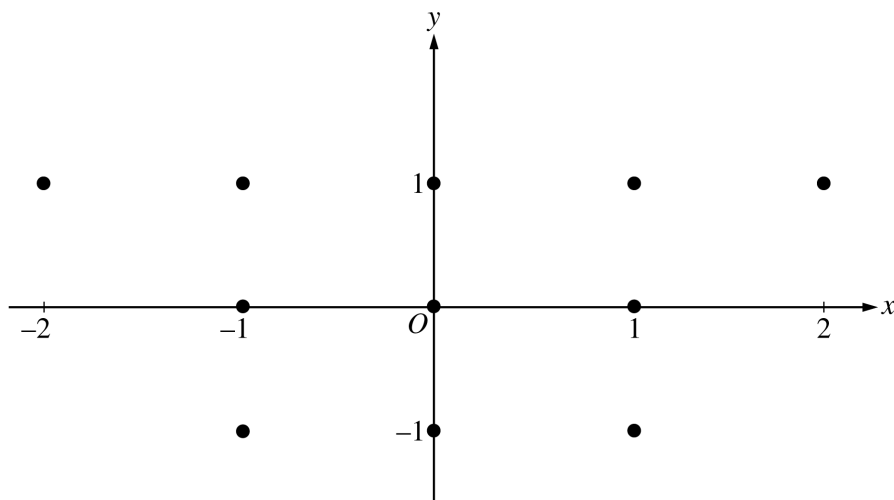


## 2000 AP® CALCULUS BC FREE-RESPONSE QUESTIONS

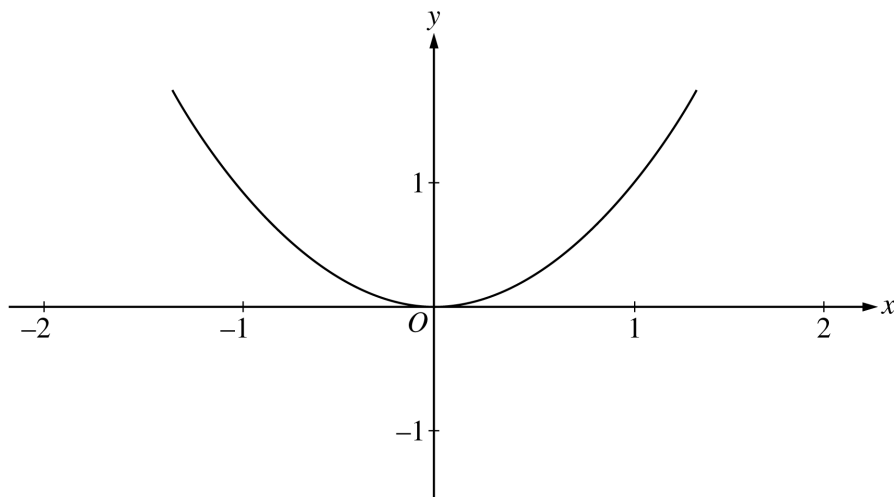
6. Consider the differential equation given by  $\frac{dy}{dx} = x(y - 1)^2$ .

(a) On the axes provided, sketch a slope field for the given differential equation at the eleven points indicated.

(Note: Use the axes provided in the pink test booklet.)



(b) Use the slope field for the given differential equation to explain why a solution could not have the graph shown below.



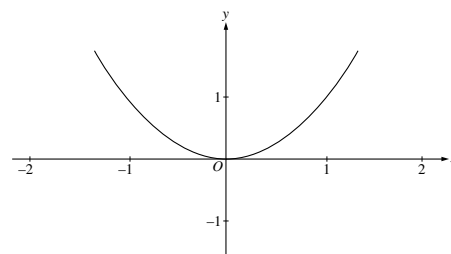
(c) Find the particular solution  $y = f(x)$  to the given differential equation with the initial condition  $f(0) = -1$ .

(d) Find the range of the solution found in part (c).

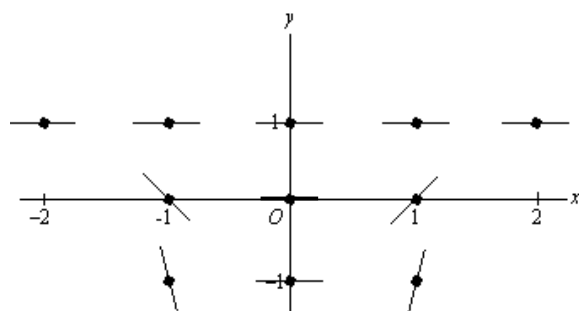
**END OF EXAMINATION**

Consider the differential equation given by  $\frac{dy}{dx} = x(y-1)^2$ .

- (a) On the axes provided, sketch a slope field for the given differential equation at the eleven points indicated.
- (b) Use the slope field for the given differential equation to explain why a solution could not have the graph shown below.
- (c) Find the particular solution  $y = f(x)$  to the given differential equation with the initial condition  $f(0) = -1$ .
- (d) Find the range of the solution found in part (c).



(a)



- 1 : zero slope at 7 points with  $y = 1$  and  $x = 0$
- 2 { 1 : negative slope at  $(-1, 0)$  and  $(-1, -1)$   
positive slope at  $(1, 0)$  and  $(1, -1)$   
steeper slope at  $y = -1$  than  $y = 0$

- (b) The graph does not have slope 0 where  $y = 1$ .  
– or –

The slope field shown suggests that solutions are asymptotic to  $y = 1$  from below, but the graph does not exhibit this behavior.

1 : reason

(c)  $\frac{1}{(y-1)^2} dy = x dx$

$$-\frac{1}{y-1} = \frac{1}{2}x^2 + C$$

$$\frac{1}{2} = 0 + C; \quad C = \frac{1}{2}$$

$$-\frac{1}{y-1} = \frac{1}{2}(x^2 + 1); \quad y = 1 - \frac{2}{x^2 + 1}$$

- 5 { 1 : separates variables  
1 : antiderivatives  
1 : constant of integration  
1 : uses initial condition  $f(0) = -1$   
1 : solves for  $y$   
0/1 if  $y$  is linear

Note: max 2/5 [1-1-0-0-0] if no constant of integration

Note: 0/5 if no separation of variables

- (d) range is  $-1 \leq y < 1$

1 : answer

0/1 if  $-1$  not in range