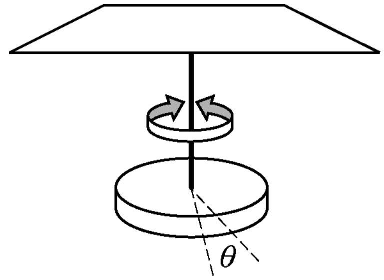


Mech. 3.

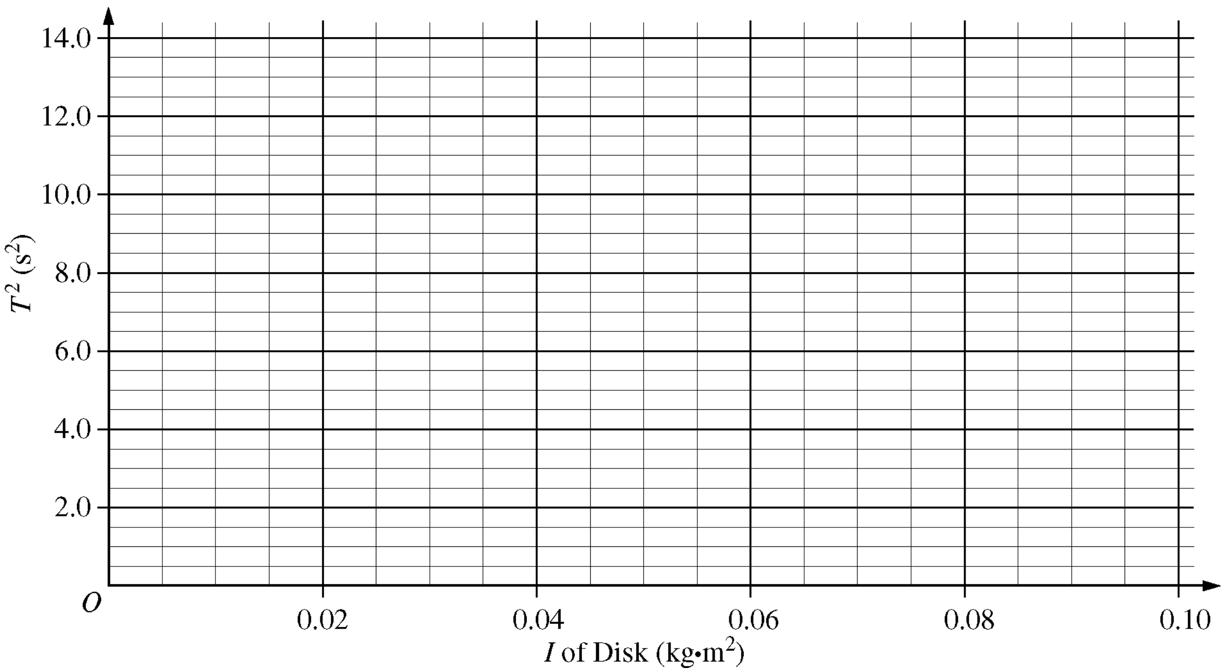
The torsion pendulum shown above consists of a disk of rotational inertia I suspended by a flexible rod attached to a rigid support. When the disk is twisted through a small angle θ, the twisted rod exerts a restoring torque τ that is proportional to the angular displacement: $\tau = -\beta\theta$, where β is a constant. The motion of a torsion pendulum is analogous to the motion of a mass oscillating on a spring.

(a) In terms of the quantities given above, write but do NOT solve the differential equation that could be used to determine the angular displacement θ of the torsion pendulum as a function of time t.
(b) Using the analogy to a mass oscillating on a spring, determine the period of the torsion pendulum in terms of the given quantities and fundamental constants, as appropriate.

To determine the torsion constant β of the rod, disks of different, known values of rotational inertia are attached to the rod, and the data below are obtained from the resulting oscillations.

Rotational Inertia I of Disk $(\text{kg}\cdot\text{m}^2)$	Average Time for Ten Oscillations (s)	Period T (s)	T^2 (s^2)
0.025	22.4	2.24	5.0
0.036	26.8	2.68	7.2
0.049	29.5	2.95	8.7
0.064	33.3	3.33	11.1
0.081	35.9	3.59	12.9

(c) On the graph below, plot the data points. Draw a straight line that best represents the data.



