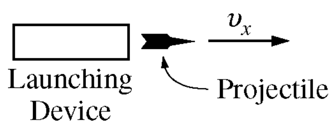
PHYSICS C: MECHANICS
SECTION II
Time—45 minutes
3 Questions

Directions: Answer all three questions. The suggested time is about 15 minutes for answering each of the questions, which are worth 15 points each. The parts within a question may not have equal weight. Show all your work in the pink booklet in the spaces provided after each part, NOT in this green insert.



Mech. 1.

A projectile is fired horizontally from a launching device, exiting with a speed v_x. While the projectile is in the launching device, the impulse imparted to it is J_p, and the average force on it is F_{avg}. Assume the force becomes zero just as the projectile reaches the end of the launching device. Express your answers to parts (a) and (b) in terms of v_x, J_p, F_{avg}, and fundamental constants, as appropriate.

(a) Determine an expression for the time required for the projectile to travel the length of the launching device.
(b) Determine an expression for the mass of the projectile.

The projectile is fired horizontally into a block of wood that is clamped to a tabletop so that it cannot move. The projectile travels a distance d into the block before it stops. Express all algebraic answers to the following in terms of d and the given quantities previously indicated, as appropriate.

(c) Derive an expression for the work done in stopping the projectile.
(d) Derive an expression for the average force F_b exerted on the projectile as it comes to rest in the block.

Now a new projectile and block are used, identical to the first ones, but the block is <u>not</u> clamped to the table. The projectile is again fired into the block of wood and travels a new distance d_n into the block while the block slides across the table a short distance D. Assume the following: the projectile enters the block with speed v_x, the average force F_b between the projectile and the block has the same value as determined in part (d), the average force of friction between the table and the block is f_T, and the collision is instantaneous so the frictional force is negligible during the collision.

(e) Derive an expression for d_n in terms of d, D, f_T, and F_b, as appropriate.
(f) Derive an expression for d_n in terms of d, the mass m of the projectile, and the mass M of the block.

Question 1

15 points total | **Distribution of points**

(a) 2 points

$\mathbf{J} = \int \mathbf{F}\, dt$

For a correct equation relating the given force, time and impulse — 1 point

$J_p = F_{avg}\Delta t$

For the correct answer — 1 point

$\Delta t = J_p / F_{avg}$

Alternate solution — *Alternate points*

For using both kinematics and Newton's second law — *1 point*

$\upsilon_x = 0 + a_{avg}\Delta t$

$\mathbf{F}_{avg} = m\mathbf{a}_{avg}$

Combining the above equations

$F_{avg} = m\left(\dfrac{\upsilon_x}{\Delta t}\right)$

$F_{avg}\Delta t = m\upsilon_x = J_p$

For the correct answer — *1 point*

$\Delta t = J_p / F_{avg}$

(b) 2 points

For the correct relationship between impulse and the change in momentum — 1 point

$\mathbf{J} = \Delta\mathbf{p} = m\Delta\mathbf{v}$

$J_p = m(\upsilon_x - 0) = m\upsilon_x$

For the correct answer — 1 point

$m = J_p / \upsilon_x$

Note: A correct kinematics and Newton's laws approach is also acceptable.

Question 1 (continued)

Distribution of points

(c) 3 points

For using the work-energy theorem — 1 point

$W = \Delta K$

$W = 0 - \frac{1}{2}m\upsilon_x^2$

For substituting the expression for m from part (b) — 1 point

$W = -\frac{1}{2}\frac{J_p}{\upsilon_x}\upsilon_x^2$

$W = -\frac{1}{2}J_p\upsilon_x$

For an indication that the work done is negative — 1 point

Alternate Solution — *Alternate points*

Using kinematics and Newton's second law to determine the average net force

$\upsilon_f^2 - \upsilon_i^2 = 2a_{avg}d$

$-\upsilon_x^2 = 2a_{avg}d$

$a_{avg} = -\frac{\upsilon_x^2}{2d}$

$\mathbf{F}_{avg} = m\mathbf{a}_{avg}$

$F_{net} = m\left(-\frac{\upsilon_x^2}{2d}\right)$

For substituting this expression for the force into the equation for work — *1 point*

