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CSCI 2824: Discrete Structures

Lecture 14: Exam 1 Review

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Time and location:

- 6:30-8 PM on Tuesday 2 October
- Rachel's section (001) is in HUMN 1B50
- Tony's section (002) is in DUAN G1B30

Please go to the correct room! Rachel's exam room does **not** have enough space for her class plus a bunch of Lost Souls from Tony's class.

Review session:
In class on Monday 1 October. Q&A format

Exam rules:

- You are allowed to use a calculator. **No smartphones** or other devices that can store large amounts of data or access the internet.
- You are allowed **one** 8.5x11-inch sheet of paper as a cheat sheet. You can write whatever you want on it and can use both sides.
- You do not need to bring blue books or anything like that.
- **Do** bring your Buff OneCard.
- **Do** bring multiple writing utensils (gotta have back-ups, right? Or should I say, *write*? I apologize for nothing.)
- Get there **early**. If you arrive late, you will not receive extra time.

Exam format:
Some combination of (a) multiple choice, (b) short answer (brief justification type problems) and (c) free response (more involved problems; think along the lines of the written homework problems).

Exam content:
Beginning of the semester through the "Set Operations and Functions" slide set from Wednesday and Friday.

Special accommodations:

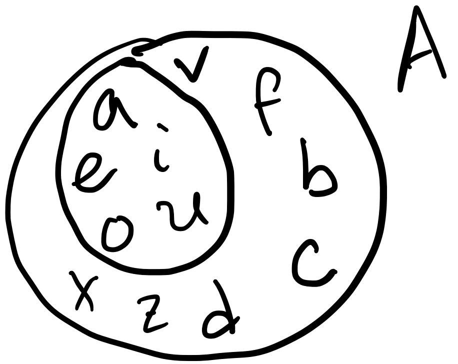
If you have a documented special need for accommodations **and have presented us with the requisite paperwork before the exam**, then you can take the exam in **DUAN G2B21 starting at 6 PM on Tuesday** (2 Oct), and ending whenever your particular accommodation indicates. It is your responsibility to keep track of your time - the proctors are instructed not to bother you, since that's kind of the point... This is a classroom with seating for about 20 people. If your particular needs require some different accommodation, just let me know in a private Piazza message and we'll sort it out. Note that if you do not have a documented need through Disability Services and **have given us your paperwork**, then this doesn't apply to you, and you must take the exam in the regularly scheduled place/time.

Set Operations

Example: Draw a Venn Diagram relating the set of all vowels to the set of all letters in the English alphabet.

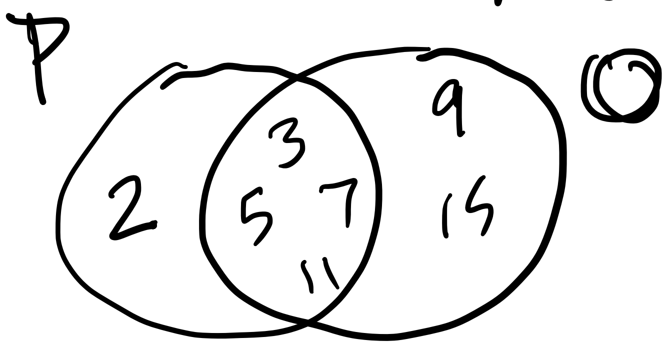
$V = \{a, e, i, o, u\}$

$A = \{a, b, c \ldots x, y, z\}$



Example: Draw a Venn diagram relating the set of all prime numbers and the set of odd numbers. (prime numbers are numbers that are only divisible by 1 and itself.

$P = \{2, 3, 5, 7, 11 \ldots\}$, $O = \{1, 3, 5, 7, 9, 11, 13 \ldots\}$



$A = \{5, \{7, K\}\}$

$P(A)$ set of all possible subsets.

$P(A) = \{\emptyset, \{5\}, \{\{7, K\}\}, \{5, \{7, K\}\}\}$

$A = \{5, \emptyset\}$

$P(A) = \{\emptyset, \{\emptyset\}$

Set Operations

Example: Consider the sets $A = \{dumbo, jumbo\}$ and $B = \{a, b, c\}$. What is $A \times B$?

