Laplacian Subdivision Surfaces

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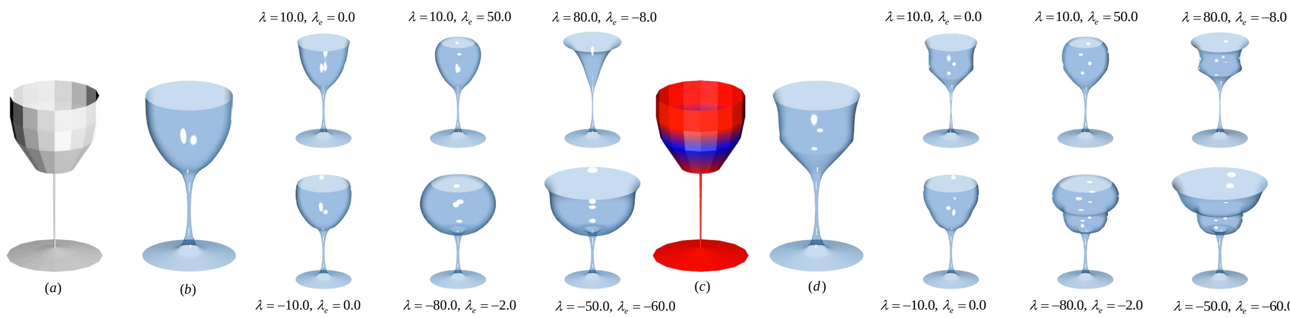

Figure 1: *Laplacian subdivision surface with weigth vertex group*

Abstract

This paper proposes a novel method for modelling poligonal mesh using subdivision surface and laplacian smoothing. This method use laplacian smooth for modelling global curvature in the model, to permit most flexible, robust and predictable results.

This method can correct traditional problems in extraordinary vertices present at catmull-clark subdivision method. The convergent rate of the laplacian smooth can be controlled by adjusting the weight in lambda parameter.

This proposal contains NN novel features

ESTE ABSTRACT NO SIRVE

(**we provide a series of examples to graphically and numerically demonstrate the quality of our results.**) debe ser cambiado copiado de desbrun 99

CR Categories: I.3.5 [Computer Graphics]: Computational Geometry and Object Modeling —Modeling packages

Keywords: laplacian smooth, subdivision surface

1 Introduction

Over the last years have been developed novel techniques of modeling that can generate a variety of shapes to look natural and realistic [Botsch et al. 2006]. Editing techniques have evolved from affine transformations to advanced tools like scuplting [Coquillart 1990; Galyean and Hughes 1991; Stanculescu et al. 2011], editing and creation from sketches [Igarashi et al. 1999; Gonen and Akleman 2012], complex interpolation techniques [Sorkine et al. 2004; Zhou et al. 2005], among others.

Traditional methods for smooth surfaces from coarse geometry like Catmull-Clark have been widely developed [Catmull and Clark 1978; Stam 1998], these works generalize uniform B-cubic splines knot insertion to meshes, some of them add control of the results with the use of creases to produce sharp edges [DeRose et al. 1998], or the modification of weights on the vertices that control locally the zone of influence [Biermann et al. 2000], instead our method performs a feature enhancement of the model allowing parameterize the curvature of the surface creating a family of different versions of the same object preserving detail and realistic natural look of the original model.

Many types of brushes have been developed to sculpt meshes, brushes that perform inflation lose detail when inflating the vertices [Stanculescu et al. 2011], our method allows inflation of the mesh vertices moving in the opposite direction to the curvature preserving the shape and sharp features of the model.

We present an extension of the Laplace Beltrami operator for meshes of arbitrary topology composed for triangles and quads representing a larger spectrum of mesh that works with today eliminating the need for preprocessing.

Overview ed nuestro metodo

Nuestro metodo usa el un bosquejo de maya le aplica una subdivision cualquiera y esa subdivision la modifica a lo largo de su curvatura de flujo usando un operador laplaciano para triangulos y cuadrados

Nuestro metodos propone tre cosas muy novedosas:

Operador Laplace beltrami para mallas de topologia arbitraria formadas por triangulos y cuadrados, para cualquier tipo de procesamiento en geometria diferencial

Permite generar una familia de formas parametrizadas.

