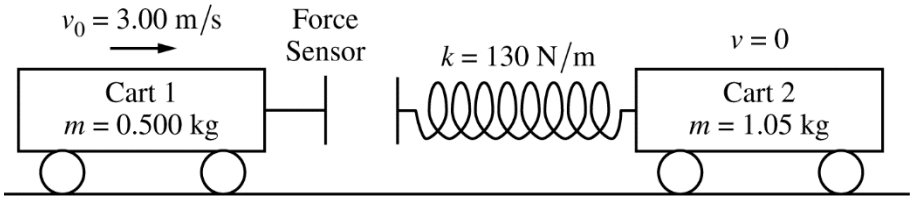
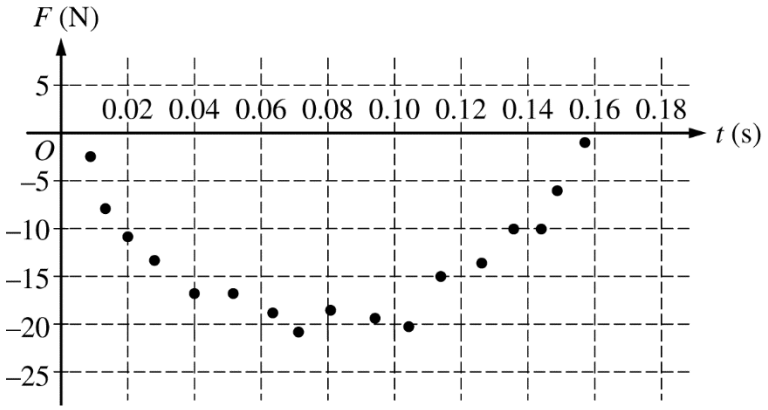


2. Two carts are on a horizontal, level track of negligible friction. Cart 1 has a sensor that measures the force exerted on it during a collision with cart 2, which has a spring attached. Cart 1 is moving with a speed of $v_0 = 3.00$ m/s toward cart 2, which is at rest, as shown in the figure above. The total mass of cart 1 and the force sensor is 0.500 kg, the mass of cart 2 is 1.05 kg, and the spring has negligible mass. The spring has a spring constant of $k = 130$ N/m. The data for the force the spring exerts on cart 1 are shown in the graph below. A student models the data as the quadratic fit $F = (3200\ \text{N/s}^2)t^2 - (500\ \text{N/s})t$.



(a) Using integral calculus, calculate the total impulse delivered to cart 1 during the collision.

(b)

i. Calculate the speed of cart 1 after the collision.

ii. In which direction does cart 1 move after the collision?

____ Left ____ Right

____ The direction is undefined, because the speed of cart 1 is zero after the collision.

(c)

i. Calculate the speed of cart 2 after the collision.

ii. Show that the collision between the two carts is elastic.

(d)

i. Calculate the speed of the center of mass of the two-cart–spring system.

ii. Calculate the maximum elastic potential energy stored in the spring.

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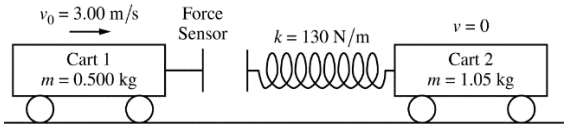
Question 2

15 points total **Distribution of points**



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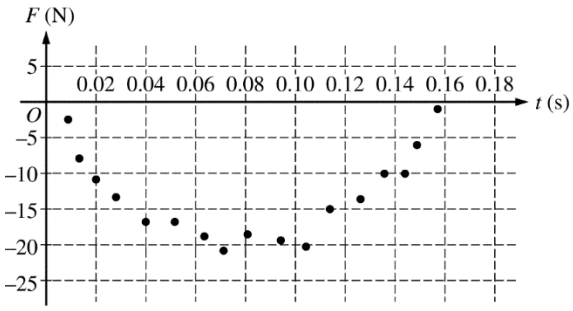

(a) 3 points

Using integral calculus, calculate the total impulse delivered to cart 1 during the collision.

For using the given force equation in the integral to determine the impulse delivered to the cart		1 point
$J = \int F \bullet dt = \int 3200t^2 - 500t\ dt$		
For integrating the force using the correct limits or constant of integration		1 point
$J = \int_{t=0}^{t=0.16\ \text{s}} \left(3200t^2 - 500t\right)dt = \left[\frac{3200}{3}t^3 - 250t^2\right]_{t=0}^{t=0.16}$		
$J = \left(\left(\frac{3200}{3}\right)(0.16)^3 - (250)(0.16)^2\right) - 0$		
For a correct answer		1 point
$J = -2.03$ N•s		

Question 2 (continued)

Distribution of points

(b)

i. 1 point

Calculate the speed of cart 1 after the collision.

For correct substitution into the impulse-momentum equation of the answer from part (a) to determine the speed of cart 1		1 point
$J = m_1\left(v_{1f} - v_{1i}\right) \therefore v_{1f} = \frac{J}{m_1} + v_{1i} = \frac{(-2.03\ \text{N•s})}{(0.500\ \text{kg})} + (3.00\ \text{m/s})$		
$v_{1f} = -1.06\ \text{m/s}$		

ii. 1 point

In which direction does cart 1 move after the collision?

