

**2003 AP<sup>®</sup> PHYSICS C: ELECTRICITY AND MAGNETISM  
FREE-RESPONSE QUESTIONS**

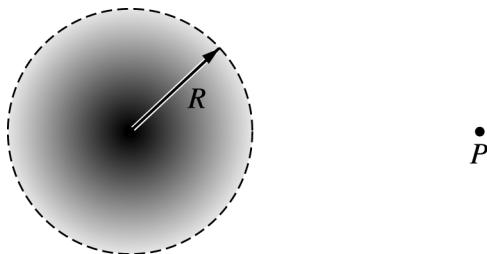
**PHYSICS C**

**Section II, ELECTRICITY AND MAGNETISM**

**Time—45 minutes**

**3 Questions**

**Directions:** Answer all three questions. The suggested time is about 15 minutes for answering each of the questions, which are worth 15 points each. The parts within a question may not have equal weight. Show all your work in the pink booklet in the spaces provided after each part, NOT in this green insert.



E&M. 1.

A spherical cloud of charge of radius  $R$  contains a total charge  $+Q$  with a nonuniform volume charge density that varies according to the equation

$$\rho(r) = \rho_0 \left( 1 - \frac{r}{R} \right) \text{ for } r \leq R \text{ and} \\ \rho = 0 \text{ for } r > R,$$

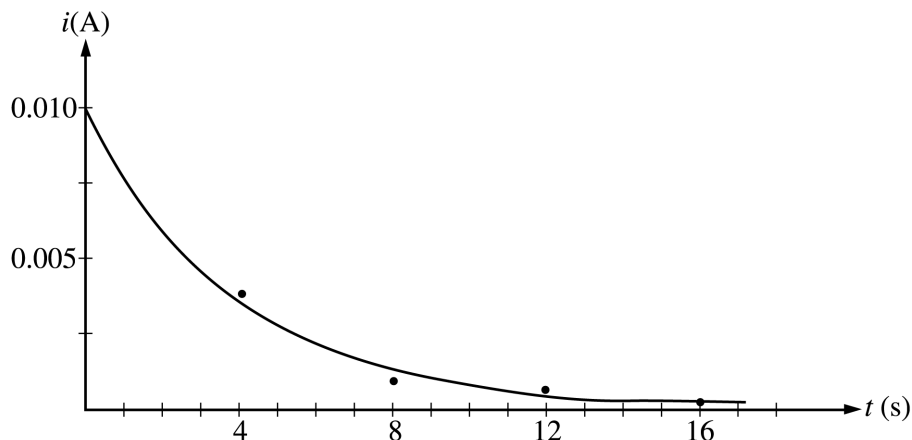
where  $r$  is the distance from the center of the cloud. Express all algebraic answers in terms of  $Q$ ,  $R$ , and fundamental constants.

- (a) Determine the following as a function of  $r$  for  $r > R$ .
  - i. The magnitude  $E$  of the electric field
  - ii. The electric potential  $V$
- (b) A proton is placed at point  $P$  shown above and released. Describe its motion for a long time after its release.
- (c) An electron of charge magnitude  $e$  is now placed at point  $P$ , which is a distance  $r$  from the center of the sphere, and released. Determine the kinetic energy of the electron as a function of  $r$  as it strikes the cloud.
- (d) Derive an expression for  $\rho_0$ .
- (e) Determine the magnitude  $E$  of the electric field as a function of  $r$  for  $r \leq R$ .

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E&M. 2.

In the laboratory, you connect a resistor and a capacitor with unknown values in series with a battery of emf  $\mathcal{E} = 12 \text{ V}$ . You include a switch in the circuit. When the switch is closed at time  $t = 0$ , the circuit is completed, and you measure the current through the resistor as a function of time as plotted below.



A data-fitting program finds that the current decays according to the equation  $i(t) = \frac{\mathcal{E}}{R} e^{-t/4}$ .

- Using common symbols for the battery, the resistor, the capacitor, and the switch, draw the circuit that you constructed. Show the circuit before the switch is closed and include whatever other devices you need to measure the current through the resistor to obtain the above plot. Label each component in your diagram.
- Having obtained the curve shown above, determine the value of the resistor that you placed in this circuit.
- What capacitance did you insert in the circuit to give the result above?

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**Question 1**

**15 points total**

**Distribution  
of points**

Answers shown here are expressed in terms of Coulomb's law constant  $k$ , but equivalent answers in terms of  $1/4\pi\epsilon_0$  were acceptable.

