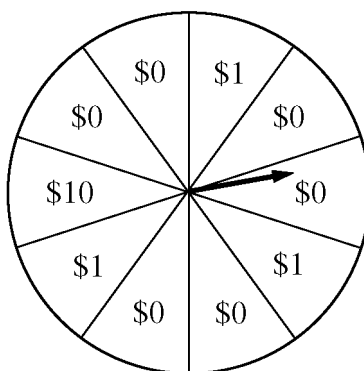


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2. A charity fundraiser has a Spin the Pointer game that uses a spinner like the one illustrated in the figure below.



A donation of \$2 is required to play the game. For each \$2 donation, a player spins the pointer once and receives the amount of money indicated in the sector where the pointer lands on the wheel. The spinner has an equal probability of landing in each of the 10 sectors.

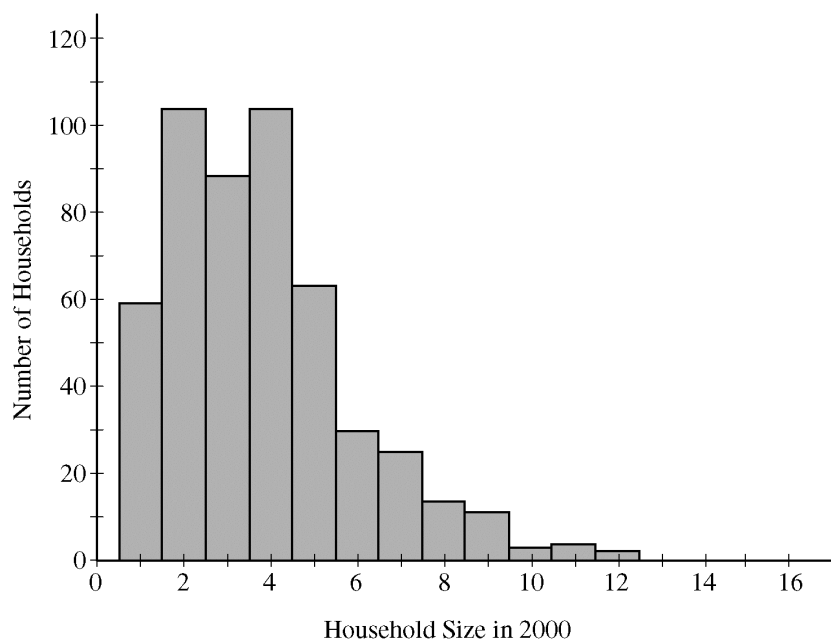
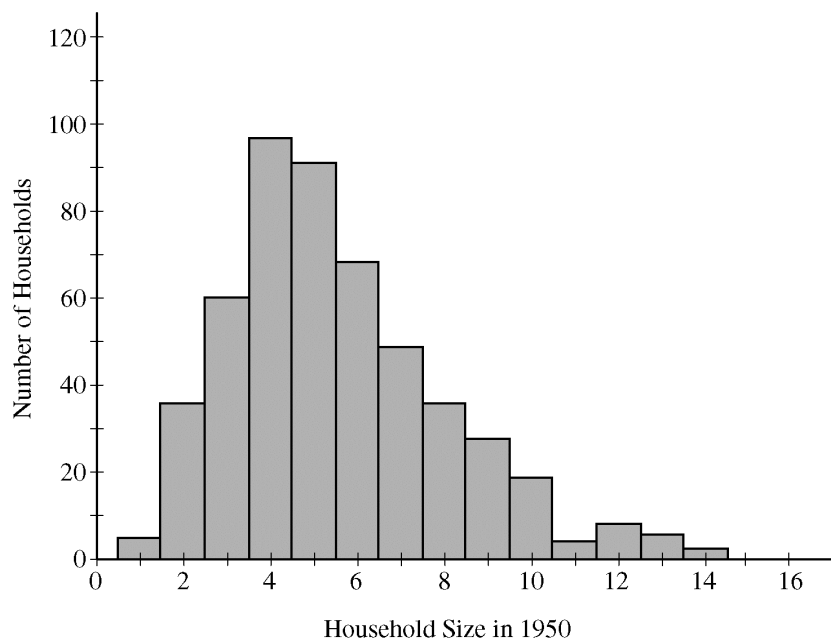
- (a) Let X represent the net contribution to the charity when one person plays the game once. Complete the table for the probability distribution of X .

x	\$2	\$1	-\$8
$P(x)$			

- (b) What is the expected value of the net contribution to the charity for one play of the game?
- (c) The charity would like to receive a net contribution of \$500 from this game. What is the fewest number of times the game must be played for the expected value of the net contribution to be at least \$500 ?
- (d) Based on last year's event, the charity anticipates that the Spin the Pointer game will be played 1,000 times. The charity would like to know the probability of obtaining a net contribution of at least \$500 in 1,000 plays of the game. The mean and standard deviation of the net contribution to the charity in 1,000 plays of the game are \$700 and \$92.79, respectively. Use the normal distribution to approximate the probability that the charity would obtain a net contribution of at least \$500 in 1,000 plays of the game.

