CALCULUS BC
SECTION II, Part B
Time—45 minutes
Number of problems—3

No calculator is allowed for these problems.

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4. Let h be a function defined for all $x \neq 0$ such that $h(4) = -3$ and the derivative of h is given by $h'(x) = \dfrac{x^2 - 2}{x}$ for all $x \neq 0$.

   (a) Find all values of x for which the graph of h has a horizontal tangent, and determine whether h has a local maximum, a local minimum, or neither at each of these values. Justify your answers.

   (b) On what intervals, if any, is the graph of h concave up? Justify your answer.

   (c) Write an equation for the line tangent to the graph of h at $x = 4$.

   (d) Does the line tangent to the graph of h at $x = 4$ lie above or below the graph of h for $x > 4$? Why?

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5. Let f be the function satisfying $f'(x) = -3xf(x)$, for all real numbers x, with $f(1) = 4$ and $\lim_{x \to \infty} f(x) = 0$.

   (a) Evaluate $\int_1^{\infty} -3xf(x)\,dx$. Show the work that leads to your answer.

   (b) Use Euler's method, starting at $x = 1$ with a step size of 0.5, to approximate $f(2)$.

   (c) Write an expression for $y = f(x)$ by solving the differential equation $\dfrac{dy}{dx} = -3xy$ with the initial condition $f(1) = 4$.

GO ON TO THE NEXT PAGE.

6. A function f is defined by

$$f(x) = \frac{1}{3} + \frac{2}{3^2}x + \frac{3}{3^3}x^2 + \cdots + \frac{n+1}{3^{n+1}}x^n + \cdots$$

for all x in the interval of convergence of the given power series.

(a) Find the interval of convergence for this power series. Show the work that leads to your answer.

(b) Find $\lim\limits_{x \to 0} \dfrac{f(x) - \frac{1}{3}}{x}$.

(c) Write the first three nonzero terms and the general term for an infinite series that represents $\int_0^1 f(x)\,dx$.

(d) Find the sum of the series determined in part (c).

END OF EXAMINATION

Question 5

Let f be the function satisfying $f'(x) = -3xf(x)$, for all real numbers x, with $f(1) = 4$ and $\lim_{x\to\infty} f(x) = 0$.

(a) Evaluate $\int_1^\infty -3xf(x)\,dx$. Show the work that leads to your answer.

(b) Use Euler's method, starting at $x = 1$ with a step size of 0.5, to approximate $f(2)$.

(c) Write an expression for $y = f(x)$ by solving the differential equation $\frac{dy}{dx} = -3xy$ with the initial condition $f(1) = 4$.

(a) $\int_1^\infty -3xf(x)\,dx$

$= \int_1^\infty f'(x)\,dx = \lim_{b\to\infty} \int_1^b f'(x)\,dx = \lim_{b\to\infty} f(x)\Big|_1^b$

$= \lim_{b\to\infty} f(b) - f(1) = 0 - 4 = -4$

$2 : \begin{cases} 1 : \text{use of FTC} \\ 1 : \text{answer from limiting process} \end{cases}$

(b) $f(1.5) \approx f(1) + f'(1)(0.5)$

$= 4 - 3(1)(4)(0.5) = -2$

$f(2) \approx -2 + f'(1.5)(0.5)$

$\approx -2 - 3(1.5)(-2)(0.5) = 2.5$

$2 : \begin{cases} 1 : \text{Euler's method equations or equivalent table} \\ 1 : \text{Euler approximation to } f(2) \\ \quad\text{(not eligible without first point)} \end{cases}$

(c) $\frac{1}{y}\,dy = -3x\,dx$

$\ln y = -\frac{3}{2}x^2 + k$

$y = Ce^{-\frac{3}{2}x^2}$

$4 = Ce^{-\frac{3}{2}}$; $C = 4e^{\frac{3}{2}}$

$y = 4e^{\frac{3}{2}}e^{-\frac{3}{2}x^2}$

$5 : \begin{cases} 1 : \text{separates variables} \\ 1 : \text{antiderivatives} \\ 1 : \text{constant of integration} \\ 1 : \text{uses initial condition } f(1) = 4 \\ 1 : \text{solves for } y \end{cases}$

Note: max 2/5 [1-1-0-0-0] if no constant of integration

Note: 0/5 if no separation of variables