(a) 3 points

The sphere of charge can be treated as a point charge located at the sphere's center.

i. (2 points)

$$E = \frac{kq}{r^2}$$

For indicating that the total charge is  $Q$

1 point

For a correct answer

1 point

$$E = \frac{kQ}{r^2}$$

ii. (1 point)

For a correct answer

1 point

$$V = \frac{kQ}{r}$$

Credit was also awarded for integrating  $E$  to obtain  $V$

(b) 3 points

For indicating that the proton will move away from the charged sphere

1 point

For indicating that the velocity of the proton will increase or reach a finite value

1 point

For indicating that the acceleration of the proton will decrease

1 point

(c) 3 points

For a correct statement of conservation of energy

1 point

$$K = U_r - U_R$$

For the substitution of an electrical potential energy with the correct form

1 point

$$K = \frac{-keQ}{r} - \frac{-keQ}{R}$$

For a correct answer

1 point

$$K = keQ \left( \frac{1}{R} - \frac{1}{r} \right)$$

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**Question 1 (cont'd.)**

	<b>Distribution of points</b>
(c) (continued)	
<i>Alternate solution</i>	<i>Alternate points</i>
For showing correct use of the work-energy theorem	1 point
$K = W = \int \mathbf{F} \cdot d\mathbf{r}$	
For setting up the correct integration	1 point
$K = \int_r^R \frac{-kQe}{r^2} dr$	
$K = -kQe \left( -\frac{1}{r} \right) \Big _r^R = -kQe \left( -\frac{1}{R} - \left( -\frac{1}{r} \right) \right) = -kQe \left( \frac{1}{r} - \frac{1}{R} \right)$	
For a correct answer	1 point
$K = keQ \left( \frac{1}{R} - \frac{1}{r} \right)$	
(d) 3 points	
$\rho_0$ can be determined by integrating the volume distribution and setting it equal to the total charge $Q$	
For indicating that an integration is necessary	1 point
$Q = \int_0^R \rho(r) dV$	
For showing a correct volume element	1 point
$dV = 4\pi r^2 dr$	
For substitution of $\rho(r)$ and $dV$	1 point
$Q = \int_0^R \rho_0 \left( 1 - \frac{r}{R} \right) 4\pi r^2 dr$	
$Q = 4\pi\rho_0 \int_0^R \left( r^2 - \frac{r^3}{R} \right) dr = 4\pi\rho_0 \left( \frac{r^3}{3} - \frac{r^4}{4R} \right) \Big _0^R = 4\pi\rho_0 \left( \frac{R^3}{3} - \frac{R^3}{4} \right) = 4\pi\rho_0 \frac{R^3}{12}$	
$\rho_0 = \frac{3Q}{\pi R^3}$	

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**Question 1 (cont'd.)**

**Distribution  
of points**

(e) 3 points

For writing Gauss's law with a charge element  $dq$  OR showing the relationship between  $E$  and  $\int dq$

1 point

$$\oint \mathbf{E} \cdot d\mathbf{A} = \frac{1}{\epsilon_0} \int dq \quad \text{OR} \quad dE = \frac{k}{r^2} dq$$

$$E 4\pi r^2 = \frac{1}{\epsilon_0} \int \rho(r) dV \quad \text{OR} \quad E = \frac{k}{r^2} \int \rho(r) dq$$

For correct substitution of  $\rho_0$ ,  $dV$ , and correct limits

1 point

$$\int_0^r \rho(r) dV = \int_0^r \frac{3Q}{\pi R^3} \left(1 - \frac{r}{R}\right) 4\pi r^2 dr$$

$$\int_0^r \rho(r) dV = \frac{12Q}{R^3} \int_0^r \left(r^2 - \frac{r^3}{R}\right) dr = \frac{12Q}{R^3} \left(\frac{r^3}{3} - \frac{r^4}{4R}\right) \Big|_0^r = \frac{Q}{R^3} \left(4r^3 - \frac{3r^4}{R}\right)$$

$$E = \frac{1}{4\pi\epsilon_0 r^2} \frac{Q}{R^3} \left(4r^3 - \frac{3r^4}{R}\right) \quad \text{OR} \quad E = \frac{k}{r^2} \frac{Q}{R^3} \left(4r^3 - \frac{3r^4}{R}\right)$$

For a correct answer

1 point

$$E = \frac{kQr}{R^3} \left(4 - \frac{3r}{R}\right)$$