

1999

The College Board
Advanced Placement Examination
CALCULUS AB
SECTION II

Time—1 hour and 30 minutes

Number of problems—6

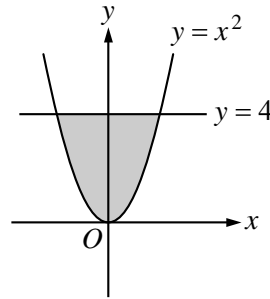
Percent of total grade—50

REMEMBER TO SHOW YOUR SETUPS AS DESCRIBED IN THE GENERAL INSTRUCTIONS.

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1. A particle moves along the y -axis with velocity given by $v(t) = t \sin(t^2)$ for $t \geq 0$.
- (a) In which direction (up or down) is the particle moving at time $t = 1.5$? Why?
 - (b) Find the acceleration of the particle at time $t = 1.5$. Is the velocity of the particle increasing at $t = 1.5$? Why or why not?
 - (c) Given that $y(t)$ is the position of the particle at time t and that $y(0) = 3$, find $y(2)$.
 - (d) Find the total distance traveled by the particle from $t = 0$ to $t = 2$.
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2. The shaded region, R , is bounded by the graph of $y = x^2$ and the line $y = 4$, as shown in the figure above.
- (a) Find the area of R .
 - (b) Find the volume of the solid generated by revolving R about the x -axis.
 - (c) There exists a number k , $k > 4$, such that when R is revolved about the line $y = k$, the resulting solid has the same volume as the solid in part (b). Write, but do not solve, an equation involving an integral expression that can be used to find the value of k .
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1. A particle moves along the y -axis with velocity given by $v(t) = t \sin(t^2)$ for $t \geq 0$.
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 - (b) Find the acceleration of the particle at time $t = 1.5$. Is the velocity of the particle increasing at $t = 1.5$? Why or why not?
 - (c) Given that $y(t)$ is the position of the particle at time t and that $y(0) = 3$, find $y(2)$.
 - (d) Find the total distance traveled by the particle from $t = 0$ to $t = 2$.

(a) $v(1.5) = 1.5 \sin(1.5^2) = 1.167$

Up, because $v(1.5) > 0$

1: answer and reason

(b) $a(t) = v'(t) = \sin t^2 + 2t^2 \cos t^2$

$a(1.5) = v'(1.5) = -2.048$ or -2.049

No; v is decreasing at 1.5 because $v'(1.5) < 0$

2 $\left\{ \begin{array}{l} 1: a(1.5) \\ 1: \text{conclusion and reason} \end{array} \right.$

(c) $y(t) = \int v(t) dt$

$$= \int t \sin t^2 dt = -\frac{\cos t^2}{2} + C$$

$$y(0) = 3 = -\frac{1}{2} + C \implies C = \frac{7}{2}$$

$$y(t) = -\frac{1}{2} \cos t^2 + \frac{7}{2}$$

$$y(2) = -\frac{1}{2} \cos 4 + \frac{7}{2} = 3.826 \text{ or } 3.827$$

3 $\left\{ \begin{array}{l} 1: y(t) = \int v(t) dt \\ 1: y(t) = -\frac{1}{2} \cos t^2 + C \\ 1: y(2) \end{array} \right.$

(d) distance $= \int_0^2 |v(t)| dt = 1.173$

or

$$v(t) = t \sin t^2 = 0$$

$$t = 0 \text{ or } t = \sqrt{\pi} \approx 1.772$$

$$y(0) = 3; \quad y(\sqrt{\pi}) = 4; \quad y(2) = 3.826 \text{ or } 3.827$$

$$[y(\sqrt{\pi}) - y(0)] + [y(2) - y(\sqrt{\pi})]$$

$$= 1.173 \text{ or } 1.174$$

3 $\left\{ \begin{array}{l} 1: \text{limits of 0 and 2 on an integral of } v(t) \text{ or } |v(t)| \\ \text{or} \\ \text{uses } y(0) \text{ and } y(2) \text{ to compute distance} \\ 1: \text{handles change of direction at student's turning point} \\ 1: \text{answer} \\ 0/1 \text{ if incorrect turning point} \end{array} \right.$