CALCULUS AB
SECTION II, Part B
Time—45 minutes
Number of problems—3

No calculator is allowed for these problems.

4. A particle moves along the *x*-axis with position at time *t* given by $x(t) = e^{-t} \sin t$ for $0 \leq t \leq 2\pi$.

   (a) Find the time *t* at which the particle is farthest to the left. Justify your answer.

   (b) Find the value of the constant *A* for which $x(t)$ satisfies the equation $Ax''(t) + x'(t) + x(t) = 0$ for $0 < t < 2\pi$.

t (minutes)	0	2	5	7	11	12
$r'(t)$ (feet per minute)	5.7	4.0	2.0	1.2	0.6	0.5

5. The volume of a spherical hot air balloon expands as the air inside the balloon is heated. The radius of the balloon, in feet, is modeled by a twice-differentiable function *r* of time *t*, where *t* is measured in minutes. For $0 < t < 12$, the graph of *r* is concave down. The table above gives selected values of the rate of change, $r'(t)$, of the radius of the balloon over the time interval $0 \leq t \leq 12$. The radius of the balloon is 30 feet when $t = 5$.

   (Note: The volume of a sphere of radius *r* is given by $V = \frac{4}{3}\pi r^3$.)

   (a) Estimate the radius of the balloon when $t = 5.4$ using the tangent line approximation at $t = 5$. Is your estimate greater than or less than the true value? Give a reason for your answer.

   (b) Find the rate of change of the volume of the balloon with respect to time when $t = 5$. Indicate units of measure.

   (c) Use a right Riemann sum with the five subintervals indicated by the data in the table to approximate $\int_0^{12} r'(t)\, dt$. Using correct units, explain the meaning of $\int_0^{12} r'(t)\, dt$ in terms of the radius of the balloon.

   (d) Is your approximation in part (c) greater than or less than $\int_0^{12} r'(t)\, dt$? Give a reason for your answer.

WRITE ALL WORK IN THE PINK EXAM BOOKLET.

GO ON TO THE NEXT PAGE.

A particle moves along the *x*-axis with position at time t given by $x(t) = e^{-t} \sin t$ for $0 \le t \le 2\pi$.

(a) Find the time t at which the particle is farthest to the left. Justify your answer.

(b) Find the value of the constant A for which $x(t)$ satisfies the equation $Ax''(t) + x'(t) + x(t) = 0$ for $0 < t < 2\pi$.

(a) $x'(t) = -e^{-t} \sin t + e^{-t} \cos t = e^{-t}(\cos t - \sin t)$

$x'(t) = 0$ when $\cos t = \sin t$. Therefore, $x'(t) = 0$ on $0 \le t \le 2\pi$ for $t = \dfrac{\pi}{4}$ and $t = \dfrac{5\pi}{4}$.

The candidates for the absolute minimum are at $t = 0, \dfrac{\pi}{4}, \dfrac{5\pi}{4},$ and 2π.

t	$x(t)$
0	$e^0 \sin(0) = 0$
$\dfrac{\pi}{4}$	$e^{-\frac{\pi}{4}} \sin\left(\dfrac{\pi}{4}\right) > 0$
$\dfrac{5\pi}{4}$	$e^{-\frac{5\pi}{4}} \sin\left(\dfrac{5\pi}{4}\right) < 0$
2π	$e^{-2\pi} \sin(2\pi) = 0$

The particle is farthest to the left when $t = \dfrac{5\pi}{4}$.

$$5: \begin{cases} 2: x'(t) \\ 1: \text{sets } x'(t) = 0 \\ 1: \text{answer} \\ 1: \text{justification} \end{cases}$$

(b) $x''(t) = -e^{-t}(\cos t - \sin t) + e^{-t}(-\sin t - \cos t)$

$\qquad = -2e^{-t} \cos t$

$Ax''(t) + x'(t) + x(t)$

$\qquad = A\left(-2e^{-t} \cos t\right) + e^{-t}(\cos t - \sin t) + e^{-t} \sin t$

$\qquad = (-2A + 1)e^{-t} \cos t$

$\qquad = 0$

Therefore, $A = \dfrac{1}{2}$.

$$4: \begin{cases} 2: x''(t) \\ 1: \text{substitutes } x''(t), x'(t), \text{ and } x(t) \\ \quad \text{into } Ax''(t) + x'(t) + x(t) \\ 1: \text{answer} \end{cases}$$