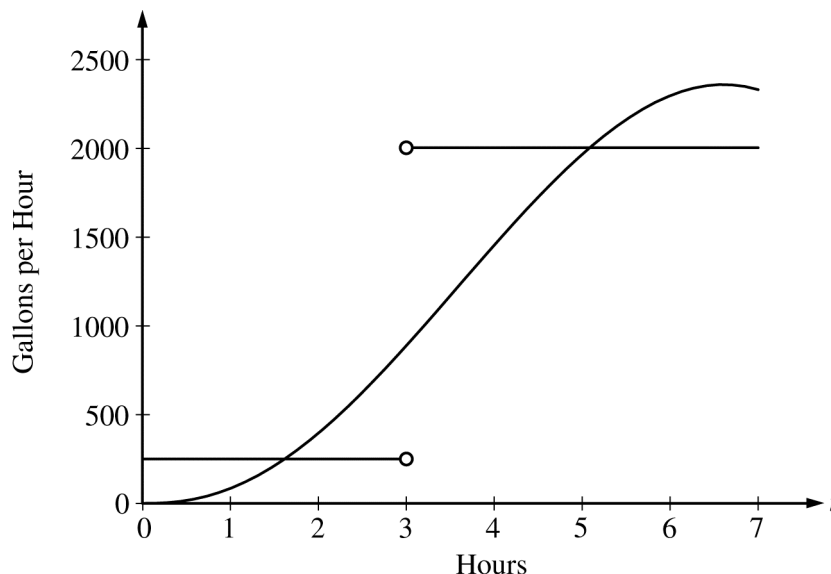


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2. The amount of water in a storage tank, in gallons, is modeled by a continuous function on the time interval  $0 \leq t \leq 7$ , where  $t$  is measured in hours. In this model, rates are given as follows:

- (i) The rate at which water enters the tank is  $f(t) = 100t^2 \sin(\sqrt{t})$  gallons per hour for  $0 \leq t \leq 7$ .  
 (ii) The rate at which water leaves the tank is

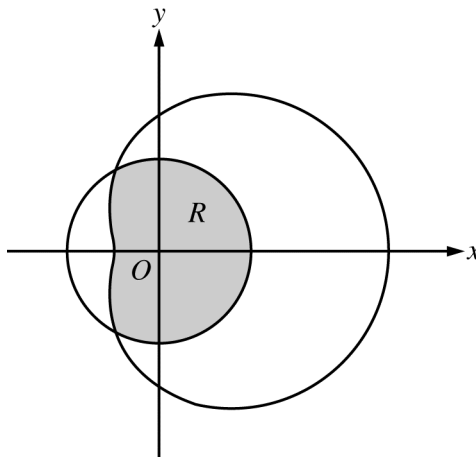
$$g(t) = \begin{cases} 250 & \text{for } 0 \leq t < 3 \\ 2000 & \text{for } 3 < t \leq 7 \end{cases} \text{ gallons per hour.}$$

The graphs of  $f$  and  $g$ , which intersect at  $t = 1.617$  and  $t = 5.076$ , are shown in the figure above. At time  $t = 0$ , the amount of water in the tank is 5000 gallons.

- (a) How many gallons of water enter the tank during the time interval  $0 \leq t \leq 7$ ? Round your answer to the nearest gallon.  
 (b) For  $0 \leq t \leq 7$ , find the time intervals during which the amount of water in the tank is decreasing. Give a reason for each answer.  
 (c) For  $0 \leq t \leq 7$ , at what time  $t$  is the amount of water in the tank greatest? To the nearest gallon, compute the amount of water at this time. Justify your answer.

