

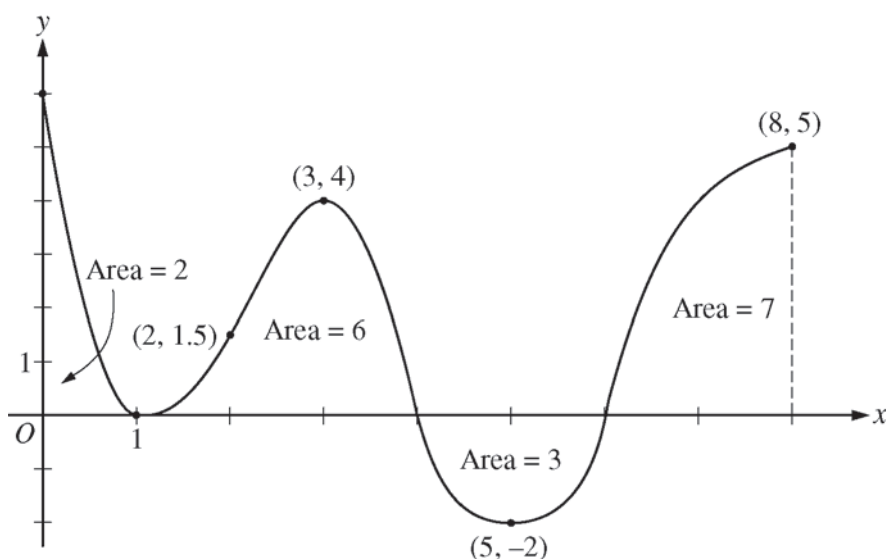
2013 AP[®] CALCULUS AB FREE-RESPONSE QUESTIONS**CALCULUS AB
SECTION II, Part B
Time—60 minutes
Number of problems—4**

No calculator is allowed for these problems.

t (minutes)	0	1	2	3	4	5	6
$C(t)$ (ounces)	0	5.3	8.8	11.2	12.8	13.8	14.5

3. Hot water is dripping through a coffeemaker, filling a large cup with coffee. The amount of coffee in the cup at time t , $0 \leq t \leq 6$, is given by a differentiable function C , where t is measured in minutes. Selected values of $C(t)$, measured in ounces, are given in the table above.
- (a) Use the data in the table to approximate $C'(3.5)$. Show the computations that lead to your answer, and indicate units of measure.
- (b) Is there a time t , $2 \leq t \leq 4$, at which $C'(t) = 2$? Justify your answer.
- (c) Use a midpoint sum with three subintervals of equal length indicated by the data in the table to approximate the value of $\frac{1}{6} \int_0^6 C(t) dt$. Using correct units, explain the meaning of $\frac{1}{6} \int_0^6 C(t) dt$ in the context of the problem.
- (d) The amount of coffee in the cup, in ounces, is modeled by $B(t) = 16 - 16e^{-0.4t}$. Using this model, find the rate at which the amount of coffee in the cup is changing when $t = 5$.
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Graph of f'

4. The figure above shows the graph of f' , the derivative of a twice-differentiable function f , on the closed interval $0 \leq x \leq 8$. The graph of f' has horizontal tangent lines at $x = 1$, $x = 3$, and $x = 5$. The areas of the regions between the graph of f' and the x -axis are labeled in the figure. The function f is defined for all real numbers and satisfies $f(8) = 4$.
- Find all values of x on the open interval $0 < x < 8$ for which the function f has a local minimum. Justify your answer.
 - Determine the absolute minimum value of f on the closed interval $0 \leq x \leq 8$. Justify your answer.
 - On what open intervals contained in $0 < x < 8$ is the graph of f both concave down and increasing? Explain your reasoning.
 - The function g is defined by $g(x) = (f(x))^3$. If $f(3) = -\frac{5}{2}$, find the slope of the line tangent to the graph of g at $x = 3$.

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Question 3

t (minutes)	0	1	2	3	4	5	6
$C(t)$ (ounces)	0	5.3	8.8	11.2	12.8	13.8	14.5

Hot water is dripping through a coffeemaker, filling a large cup with coffee. The amount of coffee in the cup at time t , $0 \leq t \leq 6$, is given by a differentiable function C , where t is measured in minutes. Selected values of $C(t)$, measured in ounces, are given in the table above.

- (a) Use the data in the table to approximate $C'(3.5)$. Show the computations that lead to your answer, and indicate units of measure.
- (b) Is there a time t , $2 \leq t \leq 4$, at which $C'(t) = 2$? Justify your answer.
- (c) Use a midpoint sum with three subintervals of equal length indicated by the data in the table to approximate the value of $\frac{1}{6} \int_0^6 C(t) dt$. Using correct units, explain the meaning of $\frac{1}{6} \int_0^6 C(t) dt$ in the context of the problem.
- (d) The amount of coffee in the cup, in ounces, is modeled by $B(t) = 16 - 16e^{-0.4t}$. Using this model, find the rate at which the amount of coffee in the cup is changing when $t = 5$.

(a) $C'(3.5) \approx \frac{C(4) - C(3)}{4 - 3} = \frac{12.8 - 11.2}{1} = 1.6$ ounces/min

2 : $\begin{cases} 1 : \text{approximation} \\ 1 : \text{units} \end{cases}$

(b) C is differentiable $\Rightarrow C$ is continuous (on the closed interval)

$$\frac{C(4) - C(2)}{4 - 2} = \frac{12.8 - 8.8}{2} = 2$$

Therefore, by the Mean Value Theorem, there is at least one time t , $2 < t < 4$, for which $C'(t) = 2$.

2 : $\begin{cases} 1 : \frac{C(4) - C(2)}{4 - 2} \\ 1 : \text{conclusion, using MVT} \end{cases}$

(c) $\frac{1}{6} \int_0^6 C(t) dt \approx \frac{1}{6} [2 \cdot C(1) + 2 \cdot C(3) + 2 \cdot C(5)]$
 $= \frac{1}{6} (2 \cdot 5.3 + 2 \cdot 11.2 + 2 \cdot 13.8)$
 $= \frac{1}{6} (60.6) = 10.1$ ounces

$\frac{1}{6} \int_0^6 C(t) dt$ is the average amount of coffee in the cup, in ounces, over the time interval $0 \leq t \leq 6$ minutes.

3 : $\begin{cases} 1 : \text{midpoint sum} \\ 1 : \text{approximation} \\ 1 : \text{interpretation} \end{cases}$

(d) $B'(t) = -16(-0.4)e^{-0.4t} = 6.4e^{-0.4t}$

$$B'(5) = 6.4e^{-0.4(5)} = \frac{6.4}{e^2} \text{ ounces/min}$$

2 : $\begin{cases} 1 : B'(t) \\ 1 : B'(5) \end{cases}$