STATISTICS
SECTION II
Part B
Question 6
Spend about 25 minutes on this part of the exam.
Percent of Section II grade—25

Directions: Show all your work. Indicate clearly the methods you use, because you will be graded on the correctness of your methods as well as on the accuracy and completeness of your results and explanations.

6. A manufacturer of thermostats is concerned that the readings of its thermostats have become less reliable (more variable). In the past, the variance has been 1.52 degrees Fahrenheit (F) squared. A random sample of 10 recently manufactured thermostats was selected and placed in a room that was maintained at 68°F. The readings for those 10 thermostats are given in the table below.

Thermostat	1	2	3	4	5	6	7	8	9	10
Temperature (°F)	66.8	67.8	70.6	69.3	65.9	66.2	68.1	68.6	67.9	67.2

(a) State the null and alternative hypotheses that the manufacturer is interested in testing.

It can be shown that if the population of thermostat temperatures is normally distributed, the sampling distribution of $\frac{(n-1)s^2}{\sigma^2}$ follows a chi-square distribution with $n-1$ degrees of freedom.

(b) Calculate the value of $\frac{(n-1)s^2}{1.52}$ for these data.

(c) Assume that the population of thermostat temperatures follows a normal distribution. Use the test statistic $\frac{(n-1)s^2}{1.52}$ from part (b) and the chi-square distribution to test the hypotheses in part (a).

(d) For the test conducted in part (c), what is the smallest value of the test statistic that would have led to the rejection of the null hypothesis at the 5 percent significance level?

Mark this value of the test statistic on the graph of the chi-square distribution below. Indicate the region that contains all of the values that would have led to the rejection of the null hypothesis.

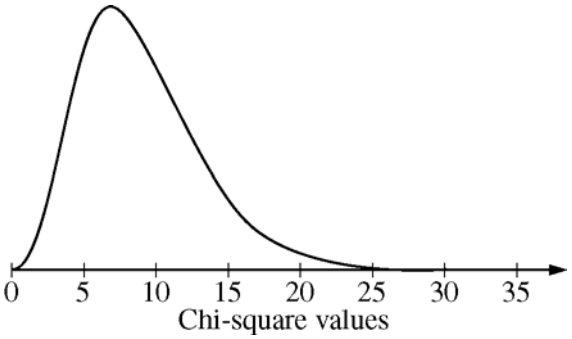

Table entry for z is the probability lying below z.

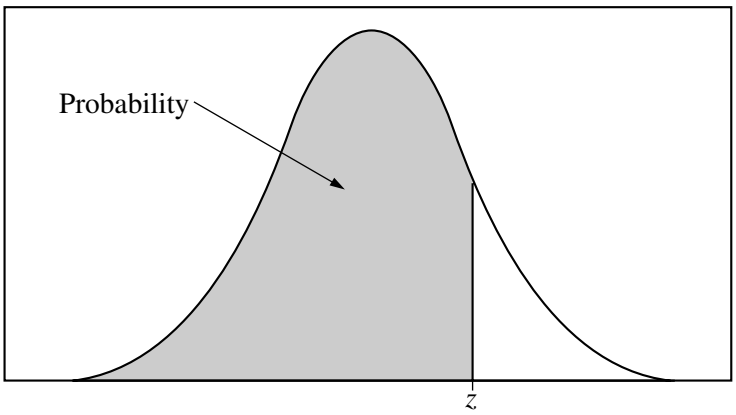

Table A (***Continued***)

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998

Question 6

Intent of Question

The primary goals of this question are to evaluate a student's ability to apply the concepts of significance testing to a new setting, in particular to: (1) state hypotheses for a parameter of interest, given a research question; (2) evaluate a new test statistic and use the probability distribution associated with that statistic to test the hypotheses of interest; (3) identify the values of the test statistic that would lead to rejection of the null hypothesis on a graph; and (4) interpret simulated sampling distributions for different populations.

Solution

Part (a):

Let σ^2 denote the variance in the temperatures measured by the thermostats recently produced by this manufacturer.

$H_0 : \sigma^2 = 1.52(^\circ F)^2$ OR Recently produced thermostats are not more variable than thermostats produced in the past.

$H_a : \sigma^2 > 1.52(^\circ F)^2$ OR Recently produced thermostats are more variable than thermostats produced in the past.

