Warm-up problem

Example: Add $11011_2 + 1101_2$. **Sanity check:** Convert to decimal and check your answer.

Announcements and Reminders

- ❒ Enroll in the class Moodle. Keys: *csci2824-Tony* or *csci2824-Rachel*
- ❒ First homework (on Moodle) is due **Friday 7 September at 12pm Noon**
  -- Meant to mimic written work: 1 attempt/problem // infinite for CodeRunner problems
  -- Clicking "Check" **locks your answer in** - don't do it
- ❒ Enroll in the class Piazza: https://piazza.com/colorado/fall2018/csci2824
  -- Message via Piazza -- Please no emails :)
- ❒ Keep track of everything via the schedule: https://goo.gl/DFuboZ

Quizlet 1 posted! Due Wed. 5 Sept

→ opens at 12p today, closes at 8a Wed

→ 3 attempts/problem

→ Linked from the course calendar, or just go to Moodle & do it

Notes about Moodle behavior:

If you type in an answer / check a box / etc,

& then navigate away w/out clicking "check",

① Moodle will remember! It's okay

② Moodle will automatically submit any such answers when the HW is due.

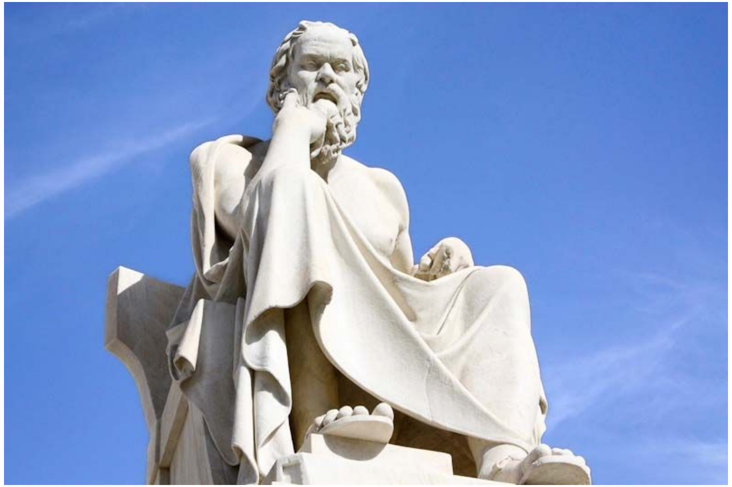
Warm-up problem

Example: Add $11011_2 + 1001_2$

```
  11011     →  2^0 + 2^1 + 2^3 + 2^4 = 1 + 2 + 8 + 16 = 27
+  1001     →  2^0 + 2^3 = 1 + 8 = 9
--------
 100100     →  2^2 + 2^5 = 4 + 32 = 36,
               and 27 + 9 does indeed = 36. Yay!
```

$$
\begin{array}{rl}
11011 & \to 2^0 + 2^1 + 2^3 + 2^4 = 1 + 2 + 8 + 16 = 27 \\
+\ 1001 & \to 2^0 + 2^3 = 1 + 8 = 9 \\
\hline
100100 & \to 2^2 + 2^5 = 4 + 32 = 36, \\
 & \text{and } 27 + 9 \text{ does indeed} = 36.\ \text{Yay!}
\end{array}
$$

Lecture 3: Propositional Logic



Propositional logic: an introduction

- Gives precise meaning to mathematical statements.
- Statements have values of either **valid** or **invalid**.

- An important learning objective of this class is to understand and construct mathematical arguments.
- So we begin with **logic**.

Propositional logic: an introduction

Definition: The basic building block of logic is a **proposition**. A proposition is a declarative statement that is either true or false, but not both.

Example: The following are propositions.

1) Boulder is a city in Colorado
2) The capital city of Colorado is Golden
3) 2+2 = 5
4) 2+3 = 5

Propositional logic: an introduction

Definition: The basic building block of logic is a **proposition**. A proposition is a declarative statement that is either true or false, but not both.

Example: The following are *not* propositions.

1) Try hard
2) How is the weather?
3) 6
4) $x+3 = 5$

Propositional logic: an introduction

Definition: The **truth value** of a proposition is **true** (denoted T) if the proposition is true; it is **false** (denoted F) if the proposition is false.

Sometimes, for brevity, we use a symbol/variable name to denote a proposition.

