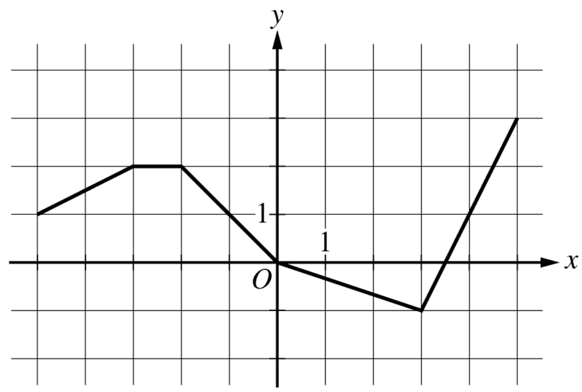
| $x$ | $g(x)$ | $g'(x)$ |
|---|---|---|
| –5 | 10 | –3 |
| –4 | 5 | –1 |
| –3 | 2 | 4 |
| –2 | 3 | 1 |
| –1 | 1 | –2 |
| 0 | 0 | –3 |




Graph of $h$

6. Let $f$ be the function defined by $f(x) = \cos(2x) + e^{\sin x}$.

Let $g$ be a differentiable function. The table above gives values of $g$ and its derivative $g'$ at selected values of $x$.

Let $h$ be the function whose graph, consisting of five line segments, is shown in the figure above.

(a) Find the slope of the line tangent to the graph of $f$ at $x = \pi$.

(b) Let $k$ be the function defined by $k(x) = h(f(x))$. Find $k'(\pi)$.

(c) Let $m$ be the function defined by $m(x) = g(-2x) \cdot h(x)$. Find $m'(2)$.

(d) Is there a number $c$ in the closed interval $[-5, -3]$ such that $g'(c) = -4$ ? Justify your answer.

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**STOP**

**END OF EXAM**

## Question 6

| Solution | Points |
|---|---|
| (a) $f'(x) = -2\sin(2x) + \cos x\, e^{\sin x}$<br><br>$f'(\pi) = -2\sin(2\pi) + \cos \pi\, e^{\sin \pi} = -1$ | $2 : f'(\pi)$ |
| (b) $k'(x) = h'(f(x)) \cdot f'(x)$<br><br>$k'(\pi) = h'(f(\pi)) \cdot f'(\pi) = h'(2) \cdot (-1)$<br>$= \left(-\frac{1}{3}\right)(-1) = \frac{1}{3}$ | $2 : \begin{cases} 1 : k'(x) \\ 1 : k'(\pi) \end{cases}$ |
| (c) $m'(x) = -2g'(-2x) \cdot h(x) + g(-2x) \cdot h'(x)$<br><br>$m'(2) = -2g'(-4) \cdot h(2) + g(-4) \cdot h'(2)$<br>$= -2(-1)\left(-\frac{2}{3}\right) + 5\left(-\frac{1}{3}\right) = -3$ | $3 : \begin{cases} 2 : m'(x) \\ 1 : m'(2) \end{cases}$ |
| (d) $g$ is differentiable. $\Rightarrow$ $g$ is continuous on the interval $[-5, -3]$.<br>$\frac{g(-3) - g(-5)}{-3 - (-5)} = \frac{2 - 10}{2} = -4$<br><br>Therefore, by the Mean Value Theorem, there is at least one value $c$, $-5 < c < -3$, such that $g'(c) = -4$. | $2 : \begin{cases} 1 : \frac{g(-3) - g(-5)}{-3 - (-5)} \\ 1 : \text{justification, using Mean Value Theorem} \end{cases}$ |