$A \times B = \{(dumbo, a), (dumbo, b), (dumbo, c),$
$(jumbo, a), (jumbo, b), (jumbo, c)\}$

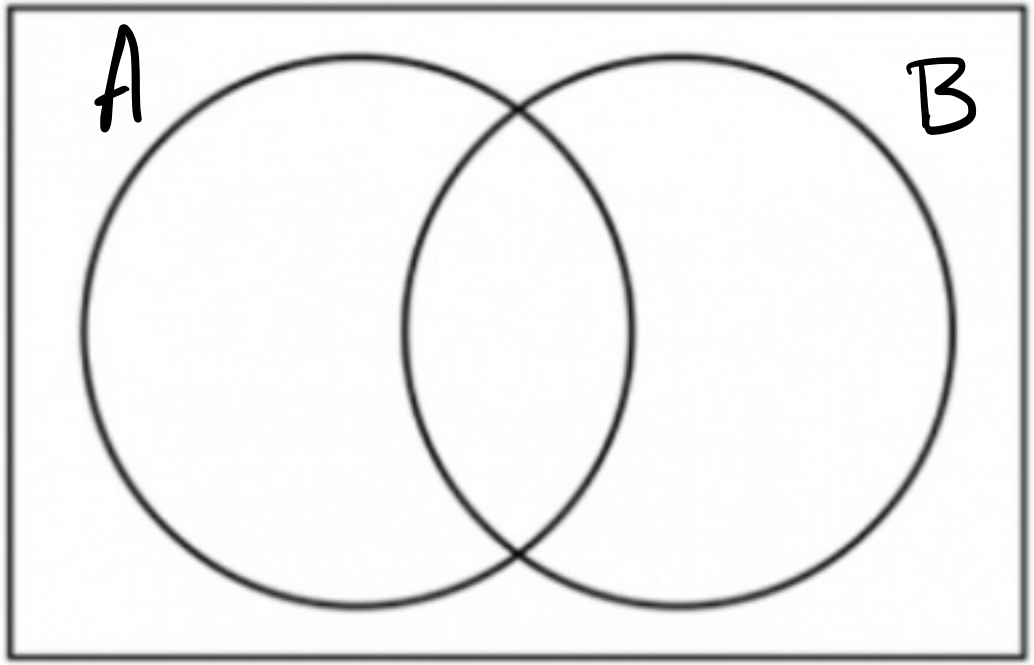
$A \times B \neq B \times A$

Set Operations

$A = \{1, 2, 3\}$, $B = \{1, 5\}$

$A \cup B = \{1, 2, 3, 5\}$

Example: How many elements are in the set $A \cup B$?



- Suppose there are n elements in A
- Suppose there are m elements in B
- Suppose there are p elements in $A \cap B$

$$\boxed{|A \cup B| = |A| + |B| - |A \cap B|}$$

Set Operations

- **Python** has some nice functionality that can help you convert lists of elements into sets, and perform some operations on them.

```
In [8]: mylist = [1,2,3,1,4]

In [9]: myset = set(mylist)

In [10]: print(myset)
{1, 2, 3, 4}
```

```
In [15]: A = set([1,2,3,4])

In [16]: B = set([3,4,5,6])

In [17]: print(set.intersection(A,B))
{3, 4}

In [18]: print(set.union(A,B))
{1, 2, 3, 4, 5, 6}

In [19]: print(set.difference(A,B))
{1, 2}

In [20]: print(set.difference(B,A))
{5, 6}
```

- **If/when** the time comes, you should feel free to explore these functions for manipulating sets…
  … he said with a knowing grin.

Set Operations

$\neg(p \vee q) \equiv \neg p \wedge \neg q$

Example: Use set identities to prove $\overline{A \cup (B \cap C)} = (\overline{C} \cup \overline{B}) \cap \overline{A}$

$$\overline{A \cup (B \cap C)} = \overline{A} \cap \overline{(B \cap C)} \quad \text{Demorgans}$$

$$= \overline{A} \cap (\overline{B} \cup \overline{C}) \quad \text{Demorgans}$$

$$= (\overline{B} \cup \overline{C}) \cap \overline{A} \quad \text{commutativity}$$

$$= (\overline{C} \cup \overline{B}) \cap \overline{A} \quad \text{commutativity}$$

$\blacksquare$

Set Operations

Example: If P is the set of prime numbers, then what is $\overline{P}$?

Assuming the universe of discourse is $\mathbb{Z}^+$ (positive integers)

$P = \{2, 3, 5, 7, 11, 13, 17, 19, \ldots\}$

$\boxed{\overline{P} = \mathbb{N} - P = \{1, 4, 6, 8, 10, 12, 14, 15, \ldots\}}$

set of composite numbers and 1

Set Operations

An assortment of good practice! I did a few as an example.

