101100001000 101100001000 101100001000 101100001000 101100001000

CSCI 2824: Discrete Structures

Lecture 9: Rules of Inference (part 2)

Rachel Cox

Department of Computer Science

101100001000 101100001000 101100001000 101100001000 101100001000

Rules of Inference

Example: What valid argument form is present in the following?

If n is a real number with $n > 3$, then $n^2 > 9$.
Suppose that $n^2 \leq 9$. Therefore $n \leq 3$.

Rules of Inference

Example: What valid argument form is present in the following?

If $\sqrt{2} > \frac{3}{2}$ then $\left(\sqrt{2}\right)^2 > \left(\frac{3}{2}\right)^2$. We know that $\sqrt{2} > \frac{3}{2}$.

Consequently, $\left(\sqrt{2}\right)^2 > \left(\frac{3}{2}\right)^2$.

Rules of Inference

When the argument is valid AND the premises are true, we call the argument **sound**.

Rules of Inference

If Socrates is a man, then Socrates is mortal.
Socrates is a man.

Therefore, Socrates is mortal.

All men are mortal.
Socrates is a man.

Therefore, Socrates is mortal.

Rules of Inference

All men are mortal.
Socrates is a man.

Therefore, Socrates is mortal.

Let $M(x)$ be "x is a man" and $D(x)$ be "x is mortal."
Then this argument has the form:

$$\forall x \left(M(x) \rightarrow D(x) \right)$$

$$M(SOCRATES)$$

$$\therefore D(SOCRATES)$$

Rules of Inference – for Quantifiers

Universal Instantiation:

$$\frac{\forall x \, P(x)}{\therefore P(c)}$$

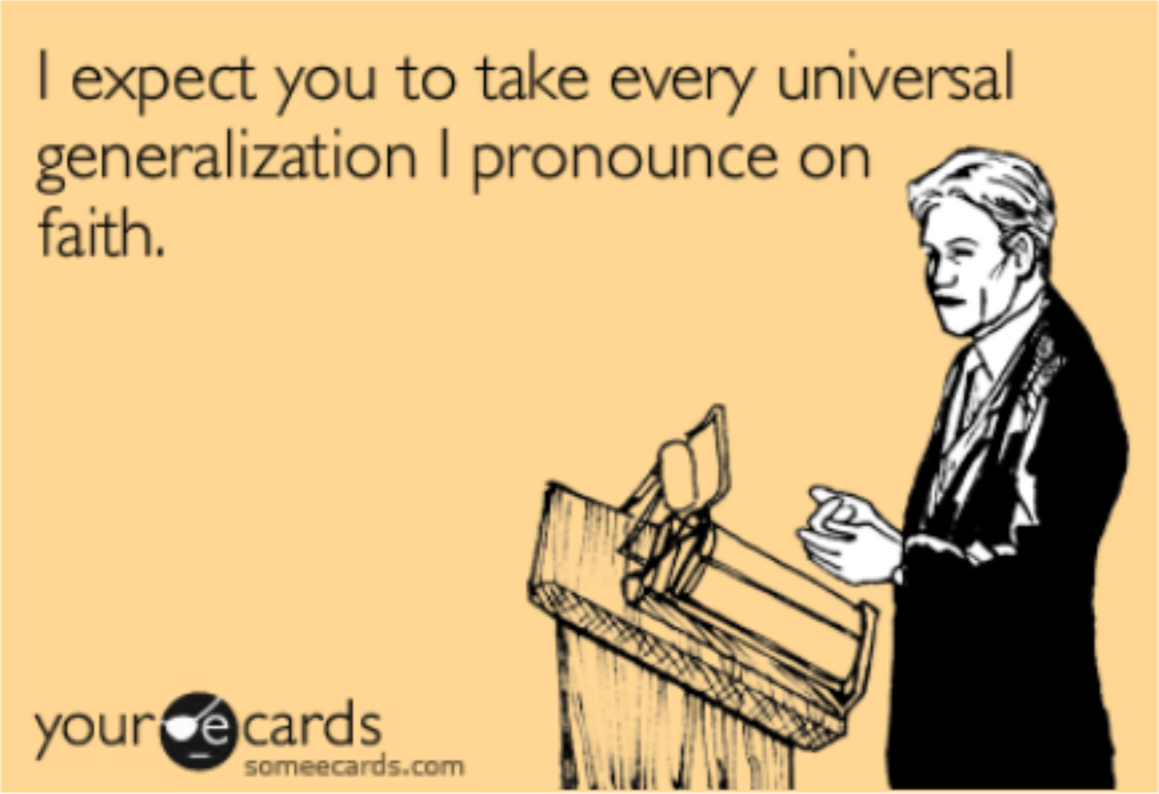
Intuition: If we know $P(x)$ is true for all x then we can insert any value of x and know that it's true.

