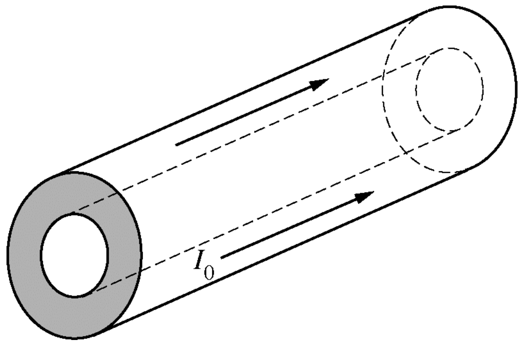
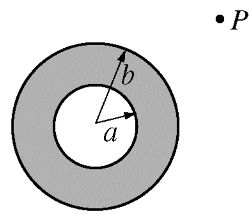




Cross-sectional View
(current into page)

E&M. 3.

A section of a long conducting cylinder with inner radius a and outer radius b carries a current I_0 that has a uniform current density, as shown in the figure above.

(a) Using Ampère's law, derive an expression for the magnitude of the magnetic field in the following regions as a function of the distance r from the central axis.

i. $r < a$

---

ii. $a < r < b$

---

iii. $r = 2b$

(b) On the cross-sectional view in the diagram above, indicate the direction of the field at point P, which is at a distance $r = 2b$ from the axis of the cylinder.

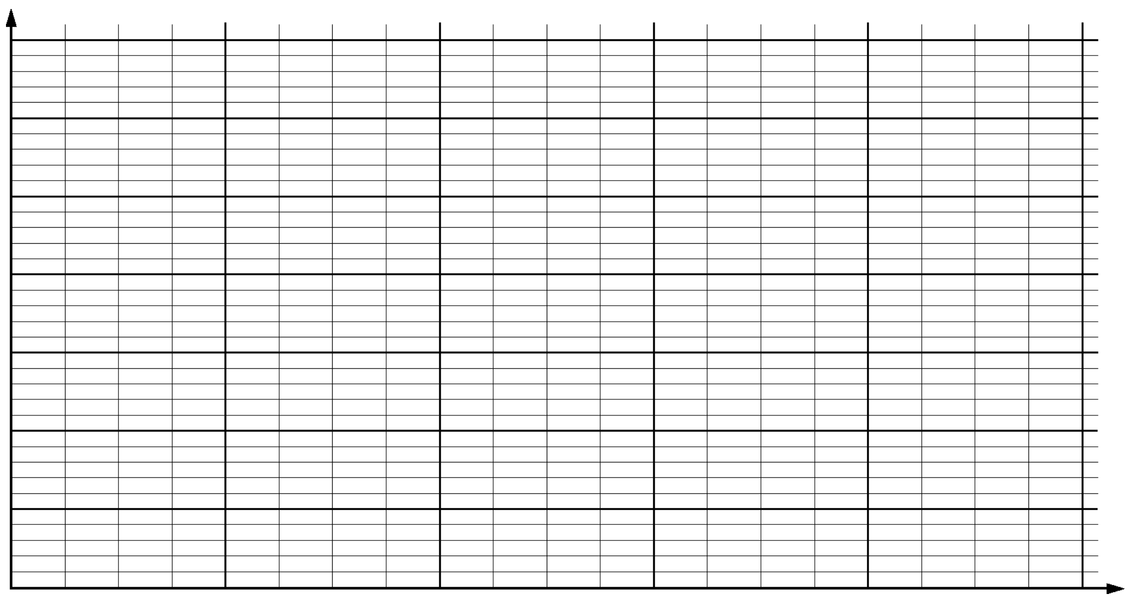
(c) An electron is at rest at point P. Describe any electromagnetic forces acting on the electron. Justify your answer.

Now consider a long, solid conducting cylinder of radius b carrying a current I_0. The magnitude of the magnetic field inside this cylinder as a function of r is given by $B = \mu_0 I_0 r / 2\pi b^2$. An experiment is conducted using a particular solid cylinder of radius 0.010 m carrying a current of 25 A. The magnetic field inside the cylinder is measured as a function of r, and the data is tabulated below.

Distance r (m)	0.002	0.004	0.006	0.008	0.010
Magnetic Field B (T)	1.2×10^{-4}	2.7×10^{-4}	3.6×10^{-4}	4.7×10^{-4}	6.4×10^{-4}

(d)

i. On the graph below, plot the data points for the magnetic field B as a function of the distance r, and label the scale on both axes. Draw a straight line that best represents the data.



ii. Use the slope of your line to estimate a value of the permeability μ_0.

END OF EXAM

Question 3

15 points total | **Distribution of points**

(a)

For all three cases, the path of integration when applying Ampere's law is a circle concentric with the cylinder and perpendicular to its axis, with a radius r in the range specified.

i. 2 points

For explicitly stating Ampere's law in at least one of parts (a)i, (a)ii or (a)iii — 1 point

$\oint \mathbf{B} \cdot d\ell = \mu_0 I_{\text{enc}}$

$I_{\text{enc}} = 0$

For the correct answer — 1 point

$B = 0$

ii. 3 points

$\oint \mathbf{B} \cdot d\ell = \mu_0 I_{\text{enc}}$

For a correct simplification of the line integral — 1 point

$\oint \mathbf{B} \cdot d\ell = B(2\pi r)$

Calculating the current density:

$J = \dfrac{I_0}{\pi b^2 - \pi a^2}$

For an expression giving I_{enc} as a fraction of I_0 — 1 point

$$I_{\text{enc}} = J \cdot (\text{area enclosed}) = J\left(\pi r^2 - \pi a^2\right) = \frac{I_0\left(\pi r^2 - \pi a^2\right)}{\left(\pi b^2 - \pi a^2\right)} = \frac{I_0\left(r^2 - a^2\right)}{\left(b^2 - a^2\right)}$$

$$B(2\pi r) = \mu_0 \frac{I_0\left(r^2 - a^2\right)}{\left(b^2 - a^2\right)}$$

For the correct expression for B — 1 point

$$B = \frac{\mu_0 I_0\left(r^2 - a^2\right)}{2\pi r\left(b^2 - a^2\right)}$$

iii. 1 point

$\oint \mathbf{B} \cdot d\ell = \mu_0 I_{\text{enc}}$

$B(2\pi b) = \mu_0 I_{\text{enc}}$

For the correct expression for B — 1 point

$$B = \frac{\mu_0 I_0}{4\pi b}$$