

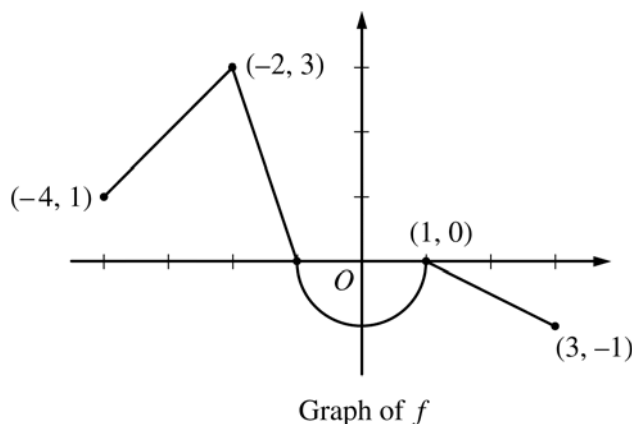
2012 AP[®] CALCULUS AB FREE-RESPONSE QUESTIONS

CALCULUS AB
SECTION II, Part B

Time—60 minutes

Number of problems—4

No calculator is allowed for these problems.



3. Let f be the continuous function defined on $[-4, 3]$ whose graph, consisting of three line segments and a semicircle centered at the origin, is given above. Let g be the function given by $g(x) = \int_1^x f(t) dt$.
- Find the values of $g(2)$ and $g(-2)$.
 - For each of $g'(-3)$ and $g''(-3)$, find the value or state that it does not exist.
 - Find the x -coordinate of each point at which the graph of g has a horizontal tangent line. For each of these points, determine whether g has a relative minimum, relative maximum, or neither a minimum nor a maximum at the point. Justify your answers.
 - For $-4 < x < 3$, find all values of x for which the graph of g has a point of inflection. Explain your reasoning.

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4. The function f is defined by $f(x) = \sqrt{25 - x^2}$ for $-5 \leq x \leq 5$.

(a) Find $f'(x)$.

(b) Write an equation for the line tangent to the graph of f at $x = -3$.

(c) Let g be the function defined by $g(x) = \begin{cases} f(x) & \text{for } -5 \leq x \leq -3 \\ x + 7 & \text{for } -3 < x \leq 5. \end{cases}$

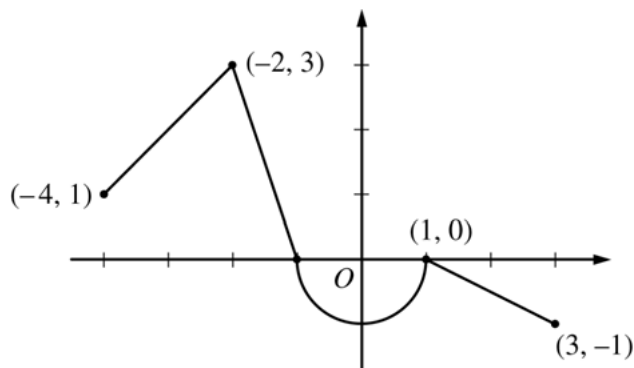
Is g continuous at $x = -3$? Use the definition of continuity to explain your answer.

(d) Find the value of $\int_0^5 x\sqrt{25 - x^2} \, dx$.

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2012 SCORING GUIDELINES

Question 3

Let f be the continuous function defined on $[-4, 3]$ whose graph, consisting of three line segments and a semicircle centered at the origin, is given above. Let g be the function given by $g(x) = \int_1^x f(t) dt$.



Graph of f

- (a) Find the values of $g(2)$ and $g(-2)$.
- (b) For each of $g'(-3)$ and $g''(-3)$, find the value or state that it does not exist.
- (c) Find the x -coordinate of each point at which the graph of g has a horizontal tangent line. For each of these points, determine whether g has a relative minimum, relative maximum, or neither a minimum nor a maximum at the point. Justify your answers.
- (d) For $-4 < x < 3$, find all values of x for which the graph of g has a point of inflection. Explain your reasoning.

(a) $g(2) = \int_1^2 f(t) dt = -\frac{1}{2}(1)\left(\frac{1}{2}\right) = -\frac{1}{4}$

$$g(-2) = \int_1^{-2} f(t) dt = -\int_{-2}^1 f(t) dt$$

$$= -\left(\frac{3}{2} - \frac{\pi}{2}\right) = \frac{\pi}{2} - \frac{3}{2}$$

$$2 : \begin{cases} 1 : g(2) \\ 1 : g(-2) \end{cases}$$

(b) $g'(x) = f(x) \Rightarrow g'(-3) = f(-3) = 2$
 $g''(x) = f'(x) \Rightarrow g''(-3) = f'(-3) = 1$

$$2 : \begin{cases} 1 : g'(-3) \\ 1 : g''(-3) \end{cases}$$

- (c) The graph of g has a horizontal tangent line where $g'(x) = f(x) = 0$. This occurs at $x = -1$ and $x = 1$.

$g'(x)$ changes sign from positive to negative at $x = -1$.
 Therefore, g has a relative maximum at $x = -1$.

$g'(x)$ does not change sign at $x = 1$. Therefore, g has neither a relative maximum nor a relative minimum at $x = 1$.

$$3 : \begin{cases} 1 : \text{considers } g'(x) = 0 \\ 1 : x = -1 \text{ and } x = 1 \\ 1 : \text{answers with justifications} \end{cases}$$

- (d) The graph of g has a point of inflection at each of $x = -2$, $x = 0$, and $x = 1$ because $g''(x) = f'(x)$ changes sign at each of these values.

$$2 : \begin{cases} 1 : \text{answer} \\ 1 : \text{explanation} \end{cases}$$