CSCI 2824: Discrete Structures

Lecture 2: Binary Arithmetic & Intro to Python

Rachel Cox

Department of Computer Science

Announcements:

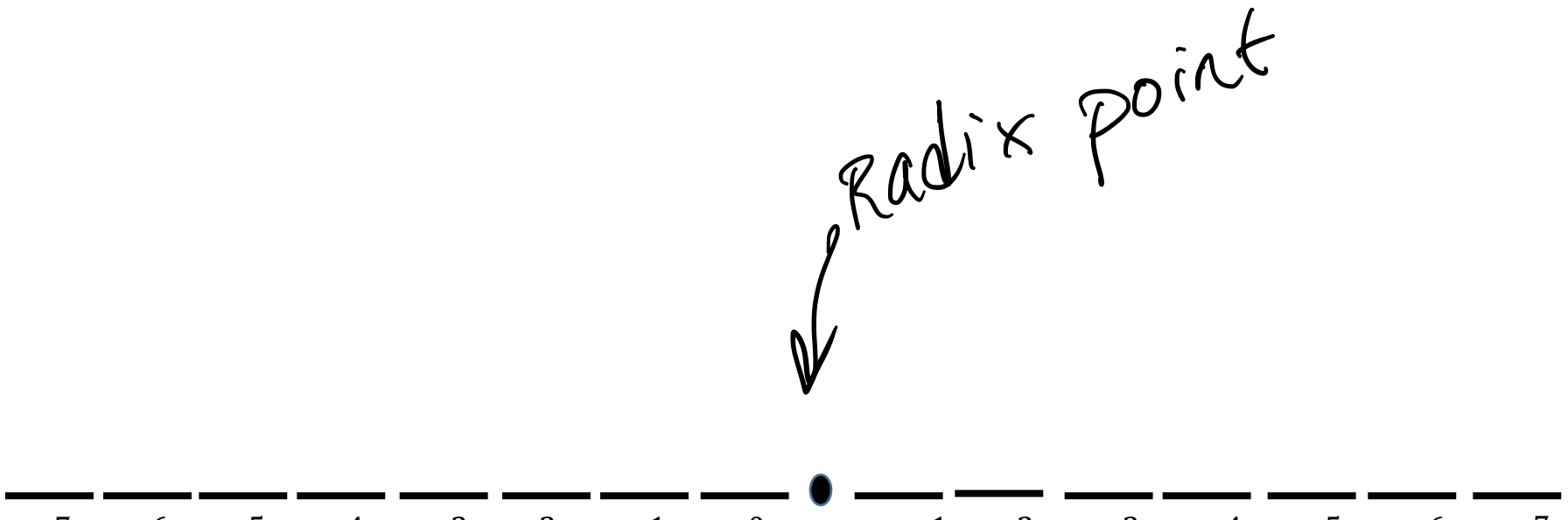
Enroll in Moodle: enrollment key is:
“csci2824-Rachel” for the 9am section, “csci2824-Tony” for the 11am section

Make sure you can access Piazza: https://piazza.com/colorado/fall2018/csci2824/home

FRAC TIONS in Binary

Powers of 2:

$$2^{-1} = \frac{1}{2}$$
$$2^{-2} = \frac{1}{4}$$
$$2^{-3} = \frac{1}{8}$$
$$2^{-4} = \frac{1}{16}$$
$$2^{-5} = \frac{1}{32}$$
$$2^{-6} = \frac{1}{64}$$
$$2^{-7} = \frac{1}{128}$$



Powers of 2:

