

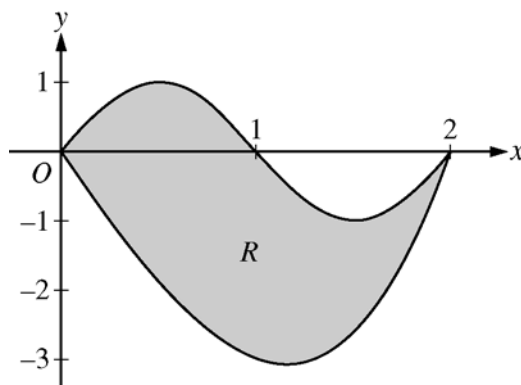
2008 AP[®] CALCULUS AB FREE-RESPONSE QUESTIONS

**CALCULUS AB
SECTION II, Part A**

Time—45 minutes

Number of problems—3

A graphing calculator is required for some problems or parts of problems.



1. Let R be the region bounded by the graphs of $y = \sin(\pi x)$ and $y = x^3 - 4x$, as shown in the figure above.
- (a) Find the area of R .
 - (b) The horizontal line $y = -2$ splits the region R into two parts. Write, but do not evaluate, an integral expression for the area of the part of R that is below this horizontal line.
 - (c) The region R is the base of a solid. For this solid, each cross section perpendicular to the x -axis is a square. Find the volume of this solid.
 - (d) The region R models the surface of a small pond. At all points in R at a distance x from the y -axis, the depth of the water is given by $h(x) = 3 - x$. Find the volume of water in the pond.
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WRITE ALL WORK IN THE PINK EXAM BOOKLET.

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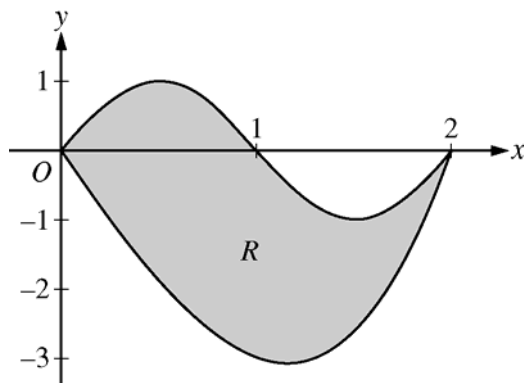
| | | | | | | | |
|-----------------|-----|-----|-----|-----|-----|----|---|
| t (hours) | 0 | 1 | 3 | 4 | 7 | 8 | 9 |
| $L(t)$ (people) | 120 | 156 | 176 | 126 | 150 | 80 | 0 |

2. Concert tickets went on sale at noon ($t = 0$) and were sold out within 9 hours. The number of people waiting in line to purchase tickets at time t is modeled by a twice-differentiable function L for $0 \leq t \leq 9$. Values of $L(t)$ at various times t are shown in the table above.
- (a) Use the data in the table to estimate the rate at which the number of people waiting in line was changing at 5:30 P.M. ($t = 5.5$). Show the computations that lead to your answer. Indicate units of measure.
- (b) Use a trapezoidal sum with three subintervals to estimate the average number of people waiting in line during the first 4 hours that tickets were on sale.
- (c) For $0 \leq t \leq 9$, what is the fewest number of times at which $L'(t)$ must equal 0? Give a reason for your answer.
- (d) The rate at which tickets were sold for $0 \leq t \leq 9$ is modeled by $r(t) = 550te^{-t/2}$ tickets per hour. Based on the model, how many tickets were sold by 3 P.M. ($t = 3$), to the nearest whole number?
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WRITE ALL WORK IN THE PINK EXAM BOOKLET.

AP[®] CALCULUS AB
2008 SCORING GUIDELINES

Question 1



Let R be the region bounded by the graphs of $y = \sin(\pi x)$ and $y = x^3 - 4x$, as shown in the figure above.

- Find the area of R .
- The horizontal line $y = -2$ splits the region R into two parts. Write, but do not evaluate, an integral expression for the area of the part of R that is below this horizontal line.
- The region R is the base of a solid. For this solid, each cross section perpendicular to the x -axis is a square. Find the volume of this solid.
- The region R models the surface of a small pond. At all points in R at a distance x from the y -axis, the depth of the water is given by $h(x) = 3 - x$. Find the volume of water in the pond.

(a) $\sin(\pi x) = x^3 - 4x$ at $x = 0$ and $x = 2$

$$\text{Area} = \int_0^2 (\sin(\pi x) - (x^3 - 4x)) dx = 4$$

$$3 : \begin{cases} 1 : \text{limits} \\ 1 : \text{integrand} \\ 1 : \text{answer} \end{cases}$$

(b) $x^3 - 4x = -2$ at $r = 0.5391889$ and $s = 1.6751309$

$$\text{The area of the stated region is } \int_r^s (-2 - (x^3 - 4x)) dx$$

$$2 : \begin{cases} 1 : \text{limits} \\ 1 : \text{integrand} \end{cases}$$

(c) $\text{Volume} = \int_0^2 (\sin(\pi x) - (x^3 - 4x))^2 dx = 9.978$

$$2 : \begin{cases} 1 : \text{integrand} \\ 1 : \text{answer} \end{cases}$$

(d) $\text{Volume} = \int_0^2 (3 - x)(\sin(\pi x) - (x^3 - 4x)) dx = 8.369 \text{ or } 8.370$

$$2 : \begin{cases} 1 : \text{integrand} \\ 1 : \text{answer} \end{cases}$$