5. Consider the differential equation $\dfrac{dy}{dx} = y^2(2x + 2)$. Let $y = f(x)$ be the particular solution to the differential equation with initial condition $f(0) = -1$.

(a) Find $\displaystyle\lim_{x \to 0} \dfrac{f(x) + 1}{\sin x}$. Show the work that leads to your answer.

(b) Use Euler's method, starting at $x = 0$ with two steps of equal size, to approximate $f\left(\dfrac{1}{2}\right)$.

(c) Find $y = f(x)$, the particular solution to the differential equation with initial condition $f(0) = -1$.

GO ON TO THE NEXT PAGE.

6. A function f has derivatives of all orders at $x = 0$. Let $P_n(x)$ denote the nth-degree Taylor polynomial for f about $x = 0$.

   (a) It is known that $f(0) = -4$ and that $P_1\left(\dfrac{1}{2}\right) = -3$. Show that $f'(0) = 2$.

   (b) It is known that $f''(0) = -\dfrac{2}{3}$ and $f'''(0) = \dfrac{1}{3}$. Find $P_3(x)$.

   (c) The function h has first derivative given by $h'(x) = f(2x)$. It is known that $h(0) = 7$. Find the third-degree Taylor polynomial for h about $x = 0$.

STOP

END OF EXAM