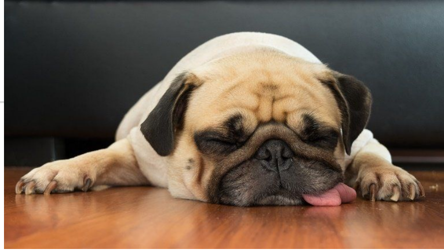
Warm-up problem



Translate the following statement using quantifiers, connectives and propositional functions. Be sure to define your domain(s)!

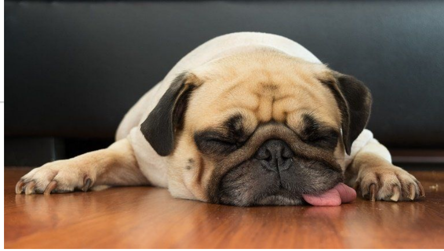
"There is someone who you can fool all of the time, if they are sleepy."

Announcements and reminders

& Quizlet 3 DUE MONDAY at 8 AM! Yay!

- Homework 2 (written) due **today at noon**.
    - **Read the instructions** at the top of the homework assignment.
    - You will be graded partially on **style** and **neatness**. (just like in life)
    - Multiple pages must be **stapled**. Do the problems **in order**.
    - **No torn-out-of-notebook fringe crap.**
    - Do **not** try to cram too much into a small space. When in doubt, start a new page.
    - **Write your full name at the top of the back page.**

Warm-up problem



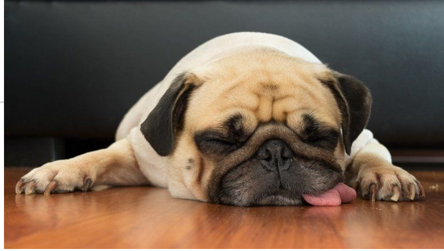
Translate the following statement using quantifiers, connectives and propositional functions. Be sure to define your domain(s)!

"There is someone who you can fool all of the time, if they are sleepy."

$\exists p \ \forall t \ (S(p,t) \rightarrow F(p,t))$

$S(p,t)$ = person p is sleepy at time t

Warm-up problem



Translate the following statement using quantifiers, connectives and propositional functions. Be sure to define your domain(s)!

"There is someone who you can fool all of the time, if they are sleepy."

Solution:

- Let $F(p, t)$ represent "you can fool person p at time t"
- Let $S(p, t)$ represent "person p is sleepy at time t"
- Let the domain for p be all people
- Let the domain for t be all time
- Then we have: $\exists p\ \forall t\ (S(p,t) \rightarrow F(p,t))$

Lecture 8: Rules of Inference

What did we do last time?

- We can represent standard propositions with quantifiers.
- We can translate statement from English to symbolic logical statements with quantifiers.
- We can prove and derive logical equivalences.

Today:

We will learn about:

1. the structure of arguments
2. identifying valid, sound and fallacious arguments
3. rules of inferences

Rules of inference

Next time we will discuss how to construct **mathematical proofs**.

- Mathematical proofs are **valid arguments** that establish the truth of a mathematical statement.

But first, we need to learn how to construct **valid arguments**

Think of an argument as a symbolic template that starts with some assumptions (called **premises**) and proceeds along a path of logical inferences to reach a **conclusion**.

Rules of inference

Next time we will discuss how to construct **mathematical proofs**.

- Mathematical proofs are **valid arguments** that establish the truth of a mathematical statement.

But first, we need to learn how to construct **valid arguments**

Think of an argument as a symbolic template that starts with some assumptions (called **premises**) and proceeds along a path of logical inferences to reach a **conclusion**.

Example: If Xerxes can bleed, then Xerxes is a mortal. Premise 1

Xerxes can bleed. Premise 2

Therefore, Xerxes is a mortal. Conclusion

This is an example of a specific valid argument.

Rules of inference

So now we need to cast this argument into a symbolic template. We can use our previous experience abstracting propositions from English to logical symbols.

