

Homework Sheet 7

Author: Abdullah Oğuz Topçuoğlu

Exercise 25

We choose the probability space

$$\begin{aligned}\Omega &= \{1, 2, 3, 4, 5, 6\}^3, \\ \mathcal{F} &= \text{Power set of } \Omega, \\ P(A) &= U_\Omega\end{aligned}$$

The events are

$$\begin{aligned}A_1 &= \{(a, b, c) \in \Omega \mid a + b < 5\}, \\ A_2 &= \{(a, b, c) \in \Omega \mid b \cdot c = 6\}, \\ A_3 &= \{(a, b, c) \in \Omega \mid b = 2\} \\ A_1 \cap A_3 &= \{(a, 2, c) \in \Omega \mid a < 3\}, \\ (A_1 \cup A_2) \setminus A_3 &= \{(a, b, c) \in \Omega \mid (a + b < 5) \text{ or } (b \cdot c = 6) \text{ and } b \neq 2\}\end{aligned}$$

Since P is uniform we count the number of elements in each event.

Calculating the probabilities:

6 many (a,b) pairs times 6 choices for c

$$P(A_1) = \frac{6 \cdot 6}{216} = \frac{36}{216} = \frac{1}{6},$$

4 many (b,c) pairs times 6 choices for a

$$P(A_2) = \frac{6 \cdot 4}{216} = \frac{24}{216} = \frac{1}{9},$$

6 choices for a times 1 choice for b times 6 choices for c

$$P(A_3) = \frac{36}{216} = \frac{1}{6},$$

2 choices for a times 1 choice for b times 6 choices for c

$$P(A_1 \cap A_3) = \frac{2 \cdot 6}{216} = \frac{1}{18},$$

For the last one we need to calculate $|A_1 \cup A_2|$ first:

$$\begin{aligned}|A_1 \cup A_2| &= |A_1| + |A_2| - |A_1 \cap A_2|, \\ |A_1| &= 36, \\ |A_2| &= 24,\end{aligned}$$

Calculating $|A_1 \cap A_2|$:

if $b = 1, c = 6$: $a + 1 < 5 \Rightarrow a < 4 \Rightarrow 3$ choices
 if $b = 2, c = 3$: $a + 2 < 5 \Rightarrow a < 3 \Rightarrow 2$ choices
 if $b = 3, c = 2$: $a + 3 < 5 \Rightarrow a < 2 \Rightarrow 1$ choice
 if $b = 6, c = 1$: $a + 6 < 5 \Rightarrow$ no choices.

So $|A_1 \cap A_2| = 3 + 2 + 1 + 0 = 6$.

$$|A_1 \cup A_2| = 36 + 24 - 6 = 54,$$

$$P(A_1 \cup A_2) = \frac{54}{216} = \frac{1}{4},$$

$$P((A_1 \cup A_2) \setminus A_3) = P(A_1 \cup A_2) - P(A_3) + P(A_1 \cap A_3) = \frac{1}{4} - \frac{1}{6} + \frac{1}{18} = \frac{5}{36}.$$

Exercise 26

Lets define a set for students

$$S := \{s \in \mathbb{N} \mid s < 30\}$$

Each number represents a student. And note that $|S| = 30$. And the first 7 students belong to tutorial A following 12 students belong to tutorial B and the last 11 students belong to tutorial C.

We choose the probability space

$$\Omega = \{(a, b, c) \mid a, b, c \in S \text{ and } a \neq b \neq c\},$$

$$\mathcal{F} = \text{Power set of } \Omega,$$

$$P(A) = U_\Omega$$

We can choose uniform distribution because each student has equal chance of being selected.

The events are:

$$A_1 = \{(a, b, c) \in \Omega \mid a \in \{7, \dots, 18\}\},$$

$$A_2 = \{(a, b, c) \in \Omega \mid |\{p \in \{a, b, c\} \mid p \in \{0, \dots, 6\}\}| = 1 \quad \wedge$$

$$|\{p \in \{a, b, c\} \mid p \in \{7, \dots, 18\}\}| = 1 \quad \wedge$$

$$|\{p \in \{a, b, c\} \mid p \in \{19, \dots, 29\}\}| = 1\}$$

Here note that $|\Omega| = 30 \cdot 29 \cdot 28$

Calculating the probabilities:

12 choices for the first paper, 29 choices for the second paper and 28 choices for the third paper

$$P(A_1) = \frac{12 \cdot 29 \cdot 28}{|\Omega|} = \frac{12 \cdot 29 \cdot 28}{30 \cdot 29 \cdot 28} = \frac{12}{30} = \frac{2}{5},$$

choose a student from tutorial A (7 choices), tutorial B (12 choices) and tutorial C (11 choices). Then permute them ($3! = 6$) since order doesn't matter

$$P(A_2) = \frac{7 \cdot 12 \cdot 11 \cdot 6}{|\Omega|} = \frac{7 \cdot 12 \cdot 11 \cdot 6}{30 \cdot 29 \cdot 28} = \frac{5544}{24360} = \frac{231}{1015}.$$