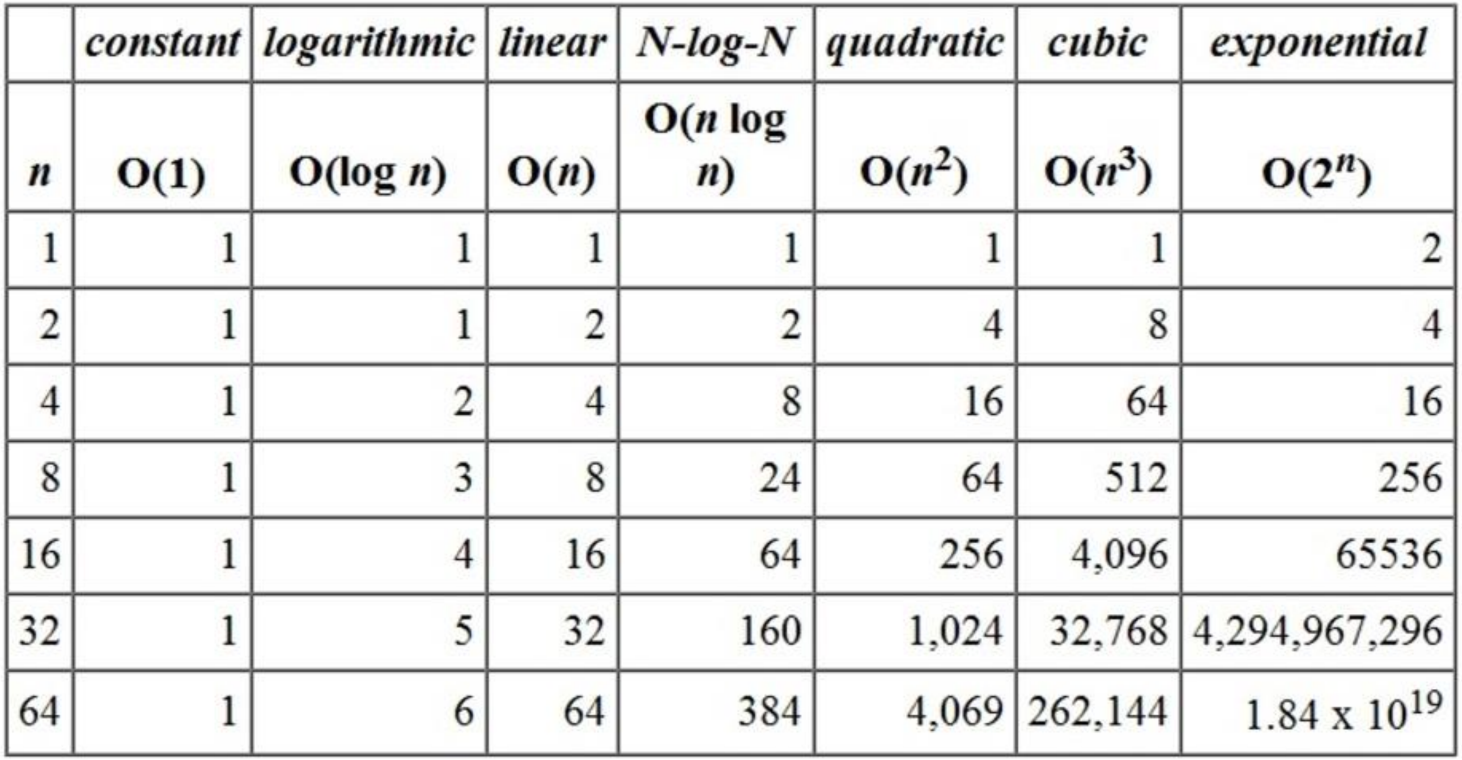
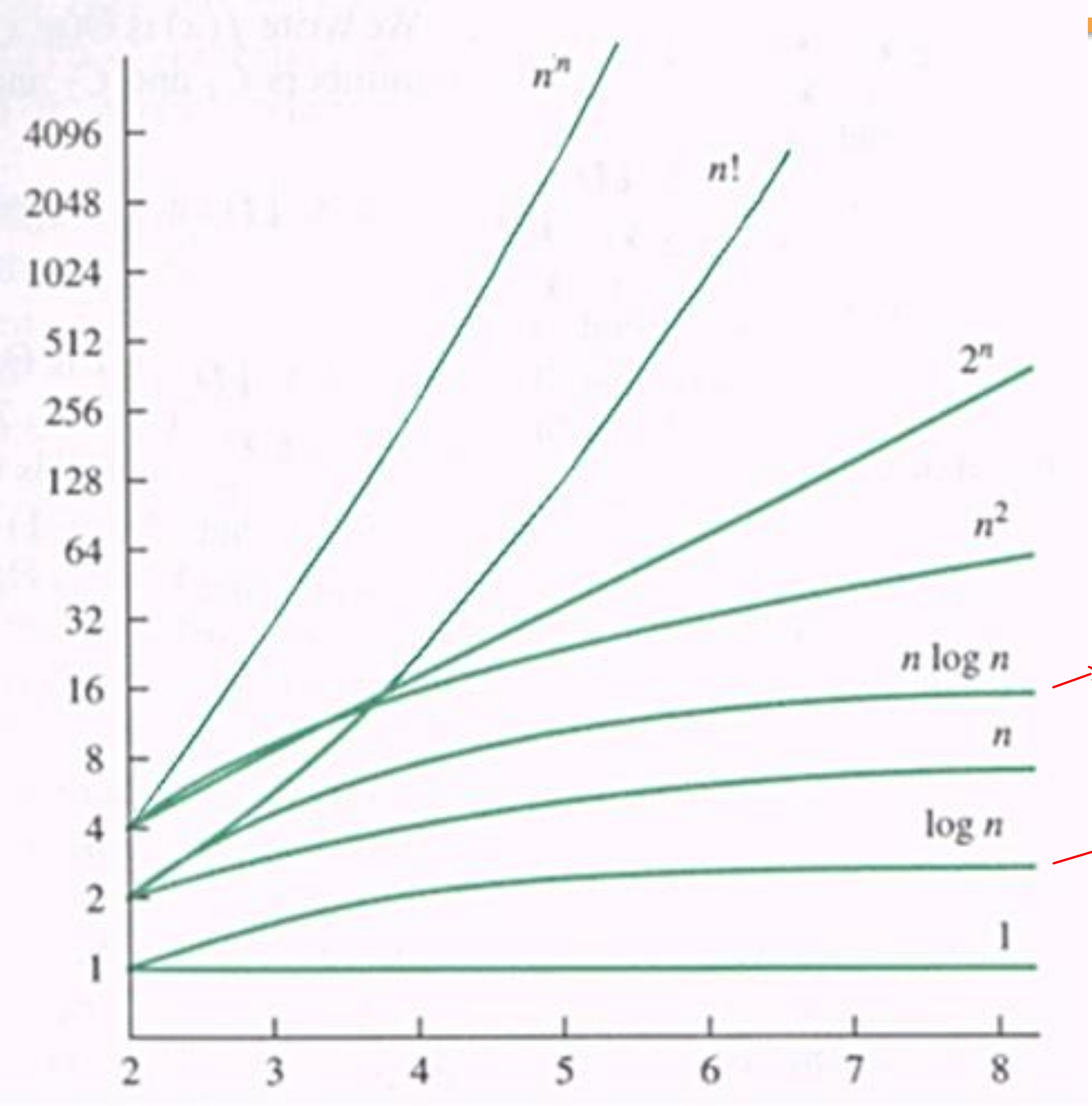
Module3—Tree

