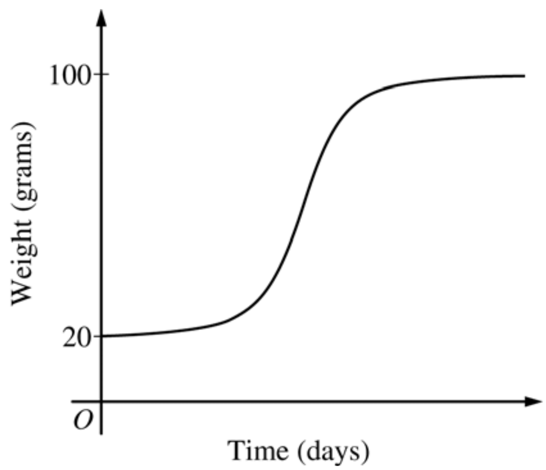
5. The rate at which a baby bird gains weight is proportional to the difference between its adult weight and its current weight. At time $t = 0,$ when the bird is first weighed, its weight is 20 grams. If $B(t)$ is the weight of the bird, in grams, at time t days after it is first weighed, then

$$\frac{dB}{dt} = \frac{1}{5}(100 - B).$$

Let $y = B(t)$ be the solution to the differential equation above with initial condition $B(0) = 20.$

(a) Is the bird gaining weight faster when it weighs 40 grams or when it weighs 70 grams? Explain your reasoning.

(b) Find $\frac{d^2B}{dt^2}$ in terms of B. Use $\frac{d^2B}{dt^2}$ to explain why the graph of B cannot resemble the following graph.



(c) Use separation of variables to find $y = B(t),$ the particular solution to the differential equation with initial condition $B(0) = 20.$

© 2012 The College Board.
Visit the College Board on the Web: www.collegeboard.org.

6. For $0 \le t \le 12$, a particle moves along the x-axis. The velocity of the particle at time t is given by $v(t) = \cos\left(\frac{\pi}{6}t\right)$. The particle is at position $x = -2$ at time $t = 0$.

   (a) For $0 \le t \le 12$, when is the particle moving to the left?

   (b) Write, but do not evaluate, an integral expression that gives the total distance traveled by the particle from time $t = 0$ to time $t = 6$.

   (c) Find the acceleration of the particle at time t. Is the speed of the particle increasing, decreasing, or neither at time $t = 4$? Explain your reasoning.

   (d) Find the position of the particle at time $t = 4$.

STOP

END OF EXAM

© 2012 The College Board.
Visit the College Board on the Web: www.collegeboard.org.

AP® CALCULUS AB
2012 SCORING GUIDELINES

Question 5

The rate at which a baby bird gains weight is proportional to the difference between its adult weight and its current weight. At time $t = 0,$ when the bird is first weighed, its weight is 20 grams. If $B(t)$ is the weight of the bird, in grams, at time t days after it is first weighed, then

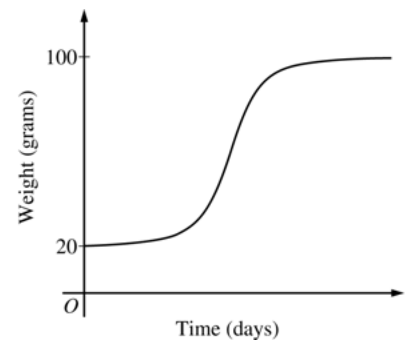
$$\frac{dB}{dt} = \frac{1}{5}(100 - B).$$

Let $y = B(t)$ be the solution to the differential equation above with initial condition $B(0) = 20.$

(a) Is the bird gaining weight faster when it weighs 40 grams or when it weighs 70 grams? Explain your reasoning.

(b) Find $\dfrac{d^2B}{dt^2}$ in terms of B. Use $\dfrac{d^2B}{dt^2}$ to explain why the graph of B cannot resemble the following graph.

(c) Use separation of variables to find $y = B(t),$ the particular solution to the differential equation with initial condition $B(0) = 20.$



(a) $\left.\dfrac{dB}{dt}\right|_{B=40} = \dfrac{1}{5}(60) = 12$

$\left.\dfrac{dB}{dt}\right|_{B=70} = \dfrac{1}{5}(30) = 6$

Because $\left.\dfrac{dB}{dt}\right|_{B=40} > \left.\dfrac{dB}{dt}\right|_{B=70},$ the bird is gaining weight faster when it weighs 40 grams.

$2 : \begin{cases} 1 : \text{uses } \dfrac{dB}{dt} \\ 1 : \text{answer with reason} \end{cases}$

(b) $\dfrac{d^2B}{dt^2} = -\dfrac{1}{5}\dfrac{dB}{dt} = -\dfrac{1}{5}\cdot\dfrac{1}{5}(100 - B) = -\dfrac{1}{25}(100 - B)$

Therefore, the graph of B is concave down for $20 \le B < 100.$ A portion of the given graph is concave up.

$2 : \begin{cases} 1 : \dfrac{d^2B}{dt^2} \text{ in terms of } B \\ 1 : \text{explanation} \end{cases}$

(c) $\dfrac{dB}{dt} = \dfrac{1}{5}(100 - B)$

$\displaystyle\int \frac{1}{100 - B}\,dB = \int \frac{1}{5}\,dt$

$-\ln|100 - B| = \dfrac{1}{5}t + C$

Because $20 \le B < 100,$ $|100 - B| = 100 - B.$

$-\ln(100 - 20) = \dfrac{1}{5}(0) + C \;\Rightarrow\; -\ln(80) = C$

$100 - B = 80e^{-t/5}$

$B(t) = 100 - 80e^{-t/5}, \quad t \ge 0$

$5 : \begin{cases} 1 : \text{separation of variables} \\ 1 : \text{antiderivatives} \\ 1 : \text{constant of integration} \\ 1 : \text{uses initial condition} \\ 1 : \text{solves for } B \end{cases}$

Note: max $2/5$ [1-1-0-0-0] if no constant of integration

Note: $0/5$ if no separation of variables

© 2012 The College Board.
Visit the College Board on the Web: www.collegeboard.org.