Let:

- p denote "Xerxes can bleed"
- q denote "Xerxes is mortal"

Then in symbolic logic, our argument becomes

$\therefore$ = "therefore"

$$
\begin{array}{ll}
 & p \rightarrow q \\
 & p \\
\hline
\therefore & q
\end{array}
$$

Note: the symbol $\therefore$ means **therefore**.
Use this to denote the conclusion of the argument.

Rules of inference

An **argument** is a set of **premises** coupled with a **conclusion**.

A **valid** argument is an argument such that **there is no circumstance in which the premises could be true and the conclusion be false**.

Your intuition probably suggests that the previous argument is valid, but let's formalize this.

Rules of inference

An **argument** is a set of **premises** coupled with a **conclusion**.

A **valid** argument is an argument such that **there is no circumstance in which the premises could be true and the conclusion be false**.

Your intuition probably suggests that the previous argument is valid, but let's formalize this.

Consider the compound proposition: $\underbrace{((p \to q) \land p)} \to q$

Note that this is a conditional: $[(\text{premise } 1) \wedge (\text{premise } 2) \wedge \ldots] \to \text{conclusion}$

- The hypothesis is the conjunction of the premises of our argument, and
- the conclusion is the conclusion of our argument.

For our argument to be valid, it must be the case that there is no situation (i.e., truth values for *p* and *q*) in which the premises of the argument are true but the conclusion false.

We joined the premises with the conclusion in a conditional (as *premises* → *conclusion*)

- So for the argument to be valid, the conditional describing it must be **always true** (i.e., it needs to be a tautology)

Check with a truth table:

p	q	$p \to q$	$(p \to q) \land p$	$((p \to q) \land p) \to q$
T	T	T	T	
T	F	F	F	
F	T	T	F	
F	F	T	F	

Rules of inference

For our argument to be valid, it must be the case that there is no situation (i.e., truth values for *p* and *q*) in which the premises of the argument are true but the conclusion false.

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p	q	$p \rightarrow q$	$(p \rightarrow q) \land p$	$((p \rightarrow q) \land p) \rightarrow q$
T	T	T	T	T
T	F	F	F	T
F	T	T	F	T
F	F	T	F	T

Rules of inference

This general form of argument is so useful and common that it has a special and fancy name, and designate it as a **rule of inference**:

Modus Ponens:
"the way that affirms by affirming"

1. $p \rightarrow q$
2. p

$\therefore \; q$

Rules of inference are common, valid mini-arguments that we can link together to construct more complex valid arguments.

Let's prove a few more rules of inference that will be handy later.

Modus Tollens:
"the way that denies by denying"

1. $p \rightarrow q$
2. $\neg q$

$\therefore \quad \neg p$

Example: If it rains today, then my basement will flood.
My basement did not flood.

$\therefore$ It did not rain today.

We could prove this using a truth table and similar strategy to Modus Ponens, but let's try something fancier.

- This other way will also be useful to prove arguments that are too unwieldy for a truth table (recall that you need 2^N rows, where N is the number of propositions).

Modus Tollens: a proof!

Step	Justification
1. $p \to q$	Premise 1
2. $\neg q$	Premise 2
3. $\neg q \to \neg p$	contrapositive of #1
4. $\therefore \neg p$	Modus Ponens, using #2 & #3

fill in the intermediate logical connections!

using Logical equiv.'s & Rules of inference

$\blacksquare$

Rules of inference

Example: deriving Modus Tollens:

	Step	Justification
1.	$p \to q$	premise
2.	$\neg q$	premise
3.	$\neg q \to \neg p$	contraposition of (1)
4.	$\therefore \neg p$	Modus Ponens of (3) and (2)

starts by listing all premises

your steps so you can refer to them

Rules of inference

Disjunctive Syllogism:
Historically: *Modus Tollendo Ponens*

1. $p \lor q$
2. $\neg p$

$\therefore \quad q$

Example: My foot is disfigured or there is a rock in my shoe
My foot is not disfigured

