Homework Sheet 3

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Task 3

We need to show that the following languages are not pumping which implies they are not regular.

We are gonna play the pumping game between us and the adversary. It is gonna look like this:

- Adversary chooses an arbitrary $N \in \mathbb{N}$.
- We partition a word $uvw \in L$ where $|v| \geq N$
- Adversary arbitrarly partitions the v into xyz where $0 < |y| \leq N$
- Finally we choose an i such that $uxy^izw \notin L$

(those game rules are copy paste from the friday lecture)

1.

We are given the language

$$A = \{a^{2n}b^{3n} \mid n \in \mathbb{N}\}$$

Lets start the game:
Adversary chooses an arbitrary $N \in \mathbb{N}$.
We partition the word $uvw \in A$ where

$$\begin{aligned} u &= a^N \\ v &= a^N \\ w &= b^{3N} \end{aligned}$$

Adversary has no choice but to partition v into xyz where

$$\begin{aligned} x &= a^k \\ y &= a^m \\ z &= a^{N-k-m} \end{aligned}$$

where $k \geq 0$, $m > 0$ and $k + m \leq N$.
We choose $i = 0$.
Now we have

$$\begin{aligned} uxy^0zw &= a^N a^k \epsilon a^{N-k-m} b^{3N} \\ &= a^{2N-m} b^{3N} \end{aligned}$$

Which is clearly not in A since $m > 0$.
Thats what we wanted to show.

2.

We are given the language

$$B = \{x \in \{0,1\}^* \mid x = x_{rev}\}$$

Lets start the game:
Adversary chooses an arbitrary $N \in \mathbb{N}$.
We partition the word $uvw \in B$ where

$$\begin{aligned} u &= \epsilon \\ v &= 0^N 1 0^N \\ w &= \epsilon \end{aligned}$$

Adversary has no choice but to partition v into xyz where x, y, z are all zeroes except one of them has exactly one 1.
If that one 1 is in x or z, we choose $i = 0$. (because when we remove the y then the 1 is no longer in the middle)
If that one 1 is in y, we choose $i = 2$.
In both cases we get a string that is not a palindrome thus not in B.
Thats what we wanted to show.