Example: Suppose $A = \{b, c, d\}$ and $B = \{a, b\}$. Find:

(a) $(A \times B) \cap (B \times B)$
(b) $(A \times B) \cup (B \times B)$
(c) $(A \times B) - (B \times B)$
(d) $(A \cap B) \times A$
(e) $(A \times B) \cap B$
(f) $\mathscr{P}(A) \cap \mathscr{P}(B)$
(g) $\mathscr{P}(A) - \mathscr{P}(B)$
(h) $\mathscr{P}(A \cap B)$
(i) $\mathscr{P}(A) \times \mathscr{P}(B)$

$A \times B = \{(b,a), (b,b), (c,a), (c,b), (d,a), (d,b)\}$

$B \times B = \{(a,a), (a,b), (b,a), (b,b)\}$

(a) $(A \times B) \cap (B \times B) = \{(b,a), (b,b)\}$

(d) $A \cap B = \{b\}$ $\quad (A \cap B) \times A = \{(b,b), (b,c), (b,d)\}$

(g) $P(A) = \{\emptyset, \{b\}, \{c\}, \{d\}, \{b,c\}, \{b,d\}, \{c,d\}, \{b,c,d\}\}$

$P(B) = \{\emptyset, \{a\}, \{b\}, \{a,b\}\}$ $\quad P(A) - P(B) = \{\{c\}, \{d\}, \{b,c\}, \{b,d\}, \{c,d\}, \{b,c,d\}\}$

Binary Representation of Numbers

Example: Represent 209 as a binary number

N		
209	1	2^0
104	0	2^1
52	0	2^2
26	0	2^3
13	1	2^4
6	0	2^5
3	1	2^6
1	1	2^7
0	0	

$$(209)_{10} = (11010001)_2$$

$$11,010,001$$

Logical Equivalences

NB: On homework and exams, Table 6 (p. 27 of Rosen) and the following 4 are the only ones you can/should invoke:

$p \to q \equiv \neg p \lor q$ **relation by implication (RBI)**

$p \to q \equiv \neg q \to \neg p$ **contraposition**

$p \Leftrightarrow q \equiv (p \to q) \land (q \to p)$ **definition of biconditional**

$p \oplus q \equiv (p \lor q) \land \neg(p \land q)$ **alt. definition of xor**

TABLE 6 Logical Equivalences.

Equivalence	*Name*
$p \land \mathbf{T} \equiv p$ $p \lor \mathbf{F} \equiv p$	Identity laws
$p \lor \mathbf{T} \equiv \mathbf{T}$ $p \land \mathbf{F} \equiv \mathbf{F}$	Domination laws
$p \lor p \equiv p$ $p \land p \equiv p$	Idempotent laws
$\neg(\neg p) \equiv p$	Double negation law
$p \lor q \equiv q \lor p$ $p \land q \equiv q \land p$	Commutative laws
$(p \lor q) \lor r \equiv p \lor (q \lor r)$ $(p \land q) \land r \equiv p \land (q \land r)$	Associative laws
$p \lor (q \land r) \equiv (p \lor q) \land (p \lor r)$ $p \land (q \lor r) \equiv (p \land q) \lor (p \land r)$	Distributive laws
$\neg(p \land q) \equiv \neg p \lor \neg q$ $\neg(p \lor q) \equiv \neg p \land \neg q$	De Morgan's laws
$p \lor (p \land q) \equiv p$ $p \land (p \lor q) \equiv p$	Absorption laws
$p \lor \neg p \equiv \mathbf{T}$ $p \land \neg p \equiv \mathbf{F}$	Negation laws

Rules of Inference

Fallacies

Fallacy of Affirming the Conclusion:
$((p \rightarrow q) \wedge q) \rightarrow p$

Fallacy of Denying the Hypothesis:
$((p \rightarrow q) \wedge \neg p) \rightarrow \neg q$

TABLE 1 Rules of Inference.

Rule of Inference	*Tautology*	*Name*
p $p \rightarrow q$ $\therefore q$	$(p \wedge (p \rightarrow q)) \rightarrow q$	Modus ponens
$\neg q$ $p \rightarrow q$ $\therefore \neg p$	$(\neg q \wedge (p \rightarrow q)) \rightarrow \neg p$	Modus tollens
$p \rightarrow q$ $q \rightarrow r$ $\therefore p \rightarrow r$	$((p \rightarrow q) \wedge (q \rightarrow r)) \rightarrow (p \rightarrow r)$	Hypothetical syllogism
$p \vee q$ $\neg p$ $\therefore q$	$((p \vee q) \wedge \neg p) \rightarrow q$	Disjunctive syllogism
p $\therefore p \vee q$	$p \rightarrow (p \vee q)$	Addition
$p \wedge q$ $\therefore p$	$(p \wedge q) \rightarrow p$	Simplification
p q $\therefore p \wedge q$	$((p) \wedge (q)) \rightarrow (p \wedge q)$	Conjunction
$p \vee q$ $\neg p \vee r$ $\therefore q \vee r$	$((p \vee q) \wedge (\neg p \vee r)) \rightarrow (q \vee r)$	Resolution