* เร็ว ——> ช้า



	constant	*logarithmic*	*linear*	*N-log-N*	*quadratic*	*cubic*	*exponential*
n	O(1)	O(log n)	O(n)	O(n log n)	O(n^2)	O(n^3)	O(2^n)
1	1	1	1	1	1	1	2
2	1	1	2	2	4	8	4
4	1	2	4	8	16	64	16
8	1	3	8	24	64	512	256
16	1	4	16	64	256	4,096	65536
32	1	5	32	160	1,024	32,768	4,294,967,296
64	1	6	64	384	4,069	262,144	1.84 x 10^{19}



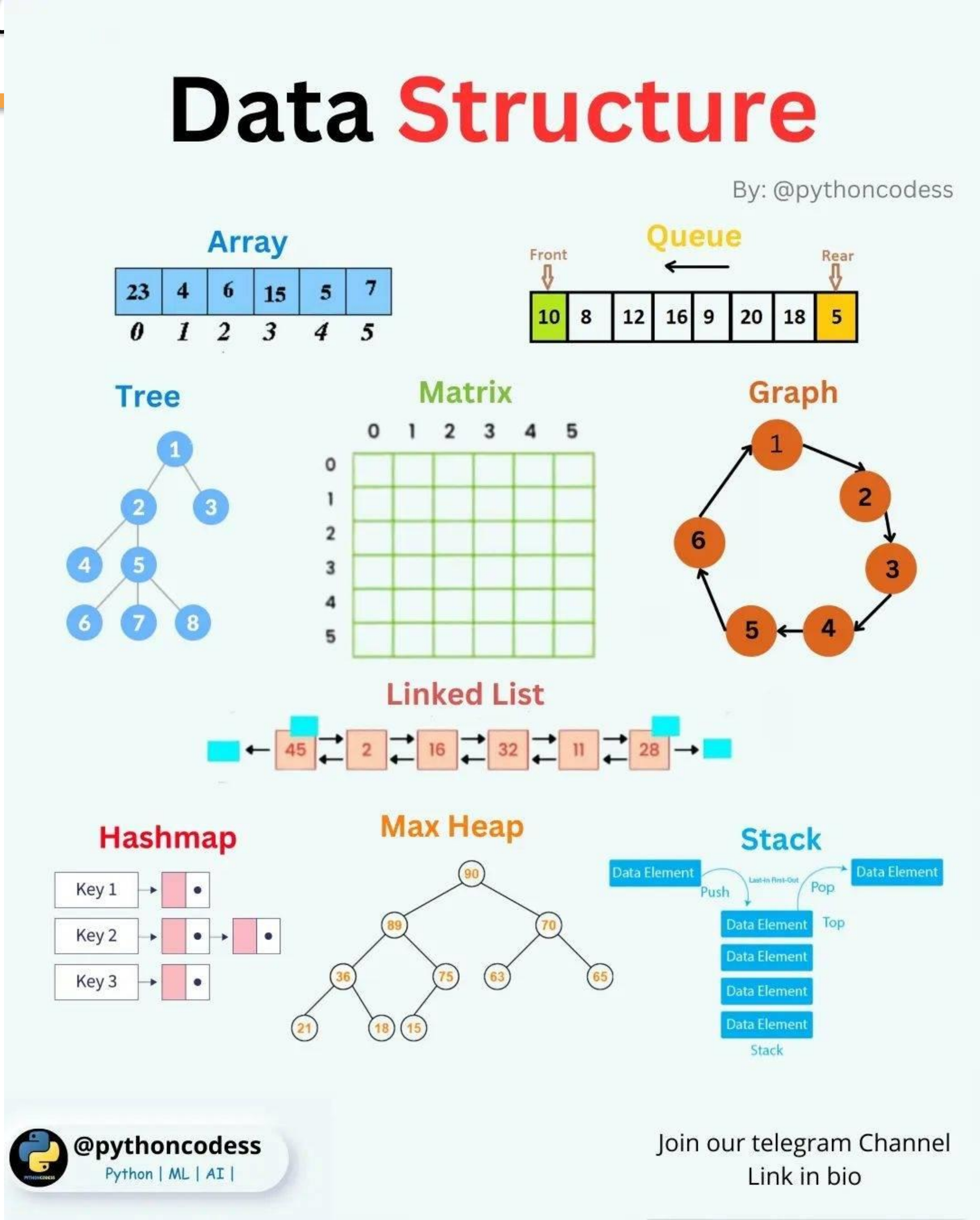
ช้า

↑

เร็ว

→ Tree

→ Tree


Data Structure
By: @pythoncodess
Array
23
4
6
15
5
7
0
1
2
3
4
5
Queue
Front
Rear
10
8
12
16
9
20
18
5
Tree
1
2
3
4
5
6
7
8
Matrix
0
1
2
3
4
5
Graph
1
2
3
4
5
6
Linked List
45
2
16
32
11
28
Hashmap
Key 1
Key 2
Key 3
Max Heap
90
89
70
36
75
63
65
21
18
15
Stack
Data Element
Push
Last-in First-Out
Pop
Data Element
Data Element
Top
Data Element
Data Element
Data Element
Stack
@pythoncodess
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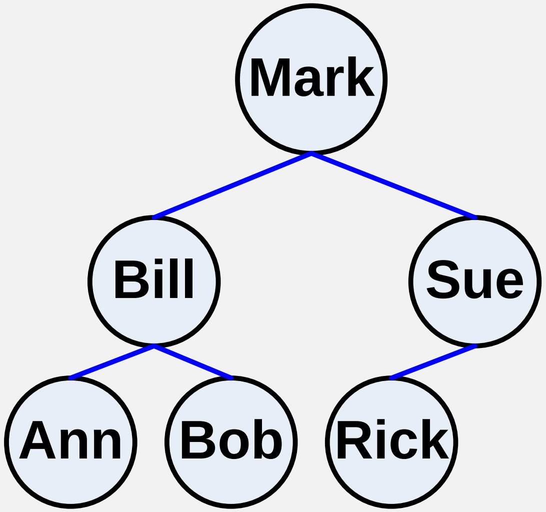
3. Trees

Problem

- Linear time access of linked list.
- Running time of operation O(n).