(d) Determine the equation for your line.

(e) Calculate the torsion constant β of the rod from your line.

(f) What is the physical significance of the intercept of your line with the vertical axis?

END OF EXAM

Question 3

15 points total | **Distribution of points**

(a) 3 points

For a statement of Newton's second law for rotation — 1 point

$\Sigma\tau = I\alpha$

For substituting the given torque expression for the net torque $\Sigma\tau$ — 1 point

$I\alpha = -\beta\theta$

For substituting the second derivative of angular position for angular acceleration — 1 point

$I\dfrac{d^2\theta}{dt^2} = -\beta\theta$

(b) 3 points

Applying Newton's second law for translation to a mass on a spring gives

$m\dfrac{d^2x}{dt^2} = -kx$, and $\omega = \sqrt{\dfrac{k}{m}}$.

For this torsion pendulum, $I\dfrac{d^2\theta}{dt^2} = -\beta\theta$.

Comparing differential equations, I is analogous to m and β is analogous to k.

For the correct expression for ω — 1 point

$\omega = \sqrt{\dfrac{\beta}{I}}$

For the correct relationship between ω and T — 1 point

$T = \dfrac{2\pi}{\omega}$

For the correct answer — 1 point

$T = 2\pi\sqrt{\dfrac{I}{\beta}}$

Alternate Solution — *Alternate points*

The period of a mass on a spring is $T = 2\pi\sqrt{\dfrac{m}{k}}$.

For recognizing that I *is analogous to* m — *1 point*

For recognizing that β *is analogous to* k — *1 point*

For the correct answer — *1 point*

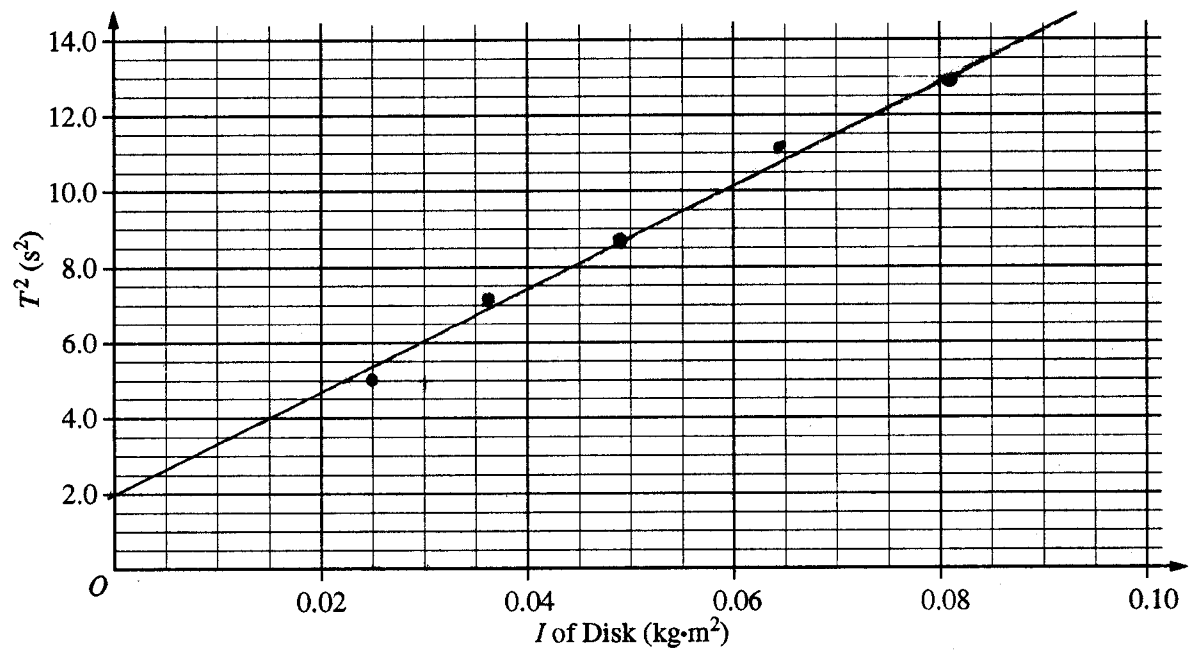
$T = 2\pi\sqrt{\dfrac{I}{\beta}}$

Question 3 (continued)

Distribution of points

(c) 2 points

Sample graph



For correctly plotting the data — 1 point

For drawing a reasonable, best-fit straight line — 1 point

Note: For correctly plotted data, a reasonable, best-fit straight line does NOT pass through the origin.

