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CSCI 2824: Discrete Structures

Lecture 1: Intro & Binary Representation of Numbers

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Course Logistics:

Fridays at noon
ECOT 732

Weekly Homework (30%)

- Half Written, Half Online

Quizlets (10%)

- Online

Moodle:
moodle.cs.colorado.edu

enrollment key:
“csci2824-Rachel”

Two Midterms (20% each)

- October 3rd, November 7th

Final Exam (20%)

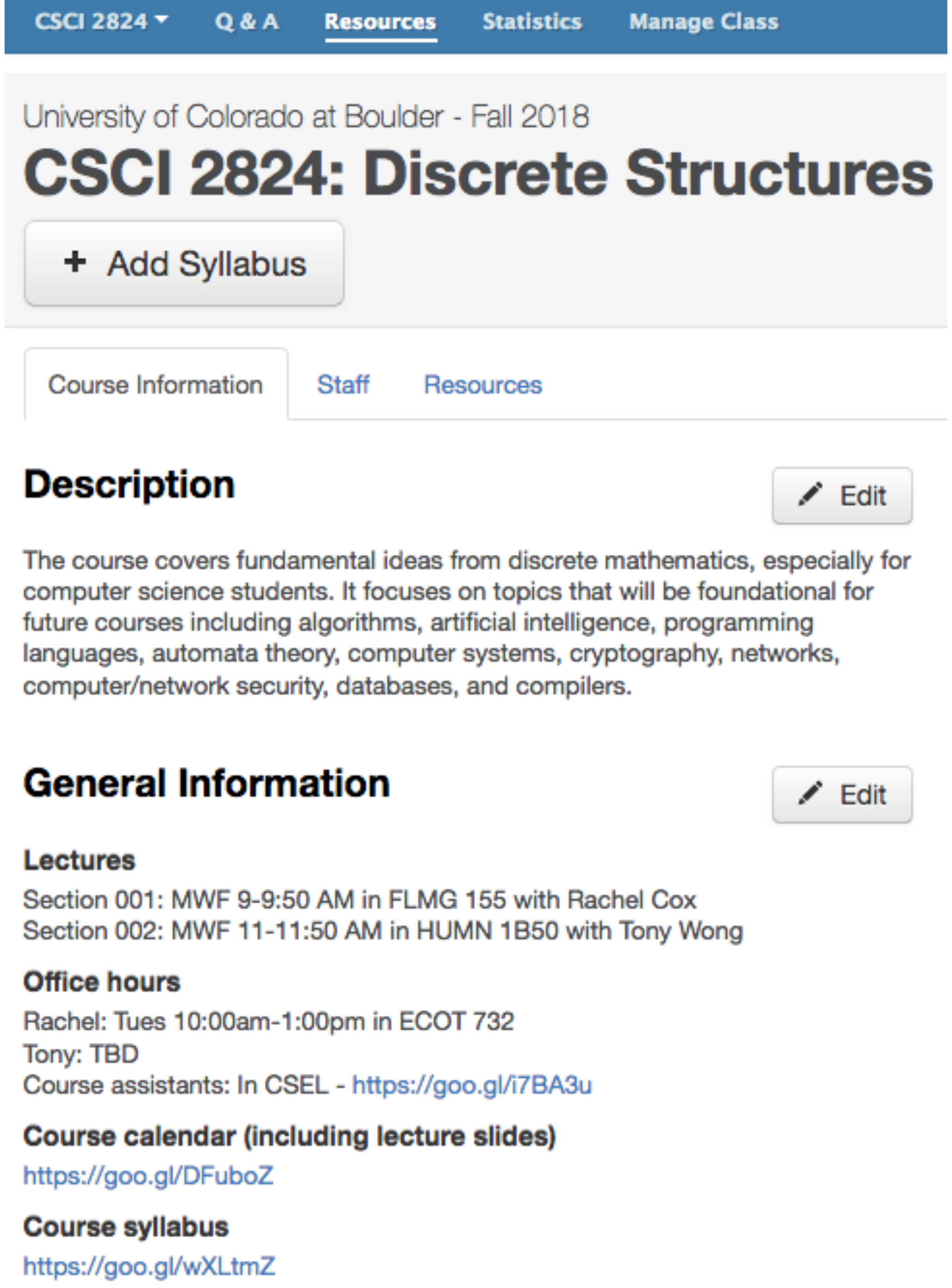
- December 19th, 1:30-4pm

Course Logistics:

