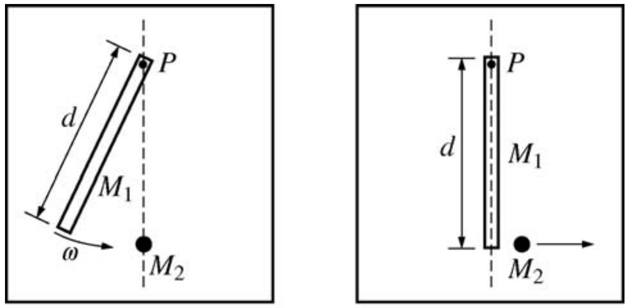


TOP VIEWS

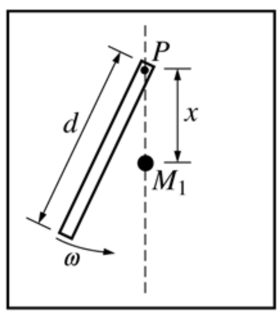
Mech. 3.

A system consists of a ball of mass M_2 and a uniform rod of mass M_1 and length d. The rod is attached to a horizontal frictionless table by a pivot at point P and initially rotates at an angular speed ω, as shown above left. The rotational inertia of the rod about point P is $\frac{1}{3}M_1 d^2$. The rod strikes the ball, which is initially at rest. As a result of this collision, the rod is stopped and the ball moves in the direction shown above right. Express all answers in terms of M_1, M_2, ω, d, and fundamental constants.

(a) Derive an expression for the angular momentum of the rod about point P before the collision.

(b) Derive an expression for the speed v of the ball after the collision.

(c) Assuming that this collision is elastic, calculate the numerical value of the ratio M_1/M_2.



Before Collision

(d) A new ball with the same mass M_1 as the rod is now placed a distance x from the pivot, as shown above. Again assuming the collision is elastic, for what value of x will the rod stop moving after hitting the ball?

END OF SECTION II, MECHANICS

Question 3

15 points total | **Distribution of points**

(a) 1 point

$L_r = I\omega$, where $I = (1/3)M_1 d^2$

For the correct expression for angular momentum before the collision — 1 point

$L_r = (1/3)M_1 d^2 \omega$

<u>Note</u>: An expression without an equation was acceptable. This point was not awarded if the expression was not given in terms of the given quantities.

(b) 4 points

For any indication of conservation of angular momentum — 1 point

$L_b = L_r$

For the correct expression for L_b — 1 point

For substitution for L_r consistent with part (a) — 1 point

$M_2 v d = \frac{1}{3} M_1 d^2 \omega$

For the correct final expression for v — 1 point

$v = \frac{1}{3}\frac{M_1}{M_2} d\omega$

(c) 4 points

For any indication of conservation of kinetic energy — 1 point

No points were awarded for conservation of mechanical energy.

$K_b = K_r$

For the correct expressions for both kinetic energies — 1 point

$\frac{1}{2} M_2 v^2 = \frac{1}{2} I\omega^2$

$M_2 v^2 = I\omega^2$

For correct substitutions for I and for v consistent with part (b) — 1 point

$$M_2\left(\frac{1}{3}\frac{M_1}{M_2} d\omega\right)^2 = \left(\frac{1}{3} M_1 d^2\right)\omega^2$$

$$M_2 \frac{1}{9}\left(\frac{M_1}{M_2}\right)^2 d^2\omega^2 = \frac{1}{3} M_1 d^2 \omega^2$$

$$\frac{1}{9}\frac{M_1^{\,2}}{M_2} = \frac{1}{3} M_1$$

For the correct final expression for the ratio — 1 point

$M_1/M_2 = 3$ (3:1 was also acceptable)

Question 3 (continued)

	Distribution of points
(d) 6 points	
For the correct equation for conservation of angular momentum	2 points
$M_1 \upsilon x = \frac{1}{3} M_1 d^2 \omega$	
For solving this equation for υ	1 point
$\upsilon = \frac{1}{3}\frac{d^2}{x}\omega$	
For the correct equation for conservation of kinetic energy	1 point
$\frac{1}{2} M_1 \upsilon^2 = \frac{1}{2} I \omega^2$	
$M_1 \upsilon^2 = \left(\frac{1}{3} M_1 d^2\right)\omega^2$	
$\upsilon^2 = \frac{1}{3} d^2 \omega^2$	
For the correct substitution of the above expression for υ from momentum conservation into the equation for conservation of kinetic energy	1 point
$\left(\frac{1}{3}\frac{d^2}{x}\omega\right)^2 = \frac{1}{3} d^2 \omega^2$	
$\frac{1}{9}\frac{d^4}{x^2}\omega^2 = \frac{1}{3} d^2 \omega^2$	
$\frac{1}{x^2} = \frac{9}{3}\frac{1}{d^2}$	
For the correct final answer	1 point
$x = \frac{d}{\sqrt{3}}$	