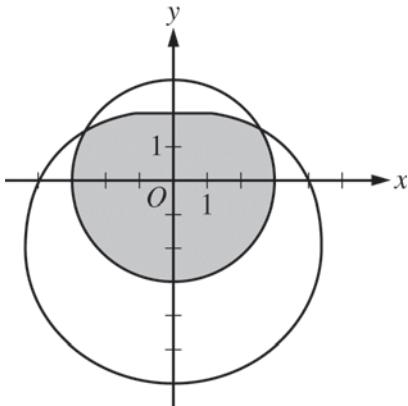


**2013 AP<sup>®</sup> CALCULUS BC FREE-RESPONSE QUESTIONS**



2. The graphs of the polar curves  $r = 3$  and  $r = 4 - 2\sin\theta$  are shown in the figure above. The curves intersect when  $\theta = \frac{\pi}{6}$  and  $\theta = \frac{5\pi}{6}$ .
- (a) Let  $S$  be the shaded region that is inside the graph of  $r = 3$  and also inside the graph of  $r = 4 - 2\sin\theta$ . Find the area of  $S$ .
- (b) A particle moves along the polar curve  $r = 4 - 2\sin\theta$  so that at time  $t$  seconds,  $\theta = t^2$ . Find the time  $t$  in the interval  $1 \leq t \leq 2$  for which the  $x$ -coordinate of the particle's position is  $-1$ .
- (c) For the particle described in part (b), find the position vector in terms of  $t$ . Find the velocity vector at time  $t = 1.5$ .
- 

**END OF PART A OF SECTION II**

**2013 AP® CALCULUS BC FREE-RESPONSE QUESTIONS**

**CALCULUS BC  
SECTION II, Part B  
Time—60 minutes  
Number of problems—4**

**No calculator is allowed for these problems.**

$t$ (minutes)	0	1	2	3	4	5	6
$C(t)$ (ounces)	0	5.3	8.8	11.2	12.8	13.8	14.5

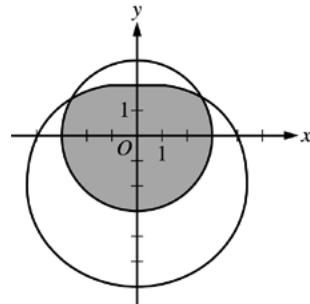
3. Hot water is dripping through a coffeemaker, filling a large cup with coffee. The amount of coffee in the cup at time  $t$ ,  $0 \leq t \leq 6$ , is given by a differentiable function  $C$ , where  $t$  is measured in minutes. Selected values of  $C(t)$ , measured in ounces, are given in the table above.
- (a) Use the data in the table to approximate  $C'(3.5)$ . Show the computations that lead to your answer, and indicate units of measure.
- (b) Is there a time  $t$ ,  $2 \leq t \leq 4$ , at which  $C'(t) = 2$ ? Justify your answer.
- (c) Use a midpoint sum with three subintervals of equal length indicated by the data in the table to approximate the value of  $\frac{1}{6} \int_0^6 C(t) dt$ . Using correct units, explain the meaning of  $\frac{1}{6} \int_0^6 C(t) dt$  in the context of the problem.
- (d) The amount of coffee in the cup, in ounces, is modeled by  $B(t) = 16 - 16e^{-0.4t}$ . Using this model, find the rate at which the amount of coffee in the cup is changing when  $t = 5$ .
-

**AP<sup>®</sup> CALCULUS BC  
2013 SCORING GUIDELINES**

**Question 2**

The graphs of the polar curves  $r = 3$  and  $r = 4 - 2\sin\theta$  are shown in the figure above. The curves intersect when  $\theta = \frac{\pi}{6}$  and  $\theta = \frac{5\pi}{6}$ .

- (a) Let  $S$  be the shaded region that is inside the graph of  $r = 3$  and also inside the graph of  $r = 4 - 2\sin\theta$ . Find the area of  $S$ .
- (b) A particle moves along the polar curve  $r = 4 - 2\sin\theta$  so that at time  $t$  seconds,  $\theta = t^2$ . Find the time  $t$  in the interval  $1 \leq t \leq 2$  for which the  $x$ -coordinate of the particle's position is  $-1$ .
- (c) For the particle described in part (b), find the position vector in terms of  $t$ . Find the velocity vector at time  $t = 1.5$ .



(a) Area =  $6\pi + \frac{1}{2} \int_{\pi/6}^{5\pi/6} (4 - 2\sin\theta)^2 d\theta = 24.709$  (or 24.708)

3 :  $\begin{cases} 1 : \text{integrand} \\ 1 : \text{limits and constant} \\ 1 : \text{answer} \end{cases}$

(b)  $x = r\cos\theta \Rightarrow x(\theta) = (4 - 2\sin\theta)\cos\theta$   
 $x(t) = (4 - 2\sin(t^2))\cos(t^2)$   
 $x(t) = -1$  when  $t = 1.428$  (or 1.427)

3 :  $\begin{cases} 1 : x(\theta) \text{ or } x(t) \\ 1 : x(\theta) = -1 \text{ or } x(t) = -1 \\ 1 : \text{answer} \end{cases}$

(c)  $y = r\sin\theta \Rightarrow y(\theta) = (4 - 2\sin\theta)\sin\theta$   
 $y(t) = (4 - 2\sin(t^2))\sin(t^2)$

3 :  $\begin{cases} 2 : \text{position vector} \\ 1 : \text{velocity vector} \end{cases}$

Position vector =  $\langle x(t), y(t) \rangle$   
 $= \langle (4 - 2\sin(t^2))\cos(t^2), (4 - 2\sin(t^2))\sin(t^2) \rangle$

$v(1.5) = \langle x'(1.5), y'(1.5) \rangle$   
 $= \langle -8.072, -1.673 \rangle$  (or  $\langle -8.072, -1.672 \rangle$ )