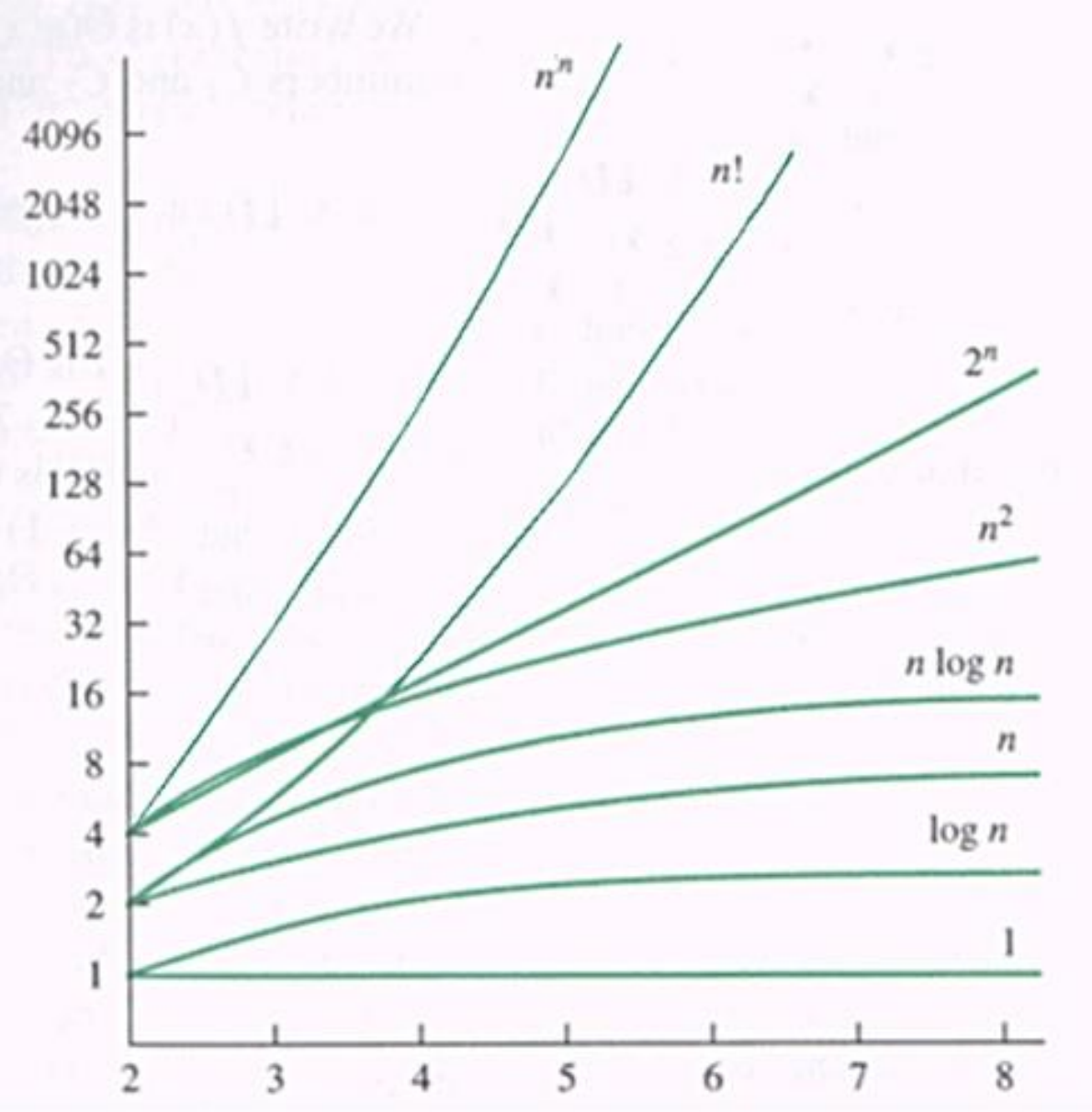
Module4—Tree

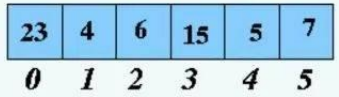
	constant	*logarithmic*	*linear*	*N-log-N*	*quadratic*	*cubic*	*exponential*
n	$O(1)$	$O(\log n)$	$O(n)$	$O(n \log n)$	$O(n^2)$	$O(n^3)$	$O(2^n)$
1	1	1	1	1	1	1	2
2	1	1	2	2	4	8	4
4	1	2	4	8	16	64	16
8	1	3	8	24	64	512	256
16	1	4	16	64	256	4,096	65536
32	1	5	32	160	1,024	32,768	4,294,967,296
64	1	6	64	384	4,069	262,144	1.84×10^{19}


n^n
$n!$
2^n
n^2
$n \log n$
n
$\log n$
1
4096
2048
1024
512
256
128
64
32
16
8
4
2
1
2
3
4
5
6
7
8

