Homework Sheet 5

Author: Abdullah Oğuz Topçuoğlu & Yousef Mostafa Farouk

Task 1

We are given the grammar

$$G = (\Sigma, V, S, P) \text{ with } \Sigma = \{a, b\}, V = \{S, A\} \text{ and the productions } P$$
$$S \to aSb \mid bSa \mid aAa \mid bAb$$
$$A \to aA \mid bA \mid \varepsilon.$$

(1)

(a) We want to derive the following word

$$w = ababb$$

The derivation:

$$\begin{aligned} S &\to aSb \\ &\to abAbb \\ &\to abaAbb \\ &\to ababb \end{aligned}$$

(b) We want to derive the following word

$$w = baaba$$

The derivation:

$$\begin{aligned} S &\to bSa \\ &\to baSba \\ &\to :) \text{ stuck} \end{aligned}$$

and from one "S" we need to derive one "a" but it is not possible. We couldnt find any other derivation too. We dont think this word is in the language.

(c) We want to derive the following word

$$w = baabb$$

The derivation:

$$\begin{aligned} S &\to bAb \\ &\to baAb \\ &\to baaAb \\ &\to baabAb \\ &\to baabb \end{aligned}$$

(2)

So at the left and right most edge of the word we can have arbitrarly many ...a...b... or ...b...a... pairs. And if we take those pairs out of the word and focus on the middle part of the word, then we have any permutation of a's and b's but enclosed by either a...a or b...b. So the language can be characterized as follows:

$$\begin{aligned} L(G) = &\{w \in \{a,b\}^* | w = xayaz \text{ where y is arbitrary word and } x = flip(rev(z))\} \\ &\cup \{w \in \{a,b\}^* | w = xbybz \text{ where y is arbitrary word and } x = flip(rev(z))\} \end{aligned}$$

Both x, y and z can be empty strings as well. Here the function rev() and flip() are intuitively defined as follows:

- rev(): reverses the string
- flip(): flips a to b and b to a