

# Homework Sheet 3

## Authors

Abdullah Oğuz Topçuoğlu  
Ahmed Waleed Ahmed Badawy Shora  
Yousef Mostafa Farouk Farag

## Tutors

Maryna Dernovaia  
Jan-Hendrik Gindorf  
Thorben Johr

## Exercise 3

We can do something similar to LsdRadix sort we saw in the lecture.

**Pseudocode:**

```
function SortGridPoints(A[1..n]) {  
    redefine key(point) := point.y  
    CountingSort(A)  
  
    redefine key(point) := point.x  
    CountingSort(A)  
}
```

**Correctness:**

- The first CountingSort sorts the points by their y coordinates.
- The second CountingSort sorts the points by their x coordinates, but since CountingSort is stable, the order of points with the same x coordinate is preserved.
- Therefore after both sorts the points are sorted lexicographically by (x, y).
- That's also what we did in the lecture for LSDRadixSort we started from the least significant digit to the most significant digit. The same idea applies here.

**Running Time Analysis:**

- Each CountingSort runs in time  $O(n + k)$  where  $k$  is the range of the keys.
- Here the keys are the x and y coordinates of the points.
- Since the points are connected, the range of x coordinates is at most  $n$  and the range of y coordinates is also at most  $n$ .
- Therefore each CountingSort runs in time  $O(n + n) = O(n)$ .
- Since we perform two CountingSorts, the total running time is  $O(n) + O(n) = O(n)$ .

## Exercise 4

(a)

We can use the LSDRadixSort again.

**Pseudocode:**

```
const ALPHABET_SIZE = 26
function SortStrings(A[1..n]) {
    for i in ALPHABET_SIZE..1 {
        redefine key(string) := string[i]
        CountingSort(A)
    }
}
```

**Correctness:**

- We sort the strings starting from the last character to the first character (least significant to most significant).
- Each CountingSort is stable, so the order of strings with the same character at position  $i$  is preserved.
- Therefore after sorting by all character positions, the strings are sorted lexicographically.
- So just like LSDRadixSort we saw in the lecture where  $d$  is alphabet size,  $U$  is the alphabet

**Running Time Analysis:**

- Each CountingSort runs in time  $O(n + k)$  where  $k$  is the range of the keys.
- Here the keys are the english letters 'a' to 'z', so  $k = 26$ .
- Therefore each CountingSort runs in time  $O(n + 26) = O(n)$ .
- Since we perform CountingSort 26 times, the total running time is  $O(26 * n) = O(n)$ .