

# Homework Sheet 5

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## Task 3

(1)

A Non deterministic Counter Push Down Automaton (NCPDA) is a 7 tuple

$$M = (Q, \Sigma, \Gamma, \Delta, q_0, -1, F)$$

where

- $Q$  is a finite set of states,
- $\Sigma$  is a finite input alphabet,
- $\Gamma$  is the stack alphabet and is equal to  $\mathbb{N} \cup \{-1\}$
- $\Delta : Q \times (\Sigma \cup \{\epsilon\}) \times \Gamma \rightarrow \mathcal{P}(Q \times \mathcal{O})$  is the transition function,
- $q_0 \in Q$  is the start state,
- $-1 \in \Gamma$  is the bottom of stack symbol,
- $F \subseteq Q$  is the set of accepting states.

Here,  $\mathcal{O}$  is the set of stack operations defined as follows:

$$\mathcal{O} = \{\text{Push}, \text{Pop}, \text{Increment}, \text{Decrement}, \text{Noop}\}$$

These operations are defined in the task description.

### Configuration space:

A configuration of an NCPDA is a triple  $(q, w, \gamma)$  where  $q \in Q$  is the current state,  $w \in \Sigma^*$  is the remaining input, and  $\gamma \in \Gamma^*$  is the current stack content similar to a PDA.

### Computing step relation:

A computing step of an NCPDA is a relation  $\vdash$  on the configuration space defined as follows:

$$(q, aw, X\gamma) \vdash (q', w, \gamma') \text{ if } \exists o \in \mathcal{O}, (q', o) \in \Delta(q, a, X)$$

for  $a \in \Sigma \cup \{\epsilon\}$ ,  $X \in \Gamma$ ,  $w \in \Sigma^*$ ,  $\gamma \in \Gamma^*$ , and  $\beta \in \Gamma^*$ .

### Acceptance of an input:

An NCPDA accepts an input string  $w \in \Sigma^*$  if there exists a sequence of configurations

$$(q_0, w, -1) \vdash^* (q_f, \epsilon, \gamma)$$

for some  $q_f \in F$  and  $\gamma \in \Gamma^*$ .

or by empty stack:

$$(q_0, w, -1) \vdash^* (q, \epsilon, \epsilon)$$

for some  $q \in Q$  (not necessarily a final state).

**NCPDA language:**

The language recognized by an NCPDA  $M$  is defined as

$$L(M) = \{w \in \Sigma^* \mid M \text{ accepts } w\}$$