Controlar el nivel de suavizado y curvatura al subdividir mallas de poligonos

Enhanced brush for sculpting modelling

1.1 Related work

Many tools have been developed for modeling based on Laplacian mesh processing. Thanks to the kindness of the Laplacian operator these tools have in common the need for preservation of the geometric details of the surface for the different processes such as: freeform deformation, fusion, morphing and other aplications [Sorkine et al. 2004].

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Methods for offset polygon meshes based on the curvature defined by the Laplace Beltrami operator have been developed. These methods allow adjusting shape offset by a constant distance with high enough precision to minimize Hausdorf error. The problem with these methods is the loss of detail caused by smoothing, which depends on the size of the offset [Zhuo and Rossignac 2012]. In volumetric approaches on computing the offset boundary that are based on distance field computation in point-based representation, this methods the topology of the offset model can be different from that the original geometry [Chen and Wang 2011].

[Gal et al. 2009] proposes automatic features detection and shape edition with feature inter-relationship preservation. In analysis step they define salient surface features how ridges and valleys with base on first and second order curvature derivates [Ohtake et al. 2004], and angle-based threshold. In feature characterization step the curves are classified by several properties as planar or non-planar, approximated by line, circle or ellipse shapes, and so on. In edit step the user define initial change over several feature and then this edit is propagated over other features with base in your inter-relationships. This method works fine with objects that have sharp edges composed of basic geometric shapes such as lines, circles or ellipses but this method has difficulties when models are smoother with organic forms and cannot find the features to edit and preserve.

Digital sculpting is divided into two principal methods: based on polygonal methods and voxel grids-based methods. Brushes for inflate operations in polygonal methods only depends on the normal at each vertex [Stanculescu et al. 2011], in grids-based some operations permit add or remove voxels and then have that processing isosurfaces from volume to produce polygonal meshes representation [Galyean and Hughes 1991]. The problem whit this type of operations is the difficult to maintain surface details during larger scale deformation.

2 Laplacian Smooth

The Laplacian Smooth techniques allows you to reduce noise on a mesh's surface with minimal changes on its shape. Computer graphics objects which have been reconstructed from real world, contain undesirable noise. A laplacian smoothing removes undesirable noise while still preserves desirable geometry as well as the shape of the original model.

The functional used in many laplacian smoothing approach to constrain energy minimization is based on a total curvature of a surface S.

$$E(S) = \int_S \kappa_1^2 + \kappa_2^2 dS \tag{1}$$

Where κ_1 and κ_2 are the two principal curvatures of the surface S.

2.1 Gradient of Voronoi Area

Consider a surface S compound by a set of triangles around vertex v_i. We can define the *Voronoi region* of v_i as show in figure 3, The change in area produced by move v_i is named gradient of *Voronoi region [Pinkall et al. 1993; Desbrun et al. 1999; Meyer et al. 2003].*

$$\nabla A = \frac{1}{2}\sum_j (\cot \alpha_j + \cot \beta_j)(v_i - v_j) \tag{2}$$

If we normalize this gradient in equation (2) by the total area in 1-ring around v_i, we have the *discrete mean curvature normal* of a

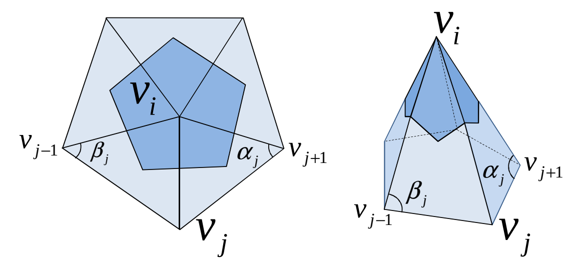

Figure 3: *Area of Voronoi region around v_i in dark blue. v_j 1-ring neighbors around v_i. α_j and β_j opposite angles to edge $\overrightarrow{v_j - v_i}$.*

surface S as shown in equation (3).

$$2\kappa\mathbf{n} = \frac{\nabla A}{A} \tag{3}$$

2.2 Laplace Beltrami Operator

The *Laplace Beltrami operator* LBO denoted $\triangle_g$ is used for measures mean curvature normal of the Surface S [Pinkall et al. 1993].

$$\triangle_g S = 2\kappa\mathbf{n} \tag{4}$$

The LBO has desirable features, one feature of the LBO is in direction of surface area minimization, allowing us to minimize energy using it on a total curvature of a surface S at equation (1).

