Mathematics Homework Sheet 3

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Problem 1

Problem 1 (a)

For i=0:

$$f_1(-1) = -1$$
$$f_1(0) = 0$$
$$f_1(1) = 0$$

Meaning that:

$$f_1(\{-1, 0, 1\}) = \{-1, 0, 0\} = \{-1, 0\}$$

Now lets take a look at the inverse:

$$f_1(-1) = -1$$
$$f_1(0) = 0$$
$$f_1(1) = 0$$
$$f_1(2) = 1$$

These are the only values we can get -1, 0, 1. Meaning that:

$$f_1^{-1}(\{-1, 0, 1\}) = \{-1, 0, 1, 2\}$$

Now, for i=2 (the second function):

$$f_2(-1) = -4$$
$$f_2(0) = -3$$
$$f_2(1) = -2$$

Meaning that:

$$f_2(\{-1, 0, 1\}) = \{-4, -3, -2\}$$

Now lets take a look at the inverse:

$$f_2(2) = -1$$
$$f_2(3) = 0$$
$$f_2(4) = 1$$

These are the only values we can get -1, 0, 1. Meaning that:

$$f_2^{-1}(\{-1, 0, 1\}) = \{2, 3, 4\}$$

Now, for i=3 (the third function):

$$\begin{aligned} f_3(-1) &= -2 \\ f_3(0) &= 0 \\ f_3(1) &= 2 \end{aligned}$$

Meaning that:

$$f_3(\{-1, 0, 1\}) = \{-2, 0, 2\}$$

Now lets take a look at the inverse:

$$\begin{aligned} &f_3(x) = -1, \text{ No such } x \text{ exists in the domain of } f_3 \\ &\qquad \text{In other words, } f_3^{-1}(\{-1\}) = \emptyset \\ &f_3(0) = 0 \\ &f_3(x) = 1, \text{ No such } x \text{ exists in the domain of } f_3 \\ &\qquad \text{In other words, } f_3^{-1}(\{1\}) = \emptyset \end{aligned}$$

These are the only values we can get -1, 0, 1. Meaning that:

$$f_3^{-1}(\{-1, 0, 1\}) = \{0\}$$

Problem 1 (b)

Lets start with f_1:
f_1 is not injective because it maps 0 and 1 to the same value, that is:

$$f_1(0) = f_1(1) = 0$$

f_1 is surjective because it maps to all the values in the codomain.
Lets take an element from the codomain $z \in Z$.
If $z \leq 0$ then we can find an element in the domain of f_3, $x \in Z$ that maps to z. Simply $x = z$ works.
If $z > 0$ then we can find an element in the domain of f_3, $x \in Z$ that maps to z. Simply $x = z + 1$ works.

Now, lets take a look at f_2:

f_2 is injective because it maps each element in the domain to a unique element in the codomain, that is:

$$f_2(x) = f_2(y) \Rightarrow x = y$$

$$\begin{aligned}&f_2(x) = x - 3, \quad f_2(y) = y - 3\\&x - 3 = y - 3 \Rightarrow x = y\end{aligned}$$

f_2 is surjective because it maps to all the values in the codomain.
Lets take an element from the codomain of f_2, $z \in Z$.
We cand find an element in the domain of f_2, $x \in Z$ that maps to z. Simply $x = z + 3$ works.

Now, lets take a look at f_3:
f_3 is injective because it maps each element in the domain to a unique element in the codomain, that is:

$$f_3(x) = f_3(y) \Rightarrow x = y$$

$$\begin{aligned}&f_3(x) = 2x, \quad f_3(y) = 2y\\&2x = 2y \Rightarrow x = y\end{aligned}$$

f_3 is not surjective because it does not map to all the values in the codomain. For example there is no element in the domain of f_3 that maps to 1. Generally all the odd numbers are not in the range of f_3.

Problem 1 (c)

By looking at Problem 1 (b),
We see that f_1 is not injective thus f_1 is not bijective.
We see that f_2 is injective and surjective thus f_2 is bijective.
We see that f_3 is not surjective thus f_3 is not bijective.

Inverse of f_2:

$$\begin{aligned}f_2 &: Z \to Z, \quad x \rightsquigarrow x - 3\\f_2^{-1} &: Z \to Z, \quad x \rightsquigarrow x + 3\end{aligned}$$