

## 2010 AP<sup>®</sup> STATISTICS FREE-RESPONSE QUESTIONS

2. A local radio station plays 40 rock-and-roll songs during each 4-hour show. The program director at the station needs to know the total amount of airtime for the 40 songs so that time can also be programmed during the show for news and advertisements. The distribution of the lengths of rock-and-roll songs, in minutes, is roughly symmetric with a mean length of 3.9 minutes and a standard deviation of 1.1 minutes.
- (a) Describe the sampling distribution of the sample mean song lengths for random samples of 40 rock-and-roll songs.
- (b) If the program manager schedules 80 minutes of news and advertisements for the 4-hour (240-minute) show, only 160 minutes are available for music. Approximately what is the probability that the total amount of time needed to play 40 randomly selected rock-and-roll songs exceeds the available airtime?
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3. A humane society wanted to estimate with 95 percent confidence the proportion of households in its county that own at least one dog.

- (a) Interpret the 95 percent confidence level in this context.

The humane society selected a random sample of households in its county and used the sample to estimate the proportion of all households that own at least one dog. The conditions for calculating a 95 percent confidence interval for the proportion of households in this county that own at least one dog were checked and verified, and the resulting confidence interval was  $0.417 \pm 0.119$ .

- (b) A national pet products association claimed that 39 percent of all American households owned at least one dog. Does the humane society's interval estimate provide evidence that the proportion of dog owners in its county is different from the claimed national proportion? Explain.
- (c) How many households were selected in the humane society's sample? Show how you obtained your answer.

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4. An automobile company wants to learn about customer satisfaction among the owners of five specific car models. Large sales volumes have been recorded for three of the models, but the other two models were recently introduced so their sales volumes are smaller. The number of new cars sold in the last six months for each of the models is shown in the table below.

Car Model	A	B	C	D	E	Total
Number of new cars sold in the last six months	112,338	96,174	83,241	3,278	2,323	297,354

The company can obtain a list of all individuals who purchased new cars in the last six months for each of the five models shown in the table. The company wants to sample 2,000 of these owners.

- For simple random samples of 2,000 new car owners, what is the expected number of owners of model E and the standard deviation of the number of owners of model E?
- When selecting a simple random sample of 2,000 new car owners, how likely is it that fewer than 12 owners of model E would be included in the sample? Justify your answer.
- The company is concerned that a simple random sample of 2,000 owners would include fewer than 12 owners of model D or fewer than 12 owners of model E. Briefly describe a sampling method for randomly selecting 2,000 owners that will ensure at least 12 owners will be selected for each of the 5 car models.

5. A large pet store buys the identical species of adult tropical fish from two different suppliers—Buy-Rite Pets and Fish Friends. Several of the managers at the pet store suspect that the lengths of the fish from Fish Friends are consistently greater than the lengths of the fish from Buy-Rite Pets. Random samples of 8 adult fish of the species from Buy-Rite Pets and 10 adult fish of the same species from Fish Friends were selected and the lengths of the fish, in inches, were recorded, as shown in the table below.

	Length of Fish	Mean	Standard Deviation
Buy-Rite Pets ( $n_B = 8$ )	3.4 2.7 3.3 4.1 3.5 3.4 3.0 3.8	3.40	0.434
Fish Friends ( $n_F = 10$ )	3.3 2.9 4.2 3.1 4.2 4.0 3.4 3.2 3.7 2.6	3.46	0.550

Do the data provide convincing evidence that the mean length of the adult fish of the species from Fish Friends is greater than the mean length of the adult fish of the same species from Buy-Rite Pets?

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### Question 3

#### **Intent of Question**

The primary goals of this question were to assess students' ability to (1) interpret the meaning of a confidence level; (2) use a confidence interval to test the plausibility of a claim about the value of a population parameter; (3) perform a sample size calculation related to a confidence interval.

#### **Solution**

##### **Part (a):**

The 95 percent confidence level means that if one were to repeatedly take random samples of the same size from the population and construct a 95 percent confidence interval from each sample, then in the long run 95 percent of those intervals would succeed in capturing the actual value of the population proportion of households in the county that own at least one dog.

