PHYSICS C: MECHANICS
SECTION II
Time—45 minutes
3 Questions

Directions: Answer all three questions. The suggested time is about 15 minutes for answering each of the questions, which are worth 15 points each. The parts within a question may not have equal weight. Show all your work in this booklet in the spaces provided after each part, NOT in the green insert.

Mech. 1.

A 3.0 kg object is moving along the x-axis in a region where its potential energy as a function of x is given as $U(x) = 4.0x^2$, where U is in joules and x is in meters. When the object passes the point $x = -0.50$ m, its velocity is +2.0 m/s. All forces acting on the object are conservative.

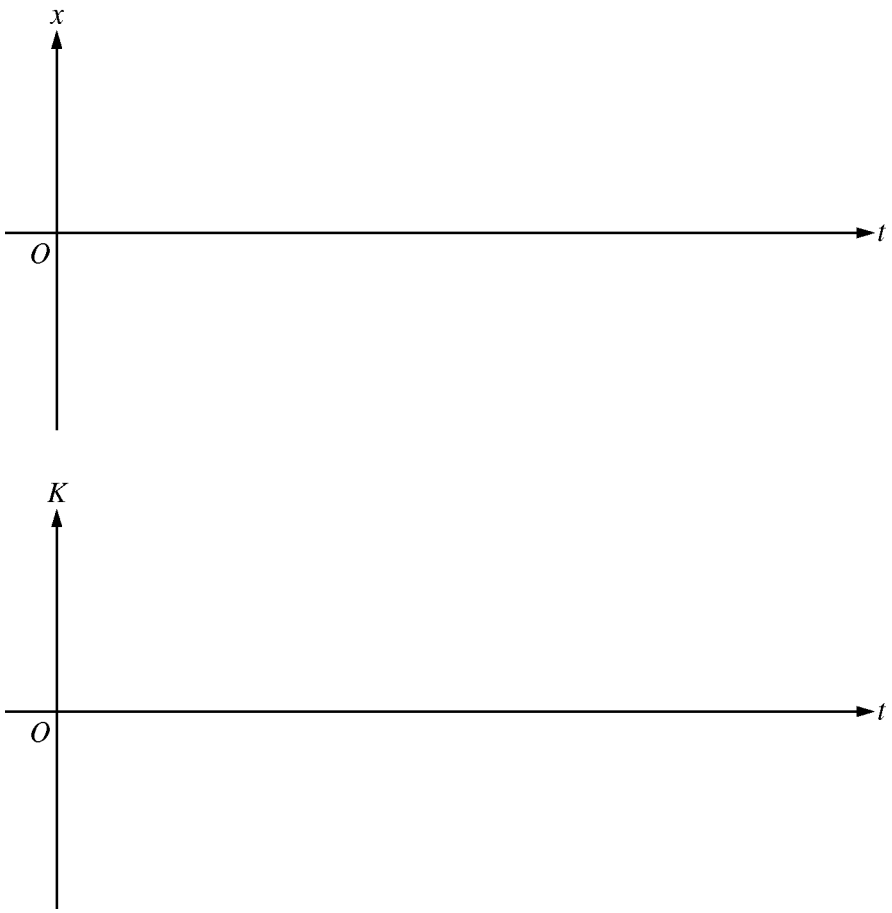
(a) Calculate the total mechanical energy of the object.

(b) Calculate the x-coordinate of any points at which the object has zero kinetic energy.

(c) Calculate the magnitude of the momentum of the object at $x = 0.60$ m.

(d) Calculate the magnitude of the acceleration of the object as it passes $x = 0.60$ m.

(e) On the axes below, sketch graphs of the object's position x versus time t and kinetic energy K versus time t. Assume that $x = 0$ at time $t = 0$. The two graphs should cover the same time interval and use the same scale on the horizontal axes.



AP® PHYSICS C: MECHANICS
2009 SCORING GUIDELINES

Question 1

15 points total **Distribution of points**

(a) 2 points

For indication that total energy is the sum of kinetic and potential energy 1 point

$E = U(x) + K(x)$

$E = 4.0x^2 + \frac{1}{2}m(v(x))^2$

$E = \left(4.0\ \text{J/m}^2\right)(-0.50\ \text{m})^2 + \frac{1}{2}(3.0\ \text{kg})(2.0\ \text{m/s})^2$

For correct calculation of the numerical value of the total energy 1 point

$E = 7.0\ \text{J}$

(b) 3 points

For indication that $E = U$ when $K = 0$ 1 point

$E = U(x)$

$E = 4.0x^2$

For substitution of E from (a) into the equation 1 point

$7.0\ \text{J} = \left(4.0\ \text{J/m}^2\right)x^2$

$x = \pm\sqrt{7.0/4.0}\ \text{m}$

For including plus and minus signs in final numerical answer 1 point

$x = \pm 1.3\ \text{m}$

(c) 3 points

For clear indication that kinetic energy is total energy minus potential energy, using total energy from (a) 1 point

$K = E_{tot} - U$

$K = 7.0\ \text{J} - \left(4.0\ \text{J/m}^2\right)(0.60\ \text{m})^2 \quad = 5.56\ \text{J}$

For using calculated K to solve for v 1 point

$K = \frac{1}{2}mv^2$

$5.56\ \text{J} = \frac{1}{2}(3.0\ \text{kg})v^2$

$v = 1.92\ \text{m/s}$

For substituting calculated value of v into expression for momentum 1 point

$p = mv = (3.0\ \text{kg})(1.92\ \text{m/s}) = 5.8\ \text{kg•m/s}$

Note: The final 2 points could also be earned by substituting the kinetic energy directly into the expression relating the kinetic energy and momentum $p = \sqrt{2mK}$.

