Mathematics Homework Sheet 5

Authors: Abdullah Oguz Topcuoglu & Ahmed Waleed Ahmed Badawy Shora

Problem 1

1. $-(x+y) = (-x) + (-y)$:
   By the definition of the additive inverse, we have:
   $$-(x+y) + (x+y) = 0$$
   Use distributivity property:
   $$\begin{aligned}(-x) + (-y) + (x+y) &= 0\\((-x) + (-y)) + (x+y) &= 0\end{aligned}$$
   So, since adding $((-x) + (-y))$ to $(x+y)$ gives 0, we can conclude that it is the additive inverse of $(x+y)$. And that's what we are trying to prove.

2. $-(x-y) = (-x) + y$:
   Apply the rule above.
   $$\begin{aligned}-(x + (-y)) &= (-x) + (-(-y))\\&= (-x) + y\end{aligned}$$

3. $x \cdot 0 = 0 \cdot x = 0$:
   $$x \cdot 0 + x \cdot 0 = x \cdot (0+0) = x \cdot 0$$
   $$\begin{aligned}x \cdot 0 + x \cdot 0 &= x \cdot 0 \qquad \text{(add additive inverse of } x \cdot 0)\\x \cdot 0 + (x \cdot 0 + -(x \cdot 0)) &= x \cdot 0 + -(x \cdot 0)\\x \cdot 0 + 0 &= 0\\x \cdot 0 &= 0\end{aligned}$$
   And by commutativity we have $0 \cdot x = 0$.

4. $(-x) \cdot y = -(x \cdot y)$:
   $$(x \cdot y) + ((-x) \cdot y) = (x + (-x)) \cdot y = 0 \cdot y = 0$$
   So, $(-x) \cdot y$ is additive inverse of $(x \cdot y)$.

5. $x \cdot (-y) = -(x \cdot y)$:

$$\begin{aligned} x \cdot (-y) &= x \cdot (-y) && \text{(commutativity)} \\ &= (-y) \cdot x && \text{(insert this into original equation)} \\ (-y) \cdot x &= -(x.y) && \text{(true by the previous rule)} \end{aligned}$$

6. $(-x) \cdot (-y) = x \cdot y$:
   Use rule (4) to get:
   $$(-x) \cdot (-y) = -(x \cdot (-y))$$
   Now use rule (5) to get:
   $$-(x \cdot (-y)) = -(-(x \cdot y))$$
   And by the definition of additive inverse, we have:
   $$-(-(x \cdot y)) = x \cdot y$$

7. $x + y = z$ if and only if $x = z - y$:
   By the definition of addition, we have:
   $$x + y = z \implies x = z - y$$
   and
   $$x = z - y \implies x + y = z$$