3 Proposed Method

Our method allow the editing of geometric features using the curvature enhancement and smoothing. Generating a parameterized family of shapes using a set of vertices representing a coarse sketch of the desired model. Our approach can be mixed with traditional or uniform subdivision surfaces methods and is iterative and converges towards a continuous and smooth version of the original model.

Unlike other methods, our method allows to use mixed arbitrary types of mesh representation as triangles and quads, exploiting the basic geometrical relationships facilitating and ensuring convergence of the algorithm and similar shapes consistent with the original shape against the other methods.

Our method allows the use of soft constraints weighting the effect of smoothing at each vertex based on a normalized weight, the weights are assigned to the control vertices of the original mesh or. The weights of the new vertices resulting from the subdivisions are calculated by interpolation, allowing to modify the behavior of the method on exact regions of the original model.

Our approach contain an extension of the Laplace Beltrami operator for meshes composed by triangles and quads. Using meshes composed by triangles and quads has been increasing in recent years due to the flexibility of modeling tools as Blender 3D [Blender-Foundation 2012]. Today many artists manually connecting vertices such that its edition allows simplest way to perform animation processes and interpolation [Mullen 2007]. For these reasons it is very important to develop an operator that allows working with this type of mesh immediately, eliminating the need to preprocess the mesh to convert to triangles and losing the original design made by users.

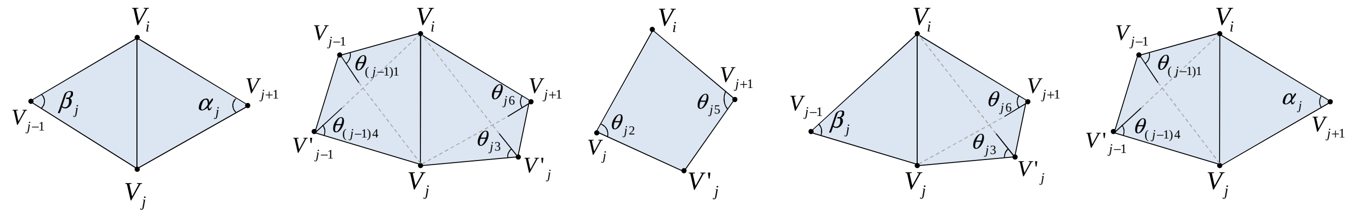

Figure 2: *The 5 basic triangle-quad cases with common vertex V_i and the relationship with V_j and V_j'. (a) Two triangles [Desbrun 1999]. (b) (c) Two quads and one quad [Xiong 2011]. (d) (e) Triangles and quads (TQLBO).*

3.1 Laplace Beltrami operator over triangular and quadrilateral meshes TQLBO

Given a mesh $M = (V, Q, T)$, with vertices V, quads Q, triangles T.

The area of 1-ring neighborhood (N_1) with shared face to vertex v_i in M is.

$$A(v_i) = A\left(Q_{N_1(v_i)}\right) + A\left(T_{N_1(v_i)}\right).$$

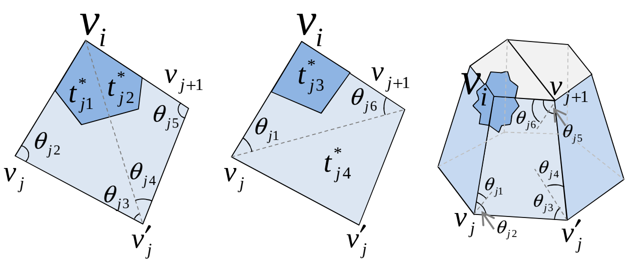

Figure 4: $t_{j1}^* \equiv \triangle\ v_i v_j v_j'$, $t_{j2}^* \equiv \triangle\ v_i v_j' v_{j+1}$, $t_{j3}^* \equiv \triangle\ v_i v_j v_{j+1}$ *Triangulations of the quad with common vertex v_i proposed by [Xiong 2011] to define Mean LBO.*

