

**2013 AP® CALCULUS AB FREE-RESPONSE QUESTIONS**

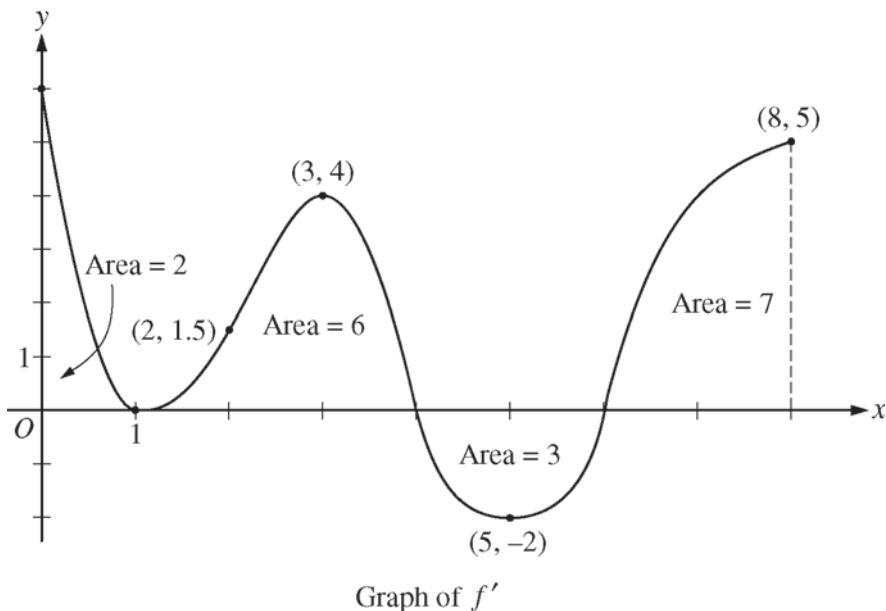
**CALCULUS AB**  
**SECTION II, Part B**  
**Time—60 minutes**  
**Number of problems—4**

**No calculator is allowed for these problems.**

$t$ (minutes)	0	1	2	3	4	5	6
$C(t)$ (ounces)	0	5.3	8.8	11.2	12.8	13.8	14.5

3. Hot water is dripping through a coffeemaker, filling a large cup with coffee. The amount of coffee in the cup at time  $t$ ,  $0 \leq t \leq 6$ , is given by a differentiable function  $C$ , where  $t$  is measured in minutes. Selected values of  $C(t)$ , measured in ounces, are given in the table above.
- (a) Use the data in the table to approximate  $C'(3.5)$ . Show the computations that lead to your answer, and indicate units of measure.
- (b) Is there a time  $t$ ,  $2 \leq t \leq 4$ , at which  $C'(t) = 2$ ? Justify your answer.
- (c) Use a midpoint sum with three subintervals of equal length indicated by the data in the table to approximate the value of  $\frac{1}{6} \int_0^6 C(t) dt$ . Using correct units, explain the meaning of  $\frac{1}{6} \int_0^6 C(t) dt$  in the context of the problem.
- (d) The amount of coffee in the cup, in ounces, is modeled by  $B(t) = 16 - 16e^{-0.4t}$ . Using this model, find the rate at which the amount of coffee in the cup is changing when  $t = 5$ .
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4. The figure above shows the graph of  $f'$ , the derivative of a twice-differentiable function  $f$ , on the closed interval  $0 \leq x \leq 8$ . The graph of  $f'$  has horizontal tangent lines at  $x = 1$ ,  $x = 3$ , and  $x = 5$ . The areas of the regions between the graph of  $f'$  and the  $x$ -axis are labeled in the figure. The function  $f$  is defined for all real numbers and satisfies  $f(8) = 4$ .
- Find all values of  $x$  on the open interval  $0 < x < 8$  for which the function  $f$  has a local minimum. Justify your answer.
  - Determine the absolute minimum value of  $f$  on the closed interval  $0 \leq x \leq 8$ . Justify your answer.
  - On what open intervals contained in  $0 < x < 8$  is the graph of  $f$  both concave down and increasing? Explain your reasoning.
  - The function  $g$  is defined by  $g(x) = (f(x))^3$ . If  $f(3) = -\frac{5}{2}$ , find the slope of the line tangent to the graph of  $g$  at  $x = 3$ .
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2013 SCORING GUIDELINES**

**Question 3**

$t$ (minutes)	0	1	2	3	4	5	6
$C(t)$ (ounces)	0	5.3	8.8	11.2	12.8	13.8	14.5

Hot water is dripping through a coffeemaker, filling a large cup with coffee. The amount of coffee in the cup at time  $t$ ,  $0 \leq t \leq 6$ , is given by a differentiable function  $C$ , where  $t$  is measured in minutes. Selected values of  $C(t)$ , measured in ounces, are given in the table above.

- (a) Use the data in the table to approximate  $C'(3.5)$ . Show the computations that lead to your answer, and indicate units of measure.
- (b) Is there a time  $t$ ,  $2 \leq t \leq 4$ , at which  $C'(t) = 2$ ? Justify your answer.
- (c) Use a midpoint sum with three subintervals of equal length indicated by the data in the table to approximate the value of  $\frac{1}{6} \int_0^6 C(t) dt$ . Using correct units, explain the meaning of  $\frac{1}{6} \int_0^6 C(t) dt$  in the context of the problem.
- (d) The amount of coffee in the cup, in ounces, is modeled by  $B(t) = 16 - 16e^{-0.4t}$ . Using this model, find the rate at which the amount of coffee in the cup is changing when  $t = 5$ .

(a)  $C'(3.5) \approx \frac{C(4) - C(3)}{4 - 3} = \frac{12.8 - 11.2}{1} = 1.6$  ounces/min

2 :  $\begin{cases} 1 : \text{approximation} \\ 1 : \text{units} \end{cases}$

(b)  $C$  is differentiable  $\Rightarrow C$  is continuous (on the closed interval)

$$\frac{C(4) - C(2)}{4 - 2} = \frac{12.8 - 8.8}{2} = 2$$

2 :  $\begin{cases} 1 : \frac{C(4) - C(2)}{4 - 2} \\ 1 : \text{conclusion, using MVT} \end{cases}$

Therefore, by the Mean Value Theorem, there is at least one time  $t$ ,  $2 < t < 4$ , for which  $C'(t) = 2$ .

(c)  $\frac{1}{6} \int_0^6 C(t) dt \approx \frac{1}{6}[2 \cdot C(1) + 2 \cdot C(3) + 2 \cdot C(5)]$   
 $= \frac{1}{6}(2 \cdot 5.3 + 2 \cdot 11.2 + 2 \cdot 13.8)$   
 $= \frac{1}{6}(60.6) = 10.1$  ounces

3 :  $\begin{cases} 1 : \text{midpoint sum} \\ 1 : \text{approximation} \\ 1 : \text{interpretation} \end{cases}$

$\frac{1}{6} \int_0^6 C(t) dt$  is the average amount of coffee in the cup, in ounces, over the time interval  $0 \leq t \leq 6$  minutes.

(d)  $B'(t) = -16(-0.4)e^{-0.4t} = 6.4e^{-0.4t}$

2 :  $\begin{cases} 1 : B'(t) \\ 1 : B'(5) \end{cases}$

$$B'(5) = 6.4e^{-0.4(5)} = \frac{6.4}{e^2} \text{ ounces/min}$$