## CALCULUS BC
## SECTION II, Part B
**Time—45 minutes**
**Number of problems—3**

**No calculator is allowed for these problems.**

4. Consider the differential equation $\frac{dy}{dx} = 6x^2 - x^2y$. Let $y = f(x)$ be a particular solution to this differential equation with the initial condition $f(-1) = 2$.

   (a) Use Euler's method with two steps of equal size, starting at $x = -1$, to approximate $f(0)$. Show the work that leads to your answer.

   (b) At the point $(-1, 2)$, the value of $\frac{d^2y}{dx^2}$ is $-12$. Find the second-degree Taylor polynomial for $f$ about $x = -1$.

   (c) Find the particular solution $y = f(x)$ to the given differential equation with the initial condition $f(-1) = 2$.

---

**WRITE ALL WORK IN THE PINK EXAM BOOKLET.**

| $x$ | 2 | 3 | 5 | 8 | 13 |
|---|---|---|---|---|---|
| $f(x)$ | 1 | 4 | –2 | 3 | 6 |

5. Let $f$ be a function that is twice differentiable for all real numbers. The table above gives values of $f$ for selected points in the closed interval $2 \le x \le 13$.

   (a) Estimate $f'(4)$. Show the work that leads to your answer.

   (b) Evaluate $\int_2^{13} (3 - 5f'(x))\,dx$. Show the work that leads to your answer.

   (c) Use a left Riemann sum with subintervals indicated by the data in the table to approximate $\int_2^{13} f(x)\,dx$. Show the work that leads to your answer.

   (d) Suppose $f'(5) = 3$ and $f''(x) < 0$ for all $x$ in the closed interval $5 \le x \le 8$. Use the line tangent to the graph of $f$ at $x = 5$ to show that $f(7) \le 4$. Use the secant line for the graph of $f$ on $5 \le x \le 8$ to show that $f(7) \ge \frac{4}{3}$.

---

6. The Maclaurin series for $e^x$ is $e^x = 1 + x + \frac{x^2}{2} + \frac{x^3}{6} + \cdots + \frac{x^n}{n!} + \cdots$. The continuous function $f$ is defined by $f(x) = \dfrac{e^{(x-1)^2} - 1}{(x-1)^2}$ for $x \ne 1$ and $f(1) = 1$. The function $f$ has derivatives of all orders at $x = 1$.

   (a) Write the first four nonzero terms and the general term of the Taylor series for $e^{(x-1)^2}$ about $x = 1$.

   (b) Use the Taylor series found in part (a) to write the first four nonzero terms and the general term of the Taylor series for $f$ about $x = 1$.

   (c) Use the ratio test to find the interval of convergence for the Taylor series found in part (b).

   (d) Use the Taylor series for $f$ about $x = 1$ to determine whether the graph of $f$ has any points of inflection.

---

**WRITE ALL WORK IN THE PINK EXAM BOOKLET.**

**END OF EXAM**

# AP® CALCULUS BC
# 2009 SCORING GUIDELINES

## Question 4

Consider the differential equation $\frac{dy}{dx} = 6x^2 - x^2y$. Let $y = f(x)$ be a particular solution to this differential equation with the initial condition $f(-1) = 2$.

(a) Use Euler's method with two steps of equal size, starting at $x = -1$, to approximate $f(0)$. Show the work that leads to your answer.

(b) At the point $(-1, 2)$, the value of $\frac{d^2y}{dx^2}$ is $-12$. Find the second-degree Taylor polynomial for $f$ about $x = -1$.

(c) Find the particular solution $y = f(x)$ to the given differential equation with the initial condition $f(-1) = 2$.

---

(a) $$f\left(-\frac{1}{2}\right) \approx f(-1) + \left(\left.\frac{dy}{dx}\right|_{(-1,\,2)}\right) \cdot \Delta x$$
$$= 2 + 4 \cdot \frac{1}{2} = 4$$

$$f(0) \approx f\left(-\frac{1}{2}\right) + \left(\left.\frac{dy}{dx}\right|_{\left(-\frac{1}{2},\,4\right)}\right) \cdot \Delta x$$
$$\approx 4 + \frac{1}{2} \cdot \frac{1}{2} = \frac{17}{4}$$

$2 : \begin{cases} 1 : \text{Euler's method with two steps} \\ 1 : \text{answer} \end{cases}$

(b) $P_2(x) = 2 + 4(x+1) - 6(x+1)^2$

1 : answer

(c) $$\frac{dy}{dx} = x^2(6 - y)$$
$$\int \frac{1}{6-y}\, dy = \int x^2\, dx$$
$$-\ln|6-y| = \frac{1}{3}x^3 + C$$
$$-\ln 4 = -\frac{1}{3} + C$$
$$C = \frac{1}{3} - \ln 4$$
$$\ln|6-y| = -\frac{1}{3}x^3 - \left(\frac{1}{3} - \ln 4\right)$$
$$|6-y| = 4e^{-\frac{1}{3}\left(x^3+1\right)}$$
$$y = 6 - 4e^{-\frac{1}{3}\left(x^3+1\right)}$$

$6 : \begin{cases} 1 : \text{separation of variables} \\ 2 : \text{antiderivatives} \\ 1 : \text{constant of integration} \\ 1 : \text{uses initial condition} \\ 1 : \text{solves for } y \end{cases}$

Note: max $3/6$ [1-2-0-0-0] if no constant of integration

Note: $0/6$ if no separation of variables