Homework Sheet 7

Author	Matriculation Number	Tutor
Abdullah Oğuz Topçuoğlu	7063561	Maryna Dernovaia
Ahmed Waleed Ahmed Badawy Shora	7069708	Jan-Hendrik Gindorf
Yousef Mostafa Farouk Farag	7073030	Thorben Johr

Exercise 1

Remember the master theorem

$$\text{If} \quad T(n) \leq a \cdot T\left(\frac{n}{b} + r\right) + c \cdot n^d \log^s n$$

$$\text{then} \quad T(n) = \begin{cases} O(n^d \log^s n) & \text{if } a < b^d \\ O(n^d \log^{s+1} n) & \text{if } a = b^d \\ O(n^{\log_b a}) & \text{if } a > b^d \end{cases}$$

(a)

We are given the recurrence

$$f(n) \leq 5 \cdot f\left(\frac{n}{2}\right) + n^2$$

Values of paramters are

$$a = 5$$
$$b = 2$$
$$d = 2$$
$$s = 0$$

Since $a > b^d$ that is $5 > 2^2$ the third case applies.

$$f(n) \in O\left(n^{\log_2 5}\right)$$

(b)

We are given the recurrence

$$g(n) \leq 9 \cdot g\left(\frac{n}{3}\right) + n^2$$

Values of parameters are

$$a = 9$$
$$b = 3$$
$$d = 2$$
$$s = 0$$

Since $a = b^d$ that is $9 = 3^2$ the second case applies.

$$g(n) \in O\left(n^2 \log n\right)$$

(c)

We are given the recurrence

$$h(n) \leq 2 \cdot h\left(\frac{n}{3}\right) + n \log^2 n$$

Values of parameters are

$$\begin{aligned} a &= 2 \\ b &= 3 \\ d &= 1 \\ s &= 2 \end{aligned}$$

Since $a < b^d$ that is $2 < 3^1$ the first case applies.

$$h(n) \in O\left(n \log^2 n\right)$$

(d)

We are given the recurrence

$$\begin{aligned} k(n) &\leq 21 \cdot k\left(\frac{n}{9}\right) + n^{1.5}\sqrt{\log n} \\ &= 21 \cdot k\left(\frac{n}{9}\right) + n^{1.5} \log^{0.5} n \end{aligned}$$

Values of parameters are

$$\begin{aligned} a &= 21 \\ b &= 9 \\ d &= 1.5 \\ s &= 0.5 \end{aligned}$$

Since $a < b^d$ that is $21 < 9^{1.5} = 27$ the first case applies.

$$k(n) \in O\left(n^{1.5} \log^{0.5} n\right)$$

(e)

We are given the recurrence

$$\ell(n) \leq 3 \cdot \ell\left(\frac{n}{9}\right) + n^{0.1} \log n$$

Values of parameters are

$$\begin{aligned} a &= 3 \\ b &= 9 \\ d &= 0.1 \\ s &= 1 \end{aligned}$$

Since $a > b^d$ that is $3 > 9^{0.1}$ the third case applies.

$$\ell(n) \in O\left(n^{\log_9 3}\right) = O\left(n^{0.5}\right) = O\left(\sqrt{n}\right)$$