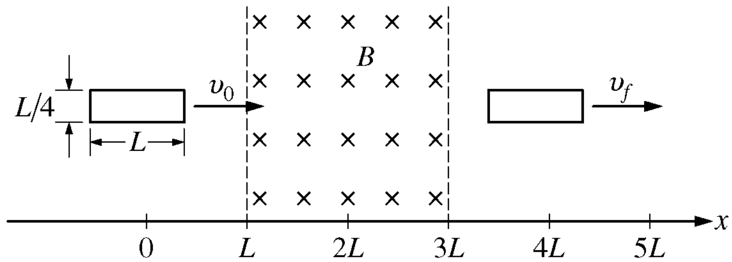

E&M. 2.

The rectangular loop of wire shown on the left in the figure above has mass M, length L, width $L/4$, and resistance R. It is initially moving to the right at constant speed v_0, with no net force acting on it. At time $t = 0$ the loop enters a region of length $2L$ that contains a uniform magnetic field of magnitude B directed into the page. The loop emerges from the field at time t_f with final speed v_f. Express all algebraic answers to the following in terms of M, L, R, B, v_0, and fundamental constants, as appropriate.

(a) Let x represent the position of the right end of the loop. Place a check mark in the appropriate box in each column in the table below to indicate whether the speed of the loop increases, decreases, or stays the same as the loop moves to the right.

	Position of Right End of Loop			
Speed of Loop	$L < x < 2L$	$2L < x < 3L$	$3L < x < 4L$	$4L < x < 5L$
Increases				
Decreases				
Stays the same				

(b) Derive an expression for the magnitude of the current induced in the loop as its right edge enters the field.

(c) What is the direction of the induced current determined in part (b) ?

____ Clockwise ____ Counterclockwise

Justify your answer.

(d) Write, but do not solve, a differential equation for the speed v as a function of time as the loop enters the field.

(e) What is the direction of the acceleration of the loop just before its left edge leaves the field?

____ Left ___ Right ____ Up ____ Down

Justify your answer.

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E&M.3.

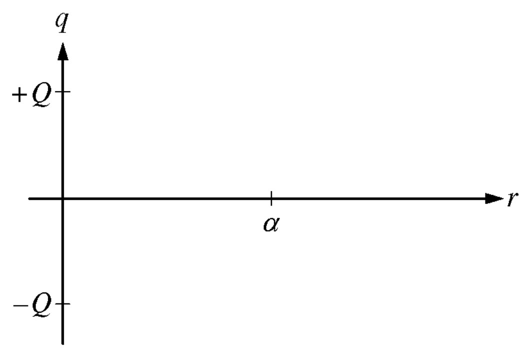
A scientist describes an electrically neutral atom with a model that consists of a nucleus that is a point particle with positive charge $+Q$ at the center of the atom and an electron volume charge density of the form

$$\rho(r) = \begin{cases} -\dfrac{\beta}{r^2} e^{-r/\alpha} & r < \alpha \\ 0 & r > \alpha \end{cases}$$

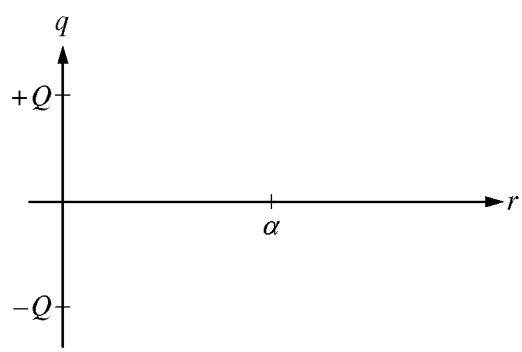
where α and β are positive constants and r is the distance from the center of the atom.

(a) On the axes below, let r stand for the radius of a Gaussian sphere. Sketch the graph for each of the following charges enclosed by the Gaussian sphere as a function of r. Explicitly label any intercepts, asymptotes, maxima, or minima with numerical values or algebraic expressions, as appropriate.

i. The nuclear charge only



ii. The electron charge only



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Question 2

15 points total **Distribution of points**

(a) 2 points

	Position of Right End of Loop			
Speed of Loop	$L < x < 2L$	$2L < x < 3L$	$3L < x < 4L$	$4L < x < 5L$
Increases				
Decreases	✓		✓	
Stays the same		✓		✓

For selecting "Decreases" in the 1st and 3rd columns — 1 point
For selecting "Stays the same" in the 2nd and 4th columns — 1 point

(b) 4 points

For using a correct equation to solve for the emf — 1 point

$$\mathcal{E} = -\frac{d\phi}{dt}$$

For a clear indication that the area is changing — 1 point

$$|\mathcal{E}| = B\frac{dA}{dt} \text{ or } |\mathcal{E}| = B\ell v$$

Note: since the question asks for a magnitude, students are not penalized for excluding the minus sign for the emf.

For relating emf and current — 1 point

$$I = \frac{V}{R} = \frac{\mathcal{E}}{R}$$

For a correct expression for the current — 1 point

$$I = \frac{BLv_0}{4R}$$

(c) 2 points

For selecting "Counterclockwise" — 1 point
For a correct justification — 1 point

Examples:

As the loop enters the magnetic field, more of its area is in a magnetic field directed into the page. According to Lenz' law, this increase in flux will create a current with an opposing magnetic field that will be out of the page. Thus, the current must be counterclockwise.

As the loop enters the magnetic field, the combination of magnetic and electric forces on the charges in the right side of the loop will create a potential difference at the right side with the top being positive. This will cause the current to flow in a counterclockwise direction.

Note: If the wrong choice is selected, then no credit can be earned.

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Question 2 (continued)

		Distribution of points
(d)	4 points	
	For a correct statement of Newton's second law	1 point
	$F = ma$	
	For using a correct expression for the magnetic force	1 point
	$F = I\ell B$	
	For substitution of variables into a correct expression for the net force, including a substitution for I consistent with the answer from part (b)	1 point
	$a = -\dfrac{B\left(\dfrac{BLv}{4R}\right)(L/4)}{m}$	
	$a = -\dfrac{B^2L^2v}{16mR}$	
	For substituting dv/dt for a	1 point
	$\dfrac{dv}{dt} = -\dfrac{B^2L^2v}{16mR}$	
(e)	3 points	
	For selecting "Left"	1 point
	For correctly stating that a clockwise current will be induced in the loop	1 point
	For correctly applying the right hand rule to the current, field, and resulting force on the loop	1 point
	Example: As the loop is leaving the field, the magnetic flux in the loop is decreasing. According to Lenz' law, a clockwise current is induced to oppose the change, which creates a magnetic field into the page. In the left end of the loop the current is up. By the right hand rule, the fingers point up in the direction of the current, the fingers cross into the direction of the magnetic field (into the page), and the thumb points left in the direction of the force.	
	Note: If the wrong choice is selected, then no credit can be earned.	

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