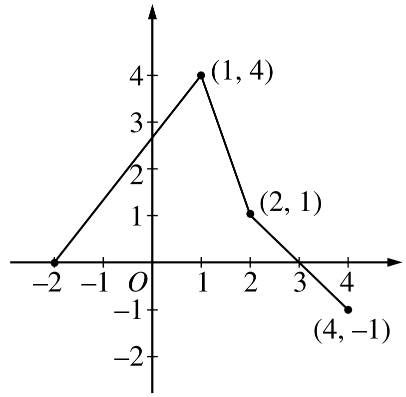

5. The graph of the function f, consisting of three line segments, is given above. Let $g(x) = \int_1^x f(t)\,dt$.

(a) Compute $g(4)$ and $g(-2)$.

(b) Find the instantaneous rate of change of g, with respect to x, at $x = 1$.

(c) Find the absolute minimum value of g on the closed interval $[-2, 4]$. Justify your answer.

(d) The second derivative of g is not defined at $x = 1$ and $x = 2$. How many of these values are x-coordinates of points of inflection of the graph of g? Justify your answer.

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6. Let f be the function whose graph goes through the point (3, 6) and whose derivative is given by $f'(x) = \dfrac{1 + e^x}{x^2}$.

   (a) Write an equation of the line tangent to the graph of f at $x = 3$ and use it to approximate $f(3.1)$.

   (b) Use Euler's method, starting at $x = 3$ with a step size of 0.05, to approximate $f(3.1)$. Use f'' to explain why this approximation is less than $f(3.1)$.

   (c) Use $\int_3^{3.1} f'(x)\,dx$ to evaluate $f(3.1)$.

END OF EXAMINATION

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5. The graph of the function f, consisting of three line segments, is given above. Let $g(x) = \int_1^x f(t)\,dt$.
   (a) Compute $g(4)$ and $g(-2)$.
   (b) Find the instantaneous rate of change of g, with respect to x, at $x = 1$.
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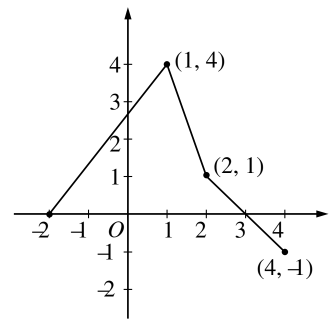
   (d) The second derivative of g is not defined at $x = 1$ and $x = 2$. How many of these values are x–coordinates of points of inflection of the graph of g? Justify your answer.



(a) $g(4) = \int_1^4 f(t)\,dt = \frac{3}{2} + 1 + \frac{1}{2} - \frac{1}{2} = \frac{5}{2}$

$g(-2) = \int_1^{-2} f(t)\,dt = -\frac{1}{2}(12) = -6$

2 $\begin{cases} 1\text{: } g(4) \\ 1\text{: } g(-2) \end{cases}$

(b) $g'(1) = f(1) = 4$

1: answer

(c) g is increasing on $[-2, 3]$ and decreasing on $[3, 4]$.

Therefore, g has absolute minimum at an endpoint of $[-2, 4]$.

Since $g(-2) = -6$ and $g(4) = \frac{5}{2}$,

the absolute minimum value is -6.

3 $\begin{cases} 1\text{: interior analysis} \\ 1\text{: endpoint analysis} \\ 1\text{: answer} \end{cases}$

(d) One; $x = 1$

On $(-2, 1)$, $g''(x) = f'(x) > 0$

On $(1, 2)$, $g''(x) = f'(x) < 0$

On $(2, 4)$, $g''(x) = f'(x) < 0$

Therefore $(1, g(1))$ is a point of inflection and $(2, g(2))$ is not.

3 $\begin{cases} 1\text{: choice of } x = 1 \text{ only} \\ 1\text{: show } (1, g(1)) \text{ is a point of inflection} \\ 1\text{: show } (2, g(2)) \text{ is not a point of inflection} \end{cases}$