

**2015 AP® CALCULUS AB FREE-RESPONSE QUESTIONS**

**CALCULUS AB**  
**SECTION II, Part B**  
**Time—60 minutes**  
**Number of problems—4**

**No calculator is allowed for these problems.**

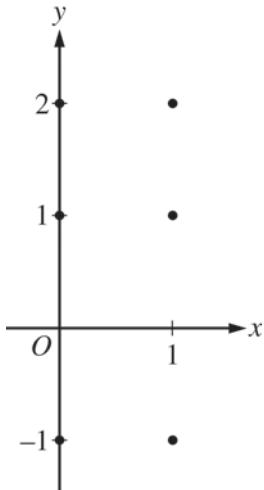
$t$ (minutes)	0	12	20	24	40
$v(t)$ (meters per minute)	0	200	240	-220	150

3. Johanna jogs along a straight path. For  $0 \leq t \leq 40$ , Johanna's velocity is given by a differentiable function  $v$ . Selected values of  $v(t)$ , where  $t$  is measured in minutes and  $v(t)$  is measured in meters per minute, are given in the table above.
- (a) Use the data in the table to estimate the value of  $v'(16)$ .
- (b) Using correct units, explain the meaning of the definite integral  $\int_0^{40} |v(t)| dt$  in the context of the problem.  
Approximate the value of  $\int_0^{40} |v(t)| dt$  using a right Riemann sum with the four subintervals indicated in the table.
- (c) Bob is riding his bicycle along the same path. For  $0 \leq t \leq 10$ , Bob's velocity is modeled by  $B(t) = t^3 - 6t^2 + 300$ , where  $t$  is measured in minutes and  $B(t)$  is measured in meters per minute.  
Find Bob's acceleration at time  $t = 5$ .
- (d) Based on the model  $B$  from part (c), find Bob's average velocity during the interval  $0 \leq t \leq 10$ .

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4. Consider the differential equation  $\frac{dy}{dx} = 2x - y$ .

(a) On the axes provided, sketch a slope field for the given differential equation at the six points indicated.



- (b) Find  $\frac{d^2y}{dx^2}$  in terms of  $x$  and  $y$ . Determine the concavity of all solution curves for the given differential equation in Quadrant II. Give a reason for your answer.
- (c) Let  $y = f(x)$  be the particular solution to the differential equation with the initial condition  $f(2) = 3$ . Does  $f$  have a relative minimum, a relative maximum, or neither at  $x = 2$ ? Justify your answer.
- (d) Find the values of the constants  $m$  and  $b$  for which  $y = mx + b$  is a solution to the differential equation.
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**AP<sup>®</sup> CALCULUS AB/CALCULUS BC  
2015 SCORING GUIDELINES**

**Question 3**

$t$ (minutes)	0	12	20	24	40
$v(t)$ (meters per minute)	0	200	240	-220	150

Johanna jogs along a straight path. For  $0 \leq t \leq 40$ , Johanna's velocity is given by a differentiable function  $v$ . Selected values of  $v(t)$ , where  $t$  is measured in minutes and  $v(t)$  is measured in meters per minute, are given in the table above.

- (a) Use the data in the table to estimate the value of  $v'(16)$ .
- (b) Using correct units, explain the meaning of the definite integral  $\int_0^{40} |v(t)| dt$  in the context of the problem.

Approximate the value of  $\int_0^{40} |v(t)| dt$  using a right Riemann sum with the four subintervals indicated in the table.

- (c) Bob is riding his bicycle along the same path. For  $0 \leq t \leq 10$ , Bob's velocity is modeled by  $B(t) = t^3 - 6t^2 + 300$ , where  $t$  is measured in minutes and  $B(t)$  is measured in meters per minute. Find Bob's acceleration at time  $t = 5$ .
- (d) Based on the model  $B$  from part (c), find Bob's average velocity during the interval  $0 \leq t \leq 10$ .

(a)  $v'(16) \approx \frac{240 - 200}{20 - 12} = 5$  meters/min<sup>2</sup>

1 : approximation

(b)  $\int_0^{40} |v(t)| dt$  is the total distance Johanna jogs, in meters, over the time interval  $0 \leq t \leq 40$  minutes.

3 :  $\begin{cases} 1 : \text{explanation} \\ 1 : \text{right Riemann sum} \\ 1 : \text{approximation} \end{cases}$

$$\begin{aligned} \int_0^{40} |v(t)| dt &\approx 12 \cdot |v(12)| + 8 \cdot |v(20)| + 4 \cdot |v(24)| + 16 \cdot |v(40)| \\ &= 12 \cdot 200 + 8 \cdot 240 + 4 \cdot 220 + 16 \cdot 150 \\ &= 2400 + 1920 + 880 + 2400 \\ &= 7600 \text{ meters} \end{aligned}$$

(c) Bob's acceleration is  $B'(t) = 3t^2 - 12t$ .  
 $B'(5) = 3(25) - 12(5) = 15$  meters/min<sup>2</sup>

2 :  $\begin{cases} 1 : \text{uses } B'(t) \\ 1 : \text{answer} \end{cases}$

(d) Avg vel =  $\frac{1}{10} \int_0^{10} (t^3 - 6t^2 + 300) dt$

$$\begin{aligned} &= \frac{1}{10} \left[ \frac{t^4}{4} - 2t^3 + 300t \right]_0^{10} \\ &= \frac{1}{10} \left[ \frac{10000}{4} - 2000 + 3000 \right] = 350 \text{ meters/min} \end{aligned}$$

3 :  $\begin{cases} 1 : \text{integral} \\ 1 : \text{antiderivative} \\ 1 : \text{answer} \end{cases}$