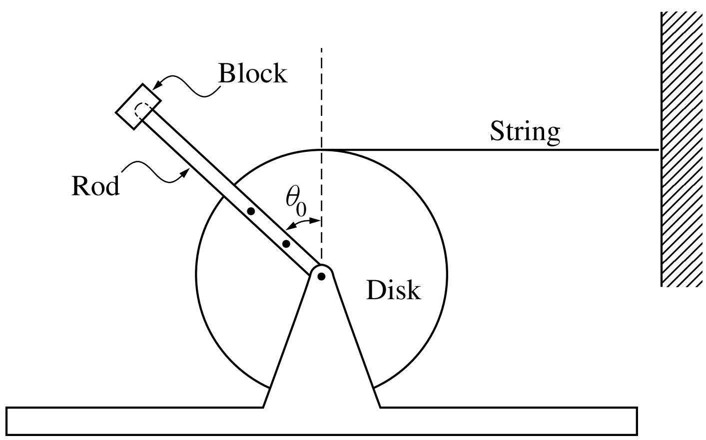
The ball is dropped into the tunnel from rest at point P at the planet's surface.

(c) Determine the work done by gravity as the ball moves from the surface to the center of the planet.

(d) Determine the speed of the ball when it reaches the center of the planet.

(e) Fully describe the subsequent motion of the ball from the time it reaches the center of the planet.

(f) Write an equation that could be used to calculate the time it takes the ball to move from point P to the center of the planet. It is not necessary to solve this equation.



Mech 3. As shown above, a uniform disk is mounted to an axle and is free to rotate without friction. A thin uniform rod is rigidly attached to the disk so that it will rotate with the disk. A block is attached to the end of the rod. Properties of the disk, rod, and block are as follows.

Disk: mass $= 3m$, radius $= R$, moment of inertia about center $I_D = \frac{3}{2}mR^2$

Rod: mass $= m$, length $= 2R$, moment of inertia about one end $I_R = \frac{4}{3}mR^2$

Block: mass $= 2m$

The system is held in equilibrium with the rod at an angle θ_0 to the vertical, as shown above, by a horizontal string of negligible mass with one end attached to the disk and the other to a wall. Express your answers to the following in terms of m, R, θ_0, and g.

(a) Determine the tension in the string.

Distribution of points

Mech. 3 (15 points)

(a) 5 points

For indicating that the net torque is zero, or that the clockwise and counterclockwise torques are equal	1 point
For a correct expression for the torque exerted by the rod	1 point
$\tau_{rod} = mgR\sin\theta_0$	
For a correct expression for the torque exerted by the block	1 point
$\tau_{block} = 2mg(2R)\sin\theta_0 = 4mgR\sin\theta_0$	
For a correct expression for the torque exerted by the string	1 point
$\tau_{string} = TR$	
For adding the counterclockwise torques and setting the sum equal to the clockwise torque (this point not awarded for just one torque)	1 point

$TR = 4mgR\sin\theta_0 + mgR\sin\theta_0$

$T = 5mg\sin\theta_0$

Only four points could be earned if the wrong trigonometric function was used.
Only three points could be earned if no trigonometric function was used.

(b)

i. 4 points

For indicating that the rotational inertia is the sum of the inertias of the disk, rod, and block	1 point
For calculating the total rotational inertia	1 point

$$I = I_{disk} + I_{rod} + I_{block}$$

$$= \frac{3}{2}mR^2 + \frac{4}{3}mR^2 + 2m(2R)^2$$

$$= \frac{65}{6}mR^2$$

$\alpha = \tau_{net}/I$

For substituting the value of torque from part (a) — 1 point

$$\alpha = \frac{5mgR\sin\theta_0}{\frac{65}{6}mR^2}$$

For an answer consistent with the values use for torque and rotational inertia — 1 point

$$\alpha = \frac{6g\sin\theta_0}{13R}$$

Mech. 3 (continued)

(b) (continued)

ii. 1 point

Expressing the linear acceleration in terms of the angular acceleration

$a = \alpha r$

For substituting the value of α and the correct radius, $2R$ — 1 point

$$a = \frac{12g\sin\theta_0}{13}$$

(c) 5 points

For indicating that energy is conserved — 1 point

For indicating that the potential energy of two bodies (the rod and the block) changes — 1 point

$\Delta U = mgh_{rod} + mgh_{block}$

For the correct expressions for these two potential energies — 1 point

$\Delta U = mgR\cos\theta_0 + 2mg(2R)\cos\theta_0$

For indicating the correct kinetic energy when the rod is horizontal — 1 point

$$K = \frac{1}{2}I\omega^2$$

Equating the kinetic and potential energies, and solving for the angular speed

$$\frac{1}{2}\left(\frac{65}{6}mR^2\right)\omega^2 = mgR\cos\theta_0 + 4mgR\cos\theta_0$$

$$\omega = \sqrt{\frac{12g\cos\theta_0}{13R}}$$

For using the relationship between linear and angular speed, and substituting ω and the correct radius, $2R$ — 1 point

$v = \omega r$

$$v = \left(\sqrt{\frac{12g\cos\theta_0}{13R}}\right)(2R) = 4\sqrt{\frac{3gR\cos\theta_0}{13}}$$

Alternate methods of solution included use of the following proper integrations

$$\omega^2 = 2\int_{\theta_0}^{\pi/2} \alpha\, d\theta$$

$$K = \int_{\theta_0}^{\pi/2} \tau\, d\theta$$