Example: Let

- *p* represent the proposition "Boulder is a city in Colorado" : $p = T$
- q = "2+2=5"

$x+3=5$ & $x=2$ $= T$

$x=2$ ← not a prop.

Propositional logic: an introduction

We can combine propositions like p and q in exciting new ways.

Definition: Let p be a proposition. The **negation** of p, denoted by $\neg p$, is the proposition "It is not the case that p". The truth value of $\neg p$ is the opposite of the truth value of p.

$\neg p$ = "it is not the case that Boulder is a city in CO"

$\neg p = F$

Propositional logic: an introduction

We can combine propositions like *p* and *q* in exciting new ways.

Definition: Let *p* be a proposition. The <u>**negation**</u> of *p*, denoted by $\neg p$, is the proposition "It is not the case that *p*". The truth value of $\neg p$ is the opposite of the truth value of *p*.

Example: Let *p* denote the proposition "Tony is wearing red shoes"

- Then $\neg p$ is the proposition "It is not the case that Tony is wearing red shoes"
- Or more simply, "Tony is not wearing red shoes"

$\neg p = T$

$p = F$

Propositional logic: an introduction

We will be combining/modifying propositions, so things could get complicated.

Definition: It is convenient to tabulate of the possible truth values for the various configurations of the propositions. This is done using a **truth table**.

Given simple propositions *p* and *q*, the truth table allows us to enumerate all possible truth values of combinations of *p* and *q*.

p	$\neg p$
T	F
F	T

Propositional logic: an introduction

We will be combining/modifying propositions, so things could get complicated.

Definition: It is convenient to tabulate of the possible truth values for the various configurations of the propositions. This is done using a **truth table**.

Given simple propositions *p* and *q*, the truth table allows us to enumerate all possible truth values of combinations of *p* and *q*.

The truth table for a proposition *p* and $q = \neg p$ is:

p	$\neg p$

Propositional logic: an introduction

We can start combining propositions. We do this using logical operators called **connectives**.

Some examples:

- **conjunction**: “and”, denoted $\wedge$
- **disjunction**: “or”, denoted $\vee$
- **conditional**: “if-then”, denoted $\rightarrow$
- **biconditional**: “if and only if”, denoted $\Leftrightarrow$

Propositional logic: an introduction

Definition: Let p and q be propositions. The **conjunction** of p and q, denoted by $p \wedge q$, is the proposition "p and q". The conjunction $p \wedge q$ has the truth value T if both p and q are T and is F otherwise.

Example:

Let p = "it is dark outside" and q = "my house is haunted"

$p \wedge q$

Propositional logic: an introduction

Definition: Let p and q be propositions. The **conjunction** of p and q, denoted by $p \wedge q$, is the proposition "p and q". The conjunction $p \wedge q$ has the truth value T if both p and q are T and is F otherwise.

Example:

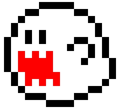
Let p = "it is dark outside" and q = "my house is haunted"

- "It is dark outside ***and*** my house is haunted" …

- "It is light outside, but my house is still haunted" … $\neg p \wedge q$

  $\neg p$ $\quad$ $\wedge$, w/ surprise! $\quad$ q

Propositional logic: an introduction



Definition: Let p and q be propositions. The **conjunction** of p and q, denoted by $p \land q$, is the proposition "p and q". The conjunction $p \land q$ has the truth value T if both p and q are T and is F otherwise.

Example:

Let p = "it is dark outside" and q = "my house is haunted"

- "It is dark outside ***and*** my house is haunted" … is the conjunction $p \land q$

- "It is light outside, but my house is still haunted" … is the conjunction $\neg p \land q$

Propositional logic: an introduction

Let's write down the truth table for the conjunction of *p* and *q*.

We need to write down all the possible combinations of truth values for *p* and *q*, and the resulting truth value for the conjunction $p \wedge q$.

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

Propositional logic: an introduction

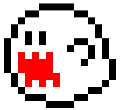
Let's write down the truth table for the conjunction of *p* and *q*.

We need to write down all the possible combinations of truth values for *p* and *q*, and the resulting truth value for the conjunction $p \wedge q$.

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

Propositional logic: an introduction

$p \vee q$ or $p \mid q$



Definition: Let p and q be propositions. The **disjunction** of p and q, denoted by $p \vee q$, is the proposition "p or q". The disjunction $p \vee q$ has the truth value T if either p or q are T and is F otherwise.

