

## 2009 AP® CALCULUS AB FREE-RESPONSE QUESTIONS

2. The rate at which people enter an auditorium for a rock concert is modeled by the function  $R$  given by  $R(t) = 1380t^2 - 675t^3$  for  $0 \leq t \leq 2$  hours;  $R(t)$  is measured in people per hour. No one is in the auditorium at time  $t = 0$ , when the doors open. The doors close and the concert begins at time  $t = 2$ .
- How many people are in the auditorium when the concert begins?
  - Find the time when the rate at which people enter the auditorium is a maximum. Justify your answer.
  - The total wait time for all the people in the auditorium is found by adding the time each person waits, starting at the time the person enters the auditorium and ending when the concert begins. The function  $w$  models the total wait time for all the people who enter the auditorium before time  $t$ . The derivative of  $w$  is given by  $w'(t) = (2 - t)R(t)$ . Find  $w(2) - w(1)$ , the total wait time for those who enter the auditorium after time  $t = 1$ .
  - On average, how long does a person wait in the auditorium for the concert to begin? Consider all people who enter the auditorium after the doors open, and use the model for total wait time from part (c).
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3. Mighty Cable Company manufactures cable that sells for \$120 per meter. For a cable of fixed length, the cost of producing a portion of the cable varies with its distance from the beginning of the cable. Mighty reports that the cost to produce a portion of a cable that is  $x$  meters from the beginning of the cable is  $6\sqrt{x}$  dollars per meter. (Note: Profit is defined to be the difference between the amount of money received by the company for selling the cable and the company's cost of producing the cable.)
- Find Mighty's profit on the sale of a 25-meter cable.
  - Using correct units, explain the meaning of  $\int_{25}^{30} 6\sqrt{x} dx$  in the context of this problem.
  - Write an expression, involving an integral, that represents Mighty's profit on the sale of a cable that is  $k$  meters long.
  - Find the maximum profit that Mighty could earn on the sale of one cable. Justify your answer.
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**WRITE ALL WORK IN THE PINK EXAM BOOKLET.**

**END OF PART A OF SECTION II**

**AP<sup>®</sup> CALCULUS AB  
2009 SCORING GUIDELINES**

**Question 2**

The rate at which people enter an auditorium for a rock concert is modeled by the function  $R$  given by  $R(t) = 1380t^2 - 675t^3$  for  $0 \leq t \leq 2$  hours;  $R(t)$  is measured in people per hour. No one is in the auditorium at time  $t = 0$ , when the doors open. The doors close and the concert begins at time  $t = 2$ .

- (a) How many people are in the auditorium when the concert begins?
- (b) Find the time when the rate at which people enter the auditorium is a maximum. Justify your answer.
- (c) The total wait time for all the people in the auditorium is found by adding the time each person waits, starting at the time the person enters the auditorium and ending when the concert begins. The function  $w$  models the total wait time for all the people who enter the auditorium before time  $t$ . The derivative of  $w$  is given by  $w'(t) = (2 - t)R(t)$ . Find  $w(2) - w(1)$ , the total wait time for those who enter the auditorium after time  $t = 1$ .
- (d) On average, how long does a person wait in the auditorium for the concert to begin? Consider all people who enter the auditorium after the doors open, and use the model for total wait time from part (c).

(a)  $\int_0^2 R(t) dt = 980$  people

2 :  $\begin{cases} 1 : \text{integral} \\ 1 : \text{answer} \end{cases}$

(b)  $R'(t) = 0$  when  $t = 0$  and  $t = 1.36296$   
The maximum rate may occur at 0,  $a = 1.36296$ , or 2.

3 :  $\begin{cases} 1 : \text{considers } R'(t) = 0 \\ 1 : \text{interior critical point} \\ 1 : \text{answer and justification} \end{cases}$

$R(0) = 0$   
 $R(a) = 854.527$   
 $R(2) = 120$

The maximum rate occurs when  $t = 1.362$  or 1.363.

(c)  $w(2) - w(1) = \int_1^2 w'(t) dt = \int_1^2 (2 - t)R(t) dt = 387.5$

2 :  $\begin{cases} 1 : \text{integral} \\ 1 : \text{answer} \end{cases}$

The total wait time for those who enter the auditorium after time  $t = 1$  is 387.5 hours.

(d)  $\frac{1}{980}w(2) = \frac{1}{980} \int_0^2 (2 - t)R(t) dt = 0.77551$   
On average, a person waits 0.775 or 0.776 hour.

2 :  $\begin{cases} 1 : \text{integral} \\ 1 : \text{answer} \end{cases}$