

**2012 AP® CALCULUS AB FREE-RESPONSE QUESTIONS**

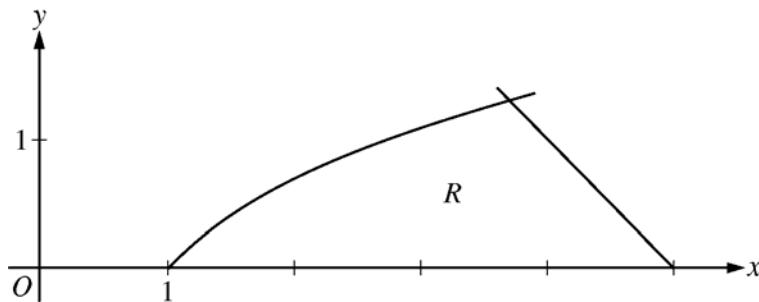
**CALCULUS AB**  
**SECTION II, Part A**  
**Time—30 minutes**  
**Number of problems—2**

**A graphing calculator is required for these problems.**

$t$ (minutes)	0	4	9	15	20
$W(t)$ (degrees Fahrenheit)	55.0	57.1	61.8	67.9	71.0

1. The temperature of water in a tub at time  $t$  is modeled by a strictly increasing, twice-differentiable function  $W$ , where  $W(t)$  is measured in degrees Fahrenheit and  $t$  is measured in minutes. At time  $t = 0$ , the temperature of the water is  $55^{\circ}\text{F}$ . The water is heated for 30 minutes, beginning at time  $t = 0$ . Values of  $W(t)$  at selected times  $t$  for the first 20 minutes are given in the table above.
- (a) Use the data in the table to estimate  $W'(12)$ . Show the computations that lead to your answer. Using correct units, interpret the meaning of your answer in the context of this problem.
- (b) Use the data in the table to evaluate  $\int_0^{20} W'(t) dt$ . Using correct units, interpret the meaning of  $\int_0^{20} W'(t) dt$  in the context of this problem.
- (c) For  $0 \leq t \leq 20$ , the average temperature of the water in the tub is  $\frac{1}{20} \int_0^{20} W(t) dt$ . Use a left Riemann sum with the four subintervals indicated by the data in the table to approximate  $\frac{1}{20} \int_0^{20} W(t) dt$ . Does this approximation overestimate or underestimate the average temperature of the water over these 20 minutes? Explain your reasoning.
- (d) For  $20 \leq t \leq 25$ , the function  $W$  that models the water temperature has first derivative given by  $W'(t) = 0.4\sqrt{t} \cos(0.06t)$ . Based on the model, what is the temperature of the water at time  $t = 25$ ?

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2. Let  $R$  be the region in the first quadrant bounded by the  $x$ -axis and the graphs of  $y = \ln x$  and  $y = 5 - x$ , as shown in the figure above.
- Find the area of  $R$ .
  - Region  $R$  is the base of a solid. For the solid, each cross section perpendicular to the  $x$ -axis is a square. Write, but do not evaluate, an expression involving one or more integrals that gives the volume of the solid.
  - The horizontal line  $y = k$  divides  $R$  into two regions of equal area. Write, but do not solve, an equation involving one or more integrals whose solution gives the value of  $k$ .
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**END OF PART A OF SECTION II**

**AP<sup>®</sup> CALCULUS AB  
2012 SCORING GUIDELINES**

**Question 1**

$t$ (minutes)	0	4	9	15	20
$W(t)$ (degrees Fahrenheit)	55.0	57.1	61.8	67.9	71.0

The temperature of water in a tub at time  $t$  is modeled by a strictly increasing, twice-differentiable function  $W$ , where  $W(t)$  is measured in degrees Fahrenheit and  $t$  is measured in minutes. At time  $t = 0$ , the temperature of the water is 55°F. The water is heated for 30 minutes, beginning at time  $t = 0$ . Values of  $W(t)$  at selected times  $t$  for the first 20 minutes are given in the table above.

- (a) Use the data in the table to estimate  $W'(12)$ . Show the computations that lead to your answer. Using correct units, interpret the meaning of your answer in the context of this problem.
- (b) Use the data in the table to evaluate  $\int_0^{20} W'(t) dt$ . Using correct units, interpret the meaning of  $\int_0^{20} W'(t) dt$  in the context of this problem.
- (c) For  $0 \leq t \leq 20$ , the average temperature of the water in the tub is  $\frac{1}{20} \int_0^{20} W(t) dt$ . Use a left Riemann sum with the four subintervals indicated by the data in the table to approximate  $\frac{1}{20} \int_0^{20} W(t) dt$ . Does this approximation overestimate or underestimate the average temperature of the water over these 20 minutes? Explain your reasoning.
- (d) For  $20 \leq t \leq 25$ , the function  $W$  that models the water temperature has first derivative given by  $W'(t) = 0.4\sqrt{t} \cos(0.06t)$ . Based on the model, what is the temperature of the water at time  $t = 25$ ?

(a) 
$$W'(12) \approx \frac{W(15) - W(9)}{15 - 9} = \frac{67.9 - 61.8}{6} = 1.017 \text{ (or } 1.016\text{)}$$

The water temperature is increasing at a rate of approximately 1.017 °F per minute at time  $t = 12$  minutes.

(b) 
$$\int_0^{20} W'(t) dt = W(20) - W(0) = 71.0 - 55.0 = 16$$

The water has warmed by 16°F over the interval from  $t = 0$  to  $t = 20$  minutes.

(c) 
$$\begin{aligned} \frac{1}{20} \int_0^{20} W(t) dt &\approx \frac{1}{20}(4 \cdot W(0) + 5 \cdot W(4) + 6 \cdot W(9) + 5 \cdot W(15)) \\ &= \frac{1}{20}(4 \cdot 55.0 + 5 \cdot 57.1 + 6 \cdot 61.8 + 5 \cdot 67.9) \\ &= \frac{1}{20} \cdot 1215.8 = 60.79 \end{aligned}$$

This approximation is an underestimate, because a left Riemann sum is used and the function  $W$  is strictly increasing.

(d) 
$$\begin{aligned} W(25) &= 71.0 + \int_{20}^{25} W'(t) dt \\ &= 71.0 + 2.043155 = 73.043 \end{aligned}$$

2 :  $\begin{cases} 1 : \text{estimate} \\ 1 : \text{interpretation with units} \end{cases}$

2 :  $\begin{cases} 1 : \text{value} \\ 1 : \text{interpretation with units} \end{cases}$

3 :  $\begin{cases} 1 : \text{left Riemann sum} \\ 1 : \text{approximation} \\ 1 : \text{underestimate with reason} \end{cases}$

2 :  $\begin{cases} 1 : \text{integral} \\ 1 : \text{answer} \end{cases}$