5. A rural county hospital offers several health services. The hospital administrators conducted a poll to determine whether the residents' satisfaction with the available services depends on their gender. A random sample of 1,000 adult county residents was selected. The gender of each respondent was recorded and each was asked whether he or she was satisfied with the services offered by the hospital. The resulting data are shown in the table below.

| | Male | Female | Total |
|---|---|---|---|
| Satisfied | 384 | 416 | 800 |
| Not Satisfied | 80 | 120 | 200 |
| Total | 464 | 536 | 1,000 |

(a) Using a significance level of 0.05, conduct an appropriate test to determine if, for adult residents of this county, there is an association between gender and whether or not they were satisfied with services offered by the hospital.

(b) Is $\frac{800}{1,000}$ a reasonable estimate for the proportion of all adult county residents who are satisfied with the services offered by this hospital? Explain why or why not.



GO ON TO THE NEXT PAGE.

**STATISTICS**
**Section II**
**Part B**
**Question 6**
**Spend about 25 minutes on this part of the exam.**
**Percent of Section II grade—25**

**Directions:** Show all your work. Indicate clearly the methods you use, because you will be graded on the correctness of your methods as well as on the accuracy of your results and explanation.

6. A pharmaceutical company has developed a new drug to reduce cholesterol. A regulatory agency will recommend the new drug for use if there is convincing evidence that the mean reduction in cholesterol level after one month of use is more than 20 milligrams/deciliter (mg/dl), because a mean reduction of this magnitude would be greater than the mean reduction for the current most widely used drug.

   The pharmaceutical company collected data by giving the new drug to a random sample of 50 people from the population of people with high cholesterol. The reduction in cholesterol level after one month of use was recorded for each individual in the sample, resulting in a sample mean reduction and standard deviation of 24 mg/dl and 15 mg/dl, respectively.

   (a) The regulatory agency decides to use an interval estimate for the population mean reduction in cholesterol level for the new drug. Provide this 95 percent confidence interval. Be sure to interpret this interval.

   (b) Because the 95 percent confidence interval includes 20, the regulatory agency is not convinced that the new drug is better than the current best-seller. The pharmaceutical company tested the following hypotheses.

   $$H_0: \ \mu = 20 \text{ versus } H_a: \ \mu > 20,$$

   where $\mu$ represents the population mean reduction in cholesterol level for the new drug.

   The test procedure resulted in a $t$-value of 1.89 and a $p$-value of 0.033. Because the $p$-value was less than 0.05, the company believes that there is convincing evidence that the mean reduction in cholesterol level for the new drug is more than 20. Explain why the confidence interval and the hypothesis test led to different conclusions.

   (c) The company would like to determine a value $L$ that would allow them to make the following statement.

   We are 95 percent confident that the true mean reduction in cholesterol level is greater than $L$.

   A statement of this form is called a one-sided confidence interval. The value of $L$ can be found using the following formula.

   $$L = \bar{x} - t^* \frac{s}{\sqrt{n}}$$

   This has the same form as the lower endpoint of the confidence interval in part (a), but requires a different critical value, $t^*$. What value should be used for $t^*$?

   Recall that the sample mean reduction in cholesterol level and standard deviation are 24 mg/dl and 15 mg/dl, respectively. Compute the value of $L$.

   (d) If the regulatory agency had used the one-sided confidence interval in part (c) rather than the interval constructed in part (a), would it have reached a different conclusion? Explain.

**END OF EXAMINATION**

# AP® STATISTICS
# 2004 SCORING GUIDELINES

## Question 5

**Solution**

**Part a:**

$H_0$ : gender and satisfaction with health services offered by the hospital are independent (OR not associated)

$H_a$ : gender and satisfaction with health services offered by the hospital are dependent (OR associated)

Chi-square test for association

Test statistic: $\chi^2 = \sum_{all\ cells} \frac{(\text{observed - expected})^2}{\text{expected}}$

Conditions: A random sample has been taken. The expected cell counts are large enough so that the chi-square approximation can be used. (See the table below for the expected cell counts.) That is, all four of the expected cell counts are at least 5 (or the smallest expected cell count is 92.8 which is greater than 5). We can use the chi-square approximation.

```
Expected counts are printed below observed counts

          Male    Female     Total
1          384       416       800
        371.20    428.80

2           80       120       200
         92.80    107.20

Total      464       536      1000

Chi-Sq =  0.441 +  0.382 + 1.766 +  1.528 = 4.117
DF = 1, P-Value = 0.042
```

Because the p-value, 0.042, is less than 0.05, we can reject $H_0$ at significance level 0.05, and conclude that there is evidence of an association between gender and satisfaction with health services offered by the hospital for adult residents of this county.

**Part b:**

Because a random sample has been taken from the population of all county residents, 0.8 is a reasonable estimate for the proportion of all county residents who are satisfied with the services offered by this hospital.

### Question 5 (cont'd.)

**Scoring**

**Part (a):** BECAUSE THE ALTERNATIVE HYPOTHESIS IS GIVEN IN THE PROBLEM, HYPOTHESES AND CONDITIONS WILL BE COMBINED INTO ONE STEP (a1) AND SCORED AS ESSENTIALLY CORRECT OR INCORRECT. Each of the OTHER 2 steps (a2 and a3) of the hypothesis test is scored either as correct or incorrect.

