



University of Colorado **Boulder**

# **CSCI 3403 INTRO TO CYBERSECURITY**

Lecture: 3-1

Topic: Asymmetric  
Encryption

Presenter: Matt  
Niemic

# Announcements

- Clarification on extra credit
- Please fill out the groups survey by January 30
- You'll be able to submit project 1 shortly after that
- Don't forget about the survey for any suggestions
  - Link: <https://forms.gle/WRUUbPkmFNsa6q3D6>
- Survey for Javascript experience

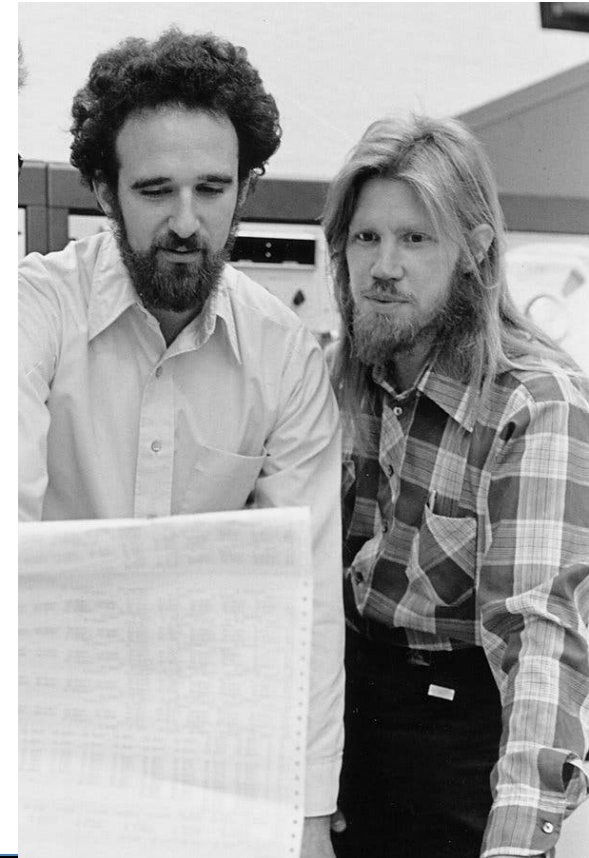


# **Asymmetric Encryption**

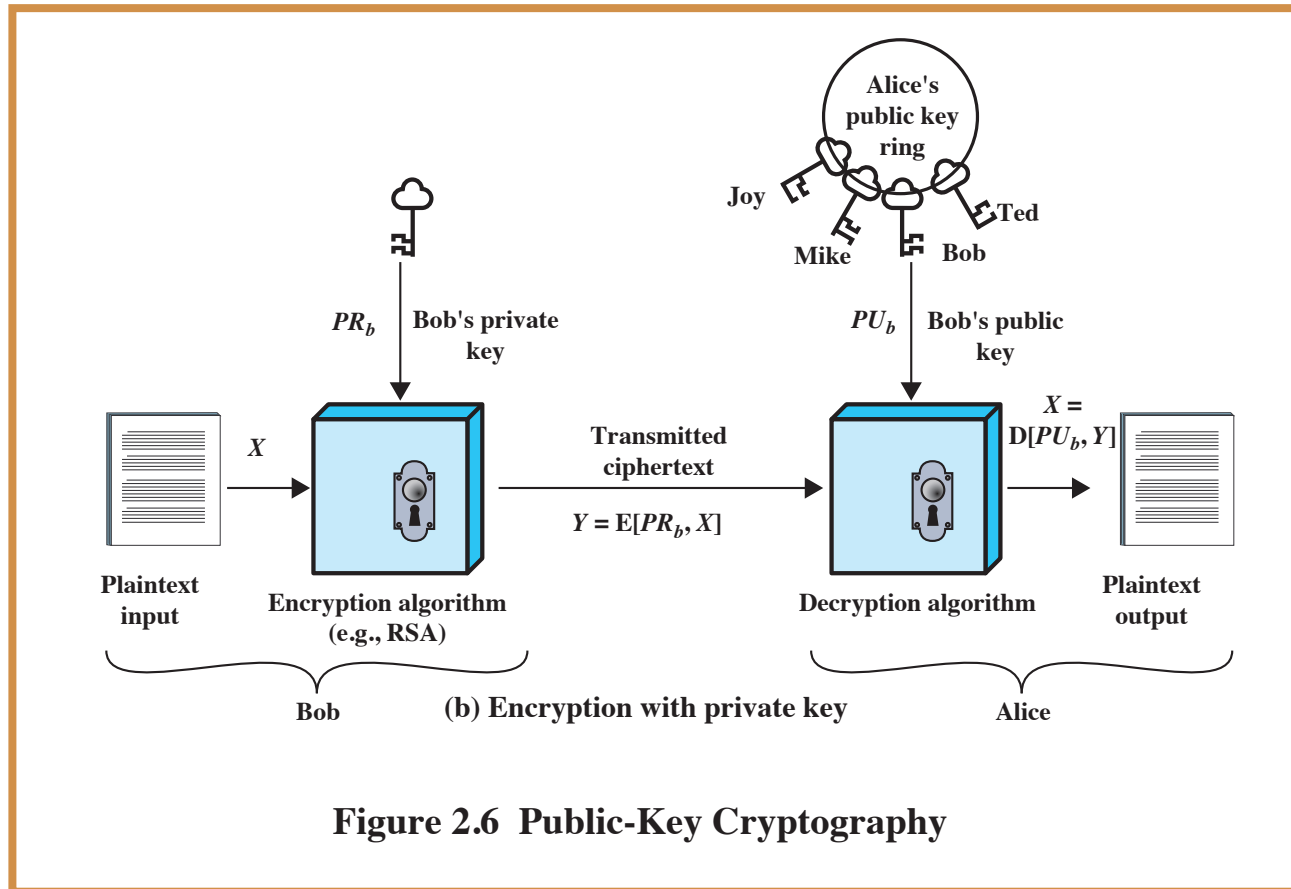


# One Simple Idea...

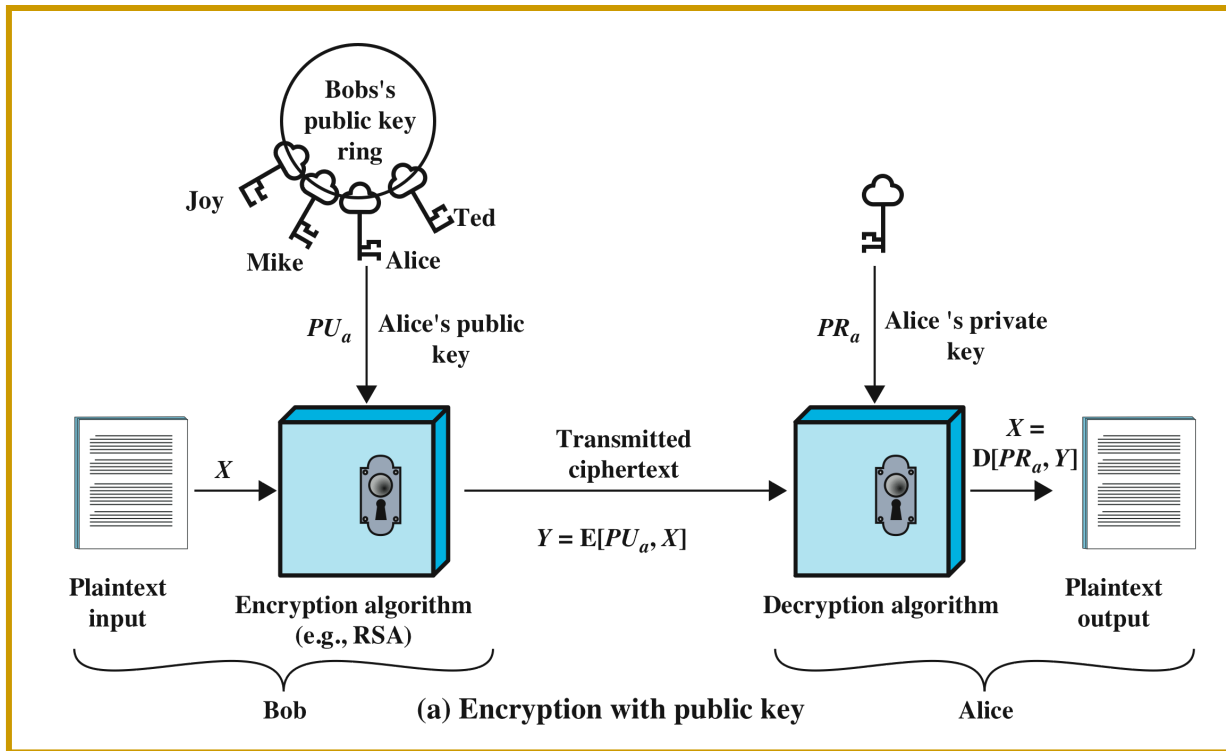
- What if the encryption key and decryption key were two different keys? - 1976
  - What if the decryption key could encrypt as well?
  - Completely changed cryptography
- Challenged somebody to find a secure way of doing this



