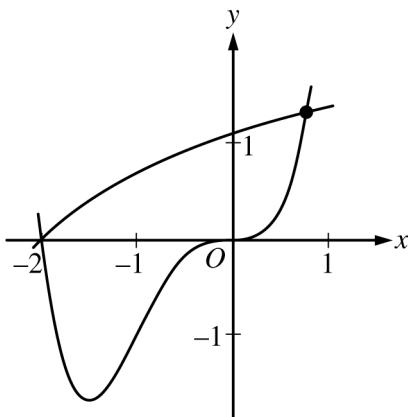


1. From 5 A.M. to 10 A.M., the rate at which vehicles arrive at a certain toll plaza is given by  $A(t) = 450\sqrt{\sin(0.62t)}$ , where  $t$  is the number of hours after 5 A.M. and  $A(t)$  is measured in vehicles per hour. Traffic is flowing smoothly at 5 A.M. with no vehicles waiting in line.
- (a) Write, but do not evaluate, an integral expression that gives the total number of vehicles that arrive at the toll plaza from 6 A.M. ( $t = 1$ ) to 10 A.M. ( $t = 5$ ).
- (b) Find the average value of the rate, in vehicles per hour, at which vehicles arrive at the toll plaza from 6 A.M. ( $t = 1$ ) to 10 A.M. ( $t = 5$ ).
- (c) Is the rate at which vehicles arrive at the toll plaza at 6 A.M. ( $t = 1$ ) increasing or decreasing? Give a reason for your answer.
- (d) A line forms whenever  $A(t) \geq 400$ . The number of vehicles in line at time  $t$ , for  $a \leq t \leq 4$ , is given by  $N(t) = \int_a^t (A(x) - 400) dx$ , where  $a$  is the time when a line first begins to form. To the nearest whole number, find the greatest number of vehicles in line at the toll plaza in the time interval  $a \leq t \leq 4$ . Justify your answer.

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**Write your responses to this question only on the designated pages in the separate Free Response booklet. Write your solution to each part in the space provided for that part.**



2. Let  $f$  and  $g$  be the functions defined by  $f(x) = \ln(x + 3)$  and  $g(x) = x^4 + 2x^3$ . The graphs of  $f$  and  $g$ , shown in the figure above, intersect at  $x = -2$  and  $x = B$ , where  $B > 0$ .
- Find the area of the region enclosed by the graphs of  $f$  and  $g$ .
  - For  $-2 \leq x \leq B$ , let  $h(x)$  be the vertical distance between the graphs of  $f$  and  $g$ . Is  $h$  increasing or decreasing at  $x = -0.5$ ? Give a reason for your answer.
  - The region enclosed by the graphs of  $f$  and  $g$  is the base of a solid. Cross sections of the solid taken perpendicular to the  $x$ -axis are squares. Find the volume of the solid.
  - A vertical line in the  $xy$ -plane travels from left to right along the base of the solid described in part (c). The vertical line is moving at a constant rate of 7 units per second. Find the rate of change of the area of the cross section above the vertical line with respect to time when the vertical line is at position  $x = -0.5$ .

**Write your responses to this question only on the designated pages in the separate Free Response booklet. Write your solution to each part in the space provided for that part.**

**Part A (AB or BC): Graphing calculator required****Question 1****9 points****General Scoring Notes**

The model solution is presented using standard mathematical notation.

Answers (numeric or algebraic) need not be simplified. Answers given as a decimal approximation should be correct to three places after the decimal point. Within each individual free-response question, at most one point is not earned for inappropriate rounding.

From 5 A.M. to 10 A.M., the rate at which vehicles arrive at a certain toll plaza is given by

$A(t) = 450\sqrt{\sin(0.62t)}$ , where  $t$  is the number of hours after 5 A.M. and  $A(t)$  is measured in vehicles per hour. Traffic is flowing smoothly at 5 A.M. with no vehicles waiting in line.

**Model Solution****Scoring**

- (a) Write, but do not evaluate, an integral expression that gives the total number of vehicles that arrive at the toll plaza from 6 A.M. ( $t = 1$ ) to 10 A.M. ( $t = 5$ ).

The total number of vehicles that arrive at the toll plaza from 6 A.M. to 10 A.M. is given by  $\int_1^5 A(t) dt$ .

Answer

**1 point****Scoring notes:**

- The response must be a definite integral with correct lower and upper limits to earn this point.
- Because  $|A(t)| = A(t)$  for  $1 \leq t \leq 5$ , a response of  $\int_1^5 |450\sqrt{\sin(0.62t)}| dt$  or  $\int_1^5 |A(t)| dt$  earns the point.
- A response missing  $dt$  or using  $dx$  is eligible to earn the point.
- A response with a copy error in the expression for  $A(t)$  will earn the point only in the presence of  $\int_1^5 A(t) dt$ .

**Total for part (a)****1 point**

- (b) Find the average value of the rate, in vehicles per hour, at which vehicles arrive at the toll plaza from 6 A.M. ( $t = 1$ ) to 10 A.M. ( $t = 5$ ).

$$\text{Average} = \frac{1}{5-1} \int_1^5 A(t) dt = 375.536966$$

The average rate at which vehicles arrive at the toll plaza from 6 A.M. to 10 A.M. is 375.537 (or 375.536) vehicles per hour.