Example:

Let p = "it is dark outside" and q = "my house is haunted"

- "It is dark outside or my house is haunted" … is the proposition $p \vee q$

Propositional logic: an introduction

Let's write down the truth table for the disjunction $p \vee q$.

Note that this is also called an "inclusive or"

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

Propositional logic: an introduction

Let’s write down the truth table for the disjunction $p \vee q$.

Note that this is also called an “inclusive or”

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

Propositional logic: an introduction

For the case of my potentially haunted house, the *inclusive or* (i.e., it could be both) made sense.

Propositional logic: an introduction

For the case of my potentially haunted house, the *inclusive or* (i.e., it could be both) made sense.

Example: What about this one?

> Karen can date Jeff or Ed.
> (Maybe we should ask Jeff and Ed how they feel about this.)

Propositional logic: an introduction

For the case of my potentially haunted house, the *inclusive or* (i.e., it could be both) made sense.

Example: What about this one?

> Karen can date Jeff or Ed.
> (Maybe we should ask Jeff and Ed how they feel about this.)

This is an example of the *exclusive or* (i.e., one or the other, but not both). $p \oplus q$

Definition: Let p and q be propositions. The **exclusive or** of p and q, denoted $p \oplus q$, is the proposition that is true when exactly one of p or q is true, and false otherwise.

- Also abbreviated as “xor” sometimes

Propositional logic: an introduction

Let's write down the truth table for the exclusive "or": $p \oplus q$

p	q	$p \oplus q$
T	T	F
T	F	T
F	T	T
F	F	F

Propositional logic: an introduction

if p, then q : $p \to q$

Definition: Let p and q be two propositions. The **conditional** "if p then q", denoted by $p \to q$**,** is false when p is true but q is false, and true otherwise.

- The conditional describes an *if-then* relationship between the two propositions.
- Think of the conditional $p \to q$ as defining a rule. What are the cases where the rule holds or where the rule is broken.

Propositional logic: an introduction

Definition: Let p and q be two propositions. The **conditional** "if p then q", denoted by $p \rightarrow q$**,** is false when p is true but q is false, and true otherwise.

- The conditional describes an *if-then* relationship between the two propositions.
- Think of the conditional $p \rightarrow q$ as defining a rule. What are the cases where the rule holds or where the rule is broken.

Example: "If I go for a run, then I am happy."

p = "I go for a run"

q = "I am happy"

In symbols: $p \rightarrow q$ if [I go for a run], then [I am happy]

Example: "If I go for a run, then I am happy."

p = "I go for a run"

q = "I am happy"

In symbols: $p \to q$

Rule:

if [I go for a run], then [I am happy]

p	q	$p \to q$
T	T	T
T	F	F
F	T	T
F	F	T

How about a truth table for $p \to q$?

p=T, q=T → I ran, & I am happy

p=F, q=F → I didn't run & I'm unhappy

→ Rule doesn't apply if I didn't run

p=F, q=T → I didn't run but I'm happy

p=T, q=F → I ran, & I'm happy

Example: “If I go for a run, then I am happy.”

p = “I go for a run”

q = “I am happy”

In symbols: $p \rightarrow q$

if [I go for a run], then [I am happy]

How about a truth table for $p \rightarrow q$?

- p=T, q=T: *I go for a run* and *I am happy*.
  - Consistent with our rule, so $p \rightarrow q$ is **true**.
- p=F, q=F: *I didn’t go for a run* and *I am not happy*.
  - Rule is not broken, so $p \rightarrow q$ is **true**.
- p=F, q=T: *I didn’t go for a run* and *I am happy.*
  - Rule is not broken (***why?***), so $p \rightarrow q$ is **true**.
- p=T, q=F: *I go for a run* and *I am not happy*.
  - If going for a run implies that I should be happy, then this is **not consistent** with our rule, so $p \rightarrow q$ is **false**.

Use those results to fill in the truth table for the conditional $p \to q$ is as follows.

p	q	$p \to q$
T	T	T
T	F	F
F	T	T
F	F	T

Example: Translate the following plain English conditionals into symbols.

