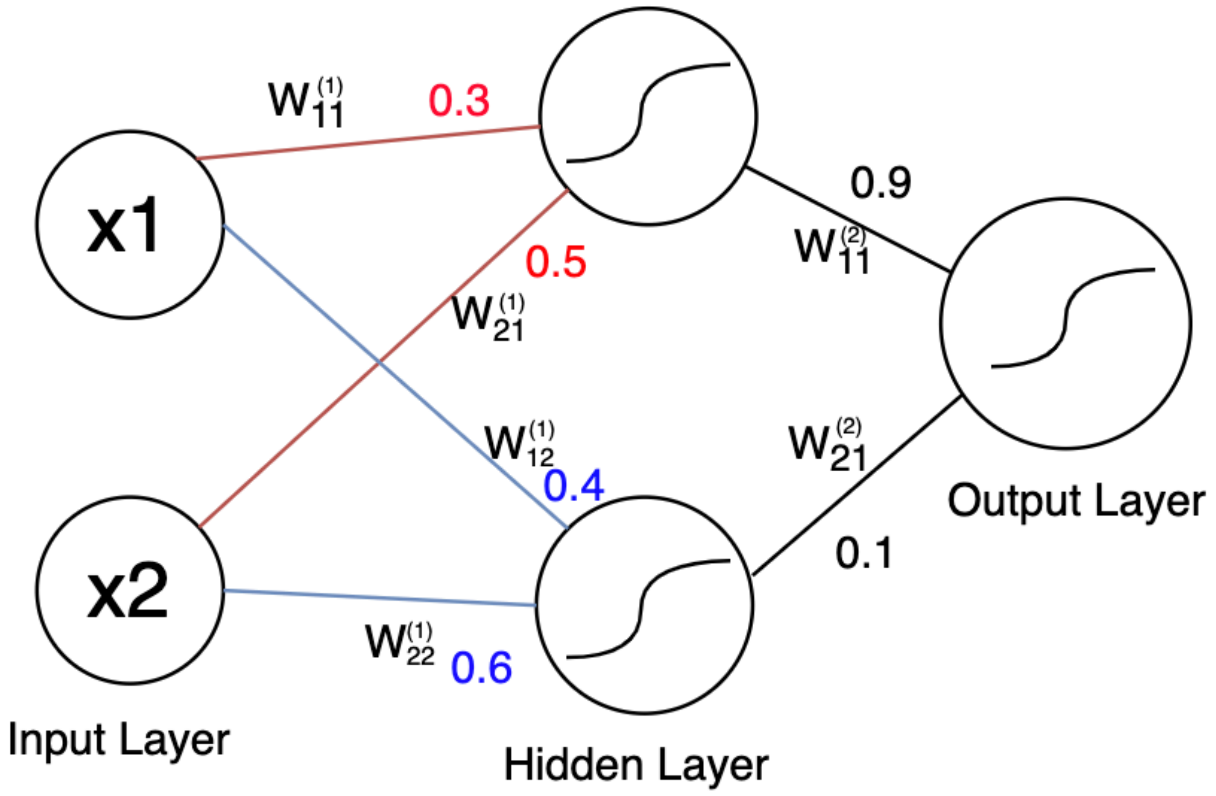
Neural Network structure



$$W^{(1)} = \begin{bmatrix} W_{11}^{(1)} & W_{12}^{(1)} \\ W_{21}^{(1)} & W_{22}^{(1)} \end{bmatrix} = \begin{bmatrix} 0.3 & 0.4 \\ 0.5 & 0.6 \end{bmatrix}$$

$$W^{(2)} = \begin{bmatrix} W_{11}^{(2)} \\ W_{21}^{(2)} \end{bmatrix} = \begin{bmatrix} 0.9 \\ 0.1 \end{bmatrix}$$

$$x = [x_1 \quad x_2] = [2 \quad 1]$$

$$\sigma(x) = \frac{1}{1+e^{-x}}$$

$$y_{true} = 0.5$$

$$\alpha = 0.5$$

Feedforward

$$\hat{y} = \sigma \circ W^{(2)} \circ \sigma \circ W^{(1)}(x)$$

$$h = \quad \sigma(xW^{(1)}) = \sigma([2 \quad 1]\begin{bmatrix} 0.3 & 0.4 \\ 0.5 & 0.6 \end{bmatrix}) = \sigma([1.1 \quad 1.4])$$

$$= [0.7503 \quad 0.8022]$$

$$output = \sigma(hW^{(2)}) = \sigma([0.7503 \quad 0.8022]\begin{bmatrix} 0.9 \\ 0.1 \end{bmatrix}) = 0.6804$$

