

## 1999 CALCULUS BC

$t$ (hours)	$R(t)$ (gallons per hour)
0	9.6
3	10.4
6	10.8
9	11.2
12	11.4
15	11.3
18	10.7
21	10.2
24	9.6

3. The rate at which water flows out of a pipe, in gallons per hour, is given by a differentiable function  $R$  of time  $t$ . The table above shows the rate as measured every 3 hours for a 24-hour period.
- (a) Use a midpoint Riemann sum with 4 subdivisions of equal length to approximate  $\int_0^{24} R(t)dt$ . Using correct units, explain the meaning of your answer in terms of water flow.
- (b) Is there some time  $t$ ,  $0 < t < 24$ , such that  $R'(t) = 0$ ? Justify your answer.
- (c) The rate of water flow  $R(t)$  can be approximated by  $Q(t) = \frac{1}{79}(768 + 23t - t^2)$ .  
Use  $Q(t)$  to approximate the average rate of water flow during the 24-hour time period.  
Indicate units of measure.
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**GO ON TO THE NEXT PAGE** 

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4. The function  $f$  has derivatives of all orders for all real numbers  $x$ . Assume  $f(2) = -3$ ,  $f'(2) = 5$ ,  $f''(2) = 3$ , and  $f'''(2) = -8$ .
- (a) Write the third-degree Taylor polynomial for  $f$  about  $x = 2$  and use it to approximate  $f(1.5)$ .
- (b) The fourth derivative of  $f$  satisfies the inequality  $|f^{(4)}(x)| \leq 3$  for all  $x$  in the closed interval  $[1.5, 2]$ . Use the Lagrange error bound on the approximation to  $f(1.5)$  found in part (a) to explain why  $f(1.5) \neq -5$ .
- (c) Write the fourth-degree Taylor polynomial,  $P(x)$ , for  $g(x) = f(x^2 + 2)$  about  $x = 0$ . Use  $P$  to explain why  $g$  must have a relative minimum at  $x = 0$ .
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**GO ON TO THE NEXT PAGE** 