

2019 AP[®] CALCULUS AB FREE-RESPONSE QUESTIONS

CALCULUS AB

SECTION II, Part A

Time—30 minutes

Number of questions—2

A GRAPHING CALCULATOR IS REQUIRED FOR THESE QUESTIONS.

1. Fish enter a lake at a rate modeled by the function E given by $E(t) = 20 + 15 \sin\left(\frac{\pi t}{6}\right)$. Fish leave the lake at a rate modeled by the function L given by $L(t) = 4 + 2^{0.1t^2}$. Both $E(t)$ and $L(t)$ are measured in fish per hour, and t is measured in hours since midnight ($t = 0$).
- (a) How many fish enter the lake over the 5-hour period from midnight ($t = 0$) to 5 A.M. ($t = 5$) ? Give your answer to the nearest whole number.
- (b) What is the average number of fish that leave the lake per hour over the 5-hour period from midnight ($t = 0$) to 5 A.M. ($t = 5$) ?
- (c) At what time t , for $0 \leq t \leq 8$, is the greatest number of fish in the lake? Justify your answer.
- (d) Is the rate of change in the number of fish in the lake increasing or decreasing at 5 A.M. ($t = 5$) ? Explain your reasoning.
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t (hours)	0	0.3	1.7	2.8	4
$v_P(t)$ (meters per hour)	0	55	-29	55	48

2. The velocity of a particle, P , moving along the x -axis is given by the differentiable function v_P , where $v_P(t)$ is measured in meters per hour and t is measured in hours. Selected values of $v_P(t)$ are shown in the table above. Particle P is at the origin at time $t = 0$.
- (a) Justify why there must be at least one time t , for $0.3 \leq t \leq 2.8$, at which $v_P'(t)$, the acceleration of particle P , equals 0 meters per hour per hour.
- (b) Use a trapezoidal sum with the three subintervals $[0, 0.3]$, $[0.3, 1.7]$, and $[1.7, 2.8]$ to approximate the value of $\int_0^{2.8} v_P(t) dt$.
- (c) A second particle, Q , also moves along the x -axis so that its velocity for $0 \leq t \leq 4$ is given by $v_Q(t) = 45\sqrt{t} \cos(0.063t^2)$ meters per hour. Find the time interval during which the velocity of particle Q is at least 60 meters per hour. Find the distance traveled by particle Q during the interval when the velocity of particle Q is at least 60 meters per hour.
- (d) At time $t = 0$, particle Q is at position $x = -90$. Using the result from part (b) and the function v_Q from part (c), approximate the distance between particles P and Q at time $t = 2.8$.
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END OF PART A OF SECTION II

AP[®] CALCULUS AB/CALCULUS BC
2019 SCORING GUIDELINES

Question 1

(a) $\int_0^5 E(t) dt = 153.457690$

To the nearest whole number, 153 fish enter the lake from midnight to 5 A.M.

2 : $\begin{cases} 1 : \text{integral} \\ 1 : \text{answer} \end{cases}$

(b) $\frac{1}{5-0} \int_0^5 L(t) dt = 6.059038$

The average number of fish that leave the lake per hour from midnight to 5 A.M. is 6.059 fish per hour.

2 : $\begin{cases} 1 : \text{integral} \\ 1 : \text{answer} \end{cases}$

- (c) The rate of change in the number of fish in the lake at time t is given by $E(t) - L(t)$.

$$E(t) - L(t) = 0 \Rightarrow t = 6.20356$$

$E(t) - L(t) > 0$ for $0 \leq t < 6.20356$, and $E(t) - L(t) < 0$ for $6.20356 < t \leq 8$. Therefore the greatest number of fish in the lake is at time $t = 6.204$ (or 6.203).

3 : $\begin{cases} 1 : \text{sets } E(t) - L(t) = 0 \\ 1 : \text{answer} \\ 1 : \text{justification} \end{cases}$

— OR —

Let $A(t)$ be the change in the number of fish in the lake from midnight to t hours after midnight.

$$A(t) = \int_0^t (E(s) - L(s)) ds$$

$$A'(t) = E(t) - L(t) = 0 \Rightarrow t = C = 6.20356$$

t	$A(t)$
0	0
C	135.01492
8	80.91998

Therefore the greatest number of fish in the lake is at time $t = 6.204$ (or 6.203).

(d) $E'(5) - L'(5) = -10.7228 < 0$

Because $E'(5) - L'(5) < 0$, the rate of change in the number of fish is decreasing at time $t = 5$.

2 : $\begin{cases} 1 : \text{considers } E'(5) \text{ and } L'(5) \\ 1 : \text{answer with explanation} \end{cases}$