**Step a1:** States a correct pair of hypotheses. Identifies a correct test (by name or by formula) and states and verifies appropriate conditions.

Note: Conditions checked must refer to expected counts. Counts must be verified as sufficiently large. A table of expected counts is sufficient to verify that the student has checked the counts. Stating that 92.8 is the smallest expected count is sufficient to verify that the student has checked counts.

The fact that the observed cell counts are from a random sample does not need to be repeated here.

**Step a2:** Correct mechanics, including the value of the test statistic ($\chi^2 = 4.117$) and p-value (0.042)
OR
the test statistic ($\chi^2 = 4.117$) and rejection region ($\chi^2$ critical value = 3.84 when $\alpha = 0.05$).

**Step a3:** Stating a correct conclusion in the context of the problem linked to both the p-value (or rejection region) and the stated hypotheses.

If an $\alpha$ and a p-value are given, the linkage is implied. If no $\alpha$ is given, the solution must be explicit about the linkage by giving a correct interpretation of the p-value or explaining how the conclusion follows from the p-value.

If the p-value in step a2 is incorrect but the conclusion is consistent with the computed p-value, step a3 can be considered as correct.

A conclusion consistent with incorrectly stated hypotheses is scored essentially correct.

NOTE: This chi-squared test can also be done as a two-sample *z* test of the difference between two proportions. The major steps of the hypothesis test are still scored as correct or incorrect. That is, the student must state the hypotheses, identify the test by name or formula, check conditions, complete the correct mechanics, and state the correct conclusion in the context of the problem.

$$H_0 : p_1 - p_2 = 0$$
$$H_a : p_1 - p_2 \neq 0$$

where $p_1$ represents the proportion of males who are satisfied, and $p_2$ represents the proportion of females who are satisfied.

### Question 5 (cont'd.)

Conditions for the two-sample $z$ hypothesis test of proportions

$n_1\hat{p} = 371.2 > 5 \quad n_1(1-\hat{p}) = 92.8 > 5 \quad$ where $\hat{p} = \dfrac{384+416}{464+536} = 0.8$

$n_2\hat{p} = 428.8 > 5 \quad n_2(1-\hat{p}) = 107.2 > 5$

OR

$n_1\hat{p}_1 = 384 > 5 \quad n_1(1-\hat{p}_1) = 80 > 5 \quad$ where $\hat{p}_1 = 0.827586$

$n_2\hat{p}_2 = 416 > 5 \quad n_2(1-\hat{p}_2) = 120 > 5 \quad$ where $\hat{p}_2 = 0.776119$

Computer output illustrating the correct mechanics for the two-sample $z$ test and confidence interval is provided below.

```
Sample      X      N  Sample p
1         384    464  0.827586
2         416    536  0.776119

Estimate for p1 − p2 :  0.0514668
95% CI for p1 − p2 :  (0.00220611, 0.100727)
Test for p1 − p2 = 0 (vs p1 − p2 ≠ 0):  Z = 2.03  P-Value = 0.042

Standard deviation for p1 − p2 = sqrt((0.8)(0.2)[1/464 + 1/536]) = 0.02536
```

This $p$-value of 0.042 is less than $\alpha = 0.05$ so we can reject the null hypothesis. We have sufficient evidence that the proportion of males who are satisfied with the hospital services is different than the proportion of females who are satisfied with the hospital services.

**Part (b)** is scored as essentially correct, partially correct, or incorrect.

Part (b) is scored as essentially correct if 0.8 is viewed as a reasonable estimate because the sample is taken from the population of interest.

> NOTE: If a student provides the correct response and then states that the estimate could be improved by conditioning then the response should be scored essentially correct.

## Question 5 (cont'd.)

Part (b) is scored as partially correct
- if the results of the hypothesis test in part (a) are used to make a statement about the applicability of the estimate. That is, 0.8 is viewed as an unreasonable estimate because there is a significant association between gender and satisfaction.
- if the estimate is unreasonable because the number of men and women are not equal in the sample.

Part (b) is scored as incorrect
- if the student simply says yes or no, without any explanation.
- if the students says yes, the sample size is large.

**4 Complete Response**

Part (b) essentially correct and all 3 steps of the hypothesis test are correct

**3 Substantial Response**

Part (b) essentially correct and 2 steps of the hypothesis test correct
OR
Part (b) partially correct or incorrect and 3 steps of the hypothesis test correct
OR
Part (b) partially correct AND either the hypotheses or conditions are correct AND both a2 and a3 are correct

**2 Developing Response**

Part (b) essentially correct and 1 step of the hypothesis test correct
OR
Part (b) partially correct or incorrect and 2 steps of the hypothesis test correct
OR
Part (b) partially correct AND either the hypotheses or conditions are correct AND either a2 or a3 is correct

**1 Minimal Response**

Part (b) essentially correct
OR
Part (b) partially correct or incorrect and 1 step of the hypothesis test is correct
OR
Part (b) partially correct AND either the hypotheses or conditions are correct