

2001 AP® STATISTICS FREE-RESPONSE QUESTIONS

2. A department supervisor is considering purchasing one of two comparable photocopy machines, *A* or *B*. Machine *A* costs \$10,000 and machine *B* costs \$10,500. This department replaces photocopy machines every three years. The repair contract for machine *A* costs \$50 per month and covers an unlimited number of repairs. The repair contract for machine *B* costs \$200 per repair. Based on past performance, the distribution of the number of repairs needed over any one-year period for machine *B* is shown below.

Number of Repairs	0	1	2	3
Probability	0.50	0.25	0.15	0.10

You are asked to give a recommendation based on overall cost as to which machine, *A* or *B*, along with its repair contract, should be purchased. What would your recommendation be? Give a statistical justification to support your recommendation.

3. Every Monday a local radio station gives coupons away to 50 people who correctly answer a question about a news fact from the previous day's newspaper. The coupons given away are numbered from 1 to 50, with the first person receiving coupon 1, the second person receiving coupon 2, and so on, until all 50 coupons are given away. On the following Saturday, the radio station randomly draws numbers from 1 to 50 and awards cash prizes to the holders of the coupons with these numbers. Numbers continue to be drawn without replacement until the total amount awarded first equals or exceeds \$300. If selected, coupons 1 through 5 each have a cash value of \$200, coupons 6 through 20 each have a cash value of \$100, and coupons 21 through 50 each have a cash value of \$50.
- (a) Explain how you would conduct a simulation using the random number table provided below to estimate the distribution of the number of prize winners each week.
- (b) Perform your simulation 3 times. (That is, run 3 trials of your simulation.) Start at the leftmost digit in the first row of the table and move across. Make your procedure clear so that someone can follow what you did. You must do this by marking directly on or above the table. Report the number of winners in each of your 3 trials.

72749 13347 65030 26128 49067 02904 49953 74674 94617 13317

81638 36566 42709 33717 59943 12027 46547 61303 46699 76423

38449 46438 91579 01907 72146 05764 22400 94490 49833 09258

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Question 2 - Solution

For A:

Total 3-year cost: $\$10,000 + 36(\$50) = \$10,000 + \$1,800 = \$11,800$

This cost is fixed.

For B:

Expected number of repairs in 3 years = $3[0(.5) + 1(.25) + 2(.15) + 3(.1)] = 3(.85) = 2.55$

Expected cost of repairs in 3 years = $3(\$200)(0.85) = \510

Expected 3-year cost = $\$10,500 + \$510 = \$11,010$

Choice:

Choose B because it has a lower expected (or average) cost. (A has a fixed cost that is \$790 (\$11,800 - \$11,010) higher than the expected cost of B.)

Scoring

The solution should include the following four elements:

1. Correct calculation of 3-year cost for A.
2. Correct calculation of a relevant expected **value** for B (expected number of repairs per year or per 3 years or expected cost of repairs per year or per 3 years). Calculation of expected value must be shown.
3. Correct calculation of expected **total cost** for B.
4. Choice of B with a *complete and coherent* explanation that is based on student's prior calculations for A & B.

"Complete and coherent " means that:

- costs for A & B are compared
- AND
- B's cost has been indicated as "expected" or "average" or "mean" or "estimated" or "approximate" or "predicted," etc.

4 Complete Response

Solution includes all four of the required elements.

3 Substantial Response

Solution includes three of the required elements.

2 Developing Response

Solution includes two of the required elements.

1 Minimal Response

Solution includes one of the required elements.

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Question 2 (cont'd.)

Notes:

1. If calculations are based on 1-year costs rather than 3-year costs, and then the student chooses A with explanation, the student can earn a score of up to 3.

Total 1-year cost for A: $\$10,000 + 12(\$50) = \$10,000 + \$600 = \$10,600$

And for B:

Expected number of repairs in 1 year = $0(.5) + 1(.25) + 2(.15) + 3(.1) = .85$

Expected cost of repairs in 1 year = $\$0(.5) + \$200(.25) + \$400(.15) + \$600(.1) = \$170$

Expected 1-year cost for B: $\$10,500 + \$170 = \$10,670$

2. If initial purchase prices are omitted from the calculations, the student can earn a score of up to 3.
3. Rounded calculations of the expected number of repairs for B: If a student rounds the expected number of repairs per year (.85) to 1, or rounds the expected number of repairs in 3 years (2.55) to 3, the maximum score is a 3. If the student identifies the rounded value as an upper bound on the expected cost, the paper may earn a maximum score of 4.
4. If choice of A or B is not based on expected cost for B, the student can still present a complete response. To earn 4 points with this solution, relevant and complete statistical reasoning must be demonstrated. This solution must include:
 - a. Decision based on break-even analysis:
 - 1 point -- Correct calculation of 3-year fixed cost for A (\$11,800)
 - 1 point -- Correct calculation that 7 or more repairs in the 3-year period would be necessary for B's cost to exceed A's 3-year cost
 - 1 point -- Says or calculates that the probability of 7 or more repairs for B is small and therefore chooses B,
OR
Chooses A because the probability of 7 or more repairs for B is not 0 and they want to guard against the possibility of paying more for B than A's fixed cost
 - 1 point -- Correctly calculates that the probability of 7 or more repairs for B is 0.01975 (or about 0.02, or about 2 percent of the time B's cost will exceed A's cost)
AND
States that this analysis depends on the assumption that repairs from year to year are independent

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Question 2 (cont'd.)

b. Decision based on minimax analysis may earn a maximum of 3 points:

1 point -- Correct calculation of 3-year fixed cost for A (\$11,800)

1 point -- Correct calculation of range of possible 3-year costs for B
 $\$10,500 \leq \text{cost of B} \leq \$12,300$

AND

Probability calculation showing that chance of observing maximum cost is small
(e.g., 0.001 or 0.1 percent that B costs \$12,500, assuming independence)

1 point -- Relevant statistical justification for choice of A or B:

Gives convincing reasoning for minimizing maximum cost (minimax) and therefore chooses A. Student might argue, for example, that a company may prefer a known fixed cost to a variable cost that could be smaller but also has the chance of being larger.