Homework Sheet 4

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Task 2

(i)

$$L_1 = \{a^n b^m c^p \mid n + 2m > 3p\}$$

We can create a PDA that accepts this langauge.

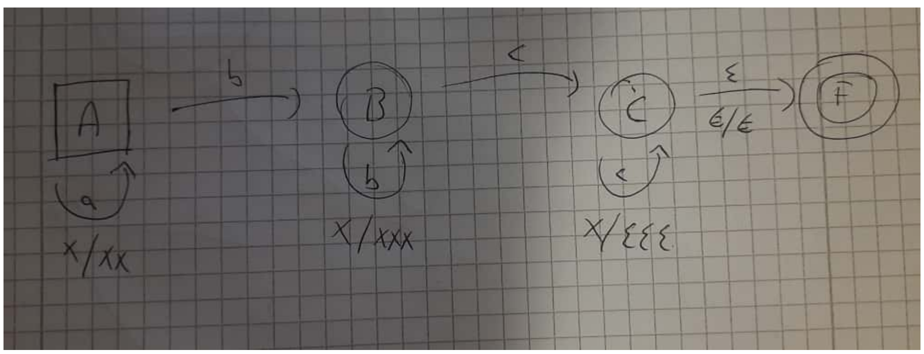

Figure 1: PDA for L_1

Where input alphabet, states and stack alphabet are

$$\Sigma = \{a, b, c\}$$
$$Q = \{A, B, C, F\}$$
$$\Gamma = \{X, \epsilon\}$$

L_1 is context free since there is a PDA that accepts it.

(ii)

$$L_2 = \{w \in \{a, b\}^* \mid \#_a(w) \leq \#_b(w)/2\}$$

(iii)

$$L_3 = \{w \in \{a,b\}^* \mid \#_a(w) \mod 5 \geq \#_b(w) \mod 3\}$$

L_3 is regular since we can create a DFA that accepts it. When we look at the condition there is a finite number of combinations of $\#_a(w) \mod 5$ and $\#_b(w) \mod 3$. We can create states for each combination and transition between them based on the input symbols.
We can define the DFA like this

$$\begin{aligned}
Q &= \{(i,j) \mid i \in \{0,1,2,3,4\}, j \in \{0,1,2\}\} \\
\Sigma &= \{a,b\} \\
\delta((i,j),a) &= ((i+1) \mod 5, j) \\
\delta((i,j),b) &= (i, (j+1) \mod 3) \\
q_0 &= (0,0) \\
F &= \{(i,j) \mid i \geq j\}
\end{aligned}$$

(iv)

$$L_4 = \{w \in \{a,b\}^* \mid \#_a(w) \mod \#_b(w) = 0\}$$

L_4 is not context free. We gonna play the pumping game to prove it. The pumping game is like this:

- Adversary chooses an N.
- We choose $z \in L, \quad |z| \geq N$
- Adversary chooses $z = uvwxy, \quad |vx| \geq 1, \quad vwx \leq N$
- We choose an i such that $uv^iwx^iy \notin L$

(these game rules are copy paste from the lecture)
Lets start the game
Adversary chooses an N.
We choose $z = a^N b^N \in L_4$
Adversary chooses $z = uvwxy, \quad |vx| \geq 1, \quad vwx \leq N$