

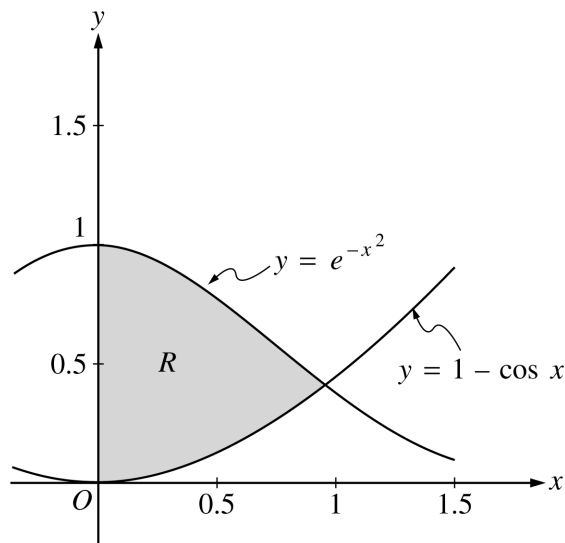
2000 AP[®] CALCULUS AB FREE-RESPONSE QUESTIONS

CALCULUS AB
SECTION II, Part A

Time—45 minutes

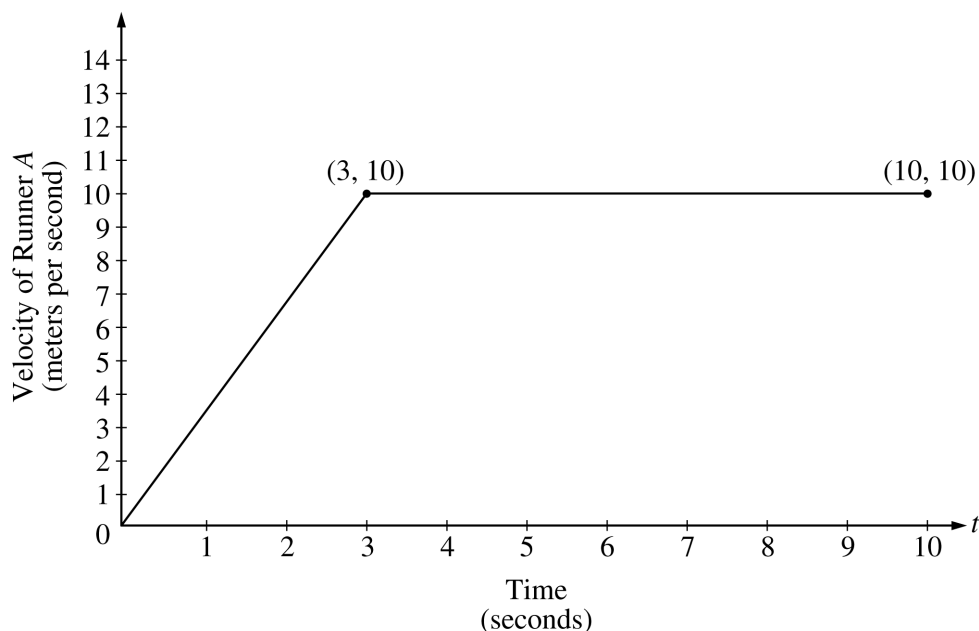
Number of problems—3

A graphing calculator is required for some problems or parts of problems.



1. Let R be the shaded region in the first quadrant enclosed by the graphs of $y = e^{-x^2}$, $y = 1 - \cos x$, and the y -axis, as shown in the figure above.
 - (a) Find the area of the region R .
 - (b) Find the volume of the solid generated when the region R is revolved about the x -axis.
 - (c) The region R is the base of a solid. For this solid, each cross section perpendicular to the x -axis is a square. Find the volume of this solid.

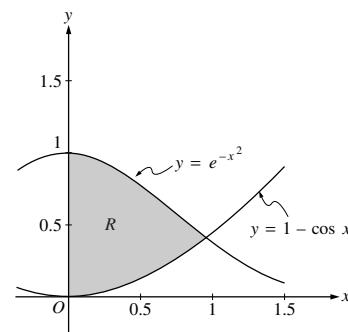
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2. Two runners, A and B , run on a straight racetrack for $0 \leq t \leq 10$ seconds. The graph above, which consists of two line segments, shows the velocity, in meters per second, of Runner A . The velocity, in meters per second, of Runner B is given by the function v defined by $v(t) = \frac{24t}{2t + 3}$.
- Find the velocity of Runner A and the velocity of Runner B at time $t = 2$ seconds. Indicate units of measure.
 - Find the acceleration of Runner A and the acceleration of Runner B at time $t = 2$ seconds. Indicate units of measure.
 - Find the total distance run by Runner A and the total distance run by Runner B over the time interval $0 \leq t \leq 10$ seconds. Indicate units of measure.

Let R be the shaded region in the first quadrant enclosed by the graphs of $y = e^{-x^2}$, $y = 1 - \cos x$, and the y -axis, as shown in the figure above.

- (a) Find the area of the region R .
- (b) Find the volume of the solid generated when the region R is revolved about the x -axis.
- (c) The region R is the base of a solid. For this solid, each cross section perpendicular to the x -axis is a square. Find the volume of this solid.



Region R

$$e^{-x^2} = 1 - \cos x \text{ at } x = 0.941944 = A$$

$$\begin{aligned} \text{(a) Area} &= \int_0^A (e^{-x^2} - (1 - \cos x)) dx \\ &= 0.590 \text{ or } 0.591 \end{aligned}$$

$$\begin{aligned} \text{(b) Volume} &= \pi \int_0^A \left((e^{-x^2})^2 - (1 - \cos x)^2 \right) dx \\ &= 0.55596\pi = 1.746 \text{ or } 1.747 \end{aligned}$$

$$\begin{aligned} \text{(c) Volume} &= \int_0^A \left(e^{-x^2} - (1 - \cos x) \right)^2 dx \\ &= 0.461 \end{aligned}$$

1 : Correct limits in an integral in (a), (b), or (c).

2 $\left\{ \begin{array}{l} 1 : \text{integrand} \\ 1 : \text{answer} \end{array} \right.$

3 $\left\{ \begin{array}{l} 2 : \text{integrand and constant} \\ < -1 > \text{ each error} \\ 1 : \text{answer} \end{array} \right.$

3 $\left\{ \begin{array}{l} 2 : \text{integrand} \\ < -1 > \text{ each error} \\ \text{Note: 0/2 if not of the form} \\ \quad k \int_c^d (f(x) - g(x))^2 dx \\ 1 : \text{answer} \end{array} \right.$