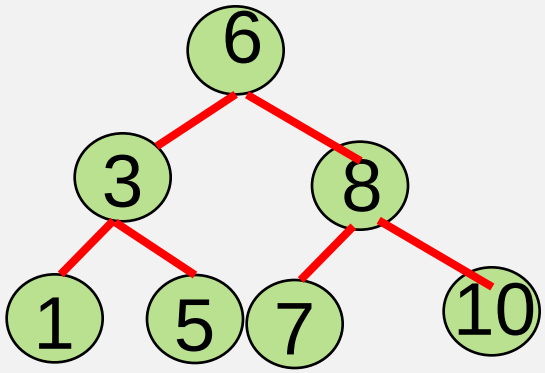
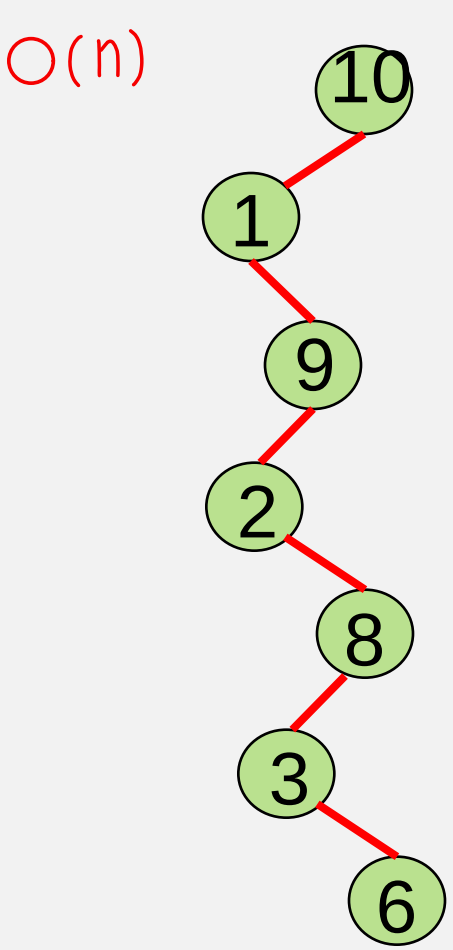
การ insert Binary Search Trees

6 3 8 1 5 7 10



Insert : 10 1 9 2 8 3 6



Big O = ?

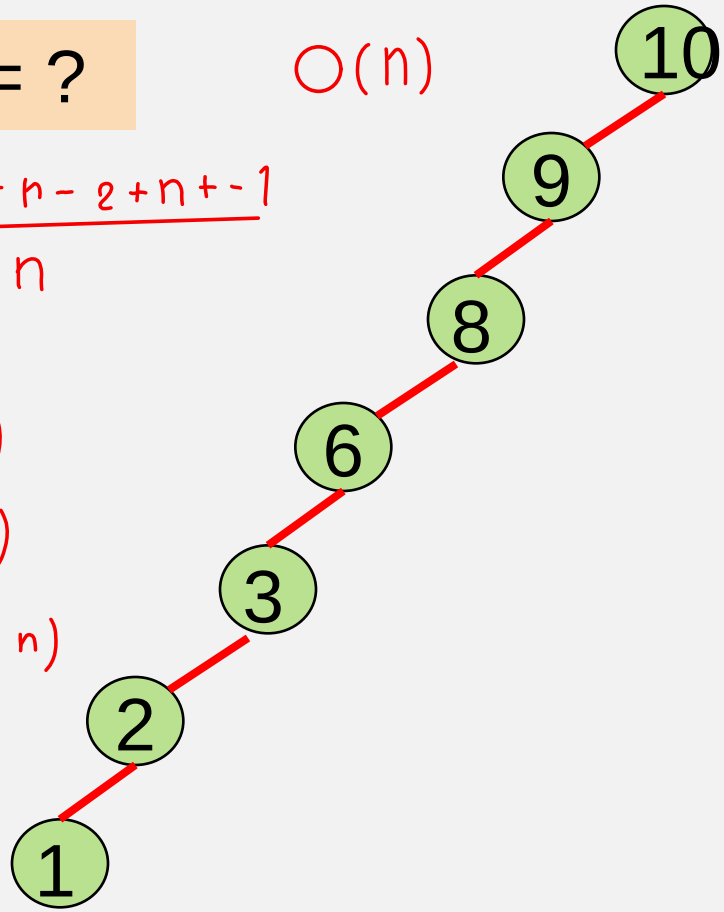
$$\frac{n + n-1 + n-2 + n+-1}{n}$$

best O(1)

Worse O(n)

Avg O(log n)

Insert : 10 9 8 6 3 2 1



ปัญหา Big O ของ Binary Search Trees

worse case O(n)

แก้ไข

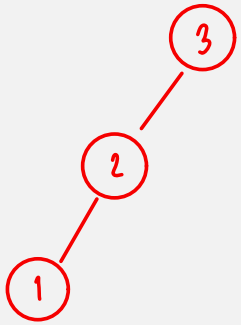
2.1 ต้องทำให้ทรี Perfect Balance Trees : ทำยาก

2.2 AVL Trees

AVL Binary Search Trees ที่มี Balance condition

(ความสูงต่างกันไม่เกิน 1)

- Single Rotation
- Double Rotation

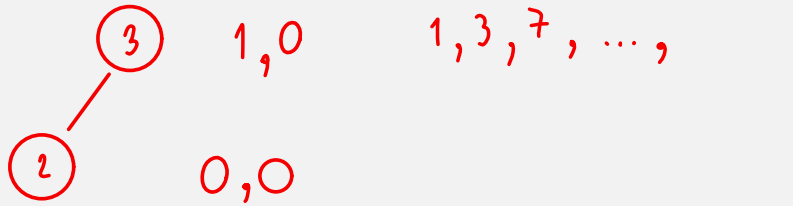


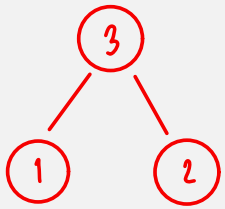
3.4 AVL Trees

A binary search tree with balance condition, it ensures that the depth of the tree is $O(\log_n)$.

Balance condition : Every node must have left and right subtree of the same height. ความสูงเท่ากัน

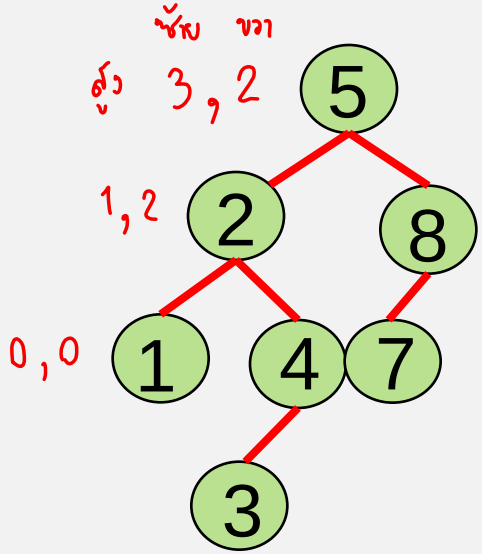
Problem : Only perfect balance trees of 2^k -1 node would satisfy this criterion.

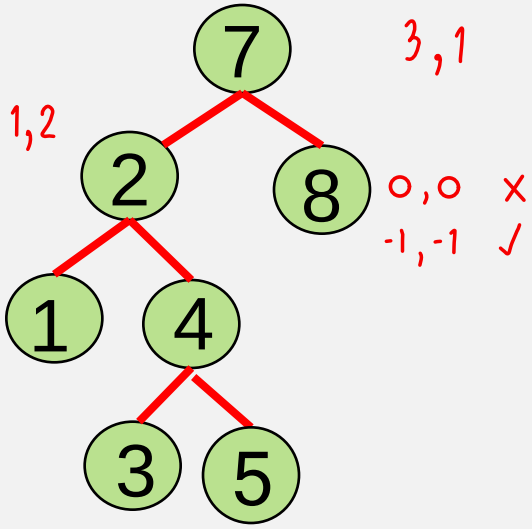


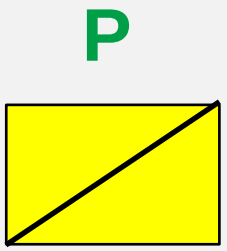


Definition : An AVL(Adelson-Velskii and Landis) tree is identical to a binary search tree, except that for every node in the tree, the height of the left and the right subtree **can differ by at most 1.**

AVL Operation All = $O(\log_N)$ except insertion







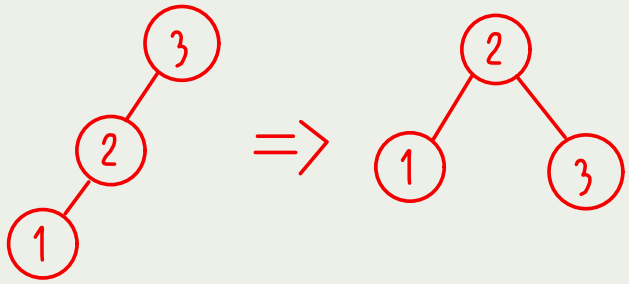
The height of an empty subtree is defined to be -1.

3.4.1 Insertion operation :

1. Update all the balancing information for the nodes on the path back to the root
2. The insertion a node could violate the AVL tree property, then property has to be restored before the insertion step. Called a *rotation*

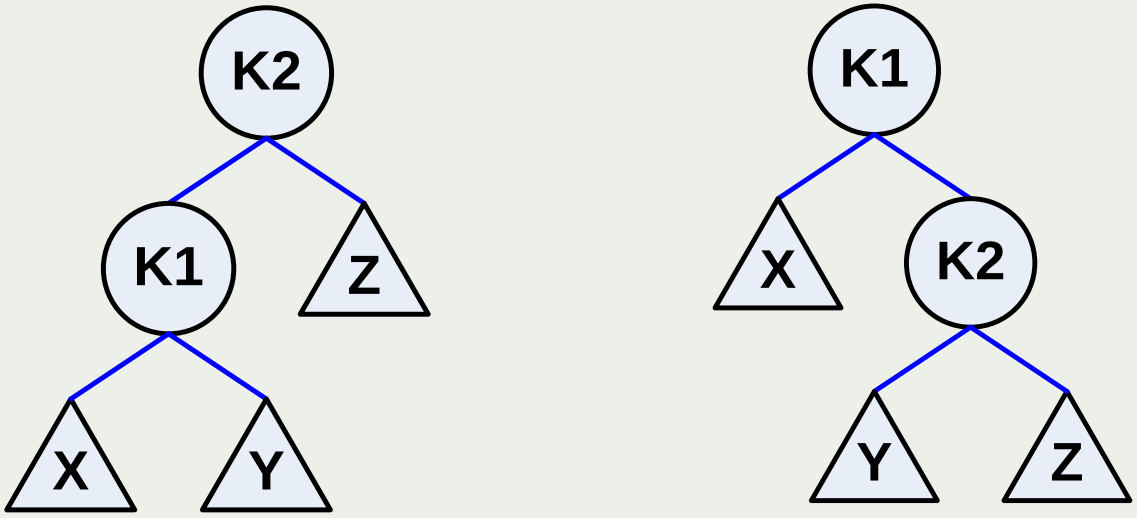
- Single rotation

- Double rotation

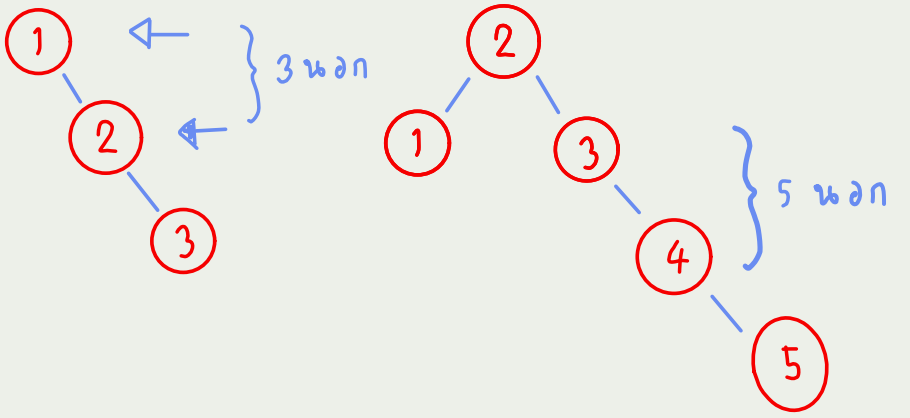


3.4.2 Single Rotation

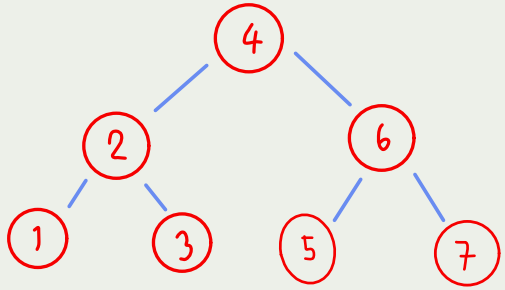
- A rotation involves only a few pointer changes, and changes the structure of the tree while preserving tree property.