Course Webpage - Piazza

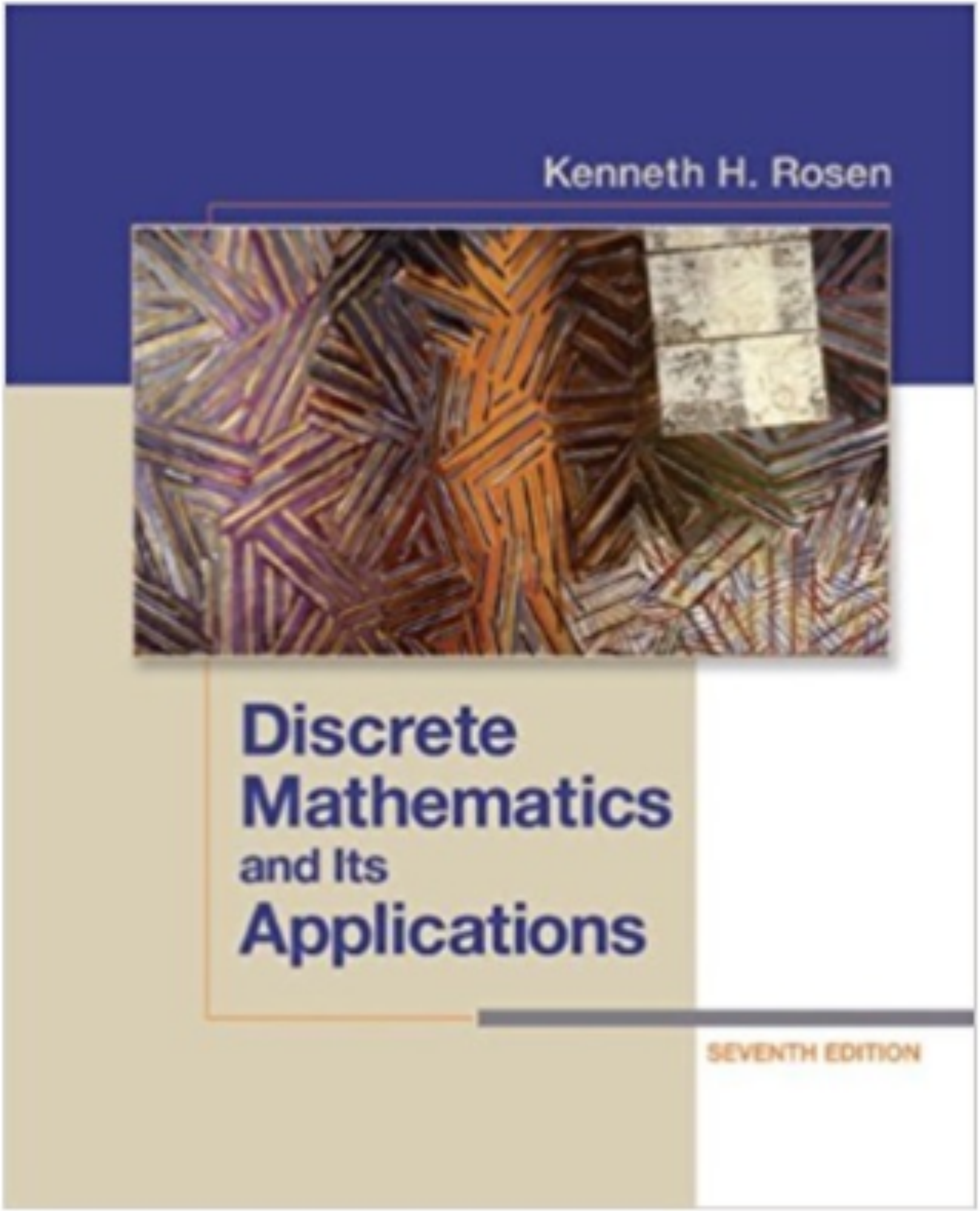
https://piazza.com/colorado/fall2018/csci2824

- Office Hours
- General Info
- Instead of emailing, post questions to Piazza – it'll be faster!
- Announcements
- Homework & Solutions



Course Logistics:

<u>Textbook</u> – *Discrete Mathematics and Its Applications*, 7th Ed. by Kenneth H. Rosen



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What is Discrete Structures?

