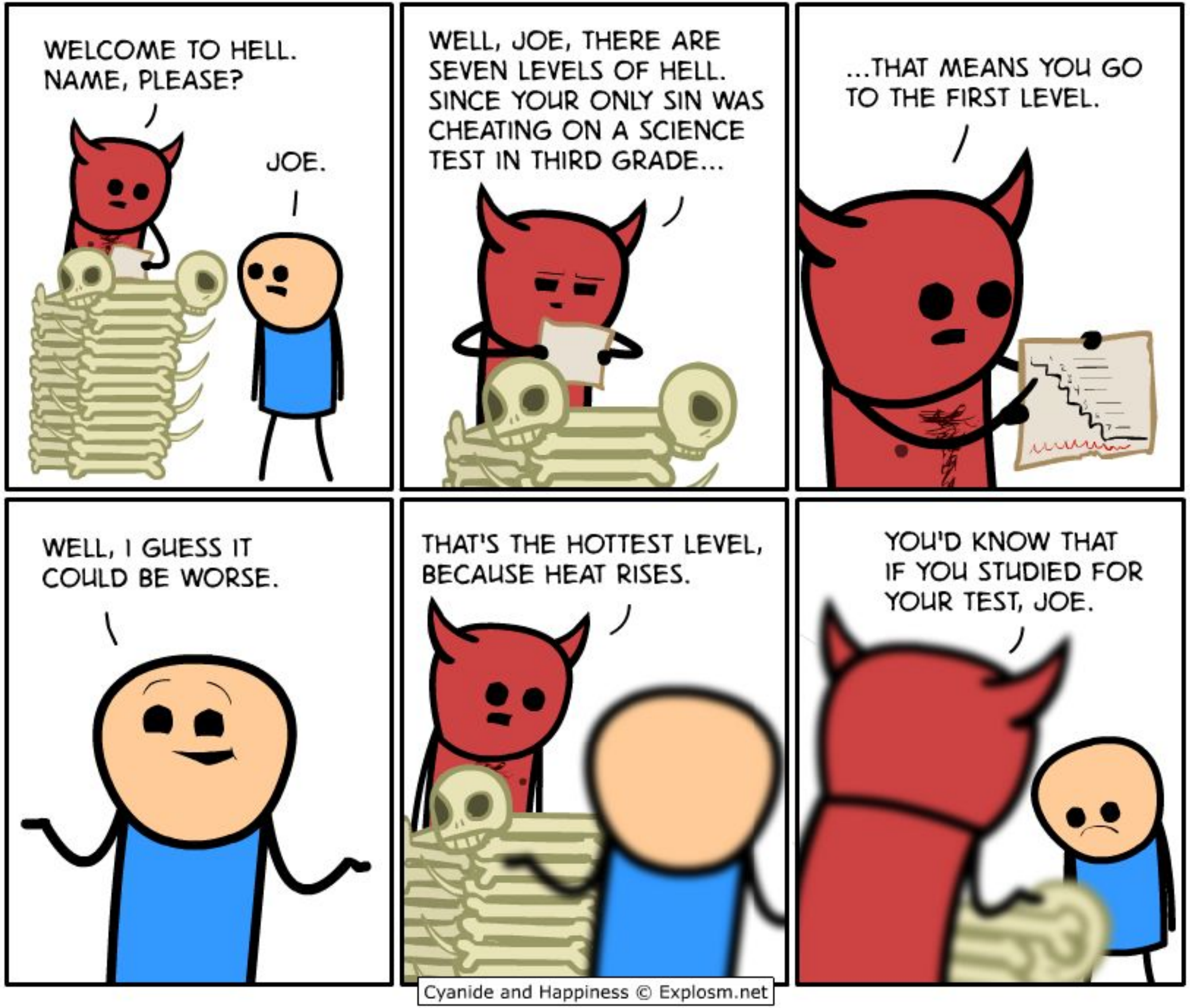
Midterm 1 Review

Midterm Review Day!

HW 5 due Monday 8 Oct

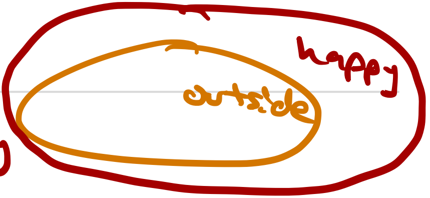
Midterm 1:

- **6:30-8p tomorrow**
- Rachel (001): HUMN 1B50
  Tony (002): DUAN G1B30
- 8.5 x 11” sheet of notes
  (cheat sheet)
- Calculators are okay.
  Smart phones are not.



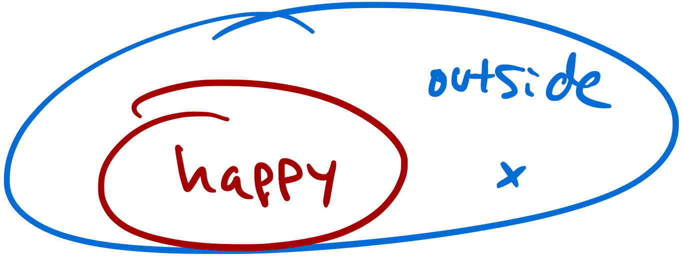
Only if stuff

Tony is happy if he is outside. outside → happy



Tony is happy only if he is outside.

