

**2007 AP<sup>®</sup> CALCULUS BC FREE-RESPONSE QUESTIONS**

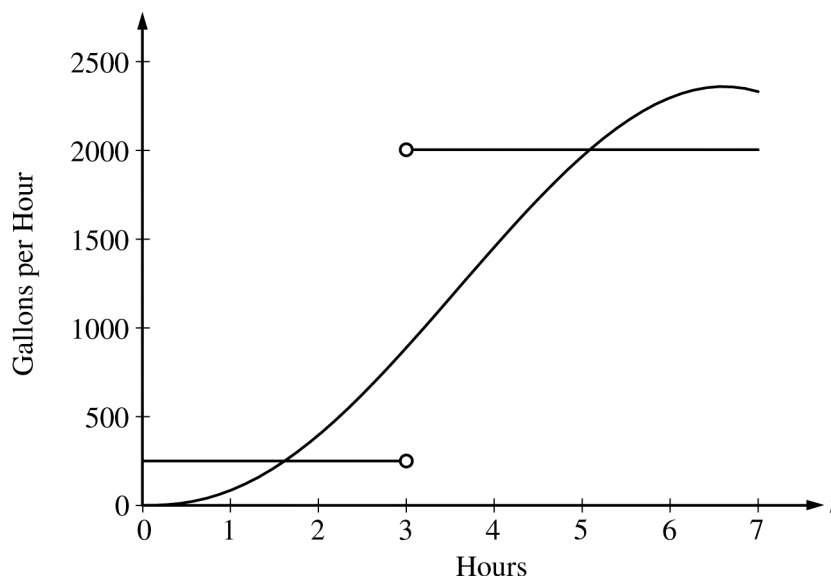
**CALCULUS BC**  
**SECTION II, Part A**  
**Time—45 minutes**  
**Number of problems—3**

**A graphing calculator is required for some problems or parts of problems.**

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1. Let  $R$  be the region in the first and second quadrants bounded above by the graph of  $y = \frac{20}{1+x^2}$  and below by the horizontal line  $y = 2$ .
- (a) Find the area of  $R$ .
- (b) Find the volume of the solid generated when  $R$  is rotated about the  $x$ -axis.
- (c) The region  $R$  is the base of a solid. For this solid, the cross sections perpendicular to the  $x$ -axis are semicircles. Find the volume of this solid.
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**WRITE ALL WORK IN THE PINK EXAM BOOKLET.**

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2. The amount of water in a storage tank, in gallons, is modeled by a continuous function on the time interval  $0 \leq t \leq 7$ , where  $t$  is measured in hours. In this model, rates are given as follows:

- (i) The rate at which water enters the tank is  $f(t) = 100t^2 \sin(\sqrt{t})$  gallons per hour for  $0 \leq t \leq 7$ .  
 (ii) The rate at which water leaves the tank is

$$g(t) = \begin{cases} 250 & \text{for } 0 \leq t < 3 \\ 2000 & \text{for } 3 < t \leq 7 \end{cases} \text{ gallons per hour.}$$

The graphs of  $f$  and  $g$ , which intersect at  $t = 1.617$  and  $t = 5.076$ , are shown in the figure above. At time  $t = 0$ , the amount of water in the tank is 5000 gallons.

- (a) How many gallons of water enter the tank during the time interval  $0 \leq t \leq 7$ ? Round your answer to the nearest gallon.  
 (b) For  $0 \leq t \leq 7$ , find the time intervals during which the amount of water in the tank is decreasing. Give a reason for each answer.  
 (c) For  $0 \leq t \leq 7$ , at what time  $t$  is the amount of water in the tank greatest? To the nearest gallon, compute the amount of water at this time. Justify your answer.

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**AP<sup>®</sup> CALCULUS BC**  
**2007 SCORING GUIDELINES**

**Question 1**

Let  $R$  be the region in the first and second quadrants bounded above by the graph of  $y = \frac{20}{1+x^2}$  and below by the horizontal line  $y = 2$ .

- (a) Find the area of  $R$ .  
 (b) Find the volume of the solid generated when  $R$  is rotated about the  $x$ -axis.  
 (c) The region  $R$  is the base of a solid. For this solid, the cross sections perpendicular to the  $x$ -axis are semicircles. Find the volume of this solid.

$$\frac{20}{1+x^2} = 2 \text{ when } x = \pm 3$$

$$(a) \text{ Area} = \int_{-3}^3 \left( \frac{20}{1+x^2} - 2 \right) dx = 37.961 \text{ or } 37.962$$

$$(b) \text{ Volume} = \pi \int_{-3}^3 \left( \left( \frac{20}{1+x^2} \right)^2 - 2^2 \right) dx = 1871.190$$

$$(c) \text{ Volume} = \frac{\pi}{2} \int_{-3}^3 \left( \frac{1}{2} \left( \frac{20}{1+x^2} - 2 \right) \right)^2 dx$$

$$= \frac{\pi}{8} \int_{-3}^3 \left( \frac{20}{1+x^2} - 2 \right)^2 dx = 174.268$$

1 : correct limits in an integral in  
 (a), (b), or (c)

2 :  $\begin{cases} 1 : \text{integrand} \\ 1 : \text{answer} \end{cases}$

3 :  $\begin{cases} 2 : \text{integrand} \\ 1 : \text{answer} \end{cases}$

3 :  $\begin{cases} 2 : \text{integrand} \\ 1 : \text{answer} \end{cases}$