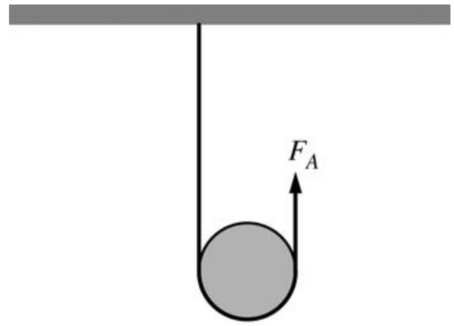

Note: Figure not drawn to scale.

Mech 3.

A disk of mass $M = 2.0$ kg and radius $R = 0.10$ m is supported by a rope of negligible mass, as shown above. The rope is attached to the ceiling at one end and passes under the disk. The other end of the rope is pulled upward with a force F_A. The rotational inertia of the disk around its center is $MR^2/2$.

(a) Calculate the magnitude of the force F_A necessary to hold the disk at rest.

At time $t = 0$, the force F_A is increased to 12 N, causing the disk to accelerate upward. The rope does not slip on the disk as the disk rotates.

(b) Calculate the linear acceleration of the disk.

(c) Calculate the angular speed of the disk at $t = 3.0$ s.

(d) Calculate the increase in total mechanical energy of the disk from $t = 0$ to $t = 3.0$ s.

(e) The disk is replaced by a hoop of the same mass and radius. Indicate whether the linear acceleration of the hoop is greater than, less than, or the same as the linear acceleration of the disk.

__ Greater than __ Less than __ The same as

Justify your answer.

STOP

END OF EXAM

Question 3

15 points total | **Distribution of points**

(a) 2 points

For a correct expression indicating that $F_{net} = 0$ — 1 point

$2F_A - Mg = 0$

$F_A = Mg/2$

$F_A = (2.0\ \text{kg})\left(9.8\ \text{m/s}^2\right)/2$

For a correct answer — 1 point

$F_A = 9.8\ \text{N}$ (or 10 N using $g = 10\ \text{m/s}^2$)

(b) 5 points

For a correct expression of Newton's second law for translational motion — 1point

$F_{net} = Ma$

$F_A + T - Mg = Ma$ (equation 1)

For using a correct expression of torque in Newton's second law for rotational motion — 1 point

$\tau = I\alpha$

$F_A R - TR = \left(MR^2/2\right)\alpha$ or $F_A R - TR = I\alpha$

For substituting for the angular acceleration in terms of the linear acceleration $(\alpha = a/R)$ — 1 point

$F_A R - TR = \left(MR^2/2\right)(a/R)$

$F_A - T = Ma/2$ (equation 2)

For combining equations 1 and 2 to solve for the linear acceleration — 1 point

Add the two equations

$2F_A - Mg = (3/2)Ma$

$a = (2/3)((2F_A/M) - g)$

$a = (2/3)\left((2(12\ \text{N})/2.0\ \text{kg}) - 9.8\ \text{m/s}^2\right)$

For a correct answer, with units — 1 point

$a = 1.47\ \text{m/s}^2$ ($1.33\ \text{m/s}^2$ using $g = 10\ \text{m/s}^2$)

(c) 2 points

For using the relationship between linear and angular acceleration in the equation for angular speed

$\omega = \omega_0 + \alpha t$ and $\alpha = a/R$ — 1 point

$\omega_0 = 0$, so $\omega = \alpha t/R$

$\omega = \left(1.47\ \text{m/s}^2\right)(3.0\ \text{s})/(0.10\ \text{m})$

1 point

Question 3 (continued)

Distribution of points

For an answer with units, consistent with previous work

$\omega = 44 \text{ rad/s}$ (40 rad/s using $g = 10 \text{ m/s}^2$)

(d) 4 points

Express the change in mechanical energy as the sum of the change in potential energy and the change in kinetic energy

$\Delta E = \Delta U_g + \Delta K$

For a correct expression for the change in kinetic energy including both translational and rotational kinetic energy — 1 point

$\Delta K = \frac{1}{2}M\left(v^2 - v_0^2\right) + \frac{1}{2}I\left(\omega^2 - \omega_0^2\right)$

For a correct expression for the change in potential energy, including a correct expression of the height h in terms of the time — 1 point

$\Delta U_g = Mgh = Mg\left(\frac{1}{2}at^2\right)$

v_0 and ω_0 are zero, so $\Delta E = Mg\left(\frac{1}{2}at^2\right) + \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$

For simplifying the expression using the relationship between linear velocity and angular velocity — 1 point

$\Delta E = Mg\left(\frac{1}{2}at^2\right) + \frac{1}{2}M(R\omega)^2 + \frac{1}{2}\left(MR^2/2\right)\omega^2$

$\Delta E = \frac{1}{2}Mgat^2 + \frac{3}{4}MR\omega^2$

For correctly substituting given values and answers from previous parts into a correct expression — 1 point

$\Delta E = \frac{1}{2}(2.0 \text{ kg})\left(9.8 \text{ m/s}^2\right)\left(1.47 \text{ m/s}^2\right)(3.0 \text{ s})^2 + \frac{3}{4}(2.0 \text{ kg})(0.10 \text{ m})^2(44 \text{ rad/s})^2$

$\Delta E = 159 \text{ J}$ (144 J using $g = 10 \text{ m/s}^2$)

(e) 2 points

For selecting "Less than" — 1 point

For a correct justification — 1 point

<u>Example</u>

The rotational inertia of a hoop is greater than that of a solid disk of the same mass and radius, therefore the acceleration of the hoop would be less.