Graph of f

1. The figure shows the graph of the function f on its domain of $-3.5 \leq x \leq 3.5$. The points $(-3, 1)$, $(0, 1)$, and $(3, 1)$ are on the graph of f . The function g is given by $g(x) = 2.916 \cdot (0.7)^x$.
- (A) (i) The function h is defined by $h(x) = (g \circ f)(x) = g(f(x))$. Find the value of $h(3)$ as a decimal approximation, or indicate that it is not defined.
- (ii) Find all values of x for which $f(x) = 1$, or indicate that there are no such values.
- (B) (i) Find all values of x , as decimal approximations, for which $g(x) = 2$, or indicate that there are no such values.
- (ii) Determine the end behavior of g as x increases without bound. Express your answer using the mathematical notation of a limit.
- (C) (i) Determine if f has an inverse function.
- (ii) Give a reason for your answer based on the definition of a function and the graph of $y = f(x)$.

Write your responses to this question only on the designated pages in the separate Free Response booklet. Write your solution to each part in the space provided for that part.

2. On the initial day of sales ($t = 0$) for a new video game, there were 40 thousand units of the game sold that day. Ninety-one days later ($t = 91$), there were 76 thousand units of the game sold that day.

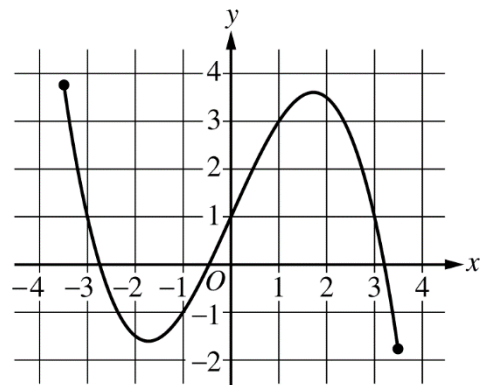
The number of units of the video game sold on a given day can be modeled by the function G given by $G(t) = a + b \ln(t + 1)$, where $G(t)$ is the number of units sold, in thousands, on day t since the initial day of sales.

- (A) (i) Use the given data to write two equations that can be used to find the values for constants a and b in the expression for $G(t)$.
- (ii) Find the values for a and b as decimal approximations.
- (B) (i) Use the given data to find the average rate of change of the number of units of the video game sold, in thousands per day, from $t = 0$ to $t = 91$ days. Express your answer as a decimal approximation. Show the computations that lead to your answer.
- (ii) Use the average rate of change found in (i) to estimate the number of units of the video game sold, in thousands, on day $t = 50$. Show the work that leads to your answer.
- (iii) Let A_t represent the estimate of the number of units of the video game sold, in thousands, using the average rate of change found in (i). For A_{50} , found in (ii), it can be shown that $A_{50} < G(50)$. Explain why, in general, $A_t < G(t)$ for all t , where $0 < t < 91$.
- (C) The makers of the video game reported that daily sales of the video game decreased each day after $t = 91$. Explain why the error in the model G increases after $t = 91$.

Write your responses to this question only on the designated pages in the separate Free Response booklet. Write your solution to each part in the space provided for that part.

Question 1: Function Concepts
Part A: Graphing calculator required

6 points



Graph of f

The figure shows the graph of the function f on its domain of $-3.5 \leq x \leq 3.5$. The points $(-3, 1)$, $(0, 1)$, and $(3, 1)$ are on the graph of f . The function g is given by $g(x) = 2.916 \cdot (0.7)^x$.

Model Solution		Scoring
(A) (i) The function h is defined by $h(x) = (g \circ f)(x) = g(f(x))$. Find the value of $h(3)$ as a decimal approximation, or indicate that it is not defined. (ii) Find all values of x for which $f(x) = 1$, or indicate that there are no such values.		
(i) $h(3) = g(f(3)) = g(1) = 2.041$	Value	1 point
(ii) From the graph, $f(x) = 1$ when $x = -3$, $x = 0$, and $x = 3$.	Values	1 point

General Scoring Notes for Question 1 Parts (A), (B), and (C):

- Decimal approximations must be correct to three places after the decimal point by rounding or truncating. Decimal values of 0 in final digits need not be reported ($2.000 = 2.00 = 2.0 = 2$).
- A **decimal presentation error** occurs when a response is complete and correct, but the answer is reported to fewer digits than required.
- The first decimal presentation error in Question 1 does not earn the point. For each additional part of Question 1 that requires a decimal approximation and contains a decimal presentation error, the response is eligible to earn the point.

