Mathematics Homework Sheet 5

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Problem 1

Problem 1(b)

Problem 1(b)(i)

We want to prove $\bigcap_{i \in I} U_i$ is closed.
We are given $(\forall i \in I \quad U_i \subseteq R)$ is closed.
A set being closed means that its complement is open. So we want to prove that $\bigcup_{i \in I} U_i^c$ is open.
Since each U_i is closed, we know that U_i^c is open.
And from the lecture we know that union or intersection of open sets is open.
Thus $\bigcup_{i \in I} U_i^c$ is open. Which means that $\bigcap_{i \in I} U_i$ is closed.
And this completes the proof.

Problem 1(b)(ii)

We want to prove $\bigcup_{i=1}^n U_i$ is closed.
We are given $(U_1, ..., U_n \subseteq R)$ are closed.
A set being closed means that its complement is open. So we want to prove that $\bigcap_{i=1}^n U_i^c$ is open.
Since each U_i is closed, we know that U_i^c is open.
And from the lecture we know that union or intersection of open sets is open.
Thus $\bigcap_{i=1}^n U_i^c$ is open. Which means that $\bigcup_{i=1}^n U_i$ is closed.
And this completes the proof.

Problem 3

Problem 3(a)

$$a_n := (-1)^n$$

a_n is not convergent. Because, for example, if we take $\epsilon = 1/10$ then there is no N that satisfies

$$|a_N - a| < 1/10$$

a_n alternates between -1 and 1. So we can't find a value a that stays in the neighborhood of both -1 and 1. For example, when $\epsilon = 1/10$ there is no $a \in R$

that satisfies

$$|1 - a| < 1/10 \quad \text{and} \quad |-1 - a| < 1/10$$

No matter what you choose N to be you will always get for some $j \in N$ $a_j = 1$ and $a_j = -1$.
Thus a_n is divergent.