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CSCI 2824: Discrete Structures

Lecture 21: Strong Induction

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Mathematical Induction

Weak Induction:

- Verify that $P(1)$ is true.
- Assume $P(k)$ is true and show that $P(k+1)$ is true.

Strong Induction:

- Verify that $P(1)$ is true.
- Assume $P(k)$ for all $k = 1, 2, \ldots, n$ and show $P(n+1)$

Argument: $P(1) \wedge P(2) \wedge \ \ldots \wedge P(k) \rightarrow P(k+1)$

Strong Induction

Example: If n is a positive integer then it can be written as the sum of distinct Fibonacci numbers.

Strong Induction

<u>Example (continued1)</u>: If n is a positive integer then it can be written as the sum of distinct Fibonacci numbers.

Strong Induction

Example (continued2): If n is a positive integer then it can be written as the sum of distinct Fibonacci numbers.

Strong Induction

Example: For any integer $n \geq 1$ there exist numbers $a, b \geq 1$ such that $5^n = a^2 + b^2$

Strong Induction

<u>Example (continued1)</u>: For any integer $n \geq 1$ there exist numbers $a, b \geq 1$ such that $5^n = a^2 + b^2$

Strong Induction

<u>Example (continued2)</u>: For any integer $n \geq 1$ there exist numbers $a, b \geq 1$ such that $5^n = a^2 + b^2$

Strong Induction

<u>Example</u>: It used to be that throughout the world McDonald's sold chicken nuggets in 4, 6, 9, and 20 piece boxes. Prove that back in the good old days, for any $n \geq 12$ you could buy exactly n nuggets.

Example (continued): It used to be that throughout the world McDonald's sold chicken nuggets in 4, 6, 9, and 20 piece boxes. Prove that back in the good old days, for any $n \geq 12$ you could buy exactly n nuggets.

Induction: *bad* examples

(Bad) Example: "Prove" that all Fibonacci numbers are even

Base case: Let $n = 0$. $F_0 = 0$, which is even ✔

Induction step: Assume that F_ℓ is even for all ℓ s.t. $0 \leq \ell \leq k$

To show: F_{k+1} is even

⇒ $F_{k+1} = F_k + F_{k-1}$

⇒ By **induction hypothesis**, F_k and F_{k-1} are both even

⇒ F_{k+1} is the sum of two evens, and therefore even "✔"

Induction: *bad* examples

(Bad) Example: "Prove" that all Fibonacci numbers are even

Mistake: We only showed F_0 is even, but then we used $F_{k+1} = F_k + F_{k-1}$ in our proof

⇒ If you think of our "proof" as a line of dominoes, each domino falling in this case requires **two** previous ones to fall

⇒ Here, we only did one base case, so we only knocked over **one** domino

Rule of thumb: If your proof requires going back *s* steps, then you need *s* base cases.

Next up: **Recursion**!