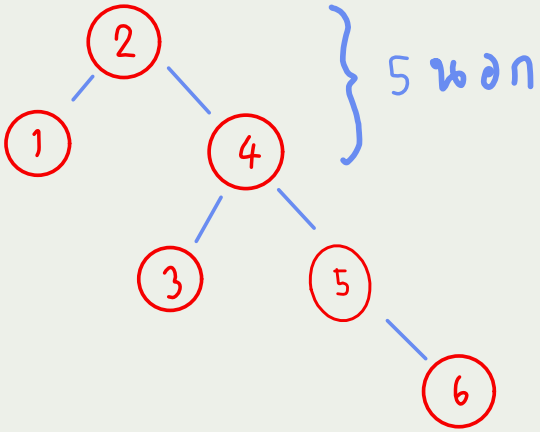


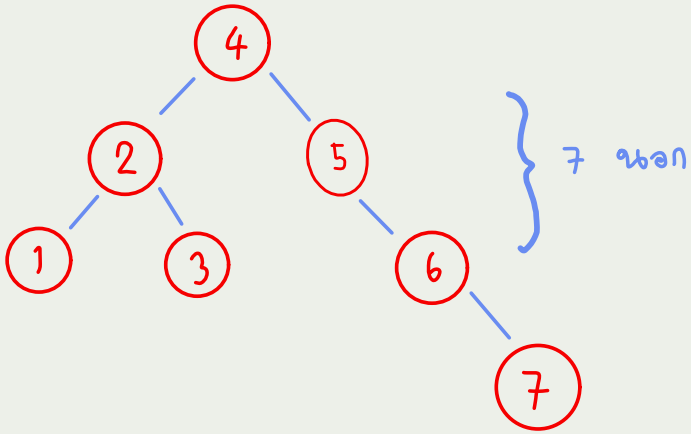
Example insert 1, 2, 3, 4, 5, 6, 7

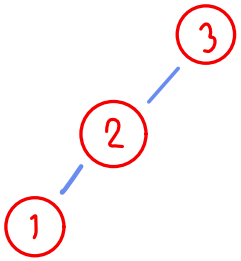


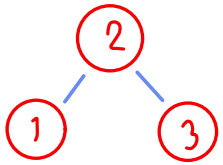
นอกช่วง

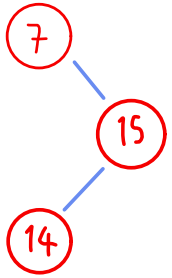


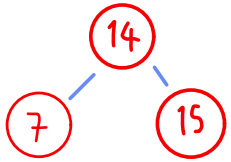




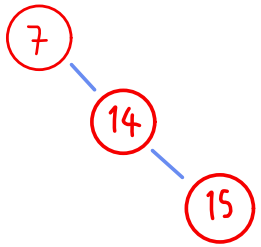


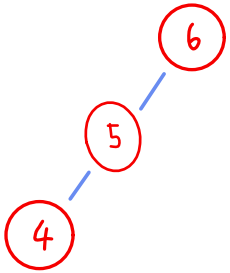


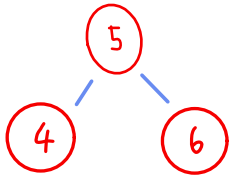




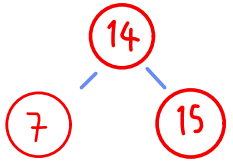
1.

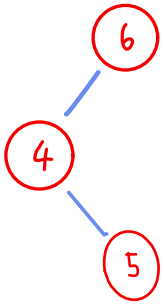


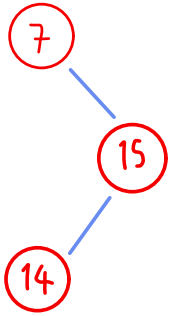




2.



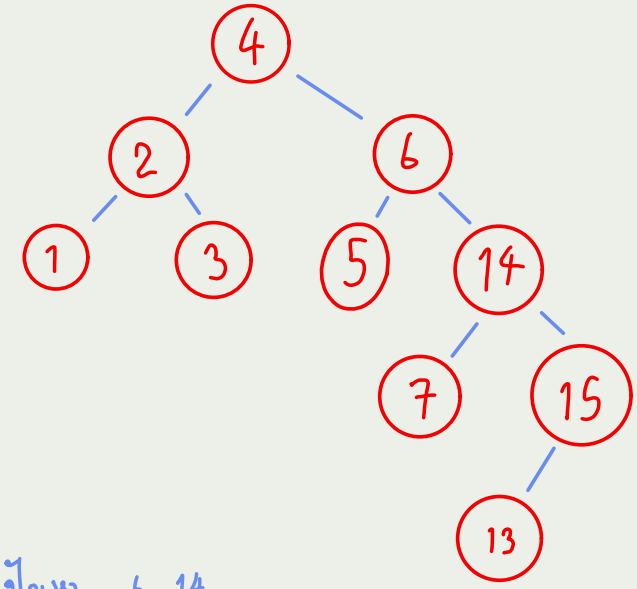




Example insert 15, 14 * นับปัญหา จาก การ Insert

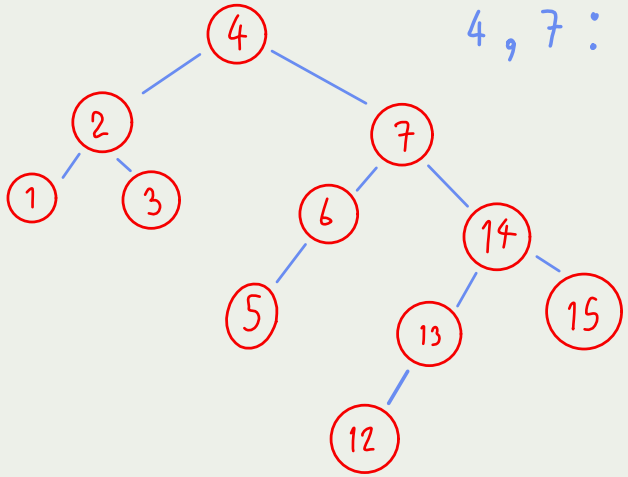
insert 13, 12, 11, 10, 9, 8, 8.5

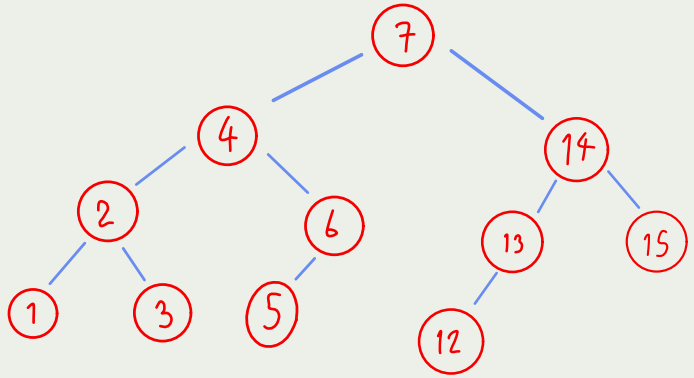
นอกช่วง

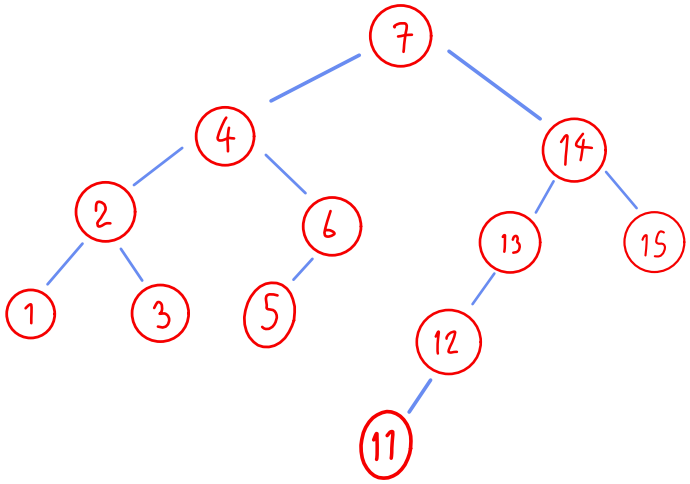


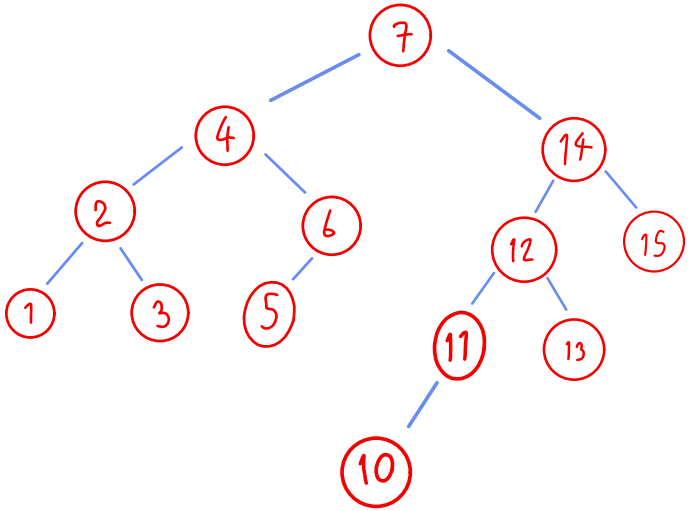
ปัญหา 6, 14
ข้อมูล 13
อยู่ในช่วง ZZ
double rotale