$2^{-1} = \frac{1}{2}$

$2^{-2} = \frac{1}{4}$

$2^{-3} = \frac{1}{8}$

$2^{-4} = \frac{1}{16}$

$2^{-5} = \frac{1}{32}$

$2^{-6} = \frac{1}{64}$

$2^{-7} = \frac{1}{128}$

Example: Convert 0.75 from decimal to binary

$0.75 = 0.50 + 0.25$

$= \frac{1}{2} + \frac{1}{4}$

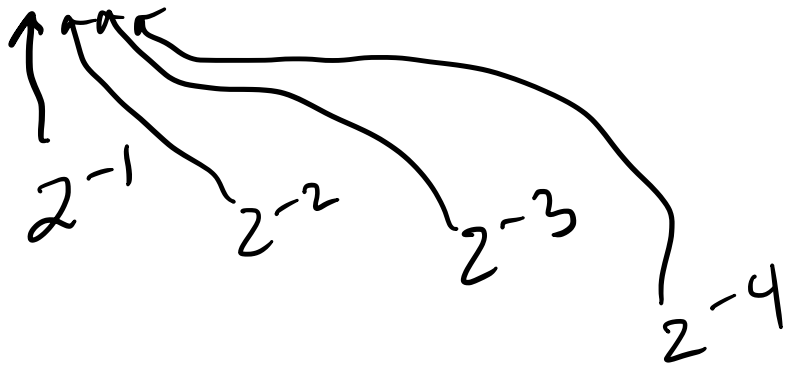
$= 2^{-1} + 2^{-2}$

$= 1 * 2^{-1} + 1 * 2^{-2}$

$(.75)_{10} = (.11)_2$

2^7	2^6	2^5	2^4	2^3	2^2	2^1	2^0	.	2^{-1}	2^{-2}	2^{-3}	2^{-4}	2^{-5}	2^{-6}	2^{-7}
								.	1	1					

Example: $(0.0001)_2 =$



$= \frac{1}{16} = .0625$

Example: $(0.10101)_2 =$ $\frac{1}{2} + \frac{1}{8} + \frac{1}{32} = \frac{21}{32}$

$= .65625$

Converting Decimal Fractions to Binary

1. Let m be a number less than 1
2. Move left to right (from radix point)
3. Multiply m by 2. Set bit to the value in the ones place.
4. Reset m to the stuff after the decimal place.
5. Continue until m=0

Example: Convert 0.84375 from decimal to binary

$m = .84375 = (.11011)_2$

$2m = 1.6875$ set 2^{-1} bit to 1

Reset $m = .6875$

$2m = 1.375$ set 2^{-2} bit to 1

Reset $m = .375$

$2m = 0.75$ set 2^{-3} bit to 0

reset $m = 0.75$

$2m = 1.5$ set 2^{-4} bit to 1

Reset $m = 0.5$

$2m = 1.0$ set 2^{-5} bit to 1

Reset $m = 0.0$

$2m = 0.0$

1. **Let m be a number less than 1**
2. **Move left to right (from radix point)**
3. **Multiply m by 2. Set bit to the value in the ones place.**
4. **Reset m to the stuff after the decimal place.**
5. **Continue until m=0**

Example: Convert 161.84375 from decimal to binary

- Convert the integer part and the fractional part separately, then stick them together with a radix point.

$$(161)_{10} = (10100001)_2$$

$$(.84375)_{10} = (.11011)_2$$

$$(161.84375)_{10} = (10100001.11011)_2$$

Binary ARITHMETIC

Example: What is 30 in binary? What is 2 in binary? What is 32 in binary?

