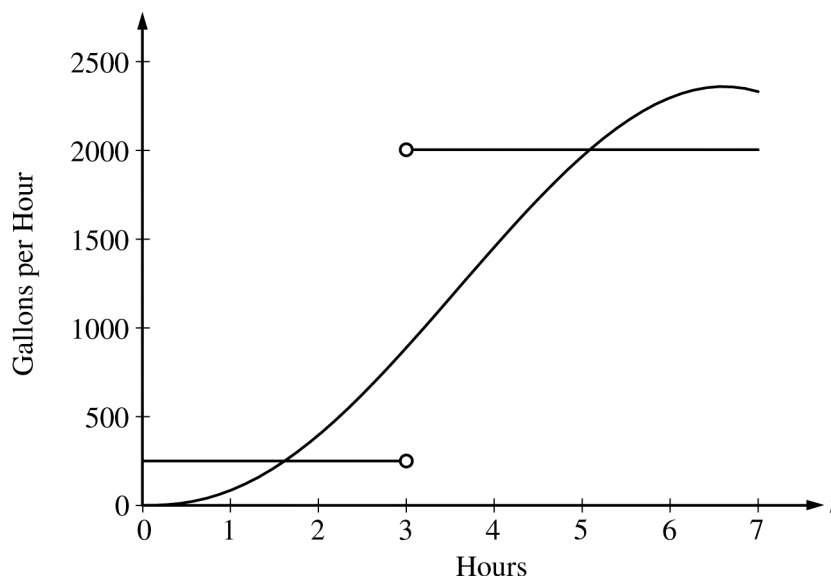**CALCULUS AB**
SECTION II, Part A
Time—45 minutes
Number of problems—3

A graphing calculator is required for some problems or parts of problems.

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1. Let R be the region in the first and second quadrants bounded above by the graph of $y = \dfrac{20}{1 + x^2}$ and below by the horizontal line $y = 2$.

   (a) Find the area of R.

   (b) Find the volume of the solid generated when R is rotated about the x-axis.

   (c) The region R is the base of a solid. For this solid, the cross sections perpendicular to the x-axis are semicircles. Find the volume of this solid.

WRITE ALL WORK IN THE PINK EXAM BOOKLET.

GO ON TO THE NEXT PAGE.

Hours

2. The amount of water in a storage tank, in gallons, is modeled by a continuous function on the time interval $0 \leq t \leq 7$, where t is measured in hours. In this model, rates are given as follows:

(i) The rate at which water enters the tank is $f(t) = 100t^2 \sin\left(\sqrt{t}\right)$ gallons per hour for $0 \leq t \leq 7$.

(ii) The rate at which water leaves the tank is

$$g(t) = \begin{cases} 250 & \text{for } 0 \leq t < 3 \\ 2000 & \text{for } 3 < t \leq 7 \end{cases} \text{ gallons per hour.}$$

The graphs of f and g, which intersect at $t = 1.617$ and $t = 5.076$, are shown in the figure above. At time $t = 0$, the amount of water in the tank is 5000 gallons.

(a) How many gallons of water enter the tank during the time interval $0 \leq t \leq 7$? Round your answer to the nearest gallon.

(b) For $0 \leq t \leq 7$, find the time intervals during which the amount of water in the tank is decreasing. Give a reason for each answer.

(c) For $0 \leq t \leq 7$, at what time t is the amount of water in the tank greatest? To the nearest gallon, compute the amount of water at this time. Justify your answer.

WRITE ALL WORK IN THE PINK EXAM BOOKLET.

GO ON TO THE NEXT PAGE.

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(a) Find the area of R.

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$\dfrac{20}{1 + x^2} = 2$ when $x = \pm 3$

(a) Area $= \displaystyle\int_{-3}^{3} \left(\dfrac{20}{1 + x^2} - 2 \right) dx = 37.961$ or 37.962

(b) Volume $= \pi \displaystyle\int_{-3}^{3} \left(\left(\dfrac{20}{1 + x^2} \right)^2 - 2^2 \right) dx = 1871.190$

(c) Volume $= \dfrac{\pi}{2} \displaystyle\int_{-3}^{3} \left(\dfrac{1}{2} \left(\dfrac{20}{1 + x^2} - 2 \right) \right)^2 dx$

$= \dfrac{\pi}{8} \displaystyle\int_{-3}^{3} \left(\dfrac{20}{1 + x^2} - 2 \right)^2 dx = 174.268$

1 : correct limits in an integral in (a), (b), or (c)

$2 : \begin{cases} 1 : \text{integrand} \\ 1 : \text{answer} \end{cases}$

$3 : \begin{cases} 2 : \text{integrand} \\ 1 : \text{answer} \end{cases}$

$3 : \begin{cases} 2 : \text{integrand} \\ 1 : \text{answer} \end{cases}$