Mathematics Homework Sheet 10

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Problem 1

We want to compute the following using the power series of sin

$$\lim_{x \to 0} \frac{sin(x) - x}{x^3}$$

Power series of sin is given as

$$sin(x) = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!}$$

Let's insert this into the limit

$$\lim_{x \to 0} \frac{\sum_{n=0}^{\infty}(-1)^n \frac{x^{2n+1}}{(2n+1)!} - x}{x^3}$$

$$= \lim_{x \to 0} \frac{\sum_{n=1}^{\infty}(-1)^n \frac{x^{2n+1}}{(2n+1)!} - x + x}{x^3}$$

$$= \lim_{x \to 0} \frac{\sum_{n=1}^{\infty}(-1)^n \frac{x^{2n+1}}{(2n+1)!}}{x^3}$$

$$= \lim_{x \to 0} \sum_{n=1}^{\infty}(-1)^n \frac{x^{2n-2}}{(2n+1)!}$$

$$= \lim_{x \to 0} \sum_{n=2}^{\infty}(-1)^n \frac{x^{2n-2}}{(2n+1)!} - \frac{1}{3!}$$

$$= \sum_{n=2}^{\infty}(-1)^n \frac{\lim_{x \to 0} x^{2n-2}}{(2n+1)!} - \frac{1}{3!}$$

$$= 0 - \frac{1}{3!}$$

$$= -\frac{1}{6}$$