Homework Sheet 5

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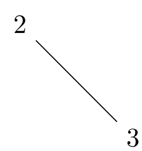
Exercise 1

We begin with an empty AVL tree and perform the following operations step by step:

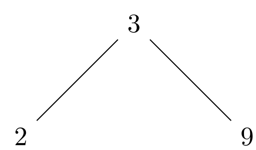
1. Insert(2)
2. Insert(3)
3. Insert(9)
4. Insert(5)
5. Insert(8)
6. Insert(6)
7. Remove(8)
8. Remove(5)
9. Insert(4)
10. Insert(5)
11. Remove(6)

Insert(2): 2

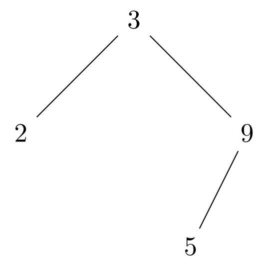
Insert(3):



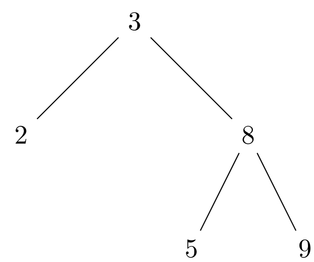
Insert(9):



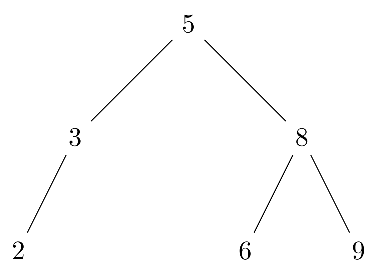
Insert(5):



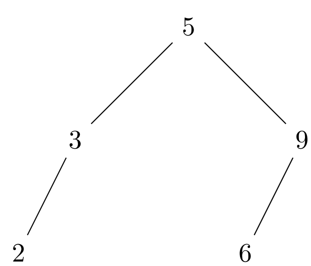
Insert(8):



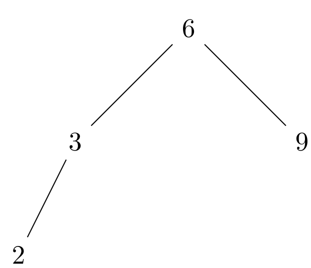
Insert(6):

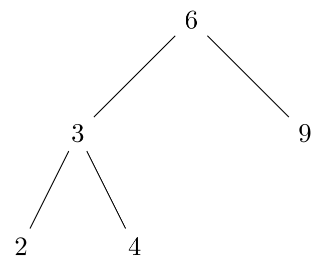


Remove(8):

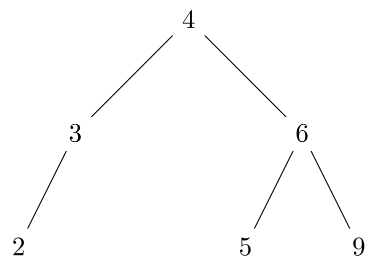


Remove(5):

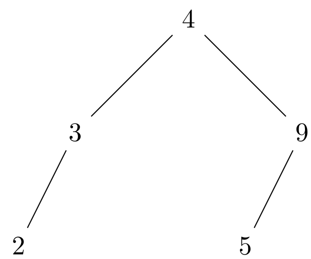




Insert(4):



Insert(5):

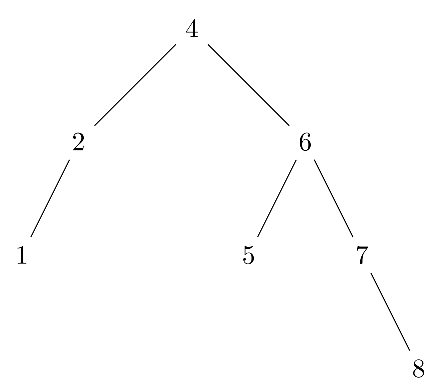


Remove(6):

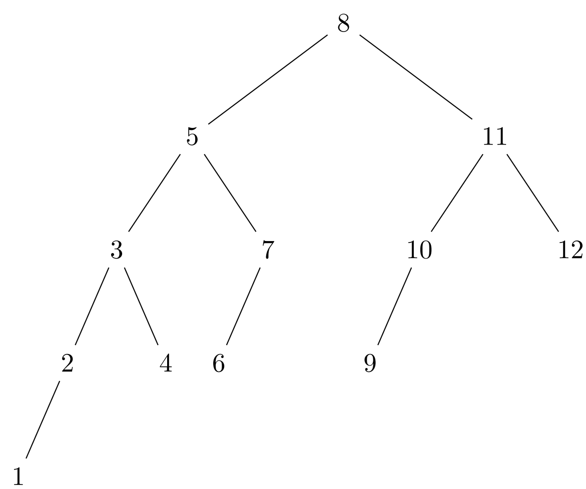
Exercise 2

(a)

An AVL tree of maximum height with 7 nodes:



An AVL tree of maximum height with 12 nodes:



(b)

Proof by Induction:

- **Base Case:** For height $h = 0$, the minimum number of nodes in an AVL tree is $F_{0+2} - 1 = F_2 - 1 = 1 - 1 = 0$, which is correct since an AVL tree of height 0 has no nodes.

- **Inductive Step:** Assume that for some height $h \geq 0$, the minimum number of nodes in an AVL tree of height h is $F_{h+2} - 1$. We need to show that this holds for height $h + 1$.

- Consider an AVL tree of height $h + 1$. To minimize the number of nodes, we can have one subtree of height h and another subtree of height $h - 1$ (since the height difference must be at most 1 for AVL trees).

- By the inductive hypothesis the minimum number of nodes in the subtree of height h is $F_{h+2} - 1$ and in the subtree of height $h-1$ is $F_{(h-1)+2} - 1 = F_{h+1} - 1$.

- Therefore the total minimum number of nodes in the AVL tree of height $h + 1$ is:

  $$N(h+1) = (F_{h+2} - 1) + (F_{h+1} - 1) + 1 = F_{h+2} + F_{h+1} - 1 = F_{(h+1)+2} - 1$$

  (using the Fibonacci property $F_n = F_{n-1} + F_{n-2}$).

- Thus by induction the statement holds for all heights $h \geq 0$.