Homework Sheet 5

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Task 2

(1)

The words in the language consist of a's whose length is a power of 2. So we need to design a grammar that somehow doubles the number of a's in each step of the derivation. So each derivation rule is going to look at the whole word generated so far and double the number of a's by placing another a next to each existing a.

We want to design a grammar G_1 for the language

$$L_1 = \{a^{2^n} \mid n \in \mathbb{N}\} \text{ over } \Sigma = \{a\}.$$

The grammar G_1 is defined as follows:

$$G_1 = (\Sigma, V, S, P) \text{ with } \Sigma = \{a\}, V = \{S, C, E\} \text{ and the productions } P$$

$$\begin{aligned} S &\to a \mid CSE \\ Ca &\to aaC \\ CE &\to \varepsilon. \end{aligned}$$

Basically how this works is that we put a cursor C and an end marker E and cursor traverses the string from left to right doubling each a it encounters until it raches the end marker E at which point both C and E are removed from the string. This way each derivation step doubles the number of a's in the string. This is not a context free grammar but this is definitely a grammar. It is not context free because the production rules doesnt follow the form $A \to \alpha$.

(2)

We want to design a grammar G_2 for the language

$$L_2 = \{a^n b^n a^n b^n \mid n \in \mathbb{N}\} \text{ over } \Sigma = \{a, b\}.$$

So the grammar is going to work in a similar fashion to the previous one. We will have a cursor C that traverses the whole word and appends an "a" before an "a" and appends a "b" before "b".

The grammar G_2 is defined as follows:

$G_2 = (\Sigma, V, S, P)$ with $\Sigma = \{a, b\}, V = \{S, C, W_b, W_e\}$ and the productions P

$$S \to W_b ababW_e \mid \varepsilon$$
$$W_b \to W_bC \mid \varepsilon \quad \text{(spawn a cursor to increase the "n" by one)}$$
$$CW_e \to W_e \quad \text{(delete the cursor that traversed the whole word)}$$
$$W_e \to \varepsilon \quad \text{(delete the word end marker when we want to terminate)}$$
$$Ca \to aaC \quad \text{(appends one a and advances the cursor)}$$
$$Cb \to bbC \quad \text{(appends one b and advances the cursor)}$$

W_b and W_e denotes the begining of the word and the end of the word respectively. C is the cursor that traverses the word appending a's and b's as it goes.
How this works is that we start with "abab" between W_b and W_e and then we spawn a cursor C using $W_b \to W_bC$. Then the cursor traverses the word appending a's and b's using the rules $Ca \to aaC$ and $Cb \to bbC$. When the cursor reaches the end marker W_e, we delete the cursor using the rule $CW_e \to W_e$. We spawn as many cursors as we want using the rule $W_b \to W_bC$ to increase the value of n by one in each derivation step.
Finally, when we want to terminate the derivation, we delete the end marker W_e using the rule $W_e \to \varepsilon$ and we also delete W_b using the rule $W_b \to \varepsilon$.
This is not a context free grammar but this is definitely a grammar. It is not context free because the production rules doesnt follow the form $A \to \alpha$.