3. A grocery store purchases melons from two distributors, J and K. Distributor J provides melons from organic farms. The distribution of the diameters of the melons from Distributor J is approximately normal with mean 133 millimeters (mm) and standard deviation 5 mm.

   (a) For a melon selected at random from Distributor J, what is the probability that the melon will have a diameter greater than 137 mm?

   Distributor K provides melons from nonorganic farms. The probability is 0.8413 that a melon selected at random from Distributor K will have a diameter greater than 137 mm. For all the melons at the grocery store, 70 percent of the melons are provided by Distributor J and 30 percent are provided by Distributor K.

   (b) For a melon selected at random from the grocery store, what is the probability that the melon will have a diameter greater than 137 mm?

   (c) Given that a melon selected at random from the grocery store has a diameter greater than 137 mm, what is the probability that the melon will be from Distributor J?

GO ON TO THE NEXT PAGE.

4. The chemicals in clay used to make pottery can differ depending on the geographical region where the clay originated. Sometimes, archaeologists use a chemical analysis of clay to help identify where a piece of pottery originated. Such an analysis measures the amount of a chemical in the clay as a percent of the total weight of the piece of pottery. The boxplots below summarize analyses done for three chemicals—X, Y, and Z—on pieces of pottery that originated at one of three sites: I, II, or III.
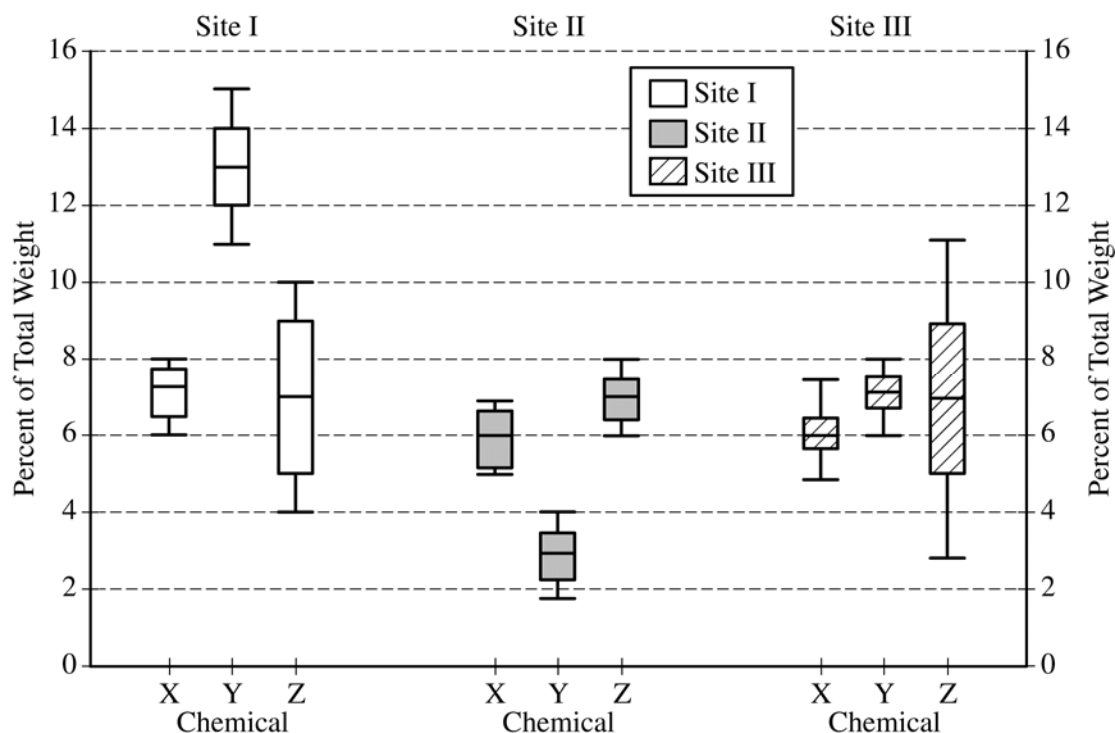
(a) For chemical Z, describe how the percents found in the pieces of pottery are similar and how they differ among the three sites.