Example: All dogs go to heaven. Therefore, Charlie B. Barkin is going to heaven.

Rules of Inference – for Quantifiers

Universal Generalization:

$$\frac{P(c) \text{ for an arbitrary } c}{\therefore \forall x\, P(x)}$$



If you prove something about an arbitrary element of your domain, then you can make a universal statement.

It is NOT that you can prove something about a specific element of your domain and then make a universal statement.

Rules of Inference – for Quantifiers

Example: Is the following argument valid?

All dogs go to heaven.

Laddie is a dog.

If Laddie goes to heaven, then he walks through the pearly gates.

Consequently, all dogs walk through the pearly gates.

Example: What about this one?

All dogs go to heaven.

"A" is an arbitrary dog.

If "A" goes to heaven, then s/he walks through the pearly gates.

Consequently, all dogs walk through the pearly gates.

Rules of Inference – for Quantifiers

Existential Instantiation:

$$\frac{\exists x\ P(x)}{\therefore P(c) \text{ for some element } c}$$

Example: There exists a man that skydives and base jumps. We don't know who he is, so we'll call him John Doe.

Rules of Inference – for Quantifiers

Existential Generalization:

$$\frac{P(c) \text{ for some element } c}{\therefore \exists x\, P(x)}$$

You can go from a statement about a specific element to an existential statement.

Example: Joe skydives and basejumps. Therefore, there exists a man who both skydives and base jumps.

