Mathematics Homework Sheet 6

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Problem 1

Problem 1 (a)

We want to compute the following limit:

$$\lim_{n\to\infty} \frac{(n+1)^4(1-4n^3)^2}{(1+2n^2)^5}$$

The top part look somethin like this:

$$(n+1)^4(1-4n^3)^2 = 16n^{10} + \sum_{i=0}^{i=9} k_i n^i$$

And the bottom part look like this:

$$(1+2n^2)^5 = 32n^{10} + \sum_{i=0}^{i=9} k_i n^i$$

When we substitute these into the limit, we get:

$$\lim_{n\to\infty} \frac{16n^{10} + \sum_{i=0}^{i=9} k_i n^i}{32n^{10} + \sum_{i=0}^{i=9} k_i n^i}$$

Divide the top and bottom by n^{10}:

$$\lim_{n\to\infty} \frac{16 + \sum_{i=0}^{i=9} k_i n^{i-10}}{32 + \sum_{i=0}^{i=9} k_i n^{i-10}}$$

$$\frac{\lim_{n\to\infty}(16 + \sum_{i=0}^{i=9} k_i n^{i-10})}{\lim_{n\to\infty}(32 + \sum_{i=0}^{i=9} k_i n^{i-10})}$$

$$\frac{16 + \sum_{i=0}^{i=9} k_i \lim_{n\to\infty} n^{i-10}}{32 + \sum_{i=0}^{i=9} k_i \lim_{n\to\infty} n^{i-10}}$$

And we know that $\lim_{n\to\infty} 1/n^i$ is zero for all $i > 0$. Therefore, the limit is:

$$\frac{16}{32} = \frac{1}{2}$$

Problem 1 (b)

We want to compute the following limit:

$$\lim_{n\to\infty} \sqrt{n+1} - \sqrt{n}$$

Multiply and divide by the conjugate:

$$\lim_{n\to\infty} \sqrt{n+1} - \sqrt{n} \cdot \frac{\sqrt{n+1} + \sqrt{n}}{\sqrt{n+1} + \sqrt{n}}$$

$$\lim_{n\to\infty} \frac{n+1-n}{\sqrt{n+1} + \sqrt{n}}$$

$$\lim_{n\to\infty} \frac{1}{\sqrt{n+1} + \sqrt{n}}$$

And observe that

$$\frac{1}{n^2} < \frac{1}{\sqrt{n+1} + \sqrt{n}} < \frac{1}{\sqrt{n}}$$

when $n > 10$ (10 is not a magic number, it is just a number that is big enough) and we only care about the tail of the sequences not the head.

And we know that:

$$\lim_{n\to\infty} \frac{1}{n^2} = 0$$

$$\lim_{n\to\infty} \frac{1}{\sqrt{n}} = 0$$

From the sandwich teorem, we can conclude that:

$$\lim_{n\to\infty} \frac{1}{\sqrt{n+1} + \sqrt{n}} = 0$$