Data Structure

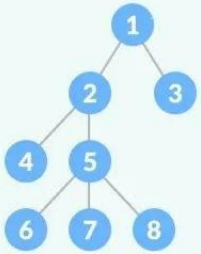
By: @pythoncodess

Array

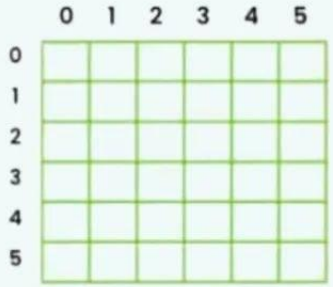


Queue

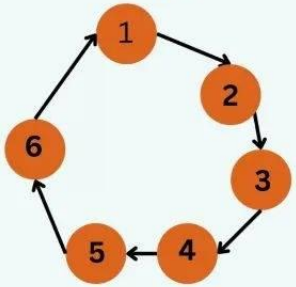
Tree



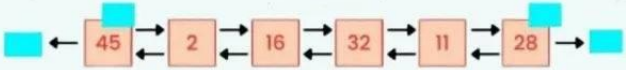
Matrix



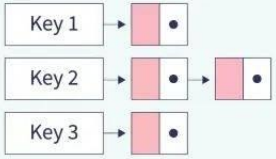
Graph



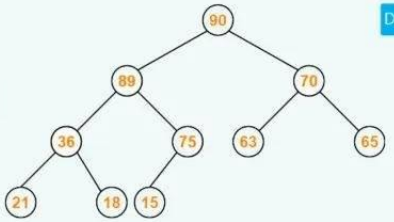
Linked List



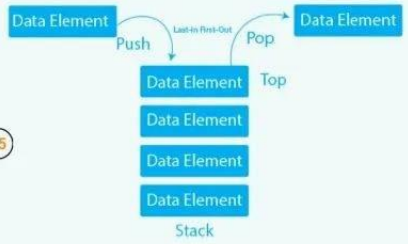
Hashmap



Max Heap



Stack



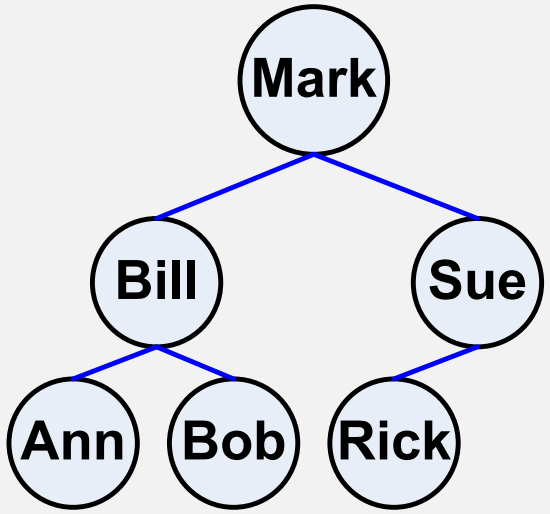
@pythoncodess
Python | ML | AI |

Join our telegram Channel
Link in bio

4 Trees

Problem

- Linear time access of linked list.
- Running time of operation O(n).

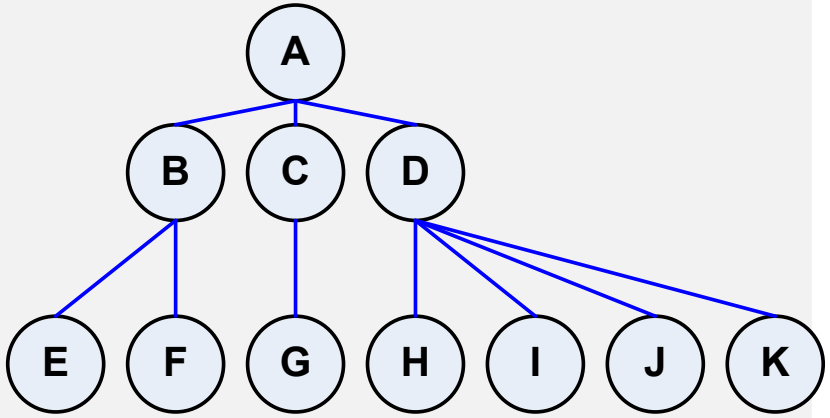


Correct : Trees

- Average time $O(\log_n)$.
- Worst case O(n).

4.1 Tree Definition

โครงสร้างข้อมูลต้นไม้ (Tree Data Structure) หรือเรียกสั้นๆว่าทรี (Tree) เป็นโครงสร้างข้อมูลรูปแบบหนึ่งในลักษณะ

