Using probability distributions in R: dnorm, pnorm, qnorm, and rnorm

Basic Statistical Concepts for Data Science 1 October 29, 2018

Distribution functions in R

Every distribution has four associated functions whose prex indicates the type of function and the sux indicates the distribution. To exemplify the use of these functions, I will limit myself to the normal (Gaussian) distribution. The four normal distribution functions are:

dnorm: **density function** of the normal distribution

pnorm: **cumulative density function** of the normal distribution

qnorm: **quantile function** of the normal distribution **r**norm: **random sampling** from the normal distribution

The probability density function: dnorm

The probability density function (PDF, in short: density) indicates the probability of observing a measurement with a specic value and x thus the integral over the density is always 1. For a value , the normal density is dened as

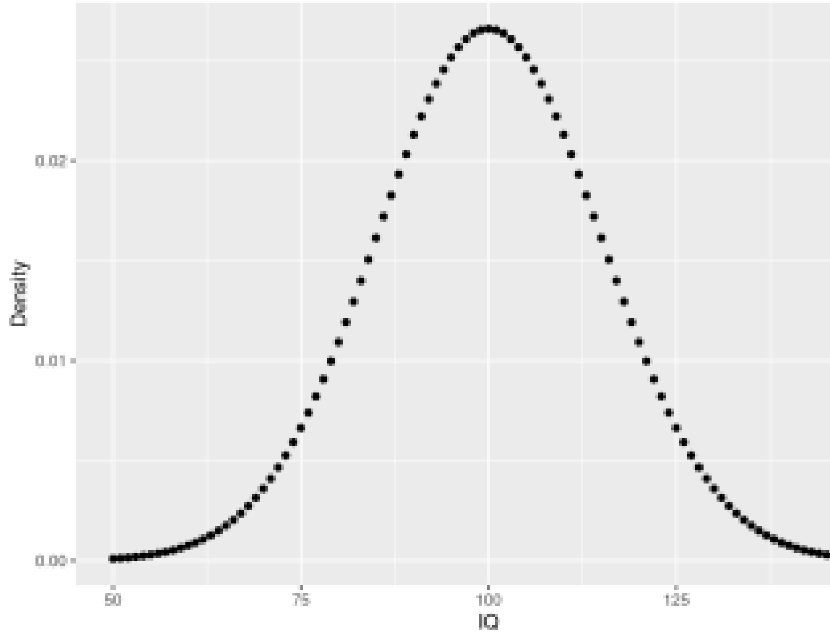
$$f(x \mid \mu, \sigma^2) = \frac{1}{\dots} \exp\left(-\frac{(x-\mu)^2}{\dots}\right)$$

$2\pi\sigma$ $\sqrt{}_2$ Other

$2\sigma^2$

μ σ σ^2
where is the mean, is the standard deviation, and is the variance.

Using the density, it is possible to determine the probabilities of events. For example, you may wonder: *What is the likelihood that a person has an IQ of exactly 140?*. In this case, you would need to retrieve the density of the IQ distribution at value 140. The IQ distribution can be modeled with a mean of 100 and a standard deviation of 15. The corresponding density is:




```
sample.range <- 50:150
iq.mean <- 100
iq.sd <- 15
iq.dist <- dnorm(sample.range, mean =
iq.mean, sd = iq.sd iq.df <-
data.frame("IQ" = sample.range, "Density" =
iq.d library(ggplot2)
ggplot(iq.df, aes(x = IQ, y = Density)) +


geom_point()
```

From these data, we can now answer the initial question as well as additional questions:

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```
pp <- function(x) {
 print(paste0(round(x * 100, 3), "%"))
}
# likelihood of IQ == 140?
pp(iq.df$Density[iq.df$IQ == 140])
```