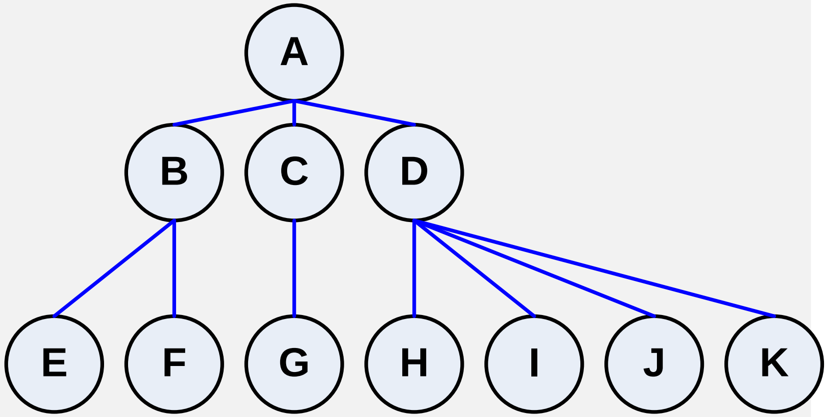
Correct : Trees

- Average time O($\log_n$). เฉลี่ย
- Worst case O(n). แย่สุด

3.1 Tree Definition

โครงสร้างข้อมูลต้นไม้ (Tree Data Structure) หรือเรียกสั้นๆว่าทรี (Tree) เป็นโครงสร้างข้อมูลรูปแบบหนึ่งในลักษณะ

- โครงสร้างข้อมูลชนิดไม่เชิงเส้น (Non-Linear)
- สมาชิกแต่ละตัวในทรีสามารถเชื่อมโยงไปยังสมาชิกตัวถัดไป (Successor) ได้มากกว่าหนึ่งตัว



- และเชื่อมโยงถึงกันในลักษณะเป็นระดับคล้ายกับการแตก กิ่งก้านสาขาออกไปของต้นไม้
- ความสัมพันธ์ของสมาชิกข้อมูลในทรี จึงมีลักษณะลำดับชั้น (Hierarchical Relationship) คือ มีการเชื่อมโยงของแต่ละโหนดเป็นแบบทางเดียวจากบนลงล่าง
- โครงสร้างข้อมูลทรีประกอบด้วย**โหนด** (Node) สำหรับจัดเก็บข้อมูล และกิ่งหรือ**เส้น**ที่เชื่อมโยง

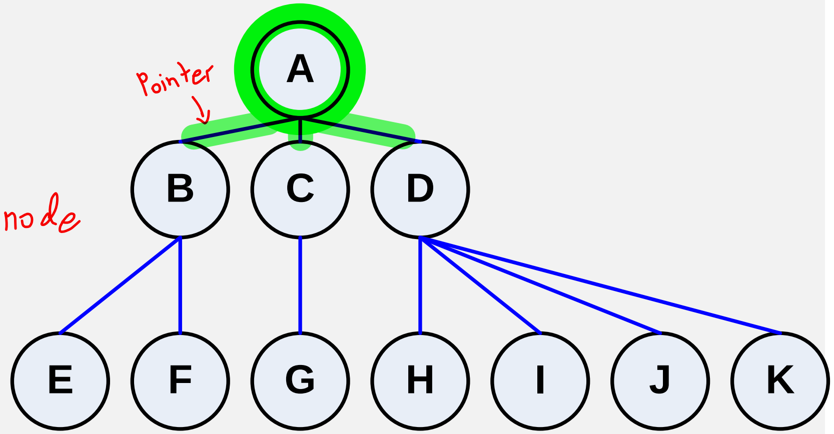
https://www2.cs.science.cmu.ac.th/courses/204251/lib/exe/fetch.php?media=tree.pdf

3.1 Tree Definition

Tree ถูกเปรียบเทียบ recursively

A tree data structure can be defined **recursively** as a collection of nodes (starting at a root node), where each **node** is a data structure consisting of a value,

ชุดหนึ่งของหลาย node

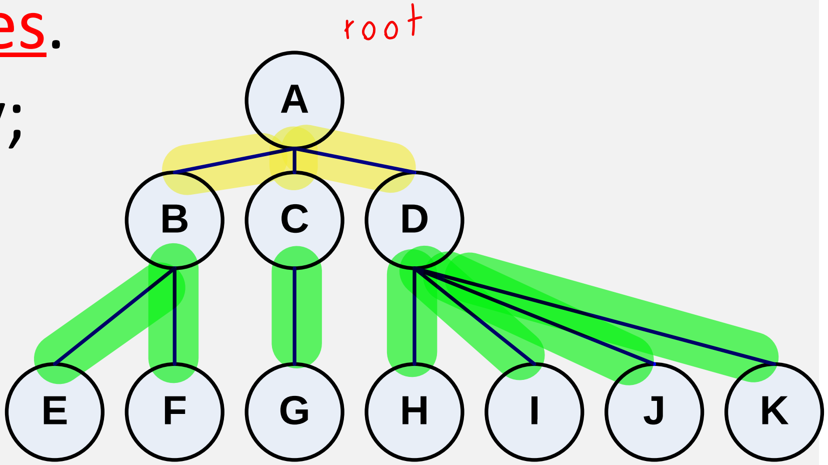


ใช้ Pointer ชี้

together with **a list of references** to nodes (the "children"), with the constraints that no reference is duplicated, and none points to the root. root ไม่ถูกชี้

- ❑ A tree is a collection of nodes.
- ❑ The collection can be empty;
- ❑ Otherwise,
    - ❑ a tree consists of a distinguished node r, called the root,
    - ❑ and zero or more nonempty (subtrees) , $T_1, T_2, \ldots, T_k$

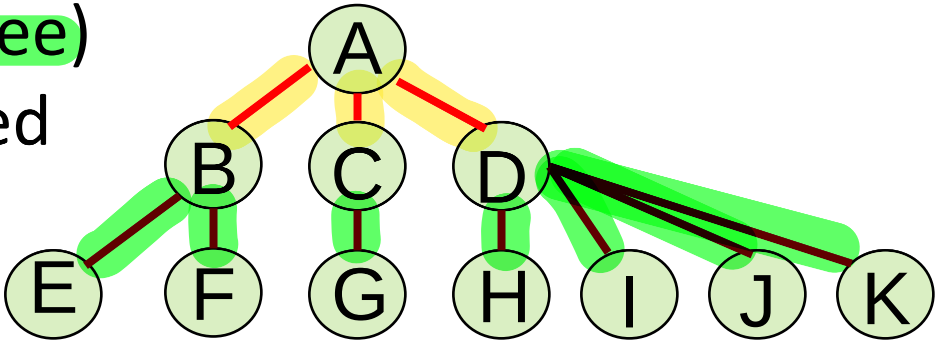
มีตั้งแต่ 0 ถึง N



เชื่อม กับ ลูก ด้วย เส้น (edge)