Appling the mean average area according to [Xiong et al. 2011] of all posible triangulations for each quad to $A\left(Q_{N_1(v_i)}\right)$ as show in figure 4.

$$A(v_i) = \tfrac{1}{2^m}\sum_{j=1}^{m} 2^{m-1} A(q_j) + \sum_{k=1}^{r} A(t_k)$$

Where $q_1, q_2, ..., q_i, ..., q_m \in Q_{N_1(v_i)}$ and $t_1, t_2, ..., t_k, ..., t_r \in T_{N_1(v_i)}$.

$$A(v_i) = \frac{1}{2}\sum_{j=1}^{m}\left[A\left(t_{j1}^*\right) + A\left(t_{j2}^*\right) + A\left(t_{j3}^*\right)\right] + \sum_{k=1}^{r} A(t_k) \quad (5)$$

Applying the gradient operator to (5).

$$\nabla A(v_i) = \frac{1}{2}\sum_{j=1}^{m}\left[\nabla A\left(t_{j1}^*\right) + \nabla A\left(t_{j2}^*\right) + \nabla A\left(t_{j3}^*\right)\right] + \sum_{k=1}^{r} \nabla A(t_k) \quad (6)$$

According to (2), we have.

$$\nabla A\left(t_{j1}^*\right) = \tfrac{\cot\theta_{j3}\left(v_j - v_i\right) + \cot\theta_{j2}\left(v_j' - v_i\right)}{2}$$

$$\nabla A\left(t_{j2}^*\right) = \tfrac{\cot\theta_{j5}\left(v_j' - v_i\right) + \cot\theta_{j4}\left(v_{j+1} - v_i\right)}{2}$$

$$\nabla A\left(t_{j3}^*\right) = \tfrac{\cot\theta_{j6}\left(v_j - v_i\right) + \cot\theta_{j1}\left(v_{j+1} - v_i\right)}{2}$$

$$\nabla A(t_k) = \tfrac{\cot\alpha_k\left(v_k - v_i\right) + \cot\beta_{k+1}\left(v_{k+1} - v_i\right)}{2}$$

All triangles and quads configurations of the 1-neighborhood faces adjacent to v_i can be simplified in five simple cases how show in figure 2.

Then according to equation (3), (4), and five simples cases defined in figure 2 the TQLBO (Triangle-Quad LBO) of v_i is.

$$\triangle_g(v_i) = 2\kappa\mathbf{n} = \frac{\nabla A}{A} = \frac{1}{2A}\sum_{v_j \in N_1(v_i)} w_{ij}\left(v_j - v_i\right) \quad (7)$$

$$w_{ij} = \begin{cases} \left(\cot\alpha_j + \cot\beta_j\right) & \text{case } a. \\ \frac{1}{2}\left(\cot\theta_{(j-1)1} + \cot\theta_{(j-1)4} + \cot\theta_{j3} + \cot\theta_{j6}\right) & \text{case } b. \\ \left(\cot\theta_{j2} + \cot\theta_{j5}\right) & \text{case } c. \\ \frac{1}{2}\left(\cot\theta_{j3} + \cot\theta_{j6}\right) + \cot\beta_j & \text{case } d. \\ \frac{1}{2}\left(\cot\theta_{(j-1)1} + \cot\theta_{(j-1)4}\right) + \cot\alpha_j & \text{case } e. \end{cases} \quad (8)$$

We define a Laplacian operator as a matrix equation

$$L(i,j) = \begin{cases} -\frac{1}{2A_i} w_{ij} & \text{if } j \in N(v_i) \\ \frac{1}{2A_i}\sum\limits_{j \in N(v_i)} w_{ij} & \text{if } i = j \\ 0 & \text{otherwise} \end{cases} \quad (9)$$

Where L is the $n \times n$, n is the number of vertices of a given mesh M, w_{ij} is the TQLBO defined in equation (8), $N(v_i)$ is the 1-ring neighbors with shared face to v_i, A_i is the ring area around v_i.

Normalized version of the TQLBO as a matrix equation

$$L(i,j) = \begin{cases} -\frac{w_{ij}}{\sum\limits_{j \in N(v_i)} w_{ij}} & \text{if } j \in N(v_i) \\ \delta_{ij} & \text{otherwise} \end{cases} \quad (10)$$

Where δ_{ij} being the Kronecker delta function.