# Public Key Visual



# Public Key Visual



# Why?

- Key exchange
- Create message for one person
- Write messages with private key



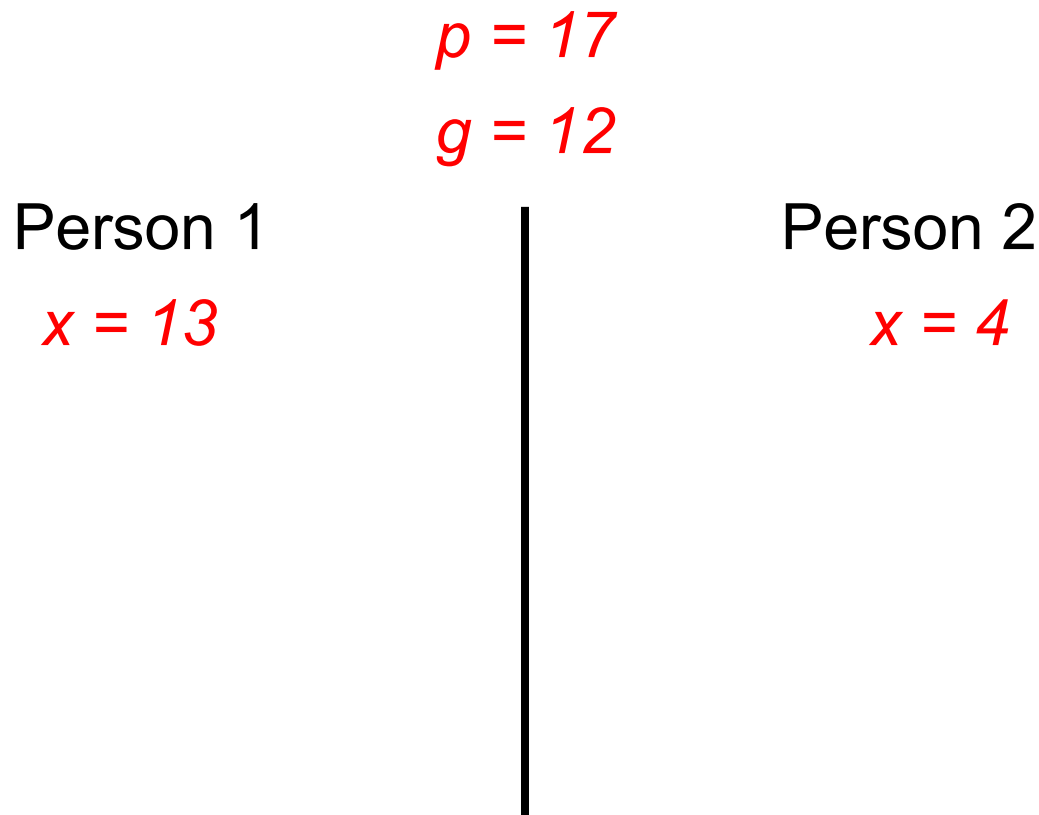
# Diffie-Hellman Key Exchange

- Simple way to exchange keys
  - Corrected formula below
- 1) Generate prime  $p$  and base  $g < p$
  - 2) Each party picks private value  $x < p-1$
  - 3) Each party calculates  $y = g^x \bmod p$  and sends it
  - 4) Calculate key  $z = y^x \bmod p$





