

Homework Sheet 6

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Task 1

The machine will work like this:

We will read the input string w from left to right until we find the first occurrence of ccd in that case we will switch to a new state where the machine will replace ccd with abd (the last letter doesn't matter since we will delete it) and after string replacement done we will move the input head to the d of abd we just replaced and switch to a new state where the machine will shift all the letters to the left by one until it reaches a blank symbol. And then finally we will move the head back to the start of the tape and switch to the final state q' .

The states of the machine would be:

```
Q := {
    q,    // starting state/searching for c
    qc,  // found the first c looking for the second
    qcc,  // found the second c looking for the d
    qback3, // found ccd, move back 3 characters
    qback2, // move back 2 characters to reach the start of ccd
    qback1, // move back 1 character to reach the start of ccd
    qreplacec1, // replace the first c with a
    qreplacec2, // replace the second c with b
    qshiftleft, // shifting left by one
    qread,
    // read the next character to be able to replace the current character. this state
    // transitions into one of the qnext states

    qnexta, // next character is a so replace this letter with a
    qnextb, // next character is b so replace this letter with b
    qnextc, // next character is c so replace this letter with c
    qnextd, // next character is d so replace this letter with d
    qshiftingdone, // shifting is done. move the head to the beginning of the input tape
    q', // final state. transitioned from the qshiftingdone state
}
```

In order to shift left by one we need to be able to read the next element so that we can replace the current element with it. For that we will create states

for each letter in the input alphabet so that we can remember what to write when we move back. We have to have states for this because we can not read the next characters in the input tape.

The transitions would be:

$$\begin{aligned}
((q, a), (q, a, +)) &\in \Delta \\
((q, b), (q, b, +)) &\in \Delta \\
((q, d), (q, d, +)) &\in \Delta \\
((q, c), (q_c, c, +)) &\in \Delta \quad (\text{read the first c}) \\
((q_c, c), (q_{cc}, c, +)) &\in \Delta \quad (\text{read the second c}) \\
((q_c, x), (q, x, +)) &\in \Delta
\end{aligned}$$

(there is no second c. transition back to the state q. here x stands for any character except c)

in general this is how the reading the ccd will work. i am not writing all the transitions of it down. i will move to replacement transitions now

$$\begin{aligned}
((q_{back3}, x), (q_{back2}, x, -)) &\in \Delta \quad (\text{move back 1}) \\
((q_{back2}, x), (q_{back1}, x, -)) &\in \Delta \quad (\text{move back 1}) \\
((q_{back1}, x), (q_{replacec_1}, x, -)) &\in \Delta \quad (\text{move back 1 to reach the first c}) \\
((q_{replacec_1}, x), (q_{replacec_2}, a, +)) &\in \Delta \quad (\text{replace first c with a and move right}) \\
((q_{replacec_2}, x), (q_{shiftright}, b, +)) &\in \Delta \quad (\text{replace second c with b and move right})
\end{aligned}$$

(now we need to start shifting left by one. so lets write the transitions for the shifting)

$$\begin{aligned}
((q_{shiftright}, x), (q_{read}, x, +)) &\in \Delta \quad (\text{go to next character}) \\
((q_{read}, a), (q_{nexta}, a, -)) &\in \Delta \quad (\text{move back and store the next character in the state}) \\
((q_{nexta}, x), (q_{shiftright}, a, +)) &\in \Delta
\end{aligned}$$

(replace the current character and move right and switch to shiftright state so every letter until the end can be shifted)

ive only written the case where the next letter is "a" but similar transitions exist for b,c,d

and when we reach the end we need to transition into $q_{shiftingdone}$ state

$$\begin{aligned}
 ((q_{shiftright}, \#), (q_{shiftingdone}, \#, -)) &\in \Delta \quad (\text{reached the end of the input tape}) \\
 ((q_{shiftingdone}, x), (q_{shiftingdone}, x, -)) &\in \Delta \quad (\text{move back to the start of the tape}) \\
 ((q_{shiftingdone}, \#), (q', \#, +)) &\in \Delta \quad (\text{reached the start of the tape. transition to final state})
 \end{aligned}$$

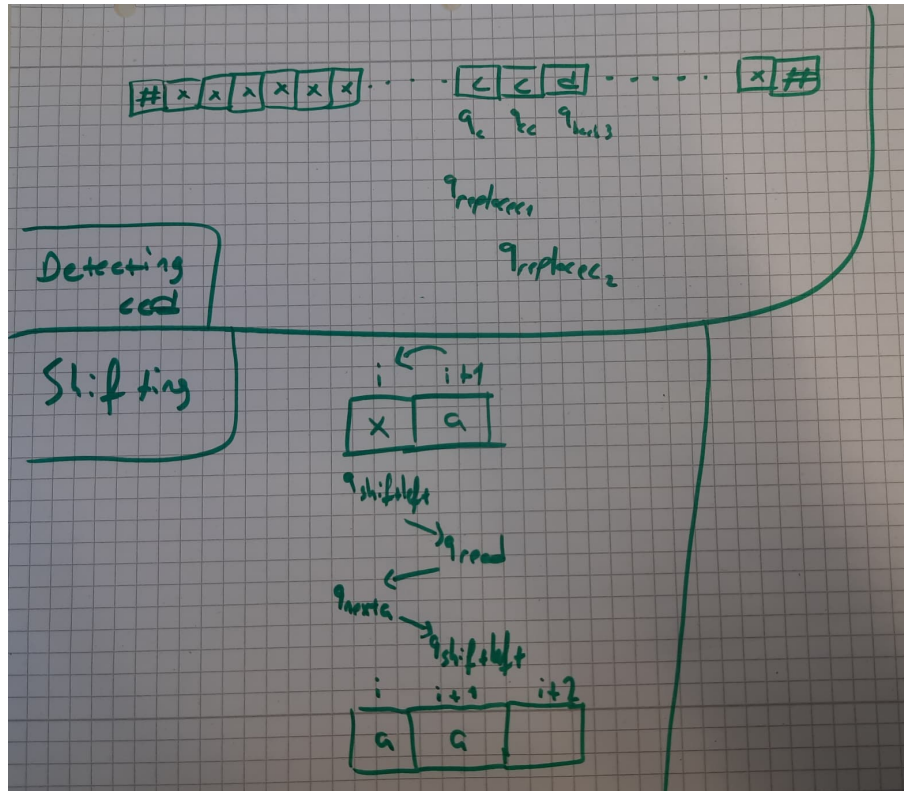


Figure 1: Graphical representation of the STM

In the graphical representation above in the shifting part i showed one step of shifting with the letter a. Similar transitions exist for b,c,d. It starts with the state $q_{shiftright}$ and ends with $q_{shiftright}$ to recurse until the end of the tape is reached.

And we never transition to final state if we don't find *ccd* in the input string because we can only transition to final state from $q_{shiftingdone}$ state which we can only reach if we found *ccd* and replaced it with *abd*.