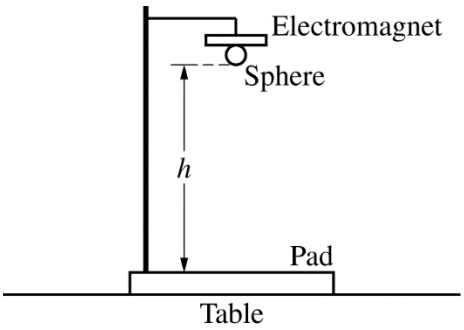
PHYSICS C: MECHANICS
SECTION II
Time—45 minutes
3 Questions

Directions: Answer all three questions. The suggested time is about 15 minutes for answering each of the questions, which are worth 15 points each. The parts within a question may not have equal weight. Show all your work in this booklet in the spaces provided after each part.



1. A student wants to determine the value of the acceleration due to gravity g for a specific location and sets up the following experiment. A solid sphere is held vertically a distance h above a pad by an electromagnet, as shown in the figure above. The experimental equipment is designed to release the sphere when the electromagnet is turned off. A timer also starts when the electromagnet is turned off, and the timer stops when the sphere lands on the pad.

   (a) While taking the first data point, the student notices that the electromagnet actually releases the sphere after the timer begins. Would the value of g calculated from this one measurement be greater than, less than, or equal to the actual value of g at the student's location?

   ____ Greater than ____ Less than ____ Equal to

   Justify your answer.

   The electromagnet is replaced so that the timer begins when the sphere is released. The student varies the distance h. The student measures and records the time Δt of the fall for each particular height, resulting in the following data table.

h (m)	0.10	0.20	0.60	0.80	1.00
Δt (s)	0.105	0.213	0.342	0.401	0.451

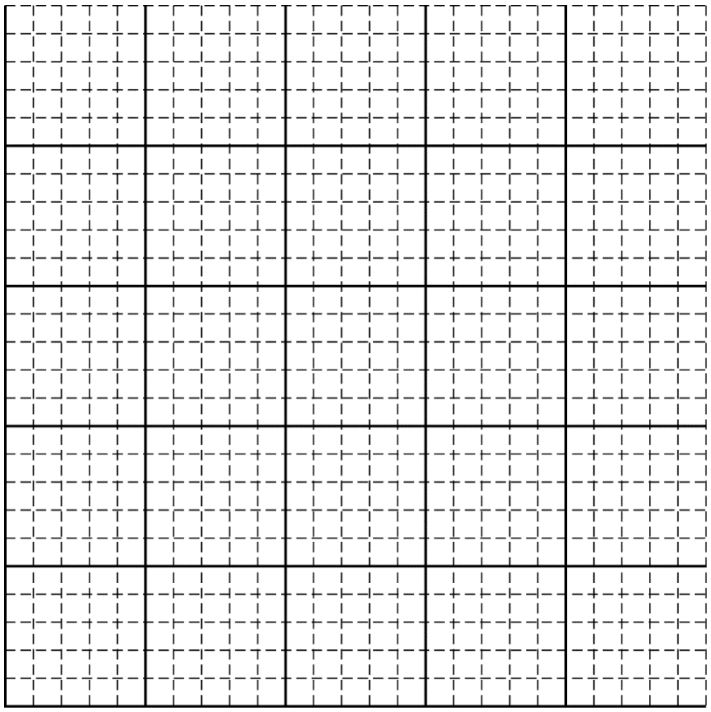
   (b) Indicate below which quantities should be graphed to yield a straight line whose slope could be used to calculate a numerical value for g.

   Vertical axis: _____

   Horizontal axis: _____

   Use the remaining rows in the table above, as needed, to record any quantities that you indicated that are not given in the table. Label each row you use and include units.

(c) Plot the data points for the quantities indicated in part (b) on the graph below. Clearly scale and label all axes, including units if appropriate. Draw a straight line that best represents the data.



(d) Using the straight line, calculate an experimental value for g.

Another student fits the data in the table to a quadratic equation. The student's equation for the distance fallen y as a function of time t is $y = At^2 + Bt + C$, where $A = 5.75\ \text{m/s}^2$, $B = -0.524\ \text{m/s}$, and $C = +0.080\ \text{m}$. Vertically down is the positive direction.

(e) Using the student's equation above, do the following.

i. Derive an expression for the velocity of the sphere as a function of time.

ii. Calculate the new experimental value for g.

iii. Using $9.81\ \text{m/s}^2$ as the accepted value for g at this location, calculate the percent error for the value found in part (e)ii.

iv. Assuming the sphere is at a height of 1.40 m at $t = 0$, calculate the velocity of the sphere just before it strikes the pad.