# Key Exchange Example



# Key Exchange Example

$$p = 17$$

$$g = 12$$

Person 1

$$x = 13$$

$$y_1 = 12^{13} \bmod 17$$

Person 2

$$x = 4$$

$$y_2 = 12^4 \bmod 17$$



# Key Exchange Example

$$p = 17$$
$$g = 12$$

Person 1

$$x = 13$$

$$y_1 = 12^{13} \bmod 17 = 14$$

Person 2

$$x = 4$$

$$y_2 = 12^4 \bmod 17 = 13$$



# Key Exchange Example

$$p = 17$$

$$g = 12$$

Person 1

$$x = 13$$

$$y_1 = 12^{13} \bmod 17 = 14$$

$$z = y_2^x \bmod p$$

Person 2

$$x = 4$$

$$y_2 = 12^4 \bmod 17 = 13$$

$$z = y_1^x \bmod p$$



# Key Exchange Example

$$p = 17$$

$$g = 12$$

Person 1

$$x = 13$$

$$y_1 = 12^{13} \bmod 17 = 14$$

$$z = y_2^x \bmod p$$

$$z = 13^{13} \bmod 17 = ?$$

Person 2

$$x = 4$$

$$y_2 = 12^4 \bmod 17 = 13$$

$$z = y_1^x \bmod p$$

$$z = 14^4 \bmod 17 = ?$$



# Key Exchange Example

$$p = 17$$

$$g = 12$$

Person 1

$$x = 13$$

$$y_1 = 12^{13} \bmod 17 = 14$$

$$z = y_2^x \bmod p$$

$$z = 13^{13} \bmod 17 = 13$$

Person 2

$$x = 4$$

$$y_2 = 12^4 \bmod 17 = 13$$

$$z = y_1^x \bmod p$$

$$z = 14^4 \bmod 17 = 13$$

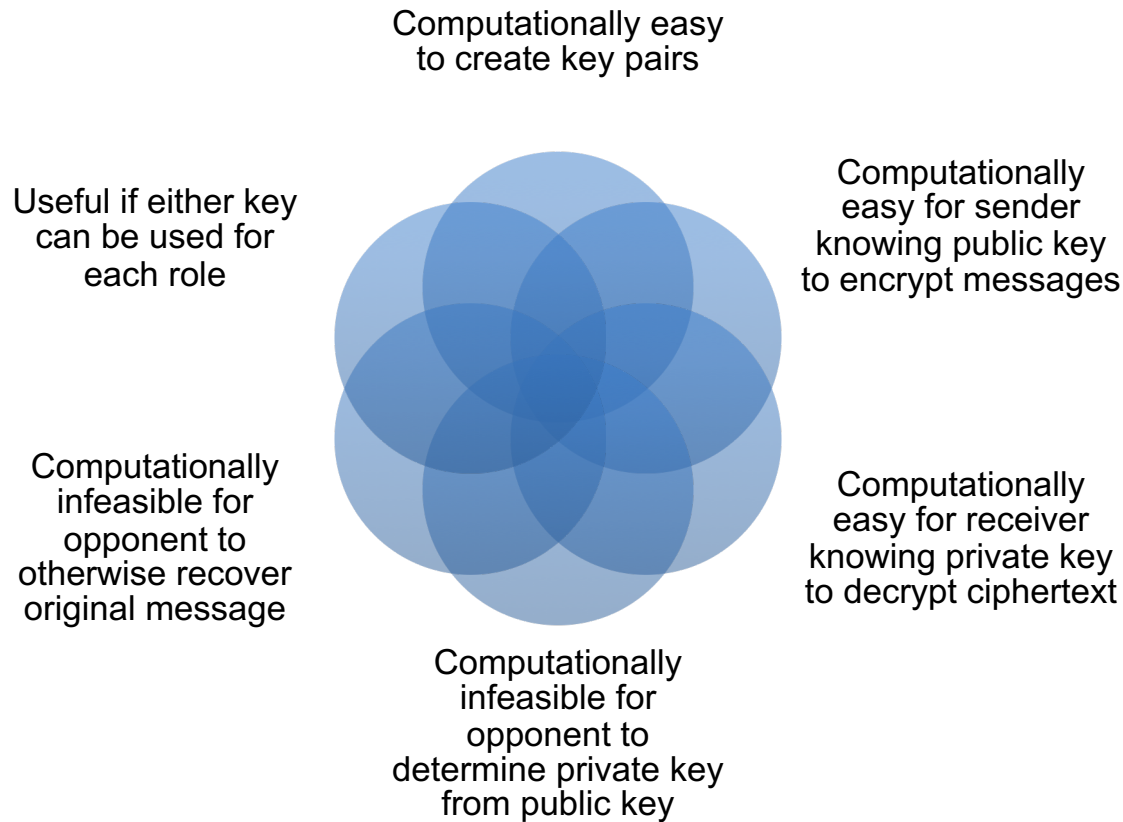


# Diffie-Hellman Key Exchange

- Both parties get the same key
- Somebody listening can't determine the key
- Problems?