##### **Part (b):**

No. The 95 percent confidence interval  $0.417 \pm 0.119$  is the interval  $(0.298, 0.536)$ . This interval includes the value 0.39 as a plausible value for the population proportion of households in the county that own at least one dog. Therefore, the confidence interval does not provide evidence that the proportion of dog owners in this county is different from the claimed national proportion.

##### **Part (c):**

The sample proportion is 0.417, and the margin of error is 0.119. Determining the sample size requires solving the equation  $0.119 = 1.96 \times \sqrt{\frac{0.417 \times (1 - 0.417)}{n}}$  for  $n$ .

Thus,  $n = \frac{1.96^2 \times 0.417 \times (1 - 0.417)}{0.119^2} \approx 65.95$ , so the humane society must have selected 66 households for its sample.

#### **Scoring**

Parts (a), (b) and (c) are scored as essentially correct (E), partially correct (P) or incorrect (I).

**Part (a)** is scored as follows:

Essentially correct (E) if the student provides a correct interpretation of the confidence level in the context of the study. A correct interpretation can take one of two approaches:

1. Based on the concept of repeated sampling, the response must fulfill the following three requirements:
  - Mentions repeated sampling or "in the long run" or "using this method"
  - Mentions that 95 percent of the intervals will capture the population proportion
  - Includes the context of the study

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### Question 3 (continued)

2. Based on probability, the response must state that there is a 0.95 probability that a random sample *selected in the future* will produce an interval that captures the actual value of the population proportion of households in the county that have at least one dog.

Partially correct (P) if the student provides an interpretation of the confidence level that includes two of the three components required for the repeated sampling interpretation *OR* provides a correct probability interpretation, but not in context.

Incorrect (I) if the student attempts to interpret a particular confidence *interval* rather than the confidence *level* (for example, by saying that we are 95 percent confident that an interval that has been obtained includes the population proportion of households in the county that have at least one dog) *OR* provides an interpretation of the confidence level that mentions at most one of the three components required for the repeated sampling interpretation.

**Part (b)** is scored as follows:

Essentially correct (E) if the student correctly states that because 0.39 (or “the claimed value”) is in the computed interval, the interval does not provide evidence that the proportion of dog owners in the county is different from the claimed national proportion.

Partially correct (P) if the student indicates that the goal is to check whether the claimed value of 0.39 is in the computed interval but makes errors in implementation. Examples of errors include the following:

- The student notes that 0.39 is within the interval but does not draw a correct conclusion.
- The student makes an arithmetic error in computing the endpoints of the interval, but the conclusion is consistent with the computed interval.

*OR*

The student correctly notes that 0.39 is in the interval and then concludes that 0.39 *is* the population proportion for the county.

Incorrect (I) if the student does not recognize how to check whether the claim is consistent with the confidence interval.

**Part (c)** is scored as follows:

Essentially correct (E) if the student provides a correct equation with correct numerical values substituted, as well as a correct integer solution.

Partially correct (P) if the student provides a correct equation (and substitutions) but makes one or more of the following errors:

- Does not complete the calculation or completes the calculation incorrectly
- Uses 0.5 or 0.39 rather than 0.417 as the sample proportion
- Uses an incorrect but plausible  $z^*$  value
- Reports the answer as a non-integer value
- Gives the calculated value of  $n$  as a lower bound for the required sample size

Incorrect (I) otherwise.

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### Question 3 (continued)

#### Notes

- It is acceptable to use  $z^* = 2$  instead of 1.96.
- It is acceptable for the response to round up or down to get an integer answer.

#### **4 Complete Response**

All three parts essentially correct

#### **3 Substantial Response**

Two parts essentially correct and one part partially correct

#### **2 Developing Response**

Two parts essentially correct and one part incorrect

OR

One part essentially correct and one or two parts partially correct

OR

Three parts partially correct

#### **1 Minimal Response**

One part essentially correct and two parts incorrect

OR

Two parts partially correct and one part incorrect