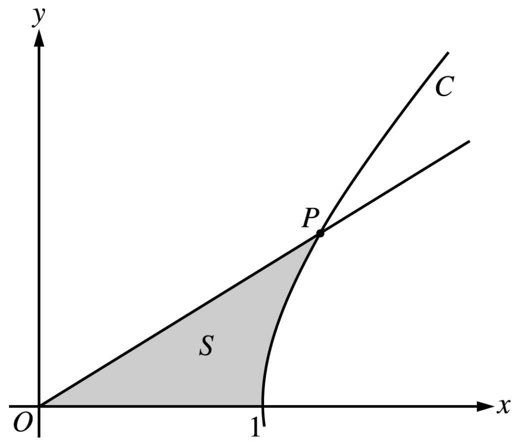


3. The figure above shows the graphs of the line $x = \frac{5}{3}y$ and the curve C given by $x = \sqrt{1 + y^2}$. Let S be the shaded region bounded by the two graphs and the x-axis. The line and the curve intersect at point P.

   (a) Find the coordinates of point P and the value of $\frac{dx}{dy}$ for curve C at point P.

   (b) Set up and evaluate an integral expression with respect to y that gives the area of S.

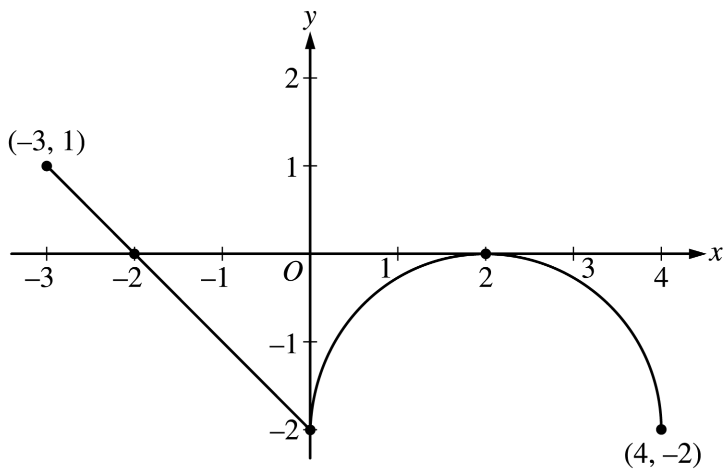
   (c) Curve C is a part of the curve $x^2 - y^2 = 1$. Show that $x^2 - y^2 = 1$ can be written as the polar equation $r^2 = \frac{1}{\cos^2\theta - \sin^2\theta}$.

   (d) Use the polar equation given in part (c) to set up an integral expression with respect to the polar angle θ that represents the area of S.

END OF PART A OF SECTION II

CALCULUS BC
SECTION II, Part B
Time—45 minutes
Number of problems—3

No calculator is allowed for these problems.



Graph of f'

4. Let f be a function defined on the closed interval $-3 \le x \le 4$ with $f(0) = 3$. The graph of f', the derivative of f, consists of one line segment and a semicircle, as shown above.

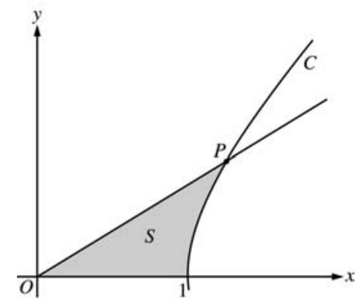
   (a) On what intervals, if any, is f increasing? Justify your answer.

   (b) Find the x-coordinate of each point of inflection of the graph of f on the open interval $-3 < x < 4$. Justify your answer.

   (c) Find an equation for the line tangent to the graph of f at the point $(0, 3)$.

   (d) Find $f(-3)$ and $f(4)$. Show the work that leads to your answers.

Question 3



The figure above shows the graphs of the line $x = \frac{5}{3}y$ and the curve C given by $x = \sqrt{1 + y^2}$. Let S be the shaded region bounded by the two graphs and the x-axis. The line and the curve intersect at point P.

(a) Find the coordinates of point P and the value of $\frac{dx}{dy}$ for curve C at point P.

(b) Set up and evaluate an integral expression with respect to y that gives the area of S.

(c) Curve C is a part of the curve $x^2 - y^2 = 1$. Show that $x^2 - y^2 = 1$ can be written as the polar equation $r^2 = \frac{1}{\cos^2\theta - \sin^2\theta}$.

(d) Use the polar equation given in part (c) to set up an integral expression with respect to the polar angle θ that represents the area of S.

(a) At P, $\frac{5}{3}y = \sqrt{1+y^2}$, so $y = \frac{3}{4}$. Since $x = \frac{5}{3}y$, $x = \frac{5}{4}$. $\frac{dx}{dy} = \frac{y}{\sqrt{1+y^2}} = \frac{y}{x}$. At P, $\frac{dx}{dy} = \frac{3/4}{5/4} = \frac{3}{5}$.	2 : { 1 : coordinates of P; 1 : $\frac{dx}{dy}$ at P }
(b) Area $= \int_0^{3/4} \left(\sqrt{1+y^2} - \frac{5}{3}y\right) dy$ $= 0.346$ or 0.347	3 : { 1 : limits; 1 : integrand; 1 : answer }
(c) $x = r\cos\theta\,;\ y = r\sin\theta$ $x^2 - y^2 = 1 \Rightarrow r^2\cos^2\theta - r^2\sin^2\theta = 1$ $r^2 = \frac{1}{\cos^2\theta - \sin^2\theta}$	2 : { 1 : substitutes $x = r\cos\theta$ and $y = r\sin\theta$ into $x^2 - y^2 = 1$; 1 : isolates r^2 }
(d) Let β be the angle that segment OP makes with the x-axis. Then $\tan\beta = \frac{y}{x} = \frac{3/4}{5/4} = \frac{3}{5}$. Area $= \int_0^{\tan^{-1}(3/5)} \frac{1}{2}r^2\,d\theta$ $= \frac{1}{2}\int_0^{\tan^{-1}(3/5)} \frac{1}{\cos^2\theta - \sin^2\theta}\,d\theta$	2 : { 1 : limits; 1 : integrand and constant }