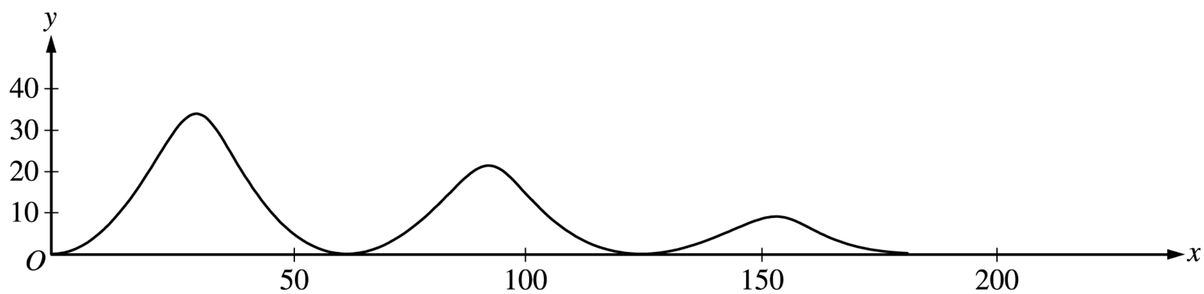


3. The figure above shows the path traveled by a roller coaster car over the time interval $0 \le t \le 18$ seconds. The position of the car at time t seconds can be modeled parametrically by

$$x(t) = 10t + 4\sin t$$
$$y(t) = (20 - t)(1 - \cos t),$$

where x and y are measured in meters. The derivatives of these functions are given by

$$x'(t) = 10 + 4\cos t$$
$$y'(t) = (20 - t)\sin t + \cos t - 1.$$

(a) Find the slope of the path at time $t = 2$. Show the computations that lead to your answer.

(b) Find the acceleration vector of the car at the time when the car's horizontal position is $x = 140$.

(c) Find the time t at which the car is at its maximum height, and find the speed, in m/sec, of the car at this time.

(d) For $0 < t < 18$, there are two times at which the car is at ground level $(y = 0)$. Find these two times and write an expression that gives the average speed, in m/sec, of the car between these two times. Do not evaluate the expression.

END OF PART A OF SECTION II

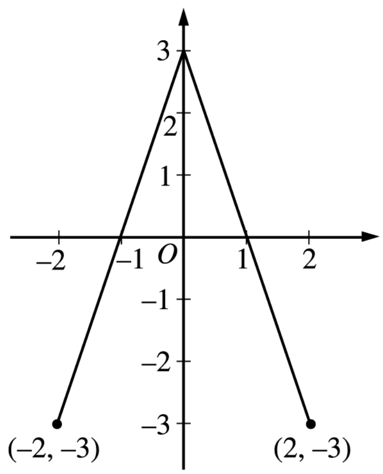
CALCULUS BC

SECTION II, Part B

Time—45 minutes

Number of problems—3

No calculator is allowed for these problems.



Graph of f

4. The graph of the function f shown above consists of two line segments. Let g be the function given by $g(x) = \int_0^x f(t)\,dt$.

   (a) Find $g(-1)$, $g'(-1)$, and $g''(-1)$.

   (b) For what values of x in the open interval $(-2, 2)$ is g increasing? Explain your reasoning.

   (c) For what values of x in the open interval $(-2, 2)$ is the graph of g concave down? Explain your reasoning.

   (d) On the axes provided, sketch the graph of g on the closed interval $[-2, 2]$.
   (Note: The axes are provided in the pink test booklet only.)

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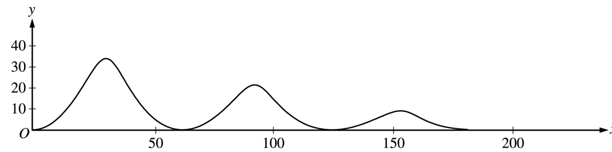
Question 3

The figure above shows the path traveled by a roller coaster car over the time interval $0 \le t \le 18$ seconds. The position of the car at time t seconds can be modeled parametrically by $x(t) = 10t + 4\sin t,\ y(t) = (20 - t)(1 - \cos t)$,



where x and y are measured in meters. The derivatives of these functions are given by

$x'(t) = 10 + 4\cos t,\ y'(t) = (20 - t)\sin t + \cos t - 1.$

(a) Find the slope of the path at time $t = 2$. Show the computations that lead to your answer.

(b) Find the acceleration vector of the car at the time when the car's horizontal position is $x = 140$.

(c) Find the time t at which the car is at its maximum height, and find the speed, in m/sec, of the car at this time.

(d) For $0 < t < 18$, there are two times at which the car is at ground level $(y = 0)$. Find these two times and write an expression that gives the average speed, in m/sec, of the car between these two times. Do not evaluate the expression.

(a) Slope $= \left.\dfrac{dy}{dx}\right|_{t=2} = \dfrac{y'(2)}{x'(2)} = \dfrac{18\sin 2 + \cos 2 - 1}{10 + 4\cos 2}$

$= 1.793$ or 1.794

1 : answer using $\dfrac{dy}{dx} = \dfrac{dy}{dt}\Big/\dfrac{dx}{dt}$

(b) $x(t) = 10t + 4\sin t = 140;\ t_0 = 13.647083$

$x''(t_0) = -3.529,\ y''(t_0) = 1.225$ or 1.226

Acceleration vector is $\langle -3.529, 1.225 \rangle$

or $\langle -3.529, 1.226 \rangle$

2 $\begin{cases} 1: \text{identifies acceleration vector as derivative of velocity vector} \\ 1: \text{computes acceleration vector when } x = 140 \end{cases}$

(c) $y'(t) = (20 - t)\sin t + \cos t - 1 = 0$

$t_1 = 3.023$ or 3.024 at maximum height

Speed $= \sqrt{\left(x'(t_1)\right)^2 + \left(y'(t_1)\right)^2} = \left|x'(t_1)\right|$

$= 6.027$ or 6.028

3 $\begin{cases} 1: \text{sets } y'(t) = 0 \\ 1: \text{selects first } t > 0 \\ 1: \text{speed} \end{cases}$

(d) $y(t) = 0$ when $t = 2\pi$ and $t = 4\pi$

Average speed $= \dfrac{1}{2\pi}\displaystyle\int_{2\pi}^{4\pi} \sqrt{\left(x'(t)\right)^2 + \left(y'(t)\right)^2}\,dt$

$= \dfrac{1}{2\pi}\displaystyle\int_{2\pi}^{4\pi} \sqrt{(10 + 4\cos t)^2 + \left((20 - t)\sin t + \cos t - 1\right)^2}\,dt$

3 $\begin{cases} 1: t = 2\pi, t = 4\pi \\ 1: \text{limits and constant} \\ 1: \text{integrand} \end{cases}$