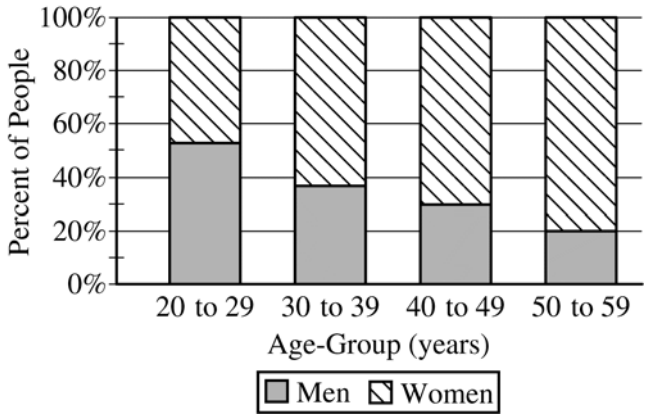
5. The table and the bar chart below summarize the age at diagnosis, in years, for a random sample of 207 men and women currently being treated for schizophrenia.

Age-Group (years)

	20 to 29	30 to 39	40 to 49	50 to 59	Total
Women	46	40	21	12	119
Men	53	23	9	3	88
Total	99	63	30	15	207



Do the data provide convincing statistical evidence of an association between age-group and gender in the diagnosis of schizophrenia?

STATISTICS
SECTION II
Part B
Question 6
Spend about 25 minutes on this part of the exam.
Percent of Section II score—25

Directions: Show all your work. Indicate clearly the methods you use, because you will be scored on the correctness of your methods as well as on the accuracy and completeness of your results and explanations.

6. Consider an experiment in which two men and two women will be randomly assigned to either a treatment group or a control group in such a way that each group has two people. The people are identified as Man 1, Man 2, Woman 1, and Woman 2. The six possible arrangements are shown below.

Arrangement A	
Treatment	Control
Man 1 Man 2	Woman 1 Woman 2

Arrangement B	
Treatment	Control
Man 1 Woman 1	Man 2 Woman 2

Arrangement C	
Treatment	Control
Man 1 Woman 2	Man 2 Woman 1

Arrangement D	
Treatment	Control
Woman 1 Woman 2	Man 1 Man 2

Arrangement E	
Treatment	Control
Man 2 Woman 2	Man 1 Woman 1

Arrangement F	
Treatment	Control
Man 2 Woman 1	Man 1 Woman 2

Two possible methods of assignment are being considered: the sequential coin flip method, as described in part (a), and the chip method, as described in part (b). For each method, the order of the assignment will be Man 1, Man 2, Woman 1, Woman 2.

(a) For the sequential coin flip method, a fair coin is flipped until one group has two people. An outcome of tails assigns the person to the treatment group, and an outcome of heads assigns the person to the control group. As soon as one group has two people, the remaining people are automatically assigned to the other group.

(i) Complete the table below by calculating the probability of each arrangement occurring if the sequential coin flip method is used.

Arrangement	A	B	C	D	E	F
Probability						

(ii) For the sequential coin flip method, what is the probability that Man 1 and Man 2 are assigned to the same group?

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Question 5

Intent of Question

The primary goal of this question was to assess a student's ability to identify, set up, perform, and interpret the results of an appropriate hypothesis test to address a particular question. More specific goals were to assess a student's ability to (1) state appropriate hypotheses; (2) identify the appropriate statistical test procedure and check appropriate conditions for inference; (3) calculate the appropriate test statistic and p-value; and (4) draw an appropriate conclusion, with justification, in the context of the study.

Solution

Step 1: State a correct pair of hypotheses.

The null hypothesis is that age group at diagnosis and gender are independent (that is, they are not associated) for the population of people currently being treated for schizophrenia.

The alternative hypothesis is that age group at diagnosis and gender are not independent for the population of people currently being treated for schizophrenia.

Step 2: Identify a correct test procedure (by name or formula) and check appropriate conditions.

The appropriate test is a chi-square test of independence.

The conditions for this test are satisfied because:

1. The question states that the sample was randomly selected.
2. The expected counts for the eight cells of the table are at least 5, as seen in the following table, with expected counts shown below observed counts.

	Age at Diagnosis				
	20 to 29	30 to 39	40 to 49	50 to 59	Total
Women	46	40	21	12	119
	56.91	36.22	17.25	8.62	
Men	53	23	9	3	88
	42.09	26.78	12.75	6.38	

Step 3: Find the value of the test statistic and the p-value.