<u>Discrete</u>

- Logic
- Combinatorics
- Discrete Probability
- Recursion
- Sets
- Sequences
- Graph Theory

.
.
.

<u>NOT Discrete</u>

- Derivatives
- Integrals
- … things that involve infinitesimals

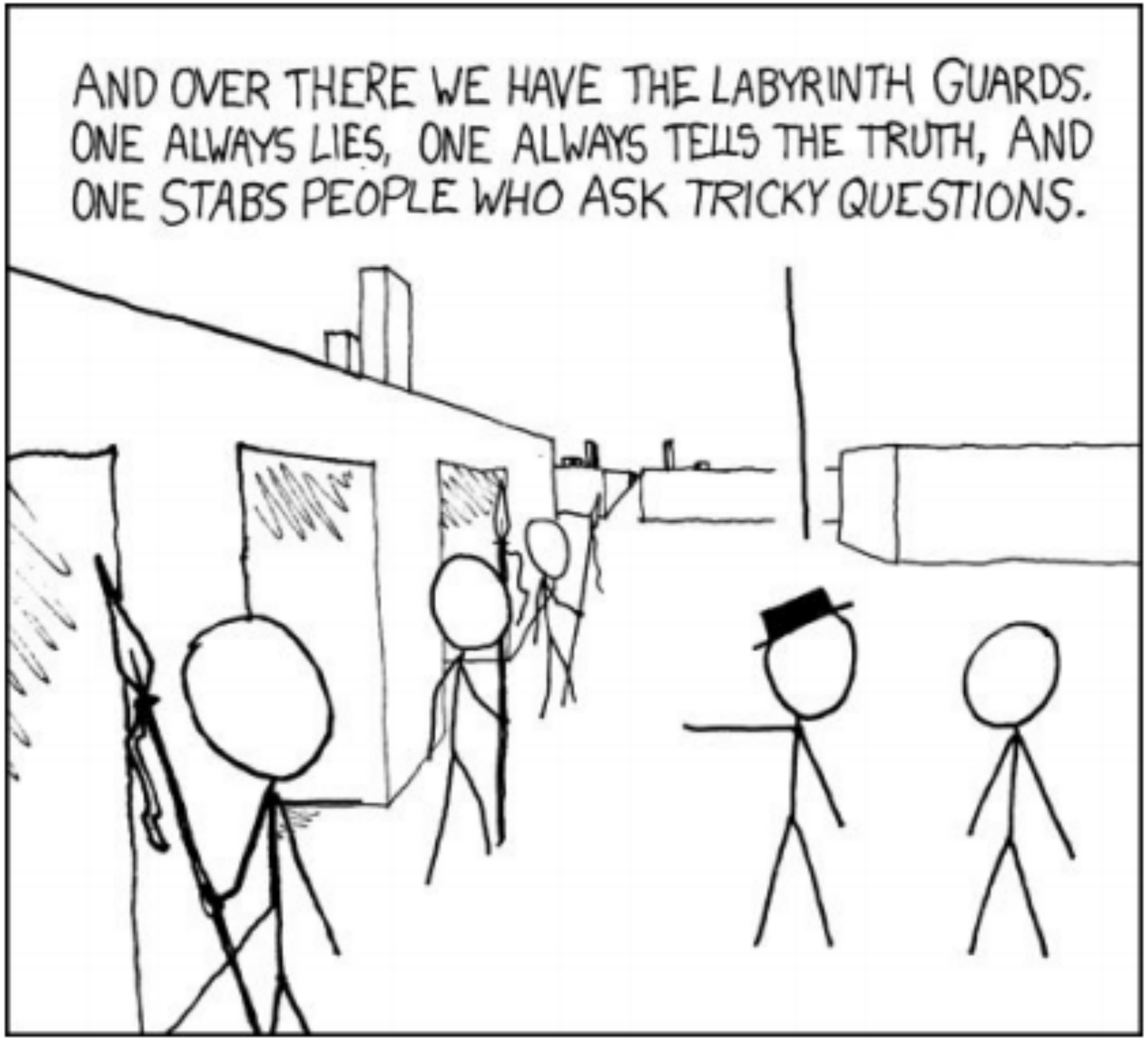
1 0 1 1 0 0 0 0 1 0 0 0 1 0 1 1 0 0 0 0 1 0 0 0 1 0 1 1 0 0 0 0 1 0 0 0 1 0 1 1 0 0 0 0 1 0 0 0 1 0 1 1 0 0 0 0 1 0 0 0

Logic:

"Two doors" riddle:

- Two doors, guarded by two guards.
- One door goes where you want to, but the other leads to certain death.
- One always lies, and one always tells the truth.

