4. The function f has derivatives of all orders for all real numbers x. Assume $f(2) = -3$, $f'(2) = 5$, $f''(2) = 3$, and $f'''(2) = -8$.

(a) Write the third-degree Taylor polynomial for f about $x = 2$ and use it to approximate $f(1.5)$.

(b) The fourth derivative of f satisfies the inequality $\left|f^{(4)}(x)\right| \le 3$ for all x in the closed interval $[1.5, 2]$. Use the Lagrange error bound on the approximation to $f(1.5)$ found in part (a) to explain why $f(1.5) \ne -5$.

(c) Write the fourth-degree Taylor polynomial, $P(x)$, for $g(x) = f\left(x^2 + 2\right)$ about $x = 0$. Use P to explain why g must have a relative minimum at $x = 0$.

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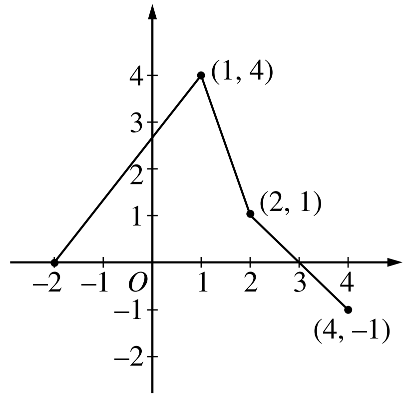

5. The graph of the function f, consisting of three line segments, is given above. Let $g(x) = \int_1^x f(t)\,dt$.

   (a) Compute $g(4)$ and $g(-2)$.

   (b) Find the instantaneous rate of change of g, with respect to x, at $x = 1$.

   (c) Find the absolute minimum value of g on the closed interval $[-2, 4]$. Justify your answer.

   (d) The second derivative of g is not defined at $x = 1$ and $x = 2$. How many of these values are x-coordinates of points of inflection of the graph of g? Justify your answer.

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(a) $T_3(f,2)(x) = -3 + 5(x-2) + \frac{3}{2}(x-2)^2 - \frac{8}{6}(x-2)^3$

$$f(1.5) \approx T_3(f,2)(1.5)$$

$$= -3 + 5(-0.5) + \frac{3}{2}(-0.5)^2 - \frac{4}{3}(-0.5)^3$$

$$= -4.958\overline{3} = -4.958$$

4 $\begin{cases} 3: T_3(f,2)(x) \\ \quad <-1> \text{ each error} \\ 1: \text{approximation of } f(1.5) \end{cases}$

(b) Lagrange Error Bound $= \frac{3}{4!}|1.5 - 2|^4 = 0.0078125$

$f(1.5) > -4.958\overline{3} - 0.0078125 = -4.966 > -5$

Therefore, $f(1.5) \neq -5$.

2 $\begin{cases} 1: \text{value of Lagrange Error Bound} \\ 1: \text{explanation} \end{cases}$

(c) $P(x) = T_4(g,0)(x)$

$$= T_2(f,2)(x^2+2) = -3 + 5x^2 + \frac{3}{2}x^4$$

The coefficient of x in $P(x)$ is $g'(0)$. This coefficient is 0, so $g'(0) = 0$.

The coefficient of x^2 in $P(x)$ is $\frac{g''(0)}{2!}$. This coefficient is 5, so $g''(0) = 10$ which is greater than 0.

Therefore, g has a relative minimum at $x = 0$.

3 $\begin{cases} 2: T_4(g,0)(x) \\ \quad <-1> \text{ each incorrect, missing,} \\ \qquad\qquad \text{or extra term} \\ 1: \text{explanation} \end{cases}$

Note:
$<-1>$ max for improper use of $+\ldots$ or equality