Uses average value formula:

$$\frac{1}{b-a} \int_a^b A(t) dt$$

Answer

**1 point**

**1 point**

**Scoring notes:**

- The use of the average value formula, indicating that  $a = 1$  and  $b = 5$ , can be presented in single or multiple steps to earn the first point. For example, the following response earns both points:

$$\int_1^5 A(t) dt = 1502.147865, \text{ so the average value is } 375.536966.$$

- A response that presents a correct integral along with the correct average value, but provides incorrect or incomplete communication, earns 1 out of 2 points. For example, the following response earns 1 out of 2 points:  $\int_1^5 A(t) dt = 1502.147865 = 375.536966$ .

- The answer must be correct to three decimal places. For example,

$$\frac{1}{5-1} \int_1^5 A(t) dt = 375.536966 \approx 376 \text{ earns only the first point.}$$

- Degree mode: A response that presents answers obtained by using a calculator in degree mode does not earn the first point it would have otherwise earned. The response is generally eligible for all subsequent points (unless no answer is possible in degree mode or the question is made simpler by using degree mode). In degree mode,  $\frac{1}{4} \int_1^5 A(t) dt = 79.416068$ .

- Special case:  $\frac{1}{5} \int_1^5 A(t) dt = 300.429573$  earns 1 out of 2 points.

**Total for part (b)      2 points**

**Scoring notes:**

- It is not necessary to indicate that  $A(t) = 400$  to earn the first point, although this statement alone would earn the first point.
- A response of “ $A(t) \geq 400$  when  $1.469372 \leq t \leq 3.597713$ ” will earn the first 2 points. A response of “ $A(t) \geq 400$ ” along with the presence of exactly one of the two numbers above will earn the first point, but not the second. A response of “ $A(t) \geq 400$ ” by itself will not earn either of the first 2 points.
- To earn the second point the values for  $a$  and  $b$  must be accurate to the number of decimals presented, with at least one and up to three decimal places. These may appear only in a candidates table, as limits of integration, or on a number line.
- A response with incorrect notation involving  $t$  or  $x$  is eligible to earn all 4 points.
- A response that does not earn the first point is still eligible for the remaining 3 points.
- To earn the third point, a response must present the greatest number of vehicles. This point is earned for answers of either 71 or 71.254\*\*\* only.
- A correct justification earns the fourth point, even if the third point is not earned because of a decimal presentation error.
- When using a Candidates Test, the response must include the values for  $N(a)$ ,  $N(b)$ , and  $N(4)$  to earn the fourth point. These values must be correct to the number of decimals presented, with up to three decimal places. (Correctly rounded integer values are acceptable.)
- Alternate solution for the third and fourth points:

For  $a \leq t \leq b$ ,  $A(t) \geq 400$ . For  $b \leq t \leq 4$ ,  $A(t) \leq 400$ .

Thus,  $N(t) = \int_a^t (A(x) - 400) dx$  is greatest at  $t = b$ .

$N(b) = 71.254129$ , and the greatest number of vehicles in line is 71.

- Degree mode: The response is only eligible to earn the first point because in degree mode  $A(t) < 400$ .

<b>Total for part (d)</b>	<b>4 points</b>
<b>Total for question 1</b>	<b>9 points</b>

- (b) Find the  $x$ -coordinates of all points of inflection of the graph of  $f$  for  $0 < x < 7$ . Justify your answer.

The graph of  $f$  has a point of inflection at each of  $x = 2$  and  $x = 6$ , because  $f'(x)$  changes from decreasing to increasing at  $x = 2$  and from increasing to decreasing at  $x = 6$ .

Answer	<b>1 point</b>
Justification	<b>1 point</b>

**Scoring notes:**

- A response that gives only one of  $x = 2$  or  $x = 6$ , along with a correct justification, earns 1 of the 2 points.
- A response that claims that there is a point of inflection at any value other than  $x = 2$  or  $x = 6$  earns neither point.
- To earn the second point a response must use correct reasoning based on the graph of  $f'$ . Examples of correct reasoning include:
  - Correctly discussing the signs of the slopes of the graph of  $f'$
  - Citing  $x = 2$  and  $x = 6$  as the locations of local extrema on the graph of  $f'$
- Examples of reasoning not (sufficiently) connected to the graph of  $f'$  include:
  - Reasoning based on sign changes in  $f''$  unless the connection is made between the sign of  $f''$  and the slopes of the graph of  $f'$
  - Reasoning based only on the concavity of the graph of  $f$
- The second point cannot be earned by use of vague or undefined terms such as “it” or “the function” or “the derivative.”
- Responses that report inflection points as ordered pairs must report the points  $(2, 3 + \pi)$  and  $(6, 5)$  in order to earn the first point. If the  $y$ -coordinates are reported incorrectly, the response remains eligible for the second point.

**Total for part (b)      2 points**

- (c) Let  $g$  be the function defined by  $g(x) = f(x) - x$ . On what intervals, if any, is  $g$  decreasing for  $0 \leq x \leq 7$ ? Show the analysis that leads to your answer.

$g'(x) = f'(x) - 1$	$g'(x) = f'(x) - 1$	<b>1 point</b>
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$f'(x) - 1 \leq 0 \Rightarrow f'(x) \leq 1$	<b>1 point</b>
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The graph of  $g$  is decreasing on the interval  $0 \leq x \leq 5$  because  $g'(x) \leq 0$  on this interval.

**Scoring notes:**

- The first point can be earned for  $f'(x) \leq 1$  or the equivalent, in words or symbols.
- Endpoints do not need to be included in the interval to be eligible for the second point.

**Total for part (c)      2 points**