$W = \int \mathbf{F}\cdot d\mathbf{r} = F_{avg}d = m\left(-\frac{\upsilon_x^2}{2d}\right)d$

$W = -m\frac{\upsilon_x^2}{2}$

For substituting the expression for m from part (b) — *1 point*

$W = -\frac{J_p}{\upsilon_x}\frac{\upsilon_x^2}{2}$

$W = -\frac{1}{2}J_p\upsilon_x$

For an indication that the work done is negative — *1 point*

Question 1 (continued)

Distribution of points

(d) 2 points

$W = \int \mathbf{F} \cdot d\mathbf{r} = \mathbf{F}_{avg} \cdot \mathbf{r}$

For using F_b as the average force in the equation for work — 1 point

$W = F_b d$

$F_b = \dfrac{W}{d}$

For substituting the expression for W from part (c), with or without a negative sign — 1 point

$F_b = \dfrac{J_p v_x}{2d}$

(e) 4 points

Applying the work-energy relationship

$K_i + W = K_f$

For correctly relating the initial kinetic energy of the projectile with the work done by the block on the projectile and the work done on the block by friction with the table — 1 point

$K_i + W_{block} + W_{friction} = 0$

For substituting for the work done by the block on the projectile (i.e., the energy lost to heat in the block-projectile collision) — 1 point

$K_i - F_b d_n + W_{friction} = 0$

For substituting the work done on the block by friction with the table (i.e., the energy lost to heat as the block slides to rest on the table) — 1 point

$K_i - F_b d_n - f_T D = 0$

The initial kinetic energy of the projectile is the same as in the first case when the block was clamped. Therefore, it can be equated to the work done in stopping the projectile from part (d).

For substituting $F_b d$ for the initial kinetic energy of the block — 1 point

$F_b d - F_b d_n - f_T D = 0$

$F_b d_n = F_b d - f_T D$

Full credit could not be earned for just writing this equation. The student needed to have some indication that the work-energy relationship was being applied, and that F_d was associated with the initial kinetic energy.

$d_n = \dfrac{F_b d - f_T D}{F_b}$

$d_n = d - \dfrac{f_T}{F_b} D$

Question 1 (continued)

Distribution of points

(f) 2 points

For a correct application of conservation of momentum to the block-projectile collision — 1 point

$m\upsilon_x = (M+m)V$

$V = \dfrac{m}{(M+m)}\upsilon_x$

The kinetic energy of the block/projectile system immediately after the collision is equal to the work done by friction in stopping it.

$\dfrac{1}{2}(M+m)V^2 = f_T D$

For substituting for V — 1 point

$\dfrac{1}{2}(M+m)\left(\dfrac{m}{(M+m)}\upsilon_x\right)^2 = f_T D$

$\dfrac{1}{2}\dfrac{m^2\upsilon_x^2}{(M+m)} = f_T D$

$\dfrac{m}{M+m}\left(\dfrac{1}{2}m\upsilon_x^2\right) = f_T D$

From part (c) the kinetic energy factor in the equation above is equal to the total work done. From part (d) that work is equal to $F_b d$.

$\dfrac{m}{M+m}F_b d = f_T D$

Using the expression $F_b d_n = F_b d - f_T D$ from part (e) to substitute for $f_T D$

$\dfrac{m}{M+m}F_b d = F_b d - F_b d_n$

$\dfrac{m}{M+m}d = d - d_n$

$d_n = d\left(1 - \dfrac{m}{M+m}\right)$

Note: Because the work for parts (e) and (f) is interrelated, the two parts are scored as a whole. Credit is earned for work related to part (f) even when it is shown in part (e) and vice versa.

Question 1

15 points total | **Distribution of points**

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