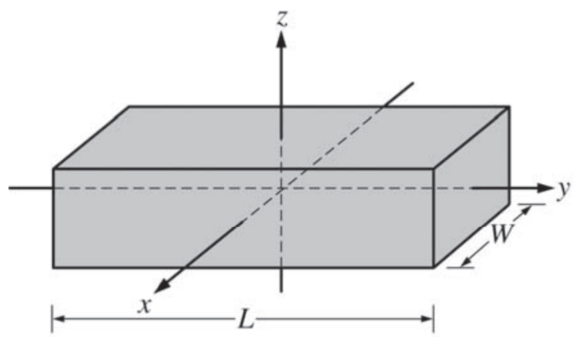
PHYSICS C: ELECTRICITY AND MAGNETISM

SECTION II

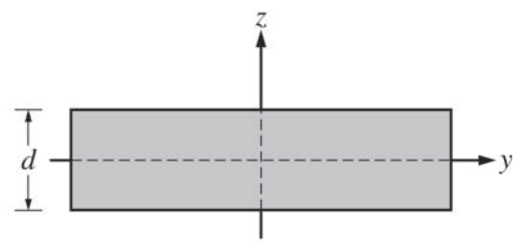
Time—45 minutes

3 Questions

Directions: Answer all three questions. The suggested time is about 15 minutes for answering each of the questions, which are worth 15 points each. The parts within a question may not have equal weight. Show all your work in this booklet in the spaces provided after each part.



Perspective View

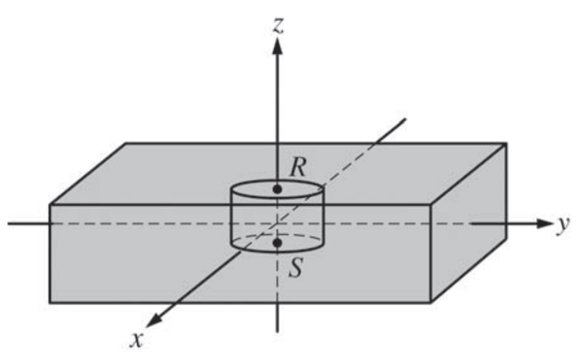


Side View

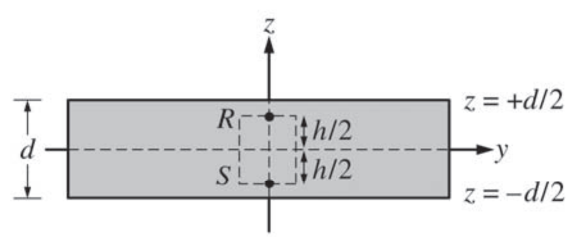
Note: Figures not drawn to scale.

1. A very large nonconducting slab with a uniform positive volume charge density ρ_0 is fixed with the origin of the *xyz*-axes at its center, as shown in the figure above. The thickness of the slab is d, the length is L, and the width is W, where $L >> d$ and $W >> d$. The large faces of the slab are parallel to the *xy*-plane.

   Consider a Gaussian cylinder with a cross-sectional area A and height h that is positioned with its axis along the *z*-axis, as shown in the figure below.



Perspective View



Side View

Note: Figures not drawn to scale.

(a) Draw a single vector on each of the dots below representing the direction of the electric field at the given points. If the electric field at either point is zero, write "$E = 0$" next to the point.

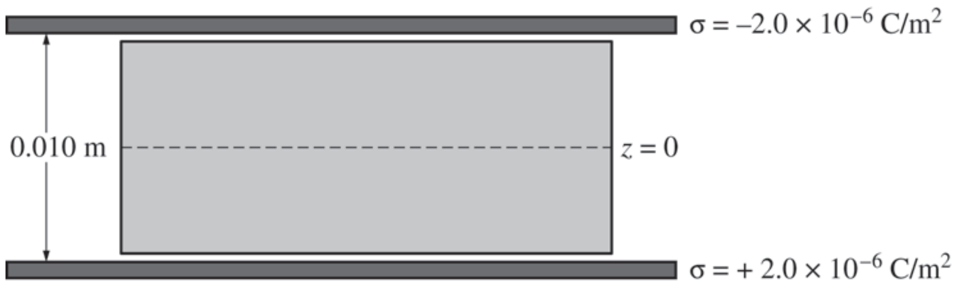
i. • R

ii. • S

b) Use Gauss's law to derive expressions for the following. Express your answers in terms of ρ_0, A, d, h, z, and physical constants, as appropriate.

