

Homework Sheet 5

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Exercise 17

We are given the function

$$f(x) := -e^x + 2x + 1.$$

We calculate the newtons method using this formula

$$x_n = x_{n-1} - \frac{f(x_{n-1})}{f'(x_{n-1})}$$

(i)

We want to find x_2 when $x_0 = -1$

Lets start by finding $f'(x)$

$$f'(x) = -e^x + 2$$

Now we can calculate x_1 and x_2

$$\begin{aligned}
 x_1 &= x_0 - \frac{f(x_0)}{f'(x_0)} \\
 &= -1 - \frac{-e^{-1} + 2(-1) + 1}{-e^{-1} + 2} \\
 &= -1 - \frac{-\frac{1}{e} - 2 + 1}{-\frac{1}{e} + 2} \\
 &= -1 - \frac{-\frac{1}{e} - 1}{-\frac{1}{e} + 2} \\
 &= -1 + \frac{\frac{1}{e} + 1}{-\frac{1}{e} + 2} \\
 &= -1 + \frac{e + 1}{-1 + 2e} \\
 &= \frac{-2e + e + 1 + 1}{-1 + 2e} \\
 &= \frac{-e + 2}{-1 + 2e} \\
 &\approx -0.16190048965915385 \quad (\text{via using a calculator})
 \end{aligned}$$

$$\begin{aligned}
 x_2 &= x_1 - \frac{f(x_1)}{f'(x_1)} \\
 &\approx -0.16190048965915385 - \frac{f(-0.16190048965915385)}{f'(-0.16190048965915385)} \\
 &\approx -0.16190048965915385 - \frac{-0.174326815763479123074}{f'(-0.16190048965915385)} \quad \text{again using a calculator} \\
 &\approx -0.16190048965915385 - \frac{-0.174326815763479123074}{1.149474163554828576926} \quad \text{again using a calculator} \\
 &\approx -0.16190048965915385 - -0.151657880873425944806890147658365522997056738243886290320790176143 \\
 &\approx -0.0102426087857279051931098523416344770029432617561137096792098238561672669 \quad \text{again using a calculator}
 \end{aligned}$$

Links to the calculator:

<https://www.wolframalpha.com/input?i=f%28x%29+%3D+-e%5Ex+%2B2x+%2B1+at+x%3D%E2%88%920.16190048965915385&assumption=%7B%22C%22%2C+%22at%22%7D+-%3E+%7B%22EnglishWord%22%7D>
<https://www.wolframalpha.com/input?i=g%28x%29+%3D+-e%5Ex+%2B2+where+x+%3D+%E2%88%920.16190048965915385>

(ii)

We want to find x_2 when $x_0 = 1$

The derivative

$$f'(x) = -e^x + 2$$

Now we can calculate x_1 and x_2

$$\begin{aligned}x_1 &= x_0 - \frac{f(x_0)}{f'(x_0)} \\&= 1 - \frac{-e^1 + 2(1) + 1}{-e^1 + 2} \\&= 1 - \frac{-e + 2 + 1}{-e + 2} \\&= 1 - \frac{-e + 3}{-e + 2} \\&= 1 + \frac{e - 3}{-e + 2} \\&= \frac{-e + 2 + e - 3}{-e + 2} \\&= \frac{-1}{-e + 2} \\&\approx 1.3922111911 \quad (\text{via using a calculator})\end{aligned}$$

$$\begin{aligned}x_2 &= x_1 - \frac{f(x_1)}{f'(x_1)} \\&= 1.3922111911 - \frac{f(1.3922111911)}{f'(1.3922111911)} \\&\approx 1.3922111911 - \frac{-0.23931509377336}{f'(1.3922111911)} \quad \text{again using a calculator} \\&\approx 1.3922111911 - \frac{-0.23931509377336}{-2.02373747597336} \quad \text{again using a calculator} \\&\approx 1.3922111911 - 0.1182540208967846811355339766932549 \quad \text{again using a calculator} \\&\approx 1.2739571702032153188644660233067451 \quad \text{again using a calculator}\end{aligned}$$