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3. Independent random samples of 500 households were taken from a large metropolitan area in the United States for the years 1950 and 2000. Histograms of household size (number of people in a household) for the years are shown below.



- (a) Compare the distributions of household size in the metropolitan area for the years 1950 and 2000.
- (b) A researcher wants to use these data to construct a confidence interval to estimate the change in mean household size in the metropolitan area from the year 1950 to the year 2000. State the conditions for using a two-sample t -procedure, and explain whether the conditions for inference are met.

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Question 2

Intent of Question

The primary goals of this question were to assess students' ability to (1) perform calculations and compute expected values related to a discrete probability distribution; (2) implement a normal approximation based on the central limit theorem.

Solution

Part (a):

By counting the number of sectors for each value and dividing by 10, the probability distribution is calculated to be:

x	\$2	\$1	-\$8
$P(x)$	0.6	0.3	0.1

Part (b):

The expected value of the net contribution for one play of the game is:

$$E(x) = \$2(0.6) + \$1(0.3) + (-\$8)(0.1) = \$0.70 \text{ (or 70 cents).}$$

Part (c):

The expected contribution after n plays is $\$0.70n$. Setting this to be at least \$500 and solving for n gives:

$$0.70n \geq 500, \text{ so } n \geq \frac{500}{0.70} \approx 714.286,$$

so 715 plays are needed for the expected contribution to be at least \$500.

Part (d):

The normal approximation is appropriate because the very large sample size ($n = 1,000$) ensures that the central limit theorem holds. Therefore, the sample mean of the contributions from 1,000 plays has an approximately normal distribution, and so the sum of the contributions from 1,000 plays also has an approximately normal distribution.

$$\text{The z-score is } \frac{500 - 700}{92.79} \approx -2.155.$$

The probability that a standard normal random variable exceeds this z-score of -2.155 is 0.9844. Therefore, the charity can be very confident about gaining a net contribution of at least \$500 from 1,000 plays of the game.

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Question 2 (continued)

Scoring

This question is scored in three sections. Section 1 consists of parts (a) and (b); section 2 consists of part (c); and section 3 consists of part (d). Sections 1, 2, and 3 are scored as essentially correct (E), partially correct (P), or incorrect (I).

Section 1 is scored as follows:

Essentially correct (E) if all three probabilities are filled in correctly in the table in part (a) *AND* the expected value is calculated correctly in part (b), with work shown.

Partially correct (P) if all three probabilities are filled in correctly in the table in part (a) *AND* the expected value is not calculated correctly in part (b),

OR

the probabilities in part (a) are not all correct *AND* the expected value in part (b) is calculated appropriately from the probabilities given in part (a) or from the correct probabilities.

Incorrect (I) if the response does not meet the criteria for E or P.

Section 2 is scored as follows:

Essentially correct (E) if the response addresses the following two components:

1. Provides a solution based on a reasonable calculation, equation, or inequality from the answer given in part (b).
2. Clearly selects the next higher integer as the answer.

Partially correct (P) if the response correctly completes component (1) listed above but not component (2).

Incorrect (I) if the response does not meet the criteria for E or P.

Section 3 is scored as follows:

Essentially correct (E) if the response correctly addresses the following three components:

1. Indicates the use of a normal distribution with the correct mean and standard deviation.
2. Uses the correct boundary and indicates the correct direction.
3. Has the correct normal probability consistent with components (1) and (2).

Partially correct (P) if the response correctly addresses exactly two of the three components listed above.

Incorrect (I) if the response does not meet the criteria for E or P.

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Question 2 (continued)

Notes

- Because the question asks students to use a normal distribution and specifies the parameter values, the response does not have to justify the normal approximation or show how to calculate the parameter values.
- If the response earns credit for component (1) but no direction has been provided for component (2), then the response earns credit for component (3) if the correct probability of 0.9844 is reported.
- If the response does not earn credit for component (1) owing to incorrect identification of the mean and/or standard deviation, then the response can still earn credit for component (2) if the boundary is calculated correctly from the mean and standard deviation indicated in component (1).

4 Complete Response

All three sections essentially correct

3 Substantial Response

Two sections essentially correct and one section partially correct

2 Developing Response

Two sections essentially correct and one section incorrect

OR

One section essentially correct and one or two sections partially correct

OR

Three sections partially correct

1 Minimal Response

One section essentially correct and two sections incorrect

OR

Two sections partially correct and one section incorrect