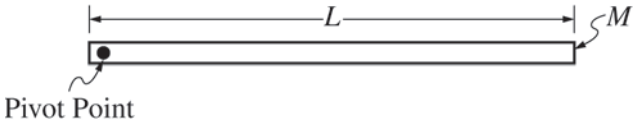
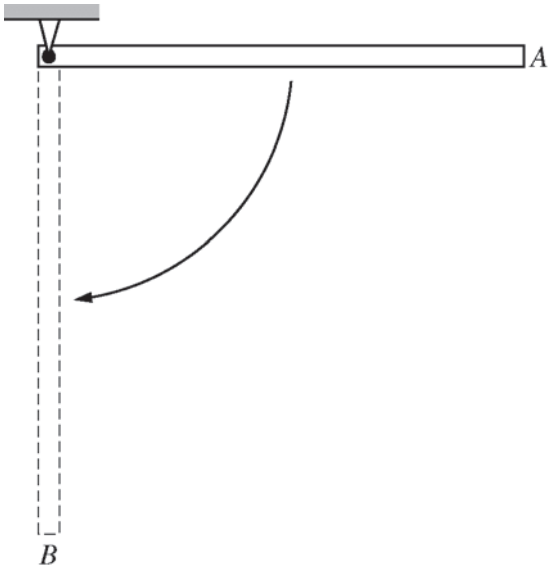



Mech.3.

A uniform, thin rod of length $L$ and mass $M$ is allowed to pivot about its end, as shown in the figure above.

(a) Using integral calculus, derive the rotational inertia for the rod around its end to show that it is $ML^2/3$.




The rod is fixed at one end and allowed to fall from the horizontal position $A$ through the vertical position $B$.

(b) Derive an expression for the velocity of the free end of the rod at position $B$. Express your answer in terms of $M$, $L$, and physical constants, as appropriate.

An experiment is designed to test the validity of the expression found in part (b). A student uses rods of various lengths that all have a uniform mass distribution. The student releases each of the rods from the horizontal position $A$ and uses photogates to measure the velocity of the free end at position $B$. The data are recorded below.

| Length (m) | 0.25 | 0.50 | 0.75 | 1.00 | 1.25 | 1.50 |
|---|---|---|---|---|---|---|
| Velocity (m/s) | 2.7 | 3.8 | 4.6 | 5.2 | 5.8 | 6.3 |
| | | | | | | |
| | | | | | | |

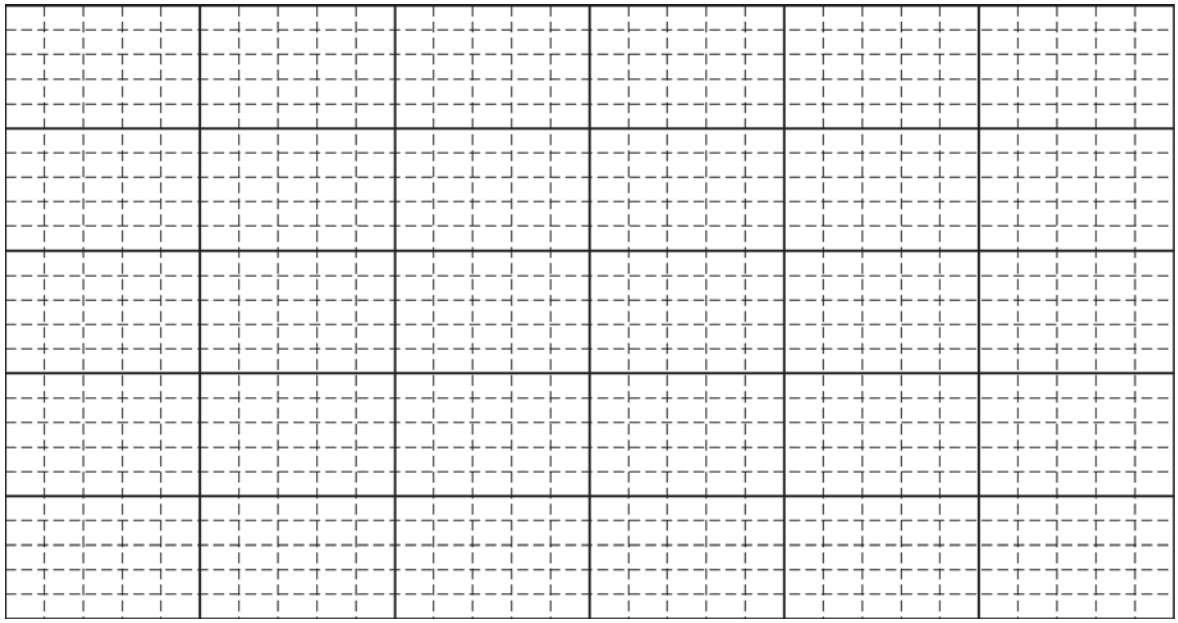
(c) Indicate below which quantities should be graphed to yield a straight line whose slope could be used to calculate a numerical value for the acceleration due to gravity $g$.

Horizontal axis: _____

Vertical axis: _____

Use the remaining rows in the table above, as needed, to record any quantities that you indicated that are not given. Label each row you use and include units.

(d) Plot the straight line data points on the grid below. Clearly scale and label all axes, including units as appropriate. Draw a straight line that best represents the data.



(e)

i. Using your straight line, determine an experimental value for $g$.

ii. Describe two ways in which the effects of air resistance could be reduced.