i. Derive an expression for the total flux Φ through the Gaussian surface shown.

ii. Derive an expression for the magnitude of the electric field as a function of z for any position inside the slab, and show that it is equal to $E = \dfrac{\rho_0 z}{\varepsilon_0}$.



Note: Figure not drawn to scale.

The charged slab is now placed between two large metal plates separated by a distance of 0.010 m, which is approximately the thickness of the slab, but the slab does not contact either metal plate. The metal plates are charged, resulting in the surface charge densities $\sigma = \pm 2.0 \times 10^{-6}\ \mathrm{C/m^2}$, as shown in the figure above. Assume the charge distribution inside the slab remains unchanged by the presence of the charged plates and that the slab's volume charge density is $\rho_0 = 1.00 \times 10^{-3}\ \mathrm{C/m^3}$.

(c)

i. The magnitude of the electric field inside the slab is zero on the z-axis at position z_0. Which of the following correctly indicates the value for z_0?

____ $z_0 > 0$ ____ $z_0 = 0$ ____ $z_0 < 0$

Justify your answer.

ii. Calculate the value z_0.

(d) Calculate the magnitude of the electric potential difference from the center of the slab to the top of the slab.

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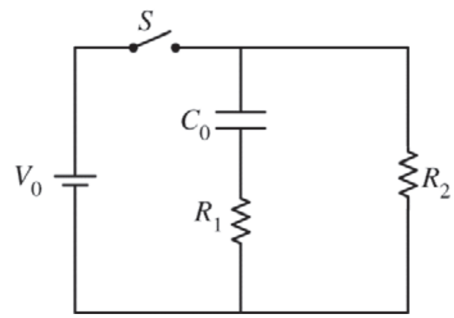
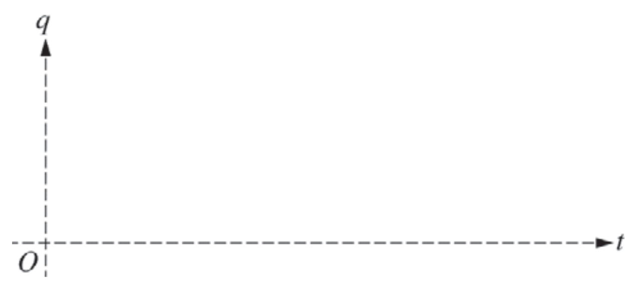
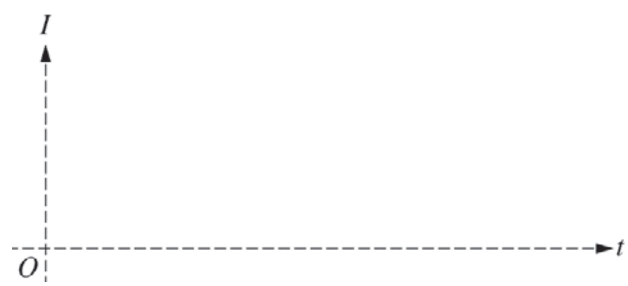


2. In the circuit above, an ideal battery of voltage V_0 is connected to a capacitor with capacitance C_0 and resistors with resistances R_1 and R_2, with $R_1 > R_2$. The switch *S* is open, and the capacitor is initially uncharged.

(a) The switch is closed at time $t = 0$. On the axes below, sketch the charge *q* on the capacitor as a function of time *t*. Explicitly label any intercepts, asymptotes, maxima, or minima with numerical values or algebraic expressions, as appropriate.



(b) On the axes below, sketch the current *I* through each resistor as a function of time *t*. Clearly label the two curves as I_1 and I_2, the currents through resistors R_1 and R_2, respectively. Explicitly label any intercepts, asymptotes, maxima, or minima with numerical values or algebraic expressions, as appropriate.



