5. Let f be the function defined by $f(x) = \dfrac{3}{2x^2 - 7x + 5}$.

(a) Find the slope of the line tangent to the graph of f at $x = 3$.

(b) Find the x-coordinate of each critical point of f in the interval $1 < x < 2.5$. Classify each critical point as the location of a relative minimum, a relative maximum, or neither. Justify your answers.

(c) Using the identity that $\dfrac{3}{2x^2 - 7x + 5} = \dfrac{2}{2x - 5} - \dfrac{1}{x - 1}$, evaluate $\int_5^{\infty} f(x)\, dx$ or show that the integral diverges.

(d) Determine whether the series $\displaystyle\sum_{n=5}^{\infty} \frac{3}{2n^2 - 7n + 5}$ converges or diverges. State the conditions of the test used for determining convergence or divergence.

$$f(0) = 0$$
$$f'(0) = 1$$
$$f^{(n+1)}(0) = -n \cdot f^{(n)}(0) \text{ for all } n \geq 1$$

6. A function f has derivatives of all orders for $-1 < x < 1$. The derivatives of f satisfy the conditions above. The Maclaurin series for f converges to $f(x)$ for $|x| < 1$.

   (a) Show that the first four nonzero terms of the Maclaurin series for f are $x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4}$, and write the general term of the Maclaurin series for f.

   (b) Determine whether the Maclaurin series described in part (a) converges absolutely, converges conditionally, or diverges at $x = 1$. Explain your reasoning.

   (c) Write the first four nonzero terms and the general term of the Maclaurin series for $g(x) = \int_0^x f(t)\,dt$.

   (d) Let $P_n\left(\frac{1}{2}\right)$ represent the nth-degree Taylor polynomial for g about $x = 0$ evaluated at $x = \frac{1}{2}$, where g is the function defined in part (c). Use the alternating series error bound to show that

   $\left| P_4\left(\frac{1}{2}\right) - g\left(\frac{1}{2}\right) \right| < \frac{1}{500}$.

STOP

END OF EXAM

Question 5

(a) $f'(x) = \dfrac{-3(4x-7)}{\left(2x^2 - 7x + 5\right)^2}$

$f'(3) = \dfrac{(-3)(5)}{(18 - 21 + 5)^2} = -\dfrac{15}{4}$

2 : $f'(3)$

(b) $f'(x) = \dfrac{-3(4x-7)}{\left(2x^2 - 7x + 5\right)^2} = 0 \Rightarrow x = \dfrac{7}{4}$

The only critical point in the interval $1 < x < 2.5$ has x-coordinate $\dfrac{7}{4}$.

f' changes sign from positive to negative at $x = \dfrac{7}{4}$.

Therefore, f has a relative maximum at $x = \dfrac{7}{4}$.

2 : { 1 : x-coordinate; 1 : relative maximum with justification }

(c) $$\int_5^\infty f(x)\,dx = \lim_{b\to\infty}\int_5^b \frac{3}{2x^2 - 7x + 5}\,dx = \lim_{b\to\infty}\int_5^b \left(\frac{2}{2x-5} - \frac{1}{x-1}\right)dx$$

$$= \lim_{b\to\infty}\Big[\ln(2x-5) - \ln(x-1)\Big]_5^b = \lim_{b\to\infty}\left[\ln\left(\frac{2x-5}{x-1}\right)\right]_5^b$$

$$= \lim_{b\to\infty}\left[\ln\left(\frac{2b-5}{b-1}\right) - \ln\left(\frac{5}{4}\right)\right] = \ln 2 - \ln\left(\frac{5}{4}\right) = \ln\left(\frac{8}{5}\right)$$

3 : { 1 : antiderivative; 1 : limit expression; 1 : answer }

(d) f is continuous, positive, and decreasing on $[5, \infty)$.

The series converges by the integral test since $\displaystyle\int_5^\infty \frac{3}{2x^2 - 7x + 5}\,dx$ converges.

— OR —

$\dfrac{3}{2n^2 - 7n + 5} > 0$ and $\dfrac{1}{n^2} > 0$ for $n \ge 5$.

Since $\displaystyle\lim_{n\to\infty}\frac{\frac{3}{2n^2 - 7n + 5}}{\frac{1}{n^2}} = \frac{3}{2}$ and the series $\displaystyle\sum_{n=5}^{\infty}\frac{1}{n^2}$ converges,

the series $\displaystyle\sum_{n=5}^{\infty}\frac{3}{2n^2 - 7n + 5}$ converges by the limit comparison test.

2 : answer with conditions