1999

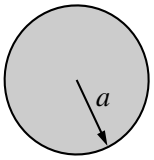
PHYSICS C

SECTION II, ELECTRICITY AND MAGNETISM

Time—45 minutes

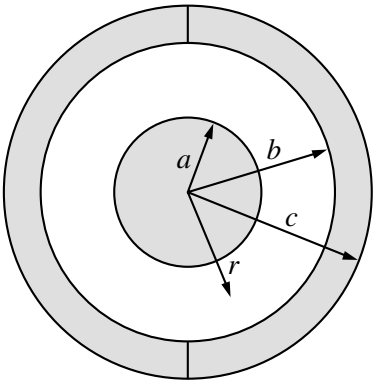
3 Questions

Directions: Answer all three questions. The suggested time is about 15 minutes for answering each of the questions, which are worth 15 points each. The parts within a question may not have equal weight. Show all your work in the pink booklet in the spaces provided after each part, NOT in this green insert.



E&M 1. An isolated conducting sphere of radius $a = 0.20\ \text{m}$ is at a potential of $-2{,}000\ \text{V}$.

(a) Determine the charge Q_0 on the sphere.



The charged sphere is then concentrically surrounded by two uncharged conducting hemispheres of inner radius $b = 0.40\ \text{m}$ and outer radius $c = 0.50\ \text{m}$, which are joined together as shown above, forming a spherical capacitor. A wire is connected from the outer sphere to ground, and then removed.

(b) Determine the magnitude of the electric field in the following regions as a function of the distance r from the center of the inner sphere.

i. $r < a$
ii. $a < r < b$
iii. $b < r < c$
iv. $r > c$

(c) Determine the magnitude of the potential difference between the sphere and the conducting shell.

(d) Determine the capacitance of the spherical capacitor.

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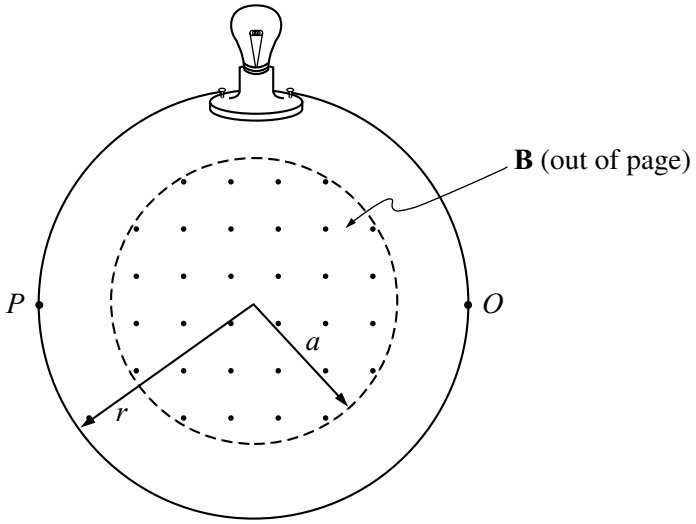
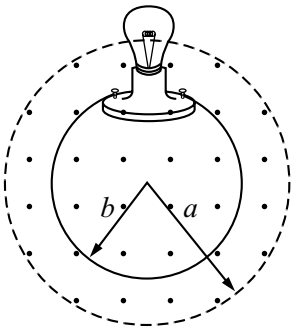


E&M 2. A uniform magnetic field **B** exists in a region of space defined by a circle of radius $a = 0.60$ m as shown above. The magnetic field is perpendicular to the page and increases out of the page at a constant rate of 0.40 T/s. A single circular loop of wire of negligible resistance and radius $r = 0.90$ m is connected to a lightbulb with a resistance $R = 5.0\ \Omega$, and the assembly is placed concentrically around the region of magnetic field.

(a) Determine the emf induced in the loop.

(b) Determine the magnitude of the current in the circuit. On the figure above, indicate the direction of the current in the loop at point O.

(c) Determine the total energy dissipated in the lightbulb during a 15 s interval.



The experiment is repeated with a loop of radius $b = 0.40$ m placed concentrically in the same magnetic field as before. The same lightbulb is connected to the loop, and the magnetic field again increases out of the page at a rate of 0.40 T/s. Neglect any direct effects of the field on the lightbulb itself.

(d) State whether the brightness of the bulb will be greater than, less than, or equal to the brightness of the bulb in part (a). Justify your answer.

