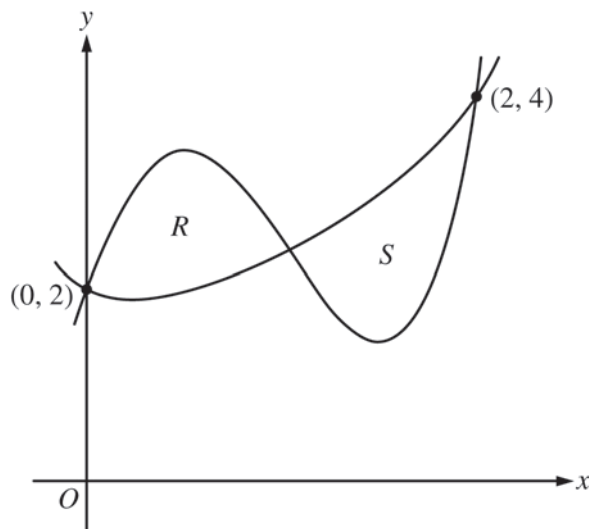


2015 AP<sup>®</sup> CALCULUS AB FREE-RESPONSE QUESTIONS



2. Let  $f$  and  $g$  be the functions defined by  $f(x) = 1 + x + e^{x^2-2x}$  and  $g(x) = x^4 - 6.5x^2 + 6x + 2$ . Let  $R$  and  $S$  be the two regions enclosed by the graphs of  $f$  and  $g$  shown in the figure above.
- Find the sum of the areas of regions  $R$  and  $S$ .
  - Region  $S$  is the base of a solid whose cross sections perpendicular to the  $x$ -axis are squares. Find the volume of the solid.
  - Let  $h$  be the vertical distance between the graphs of  $f$  and  $g$  in region  $S$ . Find the rate at which  $h$  changes with respect to  $x$  when  $x = 1.8$ .
- 

END OF PART A OF SECTION II

**2015 AP<sup>®</sup> CALCULUS AB FREE-RESPONSE QUESTIONS****CALCULUS AB  
SECTION II, Part B  
Time—60 minutes  
Number of problems—4**

No calculator is allowed for these problems.

|                               |   |     |     |      |     |
|-------------------------------|---|-----|-----|------|-----|
| $t$<br>(minutes)              | 0 | 12  | 20  | 24   | 40  |
| $v(t)$<br>(meters per minute) | 0 | 200 | 240 | −220 | 150 |

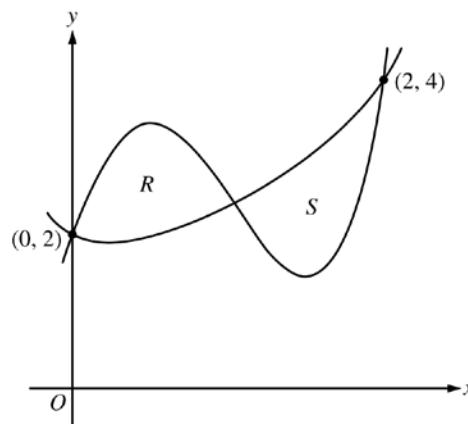
3. Johanna jogs along a straight path. For  $0 \leq t \leq 40$ , Johanna's velocity is given by a differentiable function  $v$ . Selected values of  $v(t)$ , where  $t$  is measured in minutes and  $v(t)$  is measured in meters per minute, are given in the table above.
- (a) Use the data in the table to estimate the value of  $v'(16)$ .
- (b) Using correct units, explain the meaning of the definite integral  $\int_0^{40} |v(t)| dt$  in the context of the problem.
- Approximate the value of  $\int_0^{40} |v(t)| dt$  using a right Riemann sum with the four subintervals indicated in the table.
- (c) Bob is riding his bicycle along the same path. For  $0 \leq t \leq 10$ , Bob's velocity is modeled by  $B(t) = t^3 - 6t^2 + 300$ , where  $t$  is measured in minutes and  $B(t)$  is measured in meters per minute. Find Bob's acceleration at time  $t = 5$ .
- (d) Based on the model  $B$  from part (c), find Bob's average velocity during the interval  $0 \leq t \leq 10$ .
-

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**2015 SCORING GUIDELINES**

**Question 2**

Let  $f$  and  $g$  be the functions defined by  $f(x) = 1 + x + e^{x^2 - 2x}$  and  $g(x) = x^4 - 6.5x^2 + 6x + 2$ . Let  $R$  and  $S$  be the two regions enclosed by the graphs of  $f$  and  $g$  shown in the figure above.

- (a) Find the sum of the areas of regions  $R$  and  $S$ .
- (b) Region  $S$  is the base of a solid whose cross sections perpendicular to the  $x$ -axis are squares. Find the volume of the solid.
- (c) Let  $h$  be the vertical distance between the graphs of  $f$  and  $g$  in region  $S$ . Find the rate at which  $h$  changes with respect to  $x$  when  $x = 1.8$ .



- (a) The graphs of  $y = f(x)$  and  $y = g(x)$  intersect in the first quadrant at the points  $(0, 2)$ ,  $(2, 4)$ , and  $(A, B) = (1.032832, 2.401108)$ .

$$\begin{aligned} \text{Area} &= \int_0^A [g(x) - f(x)] dx + \int_A^2 [f(x) - g(x)] dx \\ &= 0.997427 + 1.006919 = 2.004 \end{aligned}$$

- (b) Volume  $= \int_A^2 [f(x) - g(x)]^2 dx = 1.283$

- (c)  $h(x) = f(x) - g(x)$   
 $h'(x) = f'(x) - g'(x)$   
 $h'(1.8) = f'(1.8) - g'(1.8) = -3.812$  (or  $-3.811$ )

4 :  $\begin{cases} 1 : \text{limits} \\ 2 : \text{integrands} \\ 1 : \text{answer} \end{cases}$

3 :  $\begin{cases} 2 : \text{integrand} \\ 1 : \text{answer} \end{cases}$

2 :  $\begin{cases} 1 : \text{considers } h' \\ 1 : \text{answer} \end{cases}$