equivalences

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1 Equivalences

$$(1) \Leftrightarrow \forall \varepsilon > 0, \lim_{n \to \infty} Pr(|X_n - Y| < \varepsilon) = 1$$

$$(4) \Leftrightarrow \forall \varepsilon > 0, \lim_{n \to \infty} Pr(|X_n - Y| \leq \varepsilon) = 1$$

2 Proof that $(1) \Rightarrow (4)$

$$\lim_{n \to \infty} Pr(|X_n - Y| < \varepsilon) \leq \lim_{n \to \infty} Pr(|X_n - Y| \leq \varepsilon)$$

$$\lim_{n \to \infty} Pr(|X_n - Y| < \varepsilon) = 1$$

We thus have

$$1 <= \lim_{n \to \infty} Pr(|X_n - Y| \leq \varepsilon) <= 1$$

So

$$\lim_{n \to \infty} Pr(|X_n - Y| \leq \varepsilon) = 1$$

QED

3 Proof that $(4) \Rightarrow (1)$