

# 2002 AP® CALCULUS AB FREE-RESPONSE QUESTIONS

**CALCULUS AB**  
**SECTION II, Part A**  
**Time—45 minutes**  
**Number of problems—3**

**A graphing calculator is required for some problems or parts of problems.**

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1. Let  $f$  and  $g$  be the functions given by  $f(x) = e^x$  and  $g(x) = \ln x$ .
    - (a) Find the area of the region enclosed by the graphs of  $f$  and  $g$  between  $x = \frac{1}{2}$  and  $x = 1$ .
    - (b) Find the volume of the solid generated when the region enclosed by the graphs of  $f$  and  $g$  between  $x = \frac{1}{2}$  and  $x = 1$  is revolved about the line  $y = 4$ .
    - (c) Let  $h$  be the function given by  $h(x) = f(x) - g(x)$ . Find the absolute minimum value of  $h(x)$  on the closed interval  $\frac{1}{2} \leq x \leq 1$ , and find the absolute maximum value of  $h(x)$  on the closed interval  $\frac{1}{2} \leq x \leq 1$ . Show the analysis that leads to your answers.
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2. The rate at which people enter an amusement park on a given day is modeled by the function  $E$  defined by

$$E(t) = \frac{15600}{(t^2 - 24t + 160)}.$$

The rate at which people leave the same amusement park on the same day is modeled by the function  $L$  defined by

$$L(t) = \frac{9890}{(t^2 - 38t + 370)}.$$

Both  $E(t)$  and  $L(t)$  are measured in people per hour and time  $t$  is measured in hours after midnight. These functions are valid for  $9 \leq t \leq 23$ , the hours during which the park is open. At time  $t = 9$ , there are no people in the park.

- (a) How many people have entered the park by 5:00 P.M. ( $t = 17$ )? Round your answer to the nearest whole number.
- (b) The price of admission to the park is \$15 until 5:00 P.M. ( $t = 17$ ). After 5:00 P.M., the price of admission to the park is \$11. How many dollars are collected from admissions to the park on the given day? Round your answer to the nearest whole number.
- (c) Let  $H(t) = \int_9^t (E(x) - L(x)) dx$  for  $9 \leq t \leq 23$ . The value of  $H(17)$  to the nearest whole number is 3725. Find the value of  $H'(17)$ , and explain the meaning of  $H(17)$  and  $H'(17)$  in the context of the amusement park.
- (d) At what time  $t$ , for  $9 \leq t \leq 23$ , does the model predict that the number of people in the park is a maximum?