3. Let f be a function that has derivatives of all orders for all real numbers. Assume $f(0) = 5$, $f'(0) = -3$, $f''(0) = 1$, and $f'''(0) = 4$.

   (a) Write the third–degree Taylor polynomial for f about $x = 0$ and use it to approximate $f(0.2)$.

   (b) Write the fourth–degree Taylor polynomial for g, where $g(x) = f(x^2)$, about $x = 0$.

   (c) Write the third–degree Taylor polynomial for h, where $h(x) = \int_0^x f(t)\,dt$, about $x = 0$.

   (d) Let h be defined as in part (c). Given that $f(1) = 3$, either find the exact value of $h(1)$ or explain why it cannot be determined.

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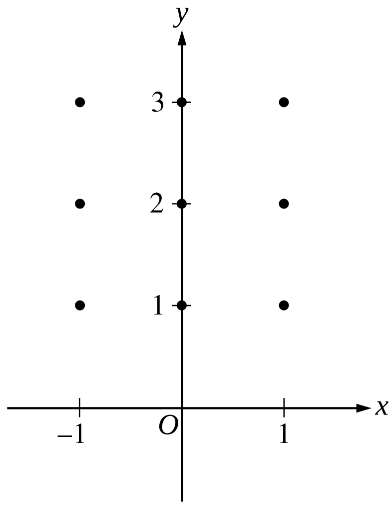
4. Consider the differential equation given by $\frac{dy}{dx} = \frac{xy}{2}$.

   (a) On the axes provided below, sketch a slope field for the given differential equation at the nine points indicated.



   (b) Let $y = f(x)$ be the particular solution to the given differential equation with the initial condition $f(0) = 3$. Use Euler's method starting at $x = 0$, with a step size of 0.1, to approximate $f(0.2)$. Show the work that leads to your answer.

   (c) Find the particular solution $y = f(x)$ to the given differential equation with the initial condition $f(0) = 3$. Use your solution to find $f(0.2)$.

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3. Let f be a function that has derivatives of all orders for all real numbers. Assume $f(0) = 5$, $f'(0) = -3$, $f''(0) = 1$, and $f'''(0) = 4$.

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(d) Let h be defined as in part (c). Given that $f(1) = 3$, either find the exact value of $h(1)$ or explain why it cannot be determined.

(a) $P_3(f)(x) = 5 - 3x + \frac{1}{2}x^2 + \frac{2}{3}x^3$

$f(0.2) \approx P_3(f)(0.2) =$

$$5 - 3(0.2) + \frac{0.04}{2} + \frac{2(0.008)}{3} =$$

4.425

$3 \begin{cases} 2\text{: } 5 - 3x + \frac{1}{2}x^2 + \frac{2}{3}x^3 \\ \quad <-1> \text{ each incorrect term,} \\ \quad \text{extra term, or } + \cdots \\ 1\text{: approximates } f(0.2) \end{cases}$

$<-1>$ for incorrect use of $=$

(b) $P_4(g)(x) = P_2(f)(x^2) = 5 - 3x^2 + \frac{1}{2}x^4$

2: $P_2(f)(x^2)$

$<-1>$ each incorrect or extra term

(c) $P_3(h)(x) = \int_0^x \left(5 - 3t + \frac{1}{2}t^2\right) dt$

$$= \left[5t - \frac{3}{2}t^2 + \frac{1}{6}t^3\right]_0^x$$

$$= 5x - \frac{3}{2}x^2 + \frac{1}{6}x^3$$

$2 \begin{cases} 1\text{: } P_3(h)(x) = \int_0^x P_2(f)(t)\,dt \\ 1\text{: answer} \\ \quad 0/1 \text{ if any incorrect or extra terms} \end{cases}$

(d) $h(1) = \int_0^1 f(t)\,dt$

cannot be determined because $f(t)$ is known only for $t = 0$ and $t = 1$

$2 \begin{cases} 1\text{: } h(1) \text{ cannot be determined} \\ 1\text{: reason} \end{cases}$