Questions from Last Time:

$.0625 = .0001$

Q: When converting from a decimal fraction to binary, will you always have either the same number of places or an infinite number of places?

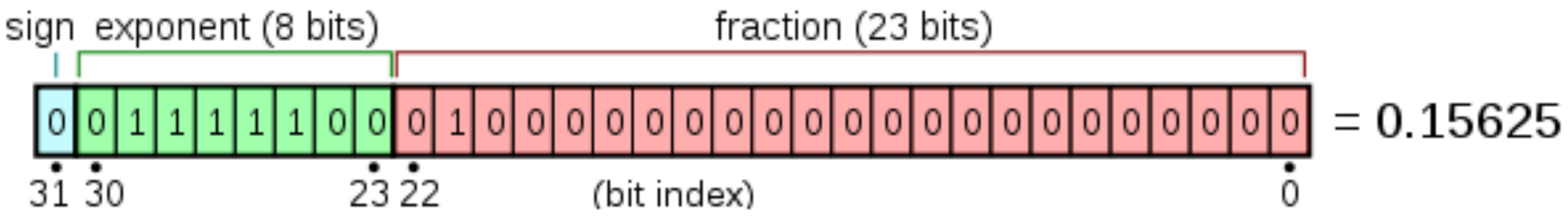
A: This actually might be the case. I cannot find a counterexample. Fun challenge: find a decimal fraction that converts to binary with a finite but different # of digits. Or prove the above.

e.g. $(.abc)_{10} = (.defg)_2$

$11/32$

Q: What the most number of places to the right of the radix point?

A: As far as I can find…it's complicated and depends on whether you are using fixed point or floating point arithmetic. Also, how many bits is your machine? Also other factors. I've posted an image from the "Single-precision floating-point format" wikipedia page under the IEEE 754 single-precision binary floating-point format: binary32



Questions from Last Time:

Q: How does a computer know if a number is less than 1?

A: It uses a “digital comparator.” I don’t have a great explanation for how that works. So for those who are curious, I will refer you to the wikipedia page: https://en.wikipedia.org/wiki/Digital_comparator

Warmup:

$29 - 7 = 22$

<u>Example:</u> Subtract $(11101)_2 - (111)_2$

$$
\begin{array}{rccccc}
 & 1 & \overset{0}{\cancel{1}} & \overset{10}{\cancel{1}} & \overset{1}{0} & 1 \\
- & & & 1 & 1 & 1 \\
\hline
 & 1 & 0 & 1 & 1 & 0
\end{array}
$$

Warmup:

Example: How many bits are needed to encode each lowercase letter of the English alphabet?

$a = 0$

$b = 1$

$c = 2$

$\vdots$

$z = 25$

$25 = (11001)_2$

$\boxed{5 \text{ bits}}$

Announcements and Reminders:

2824-001

- Enroll in the class Moodle. Keys: csci2824 – Rachel for section 001

- First homework (on Moodle) is due Friday 7 September at 12pm
  1 attempt per problem. infinite attempts for codeRunner. Clicking "Check" locks in your answer…don't do it!

- Enroll in the class Piazza: https://piazza.com/colorado/fall2018/csci2824

- Keep updated with the Schedule: https://goo.gl/DFuboZ

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CSCI 2824: Discrete Structures

Lecture 3: Propositional Logic

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Definition: The basic building block of logic is a **proposition**. A proposition is a declarative statement that is either true or false, but not both.

4 Types of Sentences

Declarative Sentence	Interrogative Sentence
• Tells something. • Ends with a period. (.)	• Asks a question. • Ends with a question mark. (?)

Exclamatory Sentence	Imperative Sentence
• Shows strong feeling. • Ends with a period. (!)	• Gives a command. • Ends with a period. (. or !)

Examples of Propositions

Boulder is a city in Colorado.

Golden is the capital of Colorado.

$2 + 2 = 5$

$1 + 2 = 3$

Examples of NOT Propositions

Don’t do that.

Where are you going?

6

$x + 2 = 3$

Definition: The **truth value** of a proposition is **true** (denoted T) if the proposition is true; it is **false** (denoted F) if the proposition is false.

1. Boulder is a city in Colorado. True T
2. $2+2 = 5$ False F

Definition: Let p be a proposition. The **negation** of p, denoted by $\neg p$, is the proposition "It is not the case that p". The truth value of $\neg p$ is the opposite of the truth value of p.

Common propositional variables: p, q, r, s

Example: Let p denote the proposition: "It is raining today."

Then, $\neg \boldsymbol{p}$ denotes: "It is **not the case that** it is raining today." or "It is **not** raining today."

Definition: It is convenient to tabulate of the possible truth values for the various configurations of the propositions. This is done using a **truth table**.

Given simple propositions p and q, the truth table allows us to enumerate all possible truth values of combinations of p and q.