3.2 Curvature Enhancing

The curvature enhancing use the change produced by laplacian smoothing in the inverse direction of the curvature flow for moves the vertices in the portions of the mesh with most curvature. In this proccess we use a diffusion proccess:

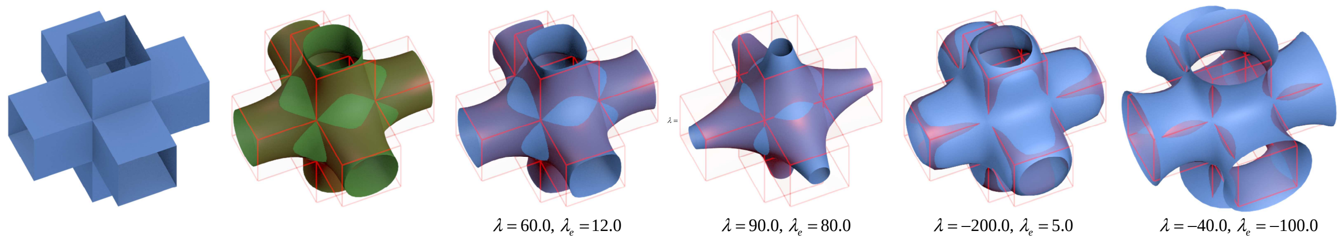

Figure 5: *Left: Original Model, in green color model with Catmull-Clark Subdivision. Models with laplacian smoothing:* $\lambda = 60.0$, $\lambda_e = 12.0$ *and* $\lambda = 90.0$, $\lambda_e = 80.0$. *Models first filter with laplacian smoothing* $\lambda = 60.0$, $\lambda_e = 12.0$ *and before applied curvature enhancing:* $\lambda = -200.0$, $\lambda_e = 12.0$ *and* $\lambda = -40.0$, $\lambda_e = -100.0$.

$$\frac{\partial V}{\partial t} = \lambda L\left(V\right)$$

For solve the equation above we use implicit integration and a normalized version of TQLBO matrix.

$$\left(I - \left|\lambda dt\right| W_p L\right) V' = V^n$$

$$V^{n+1} = V^n + \text{sign}\left(\lambda\right)\left(V' - V^n\right)$$

The vertices $V^{n+1}$are enhance along their inverse curvature normal directions by solving this simple linear system: $Ax = b$, where $A = I - \left|\lambda dt\right| W_p L$, L is the Normalized TQLBO defined in the equation (10), $x = V'$ are the smoothing vertices, $b = V^n$ are the actual vertices positions, W_p is a diagonal matrix with weigth vertex group, sign (x) is the sign function, and λdt is the enhance factor that support negative and positive values, negative for enhancing positive for smoothing.

Our method was designed for use with weigth vertex groups para especificar el grado de afectacion sobre la solucion, los pesos varian entre 0 y 1 con un valor de 0 no realiza ningun cambio y con valores de 1 se aplica el cambio total.

Los pesos pueden ser aplicados sobre el modelo tosco y luego al realizan subdivision estos pesos son suavizados de forma que produce resultados con cambios suaves en las zonas de influencia donde se aplica el laplaciano, en la formaula xy, donde Wp es una matriz diagonal con los pesos correspondientes para cada vertice, ver imagen xyz. Los pesos sobre cada vertice produciran una solucion diferente por esa razon son puestos antes de obtener la solucion del sistema lineal. Las familias que se generan pueden cambiar substancialmente con el ponderamiento de puntos de control especificos.

El manejo de los bordes con operador dependiente de escala

3.3 Sculpting

En nuestro trabajo se diseño una nueva brocha que permite realizar el realce de las curvaturas en un modelo en tiempo real.

Nuestro brocha trabajo bien con el metodo “drag drot” desarrollado en el sistema de sculptin de [Blender-Foundation 2012] el cual permite previsualizar el cambio que se produce en el modelo hasta que se libera el boton del mouse o la tableta, ademas permite mover el mouse a lo largo del modelo para ajustar el lugar exacto donde se desea realizar el realce de la curvatura.

