CSCI3656: NUMERICAL COMPUTATION
Homework 11: Due Friday, Dec 4, 5:00pm

Turn in your own writeup that includes your code. List any resources you used including collaborating with others. You shouldn't need to use the symbolic toolbox. Submit a PDF on Canvas by Friday, Dec. 4 at 5pm.

1. Implement the following numerical methods for approximating integrals: (i) trapezoidal rule, (ii) Simpson's rule, and (iii) Clenshaw-Curtis rule. Code for generating the points and weights of the Clenshaw-Curtis rule is available on Canvas.

   Consider the function

   $$f(x) = \sin(2\,x) + \cos(3\,x), \quad x \in [-1, 1].$$

   (a) Using calculus, compute the definite integral of $f(x)$ on the interval $[-1, 1]$.

   (b) For $n = 2^k + 1$ with $k = 1, \ldots, 20$, use the three numerical integration methods to estimate the integral with n points. Plot the relative error as a function of n on a log-log scale.

   (c) Identify the values of n that constitute the *asymptotic regime*. For each of the three methods, what convergence rate do you observe?

2. Repeat the previous numerical study for the function

   $$f(x) = \text{sign}\,(x - 0.2) + 1, \quad x \in [-1, 1]$$

   where

   $$\text{sign}\,(y) = \begin{cases} 1, & y > 0 \\ 0, & y = 0 \\ -1, & y < 0 \end{cases}$$

   How do the observed convergence rates differ from the first function? *Why do they differ?*