```
## [1] "0.076%"
```

```
# likelihood of IQ >= 140?
pp(sum(iq.df$Density[iq.df$IQ >= 140]))
```

```
## [1] "0.384%"
```

```
# likelihood of 50 < IQ <= 90?
pp(sum(iq.df$Density[iq.df$IQ <= 90]))
```

```
## [1] "26.284%"
```

The cumulative density function: pnorm

The cumulative density (CDF) function is a monotonically increasing function as it integrates over densities via

$$1 \qquad x - \mu$$

$$f(x|\mu, \sigma) = [1 + \mathrm{erf}(-)]$$

$$2 \qquad \sigma\sqrt{2}$$

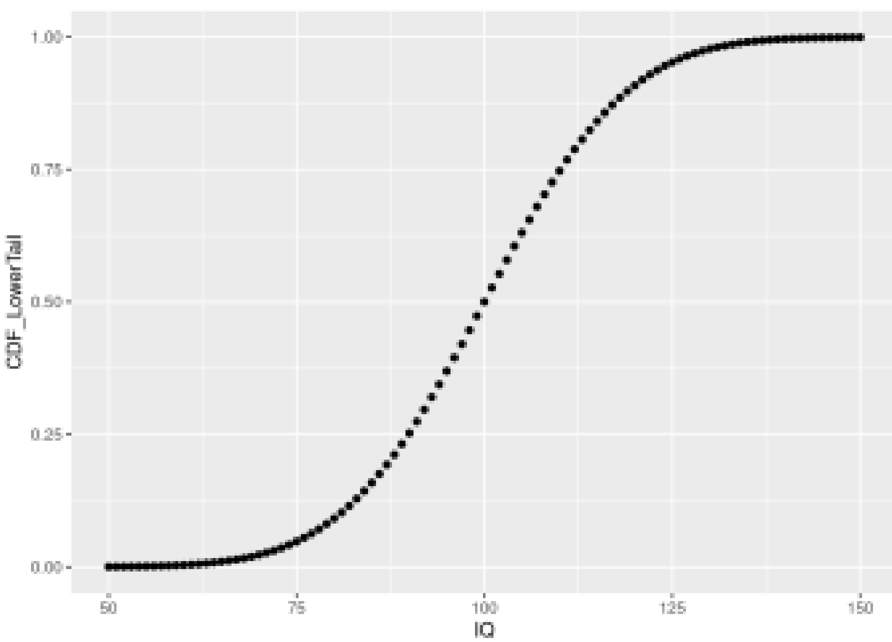
$$\sqrt{\pi}\int_{-x}^{x} e^{-t^2} 1$$

$$\mathrm{erf}(x) = dt$$

where is the error function. To get an intuition of the CDF, let’s

create a plot for the IQ data:

```
cdf <- pnorm(sample.range, iq.mean, iq.sd)
iq.df <- cbind(iq.df, "CDF_LowerTail" = cdf)
ggplot(iq.df, aes(x = IQ, y = CDF_LowerTail)) + geom_poi
```



As we can see, the depicted CDF shows the probability of having an IQ less or equal to a given value. This is because `pnorm` computes

$$P[X <= x]$$

the lower tail by default, i.e. . Using this knowledge, we can obtain answers to some of our previous questions in a slightly dierent manner:

```
# likelihood of 50 < IQ <= 90?
pp(iq.df$CDF_LowerTail[iq.df$IQ == 90])
```

```
## [1] "25.249%"
```

```
# set lower.tail to FALSE to obtain P[X >= x] cdf <-
pnorm(sample.range, iq.mean, iq.sd, lower.tail =  iq.df
<- cbind(iq.df, "CDF_UpperTail" = cdf)
# Probability for IQ >= 140? same value as before using d
pp(iq.df$CDF_UpperTail[iq.df$IQ == 140])
## [1] "0.383%"
```

Note that the results from *pnorm* are the same as those obtained from manually summing up the probabilities obtained via *dnorm*. Moreover, by setting `lower.tail = FALSE` , `dnorm` can be used to directly compute p-values, which measure how the likelihood of an observation that is at least as extreme as the obtained one.

To remember that `pnorm` does not provide the PDF but the CDF, just imagine that the function carries a *p* in its name such that *pnorm* is lexicographically close to *qnorm*, which provides the inverse of the CDF.

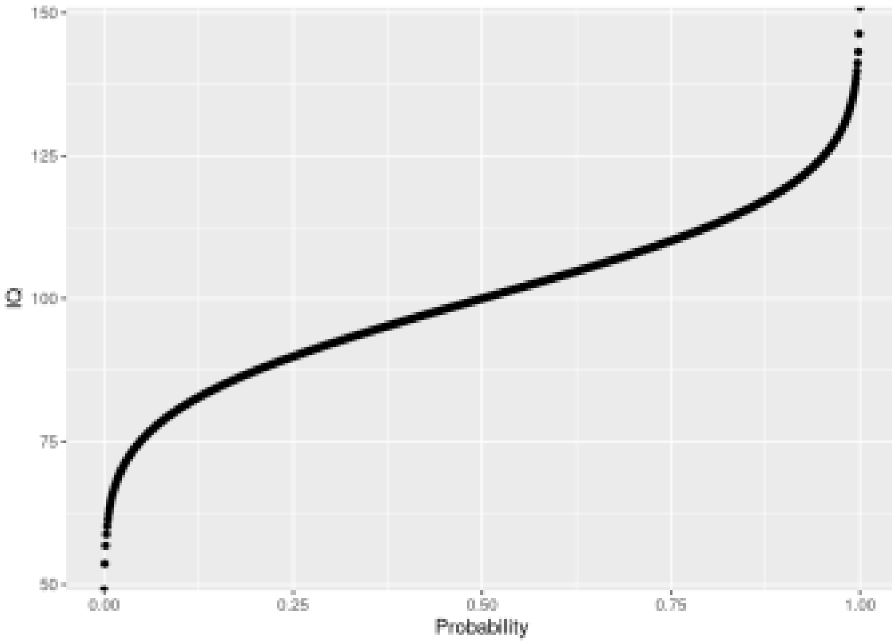
The quantile function: qnorm

The quantile function is simply the inverse of the cumulative density function (iCDF). Thus, the quantile function maps from probabilities

P[X <= x]

to values. Let's take a look at the quantile function for :

```
# input to qnorm is a vector of probabilities
prob.range <- seq(0, 1, 0.001)
icdf.df <- data.frame("Probability" = prob.range, "IQ" =
ggplot(icdf.df, aes(x = Probability, y = IQ)) + geom_poi
```



Using the quantile function, we can answer quantile-related questions:

```
# what is the 25th IQ percentile?
print(icdf.df$IQ[icdf.df$Probability == 0.25])
```

```
## [1] 89.88265
```

```
# what is the 75 IQ percentile?
print(icdf.df$IQ[icdf.df$Probability == 0.75])
```

```
## [1] 110.1173
```

```
# note: this is the same results as from the quantile fu
quantile(icdf.df$IQ)
```

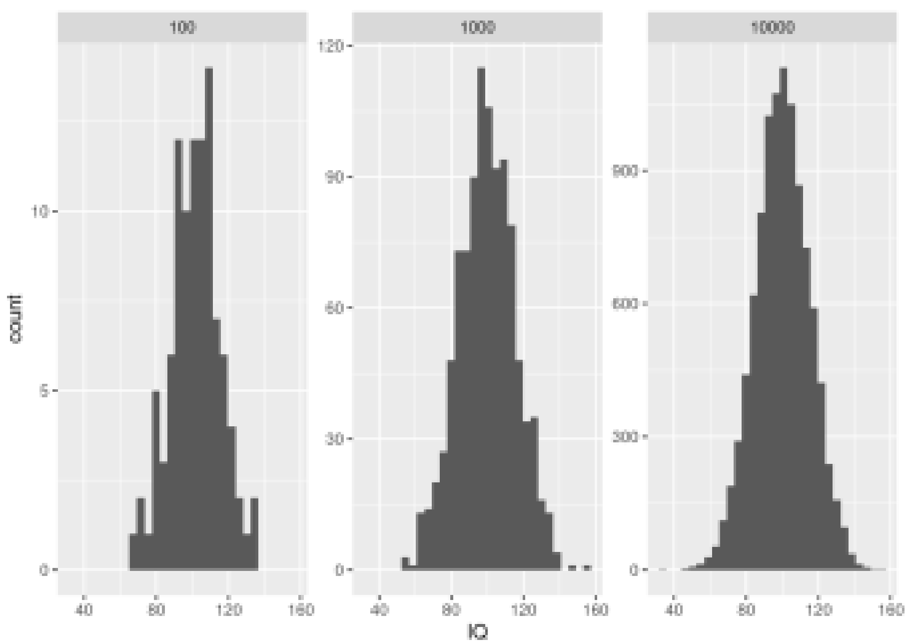
```
## 0% 25% 50% 75% 100%  ## -Inf 89.88265 100.00000
```