Brochas que realizan trabajos similares como la brocha de inflacion, crean problemas en el modelo pues al aplicarlo se pueden producir autointersecciones del modelo pues este metodo solo mueve los vertices a lo largo de la normal, y no toma en cuenta la informacion global, nuestro metodo en cambio busca la mejor manera de realizar la inflacion mientras conservar las curvaturas que le dan la apariencia caracteristica a ese objeto.

Nuestro metodo simplifica el trabajo que se requiere para realizar el realce que consistiria en usar algunos brocas diferentes algunas para inflar y otras para suavizar y estilizar. con nuestra brocha este tipo de realces se pueden realizar en un solo paso.

Para que la brocha trabaje en tiempo real es necesario que cuando se construya la matriz con los vertices solo sean tomados encuenta aquellos vertices que estan dentro del radio de afectacion definido por el usuario lo cual reduce drasticamente el tamaño de los vertices a procesar, el centro de esta esfera depende del lugar donde el usuario haga click en el canvas y el lugar tridimensional en el modelo donde el click se proyecta. Es necesario ademas realizar una cambio con los vertices que se encuentran en la frontera los cuales tienen vecinos que nos se encuentran dentro del radio de afectacion estos vertices seran marcados como de frontera y en ellos no se calculara el laplaciano perso estaran presentes en el sistema lineal de forma invariante para permitir que los vertices que tengan todos sus vecinos en el interior del radio de afectacion puedan calcular de forma correcta la curvatura, este cambio permite que los resultados sean mucho mas suaves en la frontera.

3.4 Subdivision surfaces

Metodo para generar modelos suaves y continuos desde un bosquejo como catmull clark producen resultados rapidamente debido a la simplicidad de implementacion, el unico problema es que es dificial realizar cambios en la curvatura general del modelo. Si nosotro usamos catmull clark y curvatura enhancing juntos en bosqujos con pocos vertices podemos generar familias de formas con solo la modificacion de un parametro, lo que permitiria a un diseñador escoger el modelo que mas se adapte a sus necesidades sin necesidad que tenga que modificar cada uno de los vertices de control en metodos como catmull clark.

Nuestro metodo tambien permite el uso de weigth vertex paint sobre los puntos de control desarrollado en [Blender-Foundation 2012], al realizar la subdivision estos pesos son interpolados sobre los nuevos vertices de manera que se puede pintar la zona en la cual se desea realzar la curvatura.

The Catmull-Clark subdivision transformation is used to smooth a surface as the limit of sequence of subdivision steps[Stam 1998]. This method do a recursive subdivision transformation that refines the model into a linear interpolation that is a approximate smooth surface. The process of Catmull-Clark is govern by properties of B-spline curve from multivariate spline theory[Loop 1987].

In many subdivision surfaces methods catmull clark loop so on. the smoothness of the model is autmaically guarantteed[DeRose et al. 1998].

Subdivision surfaces with catmull clark is continuos except at a extraordinary points[Loop 1987], but with our method can correct this problem

4 Results

In this section we describe the results of our curvature enhanced method that used our extension of laplace beltrami operator for triangles and quads TQLBO with several example models in figures x, y, z. We test the curvature enhance on a PC with AMD Quad-Core Processor @ 2.40 GHz and 8 GB RAM.

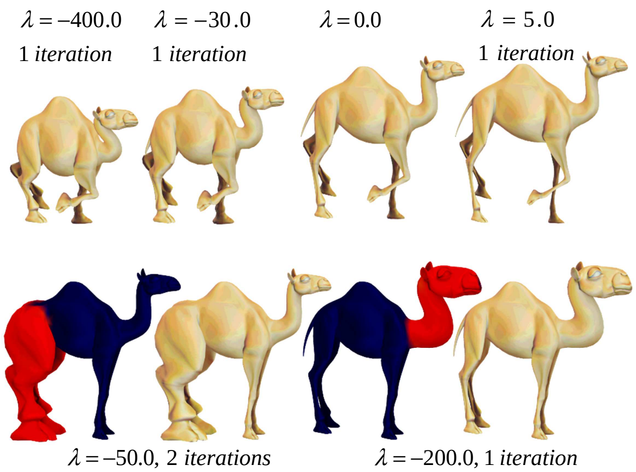

Figure 6: *Top row: Curvatture enhancing with* $\lambda = -400.0$, $\lambda = -30.0$, *Original Model* $\lambda = 0.0$, *Smoothing Model* $\lambda = 5.0$. *Bottom row: Curvature enhancing with weigth vertex group,* $\lambda = -50.0$ *and 2 iterations at legs,* $\lambda = -200.0$ *and 1 iteration in head and neck.*

