CALCULUS BC
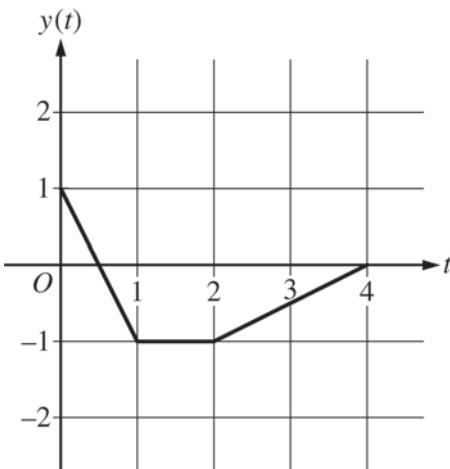**SECTION II, Part A**
Time—30 minutes
Number of problems—2

A graphing calculator is required for these problems.

t (hours)	0	1	3	6	8
$R(t)$ (liters / hour)	1340	1190	950	740	700

1. Water is pumped into a tank at a rate modeled by $W(t) = 2000e^{-t^2/20}$ liters per hour for $0 \leq t \leq 8,$ where t is measured in hours. Water is removed from the tank at a rate modeled by $R(t)$ liters per hour, where R is differentiable and decreasing on $0 \leq t \leq 8.$ Selected values of $R(t)$ are shown in the table above. At time $t = 0,$ there are 50,000 liters of water in the tank.

   (a) Estimate $R'(2).$ Show the work that leads to your answer. Indicate units of measure.

   (b) Use a left Riemann sum with the four subintervals indicated by the table to estimate the total amount of water removed from the tank during the 8 hours. Is this an overestimate or an underestimate of the total amount of water removed? Give a reason for your answer.

   (c) Use your answer from part (b) to find an estimate of the total amount of water in the tank, to the nearest liter, at the end of 8 hours.

   (d) For $0 \leq t \leq 8,$ is there a time t when the rate at which water is pumped into the tank is the same as the rate at which water is removed from the tank? Explain why or why not.

2. At time t, the position of a particle moving in the xy-plane is given by the parametric functions $(x(t), y(t))$, where $\dfrac{dx}{dt} = t^2 + \sin(3t^2)$. The graph of y, consisting of three line segments, is shown in the figure above. At $t = 0$, the particle is at position $(5, 1)$.

   (a) Find the position of the particle at $t = 3$.

   (b) Find the slope of the line tangent to the path of the particle at $t = 3$.

   (c) Find the speed of the particle at $t = 3$.

   (d) Find the total distance traveled by the particle from $t = 0$ to $t = 2$.

END OF PART A OF SECTION II

Question 1

t (hours)	0	1	3	6	8
$R(t)$ (liters / hour)	1340	1190	950	740	700

Water is pumped into a tank at a rate modeled by $W(t) = 2000e^{-t^2/20}$ liters per hour for $0 \leq t \leq 8,$ where t is measured in hours. Water is removed from the tank at a rate modeled by $R(t)$ liters per hour, where R is differentiable and decreasing on $0 \leq t \leq 8.$ Selected values of $R(t)$ are shown in the table above. At time $t = 0,$ there are 50,000 liters of water in the tank.

(a) Estimate $R'(2).$ Show the work that leads to your answer. Indicate units of measure.

(b) Use a left Riemann sum with the four subintervals indicated by the table to estimate the total amount of water removed from the tank during the 8 hours. Is this an overestimate or an underestimate of the total amount of water removed? Give a reason for your answer.

(c) Use your answer from part (b) to find an estimate of the total amount of water in the tank, to the nearest liter, at the end of 8 hours.

(d) For $0 \leq t \leq 8,$ is there a time t when the rate at which water is pumped into the tank is the same as the rate at which water is removed from the tank? Explain why or why not.

(a) $R'(2) \approx \dfrac{R(3) - R(1)}{3 - 1} = \dfrac{950 - 1190}{3 - 1} = -120$ liters/hr^2

$2 : \begin{cases} 1 : \text{estimate} \\ 1 : \text{units} \end{cases}$

(b) The total amount of water removed is given by $\int_0^8 R(t)\, dt.$

$\int_0^8 R(t)\, dt \approx 1 \cdot R(0) + 2 \cdot R(1) + 3 \cdot R(3) + 2 \cdot R(6)$

$\quad = 1(1340) + 2(1190) + 3(950) + 2(740)$

$\quad = 8050$ liters

This is an overestimate since R is a decreasing function.

$3 : \begin{cases} 1 : \text{left Riemann sum} \\ 1 : \text{estimate} \\ 1 : \text{overestimate with reason} \end{cases}$

(c) Total $\approx 50000 + \int_0^8 W(t)\, dt - 8050$

$\quad = 50000 + 7836.195325 - 8050 \approx 49786$ liters

$2 : \begin{cases} 1 : \text{integral} \\ 1 : \text{estimate} \end{cases}$

(d) $W(0) - R(0) > 0,$ $W(8) - R(8) < 0,$ and $W(t) - R(t)$ is continuous.

$2 : \begin{cases} 1 : \text{considers } W(t) - R(t) \\ 1 : \text{answer with explanation} \end{cases}$

Therefore, the Intermediate Value Theorem guarantees at least one time $t,$ $0 < t < 8,$ for which $W(t) - R(t) = 0,$ or $W(t) = R(t).$

For this value of $t,$ the rate at which water is pumped into the tank is the same as the rate at which water is removed from the tank.