Mathematics Homework Sheet 7

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Problem 1

If b_n is convergent then

$$\liminf_{n \to \infty} b_n = \limsup_{n \to \infty} b_n = \lim_{n \to \infty} b_n$$

From the first inequality we have

$$\limsup_{n \to \infty} (a_n + b_n) \le \limsup_{n \to \infty} a_n + \lim_{n \to \infty} b_n$$

And from the second inequality we have

$$\limsup_{n \to \infty} (a_n + b_n) \ge \limsup_{n \to \infty} a_n + \lim_{n \to \infty} b_n$$

Combining these two inequalities we get

$$\lim \sup_{n \to \infty} (a_n + b_n) = \lim \sup_{n \to \infty} a_n + \lim_{n \to \infty} b_n$$