4. Suppose that the function f has a continuous second derivative for all x, and that $f(0) = 2$, $f'(0) = -3$, and $f''(0) = 0$. Let g be a function whose derivative is given by $g'(x) = e^{-2x}(3f(x) + 2f'(x))$ for all x.

   (a) Write an equation of the line tangent to the graph of f at the point where $x = 0$.

   (b) Is there sufficient information to determine whether or not the graph of f has a point of inflection when $x = 0$? Explain your answer.

   (c) Given that $g(0) = 4$, write an equation of the line tangent to the graph of g at the point where $x = 0$.

   (d) Show that $g''(x) = e^{-2x}(-6f(x) - f'(x) + 2f''(x))$. Does g have a local maximum at $x = 0$? Justify your answer.

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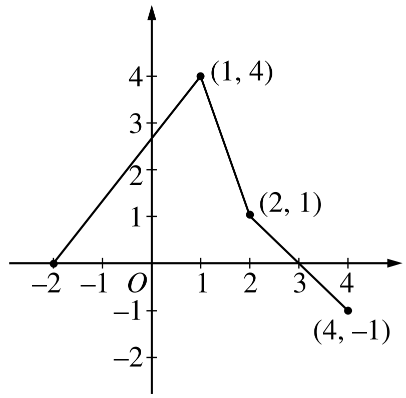

5. The graph of the function f, consisting of three line segments, is given above. Let $g(x) = \int_1^x f(t)\,dt$.

(a) Compute $g(4)$ and $g(-2)$.

(b) Find the instantaneous rate of change of g, with respect to x, at $x = 1$.

(c) Find the absolute minimum value of g on the closed interval $[-2, 4]$. Justify your answer.

(d) The second derivative of g is not defined at $x = 1$ and $x = 2$. How many of these values are x-coordinates of points of inflection of the graph of g? Justify your answer.

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4. Suppose that the function f has a continuous second derivative for all x, and that $f(0) = 2$, $f'(0) = -3$, and $f''(0) = 0$. Let g be a function whose derivative is given by $g'(x) = e^{-2x}(3f(x) + 2f'(x))$ for all x.
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(a) Slope at $x = 0$ is $f'(0) = -3$

At $x = 0$, $y = 2$

$y - 2 = -3(x - 0)$

1: equation

(b) No. Whether $f''(x)$ changes sign at $x = 0$ is unknown. The only given value of $f''(x)$ is $f''(0) = 0$.

2 $\begin{cases} 1\text{: answer} \\ 1\text{: explanation} \end{cases}$

(c) $g'(x) = e^{-2x}(3f(x) + 2f'(x))$

$$\begin{aligned} g'(0) &= e^0(3f(0) + 2f'(0)) \\ &= 3(2) + 2(-3) = 0 \end{aligned}$$

$y - 4 = 0(x - 0)$

$y = 4$

2 $\begin{cases} 1\text{: } g'(0) \\ 1\text{: equation} \end{cases}$

(d) $g'(x) = e^{-2x}(3f(x) + 2f'(x))$

$$\begin{aligned} g''(x) &= (-2e^{-2x})(3f(x) + 2f'(x)) \\ &\quad + e^{-2x}(3f'(x) + 2f''(x)) \\ &= e^{-2x}(-6f(x) - f'(x) + 2f''(x)) \end{aligned}$$

$g''(0) = e^0[(-6)(2) - (-3) + 2(0)] = -9$

Since $g'(0) = 0$ and $g''(0) < 0$, g does have a local maximum at $x = 0$.

4 $\begin{cases} 2\text{: verify derivative} \\ \quad 0/2 \text{ product or chain rule error} \\ \quad <-1> \text{ algebra errors} \\ 1\text{: } g'(0) = 0 \text{ and } g''(0) \\ 1\text{: answer and reasoning} \end{cases}$