- **How can you ask**
  <u>only one of them</u>
  <u>only one question</u>
  to discover which door is which?



Discrete Probability:

e.g. The Monte Hall Problem

Three doors problem

- There are three doors.
- One has a nice prize behind it…
- … and the other two have goats.
- You get to pick a door and will be awarded the prize behind it.
- Then the host reveals a goat behind one of the other two doors.
- You now have the option to stick with your original door or switch.

- **Should you stick with your original door or switch? Or does it not matter?**

Recursion:

e.g. The Tower of Hanoi

- Recursive solutions can be elegant and lead to more readable code
- Recursion may be frowned upon due to memory stack issues
- However, developing a recursive solution and then translating it to an iterative solution (loops) may be very helpful

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Binary Representation of Numbers

0 1

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What’s in a Number?

<u>Example</u>: Consider the numbers **235** and **1130**. Assume they are both ”decimal”.
How can we expand them as powers of 10?

$$235 = 200 + 30 + 5$$
$$= \underline{2} * 100 + \underline{3} * 10 + \underline{5} * 1 = 2*10^2 + 3*10^1 + 5*10^0$$

$$1130 = \underset{3}{1} * 10^3 + \underset{2}{1} * 10^2 + \underset{1}{3} * 10^1 + \underset{0}{0} * 10^0$$

10 digits

0, 1, 2, 3, ... 9

Decimal point

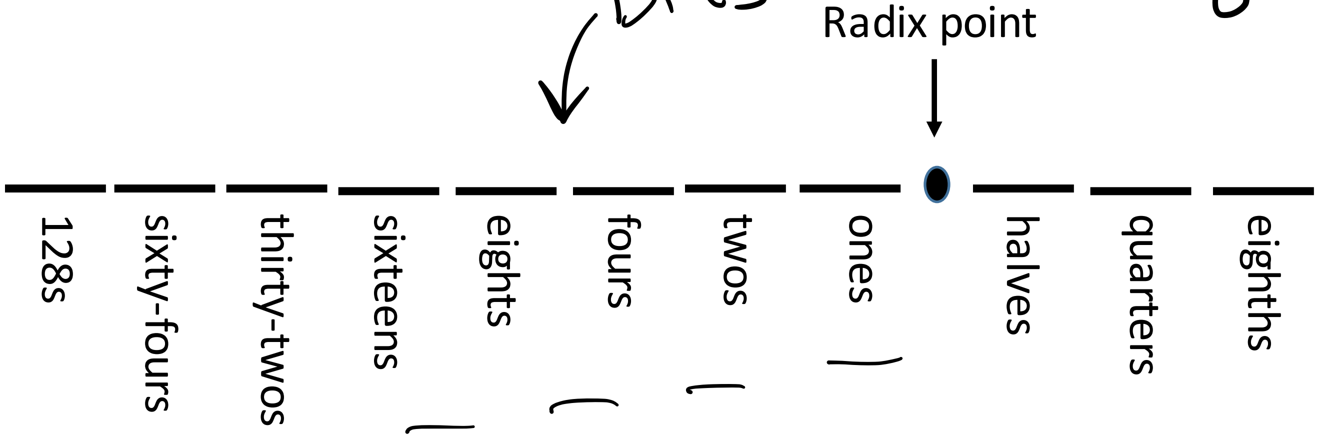
ten millions	millions	100 thousands	ten thousands	thousands	hundreds	tens	ones	.	tenths	hundredths	thousandths
					2	3	5	•			