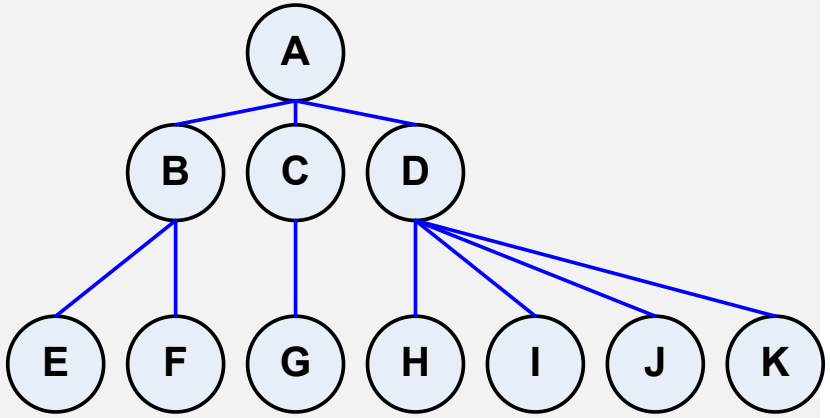


- โครงสร้างข้อมูลชนิดไม่เชิงเส้น (Non-Linear)
- สมาชิกแต่ละตัวในทรีสามารถเชื่อมโยงไปยังสมาชิกตัวถัดไป (Successor) ได้มากกว่าหนึ่งตัว
- และเชื่อมโยงถึงกันในลักษณะเป็นระดับคล้ายกับการแตก กิ่งก้านสาขาออกไปของต้นไม้
- ความสัมพันธ์ของสมาชิกข้อมูลในทรี จึงมีลักษณะลำดับชั้น (Hierarchical Relationship) คือ มีการเชื่อมโยงของแต่ละโหนดเป็นแบบทางเดียวจากบนลงล่าง
- โครงสร้างข้อมูลทรีประกอบด้วย**โหนด** (Node) สำหรับจัดเก็บข้อมูล และกิ่งหรือ**เส้น**ที่เชื่อมโยง

https://www2.cs.science.cmu.ac.th/courses/204251/lib/exe/fetch.php?media=tree.pdf

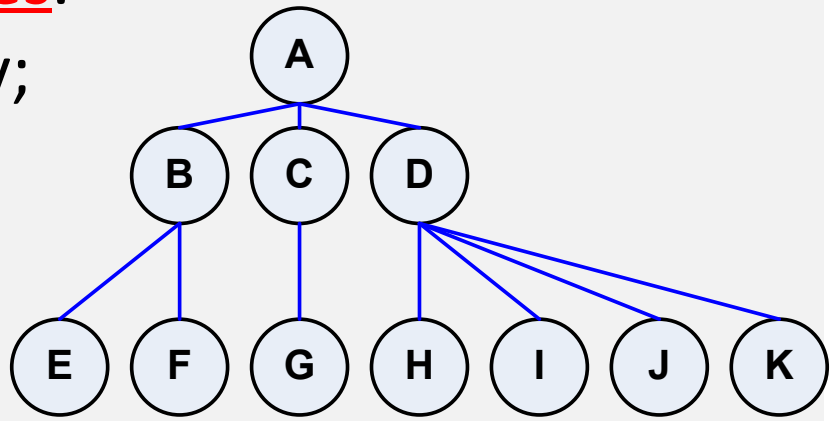
4.1 Tree Definition

A tree data structure can be defined recursively as a collection of nodes (starting at a root node), where each **node** is a data structure consisting of a value,

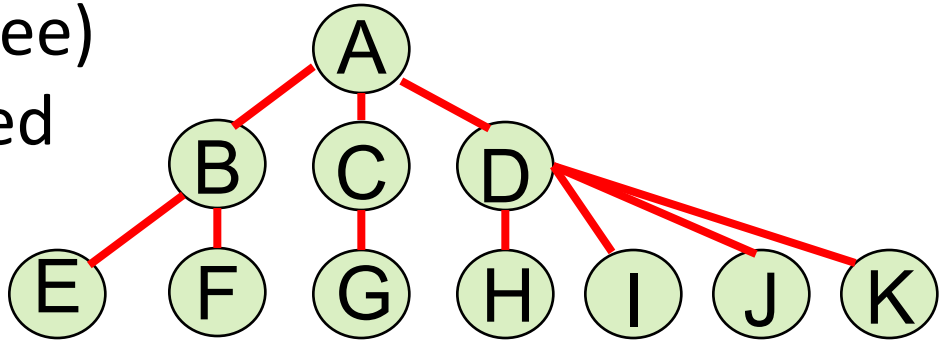


together with **a list of references** to nodes (the "children"), with the constraints that no reference is duplicated, and none points to the root.

- A tree is a collection of nodes.
- The collection can be empty;
- Otherwise,
  - a tree consists of a distinguished node r, called the root,
  - and zero or more nonempty (subtrees) , $T_1, T_2, \ldots, T_k$

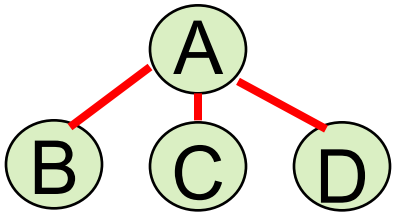


- each of whose root(Sub tree) are connected by a directed edge from r.