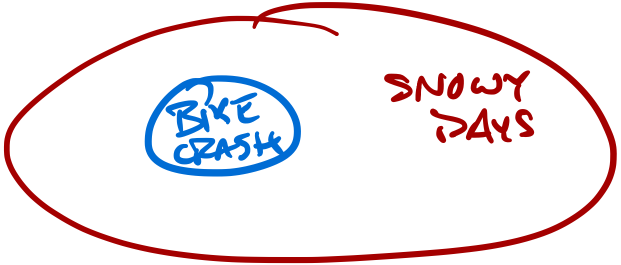
(a) *If it snows, then I crash my bicycle.*

Example: Translate the following plain English conditionals into symbols.

(a) *If it snows, then I crash my bicycle.*

Define: p = "it snows" and q = "I crash my bike"

and this statement can be expressed as $p \rightarrow q$.



(b) What about: *I crash my bicycle only if it snows.*

$$\underbrace{\text{I crash my bicycle}}_{q} \text{ only if } \underbrace{\text{it snows}}_{p}$$

$$q \rightarrow p$$

There are no bike crashes outside of snowy days

Watch out for "only if"

Example: Translate the following plain English conditionals into symbols.

(a) *If it snows, then I crash my bicycle.*

Define: p = "it snows" and q = "I crash my bike"

and this statement can be expressed as $p \rightarrow q$.

(b) What about: *I crash my bicycle only if it snows.*

Caution! Which of these events ensures the other?

Example: Transla

(a) *If it sno*

Define: p = "i

and this state

Note: Just because the "if" was attached to *it snows* does not mean that's the "if" part of the conditional.

Even though *causality* might work that direction (the snow caused me to crash), *logic/information* does not (you know it's snowing *because* I crashed).

(b) What about: *I crash my bicycle only if it snows.*

Caution! Which of these events ensures the other?

$q \to p$

Propositional logic: an introduction

HERE

Definition: Let p and q be two propositions. The **<u>biconditional</u>** "p if and only if q", denoted by $p \Leftrightarrow q$**,** or p **iff** q, is true when p and q have the same truth value, and false otherwise.

- The conditional describes an *if-and-only-if* relationship between the two propositions.

Example: *A polygon is a triangle if and only if it has exactly 3 sides.*

- p = a polygon is a triangle
- q = a polygon has exactly 3 sides
- In symbols: $p \Leftrightarrow q$

Truth table for the biconditional**:**

Propositional logic: an introduction

Definition: Let p and q be two propositions. The **biconditional** "p if and only if q", denoted by $p \Leftrightarrow q$**,** or p **iff** q, is true when p and q have the same truth value, and false otherwise.

- The conditional describes an *if-and-only-if* relationship between the two propositions.

Example: *A polygon is a triangle if and only if it has exactly 3 sides.*

- p = a polygon is a triangle
- q = a polygon has exactly 3 sides
- In symbols: $p \Leftrightarrow q$

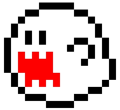
Truth table for the biconditional**:**

p	q	$p \Leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

Compound propositions

Compound propositions are constructed by linking together multiple simple propositions using connectives.

Compound propositions



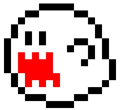
Compound propositions are constructed by linking together multiple simple propositions using connectives.

Example: Determine the truth table for the compound proposition $(p \rightarrow q) \land (q \rightarrow p)$

For example: “If think I see a ghost, then I am superstitious.”

and “If I am superstitious, then I think I see ghosts.”

Compound propositions

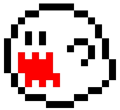


Compound propositions are constructed by linking together multiple simple propositions using connectives.

Example: Determine the truth table for the compound proposition $(p \to q) \land (q \to p)$

p	q	$p \to q$	$q \to p$	$(p \to q) \land (q \to p)$
T	T			
T	F			
F	T			
F	F			

Compound propositions



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Example: Determine the truth table for the compound proposition $(p \to q) \land (q \to p)$

p	q	$p \to q$	$q \to p$	$(p \to q) \land (q \to p)$
T	T	T	T	T
T	F	F	T	F
F	T	T	F	F
F	F	T	T	T

Compound propositions

Does the truth table for $(p \rightarrow q) \land (q \rightarrow p)$ look familiar?

p	q	$(p \rightarrow q) \land (q \rightarrow p)$
T	T	T
T	F	F
F	T	F
F	F	T

Compound propositions

Does the truth table for $(p \rightarrow q) \land (q \rightarrow p)$ look familiar?