(b) Consider a piece of pottery known to have originated at one of the three sites, but the actual site is not known.

(i) Suppose an analysis of the clay reveals that the sum of the percents of the three chemicals X, Y, and Z is 20.5%. Based on the boxplots, which site—I, II, or III—is the most likely site where the piece of pottery originated? Justify your choice.

(ii) Suppose only one chemical could be analyzed in the piece of pottery. Which chemical—X, Y, or Z— would be the most useful in identifying the site where the piece of pottery originated? Justify your choice.

Question 3

Intent of Question

The primary goals of this question were to assess a student's ability to (1) calculate a probability from a normal distribution; (2) calculate a weighted probability from two individual probabilities; and (3) calculate a conditional probability for dependent events when individual and joint probabilities are provided.

Solution

Part (a):

Let X denote the diameter of a randomly selected melon from Distributor J. X has an approximately normal distribution with mean 133 mm and standard deviation 5 mm.

The z-score for a diameter of 137 mm is $z = \dfrac{137 - 133}{5} = \dfrac{4}{5} = 0.8$.

Therefore, $P(X > 137) = P(Z > 0.8) = 1 - 0.7881 = 0.2119$.

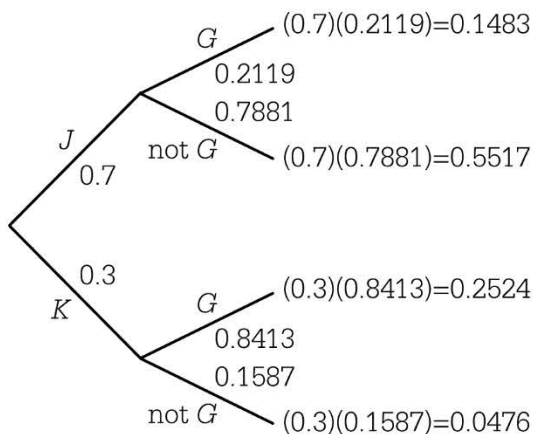**Part (b):**

Define events:
   J: melon is from Distributor J
   K: melon is from Distributor K
   G: melon diameter is greater than 137 mm

For a randomly selected melon from the grocery store,
$$\begin{aligned}
P(G) &= P(G \mid J) \times P(J) + P(G \mid K) \times P(K) \\
&= (0.2119)(0.7) + (0.8413)(0.3) \\
&= 0.1483 + 0.2524 \\
&= 0.4007
\end{aligned}$$

OR

From the tree diagram, $P(G) = P(G \text{ and } J) + P(G \text{ and } K) = 0.1483 + 0.2524 = 0.4007$.

Part (c):

Using the events defined in part (b), the requested probability is

$$P(J \mid G) = \frac{P(J \text{ and } G)}{P(G)} = \frac{P(G \mid J)P(J)}{P(G)} = \frac{(0.2119)(0.7)}{0.4007} = \frac{0.1483}{0.4007} = 0.3701.$$

Scoring

Parts (a), (b), and (c) are scored as essentially correct (E), partially correct (P), or incorrect (I).

Part (a) is scored as follows:

Essentially correct (E) if the response provides the following three components:
1. <u>Normality and parameters</u>: Indicates use of a normal (or approximately normal) distribution and clearly identifies the correct parameter values. (Showing correct components in a z-score, labeling the mean and standard deviation in a normalcdf calculator statement, or drawing a normal curve with sufficient values marked to indicate the mean and standard deviation are all sufficient methods for identifying the correct parameter values.)
2. <u>Boundary and direction</u>: Uses the correct boundary value of $x = 137$ or $z = 0.8$ and the correct direction. (Showing correct boundary and direction in a probability statement using "greater than," labeling the lower limit and upper limit in a normalcdf calculator statement, drawing a normal curve with 137 labeled and an indication that the area of interest is to the right of 137, or a conclusion in words using "greater than.")
3. <u>Probability</u>: Reports the correct normal probability consistent with the response's setup described in components 1 and 2.

