Reporte

Semana 1 y 2

Andrea Valeria Pintor Valencia A01067424

Victoria Ramirez Castro - A01640824

Integración de robótica y sistemas inteligentes

Gpo 501

04 de abril de 2025

Embedded Systems Module

System Development Life Cycle (SDLC)

The overall process for developing information systems from planning and analysis through implementation and maintenance.

SDLC phases:

- **Planning Phase:** Establishes a high-level plan of the intended project and determines project goals.
- **Analysis phase:** Involves analyzing end-user business requirements and refining project goals into defined functions and operations of the intended system.
- **Design phase:** Establishes descriptions of the desired features and operations of the system including screen layouts, business rules, process diagrams,pseudo code, and other documentation.
- **Development phase:** Involves taking all of the detailed design documents from the design phase and transforming them into the actual system.
- **Testing phase**: Involves bringing all the project pieces together into a special testing environment to eliminate errors and bugs, and verify that the system meets all of the business requirements defined in the analysis phase.
- **Implementation phase:** Involves placing the system into production so users can begin to perform actual business operations with it.
- **Maintenance phase:** Involves performing changes, corrections, additions, and upgrades to ensure the system continues to meet its business goals.

System Analysts

They are key individuals in the systems development process. They are needed to determine how people, data, processes,communications, and information technology can best accomplish improvements for the business

Neurorobotics Model

Probabilistic Methods for Robotics

- Probabilistics robotics deals with uncertainties in robot perception and action.
  - Estimates state from sensor data
- **Belief distribution.** probabilistic state estimation.
- **State.** collection of all aspects of the robot.
  - Example: velocity, sensor status.
- **Measurements.** information about a momentary state of the environment.
  - Example: range scans.

- **Control Data.** information of state of change.
    - Example: velocity of a robot.
- **State transition probability**

$$p(x_t|x_{t-1}, u_t)$$

- **Measurement probability**

$$p(z_t|x_t)$$

- **Posterior probability** of the state.

$$p(x_t|z_{1:t}, u_{1:t})$$

Probability Theory

- **Probability**. Chance that a given event occurs.
- **Random variable.** Rule (or function) that assigns a real number to every outcome of a random experiment.
- **Random process.** Rule (or function) that assigns a time function to every outcome of a random experiment.

Set theory	Probability theory	Probability symbol
Universe	Sample space (certain event)	$\mathcal{S}$
Element	Outcome (sample point)	s
Subset	Event	$A, B, or\ C \dots$
Disjoint sets	Mutually exclusive events	$E_i \cap E_j = \emptyset$
Null set	Impossible event	$\emptyset$
Simple set	Simple event	$B = \{s\}$

Basic Probability

- Develops a probabilistic model of a physical experiment.
- **Axioms.** Clear and obvious proposition accepted with no demonstrations.
- **Relative frequency.** Common sense, results of observations.
- **Trial.** Single realization of an experiment.
- **Outcome.** Result of a trial.
- **Probability of an event.** Sum of probabilities of each simple event.
- **Experiment:**
    - Procedure.
    - Observation.
    - Model.

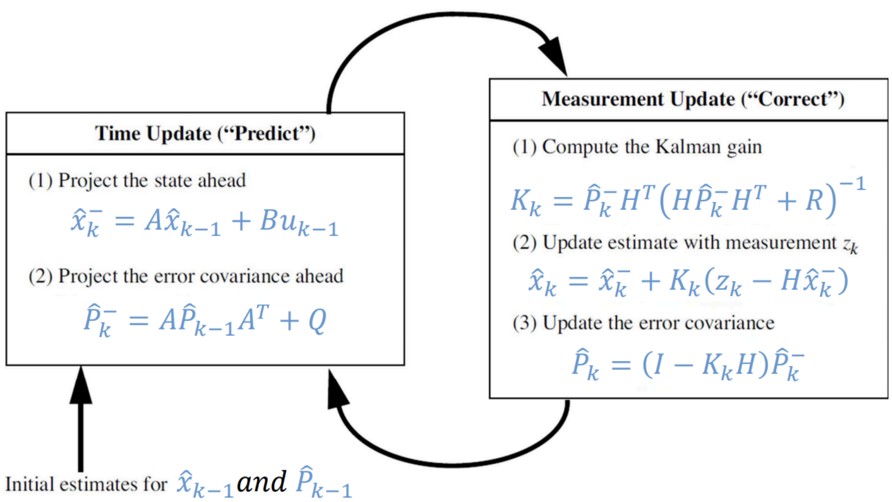
Discrete Random Variables

- **Random variable.** function that assigns a real number to each outcome of a random experiment.

	Values	PMF	$E[X]$	$\mathrm{var}(X)$
Uniform	$k=-M,\ldots,M$	$\frac{1}{2M+1}$	0	$\frac{M(M+1)}{3}$
Bernoulli	$k=0,1$	$p^k(1-p)^{1-k}$	p	$p(1-p)$
Binomial	$k=0,1,\ldots,M$	$\binom{M}{k} p^k(1-p)^{M-k}$	Mp	$Mp(1-p)$
Geometric	$k=1,2,\ldots$	$(1-p)^{k-1}p$	$\frac{1}{p}$	$\frac{1-p}{p^2}$
Poisson	$k=0,1,\ldots$	$\exp(-\lambda)\frac{\lambda^k}{k!}$	λ	λ

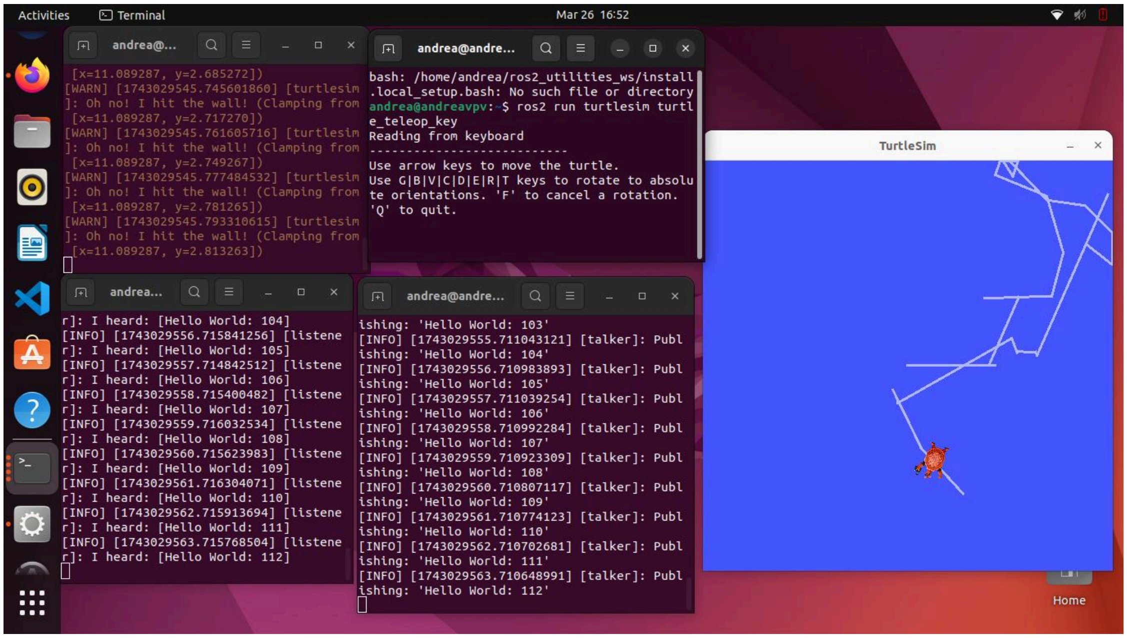
Kalman Filter

Algorithm used to fuse measurements from sensors to estimate in real time the state of a dynamical system.