… maybe to the **biconditional**, $p \Leftrightarrow q$?

p	q	$(p \rightarrow q) \land (q \rightarrow p)$
T	T	T
T	F	F
F	T	F
F	F	T

p	q	$p \Leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

Because the two propositions have the same truth values for all possible values of p and q, they are **logically equivalent**. (more on that later)

Propositional logic

Example: The island of **Knights and Knaves**. Suppose you are on an island where there are two types of people: *Knights* always tell the truth, and *Knaves* always lie

Suppose on this island, you encounter two people, Alfred and Batman. Let's call them *A* and *B* for short. Suppose *A* tells you "I am a Knave or *B* is a Knight". Use a truth table to determine what kind of people *A* and *B* are.

Propositional logic

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→ **Define propositions:** Let p = *A is a Knight* and q = *B is a Knight*

→ How can we represent *A*'s statement symbolically?

Propositional logic

Example: The island of **Knights and Knaves**. Suppose you are on an island where there are two types of people: *Knights* always tell the truth, and *Knaves* always lie

Suppose on this island, you encounter two people, Alfred and Batman. Let's call them *A* and *B* for short. Suppose *A* tells you "I am a Knave or *B* is a Knight". Use a truth table to determine what kind of people *A* and *B* are.

→ **Define propositions:** Let p = *A is a Knight* and q = *B is a Knight*

→ How can we represent *A*'s statement symbolically? … $\neg p \vee q$

→ How can we determine if *A* and *B* are knights or knaves?

Propositional logic

Example: The island of **Knights and Knaves**. Suppose you are on an island where there are two types of people: *Knights* always tell the truth, and *Knaves* always lie

Suppose on this island, you encounter two people, Alfred and Batman. Let's call them *A* and *B* for short. Suppose *A* tells you "I am a Knave or *B* is a Knight". Use a truth table to determine what kind of people *A* and *B* are.

→ **Define propositions:** Let p = *A is a Knight* and q = *B is a Knight*

→ How can we represent *A*'s statement symbolically? … $\neg p \vee q$

→ How can we determine if *A* and *B* are knights or knaves? … Look at $p \Leftrightarrow \neg p \vee q$

It **must** be T on this island that if A is a knight, then his statement is true, and if A's statement is true, then he must be a knight.

So any truth table row corresponding to ($p \Leftrightarrow \neg p \vee q$) = T is a possibility

Propositional logic

Example: So we test the proposition $p \Leftrightarrow \neg p \lor q$

p	q			
T	T			
T	F			
F	T			
F	F			

Propositional logic

Example: So we test the proposition $p \Leftrightarrow \neg p \lor q$

p	q	$\neg p$	$\neg p \lor q$	$p \Leftrightarrow \neg p \lor q$
T	T	F	T	T
T	F	F	F	F
F	T	T	T	F
F	F	T	T	F

Propositional logic

Example: So we test the proposition $p \Leftrightarrow \neg p \lor q$

- There is only one row that is consistent with what we know to be true about this island (that is, *A*'s statement is true if he is a Knight, and he is a Knight if his statement is true)
  (*note: there could be multiple possibilities with your [⇔] column = T*)
- Thus, *A* and *B* must **both be Knights**. What a relief!

p	q	$\neg p$	$\neg p \lor q$	$p \Leftrightarrow \neg p \lor q$
T	T	F	T	T
T	F	F	F	F
F	T	T	T	F
F	F	T	T	F

Propositional logic

FYOG: Suppose instead that *A* tells you “*B* is a Knight” and *B* tells you “The two of us are of different types”. Use a truth table to determine the sorts of people that *A* and *B* are. (aside from confusing)

(Extra tricky because you need to incorporate *both* person’s statements into a test proposition, which needs to ensure that each statement is consistent with that person’s type.)