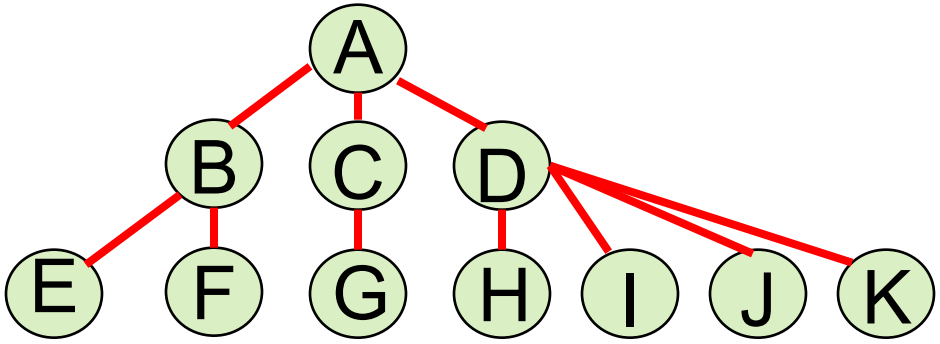


- A root of each subtree is said to be a child of r,
- And r is the parent of each subtree root.

Recursive definition

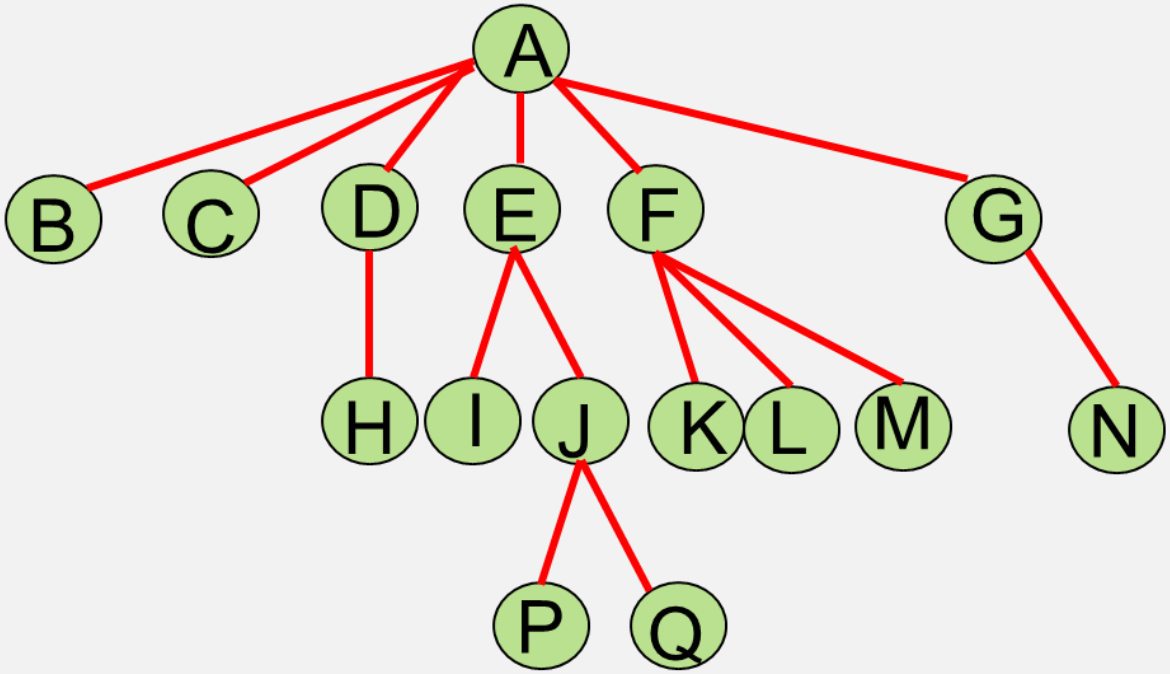


- A tree is a collection of N nodes,
- one of which is the root, and N-1 edges.
- That there are N-1 edges follows from the fact that each edge connects some node to its parents.
- And every node except the root has one parent.



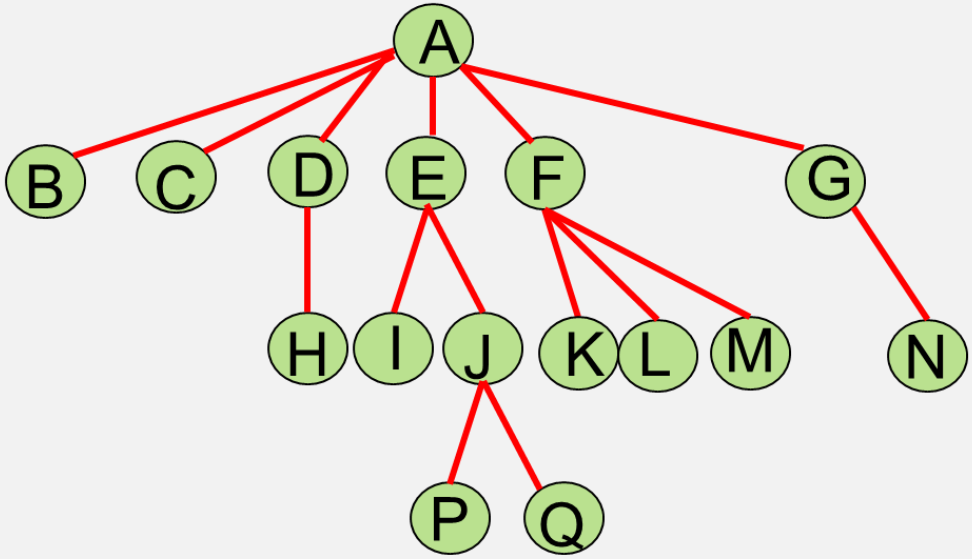
นิยามที่ใช้กับ Tree

- Leaves (Terminal)
- Parents
- Siblings
- Non Leaves (Non terminal)



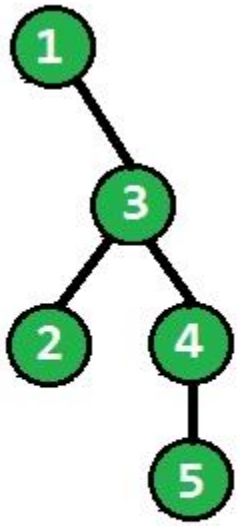
นิยามที่ใช้กับ Tree

- **Degree** :The number of children of a node x in a rooted tree T.