0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1

Binary Number System

bits ~ binary digit

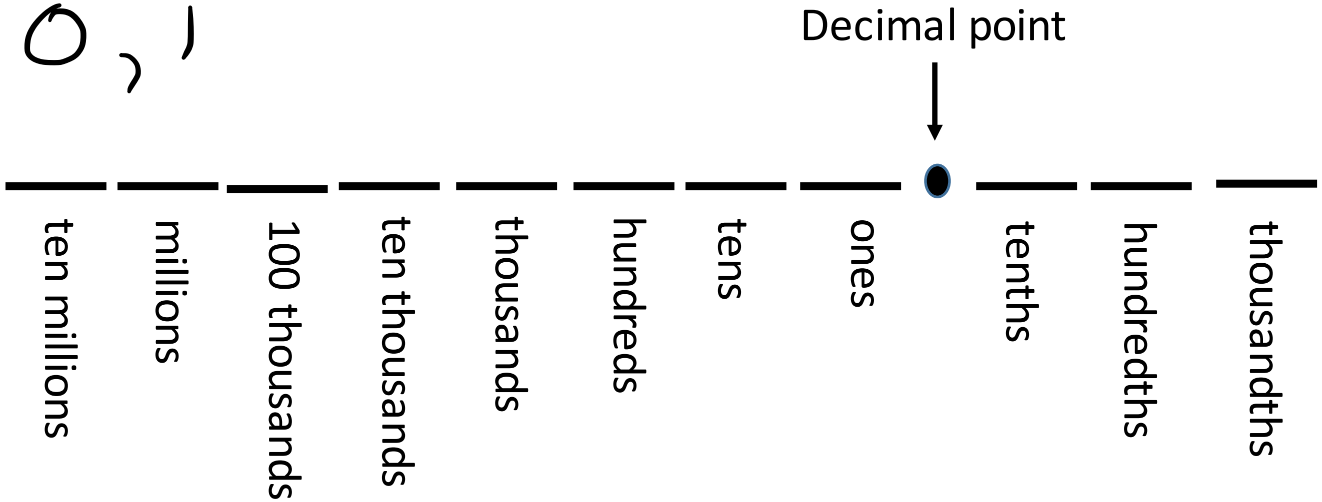
- Representing numbers in "base 2"
- Denote a binary number by $(binary\ number)_2$



Powers of 2:

$$2^0 = 1$$
$$2^1 = 2$$
$$2^2 = 4$$
$$2^3 = 8$$
$$2^4 = 16$$
$$2^5 = 32$$
$$2^6 = 64$$
$$2^7 = 128$$
$$2^8 = 256$$

Two Digits 0, 1



$(101)_2$

Powers of 10:

$$10^0 = 1$$
$$10^1 = 10$$
$$10^2 = 100$$
$$10^3 = 1000$$
$$10^4 = 10{,}000$$
$$10^5 = 100{,}000$$
$$10^6 = 1{,}000{,}000$$
$$10^7 = 10{,}000{,}000$$
$$10^8 = 100{,}000{,}000$$

0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1

Example: Convert 235 from decimal to binary

$$235 = 128 + \underline{107}$$
$$= 128 + 64 + \underline{43}$$
$$= 128 + 64 + 32 + 8 + 2 + 1$$
$$= 2^7 + 2^6 + 2^5 + 2^3 + 2^1 + 2^0$$
$$= \underline{1} * 2^7 + \underline{1} * 2^6 + \underline{1} * 2^5 + \underline{0} * 2^4 + \underline{1} * 2^3 + \underline{0} * 2^2 + \underline{1} * 2^1 + \underline{1} * 2^0$$

$$(235)_{10} = (11101011)_2$$

1	1	1	0	1	0	1	1	.			
2^7	2^6	2^5	2^4	2^3	2^2	2^1	2^0		2^{-1}	2^{-2}	2^{-3}

Powers of 2:

$2^0 = 1$
$2^1 = 2$
$2^2 = 4$
$2^3 = 8$
$2^4 = 16$
$2^5 = 32$
$2^6 = 64$
$2^7 = 128$
$2^8 = 256$

0	0	0	0	0	0	0
0	0	0	0	1		0
0	0	0	1	1		0
0	1	0	0			0
1	1	1	1			0
0	0	1				1
0		1				0
0						0
						1

0	0	0	0	0	0	0
0	0	0	0	1		0
0	0	0	1	1		0
0	1	0	0			0
1	1	1	1			0
0	0	1				1
0		1				0
0						0
						1