```
110.11735 Inf
```

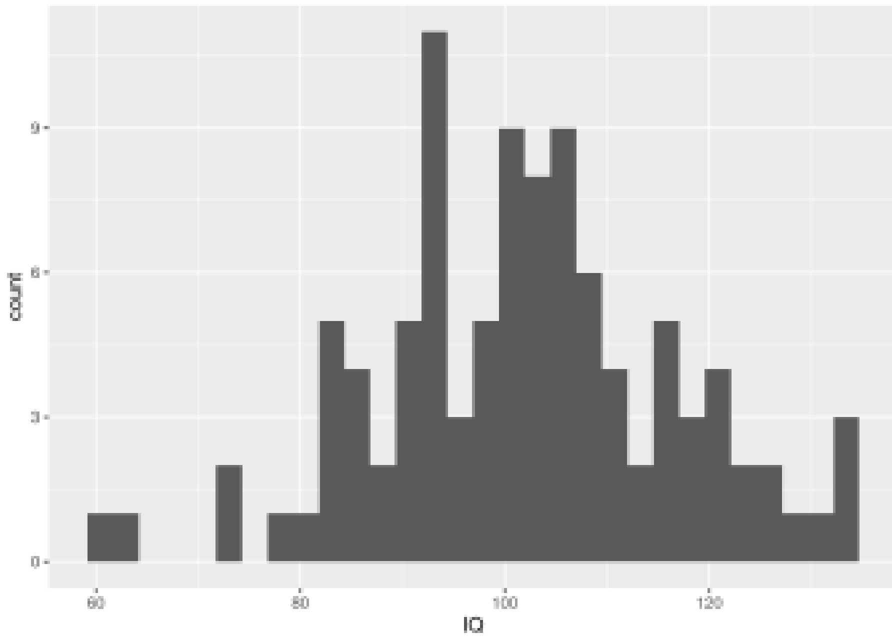
The random sampling function: rnorm

When you want to draw random samples from the normal distribution, you can use `rnorm`. For example, we could use `rnorm` to simulate random samples from the IQ distribution.

```
# fix random seed for reproducibility
set.seed(1)
# law of large numbers: mean will approach expected value
n.samples <- c(100, 1000, 10000)
my.df <- do.call(rbind, lapply(n.samples, function(x) dat
# show one facet per random sample of a given size
ggplot() + geom_histogram(data = my.df, aes(x = IQ)) + fa
```




```
# note: we can also implement our own sampler using the d
my.sample <- sample(iq.df$IQ, 100, prob = iq.df$Density,
my.sample.df <- data.frame("IQ" = my.sample)
ggplot(my.sample.df, aes(x = IQ)) + geom_histogram()
```



Note that we called `set.seed` in order to ensure that the random number generator always generates the same sequence of numbers for reproducibility.

Summary

Of the four functions dealing with distributions, `dnorm` is the most important one. This is because the values from `pnorm` , `qnorm` , and `rnorm` are based on `dnorm` . Still, `pnorm` , `qnorm` , and `rnorm` are very useful convenience functions when dealing with the normal distribution. If you would like to learn about the corresponding functions for the other distributions, you can simply call `?distribtuion` to obtain more information.

R

About Matthias Döring

Matthias Döring is a data scientist and AI architect. He is currently driving the digitization of the German railway system at DB Systel. Previously, he completed a PhD at the Max Planck Institute for Informatics in which he researched computational methods for improving treatment and prevention of viral infections.

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Perhaps I'm wrong, but it seems to me that there is a mistake in the **interpretation** of the density function. With continuous random variables, the probability of having an IQ of 140 is **not** the value of the density function at 140. Technically, the probability of having a specic value with a continuous r.v. is always zero. If one wants to makes an approximation, then he would write

```
pnorm(140.1, mean=100,sd=15)-pnorm(139.9,mean=100,s
0.0001519534
```

Which is not identical to dnorm(140,mean=100,sd=15) 0.0007597324

The small interval around 140 which we use to make our calculations with, will depend on the level of precision with which we make our measurements, I believe.

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