Laplacian Subdivision Surfaces

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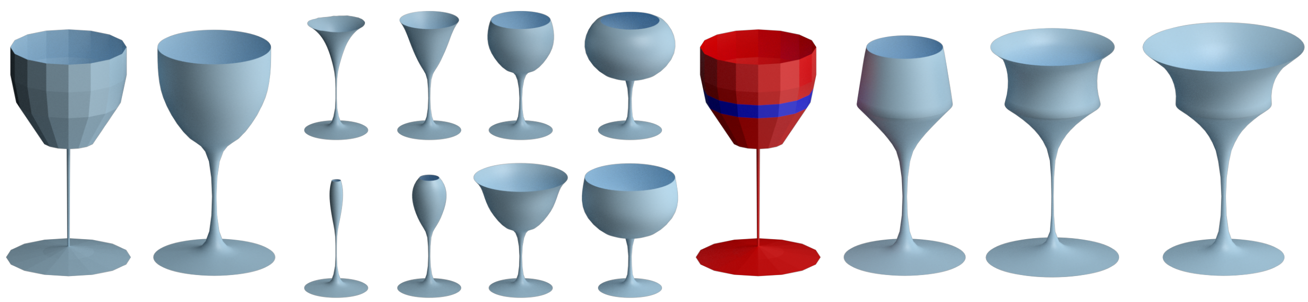

Figure 1: *Laplacian subdivision surface with weigth vertex group*

Abstract

This paper proposes a novel method for modelling poligonal mesh using subdivision surface and laplacian smoothing. This method use laplacian smooth for modelling global curvature in the model, to permit most flexible, robust and predictable results.

This method can correct traditional problems in extraordinary vertices present at catmull-clark subdivision method. The convergent rate of the laplacian smooth can be controlled by adjusting the weight in lambda parameter.

This proposal contains NN novel features

ESTE ABSTRACT NO SIRVE

(**we provide a series of examples to graphically and numerically demonstrate the quality of our results.**) debe ser cambiado copiado de desbrun 99

CR Categories: I.3.5 [Computer Graphics]: Computational Geometry and Object Modeling —Modeling packages

Keywords: laplacian smooth, subdivision surface

1 Introduction

The polygon meshes are used to represent real-world objects in three-dimensional space, these objects are captured with scanners or created with novel techniques of modeling that can generate a variety of shapes to look natural and realistic [Botsch2006]. Editing techniques have evolved from affine transformations to advanced tools like scuplting [Coquillart1990, Galyean1991, Stanculescu2011], editing and creation from sketches [Igarashi1999, Gonen2012], complex interpolation techniques [Sorkine2004, Zhou2005], and so on .

Traditional methods for smooth surfaces from coarse geometry like Catmull-Clark have been widely developed [Catmull and Clark 1978; Stam 1998], these works generalize uniform B-cubic splines knot insertion to meshes, some of them add control of the results with the use of creases to produce sharp edges [DeRose et al. 1998], or the modification of weights on the vertices that control locally the zone of influence [Biermann et al. 2000], instead our method performs a feature enhancement of the model allowing parameterize the curvature of the surface creating a family of different versions of the same object preserving detail and realistic natural look of the original model.

Many types of brushes have been developed to sculpt meshes, brushes that perform inflation lose detail when inflating the vertices [Stanculescu et al. 2011], our method allows inflation of the mesh vertices moving in the opposite direction to the curvature preserving the shape and sharp features of the model.

We present an extension of the Laplace Beltrami operator for meshes of arbitrary topology composed for triangles and quads representing a larger spectrum of mesh that works with today eliminating the need for preprocessing.

1.1 Previous work

Many tools have been developed for modeling based on Laplacian mesh processing. Thanks to the kindness of the Laplacian operator these tools have in common the need for preservation of the geometric details of the surface for the different processes such as: freeform deformation, fusion, morphing and other aplications [Sorkine et al. 2004].

Methods for offset, and dilation of polygon meshes based on the curvature defined by the Laplace Beltrami operator have been developed. These methods allow adjusting shape offset by a constant distance with high enough precision to minimize Hausdorf error. The problem with these methods is the loss of detail caused by smoothing, which depends on the size of the offset [Zhuo and Rossignac 2012]. In volumetric approaches on computing the offset boundary that are based on distance field computation in point-based representation, this methods the topology of the offset model can be different from that the original geometry [Chen and Wang 2011].

[Gal et al. 2009] proposes automatic features detection and shape edition with feature inter-relationship preservation. In analysis step they define salient surface features how ridges and valleys with base on first and second order curvature derivates [Ohtake et al. 2004], and angle-based threshold. In feature characterization step the curves are classified by several properties as planar or non-

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planar, approximated by line, circle or ellipse shapes, and so on. In edit step the user define initial change over several feature and then this edit is propagated over other features with base in your interrelationships. This method works fine with objects that have sharp edges composed of basic geometric shapes such as lines, circles or ellipses but this method has difficulties when models are smoother with organic forms and cannot find the features to edit and preserve.

