

2006 AP[®] CALCULUS AB FREE-RESPONSE QUESTIONS**CALCULUS AB
SECTION II, Part B****Time—45 minutes****Number of problems—3****No calculator is allowed for these problems.**

t (seconds)	0	10	20	30	40	50	60	70	80
$v(t)$ (feet per second)	5	14	22	29	35	40	44	47	49

4. Rocket A has positive velocity $v(t)$ after being launched upward from an initial height of 0 feet at time $t = 0$ seconds. The velocity of the rocket is recorded for selected values of t over the interval $0 \leq t \leq 80$ seconds, as shown in the table above.
- (a) Find the average acceleration of rocket A over the time interval $0 \leq t \leq 80$ seconds. Indicate units of measure.
- (b) Using correct units, explain the meaning of $\int_{10}^{70} v(t) \, dt$ in terms of the rocket's flight. Use a midpoint Riemann sum with 3 subintervals of equal length to approximate $\int_{10}^{70} v(t) \, dt$.
- (c) Rocket B is launched upward with an acceleration of $a(t) = \frac{3}{\sqrt{t+1}}$ feet per second per second. At time $t = 0$ seconds, the initial height of the rocket is 0 feet, and the initial velocity is 2 feet per second. Which of the two rockets is traveling faster at time $t = 80$ seconds? Explain your answer.
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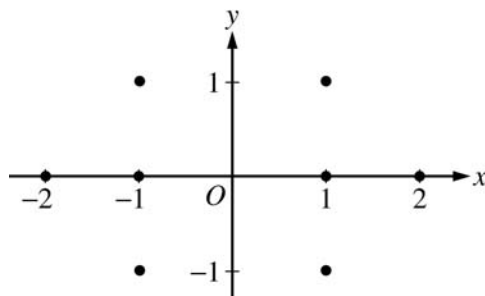
WRITE ALL WORK IN THE PINK EXAM BOOKLET.

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5. Consider the differential equation $\frac{dy}{dx} = \frac{1+y}{x}$, where $x \neq 0$.

(a) On the axes provided, sketch a slope field for the given differential equation at the eight points indicated.

(Note: Use the axes provided in the pink exam booklet.)



(b) Find the particular solution $y = f(x)$ to the differential equation with the initial condition $f(-1) = 1$ and state its domain.

6. The twice-differentiable function f is defined for all real numbers and satisfies the following conditions:

$$f(0) = 2, \quad f'(0) = -4, \quad \text{and} \quad f''(0) = 3.$$

(a) The function g is given by $g(x) = e^{ax} + f(x)$ for all real numbers, where a is a constant. Find $g'(0)$ and $g''(0)$ in terms of a . Show the work that leads to your answers.

(b) The function h is given by $h(x) = \cos(kx)f(x)$ for all real numbers, where k is a constant. Find $h'(x)$ and write an equation for the line tangent to the graph of h at $x = 0$.

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END OF EXAM

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2006 SCORING GUIDELINES

Question 4

t (seconds)	0	10	20	30	40	50	60	70	80
$v(t)$ (feet per second)	5	14	22	29	35	40	44	47	49

Rocket A has positive velocity $v(t)$ after being launched upward from an initial height of 0 feet at time $t = 0$ seconds. The velocity of the rocket is recorded for selected values of t over the interval $0 \leq t \leq 80$ seconds, as shown in the table above.

(a) Find the average acceleration of rocket A over the time interval $0 \leq t \leq 80$ seconds. Indicate units of measure.

(b) Using correct units, explain the meaning of $\int_{10}^{70} v(t) \, dt$ in terms of the rocket's flight. Use a midpoint Riemann sum with 3 subintervals of equal length to approximate $\int_{10}^{70} v(t) \, dt$.

(c) Rocket B is launched upward with an acceleration of $a(t) = \frac{3}{\sqrt{t+1}}$ feet per second per second. At time $t = 0$ seconds, the initial height of the rocket is 0 feet, and the initial velocity is 2 feet per second. Which of the two rockets is traveling faster at time $t = 80$ seconds? Explain your answer.

(a) Average acceleration of rocket A is

$$\frac{v(80) - v(0)}{80 - 0} = \frac{49 - 5}{80} = \frac{11}{20} \text{ ft/sec}^2$$

(b) Since the velocity is positive, $\int_{10}^{70} v(t) \, dt$ represents the distance, in feet, traveled by rocket A from $t = 10$ seconds to $t = 70$ seconds.

$$\begin{aligned} \text{A midpoint Riemann sum is} \\ 20[v(20) + v(40) + v(60)] \\ = 20[22 + 35 + 44] = 2020 \text{ ft} \end{aligned}$$

(c) Let $v_B(t)$ be the velocity of rocket B at time t .

$$\begin{aligned} v_B(t) &= \int \frac{3}{\sqrt{t+1}} \, dt = 6\sqrt{t+1} + C \\ 2 &= v_B(0) = 6 + C \\ v_B(t) &= 6\sqrt{t+1} - 4 \\ v_B(80) &= 50 > 49 = v(80) \end{aligned}$$

Rocket B is traveling faster at time $t = 80$ seconds.

Units of ft/sec^2 in (a) and ft in (b)

1 : answer

3 : $\begin{cases} 1 : \text{explanation} \\ 1 : \text{uses } v(20), v(40), v(60) \\ 1 : \text{value} \end{cases}$

4 : $\begin{cases} 1 : 6\sqrt{t+1} \\ 1 : \text{constant of integration} \\ 1 : \text{uses initial condition} \\ 1 : \text{finds } v_B(80), \text{ compares to } v(80), \\ \text{and draws a conclusion} \end{cases}$

1 : units in (a) and (b)