

2011 AP[®] CALCULUS BC FREE-RESPONSE QUESTIONS

**CALCULUS BC
SECTION II, Part A
Time—30 minutes
Number of problems—2**

A graphing calculator is required for these problems.

1. At time t , a particle moving in the xy -plane is at position $(x(t), y(t))$, where $x(t)$ and $y(t)$ are not explicitly given. For $t \geq 0$, $\frac{dx}{dt} = 4t + 1$ and $\frac{dy}{dt} = \sin(t^2)$. At time $t = 0$, $x(0) = 0$ and $y(0) = -4$.
 - (a) Find the speed of the particle at time $t = 3$, and find the acceleration vector of the particle at time $t = 3$.
 - (b) Find the slope of the line tangent to the path of the particle at time $t = 3$.
 - (c) Find the position of the particle at time $t = 3$.
 - (d) Find the total distance traveled by the particle over the time interval $0 \leq t \leq 3$.

WRITE ALL WORK IN THE EXAM BOOKLET.

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| | | | | | |
|-----------------------------|----|----|----|----|----|
| t (minutes) | 0 | 2 | 5 | 9 | 10 |
| $H(t)$ (degrees Celsius) | 66 | 60 | 52 | 44 | 43 |

2. As a pot of tea cools, the temperature of the tea is modeled by a differentiable function H for $0 \leq t \leq 10$, where time t is measured in minutes and temperature $H(t)$ is measured in degrees Celsius. Values of $H(t)$ at selected values of time t are shown in the table above.
- (a) Use the data in the table to approximate the rate at which the temperature of the tea is changing at time $t = 3.5$. Show the computations that lead to your answer.
- (b) Using correct units, explain the meaning of $\frac{1}{10} \int_0^{10} H(t) dt$ in the context of this problem. Use a trapezoidal sum with the four subintervals indicated by the table to estimate $\frac{1}{10} \int_0^{10} H(t) dt$.
- (c) Evaluate $\int_0^{10} H'(t) dt$. Using correct units, explain the meaning of the expression in the context of this problem.
- (d) At time $t = 0$, biscuits with temperature 100°C were removed from an oven. The temperature of the biscuits at time t is modeled by a differentiable function B for which it is known that $B'(t) = -13.84e^{-0.173t}$. Using the given models, at time $t = 10$, how much cooler are the biscuits than the tea?

WRITE ALL WORK IN THE EXAM BOOKLET.**END OF PART A OF SECTION II**

**AP[®] CALCULUS BC
2011 SCORING GUIDELINES**

Question 1

At time t , a particle moving in the xy -plane is at position $(x(t), y(t))$, where $x(t)$ and $y(t)$ are not explicitly given. For $t \geq 0$, $\frac{dx}{dt} = 4t + 1$ and $\frac{dy}{dt} = \sin(t^2)$. At time $t = 0$, $x(0) = 0$ and $y(0) = -4$.

- (a) Find the speed of the particle at time $t = 3$, and find the acceleration vector of the particle at time $t = 3$.
- (b) Find the slope of the line tangent to the path of the particle at time $t = 3$.
- (c) Find the position of the particle at time $t = 3$.
- (d) Find the total distance traveled by the particle over the time interval $0 \leq t \leq 3$.

(a) Speed = $\sqrt{(x'(3))^2 + (y'(3))^2} = 13.006$ or 13.007
 Acceleration = $\langle x''(3), y''(3) \rangle$
 $= \langle 4, -5.466 \rangle$ or $\langle 4, -5.467 \rangle$

2 : $\begin{cases} 1 : \text{speed} \\ 1 : \text{acceleration} \end{cases}$

(b) Slope = $\frac{y'(3)}{x'(3)} = 0.031$ or 0.032

1 : answer

(c) $x(3) = 0 + \int_0^3 \frac{dx}{dt} dt = 21$

$y(3) = -4 + \int_0^3 \frac{dy}{dt} dt = -3.226$

At time $t = 3$, the particle is at position $(21, -3.226)$.

4 : $\begin{cases} 2 : x\text{-coordinate} \\ 1 : \text{integral} \\ 1 : \text{answer} \\ 2 : y\text{-coordinate} \\ 1 : \text{integral} \\ 1 : \text{answer} \end{cases}$

(d) Distance = $\int_0^3 \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} dt = 21.091$

2 : $\begin{cases} 1 : \text{integral} \\ 1 : \text{answer} \end{cases}$