

## **2012 AP<sup>®</sup> CALCULUS AB FREE-RESPONSE QUESTIONS**

6. For  $0 \leq t \leq 12$ , a particle moves along the  $x$ -axis. The velocity of the particle at time  $t$  is given by  $v(t) = \cos\left(\frac{\pi}{6}t\right)$ . The particle is at position  $x = -2$  at time  $t = 0$ .
- (a) For  $0 \leq t \leq 12$ , when is the particle moving to the left?
- (b) Write, but do not evaluate, an integral expression that gives the total distance traveled by the particle from time  $t = 0$  to time  $t = 6$ .
- (c) Find the acceleration of the particle at time  $t$ . Is the speed of the particle increasing, decreasing, or neither at time  $t = 4$ ? Explain your reasoning.
- (d) Find the position of the particle at time  $t = 4$ .
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**STOP**

**END OF EXAM**

**AP<sup>®</sup> CALCULUS AB  
2012 SCORING GUIDELINES**

**Question 6**

For  $0 \leq t \leq 12$ , a particle moves along the  $x$ -axis. The velocity of the particle at time  $t$  is given by

$$v(t) = \cos\left(\frac{\pi}{6}t\right). \text{ The particle is at position } x = -2 \text{ at time } t = 0.$$

- (a) For  $0 \leq t \leq 12$ , when is the particle moving to the left?
- (b) Write, but do not evaluate, an integral expression that gives the total distance traveled by the particle from time  $t = 0$  to time  $t = 6$ .
- (c) Find the acceleration of the particle at time  $t$ . Is the speed of the particle increasing, decreasing, or neither at time  $t = 4$ ? Explain your reasoning.
- (d) Find the position of the particle at time  $t = 4$ .

(a)  $v(t) = \cos\left(\frac{\pi}{6}t\right) = 0 \Rightarrow t = 3, 9$

The particle is moving to the left when  $v(t) < 0$ .

This occurs when  $3 < t < 9$ .

(b)  $\int_0^6 |v(t)| dt$

(c)  $a(t) = -\frac{\pi}{6} \sin\left(\frac{\pi}{6}t\right)$

$$a(4) = -\frac{\pi}{6} \sin\left(\frac{2\pi}{3}\right) = -\frac{\sqrt{3}\pi}{12} < 0$$

$$v(4) = \cos\left(\frac{2\pi}{3}\right) = -\frac{1}{2} < 0$$

The speed is increasing at time  $t = 4$ , because velocity and acceleration have the same sign.

$$\begin{aligned} (d) \quad x(4) &= -2 + \int_0^4 \cos\left(\frac{\pi}{6}t\right) dt \\ &= -2 + \left[ \frac{6}{\pi} \sin\left(\frac{\pi}{6}t\right) \right]_0^4 \\ &= -2 + \frac{6}{\pi} \left[ \sin\left(\frac{2\pi}{3}\right) - 0 \right] \\ &= -2 + \frac{6}{\pi} \cdot \frac{\sqrt{3}}{2} = -2 + \frac{3\sqrt{3}}{\pi} \end{aligned}$$

2 :  $\begin{cases} 1 : \text{considers } v(t) = 0 \\ 1 : \text{interval} \end{cases}$

1 : answer

3 :  $\begin{cases} 1 : a(t) \\ 2 : \text{conclusion with reason} \end{cases}$

3 :  $\begin{cases} 1 : \text{antiderivative} \\ 1 : \text{uses initial condition} \\ 1 : \text{answer} \end{cases}$