

Mathematics Homework Sheet 7

Authors: Abdullah Oguz Topcuoglu & Ahmed Waleed Ahmed Badawy
Shora

Problem 1

A transformation T is linear if and only if it satisfies the following two properties for all vectors u, v and scalar c :

1. $T(u + v) = T(u) + T(v)$
 2. $T(cu) = cT(u)$
1. $T : \mathbb{R}^2 \rightarrow \mathbb{R}, \begin{pmatrix} x \\ y \end{pmatrix} \mapsto x + 2y$ is linear.
 2. $T : \mathbb{R}^2 \rightarrow \mathbb{R}, \begin{pmatrix} x \\ y \end{pmatrix} \mapsto x + y^2$ is not linear.
Because rule (1) is not satisfied.
 3. $T : \mathbb{R}^2 \rightarrow \mathbb{R}, \begin{pmatrix} x \\ y \end{pmatrix} \mapsto xy$ is not linear.
Because rule (2) is not satisfied.
 4. $T : \mathbb{C} \rightarrow \mathbb{C}, z \mapsto \bar{z}$ is linear.
 5. $T : \mathbb{R}^2 \rightarrow \mathbb{R}^2, \begin{pmatrix} x \\ y \end{pmatrix} \mapsto \begin{pmatrix} x + 1 \\ y - 1 \end{pmatrix}$ is not linear.
Because rule (1) is not satisfied.
 6. $T : \mathbb{R}^2 \rightarrow \mathbb{R}^2, \begin{pmatrix} x \\ y \end{pmatrix} \mapsto \begin{pmatrix} x - y \\ x + 2y \end{pmatrix}$ is linear.
 7. $T : \mathbb{R}^n[x] \rightarrow \mathbb{R}, p(x) \mapsto p(1)$ is linear.
 8. $T : \mathbb{R}^n[x] \rightarrow \mathbb{R}^{n+2}[x], p(x) \mapsto x^2 p(x)$ is linear.