3. A particle moves along the y-axis so that its velocity v at time $t \geq 0$ is given by $v(t) = 1 - \tan^{-1}\left(e^t\right)$. At time $t = 0$, the particle is at $y = -1$. (Note: $\tan^{-1} x = \arctan x$)

   (a) Find the acceleration of the particle at time $t = 2$.

   (b) Is the speed of the particle increasing or decreasing at time $t = 2$? Give a reason for your answer.

   (c) Find the time $t \geq 0$ at which the particle reaches its highest point. Justify your answer.

   (d) Find the position of the particle at time $t = 2$. Is the particle moving toward the origin or away from the origin at time $t = 2$? Justify your answer.

END OF PART A OF SECTION II

CALCULUS AB
SECTION II, Part B
Time—45 minutes
Number of problems—3

No calculator is allowed for these problems.

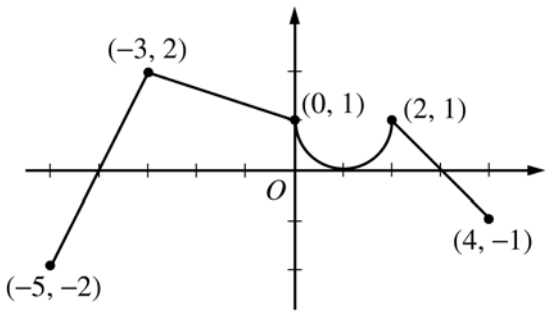
---

4. Consider the curve given by $x^2 + 4y^2 = 7 + 3xy$.

   (a) Show that $\frac{dy}{dx} = \frac{3y - 2x}{8y - 3x}$.

   (b) Show that there is a point P with x-coordinate 3 at which the line tangent to the curve at P is horizontal. Find the y-coordinate of P.

   (c) Find the value of $\frac{d^2y}{dx^2}$ at the point P found in part (b). Does the curve have a local maximum, a local minimum, or neither at the point P ? Justify your answer.



Graph of f

5. The graph of the function f shown above consists of a semicircle and three line segments. Let g be the function given by $g(x) = \int_{-3}^{x} f(t)\,dt$.

   (a) Find $g(0)$ and $g'(0)$.

   (b) Find all values of x in the open interval $(-5, 4)$ at which g attains a relative maximum. Justify your answer.

   (c) Find the absolute minimum value of g on the closed interval $[-5, 4]$. Justify your answer.

   (d) Find all values of x in the open interval $(-5, 4)$ at which the graph of g has a point of inflection.

GO ON TO THE NEXT PAGE.

AP® CALCULUS AB
2004 SCORING GUIDELINES

Question 3

A particle moves along the y-axis so that its velocity v at time $t \geq 0$ is given by $v(t) = 1 - \tan^{-1}\left(e^t\right)$.

At time $t = 0$, the particle is at $y = -1$. (Note: $\tan^{-1} x = \arctan x$)

(a) Find the acceleration of the particle at time $t = 2$.

(b) Is the speed of the particle increasing or decreasing at time $t = 2$? Give a reason for your answer.

(c) Find the time $t \geq 0$ at which the particle reaches its highest point. Justify your answer.

(d) Find the position of the particle at time $t = 2$. Is the particle moving toward the origin or away from the origin at time $t = 2$? Justify your answer.

(a) $a(2) = v'(2) = -0.132$ or -0.133

1 : answer

(b) $v(2) = -0.436$
Speed is increasing since $a(2) < 0$ and $v(2) < 0$.

1 : answer with reason

(c) $v(t) = 0$ when $\tan^{-1}\left(e^t\right) = 1$
$t = \ln(\tan(1)) = 0.443$ is the only critical value for y.

$v(t) > 0$ for $0 < t < \ln(\tan(1))$
$v(t) < 0$ for $t > \ln(\tan(1))$

$y(t)$ has an absolute maximum at $t = 0.443$.

3 : { 1 : sets $v(t) = 0$; 1 : identifies $t = 0.443$ as a candidate; 1 : justifies absolute maximum }

(d) $y(2) = -1 + \int_0^2 v(t)\,dt = -1.360$ or -1.361
The particle is moving away from the origin since $v(2) < 0$ and $y(2) < 0$.

4 : { 1 : $\int_0^2 v(t)\,dt$; 1 : handles initial condition; 1 : value of $y(2)$; 1 : answer with reason }