- **Path** from node n_1 to n_k : Sequence of nodes $n_1, n_2, .., n_k$ such that n_i is the parent of n_{i+1} for $1 <= i <= k$.

- **Depth** : For Any node n_i, the **depth** of n_i is the length of the unique path from the root to n_i.
- **Height** : Is the longest path from n_i to a leaf. All leaves are at height 0. The height of a tree is equal to the Height of the root.



Depth 0

Depth 1

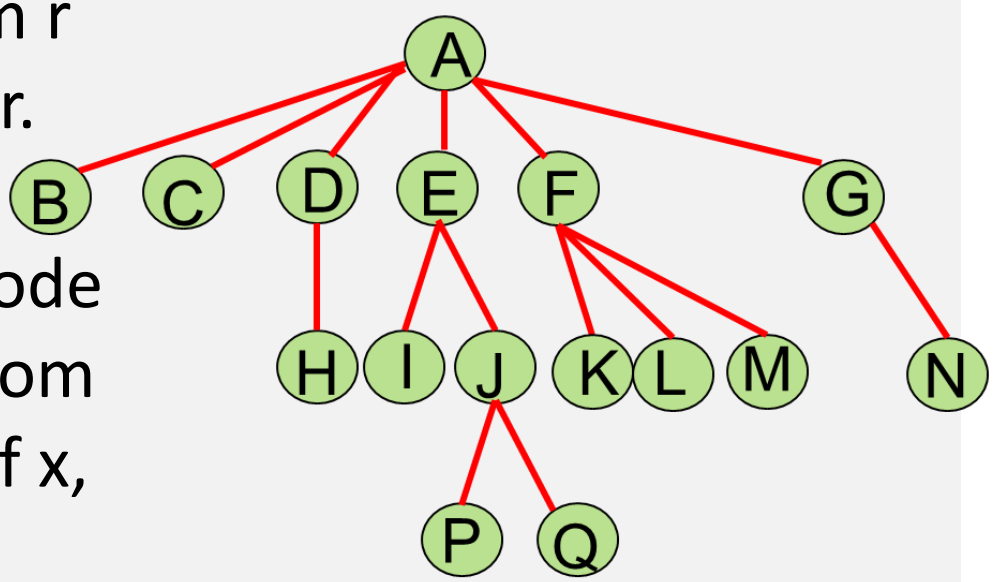
Depth 2

Depth 3

Height 3

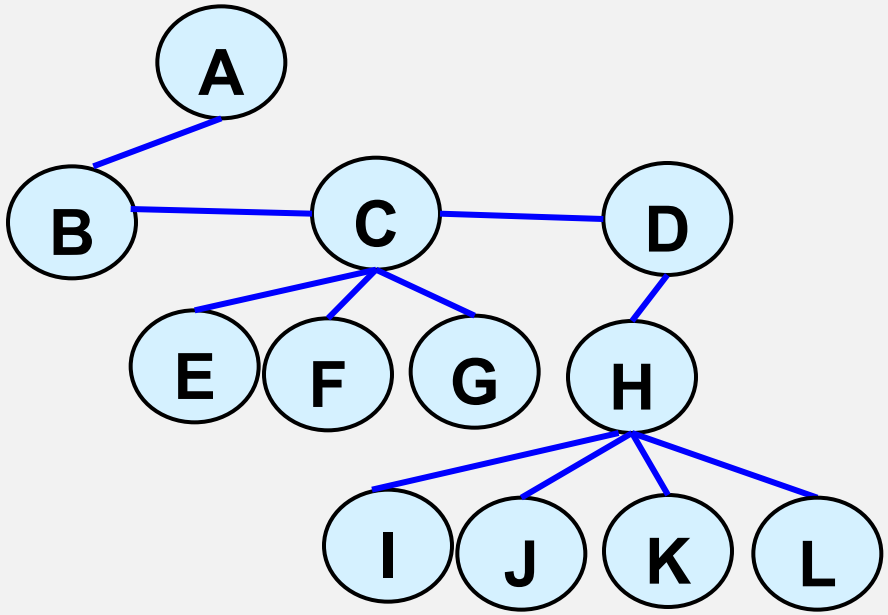
Notice that in a tree there is exactly one path from the root to each node.

