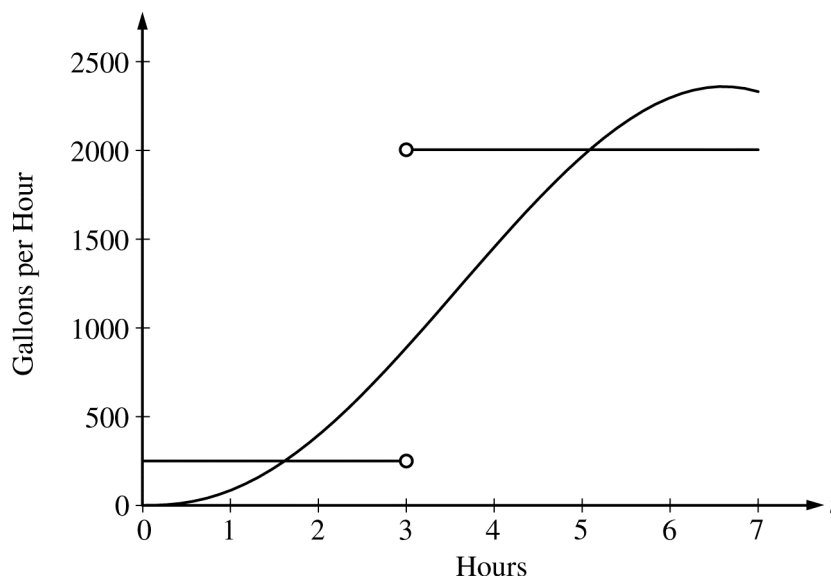


**2007 AP® CALCULUS AB FREE-RESPONSE QUESTIONS**



2. The amount of water in a storage tank, in gallons, is modeled by a continuous function on the time interval  $0 \leq t \leq 7$ , where  $t$  is measured in hours. In this model, rates are given as follows:

- (i) The rate at which water enters the tank is  $f(t) = 100t^2 \sin(\sqrt{t})$  gallons per hour for  $0 \leq t \leq 7$ .  
 (ii) The rate at which water leaves the tank is

$$g(t) = \begin{cases} 250 & \text{for } 0 \leq t < 3 \\ 2000 & \text{for } 3 < t \leq 7 \end{cases} \text{ gallons per hour.}$$

The graphs of  $f$  and  $g$ , which intersect at  $t = 1.617$  and  $t = 5.076$ , are shown in the figure above. At time  $t = 0$ , the amount of water in the tank is 5000 gallons.

- (a) How many gallons of water enter the tank during the time interval  $0 \leq t \leq 7$ ? Round your answer to the nearest gallon.  
 (b) For  $0 \leq t \leq 7$ , find the time intervals during which the amount of water in the tank is decreasing. Give a reason for each answer.  
 (c) For  $0 \leq t \leq 7$ , at what time  $t$  is the amount of water in the tank greatest? To the nearest gallon, compute the amount of water at this time. Justify your answer.

**WRITE ALL WORK IN THE PINK EXAM BOOKLET.**

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$x$	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
1	6	4	2	5
2	9	2	3	1
3	10	-4	4	2
4	-1	3	6	7

3. The functions  $f$  and  $g$  are differentiable for all real numbers, and  $g$  is strictly increasing. The table above gives values of the functions and their first derivatives at selected values of  $x$ . The function  $h$  is given by  $h(x) = f(g(x)) - 6$ .
- (a) Explain why there must be a value  $r$  for  $1 < r < 3$  such that  $h(r) = -5$ .
- (b) Explain why there must be a value  $c$  for  $1 < c < 3$  such that  $h'(c) = -5$ .
- (c) Let  $w$  be the function given by  $w(x) = \int_1^{g(x)} f(t) dt$ . Find the value of  $w'(3)$ .
- (d) If  $g^{-1}$  is the inverse function of  $g$ , write an equation for the line tangent to the graph of  $y = g^{-1}(x)$  at  $x = 2$ .
- 

**WRITE ALL WORK IN THE PINK EXAM BOOKLET.**

**END OF PART A OF SECTION II**

**AP<sup>®</sup> CALCULUS AB**  
**2007 SCORING GUIDELINES**

**Question 2**

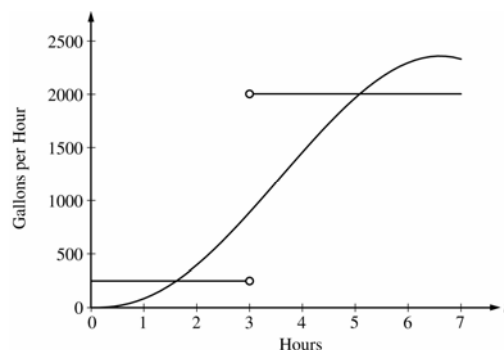
The amount of water in a storage tank, in gallons, is modeled by a continuous function on the time interval  $0 \leq t \leq 7$ , where  $t$  is measured in hours. In this model, rates are given as follows:

- (i) The rate at which water enters the tank is

$$f(t) = 100t^2 \sin(\sqrt{t}) \text{ gallons per hour for } 0 \leq t \leq 7.$$

- (ii) The rate at which water leaves the tank is

$$g(t) = \begin{cases} 250 & \text{for } 0 \leq t < 3 \\ 2000 & \text{for } 3 < t \leq 7 \end{cases} \text{ gallons per hour.}$$



The graphs of  $f$  and  $g$ , which intersect at  $t = 1.617$  and  $t = 5.076$ , are shown in the figure above. At time  $t = 0$ , the amount of water in the tank is 5000 gallons.

- (a) How many gallons of water enter the tank during the time interval  $0 \leq t \leq 7$ ? Round your answer to the nearest gallon.
- (b) For  $0 \leq t \leq 7$ , find the time intervals during which the amount of water in the tank is decreasing. Give a reason for each answer.
- (c) For  $0 \leq t \leq 7$ , at what time  $t$  is the amount of water in the tank greatest? To the nearest gallon, compute the amount of water at this time. Justify your answer.

(a)  $\int_0^7 f(t) dt \approx 8264$  gallons

2 :  $\begin{cases} 1 : \text{integral} \\ 1 : \text{answer} \end{cases}$

- (b) The amount of water in the tank is decreasing on the intervals  $0 \leq t \leq 1.617$  and  $3 \leq t \leq 5.076$  because  $f(t) < g(t)$  for  $0 \leq t < 1.617$  and  $3 < t < 5.076$ .

2 :  $\begin{cases} 1 : \text{intervals} \\ 1 : \text{reason} \end{cases}$

- (c) Since  $f(t) - g(t)$  changes sign from positive to negative only at  $t = 3$ , the candidates for the absolute maximum are at  $t = 0$ , 3, and 7.

5 :  $\begin{cases} 1 : \text{identifies } t = 3 \text{ as a candidate} \\ 1 : \text{integrand} \\ 1 : \text{amount of water at } t = 3 \\ 1 : \text{amount of water at } t = 7 \\ 1 : \text{conclusion} \end{cases}$

$t$ (hours)	gallons of water
0	5000
3	$5000 + \int_0^3 f(t) dt - 250(3) = 5126.591$
7	$5126.591 + \int_3^7 f(t) dt - 2000(4) = 4513.807$

The amount of water in the tank is greatest at 3 hours. At that time, the amount of water in the tank, rounded to the nearest gallon, is 5127 gallons.