

# Mathematics Homework Sheet 4

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## Problem 1

We want to prove

$$(\forall n \in \mathbb{N}) \wedge (x \in \mathbb{R}) \wedge (x \geq -1) \quad (1+x)^n \geq 1+nx$$

by using mathematical induction.

**Base Case:** For  $n = 1$ , we have

$$(1+x)^1 = 1+x \geq 1+1 \cdot x$$

Which is true for every  $x \in \mathbb{R}$  so it means it is also true for  $x \in [-1, \infty]$

**Inductive Step:** Assume that the statement is true for  $n = k$ , i.e.

$$(1+x)^k \geq 1+kx$$

is true for every  $x \in [-1, \infty]$ .

We want to prove that the statement is also true for  $n = k+1$ , i.e.

$$(1+x)^{k+1} \geq 1+(k+1)x$$

for every  $x \in [-1, \infty]$ .

$$(1+x)^{k+1} = (1+x)^k \cdot (1+x) \tag{1}$$

$$\geq (1+kx) \cdot (1+x) \quad (\text{Using inductive step}) \tag{2}$$

$$= 1+kx+x+kx^2 \tag{3}$$

$$= 1+(k+1)x+kx^2 \tag{4}$$

$$\geq 1+(k+1)x \quad (\text{Since } kx^2 \geq 0) \tag{5}$$

And this completes the proof.

**Give a counterexample to show that the condition  $x \geq -1$  is necessary:**

Let's take  $x = -2$  and  $n = 2$ . Then we have

$$(1-2)^2 \geq 1+2 \cdot (-2)$$

$$(-1)^2 \geq 1-4$$

$$1 \geq -3$$

Which is not true. In fact it would be false when  $n$  is even. So the condition  $x \geq -1$  is necessary. Because that way  $1 + x$  is never negative