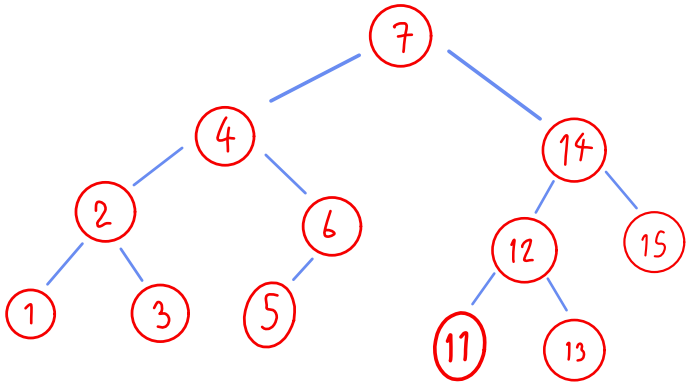
4, 7 : 12

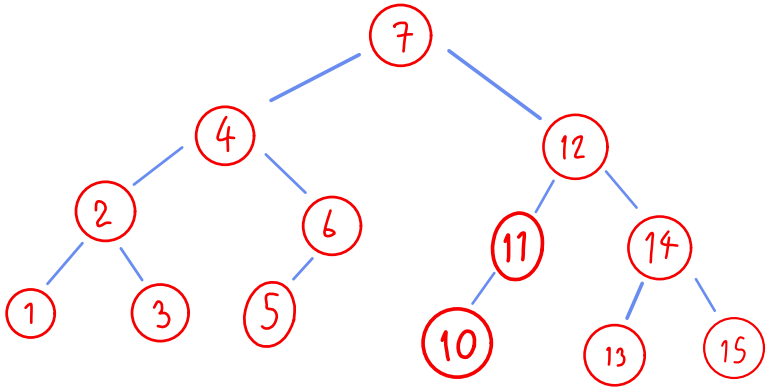





7
4
14
2
6
13
15
1
3
5
12
11

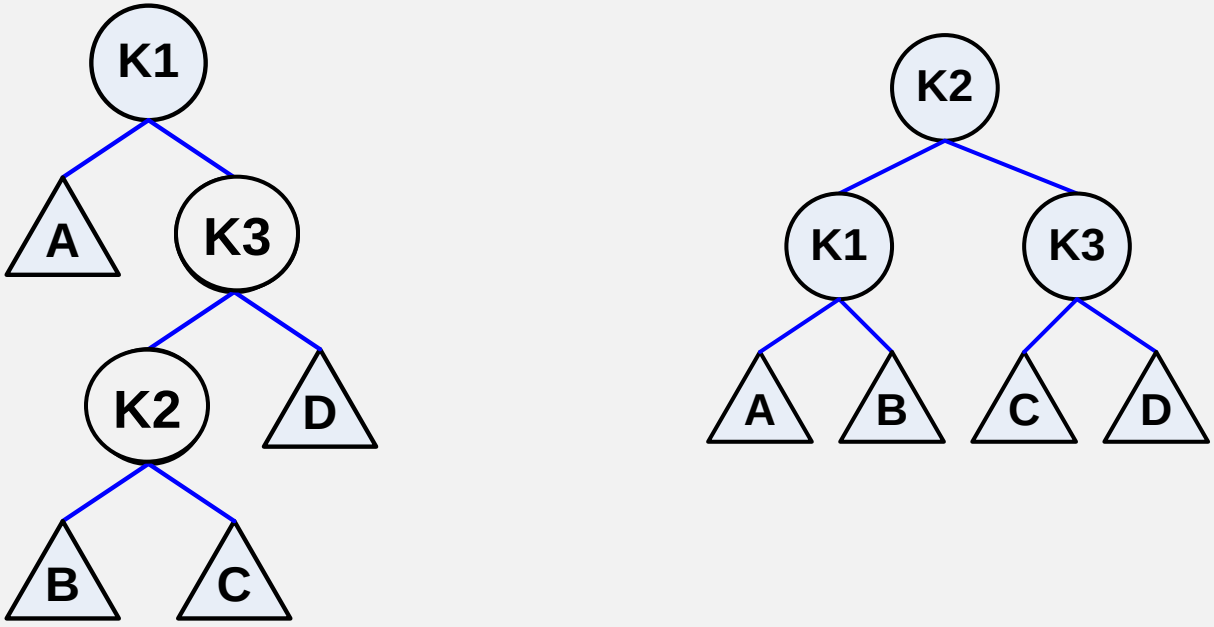

7
4
14
2
6
12
15
1
3
5
11
13
10

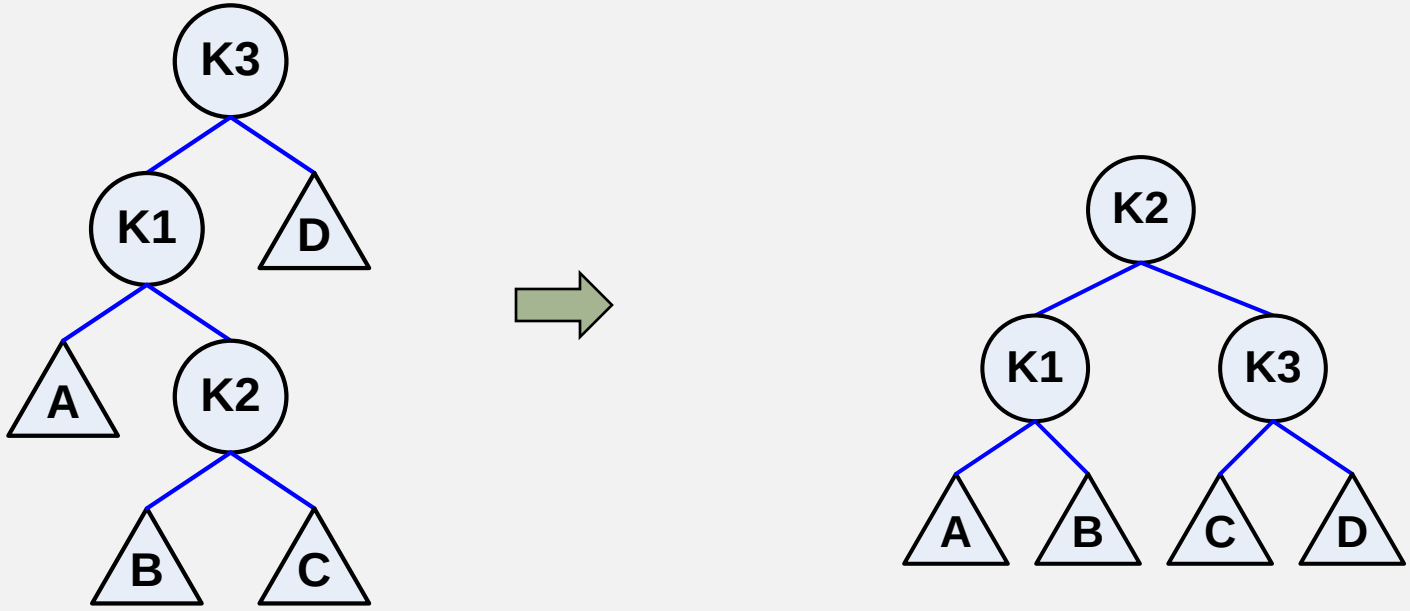

7
4
14
2
6
12
15
1
3
5
11
13


7
4
12
2
6
11
14
1
3
5
10
13
15

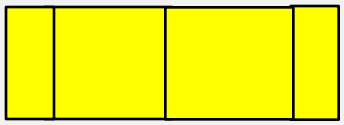
3.4.3 Double rotation

Problem : Single rotation has not fixed the height imbalance by a node inserted into the tree containing the middle elements.

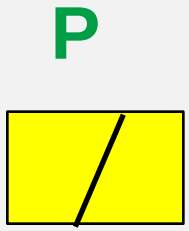


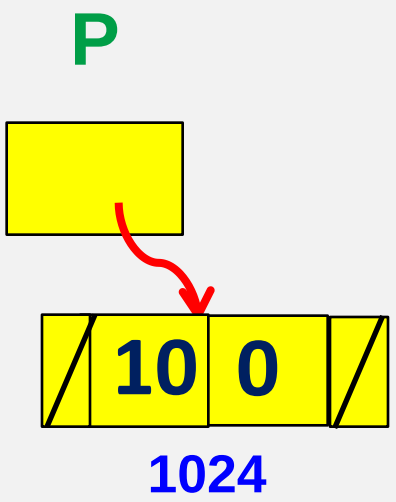

K3
K1
D
A
K2
B
C
K2
K1
K3
A
B
C
D

```
struct node
{     int value;
      int height;
      struct node *left;
      struct node *right;
};
```



```
1. int fheight( struct node *P)
2. {   if ( P==NULL)
3.        return  -1;
4.     else
5.        return P->height;
   }
```





Code Insert

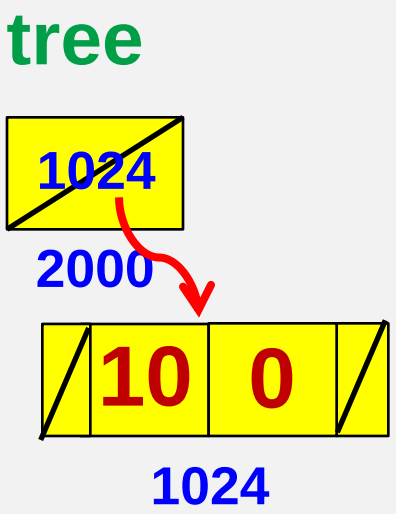
```
ถ้า tree == NULL
    สร้าง node ใหม่ ใส่ค่า

else
    ถ้า x น้อยกว่า tree->value  ให้ recursive ลงไปทางซ้าย
        tree->left = insert (tree->left)
    ถ้าความสูงของทรีทางซ้ายและทางขวาต่างกันเกิน 2
            ถ้า x น้อยกว่า tree->left->value
                single rotation  (อยู่นอกช่วง)
            else
                double rotation (อยู่ในช่วง)

update ความสูง
return tree
```

เมื่อ insert แล้ว return กลับมาเช็คความสูง

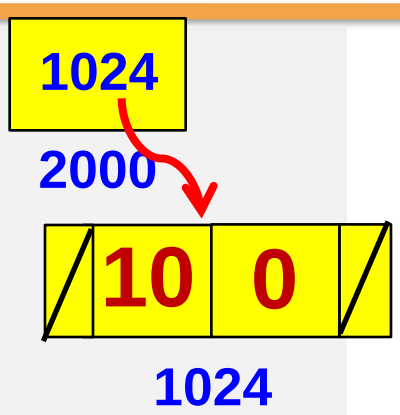
```
struct node *insert(int x, struct node *T)
1{     if(T == NULL)
2      {   T=new struct node;
3          T->value=x;
4          T->left=T->right=NULL;
5          T->height=0;
6      }
       else
```



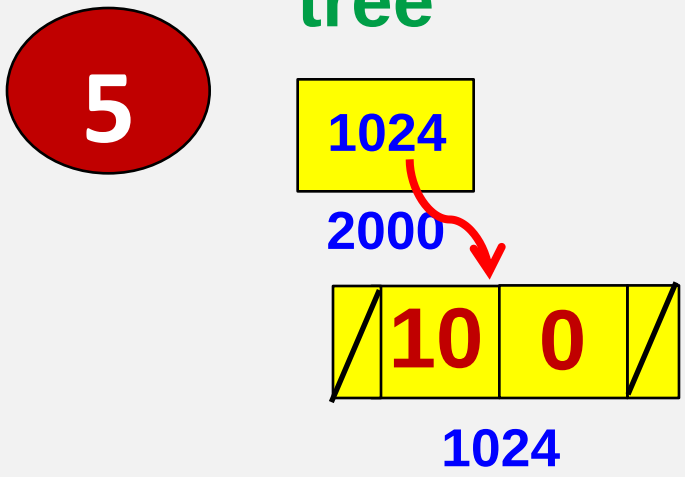
```
8 if(x < T->value )
9       {   T->left = insert(x,T->left);
10           if(fheight(T->left) - fheight(T->right) == 2)
11               if( x < T->left->value)
12                  T=srleft(T);
13               else
14                   T=drleft(T);
15       }
16   else
17   ถึง 29 if(x > T->value )
         {  ข้างขวาบ้าง  ให้เขียนเอง ... }
```

max(-1,-1) = -1 +1

```
30   T->height = max(fheight(T->left), fheight(T->right)) + 1;
31   return T;
 }
```



```
struct node *insert(int x, struct node *T)
1{    if(T == NULL)
2       {    T=new struct node;
3            T->value=x;
4            T->left=T->right=NULL;
5            T->height=0;
       }
7      else
```

