Homework Sheet 1

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Problem 1

(1.)

The given definition of antisymmetry is not sensible because it implies that for any elements $a, a' \in A$, if $(a, a') \in R$, then $(a', a) \notin R$. When I hear "antisymmetric" i think of something that is the opposite of symmetric and symmetric means if $(a, a') \in R$ then $(a', a) \in R$. And opposite of that would be if $(a, a') \in R$, then $(a', a) \notin R$ as in the question but with a difference that it only applies when $a \neq a'$. Because otherwise in the current definition if we plug in $a = a'$, we get $(a, a) \in R \iff (a, a) \notin R$ which is a contradiction.

(2.)

(a) True.
If $f : A \to B$ and $g : B \to C$ are bijective functions.
To prove that $g \circ f$ is bijective, we need to show that it is both injective and surjective.
Since f and g are injective their composition is also injective. And since f and g are surjective their composition is also surjective.
Therefore, $g \circ f$ is bijective.

(b) False.
Let $A = \{1, 2\}, B = \{1, 2\}, C = \{1\}$.
Define $f : A \to B$ by $f(1) = 1$, $f(2) = 2$ (which is injective) and $g : B \to C$ by $g(1) = 1, g(2) = 1$ (which is surjective).
Then, $g \circ f(1) = g(f(1)) = g(1) = 1$.
Also, $g \circ f(2) = g(f(2)) = g(2) = 1$.
Therefore $g \circ f$ is not injective since $g \circ f(1) = g \circ f(2)$.

(c) False.
Consider this example: Let $A = \{1\}, B = \{1, 2\}, C = \{1, 2\}$.
Define $f : A \to B$ by $f(1) = 1$ (which is injective) and $g : B \to C$ by $g(1) = 1, g(2) = 2$ (which is surjective).
In this configuration there is no element in A that maps to 2 in C through $g \circ f$.
Thus, $g \circ f$ is not surjective.