Part (b):

$$\frac{(n-1)s^2}{1.52} = \frac{9 \times (1.4277)^2}{1.52} = \frac{9 \times (2.0383)}{1.52} = 12.069$$

Part (c):

The test statistic has a χ^2 distribution with 9 degrees of freedom under H_0. The chance of exceeding the observed value of 12.069, under H_0, is

$$p\text{−value} = P(\chi_9^2 \geq 12.069) = 0.2094 .$$

(or, from the table, .20 < *p*-value < .25). Since the *p*-value is greater than 0.05, we cannot reject the null hypothesis. That is, we do not have statistically significant evidence that recent thermostats are less reliable (more variable) than in the past.

Part (d):

The smallest value that would have led to the rejection of the null hypothesis is the 95th percentile of the χ^2 distribution with 9 degrees of freedom, which is 16.92. The rejection region contains all values greater than or equal to 16.92. This region should be identified on the graph by indicating the approximate location of 16.92 on the axis and shading the region that is bounded by the vertical line through 16.92, the horizontal axis, and the χ^2 curve.

Question 6 (continued)

Part (e)

Indicate the region to the right of 16.92 on all three histograms.

Part (f)

The population with the largest variance will tend to produce the largest values of s^2 in the simulation and hence the largest test statistics. Histogram III has the largest probability of producing a sample that would lead to the rejection of H_0 so Histogram III corresponds to the population with the largest variance.

Similarly, the test statistics will tend to be smallest for the population with variance closest to 1.52. Histogram II has the smallest probability of producing a sample that would lead to the rejection of H_0 so Histogram II corresponds to the population with the smallest variance.

Scoring

Each of four components are scored as essentially correct (E), partially correct (P), or incorrect (I).

I. **Parts (a) and (b)** are combined into one component and scored as essentially correct (E) if both part (a) and part (b) are correct.

Parts (a) and (b) are partially correct (P) if one of the two parts is correct.

Notes*:*

1. If a two-sided alternative is used or the hypotheses involve a mean, then part (a) is not correct.
2. Nonstandard notation for the population variance must be defined.
3. If the value of s (or of s^2) is not shown in part (b), then part (b) is incorrect.

II. **Part (c)** is scored as essentially correct (E) if both:

- The p-value is given (or the test statistic compared to the critical value).
- The conclusion is written in context and linked to the p-value.

Part (c) is partially correct (P) if one of the two bulleted items is correct.

Notes*:*

1. Conditions (SRS, normal population) are given in the problem so it is not necessary to restate them. However, if incorrect conditions are given, the first bullet is incorrect.
2. If the null hypothesis is "accepted" or equivalent, the second bullet is incorrect.
3. If both an α and a p-value are given, the linkage is implied. If no α is given, the solution must be explicit about the linkage by giving a correct interpretation of the p-value or explaining how the conclusion follows from the p-value.

III. **Parts (d) and (e)** are combined into one component and scored as essentially correct (E) if both:

- The critical value is identified as 16.92.
- The region to the right of a cut-off point of between 15 and 20 is identified in part (d). AND the same region is identified in each of the three histograms in part (e).

Question 6 (continued)

Parts (d) and (e) are partially correct (P) if one of the two bulleted items above is correct.

IV. **Part (f)** is essentially correct (E) if both:

- Histograms III and II are identified as the simulated sampling distributions for the populations with the largest and smallest variances, respectively.
- The justification refers to fact that Histogram III came from the population with the largest variance because the identified region is largest, and so it will be more likely to reject the null hypothesis. (Similarly for the smallest variance.)

Part (f) is partially correct (P) if both:

- Histograms III and II are identified as the simulated sampling distributions for the populations with the largest and smallest variances, respectively.
- The justification says only that Histogram III represents the population with the largest variance because the identified region is largest. OR The justification refers to the fact that the simulated sampling distribution for the population with the largest variance should result in sample variances—and hence test statistics—that are centered at the largest values. (Similarly for the smallest variance.) OR The justification refers to the fact that the simulated sampling distribution for the population with the largest variance should result in sample variances—and hence test statistics—that are more variable and Histogram III has the more variable values of the test statistic. (Similarly for the smallest variance.)

Part (f) is incorrect (I) if

- Histograms III and II are identified as the simulated sampling distributions for the populations with the largest and smallest variances, respectively, but the justification refers only to the fact that these histograms themselves have the largest and smallest variability.

Note*:*

1. If only one of Histogram III or Histogram II is identified and correctly justified, the response is scored partially correct.

For each of the four components,

Essentially Correct (E) = 1
Partially Correct (P) = 1/2
Incorrect (I) = 0

4 Complete Response

3 Substantial Response

2 Developing Response

1 Minimal Response

If a response is between two scores (for example, 2½ points), use a holistic approach to determine whether to score up or down depending on the strength of the response and communication.