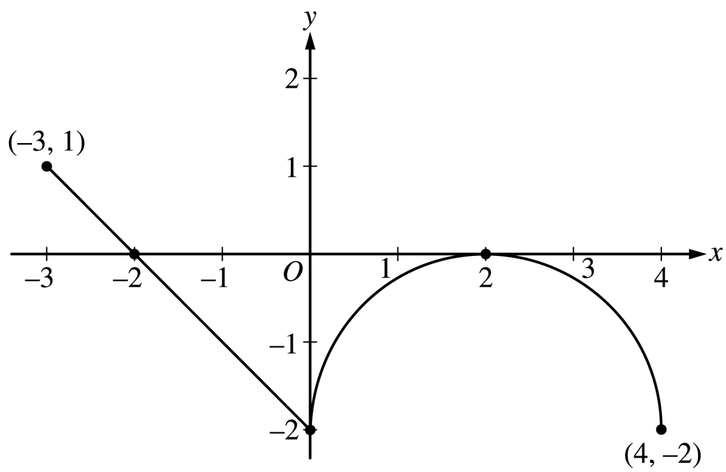
**CALCULUS AB**
**SECTION II, Part B**
**Time—45 minutes**
**Number of problems—3**

**No calculator is allowed for these problems.**




Graph of $f'$

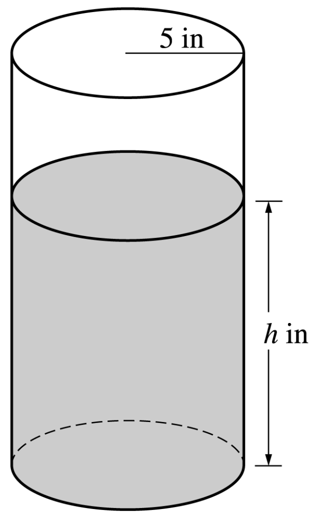
4. Let $f$ be a function defined on the closed interval $-3 \le x \le 4$ with $f(0) = 3$. The graph of $f'$, the derivative of $f$, consists of one line segment and a semicircle, as shown above.

(a) On what intervals, if any, is $f$ increasing? Justify your answer.

(b) Find the $x$-coordinate of each point of inflection of the graph of $f$ on the open interval $-3 < x < 4$. Justify your answer.

(c) Find an equation for the line tangent to the graph of $f$ at the point $(0, 3)$.

(d) Find $f(-3)$ and $f(4)$. Show the work that leads to your answers.




5. A coffeepot has the shape of a cylinder with radius 5 inches, as shown in the figure above. Let $h$ be the depth of the coffee in the pot, measured in inches, where $h$ is a function of time $t$, measured in seconds. The volume $V$ of coffee in the pot is changing at the rate of $-5\pi\sqrt{h}$ cubic inches per second. (The volume $V$ of a cylinder with radius $r$ and height $h$ is $V = \pi r^2 h$.)

   (a) Show that $\frac{dh}{dt} = -\frac{\sqrt{h}}{5}$.

   (b) Given that $h = 17$ at time $t = 0$, solve the differential equation $\frac{dh}{dt} = -\frac{\sqrt{h}}{5}$ for $h$ as a function of $t$.

   (c) At what time $t$ is the coffeepot empty?

---

6. Let $f$ be the function defined by

$$f(x) = \begin{cases} \sqrt{x+1} & \text{for } 0 \le x \le 3 \\ 5 - x & \text{for } 3 < x \le 5. \end{cases}$$

   (a) Is $f$ continuous at $x = 3$? Explain why or why not.

   (b) Find the average value of $f(x)$ on the closed interval $0 \le x \le 5$.

   (c) Suppose the function $g$ is defined by

$$g(x) = \begin{cases} k\sqrt{x+1} & \text{for } 0 \le x \le 3 \\ mx + 2 & \text{for } 3 < x \le 5, \end{cases}$$

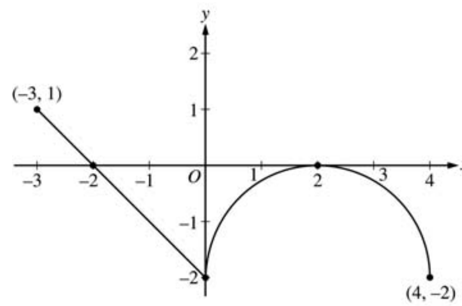
   where $k$ and $m$ are constants. If $g$ is differentiable at $x = 3$, what are the values of $k$ and $m$?

---

**END OF EXAMINATION**

## Question 4

Let $f$ be a function defined on the closed interval $-3 \le x \le 4$ with $f(0) = 3$. The graph of $f'$, the derivative of $f$, consists of one line segment and a semicircle, as shown above.



Graph of $f'$

(a) On what intervals, if any, is $f$ increasing? Justify your answer.

(b) Find the $x$-coordinate of each point of inflection of the graph of $f$ on the open interval $-3 < x < 4$. Justify your answer.

(c) Find an equation for the line tangent to the graph of $f$ at the point $(0, 3)$.

(d) Find $f(-3)$ and $f(4)$. Show the work that leads to your answers.

---

| | |
|---|---|
| (a) The function $f$ is increasing on $[-3, -2]$ since $f' > 0$ for $-3 \le x < -2$. | $2 : \begin{cases} 1 : \text{interval} \\ 1 : \text{reason} \end{cases}$ |
| (b) $x = 0$ and $x = 2$<br>$f'$ changes from decreasing to increasing at $x = 0$ and from increasing to decreasing at $x = 2$ | $2 : \begin{cases} 1 : x = 0 \text{ and } x = 2 \text{ only} \\ 1 : \text{justification} \end{cases}$ |
| (c) $f'(0) = -2$<br>Tangent line is $y = -2x + 3$. | 1 : equation |
| (d) $f(0) - f(-3) = \int_{-3}^{0} f'(t)\,dt = \frac{1}{2}(1)(1) - \frac{1}{2}(2)(2) = -\frac{3}{2}$<br>$f(-3) = f(0) + \frac{3}{2} = \frac{9}{2}$<br>$f(4) - f(0) = \int_{0}^{4} f'(t)\,dt = -\left(8 - \frac{1}{2}(2)^2\pi\right) = -8 + 2\pi$<br>$f(4) = f(0) - 8 + 2\pi = -5 + 2\pi$ | $4 : \begin{cases} 1 : \pm\left(\frac{1}{2} - 2\right) \text{ (difference of areas of triangles)} \\ 1 : \text{answer for } f(-3) \text{ using FTC} \\ 1 : \pm\left(8 - \frac{1}{2}(2)^2\pi\right) \text{ (area of rectangle} - \text{area of semicircle)} \\ 1 : \text{answer for } f(4) \text{ using FTC} \end{cases}$ |