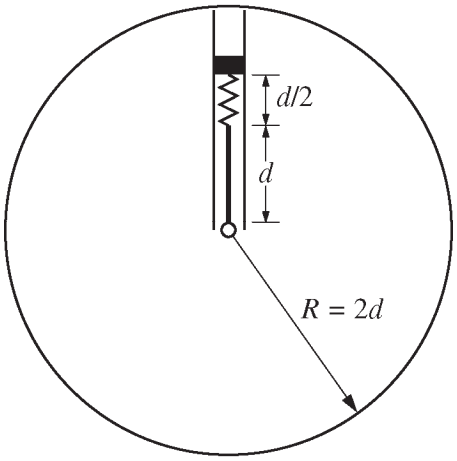



Figure 1

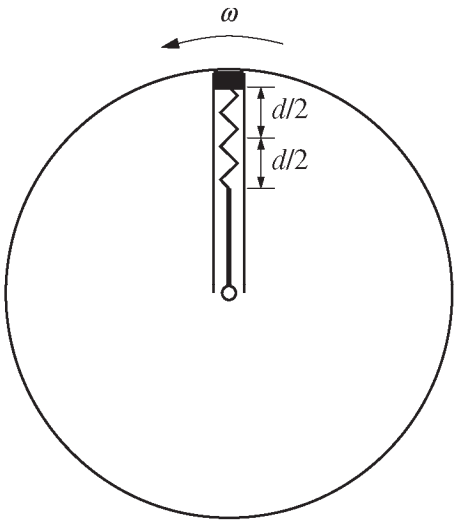



Figure 2

Mech.3.

A uniform rod of length $d$ has one end fixed to the central axis of a horizontal, frictionless circular platform of radius $R = 2d$. Fixed at the other end of the rod is an ideal spring of negligible mass to which a block is attached. The block is set in frictionless grooves so that it can only move along a radius of the platform, as shown in Figure 1 above. The equilibrium length of the spring is $d/2$. Below is a table showing the mass of the block and the masses and rotational inertias of the rod and platform.

| | Mass | Rotational Inertia |
|---|---|---|
| Block | $m$ | |
| Rod | $m_R = 3m$ | $\frac{m_R d^2}{3}$ (about the end of the rod) |
| Platform | $m_P = 5m$ | $\frac{m_P R^2}{2}$ (about the central axis) |

A motor begins to slowly rotate the platform counterclockwise as viewed from above until the platform reaches a constant angular speed $\omega$. Under these conditions, the spring has stretched by an additional length $d/2$, as shown in Figure 2.

Answer the following questions for the platform rotating at constant angular speed $\omega$. Express all algebraic answers in terms of $m$, $d$, $\omega$, and physical constants, as appropriate.

(a) Derive an expression for the spring constant of the spring.

(b)

i. Determine an expression for the rotational inertia of the block around the axis of the platform.

ii. Derive an expression for the rotational inertia of the entire system about the axis of the platform.

(c) Determine an expression for the angular momentum of the entire system about the axis of the platform.

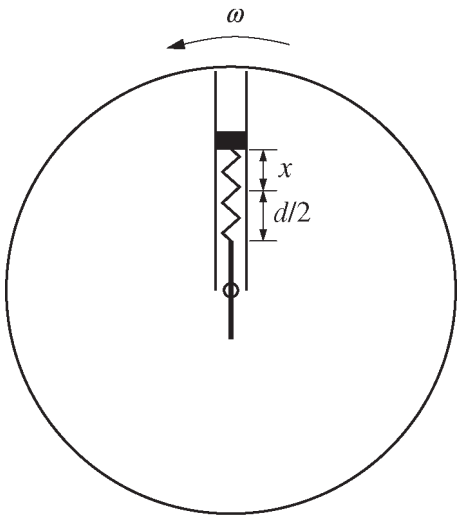



Figure 3

While the system continues to rotate, a small mechanism in the pivot moves the rod slowly until the center of the rod is positioned on the axis, as shown in Figure 3 above. The same constant angular speed $\omega$ is maintained by the motor driving the platform.

(d) Derive an expression for the distance $x$ that the spring is stretched when the rod reaches the position shown in Figure 3 above.

For parts (e), (f), and (g), assume the center of the rod is still moving toward the axis of the platform.

(e) Is the angular momentum of the entire system increasing, decreasing, or staying the same?