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1999 Physics C Solutions | **Distribution of points**

E & M 1 (15 points)

(a) 4 points

For using the relationship between potential and charge — 1 point

$V = \frac{1}{4\pi\epsilon_0}\frac{Q}{r}$

Solving for Q:

$Q = 4\pi\epsilon_0 Vr$

For correct substitutions for the potential and radius — 1 point

$Q_0 = 4\pi\epsilon_0(-2000\text{ V})(0.20\text{ m})$ or $(-2000\text{ V})(0.20\text{ m})/(9\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2)$

$Q_0 = -1600\pi\epsilon_0\text{ C}$ or $-4.4\times10^{-8}\text{ C}$

For the correct magnitude of Q_0 — 1 point

For the negative sign — 1 point

(b) 5 points

i. For indicating that the electric field is zero — 1 point

ii. The charge on the sphere can be treated as a point charge at its center

$E = \frac{1}{4\pi\epsilon_0}\frac{Q_0}{r^2}$

$E = (9\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2)\left(\frac{4.4\times10^{-8}\text{ C}}{r^2}\right)$

$E = \frac{396}{r^2}\frac{\text{N}}{\text{C}}$ or $\frac{400}{r^2}\frac{\text{N}}{\text{C}}$ where r is in meters

For any of the above expressions for E — 1 point

iii. For indicating that the electric field is zero — 1 point

iv. For indicating that the electric field is zero — 1 point

For having all four answers correct OR for some mention of using the enclosed charge OR for some mention of Gauss' law — 1 point

Distribution of points

E & M 1 (continued)

(c) 3 points

$$\Delta V = V_b - V_a = -\int_a^b E\, dr$$

For recognition of the need to take the difference of the potentials at radii a and b, or for writing the definite integral (with limits) 1 point

$$|\Delta V| = \frac{Q_0}{4\pi\epsilon_0}\int_a^b \frac{dr}{r^2}$$

$$= \frac{Q_0}{4\pi\epsilon_0}\left(\frac{1}{r}\right)\Big|_a^b$$

$$|\Delta V| = \frac{Q_0}{4\pi\epsilon_0}\left(\frac{1}{b} - \frac{1}{a}\right)$$

For correct substitution of variables or numerical values for Q_0, a, and b 1 point

For the correct answer 1 point

$$|\Delta V| = \frac{5Q_0}{8\pi\epsilon_0} \quad \text{or} \quad 1000 \text{ V}$$

(Alternate solution) *(Alternate points)*

For recognition of the need to take the difference of the potentials at radii a and b *1 point*

$$\Delta V = V_b - V_a$$

$$\Delta V = \frac{Q_0}{4\pi\epsilon_0}\left(\frac{1}{r_b}\right) - \frac{Q_0}{4\pi\epsilon_0}\left(\frac{1}{r_a}\right)$$

For correct substitution of Q_0, a, and b *1 point*

$$\Delta V = \frac{Q_0}{4\pi\epsilon_0}\left(\frac{1}{b} - \frac{1}{a}\right)$$

For the correct answer *1 point*

$$|\Delta V| = \frac{5Q_0}{8\pi\epsilon_0} \quad \text{or} \quad 1000 \text{ V}$$

(Alternate solution) *(Alternate points)*

$$V = \frac{Q}{C}$$

For using the above relationship *1 point*

For substituting Q_0 from part (a) and C from part (d) alternate solution *1 point*

For the correct answer *1 point*

$$|\Delta V| = \frac{5Q_0}{8\pi\epsilon_0} \quad \text{or} \quad 1000 \text{ V}$$

Distribution of points

E & M 1 (continued)

(d) 2 points

$$C = \frac{Q_0}{V}$$

For using the above relationship — 1 point

For substituting Q_0 from part (a) and ΔV from part (c) — 1 point

$$C = \frac{4.4 \times 10^{-8}\ \text{C}}{1000\ \text{V}}$$

$$C = 4.4 \times 10^{-11}\ \text{F}$$

(Alternate solution) — *(Alternate points)*

For writing the equation for the capacitance of the spherical capacitor — *1 point*

$$C = \frac{4\pi\epsilon_0 ab}{b - a}$$

$$C = \frac{(0.02\ \text{m})(0.04\ \text{m})}{(9 \times 10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(0.04\ \text{m} - 0.02\ \text{m})}$$

For the correct answer — *1 point*

$C = 4.4 \times 10^{-11}$ F

For correct units on two answers and no incorrect units — 1 point