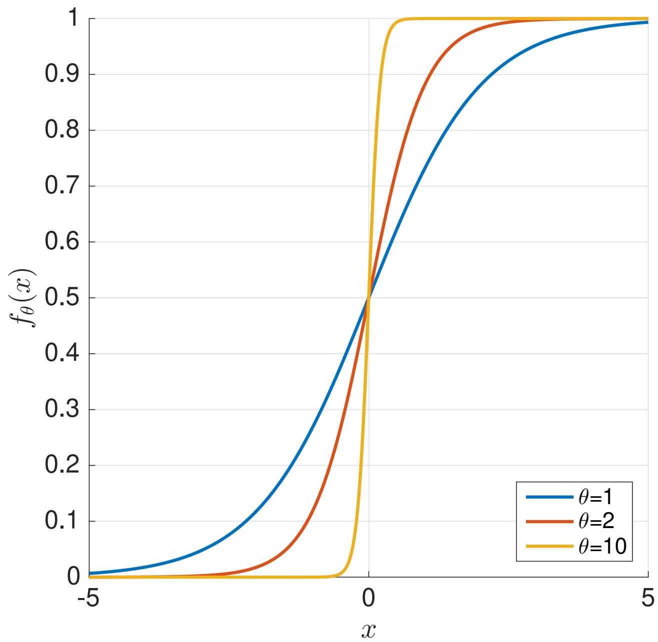
CSCI3656: NUMERICAL COMPUTATION
Homework 5: Due Friday, Oct. 30, 3:00pm

Turn in your own writeup that includes your code. List any resources you used including collaborating with others. You shouldn't need to use the symbolic toolbox. Submit a PDF on Canvas by Friday, Oct. 30 at 5pm.

Consider the parameterized family of functions,

$$f_\theta(x) \;=\; \frac{1}{1+\exp(-\theta x)}, \quad x \in [-5, 5].$$

The parameter θ controls how smooth f_θ is near $x = 0$, as shown:



To start this homework, let $\theta = 1$.

1. **Generate training data:** Create a vector with $n = 7$ evenly spaced points in the interval $[-5, 5]$. (In Matlab, you can do this with `linspace`.) For each point x_i in this vector, compute $y_i = f_\theta(x_i)$. You should now have 7 pairs (x_i, y_i). Provide a printout of the 7 pairs (e.g., in a table).

2. **Train the model:** Construct the Vandermonde system and solve for the coefficients of the unique degree 6 interpolating polynomial $p_6(x)$. Provide a printout of the 6 coefficients.

3. **Generate testing data:** Create a new vector with 101 evenly spaced points in $[-5, 5]$. For each point x_i', compute $y_i' = f_\theta(x_i')$. Report the mean (`mean` in Matlab) and standard deviation (`std` in Matlab) from the set of points $y_1', \ldots, y_{101}'$.

4. **Compute the testing error:** Compute and report the the absolute testing error:

$$\text{error} \;=\; \text{error}_{\theta=1,n=7} \;=\; \underset{1\le i\le 101}{\text{maximum}} \; |\, y_i' - p_6(x_i')\,|$$

Note that the error depends on the value of θ and the number of training points / the degree of the polynomial.

5. Repeat steps 1-4 with $\theta = 10$. How does the error change? What does that tell you about the quality of the polynomial approximation for the two functions?

6. EXTRA CREDIT (15pts): Repeat steps 1-4 with $\theta = 10$ and $n = 8, 9, \ldots, 15$. Plot $\text{error}_{\theta=10,n}$ versus n on a semilog scale. How does the polynomial approximation converge as n increases?