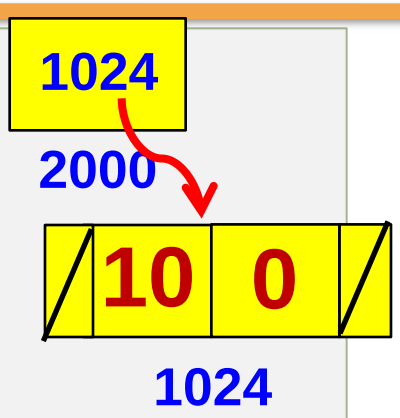


ถ้า x น้อยกว่า tree->value
ให้ recursive ลงไปทางซ้าย

5, NULL

5

tree




```
8 if(x < T->value )
9       {   T->left = insert(x,T->left);
10                                  ght(T->right) == 2)
11                                  )
12                T=srleft(T);
13              else
14                 T=drleft(T);
15      }

16   else
17   ถึง 29 if(x > T->value )
          {  ข้างขวาบ้าง   ให้เขียนเอง ... }
30  T->height = max(fheight(T->left), fheight(T->right)) + 1;
31  return T;
 }
```

ค้างบรรทัด 9 1024

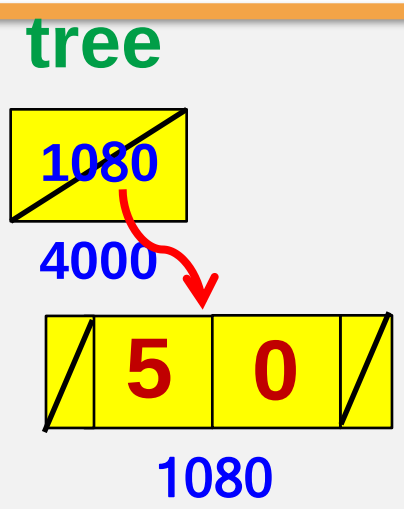
Recursive 1

5, NULL

```
struct node *insert(int x, struct node *T)
1{     if(T == NULL)
2      {    T=new struct node;
3           T->value=x;
4           T->left=T->right=NULL;
5           T->height=0;
       }
7    else
            .....
30   T->height = max(fheight(T->left), fheight(T->right)) + 1;
31    return T;
 }
```

max(-1,-1) = -1 +1

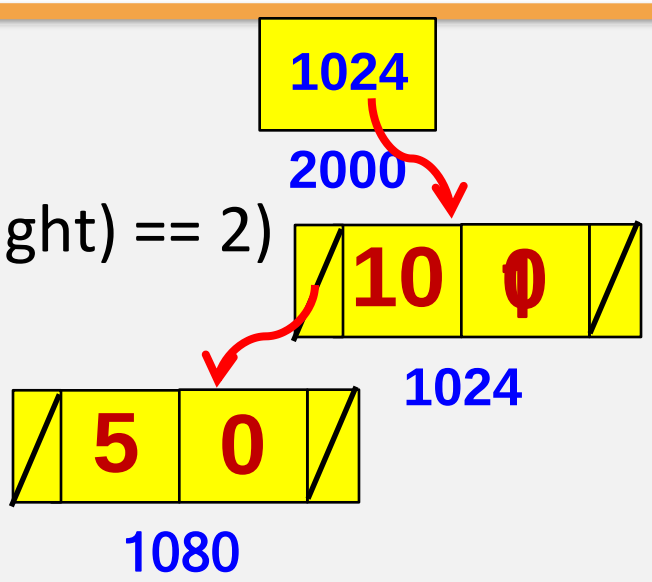
1080



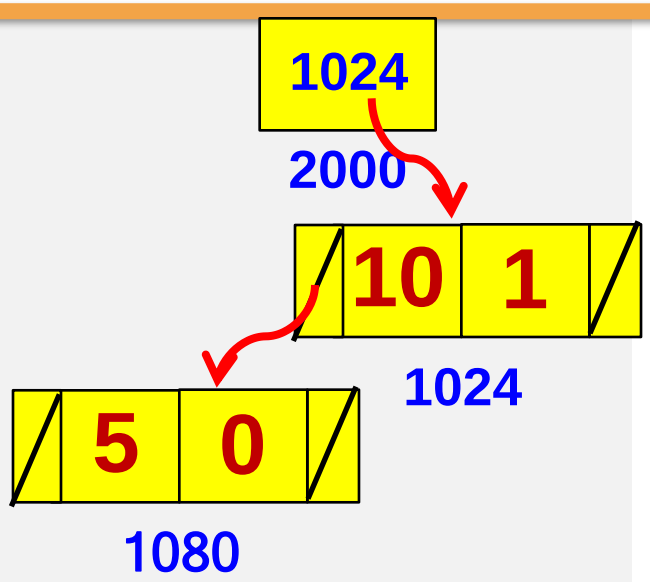
ค้างบรรทัด 9 1024

1080

```
8 if(x < T->value )
9       {   T->left = insert(x,T->left);
10           if(fheight(T->left) - fheight(T->right) == 2)
11               if( x < T->left->value)
12                  T=srleft(T);
13               else
14                   T=drleft(T);
15       }
16        29 if(x > T->value )
          { ข้างขวาบ้าง ให้เขียนเอง … }
30   T->height = max(fheight(T->left), fheight(T->right)) + 1;
31   return T;
 }
```



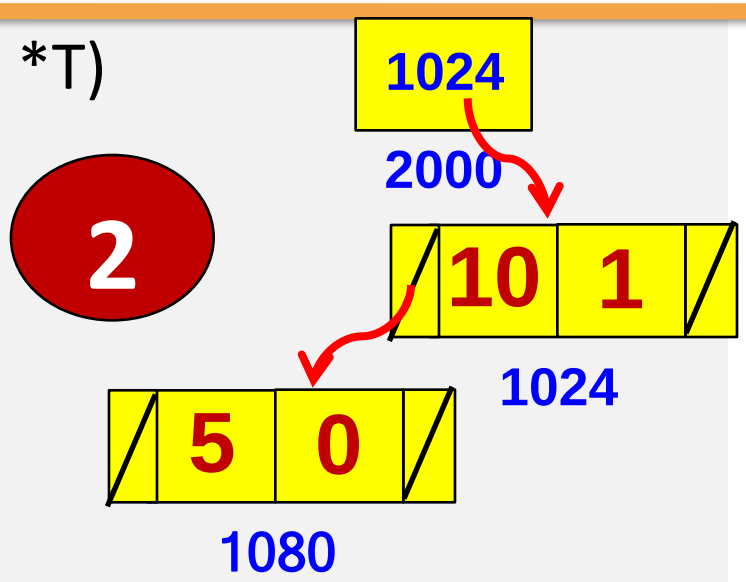
```
main()
{
    ……
    tree=insert(2,tree);
```



จาก main

2, 1024

```
struct node *insert(int x, struct node *T)
1{     if(T == NULL)
2      {   T=new struct node;
3          T->value=x;
4          T->left=T->right=NULL;
5          T->height=0;
       }
7   else
           .....
```

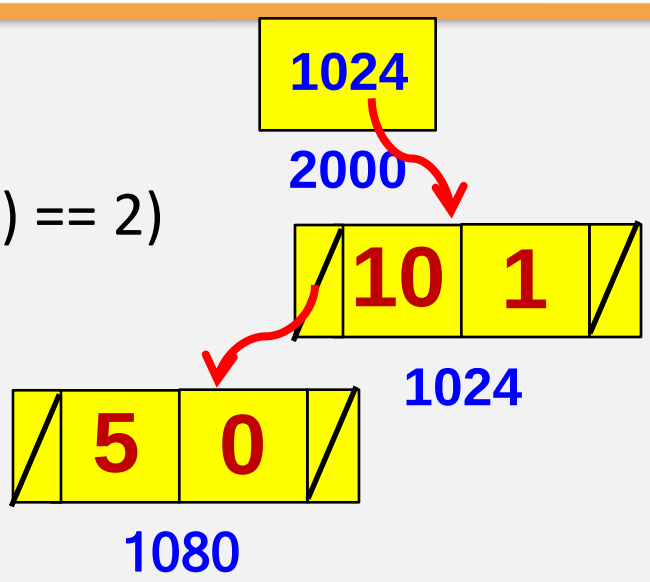


ถ้า x น้อยกว่า tree->value
ให้ recursive ลงไปทางซ้าย

2, 1080

A ค้างบรรทัด 9 1024

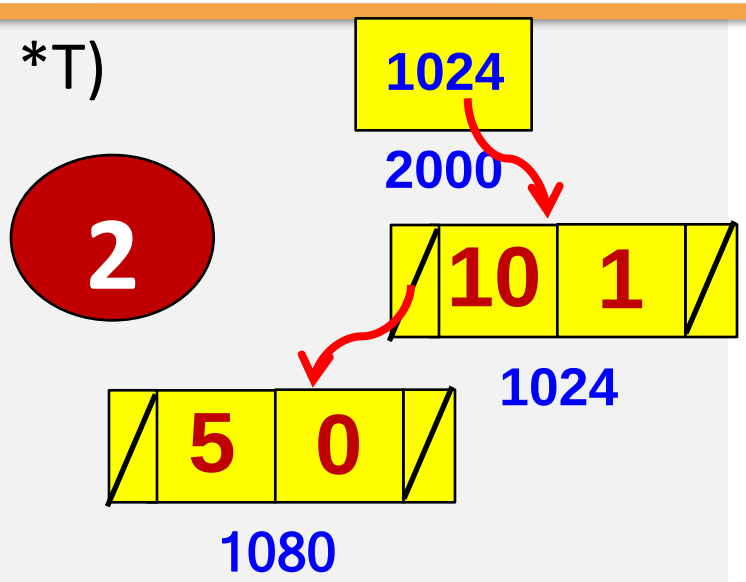
```
8 if(x < T->value )
9       {   T->left = insert(x,T->left);
10          if(fheight(T->left) - fheight(T->right) == 2)
11              {
12                  T=srleft(T);
13              else
14                  T=drleft(T);
15      }
16  ถึง 29 if(x > T->value )
           {  ข้างขวาบ้าง  ให้เขียนเอง … }
30  T->height = max(fheight(T->left), fheight(T->right)) + 1;
31    return T;
 }
```



Recursive 1

2, 1080

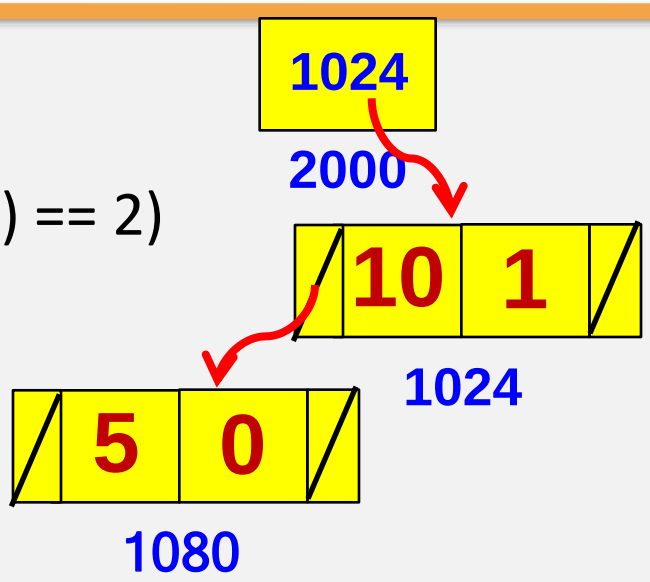
```
struct node *insert(int x, struct node *T)
1{     if(T == NULL)
2      {   T=new struct node;
3          T->value=x;
4          T->left=T->right=NULL;
5          T->height=0;
       }
7   else
           .....
```