Scoring notes:

- The first point is earned for a correct decimal approximation of 2.041.
- The second point does not require supporting work.

- A response that does not earn either point in Part (A) is eligible for **partial credit** in Part (A) if the response has one criteria from the first column AND one criteria from the second column. Partial credit response is scored **0-1** in Part (A).

First Column	Second Column
Correct value in (i) that is not expressed as a decimal approximation	Only one correct value in (ii) with no incorrect values included
Correct value in (i) with a decimal presentation error	Only two correct values in (ii) with no incorrect values included

- (B) (i) Find all values of x , as decimal approximations, for which $g(x) = 2$, or indicate that there are no such values.
- (ii) Determine the end behavior of g as x increases without bound. Express your answer using the mathematical notation of a limit.

(i) $g(x) = 2 \Rightarrow 2.916(0.7)^x = 2$ $x = 1.057$	Value 1 point
(ii) As x increases without bound, the output values of g get arbitrarily close to 0. Therefore, $\lim_{x \rightarrow \infty} g(x) = 0$.	End behavior with limit notation 1 point

Scoring notes:

- The first point is earned for a correct decimal approximation of 1.057. No incorrect values may be included.
- The second point requires a correct limit statement with four components: “lim,” “ $x \rightarrow \infty$,” the function g , and 0. Examples that earn the point include:
 - $\lim_{x \rightarrow \infty} g(x) = 0$ OR $\lim_{x \rightarrow \infty} g = 0$
 - $\lim_{x \rightarrow \infty} g(x) \rightarrow 0$ OR $\lim_{x \rightarrow \infty} g \rightarrow 0$
 - $\lim_{x \rightarrow \infty} g(x) \quad 0$ OR $\lim_{x \rightarrow \infty} g \quad 0$

If the response includes an additional, complete limit statement (e.g., $\lim_{x \rightarrow -\infty} g(x) = \infty$), the value of the limit must be correct.

- A response that does not earn either point in Part (B) is eligible for **partial credit** in Part (B) if the response has one criteria from the first column AND one criteria from the second column. Partial credit response is scored **1-0** in Part (B).

First Column	Second Column
Correct answer in (i) that is not expressed as a decimal approximation	Correct end behavior statement in (ii) without use of limit notation
Correct value in (i) with a decimal presentation error	Correct end behavior statement in (ii) with incorrect limit notation
	Correct limit statement in (ii) that is missing “ $x \rightarrow \infty$ ”

- (C) (i) Determine if f has an inverse function.
(ii) Give a reason for your answer based on the definition of a function and the graph of $y = f(x)$.

(i) f does not have an inverse function on its domain of $-3.5 \leq x \leq 3.5$.	Answer	1 point
(ii) There are output values of f that are not mapped from unique input values; for example, $f(-3) = f(0) = f(3) = 1$.	Reason	1 point

Scoring notes:

- The first point is earned for a correct answer.
- Both points may be earned in (ii) provided there is no incorrect response in (i).
- The second point requires an implicit or explicit reference to the definition of a function AND support for the reason by referencing specific function values.
- A response such as “ f does not have an inverse function because $f(-3) = f(0) = 1$ ” OR “ f does not have an inverse function because there are two input values mapped to 1” earns both points.
- A response such as “ f is not one-to-one” OR “ f fails the horizontal line test” OR “There are output values that are not mapped from unique input values” is not sufficient to earn the second point.
- The second point cannot be earned if there are any errors in Part (C) (ii).
- A response that indicates that f **has** an inverse function in Part (C) (i) without a reason in Part (C) (i) combined with a response in Part (C) (ii) that provides both the correct answer and a correct reason is scored **0-1**.

Total for question 1 6 points