3. The depth from the surface of Earth to a refracting layer beneath the surface can be estimated using methods developed by seismologists. One method is based on the time required for vibrations to travel from a distant explosion to a receiving point. The depth measurement (M) is the sum of the true depth (D) and the random measurement error (E). That is, $M = D + E$. The measurement error (E) is assumed to be normally distributed with mean 0 feet and standard deviation 1.5 feet.

   (a) If the true depth at a certain point is 2 feet, what is the probability that the depth measurement will be negative?

   (b) Suppose three independent depth measurements are taken at the point where the true depth is 2 feet. What is the probability that <u>at least one</u> of these measurements will be negative?

   (c) What is the probability that the <u>mean</u> of the three independent depth measurements taken at the point where the true depth is 2 feet will be negative?

4. Patients with heart-attack symptoms arrive at an emergency room either by ambulance or self-transportation provided by themselves, family, or friends. When a patient arrives at the emergency room, the time of arrival is recorded. The time when the patient's diagnostic treatment begins is also recorded.

   An administrator of a large hospital wanted to determine whether the mean wait time (time between arrival and diagnostic treatment) for patients with heart-attack symptoms differs according to the mode of transportation. A random sample of 150 patients with heart-attack symptoms who had reported to the emergency room was selected. For each patient, the mode of transportation and wait time were recorded. Summary statistics for each mode of transportation are shown in the table below.

Mode of Transportation	Sample Size	Mean Wait Time (in minutes)	Standard Deviation of Wait Times (in minutes)
Ambulance	77	6.04	4.30
Self	73	8.30	5.16

   (a) Use a 99 percent confidence interval to estimate the difference between the mean wait times for ambulance-transported patients and self-transported patients at this emergency room.

   (b) Based only on this confidence interval, do you think the difference in the mean wait times is statistically significant? Justify your answer.

Question 3

Intent of Question

The primary goals of this question are to assess a student's ability to: (1) recognize the random variable of interest, identify its probability distribution, and calculate a probability for a linear combination of a normal random variable and a constant; (2) use basic probability rules to find a different probability; and (3) use the sampling distribution of the sample mean to find a probability about the mean of three observations.

Solution

Part (a):

Since $M = D + E$ (a normal random variable plus a constant is a normal random variable), we know that M is normally distributed with a mean of 2 feet and a standard deviation of 1.5 feet. Thus,

$P(M < 0) = P\left(Z < \frac{0-2}{1.5}\right) < P(Z < -1.33) = 0.0918$, where $Z = \frac{M - \mu}{\sigma}$.

Part (b):

$$\begin{aligned} P(\text{at least one measurement } < 0) &= 1 - P(\text{all three measurements } \geq 0) \\ &= 1 - (1 - 0.0918)^3 \\ &= 1 - (0.9082)^3 \\ &= 1 - 0.7491 \\ &= 0.2509 \end{aligned}$$

Part (c):

Let $\bar{X}$ denote the mean of three independent depth measurements taken at a point where the true depth is 2 feet. Since each measurement comes from a normal distribution, the distribution of $\bar{X}$ is normal with a mean of 2 feet and a standard deviation of $\frac{1.5}{\sqrt{3}} = 0.8660$ feet. Thus,

$P(\bar{X} < 0) = P\left(Z < \frac{0-2}{\frac{1.5}{\sqrt{3}}}\right) < P(Z < -2.31) = 0.0104$, where $Z = \frac{\bar{X} - \mu}{\frac{\sigma}{\sqrt{n}}}$.

Question 3 (continued)

Scoring

Parts (a), (b), and (c) are scored as essentially correct (E), partially correct (P), or incorrect (I).

Part (a) is essentially correct (E) if the student clearly does ALL three of the following:

- identifies the distribution as normal;
- specifies BOTH μ and σ; AND
- calculates the correct probability.

Part (a) is partially correct (P) if the student:

- calculates the correct probability but fails to identify the distribution as normal with BOTH μ and σ specified;

  OR
- correctly identifies the distribution as normal with BOTH μ and σ specified but fails to calculate the correct probability.

Part (a) is incorrect (I) if any of the following occur:

- the student indicates the probability is 0.5 because the random error is symmetric about zero;

  OR
- the student uses a mean of zero and a standard deviation of 1;

  OR
- the student conducts a hypothesis test.

