

Begin your response to **QUESTION 2** on this page.

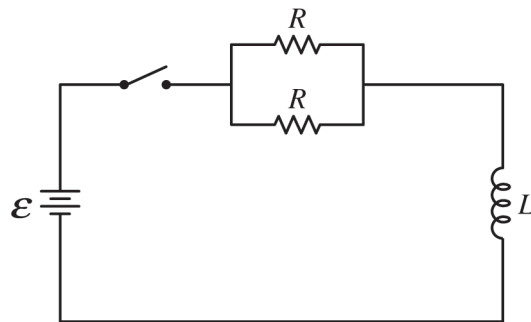


Figure 1

2. Students are asked to determine the resistance  $R$  of two identical resistors. The resistors are in parallel with each other and are connected in series to a battery of known emf  $\mathcal{E}$ , an inductor of known inductance  $L$ , and a switch, as shown in Figure 1. The students have access to a voltmeter that can measure potential difference as a function of time. The students are required to measure a quantity that decreases with time to determine  $R$ .

(a)

- i. On the circuit diagram shown in Figure 1, **draw** the voltmeter, using the following symbol, with connections that would allow the students to correctly measure a potential difference that decreases with time.



Voltmeter Symbol

- ii. **Describe** a procedure for collecting data that would allow the students to graphically determine the experimental value for  $R$  using the measured quantity that decreases with time. Provide enough detail so that another student could replicate the experiment.

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(b)

i. On the axes shown in Figure 2, produce a graph that represents the expected trend of the data by completing the following tasks.

- **Label** the quantities graphed on the vertical and horizontal axes.
- **Sketch** a line or curve that represents the expected trend of the collected data.
- **Label** any appropriate intercepts and/or asymptotes in terms of the quantities provided.



Figure 2

ii. **Describe** how the information from the graph in part (b)(i) would be used to determine the experimental value for  $R$ .

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- (c) Starting with an appropriate application of Kirchhoff's loop rule, **derive**, but do NOT solve, a differential equation that can be used to determine the current  $I$  in the inductor at time  $t$  after the switch is closed. Express your answer in terms of  $R$ ,  $\mathcal{E}$ ,  $L$ ,  $t$ , and physical constants, as appropriate.

After reaching steady state, the absolute value of the potential difference across the inductor is  $|\Delta V_1|$ . The students replace the original inductor with a new inductor that has nonnegligible resistance. The experiment is repeated. After a long time, the absolute value of the potential difference across the new inductor is  $|\Delta V_2|$ .

- (d) **Indicate** whether  $|\Delta V_2|$  is greater than, less than, or equal to  $|\Delta V_1|$ .

\_\_\_\_\_  $|\Delta V_2| > |\Delta V_1|$       \_\_\_\_\_  $|\Delta V_2| < |\Delta V_1|$       \_\_\_\_\_  $|\Delta V_2| = |\Delta V_1|$

**Justify** your answer.

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Begin your response to **QUESTION 3** on this page.

