

Homework Sheet 1

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Exercise 1

(a)

The algorithm is correct because when we sort the numbers, if there is no missing number, then each number has an index one more than its value. (that's the case because the array given is 1-indexed but the values we are given start from 0) And the algorithm starts from the first element and when it encounters the first number that is equal to its index (rather than being equal to index + 1) it means we just skipped a number and we return it. And if we haven't encountered any number in process then it means it has to be the last element in the sorted array which has the value "n" always (because it is sorted)

(b)

So we have an accumulator "y" and in each iteration we add the current index and we subtract the current value in the array. So the value of "y" at the end is:

$$\begin{aligned}\sum_{i=1}^n i - A[i] &= \sum_{i=1}^n i - \sum_{i=1}^n A[i] \\ &= \frac{n(n+1)}{2} - (0 + 1 + 2 + \dots + n - x) \quad (\text{where } x \text{ is the missing number}) \\ &= \frac{n(n+1)}{2} - \left(\frac{n(n+1)}{2} - x \right) \\ &= x\end{aligned}$$

So "y" is equal to the missing number at the end of the algorithm. And we return "y" at the end.