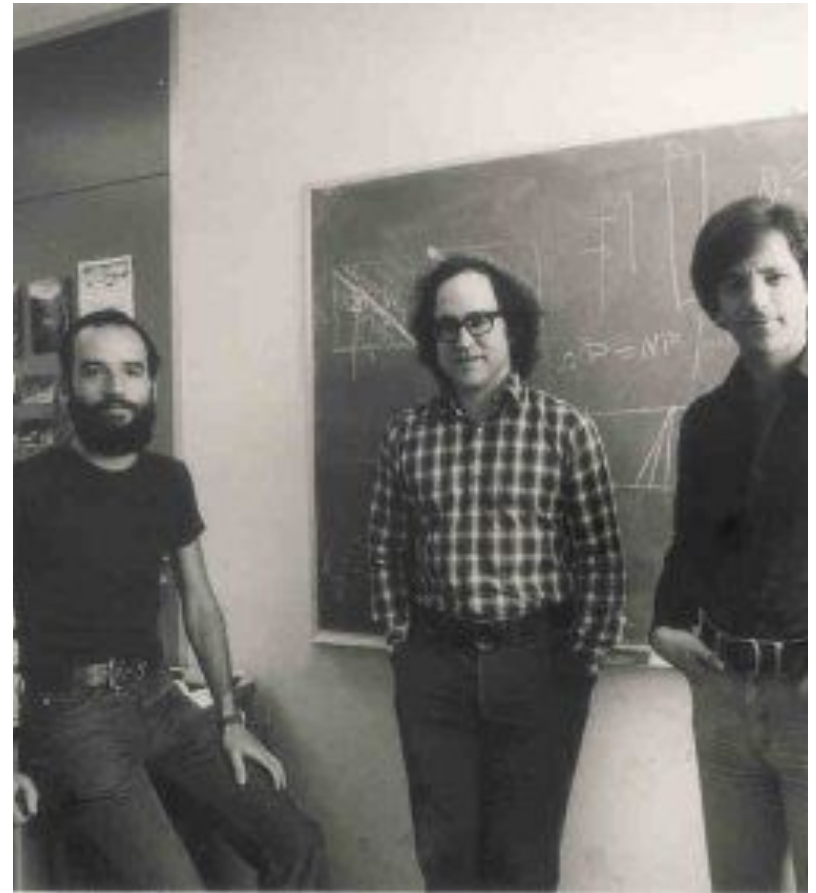
# Requirements of Public Key Cryptosystems





# RSA

- Rivest, Shamir, and Adleman: “Hold my beer” – 1977
- Most commonly used today



# RSA Key Generation Algorithm

- 1) Select two primes,  $p$  and  $q$ ,  $p \neq q$
- 2) Calculate  $n = p \times q$
- 3) Calculate  $\varphi(n) = (p-1)(q-1)$
- 4) Select  $e$  s.t.  $\text{GCD}(\varphi(n), e) = 1$  and  $e < \varphi(n)$
- 5) Calculate  $d$  in  $de \bmod \varphi(n) = 1$
- 6) Your public key is  $\{e, n\}$
- 7) Your private key is  $\{d, n\}$



# RSA Encryption/Decryption Algorithm

## Encryption

- 1) Plaintext is  $M < n$
- 2) Ciphertext is  $C = M^e \bmod n$

## Decryption

- 1) Ciphertext is  $C$
- 2) Plaintext is  $M = C^d \bmod n$



# Elliptic Curve Cryptography (ECC)

- Considered as secure as RSA
- Much newer and coming out
  - Smaller key size
  - Not as trusted
- The future of encryption?



# Shortcomings of Public Key Crypto

- This seems amazing!
- Is sloooooow...
- Relies on unsolved problems in math
- What else?



