

2009 AP® STATISTICS FREE-RESPONSE QUESTIONS

- (b) Write a few sentences summarizing what the display in part (a) reveals about the association between gender and job experience for the students in the sample.
- (c) Which test of significance should be used to test if there is an association between gender and job experience for the population of high school seniors in the district?

State the null and alternative hypotheses for the test, but do not perform the test.

2. A tire manufacturer designed a new tread pattern for its all-weather tires. Repeated tests were conducted on cars of approximately the same weight traveling at 60 miles per hour. The tests showed that the new tread pattern enables the cars to stop completely in an average distance of 125 feet with a standard deviation of 6.5 feet and that the stopping distances are approximately normally distributed.

- (a) What is the 70th percentile of the distribution of stopping distances?
 - (b) What is the probability that at least 2 cars out of 5 randomly selected cars in the study will stop in a distance that is greater than the distance calculated in part (a) ?
 - (c) What is the probability that a randomly selected sample of 5 cars in the study will have a mean stopping distance of at least 130 feet?
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3. Before beginning a unit on frog anatomy, a seventh-grade biology teacher gives each of the 24 students in the class a pretest to assess their knowledge of frog anatomy. The teacher wants to compare the effectiveness of an instructional program in which students physically dissect frogs with the effectiveness of a different program in which students use computer software that only simulates the dissection of a frog. After completing one of the two programs, students will be given a posttest to assess their knowledge of frog anatomy. The teacher will then analyze the changes in the test scores (score on posttest minus score on pretest).

- (a) Describe a method for assigning the 24 students to two groups of equal size that allows for a statistically valid comparison of the two instructional programs.
- (b) Suppose the teacher decided to allow the students in the class to select which instructional program on frog anatomy (physical dissection or computer simulation) they prefer to take, and 11 students choose actual dissection and 13 students choose computer simulation. How might that self-selection process jeopardize a statistically valid comparison of the changes in the test scores (score on posttest minus score on pretest) for the two instructional programs? Provide a specific example to support your answer.

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Question 2

Intent of Question

The primary goals of this question were to assess a student's ability to (1) calculate a percentile value from a normal probability distribution; (2) recognize a binomial scenario and calculate an appropriate probability; and (3) use the sampling distribution of the sample mean to find a probability for the mean of five observations.

Solution

Part (a):

Let X denote the stopping distance of a car with new tread tires where X is normally distributed with a mean of 125 feet and a standard deviation of 6.5 feet. The z-score corresponding to a cumulative probability of 70 percent is $z = 0.52$. Thus, the 70th percentile value can be computed as:

$$x = \mu_X + z\sigma_X = 125 + 0.52(6.5) = 128.4 \text{ feet.}$$

Part (b):

From part (a), it was found that a stopping distance of 128.4 feet has a cumulative probability of 0.70. Thus the probability of a stopping distance greater than 128.4 is $1 - 0.70 = 0.30$.

Let Y denote the number of cars with the new tread pattern out of five cars that stop in a distance greater than 128.4 feet. Y is a binomial random variable with $n = 5$ and $p = 0.30$.

$$\begin{aligned} P(Y \geq 2) &= 1 - P(Y \leq 1) = 1 - \left[\binom{5}{0}(0.3)^0(0.7)^5 + \binom{5}{1}(0.3)^1(0.7)^4 \right] \\ &= 1 - 0.5282 = 0.4718. \end{aligned}$$

Part (c):

Let \bar{X} denote the mean of the stopping distances of five randomly selected cars. All tires have the new tread pattern. Because the stopping distance for each of the five cars has a normal distribution, the distribution of \bar{X} is normal with a mean of 125 feet and a standard deviation of $\frac{6.5}{\sqrt{5}} = 2.91$ feet. Thus,

$$P(\bar{X} > 130) = P\left(Z > \frac{130 - 125}{6.5/\sqrt{5}}\right) \approx P(Z > 1.72) = 0.0427.$$

Scoring

Parts (a), (b), and (c) are scored as essentially correct (E), partially correct (P), or incorrect (I).

