Mech. 2.

In March 1999 the Mars Global Surveyor (GS) entered its final orbit about Mars, sending data back to Earth. Assume a circular orbit with a period of 1.18×10^2 minutes $= 7.08 \times 10^3$ s and orbital speed of 3.40×10^3 m/s. The mass of the GS is 930 kg and the radius of Mars is 3.43×10^6 m.

(a) Calculate the radius of the GS orbit.

(b) Calculate the mass of Mars.

(c) Calculate the total mechanical energy of the GS in this orbit.

(d) If the GS was to be placed in a lower circular orbit (closer to the surface of Mars), would the new orbital period of the GS be greater than or less than the given period?

_______Greater than _________Less than

Justify your answer.

(e) In fact, the orbit the GS entered was slightly elliptical with its closest approach to Mars at 3.71×10^5 m above the surface and its furthest distance at 4.36×10^5 m above the surface. If the speed of the GS at closest approach is 3.40×10^3 m/s, calculate the speed at the furthest point of the orbit.

Question 2

15 points total | **Distribution of points**

(a) 2 points

For a correct expression of the relationship among T, R, and υ — 1 point

$$T = \frac{2\pi R}{\upsilon}$$

$$R = \frac{\upsilon T}{2\pi}$$

$$R = \frac{\left(3.40\times10^{3}\ \text{m/s}\right)\left(7.08\times10^{3}\ \text{s}\right)}{2\pi}$$

For the correct numerical answer — 1 point

$$R = 3.83\times10^{6}\ \text{m}$$

(b) 2 points

For correctly equating centripetal force and gravitational force — 1 point

$$\frac{m_s\upsilon^2}{R} = \frac{Gm_sM_M}{R^2}$$

$$M_M = \frac{\upsilon^2 R}{G}$$

For substituting the value of R from (a) into either the original equation or the simplified expression for M_M above — 1 point

$$M_M = \frac{\left(3.40\times10^{3}\ \text{m/s}\right)^2\left(3.83\times10^{6}\ \text{m}\right)}{6.67\times10^{-11}\ \text{m}^3/\text{kg}\cdot\text{s}^2}$$

$$M_M = 6.64\times10^{23}\ \text{kg}$$

Question 2 (continued)

Distribution of points

(c) 4 points

For a correct expression that equates E_{tot} to the sum of kinetic and gravitational potential energies — 1 point

$E_{tot} = K + U$

For a negative sign on a correct expression for U_G — 1 point

$E_{tot} = \frac{1}{2}m_s\upsilon^2 - \frac{Gm_sM_M}{R}$

For explicit substitution of the value of *R* from (a) and the value of M_M from (b) in the equation above <u>or</u> for correct numerical answer if worked as follows: — 1 point

From (b), $M_M = \frac{\upsilon^2 R}{G}$

$E_{tot} = \frac{1}{2}m_s\upsilon^2 - \frac{Gm_s}{R}\frac{\upsilon^2 R}{G} = \frac{1}{2}m_s\upsilon^2 - m_s\upsilon^2 = -\frac{1}{2}m_s\upsilon^2$

$E_{tot} = -\frac{1}{2}(930\text{ kg})\left(3.40\times10^3\text{ m/s}\right)^2$

For a negative sign on the final answer — 1 point

$E_{tot} = -5.38\times10^9\text{ J}$

(d) 3 points

For correct selection of "Less than" check space — 1 point

For a correct justification — 2 points

Example 1: From Kepler's third law ($r^3/T^2 = \text{constant}$), if *r* decreases, then *T* must also decrease

Example 2: Use relationships among *R*, υ, and *T* with no incorrect physics such as the following: From (b), $M_M = \frac{\upsilon^2 R}{G}$, so as *R* decreases, υ must increase. From (a), $T = \frac{2\pi R}{\upsilon}$, so both a decrease in *R* and an increase in υ contribute to a decrease in *T*.

<u>Note</u>: 1 point partial credit was awarded for using only $T = \frac{2\pi R}{\upsilon}$, unless it was stated that υ was constant, in which case no credit was awarded.

Question 2 (continued)

Distribution of points

(e) 2 points

For a correct expression of conservation of angular momentum — 1 point

$m_s \upsilon_1 r_1 = m_s \upsilon_2 r_2$ or equivalent such as $I_1 \omega_1 = I_2 \omega_2$ or $\upsilon_1 r_1 = \upsilon_2 r_2$

$\upsilon_2 = \upsilon_1 \dfrac{r_1}{r_2} = \upsilon_1 \dfrac{R_C + R_M}{R_F + R_M}$, where R_C and R_F are the distances of closest and farthest approaches, respectively, and R_M is the radius of Mars

For explicit substitution of radii (not altitudes) into the equation <u>or</u> for the correct numerical answer — 1 point

$$\upsilon_2 = \left(3.40 \times 10^3 \text{ m/s}\right) \frac{3.71 \times 10^5 \text{ m} + 34.3 \times 10^5 \text{ m}}{4.36 \times 10^5 \text{ m} + 34.3 \times 10^5 \text{ m}}$$

$$\upsilon_2 = 3.34 \times 10^3 \text{ m/s}$$

Alternatively, if the longer approach using conservation of energy was taken, 1 point was awarded for a correct statement of conservation of energy if explicitly written as $\frac{1}{2} m_s \upsilon_1^2 - \frac{G m_s M_M}{r_1} = \frac{1}{2} m_s \upsilon_2^2 - \frac{G m_s M_M}{r_2}$ *, and 1 point was awarded for the explicit substitution of radii (not altitudes)* <u>*or*</u> *for a correct numerical answer.*

Units point

For including correct units on at least three numerical answers — 1 point

Significant figures point

For including less than five significant digits on at least three numerical answers for which a calculation was shown — 1 point