Propositional logic

Recap:

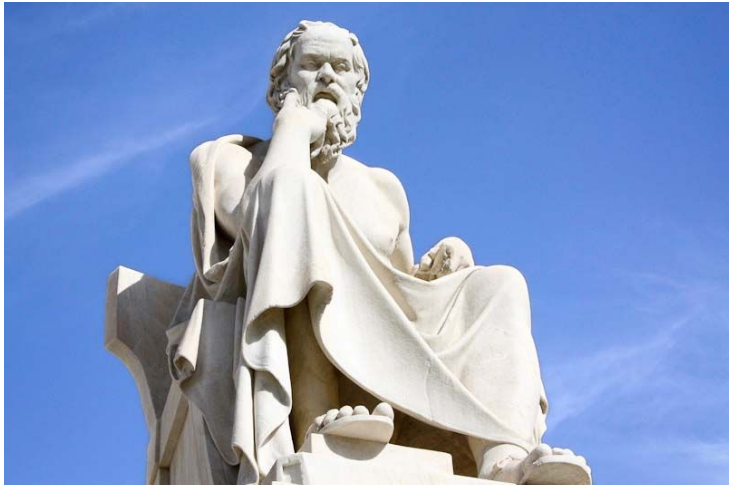
Today, we learned about…

- **Connectives**: *and*, *or*, *if-then (conditional)*, *if-and-only-if (biconditional)*
- **Truth tables**: got more complex, with compound propositions
- **Solving riddles using truth tables**: Knights and Knaves

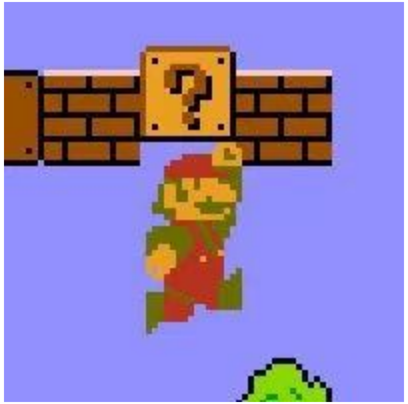
Next time:

- We talk **logical equivalence**

  → do *these* or *those* statements mean
  the same thing?

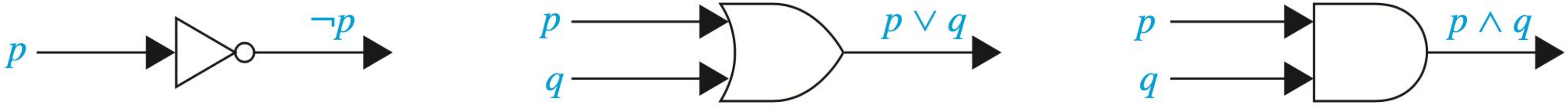


Bonus material!



Application: logic circuits ← not required material, but interesting

- Using propositional logic to design computer hardware.
- A logic circuit is an idealized circuit where either voltage (T) or no voltage (F) travel along inputs ("wires") into logical gates with varying outputs.
- There are three main types of logical gates:



Inverter OR gate AND gate

Application: logic circuits

- Using propositional logic to design computer hardware.
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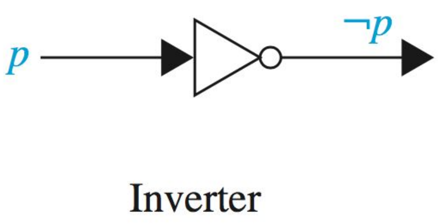
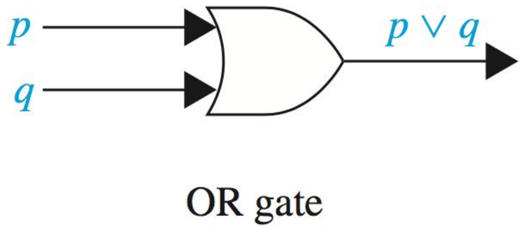
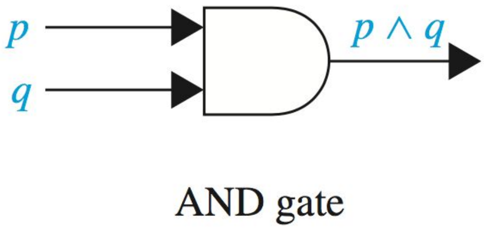
- There are three main types of logical gates:







Example: If the input to an *Inverter* has no voltage, that the output will have voltage.

Example: If either of the inputs to an *AND* gate has no voltage, that the output will have no voltage.