Figure 6 show the generation of diferent version of camel with the variation of parameter lambda. In the top row you can see results of do curvature enhance over all model, a medida que el lambda se hace mas negativo el modelo tiende a colapsarse. El realce de curvatura con el laplaciano normalizado tiene un comportamiento regular y predecible e invariante frente a transformaciones isometricas como las animaciones de caminatas por ejemplo, pues a medida que se varia el lambda cambia el modelo, entre mas negativo sea mas realce en las curvaturas realiza,

Nosotros realizamos pruebas del operador laplaciano 9 y su version normalizada 10, las dos producen resultados similares si los triangulos que componen la malla son del mismo tama;o en promedio, pero la version normalizada es mucho mas estable y predecible, debido a que no es dividida por el area del anillo que puede producir problemas de calculo debido a erroreres de punto flotante como se observa en la figura 7 (c) bottom row en la cual la malla se deformo pues el TQLBO es suceptible al tamaño de los triangulo. El modelo se puede deformar en la version normalizada de l TQLBO con lambdas grandes > 400 se autointersecta, pero no produce los picos que se observan con el TQLBO .

Se trabajo con blender software blblbsd

se hicieron pruebas de rendimiento asdadasd

Pruebas de catmull clark vs nuestro metodo

pruebas de T Q, y nuestro metodos de Ty Q

Pruebas generando familias de objetos, que involucran y no CatCl.

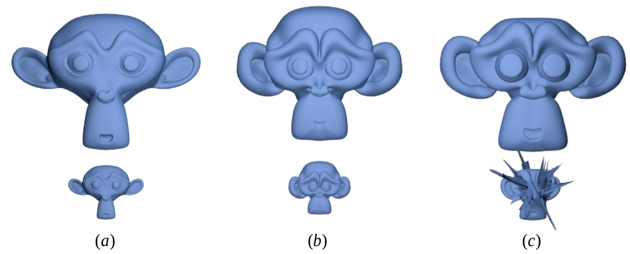

Figure 7: *(a) Top row: Original model scaled by 4. Bottom row: Original Model (b) Top and bottom row: enhancing with Normalized-TQLBO* $\lambda = -50$ *(c) Top and bottom row: enhancing with TQLBO* $\lambda = -50$.

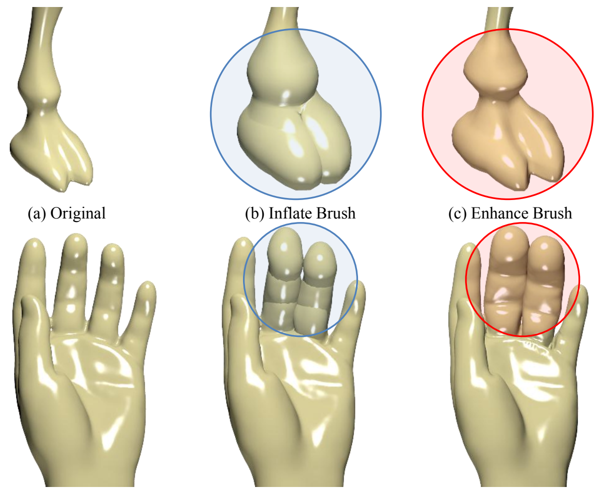

Figure 8: *Top row: (a) Leg Camel, (b) Inflate brush for leg into blue circle, (c) Enhance curvature brush for leg into red circle. Bottom row: (a) Hand, (b) Inflate brush for fingers into blue circle, (c) Enhance curvature brush for fingers in red circle.*

4.1 Implementation

We implementer our algorithm in C and C++ on the blender platform version 2.56[Blender-Foundation 2012],

c y c++ blender.

4.2 Sparse linear system

superlu opennl

5 Conclusion and future work

optimizacion del metodo de solucion

aplicarlo en otras areas

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