CSCI3656: NUMERICAL COMPUTATION
Homework 10: Due Friday, Nov. 20, 5:00pm

Turn in your own writeup that includes your code. List any resources you used including collaborating with others. You shouldn't need to use the symbolic toolbox. Submit a PDF on Canvas by Friday, Nov. 20 at 5pm.

1. Implement the following numerical methods for approximating first derivatives: (i) one-sided forward difference, (ii) one-sided backward difference, and (iii) central difference. Consider the function
$$f(x) = \sin(4.8\,\pi\,x).$$
Use the numerical methods to estimate the derivative at $x = 0.2$ using the following values for h:
$$h \;=\; 2^{-k}, \quad k = 5, 6, \ldots, 24.$$
Plot the error versus h on a log-log scale. (You can compute the derivative by hand to get the *truth*.) For each of the three methods, what is the rate of convergence you observe?

2. Consider the finite difference approximation
$$f'(x) \;\approx\; \frac{1}{6h}\left[2f(x+h) + 3f(x) - 6f(x-h) + f(x-2h)\right].$$
Using a numerical study similar to Problem 1, identify the rate of convergence for this approximation. Produce a plot that justifies your computed rate. Identify the asymptotic regime in your plot.

BONUS: Construct and implement a 4th-order finite difference approximation of a first derivative.

1. (50 points) Use a Taylor series argument to prove that your method is 4th order.
2. (50 points) Run a numerical experiment to demonstrate that your method is 4th order.