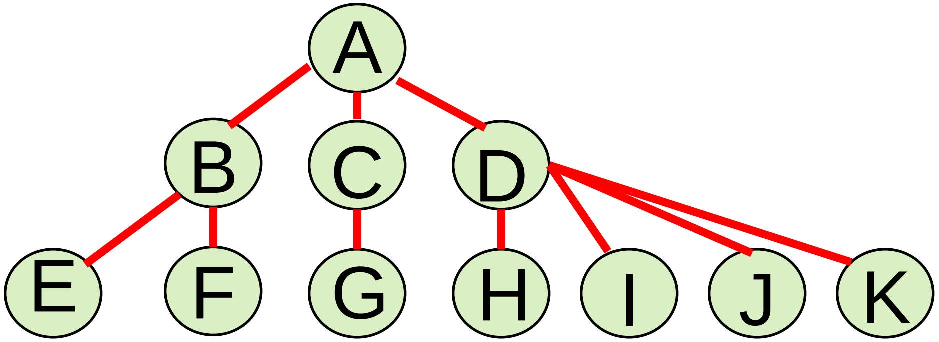
- ❑ each of whose root(Sub tree) are connected by a directed edge from r.

เส้น



- ❑ A root of each subtree is said to be a child of r,
- ❑ And r is the parent of each subtree root.

Recursive definition

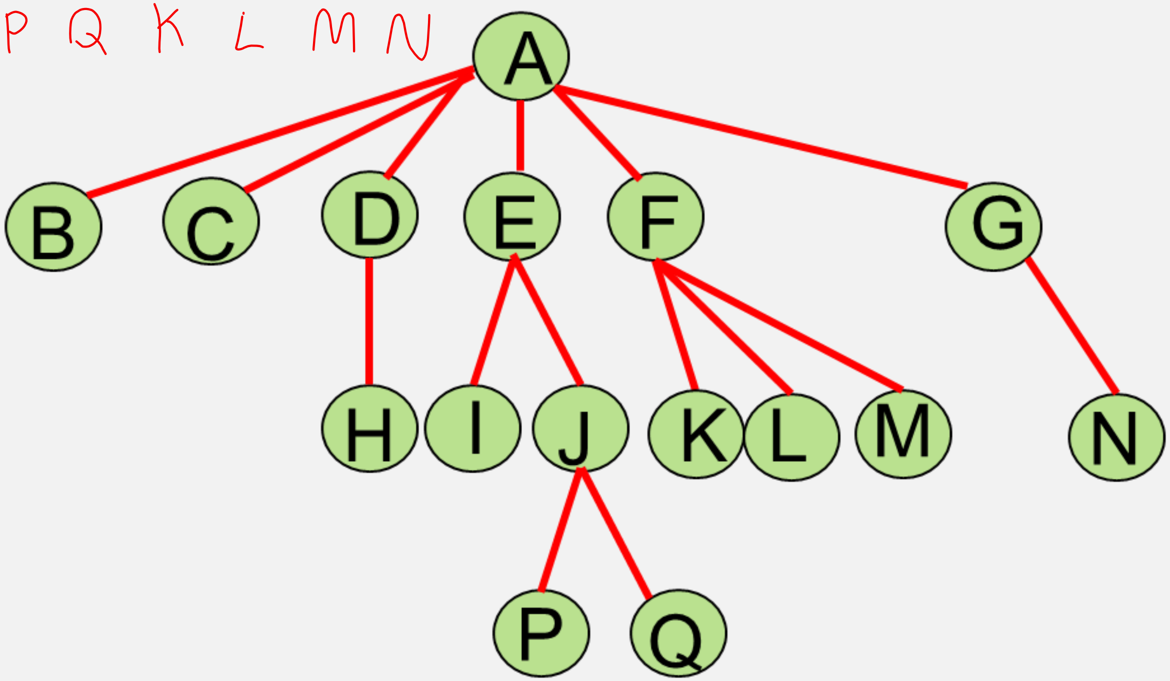


- ❑ A tree is a collection of N nodes,
- ❑ one of which is the root, and N-1 edges. ไม่ชี้ root
- ❑ That there are N-1 edges follows from the fact that
  ทุก เส้น ต่อ เชื่อม
  each edge connects some node to its parents.
- ❑ And every node except the root has one parent.

นิยามที่ใช้กับ Tree

- ❑ Leaves B C H I P Q K L M N
  (Terminal)
- ❑ Parents พ่อแม่
- ❑ Siblings พี่น้อง
- ❑ Non Leaves
  (Non terminal)

  A D E F G J

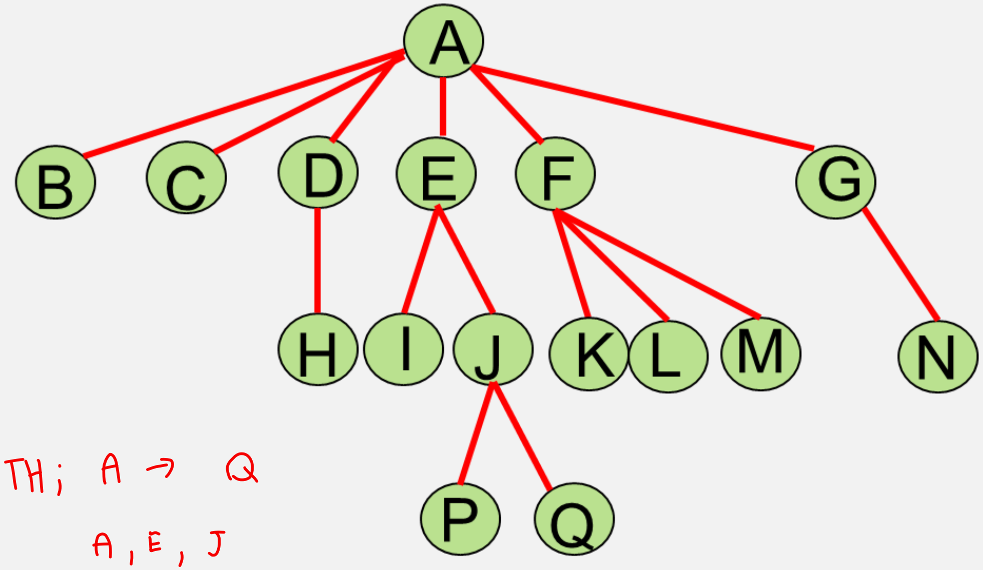


นิยามที่ใช้กับ Tree

- ❑ **Degree** :The number of children of a node x in a rooted tree T. จำนวนลูก

เส้นทาง

- ❑ **Path** from node n_1 to n_k : Sequence of nodes $n_1, n_2, .., n_k$ such that n_i is the parent of n_{i+1} for $1 <= i <=k$.



