3. A shopping mall has three automated teller machines (ATMs). Because the machines receive heavy use, they sometimes stop working and need to be repaired. Let the random variable X represent the number of ATMs that are working when the mall opens on a randomly selected day. The table shows the probability distribution of X.

Number of ATMs working when the mall opens	0	1	2	3
Probability	0.15	0.21	0.40	0.24

(a) What is the probability that at least one ATM is working when the mall opens?

(b) What is the expected value of the number of ATMs that are working when the mall opens?

(c) What is the probability that all three ATMs are working when the mall opens, given that at least one ATM is working?

(d) Given that at least one ATM is working when the mall opens, would the expected value of the number of ATMs that are working be less than, equal to, or greater than the expected value from part (b) ? Explain.

4. A researcher conducted a medical study to investigate whether taking a low-dose aspirin reduces the chance of developing colon cancer. As part of the study, 1,000 adult volunteers were randomly assigned to one of two groups. Half of the volunteers were assigned to the experimental group that took a low-dose aspirin each day, and the other half were assigned to the control group that took a placebo each day. At the end of six years, 15 of the people who took the low-dose aspirin had developed colon cancer and 26 of the people who took the placebo had developed colon cancer. At the significance level $\alpha = 0.05$, do the data provide convincing statistical evidence that taking a low-dose aspirin each day would reduce the chance of developing colon cancer among all people similar to the volunteers?

Question 3

Intent of Question

The primary goals of this question were to assess a student's ability to (1) perform a probability calculation from a discrete random variable; (2) calculate the expected value of a discrete random variable; (3) perform a conditional probability calculation from a discrete random variable; and (4) use probabilistic thinking to make a prediction about how an expected value will change given a condition about the random variable.

Solution

Part (a):

The probability that at least one ATM is working when the mall opens is:

$$P(X \geq 1) = 0.21 + 0.40 + 0.24 = 0.85.$$

Part (b):

The expected value of the number of ATMs that are working when the mall opens is:

$$E(X) = 0(0.15) + 1(0.21) + 2(0.40) + 3(0.24) = 1.73 \text{ machines.}$$

Part (c):

The probability that all three ATMs are working when the mall opens, given that at least one ATM is working is:

$$P(X = 3 \mid X \geq 1) = \frac{P(X = 3 \text{ and } X \geq 1)}{P(X \geq 1)} = \frac{P(X = 3)}{P(X \geq 1)} = \frac{0.24}{0.85} \approx 0.282$$

Part (d):

Given that at least one ATM is working when the mall opens, the expected value of the number of working ATMs would be greater than the expected value calculated in part (b). By eliminating the possibility of 0 working ATMs, the probabilities for 1, 2, and 3 working ATMs all increase proportionally, so the expected value must increase.

Scoring

Parts (a), (b), (c), and (d) are scored as essentially correct (E), partially correct (P), or incorrect (I).

Part (a) is scored as follows:

Essentially correct (E) if the probability is computed correctly with work shown.

Partially correct (P) if the correct answer is given, but no work is shown;
OR
if appropriate work is shown but the answer is incorrect or missing;
OR

Question 3 (continued)

if one of the incorrect cumulative probabilities $P(X < 1)$, $P(X \le 1)$, or $P(X > 1)$ is stated *AND* computed correctly.

Incorrect (I) if the response does not meet the criteria for E or P.

Note: The probability can be calculated as $1 - P(X = 0) = 1 - 0.15 = 0.85$.

Part (b) is scored as follows:

Essentially correct (E) if the expected value is computed correctly with work shown.

Partially correct (P) if the correct answer is given, but no work is shown;
OR
if appropriate work is shown but the answer is incorrect or missing.

Incorrect (I) if the response does not meet the criteria for E or P.

Part (c) is scored as follows:

Essentially correct (E) if the probability is computed correctly, with work shown that includes correct numerical values for both the numerator and denominator.

Partially correct (P) if the response includes a numerator and denominator in calculating the conditional probability, with one (numerator or denominator) correct in numerical value and the other incorrect;
OR
if the correct answer is given, but no work is shown.

Incorrect (I) if the response does not meet the criteria for E or P.

Part (d) is scored as follows:

Essentially correct (E) if the response provides the correct answer (greater than) with a reasonable explanation based on the fact that with $X = 0$ eliminated, the probabilities for $X = 1$, $X = 2$, and $X = 3$ *all* increase;
OR
if the response provides the correct answer (greater than) with a reasonable explanation based on the fact that with $X = 0$ eliminated, the balance point of the distribution increases;
OR
if the response provides the correct answer (greater than) and the conditional expected value is computed correctly with work shown.

Partially correct (P) if the response provides the correct answer (greater than) with a weak explanation, such as "Yes, because 0 is eliminated";
OR

Question 3 (continued)

if the response provides the correct answer (greater than) with a reasonable but incorrect attempt to calculate the conditional expected value using a revised probability distribution with only the values $X = 1$, $X = 2$, and $X = 3$, and corresponding probabilities greater than or equal to those in the original probability distribution.

Incorrect (I) if the response does not provide the correct answer (greater than);
OR
if the response provides the correct answer (greater than) with an incorrect explanation or no explanation.

Note: The conditional expected value is:

$$1\left(\frac{0.21}{0.85}\right) + 2\left(\frac{0.40}{0.85}\right) + 3\left(\frac{0.24}{0.85}\right) \approx 1(0.247) + 2(0.471) + 3(0.282) = 2.04 \text{ machines.}$$

Each essentially correct (E) part counts as 1 point. Each partially correct (P) part counts as ½ point.

4 Complete Response

3 Substantial Response

2 Developing Response

1 Minimal Response

If a response is between two scores (for example, 2½ points), use a holistic approach to decide whether to score up or down, depending on the overall strength of the response and communication.