

**2008 AP<sup>®</sup> CALCULUS BC FREE-RESPONSE QUESTIONS**

$t$ (hours)	0	1	3	4	7	8	9
$L(t)$ (people)	120	156	176	126	150	80	0

2. Concert tickets went on sale at noon ( $t = 0$ ) and were sold out within 9 hours. The number of people waiting in line to purchase tickets at time  $t$  is modeled by a twice-differentiable function  $L$  for  $0 \leq t \leq 9$ . Values of  $L(t)$  at various times  $t$  are shown in the table above.
- (a) Use the data in the table to estimate the rate at which the number of people waiting in line was changing at 5:30 P.M. ( $t = 5.5$ ). Show the computations that lead to your answer. Indicate units of measure.
- (b) Use a trapezoidal sum with three subintervals to estimate the average number of people waiting in line during the first 4 hours that tickets were on sale.
- (c) For  $0 \leq t \leq 9$ , what is the fewest number of times at which  $L'(t)$  must equal 0? Give a reason for your answer.
- (d) The rate at which tickets were sold for  $0 \leq t \leq 9$  is modeled by  $r(t) = 550te^{-t/2}$  tickets per hour. Based on the model, how many tickets were sold by 3 P.M. ( $t = 3$ ), to the nearest whole number?
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**WRITE ALL WORK IN THE PINK EXAM BOOKLET.**

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$x$	$h(x)$	$h'(x)$	$h''(x)$	$h'''(x)$	$h^{(4)}(x)$
1	11	30	42	99	18
2	80	128	$\frac{488}{3}$	$\frac{448}{3}$	$\frac{584}{9}$
3	317	$\frac{753}{2}$	$\frac{1383}{4}$	$\frac{3483}{16}$	$\frac{1125}{16}$

3. Let  $h$  be a function having derivatives of all orders for  $x > 0$ . Selected values of  $h$  and its first four derivatives are indicated in the table above. The function  $h$  and these four derivatives are increasing on the interval  $1 \leq x \leq 3$ .
- (a) Write the first-degree Taylor polynomial for  $h$  about  $x = 2$  and use it to approximate  $h(1.9)$ . Is this approximation greater than or less than  $h(1.9)$ ? Explain your reasoning.
- (b) Write the third-degree Taylor polynomial for  $h$  about  $x = 2$  and use it to approximate  $h(1.9)$ .
- (c) Use the Lagrange error bound to show that the third-degree Taylor polynomial for  $h$  about  $x = 2$  approximates  $h(1.9)$  with error less than  $3 \times 10^{-4}$ .
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**END OF PART A OF SECTION II**

**AP<sup>®</sup> CALCULUS BC**  
**2008 SCORING GUIDELINES**

**Question 2**

$t$ (hours)	0	1	3	4	7	8	9
$L(t)$ (people)	120	156	176	126	150	80	0

Concert tickets went on sale at noon ( $t = 0$ ) and were sold out within 9 hours. The number of people waiting in line to purchase tickets at time  $t$  is modeled by a twice-differentiable function  $L$  for  $0 \leq t \leq 9$ . Values of  $L(t)$  at various times  $t$  are shown in the table above.

- (a) Use the data in the table to estimate the rate at which the number of people waiting in line was changing at 5:30 P.M. ( $t = 5.5$ ). Show the computations that lead to your answer. Indicate units of measure.
- (b) Use a trapezoidal sum with three subintervals to estimate the average number of people waiting in line during the first 4 hours that tickets were on sale.
- (c) For  $0 \leq t \leq 9$ , what is the fewest number of times at which  $L'(t)$  must equal 0? Give a reason for your answer.
- (d) The rate at which tickets were sold for  $0 \leq t \leq 9$  is modeled by  $r(t) = 550te^{-t/2}$  tickets per hour. Based on the model, how many tickets were sold by 3 P.M. ( $t = 3$ ), to the nearest whole number?

(a)  $L'(5.5) \approx \frac{L(7) - L(4)}{7 - 4} = \frac{150 - 126}{3} = 8$  people per hour

2 :  $\begin{cases} 1 : \text{estimate} \\ 1 : \text{units} \end{cases}$

- (b) The average number of people waiting in line during the first 4 hours is approximately

$$\frac{1}{4} \left( \frac{L(0) + L(1)}{2}(1 - 0) + \frac{L(1) + L(3)}{2}(3 - 1) + \frac{L(3) + L(4)}{2}(4 - 3) \right) = 155.25 \text{ people}$$

2 :  $\begin{cases} 1 : \text{trapezoidal sum} \\ 1 : \text{answer} \end{cases}$

- (c)  $L$  is differentiable on  $[0, 9]$  so the Mean Value Theorem implies  $L'(t) > 0$  for some  $t$  in  $(1, 3)$  and some  $t$  in  $(4, 7)$ . Similarly,  $L'(t) < 0$  for some  $t$  in  $(3, 4)$  and some  $t$  in  $(7, 8)$ . Then, since  $L'$  is continuous on  $[0, 9]$ , the Intermediate Value Theorem implies that  $L'(t) = 0$  for at least three values of  $t$  in  $[0, 9]$ .

3 :  $\begin{cases} 1 : \text{considers change in sign of } L' \\ 1 : \text{analysis} \\ 1 : \text{conclusion} \end{cases}$

OR

The continuity of  $L$  on  $[1, 4]$  implies that  $L$  attains a maximum value there. Since  $L(3) > L(1)$  and  $L(3) > L(4)$ , this maximum occurs on  $(1, 4)$ . Similarly,  $L$  attains a minimum on  $(3, 7)$  and a maximum on  $(4, 8)$ .  $L$  is differentiable, so  $L'(t) = 0$  at each relative extreme point on  $(0, 9)$ . Therefore  $L'(t) = 0$  for at least three values of  $t$  in  $[0, 9]$ .

OR

3 :  $\begin{cases} 1 : \text{considers relative extrema of } L \text{ on } (0, 9) \\ 1 : \text{analysis} \\ 1 : \text{conclusion} \end{cases}$

[Note: There is a function  $L$  that satisfies the given conditions with  $L'(t) = 0$  for exactly three values of  $t$ .]

(d)  $\int_0^3 r(t) dt = 972.784$

There were approximately 973 tickets sold by 3 P.M.

2 :  $\begin{cases} 1 : \text{integrand} \\ 1 : \text{limits and answer} \end{cases}$