

2004 AP[®] STATISTICS FREE-RESPONSE QUESTIONS

3. At an archaeological site that was an ancient swamp, the bones from 20 brontosaur skeletons have been unearthed. The bones do not show any sign of disease or malformation. It is thought that these animals wandered into a deep area of the swamp and became trapped in the swamp bottom. The 20 left femur bones (thigh bones) were located and 4 of these left femurs are to be randomly selected without replacement for DNA testing to determine gender.
- (a) Let X be the number out of the 4 selected left femurs that are from males. Based on how these bones were sampled, explain why the probability distribution of X is not binomial.
 - (b) Suppose that the group of 20 brontosaurs whose remains were found in the swamp had been made up of 10 males and 10 females. What is the probability that all 4 in the sample to be tested are male?
 - (c) The DNA testing revealed that all 4 femurs tested were from males. Based on this result and your answer from part (b), do you think that males and females were equally represented in the group of 20 brontosaurs stuck in the swamp? Explain.
 - (d) Is it reasonable to generalize your conclusion in part (c) pertaining to the group of 20 brontosaurs to the population of all brontosaurs? Explain why or why not.
4. Two antibiotics are available as treatment for a common ear infection in children.
- Antibiotic A is known to effectively cure the infection 60 percent of the time. Treatment with antibiotic A costs \$50.
 - Antibiotic B is known to effectively cure the infection 90 percent of the time. Treatment with antibiotic B costs \$80.

The antibiotics work independently of one another. Both antibiotics can be safely administered to children. A health insurance company intends to recommend one of the following two plans of treatment for children with this ear infection.

- Plan I: Treat with antibiotic A first. If it is not effective, then treat with antibiotic B.
 - Plan II: Treat with antibiotic B first. If it is not effective, then treat with antibiotic A.
- (a) If a doctor treats a child with an ear infection using plan I, what is the probability that the child will be cured?
- If a doctor treats a child with an ear infection using plan II, what is the probability that the child will be cured?
- (b) Compute the expected cost per child when plan I is used for treatment.
- Compute the expected cost per child when plan II is used for treatment.
- (c) Based on the results in parts (a) and (b), which plan would you recommend?
- Explain your recommendation.

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2004 SCORING GUIDELINES

Question 3

Solution

Part (a):

X is not binomial since the trials are not independent and the conditional probabilities of selecting a male change at each trial depending on the previous outcome(s), due to the sampling without replacement.

Part (b):

$$P(X = 4) = \left(\frac{10}{20}\right)\left(\frac{9}{19}\right)\left(\frac{8}{18}\right)\left(\frac{7}{17}\right) = \frac{5040}{116280} = 0.043$$

Part (c):

No. If males and females were equally represented, the probability of observing four males is small (0.043).

Part (d):

No, we can't generalize to the population of all brontosaurs because it is not reasonable to regard this sample as a random sample from the population of all brontosaurs; there is reason to suspect that this sampling method might cause bias.

Scoring

Parts (a), (b), and (c) are scored as essentially correct, partially correct, or incorrect. Part (d) is scored as essentially correct or incorrect.

Part (a): Score as:

Essentially correct if the response indicates that

- (i) trials are not independent, with an explanation that independence means the outcome on any trial will not impact the probability of success on future trials OR
- (ii) the probability of selecting a male on any given trial depends on the results of previous trials.

Partially correct if the response indicates that

- (i) the student is focusing on one of the concepts above, but discussion is weak OR
- (ii) there is sampling without replacement without connection to one of the concepts under Essentially Correct above.

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Question 3 (cont'd.)

Part (b): Score as:

Essentially correct if the probability is correctly computed (with minor arithmetic errors being overlooked), with supporting work or rationale. A statement that this is a hypergeometric distribution (either in Part(a) or Part(b)) will suffice. It is OK if the student leaves the answer as the product of fractions. The probability that all four femurs belong to males can also be

computed by using the formula
$$P(X = 4) = \frac{\binom{10}{4} \binom{10}{0}}{\binom{20}{4}} = .043.$$

Partially correct if there is a correct answer (to 3 decimal places) with incomplete justification.

Incorrect if arithmetic errors result in a probability that is negative or greater than one.

Part (c): Score as:

Essentially correct if the probability provided in Part (b) is interpreted correctly.

Partially correct if it is not clear that the student used the probability from Part (b).

Incorrect if just a “Yes” or “No” is given without an explanation.

Part (d): Score as:

Essentially correct if the response indicates that generalization is not possible because this sample

- (i) cannot be viewed as a random sample of all brontosaurus OR
- (ii) there is reason to suspect that this sample might not be representative of the population at large.

Incorrect if “No” is given without an explanation.

Note: Discussions about conditions for inference are irrelevant.

Each essentially correct response is worth 1 point; each partially correct answer is worth ½ point.

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Question 3 (cont'd.)

- 4 Complete Response**
- 3 Substantial Response**
- 2 Developing Response**
- 1 Minimal Response**

IF A PAPER IS BETWEEN TWO SCORES (FOR EXAMPLE, 2½ POINTS) USE A HOLISTIC APPROACH TO DETERMINE WHETHER TO SCORE UP OR DOWN DEPENDING ON THE STRENGTH OF THE RESPONSE AND COMMUNICATION.