# Solution: Certificate Authority

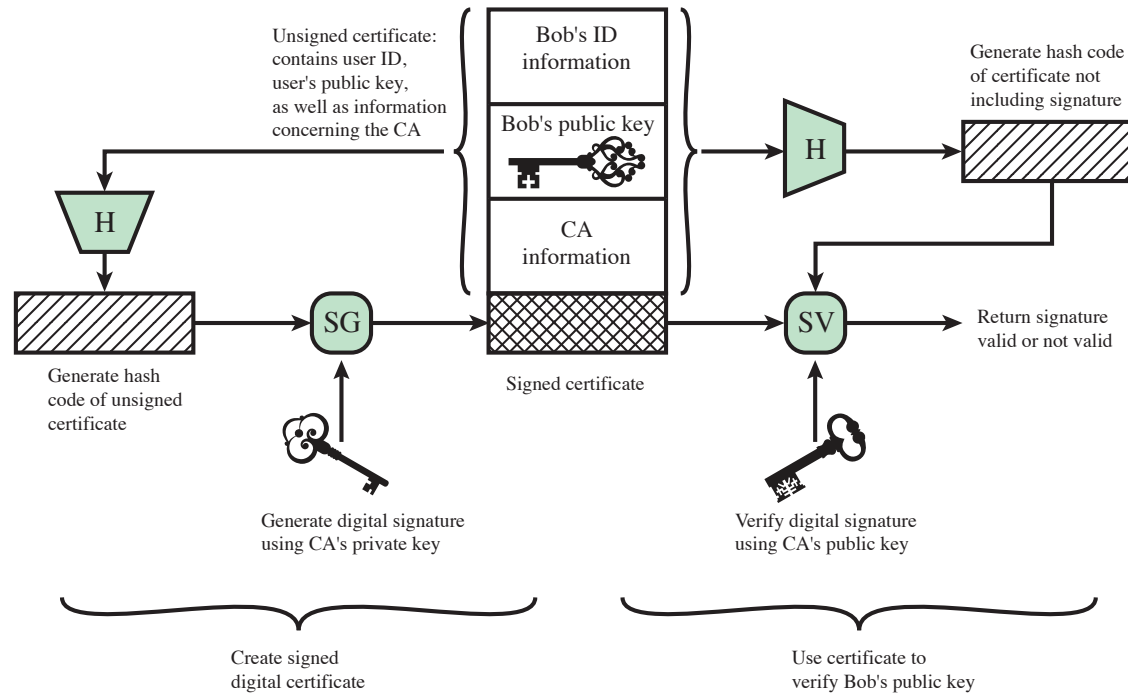


Figure 2.8 Public-Key Certificate Use



# Problem With Solution

## CRYPTOGRAPHY

# Ten Risks of PKI: What You're not Being Told about Public Key Infrastructure

**By Carl Ellison and Bruce Schneier**

Computer security has been victim of the “year of the...” syndrome. First it was firewalls, then intrusion detection systems, then VPNs, and now certification authorities (CAs) and public-key infrastructure (PKI). “If you only buy X,”

Open any article on PKI in the popular or technical press and you're likely to find the statement that a PKI is desperately needed for e-commerce to flourish. This statement is patently false. E-commerce is already flourish-



# Uses of Public Key Crypto





# Key Exchange

- Want to communicate over insecure channel
  - Need a way to communicate
- Key encryption vs key distribution



