6. A particle moves along the curve defined by the equation $y = x^3 - 3x$. The x–coordinate of the particle, $x(t)$, satisfies the equation $\frac{dx}{dt} = \frac{1}{\sqrt{2t+1}}$, for $t \geq 0$ with initial condition $x(0) = -4$.

   (a) Find $x(t)$ in terms of t.

   (b) Find $\frac{dy}{dt}$ in terms of t.

   (c) Find the location and speed of the particle at time $t = 4$.

END OF EXAMINATION

6. A particle moves along the curve defined by the equation $y = x^3 - 3x$. The x-coordinate of the particle, $x(t)$, satisfies the equation $\frac{dx}{dt} = \frac{1}{\sqrt{2t+1}}$, for $t \geq 0$ with initial condition $x(0) = -4$.

(a) Find $x(t)$ in terms of t.

(b) Find $\frac{dy}{dt}$ in terms of t.

(c) Find the location and speed of the particle at time $t = 4$.

(a) $x(t) = \int \frac{1}{\sqrt{2t+1}}\, dt$

$x(t) = \sqrt{2t+1} + C$

$x(0) = -4 = 1 + C \implies C = -5$

$x(t) = \sqrt{2t+1} - 5$

$$3 \begin{cases} 1\text{: } x(t) = \int \frac{dt}{\sqrt{2t+1}} \\ 1\text{: } x(t) = \sqrt{2t+1} + C \\ 1\text{: evaluates } C \end{cases}$$

(b) $y = x^3 - 3x$

$\frac{dy}{dt} = 3x^2 \frac{dx}{dt} - 3\frac{dx}{dt}$

$= (3x^2 - 3)\frac{dx}{dt}$

$= \left[3\left(\sqrt{2t+1} - 5\right)^2 - 3\right]\left[\frac{1}{\sqrt{2t+1}}\right]$

2: answer

<−1> each error

Note: failure to express $\frac{dy}{dt}$ solely in terms of t is a single error

(c) $x(4) = \sqrt{9} - 5 = -2$

$y(4) = (-2)^3 - 3(-2) = -2$

Location at $t = 4$ is $(-2, -2)$

$\left.\frac{dx}{dt}\right|_{t=4} = \frac{1}{3}$

$\left.\frac{dy}{dt}\right|_{t=4} = \frac{3(3-5)^2 - 3}{3} = 3$

Speed $= \sqrt{\left(\frac{1}{3}\right)^2 + 3^2} = \sqrt{\frac{82}{9}} = 3.018$

$$4 \begin{cases} 1\text{: position} \\ 1\text{: evaluates } \frac{dx}{dt} \text{ and } \frac{dy}{dt} \text{ at } t = 4 \\ 1\text{: uses speed formula} \\ 1\text{: answer} \end{cases}$$