3. Ms. Fey is a manager at a restaurant. To improve the dining experience for her customers, she uses a digital music service to create a playlist of songs that will be played in the restaurant. The playlist contains 1,000 songs and consists of four different types of music in the following quantities: 200 country songs, 400 pop songs, 100 rock songs, and 300 jazz songs. The digital music service will select songs at random from the playlist to be played in the restaurant. Any song can be replayed at any time.

A.

i. Suppose one song is selected at random to be played. What is the probability that the song is a rock song? Show your work.

ii. Suppose two songs are selected at random to be played. What is the probability that both songs are rock songs? Show your work.

B. In every one-hour period, 20 songs will be played at random and any song can be replayed at any time. Ms. Fey is interested in how many rock songs will be played in a typical one-hour period.

i. Define the random variable of interest to Ms. Fey, and state how the random variable is distributed.

ii. What is the expected value for the random variable in part B (i)? Show your work.

C. Recall that in every one-hour period, 20 songs will be played at random and any song can be replayed at any time.

i. Determine the probability that 4 or more rock songs in a particular one-hour period will be played. Show your work.

ii. Suppose 4 rock songs are played during a particular one-hour period. Does this provide strong evidence that the song selection process was not truly random? Justify your answer without performing an inference procedure.

4. A software application (app) lets users enter questions to receive answers in the form of images, texts, or videos. Research indicates that 22 percent of high school students in Country W use the app to help them with their homework at least once per week. Karen is an AP Statistics student in Country W at a high school that has more than 2,000 students. She believes the proportion of all students at her school who use the app to help them with their homework at least once per week is greater than the proportion for her country. To investigate her belief, she took a simple random sample of 130 students from her school and found that 38 of the sampled students use the app to help them with their homework at least once per week.

Is there convincing statistical evidence, at a 0.05 significance level, to support Karen's belief? Justify your answer with the appropriate inference procedure.

Question 3: Focus on Probability and Sampling Distributions — 4 points

General Scoring Notes

- Each part of the question (indicated by a letter) is initially scored by determining if it meets the criteria for essentially correct (E), partially correct (P), or incorrect (I). The response is then categorized based on the scores assigned to each letter part and awarded an integer score between 0 and 4 (see the table at the end of the question).
- The model solution represents an ideal response to each part of the question, and the scoring criteria identify the specific components of the model solution that are used to determine the score.

Model Solution	Scoring
A i. $P(\text{Rock Song}) = \dfrac{100}{1{,}000} = 0.10$ ii. $P(\text{Both Rock Songs}) = (0.10)(0.10) = 0.01$	**Essentially correct (E)** if the response satisfies the following four components: 1. In part A (i) the response calculates the correct probability. 2. In part A (i) the response provides supporting work for the correct probability. 3. In part A (ii) the response calculates the correct probability consistent with the answer in part A (i). 4. In part A (ii) the response provides supporting work consistent with the probability calculated in part A (i). **Partially correct (P)** if the response satisfies two or three of the four components required for E. **Incorrect (I)** if the response does not meet the criteria for E or P.

Scoring Notes:

- In part A (ii) a response that does not consider independence when solving (e.g., $\left(\dfrac{100}{1000}\right)\left(\dfrac{99}{999}\right) = 0.0099$) does not satisfy component 3 but may satisfy component 4.
- If probabilities are not labeled but are given in the order they are asked, then the response may earn an E. If the probabilities are presented in a different order, there must be a label to satisfy the components. In this case, sufficient labels include probability notation, context, work, or identification of the subparts (i) and (ii).

Model Solution	Scoring
B i. Let the random variable of interest, X, represent the number of the 20 songs played in one hour that are rock songs. It is stated that any song can be replayed at any time, which establishes that each rock song has probability $\frac{100}{1,000} = 0.10$ of being selected each hour and each song is independent from every other song. Therefore, X has a binomial distribution with $n = 20$ independent trials and probability of success $p = 0.10$ for each trial. ii. The expected value for the number of rock songs played in one hour is $np = 20(0.10) = 2$ songs.	**Essentially correct (E)** if the response satisfies at least three of the following four components: 1. In part B (i) the response defines the random variable as the number of rock songs played in one hour. 2. In part B (i) the response describes the distribution as binomial. 3. In part B (i) or B (ii) the response states that $n = 20$ and $p = 0.10$. 4. In part B (ii) the response correctly calculates the expected value AND provides supporting work for the calculation of the correct expected value. **Partially correct (P)** if the response satisfies only two of the four components required for E. **Incorrect (I)** if the response does not meet the criteria for E or P.