_____ Increasing _____ Decreasing _____ Staying the same

Justify your answer.

(f) In order to keep the system rotating with constant angular speed $\omega$, is the motor doing positive work, negative work, or no work on the rotating system?

_____ Positive _____ Negative _____ No work

Justify your answer.

(g) On the block in Figure 4 below, draw a single vector representing the direction of the acceleration of the block. Draw the vector so that it is starting on, and pointing away from, the block.

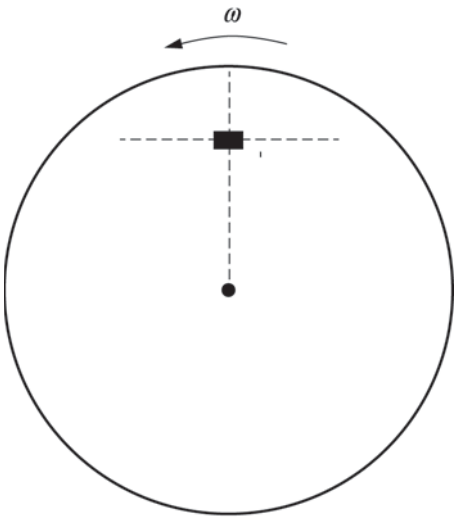



Figure 4

**STOP**

**END OF EXAM**

# AP® PHYSICS C: MECHANICS
# 2016 SCORING GUIDELINES

## Question 3

**15 points total** **Distribution of points**

(a) 3 points

For an indication that the spring force equals the mass times the centripetal acceleration — 1 point

$F_S = F_C = ma_C$

$kx = mr\omega^2$

For a correct substitution for $x$ in the equation — 1 point

For a correct substitution for $r$ in the equation that can be used to solve for $k$ — 1 point

$k\left(\frac{d}{2}\right) = m(2d)\omega^2$

Correct answer:

$k = 4m\omega^2$

(b)

i. 1 point

Substitute values for the block into the equation for rotational inertia

$I = \sum mr^2 = m(2d)^2$

For a correct answer or an answer consistent with the radius used in part (a) — 1 point

$I = 4md^2$

ii. 2 points

For an indication that the rotational inertia of the system must include the platform, the rod, and the object — 1 point

$I = I_P + I_R + I_O$

$I = \frac{m_P R^2}{2} + \frac{m_R d^2}{3} + mr^2$

For correctly substituting into the equation the rotational inertia of the platform, the rod, and the object consistent with (b) i. — 1 point

$I = \frac{5m(2d)^2}{2} + \frac{3m\,d^2}{3} + 4md^2$

$I = (10 + 1 + 4)md^2$

Correct answer:

$I = 15md^2$

(c) 1 point

Using a correct expression of angular momentum

$L = I\omega$

For an answer consistent with the answer from part (b) ii. — 1 point

$L = 15md^2\omega$

## Question 3 (continued)

**Distribution of points**

(d) 3 points

For indicating that the spring force equals the mass times the centripetal acceleration — 1 point

$F_S = F_C = ma_C$

$kx = mr\omega^2$

For a correct substitution for $k$, or a substitution consistent with part (a), in the equation — 1 point

For a correct substitution for $r$ in the equation that can be used to solve for $x$ — 1 point

$\left(4m\omega^2\right)x = m(d + x)\omega^2$

$4x = d + x$

$3x = d$

Correct answer:

$x = d/3$

(e) 2 points

For selecting "Decreasing" — 1 point

If the wrong selection is made, no points are earned for the justification.

For a correct justification — 1 point

Example: The rotational inertia $I$ of the system decreases while the angular velocity $w$ stays the same. Because the angular momentum is $Iw$, it decreases.

(f) 2 points

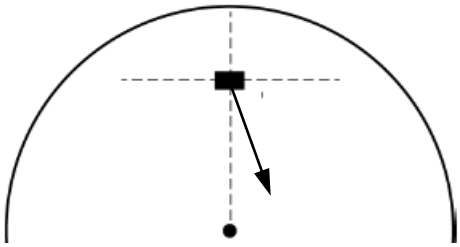
For selecting "Negative" — 1 point

If the wrong selection is made, no points are earned for the justification.

For a correct justification — 1 point

Example: Because the angular velocity is constant and the rotational inertia has decreased, the rotational kinetic energy has decreased. Therefore, work must be negative.

(g) 1 point



For an arrow starting on the box and pointing down and to the right — 1 point