CALCULUS AB
SECTION II, Part B
Time—45 minutes
Number of problems—3

No calculator is allowed for these problems.

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4. Let h be a function defined for all $x \neq 0$ such that $h(4) = -3$ and the derivative of h is given by $h'(x) = \dfrac{x^2 - 2}{x}$ for all $x \neq 0$.

   (a) Find all values of x for which the graph of h has a horizontal tangent, and determine whether h has a local maximum, a local minimum, or neither at each of these values. Justify your answers.

   (b) On what intervals, if any, is the graph of h concave up? Justify your answer.

   (c) Write an equation for the line tangent to the graph of h at $x = 4$.

   (d) Does the line tangent to the graph of h at $x = 4$ lie above or below the graph of h for $x > 4$? Why?

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5. A cubic polynomial function f is defined by

$$f(x) = 4x^3 + ax^2 + bx + k$$

   where a, b, and k are constants. The function f has a local minimum at $x = -1$, and the graph of f has a point of inflection at $x = -2$.

   (a) Find the values of a and b.

   (b) If $\int_0^1 f(x)\,dx = 32$, what is the value of k ?

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6. The function f is differentiable for all real numbers. The point $\left(3, \frac{1}{4}\right)$ is on the graph of $y = f(x)$, and the slope at each point (x, y) on the graph is given by $\dfrac{dy}{dx} = y^2(6 - 2x)$.

   (a) Find $\dfrac{d^2y}{dx^2}$ and evaluate it at the point $\left(3, \frac{1}{4}\right)$.

   (b) Find $y = f(x)$ by solving the differential equation $\dfrac{dy}{dx} = y^2(6 - 2x)$ with the initial condition $f(3) = \dfrac{1}{4}$.

END OF EXAMINATION

Question 4

Let h be a function defined for all $x \neq 0$ such that $h(4) = -3$ and the derivative of h is given by $h'(x) = \dfrac{x^2 - 2}{x}$ for all $x \neq 0$.

(a) Find all values of x for which the graph of h has a horizontal tangent, and determine whether h has a local maximum, a local minimum, or neither at each of these values. Justify your answers.

(b) On what intervals, if any, is the graph of h concave up? Justify your answer.

(c) Write an equation for the line tangent to the graph of h at $x = 4$.

(d) Does the line tangent to the graph of h at $x = 4$ lie above or below the graph of h for $x > 4$? Why?

(a) $h'(x) = 0$ at $x = \pm\sqrt{2}$

$$\begin{array}{c|ccccccc} h'(x) & - & 0 & + & \text{und} & - & 0 & + \\ \hline x & & -\sqrt{2} & & 0 & & \sqrt{2} & \end{array}$$

Local minima at $x = -\sqrt{2}$ and at $x = \sqrt{2}$

$$4: \begin{cases} 1: x = \pm\sqrt{2} \\ 1: \text{analysis} \\ 2: \text{conclusions} \\ \quad < -1 > \text{ not dealing with discontinuity at } 0 \end{cases}$$

(b) $h''(x) = 1 + \dfrac{2}{x^2} > 0$ for all $x \neq 0$. Therefore, the graph of h is concave up for all $x \neq 0$.

$$3: \begin{cases} 1: h''(x) \\ 1: h''(x) > 0 \\ 1: \text{answer} \end{cases}$$

(c) $h'(4) = \dfrac{16 - 2}{4} = \dfrac{7}{2}$

$y + 3 = \dfrac{7}{2}(x - 4)$

1 : tangent line equation

(d) The tangent line is below the graph because the graph of h is concave up for $x > 4$.

1 : answer with reason