Example: Give the truth table for p and $\neg p$

p	$\neg p$
T	F
F	T

Connectives – Logical Operators

With connectives, we can begin combining propositions to make compound and far more complex statements.

conjunction:	"and"	denoted $\wedge$	
disjunction:	"or"	denoted $\vee$	inclusive or
conditional:	"if-then"	denoted $\rightarrow$	
biconditional:	"if and only if"	denoted $\Leftrightarrow$	

Definition: Let p and q be propositions. The **conjunction** of p and q, denoted by $p \wedge q$, is the proposition "p and q". The conjunction $p \wedge q$ has the truth value T if both p and q are T and is F otherwise.

<u>Example:</u> Let p = "it is dark outside" and q = "my house is haunted"

It is dark out side ***and*** my house is haunted.

It is light outside, but my house is still haunted.

Truth Table for a Conjunction: $p \wedge q$

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

True when p and q are true, false otherwise

Definition: Let *p* and *q* be propositions. The **disjunction** of *p* and *q*, denoted by $p \vee q$, is the proposition "*p* or *q*". The disjunction $p \vee q$ has the truth value T if either *p* or *q* are T and is F otherwise.

Example: Let p = "it is dark outside" and q = "my house is haunted"

It is dark out side ***or*** my house is haunted.

This is an "inclusive or". So one proposition could be true, or they both could be true and the overall statement would be true.

Truth Table for a Disjunction: $\boldsymbol{p} \vee \boldsymbol{q}$

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

False when p and q are both false,
True otherwise

Definition: Let p and q be propositions. The **exclusive or** of p and q, denoted $p \oplus q$, is the proposition that is true when exactly one of p or q is true, and false otherwise.

- Also abbreviated as "xor" sometimes

Example: Let p = "It is daytime." and q = "It is nighttime."

It is daytime ***or*** it is nighttime.

"Exclusive or" means that we could have either one of the propositions be true, but not both.

Truth Table for an Exclusive Or: $\boldsymbol{p} \oplus \boldsymbol{q}$

p	q	$p \oplus q$
T	T	F
T	F	T
F	T	T
F	F	F

True when exactly one of p and q is true and false otherwise.

Definition: Let *p* and *q* be two propositions. The **conditional** "if *p* then *q*", denoted by $p \to q$, is false when *p* is true but *q* is false, and true otherwise.

- The conditional describes an *if-then* relationship between the two propositions.
- Think of the conditional $p \to q$ as defining a rule. What are the cases where the rule holds or where the rule is broken.

Other ways to express $p \longrightarrow q$:

If p, then q.
If p, q.
p is sufficient for q.
q if p.
q when p.
q unless $\neg p$.
p implies q
p only if q
A sufficient condition for q is p.
q whenever p
q is necessary for p
q follows from p

Example: If you get a 100% on the final, then you will get an A.

Truth Table for a Conditional: $p \longrightarrow q$

p	q	$p \longrightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

False when p is True and q is False, true otherwise

Definition: Let p and q be two propositions. The **<u>biconditional</u>** "p if and only if q", denoted by $p \Leftrightarrow q$**,** or p **iff** q, is true when p and q have the same truth value, and false otherwise.

- The conditional describes an *if-and-only-if* relationship between the two propositions.

<u>Example:</u> Let p = "A polygon has 3 sides." and q = "It is a triangle."

Truth Table for a biconditional: $p \iff q$

p	q	$p \iff q$
T	T	T
T	F	F
F	T	F
F	F	T

True when p and q have the same truth values, false otherwise.

Compound propositions:

Compound propositions are constructed by linking together multiple simple propositions using connectives.

Example: Construct a truth table for $(p \longrightarrow q) \wedge (\neg p \longrightarrow r)$

Order of Operations/Precedence of Logical Operators

1. Negation

2. Conjunction over disjunction $p \wedge q \vee r$ means $(p \wedge q) \vee r$

3. Conditionals, biconditionals $p \vee q \longrightarrow r$ means $(p \vee q) \longrightarrow r$

Example: Construct a truth table for $(p \longrightarrow q) \wedge (\neg p \longrightarrow r)$

p	q	r	$\neg p$	$p \to q$	$\neg p \to r$	$(p \to q) \wedge (\neg p \to r)$
T	T	T	F	T	T	
T	T	F	F	T	T	
T	F	T	F	F	T	
T	F	F	F	F	T	
F	T	T	T	T	T	
F	T	F	T	T	F	
F	F	T	T	T	T	
F	F	F	T	T	F	

Example: The island of Knights and Knaves. Suppose you are on an island where there are two types of people: Knights always tell the truth, and Knaves always lie.

Suppose on this island, you encounter two people, Alfred and Batman. Let's call them A and B for short. Suppose A tells you "I am a Knave or B is a Knight." Use a truth table to determine what kind of people A and B are.

p: Alfred is a Knight.

q: Batman is a Knight.

A's statement: $\neg p \vee q$

$$p \Leftrightarrow (\neg p \vee q)$$

It must be True on this island that is A is a Knight, then his statement is true and if A's statement is true, then he must be a knight.

p	q			
T	T			
T	F			
F	T			
F	F			