Extra credit opportunity!   Purely attendance-based.

People respond to incentives and we **know attendance correlates with course grade**

- Consider this carefully if the exams/homeworks are not going as well as you'd like
- Download **Arkaive** -- attendance app (or go to https://arkaive.com/login)
- Enroll in this course with enrollment code: **1KG4** (9 AM w/ Rachel)
  R422 (11 AM w/ Tony)
- During first 15 minutes of each class, check in on Arkaive. You can get credit for attending either section, but not both.
- Remainder of course will be worth **2% pts extra credit** (*before* factored into any final curve)

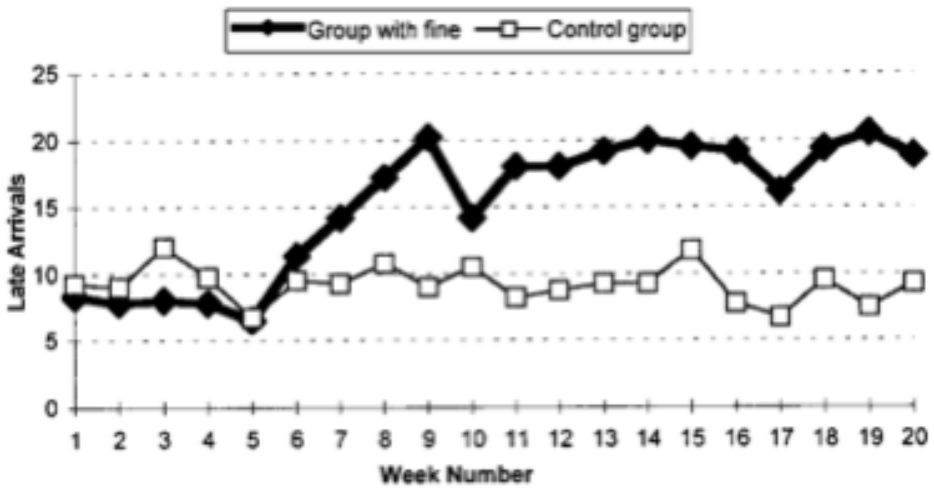

FIGURE 1.—Average number of late-coming parents, per week

1 0 1 1 0 0 0 0 1 0 0 0 1 0 1 1 0 0 0 0 1 0 0 0 1 0 1 1 0 0 0 0 1 0 0 0 1 0 1 1 0 0 0 0 1 0 0 0 1 0 1 1 0 0 0 0 1 0 0 0

CSCI 2824: Discrete Structures

Lecture 20: Weak Induction

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Weak Induction

Suppose we have an infinite line of dominoes.

Prove that if we tip over the first domino, the rest of them will fall.

<u>Argument could go like this</u>:
Base Case: The first domino falls (because we knock it over)
Induction Step: Whenever the kth domino falls, then its successor k+1 also falls.
Therefore, we conclude that all the dominoes will fall.

Weak Induction

This argument is an example of **induction**!

To prove a property over all natural numbers k, we may argue as follows:

The property is true for *k=0* (or *k=1*, etc).
If the property is true for some natural number *k*, then it is true for natural number *k+1*

Weak Induction

Example: Prove that $1 + 2 + 3 + \cdots + n = \frac{n(n+1)}{2}$

Weak Induction

Example(continued): Prove that $1 + 2 + 3 + \cdots + n = \frac{n(n+1)}{2}$

Mathematical Induction

Let $P(n)$ be the property that we're trying to prove.

An inductive argument goes as follows:

Base Case: Verify that $P(0)$ holds

Induction Step: $(\forall k \geq 0)$ If $P(k)$, then $\mathrm{P(k+1)}$

Conclusion: $(\forall n \geq 0)\ P(n)$

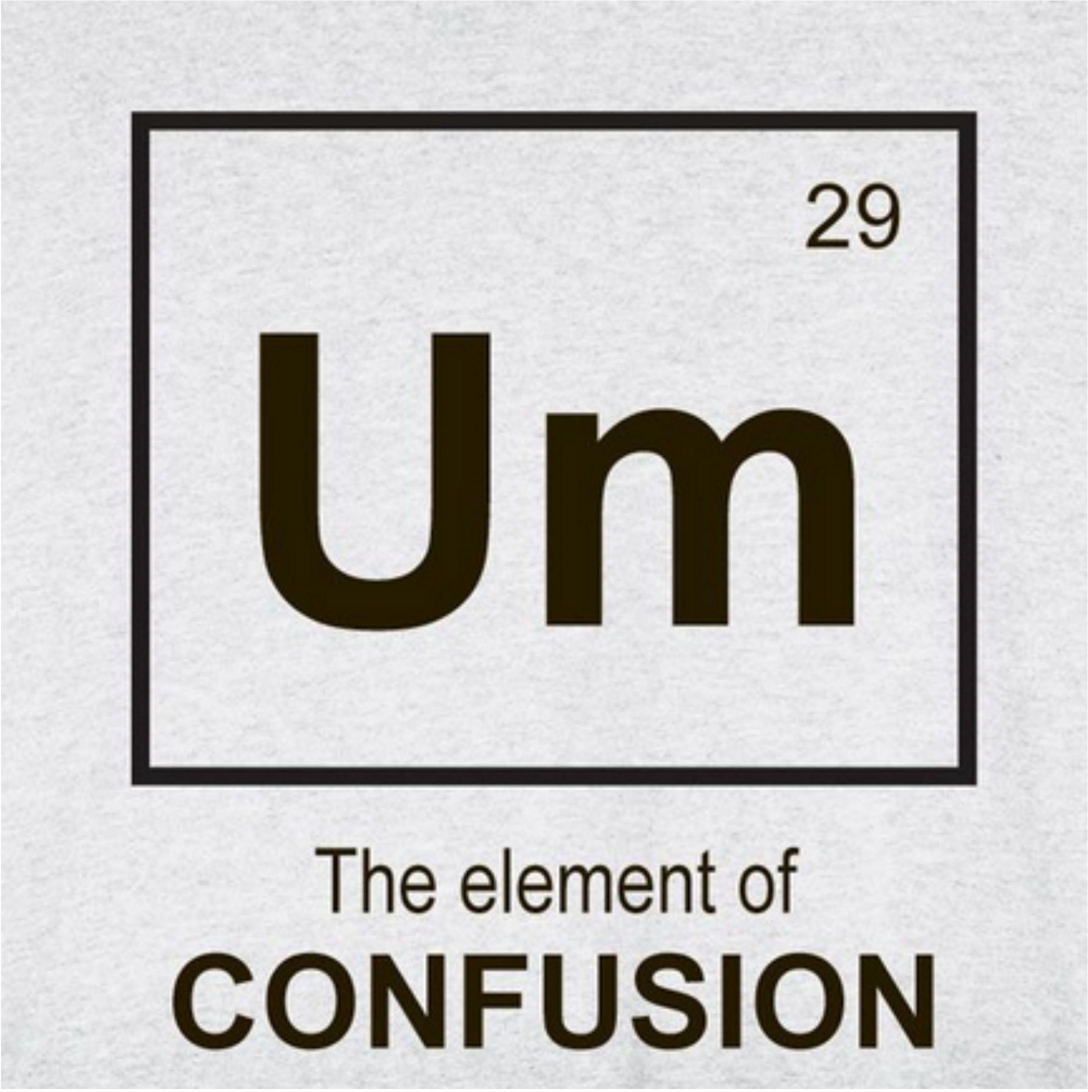
Mathematical Induction

Weak Induction:

- Verify that $P(1)$ is true.
- Assume $P(k)$ is true and show that $P(k+1)$ is true.

Strong Induction:

- Verify that $P(1)$ is true.
- Assume $P(k)$ for all $k = 1, 2, \dots, n$ and show $P(n+1)$



Mathematical Induction

ELIF (great analogies from reddit users TorsionFree & wildgurularry):

In both strong and weak induction, you must prove that the first domino in the line falls. i.e. the first logical proposition is true – base case.

Analogy 1: To prove that ALL the other dominoes fall, you either show why (1) each falling domino by itself causes the next one to fall, or (2) all the dominoes that have fallen up to a point will cause the next one to fall. Tactic 1 is called weak induction, tactic 2 is called strong induction.

Analogy 2: Weak induction only cares about the ladder rung you are currently standing on. As long as that one exists, you know that you can step up to the next rung. For strong induction, you need all the previous rungs to still exist before you can safely move up to the next rung.

Weak Induction

Example: Propose a formula for the sum of the first n odd positive integers. Then prove it with induction.

Weak Induction

Example (continued): Propose a formula for the sum of the first n odd positive integers. Then prove it with induction.

Weak Induction

Example: Prove that if n is an integer and $n \geq 4$ then $2^n < n!$.

Weak Induction

Example (continued): Prove that if n is an integer and $n \geq 4$ then $2^n < n!$.

What we’ve done:

- ❖ Weak Induction

Next:

Strong Induction!