## Mathematics Homework Sheet 2

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## Problem 1

(a)

 $Z_5$ :

| $\oplus$    | 0           | 1           | 2           | 3           | 4           |
|-------------|-------------|-------------|-------------|-------------|-------------|
| 0           | 0           | 1           | 2           | 3           | 4           |
| 1           | 1           | 2           | 3           | 4           | 0           |
| 2           | 2           | 3           | 4           | 0           | 1           |
| 3           | 3           | 4           | 0           | 1           | 2           |
| 4           | 4           | 0           | 1           | 2           | 3           |
|             |             |             |             |             |             |
|             |             |             |             |             |             |
| $\times$    | 0           | 1           | 2           | 3           | 4           |
| × 0         | 0           | 1 0         | 0           | 0           | 0           |
|             |             |             |             |             |             |
| 0           | 0           | 0           | 0           | 0           | 0           |
| 0           | 0           | 0           | 0 2         | 0 3         | 0 4         |
| 0<br>1<br>2 | 0<br>0<br>0 | 0<br>1<br>2 | 0<br>2<br>4 | 0<br>3<br>1 | 0<br>4<br>3 |

 $Z_7$ :

| $\oplus$                   | 0                     | 1                     | 2                     | 3                     | 4                     | 5                     | 6                     |
|----------------------------|-----------------------|-----------------------|-----------------------|-----------------------|-----------------------|-----------------------|-----------------------|
| 0                          | 0                     | 1                     | 2                     | 3                     | 4                     | 5                     | 6                     |
| 1                          | 1                     | 2                     | 3                     | 4                     | 5                     | 6                     | 0                     |
| 2                          | 2                     | 3                     | 4                     | 5                     | 6                     | 0                     | 1                     |
| 3                          | 3                     | 4                     | 5                     | 6                     | 0                     | 1                     | 2                     |
| 4                          | 4                     | 5                     | 6                     | 0                     | 1                     | 2                     | 3                     |
| 5                          | 5                     | 6                     | 0                     | 1                     | 2                     | 3                     | 4                     |
| 6                          | 6                     | 0                     | 1                     | 2                     | 3                     | 4                     | 5                     |
|                            |                       |                       |                       |                       |                       |                       |                       |
|                            |                       |                       |                       |                       |                       |                       |                       |
| ×                          | 0                     | 1                     | 2                     | 3                     | 4                     | 5                     | 6                     |
| × 0                        | 0                     | 1                     | 2                     | 3                     | 0                     | 5                     | 6                     |
| 0                          |                       |                       |                       |                       |                       |                       |                       |
| 0                          | 0                     | 0                     | 0                     | 0                     | 0                     | 0                     | 0                     |
| 0<br>1<br>2<br>3           | 0                     | 0                     | 0 2                   | 0                     | 0 4                   | 0<br>5                | 0<br>6                |
| 0<br>1<br>2                | 0<br>0<br>0           | 0<br>1<br>2           | 0<br>2<br>4           | 0<br>3<br>6           | 0<br>4<br>1           | 0<br>5<br>3           | 0<br>6<br>5           |
| 0<br>1<br>2<br>3<br>4<br>5 | 0<br>0<br>0<br>0      | 0<br>1<br>2<br>3      | 0<br>2<br>4<br>6      | 0<br>3<br>6<br>2      | 0<br>4<br>1<br>5      | 0<br>5<br>3<br>1      | 0<br>6<br>5<br>4      |
| 0<br>1<br>2<br>3<br>4      | 0<br>0<br>0<br>0<br>0 | 0<br>1<br>2<br>3<br>4 | 0<br>2<br>4<br>6<br>1 | 0<br>3<br>6<br>2<br>5 | 0<br>4<br>1<br>5<br>2 | 0<br>5<br>3<br>1<br>6 | 0<br>6<br>5<br>4<br>3 |

(b)

 $Z_4$ :

| $\oplus$           | 0   | 1 | 2   | 3   |
|--------------------|-----|---|-----|-----|
| 0                  | 0   | 1 | 2   | 3   |
| 1                  | 1   | 2 | 3   | 0   |
| 2                  | 2   | 3 | 0   | 1   |
| 3                  | 3   | 0 | 1   | 2   |
|                    |     |   |     |     |
|                    |     |   |     |     |
| ×                  | 0   | 1 | 2   | 3   |
| $\frac{\times}{0}$ | 0   | 1 | 0   | 3   |
|                    | _   |   |     |     |
| 0                  | 0   | 0 | 0   | 0   |
| 0<br>1             | 0 0 | 0 | 0 2 | 0 3 |

 $(Z_4, \oplus, \times)$  is not a field because [2] does not have a multiplicative inverse.