# Digital Signatures

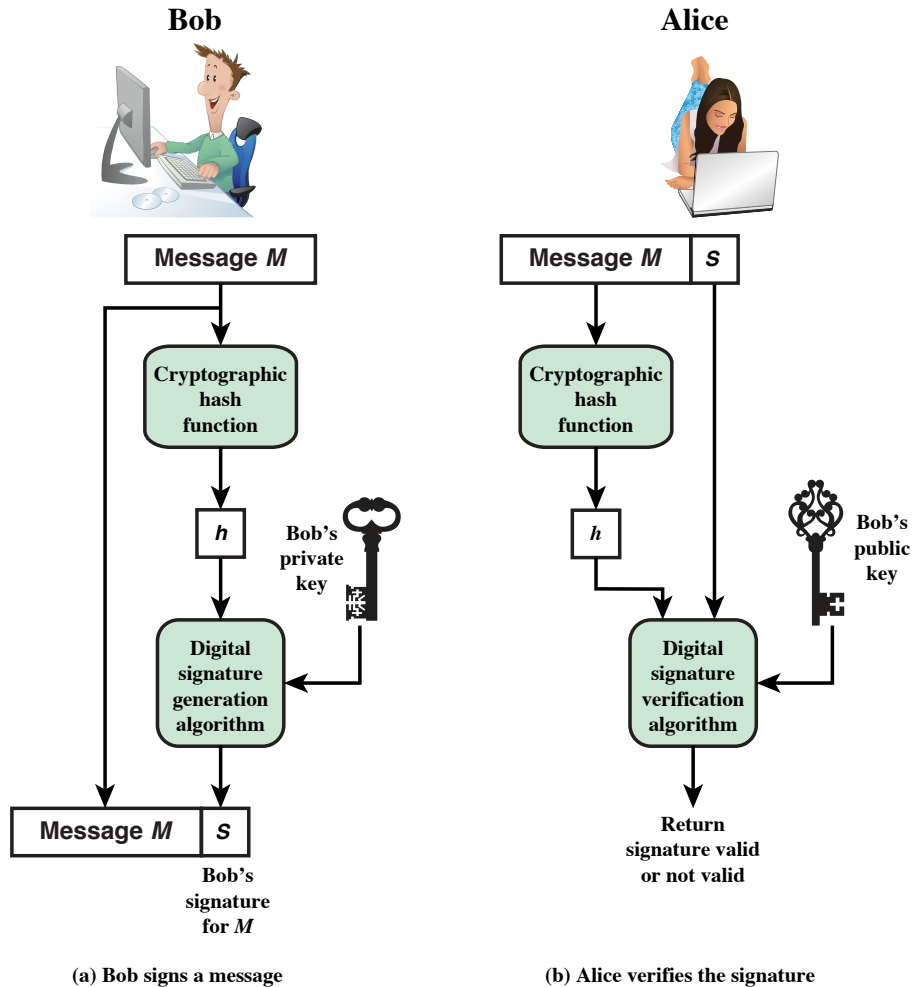


Figure 2.7 Simplified Depiction of Essential Elements of Digital Signature Process



# Digital Envelopes

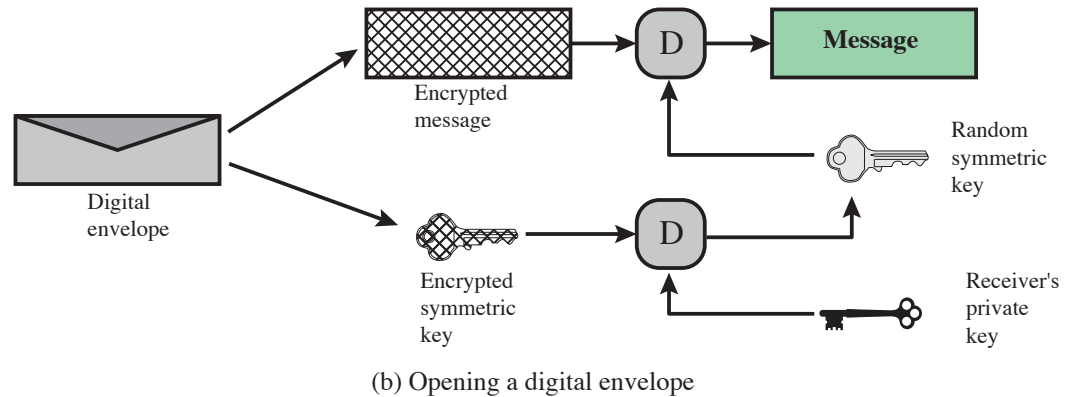
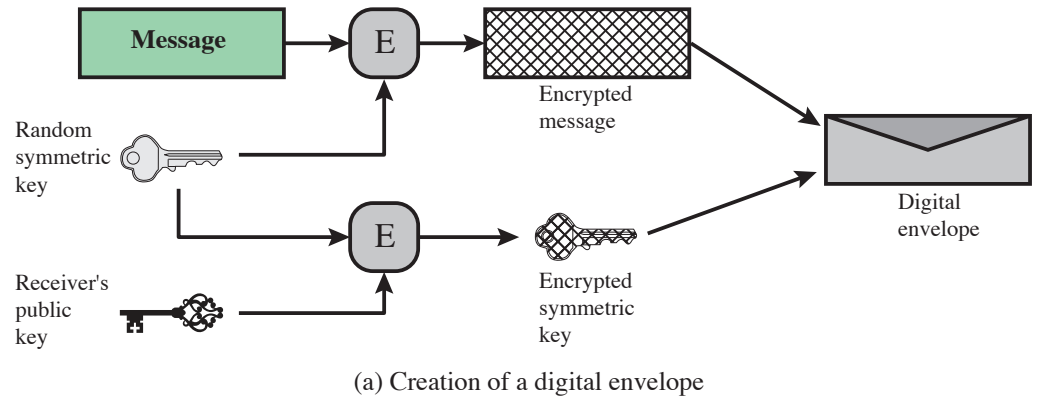


Figure 2.9 Digital Envelopes



# Two-Way Communication

- How would we make this work?

