Information and coding theory assignment 2

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1 $H(X, Y|Z)$

$$H(X, Y|Z) = -\sum_{x,y,z} \mathcal{P}(x, y, z) \log(\mathcal{P}(x, y|z))$$

$$H(X, Y|Z) = -\sum_{x,y,z} \mathcal{P}(x, y, z) \log(\frac{\mathcal{P}(x, y, z)}{\mathcal{P}(z)})$$

$$H(X, Y|Z) = -(1/4 \log(\frac{1/4}{1/2}) + 1/4 \log(\frac{1/4}{1/2}) + 1/4 \log(\frac{1/4}{1/2}) + 1/4 \log(\frac{1/4}{1/2}) + 0 + 0 + 0 + 0)$$

$$H(X, Y|Z) = -\log(\frac{1/4}{1/2})$$

$$H(X, Y|Z) = \log(2)$$

2 $H(X, Y|Z = 0)$

$$H(X, Y|Z = 0) = -\sum_{x,y,z=0} \mathcal{P}(x, y, z = 0) \log(\mathcal{P}(x, y|z = 0))$$

$$H(X, Y|Z = 0) = -\sum_{x,y,z=0} \mathcal{P}(x, y, z = 0) \log(\frac{\mathcal{P}(x, y, z = 0)}{\mathcal{P}(z = 0)})$$

$$H(X, Y|Z = 0) = -(1/4 \log(\frac{1/4}{1/2}) + 1/4 \log(\frac{1/4}{1/2}) + 0 + 0)$$

$$H(X, Y|Z = 0) = -1/2 \log(\frac{1/4}{1/2})$$

$$H(X, Y|Z = 0) = 1/2 \log(2)$$

3 $I(X, Y|Z)$

$$I(X, Y|Z) = \sum_{x,y,z} \mathcal{P}(x, y, z) \log(\frac{\mathcal{P}(x, y|z)}{\mathcal{P}(x|z)\mathcal{P}(y|z)})$$

$$I(X, Y|Z) = 4 * 1/4 \log(\frac{1/2}{1/4 * 1/4})$$

$$I(X, Y|Z) = 3$$

4 $I(X, Y|Z = 0)$

$$I(X, Y|Z = 0) = \sum_{x,y,z=0} \mathcal{P}(x, y, z = 0) \log(\frac{\mathcal{P}(x, y|z = 0)}{\mathcal{P}(x|z = 0)\mathcal{P}(y|z = 0)})$$

$$I(X, Y|Z = 0) = 2 * 1/4 \log(\frac{1/2}{1/4 * 1/4})$$

$$I(X, Y|Z) = 3/2$$

5 Proof of $I(X, Y) = 0$

$$I(X, Y) = \sum_{x,y} \mathcal{P}(x, y) \log(\frac{\mathcal{P}(x, y)}{\mathcal{P}(x)\mathcal{P}(y)})$$

$$I(X, Y) = 4 * 1/4 \log(\frac{1/4}{1/2 * 1/2})$$

$$I(X, Y) = \log(1)$$

$$I(X, Y) = 0$$

QED