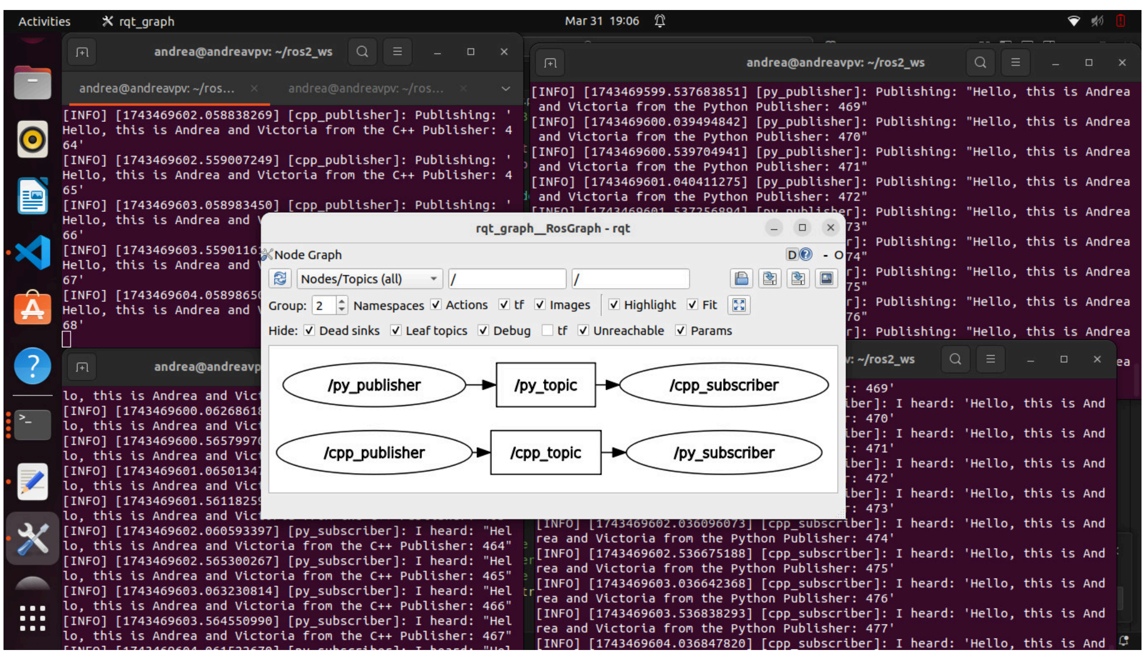


ROS2 Module

Activity 1. Introduction to ROS2



Activity 2. ROS2 Nodes



Autonomous Robots Module

Linear Algebra

Eigenvalues and Eigenvector

- **Eigenvalues.** Let $A \epsilon R^{NxN}$ then a scalar λ is an eigenvalue if there exists a vector $x \neq 0$ such that

$$Ax = \lambda x \text{ for } x \neq 0$$

  - ➔ (A-λI) no es invertible.
  - ➔ det(A-λI) = 0
  - ➔ If an eigenvalue exists, it means that an invariant subspace exists.
  - ➔ For each eigenvalue, we have an eigenvector.
  - ➔ When a matrix is non-invertible, there are ∞ solutions.
- **Eigenspace.** Collection of all eigenvectors corresponding to λ

$$E\lambda$$

Dynamic Systems

- **Dynamics.** Study of motion through phase space.
- **Vector field.** is a function $f : \mathbb{R} \times \mathbb{R} \times \cdots \times \mathbb{R} \rightarrow \mathbb{R}^n$ that assigns a vector $v \in \mathbb{R}n$ to each point in space. That is

$$f(x1, x2, \ldots, xn)$$

- **Dynamical System.** Transformation or function f normally applied to a vector field x. This vector field is said to b e the states of the dynamical system.

$$\boldsymbol{x}' = \boldsymbol{fx}, \boldsymbol{x} \in \mathbb{R}^n$$

(autonomous system)

Equilibrium or Fixed Points

- **Equilibrium point.** Particular point in phase space where the dynamical system stays stationary.

$$\boldsymbol{x}' = \boldsymbol{fx} = 0$$

Flows

This is a way of viewing an equation in vector field space.

- **Flow.** Interpretation of a differential equation as a vector field. Where the vector field moves in an arbitrary initial condition.
  - ○ The flow goes to the right if x' > 0
  - ○ The flow goes to the left if $x' < 0$

- There is no flow if $x' = 0$
- Fixed point **stable** if the flow goes towards it.
- Fixed point **unstable** if the flow goes outwards.

Linearization

- Approximate a nonlinear function or a nonlinear system with a linear function at a specific fixed point
- **Jacobian Linearization**

$$A := \frac{\partial f(x)}{\partial x}\Big|_{\substack{x=x^* \\ u=u^*}}$$

$$B := \frac{\partial f(u)}{\partial u}\Big|_{\substack{x=x^* \\ u=u^*}}$$

$$\dot{\delta}_x(t) = A\,\delta_x(t) + B\,\delta_u(t)$$

Example:

ejemplo 1. $\dot{x}_1 = \begin{bmatrix} x_2 \\ -\frac{g}{L}\sin(x_1) \end{bmatrix}$

$B = \varnothing$ ¡u no existe → derivada parcial es $\varnothing$

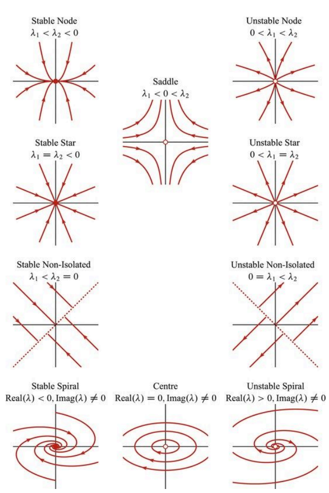
$A = \begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} \\ \frac{\partial f_2}{\partial x_1} & \frac{\partial f_2}{\partial x_2} \end{bmatrix}$! $f_1 = x_2$, $f_2 = -\frac{g}{L}\sin(x_1)$

$A = \begin{bmatrix} \varnothing & 1 \\ -\frac{g}{L}\cos(x_1) & \varnothing \end{bmatrix}$ Paso 1. Derivar con respecto a x_n

$A = \begin{bmatrix} 0 & 1 \\ -\frac{g}{L} & \varnothing \end{bmatrix}_{\overset{*}{x}_1 = 0}$ Paso 2. Evaluar en 1 punto fijo

$\dot{x} = \begin{bmatrix} 0 & 1 \\ -\frac{g}{L} & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$ Paso 3. Modelo Dinámico linearizado

Linear Control Theory



Linear control theory is used to view the $\mathbb{R}^2$ flow of linear systems. An uncoupled system is a system that can be developed in two equations. Equation 1 is only in terms of x, and equation 2 is only in terms of y. To determine whether a system is stable or not, consider the diagonal matrix or the eigenvalues. It is important to remember that in a diagonal matrix, the values on the diagonal are the eigenvalues.

