PHYSICS C
Section II, MECHANICS
Time—45 minutes
3 Questions

Directions: Answer all three questions. The suggested time is about 15 minutes for answering each of the questions, which are worth 15 points each. The parts within a question may not have equal weight. Show all your work in the pink booklet in the spaces provided after each part, NOT in this green insert.

Mech 1.

A crash test car of mass 1,000 kg moving at constant speed of 12 m/s collides completely inelastically with an object of mass M at time $t = 0$. The object was initially at rest. The speed v in m/s of the car-object system after the collision is given as a function of time t in seconds by the expression

$$v = \frac{8}{1 + 5t}.$$

(a) Calculate the mass M of the object.

(b) Assuming an initial position of $x = 0$, determine an expression for the position of the car-object system after the collision as a function of time t.

(c) Determine an expression for the resisting force on the car-object system after the collision as a function of time t.

(d) Determine the impulse delivered to the car-object system from $t = 0$ to $t = 2.0$ s.

GO ON TO THE NEXT PAGE.

AP® PHYSICS C: MECHANICS
2002 SCORING GUIDELINES

Question 1

15 points total **Distribution of points**

(a) 4 points

For any statement of conservation of momentum 1 point

$p_{before} = p_{after}$

For correctly substituting quantities into the left side of the above equation 1 point

For correctly substituting quantities into the right side of the above equation 1 point

$m_{car}\, v_{before} = (M + m_{car})\, v_{after}$

$(1{,}000 \text{ kg})(12 \text{ m/s}) = (M + 1{,}000 \text{ kg})\left(\frac{8}{1+5t} \text{ m/s}\right)$

p_{after} occurs just after the collision, i.e. at $t = 0$

$(1{,}000 \text{ kg})(12 \text{ m/s}) = (M + 1{,}000 \text{ kg})(8 \text{ m/s})$

For the correct answer 1 point

$M = 500$ kg

(b) 3 points

For any indication that the velocity v is the time derivative of position x 1 point

$\frac{dx}{dt} = \frac{8}{1+5t}$

For correctly expressing the above equation as an integral, with or without using limits 1 point

$x = \int \frac{8}{1+5t}\, dt$

Substituting $u = 1 + 5t$ and $du = 5dt$

$x = \frac{8}{5}\int \frac{du}{u} = \frac{8}{5}\ln u + C = \frac{8}{5}\ln(1+5t) + C$ (or equivalent using limits)

The condition $x = 0$ at $t = 0$ gives $C = 0$

For the correct answer 1 point

$x = \frac{8}{5}\ln(1+5t)$

Question 1 (cont'd.)

Distribution of points

(c) 4 points

For use of Newton's second law — 1 point

$F = ma$

For any indication that the acceleration a is the time derivative of velocity v — 1 point

$$a = \frac{dv}{dt} = \frac{d}{dt}\left(\frac{8}{1+5t}\right)$$

$$a = -\frac{40}{(1+5t)^2}$$

$$F = (1500)\left(-\frac{40}{(1+5t)^2}\right)$$

For the including the minus sign in the final expression for the force — 1 point

For having the proper expression for the magnitude of the force — 1 point

$$F = -\frac{60,000}{(1+5t)^2}$$

Alternate solution — *Alternate points*

For any indication that the force F is the time derivative of momentum p — *1 point*

$$F = \frac{dp}{dt}$$

For expressing the force in terms of the derivative of the velocity — *1 point*

$$F = m\frac{dv}{dt} = m\frac{d}{dt}\left(\frac{8}{1+5t}\right) = (1500)\left(-\frac{40}{(1+5t)^2}\right)$$

For the including the minus sign in the final expression for the force — *1 point*

For having the proper expression for the magnitude of the force — *1 point*

$$F = -\frac{60,000}{(1+5t)^2}$$

Question 1 (cont'd.)

Distribution of points

(d) 3 points

For a correct expression for the impulse — 1 point
$J = \Delta p$ or $m\,\Delta v$

For correct substitution into the equation above — 1 point
$J = m_{total}\left[v(2) - v(0)\right] = (1500\text{ kg})\left(\frac{8}{11} - 8\right)\text{m/s}$

For the correct sign on the final answer — 1 point
$J = -10{,}909\,\text{kg}\cdot\text{m/s}$ or $\text{N}\cdot\text{s}$

Alternate solution — *Alternate points*

For a correct expression for the impulse — *1 point*
$J = \int F\,dt$ or $F_{average}\,\Delta t$

For correct substitution into the equation above — *1 point*
$J = \int_0^2 -\frac{60{,}000}{(1+5t)^2}\,dt$

For the correct sign on the final answer — *1 point*
$J = -10{,}909\,\text{kg}\cdot\text{m/s}$ or $\text{N}\cdot\text{s}$

Units point

For correct units on the answers to both part (a) and part (d) — 1 point