2, NULL

```
8 if(x < T->value )
9       {   T->left = insert(x,T->left);
10          if(fheight(T->left) - fheight(T->right) == 2)
11              {
12                  T=srleft(T);
13              else
14                  T=drleft(T);
15      }
16  ถึง 29 if(x > T->value )
          {  ข้างขวาบ้าง  ให้เขียนเอง … }
30  T->height = max(fheight(T->left), fheight(T->right)) + 1;
31    return T;
 }
```

B ค้างบรรทัด 9 1080



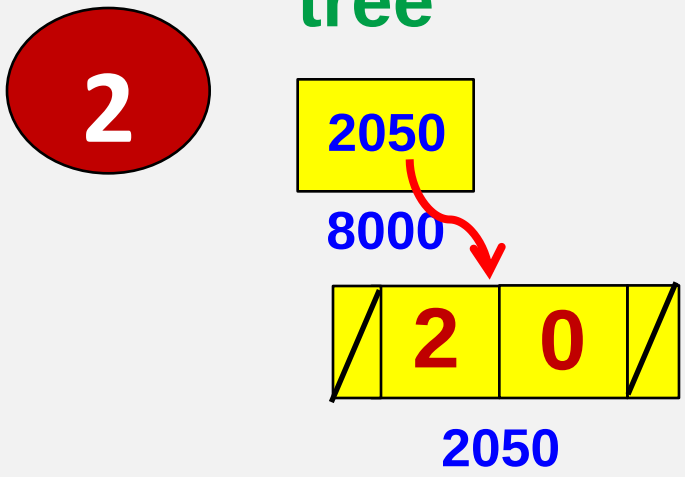
Recursive 2

2, NULL

```
struct node *insert(int x, struct node *T)
1{     if(T == NULL)
2      {    T=new struct node;
3           T->value=x;
4           T->left=T->right=NULL;
5           T->height=0;
       }
7    else
            .....

30   T->height = max(fheight(T->left), fheight(T->right)) + 1;
31     return T;
 }
```

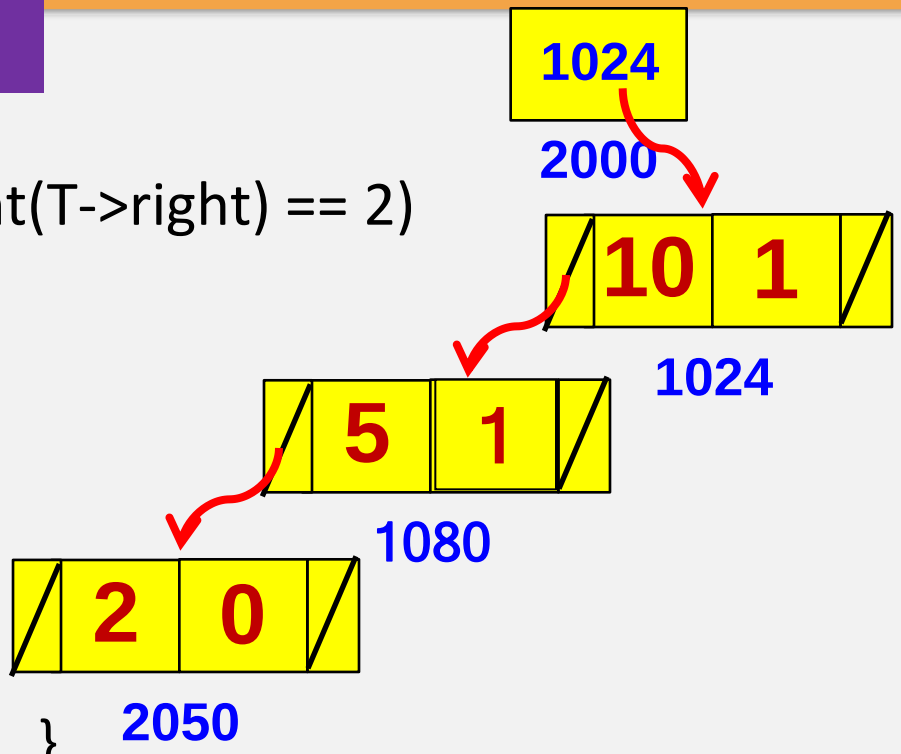
max(-1,-1) = -1 **+1**



กลับไปที่ B ค้างบรรทัด 9 1080

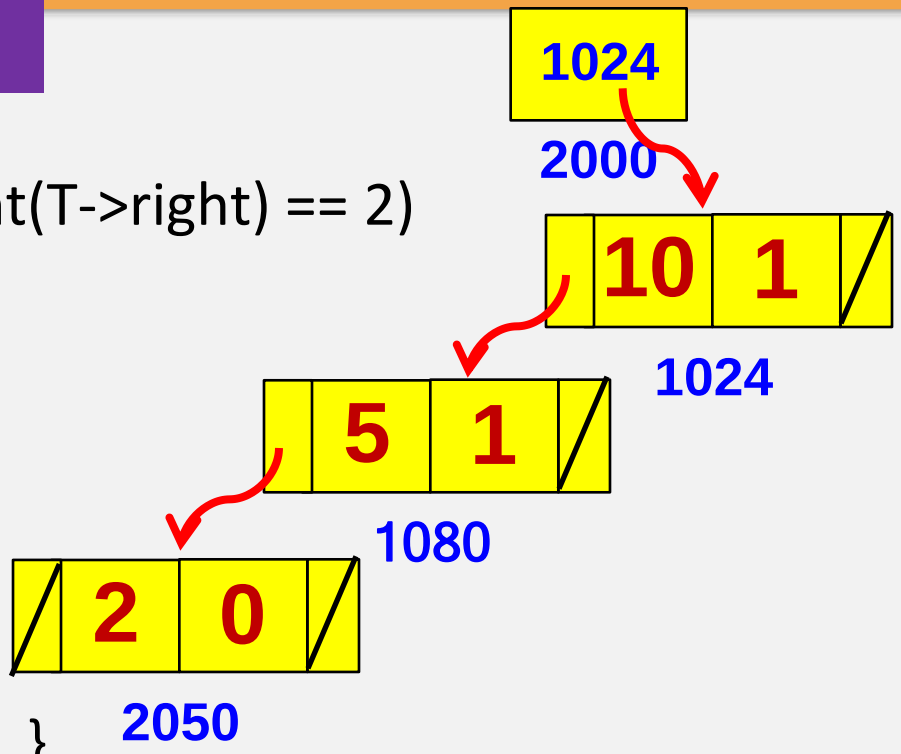
2050

```
8 if(x < T->value )
9       {   T->left = insert(x,T->left);
10           if(fheight(T->left) - fheight(T->right) == 2)
11               if( x < T->left->value)
12                  T=srleft(T);
13               else
14                   T=drleft(T);
15      }
16  ถึง 29 if(x > T->value )
          {  ข้างขวาบ้าง  ให้เขียนเอง ... }
30  T->height = max(fheight(T->left), fheight(T->right)) + 1;
31    return T;
 }
```

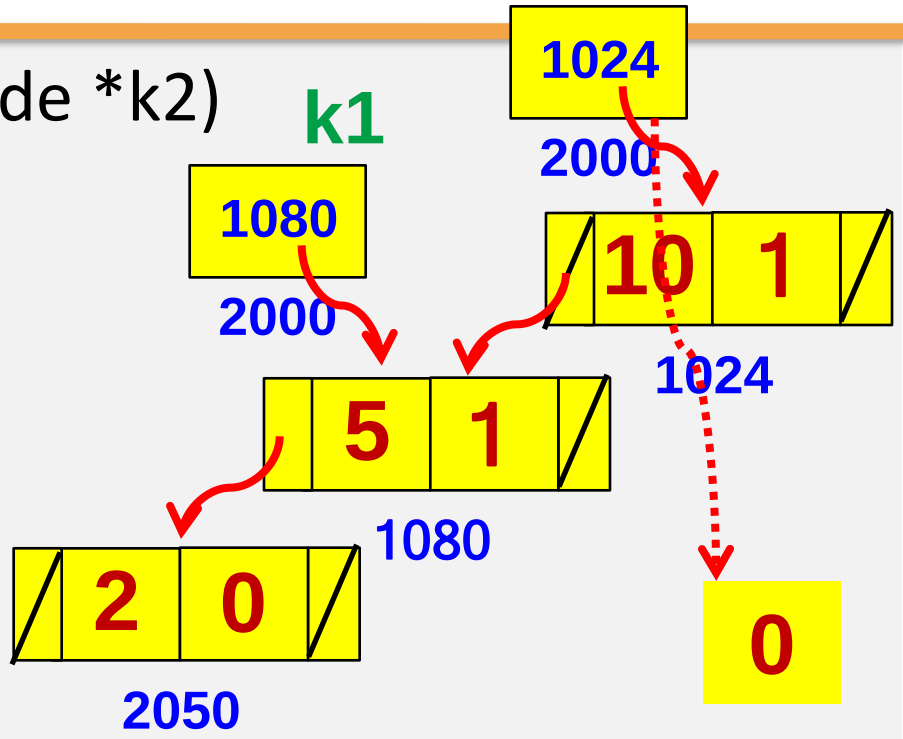


1080

```
8 if(x < T->value )
9       {   T->left = insert(x,T->left);
10           if(fheight(T->left) - fheight(T->right) == 2)
11               if( x < T->left->value)
12                  T=srleft(T);
13               else
14                   T=drleft(T);
15       }
16ถึง 29 if(x > T->value )
          {  ข้างขวาบ้าง  ให้เขียนเอง ... }
30   T->height = max(fheight(T->left), fheight(T->right)) + 1;
31    return T;
 }
```