If the values on the main diagonal are:

- **Negative:** the system is stable.
- **Positive:** the system is unstable.
- **Positive and negative**: the system is unstable.

Activity Eigenvalues and Eigenvectors

a) $A = \begin{bmatrix} 2 & 2 \\ 2 & -1 \end{bmatrix}$

$$A - \lambda I = \begin{bmatrix} 2 & 2 \\ 2 & -1 \end{bmatrix} - \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix} = \begin{bmatrix} 2-\lambda & 2 \\ 2 & -1-\lambda \end{bmatrix}$$

$$\det(A - \lambda I) = (2-\lambda)(-1-\lambda) - 4$$
$$= -2 - 2\lambda + \lambda + \lambda^2 - 4$$
$$= \lambda^2 - \lambda - 6 = (\lambda - 3)(\lambda + 2)$$

$\lambda_1 = 3$ $\quad$ $\lambda_2 = -2$

para $\lambda_1 = 3$

$$(A - \lambda I)x = 0$$

$$\begin{bmatrix} 2-\lambda_1 & 2 \\ 2 & -1-\lambda_1 \end{bmatrix} x = 0 \Rightarrow \begin{bmatrix} -1 & 2 \\ 2 & -4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0$$

$$-x_1 + 2x_2 = 0$$
$$2x_1 - 4x_2 = 0$$
$$x_1 = 2x_2$$

$$\boxed{x = s \begin{bmatrix} 1 \\ 1/2 \end{bmatrix}, \; s \in \mathbb{R}}$$

para $\lambda_2 = -2$

$$(A - \lambda I)x = 0$$

$$\begin{bmatrix} 2-\lambda_2 & 2 \\ 2 & -1-\lambda_2 \end{bmatrix} x = 0 \Rightarrow \begin{bmatrix} 4 & 2 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0$$

$$4x_1 + 2x_2 = 0$$
$$2x_1 + x_2 = 0$$
$$x_2 = -2x_1$$
$$x_1 = -\frac{1}{2} x_2$$

$$\boxed{x = \begin{bmatrix} 1 \\ -2 \end{bmatrix}, \; s \in \mathbb{R}}$$

b) $B = \begin{bmatrix} 0 & 4 \\ -1 & 5 \end{bmatrix}$

$$A - \lambda I = \begin{bmatrix} 0 & 4 \\ -1 & 5 \end{bmatrix} - \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix} = \begin{bmatrix} -\lambda & 4 \\ -1 & 5-\lambda \end{bmatrix}$$

$$\det(A - \lambda I) = (-\lambda)(5-\lambda) - (-4)$$
$$= -5\lambda + \lambda^2 + 4 = (\lambda - 4)(\lambda - 1)$$

$\lambda_1 = 4$ $\quad$ $\lambda_2 = 1$

para $\lambda_1 = 4$

$$(A - \lambda I)x = 0$$

$$\begin{bmatrix} -\lambda_1 & 4 \\ -1 & 5-\lambda_1 \end{bmatrix} x = 0 \Rightarrow \begin{bmatrix} -4 & 4 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0$$

1

$$-4x_1 + 4x_2 = 0$$
$$-x_1 + x_2 = 0$$
$$x_2 = x_1$$

$\boxed{x = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, s \in \mathbb{R}}$

para $\lambda_2 = 1$

$$(A - \lambda I)x = 0$$
$$\begin{bmatrix} -\lambda_2 & 4 \\ -1 & 5-\lambda_2 \end{bmatrix} x = 0 \implies \begin{bmatrix} -1 & 4 \\ -1 & 4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0$$
$$-x_1 + 4x_2 = 0$$
$$-x_1 + 4x_2 = 0$$
$$x_2 = \frac{x_1}{4}$$

$\boxed{x = \begin{bmatrix} 1 \\ 1/4 \end{bmatrix}, s \in \mathbb{R}}$

c) $C = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$

$$A - \lambda I = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} - \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix} = \begin{bmatrix} -\lambda & -1 \\ 1 & -\lambda \end{bmatrix} = 0$$
$$\det(A - \lambda I) = (-\lambda)(-\lambda) - (-1)$$
$$= \lambda^2 + 1$$

$\lambda_1 = i$ $\qquad$ $\lambda_2 = -i$

para $\lambda_1 = i$

$$(A - \lambda I)x = 0$$
$$\begin{bmatrix} -\lambda_1 & -1 \\ 1 & -\lambda_1 \end{bmatrix} x = 0 \implies \begin{bmatrix} -i & -1 \\ 1 & -i \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0$$
$$-ix_1 - x_2 = 0$$
$$x_1 - ix_2 = 0$$
$$x_1 = ix_2$$
$$x_2 = -ix_1$$

$\boxed{x = \begin{bmatrix} 1 \\ -i \end{bmatrix}, s \in \mathbb{R}}$

para $\lambda_2 = -i$

$$(A - \lambda I)x = 0$$
$$\begin{bmatrix} -\lambda_2 & -1 \\ 1 & -\lambda_2 \end{bmatrix} x = 0 \implies \begin{bmatrix} i & -1 \\ 1 & i \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0$$
$$ix_1 - x_2 = 0$$
$$x_1 + ix_2 = 0$$
$$x_1 = -ix_2$$
$$ix_1 = x_2$$

$\boxed{x = \begin{bmatrix} 1 \\ i \end{bmatrix}, s \in \mathbb{R}}$

d) $D = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$

$$A - \lambda I = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} - \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix} = \begin{bmatrix} 1-\lambda & 0 \\ 0 & -1-\lambda \end{bmatrix} = 0$$
$$\det(A - \lambda I) = (1-\lambda)(-1-\lambda) - 0$$
$$= -1 - \lambda + \lambda + \lambda^2$$
$$= \lambda^2 - 1 = (\lambda - 1)(\lambda + 1)$$

$\lambda_1 = 1$ $\qquad$ $\lambda_2 = -1$

2

para $\lambda_1 = 1$

$$(A - \lambda I)x = 0$$
$$\begin{bmatrix} 1-\lambda_1 & 0 \\ 0 & -1-\lambda_1 \end{bmatrix} x = 0 \implies \begin{bmatrix} 0 & 0 \\ 0 & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0$$
$$-2x_2 = 0$$
$$x_2 = 0$$

$\boxed{x = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, s \in \mathbb{R}}$

para $\lambda_2 = -1$

$$(A - \lambda I)x = 0$$
$$\begin{bmatrix} 1-\lambda_2 & 0 \\ 0 & -1-\lambda_2 \end{bmatrix} x = 0 \implies \begin{bmatrix} 2 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0$$
$$2x_1 = 0$$
$$x_1 = 0$$

$\boxed{x = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, s \in \mathbb{R}}$