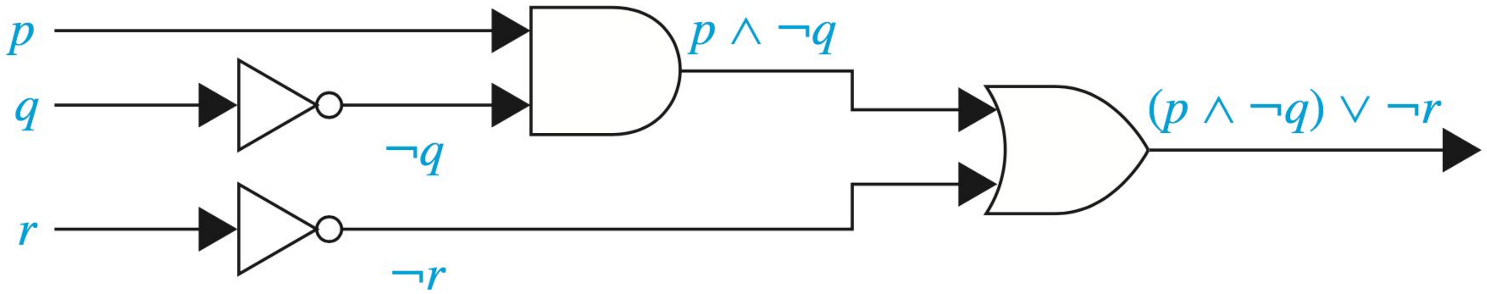
Application: logic circuits

- You can cook up more complex circuits with varying outputs, based on the flow of inputs.
- For example, *p*, *q* and *r* are each binary (voltage or no voltage).

  Say: *p* = user is pressing the "Ctrl" key

  q = user is pressing the "Alt" key

  r = user is pressing the "Delete" key

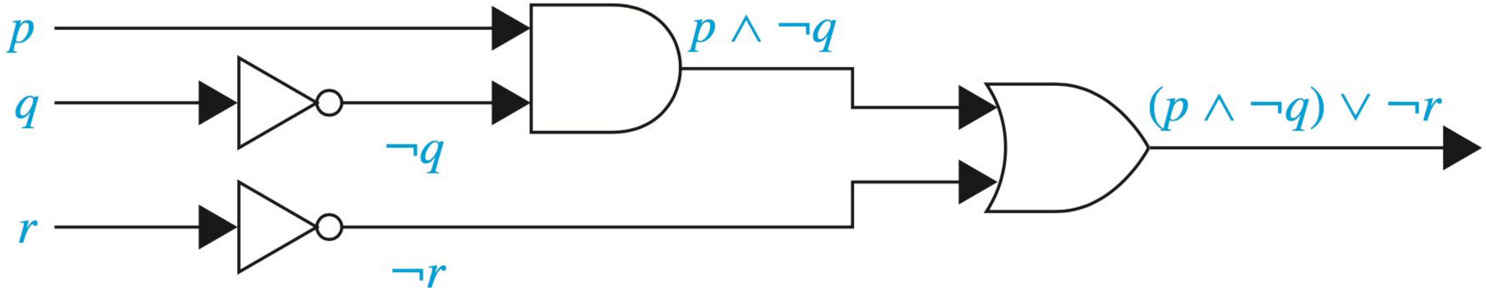


Example: What is a set of inputs for this circuit that leads to the last logic gate having an output with a voltage?

Application: logic circuits

Example: What is a set of inputs for this circuit that leads to the last logic gate having an output with a voltage?

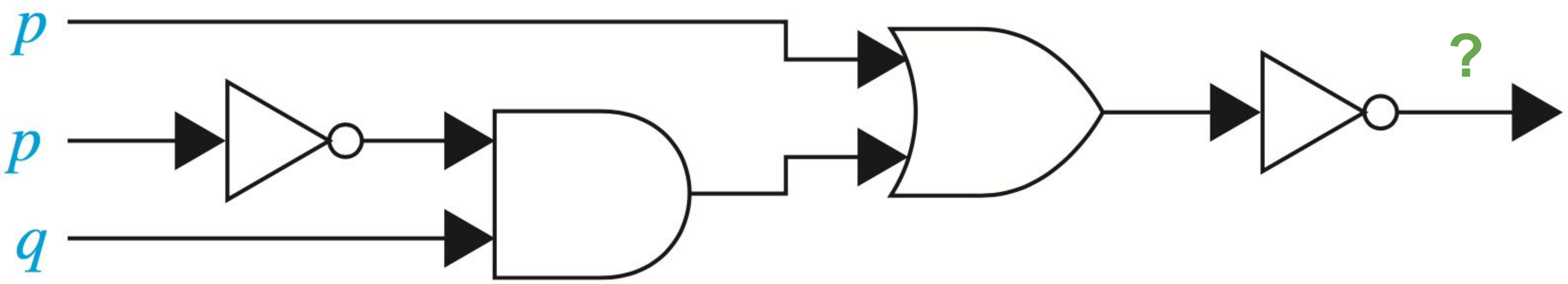
- **Easy answer:** Last connective has … $\vee \neg r$ on the end.
  ⇒ Anything where *r* has no voltage works
  because r is negated at its first gate, so if it is OFF to begin, then this line has voltage after the gate, and flows into the OR gate, which is ON if either of its inputs has voltage



- **More challenging answer:** Use a truth table!

