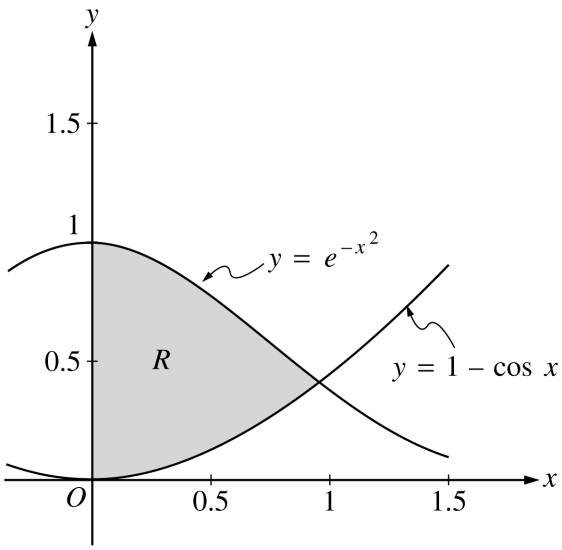
CALCULUS BC

SECTION II, Part A

Time—45 minutes

Number of problems—3

A graphing calculator is required for some problems or parts of problems.



1. Let R be the shaded region in the first quadrant enclosed by the graphs of $y = e^{-x^2}$, $y = 1 - \cos x$, and the y-axis, as shown in the figure above.

   (a) Find the area of the region R.

   (b) Find the volume of the solid generated when the region R is revolved about the x-axis.

   (c) The region R is the base of a solid. For this solid, each cross section perpendicular to the x-axis is a square. Find the volume of this solid.

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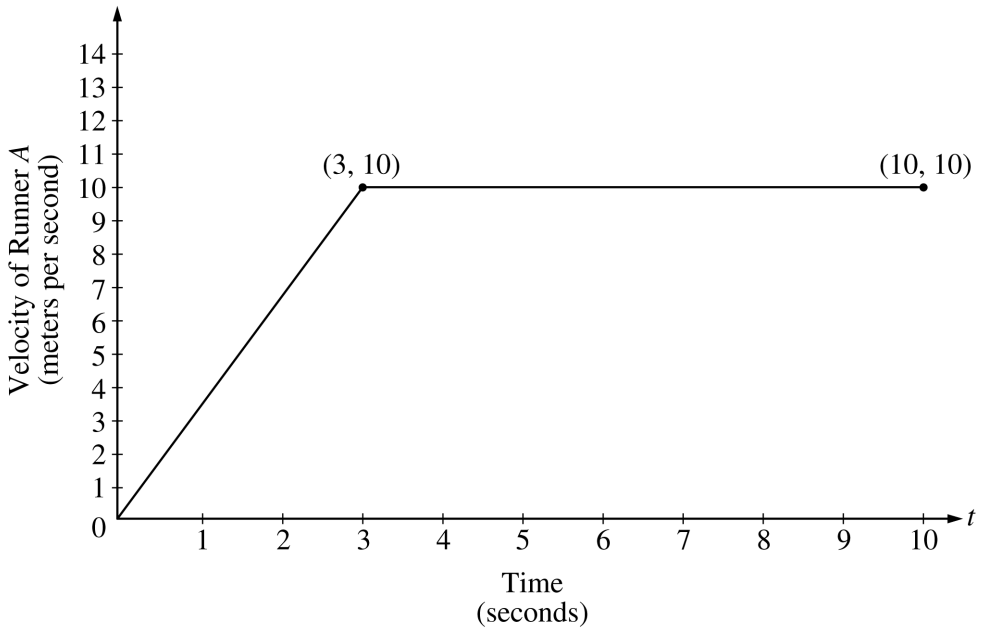

2. Two runners, A and B, run on a straight racetrack for $0 \le t \le 10$ seconds. The graph above, which consists of two line segments, shows the velocity, in meters per second, of Runner A. The velocity, in meters per second, of Runner B is given by the function v defined by $v(t) = \dfrac{24t}{2t + 3}$.

   (a) Find the velocity of Runner A and the velocity of Runner B at time $t = 2$ seconds. Indicate units of measure.

   (b) Find the acceleration of Runner A and the acceleration of Runner B at time $t = 2$ seconds. Indicate units of measure.

   (c) Find the total distance run by Runner A and the total distance run by Runner B over the time interval $0 \le t \le 10$ seconds. Indicate units of measure.

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3. The Taylor series about $x = 5$ for a certain function f converges to $f(x)$ for all x in the interval of convergence. The nth derivative of f at $x = 5$ is given by $f^{(n)}(5) = \dfrac{(-1)^n n!}{2^n (n + 2)}$, and $f(5) = \dfrac{1}{2}$.

   (a) Write the third-degree Taylor polynomial for f about $x = 5$.

   (b) Find the radius of convergence of the Taylor series for f about $x = 5$.

   (c) Show that the sixth-degree Taylor polynomial for f about $x = 5$ approximates $f(6)$ with error less than $\dfrac{1}{1000}$.

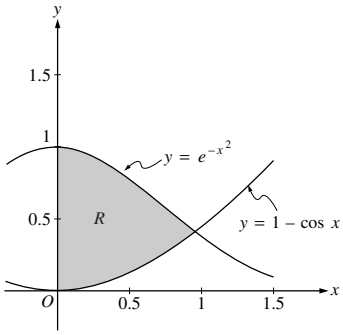
END OF PART A OF SECTION II

Let R be the shaded region in the first quadrant enclosed by the graphs of $y = e^{-x^2}$, $y = 1 - \cos x$, and the y-axis, as shown in the figure above.

(a) Find the area of the region R.

(b) Find the volume of the solid generated when the region R is revolved about the x-axis.

(c) The region R is the base of a solid. For this solid, each cross section perpendicular to the x-axis is a square. Find the volume of this solid.



Region R

$e^{-x^2} = 1 - \cos x$ at $x = 0.941944 = A$

(a) Area $= \int_0^A (e^{-x^2} - (1 - \cos x))\,dx$

$= 0.590$ or 0.591

2 { 1 : integrand; 1 : answer }

(b) Volume $= \pi \int_0^A \left(\left(e^{-x^2} \right)^2 - (1 - \cos x)^2 \right) dx$

$= 0.55596\pi = 1.746$ or 1.747

3 { 2 : integrand and constant; $< -1 >$ each error; 1 : answer }

(c) Volume $= \int_0^A \left(e^{-x^2} - (1 - \cos x) \right)^2 dx$

$= 0.461$

3 { 2 : integrand; $< -1 >$ each error; Note: 0/2 if not of the form $k \int_c^d (f(x) - g(x))^2\,dx$; 1 : answer }

1 : Correct limits in an integral in (a), (b), or (c).