Truth Tables & Knights / Knaves

Example: The Island of Knights and Knaves has two types of inhabitants: Knights, who always tell the truth, and Knaves, who always lie. As you are exploring the Island of Knights and Knaves you encounter two people named A and B.

A tells you “Of B and myself, exactly one of us is a Knight.” $p \oplus q$

B tells you “A is a Knave.” $\neg p$

p: A is a Knight.

q: B is a Knight.

Determine the nature of A and B, if you can.

$$(p \leftrightarrow (p \oplus q)) \wedge (q \leftrightarrow (\neg p))$$

p	q	$p \oplus q$	$p \leftrightarrow p \oplus q$	$\neg p$	$q \leftrightarrow \neg p$	$(p \leftrightarrow p \oplus q) \wedge (q \leftrightarrow \neg p)$
T	T					F
$\boxed{T}$	$\boxed{F}$					T
F	T					F
F	F					F

Rules of Inference

Example: If it snows and it is dark out, then Tony will crash his bicycle. Suppose you see Tony and he has crashed his bicycle. What then, do you know must also be true?

a) It is snowing.
b) It is dark out.
c) It is snowing and it is dark out.
d) Nothing
e) It is not snowing and it is not dark out.

Fallacy of Affirming the Conclusion.

Quantifiers

Select the quantifier translation that best matches the following English statement.

All computer scientists like pizza, and some computer scientists like soda, but no computer scientists like asparagus.

Let $L(x, y)$ denote "x *likes* y", let $C(x)$ denote "x *is a computer scientist*" and suppose the domain for x is all people.

$\forall x\,(C(x) \rightarrow L(x,\text{pizza})) \wedge \exists x\,(C(x) \wedge L(x,\text{soda})) \wedge \underline{\neg\exists x\,(C(x) \wedge L(x,\text{asparagus}))}$

Select one:

a. $\boxed{\forall x\ [C(x) \rightarrow (L(x,\text{pizza}) \wedge \neg L(x,\text{asparagus}))] \wedge \exists x\ (C(x) \wedge L(x,\text{soda}))}$

change this part.

b. $\forall x\ (C(x) \wedge L(x,\text{pizza}) \wedge \neg L(x,\text{asparagus})) \wedge \exists x\ (C(x) \wedge L(x,\text{soda}))$

→ next page

c. $\neg\exists x\ [(C(x) \wedge \neg L(x,\text{pizza})) \wedge (C(x) \wedge L(x,\text{asparagus}))] \wedge \exists x\ (C(x) \wedge L(x,\text{soda}))$

d. $\forall x\ [C(x) \rightarrow (L(x,\text{pizza}) \wedge \neg L(x,\text{asparagus}))] \wedge \neg\forall x\ (C(x) \wedge \neg L(x,\text{soda}))$

$$\neg \exists x (C(x) \wedge L(x, \text{asparagus}))$$

$$\equiv \forall x \neg (C(x) \wedge L(x, \text{asparagus})) \quad \text{negation of quantifiers}$$

$$\equiv \forall x (\neg C(x) \vee \neg L(x, \text{aspargus})) \quad \text{demorgans}$$

$$\equiv \forall x (C(x) \rightarrow \neg L(x, \text{asparagus})) \quad \text{RBI}$$

Combining this 3^{rd} term with the first two and switching the order, we have:

$$\forall x (C(x) \rightarrow L(x, \text{pizza})) \wedge \forall x (C(x) \rightarrow \neg L(x, \text{asparagus}))$$

$$\wedge \exists x (C(x) \wedge L(x, \text{soda}))$$

.... If we combine the first two quantified statements, we must choose (a)

Valid Arguments

Example: Is the following argument valid?

1. If an animal is a tapir, then it has short legs.
2. This animal is not a tapir.
3. Therefore, this animal does not have short legs.

$$\begin{array}{l} p \to q \\ \neg p \\ \hline \therefore \neg q \end{array}$$

Nope. This is an example of the fallacy of denying the hypothesis.