**STOP**

**END OF EXAM**

# AP® PHYSICS C: MECHANICS
# 2015 SCORING GUIDELINES

## Question 3

**15 points total** **Distribution of points**

(a) 3 points

Writing an integral to derive the rotational inertia of the rod

$I = \int r^2 dm$

For a correct expression for $dm$ 1 point

$\lambda = M/L$, $M = \lambda L$, $dm = \lambda dr$

For using the correct limits of integration or a correct constant of integration 1 point

$I = \int_{r=0}^{r=L} \lambda r^2 dr$

For correctly evaluating the integral above, leading to the answer $ML^2/3$ 1 point

$I = \left[\frac{\lambda r^3}{3}\right]_{r=0}^{r=L} = \frac{1}{3}\lambda\left(L^3 - 0\right) = \frac{1}{3}\left(\frac{M}{L}\right)\left(L^3\right) = \frac{1}{3}ML^2$

(b) 4 points

For using any expression of conservation of energy 1 point

$K_1 + U_{g1} = K_2 + U_{g2}$

For a correct energy expression relating gravitational potential energy to rotational kinetic energy 1 point

$mgh_1 = \frac{1}{2}I\omega_2^2$

For correctly substituting $L/2$ for the change in height 1 point

$Mg(L/2) = \frac{1}{2}\left(\frac{1}{3}ML^2\right)\omega^2$

For using $v = r\omega$ with $r = L$ to solve for the velocity of the end of the rod 1 point

$\frac{MgL}{2} = \frac{1}{6}ML^2\left(\frac{v}{L}\right)^2$

$v = \sqrt{3gL}$

(c) 1 point

For correctly identifying a relationship between length and velocity that will result in a straight line 1 point

Example 1: Horizontal axis: $\text{velocity}$

Vertical axis: $\sqrt{\text{length}}$

Example 2: Horizontal axis: $(\text{velocity})^2$

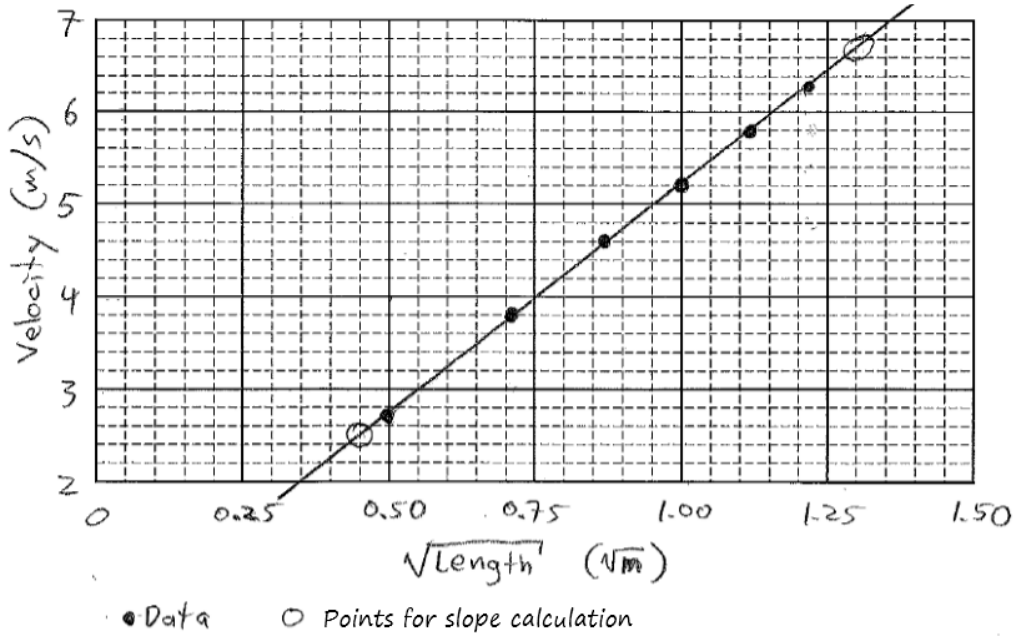
Vertical axis: $\text{length}$

Note: Each of the above axis choices can also be switched to yield a straight line.

**Question 3 (continued)**

**Distribution of points**

(d) 3 points




For a correct scale that uses at least half the grid and for correctly labeling the axes, including units — 1 point

For plotting data consistent with quantities in the data table in part (c) — 1 point

For drawing a straight line consistent with the data in part (c) — 1 point

(e)

i. 2 points

For correctly calculating the slope using the straight line drawn in part (d), and not using data points unless the points lie on the line — 1 point

$$m = \frac{(y_2 - y_1)}{(x_2 - x_1)} = \frac{(6.70 - 2.50)}{(1.30 - 0.45)} = 4.94 \ \sqrt{\text{m}}/\text{s}$$

For correctly calculating $g$ using the slope — 1 point

$$m = \sqrt{3g}$$

$$g = m^2/3 = \left(4.94 \ \sqrt{\text{m}}/\text{s}\right)^2/3 = 8.1 \ \text{m}/\text{s}^2$$

*Alternate Solution* — *Alternate points*

*For stating that linear regression was used and getting one of the results noted below* — *1 point*

*For correctly calculating g using the slope* — *1 point*

*When plotting velocity as a function of* $\sqrt{\text{length}}$ *, the slope is* $4.94 \ \sqrt{\text{m}}/\text{s}$ *and* $g = 8.14 \ \text{m}/\text{s}^2$ .

*When plotting the square of velocity as a function of length, the slope is* $25.77 \ \text{m}/\text{s}^2$ *and* $g = 8.59 \ \text{m}/\text{s}^2$ .

**Question 3 (continued)**

**Distribution of points**

(e) (continued)

ii. 2 points

For one example that directly decreases the effect of air resistance — 1 point
For another example that directly decreases the effect of air resistance — 1 point
Some examples include:
- Do the experiment in a vacuum
- Use shorter rod lengths
- Use more massive (or denser) rods
- Use a more aerodynamic shape for the rods