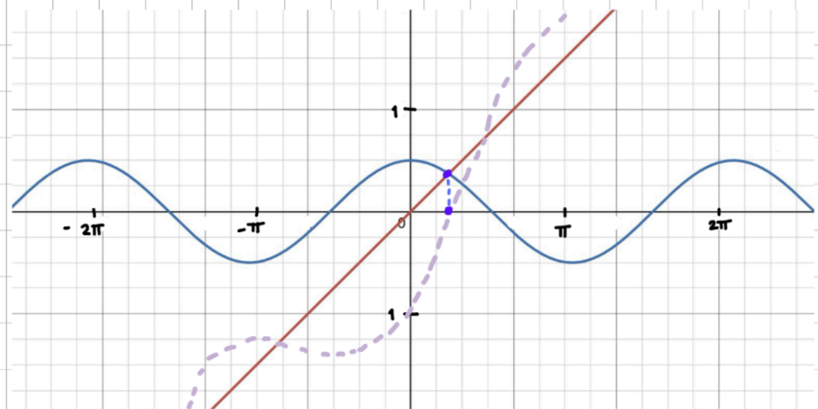
Activity Fixed Points and Equilibrium

Flujo inestable

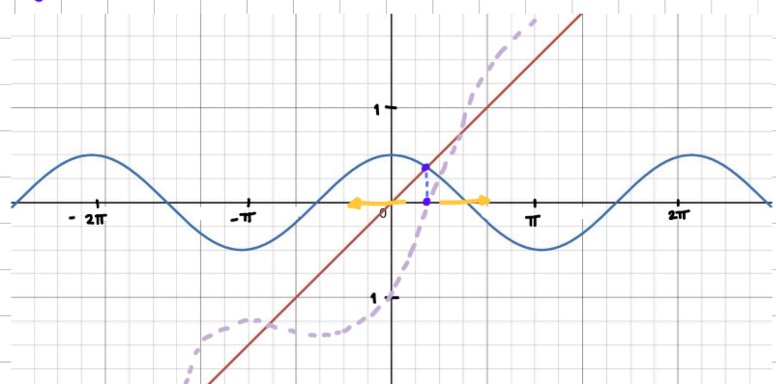
1 $\dot{x} = x - \cos x$

fase

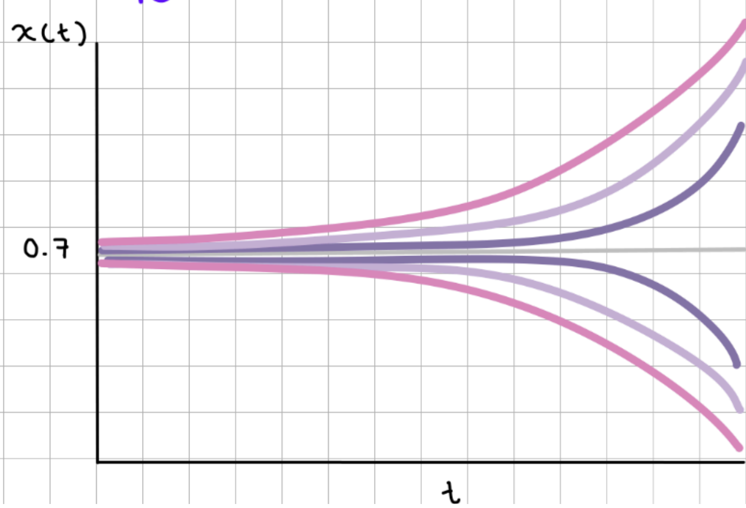
$x \approx 0.7$



flujo



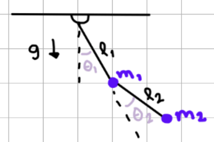
tiempo



Activity Double Pendulum Model

System and linearization

Modelo Dinámico del Doble péndulo



Lagrangian dynamic questions [from Newton's Eq.]

(1) $$\frac{d}{dt}\cdot\frac{\partial L}{\partial \dot{q}_i} - \frac{\partial L}{\partial q_i} = \tau_i$$

(2) $$\begin{cases} p_1 = l_1 \begin{bmatrix} S_{q_1} \\ -C_{q_1} \end{bmatrix}, \quad p_2 = p_1 + l_2 \begin{bmatrix} S_{q_1+q_2} \\ -C_{q_1+q_2} \end{bmatrix} \\ \dot{p}_1 = l_1 \dot{q}_1 \begin{bmatrix} C_{q_1} \\ S_{q_1} \end{bmatrix}, \quad \dot{p}_2 = \dot{p}_1 + l_2 (\dot{q}_1 + \dot{q}_2) \begin{bmatrix} C_{q_1+q_2} \\ S_{q_1+q_2} \end{bmatrix} \end{cases}$$

Kinetic & Potential Energy

$$T = \frac{1}{2} m_1 \dot{p}_1^T \dot{p}_1 + \frac{1}{2} m_2 \dot{p}_2^T \dot{p}_2$$

(3) $$T = \frac{1}{2}(m_1 + m_2)\, l_1^2 \dot{q}_1^2 + \frac{1}{2} m_2 l_2^2 (\dot{q}_1 + \dot{q}_2)^2 + m_2 l_1 l_2 \dot{q}_1 (\dot{q}_1 + q_2)\, C_{q_2}$$

(4) $$U = m_1 g y_1 + m_2 g y_2 = -(m_1 + m_2) g l_1 C_{q_1} - m_2 g l_2 C_{q_1+q_2}$$

After partial derivatives & substitute in (1)

(5) $$(m_1 + m_2) l_1^2 \ddot{q}_1 + m_2 l_2^2 (\ddot{q}_1 + \ddot{q}_2) + m_2 l_1 l_2 (2\ddot{q}_1 + q_2) C_{q_2} - m_2 l_1 l_2 (2\dot{q}_1 + \dot{q}_2)\dot{q}_2 S_{q_2} + (m_1 + m_2) l_1 g S_{q_1} + m_2 g l_2 S_{q_1+q_2} = \tau_1$$

(6) $$m_2 l_2^2 (\ddot{q}_1 + q_2) + m_2 l_1 l_2 \ddot{q}_1 C_{q_2} + m_2 l_1 l_2 \dot{q}_1^2 S_{q_2} + m_2 g l_2 S_{q_1+q_2} = \tau_2$$

(5) & (6) are equations of Motion

can be rewritten as

(7) $$M(q)\ddot{q} + C(q,\dot{q})\dot{q} = \tau_g(q) + Bu$$

Vector de estados $q[\theta_1, \theta_2]$ where

$$M(q) = \begin{bmatrix} (m_1+m_2) l_1^2 + m_2 l_2 + 2 m_2 l_1 l_2 C_2 & m_2 l_2^2 + m_2 l_1 l_2 C_2 \\ m_2 l_2^2 + m_2 l_1 l_2 C_2 & m_2 l_2 \end{bmatrix}$$

$$C(q,\dot{q}) = \begin{bmatrix} 0 & -m_2 l_1 l_2 (2\dot{\theta}_1 + \dot{\theta}_2) S_2 \\ \frac{1}{2} m_2 l_1 l_2 (2\dot{\theta}_1 + \dot{\theta}_2) S_2 & -\frac{1}{2} m_2 l_1 l_2 \dot{\theta}_1 S_2 \end{bmatrix}$$

$$\tau_g(q) = -g \begin{bmatrix} (m_1+m_2) l_1 S_1 + m_2 l_2 S_{1+2} \\ m_2 l_2 S_{1+2} \end{bmatrix} \qquad B = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

reescribir (7)

$$M\ddot{q} + C\dot{q} = \tau + Bu$$

cambio de variable

$$q = q_1$$
$$\dot{q} = \dot{q}_1 = q_2$$
$$\ddot{q} = \ddot{q}_1 = \dot{q}_2$$

despejar $\ddot{q}$ de (7)

$$\ddot{q} = M^{-1}[\tau + BU - C\dot{q}] \quad (8)$$

$$\bar{q} = \begin{bmatrix} q_1 \\ q_2 \end{bmatrix}$$

reescribir (8) en forma matricial

$$\dot{\bar{q}} = \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \end{bmatrix} = \begin{bmatrix} q_2 \\ M^{-1}[\tau + BU - Cq_2] \end{bmatrix} \quad (9)$$

hacer multiplicación y hacer expansión de (9)

$$\begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \end{bmatrix} = \begin{bmatrix} q_2 \\ M^{-1}\tau + M^{-1}BU - M^{-1}Cq_2 \end{bmatrix}$$

$$\begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \end{bmatrix} = \underbrace{\begin{bmatrix} q_2 \\ M^{-1}\tau - M^{-1}Cq_2 \end{bmatrix}}_{A} + \underbrace{\begin{bmatrix} 0 \\ M^{-1}BU \end{bmatrix}}_{B}$$

