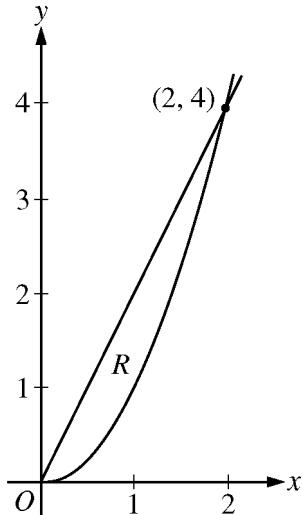


2009 AP® CALCULUS AB FREE-RESPONSE QUESTIONS

CALCULUS AB
SECTION II, Part B
Time—45 minutes
Number of problems—3

No calculator is allowed for these problems.



4. Let R be the region in the first quadrant enclosed by the graphs of $y = 2x$ and $y = x^2$, as shown in the figure above.
- Find the area of R .
 - The region R is the base of a solid. For this solid, at each x the cross section perpendicular to the x -axis has area $A(x) = \sin\left(\frac{\pi}{2}x\right)$. Find the volume of the solid.
 - Another solid has the same base R . For this solid, the cross sections perpendicular to the y -axis are squares. Write, but do not evaluate, an integral expression for the volume of the solid.

WRITE ALL WORK IN THE PINK EXAM BOOKLET.

2009 AP® CALCULUS AB FREE-RESPONSE QUESTIONS

x	2	3	5	8	13
$f(x)$	1	4	-2	3	6

5. Let f be a function that is twice differentiable for all real numbers. The table above gives values of f for selected points in the closed interval $2 \leq x \leq 13$.
- (a) Estimate $f'(4)$. Show the work that leads to your answer.
- (b) Evaluate $\int_2^{13} (3 - 5f'(x)) dx$. Show the work that leads to your answer.
- (c) Use a left Riemann sum with subintervals indicated by the data in the table to approximate $\int_2^{13} f(x) dx$. Show the work that leads to your answer.
- (d) Suppose $f'(5) = 3$ and $f''(x) < 0$ for all x in the closed interval $5 \leq x \leq 8$. Use the line tangent to the graph of f at $x = 5$ to show that $f(7) \leq 4$. Use the secant line for the graph of f on $5 \leq x \leq 8$ to show that $f(7) \geq \frac{4}{3}$.
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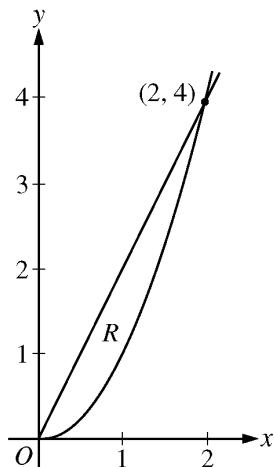
WRITE ALL WORK IN THE PINK EXAM BOOKLET.

**AP[®] CALCULUS AB
2009 SCORING GUIDELINES**

Question 4

Let R be the region in the first quadrant enclosed by the graphs of $y = 2x$ and $y = x^2$, as shown in the figure above.

- (a) Find the area of R .
- (b) The region R is the base of a solid. For this solid, at each x the cross section perpendicular to the x -axis has area $A(x) = \sin\left(\frac{\pi}{2}x\right)$. Find the volume of the solid.
- (c) Another solid has the same base R . For this solid, the cross sections perpendicular to the y -axis are squares. Write, but do not evaluate, an integral expression for the volume of the solid.



$$\begin{aligned}(a) \text{ Area} &= \int_0^2 (2x - x^2) dx \\&= x^2 - \frac{1}{3}x^3 \Big|_{x=0}^{x=2} \\&= \frac{4}{3}\end{aligned}$$

3 : $\begin{cases} 1 : \text{integrand} \\ 1 : \text{antiderivative} \\ 1 : \text{answer} \end{cases}$

$$\begin{aligned}(b) \text{ Volume} &= \int_0^2 \sin\left(\frac{\pi}{2}x\right) dx \\&= -\frac{2}{\pi} \cos\left(\frac{\pi}{2}x\right) \Big|_{x=0}^{x=2} \\&= \frac{4}{\pi}\end{aligned}$$

3 : $\begin{cases} 1 : \text{integrand} \\ 1 : \text{antiderivative} \\ 1 : \text{answer} \end{cases}$

$$(c) \text{ Volume} = \int_0^4 \left(\sqrt{y} - \frac{y}{2} \right)^2 dy$$

3 : $\begin{cases} 2 : \text{integrand} \\ 1 : \text{limits} \end{cases}$