5. A growing number of employers are trying to hold down the costs that they pay for medical insurance for their employees. As part of this effort, many medical insurance companies are now requiring clients to use generic brand medicines when filling prescriptions. An independent consumer advocacy group wanted to determine if there was a difference, in milligrams, in the amount of active ingredient between a certain "name" brand drug and its generic counterpart. Pharmacies may store drugs under different conditions. Therefore, the consumer group randomly selected ten different pharmacies in a large city and filled two prescriptions at each of these pharmacies, one for the "name" brand and the other for the generic brand of the drug. The consumer group's laboratory then tested a randomly selected pill from each prescription to determine the amount of active ingredient in the pill. The results are given in the following table.

ACTIVE INGREDIENT
(in milligrams)

Pharmacy	1	2	3	4	5	6	7	8	9	10
Name brand	245	244	240	250	243	246	246	246	247	250
Generic brand	246	240	235	237	243	239	241	238	238	234

Based on these results, what should the consumer group's laboratory report about the difference in the active ingredient in the two brands of pills? Give appropriate statistical evidence to support your response.

STATISTICS
SECTION II
Part B
Question 6
Spend about 25 minutes on this part of the exam.
Percent of Section II grade—25

Directions: Show all your work. Indicate clearly the methods you use, because you will be graded on the correctness of your methods as well as on the accuracy of your results and explanation.

6. The statistics department at a large university is trying to determine if it is possible to predict whether an applicant will successfully complete the Ph.D. program or will leave before completing the program. The department is considering whether GPA (grade point average) in undergraduate statistics and mathematics courses (a measure of performance) and mean number of credit hours per semester (a measure of workload) would be helpful measures. To gather data, a random sample of 20 entering students from the past 5 years is taken. The data are given below.

Successfully Completed Ph.D. Program

Student	A	B	C	D	E	F	G	H	I	J	K	L	M
GPA	3.8	3.5	4.0	3.9	2.9	3.5	3.5	4.0	3.9	3.0	3.4	3.7	3.6
Credit hours	12.7	13.1	12.5	13.0	15.0	14.7	14.5	12.0	13.1	15.3	14.6	12.5	14.0

Did Not Complete Ph.D. Program

Student	N	O	P	Q	R	S	T
GPA	3.6	2.9	3.1	3.5	3.9	3.6	3.3
Credit hours	11.1	14.5	14.0	10.9	11.5	12.1	12.0

The regression output at the top of the next page resulted from fitting a line to the data in each group. The residual plots (not shown) indicated no unusual patterns, and the assumptions necessary for inference were judged to be reasonable.

GO ON TO THE NEXT PAGE.

Question 5 - Solution

Part 1: States a correct pair of hypotheses

μ_G = mean amount of active ingredient for generic
μ_B = mean amount of active ingredient for name brand
μ_D = mean difference in amount of active ingredient

$H_o: \mu_D = 0$ OR $H_o: \mu_G - \mu_B = 0$ OR $H_o: \mu_B - \mu_G = 0$
$H_a: \mu_D \neq 0$ $H_a: \mu_G - \mu_B \neq 0$ $H_a: \mu_B - \mu_G \neq 0$

Part 2: Identifies correct test by name or by formula and checks appropriate assumptions.

Paired t test

$$t = \frac{\bar{x}_D - 0}{\frac{s_D}{\sqrt{n_D}}}$$

Differences: 1, -4, -5, -13, 0, -7, -5, -8, -9, -16

Assumptions: (1) random samples (given) and (2) the population of differences is approximately normal.

Stem and Leaf of differences

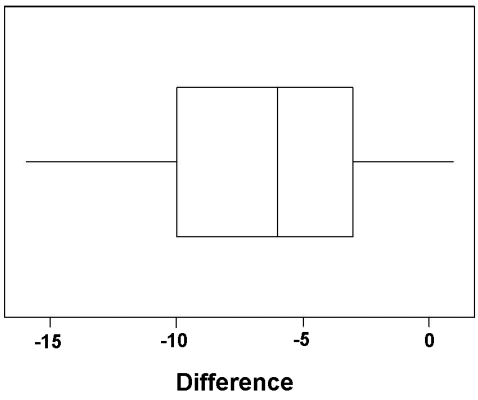
-1 | 6
-1 | 3
-0 | 9 8 7 5 5
-0 | 4
 0 | 0 1

Question 5 (cont'd.)

It is reasonable to assume that the population of differences is approximately normal since the stem-and-leaf plot is roughly symmetric with no apparent outliers.