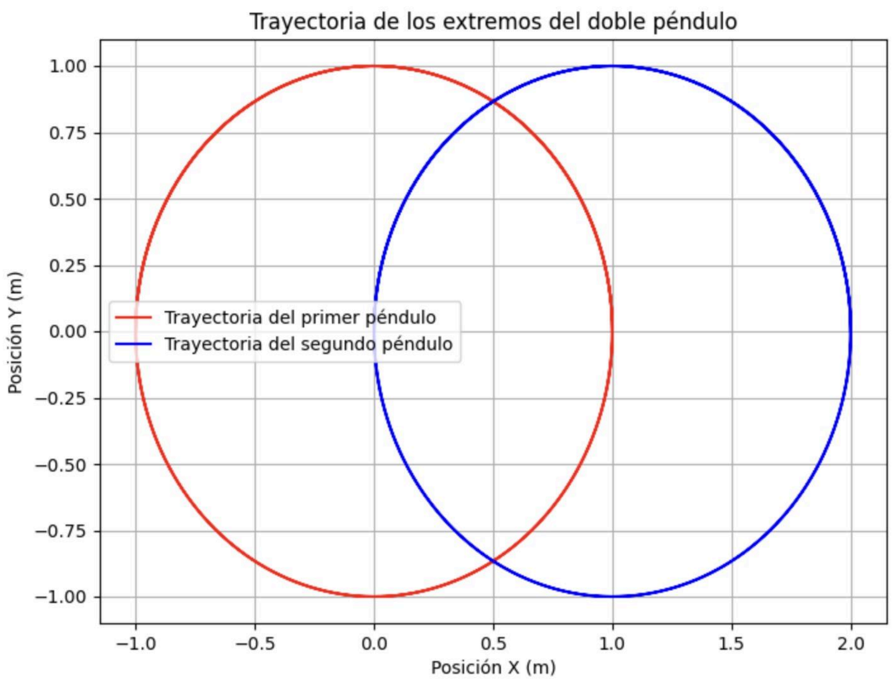
$$A = \begin{bmatrix} q_2 \\ M^{-1}\tau - M^{-1}Cq_2 \end{bmatrix} \qquad B = \begin{bmatrix} 0 \\ M^{-1}B \end{bmatrix}$$

$$A = \begin{bmatrix} \frac{\partial q_2}{\partial q_1} & \frac{\partial q_2}{\partial q_2} \\ \frac{\partial M^{-1}\tau - M^{-1}Cq_2}{\partial q_1} & \frac{\partial M^{-1}\tau - M^{-1}Cq_2}{\partial q_2} \end{bmatrix} \qquad B = \begin{bmatrix} \frac{\partial 0}{\partial U} \\ \frac{\partial M^{-1}BU}{\partial U} \end{bmatrix}$$

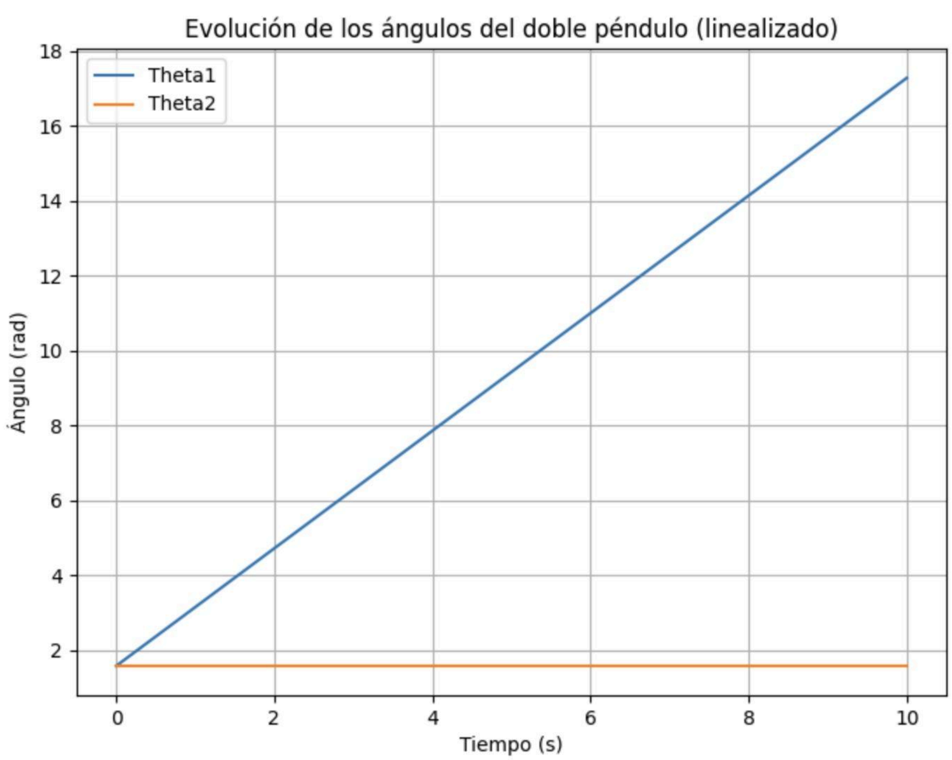
$$\boxed{A = \begin{bmatrix} 0 & 1 \\ 0 & -M^{-1}C \end{bmatrix} \quad B = \begin{bmatrix} 0 \\ M^{-1}B \end{bmatrix}}$$

linealizado

System simulation



Linearization simulation



Activity Dynamic Model RDK X3

System and linearization

Modelo dinamico del robot

$$X = \begin{bmatrix} x \\ y \\ \theta \end{bmatrix} \qquad \dot{x} = \begin{bmatrix} V_x \\ V_y \\ \omega \end{bmatrix} = \begin{bmatrix} V\cos\theta \\ V\,\mathrm{sen}\,\theta \\ \frac{V}{L}\tan\theta \end{bmatrix}$$

