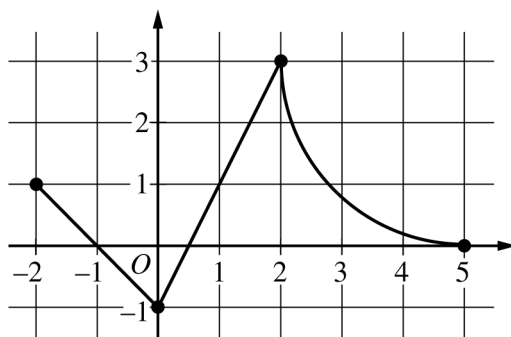


**2019 AP<sup>®</sup> CALCULUS AB FREE-RESPONSE QUESTIONS****CALCULUS AB  
SECTION II, Part B****Time—1 hour****Number of questions—4****NO CALCULATOR IS ALLOWED FOR THESE QUESTIONS.**Graph of  $f$ 

3. The continuous function  $f$  is defined on the closed interval  $-6 \leq x \leq 5$ . The figure above shows a portion of the graph of  $f$ , consisting of two line segments and a quarter of a circle centered at the point  $(5, 3)$ . It is known that the point  $(3, 3 - \sqrt{5})$  is on the graph of  $f$ .

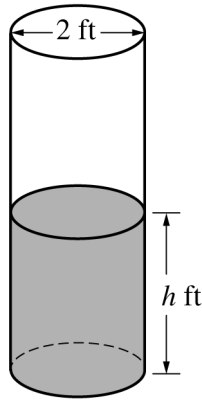
(a) If  $\int_{-6}^5 f(x) \, dx = 7$ , find the value of  $\int_{-6}^{-2} f(x) \, dx$ . Show the work that leads to your answer.

(b) Evaluate  $\int_3^5 (2f'(x) + 4) \, dx$ .

(c) The function  $g$  is given by  $g(x) = \int_{-2}^x f(t) \, dt$ . Find the absolute maximum value of  $g$  on the interval  $-2 \leq x \leq 5$ . Justify your answer.

(d) Find  $\lim_{x \rightarrow 1} \frac{10^x - 3f'(x)}{f(x) - \arctan x}$ .

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4. A cylindrical barrel with a diameter of 2 feet contains collected rainwater, as shown in the figure above. The water drains out through a valve (not shown) at the bottom of the barrel. The rate of change of the height  $h$  of the water in the barrel with respect to time  $t$  is modeled by  $\frac{dh}{dt} = -\frac{1}{10}\sqrt{h}$ , where  $h$  is measured in feet and  $t$  is measured in seconds. (The volume  $V$  of a cylinder with radius  $r$  and height  $h$  is  $V = \pi r^2 h$ .)
- (a) Find the rate of change of the volume of water in the barrel with respect to time when the height of the water is 4 feet. Indicate units of measure.
- (b) When the height of the water is 3 feet, is the rate of change of the height of the water with respect to time increasing or decreasing? Explain your reasoning.
- (c) At time  $t = 0$  seconds, the height of the water is 5 feet. Use separation of variables to find an expression for  $h$  in terms of  $t$ .
-

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**2019 SCORING GUIDELINES**

**Question 3**

(a)  $\int_{-6}^5 f(x) dx = \int_{-6}^{-2} f(x) dx + \int_{-2}^5 f(x) dx$   
 $\Rightarrow 7 = \int_{-6}^{-2} f(x) dx + 2 + \left(9 - \frac{9\pi}{4}\right)$   
 $\Rightarrow \int_{-6}^{-2} f(x) dx = 7 - \left(11 - \frac{9\pi}{4}\right) = \frac{9\pi}{4} - 4$

(b)  $\int_3^5 (2f'(x) + 4) dx = 2\int_3^5 f'(x) dx + \int_3^5 4 dx$   
 $= 2(f(5) - f(3)) + 4(5 - 3)$   
 $= 2(0 - (3 - \sqrt{5})) + 8$   
 $= 2(-3 + \sqrt{5}) + 8 = 2 + 2\sqrt{5}$

— OR —

$$\begin{aligned} \int_3^5 (2f'(x) + 4) dx &= [2f(x) + 4x]_{x=3}^{x=5} \\ &= (2f(5) + 20) - (2f(3) + 12) \\ &= (2 \cdot 0 + 20) - (2(3 - \sqrt{5}) + 12) \\ &= 2 + 2\sqrt{5} \end{aligned}$$

(c)  $g'(x) = f(x) = 0 \Rightarrow x = -1, x = \frac{1}{2}, x = 5$

$x$	$g(x)$
-2	0
-1	$\frac{1}{2}$
$\frac{1}{2}$	$-\frac{1}{4}$
5	$11 - \frac{9\pi}{4}$

On the interval  $-2 \leq x \leq 5$ , the absolute maximum value of  $g$  is  $g(5) = 11 - \frac{9\pi}{4}$ .

(d)  $\lim_{x \rightarrow 1} \frac{10^x - 3f'(x)}{f(x) - \arctan x} = \frac{10^1 - 3f'(1)}{f(1) - \arctan 1}$   
 $= \frac{10 - 3 \cdot 2}{1 - \arctan 1} = \frac{4}{1 - \frac{\pi}{4}}$

3 :  $\begin{cases} 1 : \int_{-6}^5 f(x) dx = \int_{-6}^{-2} f(x) dx + \int_{-2}^5 f(x) dx \\ 1 : \int_{-2}^5 f(x) dx \\ 1 : \text{answer} \end{cases}$

2 :  $\begin{cases} 1 : \text{Fundamental Theorem of Calculus} \\ 1 : \text{answer} \end{cases}$

3 :  $\begin{cases} 1 : g'(x) = f(x) \\ 1 : \text{identifies } x = -1 \text{ as a candidate} \\ 1 : \text{answer with justification} \end{cases}$

1 : answer