

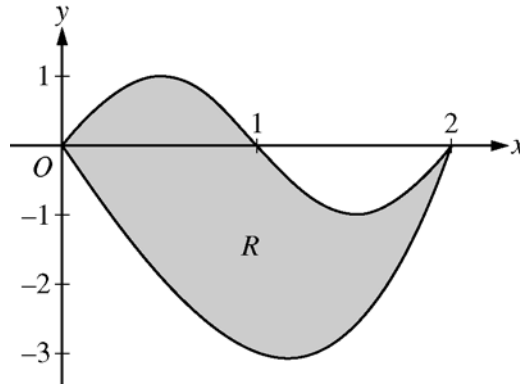
**2008 AP<sup>®</sup> CALCULUS BC FREE-RESPONSE QUESTIONS**

**CALCULUS BC  
SECTION II, Part A**

**Time—45 minutes**

**Number of problems—3**

**A graphing calculator is required for some problems or parts of problems.**



1. Let  $R$  be the region bounded by the graphs of  $y = \sin(\pi x)$  and  $y = x^3 - 4x$ , as shown in the figure above.
- (a) Find the area of  $R$ .
  - (b) The horizontal line  $y = -2$  splits the region  $R$  into two parts. Write, but do not evaluate, an integral expression for the area of the part of  $R$  that is below this horizontal line.
  - (c) The region  $R$  is the base of a solid. For this solid, each cross section perpendicular to the  $x$ -axis is a square. Find the volume of this solid.
  - (d) The region  $R$  models the surface of a small pond. At all points in  $R$  at a distance  $x$  from the  $y$ -axis, the depth of the water is given by  $h(x) = 3 - x$ . Find the volume of water in the pond.
- 

**WRITE ALL WORK IN THE PINK EXAM BOOKLET.**

**2008 AP<sup>®</sup> CALCULUS BC FREE-RESPONSE QUESTIONS**

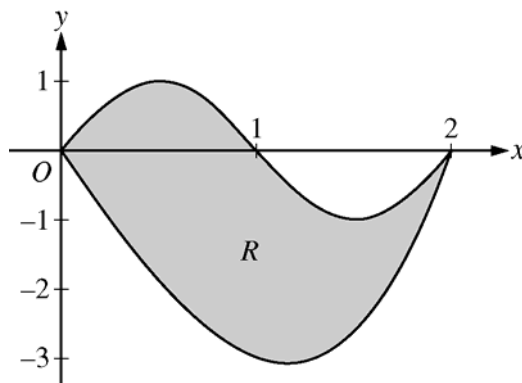
$t$ (hours)	0	1	3	4	7	8	9
$L(t)$ (people)	120	156	176	126	150	80	0

2. Concert tickets went on sale at noon ( $t = 0$ ) and were sold out within 9 hours. The number of people waiting in line to purchase tickets at time  $t$  is modeled by a twice-differentiable function  $L$  for  $0 \leq t \leq 9$ . Values of  $L(t)$  at various times  $t$  are shown in the table above.
- (a) Use the data in the table to estimate the rate at which the number of people waiting in line was changing at 5:30 P.M. ( $t = 5.5$ ). Show the computations that lead to your answer. Indicate units of measure.
- (b) Use a trapezoidal sum with three subintervals to estimate the average number of people waiting in line during the first 4 hours that tickets were on sale.
- (c) For  $0 \leq t \leq 9$ , what is the fewest number of times at which  $L'(t)$  must equal 0? Give a reason for your answer.
- (d) The rate at which tickets were sold for  $0 \leq t \leq 9$  is modeled by  $r(t) = 550te^{-t/2}$  tickets per hour. Based on the model, how many tickets were sold by 3 P.M. ( $t = 3$ ), to the nearest whole number?
- 

**WRITE ALL WORK IN THE PINK EXAM BOOKLET.**

**AP<sup>®</sup> CALCULUS BC**  
**2008 SCORING GUIDELINES**

**Question 1**



Let  $R$  be the region bounded by the graphs of  $y = \sin(\pi x)$  and  $y = x^3 - 4x$ , as shown in the figure above.

- Find the area of  $R$ .
- The horizontal line  $y = -2$  splits the region  $R$  into two parts. Write, but do not evaluate, an integral expression for the area of the part of  $R$  that is below this horizontal line.
- The region  $R$  is the base of a solid. For this solid, each cross section perpendicular to the  $x$ -axis is a square. Find the volume of this solid.
- The region  $R$  models the surface of a small pond. At all points in  $R$  at a distance  $x$  from the  $y$ -axis, the depth of the water is given by  $h(x) = 3 - x$ . Find the volume of water in the pond.

(a)  $\sin(\pi x) = x^3 - 4x$  at  $x = 0$  and  $x = 2$

$$\text{Area} = \int_0^2 (\sin(\pi x) - (x^3 - 4x)) dx = 4$$

$$3 : \begin{cases} 1 : \text{limits} \\ 1 : \text{integrand} \\ 1 : \text{answer} \end{cases}$$

(b)  $x^3 - 4x = -2$  at  $r = 0.5391889$  and  $s = 1.6751309$

$$\text{The area of the stated region is } \int_r^s (-2 - (x^3 - 4x)) dx$$

$$2 : \begin{cases} 1 : \text{limits} \\ 1 : \text{integrand} \end{cases}$$

(c)  $\text{Volume} = \int_0^2 (\sin(\pi x) - (x^3 - 4x))^2 dx = 9.978$

$$2 : \begin{cases} 1 : \text{integrand} \\ 1 : \text{answer} \end{cases}$$

(d)  $\text{Volume} = \int_0^2 (3 - x)(\sin(\pi x) - (x^3 - 4x)) dx = 8.369 \text{ or } 8.370$

$$2 : \begin{cases} 1 : \text{integrand} \\ 1 : \text{answer} \end{cases}$$