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3. The graphs of the polar curves  $r = 2$  and  $r = 3 + 2\cos \theta$  are shown in the figure above. The curves intersect when  $\theta = \frac{2\pi}{3}$  and  $\theta = \frac{4\pi}{3}$ .
- (a) Let  $R$  be the region that is inside the graph of  $r = 2$  and also inside the graph of  $r = 3 + 2\cos \theta$ , as shaded in the figure above. Find the area of  $R$ .
- (b) A particle moving with nonzero velocity along the polar curve given by  $r = 3 + 2\cos \theta$  has position  $(x(t), y(t))$  at time  $t$ , with  $\theta = 0$  when  $t = 0$ . This particle moves along the curve so that  $\frac{dr}{dt} = \frac{dr}{d\theta}$ . Find the value of  $\frac{dr}{dt}$  at  $\theta = \frac{\pi}{3}$  and interpret your answer in terms of the motion of the particle.
- (c) For the particle described in part (b),  $\frac{dy}{dt} = \frac{dy}{d\theta}$ . Find the value of  $\frac{dy}{dt}$  at  $\theta = \frac{\pi}{3}$  and interpret your answer in terms of the motion of the particle.
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**WRITE ALL WORK IN THE PINK EXAM BOOKLET.**

**END OF PART A OF SECTION II**

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**2007 SCORING GUIDELINES**

**Question 2**

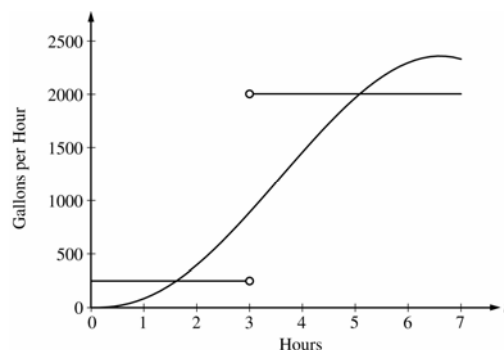
The amount of water in a storage tank, in gallons, is modeled by a continuous function on the time interval  $0 \leq t \leq 7$ , where  $t$  is measured in hours. In this model, rates are given as follows:

- (i) The rate at which water enters the tank is

$$f(t) = 100t^2 \sin(\sqrt{t}) \text{ gallons per hour for } 0 \leq t \leq 7.$$

- (ii) The rate at which water leaves the tank is

$$g(t) = \begin{cases} 250 & \text{for } 0 \leq t < 3 \\ 2000 & \text{for } 3 < t \leq 7 \end{cases} \text{ gallons per hour.}$$



The graphs of  $f$  and  $g$ , which intersect at  $t = 1.617$  and  $t = 5.076$ , are shown in the figure above. At time  $t = 0$ , the amount of water in the tank is 5000 gallons.

- (a) How many gallons of water enter the tank during the time interval  $0 \leq t \leq 7$ ? Round your answer to the nearest gallon.
- (b) For  $0 \leq t \leq 7$ , find the time intervals during which the amount of water in the tank is decreasing. Give a reason for each answer.
- (c) For  $0 \leq t \leq 7$ , at what time  $t$  is the amount of water in the tank greatest? To the nearest gallon, compute the amount of water at this time. Justify your answer.

(a)  $\int_0^7 f(t) dt \approx 8264$  gallons

2 :  $\begin{cases} 1 : \text{integral} \\ 1 : \text{answer} \end{cases}$

- (b) The amount of water in the tank is decreasing on the intervals  $0 \leq t \leq 1.617$  and  $3 \leq t \leq 5.076$  because  $f(t) < g(t)$  for  $0 \leq t < 1.617$  and  $3 < t < 5.076$ .

2 :  $\begin{cases} 1 : \text{intervals} \\ 1 : \text{reason} \end{cases}$

- (c) Since  $f(t) - g(t)$  changes sign from positive to negative only at  $t = 3$ , the candidates for the absolute maximum are at  $t = 0$ , 3, and 7.

5 :  $\begin{cases} 1 : \text{identifies } t = 3 \text{ as a candidate} \\ 1 : \text{integrand} \\ 1 : \text{amount of water at } t = 3 \\ 1 : \text{amount of water at } t = 7 \\ 1 : \text{conclusion} \end{cases}$

$t$ (hours)	gallons of water
0	5000
3	$5000 + \int_0^3 f(t) dt - 250(3) = 5126.591$
7	$5126.591 + \int_3^7 f(t) dt - 2000(4) = 4513.807$

The amount of water in the tank is greatest at 3 hours. At that time, the amount of water in the tank, rounded to the nearest gallon, is 5127 gallons.