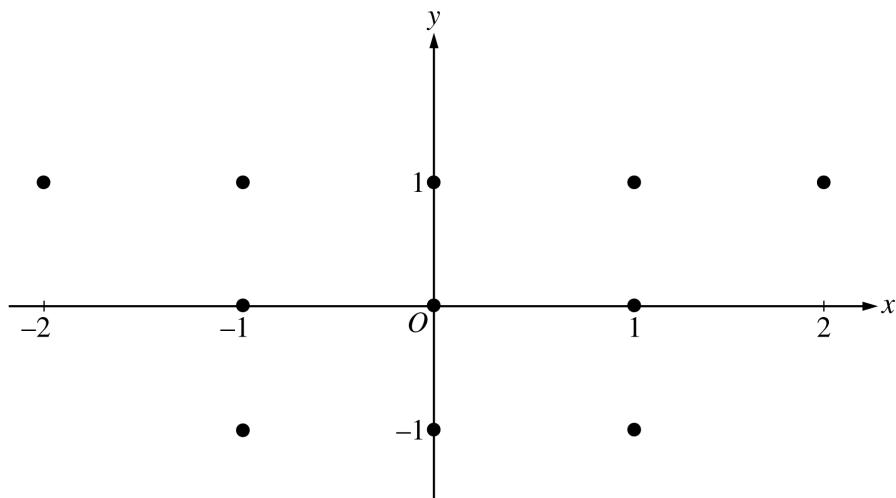


2000 AP® CALCULUS BC FREE-RESPONSE QUESTIONS

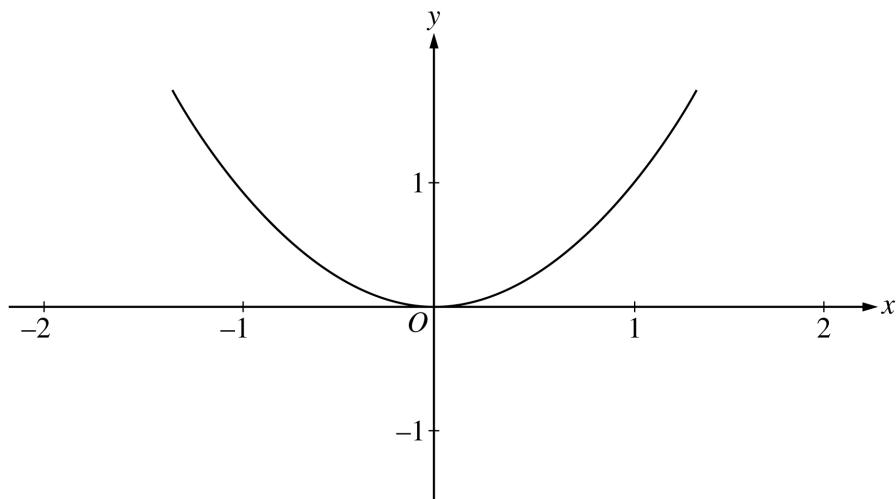
6. Consider the differential equation given by $\frac{dy}{dx} = x(y - 1)^2$.

(a) On the axes provided, sketch a slope field for the given differential equation at the eleven points indicated.

(Note: Use the axes provided in the pink test booklet.)



(b) Use the slope field for the given differential equation to explain why a solution could not have the graph shown below.



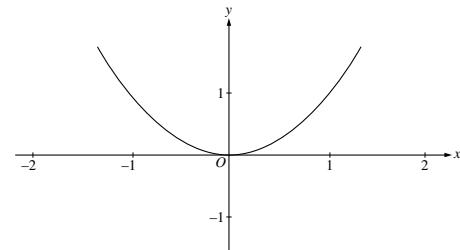
(c) Find the particular solution $y = f(x)$ to the given differential equation with the initial condition $f(0) = -1$.

(d) Find the range of the solution found in part (c).

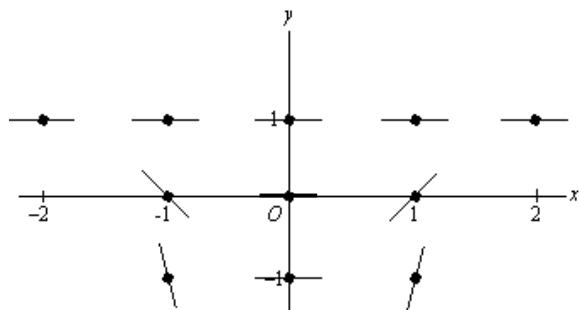
END OF EXAMINATION

Consider the differential equation given by $\frac{dy}{dx} = x(y - 1)^2$.

- On the axes provided, sketch a slope field for the given differential equation at the eleven points indicated.
- Use the slope field for the given differential equation to explain why a solution could not have the graph shown below.
- Find the particular solution $y = f(x)$ to the given differential equation with the initial condition $f(0) = -1$.
- Find the range of the solution found in part (c).



(a)



- | | |
|-----|--------------------------------------------------------------------------------------------------------------------------------------|
| 1 : | zero slope at 7 points with $y = 1$ and $x = 0$ |
| 2 : | 1 : negative slope at $(-1, 0)$ and $(-1, -1)$ positive slope at $(1, 0)$ and $(1, -1)$ steeper slope at $y = -1$ than $y = 0$ |

(b) The graph does not have slope 0 where $y = 1$.

– or –

The slope field shown suggests that solutions are asymptotic to $y = 1$ from below, but the graph does not exhibit this behavior.

1 : reason

$$(c) \frac{1}{(y-1)^2} dy = x dx$$

$$-\frac{1}{y-1} = \frac{1}{2}x^2 + C$$

$$\frac{1}{2} = 0 + C; \quad C = \frac{1}{2}$$

$$-\frac{1}{y-1} = \frac{1}{2}(x^2 + 1); \quad y = 1 - \frac{2}{x^2 + 1}$$

- | | |
|-----|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| 5 : | 1 : separates variables 1 : antiderivatives 1 : constant of integration 1 : uses initial condition $f(0) = -1$ 1 : solves for y 0/1 if y is linear |
|-----|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------|

Note: max 2/5 [1-1-0-0-0] if no constant of integration

Note: 0/5 if no separation of variables

(d) range is $-1 \leq y < 1$

1 : answer

0/1 if -1 not in range