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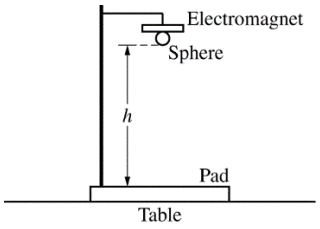
2018 SCORING GUIDELINES

Question 1

15 points total **Distribution of points**



A student wants to determine the value of the acceleration due to gravity g for a specific location and sets up the following experiment. A solid sphere is held vertically a distance h above a pad by an electromagnet, as shown in the figure above. The experimental equipment is designed to release the sphere when the electromagnet is turned off. A timer also starts when the electromagnet is turned off, and the timer stops when the sphere lands on the pad.

(a) 2 points

While taking the first data point, the student notices that the electromagnet actually releases the sphere after the timer begins. Would the value of g calculated from this one measurement be greater than, less than, or equal to the actual value of g at the student's location?

____ Greater than ____ Less than ____ Equal to

Justify your answer.

For selecting "Less than" with an attempt at a relevant justification		1 point
For a correct justification		1 point
Example: Because the measured time to fall the same distance will be larger, the acceleration must be less.		
Example: Because the time is larger and $a \propto 1/t^2$ *, then g must decrease.*		

Question 1 (continued)

Distribution of points

The electromagnet is replaced so that the timer begins when the sphere is released. The student varies the distance h. The student measures and records the time Δt of the fall for each particular height, resulting in the following data table.

h (m)	0.10	0.20	0.60	0.80	1.00
Δt (s)	0.105	0.213	0.342	0.401	0.451

(b) 1 point

Indicate below which quantities should be graphed to yield a straight line whose slope could be used to calculate a numerical value for g.

Vertical axis: _____

Horizontal axis: _____

Use the remaining rows in the table above, as needed, to record any quantities that you indicated that are not given in the table. Label each row you use and include units.

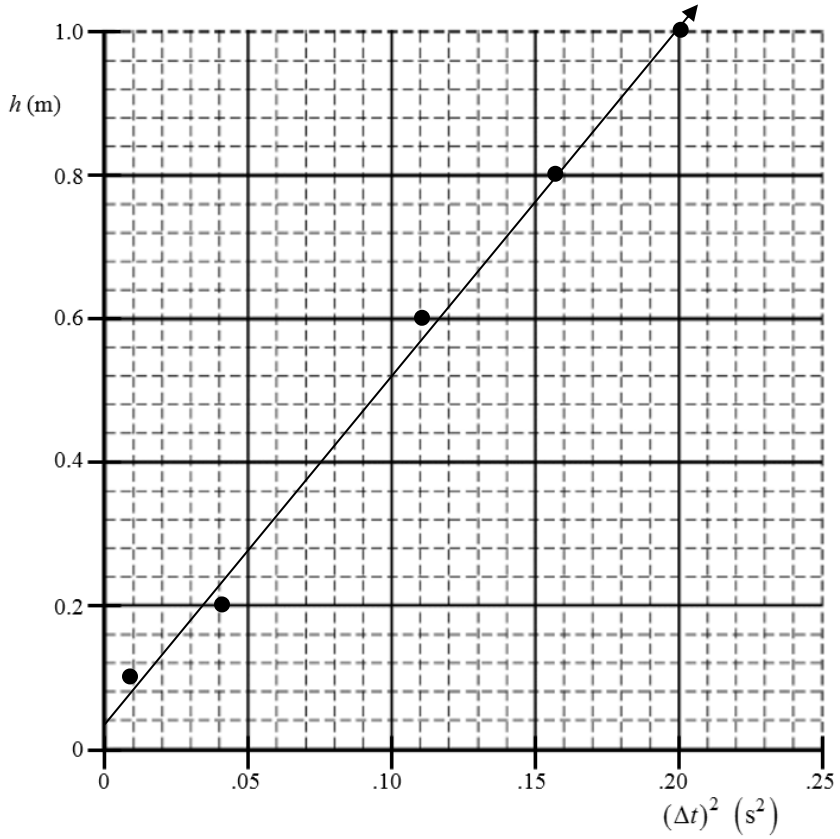
For correctly indicating two variables that will yield a straight line that could be used to determine a value for g		1 point
Example: Vertical Axis: h *Horizontal Axis:* $(\Delta t)^2$		
Note: Student earns full credit if axes are reversed or if they use another acceptable combination.		

Question 1 (continued)

Distribution of points

(c) 4 points

Plot the data points for the quantities indicated in part (b) on the graph below. Clearly scale and label all axes, including units if appropriate. Draw a straight line that best represents the data.



For a correct scale that uses more than half the grid		1 point
For correctly labeling the axis with variables and units consistent with part (b)		1 point
For correctly plotting data consistent with part (b)		1 point
For drawing a straight line consistent with the plotted data		1 point

Question 1 (continued)

Distribution of points

(d) 2 points

Using the straight line, calculate an experimental value for g.

