

Homework Sheet 6

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Task 2

First notice that

$$\begin{aligned}\binom{n}{2} &= \frac{n!}{2!(n-2)!} \\ &= \frac{n(n-1)}{2} \\ &= \sum_{i=1}^{n-1} i\end{aligned}$$

The trick i will do to detect if the input contains $\binom{n}{2}$ many a's is that i will try to subtract 1, 2, 3, ... from the total number of a's until i reach 0. If i reach 0 exactly then the input contains $\binom{n}{2}$ many a's for some n, otherwise it does not.

We need only one letter in the work tape. Lets call it p . At the start we have an empty work tape. And then we will move the work tape cursor to the right by one as well as the input head by one and if any point we reach the end of input tape we stop and dont accept the input word.

If we reach the end of the work tape at the same time as the input head reaches its end then we accept the input word.

If work tape head reaches the end but there is still some letters left in the input tape then we will add one more p to the work tape and rewind work tape cursor to the start of the work tape and continue the process.

The states would be:

$Q = \{$
 $q_{continue},$ (start state, continue reading from the work tape and input tape)
 $q_{abort},$

(we switch to this state when there is no more letters left in the input tape but work tape has not reached to its end)

$q_{addone},$ (work tape reached its end.)
 $q_{rewind},$ (rewinding the work tape head)
 $q_{final},$ (final state)
}

The transitions would be:

- $((q_{continue}, a, p), (q_{continue}, a, p, +, +)) \in \Delta$ (consume one letter)
- $((q_{continue}, a, \#), (q_{addone}, a, \#, 0, 0)) \in \Delta$ (reached the end of work tape)
- $((q_{addone}, a, \#), (q_{rewind}, a, p, 0, 0)) \in \Delta$ (add one p and start rewinding)
- $((q_{rewind}, a, p), (q_{rewind}, a, p, 0, -)) \in \Delta$ (continue rewinding)
- $((q_{rewind}, a, \#), (q_{continue}, a, \#, 0, +)) \in \Delta$ (rewinding done)
- $((q_{addone}, \#, \#), (q_{final}, \#, \#, 0, 0)) \in \Delta$ (thats a word we are looking for)
- $((q_{continue}, \#, p), (q_{abort}, \#, p, 0, 0)) \in \Delta$ (not enough letters in the input. abort)

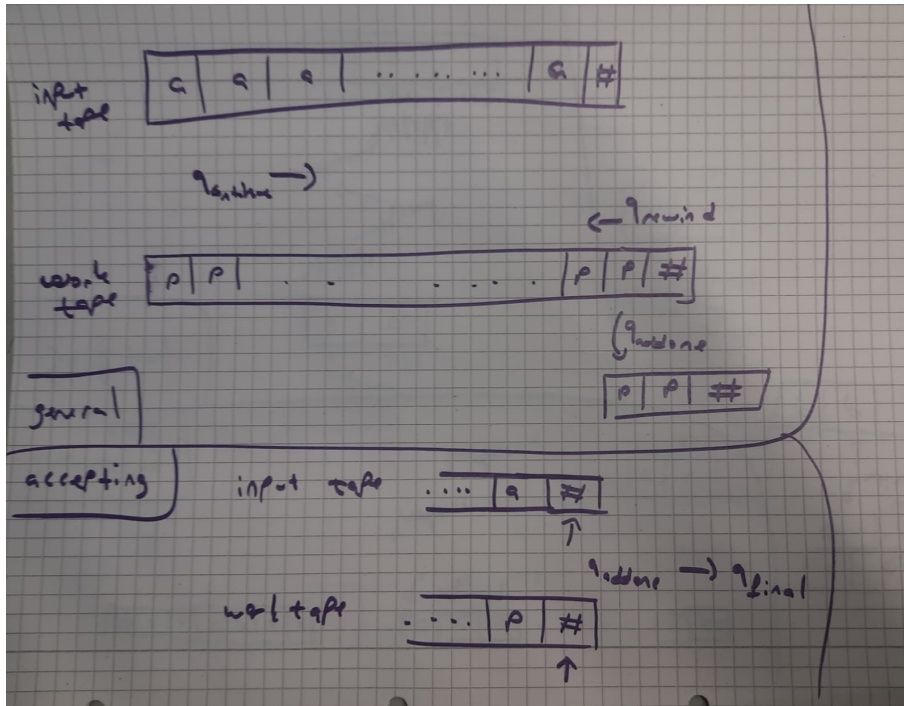


Figure 1: Graphical representation of the TM