Section II

Part B

Question 6

Spend about 25 minutes on this part of the exam.

Percent of Section II grade—25

6. The manager of a cultured pearl farm has received a special order for two pearls between 7 millimeters and 9 millimeters in diameter. From past experience, the manager knows that the pearls found in his oyster bed have diameters that are normally distributed with a mean of 8 millimeters and a standard deviation of 2 millimeters. Assume that every oyster contains one pearl.

   The manager wants to know how many oysters he should expect to open to find two pearls of the appropriate size for this special order. Complete the following parts to design a simulation to answer the manager's question.

   (a) Determine the probability of finding a pearl of the appropriate size in an oyster selected at random. (Express this probability as a number between 0 and 1. Round this probability to the nearest tenth.)

   (b) Describe how you would use a table of random digits to carry out a simulation to determine the number of oysters needed to find two pearls of the appropriate size. Include a description of what each of the digits 0, 1, 2, 3, 4, 5, 6, 7, 8, and 9 will represent in your simulation.

   (c) Perform your simulation 3 times. (That is, run 3 trials of you simulation.) Start at the upper left most digit in the first row of the table and move across. Make your procedure clear so that someone can follow what you did. You must do this by marking directly on or above the table.

48747	76595	32588	38392	84422	80016	37890
71950	22494	00369	51269	87073	73694	97751
17857	52352	21392	22930	43776	10503	58249
80993	52010	88856	23882	73613	57648	47051
63016	73572	22684	02409	37565	52457	01257
40615	63910	09596	10241	03413	77576	74872
57431	29251	77848	98037	81230	38561	69580
06181	97842	48327	37976	81333	10264	77769

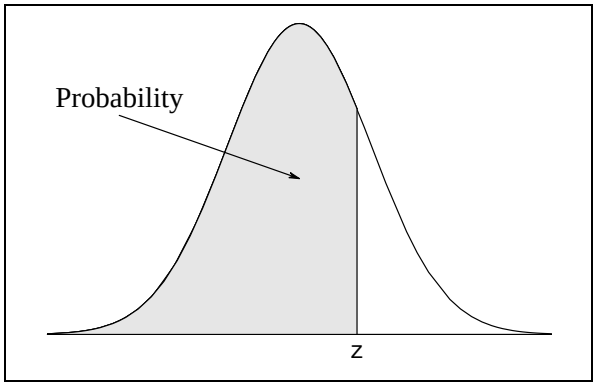

Table entry for z is the probability lying below z

Table A (Continued)

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998

Free-Response Scoring Guidelines: Question 6

4 Complete Response

The response is substantially correct for all three sections, I, II, and III.

3 Substantial Response

The response is substantially correct for two of the three sections, I, II, and III, or the response is substantially correct for one of the three sections and nearly correct for each of the other two.

2 Developing Response

The response is substantially correct for one of the three sections, I, II, and III, and shows an acceptable approach in at least one of the other sections.

1 Minimal Response

The response is substantially correct for one of the three sections, I, II, and III, or shows an acceptable approach in at least one of the three sections.

Solutions and Scoring

Section 1: Finding a probability

(a) Using the normal density function, determines that $P(7 < X < 9) = 0.38$, which rounds to 0.4.

A "substantially correct" response must show some indication of how this value was determined. For example, either

$P(7 < X < 9) = P(-0.5 < Z < 0.5) = 0.6915 - 0.3085 = 0.3830$

or

Normalcdf$(7, 9, 8, 2)$

is fine. If there is no explanation for the 0.38, this is considered only an "acceptable approach."

Other acceptable approaches, but not complete responses, include answers where normal curve calculations or geometry is incorrect. Such acceptable approaches include answers of $1 - 0.3830 = 0.6170$ and 0.1915. Note that the answer $0.5(0.68) = 0.34$ should receive no credit.

Section 2: Setting up and running a simulation

(b) Shows a listing of four of the ten digits defined to be equivalent to finding a pearl of the correct size (a "success") and states that the other digits represent finding no pearl. The value used in (b) must match the value calculated in (a). If no solution is given in (a), a value can be "made up" and the problem completed. It is OK to use a two digit simulation with $p = 0.38$.

Describes a correct method of sampling digits from a random digit table until two successes are found.

To get any credit for Section II, the student must describe a waiting time simulation. A correct assignment of digits in (b) is not sufficient to get any credit if the waiting time component is missing.

(c) Correctly identifies a success, based on the definition in part (b) and counts the number of digits it took to get two successes.

Correctly runs three trials (more is OK) and records the number of digits it took to get two

successes. (Starting points for the trials will vary.)

If the assignment of digits in (b) is incorrect, the response should be considered an "acceptable approach" if a waiting time simulation is carried out correctly in part (c) according to the assignments of digits in part (b).

Section 3: Interpreting results of a simulation

(d) Identifies A as the correct distribution for $P = 0.4$ by noting that the probability of finding a pearl between 4 and 6.5 mm is smaller than the probability of finding a pearl between 7 and 9 mm, and it therefore takes longer to find two pearls between 4 and 6.5 than two pearls between 7 and 9. If the student computes the probability of finding a smaller pearl, this should be considered a bonus. ($P(4 < X < 6.5) = 0.2039$.) If the argue that a pearl between 7 and 9 is more likely because this interval contains the mean or is closer to the mean than $[4, 6.5]$, this is not a major error but is considered a minus. If the student selects Distribution A with no reason or an incorrect reason, they get no credit on part (d). If the student selects a distribution based on how well it matches their simulation, this is an "acceptable approach."

Another correct response is to observe that the mean should be $2(1/0.38) = 5.26$, which is closer to 5 than to 10.

(e) The best estimate of the expected number of pearls is the sample mean of 5.16, for distribution A. This can be arrived at by direct computation using the formula

$$\overline{x} = \frac{\sum x \cdot f}{100} = 5.16$$

by estimating the balance point of the distribution, or by using the formula $2(1/0.38) = 5.26$. [The standard deviation is 2.56 in case anyone uses it.] A computational mistake using the right formula is okay, if the answer is reasonable.

If no indication is given of how the answer was determined, this is an "acceptable approach," but not a "complete response." However, if no work was shown in part (a) as well, don't count off again.

If a student uses the median of 4 or the mode of 3 as the best measure of center with justification, this is OK. Without justification, it is an "acceptable approach."

If the student selects distribution B in part (d), the mean should be 9.36, the median 8, and the mode 7 (and the standard deviation is 5.27).