For using points on the line rather than the data to calculate the slope		1 point
$\text{slope} = \dfrac{\Delta h}{\Delta(\Delta t)^2} = \dfrac{(0.80 - 0.20)\ \text{m}}{(0.160 - 0.039)\ \text{s}^2} = 4.96\ \text{m/s}^2$ (Linear regression = $4.83\ \text{m/s}^2$)		
For correctly relating the slope to the acceleration due to gravity		1 point
$\text{slope} = \dfrac{1}{2}g\ \therefore\ g = 2 \times \text{slope} = 2 \times \left(4.96\ \text{m/s}^2\right)$		
$g = 9.92\ \text{m/s}^2$ (Linear regression = $9.66\ \text{m/s}^2$)		

Another student fits the data in the table to a quadratic equation. The student's equation for the distance fallen y as a function of time t is $y = At^2 + Bt + C$, where $A = 5.75\ \text{m/s}^2$, $B = -0.524\ \text{m/s}$, and $C = +0.080\ \text{m}$. Vertically down is the positive direction.

(e) Using the student's equation above, do the following.

i. 1 point

Derive an expression for the velocity of the sphere as a function of time.

For correctly taking the time derivative of the given equation		1 point
$y = y_0 + v_1 t + \dfrac{1}{2}at^2 = At^2 + Bt + C$		
$v(t) = \dfrac{dy}{dt} = 2At + B$		
Note: Credit is earned for substituting numbers: $v(t) = \left(11.5\ \text{m/s}^2\right)t - 0.524\ \text{m/s}$.		

ii. 2 points

Calculate the new experimental value for g.

For correctly relating the given equation to a correct kinematic equation		1 point
$\Delta y = y - y_0 = v_1 t + \dfrac{1}{2}at^2$		
$y = y_0 + v_1 t + \dfrac{1}{2}at^2 = At^2 + Bt + C$		
For correctly relating the equation to the value of g		1 point
$A = \dfrac{1}{2}a = \dfrac{1}{2}g$		
$g = 2A = (2)\left(5.75\ \text{m/s}^2\right) = 11.5\ \text{m/s}^2$		

Question 1 (continued)

Distribution of points

iii. 1 point

Using $9.81\ \text{m/s}^2$ as the accepted value for g at this location, calculate the percent error for the value found in part (e)(ii).

For correctly calculating the percent error		1 point
$\%\text{error} = \frac{\lvert\text{acc} - \text{exp}\rvert}{\text{acc}} \times 100\% = \frac{\left\lvert\left(11.5\ \text{m/s}^2\right) - \left(9.81\ \text{m/s}^2\right)\right\rvert}{\left(9.81\ \text{m/s}^2\right)} \times 100\%$		
$\%\text{error} = 17.2\%$		
Note: Credit is earned if percent error is expressed as positive or negative.		

iv. 2 points

Assuming the sphere is at a height of 1.40 m at $t = 0$, calculate the velocity of the sphere just before it strikes the pad.

For relating the coefficients of the equation to the kinematic variables		1 point
$y = At^2 + Bt + C \ \therefore v_1 = B = -0.524\ \text{m/s}$		
$a = 2A = 2 \times \left(5.75\ \text{m/s}^2\right) = 11.5\ \text{m/s}^2$		
$y_0 = C = 0.080\ \text{m}$		
For correctly using an appropriate kinematics equation to determine the velocity of the sphere		1 point
$v_2^2 = v_1^2 + 2a\Delta y$		
$v = \sqrt{v_1^2 + 2a\Delta y} = \sqrt{(-0.524\ \text{m/s})^2 + (2)\left(11.5\ \text{m/s}^2\right)(1.40\ \text{m} - 0.080\ \text{m})}$		
$v = 5.54\ \text{m/s}$		
Alternate solution:		*Alternate Points*
For correctly determining the time of fall for the sphere		*1 point*
$y = At^2 + Bt + C$		
$1.40 = \left(5.75\ \text{m/s}^2\right)t^2 + (-0.524\ \text{m/s})t + (0.080\ \text{m})$		
$5.75t^2 - 0.524t - 1.32 = 0$		
$t = 0.53\ \text{s}$ or $-0.44\ \text{s} \ \therefore t = 0.53\ \text{s}$		
For correctly using the equation from part (e)(i)		*1 point*
$v(t) = \left(11.5\ \text{m/s}^2\right)(0.53\ \text{s}) - 0.524\ \text{m/s}$		
$v = 5.54\ \text{m/s}$		

Question 1 (continued)

Distribution of points

(e)

iv. (continued)

Alternate Solution — Conservation of Energy		
For relating the coefficients of the equation to the initial height and speed		1 point
$y = At^2 + Bt + C \ \therefore v_1 = B = -0.524$ m/s		
$y_0 = C = 0.080$ m		
For correctly using conservation of energy to determine the velocity of the sphere		1 point
$U_i + K_i = U_f + K_f$		
$mgh + \frac{1}{2}mv_1^2 = \frac{1}{2}mv^2$		
$v = \sqrt{2(11.5)(1.40 - 0.080) + (-0.524)^2} = 5.54$ m/s		