(d) 3 points

The general equation for a straight line is $y(x) = mx + b$, where m is the slope and b is the y-intercept.

$T^2 = mI + b$

$m = \Delta\left(T^2\right)/\Delta I$

For using two points from the best-fit line to calculate the slope — 1 point

Example from the graph shown: $m = \dfrac{\left(11.5\ \text{s}^2 - 2.0\ \text{s}^2\right)}{\left(0.07\ \text{kg}\bullet\text{m}^2 - 0.00\ \text{kg}\bullet\text{m}^2\right)}$

$m = 135\ \text{s}^2/\text{kg}\bullet\text{m}^2$

For an intercept calculated or directly read from the graph — 1 point

$b = 2.0\ \text{s}^2$

For using the variables T^2 and I in the equation — 1 point

$T^2 = \left(135\ \text{s}^2/\text{kg}\bullet\text{m}^2\right)I + 2.0\ \text{s}^2$

Question 3 (continued)

Distribution of points

(e) 3 points

Using the equation from part (b)

$$T = 2\pi\sqrt{\frac{I}{\beta}}$$

$$T^2 = 4\pi^2 \frac{I}{\beta} = \frac{4\pi^2}{\beta} I$$

For comparing this to part (d) and noting that $\frac{4\pi^2}{\beta}$ is the slope of the line — 1 point

$$\frac{4\pi^2}{\beta} = m$$

For using the value of the slope determined in part (d) — 1 point

$$\beta = \frac{4\pi^2}{m} = \frac{4\pi^2}{135\ \text{s}^2/\text{kg}{\bullet}\text{m}^2}$$

$$\beta = 0.292\ \text{kg}{\bullet}\text{m}^2/\text{s}^2$$

For the correct units on the numerical answer — 1 point

(f) 1 point

For a correct physical explanation for the intercept that mentions the effect of the flexible rod — 1 point

Example: The intercept is the square of the period of oscillation of the flexible rod.

Question 3

15 points total | **Distribution of points**

(a) 3 points

For a statement of Newton's second law for rotation — 1 point

$\Sigma\tau = I\alpha$

For substituting the given torque expression for the net torque $\Sigma\tau$ — 1 point

$I\alpha = -\beta\theta$

For substituting the second derivative of angular position for angular acceleration — 1 point

$I\dfrac{d^2\theta}{dt^2} = -\beta\theta$

(b) 3 points

Applying Newton's second law for translation to a mass on a spring gives

$m\dfrac{d^2x}{dt^2} = -kx$, and $\omega = \sqrt{\dfrac{k}{m}}$.

For this torsion pendulum, $I\dfrac{d^2\theta}{dt^2} = -\beta\theta$.

Comparing differential equations, I is analogous to m and β is analogous to k.

For the correct expression for ω — 1 point

$\omega = \sqrt{\dfrac{\beta}{I}}$

For the correct relationship between ω and T — 1 point

$T = \dfrac{2\pi}{\omega}$

For the correct answer — 1 point

$T = 2\pi\sqrt{\dfrac{I}{\beta}}$

Alternate Solution | *Alternate points*

The period of a mass on a spring is $T = 2\pi\sqrt{\dfrac{m}{k}}$.