V: Velocidad lineal
L: Distancia entre ejes

Espacio de estados

$$q = [x, y, \theta]^T$$

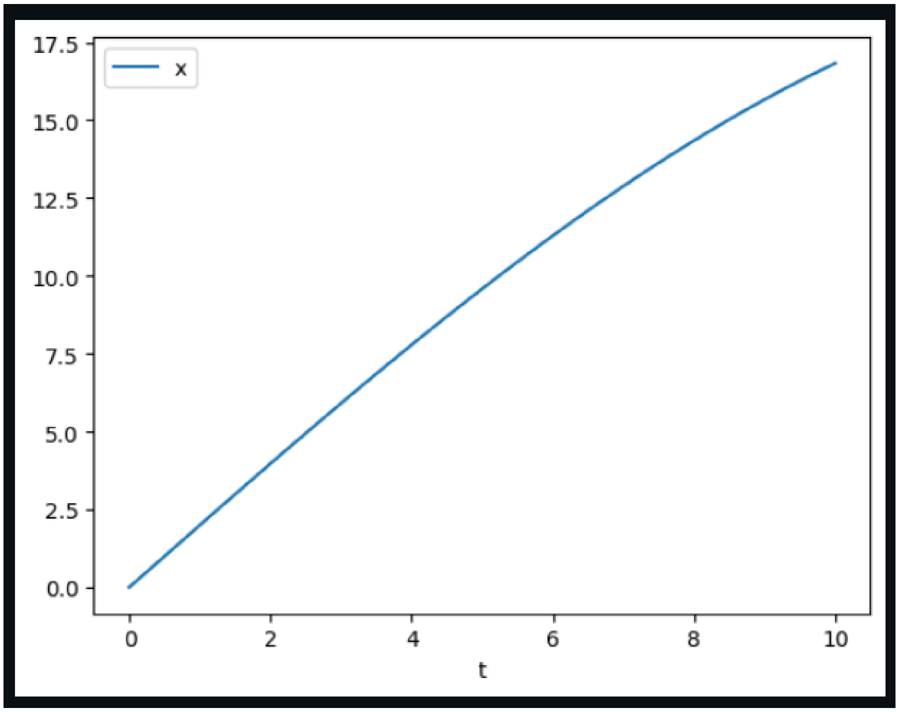
$$\dot{q} = \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} V\cos\theta \\ V\,\mathrm{sen}\,\theta \\ \omega \end{bmatrix} = \underbrace{\begin{bmatrix} \cos\theta & 0 \\ \mathrm{sen}\,\theta & 0 \\ 0 & 1 \end{bmatrix}}_{g(x)} \underbrace{\begin{bmatrix} V \\ \omega \end{bmatrix}}_{U}$$

Linealización:

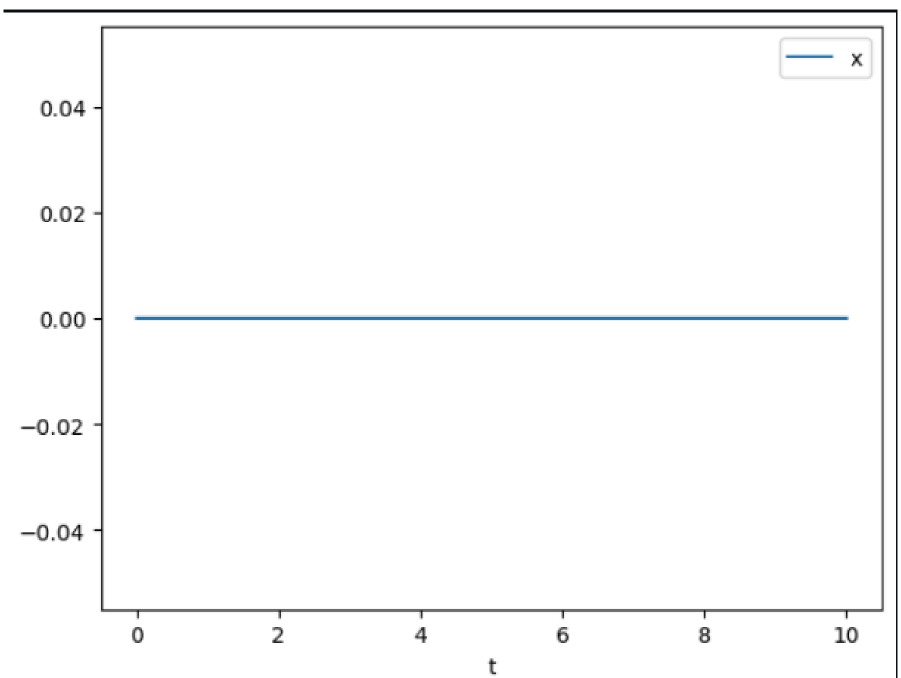
$$A = 0 \qquad B = \begin{bmatrix} \frac{\partial f_1}{\partial V} & \frac{\partial f_1}{\partial \omega} \\ \frac{\partial f_2}{\partial V} & \frac{\partial f_2}{\partial \omega} \\ \frac{\partial f_3}{\partial V} & \frac{\partial f_3}{\partial \omega} \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix}$$

$$q = g(x)\,U = \frac{q_k - q_{k-1}}{\tau} \longrightarrow \underbrace{q_k = q_{k-1} + \tau\, g(x)\,U}_{\text{Odometria}}$$

System simulation



Linearization simulation



Activity Lineal Control

$$A = \begin{bmatrix} 1 & 1 \\ 4 & -2 \end{bmatrix}$$

$$A - \lambda I = \begin{bmatrix} 1 & 1 \\ 4 & -2 \end{bmatrix} - \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix} = \begin{bmatrix} 1-\lambda & 1 \\ 4 & -2-\lambda \end{bmatrix}$$

$$\det(A - \lambda I) = (1-\lambda)(-2-\lambda) - 4$$
$$= -2 - \lambda + 2\lambda + \lambda^2 - 4$$
$$= \lambda^2 + \lambda - 6 = (\lambda + 3)(\lambda - 2)$$

$\lambda_1 = -3$ $\lambda_2 = 2$

Para $\lambda_1 = -3$

$$(A - \lambda I)x = 0$$

$$\begin{bmatrix} 1-\lambda_1 & 1 \\ 4 & -2-\lambda_1 \end{bmatrix} x = 0 \implies \begin{bmatrix} 4 & 1 \\ 4 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0$$

$$4x_1 + x_2 = 0$$
$$4x_1 + x_2 = 0$$
$$x_2 = -4x_1$$

$$x = s \begin{bmatrix} -1/4 \\ 1 \end{bmatrix}, \; s \in \mathbb{R}$$

Para $\lambda_2 = 2$

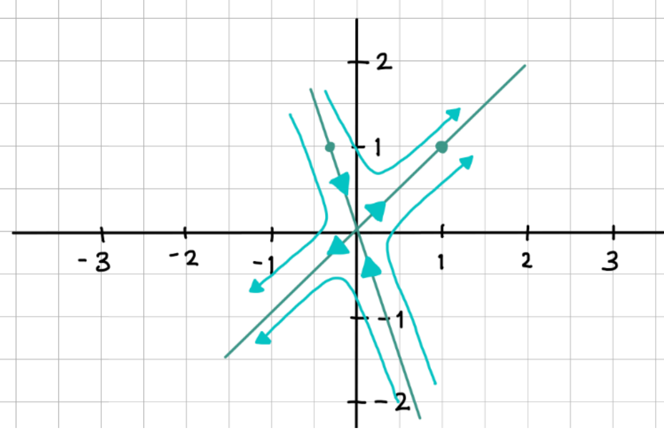
$$(A - \lambda I)x = 0$$

$$\begin{bmatrix} 1-\lambda_2 & 1 \\ 4 & -2-\lambda_2 \end{bmatrix} x = 0 \implies \begin{bmatrix} -1 & 1 \\ 4 & -4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0$$

$$-x_1 + x_2 = 0$$
$$4x_1 - 4x_2 = 0$$
$$x_2 = x_1$$

$$x = s \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \; s \in \mathbb{R}$$