30	even	0
15	odd	1
7	odd	1
3	odd	1
1	odd	1
0	even	0
0		0
0		0

$30_{10} = (11110)_2$

$+ \; 2_{10} = (10)_2$

$\frac{1-1}{2} = 0$

$32 = (100000)_2$

Adding two numbers in binary:
Proceed right to left

In decimal: If the column exceeds 10, we carry a 1 to the left

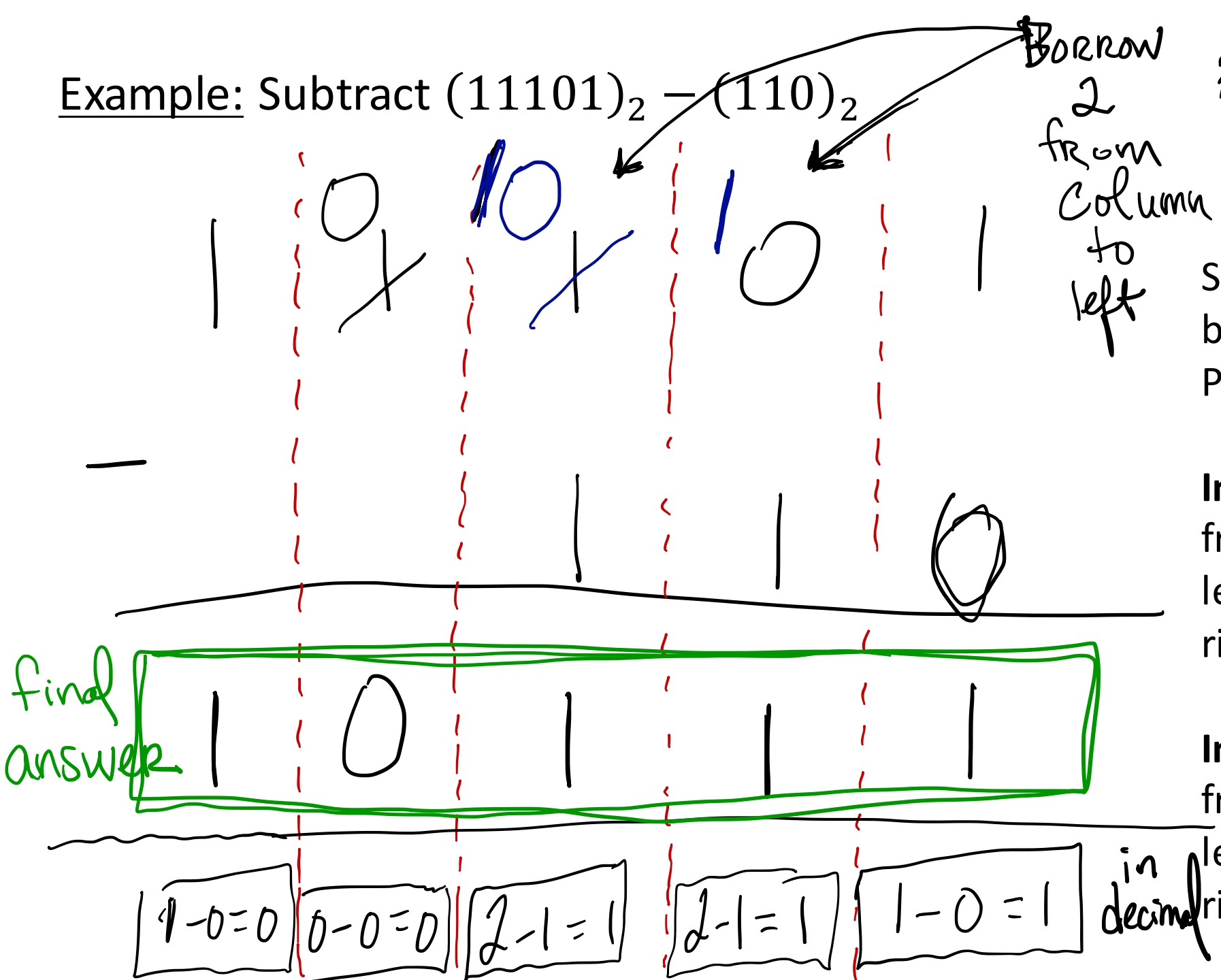
In binary: If our column exceeds 2, we carry a 1 to the left

Example: Add $(11110)_2 + (10)_2$

$2 = (10)_2$

$$\begin{array}{r} 1\ 1\ 1\ 1\ 0 \\ +\quad\ \ 1\ 0 \\ \hline 1\ 0\ 0\ 0\ 0\ 0\ 0. \end{array}$$

Example: Subtract $(11101)_2 - (110)_2$



Subtracting two numbers in binary:
Proceed right to left.

