CALCULUS BC

SECTION II, Part B

Time—45 minutes

Number of problems—3

No calculator is allowed for these problems.

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4. A moving particle has position $(x(t),\ y(t))$ at time t. The position of the particle at time $t = 1$ is $(2,6)$, and the velocity vector at any time $t > 0$ is given by $\left(1 - \frac{1}{t^2}, 2 + \frac{1}{t^2}\right)$.

   (a) Find the acceleration vector at time $t = 3$.

   (b) Find the position of the particle at time $t = 3$.

   (c) For what time $t > 0$ does the line tangent to the path of the particle at $(x(t),\ y(t))$ have a slope of 8 ?

   (d) The particle approaches a line as $t \to \infty$. Find the slope of this line. Show the work that leads to your conclusion.

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5. Consider the curve given by $xy^2 - x^3y = 6$.

   (a) Show that $\frac{dy}{dx} = \frac{3x^2y - y^2}{2xy - x^3}$.

   (b) Find all points on the curve whose x-coordinate is 1, and write an equation for the tangent line at each of these points.

   (c) Find the x-coordinate of each point on the curve where the tangent line is vertical.

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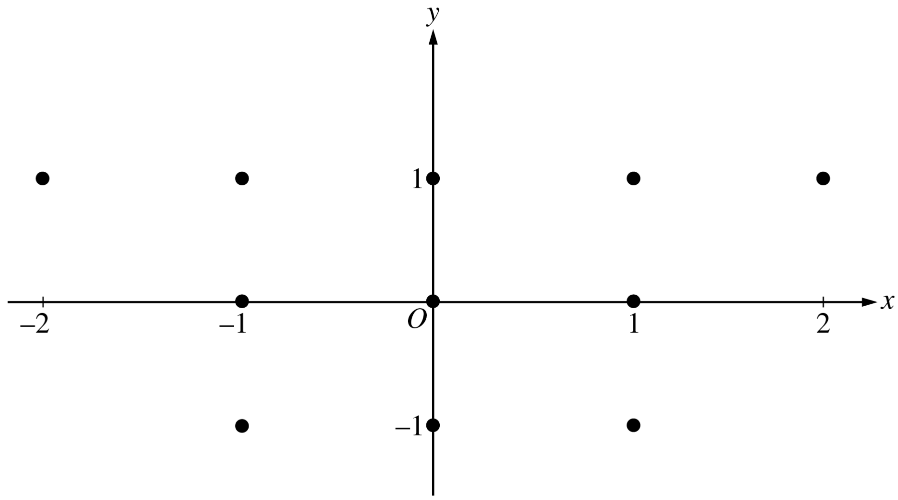
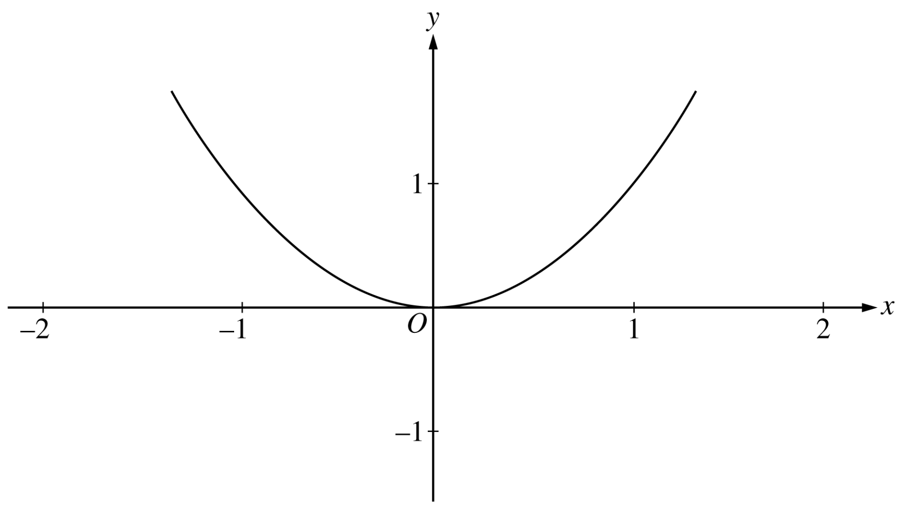
6. Consider the differential equation given by $\frac{dy}{dx} = x(y-1)^2$.

   (a) On the axes provided, sketch a slope field for the given differential equation at the eleven points indicated.

   (Note: Use the axes provided in the pink test booklet.)



   (b) Use the slope field for the given differential equation to explain why a solution could not have the graph shown below.



   (c) Find the particular solution $y = f(x)$ to the given differential equation with the initial condition $f(0) = -1$.

   (d) Find the range of the solution found in part (c).

END OF EXAMINATION

Consider the curve given by $xy^2 - x^3y = 6$.

(a) Show that $\dfrac{dy}{dx} = \dfrac{3x^2y - y^2}{2xy - x^3}$.

(b) Find all points on the curve whose x-coordinate is 1, and write an equation for the tangent line at each of these points.

(c) Find the x-coordinate of each point on the curve where the tangent line is vertical.

(a) $y^2 + 2xy\dfrac{dy}{dx} - 3x^2y - x^3\dfrac{dy}{dx} = 0$

$\dfrac{dy}{dx}\left(2xy - x^3\right) = 3x^2y - y^2$

$\dfrac{dy}{dx} = \dfrac{3x^2y - y^2}{2xy - x^3}$

$2\begin{cases} 1: \text{implicit differentiation} \\ 1: \text{verifies expression for } \dfrac{dy}{dx} \end{cases}$

(b) When $x = 1$, $y^2 - y = 6$

$y^2 - y - 6 = 0$

$(y-3)(y+2) = 0$

$y = 3,\ y = -2$

At $(1,3)$, $\dfrac{dy}{dx} = \dfrac{9-9}{6-1} = 0$

Tangent line equation is $y = 3$

At $(1,-2)$, $\dfrac{dy}{dx} = \dfrac{-6-4}{-4-1} = \dfrac{-10}{-5} = 2$

Tangent line equation is $y + 2 = 2(x-1)$

$4\begin{cases} 1: & y^2 - y = 6 \\ 1: & \text{solves for } y \\ 2: & \text{tangent lines} \end{cases}$

Note: 0/4 if not solving an equation of the form $y^2 - y = k$

(c) Tangent line is vertical when $2xy - x^3 = 0$

$x\left(2y - x^2\right) = 0$ gives $x = 0$ or $y = \dfrac{1}{2}x^2$

There is no point on the curve with x-coordinate 0.

When $y = \dfrac{1}{2}x^2$, $\dfrac{1}{4}x^5 - \dfrac{1}{2}x^5 = 6$

$-\dfrac{1}{4}x^5 = 6$

$x = \sqrt[5]{-24}$

$3\begin{cases} 1: & \text{sets denominator of } \dfrac{dy}{dx} \text{ equal to 0} \\ 1: & \text{substitutes } y = \dfrac{1}{2}x^2 \text{ or } x = \pm\sqrt{2y} \\ & \text{into the equation for the curve} \\ 1: & \text{solves for } x\text{-coordinate} \end{cases}$