2. A local radio station plays 40 rock-and-roll songs during each 4-hour show. The program director at the station needs to know the total amount of airtime for the 40 songs so that time can also be programmed during the show for news and advertisements. The distribution of the lengths of rock-and-roll songs, in minutes, is roughly symmetric with a mean length of 3.9 minutes and a standard deviation of 1.1 minutes.

   (a) Describe the sampling distribution of the sample mean song lengths for random samples of 40 rock-and-roll songs.

   (b) If the program manager schedules 80 minutes of news and advertisements for the 4-hour (240-minute) show, only 160 minutes are available for music. Approximately what is the probability that the total amount of time needed to play 40 randomly selected rock-and-roll songs exceeds the available airtime?

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3. A humane society wanted to estimate with 95 percent confidence the proportion of households in its county that own at least one dog.

   (a) Interpret the 95 percent confidence level in this context.

   The humane society selected a random sample of households in its county and used the sample to estimate the proportion of all households that own at least one dog. The conditions for calculating a 95 percent confidence interval for the proportion of households in this county that own at least one dog were checked and verified, and the resulting confidence interval was 0.417 ± 0.119.

   (b) A national pet products association claimed that 39 percent of all American households owned at least one dog. Does the humane society's interval estimate provide evidence that the proportion of dog owners in its county is different from the claimed national proportion? Explain.

   (c) How many households were selected in the humane society's sample? Show how you obtained your answer.

GO ON TO THE NEXT PAGE.

Question 2

Intent of Question

The primary goals of this question were to assess students' ability to (1) describe a sampling distribution of a sample mean; (2) set up and perform a normal probability calculation based on the sampling distribution.

Solution

Part (a):

The sampling distribution of the sample mean song length has mean $\mu_{\overline{X}} = \mu = 3.9$ minutes and standard deviation $\sigma_{\overline{X}} = \frac{\sigma}{\sqrt{n}} = \frac{1.1}{\sqrt{40}} \approx 0.174$ minutes. The central limit theorem (CLT) applies in this case because the sample size ($n = 40$) is fairly large, especially with the population of song lengths having a roughly symmetric distribution. Thus, the sampling distribution of the sample mean song length is approximately normal.

Part (b):

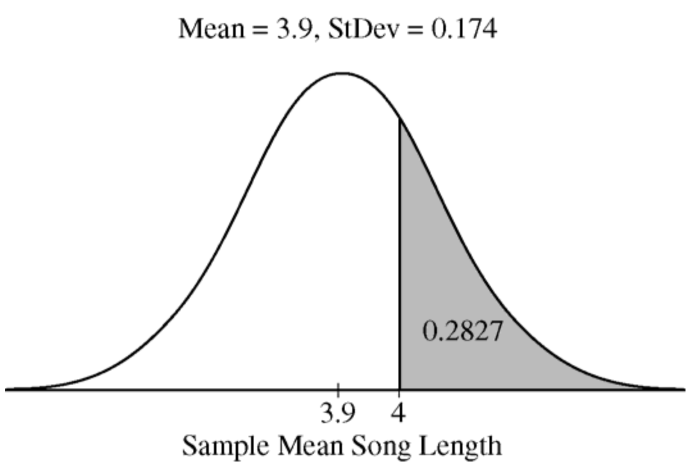
The probability that the total airtime of 40 randomly selected songs exceeds the available time (that is, the probability that the total airtime of 40 randomly selected songs is greater than 160 minutes) is equivalent to the probability that the sample mean length of the 40 songs is greater than $\frac{160}{40} = 4.0$ minutes.

According to part (a), the distribution of the sample mean length $\overline{X}$ is approximately normal. Therefore,

$$P\left(\overline{X} > 4.0\right) \approx P\left(Z > \frac{4.0 - 3.9}{0.174}\right) = P(Z > 0.57) = 1 - 0.7157 = 0.2843.$$

(The calculator gives the answer as 0.2827.)

The approximate sampling distribution of the sample mean song length and the desired probability are displayed below.



Question 2 (continued)

Part (b) (*alternative*):

An equivalent approach is to note that the sampling distribution of the total airtime, T, for the 40 songs is approximately normal, with mean $40(3.9) = 156$ minutes and standard deviation $\sqrt{40}(1.1) \approx 6.96$ minutes. The z-score for a total airtime of 160 minutes is then $z = \frac{160-156}{6.96} \approx 0.57$, and the calculation proceeds as above.

<u>Scoring</u>

Parts (a) and (b) are scored as essentially correct (E), partially correct (P) or incorrect (I).

Part (a) is scored as follows:

Essentially correct (E) if the student correctly provides all three components of the sampling distribution: shape (*approximately* normal), center (mean 3.9) and spread (standard deviation $\frac{1.1}{\sqrt{40}} \approx 0.174$).

Partially correct (P) if the student correctly provides only two of the three components.

Incorrect (I) if the student correctly provides only one or none of the components.

Notes

- Describing the sampling distribution as normal instead of approximately normal does not earn credit for the shape component.
- To earn credit for the spread component, the response must show how the standard deviation is calculated.
- If a response contains incorrect notation or terminology, it can at best be scored as partially correct (P).

Part (b) is scored as follows:

Essentially correct (E) if the student sets up and performs a correct normal probability calculation.

Partially correct (P) if the student sets up the normal probability calculation correctly but does not carry it through correctly *OR* sets up an incorrect but plausible calculation (for example, by using an incorrect standard deviation) but carries it through correctly.

Incorrect (I) if the student does not set up or perform the normal probability calculation correctly.

Question 2 (continued)

Notes

- A student can earn a score of essentially correct (E) in part (b) even with incorrect parameter values in part (a) by providing a correct calculation that uses the mean and standard deviation from part (a).
- Calculator syntax: An answer containing "normalcdf(…)" with no additional work or labeling is at best partially correct (P). If an appropriate sketch with the mean and standard deviation correctly labeled accompanies the calculator command, *OR* if the mean and standard deviation used in the calculator command are clearly identified in part (a) or part (b), then the response should be scored as essentially correct (E).
- If a student uses the sampling distribution of the total amount of time, T, needed to play the 40 randomly selected songs to do the probability calculation, the student must show how the standard deviation is calculated — unless this value is carried forward from part (a) — for the response to be scored as essentially correct (E). For example,

$$\sigma_T = \sqrt{40}\sigma_X = \sqrt{40}(1.1) \approx 6.96 \qquad \textit{OR} \qquad \sigma_T = 40\sigma_{\bar{X}} = 40(0.174) \approx 6.96.$$

4 **Complete Response**

Both parts essentially correct

3 **Substantial Response**

One part essentially correct and one part partially correct

2 **Developing Response**

One part essentially correct and one part incorrect

OR

Both parts partially correct

1 **Minimal Response**

One part partially correct and one part incorrect