# Solow-Swan Model
# with Dynare

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## 1. Introduction

The Solow-Swan model was developed by Solow (1956) and Swan (1956) in the same period. Here we will explore a simple example of this model using Dynare, which is an optimal tool for solving dynamic systems and for handling shocks.

Once you have downloaded and installed Dynare, you can open Matlab or Octave to work with it. Before coding, you need to add the following line.[1]

```
addpath /Applications/Dynare/x.y/matlab //For Matlab
addpath /usr/local/lib/dynare/matlab //For Octave
```

## 2. Endogenous Variables

First of all, we need to declare three endogenous variables: per capita income ($y_t$), per capita consumption ($c_t$ ) and per capita capital stock ($k_t$).

```
% -------------- | ENDOGENEOUS VARIABLES |----------------%
var
k // per capita capital stock
c // per capita consumption
y // per capita income
;
```

## 3. Exogenous Variables

Since agents save a fixed fraction of their income, the only exogenous variable is the saving rate, $s \in (0, 1)$.

```
% ---------------| EXOGENEOUS VARIABLES |-----------------%
varexo
s //saving rate
;
```

## 4. Parameters

In our model, all the parameters are: the capital depreciation rate ($\delta$), the capital share ($\alpha$), the population growth ($n$) and the technological progress ($\mu$). Once we have defined them, we need to specify their values. Remember that the parameters can not be shocked. For example, if we want to know how income changes when $\delta$ changes, we must declare $\delta$ in the exogenous variables section.

```
%--------------------| PARAMETERS |-----------------------%
parameters
ALPHA // capital share
DELTA // depreciation rate of capital stock
```

[1] For more information on installing and configuring Dynare, visit the Dynare website at https://www.dynare.org.

```
n // population growth
MU //technological progress
;

%----------------------| VALUES |------------------------%
ALPHA = 0.33;
DELTA = 0.02;
n = 0; //no population growth
MU = 0; //no technological progress
```

## 5. Model

We consider a one-good, closed economy without government, where the national income identity is

$$Y_t = I_t + C_t. \tag{1}$$

Capital stock changes over time according to

$$K_{t+1} = I_t + (1-\delta)K_t, \tag{2}$$

where $\delta \in (0,1)$ is the capital depreciation rate. Firms produce the output $Y_t$ by the following Cobb-Douglas production function with a labour augmenting technological progress (Harrold-neutral):[2]

$$Y_t \equiv F(K_t, A_tN_t) = (K_t)^{\alpha}(N_tA_t)^{1-\alpha} = K_t^{\alpha}L_t^{1-\alpha} \qquad \text{where } \; L_t \equiv A_tN_t, \;\; \alpha \in (0,1) \tag{3}$$

Using CRTS, we can rewrite the (3) in per capita terms as:

$$y_t = \frac{F(K_t, L_t)}{L_t} = f(k_t) = k_t^{\alpha} \qquad \text{where } \; y_t \equiv \frac{Y_t}{L_t} \tag{4}$$

Firms maximize their profits by choosing the optimal amount of capital stock and labour according to the following FOCs:[3]

$$R = F_K \tag{5}$$
$$w = F_L \tag{6}$$

Since households save a fixed fraction of output $s$, savings are equal to $sf(k_t)$. Therefore, the relation (2) can be rewritten as

$$k_{t+1}(1+n+\mu) = sk_t^{\alpha} + (1-\delta)k_t \tag{7}$$

[2]As we can easily show, the production function exhibits decreasing marginal products of both inputs and constant returns to scale (CTRS).

[3]Thanks to the CTRS, Euler's theorem holds, which means that output is fully exhausted by payments to capital and labour. In other words, we have

$$Y = RK + wL.$$

```
%------------------------| MODEL |------------------------%
model;

//law of motion of capital
k = (1/(1 + n + MU))*(s*k(-1)^ALPHA + (1 - DELTA)*k(-1));

//per capita production function
y = k^ALPHA;

//per capita consumption
c = (1 - s)*y;

end;
```

## 6. Steady State

A steady state equilibrium is an equilibrium path where, for all $t \in \mathbb{T}$,

$$k_t = k_{t+1} = k_{ss}.$$

From (7), the corresponding steady state equilibrium is determined by the following equation

$$k_{ss}(1 + n + \mu) = sk_{ss}^{\alpha} + k_{ss} - \delta k_{ss}$$

Finally, the steady state equilibrium of capital, production and consumption will be

$$\begin{aligned} k_{ss} &= \left(\frac{s}{\delta + n + \mu}\right)^{\frac{1}{1-\alpha}} \\ y_{ss} &= k_{ss}^{\alpha} \\ c_{ss} &= (1 - s)y_{ss} \end{aligned}$$

However, we need to specify the initial values of all endogenous and exogenous variables. We can replace the initial values of capital, consumption and output with their steady state values.

```
%---------------------| INITIAL VALUES |------------------%
initval;
s = 0.3;
k = (s/(DELTA + MU + n))^(1/(1 - ALPHA));
y = k^ALPHA;
c = (1 - s)*y;
end;
```

