

2007 AP® CALCULUS AB FREE-RESPONSE QUESTIONS

**CALCULUS AB
SECTION II, Part B
Time—45 minutes
Number of problems—3**

No calculator is allowed for these problems.

4. A particle moves along the x -axis with position at time t given by $x(t) = e^{-t} \sin t$ for $0 \leq t \leq 2\pi$.
- Find the time t at which the particle is farthest to the left. Justify your answer.
 - Find the value of the constant A for which $x(t)$ satisfies the equation $Ax''(t) + x'(t) + x(t) = 0$ for $0 < t < 2\pi$.
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t (minutes)	0	2	5	7	11	12
$r'(t)$ (feet per minute)	5.7	4.0	2.0	1.2	0.6	0.5

5. The volume of a spherical hot air balloon expands as the air inside the balloon is heated. The radius of the balloon, in feet, is modeled by a twice-differentiable function r of time t , where t is measured in minutes. For $0 < t < 12$, the graph of r is concave down. The table above gives selected values of the rate of change, $r'(t)$, of the radius of the balloon over the time interval $0 \leq t \leq 12$. The radius of the balloon is 30 feet when $t = 5$.
(Note: The volume of a sphere of radius r is given by $V = \frac{4}{3}\pi r^3$.)
- Estimate the radius of the balloon when $t = 5.4$ using the tangent line approximation at $t = 5$. Is your estimate greater than or less than the true value? Give a reason for your answer.
 - Find the rate of change of the volume of the balloon with respect to time when $t = 5$. Indicate units of measure.
 - Use a right Riemann sum with the five subintervals indicated by the data in the table to approximate $\int_0^{12} r'(t) dt$. Using correct units, explain the meaning of $\int_0^{12} r'(t) dt$ in terms of the radius of the balloon.
 - Is your approximation in part (c) greater than or less than $\int_0^{12} r'(t) dt$? Give a reason for your answer.
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WRITE ALL WORK IN THE PINK EXAM BOOKLET.

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6. Let f be the function defined by $f(x) = k\sqrt{x} - \ln x$ for $x > 0$, where k is a positive constant.
- Find $f'(x)$ and $f''(x)$.
 - For what value of the constant k does f have a critical point at $x = 1$? For this value of k , determine whether f has a relative minimum, relative maximum, or neither at $x = 1$. Justify your answer.
 - For a certain value of the constant k , the graph of f has a point of inflection on the x -axis. Find this value of k .
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END OF EXAM

**AP[®] CALCULUS AB
2007 SCORING GUIDELINES**

Question 5

t (minutes)	0	2	5	7	11	12
$r'(t)$ (feet per minute)	5.7	4.0	2.0	1.2	0.6	0.5

The volume of a spherical hot air balloon expands as the air inside the balloon is heated. The radius of the balloon, in feet, is modeled by a twice-differentiable function r of time t , where t is measured in minutes. For $0 < t < 12$, the graph of r is concave down. The table above gives selected values of the rate of change, $r'(t)$, of the radius of the balloon over the time interval $0 \leq t \leq 12$. The radius of the balloon is 30 feet when

$t = 5$. (Note: The volume of a sphere of radius r is given by $V = \frac{4}{3}\pi r^3$.)

- (a) Estimate the radius of the balloon when $t = 5.4$ using the tangent line approximation at $t = 5$. Is your estimate greater than or less than the true value? Give a reason for your answer.
- (b) Find the rate of change of the volume of the balloon with respect to time when $t = 5$. Indicate units of measure.
- (c) Use a right Riemann sum with the five subintervals indicated by the data in the table to approximate $\int_0^{12} r'(t) dt$. Using correct units, explain the meaning of $\int_0^{12} r'(t) dt$ in terms of the radius of the balloon.
- (d) Is your approximation in part (c) greater than or less than $\int_0^{12} r'(t) dt$? Give a reason for your answer.

(a) $r(5.4) \approx r(5) + r'(5)\Delta t = 30 + 2(0.4) = 30.8$ ft
Since the graph of r is concave down on the interval $5 < t < 5.4$, this estimate is greater than $r(5.4)$.

(b) $\frac{dV}{dt} = 3\left(\frac{4}{3}\right)\pi r^2 \frac{dr}{dt}$

$$\left.\frac{dV}{dt}\right|_{t=5} = 4\pi(30)^2 2 = 7200\pi \text{ ft}^3/\text{min}$$

(c) $\int_0^{12} r'(t) dt \approx 2(4.0) + 3(2.0) + 2(1.2) + 4(0.6) + 1(0.5)$
 $= 19.3$ ft
 $\int_0^{12} r'(t) dt$ is the change in the radius, in feet, from $t = 0$ to $t = 12$ minutes.

(d) Since r is concave down, r' is decreasing on $0 < t < 12$. Therefore, this approximation, 19.3 ft, is less than $\int_0^{12} r'(t) dt$.

Units of ft^3/min in part (b) and ft in part (c)

2 : $\begin{cases} 1 : \text{estimate} \\ 1 : \text{conclusion with reason} \end{cases}$

3 : $\begin{cases} 2 : \frac{dV}{dt} \\ 1 : \text{answer} \end{cases}$

2 : $\begin{cases} 1 : \text{approximation} \\ 1 : \text{explanation} \end{cases}$

1 : conclusion with reason

1 : units in (b) and (c)