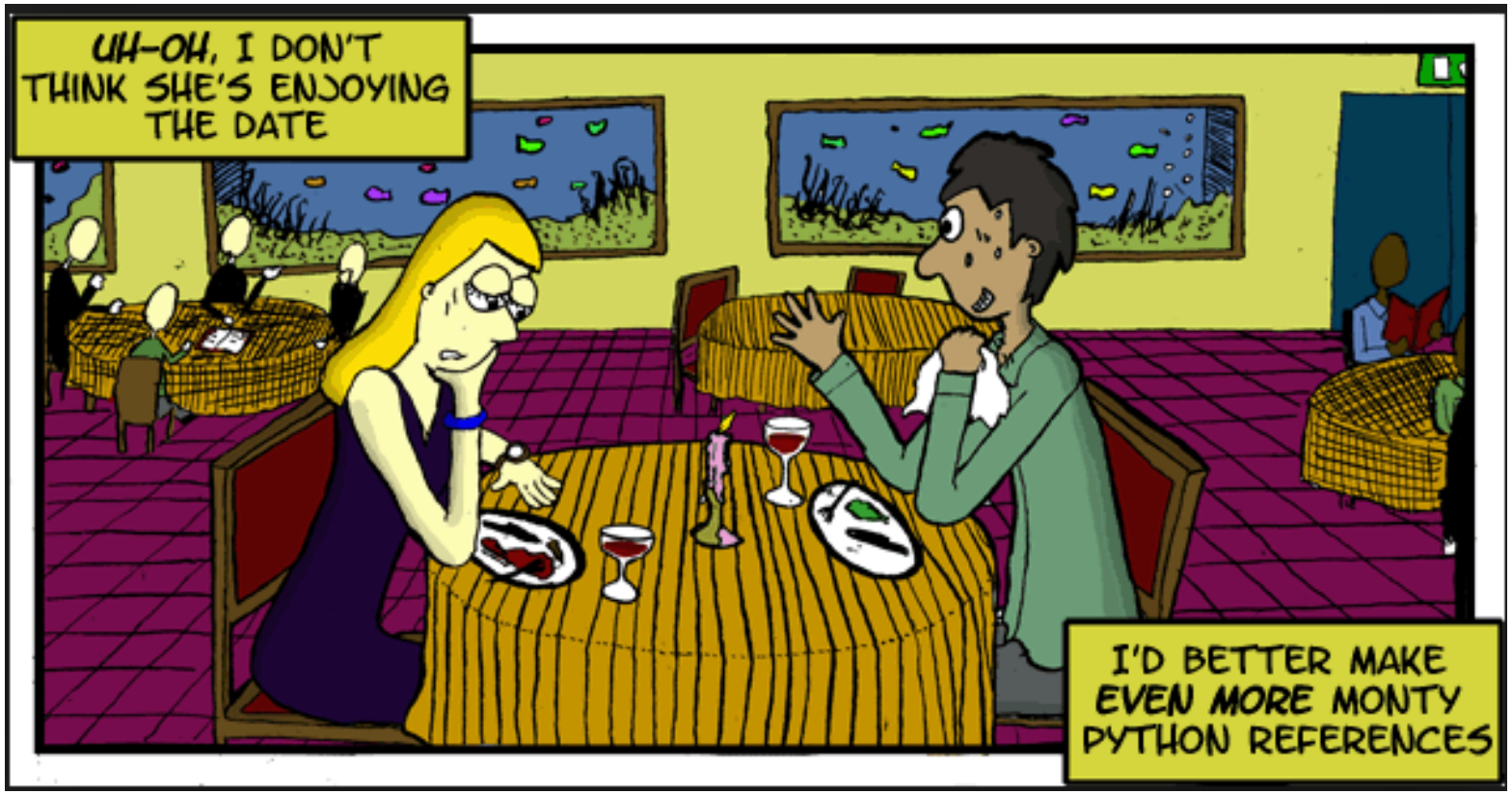
In decimal: We can take 1 from the column to the left, and bring 10 to the right.

In binary: We can take 1 from the column to the left, and bring 2 to the right.