<u>Example:</u> Convert 235 from decimal to binary in a more systematic way.

$$
\begin{aligned}
235 &= 234 + 1 \\
&= 2 \cdot 117 + 1 \\
&= 2(116 + 1) + 1 \\
&= 2 \cdot 116 + 2 + 1 \\
&= 2^2 \cdot 58 + 2 + 1 \\
&= 2^3 \cdot 29 + 2 + 1 \\
&= 2^3(28 + 1) + 2 + 1 \\
&= 2^3 \cdot 28 + 2^3 + 2 + 1 \\
&= 2^4 \cdot 14 + 2^3 + 2 + 1
\end{aligned}
$$

				1	0	1	1	.			
2^7	2^6	2^5	2^4	2^3	2^2	2^1	2^0		2^{-1}	2^{-2}	2^{-3}

0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1

An Algorithm for Converting Decimal Integers to Binary

Let N be a nonnegative integer. Move from right to left.

Is N even? or odd?

If N is even, set bit to 0, reset $N = \frac{N}{2}$

If N is odd, set bit to 1, reset $N = \frac{N-1}{2}$

Move left to the next bit

Repeat until N = 0

0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1

$(1130)_{10} = (10001101010)_2$

<u>Example:</u> Convert 1130 from decimal to binary using the algorithm we just defined.

N				
1130	even	set 2^0 to	0	$N = \frac{1130}{2} = 565$
565	odd	set 2^1 to	1	$N = \frac{565-1}{2} = 282$
282	even	2^2	0	$N = 141$
141	odd	2^3	1	$N = \frac{141-1}{2} = 70$
70	even		0	$N = 35$
35	odd		1	$N = \frac{35-1}{2} = 17$
17	odd		1	$N = 8$
8	even		0	$N = 4$
4	even		0	$N = 2$
2	even		0	$N = 1$
1	odd		1	

0 0 0 0 0 0 0
0 0 0 0 1 0
0 0 0 1 1 0
0 1 0 0 0
1 1 1 1 0
0 0 1 1
0 1 0
0 0
1

0	0	0	0	0	0	0
0	0	0	0	1		0
0	0	0	1	1		0
0	1	0	0			0
1	1	1	1			0
0	0	1				1
0		1				0
0						0
						1

<u>Example:</u> Convert 160 and 161 from decimal to binary using the algorithm we just defined.

$$160 \quad (10100000)_2$$

$$(161)_{10} = (10100001)_2$$

0	0	0	0	0	0	0
0	0	0	0	1		0
0	0	0	1	1		0
0	1	0	0			0
1	1	1	1			0
0	0	1				1
0		1				0
0						0
						1

<u>Example:</u> Convert 1100101 from binary to decimal.

$$\begin{array}{ccccccc} 1 & 1 & 0 & 0 & 1 & 0 & 1. \\ \overline{2^6} & \overline{2^5} & \overline{2^4} & \overline{2^3} & \overline{2^2} & \overline{2^1} & \overline{2^0} \end{array}$$

$$N = 1 * 2^6 + 1 * 2^5 + 1 * 2^2 + 1 \times 2^0$$

$$= 64 + 32 + 4 + 1$$

$$= (101)_{10}$$

Example: What's the largest number you can store as a 32-bit signed int?

$$\underline{+/-}\ \underline{1}\ \underline{1}\ \underline{1}\ \underline{1}\ \underline{1}\ \ldots\ \underline{1}\ \underline{1}\ \underline{1}\,.$$

$$2^{31}\quad 2^{30}\quad 2^{29}\quad 2^{28}\quad 2^{27}\ldots\quad\ldots\ 2^{4}\quad 2^{1}\quad 2^{0}$$

$$N = 2^{30} + 2^{29} + \ldots \quad + 2^4 + 2^3 + 2^2 + 2^1 + 2^0$$

$$2N = 2^{31} + 2^{30} + \ldots \quad + 2^5 + 2^4 + 2^3 + 2^2 + 2^1$$

$$-N = -2^{31} \quad + 2^0$$

Marin Mersenne

Born	8 September 1588 Oizé, Maine
Died	1 September 1648 (aged 59) Paris
Nationality	French
Known for	Acoustics, Mersenne primes