OR

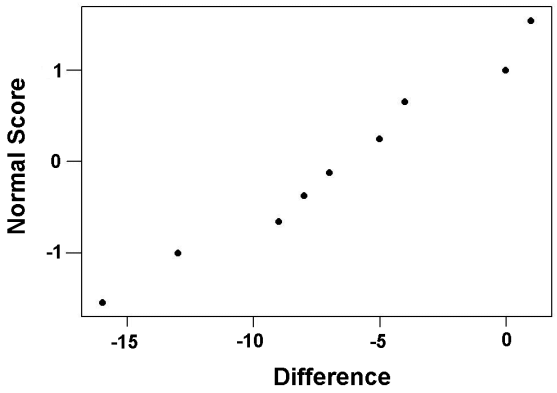
Box Plot of differences



The boxplot is roughly symmetric and shows no outliers. So, it is not unreasonable to assume that the distribution of differences is approximately normal.

OR

Normal probability plot of differences

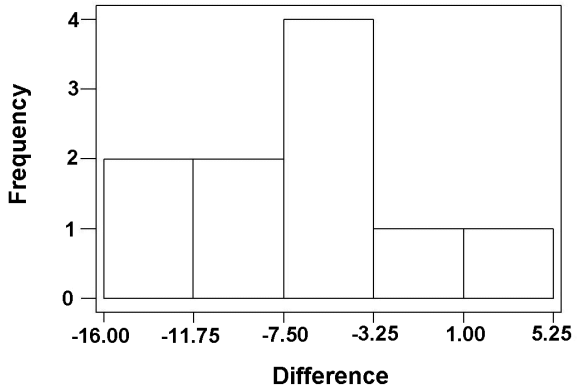


Normal probability plot is reasonably straight, so it is reasonable to assume that the population of differences is approximately normal.

OR

Histogram of differences (using the same intervals as the calculator)

Question 5 (cont'd.)



It is not unreasonable to assume that the population of differences is approximately normal since the histogram is roughly symmetric (especially considering the small sample size).

Note:

- It is acceptable for a student to comment, based on an appropriate graph, that there are no apparent outliers or extreme skewness, without having to mention normality.
- The student should recognize that the assumptions are about the difference distribution.
- If they look at plots of the two samples individually, they must comment on the fact that if it is reasonable to assume that the two population distributions are each approximately normal, then the distribution of the differences will also be approximately normal.

Part 3: Correct mechanics, including value of the test statistic, df, and P-value (or rejection region).

$\overline{x_D} = -6.6$ $\quad s_D = 5.27$

$$t = \frac{-6.6 - 0}{\frac{5.27}{\sqrt{10}}} = \frac{-6.6}{1.67} = -3.96$$ $\quad df = n_D - 1 = 9$ $\quad$ P-value = .00332

(Calculator: t = -3.956835797, df = 9, P-value = .0033201462)
(OR rejection region: $\alpha = .05$, t critical value = $\pm$ 2.262
$\alpha = .01$, t critical value = $\pm$ 3.250) (OR 95% C.I. : (-10.37 , -2.827))

Note: If the differences are formed using Name Brand – Generic, the differences, the mean of the differences, and the value of the test statistic will be opposite in sign, but the conclusion will be the same.

Question 5 (cont'd.)

Part 4: Stating a correct conclusion in the context of the problem and making a clear statement of linkage to the results of the statistical test.

Because the P-value is so small (or because P-value < α, or because t is in the rejection region, or because 0 is not contained in the confidence interval), reject H_o. There is evidence that the mean amount of active ingredient is not the same for the name brand and generic drugs.

The consumer group should report that the mean amount of active ingredient is not the same for the name brand and generic drugs.

Note: It is OK if the student says that the consumer group should report that the mean amount of active ingredient is lower for the generic drug **as long as it follows a two-sided conclusion**.

Notes for an incorrect test for means

2 sample t test (unequal variances): t = -4.438 ; df = 17.46; p =.0003391;
95% C.I. (-9.7309,-3.4691)
2 sample t test (equal variances): t = -4.439 ; df= 18; p = .0003172 ;
95% C.I. (-9.724,-3.476)
2 sample t test (conservative approach): t =-4.438 ; df= 9; p =.001627;
95% C.I. (-9.963,-3.236)
2 sample z test: z= -4.44 ; p=.000009 ; 95% C.I. (-9.512,-3.687)

1 sample z test on differences: z = -4.17 ; p = .00003; 95% C.I. (-9.866, -3.334)

Note: A student cannot receive credit for using a Chi-Square test or a test for proportions.

Question 5 (cont'd.)

Scoring for matched pairs t test

Each part of the hypothesis test is either completely correct or incorrect.

4 **Complete Response**
All four parts correct.

3 **Substantial Response**
Three parts correct.

2 **Developing Response**
Two parts correct.

1 **Minimal Response**
One part correct.

Scoring for an incorrect test for means

Each part of the hypothesis test is either completely correct or incorrect.

4 **Complete Response**
Not possible

3 **Substantial Response**
All four parts correct

2 **Developing Response**
Three parts correct

1 **Minimal Response**
Two parts correct