

2004 AP[®] CALCULUS BC FREE-RESPONSE QUESTIONS

CALCULUS BC
SECTION II, Part A
Time—45 minutes
Number of problems—3

A graphing calculator is required for some problems or parts of problems.

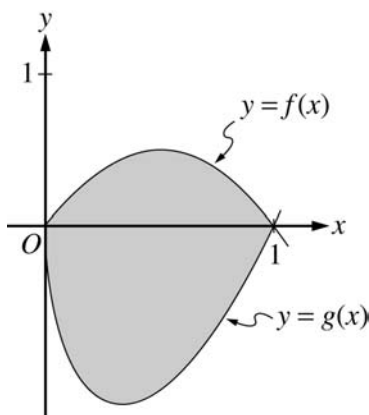
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1. Traffic flow is defined as the rate at which cars pass through an intersection, measured in cars per minute. The traffic flow at a particular intersection is modeled by the function F defined by

$$F(t) = 82 + 4\sin\left(\frac{t}{2}\right) \text{ for } 0 \leq t \leq 30,$$

where $F(t)$ is measured in cars per minute and t is measured in minutes.

- (a) To the nearest whole number, how many cars pass through the intersection over the 30-minute period?
- (b) Is the traffic flow increasing or decreasing at $t = 7$? Give a reason for your answer.
- (c) What is the average value of the traffic flow over the time interval $10 \leq t \leq 15$? Indicate units of measure.
- (d) What is the average rate of change of the traffic flow over the time interval $10 \leq t \leq 15$? Indicate units of measure.
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2. Let f and g be the functions given by $f(x) = 2x(1 - x)$ and $g(x) = 3(x - 1)\sqrt{x}$ for $0 \leq x \leq 1$. The graphs of f and g are shown in the figure above.
- Find the area of the shaded region enclosed by the graphs of f and g .
 - Find the volume of the solid generated when the shaded region enclosed by the graphs of f and g is revolved about the horizontal line $y = 2$.
 - Let h be the function given by $h(x) = kx(1 - x)$ for $0 \leq x \leq 1$. For each $k > 0$, the region (not shown) enclosed by the graphs of h and g is the base of a solid with square cross sections perpendicular to the x -axis. There is a value of k for which the volume of this solid is equal to 15. Write, but do not solve, an equation involving an integral expression that could be used to find the value of k .
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AP[®] CALCULUS BC
2004 SCORING GUIDELINES

Question 1

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- (d) What is the average rate of change of the traffic flow over the time interval $10 \leq t \leq 15$? Indicate units of measure.

(a) $\int_0^{30} F(t) dt = 2474 \text{ cars}$

$$3 : \begin{cases} 1 : \text{limits} \\ 1 : \text{integrand} \\ 1 : \text{answer} \end{cases}$$

(b) $F'(7) = -1.872$ or -1.873
 Since $F'(7) < 0$, the traffic flow is decreasing
 at $t = 7$.

1 : answer with reason

(c) $\frac{1}{5} \int_{10}^{15} F(t) dt = 81.899 \text{ cars/min}$

$$3 : \begin{cases} 1 : \text{limits} \\ 1 : \text{integrand} \\ 1 : \text{answer} \end{cases}$$

(d) $\frac{F(15) - F(10)}{15 - 10} = 1.517$ or 1.518 cars/min^2

1 : answer

Units of cars/min in (c) and cars/min² in (d)

1 : units in (c) and (d)