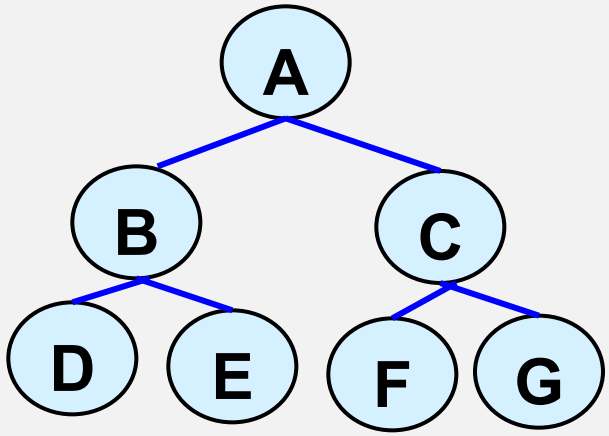
- **Ancestor** of x : Any node y on the unique path from r to x is called an ancestor.
- **Descendant** of y :Any node y on the unique path from r to x, y is descendant of x, Every node is both an ancestor of and a descendant of itself.



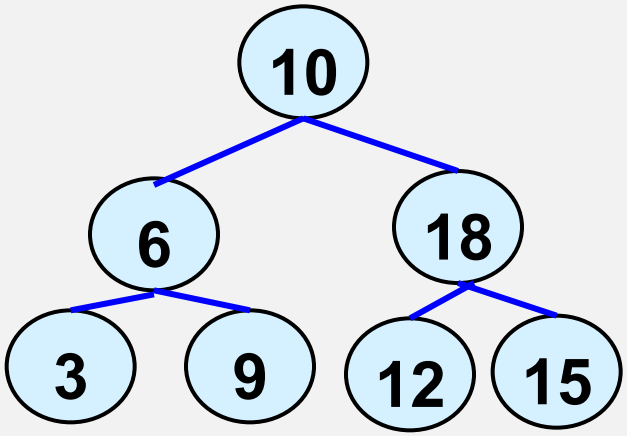
4.2 Binary tree

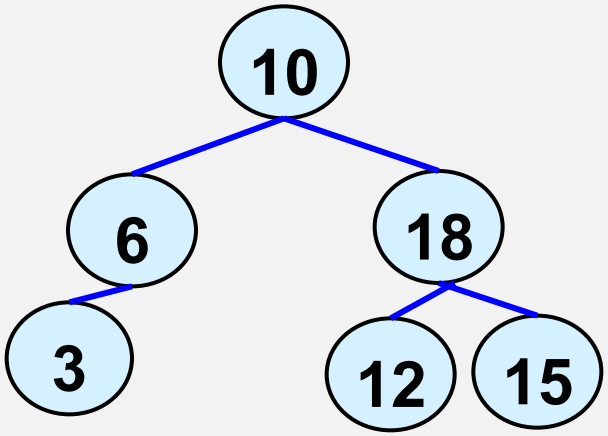
1) A Binary tree is a tree in which no node can have more than two children.



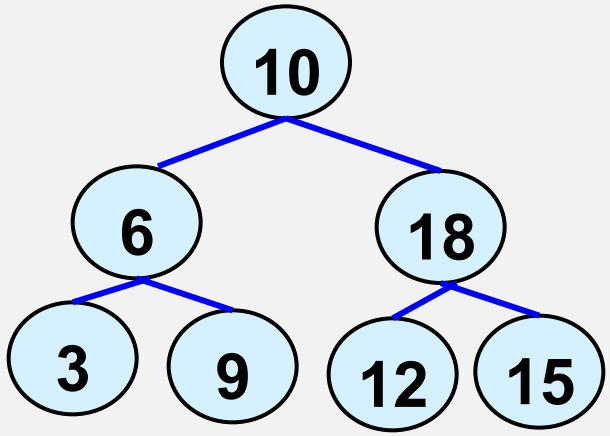


2) Full Binary tree (Complete Binary tree) :Binary tree which each node is either a leaf or has a degree exactly2





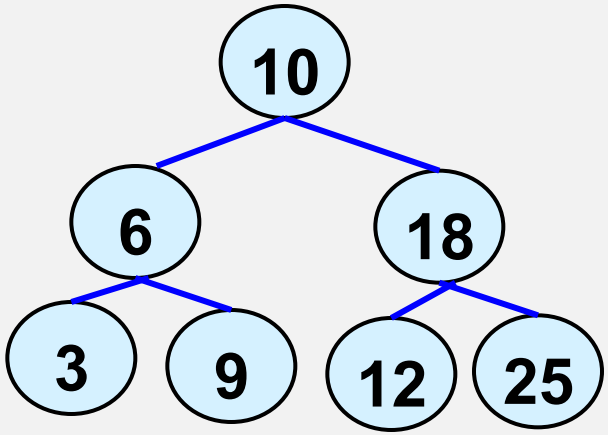
4.3 Binary search tree



- Special type of binary tree,
- The keys in a binary search tree are always stored in such a way as to satisfy the binary search tree property:
  - Let x be a node in a binary search tree.
  - If y is a node in the left subtree of x, then key y <= key x. If y is a node in the right subtree of x, then key x <= key y.

4.3.1 Tree Traversal

Binary search tree property allow us to print out all the keys in a tree in sorted order by a simple recursive algorithm called inorder **tree walk**.