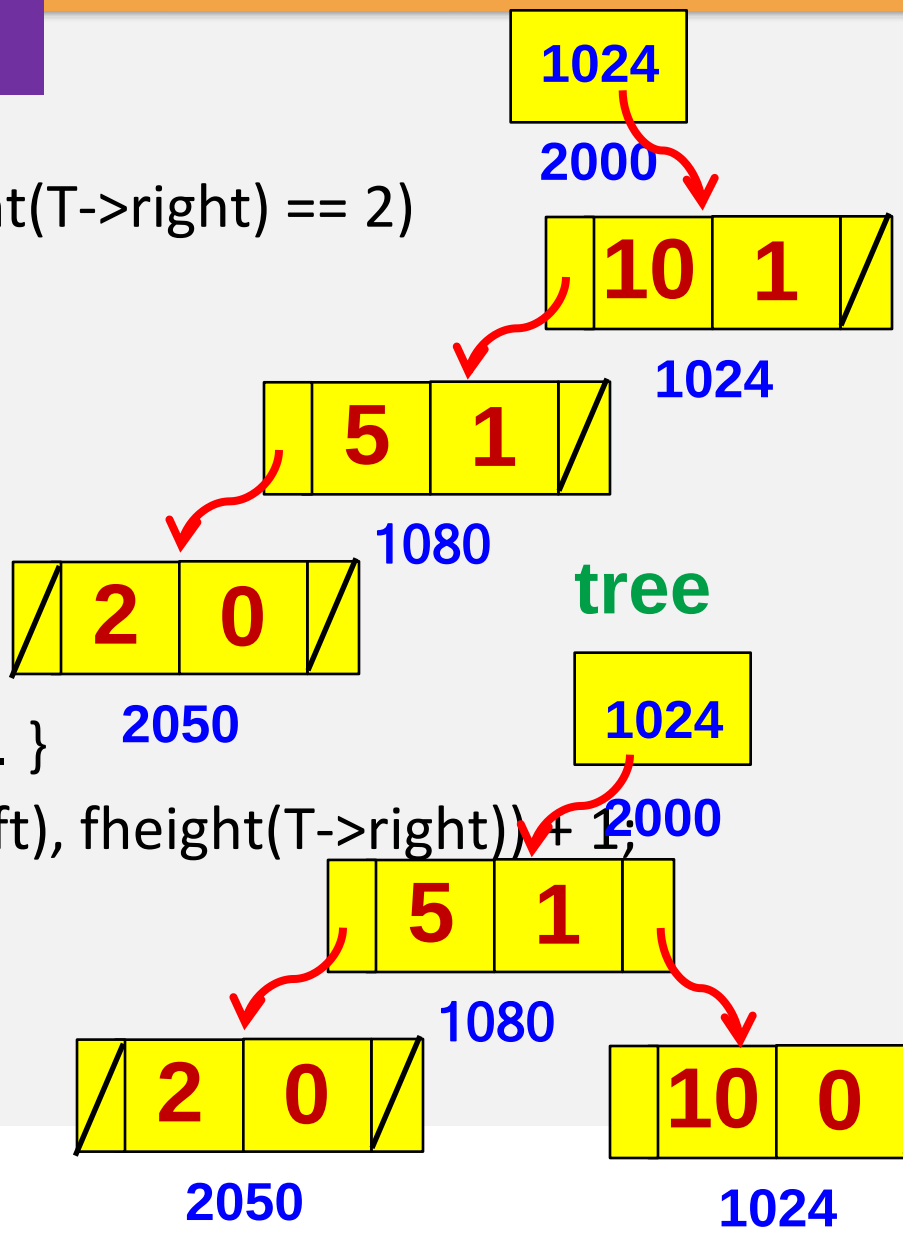
```
struct node *srleft(struct node *k2)
{    struct node *k1;
     k1=k2->left;
     k2->left = k1->right;
     k1->right = k2;
```




```
     k2->height = max(fheight(k2->left), fheight(k2->right)) + 1;
     k1->height = max(fheight(k1->left), k2->height) + 1;
     return k1;
}
```

1080

```
8 if(x < T->value )
9       {   T->left = insert(x,T->left);
10          if(fheight(T->left) - fheight(T->right) == 2)
11              if( x < T->left->value)
12                 T=srleft(T);
13              else
14                 T=drleft(T);
15      }
16ถึง 29 if(x > T->value )
          { ข้างขวาบ้าง ให้เขียนเอง ... }
30   T->height = max(fheight(T->left), fheight(T->right)) + 1;
31    return T;
 }
```

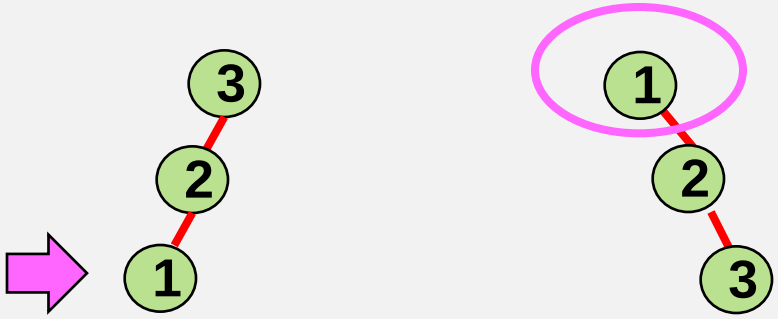


```
struct node *drleft(struct node *k3)
{    k3->left = srright(k3->left);
     return srleft(k3);
}
```

มีทรีอยู่แล้ว
แต่ไม่ Balance

3.5 Splay Trees

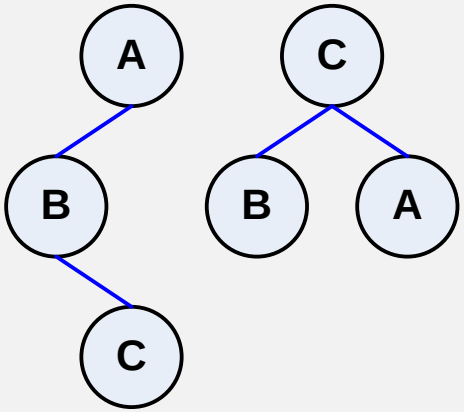
- O(N) worst-case time per operation for binary search trees.
- After a node is accessed, it is pushed to the root



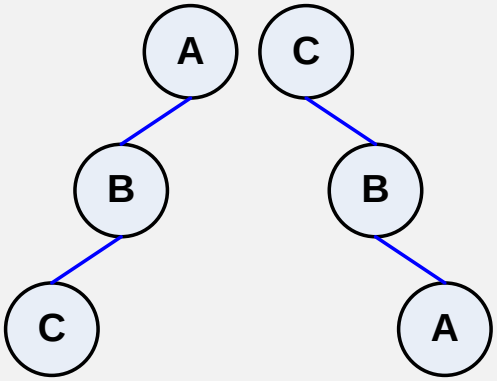
Rotation idea

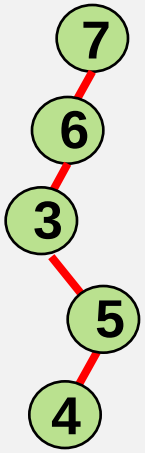
Transform

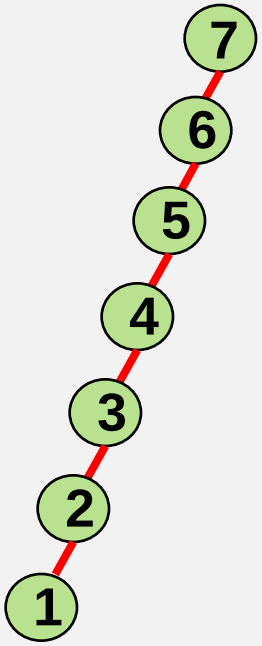
1. zig-zag



2. zig-zig



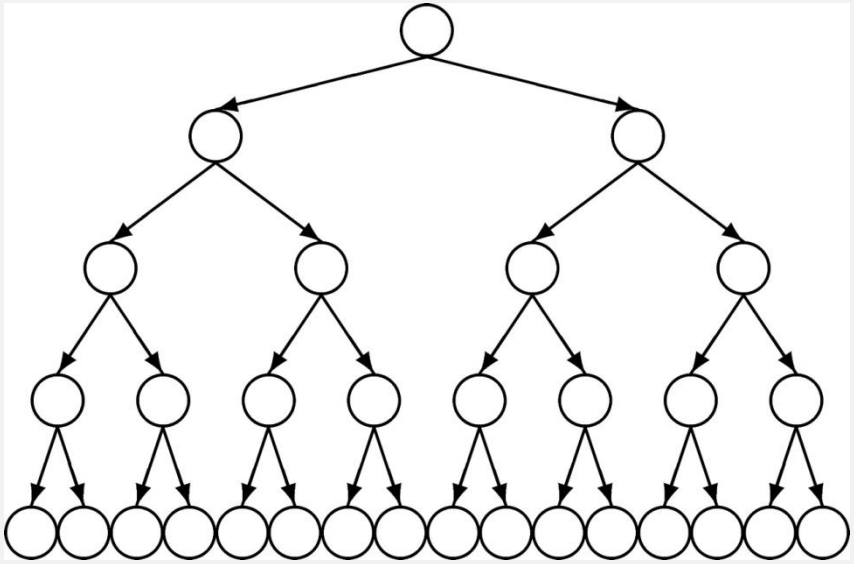

7
6
3
5
4


7
6
5
4
3
2
1

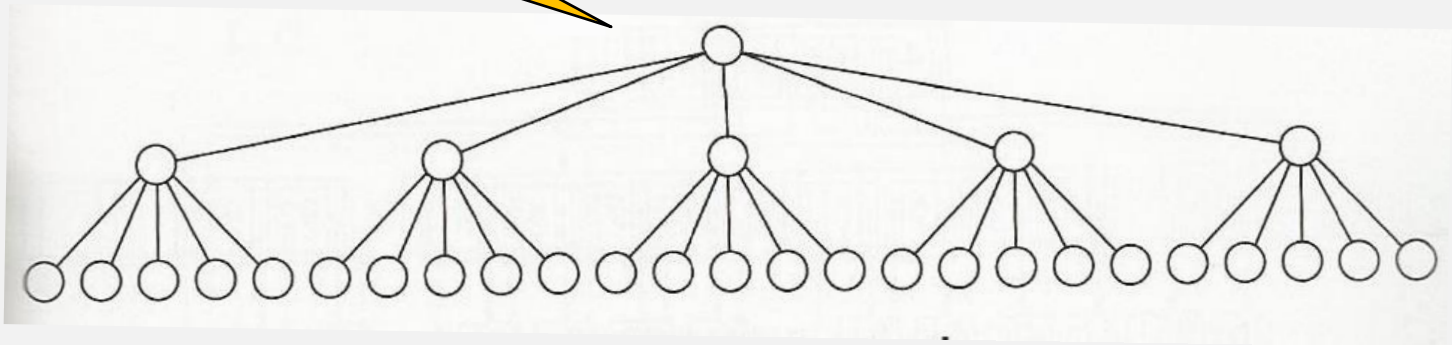
3.6 B-Trees

Complete binary

Tree 31 nodes



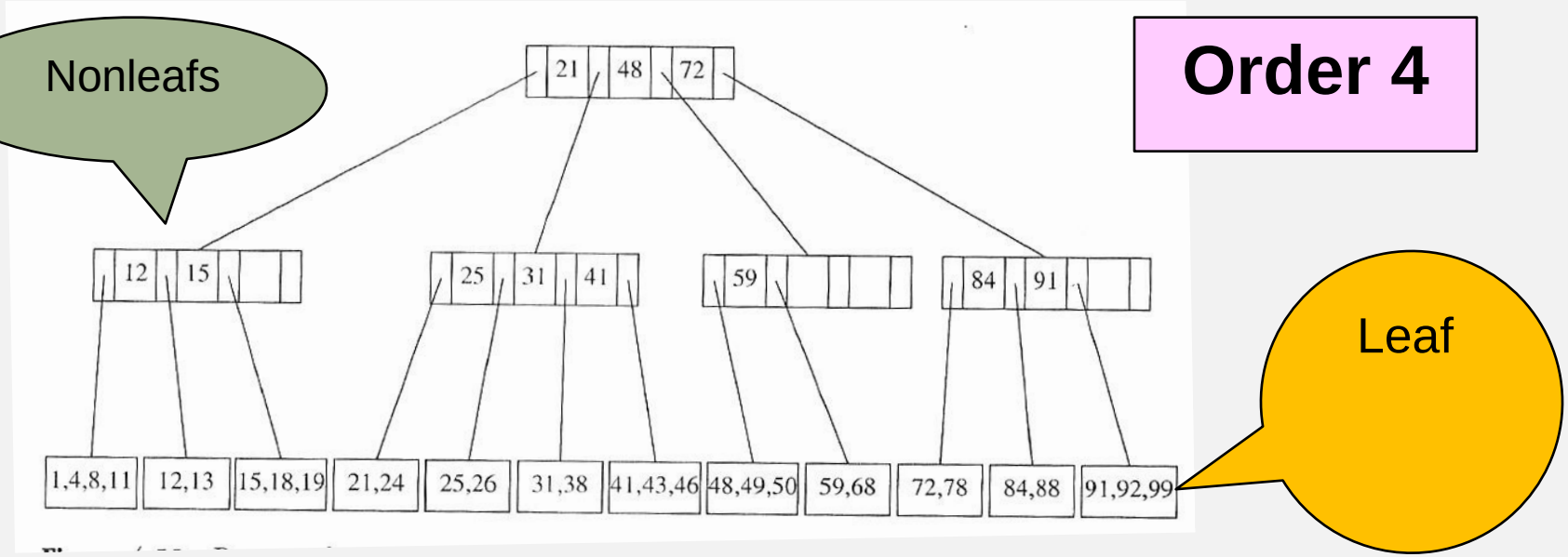
สร้างทรีรูปแบบใหม่ ให้มีคุณสมบัติตามที่เราต้องการ ที่ไม่ใช่ Binary Search, AVL