Notes:

- The student may use the distribution of the error, E, to solve the problem. That is, finding the area below -2 for a normal distribution with mean 0 and standard deviation 1.5 should be scored essentially correct (E).

  Thus $P(E < -2) = P\left(Z < \frac{-2-0}{1.5}\right) < P(Z < -1.33) = 0.0918$, where $Z = \frac{E-\mu}{\sigma}$.
- If only the calculator command normalcdf $(-\infty, 0, 2, 1.5)$ is provided along with the probability 0.0912, then the response should be scored as partially correct (P).

Part (b) is essentially correct (E) if the student calculates the correct probability AND:

- correctly applies complement and probability rules using the value obtained in part (a);

  OR
- clearly identifies the distribution as binomial AND specifies BOTH n and p using the value obtained in part (a).

Part (b) is partially correct (P) if the student:

- clearly identifies the distribution as binomial AND specifies BOTH n and p, using the value obtained in part (a), but does not calculate the correct probability;

  OR

Question 3 (continued)

- clearly identifies the distribution as binomial AND specifies BOTH n and p using a value of p that is unrelated to the value obtained in part (a) and calculates the correct probability based on their value of p;

  OR
- calculates the correct probability using the value obtained in part (a) but fails to correctly identify the distribution as binomial with BOTH n and p specified;

  OR
- recognizes the solution as the sum of the product of the probabilities of successes and failures, using the answer from part (a), but omits only the binomial coefficients.

Part (b) is incorrect (I) if the student calculates $P(\text{at least one measurement} < 0) = 1 - p^3$, where p is the solution to part (a).

Notes:

- The solution using the binomial distribution with $p = 0.0918$ is:

$$\begin{aligned} P(\text{at least one measurement} < 0) &= P(B=1) + P(B=2) + P(B=3) \\ &= \binom{3}{1} 0.0918^1 (1-0.0918)^2 + \\ &\quad \binom{3}{2} 0.0918^2 (1-0.0918)^1 + \binom{3}{3} 0.0918^3 \\ &= 0.2272 + 0.0230 + 0.0008 \\ &= 0.2510 \end{aligned}$$

- If only the calculator command $1 - \text{binomcdf}(3, 0.0918, 0)$ is provided along with the probability 0.2509, then the response should be scored as partially correct (P).

Part (c) is essentially correct (E) if the student clearly does ALL three of the following:

- identifies the distribution of the sample mean as normal;
- specifies BOTH $\mu_{\bar{x}}$ and $\sigma_{\bar{x}}$; AND
- calculates the correct probability.

Part (c) is partially correct (P) if the student:

- calculates the correct probability, but fails to identify the distribution of the sample mean as normal with BOTH $\mu_{\bar{x}}$ and $\sigma_{\bar{x}}$ specified;

  OR
- correctly identifies the distribution of the sample mean as normal with BOTH $\mu_{\bar{x}}$ and $\sigma_{\bar{x}}$ specified, but fails to calculate the correct probability.

Part (c) is incorrect (I) if any of the following occur:

- the student uses the same calculation as in part (a);

  OR

Question 3 (continued)

- the student uses an incorrect standard deviation (e.g., $\frac{1.5}{\sqrt{2}}$ or $\sqrt{3(1.5)}$;

  OR
- the student conducts a hypothesis test.

Notes:

- An alternate solution using the sum instead of the mean is:
  Let $\bar{X}$ denote the mean of three independent depth measurements taken at a point where the true depth is 2 feet. Since each measurement comes from a normal distribution, the distribution of the sum of the three measurements, $S = (X_1 + X_2 + X_3)$, is normal with a mean $\mu_S = 6$ feet and a standard deviation $\sigma_S = 2.598$ feet $\left(\sigma_S = 3\left(\frac{1.5}{\sqrt{3}}\right)\right.$, often calculated as $\left.\sqrt{(1.5)^2 + (1.5)^2 + (1.5)^2}\right)$.

  Thus $P(S < 0) = P\left(Z < \frac{0-6}{2.598}\right) = P(Z < -2.31) = 0.0104$, where $Z = \frac{S - \mu_S}{\sigma_S}$.

- If only the calculator command normalcdf $(-\infty, 0, 2, 0.866)$ is provided along with the probability 0.01046, then the response should be scored as partially correct (P).
- If the student does not consistently specify a correct μ and σ from the same distribution, i.e., for the mean or the sum, the response should be scored at most partially correct (P).