Part 1

1. $W(n) = 2W(n/3) + 1$

$= 2(2W(\frac{n}{9}) + 1) + 1 = 4W(n/9) + 2 + 1$

$= 8W(n/27) + 4 + 2 + 1$

i levels: $2^i W(n/3^i) +$ some constant

$i = \log_3 n \quad W(n) = 2^{\log_3 n} = n^{\log_3 2}$

$W(n) = \boxed{O(n^{\log_3 2})}$

2. $W(n) = 5W(n/4) + n$

$= 5(5W(n/16) + n/4) + n$

$= 25W(n/16) + n + n/4$

i levels: $5^i W(n/4^i) \dots \quad i = \log_4 n$

$5^{\log_4 n} = n^{\log_4 5}$

$\boxed{O(n^{\log_4 5})}$

3. $W(n) = 7W(n/7) + n$

$= 7(7W(n/49) + \frac{n}{7}) + n$

$i = \log_7 n \rightarrow 7^{\log_7 n} = \log n$

the extra work at each step is geometric and $O(n)$. Hence total work is $\boxed{O(n \log n)}$

4. $W(n) = 9W(n/3) + n^2$

- each level does n^2 total work with $\log n$ levels

Hence $\boxed{O(n^2 \log n)}$

5. $W(n) = 8W(n/2) + n^3$

each level does n^3 work and there are $\log n$ levels

Hence $\boxed{O(n^3 \log n)}$

6. $W(n) = 49W(n/25) + n^{3/2} \log n$

each level does $n^{3/2}$ work with $\log n$ levels

Hence $\boxed{O(n^{3/2} \log n)}$

7. $W(n) = W(n-1) + 2$

$= W(n-2) + 2 + 2$

$= W(n-3) + 2 + 2 + 2$

At n: $W(n) = W(1) + 2(n-1)$

Hence $\boxed{O(n)}$

8. $W(n) = W(n-1) + n^c$; $c \geq 1$

$= W(n-2) + (n-1)^c + n^c$

$= W(n-3) + (n-2)^c + (n-1)^c + n^c$

$W(1) + \sum_{i=1}^{n} i^c$

$\approx \boxed{O(n^{c+1})}$

9. $W(n) = W(\sqrt{n}) + 1$

n reduces to $\sqrt{n}$ at each level and stops at 1

$\sqrt[i]{n} = 1 \rightarrow (\frac{1}{2})^i \log_2 n = 0$

$i = \log_2 \log_2 4 \Rightarrow \boxed{O(\log \log n)}$

Part 2: Algorithm comparison

A: $W(n) = 5W(\frac{n}{2}) + O(n)$

$\boxed{O(n^{\log_2 5})}$

B: $W(n) = 2W(n-1) + O(1)$

$= 2(2W(n-2)-1+1 = 4W(n-2)+2+1$

$= 8W(n-3)+4+2+1$

at n level: $\boxed{O(2^n)}$

C: $W(n) = 9W(n/3) + O(n^2)$

$\boxed{O(n^2 \log n)}$

I will choose $\boxed{C}$: $O(n^2 \log n)$

because it has the less run time

$2^n > n^{\log_2 5} > n^2 \log n$

Part 3:

3b. $W(n) = W(n-1) + O(1)$

total work : $O(n)$

$S(n) = S(n-1) + O(1)$

span is $O(n)$

3d: work : map : n $\quad w(n) = n + n + n$

scan : n $\quad = 3n$

reduce : n $\quad \Rightarrow \boxed{O(n)}$

span: runs in parallele ~~hence~~ $\boxed{O(\log n)}$

3f: $W(n) = 2W(n/2) + O(1)$

$\boxed{O(n)}$

$S(n) = S(n/2) + O(1)$

$\boxed{O(\log n)}$