

Homework Sheet 4

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Exercise 2

(a)

We will have the Stack structure like this

```
struct Stack {
    int top; // index of the top element
    int A[N]; // array of size N
};
```

So removing k many elements from stack can be done by just decrementing the top index by k . Here is the pseudocode:

```
function ManyPop(Stack S, int k)
    if S.top < k then
        S.top = 0; // stack becomes empty
    else
        S.top -= k; // remove top k elements
```

This implementation runs in $O(1)$ time since it only involves a couple of arithmetic operations and a conditional check.

(b)

We will have the Stack structure like this

```
struct Node {
    int data;
    Node* next;
};

struct Stack {
    Node* top; // pointer to the top node
};
```

Every element on the stack points to the elements that's below it. So the first element that's pushed to the stack has its next pointer as null. Here is the pseudocode for ManyPop:

```

function ManyPop(Stack S, int k)
    count = 0;
    while S.top != null and count < k do
        temp = S.top;
        S.top = S.top.next; // move top to the next element
        delete temp; // free memory of the popped element
        count += 1;

```

This implementation runs in $O(k)$ time since it involves a loop that iterates k times, performing constant time operations in each iteration.

(c)

To prove that the amortized running time of Push and ManyPop is $O(1)$, we will use the bank accounting method from the lecture. We will assign an amortized cost to each operation as follows:

- Push operation: Lets say we get 2 euros every time Push() is called. We spend 1 euro for the constant time operation in Push() function and store the remaining 1 euro in the bank.
- ManyPop operation: Lets say we get 0 euro every time ManyPop() is called. We need k euros to remove k elements from the stack. Meaning that we need 1 euro to remove one element. Luckily enough we saved 1 euro per element during the Push() operations so we can spend them here.