

4. The graph of the differentiable function  $f$ , shown for  $-6 \leq x \leq 7$ , has a horizontal tangent at  $x = -2$  and is linear for  $0 \leq x \leq 7$ . Let  $R$  be the region in the second quadrant bounded by the graph of  $f$ , the vertical line  $x = -6$ , and the  $x$ - and  $y$ -axes. Region  $R$  has area 12.
- The function  $g$  is defined by  $g(x) = \int_0^x f(t) dt$ . Find the values of  $g(-6)$ ,  $g(4)$ , and  $g(6)$ .
  - For the function  $g$  defined in part (a), find all values of  $x$  in the interval  $0 \leq x \leq 6$  at which the graph of  $g$  has a critical point. Give a reason for your answer.
  - The function  $h$  is defined by  $h(x) = \int_{-6}^x f'(t) dt$ . Find the values of  $h(6)$ ,  $h'(6)$ , and  $h''(6)$ . Show the work that leads to your answers.

**Write your responses to this question only on the designated pages in the separate Free Response booklet. Write your solution to each part in the space provided for that part.**

$x$	0	$\pi$	$2\pi$
$f'(x)$	5	6	0

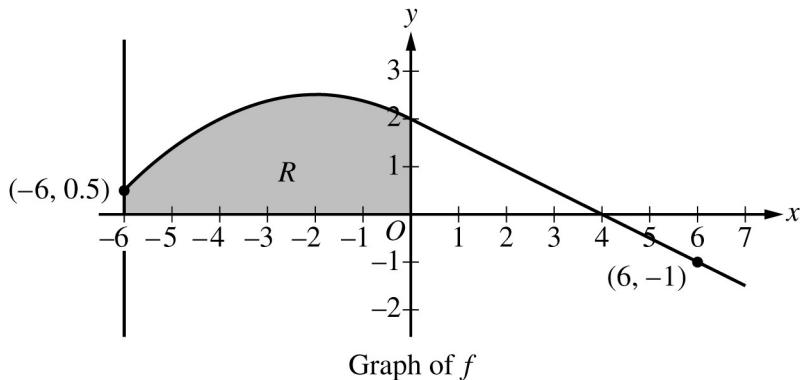
5. The function  $f$  is twice differentiable for all  $x$  with  $f(0) = 0$ . Values of  $f'$ , the derivative of  $f$ , are given in the table for selected values of  $x$ .
- (a) For  $x \geq 0$ , the function  $h$  is defined by  $h(x) = \int_0^x \sqrt{1 + (f'(t))^2} dt$ . Find the value of  $h'(\pi)$ . Show the work that leads to your answer.
- (b) What information does  $\int_0^\pi \sqrt{1 + (f'(x))^2} dx$  provide about the graph of  $f$ ?
- (c) Use Euler's method, starting at  $x = 0$  with two steps of equal size, to approximate  $f(2\pi)$ . Show the computations that lead to your answer.
- (d) Find  $\int (t+5)\cos\left(\frac{t}{4}\right) dt$ . Show the work that leads to your answer.

**Write your responses to this question only on the designated pages in the separate Free Response booklet. Write your solution to each part in the space provided for that part.**

**Part B (AB or BC): Graphing calculator not allowed****Question 4****9 points****General Scoring Notes**

The model solution is presented using standard mathematical notation.

Answers (numeric or algebraic) need not be simplified. Answers given as a decimal approximation should be correct to three places after the decimal point. Within each individual free-response question, at most one point is not earned for inappropriate rounding.



The graph of the differentiable function  $f$ , shown for  $-6 \leq x \leq 7$ , has a horizontal tangent at  $x = -2$  and is linear for  $0 \leq x \leq 7$ . Let  $R$  be the region in the second quadrant bounded by the graph of  $f$ , the vertical line  $x = -6$ , and the  $x$ - and  $y$ -axes. Region  $R$  has area 12.

**Model Solution****Scoring**

- (a) The function  $g$  is defined by  $g(x) = \int_0^x f(t) dt$ . Find the values of  $g(-6)$ ,  $g(4)$ , and  $g(6)$ .

$g(-6) = \int_0^{-6} f(t) dt = -\int_{-6}^0 f(t) dt = -12$	$g(-6)$	<b>1 point</b>
$g(4) = \int_0^4 f(t) dt = \frac{1}{2} \cdot 4 \cdot 2 = 4$	$g(4)$	<b>1 point</b>
$g(6) = \int_0^6 f(t) dt = \frac{1}{2} \cdot 4 \cdot 2 - \frac{1}{2} \cdot 2 \cdot 1 = 3$	$g(6)$	<b>1 point</b>

- (c) The function  $h$  is defined by  $h(x) = \int_{-6}^x f'(t) dt$ . Find the values of  $h(6)$ ,  $h'(6)$ , and  $h''(6)$ . Show the work that leads to your answers.

$h(6) = \int_{-6}^6 f'(t) dt = f(6) - f(-6) = -1 - 0.5 = -1.5$	Uses Fundamental Theorem of Calculus <b>1 point</b>
	$h(6)$ with supporting work <b>1 point</b>
$h'(x) = f'(x)$ , so $h'(6) = f'(6) = -\frac{1}{2}$ .	$h'(6)$ <b>1 point</b>
$h''(x) = f''(x)$ , so $h''(6) = f''(6) = 0$ .	$h''(6)$ <b>1 point</b>

**Scoring notes:**

- Labeled values may be presented in any order.
- Unlabeled values are read from left to right and from top to bottom as  $h(6)$ ,  $h'(6)$ , and  $h''(6)$ , respectively. A response that presents only 1 or 2 values must label them in order to earn any points.
- A response of  $h(6) = -1.5$  does not earn either of the first 2 points. A response of  $h(6) = f(6) - f(-6)$  earns the first point but not yet the second point.
- A response of  $h(6) = -1 - 0.5$  is the minimum work required to earn both of the first 2 points.
- To earn the third point a response must state either  $h'(x) = f'(x)$  or  $h'(6) = f'(6)$ , and provide an answer of  $-\frac{1}{2}$ .
- The fourth point is earned for a response of  $h''(6) = 0$ , with or without supporting work.
- A response that has one or more linkage errors does not earn the first point it would have otherwise earned. For example,  $h'(x) = f'(6) = -\frac{1}{2}$  does not earn the third point but is eligible for the fourth point even in the presence of another linkage error, such as  $h''(x) = f''(6) = 0$ .

**Total for part (c)    4 points**

**Total for question 4    9 points**