PATH; A → Q

A, E, J

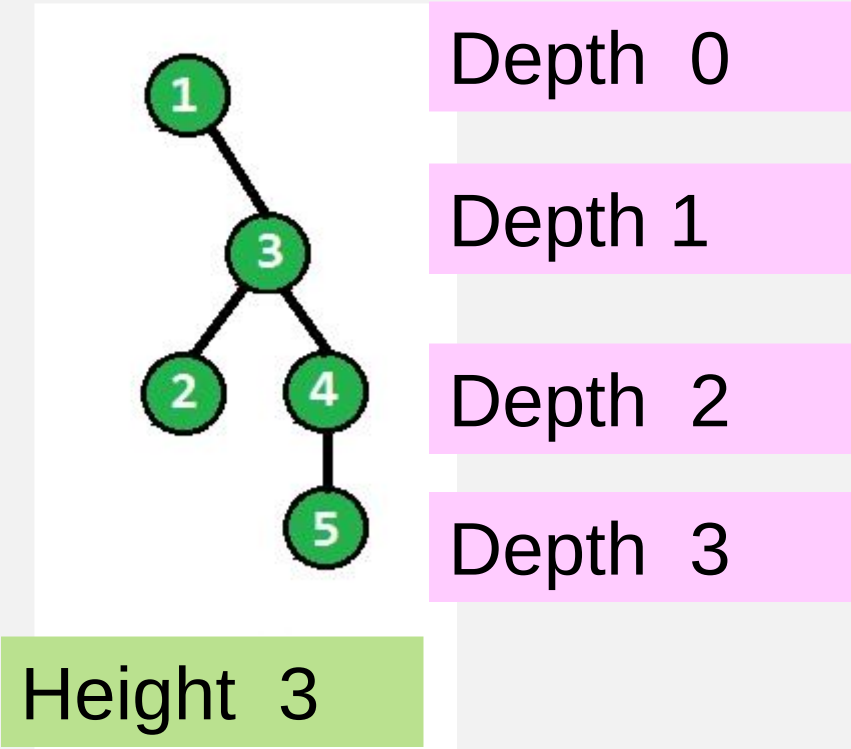
Length ; เส้นที่ยาวที่สุด

ความลึก

- ❑ **Depth** : For Any node n_i, the **depth** of n_i is the length of the unique path from the root to n_i.

ความสูง

- ❑ **Height** : Is the longest path from n_i to a leaf. All leaves are at height 0. The height of a tree is equal to the Height of the root.



Depth 0

Depth 1

Depth 2

Depth 3

Height 3

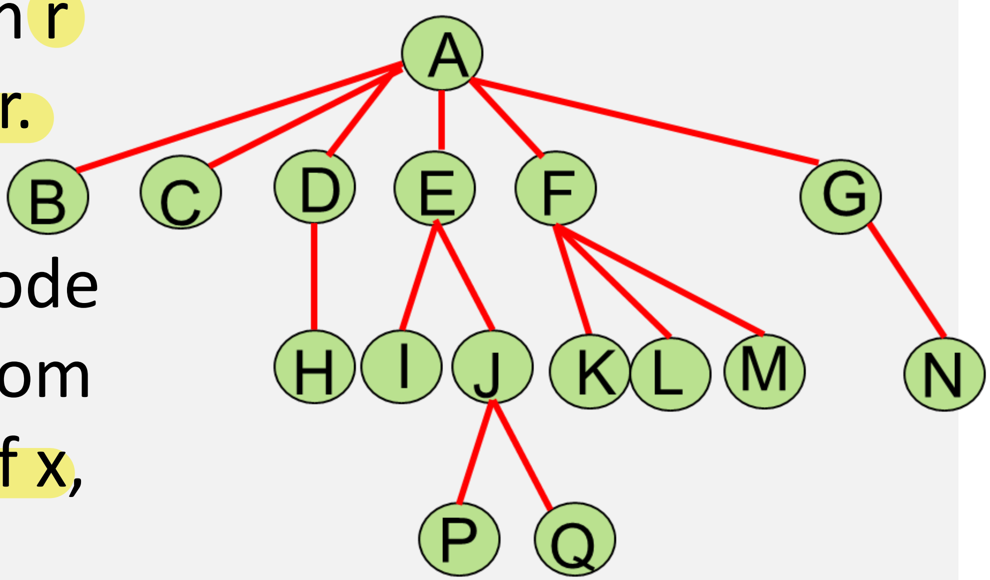
__Notice that__ in a tree there is exactly one path from the root to each node.

บรรพ บุรุษ

- **Ancestor** of x : Any node y on the unique path from r to x is called an ancestor.

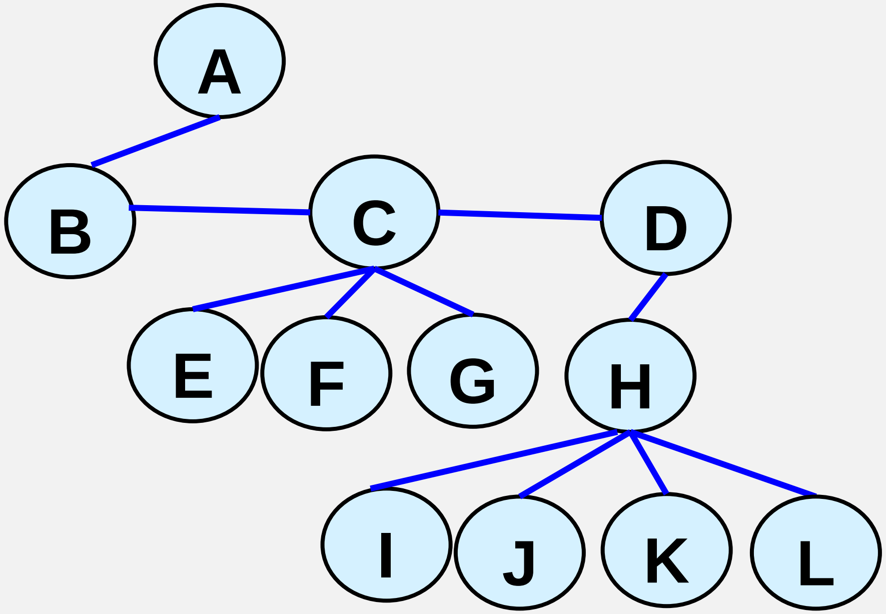
ลูกหลาน

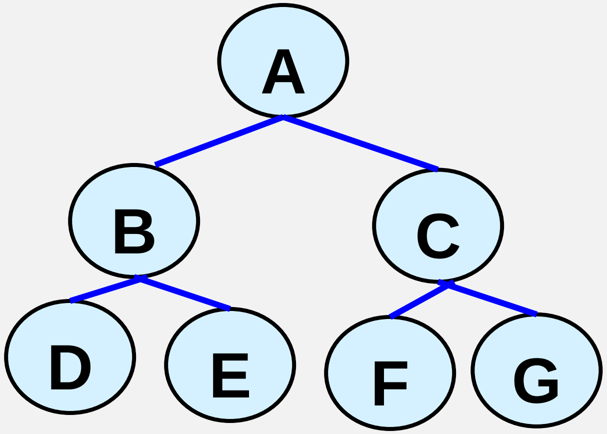
- **Descendant** of y :Any node y on the unique path from r to x, y is descendant of x, Every node is both an ancestor of and a descendant of itself.



