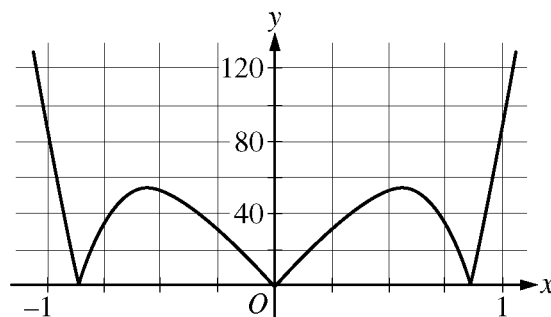


2011 AP[®] CALCULUS BC FREE-RESPONSE QUESTIONS



Graph of $y = |f^{(5)}(x)|$

6. Let $f(x) = \sin(x^2) + \cos x$. The graph of $y = |f^{(5)}(x)|$ is shown above.
- Write the first four nonzero terms of the Taylor series for $\sin x$ about $x = 0$, and write the first four nonzero terms of the Taylor series for $\sin(x^2)$ about $x = 0$.
 - Write the first four nonzero terms of the Taylor series for $\cos x$ about $x = 0$. Use this series and the series for $\sin(x^2)$, found in part (a), to write the first four nonzero terms of the Taylor series for f about $x = 0$.
 - Find the value of $f^{(6)}(0)$.
 - Let $P_4(x)$ be the fourth-degree Taylor polynomial for f about $x = 0$. Using information from the graph of $y = |f^{(5)}(x)|$ shown above, show that $\left| P_4\left(\frac{1}{4}\right) - f\left(\frac{1}{4}\right) \right| < \frac{1}{3000}$.

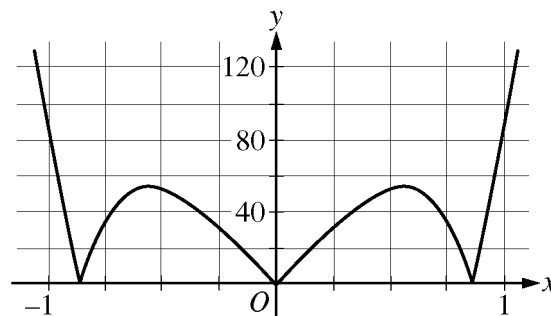
WRITE ALL WORK IN THE EXAM BOOKLET.

END OF EXAM

AP[®] CALCULUS BC
2011 SCORING GUIDELINES

Question 6

Let $f(x) = \sin(x^2) + \cos x$. The graph of $y = |f^{(5)}(x)|$ is shown above.



Graph of $y = |f^{(5)}(x)|$

- (a) Write the first four nonzero terms of the Taylor series for $\sin x$ about $x = 0$, and write the first four nonzero terms of the Taylor series for $\sin(x^2)$ about $x = 0$.
- (b) Write the first four nonzero terms of the Taylor series for $\cos x$ about $x = 0$. Use this series and the series for $\sin(x^2)$, found in part (a), to write the first four nonzero terms of the Taylor series for f about $x = 0$.
- (c) Find the value of $f^{(6)}(0)$.
- (d) Let $P_4(x)$ be the fourth-degree Taylor polynomial for f about $x = 0$. Using information from the graph of $y = |f^{(5)}(x)|$ shown above, show that $\left|P_4\left(\frac{1}{4}\right) - f\left(\frac{1}{4}\right)\right| < \frac{1}{3000}$.

(a) $\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \dots$
 $\sin(x^2) = x^2 - \frac{x^6}{3!} + \frac{x^{10}}{5!} - \frac{x^{14}}{7!} + \dots$

3 : $\begin{cases} 1 : \text{series for } \sin x \\ 2 : \text{series for } \sin(x^2) \end{cases}$

(b) $\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \dots$
 $f(x) = 1 + \frac{x^2}{2} + \frac{x^4}{4!} - \frac{121x^6}{6!} + \dots$

3 : $\begin{cases} 1 : \text{series for } \cos x \\ 2 : \text{series for } f(x) \end{cases}$

(c) $\frac{f^{(6)}(0)}{6!}$ is the coefficient of x^6 in the Taylor series for f about $x = 0$. Therefore $f^{(6)}(0) = -121$.

1 : answer

(d) The graph of $y = |f^{(5)}(x)|$ indicates that $\max_{0 \leq x \leq \frac{1}{4}} |f^{(5)}(x)| < 40$.

Therefore

$$\left|P_4\left(\frac{1}{4}\right) - f\left(\frac{1}{4}\right)\right| \leq \frac{\max_{0 \leq x \leq \frac{1}{4}} |f^{(5)}(x)|}{5!} \cdot \left(\frac{1}{4}\right)^5 < \frac{40}{120 \cdot 4^5} = \frac{1}{3072} < \frac{1}{3000}.$$

2 : $\begin{cases} 1 : \text{form of the error bound} \\ 1 : \text{analysis} \end{cases}$