Homework Sheet 5

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Exercise 17

We are given the function

$$f(x) := -e^x + 2x + 1.$$

We calculte the newtons method using this formula

$$x_n = x_{n-1} - \frac{f(x_{n-1})}{f'(x_{n-1})}$$

(i)

We want to find x_2 when $x_0 = -1$

Lets start by finding $f'(x)$

$$f'(x) = -e^x + 2$$

Now we can calculate x_1 and x_2

$$
\begin{aligned}
x_1 &= x_0 - \frac{f(x_0)}{f'(x_0)} \\
&= -1 - \frac{-e^{-1} + 2(-1) + 1}{-e^{-1} + 2} \\
&= -1 - \frac{-\frac{1}{e} - 2 + 1}{-\frac{1}{e} + 2} \\
&= -1 - \frac{-\frac{1}{e} - 1}{-\frac{1}{e} + 2} \\
&= -1 + \frac{\frac{1}{e} + 1}{-\frac{1}{e} + 2} \\
&= -1 + \frac{e + 1}{-1 + 2e} \\
&= \frac{-2e + e + 1 + 1}{-1 + 2e} \\
&= \frac{-e + 2}{-1 + 2e} \\
&\approx -0.16190048965915385 \quad \text{(via using a calculator)}
\end{aligned}
$$

$$
\begin{aligned}
x_2 &= x_1 - \frac{f(x_1)}{f'(x_1)} \\
&\approx -0.16190048965915385 - \frac{f(-0.16190048965915385)}{f'(-0.16190048965915385)} \\
&\approx -0.16190048965915385 - \frac{-0.174326815763479123074}{f'(-0.16190048965915385)} \quad \text{again using a calculator} \\
&\approx -0.16190048965915385 - \frac{-0.174326815763479123074}{1.14947416355482857 6926} \quad \text{again using a calculator} \\
&\approx -0.16190048965915385 - -0.151657880873425944806890147658365522997056738243886290320790176143 \\
&\approx -0.0102426087857279051931098523416344770029432617561137096792098238561672669 \quad \text{again using a c}
\end{aligned}
$$

Links to the calculator:
`https://www.wolframalpha.com/input?i=f%28x%29+%3D+-e%5Ex+%2B2x+%2B1+at+x%3D%E2%88%920.16190048965915385&assumption=%7B%22C%22%2C+%22at%22%7D+-%3E+%7B%22EnglishWord%22%7D`
`https://www.wolframalpha.com/input?i=g%28x%29+%3D+-e%5Ex+%2B2+where+x+%3D+%E2%88%920.16190048965915385`

(ii)

We want to find x_2 when $x_0 = 1$
The derivatie

$$f'(x) = -e^x + 2$$

Now we can calculate x_1 and x_2

$$\begin{aligned}
x_1 &= x_0 - \frac{f(x_0)}{f'(x_0)} \\
&= 1 - \frac{-e^1 + 2(1) + 1}{-e^1 + 2} \\
&= 1 - \frac{-e + 2 + 1}{-e + 2} \\
&= 1 - \frac{-e + 3}{-e + 2} \\
&= 1 + \frac{e - 3}{-e + 2} \\
&= \frac{-e + 2 + e - 3}{-e + 2} \\
&= \frac{-1}{-e + 2} \\
&\approx 1.3922111911 \quad \text{(via using a calculator)}
\end{aligned}$$

$$\begin{aligned}
x_2 &= x_1 - \frac{f(x_1)}{f'(x_1)} \\
&= 1.3922111911 - \frac{f(1.3922111911)}{f'(1.3922111911)} \\
&\approx 1.3922111911 - \frac{-0.23931509377336}{f'(1.3922111911)} \quad \text{again using a calculator} \\
&\approx 1.3922111911 - \frac{-0.23931509377336}{-2.02373747597336} \quad \text{again using a calculator} \\
&\approx 1.3922111911 - 0.11825402089678468113553397669325 49 \quad \text{again using a calculator} \\
&\approx 1.27395717020321531886446602330674 51 \quad \text{again using a calculator}
\end{aligned}$$

Exercise 18

We are given the function

$$f(x_1, x_2) := \begin{pmatrix} x_1^3 + x_2 - 2 \\ x_1 + x_2^3 - 2 \end{pmatrix}$$

The newtons method for multivariable functions is given by

$$\mathbf{x_n} = \mathbf{x_{n-1}} - J_f(\mathbf{x_{n-1}})^{-1} f(\mathbf{x_{n-1}})$$

The Jacobian matrix J_f is

$$\begin{aligned} J_f(x_1, x_2) &= \begin{pmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} \\ \\ \frac{\partial f_2}{\partial x_1} & \frac{\partial f_2}{\partial x_2} \end{pmatrix} \\ &= \begin{pmatrix} 3x_1^2 & 1 \\ 1 & 3x_2^2 \end{pmatrix} \end{aligned}$$

We start with $\mathbf{x_0} = (1, 1)$

$$\begin{aligned} \mathbf{x_1} &= \mathbf{x_0} - J_f(\mathbf{x_0})^{-1} f(\mathbf{x_0}) \\ &= \begin{pmatrix} 1 \\ 1 \end{pmatrix} - \begin{pmatrix} 3(1)^2 & 1 \\ 1 & 3(1)^2 \end{pmatrix}^{-1} \begin{pmatrix} (1)^3 + (1) - 2 \\ (1) + (1)^3 - 2 \end{pmatrix} \\ &= \begin{pmatrix} 1 \\ 1 \end{pmatrix} - \begin{pmatrix} 3 & 1 \\ 1 & 3 \end{pmatrix}^{-1} \begin{pmatrix} 0 \\ 0 \end{pmatrix} \\ &= \begin{pmatrix} 1 \\ 1 \end{pmatrix} - \begin{pmatrix} 0 \\ 0 \end{pmatrix} \\ &= \begin{pmatrix} 1 \\ 1 \end{pmatrix} \end{aligned}$$

Doing the same calculation $\mathbf{x_2}$ will also be $\begin{pmatrix} 1 \\ 1 \end{pmatrix}$. It turns out that i didnt even need to calculate the jacobian matrix :)