PHYSICS C: ELECTRICITY AND MAGNETISM
SECTION II
Time—45 minutes
3 Questions

Directions: Answer all three questions. The suggested time is about 15 minutes for answering each of the questions, which are worth 15 points each. The parts within a question may not have equal weight. Show all your work in the pink booklet in the spaces provided after each part, NOT in this green insert.

E&M. 1.

A nonconducting, thin, spherical shell has a uniform surface charge density σ on its outside surface and no charge anywhere else inside.

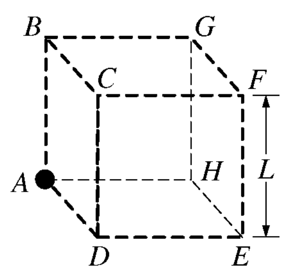
(a) Use Gauss's law to prove that the electric field inside the shell is zero everywhere. Describe the Gaussian surface that you use.

(b) The charges are now redistributed so that the surface charge density is no longer uniform. Is the electric field still zero everywhere inside the shell?

____ Yes ____ No ____ It cannot be determined from the information given.

Justify your answer.

Now consider a small conducting sphere with charge $+Q$ whose center is at corner A of a cubical surface, as shown below.



(c) For which faces of the surface, if any, is the electric flux through that face equal to zero?

____ *ABCD* ____ *CDEF* ____ *EFGH* ____ *ABGH* ____ *BCFG* ____ *ADEH*

Explain your reasoning.

(d) At which corner(s) of the surface does the electric field have the least magnitude?

(e) Determine the electric field strength at the position(s) you have indicated in part (d) in terms of Q, L, and fundamental constants, as appropriate.

(f) Given that one-eighth of the sphere at point A is inside the surface, calculate the electric flux through face $CDEF$.

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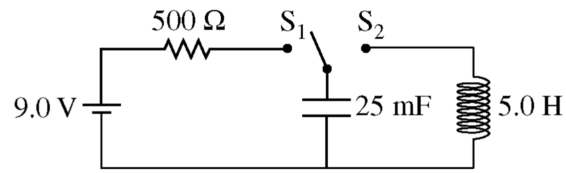


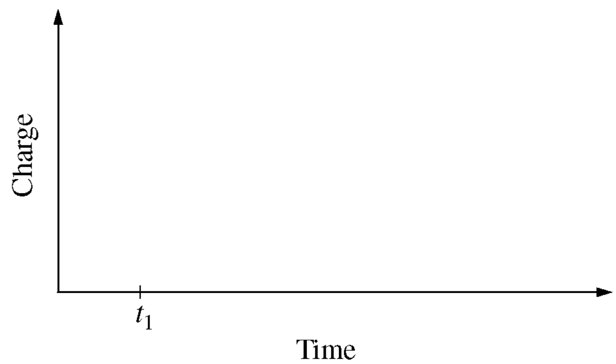
E&M. 2.

The circuit represented above contains a 9.0 V battery, a 25 mF capacitor, a 5.0 H inductor, a 500 Ω resistor, and a switch with two positions, S_1 and S_2. Initially the capacitor is uncharged and the switch is open.

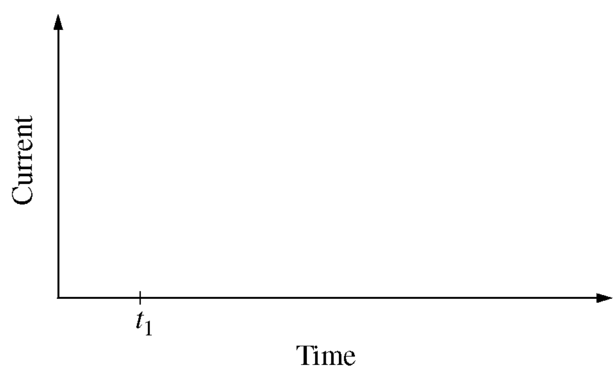
(a) In experiment 1 the switch is closed to position S_1 at time t_1 and left there for a long time.

i. Calculate the value of the charge on the bottom plate of the capacitor a long time after the switch is closed.

ii. On the axes below, sketch a graph of the magnitude of the charge on the bottom plate of the capacitor as a function of time. On the axes, explicitly label any intercepts, asymptotes, maxima, or minima with numerical values or algebraic expressions, as appropriate.



iii. On the axes below, sketch a graph of the current through the resistor as a function of time. On the axes, explicitly label any intercepts, asymptotes, maxima, or minima with numerical values or algebraic expressions, as appropriate.



Question 1

15 points total | **Distribution of points**

(a) 3 points

$$\oint \mathbf{E} \cdot d\mathbf{A} = \frac{Q}{\epsilon_0}$$

For a proper application of Gauss's Law using spherical symmetry — 1 point

$$E\left(4\pi r^2\right) = \frac{Q_{\text{enc}}}{\epsilon_0}$$

For a proper description of the correct Gaussian surface — 1 point

A proper description of the Gaussian surface should indicate that it is a sphere, concentric with the charged shell, and with a radius less than the radius of the shell. Drawing a proper Gaussian surface is acceptable.

For completing the response with an indication that $E = 0$, consistent with previous work — 1 point

The enclosed charge Q is zero for all radii of the Gaussian surface; therefore, the electric field E is also zero everywhere inside the sphere.

(b) 2 points

For selecting the correct answer of "No" — 1 point

For a correct justification — 1 point

Example: With a nonsymmetric distribution, the fields from individual charges no longer have the net effect of completely canceling inside the shell.

(c) 5 points

For correctly selecting face *ABCD* — 1 point

For correctly selecting face *ABGH* — 1 point

For correctly selecting face *ADEH* — 1 point

One earned point is deducted for each incorrect face selected.

For a correct and complete justification of the correctly checked choices — 2 points

Examples:

The electric field from the sphere is radial, so it is parallel to the three correct faces.

The electric field vector does not penetrate the area of any of the three correct faces.

Note: One point can be earned for a partial explanation or an explanation with a minor factual error.

Question 1 (continued)

Distribution of points

(d) 1 point

For correctly identifying corner A as having the smallest magnitude of electric field. — 1 point

Corner A is inside the small conducting sphere, so the electric field there is zero. All other corners have a nonzero electric field.

(e) 1 point

For correctly determining the electric field strength at the position indicated in part (d). — 1 point

As explained above, the electric field at point A is zero. A correct calculation for whatever point is indicated in part (d) also receives full credit.

(f) 3 points

For proper use of Gauss's Law that recognizes that the flux is a constant — 1 point

Total electric flux $= \phi_{\text{total}} = \dfrac{Q_{\text{enc}}}{\epsilon_0}$. The cube encloses $\dfrac{1}{8}$ of the charge, i.e. $Q_{\text{enc}} = \dfrac{Q}{8}$.

For recognizing that the flux is the same through each of the three nonzero flux sides of the cube and is equal to $1/3$ of the total flux through the cube. — 1 point

For proper reasoning leading to the final correct answer — 1 point

$$\phi_{\text{total}} = 3\phi_{CDEF} = \frac{Q/8}{\epsilon_0}$$

$$\phi_{CDEF} = \frac{Q}{24\epsilon_0}$$