Application: logic circuits

FYOG: What is the symbolic output for the following logic circuit?



Propositional logic

FYOG: Suppose instead that *A* tells you "*B* is a Knight" and *B* tells you "The two of us are of different types". Use a truth table to determine the sorts of people that *A* and *B* are. (aside from confusing)

Let p = "A is a knight" and let q = "B is a knight"

Then A's statement is q, which holds if and only if A is telling the truth: $p \Leftrightarrow q$

and B's statement can be represented in a few different ways.

- The simplest way is probably: $(p \wedge \neg q) \vee (\neg p \wedge q)$
- The most compact way is with the exclusive-or: $p \oplus q$

So to represent the fact that B's statement is true **if and only if** B is a knight (i.e., telling the truth), we have: $q \Leftrightarrow p \oplus q$

Now, *both* A and B have made statements, and we want to include *both* of them in our truth table test. Since we want to test both, we link them together with a conjunction:

FYOG (continued): Suppose instead that *A* tells you "*B* is a Knight" and *B* tells you "The two of us are of different types". Use a truth table to determine the sorts of people that *A* and *B* are.

Now, *both* A and B have made statements, and we want to include *both* of them in our truth table test. Since we want to test both, we link them together with a conjunction:

→ Test when this compound proposition is true: $(p \Leftrightarrow q) \wedge (q \Leftrightarrow p \oplus q)$

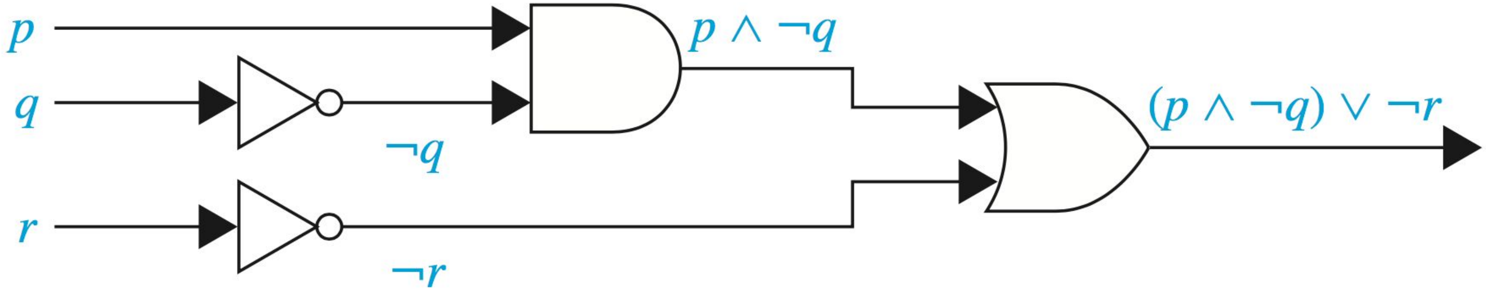
p	q	$p \Leftrightarrow q$	$p \oplus q$	$q \Leftrightarrow p \oplus q$	$(p \Leftrightarrow q) \wedge (q \Leftrightarrow p \oplus q)$
T	T	T	F	F	F
T	F	F	T	F	F
F	T	F	T	T	F
F	F	T	F	T	T

The only row where our compound proposition is T is the last one, so both A and B must be knaves.

Application: logic circuits

Example: What is a set of inputs for this circuit that leads to the last logic gate having an output with a voltage?

- **Easy answer:** Last connective has … $\vee \neg r$ on the end.
  $\Rightarrow$ Anything where *r* has no voltage works
  because r is negated at its first gate, so if it is OFF to begin, then this line has voltage after the gate, and flows into the OR gate, which is ON if either of its inputs has voltage



- **More challenging answer:** Use a truth table! ***r* ON, *p* ON, *q* OFF.**