Question 1 (continued)

Distribution of points

(d) 3 points

For a correct relationship between force and potential energy 1 point
Note: This point was awarded if the negative sign was not included, since the question asks for magnitude of the acceleration.

$$F = -\frac{dU(x)}{dx}$$

For an expression or calculated value for force consistent with relationship above 1 point

$$F = -\left(\frac{d}{dx}4.0x^2\right) = -8.0x$$

For application of Newton's second law using a derived expression or calculated value for force 1 point

$$a = \frac{F}{m} = \frac{8.0x}{m}$$

$$a = \frac{\left(8.0\ \text{kg/s}^2\right)(0.60\ \text{m})}{3.0\ \text{kg}}$$

$$a = 1.6\ \text{m/s}^2$$

Alternate Solution *Alternate Points*

$E = U + K$

For a correct energy relationship with potential and kinetic energy substituted *1 point*

$$7 = 4x^2 + \frac{1}{2}mv^2$$

$$14 - 8x^2 = mv^2$$

For taking the derivative with respect to time of each side of this equation *1 point*

$$-16x\frac{dx}{dt} = 2mv\frac{dv}{dt}$$

$$-16xv = 2mva$$

$$-8x = ma$$

For algebraically solving for acceleration *1 point*

$$a = -\frac{8x}{m}$$

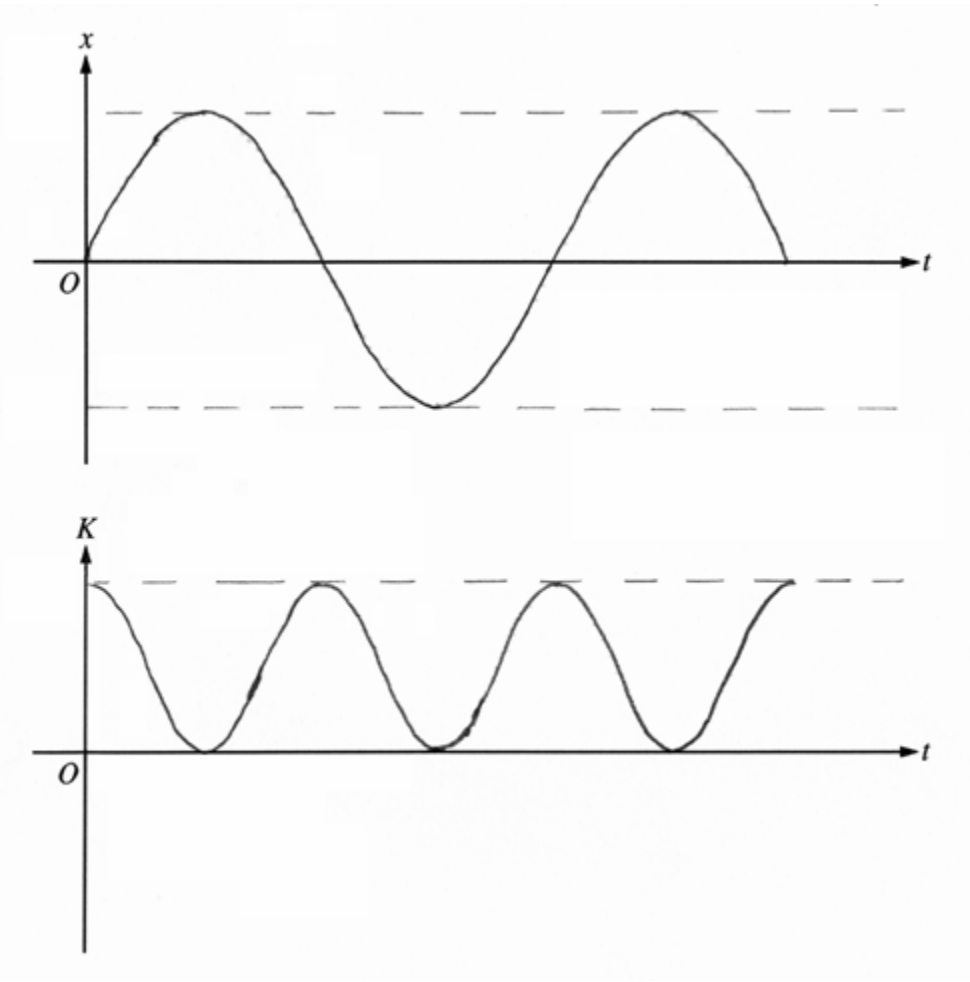
$$a = \frac{\left(8.0\ \text{kg/s}^2\right)(0.60\ \text{m})}{3.0\ \text{kg}}$$

$$a = 1.6\ \text{m/s}^2$$

Question 1 (continued)

Distribution of points

(e) 3 points



For a minimum of one complete cycle of a sine curve starting at the origin on the x versus t graph — 1 point

For a minimum of one complete cycle of a cosine squared curve starting at the maximum value on the K versus t graph — 1 point

For maxima and minima of the x graph matching the zeroes of the K graph — 1 point

Units point

For correct units on all completed nonzero numerical answers — 1 point