

**2018 AP<sup>®</sup> CALCULUS AB FREE-RESPONSE QUESTIONS**

**CALCULUS AB**

**SECTION II, Part A**

**Time—30 minutes**

**Number of questions—2**

**A GRAPHING CALCULATOR IS REQUIRED FOR THESE QUESTIONS.**

1. People enter a line for an escalator at a rate modeled by the function  $r$  given by

$$r(t) = \begin{cases} 44\left(\frac{t}{100}\right)^3\left(1 - \frac{t}{300}\right)^7 & \text{for } 0 \leq t \leq 300 \\ 0 & \text{for } t > 300, \end{cases}$$

where  $r(t)$  is measured in people per second and  $t$  is measured in seconds. As people get on the escalator, they exit the line at a constant rate of 0.7 person per second. There are 20 people in line at time  $t = 0$ .

- (a) How many people enter the line for the escalator during the time interval  $0 \leq t \leq 300$  ?
  - (b) During the time interval  $0 \leq t \leq 300$ , there are always people in line for the escalator. How many people are in line at time  $t = 300$  ?
  - (c) For  $t > 300$ , what is the first time  $t$  that there are no people in line for the escalator?
  - (d) For  $0 \leq t \leq 300$ , at what time  $t$  is the number of people in line a minimum? To the nearest whole number, find the number of people in line at this time. Justify your answer.
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2. A particle moves along the  $x$ -axis with velocity given by  $v(t) = \frac{10 \sin(0.4t^2)}{t^2 - t + 3}$  for time  $0 \leq t \leq 3.5$ .

The particle is at position  $x = -5$  at time  $t = 0$ .

(a) Find the acceleration of the particle at time  $t = 3$ .

(b) Find the position of the particle at time  $t = 3$ .

(c) Evaluate  $\int_0^{3.5} v(t) \, dt$ , and evaluate  $\int_0^{3.5} |v(t)| \, dt$ . Interpret the meaning of each integral in the context of the problem.

(d) A second particle moves along the  $x$ -axis with position given by  $x_2(t) = t^2 - t$  for  $0 \leq t \leq 3.5$ . At what time  $t$  are the two particles moving with the same velocity?

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**END OF PART A OF SECTION II**

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**2018 SCORING GUIDELINES**

**Question 1**

(a)  $\int_0^{300} r(t) dt = 270$

According to the model, 270 people enter the line for the escalator during the time interval  $0 \leq t \leq 300$ .

2 :  $\begin{cases} 1 : \text{integral} \\ 1 : \text{answer} \end{cases}$

(b)  $20 + \int_0^{300} (r(t) - 0.7) dt = 20 + \int_0^{300} r(t) dt - 0.7 \cdot 300 = 80$

According to the model, 80 people are in line at time  $t = 300$ .

2 :  $\begin{cases} 1 : \text{considers rate out} \\ 1 : \text{answer} \end{cases}$

(c) Based on part (b), the number of people in line at time  $t = 300$  is 80.

The first time  $t$  that there are no people in line is

$$300 + \frac{80}{0.7} = 414.286 \text{ (or 414.285) seconds.}$$

1 : answer

(d) The total number of people in line at time  $t$ ,  $0 \leq t \leq 300$ , is modeled by  $20 + \int_0^t r(x) dx - 0.7t$ .

$$r(t) - 0.7 = 0 \Rightarrow t_1 = 33.013298, t_2 = 166.574719$$

4 :  $\begin{cases} 1 : \text{considers } r(t) - 0.7 = 0 \\ 1 : \text{identifies } t = 33.013 \\ 1 : \text{answers} \\ 1 : \text{justification} \end{cases}$

$t$	People in line for escalator
0	20
$t_1$	3.803
$t_2$	158.070
300	80

The number of people in line is a minimum at time  $t = 33.013$  seconds, when there are 4 people in line.