Title: Shape Enhanced With and Adapted Laplacian Operator For Hybrid Quad/Triangle Meshes

Reviewer #3:

1) **Description**

   This paper describes a Laplace-Beltrami operator for mixed quadrilateral-triangle meshes. It also describes the use of this operator for smoothing and frequency enhancement (negative smoothing). These are nice modeling techniques - unfortunately, they have already been developed in previous work.

2) **Clarity of Exposition**

   The exposition (flow) is relatively clear. The english (grammar), however, could use another pass. It was often difficult to understand some sentences not because the concept was difficult or poorly explained, but simply because the english was incorrect.

3) **Quality of References**

   An important reference is missing - "Multiresolution Signal Processing for Meshes" by Igor Guskov, Wim Sweldens and Peter Schroeder (SIGGRAPH '99) covers most of these concepts including enhancement of meshes.

4) **Reproducibility**

   The work could easily be reproduced given the discussion in the paper. I'd like to see the limitation regarding numerical stability discussed more as curvature enhancement (negative curvature flow) is inherently an unstable process. The current discussion of normalized vs unnormalized Laplacian operators doesn't seem to examine this deeply enough.

5) **Rating 2.0**

6) **Explanation of Rating**

   This paper presents some nice modeling techinques based on a Laplace Beltrami operator. These modeling techniques are quite useful in practice. Unfortunately, similar operations and operators have been examined in previous work. Therefore, I do not believe that this paper has enough contribution for SIGGRAPH - and I recommend rejection.

   One contribution of the paper is the mixed quad-triangle Laplace Beltrami operator. The triangle version has been previously proposed [Pinkall, Desbrun, Meyer]. The quad version is a relatively straightforward extension based on the triangle operator and was previously presented in [Xiong 2011]. The mixed quad-triangle version is simply applying the triangle rule on triangles and the quad rule on quads - and as such I would not consider it a major contribution.

The smoothing and enhancement using the Laplace Beltrami operator is a straightforward application of the operator in known curvature flow equations. This type of mesh modification is well known and has been presented in previous work for smoothing [Desbrun, Meyer, Guskov] as well as enhancement [Guskov]. In particular, "Multiresolution Signal Processing for Meshes" by Guskov et al describes these smoothing and enhancement techniques for meshes - reducing the novelty and contribution of this portion of the paper.

As the operator is a straightforward combination of prior art, and the smoothing and enhancement operations have been detailed in previous works, the contribution of this paper is not enough for acceptance to SIGGRAPH.

7) **Detailed questions:**

- For the quads, why don't you simply use the bilinear function of the quad to compute the laplace beltrami operator?

- On page 6, you claim that "... suitable in real applications since an artist sculpts a model ... and each part is represented by an average of about 1800 vertices". I would probably reduce this claim to "what the artists you have observed using" - as many artists in production are sculpting and manipulating models with much higher than 1800 vertices per part on a daily basis.

- Figure 11c: What is supposed to be shown in the top versus the bottom? I understand that the process was unstable - producing the bottom image, but what is different that produced the top?

- You may want to discuss and analyze stability a bit more as negative curvature flow is inherently an unstable process. You should look at the valid time step sizes for both the normalized and unnormalized versions (noting that the unnormalized is simply a scale on lambda if all neighborhood areas are approximately the same).

Reviewer #33:

1) **Description**

   This paper introduces a Laplace-Beltrami operator for mixed quad-triangle meshes and shows applications to shape smoothing, shape enhancement, and brush-assisted sculpting. Additionally, a combination of Catmull-Clark subdivision and shape enhancement is presented.

   Unfortunately, the paper does not provide significant contributions in the view of this reviewer. Partially, this is due to missing comparisons to previous work for constructing LBOs on mixed quad triangle meshes, such as the approach proposed in Alexa/Wardetzky (2011). Additionally, I did not find the applications convincing enough -- they appear to be a somewhat random collection of what one can do when equipped with a Laplacian.

