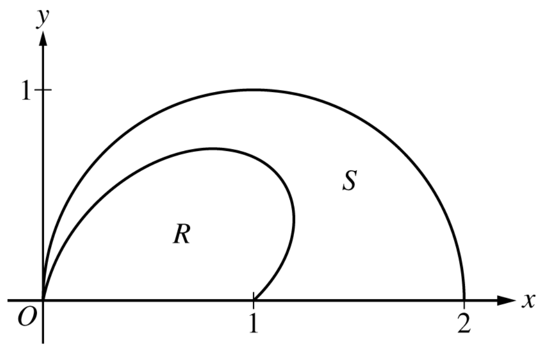


2. The figure above shows the polar curves $r = f(\theta) = 1 + \sin\theta\cos(2\theta)$ and $r = g(\theta) = 2\cos\theta$ for $0 \le \theta \le \frac{\pi}{2}$. Let $R$ be the region in the first quadrant bounded by the curve $r = f(\theta)$ and the $x$-axis. Let $S$ be the region in the first quadrant bounded by the curve $r = f(\theta)$, the curve $r = g(\theta)$, and the $x$-axis.

   (a) Find the area of $R$.

   (b) The ray $\theta = k$, where $0 < k < \frac{\pi}{2}$, divides $S$ into two regions of equal area. Write, but do not solve, an equation involving one or more integrals whose solution gives the value of $k$.

   (c) For each $\theta$, $0 \le \theta \le \frac{\pi}{2}$, let $w(\theta)$ be the distance between the points with polar coordinates $(f(\theta), \theta)$ and $(g(\theta), \theta)$. Write an expression for $w(\theta)$. Find $w_A$, the average value of $w(\theta)$ over the interval $0 \le \theta \le \frac{\pi}{2}$.

   (d) Using the information from part (c), find the value of $\theta$ for which $w(\theta) = w_A$. Is the function $w(\theta)$ increasing or decreasing at that value of $\theta$ ? Give a reason for your answer.

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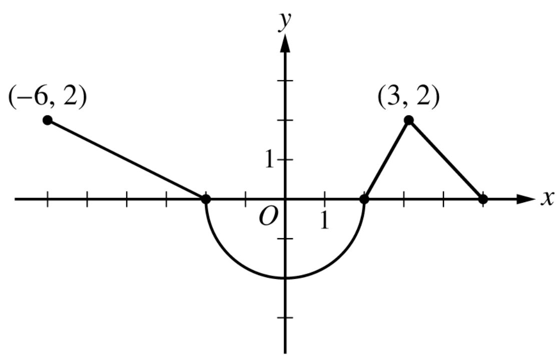
**END OF PART A OF SECTION II**

## CALCULUS BC
## SECTION II, Part B
### Time—1 hour
### Number of questions—4

**NO CALCULATOR IS ALLOWED FOR THESE QUESTIONS.**




Graph of $f'$

3. The function $f$ is differentiable on the closed interval $[-6, 5]$ and satisfies $f(-2) = 7$. The graph of $f'$, the derivative of $f$, consists of a semicircle and three line segments, as shown in the figure above.

   (a) Find the values of $f(-6)$ and $f(5)$.

   (b) On what intervals is $f$ increasing? Justify your answer.

   (c) Find the absolute minimum value of $f$ on the closed interval $[-6, 5]$. Justify your answer.

   (d) For each of $f''(-5)$ and $f''(3)$, find the value or explain why it does not exist.

**Question 2**

| | |
|---|---|
| (a) $\frac{1}{2}\int_0^{\pi/2} (f(\theta))^2 \, d\theta = 0.648414$<br><br>The area of $R$ is 0.648. | $2 : \begin{cases} 1 : \text{integral} \\ 1 : \text{answer} \end{cases}$ |
| (b) $\int_0^k \left((g(\theta))^2 - (f(\theta))^2\right) d\theta = \frac{1}{2}\int_0^{\pi/2} \left((g(\theta))^2 - (f(\theta))^2\right) d\theta$<br><br>— OR —<br><br>$\int_0^k \left((g(\theta))^2 - (f(\theta))^2\right) d\theta = \int_k^{\pi/2} \left((g(\theta))^2 - (f(\theta))^2\right) d\theta$ | $2 : \begin{cases} 1 : \text{integral expression for one region} \\ 1 : \text{equation} \end{cases}$ |
| (c) $w(\theta) = g(\theta) - f(\theta)$<br><br>$w_A = \dfrac{\int_0^{\pi/2} w(\theta) \, d\theta}{\frac{\pi}{2} - 0} = 0.485446$<br><br>The average value of $w(\theta)$ on the interval $\left[0, \frac{\pi}{2}\right]$ is 0.485. | $3 : \begin{cases} 1 : w(\theta) \\ 1 : \text{integral} \\ 1 : \text{average value} \end{cases}$ |
| (d) $w(\theta) = w_A$ for $0 \le \theta \le \frac{\pi}{2} \Rightarrow \theta = 0.517688$<br><br>$w(\theta) = w_A$ at $\theta = 0.518$ (or 0.517).<br><br>$w'(0.518) < 0 \Rightarrow w(\theta)$ is decreasing at $\theta = 0.518$. | $2 : \begin{cases} 1 : \text{solves } w(\theta) = w_A \\ 1 : \text{answer with reason} \end{cases}$ |