

## 2012 AP<sup>®</sup> CALCULUS BC FREE-RESPONSE QUESTIONS

6. The function  $g$  has derivatives of all orders, and the Maclaurin series for  $g$  is

$$\sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{2n+3} = \frac{x}{3} - \frac{x^3}{5} + \frac{x^5}{7} - \dots$$

- (a) Using the ratio test, determine the interval of convergence of the Maclaurin series for  $g$ .
- (b) The Maclaurin series for  $g$  evaluated at  $x = \frac{1}{2}$  is an alternating series whose terms decrease in absolute value to 0. The approximation for  $g\left(\frac{1}{2}\right)$  using the first two nonzero terms of this series is  $\frac{17}{120}$ . Show that this approximation differs from  $g\left(\frac{1}{2}\right)$  by less than  $\frac{1}{200}$ .
- (c) Write the first three nonzero terms and the general term of the Maclaurin series for  $g'(x)$ .
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**STOP**

**END OF EXAM**

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2012 SCORING GUIDELINES**

**Question 6**

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(a)  $\left| \frac{x^{2n+3}}{2n+5} \cdot \frac{2n+3}{x^{2n+1}} \right| = \left( \frac{2n+3}{2n+5} \right) \cdot x^2$

$$\lim_{n \rightarrow \infty} \left( \frac{2n+3}{2n+5} \right) \cdot x^2 = x^2$$

$$x^2 < 1 \Rightarrow -1 < x < 1$$

The series converges when  $-1 < x < 1$ .

When  $x = -1$ , the series is  $-\frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \frac{1}{9} - \dots$

This series converges by the Alternating Series Test.

When  $x = 1$ , the series is  $\frac{1}{3} - \frac{1}{5} + \frac{1}{7} - \frac{1}{9} + \dots$

This series converges by the Alternating Series Test.

Therefore, the interval of convergence is  $-1 \leq x \leq 1$ .

5 :  $\begin{cases} 1 : \text{sets up ratio} \\ 1 : \text{computes limit of ratio} \\ 1 : \text{identifies interior of} \\ \quad \text{interval of convergence} \\ 1 : \text{considers both endpoints} \\ 1 : \text{analysis and interval of convergence} \end{cases}$

(b)  $\left| g\left(\frac{1}{2}\right) - \frac{17}{120} \right| < \frac{\left(\frac{1}{2}\right)^5}{7} = \frac{1}{224} < \frac{1}{200}$

2 :  $\begin{cases} 1 : \text{uses the third term as an error bound} \\ 1 : \text{error bound} \end{cases}$

(c)  $g'(x) = \frac{1}{3} - \frac{3}{5}x^2 + \frac{5}{7}x^4 + \dots + (-1)^n \left( \frac{2n+1}{2n+3} \right) x^{2n} + \dots$

2 :  $\begin{cases} 1 : \text{first three terms} \\ 1 : \text{general term} \end{cases}$