Mathematics Homework Sheet 10

Authors: Abdullah Oguz Topcuoglu & Ahmed Waleed Ahmed Badawy
Shora

Problem 1

Lets calculate the characteristic function of each matrix.

$$\text{For } A_1 = \begin{pmatrix} 1 & 0 & -1 \\ 1 & 2 & 1 \\ 2 & 2 & 3 \end{pmatrix}, \quad \text{the characteristic polynomial is}$$

$$c(\lambda) = \det(A_1 - \lambda I) = \det \begin{pmatrix} 1 - \lambda & 0 & -1 \\ 1 & 2 - \lambda & 1 \\ 2 & 2 & 3 - \lambda \end{pmatrix}$$

$$= (1 - \lambda) \det \begin{pmatrix} 2 - \lambda & 1 \\ 2 & 3 - \lambda \end{pmatrix} - 0 + (-1) \det \begin{pmatrix} 1 & 2 - \lambda \\ 2 & 2 \end{pmatrix}$$

$$= (1 - \lambda)((2 - \lambda)(3 - \lambda) - 2) - (1(2) - 2(2 - \lambda))$$

$$= (1 - \lambda)(\lambda^2 - 5\lambda + 4) - (2 - 4 + 2\lambda)$$

$$= (1 - \lambda)(\lambda^2 - 5\lambda + 4) - (2\lambda - 2)$$

$$= (1 - \lambda)(\lambda - 4)(\lambda - 1) - 2(\lambda - 1)$$

$$= (\lambda - 1)(-\lambda^2 + 5\lambda - 4 - 2)$$

$$= (1 - \lambda)(\lambda^2 - 5\lambda + 6)$$

$$= (1 - \lambda)(\lambda - 2)(\lambda - 3)$$

The eigenvalues are $\lambda_1 = 1$, $\lambda_2 = 2$, and $\lambda_3 = 3$. The eigenspaces are:

$$E_1 = \ker \begin{pmatrix} 1 - 1 & 0 & -1 \\ 1 & 2 - 1 & 1 \\ 2 & 2 & 3 - 1 \end{pmatrix}$$

$$= \ker \begin{pmatrix} 0 & 0 & -1 \\ 1 & 1 & 1 \\ 2 & 2 & 2 \end{pmatrix}$$

$$= \ker \begin{pmatrix} 1 & 1 & 1 \\ 0 & 0 & -1 \\ 0 & 0 & 0 \end{pmatrix}$$

$$= \text{span} \left\{ \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix} \right\}$$

$$E_2 = \ker \begin{pmatrix} 1-2 & 0 & -1 \\ 1 & 2-2 & 1 \\ 2 & 2 & 3-2 \end{pmatrix}$$

$$= \ker \begin{pmatrix} -1 & 0 & -1 \\ 1 & 0 & 1 \\ 2 & 2 & 1 \end{pmatrix}$$

$$= \ker \begin{pmatrix} 1 & 0 & 1 \\ 0 & 2 & -1 \\ 0 & 0 & 0 \end{pmatrix}$$

$$= \mathrm{span} \left\{ \begin{pmatrix} -1 \\ 1/2 \\ 1 \end{pmatrix} \right\}$$

$$E_3 = \ker \begin{pmatrix} 1-3 & 0 & -1 \\ 1 & 2-3 & 1 \\ 2 & 2 & 3-3 \end{pmatrix}$$

$$= \ker \begin{pmatrix} -2 & 0 & -1 \\ 1 & -1 & 1 \\ 2 & 2 & 0 \end{pmatrix}$$

$$= \ker \begin{pmatrix} 1 & -1 & 1 \\ 0 & 2 & -1 \\ 0 & 0 & 0 \end{pmatrix}$$

$$= \mathrm{span} \left\{ \begin{pmatrix} -1/2 \\ 1/2 \\ 1 \end{pmatrix} \right\}$$