## 7. Shock

Dynare can handle both deterministic and stochastic shocks. Here we will see a deterministic shock (i.e., we know the exact evolution of the variable) to the saving rate. At the very beginning, the saving rate is 0.3, but after 10 quarters it rises to 0.5 by the end of our simulation.

```
%----------------------| SHOCKS |--------------------------%
shocks;
```

```
var s;
periods 10:50;
values 0.5;
end;

%--------------| NON STOCHASTIC SIMULATION |----------------%
perfect_foresight_setup(periods=50); //number of periods is set to 50
perfect_foresight_solver;
```

## 8. Results

We can plot the results of our simulation using the command "rplot" followed by the name of the variables. We can also customise the plots using the usual Matlab commands.

```
%--------------------| PLOT SIMULATION |--------------------%
rplot k;
rplot c;
rplot y;
rplot s;
```

Since we can code in a Dynare script in the same way as in Matlab (or Octave), we can produce a more complete graphical output. We can calculate all the steady states by hand.

```
%--------------| NON STOCHASTIC SIMULATION |----------------%
perfect_foresight_setup(periods=50); //number of periods is set to 50
perfect_foresight_solver;

%---------------| STEADY STATE COMPUTATIONS| ---------------%
k_ss = ((M_.det_shocks.value)/(DELTA + MU + n))^(1/(1 - ALPHA));
y_ss = k_ss^ALPHA;
c_ss = (1 - M_.det_shocks.value)*y_ss;

%--------------------| PLOT SIMULATION |--------------------%
figure('Name','Dynamics');

%CAPITAL
subplot(2,2,1);
plot(k,"LineWidth", 1.5);
title("Capital Stock (k)");
xlabel("Time");
ylabel("k");
hold on;
plot([xlim], [k_ss,k_ss],"r--","LineWidth",1.2); //steady state line

%CONSUMPTION
subplot(2,2,2);
plot(c,"LineWidth", 1.5);
title("Consumption (c)");
xlabel("Time");
ylabel("c");
hold on;
plot([xlim], [c_ss,c_ss],"r--","LineWidth",1.2); //steady state line

%OUTPUT
subplot(2,2,3);
plot(y,"LineWidth", 1.5);
title("Output (y)");
xlabel("Time");
ylabel("y");
hold on;
plot([xlim], [y_ss,y_ss],"r--","LineWidth",1.2); //steady state line

%SAVING RATE
```

```
subplot(2,2,4);
plot(oo_.exo_simul,"LineWidth", 1.5);
title("Saving Rate (s)");
xlabel("Time");
ylabel("s");

saveas(gcf,"Solow_Dynamics.pdf"); //save all dynamics
```

When you run the code, you get the impulse response function to the saving rate shock. The figure (1) captures the evolution of the main economic variables: per capita capital stock, consumption per capita and output per capita, and there is another graph to capture the evolution of the saving rate.

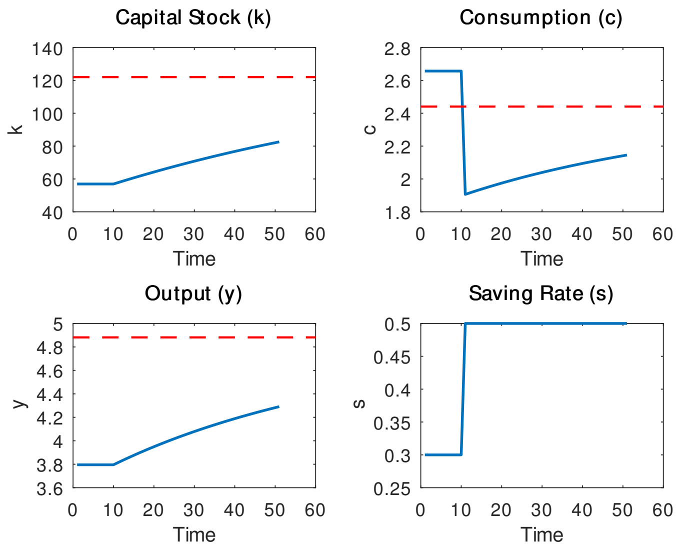


Figure 1. Dynamics of all economic variables starting from their steady state.

The interpretation is quite straightforward. The economy starts from its steady state position, after 10 quarters the saving rate rises, consumption immediately falls and the capital stock starts to accumulate. At the end of our simulation, we have an economy with a lower steady-state level of consumption, while the capital stock and output have a higher steady-state level.

## References


Adjemian, Stéphane et al. (2011). “Dynare: Reference manual, version 4”. In: ed. by CEPREMAP Dynare working papers 1.

Romer, David (2018). *Advanced Macroeconomics*. Ed. by Berkeley University of California.

Solow, Robert M (1956). “A contribution to the theory of economic growth”. In: *The quarterly journal of economics* 70(1), pp. 65–94.

Solow, Robert M (1957). “Technical change and the aggregate production function”. In: *The review of Economics and Statistics* 39(3), pp. 312–320.

Swan, Trevor W (1956). “Economic growth and capital accumulation”. In: *Economic record* 32(2), pp. 334–361.