3.6 B-Trees

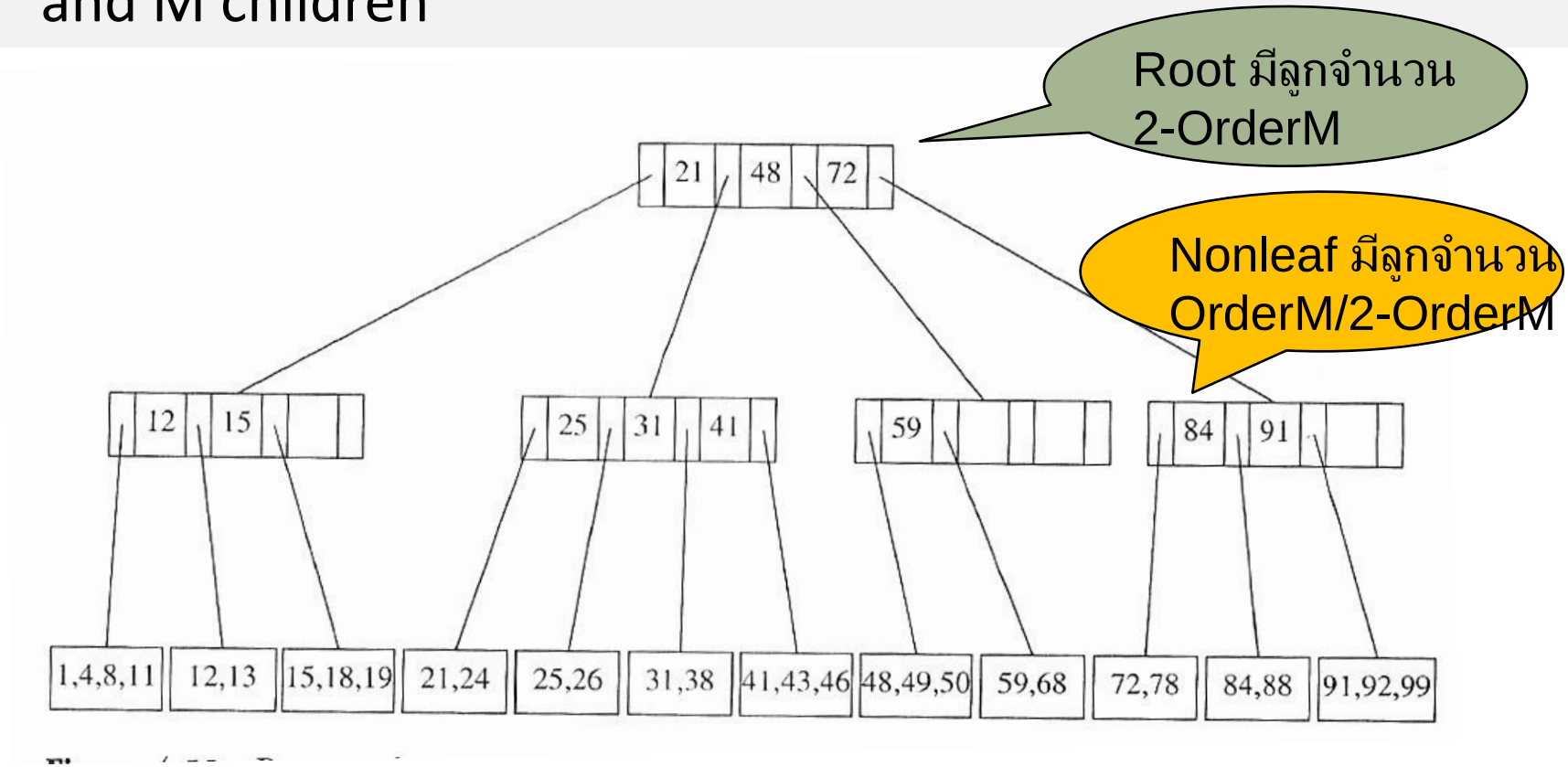
A B-tree of ***order m*** is a tree with the following structural properties:

1) The data items are stores at leaves.
2) The nonleafs nodes store up to M-1 keys to guide the searching; key i represents the smallest key in subtrees i+1

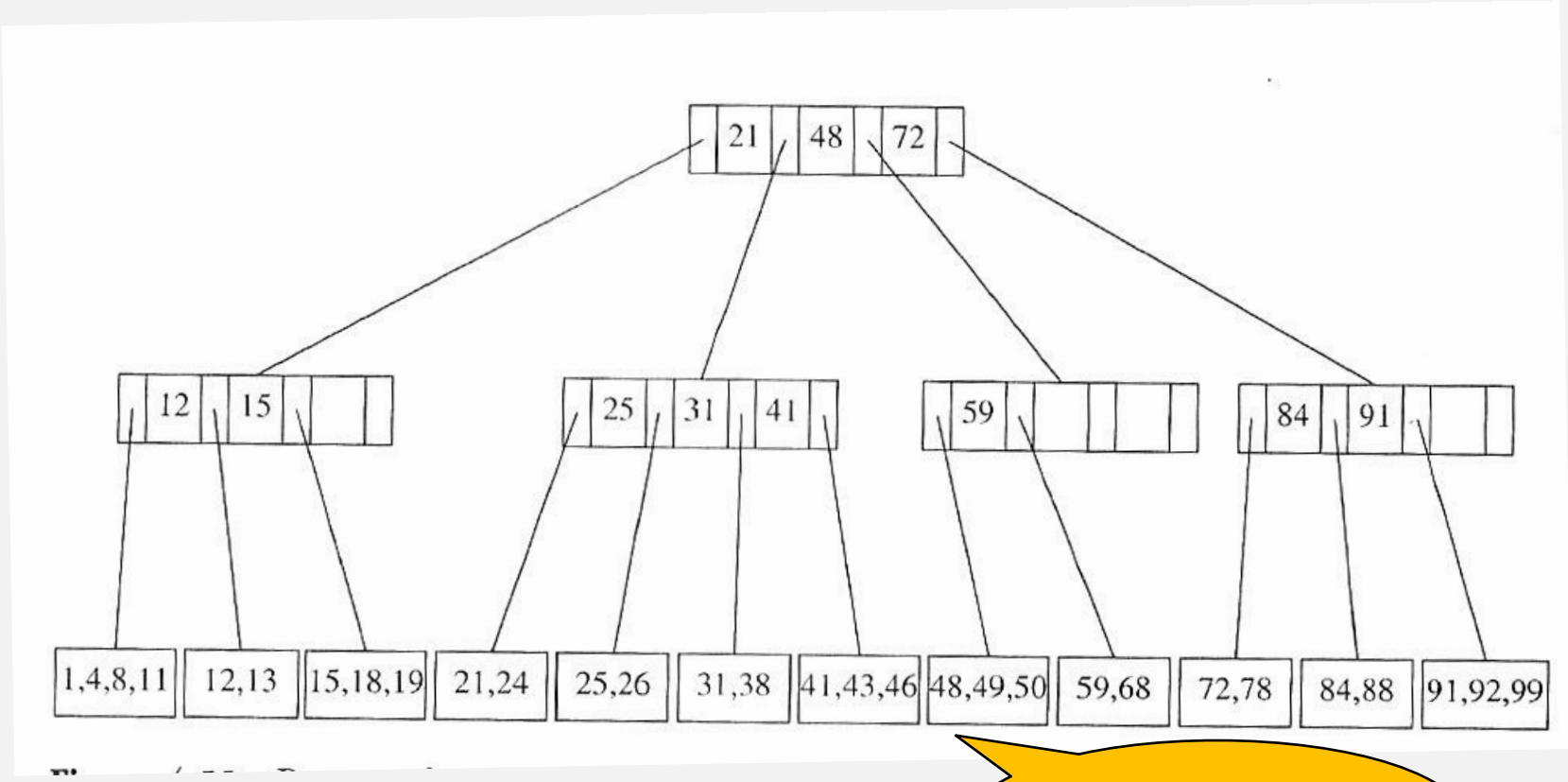


3) The root is either a leaf or has between two and M children.

4) All nonleaf nodes(Except the root) have between ⌈M/2⌉ and M children



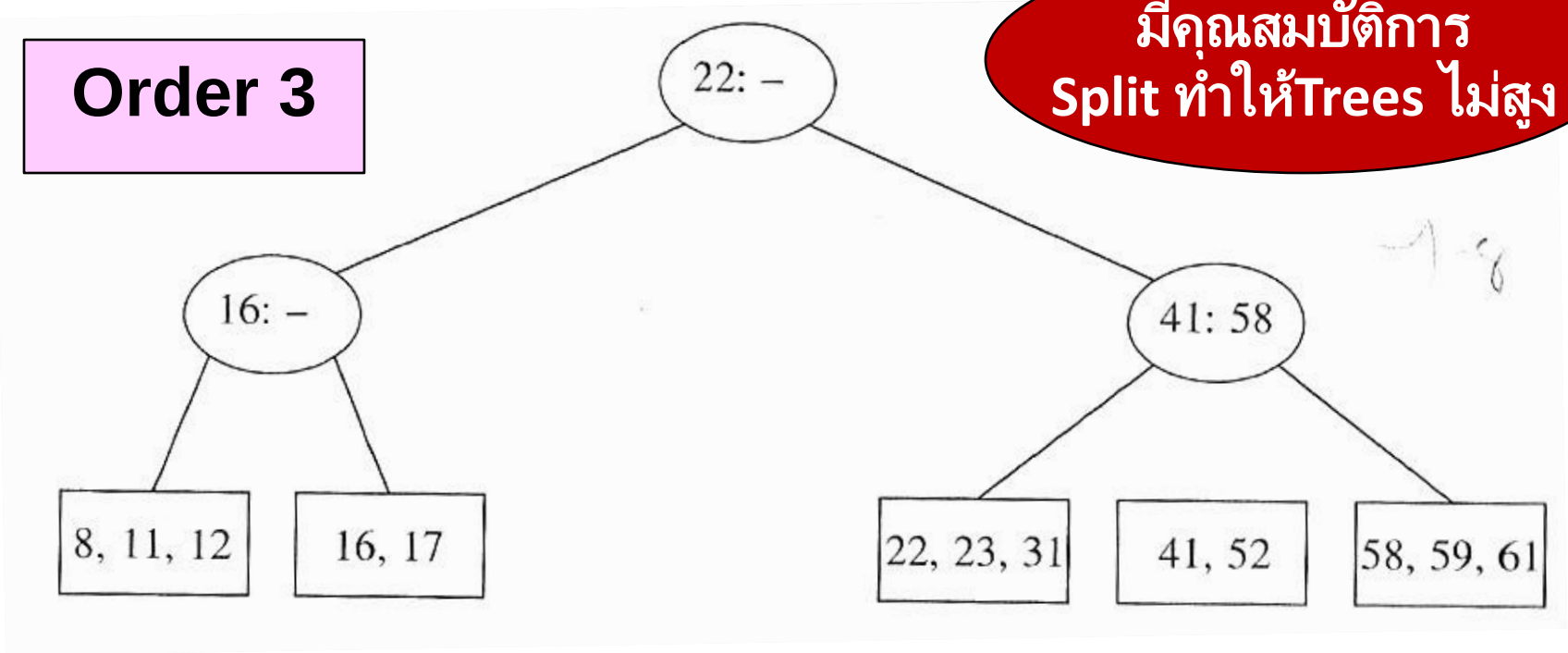
5) All leaves are at the same depth and have between ⌈L/2⌉ and L children.



