Laplacian Subdivision Surface

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Outline

Subdivision Surface

System

A subdivision surface, in the field of 3D computer graphics, is a method of representing a smooth surface via the specification of a coarser piecewise linear polygon mesh.

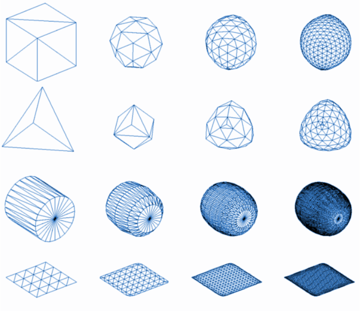


Figure: Catmull Clark Subdivision Surface

Subdivision Surface Methods

- Catmull–Clark (1978) generalized bi-cubic uniform B-spline to produce their subdivision scheme.
- In computer graphics, Doo–Sabin subdivision surface is a type of subdivision surface based on a generalization of bi-quadratic uniform B-splines. It was developed in 1978 by Daniel Doo and Malcolm Sabin.
- Loop, Triangles - Loop (1987) proposed his subdivision scheme based on a quartic box-spline of six direction vectors to provide a rule to generate C2 continuous limit surfaces everywhere except at extraordinary vertices where they are C1 continuous.
- Mid-Edge subdivision scheme - The mid-edge subdivision scheme was proposed independently by Peters–Reif (1997) and Habib–Warren (1999). The former used the mid-point of each edge to build the new mesh.
- $\sqrt{3}$ subdivision scheme - This scheme has been developed by Kobbelt (2000) and offers several interesting features: it handles arbitrary triangular meshes, it is C2 continuous everywhere except at extraordinary vertices where it is C1 continuous and it offers a natural adaptive refinement when required.

Laplacian Smoothing

The basic idea is that a vertex of a mesh is incrementally moved towards the Laplacian direction [Bray2004].

$$\frac{\partial X}{\partial t} = \lambda L(X),$$

which is implemented as the forward difference equation:

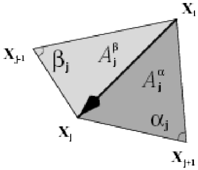
$X_{t+1} = (I + \lambda L) X_t$

where X is the set of vertices, L is the Laplacian, and $\lambda \in \mathbb{R}$ is a diffusion speed.

and the discrete Approximation reads as:

$L(x_i) = \sum w_{ij}(x_j - x_i), \qquad x_j \in Neighbor(x_i)$

The Laplacian can be approximated as



- Simple Laplacian $w_{ij} = \frac{1}{m}$
- Scale-dependent Laplacian $w_{ij} = \frac{1}{\|x_i - x_j\|}$
- Normal Curvature $w_{ij} = \cot \alpha_j + \cot \beta_j$

Laplacian Subdivision Surface

Simple subdivision

- For every edge create new vertex
- For every face create new vertex

Smooth the surface

- Laplacian smooth based on .

Linear system

$$W_L L \cdot X_{t+1} = X_t$$

Where N is number of vertices ($X \approx 30.000$)
We need to solve the system $Ax = B$
We solve it in the least-squares sense

$$minimize \, \|Ax - b\|^2$$

which is equivalent to minimizing the following quadratic energy.

$$\|W_L L X_{t+1}\|^2 + \sum_i W_{H,i}^2 \, \|x_{t+1} - x_t\|^2$$

Blender

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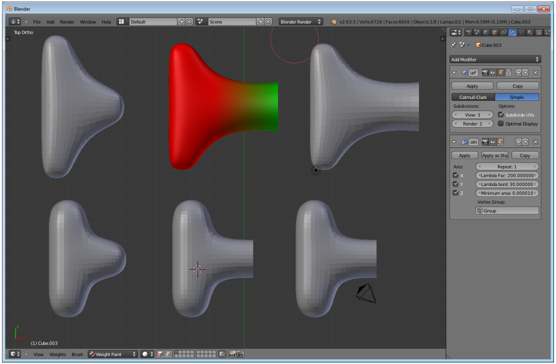
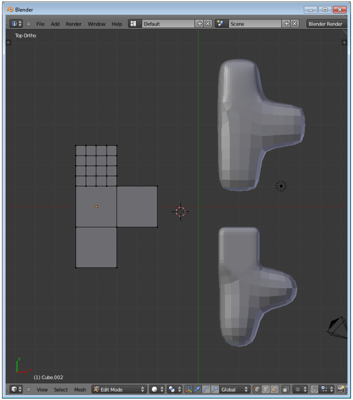


- Blender is the free open source 3D content creation suite, available for all major operating systems under the GNU General Public License..
- Use OpenNL for solve sparse system

Results


Blender
File Add Render Window Help
Default
Scene
Blender Render
Top Ortho
(1) Cube.002
View Select Mesh
Edit Mode
Global

Bibliography

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