$\boxed{\text{Tony happy}} \to \boxed{\text{outside}}$

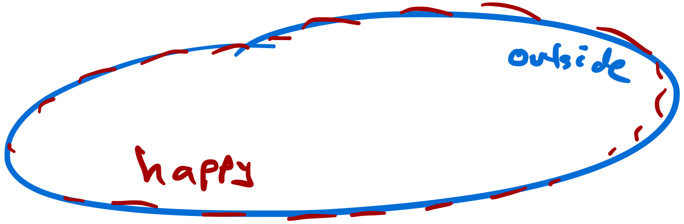


outside is necessary for Tony's happiness

Tony being happy is sufficient to know Tony is outside

Biconditional

Tony happy if & only if Tony is outside



$$\overline{A \cap B} = \bar{A} \cup \bar{B} \rightsquigarrow \neg(A \wedge B) \equiv \neg A \vee \neg B$$

Power sets: the set of all subsets of a given set

$A = \{\underbrace{0, 1}\}$ $\rightarrow$ $P(A) = \underbrace{\{\phi, \{0\}, \{1\}, \{0,1\}\}}$

- 2 elements
- each one is either in or out (but not both!) of a given subset of A $\rightarrow$ 2 options for each of 2 elts.
- $\underline{\underline{2^{2}}}$ total subsets of A (in general: $2^{|A|}$)

$$\underbrace{P(P(A))}_{\uparrow} = \{\phi, \underbrace{\{\phi\}, \{\{0\}\}, \{\{1\}\}, \{\{0,1\}\}}_{\text{contain only 1 elt.}}, \underbrace{\{\phi, \{0\}\}, \{\phi, \{1\}\}, \{\phi, \{0,1\}\}, \{\{0\}, \{1\}\}, \{\{0\}, \{0,1\}\}, \{\{1\}, \{0,1\}\}}_{\text{subsets containing 2 elts. of } P(A)}, \ldots$$

$2^{|P(A)|} = 2^{2^2} = 16$ elts.

Example: Prove that a solution to $x^2 - 2x + 1 = 0$ exists.

Proof: (by construction)

S'pose x solves $x^2 - 2x + 1 = 0$. Let's find it!

$$\Rightarrow (x-1)^2 = 0 \qquad x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(1)}}{2(1)} = \frac{2 \pm \sqrt{\cancel{4-4}}^{\,0}}{2} = 1$$

$$\Rightarrow \underline{\underline{x = 1}}$$

$\therefore$ a solution $(x=1)$ exists! ✓

Proof of uniqueness:

S'pose z & $\cancel{y}$ are both solutions to $x^2 - 2x + 1 = 0$

$$\Rightarrow 0 = \cancel{z^2 - 2z + 1} \overset{\text{AND}}{=} y^2 - 2y + 1$$

$$\Rightarrow (z-1)^2 = (y-1)^2 \Rightarrow \pm(z-1) = \pm(y-1)$$

Example of uniqueness: Prove that a solution to $2x + 4 = 0$ is unique.

Proof: (of uniqueness)

S'pose y & z both are solutions to $2x+4=0$

$\longrightarrow$ $2y + 4 = 0 \overset{\&}{=} 2z + 4$

$\longrightarrow$ $2y = 2z$

$\longrightarrow$ $\underline{y = z}$

$\therefore$ the solution must be unique.

From HW 4:

Prove that if a, b, c, d, e, f are real #'s such that $ad - bc \neq 0$, then $\begin{cases} ax + by = e \\ cx + dy = f \end{cases}$ has a solution (x, y) where x & $y \in \mathbb{R}$.

Proof: S'pose a–f are real #'s s.t. $ad - bc \neq 0$.

Now let's construct our solution x, y ...

$\vdots$

$$x = \frac{\sim\sim\sim}{ad - bc} \quad \& \quad y = \frac{\sim\sim\sim}{ad - bc}$$

$\leftarrow$ now we know x & y $\in \mathbb{R}$ b/c den. $\neq 0$

Prove that $2^{3/2}$ is irrational.

$\mathbb{Z}$ integers

Proof: (by contradiction)

S'pose (FSOC) that $2^{3/2}$ is rational.

$\rightarrow \exists\, a,b \in \mathbb{Z}$ ($b \neq 0$ & a,b have no common factors) s.t. $2^{\frac{3}{2}} = \frac{a}{b}$

$\rightarrow 2^{3/2} b = a$

$\rightarrow 2^3 b^2 = a^2$

$\rightarrow 8b^2 = a^2$

$\rightarrow a^2$ is even

$\rightarrow a$ is also even

$\rightarrow \exists\, k \in \mathbb{Z}$ s.t. $a = 2k$

$\rightarrow 8b^2 = (2k)^2 = 4k^2$

$\rightarrow 2b^2 = k^2$

$\rightarrow k^2$ even $\rightarrow k$ even $\rightarrow k = 2l$ (some $l \in \mathbb{Z}$)

$\rightarrow 2b^2 = (2l)^2 = 4l^2$

(cont.)

n is prime iff n is only divisible by 1 & itself

$\rightarrow b^2 = 2\ell^2$

$\rightarrow b^2$ even

$\rightarrow b$ even $\rightarrow$ $\underline{b = 2m}$ for some integer m

$\rightarrow\leftarrow$ contradiction! b/c a & b both have a factor of 2.

$\therefore 2^{3/2}$ is irrational

Useful starting points

- even/odd
- rational #'s
- prime / not prime
- divisibility / multiple of