1) **Preorder** Root Left Right
2) **Inorder** Left Root Right
3) **Postorder** Left Right Root

4.3.2 Operation

1. **Insert**
2. **Delete**
3. **Print :**
   - Preorder,
   - Inorder,
   - Postorder
4. **Find**

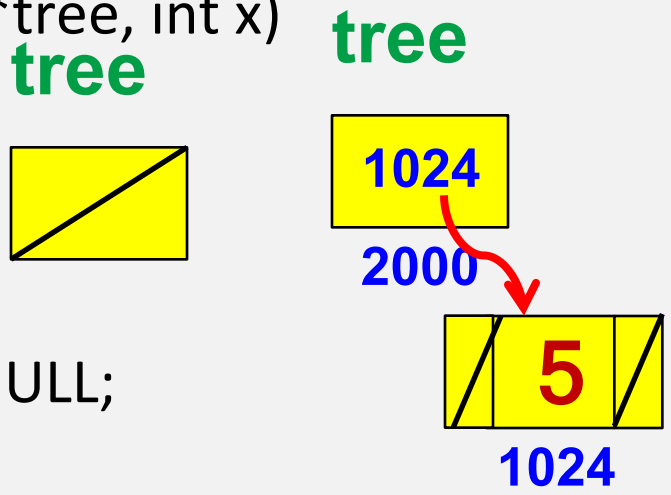
Example 1

```
#include <iostream>
#include <stdio.h>
using namespace std;
struct node
{ int value;
  struct node *left;
  struct node *right;
};
```

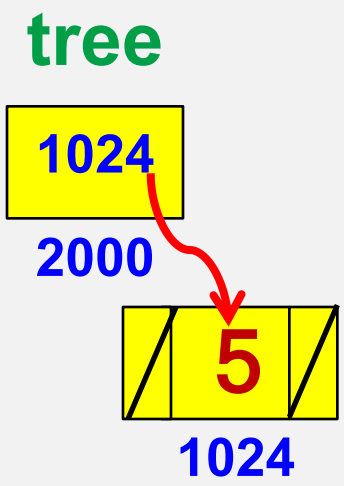
```
struct node *insert(struct node *tree, int x)
{     ……….
}
int main()
{         struct record *tree=NULL;
          tree=insert(tree,5);
}
```

NULL 5

```
   struct node *insert(struct node *tree, int x)
1  {   if(tree==NULL)
2      {   tree = new struct node;
3          tree->value = x;
4          tree->left =  tree->right = NULL;
       }
6      else
7      {     if( x < tree->value )
8                  tree->left = insert(tree->left, x);
9              else  if(x > tree->value)
10                 tree->right = insert(tree->right, x);
       }
11 return tree;  }
```

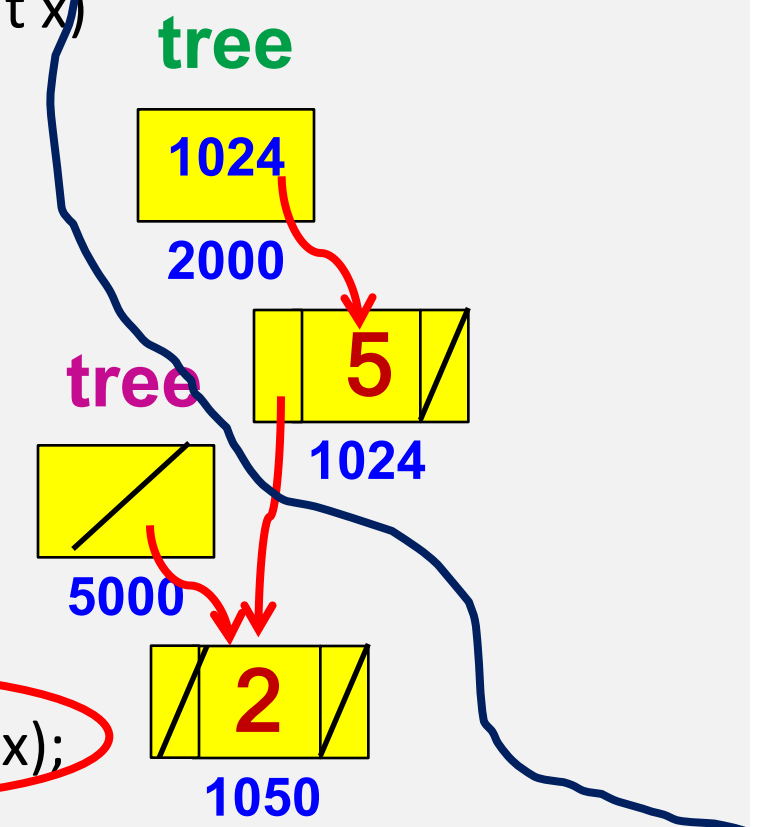


```
   struct node *insert(struct node *tree, int x)
1  {   if(tree==NULL)
2      {   tree = new struct node;
3          tree->value = x;
4          tree->left =  tree->right = NULL;
       }
6      else
7      {     if( x < tree->value )
8                tree->left = insert(tree->left, x);
9            else  if(x > tree->value)
10               tree->right = insert(tree->right, x);
       }
11     return tree;  }
```