For recognizing that I is analogous to m — *1 point*

For recognizing that β *is analogous to k* — *1 point*

For the correct answer — *1 point*

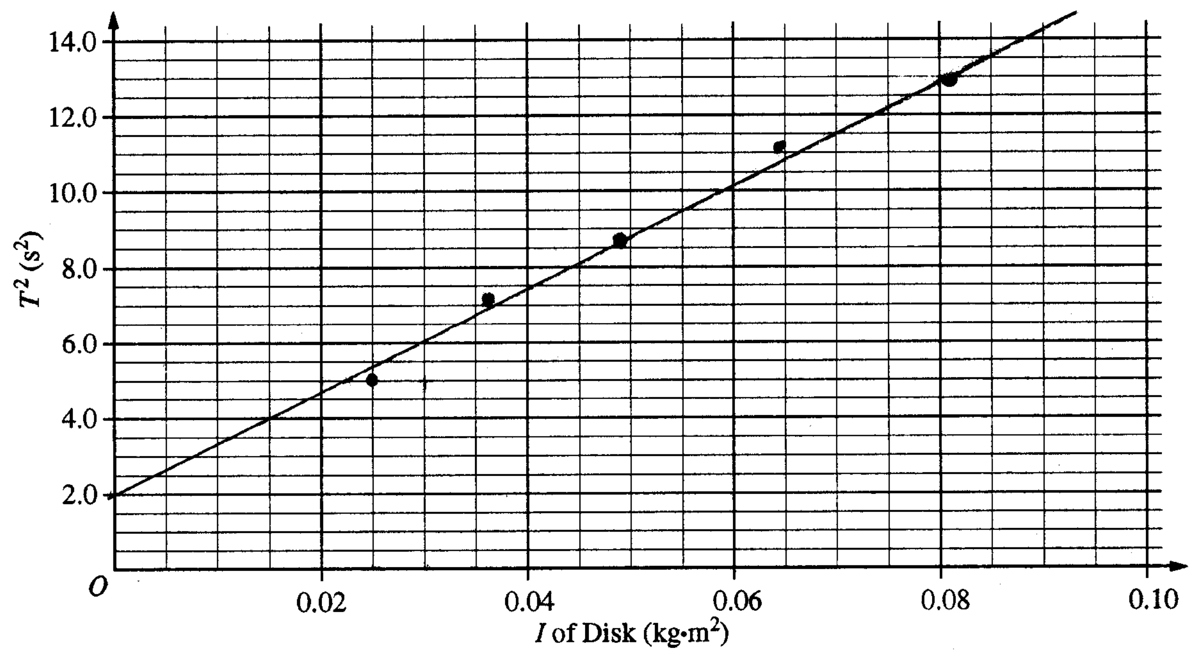
$T = 2\pi\sqrt{\dfrac{I}{\beta}}$

Question 3 (continued)

Distribution of points

(c) 2 points

Sample graph



For correctly plotting the data — 1 point
For drawing a reasonable, best-fit straight line — 1 point
Note: For correctly plotted data, a reasonable, best-fit straight line does NOT pass through the origin.

(d) 3 points

The general equation for a straight line is $y(x) = mx + b$, where m is the slope and b is the y-intercept.

$T^2 = mI + b$

$m = \Delta\left(T^2\right)/\Delta I$

For using two points from the best-fit line to calculate the slope — 1 point

Example from the graph shown: $m = \dfrac{\left(11.5\ \text{s}^2 - 2.0\ \text{s}^2\right)}{\left(0.07\ \text{kg}\bullet\text{m}^2 - 0.00\ \text{kg}\bullet\text{m}^2\right)}$

$m = 135\ \text{s}^2/\text{kg}\bullet\text{m}^2$

For an intercept calculated or directly read from the graph — 1 point

$b = 2.0\ \text{s}^2$

For using the variables T^2 and I in the equation — 1 point

$T^2 = \left(135\ \text{s}^2/\text{kg}\bullet\text{m}^2\right)I + 2.0\ \text{s}^2$

Question 3 (continued)

Distribution of points

(e) 3 points

Using the equation from part (b)

$$T = 2\pi\sqrt{\frac{I}{\beta}}$$

$$T^2 = 4\pi^2\frac{I}{\beta} = \frac{4\pi^2}{\beta}I$$

For comparing this to part (d) and noting that $\frac{4\pi^2}{\beta}$ is the slope of the line — 1 point

$$\frac{4\pi^2}{\beta} = m$$

For using the value of the slope determined in part (d) — 1 point

$$\beta = \frac{4\pi^2}{m} = \frac{4\pi^2}{135\ \mathrm{s^2/kg{\bullet}m^2}}$$

$$\beta = 0.292\ \mathrm{kg{\bullet}m^2/s^2}$$

For the correct units on the numerical answer — 1 point

(f) 1 point

For a correct physical explanation for the intercept that mentions the effect of the flexible rod — 1 point

Example: The intercept is the square of the period of oscillation of the flexible rod.