5.2 Binary tree

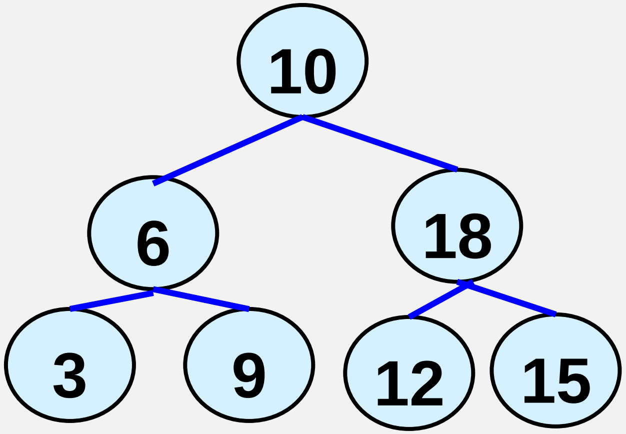
1) **A Binary tree** is a tree in which no node can have more than two children. *มีลูก 2 คน ได้ตั้งแต่ 0 – 2 คน



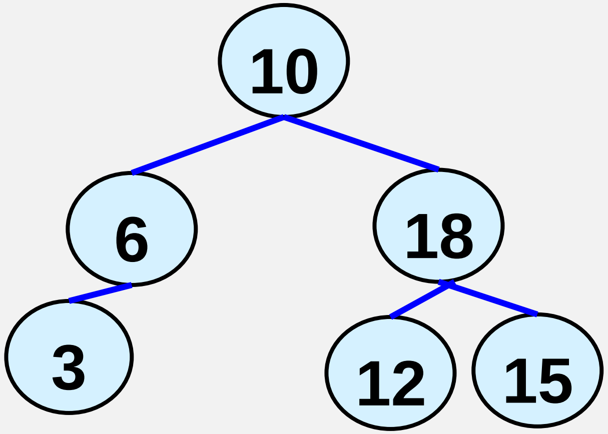


2) Full Binary tree (Complete Binary tree) :Binary tree which each node is either a leaf or has a degree exactly2

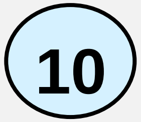
อย่างใดอย่างหนึ่ง ไม่ 0 ก็ 2 แท้จริง



A ✓



B ✗

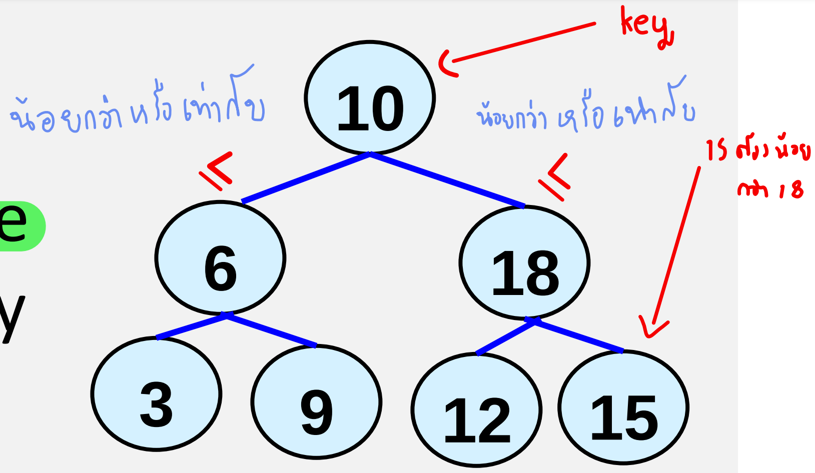


C ✓

5.3 Binary search tree

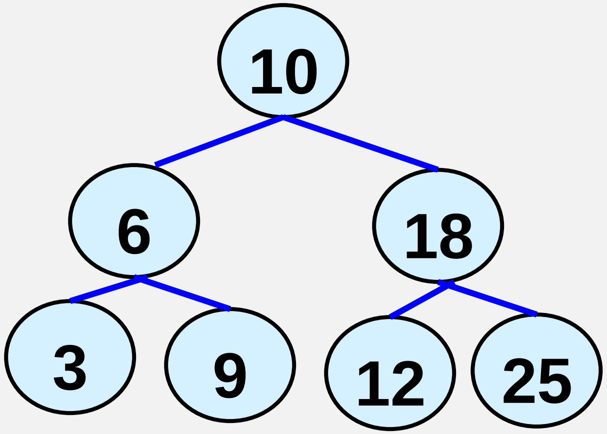
พิเศษ

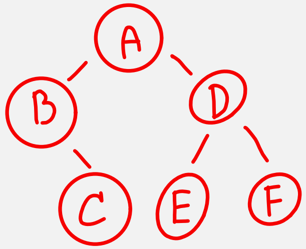
- ○ Special type of binary tree,
- ○ The keys in a binary search tree are always stored in such a way as to satisfy the binary search tree property:
  - Let x be a node in a binary search tree.
  - If y is a node in the left subtree of x, then key y <= key x. If y is a node in the right subtree of x, then key x <= key y.



5.3.1 Tree Traversal

Binary search tree property allow us to print out all the keys in a tree in sorted order by a simple recursive algorithm called inorder **tree walk**.