____ Left ____ Right

____ The direction is undefined, because the speed of cart 1 is zero after the collision.

For correctly selecting "Left"		1 point

(c)

i. 2 points

Calculate the speed of cart 2 after the collision.

For using a correct equation to determine the speed of cart 2		1 point
$-J = -m_1\left(v_{1f} - v_{1i}\right) = m_2 v_{2f} \therefore v_{2f} = \frac{-J}{m_2}$		
For correct substitution of the answer from part (a)		1 point
$v_{2f} = \frac{(2.03\ \text{N•s})}{(1.05\ \text{kg})}$		
$v_{2f} = 1.93\ \text{m/s}$		

Question 2 (continued)

Distribution of points

(c)

i. (continued)

Alternate solution		*Alternate Points*
For using the conservation of momentum to calculate the speed of cart 2 after the collision		*1 point*
$m_1 v_{1i} = m_1 v_{1f} + m_2 v_{2f} \therefore v_{2f} = \dfrac{m_1 \left(v_{1i} - v_{1f} \right)}{m_2}$		
For correct substitution of the answer from part (b)(i)		*1 point*
$v_{2f} = \dfrac{(0.500 \text{ kg})((3.00 \text{ m/s}) - (-1.06 \text{ m/s}))}{(1.05 \text{ kg})} = 1.93 \text{ m/s}$		

ii. 2 points

Show that the collision between the two carts is elastic.

For indicating that the initial and final kinetic energies must be equal		1 point
$K_{1i} = K_{1f} + K_{2f}$		
For correct substitutions of answers from parts (b)(i) and (c)(i) into the calculations of the initial and final kinetic energies		1 point
$\left(\frac{1}{2}\right)(0.500 \text{ kg})(3.00 \text{ m/s})^2 = \left(\frac{1}{2}\right)(0.500 \text{ kg})(-1.06 \text{ m/s})^2 + \left(\frac{1}{2}\right)(1.05 \text{ kg})(1.93 \text{ m/s})^2$		
$2.25 \text{ J} \approx 2.24 \text{ J}$		

(d)

i. 2 points

Calculate the speed of the center of mass of the two-cart-spring system.

For using the equation for the conservation of momentum to calculate the speed for the center of mass of the system		1 point
$m_1 v_{1i} = (m_1 + m_2) v_{cm} \therefore v_{cm} = \dfrac{m_1 v_{1i}}{(m_1 + m_2)}$		
For correct substitution into the equation above		1 point
$v_{cm} = \dfrac{(0.500 \text{ kg})(3.00 \text{ m/s})}{(0.500 \text{ kg} + 1.05 \text{ kg})} = 0.97 \text{ m/s}$		

Question 2 (continued)

Distribution of points

(d) (continued)

ii. 3 points

Calculate the maximum elastic potential energy stored in the spring.

For using conservation of energy to calculate the maximum elastic potential energy stored in the spring		1 point
$K_i = K_f + U_{Sf}$		
For using the speed of the center of mass of the system for kinetic energy		1 point
$\frac{1}{2}m_1 v_{1i}^2 = \frac{1}{2}(m_1 + m_2)v_{cm}{}^2 + U_{Sf}$		
$U_{Sf} = \frac{1}{2}m_1 v_{1i}^2 - \frac{1}{2}(m_1 + m_2)v_{cm}{}^2$		
For correct substitution into above equation		1 point
$U_S = \left(\frac{1}{2}\right)(0.500\text{ kg})(3.00\text{ m/s})^2 - \frac{1}{2}(0.500\text{ kg} + 1.05\text{ kg})(0.97\text{ m/s})^2$		
$U_S = 1.52\text{ J}$		
Alternate Solution:		*Alternate Points*
For correctly determining the magnitude of the maximum force exerted between the carts		1 point
Set $\frac{dF}{dt} = 0 = 6400t - 500 \therefore 6400t = 500 \therefore t = 0.078\text{ s}$		
$F_{MAX} = 3200(0.078)^2 - 500(0.078) = -19.5$		
$\lvert F_{MAX}\rvert = 19.5\text{ N}$		
Note: Can estimate from the graph, and accept the range of 20 N to 21 N.		
For calculating the maximum compression of the spring		1 point
$F_{MAX} = -kx_{MAX}$		
$x_{MAX} = -\frac{F_{MAX}}{k} = -\frac{(-19.5\text{ N})}{(130\text{ N/m})} = 0.15\text{ m}$		
Note: If estimating from the graph, accept the range from 0.15 m to 0.16 m.		
For substituting into an equation for the elastic potential energy at maximum spring compression		1 point
$U_S = \frac{1}{2}kx_{MAX}^2 = \left(\frac{1}{2}\right)(130\text{ N/m})(0.15\text{ m})^2$		
$U_S = 1.46\text{ J}$		
Note: If estimating from the graph, accept range from 1.46 J to 1.70 J.		

Units point: 1 point for correct units on all calculated answers