Homework Sheet 3

Author: Abdullah Oğuz Topçuoğlu & Yousef Mostafa Farouk

Task 4

The idea is we are going to define a set S_i for each i from 0 to n where S_i contains all the states that can be reached after reading the first i characters of the input string x. For example S_0 would only contain the starting state (assuming there are no epslion transitions). We can construct these sets iteratively as follows:

- Initialize S_0 to contain only the start state of the NFA M and the states with epsilon transitions if exists any.
- For each i from 1 to n:
  - Initialize S_i to be an empty set.
  - For each state p in S_{i-1}:
    - Determine the set of all states q such that there is a transition from p to q labeled with the character x_i.
    - Add all such states q to S_i.

At the end we would check if any of the states in S_n is an accepting state. If yes then the NFA accepts the input string x otherwise it doesnt accept it.

Running Time: We iterate over each character. So our algorithm runs in linear time with respect to n.
And in the inner loop we iterate over states (linear in time with respect to s) in the current set and then we decide if there is an arc between two states which is linear with respect to s (this is given in the problem). So the overall time complexity of the algorithm is $O(n \cdot s^2)$.