CALCULUS BC
SECTION II, Part A
Time—45 minutes
Number of problems—3

A graphing calculator is required for some problems or parts of problems.

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1. An object moving along a curve in the xy-plane has position $(x(t), y(t))$ at time t with

$$\frac{dx}{dt} = \cos\left(t^3\right) \text{ and } \frac{dy}{dt} = 3\sin\left(t^2\right)$$

for $0 \le t \le 3$. At time $t = 2$, the object is at position $(4, 5)$.

(a) Write an equation for the line tangent to the curve at $(4, 5)$.

(b) Find the speed of the object at time $t = 2$.

(c) Find the total distance traveled by the object over the time interval $0 \le t \le 1$.

(d) Find the position of the object at time $t = 3$.

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t (days)	$W(t)$ (°C)
0	20
3	31
6	28
9	24
12	22
15	21

2. The temperature, in degrees Celsius (°C), of the water in a pond is a differentiable function W of time t. The table above shows the water temperature as recorded every 3 days over a 15-day period.

   (a) Use data from the table to find an approximation for $W'(12)$. Show the computations that lead to your answer. Indicate units of measure.

   (b) Approximate the average temperature, in degrees Celsius, of the water over the time interval $0 \le t \le 15$ days by using a trapezoidal approximation with subintervals of length $\Delta t = 3$ days.

   (c) A student proposes the function P, given by $P(t) = 20 + 10te^{(-t/3)}$, as a model for the temperature of the water in the pond at time t, where t is measured in days and $P(t)$ is measured in degrees Celsius. Find $P'(12)$. Using appropriate units, explain the meaning of your answer in terms of water temperature.

   (d) Use the function P defined in part (c) to find the average value, in degrees Celsius, of $P(t)$ over the time interval $0 \le t \le 15$ days.

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Question 1

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(d) Find the position of the object at time $t = 3$.

(a) $\frac{dy}{dx} = \frac{3\sin\left(t^2\right)}{\cos\left(t^3\right)}$ $\left.\frac{dy}{dx}\right\|_{t=2} = \frac{3\sin\left(2^2\right)}{\cos\left(2^3\right)} = 15.604$ $y - 5 = 15.604(x - 4)$	1 : tangent line
(b) Speed $= \sqrt{\cos^2(8) + 9\sin^2(4)} = 2.275$	1 : answer
(c) Distance $= \int_0^1 \sqrt{\cos^2\left(t^3\right) + 9\sin^2\left(t^2\right)}\, dt$ $= 1.458$	3 : { 2 : distance integral < −1 > each integrand error < −1 > error in limits 1 : answer
(d) $x(3) = 4 + \int_2^3 \cos\left(t^3\right) dt = 3.953$ or 3.954 $y(3) = 5 + \int_2^3 3\sin\left(t^2\right) dt = 4.906$	4 : { 1 : definite integral for x 1 : answer for $x(3)$ 1 : definite integral for y 1 : answer for $y(3)$