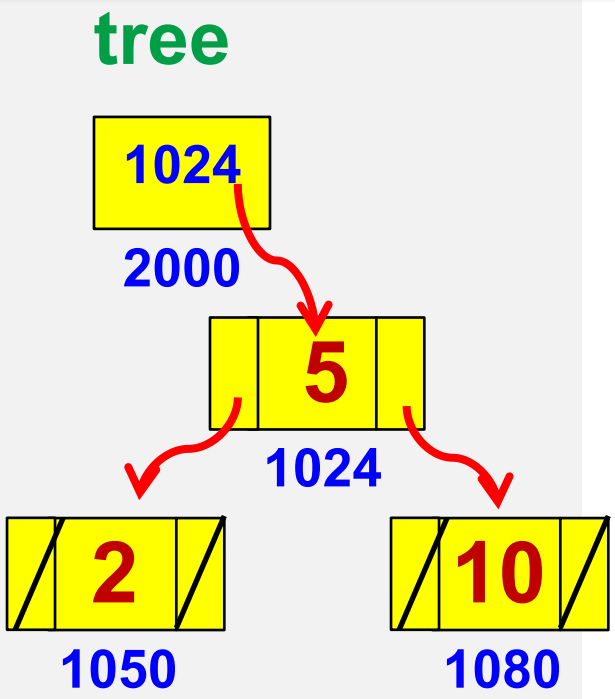


1024	2
NULL	2

```
struct node *insert(struct node *tree, int x)
1 {   if(tree==NULL)
2     {   tree = new struct node;
3         tree->value = x;
4         tree->left =  tree->right = NULL;
      }
6     else
7     {     if( x < tree->value )
8               tree->left = insert(tree->left, x);
9           else  if(x > tree->value)
10              tree->right = insert(tree->right, x);
      }
11    return tree;  }
```



```
  void print(struct node *tree)
1{    if ( tree == NULL )
2         return;
3     else
4     {   cout << tree->value << endl;
5         print(tree->left);
6         print(tree->right);
      }
7    return;
  }
```



Time complexity:

Best case: O(1)

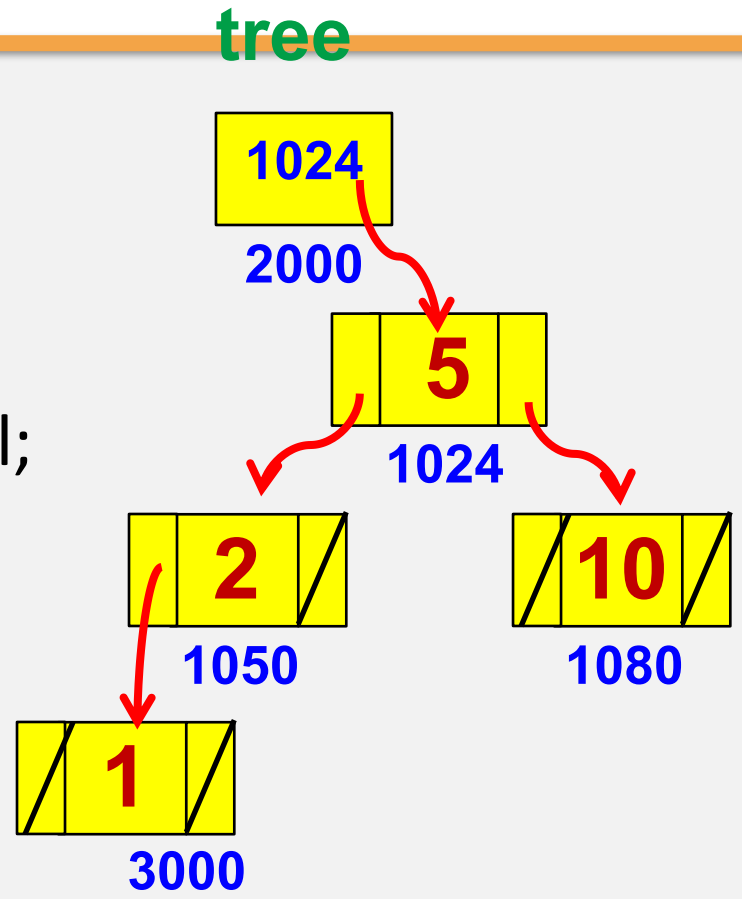
Average case: When there is a balanced binary search tree(a binary search tree is called balanced if height difference of nodes on left and right subtree is not more than one), so height becomes logN where N is number of nodes in a tree.
searching is O(logN)

Worst case: O(N)

Trees

- Insert
- Print
- Search
- Find Min
- Delete

```
void print(struct node *tree)
{    if ( tree == NULL )
         return;
     else
     {   cout << tree->value << endl;
         print(tree->left);
         print(tree->right);
     }
    return;
}
```



Preorder

```
  void print(struct node *tree)
1{    if ( tree == NULL )
2         return;
3     else
4     {   print(tree->left);
5         cout << tree->value << endl;
6          print(tree->right);
      }
7    return;
  }
```