The work by Finn et al further investigating our system is indeed relevant. They calculate maximum Lyapunov exponents (λ_m) showing monotonic suppression of chaos as β is increased for $\Gamma = 0.125$, and no chaos for $\Gamma = 0.3$. Their results are extremely puzzling, particularly the complete absence of chaos for the $\Gamma = 0.3$ case, and definitely inconsistent with us.

We worked with the in-principle experimentally accessible time-series. Poincaré sections indicate, for $\Gamma = 0.125$, a chaotic attractor at $\beta = 0.01$, which is altered but persists for $\beta = 0.3$ and disappears for $\beta = 0.1$. The power spectra for $\langle \hat{X}(t) \rangle$ agree, indicating chaos for the first two cases and no chaos for the last. For $\Gamma = 0.3$, we see no $\beta = 0.01$ chaos, an attractor for $\beta = 0.3$, which disappears for $\beta = 0.1$, in agreement with the power spectra. Further, the $\beta = 0.3$ results look extremely similar for the two Γ cases. The specific disagreement with Finn et al is the $\Gamma = 0.3, \beta = 0.3$ case, and also that their results are so different for the two $\beta = 0.3$ cases.

We have since worked with the TISEAN package, using phase-space delay reconstruction of $\langle \hat{X}(t) \rangle$ to obtain λ_m, as shown in the attached figure. Each plot shows the average divergence of nearby points in the reconstructed phase space for X, on a log scale. Exponential growth appears as linearity before the trajectories reach saturation; the slope is proportional to λ_m. The delay embedding dimension is indicated by m. For each dimension, several graphs are shown, corresponding to different neighborhood choices in the reconstructed phase space. We see qualitative agreement with our previous results. Because of the form of the sampling in TISEAN, a raw slope from these figures needs to be divided by (interval * modulo) $= (0.01 * 11)$ to find λ_m. We estimate λ_m for respective (Γ, β) pairs (approximately, since they derive from finding the slope of the straight line parts of these curves) as: $(0.125, 0.01) = 0.1, (0.125, 0.3) = 0.16, (0.125, 1.0) < 0.05, (0.3, 0.01) < 0.03, (0.3, 0.3) = 0.13, (0.3, 1.0) < 0.05$. In short, this agrees with our previous conclusions about where chaos exists. Interestingly, using λ_m, the transition from quantum to classical behavior appears to be non-monotonic for BOTH instances of Γ.

Our three methods of analysis (Poincaré sections, power spectra, and time-series Lyapunov exponents) are all consistent with each other, and consistent with our physical understanding of how the chaos emerges and/or is swamped by quantum effects, and Finn et al's calculation is inconsistent with this for the one 'mesoscopic' case of $(\Gamma, \beta) = (0.3, 0.3)$. The difference is intriguing. We expect that understanding the source of this difference – provided it is not due to technical errors – will help us in understanding something deeper about the physics, or about the methods of analysis. Behind the immediate questions about the behavior of this model system stands the larger and fundamental question of whether quantum corrections always regularize and suppress chaotic dynamics. We believe that this, while often true, is not universal. For the QSD equations (or equivalent stochastic Schrodinger equations), that such a highly nonlinear equation has a priori a monotonic parameter landscape is extremely unlikely. Our perspective is supported by Bhattacharyya et al [our ref 14] showing that the diffusion rate for the kicked rotor (a measure of the chaos in the system) can increase into the quantum regime. It is only a matter of more systematic investigation to find other such counter-examples to the folklore.

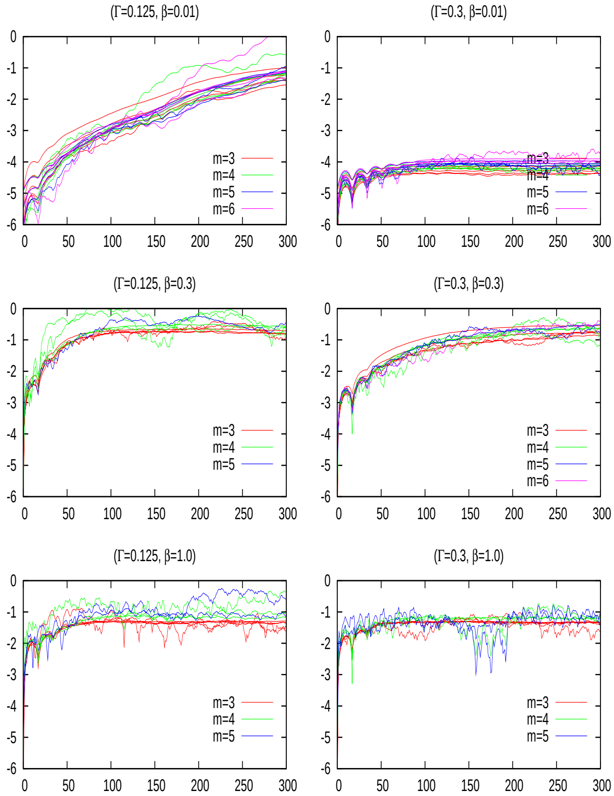

FIG. 1:

Kyle Kingsbury[(a)]
Chris Amey[(a)]
Arie Kapulkin[(a)]
Arjendu Pattanayak[(a)]
(a) Department of Physics and Astronomy, Carleton College, Northfield, Minnesota 55057
(b) 128 Rockwood Cr, Thornhill, Ont L4J 7W1 Canada

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