The circuit is constructed using an ideal 1.5 V battery, an 80 μF capacitor, and resistors $R_1 = 150\ \Omega$ and $R_2 = 100\ \Omega$. The switch is closed, allowing the capacitor to fully charge. The switch is then opened, allowing the capacitor to discharge.

(c) The time it takes to charge the capacitor to 50% of its maximum charge is Δt_C. The time it takes for the capacitor to discharge to 50% of its maximum charge is Δt_D. Which of the following correctly relates the two time intervals?

____ $\Delta t_C > \Delta t_D$ ____ $\Delta t_C = \Delta t_D$ ____ $\Delta t_C < \Delta t_D$

Justify your answer.

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Question 1 (continued)

Distribution of points

(c)

i. 1 point

Selecting “$z_0 < 0$”

For a correct justification — 1 point

Example: The electric field due to the plates alone is directed toward the top of the page. Therefore, the net electric field can only be zero where the electric field of the slab is directed toward the bottom of the page. So the net electric field can only be zero when $z_0 < 0$.

ii. 3 points

For setting the net electric field between the plates equal to zero and relating it to the sum of the electric fields of the plates and the slab — 1 point

$$E_{total} = 0 = E_{plate} + E_{slab}$$

For using the correct formula for the electric field between the plates regardless of sign — 1 point

$$\frac{\sigma}{\varepsilon_0} = -\frac{\rho_0 z_0}{\varepsilon_0}$$

$$z_0 = -\frac{\sigma}{\rho_0}$$

Substitute into equation above

$$z_0 = -\frac{\left(2.0 \times 10^{-6}\ \mathrm{C/m^2}\right)}{\left(1.0 \times 10^{-3}\ \mathrm{C/m^3}\right)}$$

For an answer that is consistent with the formula provided by the student above (must include units) — 1 point

$$z_0 = -2.0 \times 10^{-3}\ \mathrm{m}$$

Question 1 (continued)

Distribution of points

(d) 4 points

For using the sum of the electric potential of the slab and the plates as the total potential difference between the center and top of the slab — 1 point

$$\Delta V = \Delta V_{slab} + \Delta V_{plate}$$

$$\Delta V = \int E_{total}\, dr = \int \left(E_{slab} + E_{plate}\right) dr$$

$$\Delta V = \int E_{slab} dr + \int E_{plate}\, dr$$

For integrating with proper limits or constants of integration — 1 point

$$\Delta V = \int_{z=0}^{z=0.005} \frac{\rho_0 z}{\varepsilon_0} dz + \int_{z=0}^{z=0.005} \frac{\sigma}{\varepsilon_0} dz$$

For correctly integrating each term regardless of whether they contain the correct expressions — 1 point

$$\Delta V = \frac{\rho_0}{\varepsilon_0}\left[\frac{z^2}{2}\right]_{z=0}^{z=0.005} + \frac{\sigma}{\varepsilon_0}[z]_{z=0}^{z=0.005} = \frac{\rho_0}{2\varepsilon_0}\left(0.005^2 - 0\right) + \frac{\sigma}{\varepsilon_0}(0.005 - 0)$$

$$\Delta V = \frac{\rho_0 z^2}{2\varepsilon_0} + \frac{\sigma z}{\varepsilon_0}$$

$$\Delta V = \frac{\left(1.0 \times 10^{-3}\ \mathrm{C/m^3}\right)\left(5.0 \times 10^{-3}\ \mathrm{m}\right)^2}{(2)\left(8.85 \times 10^{-12}\ \mathrm{C^2/N{\bullet}m^2}\right)} + \frac{\left(2.0 \times 10^{-6}\ \mathrm{C/m^2}\right)\left(5.0 \times 10^{-3}\ \mathrm{m}\right)}{\left(8.85 \times 10^{-12}\ \mathrm{C^2/N{\bullet}m^2}\right)}$$

For a correct answer — 1 point

$$\Delta V = 2540\ \mathrm{V}$$