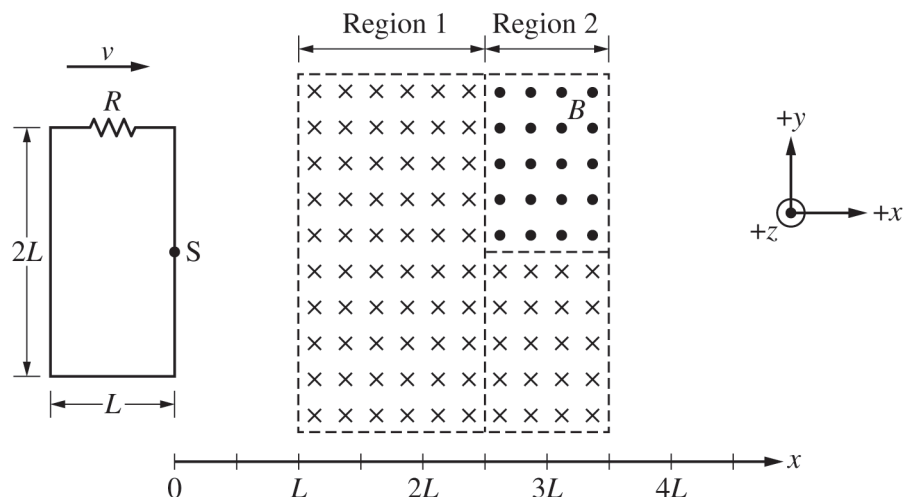


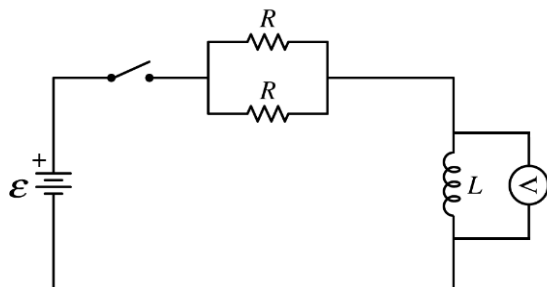
Figure 1

3. A wire is connected to a resistor of resistance  $R$  to form a rigid rectangular loop of width  $L$  and height  $2L$ . An external force is exerted on the loop so that the loop always moves with constant speed  $v$  in the  $+x$ -direction, as shown in Figure 1. The loop then enters Region 1 of external uniform magnetic field of magnitude  $B$  that is directed in the  $-z$ -direction. Region 1 has boundaries  $x = L$  and  $x = 2.5L$ . The loop later enters Region 2 with two external, uniform magnetic fields, each of magnitude  $B$ , that are parallel but are directed in opposite  $z$ -directions. Region 2 has boundaries  $x = 2.5L$  and  $x = 3.5L$ . Point S is the midpoint of the leading edge of the loop and is aligned with the horizontal boundary in Region 2 that separates the two magnetic fields.

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**Question 2: Free-Response Question****15 points**

- (a)(i)** For correctly placing the voltmeter in parallel with the inductor **1 point**

**Example Response**

- (a)(ii)** For a procedure that indicates that the voltmeter should be used to measure the potential difference for at least one time **1 point**

For measuring the potential difference from immediately after the switch is closed to when steady-state conditions have been established or during a time interval that would allow the time constant to be determined **1 point**

**Example Response**

*Close the switch. Using the voltmeter, record the potential difference as a function of time until steady-state conditions are established.*

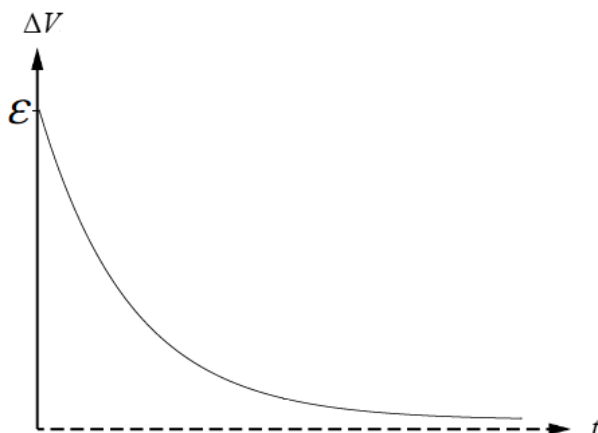
**Total for part (a) 3 points**

- (b)(i)** For correctly labeling potential difference on the vertical axis and time on the horizontal axis **1 point**

For a concave-up and decreasing curve **1 point**

For a curve that asymptotically approaches zero, or a curve that starts at the origin and approaches a horizontal asymptote that is consistent with the placement of the voltmeter in the response in part (a)(i) **1 point**

For including a vertical intercept or a horizontal asymptote that is consistent with the placement of the voltmeter in the response in part (a)(i) that is correctly labeled as  $\mathcal{E}$  **1 point**

**Example Response**

- (b)(ii) For indicating that a curve fit to the graph is that of an exponential function or for correctly relating the area under the curve to the current in the circuit **1 point**

*Alternate Solution*

For indicating the time on the graph where the potential difference is approximately  $0.37\mathcal{E}$ , or  $0.63\mathcal{E}$  for a curve that starts at the origin and approaches a horizontal asymptote that is consistent with the placement of the voltmeter in the response in part (a)(i)

- For indicating that the coefficient in front of the  $t$  of the curve-fit equation is equal to  $\frac{R}{2L}$  **1 point**  
or for correctly relating  $R$  to  $L$  and the current

*Alternate Solution*

For indicating that the time constant is equal to  $\frac{2L}{R}$

**Example Response**

The data in the graph should be fit with an exponential function for the equation

$$V_L = \mathcal{E} \left( e^{-t \frac{R}{2L}} \right). \text{ Because } \mathcal{E} \text{ and } L \text{ are known, } R \text{ can be calculated.}$$

*Alternate Example Response*

The potential difference at  $0.37\mathcal{E}$  along the vertical axis corresponds to the time constant along the horizontal axis. Because the time constant is equal to  $\frac{2L}{R}$ , and  $L$  is known,  $R$  can be calculated.

**Total for part (b) 6 points**