Mathematics Homework Sheet 1

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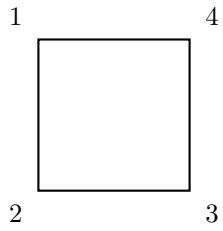
Problem 1

Symmetry group S will consist of rotations and reflections.

- Rotations: R_{90}, R_{180}, R_{270}
- Reflections: T_x, T_y, T_d, $T_{d'}$
- Identity: I



R_i rotates i degrees clockwise.
T_x reflects over the x-axis, T_y reflects over the y-axis, T_d reflects diagonally, and $T_{d'}$ reflects over the other diagonal.
When we take a look at S_4, S_4 has $4! = 24$ elements.
Our group S has 8 elements.
Lets start with identity I.

- ()

Rotations:

- $R_{90} = (1, 2, 3, 4)$
- $R_{180} = (1, 3)(2, 4)$
- $R_{270} = (1, 4, 3, 2)$

Reflections:

- $T_x = (1, 2)(3, 4)$
- $T_y = (1, 4)(2, 3)$
- $T_d = (1, 3)$
- $T_{d'} = (2, 4)$

So, when combined, S can be identified with this subset of S_4:

$$\{(), (1, 2, 3, 4), (1, 3)(2, 4), (1, 4, 3, 2), (1, 2)(3, 4), (1, 4)(2, 3), (1, 3), (2, 4)\}$$