

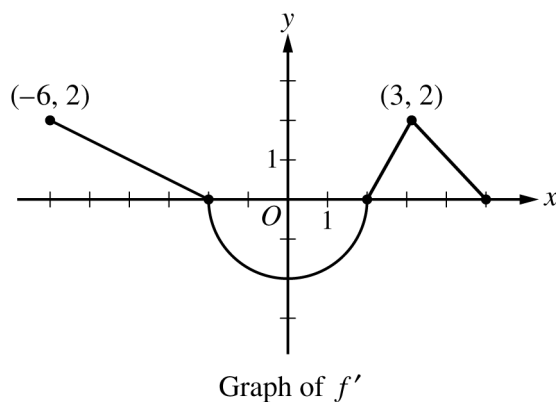
**2017 AP<sup>®</sup> CALCULUS AB FREE-RESPONSE QUESTIONS**

**CALCULUS AB  
SECTION II, Part B**

**Time—1 hour**

**Number of questions—4**

**NO CALCULATOR IS ALLOWED FOR THESE QUESTIONS.**



3. The function  $f$  is differentiable on the closed interval  $[-6, 5]$  and satisfies  $f(-2) = 7$ . The graph of  $f'$ , the derivative of  $f$ , consists of a semicircle and three line segments, as shown in the figure above.
- (a) Find the values of  $f(-6)$  and  $f(5)$ .
  - (b) On what intervals is  $f$  increasing? Justify your answer.
  - (c) Find the absolute minimum value of  $f$  on the closed interval  $[-6, 5]$ . Justify your answer.
  - (d) For each of  $f''(-5)$  and  $f''(3)$ , find the value or explain why it does not exist.
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4. At time  $t = 0$ , a boiled potato is taken from a pot on a stove and left to cool in a kitchen. The internal temperature of the potato is 91 degrees Celsius ( $^{\circ}\text{C}$ ) at time  $t = 0$ , and the internal temperature of the potato is greater than  $27^{\circ}\text{C}$  for all times  $t > 0$ . The internal temperature of the potato at time  $t$  minutes can be modeled by the function  $H$  that satisfies the differential equation  $\frac{dH}{dt} = -\frac{1}{4}(H - 27)$ , where  $H(t)$  is measured in degrees Celsius and  $H(0) = 91$ .
- (a) Write an equation for the line tangent to the graph of  $H$  at  $t = 0$ . Use this equation to approximate the internal temperature of the potato at time  $t = 3$ .
- (b) Use  $\frac{d^2H}{dt^2}$  to determine whether your answer in part (a) is an underestimate or an overestimate of the internal temperature of the potato at time  $t = 3$ .
- (c) For  $t < 10$ , an alternate model for the internal temperature of the potato at time  $t$  minutes is the function  $G$  that satisfies the differential equation  $\frac{dG}{dt} = -(G - 27)^{2/3}$ , where  $G(t)$  is measured in degrees Celsius and  $G(0) = 91$ . Find an expression for  $G(t)$ . Based on this model, what is the internal temperature of the potato at time  $t = 3$  ?
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**Question 3**

(a)  $f(-6) = f(-2) + \int_{-2}^{-6} f'(x) dx = 7 - \int_{-6}^{-2} f'(x) dx = 7 - 4 = 3$

$$f(5) = f(-2) + \int_{-2}^5 f'(x) dx = 7 - 2\pi + 3 = 10 - 2\pi$$

$$3 : \begin{cases} 1 : \text{uses initial condition} \\ 1 : f(-6) \\ 1 : f(5) \end{cases}$$

(b)  $f'(x) > 0$  on the intervals  $[-6, -2]$  and  $(2, 5]$ .

Therefore,  $f$  is increasing on the intervals  $[-6, -2]$  and  $[2, 5]$ .

2 : answer with justification

(c) The absolute minimum will occur at a critical point where  $f'(x) = 0$  or at an endpoint.

$$f'(x) = 0 \Rightarrow x = -2, x = 2$$

$x$	$f(x)$
-6	3
-2	7
2	$7 - 2\pi$
5	$10 - 2\pi$

$$2 : \begin{cases} 1 : \text{considers } x = 2 \\ 1 : \text{answer with justification} \end{cases}$$

The absolute minimum value is  $f(2) = 7 - 2\pi$ .

(d)  $f''(-5) = \frac{2 - 0}{-6 - (-2)} = -\frac{1}{2}$

$$2 : \begin{cases} 1 : f''(-5) \\ 1 : f''(3) \text{ does not exist,} \\ \text{with explanation} \end{cases}$$

$$\lim_{x \rightarrow 3^-} \frac{f'(x) - f'(3)}{x - 3} = 2 \text{ and } \lim_{x \rightarrow 3^+} \frac{f'(x) - f'(3)}{x - 3} = -1$$

$f''(3)$  does not exist because

$$\lim_{x \rightarrow 3^-} \frac{f'(x) - f'(3)}{x - 3} \neq \lim_{x \rightarrow 3^+} \frac{f'(x) - f'(3)}{x - 3}.$$