$\therefore$ I have a rock in my shoe

Other sort-of example: "*Once you eliminate the impossible, whatever remains, no matter how improbable, must be the truth.*"

- Sherlock Holmes (Sir Arthur Conan Doyle, 1890:
The Sign of the Four, ch. 6)

Rules of inference

Example: deriving disjunctive syllogism:

	Step	Justification
1.	$p \lor q$	premise
2.	$\neg p$	premise
3.	$\neg p \to q$	RBD (#1)
4.	$\therefore$ q	Modus Ponens (#2 & 3)

Example: deriving disjunctive syllogism:

	Step	Justification
1.	$p \vee q$	premise
2.	$\neg p$	premise
3.	$\neg p \to q$	relation by implication, using (1)
4.	$\therefore q$	modus ponens, using (2) and (3)

FYOG: Prove that this is a valid argument using a truth table. : $[(p \vee q) \wedge (\neg p)] \to q$

FYOG: Prove that this is a valid argument by using a *different* sequence of inference rules and logical equivalences.

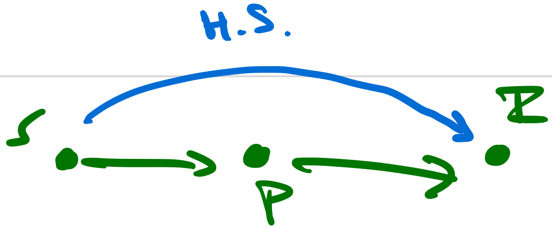
Example: What can you conclude from the following?

If it is sunny outside, then I will go to the park.

If I go to the park, then I will get ice cream

$\therefore$?

Rules of inference



Example: What can you conclude from the following?

If it is sunny outside, then I will go to the park.

If I go to the park, then I will get ice cream

$\therefore$ If it is sunny outside, then I will get ice cream

This is called **hypothetical syllogism**:

1. $p \to q$
2. $q \to r$

$\therefore \quad p \to r$

FYOG: Show that hypothetical syllogism is a valid rule of inference by showing that $((p \to q) \land (q \to r)) \to (p \to r)$ is a tautology.

Rules of inference

Here are a few more simple ones…

Simplification:

$$\frac{p \land q}{\therefore\ p}$$

(or $\dfrac{p \land q}{\therefore\ q}$)

Addition:

$$\frac{p}{\therefore\ p \lor q}$$

Conjunction:

$$\frac{\begin{matrix} p \\ q \end{matrix}}{\therefore\ p \land q}$$

… and one tricky one rule.

Argument :

Resolution:

$$\begin{array}{rl} & p \vee q \\ & \neg p \vee r \\ \hline \therefore & q \vee r \end{array}$$

$$[(p \vee q) \wedge (\neg p \vee r)] \rightarrow (q \vee r)$$

Intuition: *p* can either be true or false

- If *p* is true, then *r* must be true.
- If *p* is false, then *q* must be true.
- So either way, at least one of *q* or *r* must be true (or both).

H.S., which req. conditionals

Can we derive **resolution** using the rules of inference we already know?

	Step	Justification
1.	$p \vee q$	premise
2.	$\neg p \vee r$	premise
3.	$\neg p \to q$	RBE (1)
4.	$p \to r$	RBE (2)
5.	$\neg q \to p$	contrapos. (3)
6.	$\neg q \to r$	Hyp-Syll (4 & 5)
7.	$\therefore q \vee r$	RBE (6)

WHERE

Can we derive **resolution** using the rules of inference we already know?

