

### 1998 Calculus BC Free-Response Questions

2. Let  $f$  be the function given by  $f(x) = 2xe^{2x}$ .
- Find  $\lim_{x \rightarrow -\infty} f(x)$  and  $\lim_{x \rightarrow \infty} f(x)$ .
  - Find the absolute minimum value of  $f$ . Justify that your answer is an absolute minimum.
  - What is the range of  $f$ ?
  - Consider the family of functions defined by  $y = bxe^{bx}$ , where  $b$  is a nonzero constant. Show that the absolute minimum value of  $bxe^{bx}$  is the same for all nonzero values of  $b$ .
- 

GO ON TO THE NEXT PAGE 

### 1998 Calculus BC Free-Response Questions

3. Let  $f$  be a function that has derivatives of all orders for all real numbers. Assume  $f(0) = 5$ ,  $f'(0) = -3$ ,  $f''(0) = 1$ , and  $f'''(0) = 4$ .
- Write the third-degree Taylor polynomial for  $f$  about  $x = 0$  and use it to approximate  $f(0.2)$ .
  - Write the fourth-degree Taylor polynomial for  $g$ , where  $g(x) = f(x^2)$ , about  $x = 0$ .
  - Write the third-degree Taylor polynomial for  $h$ , where  $h(x) = \int_0^x f(t) dt$ , about  $x = 0$ .
  - Let  $h$  be defined as in part (c). Given that  $f(1) = 3$ , either find the exact value of  $h(1)$  or explain why it cannot be determined.
- 

GO ON TO THE NEXT PAGE 