

2012 AP[®] CALCULUS BC FREE-RESPONSE QUESTIONS

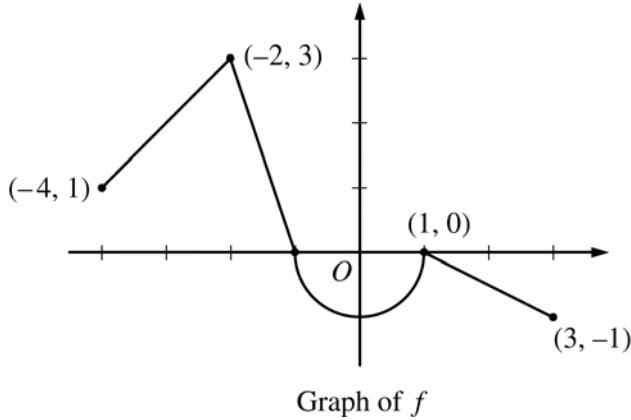
2. For $t \geq 0$, a particle is moving along a curve so that its position at time t is $(x(t), y(t))$. At time $t = 2$, the particle is at position $(1, 5)$. It is known that $\frac{dx}{dt} = \frac{\sqrt{t+2}}{e^t}$ and $\frac{dy}{dt} = \sin^2 t$.
- (a) Is the horizontal movement of the particle to the left or to the right at time $t = 2$? Explain your answer.
Find the slope of the path of the particle at time $t = 2$.
- (b) Find the x -coordinate of the particle's position at time $t = 4$.
- (c) Find the speed of the particle at time $t = 4$. Find the acceleration vector of the particle at time $t = 4$.
- (d) Find the distance traveled by the particle from time $t = 2$ to $t = 4$.
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END OF PART A OF SECTION II

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**CALCULUS BC
SECTION II, Part B
Time—60 minutes
Number of problems—4**

No calculator is allowed for these problems.



Graph of f

3. Let f be the continuous function defined on $[-4, 3]$ whose graph, consisting of three line segments and a semicircle centered at the origin, is given above. Let g be the function given by $g(x) = \int_1^x f(t) dt$.
- Find the values of $g(2)$ and $g(-2)$.
 - For each of $g'(-3)$ and $g''(-3)$, find the value or state that it does not exist.
 - Find the x -coordinate of each point at which the graph of g has a horizontal tangent line. For each of these points, determine whether g has a relative minimum, relative maximum, or neither a minimum nor a maximum at the point. Justify your answers.
 - For $-4 < x < 3$, find all values of x for which the graph of g has a point of inflection. Explain your reasoning.
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Question 2

For $t \geq 0$, a particle is moving along a curve so that its position at time t is $(x(t), y(t))$. At time $t = 2$, the particle is at position $(1, 5)$. It is known that $\frac{dx}{dt} = \frac{\sqrt{t+2}}{e^t}$ and $\frac{dy}{dt} = \sin^2 t$.

- (a) Is the horizontal movement of the particle to the left or to the right at time $t = 2$? Explain your answer.
 Find the slope of the path of the particle at time $t = 2$.
- (b) Find the x -coordinate of the particle's position at time $t = 4$.
- (c) Find the speed of the particle at time $t = 4$. Find the acceleration vector of the particle at time $t = 4$.
- (d) Find the distance traveled by the particle from time $t = 2$ to $t = 4$.

(a) $\left. \frac{dx}{dt} \right|_{t=2} = \frac{2}{e^2}$

Because $\left. \frac{dx}{dt} \right|_{t=2} > 0$, the particle is moving to the right
 at time $t = 2$.

$$\left. \frac{dy}{dx} \right|_{t=2} = \left. \frac{dy/dt}{dx/dt} \right|_{t=2} = 3.055 \text{ (or } 3.054\text{)}$$

(b) $x(4) = 1 + \int_2^4 \frac{\sqrt{t+2}}{e^t} dt = 1.253 \text{ (or } 1.252\text{)}$

(c) Speed $= \sqrt{(x'(4))^2 + (y'(4))^2} = 0.575 \text{ (or } 0.574\text{)}$

$$\begin{aligned} \text{Acceleration} &= \langle x''(4), y''(4) \rangle \\ &= \langle -0.041, 0.989 \rangle \end{aligned}$$

(d) Distance $= \int_2^4 \sqrt{(x'(t))^2 + (y'(t))^2} dt$
 $= 0.651 \text{ (or } 0.650\text{)}$

3 : $\begin{cases} 1 : \text{moving to the right with reason} \\ 1 : \text{considers } \frac{dy/dt}{dx/dt} \\ 1 : \text{slope at } t = 2 \end{cases}$

2 : $\begin{cases} 1 : \text{integral} \\ 1 : \text{answer} \end{cases}$

2 : $\begin{cases} 1 : \text{speed} \\ 1 : \text{acceleration} \end{cases}$

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