tangent line is vertical.

A moving particle has position $(x(t), y(t))$ at time t. The position of the particle at time $t = 1$ is (2,6) and the velocity vector at any time $t > 0$ is given by $\left(1 - \frac{1}{t^2}, 2 + \frac{1}{t^2}\right)$.

(a) Find the acceleration vector at time $t = 3$.

(b) Find the position of the particle at time $t = 3$.

(c) For what time $t > 0$ does the line tangent to the path of the particle at $(x(t), y(t))$ have a slope of 8?

(d) The particle approaches a line as $t \to \infty$. Find the slope of this line. Show the work that leads to your conclusion.

(a) acceleration vector $= (x''(t), y''(t)) = \left(\frac{2}{t^3}, -\frac{2}{t^3}\right)$

$(x''(3), y''(3)) = \left(\frac{2}{27}, -\frac{2}{27}\right)$

$2 \begin{cases} 1: & \text{components of acceleration vector as a function of } t \\ 1: & \text{acceleration vector at } t = 3 \end{cases}$

(b) $(x(t), y(t)) = \left(t + \frac{1}{t} + C_1,\ 2t - \frac{1}{t} + C_2\right)$

$(2,6) = (x(1), y(1)) = (2 + C_1, 1 + C_2)$

$C_1 = 0,\ C_2 = 5$

$(x(3), y(3)) = \left(3 + \frac{1}{3}, 6 - \frac{1}{3} + 5\right) = \left(\frac{10}{3}, \frac{32}{3}\right)$

$3 \begin{cases} 1: & \text{antidifferentiation} \\ 1: & \text{uses initial condition at } t = 1 \\ 1: & \text{position at } t = 3 \end{cases}$

Note: max 1/3 [1–0–0] if no constants of integration

(c) $\frac{dy}{dx} = \frac{2 + \frac{1}{t^2}}{1 - \frac{1}{t^2}} = 8$

$2 + \frac{1}{t^2} = 8\left(1 - \frac{1}{t^2}\right); \quad t^2 = \frac{9}{6}$

$t = \sqrt{\frac{3}{2}}$

$2 \begin{cases} 1: & \frac{dy}{dx} = 8 \text{ as equation in } t \\ 1: & \text{solution for } t \end{cases}$

(d) $\lim\limits_{t\to\infty} \frac{dy}{dx} = \lim\limits_{t\to\infty} \frac{2 + \frac{1}{t^2}}{1 - \frac{1}{t^2}} = 2$

– or –

Since $x(t) \to \infty$ as $t \to \infty$, the slope of the line is

$\lim\limits_{t\to\infty} \frac{y(t)}{x(t)} = \lim\limits_{t\to\infty} \frac{2t - \frac{1}{t} + 5}{t + \frac{1}{t}} = 2$

$2 \begin{cases} 1: & \text{considers limit of } \frac{dy}{dx} \text{ or } \frac{y(t)}{x(t)} \\ 1: & \text{answer} \end{cases}$

Note: 0/2 if no consideration of limit