Mathematics Homework Sheet 5

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Problem 1

Problem 1(b)

Problem 1(b)(i)

We want to prove $\bigcap_{i \in I} U_i$ is closed.
We are given $(\forall i \in I \quad U_i \subseteq R)$ is closed.
A set being closed means that its complement is open. So we want to prove that $\bigcup_{i \in I} U_i^c$ is open.
Since each U_i is closed, we know that U_i^c is open.
And from the lecture we know that union or intersection of open sets is open.
Thus $\bigcup_{i \in I} U_i^c$ is open. Which means that $\bigcap_{i \in I} U_i$ is closed.
And this completes the proof.

Problem 1(b)(ii)

We want to prove $\bigcup_{i=1}^{n} U_i$ is closed.
We are given $(U_1, ..., U_n \subseteq R)$ are closed.
A set being closed means that its complement is open. So we want to prove that $\bigcap_{i=1}^{n} U_i^c$ is open.
Since each U_i is closed, we know that U_i^c is open.
And from the lecture we know that union or intersection of open sets is open.
Thus $\bigcap_{i=1}^{n} U_i^c$ is open. Which means that $\bigcup_{i=1}^{n} U_i$ is closed.
And this completes the proof.