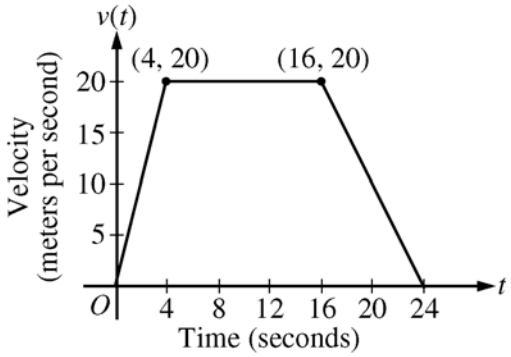

5. A car is traveling on a straight road. For $0 \le t \le 24$ seconds, the car's velocity $v(t)$, in meters per second, is modeled by the piecewise-linear function defined by the graph above.

   (a) Find $\int_0^{24} v(t)\,dt$. Using correct units, explain the meaning of $\int_0^{24} v(t)\,dt$.

   (b) For each of $v'(4)$ and $v'(20)$, find the value or explain why it does not exist. Indicate units of measure.

   (c) Let $a(t)$ be the car's acceleration at time t, in meters per second per second. For $0 < t < 24$, write a piecewise-defined function for $a(t)$.

   (d) Find the average rate of change of v over the interval $8 \le t \le 20$. Does the Mean Value Theorem guarantee a value of c, for $8 < c < 20$, such that $v'(c)$ is equal to this average rate of change? Why or why not?

WRITE ALL WORK IN THE TEST BOOKLET.

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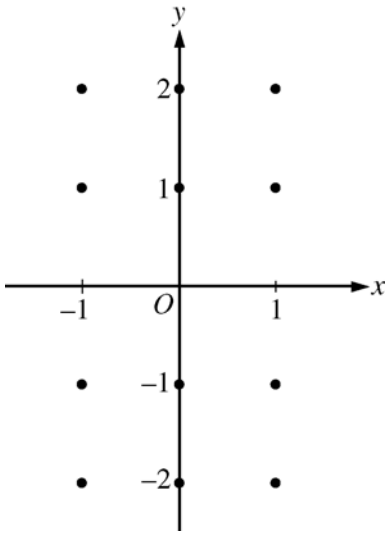
6. Consider the differential equation $\frac{dy}{dx} = -\frac{2x}{y}$.

(a) On the axes provided, sketch a slope field for the given differential equation at the twelve points indicated. (**Note: Use the axes provided in the pink test booklet.**)



(b) Let $y = f(x)$ be the particular solution to the differential equation with the initial condition $f(1) = -1$. Write an equation for the line tangent to the graph of f at $(1, -1)$ and use it to approximate $f(1.1)$.

(c) Find the particular solution $y = f(x)$ to the given differential equation with the initial condition $f(1) = -1$.

WRITE ALL WORK IN THE TEST BOOKLET.

END OF EXAM

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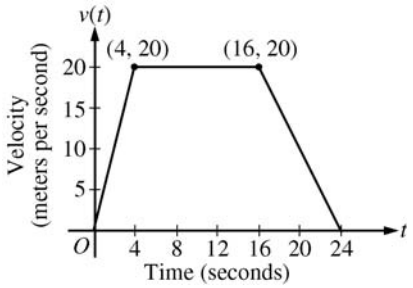
2005 SCORING GUIDELINES

Question 5

A car is traveling on a straight road. For $0 \le t \le 24$ seconds, the car's velocity $v(t)$, in meters per second, is modeled by the piecewise-linear function defined by the graph above.



(a) Find $\int_0^{24} v(t)\,dt$. Using correct units, explain the meaning of $\int_0^{24} v(t)\,dt$.

(b) For each of $v'(4)$ and $v'(20)$, find the value or explain why it does not exist. Indicate units of measure.

(c) Let $a(t)$ be the car's acceleration at time t, in meters per second per second. For $0 < t < 24$, write a piecewise-defined function for $a(t)$.

(d) Find the average rate of change of v over the interval $8 \le t \le 20$. Does the Mean Value Theorem guarantee a value of c, for $8 < c < 20$, such that $v'(c)$ is equal to this average rate of change? Why or why not?

(a) $\int_0^{24} v(t)\,dt = \frac{1}{2}(4)(20) + (12)(20) + \frac{1}{2}(8)(20) = 360$

The car travels 360 meters in these 24 seconds.

$2 : \begin{cases} 1 : \text{value} \\ 1 : \text{meaning with units} \end{cases}$

(b) $v'(4)$ does not exist because

$\lim_{t \to 4^-}\left(\frac{v(t) - v(4)}{t - 4}\right) = 5 \ne 0 = \lim_{t \to 4^+}\left(\frac{v(t) - v(4)}{t - 4}\right)$.

$v'(20) = \frac{20 - 0}{16 - 24} = -\frac{5}{2}\ \text{m/sec}^2$

$3 : \begin{cases} 1 : v'(4) \text{ does not exist, with explanation} \\ 1 : v'(20) \\ 1 : \text{units} \end{cases}$

(c) $a(t) = \begin{cases} 5 & \text{if } 0 < t < 4 \\ 0 & \text{if } 4 < t < 16 \\ -\frac{5}{2} & \text{if } 16 < t < 24 \end{cases}$

$a(t)$ does not exist at $t = 4$ and $t = 16$.

$2 : \begin{cases} 1 : \text{finds the values } 5,\ 0,\ -\frac{5}{2} \\ 1 : \text{identifies constants with correct intervals} \end{cases}$

(d) The average rate of change of v on $[8, 20]$ is $\frac{v(20) - v(8)}{20 - 8} = -\frac{5}{6}\ \text{m/sec}^2$.

No, the Mean Value Theorem does not apply to v on $[8, 20]$ because v is not differentiable at $t = 16$.

$2 : \begin{cases} 1 : \text{average rate of change of } v \text{ on } [8, 20] \\ 1 : \text{answer with explanation} \end{cases}$

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