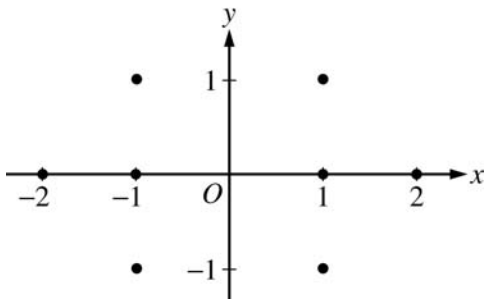
5. Consider the differential equation $\frac{dy}{dx} = \frac{1+y}{x}$, where $x \neq 0$.

(a) On the axes provided, sketch a slope field for the given differential equation at the eight points indicated.

(Note: Use the axes provided in the pink exam booklet.)



(b) Find the particular solution $y = f(x)$ to the differential equation with the initial condition $f(-1) = 1$ and state its domain.

---

6. The twice-differentiable function f is defined for all real numbers and satisfies the following conditions:

$$f(0) = 2, \quad f'(0) = -4, \text{ and } f''(0) = 3.$$

(a) The function g is given by $g(x) = e^{ax} + f(x)$ for all real numbers, where a is a constant. Find $g'(0)$ and $g''(0)$ in terms of a. Show the work that leads to your answers.

(b) The function h is given by $h(x) = \cos(kx) f(x)$ for all real numbers, where k is a constant. Find $h'(x)$ and write an equation for the line tangent to the graph of h at $x = 0$.

WRITE ALL WORK IN THE PINK EXAM BOOKLET.

END OF EXAM

© 2006 The College Board. All rights reserved.
Visit apcentral.collegeboard.com (for AP professionals) and www.collegeboard.com/apstudents (for students and parents).

Question 6

The twice-differentiable function f is defined for all real numbers and satisfies the following conditions:
$f(0) = 2$, $f'(0) = -4$, and $f''(0) = 3$.

(a) The function g is given by $g(x) = e^{ax} + f(x)$ for all real numbers, where a is a constant. Find $g'(0)$ and $g''(0)$ in terms of a. Show the work that leads to your answers.

(b) The function h is given by $h(x) = \cos(kx) f(x)$ for all real numbers, where k is a constant. Find $h'(x)$ and write an equation for the line tangent to the graph of h at $x = 0$.

(a) $g'(x) = ae^{ax} + f'(x)$
$g'(0) = a - 4$

$g''(x) = a^2 e^{ax} + f''(x)$
$g''(0) = a^2 + 3$

$4 : \begin{cases} 1 : g'(x) \\ 1 : g'(0) \\ 1 : g''(x) \\ 1 : g''(0) \end{cases}$

(b) $h'(x) = f'(x)\cos(kx) - k\sin(kx) f(x)$
$h'(0) = f'(0)\cos(0) - k\sin(0) f(0) = f'(0) = -4$
$h(0) = \cos(0) f(0) = 2$
The equation of the tangent line is $y = -4x + 2$.

$5 : \begin{cases} 2 : h'(x) \\ 3 : \begin{cases} 1 : h'(0) \\ 1 : h(0) \\ 1 : \text{equation of tangent line} \end{cases} \end{cases}$

© 2006 The College Board. All rights reserved.
Visit apcentral.collegeboard.com (for AP professionals) and www.collegeboard.com/apstudents (for AP students and parents).