Mathematics Homework Sheet 6

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Problem 2

(a)

U_1:
U_1 is subspace of $R[x]$.
Not empty:
U_1 is not empty because $0 \in U_1$ (zero polynomial).
Closed under addition:
Let $p(x), q(x) \in U_1$. Then $p(0) = 0$ and $q(0) = 0$.
Then, $(p+q)(0) = p(0) + q(0) = 0 + 0 = 0$.
Closed under scalar multiplication:
Let $p(x) \in U_1$ and $c \in R$. Then, $(cp)(0) = c(p(0)) = c(0) = 0$.
Thus, U_1 is closed under scalar multiplication.

U_2:
U_2 is not a subspace of $R[x]$. Because U_2 doesn't contain the zero polynomial. (every vector space has to contain the zero vector which is the zero polynomial in this case)

U_3:
U_3 is subspace of $R[x]$.
Not empty:
U_3 is not empty because $0 \in U_3$ (zero polynomial).
Closed under addition:
Let $p(x), q(x) \in U_3$. Then $p(1) = 0$ and $q(1) = 0$.
Then, $(p+q)(1) = p(1) + q(1) = 0 + 0 = 0$.
Closed under scalar multiplication:
Let $p(x) \in U_3$ and $c \in R$. Then, $(cp)(1) = c(p(1)) = c(0) = 0$.
Thus, U_3 is closed under scalar multiplication.

U_4:
U_4 is subspace of $R[x]$.
Not empty:
U_4 is not empty because $0 \in U_4$ (zero polynomial).
Closed under addition:
Let $p(x), q(x) \in U_4$. Then $\int_0^1 p(x)dx = 0$ and $\int_0^1 q(x)dx = 0$.
Then, $\int_0^1 (p+q)(x)dx = \int_0^1 p(x)dx + \int_0^1 q(x)dx = 0 + 0 = 0$.
Closed under scalar multiplication:

Let $p(x) \in U_4$ and $c \in R$. Then, $\int_0^1 (cp)(x)dx = c\int_0^1 p(x)dx = c(0) = 0$.
Thus, U_4 is closed under scalar multiplication.

U_5:
U_5 is subspace of $R[x]$.
Not empty:
U_5 is not empty because $0 \in U_5$ (zero polynomial).
Closed under addition:
Let $p(x), q(x) \in U_5$. Then $p'(0) + p''(0) = 0$ and $q'(0) + q''(0) = 0$.
Then, $(p+q)'(0) + (p+q)''(0) = p'(0) + q'(0) + p''(0) + q''(0) = 0 + 0 = 0$.
Closed under scalar multiplication:
Let $p(x) \in U_5$ and $c \in R$. Then, $(cp)'(0) + (cp)''(0) = c(p'(0)) + c(p''(0)) = c(p'(0) + p''(0)) = c(0) = 0$.
Thus, U_5 is closed under scalar multiplication.

U_6:
U_6 is not a subspace of $R[x]$. Becuase it is not closed under addition
Let $p(x), q(x) \in U_6$. Then $p'(0)p''(0) = 0$ and $q'(0)q''(0) = 0$.
Then, $(p+q)'(0)(p+q)''(0) = (p'(0) + q'(0))(p''(0) + q''(0)) = p'(0)p''(0) + p'(0)q''(0) + q'(0)p''(0) + q'(0)q''(0) = p'(0)q''(0) + q'(0)p''(0)$
Which is not necessarily equal to 0. Thus U_6 is not closed under addition.