2) **Clarity of Exposition**

   The exposition is mostly clear.

3) **Quality of References**

   Serious omission: Alexa/Wardetzky "Discrete Laplacians on General Polygonal Meshes".

4) **Reproducibility**

   Limitations are hardly discussed. Most seriously, I would expect that the inverse curvature flow (used for enhancement) is numerically unstable. More details are required to support stability.

5) **Rating 2.0**

6) **Explanation of Rating**

   I don't think that the paper is ready yet for publication. If you consider to re-submit at some future time, then please add thorough comparisons to the LBO considered in Alexa/Wardetzky and provide evidence that the inverse curvature flow is sufficiently stable in practice.

Reviewer #87:

1) **Description**

The paper proposes a discrete Laplace-Beltrami operator for meshes composed of both triangles and quads. Using this discrete Laplacian, some mesh editing operations are explored, such as shape inflation and smoothing (and subdivision).

A Laplacian for general polygonal meshes is a useful discreet tool to have in order to enjoy all the geometry processing techniques developed for triangle meshes using discrete Laplace operators. Especially since quad meshes (or quad-triangle meshes) are popular with artists and designers, extending all the triangle mesh editing techniques to such meshes is useful. Unfortunately the submission omits previous work on discrete Laplacians on polygonal meshes, so the magnitude of the contribution and the improvement over previous work is unclear.

2) **Clarity of Exposition**

The introduction section lacks focus - it is not clear what the main goal, or the solved problem is, until one reaches the contribution section. I also find the term "shape enhancement" confusing, because to me, enhancement means improvement, whereas I think the paper means just shape inflation (perhaps improved inflation).

3) **Quality of References**

The paper misses the following work:
Discrete Laplacians on General Polygonal Meshes
Marc Alexa and Max Wardetzky
SIGGRAPH 2011.

That work defined a discrete Laplacian for quads, or mixed quad-triangle meshes and general polygonal meshes.

Also, I was missing references to standard finite element works - probably, a discretization of the Laplace operator over polygonal finite elements is well possible within the FEM framework.

Additional references for the modeling context mentioned in the paper:
Sketch-based modeling:
FiberMesh: Designing Freeform Surfaces with 3D Curves
SIGGRAPH 2007 Nealen et al.
Modeling using curvature-based optimization:
Positional, Metric, and Curvature Control for Constraint-Based Surface Deformation
EUROGRAPHICS 2009
Eigensatz et al.

Feature-Based Mesh Editing
EUROGRAPHICS 2011 short paper
Zhou et al.
Subdivision surface modeling using Laplacian smoothing: the seminal paper of Taubin should be cited:
A Signal Processing Approach To Fair Surface Design
SIGGRAPH 1995

4) Reproducibility

The paper can be reproduced. The limitations are not entirely clear, in particular the convergence properties of the proposed Laplacian, and its mesh-independence. Can something be said about maintaining the overall shape resulting from the proposed modeling operations if we refine the mesh?

5) Rating 2.9

6) Explanation of Rating

The main contribution of the paper is the hybrid Laplace operator for quad-triangle meshes, relying on the averaging construction of Xiong et al. The editing operations relying on this operator are only secondary contributions, because very similar things have been already described, perhaps only in the context of triangle meshes (for example, the inflation, or "enhancement", is a variant of the lambda-mu process proposed by Taubin in his 1995 paper).

Unfortunately, the paper does not mention nor compare the proposed operator to that of Wardetzky and Alexa. The latter operator is more general (any polygonal mesh) and fulfills certain properties one expects from a discrete Laplacian. The currently proposed operator should be analyzed in that respect, and it would be nice to know something about the convergences properties. Otherwise, one could continue relying on the well-studied operators for triangle meshes, convert the input quad-triangle mesh to triangle mesh for the purpose of the editing operation, and then convert back to the original connectivity. After all, if only the geometry is edited.

Reviewer #26:

1) **Description**

This paper proposes an extension of the Laplace operator that supports hybrid meshes comprised of both quads and triangles and describes its application in the context of shape modification via geometry sharpening.