→ Inestable



$$B = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

$$A - \lambda I = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} - \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix} = \begin{bmatrix} 1-\lambda & 0 \\ 0 & -1-\lambda \end{bmatrix} = 0$$

$$\det(A - \lambda I) = (1-\lambda)(-1-\lambda) - 0$$
$$= -1 - \lambda + \lambda + \lambda^2$$
$$= \lambda^2 - 1 = (\lambda - 1)(\lambda + 1)$$
$$\lambda_1 = 1 \qquad \lambda_2 = -1$$

para $\lambda_1 = 1$

$$(A - \lambda I)x = 0$$

$$\begin{bmatrix} 1-\lambda_1 & 0 \\ 0 & -1-\lambda_1 \end{bmatrix} x = 0 \implies \begin{bmatrix} 0 & 0 \\ 0 & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0$$

$$-2x_2 = 0$$
$$x_2 = 0$$

$$x = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \ s \in \mathbb{R}$$

para $\lambda_2 = -1$

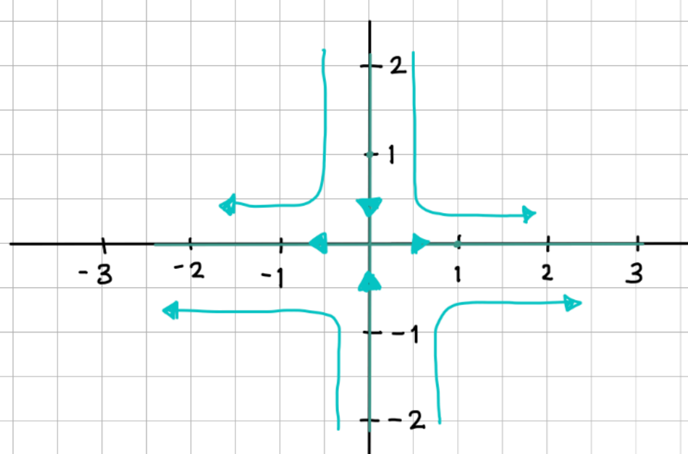
$$(A - \lambda I)x = 0$$

$$\begin{bmatrix} 1-\lambda_2 & 0 \\ 0 & -1-\lambda_2 \end{bmatrix} x = 0 \implies \begin{bmatrix} 2 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0$$

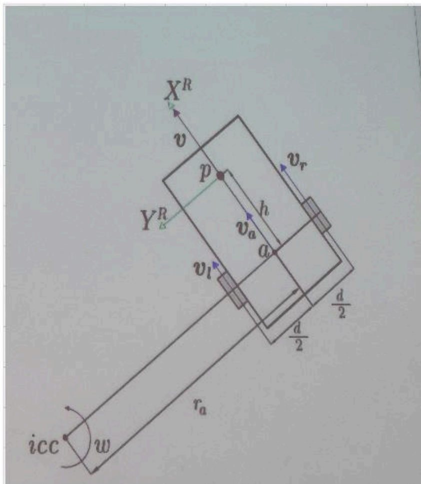
$$2x_1 = 0$$
$$x_1 = 0$$

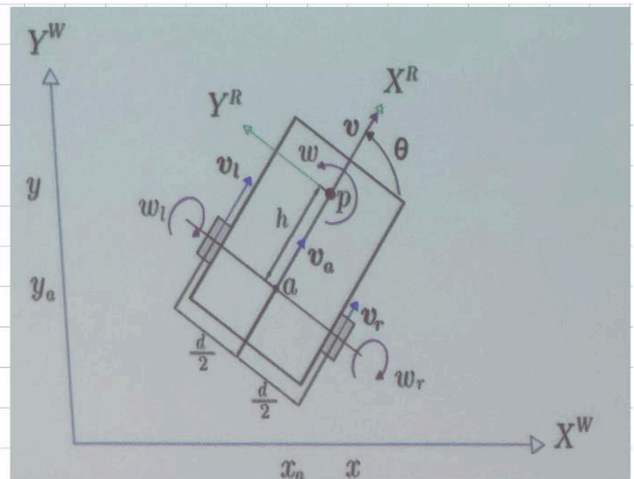
$$x = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \ s \in \mathbb{R}$$

→ Inestable



Activity Rover Feedback Control





$$x = x_a + h\cos\theta$$
$$y = y_a + h\sin\theta$$

$$\dot{\theta} = \omega$$

$$\dot{x} = \dot{x}_a - h\sin\theta \cdot \dot{\theta}$$
$$\dot{y} = \dot{y}_a + h\cos\theta \cdot \dot{\theta}$$

$$\dot{x}_a = V_a \cos\theta$$
$$\dot{y}_a = V_a \sin\theta$$

$$\dot{x} = V\cos\theta - h\sin\theta\,\omega$$
$$\dot{y} = V\sin\theta + h\cos\theta\,\omega$$

$$\longrightarrow \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} c\theta & -hs\theta \\ s\theta & hc\theta \\ 0 & 1 \end{bmatrix} \begin{bmatrix} V \\ \omega \end{bmatrix}$$

si $h = 0$

$$\downarrow$$

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} \cos\theta & 0 \\ \text{sen}\,\theta & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} V \\ W \end{bmatrix}$$

$$\begin{bmatrix} V \\ W \end{bmatrix} = \begin{bmatrix} \frac{1}{2}r & \frac{1}{2}r \\ \frac{r}{d} & -\frac{r}{d} \end{bmatrix} \begin{bmatrix} \omega_r \\ \omega_l \end{bmatrix}$$

r : radio de la llanta
d : distancia entre llantas

$$u = \begin{bmatrix} v \\ w \end{bmatrix} \qquad z = \begin{bmatrix} x \\ y \\ \theta \end{bmatrix} = \begin{bmatrix} q \\ \theta \end{bmatrix} \qquad \dot{z} = \begin{bmatrix} D \\ \phi^T \end{bmatrix} u$$

$$D = \begin{bmatrix} c\theta & -hs\theta \\ s\theta & hc\theta \end{bmatrix}$$

$$\phi^T = \begin{bmatrix} 0 & 1 \end{bmatrix}$$

Sistemas dinamicos:

$$\dot{q} = Du$$
$$\dot{\theta} = \phi^T u$$

Seguimiento de trayectoria:

$$e = q - q_d$$

$$\dot{e} = \dot{q} - \dot{q}_d = -ke \quad , \quad k > 0$$

$$= Du - \dot{q}_d = -ke$$

$$u = D^{-1}[-ke + \dot{q}_d]$$

← Tiene que existir la inversa.

$$A^{-1} = \frac{1}{\det A}[\quad]$$

$$\det A \neq 0$$

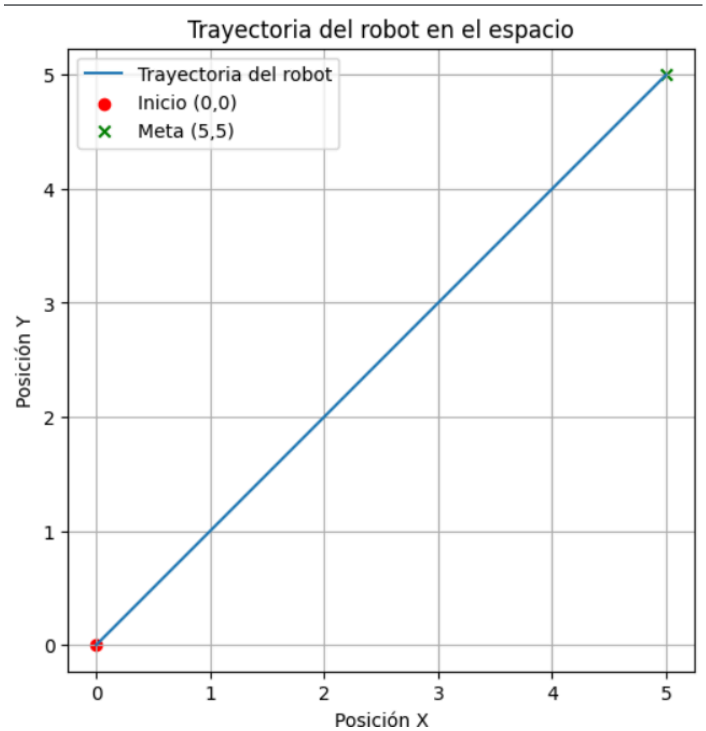
* La h de la D tiene que ser diferente de 0