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Question 2 (continued)

Part (a) is scored as follows:

Essentially correct (E) if the student clearly indicates which distribution is being used, along with the parameters (μ_x and σ_x), and correctly calculates the percentile value with appropriate justification (except for minor arithmetic or transcription errors). There are three components: distribution, parameters, and calculation of distance.

Notes:

- The standard notation $N(125, 6.5)$ defines distribution and parameters. Also, the z-score formula setup implies distribution and parameters. This applies only in part (a), because approximate normality is given in the stem of the problem.
- If the calculator command $invNorm(0.70, 125, 6.5)$ is provided along with 128.4 feet *AND* an appropriately labeled sketch of a normal distribution is supplied, then the response should be scored as essentially correct (E). An appropriately labeled sketch must include correct labels for center and spread.
- If the calculator command $invNorm(0.70, \mu = 125, \sigma = 6.5)$ is provided along with 128.4 feet, then the response should be scored as essentially correct (E).

Partially correct (P) if the student correctly supplies only two out of the three components.

Note: If the calculator command $invNorm(0.70, 125, 6.5)$ is provided along with 128.4 feet, then the response should be scored as partially correct (P).

Incorrect (I) if the student correctly supplies at most one of the components.

Part (b) is scored as follows:

Essentially correct (E) if the student recognizes this probability as an application of the binomial distribution and sets up the problem correctly by first finding the probability for p , the probability of a success, and then using this p to find the correct binomial probability. There are three components: distribution, parameters, and calculation.

Note: If the calculator command $1\text{-binomcdf}(5, 0.3, 1)$ is provided along with 0.4718 and an identification of the distribution and its parameters—e.g., by the standard notation $B(5,0.3)$ or $\text{Bin}(5, 0.3)$ —then the response should be scored as essentially correct (E).

Partially correct (P) if the student correctly supplies only two out of the three components.

Notes:

- As long as the student identifies the distribution and parameters—e.g., by the standard notation $B(5, 0.3)$ or $\text{Bin}(5, 0.3)$ —the binomial formula does not need to be set up to receive full credit. However, the binomial formula setup can suffice for identifying two of the three components: distribution and parameters.
- If the calculator command $1\text{-binomcdf}(5, 0.3, 1)$ is provided along with 0.4718, then the response should be scored as partially correct (P).

Incorrect (I) if the student correctly supplies at most one of the components.

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Question 2 (continued)

Part (c) is scored as follows:

Essentially correct (E) if the student recognizes that the distribution of the sample mean will be approximately normal with the appropriate mean and standard deviation and calculates the probability correctly. There are three components: sampling distribution, parameters, and calculation.

Notes:

- The z-score formula setup suffices only for parameters in part (c).
- If the calculator command $\text{Normalcdf}(130, \infty, 125, 2.91)$ AND an appropriately labeled sketch of a normal distribution are provided along with the value obtained using the calculator, 0.0428, then the response should be scored as essentially correct (E). An appropriately labeled sketch must include correct labels for center and spread.
- If the calculator command $\text{Normalcdf}(130, \infty, \mu = 125, \sigma = 2.91)$ is provided along with 0.0428, then the response should be scored as essentially correct (E).

Partially correct (P) if the student correctly supplies only two out of the three components.

Note: If the calculator command $\text{Normalcdf}(130, \infty, 125, 2.91)$ is provided along with 0.0428, then the response should be scored as partially correct (P).

Incorrect (I) if the student correctly supplies at most one of the components.

Notes:

- The calculator solution is 0.0428. If this is the only information provided, the response is scored as incorrect (I).
- If a *t* distribution is used, then the response should be scored as incorrect (I).

A student should be penalized only once for using calculator syntax—that is, look at parts (a), (b), and (c) together.

4 Complete Response

All three parts essentially correct

3 Substantial Response

Two parts essentially correct and one part partially correct

2 Developing Response

Two parts essentially correct and no part partially correct

OR

One part essentially correct and one or two parts partially correct

OR

Three parts partially correct