Backpropagation

$$E = \frac{1}{2n}\sum_i^n (y_i - \hat{y}_i)^2$$

$$\nabla E = \begin{bmatrix} \frac{\partial E}{\partial W_{11}^{(1)}} & \frac{\partial E}{\partial W_{12}^{(1)}} & \frac{\partial E}{\partial W_{11}^{(2)}} \\ \frac{\partial E}{\partial W_{21}^{(1)}} & \frac{\partial E}{\partial W_{22}^{(1)}} & \frac{\partial E}{\partial W_{21}^{(2)}} \end{bmatrix} \qquad \sigma'(x) = \sigma(x)(1-\sigma(x))$$

$$\frac{\partial E}{\partial W_{11}^{(1)}} = \frac{\partial E}{\partial \hat{y}}\frac{\partial \hat{y}}{\partial h}\frac{\partial h}{\partial h_1}\frac{\partial h_1}{W_{11}^{(1)}} = 0.1804*0.2174*0.1686*2=0.0132$$

$$\frac{\partial E}{\partial \hat{y}} = \frac{\partial}{\partial \hat{y}}(\frac{1}{2}(y-\hat{y})^2) = \hat{y} - y = 0.6804-0.5=0.1804$$

$$\frac{\partial \hat{y}}{\partial h} = \frac{\partial}{\partial h}(\sigma(h)) = \sigma(h)(1-\sigma(h)) = 0.2174$$

$$\frac{\partial h}{\partial h_1} = \frac{\partial}{\partial h_1}(W_{11}^{(2)}\sigma(h_1) + W_{21}^{(2)}\sigma(h_2)) = W_{11}^{(2)}\sigma(h_1)(1-\sigma(h_1)) = 0.1686$$

$$\frac{\partial h_1}{\partial W_{11}^{(1)}} = \frac{\partial}{\partial W_{11}^{(1)}}(W_{11}^{(1)}x_1 + W_{21}^{(1)}x_2) = x_1$$

$$h_1 = W_{11}^{(1)}x_1 + W_{21}^{1}x_2 = 1.1$$

$$h_2 = W_{12}^{(1)}x_1 + W_{22}^{1}x_2 = 1.4$$

$$\frac{\partial E}{\partial W_{21}^{(1)}} = \frac{\partial E}{\partial \hat{y}}\frac{\partial \hat{y}}{\partial h}\frac{\partial h}{\partial h_1}\frac{\partial h_1}{W_{21}^{(1)}} = 0.0066$$

$$\frac{\partial E}{\partial W_{12}^{(1)}} = \frac{\partial E}{\partial \hat{y}}\frac{\partial \hat{y}}{\partial h}\frac{\partial h}{\partial h_2}\frac{\partial h_2}{W_{12}^{(1)}} = 0.0012$$

$$\frac{\partial E}{\partial W_{22}^{(1)}} = \frac{\partial E}{\partial \hat{y}}\frac{\partial \hat{y}}{\partial h}\frac{\partial h}{\partial h_2}\frac{\partial h_2}{W_{22}^{(1)}} = 0.0006$$

$$\frac{\partial E}{\partial W_{11}^{(2)}} = \frac{\partial E}{\partial \hat{y}}\frac{\partial \hat{y}}{\partial h}\frac{\partial h}{\partial W_{11}^{(2)}} = 0.0294$$

$$\frac{\partial h}{\partial W_{11}^{(2)}} = \frac{\partial}{\partial W_{11}^{(2)}}(W_{11}^{(2)}\sigma(h_1) + W_{21}^{(2)}\sigma(h_2)) = \sigma(h_1)$$

$$\frac{\partial E}{\partial W_{21}^{(2)}} = \frac{\partial E}{\partial \hat{y}}\frac{\partial \hat{y}}{\partial h}\frac{\partial h}{\partial W_{21}^{(2)}} = 0.0314$$

$$W_{i,j}^{(k)} \longleftarrow W_{i,j}^{(k)} - \alpha\frac{\partial E}{\partial W_{i,j}^{(k)}}$$

$$W = \begin{bmatrix} 0.3 & 0.4 & 0.9 \\ 0.5 & 0.6 & 0.1 \end{bmatrix} \qquad W' = \begin{bmatrix} 0.2934 & 0.3994 & 0.8853 \\ 0.4967 & 0.5997 & 0.0843 \end{bmatrix}$$

Xiaoni Cai, Sneha Mohanty, Anh Phuong Le@neural-bauhaus-style-transfer SS19