Entonces:

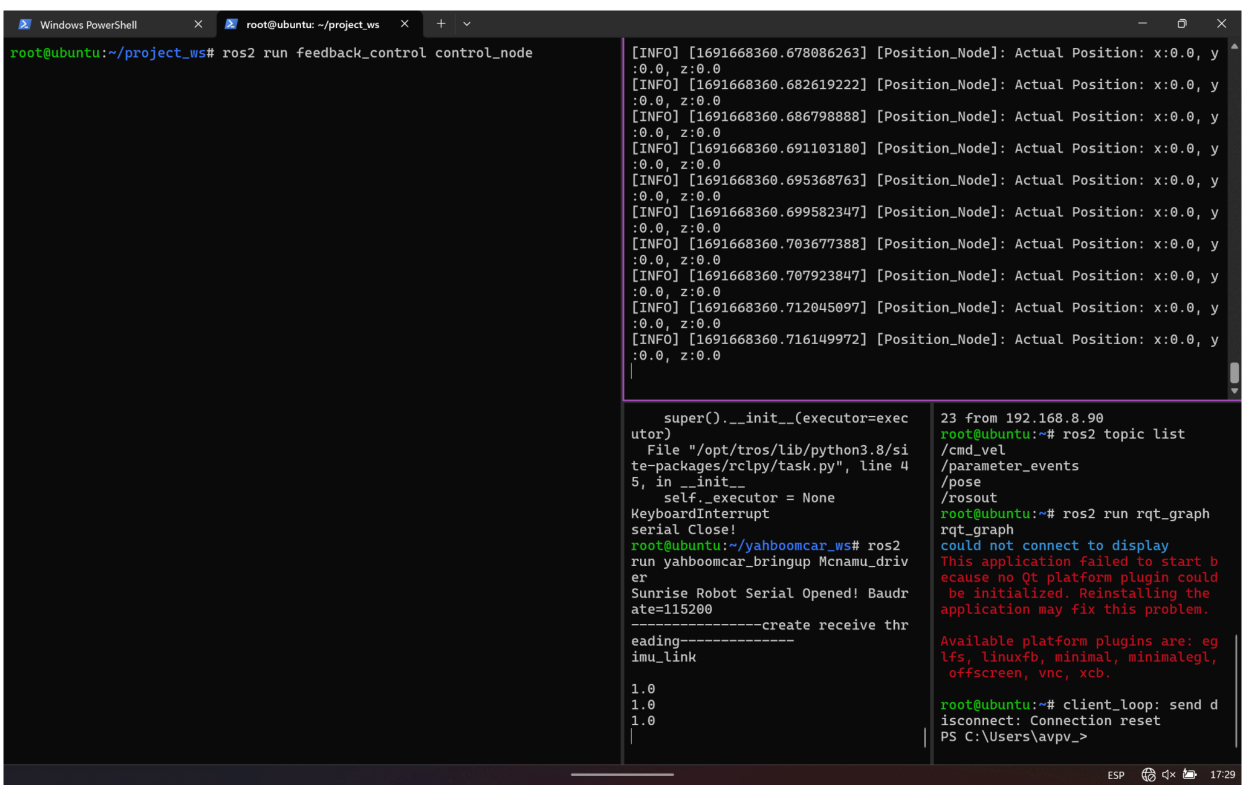
$$\dot{\theta} = \phi^T \cdot D^{-1}[-ke + \dot{q}_d]$$

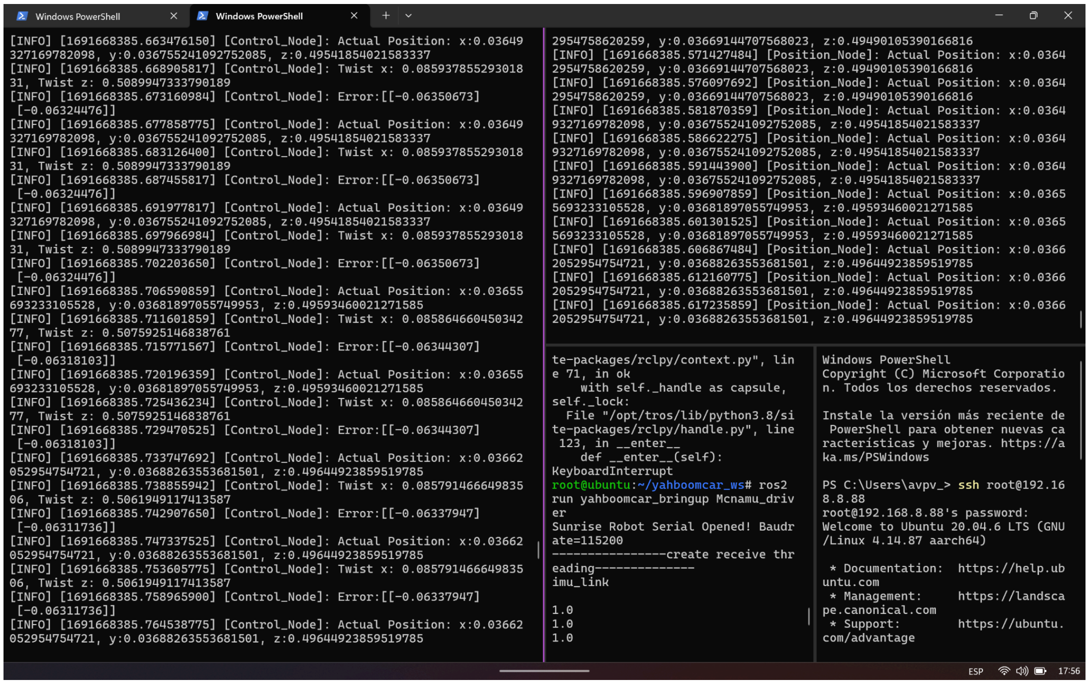
! No puedo controlar la orientación y la posicion al mismo tiempo

Simulation



ROS2 Nodes





Vision Module

Deep Learning

Multilayer neural networks are used to model and solve complex problems. A piece of data enters a neuron multiplied by a weight indicating the importance of the data. All inputs are summed, and then an activation function is applied.

Training

- Forward propagation: Inputs pass through the network to produce an output.
- Error calculation: Compares the predicted output with the actual value.
- Backpropagation: Adjusts weights using gradient descent to minimize error.

Types of Neural Layers

- Input: Defines the shape of the data.
- Dense: Full connections for global learning.
- Convolutional: Extracts local features.
- Pooling: Reduces spatial dimensionality.
- Dropout: To avoid overfitting.
- Output: Generates the prediction.