CSCI3656: NUMERICAL COMPUTATION
Homework 9: Due Friday, Nov. 13, 5:00pm

Turn in your own writeup that includes your code. List any resources you used including collaborating with others. You shouldn't need to use the symbolic toolbox. Submit a PDF on Canvas by Friday, Nov. 13 at 5pm.

1. Consider the function
$$f(x) \;=\; \sin(2\pi x) + \cos(3\pi x), \quad x \in [-1, 1]. \tag{1}$$
Compute the coefficients of a least-squares-fit degree-7 polynomial from $n = 33$ evenly spaced points. In other words, your training data are pairs (x_i, y_i) with $i = 1, \ldots, 33$ where the x_i's are evenly spaced points in $[-1, 1]$ (like, `linspace`) and $y_i = f(x_i)$. Make a plot of both $f(x)$ and the degree-7 polynomial approximation.

2. Create *testing data* by (i) choosing 100 random points in the interval $[-1, 1]$ and (ii) evaluating the function at each of those points. This gives you a new set of data $(x'_1, y'_1), \ldots, (x'_{100}, y'_{100})$.
For d from 1 to 31, compute the least-squares coefficients of a polynomial of degree d with the same training data as in the last problem using both the QR method and the normal equations.
For each trained polynomial $p_d(x)$, compute the normalized testing error:
$$e_d \;=\; \left(\frac{\sum_{i=1}^{100} (y'_i - p(x'_i))^2}{\sum_{i=1}^{100} (y'_i)^2} \right)^{1/2} \tag{2}$$
Plot the error e_d versus d on a log scale (that is, use `semilogy`). Make sure to include both (i) the error computed using the QR decomposition and (ii) the error computed using the normal equations. Interpret the error behavior.

BONUS (50 POINTS): Here some US COVID case counts from back in March.

Days since Feb 29	Case count
1	89
2	105
3	125
4	159
5	227
6	331
7	444
8	564
9	728
10	1000
11	1267
12	1645
13	2204
14	2826
15	3485
17	7038

Derive the linear least-squares system whose solution contains the coefficients of a log-linear model for case count over time. Plot the data on top of the model on a log scale. How well does the log-linear model (which represents exponential growth) appear to model the growth in the case count? What was the case doubling time over this roughly two week period?