Homework Sheet 5

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Task 3

(1)

A Non deterministic Counter Push Down Automaton (NCPDA) is a 7 tuple

$$M = (Q, \Sigma, \Gamma, \Delta, q_0, -1, F)$$

where

- Q is a finite set of states,
- Σ is a finite input alphabet,
- Γ is the stack alphabet and is equal to $\mathbb{N} \cup \{-1\}$
- $\Delta : Q \times (\Sigma \cup \{\epsilon\}) \times \Gamma \to \mathcal{P}(Q \times \Gamma^*)$ is the transition function,
- $q_0 \in Q$ is the start state,
- $-1 \in \Gamma$ is the bottom of stack symbol,
- $F \subseteq Q$ is the set of accepting states.

Configuration space:
A configuration of an NCPDA is a triple (q, w, γ) where $q \in Q$ is the current state, $w \in \Sigma^*$ is the remaining input, and $\gamma \in \Gamma^*$ is the current stack content similar to a PDA.

Computing step relation:
A computing step of an NCPDA is a relation $\vdash$ on the configuration space defined as follows:

$$(q, aw, X\gamma) \vdash (p, w, \beta\gamma) \text{ if } (p, \beta) \in \Delta(q, a, X)$$

for $a \in \Sigma \cup \{\epsilon\}$, $X \in \Gamma$, $w \in \Sigma^*$, $\gamma \in \Gamma^*$, and $\beta \in \Gamma^*$.

Acceptance of an input:
An NCPDA accepts an input string $w \in \Sigma^*$ if there exists a sequence of configurations

$$(q_0, w, -1) \vdash^* (q_f, \epsilon, \gamma)$$

for some $q_f \in F$ and $\gamma \in \Gamma^*$.
or by empty stack:

$$(q_0, w, -1) \vdash^* (q, \epsilon, \epsilon)$$

for some $q \in Q$ (not necessarily a final state).

NCPDA language:
The language recognized by an NCPDA M is defined as

$$L(M) = \{w \in \Sigma^* \mid M \text{ accepts } w\}$$