* เดินให้สุด

1) **Preorder** Root Left Right 10 5 1 7 14 12 20

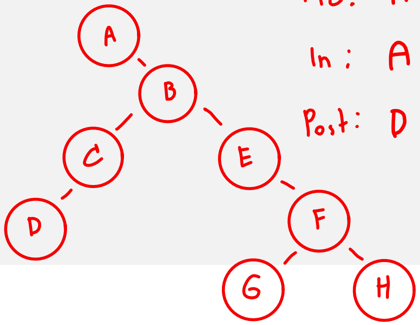
2) **Inorder** น้อยไปมาก Left Root Right 1 5 7 10 12 14 20

3) **Postorder** Left Right Root 1 7 5 12 20 14 10

Pre: A B C D E F

In: B C A E D F

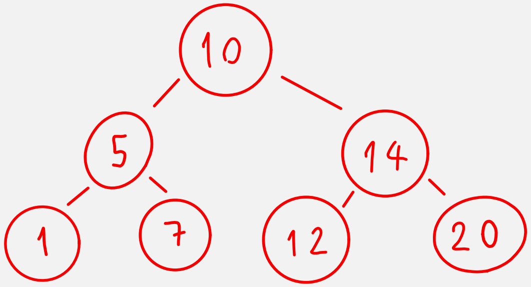
Post: C B E F D A



Pre: A B C D E F G H

In: A D C B E G F H

Post: D C G H F E B A



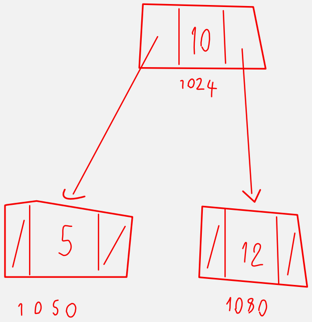
5.3.2 Operation

1. **Insert**
2. **Delete**
3. **Print :**
   - Preorder,
   - Inorder,
   - Postorder
4. **Find**

Example 1

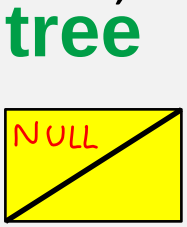
* สร้าง node

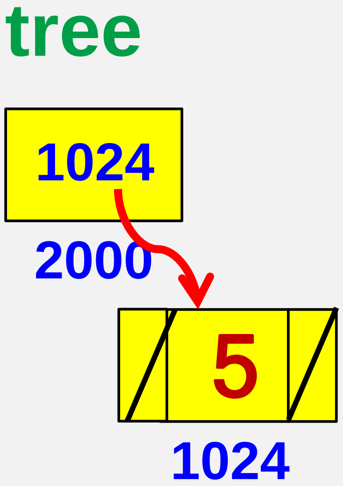
```
#include <iostream>
#include <stdio.h>
using namespace std;
struct node
{ int value;
  struct node *left;
  struct node *right;
};
  int main () {
      struct node *tree = NULL;
      tree = insert (tree,5);
  }
```



NULL 5

```
struct node *insert(struct node *tree, int x)
{   if(tree==NULL)                                              ...... 1
                        สร้าง กล่อง 20 byte
  {   tree = new struct node;                                   ...... 2
       tree->value = x;                                         ...... 3
       tree->left =  tree->right = NULL;                        ... 4,5
  }
 else
 {     if( x < tree->value )                                    ...... 6
          tree->left = insert(tree->left, x); recursively       ...... 7
       else  if(x > tree->value)                                ...... 8
          tree->right = insert(tree->right, x); recursively     ....... 9
 }
            pointer   * ตำแหน่ง เช่น 1024
 return tree; }                                                 ...... 10
```



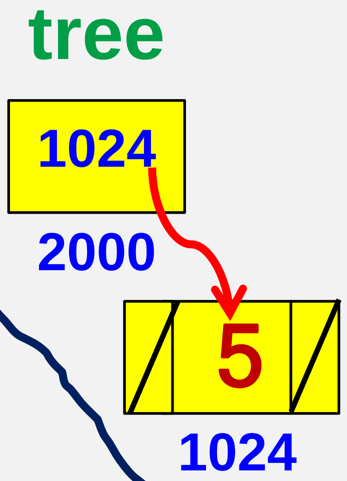


1024 2

```
struct node *insert(struct node *tree, int x)
{   if(tree==NULL)
    {   tree = new struct node;                      ...... 1
        tree->value = x;                             ...... 2
                                                     ...... 3
        tree->left =  tree->right = NULL;            ... 4,5
    }
    else
    {     if( x < tree->value )                      ...... 6
              tree->left = insert(tree->left, x);    ...... 7
          else  if(x > tree->value)                  ...... 8
              tree->right = insert(tree->right, x);  ....... 9
    }
return tree;  }                                      ...... 10
```

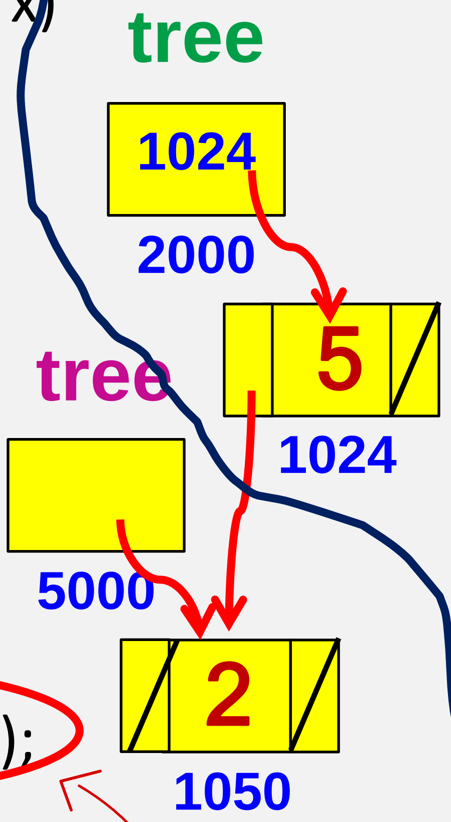
ส่ง NULL, 2

ค้างบรรทัดนี้



1024	2
NULL	2

```
struct node *insert(struct node *tree, int x)
{    if(tree==NULL)                          สร้างกล่อง
     {    tree = new struct node;
          tree->value = x;    2
          tree->left =  tree->right = NULL;
     }
     else
     {     if( x < tree->value )                    1050
                tree->left = insert(tree->left, x);
           else  if(x > tree->value)
                tree->right = insert(tree->right, x);
     }
  return tree; }  1050
```



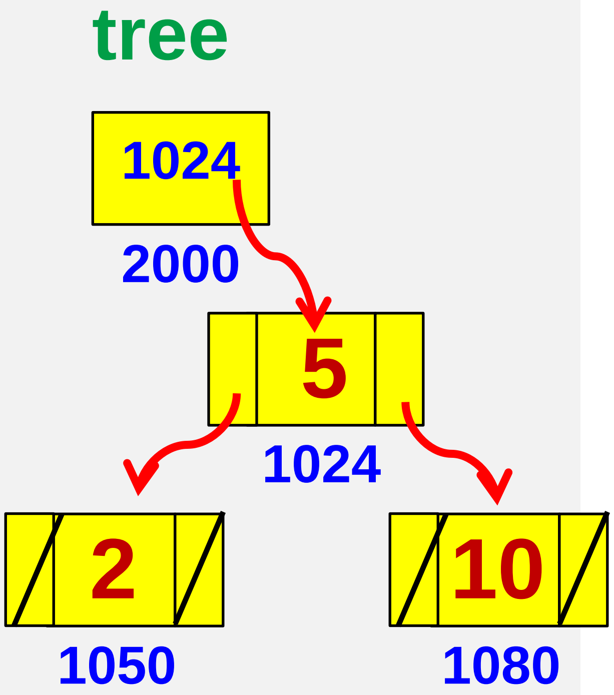
* พอทำ recursively เสร็จ จะส่ง tree ที่มีค่า 1024 กลับไป

```
void print(struct node *tree)
{  1 if ( tree == NULL )
   2       return;
   3 else
   4 {    cout << tree->value << endl;
   5       print(tree->left);
   6       print(tree->right);
     }
  7 return;
}
```

