Information and coding theory assignment 3

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1 Inequalities

Given $U \to X \to Y \to V$ a Markov chain, we can find :

$$I(U,V) \leq I(U,Y)$$

$$I(U,V) \leq I(X,V)$$

$$I(U,V) \leq I(U,X)$$

$$I(U,V) \leq I(X,Y)$$

$$I(U,V) \leq I(Y,V)$$

$$I(U,Y) \leq I(U,X)$$

$$I(U,Y) \leq I(X,Y)$$

$$I(X,V) \leq I(X,Y)$$

$$I(X,V) \leq I(Y,V)$$

2 Fano's inequality

3 Kraft's inequality

We want to prove that for a uniquely decodable code, then $\sum_{k=1}^{m} D^{-l_k} \leq 1$ With D the size of the size of the alphabet, m the number of words, and l_k the length of the k^{th} word.

For the code to be uniquely decodable, it can't have a code that is the beginning of a following code. For a D-ary code, this means that if there are D words of code length l, there can't be words of length superior to l.

Thus, for the D-ary code to be uniquely decodable, we can at most have $D-1$ words of code length $l < l_{max}$, and D words of code length l_{max}.

This means that we have :

$$\sum_{k=1}^{m} D^{-l_k} = (D-1)\sum_{i=1}^{l_{max-1}} D^{-i} + D * D^{-l_{max}}$$

$$\sum_{k=1}^{m} D^{-l_k} = (D-1)\sum_{i=1}^{l_{max}} D^{-i} + D^{-l_{max}}$$

$$\sum_{k=1}^{m} D^{-l_k} = (D-1)\frac{D^{-l_{max}}(D^{l_{max}-1})}{D-1} + D^{-l_{max}}$$

$$\sum_{k=1}^{m} D^{-l_k} = 1$$

Since this is at the most, a code is uniquely decodable if

$$\sum_{k=1}^{m} D^{-l_k} \leq 1$$

QED