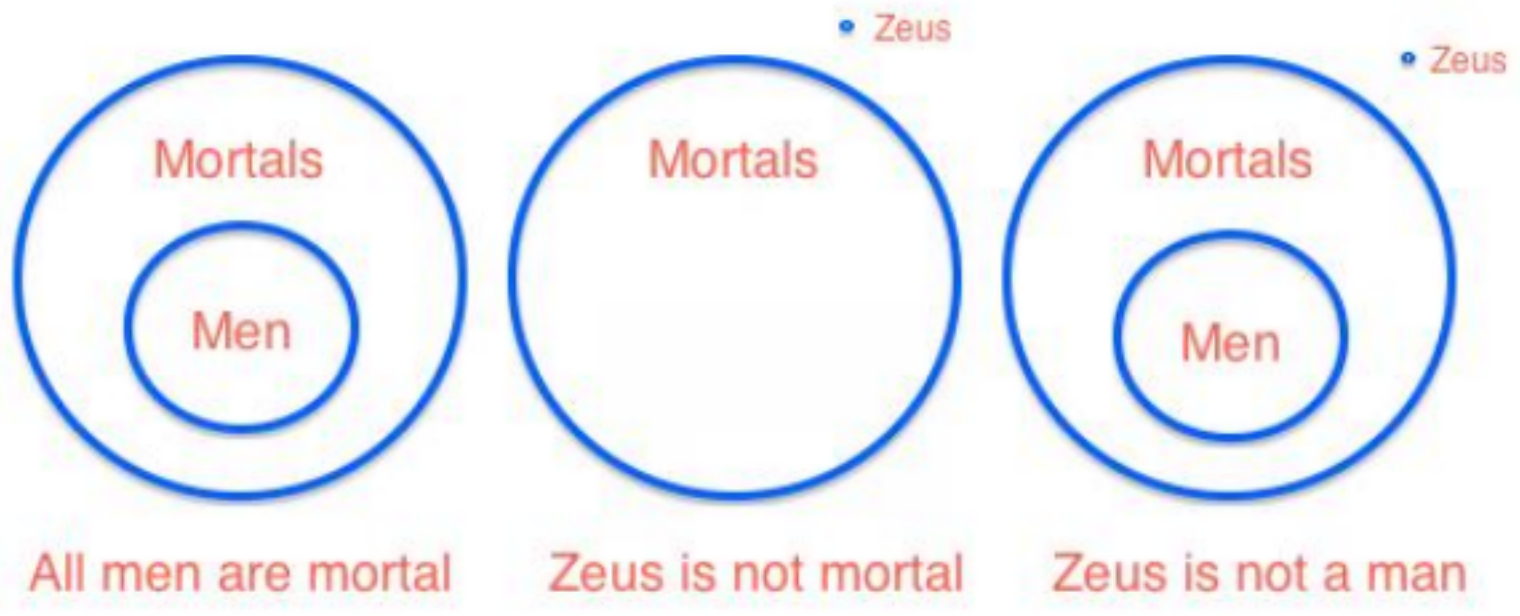
Rules of Inference – for Quantifiers

Universal Modus Tollens

$\forall x\,(P(x) \rightarrow Q(x))$

$\neg Q(a)$, where a is a particular element in the domain

$\therefore \neg P(a)$



Rules of Inference – for Quantifiers

Universal Modus Ponens

$\forall x\big(P(x) \to Q(x)\big)$

$P(a)$, where a is a particular element in the domain

$\therefore Q(a)$

Example:

All men are mortal.
Socrates is a man.

Therefore, Socrates is mortal.

	step	justification
1.	$\forall x\,(M(x) \to D(x))$	premise
2.	$M(SOCRATES)$	premise
3.	$M(SOCRATES) \to D(SOCRATES)$	universal inst. (1)
4.	$\therefore D(SOCRATES)$	mp (2) and (3)

Rules of Inference – for Quantifiers

Example: Translate the following argument using quantifiers. Show that the conclusion follows from the premises by logical inference.

All lions are fierce.

Some lions do not drink coffee.

Consequently, some fierce creatures do not drink coffee.

Rules of Inference – for Quantifiers

(Previous example continued)

Rules of Inference – for Quantifiers

Example: For the following argument, explain which rules of inference are used for each step.

Somebody in this class enjoys whale watching.

Every person who enjoys whale watching cares about ocean pollution.

Therefore, there is a person in this class who cares about ocean pollution.

Rules of Inference – for Quantifiers

(Previous example continued)

Rules of Inference – Fallacies

Fallacy of Affirming the Conclusion

$$\begin{array}{l} p \to q \\ q \\ \hline \therefore p \end{array}$$

Note that is $p = F$ and $q = T$, then both the premises are true and the conclusion is false.

Example: If you travel to Thailand, you will eat fantastic Pad Thai. You ate fantastic Pad Thai. Therefore, you traveled to Thailand.

- People often make invalid arguments when conditionals are involved.

Rules of Inference – Fallacies

Fallacy of Denying the Hypothesis

$$\begin{array}{r} p \to q \\ \neg p \\ \hline \therefore \neg q \end{array}$$

Note that if $p = F$ and $q = F$, then both of the premises are true while the conclusion is false.

Example: If a Run. Hide. Fight email is sent out, then you go hide behind a tree. A Run. Hide. Fight email is not sent out. Therefore, you do not hide behind a tree.