* Preorder

Pre Order:

Root Left Right



tree 1024

1
3
4
5
print (1050)

tree 1050

1
3
4
5 ↩
6 ↩
7
reture

tree 1080

1
3
4
5 ↩
6 ↩
7

Time complexity:

Best case: O(1)

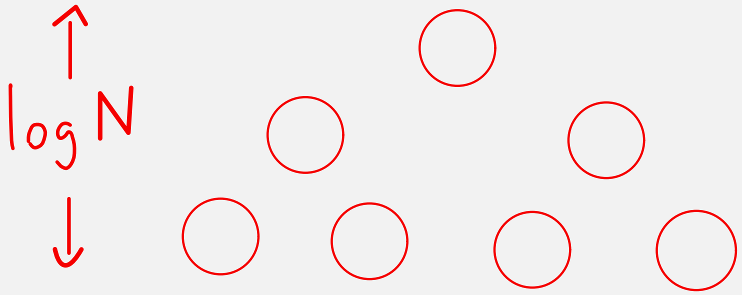
Average case: When there is a balanced binary search tree(a binary search tree is called balanced if height difference of nodes on left and right subtree is not more than one), so height becomes logN where N is number of nodes in a tree.

searching is O(logN)

Worst case: O(N)

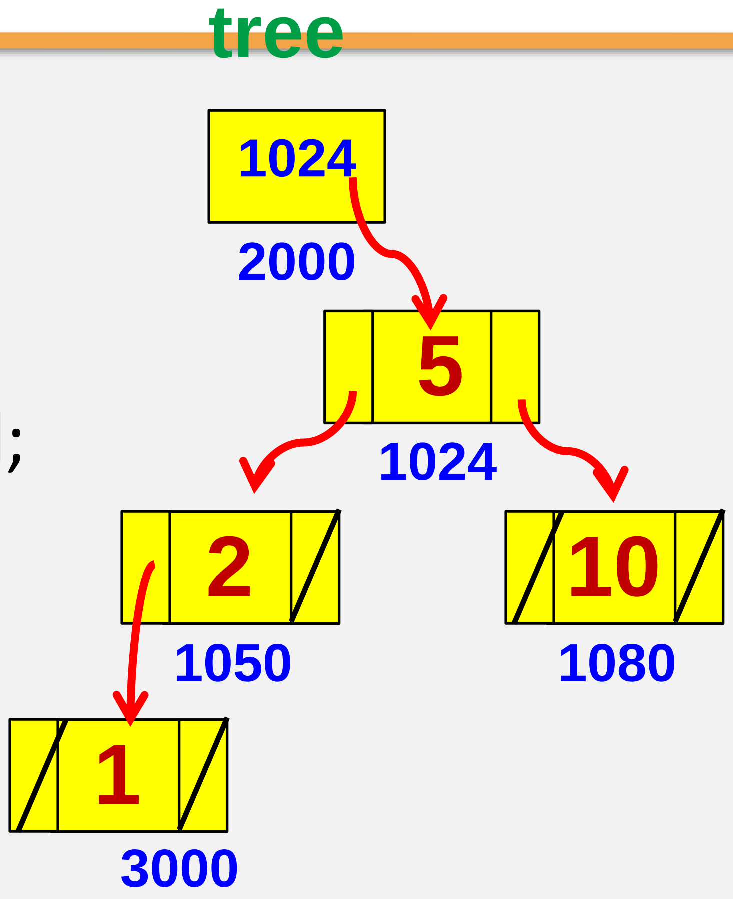
Trees

- Insert
- Print
- Search
- Find Min
- Delete

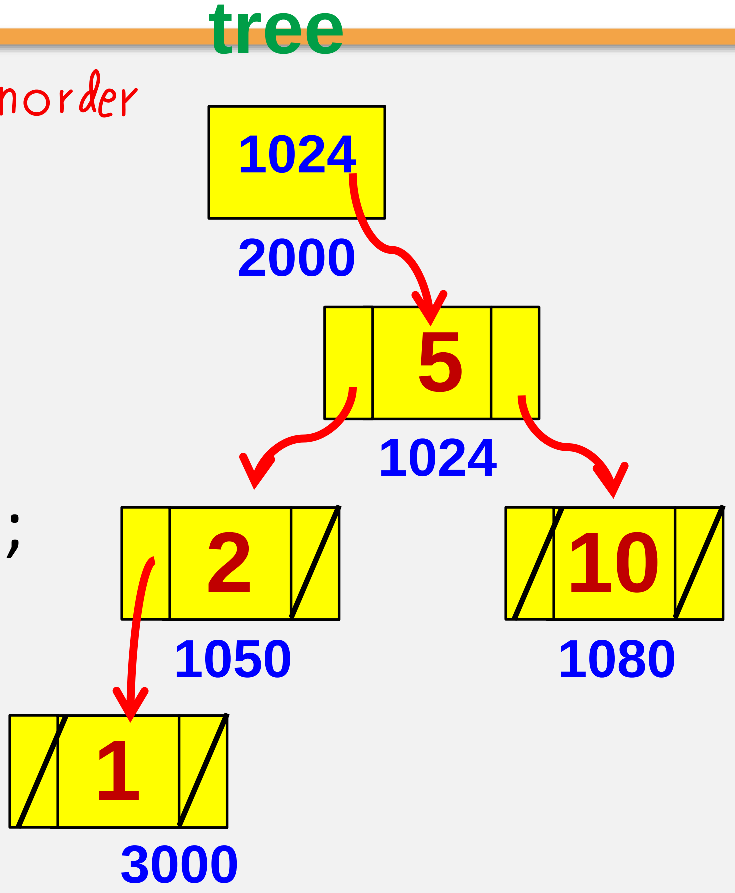


```
void print(struct node *tree)
{     if ( tree == NULL )
           return;
      else
      {    cout << tree->value << endl;
            print(tree->left);
            print(tree->right);
      }
     return;
}
```

Preorder



```
void print(struct node *tree)   * Inorder
{ 1  if ( tree == NULL )
  2      return;
  3  else
  4  {   print(tree->left);
  5      cout << tree->value << endl;
  6      print(tree->right);
     }
 7  return;
}
```


tree
1024
2000
5
1024
2
1050
10
1080
1
3000