	Step	Justification
1.	$p \lor q$	premise
2.	$\neg p \lor r$	premise
3.	$q \lor p$	commutativity, using (1)
4.	$\neg q \rightarrow p$	relation by implication, using (3)
5.	$p \rightarrow r$	relation by implication, using (2)
6.	$\neg q \rightarrow r$	hypothetical syllogism, using (4) and (5)
7.	$\therefore q \lor r$	relation by implication, using (6)

Rules of inference

It is raining $\therefore$ It is raining or snowing

Example: Use the rules of inference that you know so far to show that the following argument is valid.

$$\left.\begin{array}{l}(p \vee q) \rightarrow \neg r \\ \neg r \rightarrow s \\ p\end{array}\right\} \text{Premises}$$

$$\boxed{\therefore s} \quad \text{conclusion}$$

$$\left.\begin{array}{l}A \rightarrow B \\ A \\ \hline \therefore B\end{array}\right\} \text{Modus Ponens}$$

	Step	Justification
1.	$(p \vee q) \rightarrow \neg r$	Premise
2.	$\neg r \rightarrow s$	Premise
3.	p	Premise
4.	$(p \vee q) \rightarrow s$	Hypothetical Syllogism (1 & 2)
5.	$p \vee q$	Addition (3)
6.	$\therefore s$	Modus Ponens (4 & 5)

$$
\begin{array}{ll}
 & (p \vee q) \to \neg r \\
 & \neg r \to s \\
 & p \\
\hline
\therefore & s
\end{array}
$$

Example: Use the rules of inference that you know so far to show that the following argument is valid.

Start from the premises to end up at the conclusion:

	Step	Justification
1.	$(p \vee q) \to \neg r$	premise
2.	$\neg r \to s$	premise
3.	p	premise
??	(math occurs)	??
	$\therefore s$	magic?

	Step	Justification
1.	$(p \vee q) \to \neg r$	premise
2.	$\neg r \to s$	premise
3.	p	premise
4.		
5.		
6.	$\therefore s$	

Rules of inference

	Step	Justification
1.	$(p \vee q) \rightarrow \neg r$	premise
2.	$\neg r \rightarrow s$	premise
3.	p	premise
4.	$p \vee q$	addition, using (3)
5.	$\neg r$	modus ponens, using (1) and (4)
6.	$\therefore s$	modus ponens, using (2) and (5)

FYOG: Use the rules of inference that you know so far to show that the following argument is valid.

premises
$$\begin{array}{l} p \wedge q \\ p \rightarrow \neg r \\ q \rightarrow \neg s \\ \hline \therefore \; \neg r \wedge \neg s \end{array}$$
conclusion

$\neg n^2$ $q = F,\ \neg q = T$ $\neg n^2$ $q : T$

q

Example: What valid argument form is present in the following?

If $\overbrace{n \text{ is a real number with } n > 3}^{p}$, then $\overbrace{n^2 > 9}^{q}$.

Suppose that $\underbrace{n^2 \le 9}_{\neg q}$. Then $n \le 3$.

$p \to q$ premise 1

$\neg q$ premise 2

$\therefore \neg p$ conclusion

Rules of inference

Example: What valid argument form is present in the following?

If n is a real number with $n > 3$, then $n^2 > 9$. $p \to q$

Suppose that $n^2 \leq 9$. Then $n \leq 3$.

Solution:

Let p represent the statement n is a real number with $n > 3$

Let q represent the statement $n^2 > 9$

if $n = -4 \to p = F \to \neg p = T$

$\to \neg q = F \to q = T$

Then we have:

$$\boxed{\begin{array}{l} p \to q \\ \neg q \\ \text{-----------------} \\ \therefore \neg p \end{array}}$$

⇐ This is **Modus Tollens**

Rules of inference

Example: What valid argument form is present in the following?

If $\overbrace{\sqrt{2} > 3/2}^{p}$, then $\overbrace{(\sqrt{2})^2 > (3/2)^2}^{q}$. We know that $\sqrt{2} > 3/2$.

Consequently, $(\sqrt{2})^2 = 2 > (3/2)^2 = 9/4$.

Premise 1: $p \to q$

Premise 2: p

$\therefore \quad q$

Modus Ponens!

Example: What valid argument form is present in the following?

If $\sqrt{2} > 3/2$, then $(\sqrt{2})^2 > (3/2)^2$. We know that $\sqrt{2} > 3/2$.

