3. An object moving along a curve in the xy-plane has position $(x(t),\ y(t))$ at time $t \geq 0$ with $\dfrac{dx}{dt} = 3 + \cos\left(t^2\right)$. The derivative $\dfrac{dy}{dt}$ is not explicitly given. At time $t = 2$, the object is at position $(1, 8)$.

   (a) Find the x-coordinate of the position of the object at time $t = 4$.

   (b) At time $t = 2$, the value of $\dfrac{dy}{dt}$ is -7. Write an equation for the line tangent to the curve at the point $(x(2),\ y(2))$.

   (c) Find the speed of the object at time $t = 2$.

   (d) For $t \geq 3$, the line tangent to the curve at $(x(t),\ y(t))$ has a slope of $2t + 1$. Find the acceleration vector of the object at time $t = 4$.

END OF PART A OF SECTION II

CALCULUS BC
SECTION II, Part B
Time—45 minutes
Number of problems—3

No calculator is allowed for these problems.

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4. Consider the curve given by $x^2 + 4y^2 = 7 + 3xy$.

(a) Show that $\frac{dy}{dx} = \frac{3y - 2x}{8y - 3x}$.

(b) Show that there is a point P with x-coordinate 3 at which the line tangent to the curve at P is horizontal. Find the y-coordinate of P.

(c) Find the value of $\frac{d^2y}{dx^2}$ at the point P found in part (b). Does the curve have a local maximum, a local minimum, or neither at the point P? Justify your answer.

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5. A population is modeled by a function P that satisfies the logistic differential equation

$$\frac{dP}{dt} = \frac{P}{5}\left(1 - \frac{P}{12}\right).$$

(a) If $P(0) = 3$, what is $\lim_{t\to\infty} P(t)$?
If $P(0) = 20$, what is $\lim_{t\to\infty} P(t)$?

(b) If $P(0) = 3$, for what value of P is the population growing the fastest?

(c) A different population is modeled by a function Y that satisfies the separable differential equation

$$\frac{dY}{dt} = \frac{Y}{5}\left(1 - \frac{t}{12}\right).$$

Find $Y(t)$ if $Y(0) = 3$.

(d) For the function Y found in part (c), what is $\lim_{t\to\infty} Y(t)$?