Partially correct (P) if the response correctly provides only two of the three components.

Incorrect (I) if the response gives the answer with no work shown or otherwise does not satisfy the criteria for E or P.

Notes:
- An inconsistency in calculations lowers the score for part (a) by one level (from E to P, or from P to I). For instance, if the response states the requested probability as $P(X > 137)$, but actually computes $P(X < 137)$.
- An error in statistical notation, such as using s instead of σ for the standard deviation or $\overline{x}$ instead of μ for the mean, does not satisfy component 1.
- A correct direction can be obtained by using a left direction and an upper bound of 137 and then subtracting the value of the cumulative probability from 1. However, simply showing the calculation $1 - 0.7881 = 0.2119$ does NOT give a correct direction.
- Consider any steps of a hypothesis test as extraneous work.
- Standard notations such as N(133,5) or N(133,25) satisfy the first component.
- A sketch of the normal curve with 133 and 138 in the appropriate positions satisfies the first component.
- Use of a value other than 137 does not satisfy the second component.
- If the only error in part (a) is the reversal of the numerator for the z-score $(133 - 137)$, the response is scored as P.

- For calculator notation:
  - ncdf is sufficient to indicate use of the normal distribution.
  - Stating the lower and upper bounds is sufficient for identifying boundary and direction.
  - Minimum notation, such as "u" for upper, is sufficient for identifying boundary.
  - The minimum value for an upper bound must be at least 4 standard deviations above the mean.

Part (b) is scored as follows:

Essentially correct (E) if the probability is computed correctly, *AND* work is shown that includes correct numerical values using a formula, end results from a tree diagram, or some other appropriate method.

Partially correct (P) if the response provides a correct strategy for finding the probability, such as a formula or tree diagram, but uses one or more incorrect values or does not use the end result from the tree;
  OR
if the response gives the correct probability but not enough work is shown to determine how it was found.

Incorrect if the response does not meet the criteria for E or P.

Notes:

- A response that uses 0.3 for J and 0.7 for K earns a P if no other mistakes are made.
- Tree diagrams:
  - If used, the multiplication is implied, but the addition must be indicated.
  - If not used, both the multiplication and the addition must be indicated.
- The shown work might be found in part (c).
- Probabilities can be shown in a table. Addition is implied if "Total" is labeled, as shown.

Intersection	G	Not G	Total
J	0.1483	0.5517	0.7
K	0.2524	0.0576	0.3
Total	0.4007	0.5993	

Part (c) is scored as follows:

Essentially correct (E) if the probability is computed correctly *AND* work is shown that illustrates how the probability was found.

Partially correct (P) if the response provides a strategy for finding the probability for dependent variables, such as the appropriate formula or end results from the correct tree diagram, but uses one or more incorrect values or does not use the end result from the tree;
  OR
if the response gives the correct probability but not enough work is shown to determine how it was found.

Incorrect (I) if the response does not meet the criteria for E or P.

Notes:

- Part (c) earns an E if a correct answer follows from incorrect values used from part (a) and/or part (b), provided all values are between 0 and 1, inclusive.
- A transcription error in a response can be ignored if correct work is shown.
- For any part, if the resulting probability or part of the calculation of the probability uses a value that is not between 0 and 1, inclusive, the score is lowered by one level (from E to P, or from P to I).
- In any part, unsupported answers are scored as I.

4   Complete Response

Three parts essentially correct

3   Substantial Response

Two parts essentially correct and one part partially correct

2   Developing Response

Two parts essentially correct and no parts partially correct

OR

One part essentially correct and one or two parts partially correct

OR

Three parts partially correct

1   Minimal Response

One part essentially correct

OR

No parts essentially correct and two parts partially correct