

2013 AP[®] CALCULUS AB FREE-RESPONSE QUESTIONS

2. A particle moves along a straight line. For $0 \leq t \leq 5$, the velocity of the particle is given by $v(t) = -2 + (t^2 + 3t)^{6/5} - t^3$, and the position of the particle is given by $s(t)$. It is known that $s(0) = 10$.
- (a) Find all values of t in the interval $2 \leq t \leq 4$ for which the speed of the particle is 2.
 - (b) Write an expression involving an integral that gives the position $s(t)$. Use this expression to find the position of the particle at time $t = 5$.
 - (c) Find all times t in the interval $0 \leq t \leq 5$ at which the particle changes direction. Justify your answer.
 - (d) Is the speed of the particle increasing or decreasing at time $t = 4$? Give a reason for your answer.
-

END OF PART A OF SECTION II

2013 AP[®] CALCULUS AB FREE-RESPONSE QUESTIONS**CALCULUS AB
SECTION II, Part B
Time—60 minutes
Number of problems—4**

No calculator is allowed for these problems.

t (minutes)	0	1	2	3	4	5	6
$C(t)$ (ounces)	0	5.3	8.8	11.2	12.8	13.8	14.5

3. Hot water is dripping through a coffeemaker, filling a large cup with coffee. The amount of coffee in the cup at time t , $0 \leq t \leq 6$, is given by a differentiable function C , where t is measured in minutes. Selected values of $C(t)$, measured in ounces, are given in the table above.
- (a) Use the data in the table to approximate $C'(3.5)$. Show the computations that lead to your answer, and indicate units of measure.
- (b) Is there a time t , $2 \leq t \leq 4$, at which $C'(t) = 2$? Justify your answer.
- (c) Use a midpoint sum with three subintervals of equal length indicated by the data in the table to approximate the value of $\frac{1}{6} \int_0^6 C(t) dt$. Using correct units, explain the meaning of $\frac{1}{6} \int_0^6 C(t) dt$ in the context of the problem.
- (d) The amount of coffee in the cup, in ounces, is modeled by $B(t) = 16 - 16e^{-0.4t}$. Using this model, find the rate at which the amount of coffee in the cup is changing when $t = 5$.
-

AP[®] CALCULUS AB
2013 SCORING GUIDELINES

Question 2

A particle moves along a straight line. For $0 \leq t \leq 5$, the velocity of the particle is given by $v(t) = -2 + (t^2 + 3t)^{6/5} - t^3$, and the position of the particle is given by $s(t)$. It is known that $s(0) = 10$.

- (a) Find all values of t in the interval $2 \leq t \leq 4$ for which the speed of the particle is 2.
- (b) Write an expression involving an integral that gives the position $s(t)$. Use this expression to find the position of the particle at time $t = 5$.
- (c) Find all times t in the interval $0 \leq t \leq 5$ at which the particle changes direction. Justify your answer.
- (d) Is the speed of the particle increasing or decreasing at time $t = 4$? Give a reason for your answer.

- (a) Solve $|v(t)| = 2$ on $2 \leq t \leq 4$.
 $t = 3.128$ (or 3.127) and $t = 3.473$

2 : $\begin{cases} 1 : \text{considers } |v(t)| = 2 \\ 1 : \text{answer} \end{cases}$

- (b) $s(t) = 10 + \int_0^t v(x) \, dx$

$$s(5) = 10 + \int_0^5 v(x) \, dx = -9.207$$

2 : $\begin{cases} 1 : s(t) \\ 1 : s(5) \end{cases}$

- (c) $v(t) = 0$ when $t = 0.536033, 3.317756$
 $v(t)$ changes sign from negative to positive at time $t = 0.536033$.
 $v(t)$ changes sign from positive to negative at time $t = 3.317756$.

Therefore, the particle changes direction at time $t = 0.536$ and time $t = 3.318$ (or 3.317).

3 : $\begin{cases} 1 : \text{considers } v(t) = 0 \\ 2 : \text{answers with justification} \end{cases}$

- (d) $v(4) = -11.475758 < 0$, $a(4) = v'(4) = -22.295714 < 0$

The speed is increasing at time $t = 4$ because velocity and acceleration have the same sign.

2 : conclusion with reason