

# 2001 AP® CALCULUS BC FREE-RESPONSE QUESTIONS

**CALCULUS BC**  
**SECTION II, Part B**  
**Time—45 minutes**  
**Number of problems—3**

**No calculator is allowed for these problems.**

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4. Let  $h$  be a function defined for all  $x \neq 0$  such that  $h(4) = -3$  and the derivative of  $h$  is given by

$$h'(x) = \frac{x^2 - 2}{x} \text{ for all } x \neq 0.$$

- (a) Find all values of  $x$  for which the graph of  $h$  has a horizontal tangent, and determine whether  $h$  has a local maximum, a local minimum, or neither at each of these values. Justify your answers.  
(b) On what intervals, if any, is the graph of  $h$  concave up? Justify your answer.  
(c) Write an equation for the line tangent to the graph of  $h$  at  $x = 4$ .  
(d) Does the line tangent to the graph of  $h$  at  $x = 4$  lie above or below the graph of  $h$  for  $x > 4$ ? Why?
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5. Let  $f$  be the function satisfying  $f'(x) = -3xf(x)$ , for all real numbers  $x$ , with  $f(1) = 4$  and  $\lim_{x \rightarrow \infty} f(x) = 0$ .

- (a) Evaluate  $\int_1^\infty -3xf(x)dx$ . Show the work that leads to your answer.  
(b) Use Euler's method, starting at  $x = 1$  with a step size of 0.5, to approximate  $f(2)$ .  
(c) Write an expression for  $y = f(x)$  by solving the differential equation  $\frac{dy}{dx} = -3xy$  with the initial condition  $f(1) = 4$ .
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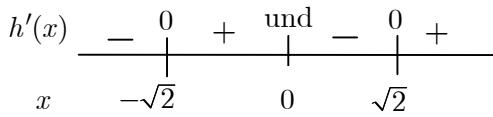
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**Question 4**

Let  $h$  be a function defined for all  $x \neq 0$  such that  $h(4) = -3$  and the derivative of  $h$  is given by  $h'(x) = \frac{x^2 - 2}{x}$  for all  $x \neq 0$ .

- (a) Find all values of  $x$  for which the graph of  $h$  has a horizontal tangent, and determine whether  $h$  has a local maximum, a local minimum, or neither at each of these values. Justify your answers.
- (b) On what intervals, if any, is the graph of  $h$  concave up? Justify your answer.
- (c) Write an equation for the line tangent to the graph of  $h$  at  $x = 4$ .
- (d) Does the line tangent to the graph of  $h$  at  $x = 4$  lie above or below the graph of  $h$  for  $x > 4$ ? Why?

(a)  $h'(x) = 0$  at  $x = \pm\sqrt{2}$



4 :  $\begin{cases} 1 : x = \pm\sqrt{2} \\ 1 : \text{analysis} \\ 2 : \text{conclusions} \\ <-1> \text{not dealing with discontinuity at } 0 \end{cases}$

Local minima at  $x = -\sqrt{2}$  and at  $x = \sqrt{2}$

- (b)  $h''(x) = 1 + \frac{2}{x^2} > 0$  for all  $x \neq 0$ . Therefore, the graph of  $h$  is concave up for all  $x \neq 0$ .

3 :  $\begin{cases} 1 : h''(x) \\ 1 : h''(x) > 0 \\ 1 : \text{answer} \end{cases}$

(c)  $h'(4) = \frac{16 - 2}{4} = \frac{7}{2}$

$$y + 3 = \frac{7}{2}(x - 4)$$

1 : tangent line equation

- (d) The tangent line is below the graph because the graph of  $h$  is concave up for  $x > 4$ .

1 : answer with reason