leaf มีลูกจำนวน L/2-OrderM

Order 3

มีคุณสมบัติการ Split ทำให้Trees ไม่สูง



Insert 18, 1, 19, 28

Order 5

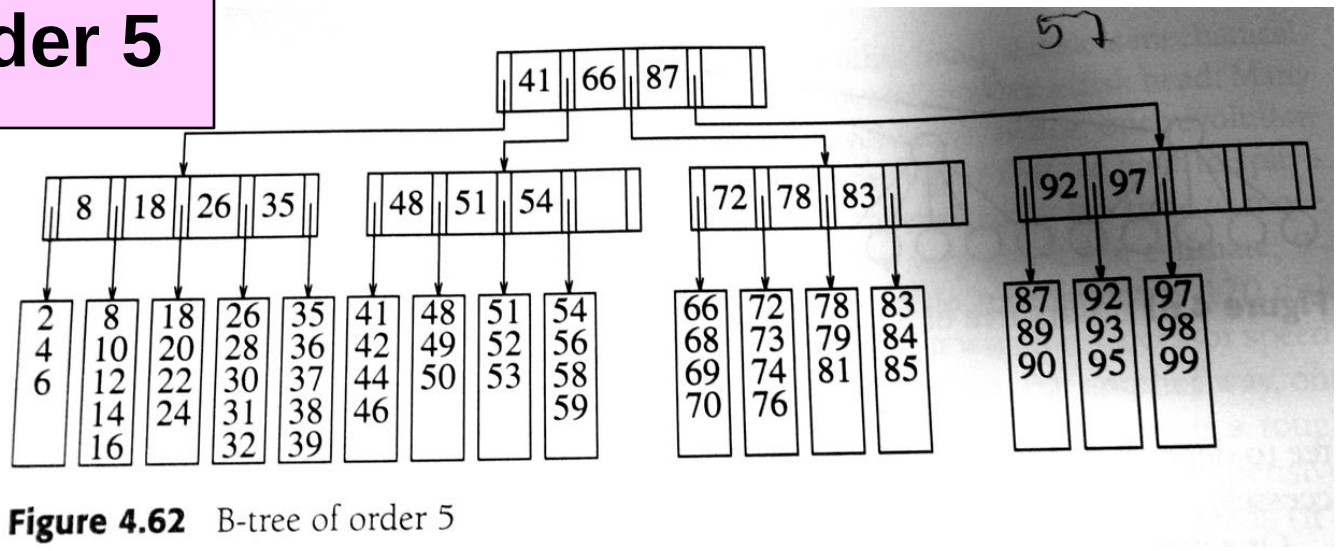
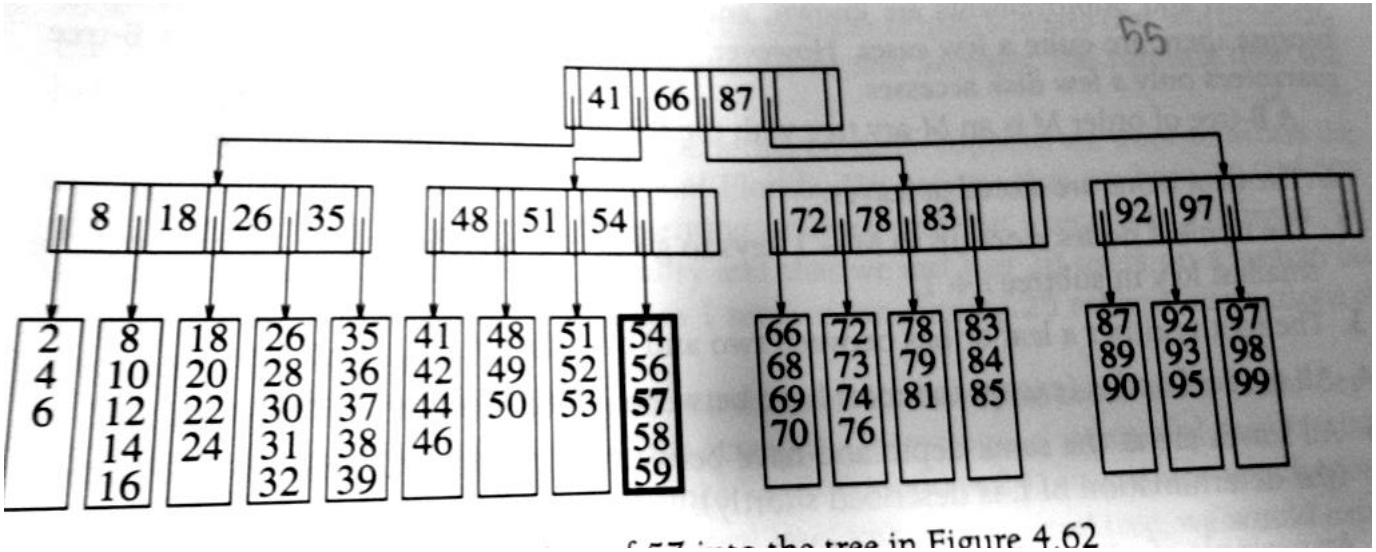
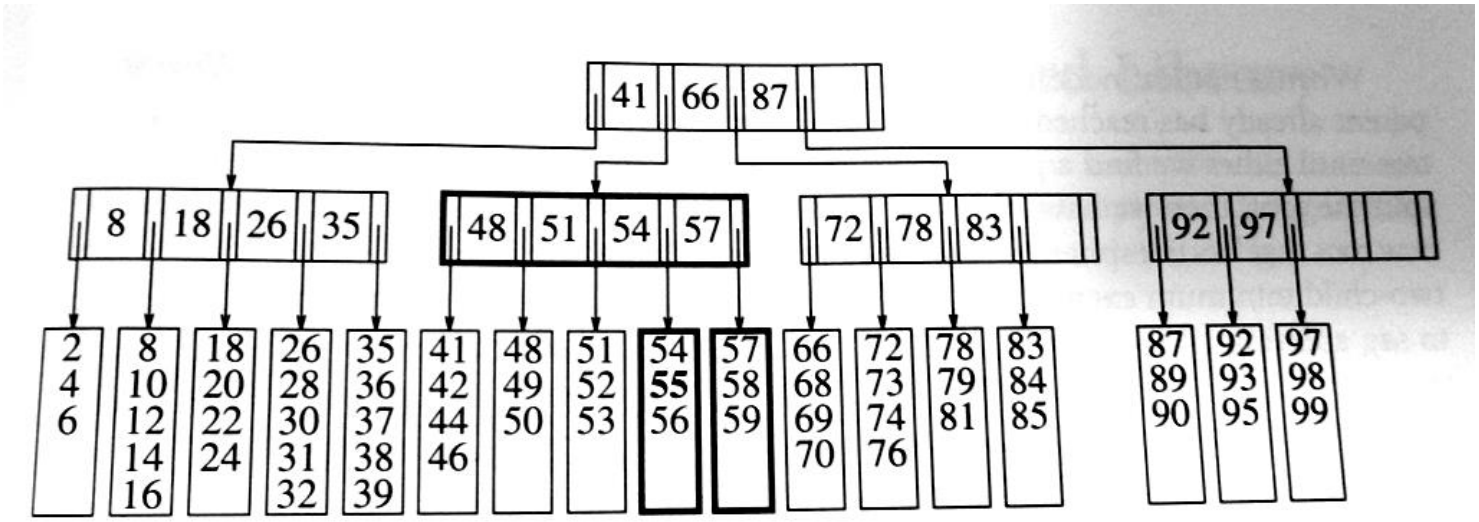
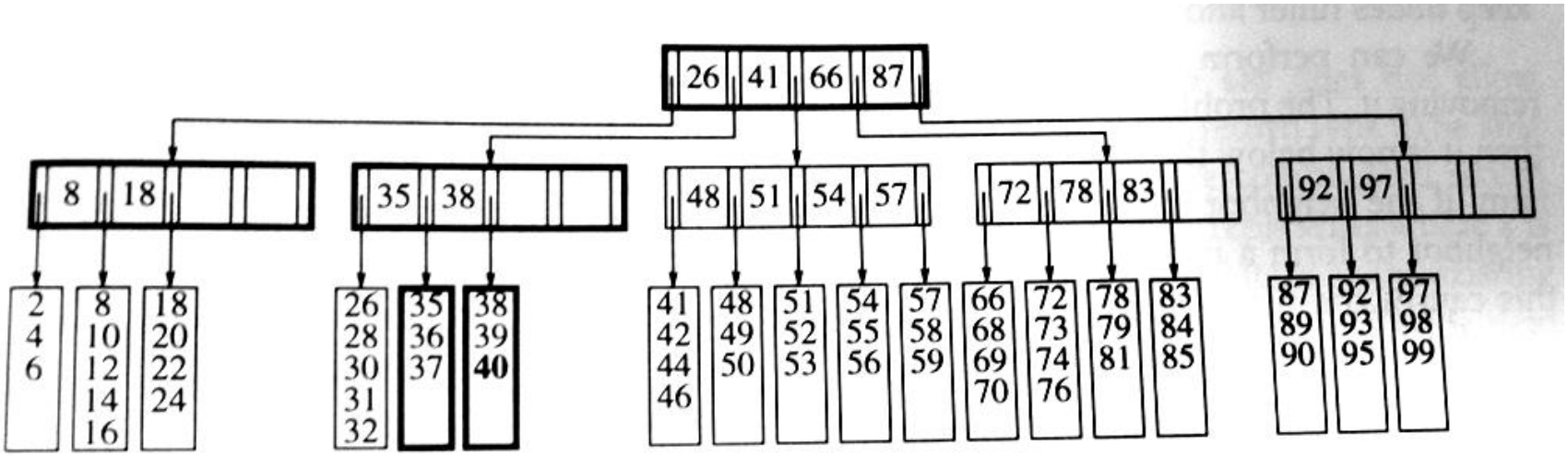


Figure 4.62 B-tree of order 5




41 66 87
8 18 26 35
48 51 54 57
72 78 83
92 97
2 4 6
8 10 12 14 16
18 20 22 24
26 28 30 31 32
35 36 37 38 39
41 42 44 46
48 49 50
51 52 53
54 55 56
57 58 59
66 68 69 70
72 73 74 76
78 79 81
83 84 85
87 89 90
92 93 95
97 98 99


26 41 66 87
8 18
35 38
48 51 54 57
72 78 83
92 97
2 4 6
8 10 12 14 16
18 20 22 24
26 28 30 31 32
35 36 37
38 39 40
41 42 44 46
48 49 50
51 52 53
54 55 56
57 58 59
66 68 69 70
72 73 74 76
78 79 81
83 84 85
87 89 90
92 93 95
97 98 99