Consequently, $(\sqrt{2})^2 = 2 > (3/2)^2 = 9/4$.

$2 > 2.25$ ... ??

Solution:

- This is demonstrating "if *p*, then *q*; *p*, therefore *q*"
  - Use *p* = the statement $\sqrt{2} > 3/2$
  - Use *q* = the statement $(\sqrt{2})^2 > (3/2)^2$
- That is **Modus Ponens** at work!

Question: Does something here not look quite right…?

Rules of inference

A **valid** argument is one where there is no way the conclusion can be false **if the premises are true** → are our premises true?

Valid arguments are patterns of logical reasoning.

A **valid** argument is one where there is no way the conclusion can be false **if the premises are true**

Valid arguments are patterns of logical reasoning.

But just because an argument is valid does not mean you can trust the conclusion.

In the previous example, the conclusion that 2 > 9/4 is very, very **false**.

The problem arises because the premise that $\underline{\sqrt{2} > 3/2}$ is **false**.

$1.4 \overset{?}{>} 1.5$

SOUND = VALID + PREMISES ARE TRUE

A **valid** argument is one where there is no way the conclusion can be false **if the premises are true**

Valid arguments are patterns of logical reasoning.

But just because an argument is valid does not mean you can trust the conclusion.

VALID, BUT NOT SOUND:

In the previous example, the conclusion that $2 > 9/4$ is very, very **false**.

The problem arises because the premise that $\sqrt{2} > 3/2$ is **false**.

- If 5 is even, then 5 is a prime #.
- 5 is even.

∴ 5 is prime

premises are False

So even though this argument is valid, it is not "useful" or "nice".

- We want to be able to tell which arguments are not only valid, but "nice" too.

When an argument is both **valid** *and* the **premises are true**, we call the argument <u>**sound**</u>.

Recap:

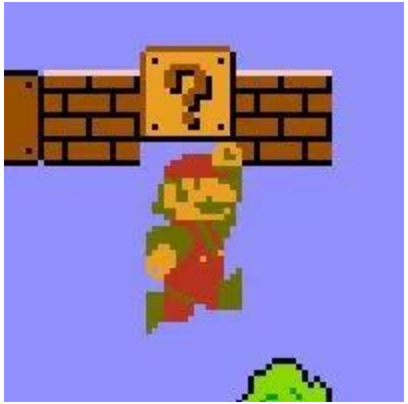
- We have learned the rules of inference, and how to use them to construct **valid** arguments. (good arguments)
- We have learned how to identify a **sound** argument. (great arguments)
- We have learned how to recognize common **fallacious** arguments. (awful arguments)

Next time:

- Rules of inference, continued
- We bring **quantifiers** into the mix!

  ("*for all*", "*there exist*")

Bonus material!



FYOG: Show that hypothetical syllogism is a valid rule of inference by showing that $((p \to q) \wedge (q \to r)) \to (p \to r)$ is a tautology.

Use a truth table -- what should the entire last column be, to show this is a tautology?

p	q	r	$p \to q$	$q \to r$	$(p \to q) \wedge (q \to r)$	$p \to r$	$((p \to q) \wedge (q \to r)) \to (p \to r)$
T	T	T					
T	T	F					
T	F	T					
T	F	F					
F	T	T					
F	T	F					
F	F	T					
F	F	F					

FYOG: Use the rules of inference that you know so far to show that the following argument is valid.

$$\begin{array}{l} p \land q \\ p \rightarrow \neg r \\ q \rightarrow \neg s \\ \hline \therefore \neg r \land \neg s \end{array}$$

	Step	Justification
1.	$p \land q$	premise
2.	$p \rightarrow \neg r$	premise
3.	$q \rightarrow \neg s$	premise
4.	p	Simplication (1)
5.	$\neg r$	Modus ponens (2, 4)
6.	q	Simplification (1)
7.	$\neg s$	Modus ponens (3, 6)
8.	$\therefore \neg r \land \neg s$	Conjunction (5, 7)