Scoring Notes:

- When defining the random variable, the response must include both "the number of rock songs" and "in one hour" or "out of 20 songs."
- A response that states $X \sim B(20, 0.1)$ satisfies components 2 and 3.
- If a response states the random variable has a distribution other than binomial (e.g., normal, left skewed, or uniform), part B cannot be scored E.
- Stating that songs are distributed randomly is not a distribution and should be considered extraneous.
- Examples that satisfy components 3 and 4 include:
  - $np = 20(0.10) = 2$
  - $np = 20\left(\frac{100}{1,000}\right) = 2$
  - $n = 20, p = 0.10, np = 2$
- An example that satisfies component 4 only:
  - $20(0.10) = 2$
- An arithmetic or transcription error in a response can be ignored if correct work is shown.

Model Solution	Scoring
C i. The probability that in a particular hour 4 or more rock songs will be played is $P(X \geq 4) = 1 - P(X \leq 3)$ $P(X \geq 4) = 1 - \left[\binom{20}{0}(0.10)^0 (0.90)^{20} + \right.$ $\binom{20}{1}(0.10)^1 (0.90)^{19} + \binom{20}{2}(0.10)^2 (0.90)^{18} +$ $\left. \binom{20}{3}(0.10)^3 (0.90)^{17} \right]$ $P(X \geq 4) = 1 - 0.867 = 0.133$. ii. No, the probability that 4 or more rock songs would be played in an hour is 0.133, which is high enough to be reasonably attributed to chance alone. This probability is not small enough to provide evidence that the selection process was not truly random.	**Essentially correct (E)** if the response satisfies the following four components: 1. In part C (i) the response provides a correct probability. 2. In part C (i) the response shows work that supports the correct probability. 3. In part C (ii) the response indicates that there is not a strong reason to believe that the selection process was not truly random. 4. In part C (ii) the response provides an explanation that correctly links the probability to the decision. **Partially correct (P)** if the response satisfies only two or three of the four components required for E. **Incorrect (I)** if the response does not meet the criteria for E or P.

Scoring Notes:

- A response may satisfy component 2 by any of the following:
  - <u>Graphical display</u>: Displaying a bar graph of binomial probabilities including axes with scale with appropriate bars shaded.
  - <u>Probability formula</u>: For example,
    $$1 - \binom{20}{0}(0.10)^0 (0.90)^{20} - \binom{20}{1}(0.10)^1 (0.90)^{19} - \binom{20}{2}(0.10)^2 (0.90)^{18} - \binom{20}{3}(0.10)^3 (0.90)^{17}.$$
  - <u>Calculator function notation</u>: Using calculator function notation with clearly defined arguments. For example:
    - $1 - \text{binomcdf}\,(n = 20,\ p = 0.10,\ \text{upper bound} = 3)$ satisfies component 2 because the boundary value is clearly labeled.
    - $1 - \text{binomcdf}\,(n = 20,\ p = 0.10,\ 3)$ does not satisfy component 2 because the boundary value is not labeled.
    - $\text{Binomcdf}\,(n = 20,\ p = 0.10,\ \text{lower bound} = 4,\ \text{upper bound} = 20)$ satisfies component 2 because the boundary value is clearly labeled.
  - <u>Random Variable</u>: $P(X \geq 4)$ or $1 - P(X \leq 3)$ with identification of the binomial distribution with correct parameters ($n = 20$ and $p = 0.10$) included in part C satisfies component 2.
- An arithmetic or transcription error in a response can be ignored if correct work is shown.
- A response that indicates that the manager does have a strong argument that the selection process was not truly random (or responds "yes") that is adequately supported by an explanation based on an incorrectly calculated probability in part C (i) may satisfy components 3 and 4.
- A response that indicates that the manager does have a strong argument that the selection process was not truly random (or responds "yes") that is supported by a statement claiming the probability is low may satisfy components 3 and 4.

- A response that indicates that the manager does not have a strong argument that the selection process was not random supported by the calculation of the standard deviation of the binomial distribution, 1.342, and an explanation based on 4 being within two standard deviations of the expected value (mean) may earn credit for components 3 and 4.
- If a response gives two arguments, treat them as parallel solutions and score the weaker solution.
- A response that finds the probability of exactly 4 songs playing (e.g., $\text{binompdf}(20, 0.1, 4) = 0.089$) and explains that this is not strong evidence that the selection process was not truly random may still satisfy components 3 and 4.

Scoring for Question 3	Score
Complete Response Three parts essentially correct	**4**
Substantial Response Two parts essentially correct and one part partially correct	**3**

Developing Response
Two parts essentially correct and no part partially correct
    OR
One part essentially correct and one or two parts partially correct
    OR
Three parts partially correct
	2

Minimal Response One part essentially correct and no parts partially correct *OR* No part essentially correct and two parts partially correct	**1**