2) **Clarity of Exposition**

The paper was somewhat difficult to follow.

Independent of local grammatical errors:

The authors repeatedly refer to the editing of curvature, but I do not believe that's not precise. Biharmonic flow can be related to minimizing curvature, but I do not believe that's the case for harmonic/MCF flow on (closed) surfaces.

Do the authors mean "boundary" when they say "silhouette"?

Line 84: Hausdorff error of what?

Line 95: Why is Ohtake the source for curvature derivatives?

Equation 1: How is this related to Laplacian flows?

Line 132: the "g" in Δ_g likely refers to the fact that the Laplace-Beltrami operator is defined in terms of the metric on the manifold, but g is never defined.

Line 136: The LBO does not minimize the total surface curvature.

Figure 2(c): Why is this quad not split into the different triangles containing V_i?

Line 190: What is a "weight vertex group"?

Line 225: I did not see how the method proposed here avoids self-intersection.

Line 249: Why is Stam the reference for Catmull-Clark subdivision?

In Section 4.4: What is the motivation for smoothing the surface? Isn't the whole point to increase the sharpness?

3) **Quality of References**

With regards to extending the Laplacian, the authors should reference/discuss the work of Alexa et al. "Discrete Laplacians on General Polygonal Meshes".

With regards to sharpening, the approach seems very similar to the method proposed by Chuang et al. "Interactive and Anisotropic Geometry Processing Using the Screened Poisson Equation". In particular, one can reformulate the proposed method as a screened Poisson equation:

To discretize the PDE the authors use:

$d \phi_t / dt = (1-\lambda)\Delta \phi_t$

one can write out:

$\phi_{t+1} - \phi_t \approx \Delta(\phi_{t+1} - \lambda\phi_t)$

where the derivative is approximated by the difference and the instantaneous value ϕ_t is expressed as the average of the values at the two time-steps. Then one gets:

$(1-\Delta)\phi_{t+1} = (1-\lambda\Delta)\phi_t$

which is essentially the screened-Poisson equation solved by the authors. In their work, Chuang et al. describe a system that supports the type of editing the authors describe, including general smoothing/sharpening and localization of editing constraints.

4) Reproducibility

Seems so.

5) Rating   1.5

6) Explanation of Rating

Given the related work in this area, it is difficult for me to see where the contribution of this work lies.

From the perspective of defining a general Laplace-Beltrami operator, the authors need to describe how their method differs from that of Alexa et al. (which I believe also uses the area gradient to define a Lapalcian).

From the perspective of surface editing, the authors need to describe how their approach differs from that of Chuang et al. (which I believe is more general as it also supports anisotropy).

Reviewer #73:

1) **Description**

The paper describes a method for deformation of triangular or quad meshe based on heat-flow. The idea is to mark a region of interest and to perform either forward or backward heat flow using some chosen discretization of the Laplace-Beltrami operator.

Using the heat flow for mesh processing is common practice by now and the contribution offered by this paper is rather incremental. Furthermore, relevant previous works are not discussed, cited or compared to.

2) **Clarity of Exposition**

The paper is hard to read and parse.

3) **Quality of References**

The work by Ming and Kazhdan from Siggraph 2011 "Interactive and anisotropic geometry processing using the screened Poisson equation" uses the screened Poisson equation to build operators similar to the ones discussed in this paper.

Another relevant work is "Spectral Geometry Processing with Manifold Harmonics" by Bruno Vallet and Bruno Lévy.

4) **Reproducibility**

OK.

5) **Rating**   **2.2**

6) **Explanation of Rating**

The paper suggests performing inverse time heat-flow for mesh enhancement. This process is equivalent to certain filtering in the spectral domain. There are several papers that used filters on the spectral domain of the Laplace-Beltrami operator to achieve similar effects but are not discussed in this paper. I give two examples above. The contribution of this paper over these previous works seems rather incremental.