The test statistic is calculated as $\chi^2 = \sum \frac{(O-E)^2}{E}$, or

$$\begin{aligned}\chi^2 &= 2.093 + 0.395 + 0.817 + 1.322 \\ &\quad + 2.830 + 0.534 + 1.105 + 1.788 \\ &= 10.884.\end{aligned}$$

The p-value is $P(\chi^2 \geq 10.884) = 0.012$, based on $(4-1) \times (2-1) = 3$ degrees of freedom.

Question 5 (continued)

Step 4: State the conclusion in context, with linkage to the p-value.

Because the p-value is very small (for instance much smaller than $\alpha = 0.05$), we reject the null hypothesis and conclude that the sample data provide strong evidence that there is an association between age group at diagnosis and gender for the population currently being treated for schizophrenia.

Scoring

This question is scored in three sections. Section 1 consists of steps 1 and 2 (stating the correct hypotheses, identifying the appropriate test procedure, and checking the technical conditions); section 2 consists of step 3 (performing the correct mechanics); and section 3 consists of step 4 (stating a correct conclusion with justification). Sections 1, 2, and 3 are scored as essentially correct (E), partially correct (P), or incorrect (I).

Section 1 is scored as follows:

Essentially correct (E) if the response correctly includes the following three components:
1. States BOTH hypotheses correctly with context included in at least one of them
2. Identifies a chi-square test of independence by name or formula
3. Verifies appropriate conditions that minimally include the condition for the expected counts and do not include any incorrect conditions (such as normality)

Partially correct (P) if the response includes only two of the three components.

Incorrect (I) if the response includes at most one of the three components.

Notes:
- Stating the expected count condition is not sufficient; the condition must be checked by reporting the expected counts, or minimally by showing that the smallest expected count is at least 5.
- The random sample condition was stated in the stem so need not be explicitly checked.
- If the null and alternative hypothesis are correctly stated in terms of population proportions, component 1 is satisfied. For example:
  $H_0 : p_1 = p_2 = p_3 = p_4$, where p_i is the population proportion of women at each indicated age group, 1, 2, 3, or 4, who are currently being treated for schizophrenia.
  H_a : At least one of the population proportions, p_1, p_2, p_3, p_4, differs from the other three.
  OR
  H_a : The population proportions for the four age groups are not all the same.

Section 2 is scored as follows.

Essentially correct (E) if the response correctly calculates the following two values:
1. The value of the chi-square test statistic
2. The p-value, critical value, or p-value range from chi-square table

Partially correct (P) if the response correctly calculates only one of the two values.

Question 5 (continued)

Incorrect (I) if the response does not satisfy the criteria for E or P.

Notes:

- If the response makes an error in one calculation, subsequent calculations are considered correct if they follow correctly from the initial miscalculation.
- With 3 degrees of freedom, the correct critical value is 7.81 for a significance level of 0.05 and 11.34 for a significance level of 0.01.
- Work does not have to be shown for calculations of test statistic or p-value. However, if incorrect work (other than minor arithmetic/transcription errors) is shown it is considered to be an incorrect calculation of the respective component, even if the correct value is given.
- If a response provides a test statistic that is not a chi-square test statistic, section 2 is scored I.

Section 3 is scored as follows.

Essentially correct (E) if the response includes the following three components:

1. A correct conclusion about the alternative hypothesis.
2. Justification of the conclusion based on linkage between the p-value and a reasonable alpha (or linkage between test statistic and critical value).
3. The conclusion is stated in context.

Partially correct (P) if the response includes only two of the three components.

Incorrect (I) if the response includes at most one of the three components.

Notes:

- If the response provides a correct decision, in context, with linkage to the p-value, but the decision is stated in terms of the null hypothesis with no conclusion about the alternative hypothesis, component 1 is not satisfied.
- Incorrect statistical statements are considered incorrect conclusions for the hypothesis test and do not satisfy component 1.
- If the conclusion is consistent with the p-value from section 2, and also in context with justification based on linkage to the p-value, section 3 is scored E.
- If no alpha level is given, the solution must be explicit about the linkage by giving a correct interpretation of the p-value or explaining how the conclusion follows from the p-value. For example, stating that because the p-value is small, we reject the null hypothesis or stating that because the p-value is large, we do not reject the null hypothesis.
- A decision about the null hypothesis (reject H_0 or fail to reject H_0) is not required, but if such a statement is given the scoring of the decision is considered in component 2.

Question 5 (continued)

4 Complete Response

Three sections essentially correct

3 Substantial Response

Two sections essentially correct and one section partially correct

2 Developing Response

Two sections essentially correct and no sections partially correct

OR

One section essentially correct and one or two sections partially correct

OR

Three sections partially correct

1 Minimal Response

One section essentially correct

OR

No sections essentially correct and two sections partially correct

OR

Section 1 partially correct and the other two sections incorrect