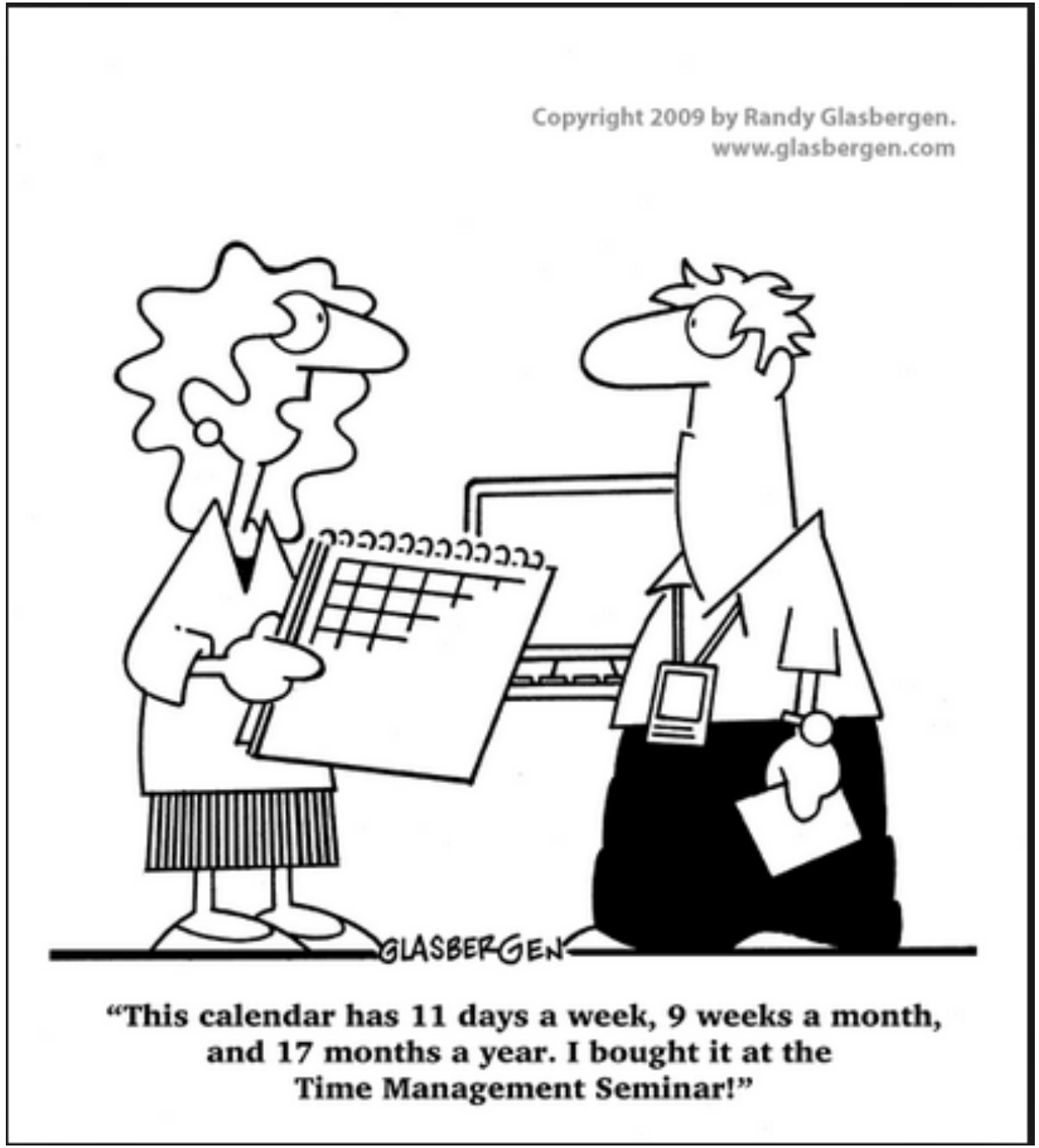
Intro to Python (Python 3)

Easiest way to get Python: https://www.anaconda.com/download

Good Practice: https://www.hackerrank.com/domains/python



More Examples
IF WE HAVE TIME



Example: Convert 0.2 from decimal to binary

Example: **Truncation error**.

$$f(a,b) = 333.75b^6 + a^2(11a^2b^2 - b^6 - 121b^4 - 2) + 5.5b^8 + \frac{a}{2b}$$

$$a = 77617$$
$$b = 33096$$

If this is run on a 64–bit machine,

$$f(a,b) = -44450695952321879337122922496.000$$

OR

$$f(a,b) = -1.180592e + 21$$

Real Answer: $f(a,b) \approx -0.82739605$