1.2 Overview of our method

Nuestro metodo usa el un bosquejo de maya le aplica una subdivision cualquiera y esa subdivision la modifica a lo largo de su curvatura de flujo usando un operador laplaciano para triangulos y cuadrados

Nuestro metodos propone tre cosas muy novedosas:

Operador Laplace beltrami para mallas de topologia arbitraria formadas por triangulos y cuadrados, para cualquier tipo de procesamiento en geometria diferencial

Permite generar una familia de formas parametrizadas.

Controlar el nivel de suavizado y curvatura al subdividir mallas de poligonos

Enhanced brush for sculpting modelling

2 Laplacian Smooth

The Laplacian Smooth techniques allows you to reduce noise on a mesh's surface with minimal changes on its shape. Computer graphics objects which have been reconstructed from real world, contain undesirable noise. A laplacian smoothing removes undesirable noise while still preserves desirable geometry as well as the shape of the original model.

The functional used in many laplacian smoothing approach to constrain energy minimization is based on a total curvature of a surface S.

$$E\left(S\right)=\int_{S}\kappa_1^2+\kappa_2^2 dS \tag{1}$$

Where κ_1 and κ_2 are the two principal curvatures of the surface S.

2.1 Gradient of Voronoi Area

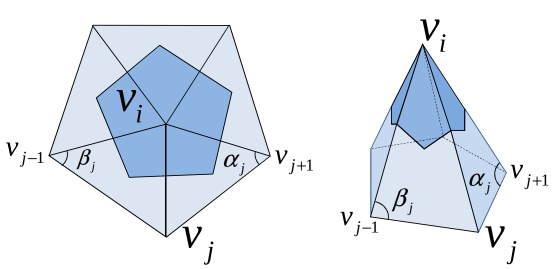

Figure 2: *Area of Voronoi region around v_i in dark blue. v_j 1-ring neighbors around v_i. α_j and β_j opposite angles to edge $\overrightarrow{v_j - v_i}$.*

Consider a surface S compound by a set of triangles around vertex v_i. We can define the *Voronoi region* of v_i as show in figure 2, The change in area produced by move v_i is named gradient of *Voronoi region [Pinkall et al. 1993; Desbrun et al. 1999; Meyer et al. 2003].*

$$\nabla A=\frac{1}{2}\sum_j\left(\cot\alpha_j+\cot\beta_j\right)\left(v_i-v_j\right) \tag{2}$$

If we normalize this gradient in equation (2) by the total area in 1-ring around v_i, we have the *discrete mean curvature normal* of a surface S as shown in equation (3).

$$2\kappa\mathbf{n}=\frac{\nabla A}{A} \tag{3}$$

2.2 Laplace Beltrami Operator

The *Laplace Beltrami operator* LBO denoted $\triangle_g$ is used for measures mean curvature normal of the Surface S [Pinkall et al. 1993].

$$\triangle_g S=2\kappa\mathbf{n} \tag{4}$$

The LBO has desirable features, one feature of the LBO is in direction of surface area minimization, allowing us to minimize energy using it on a total curvature of a surface S equation (1).

3 Proposed Method

Our method simplifies the design of irregular polygon meshes, generating a parameterized family of shapes using a set of vertices representing a coarse sketch of the desired model. Our method is iterative and converges towards a continuous and smooth version of the original model.

Unlike other methods, our method allows use arbitrary topologys types of representation as triangles and quads, exploiting the basic geometrical relationships facilitating and ensuring convergence of the algorithm and similar shapes consistent with the original shape against the other methods.

Our method allows the use of soft constraints weighting the effect of smoothing at each vertex based on a normalized weight, the weights are assigned to the control vertices of the original mesh or. The weights of the new vertices resulting from the iterations are calculated by interpolation, allowing to modify the behavior of the method on exact regions of the original model.

3.1 Laplace Beltrami operator over triangular and quadrilateral meshes TQLBO

Given a mesh $M=(V,Q,T)$, with vertices V, quads Q, triangles T.

The area of 1-ring neighborhood (N_1) with shared face to vertex v_i in M is.

$$A\left(v_i\right)=A\left(Q_{N_1\left(v_i\right)}\right)+A\left(T_{N_1\left(v_i\right)}\right).$$

Appling the mean average area according to [Xiong et al. 2011] of all posible triangulations for each quad to $A\left(Q_{N_1\left(v_i\right)}\right)$ as show in figure 4.

$$A\left(v_i\right)=\tfrac{1}{2^m}\sum_{j=1}^{m}2^{m-1}A\left(q_j\right)+\sum_{k=1}^{r}A\left(t_k\right)$$

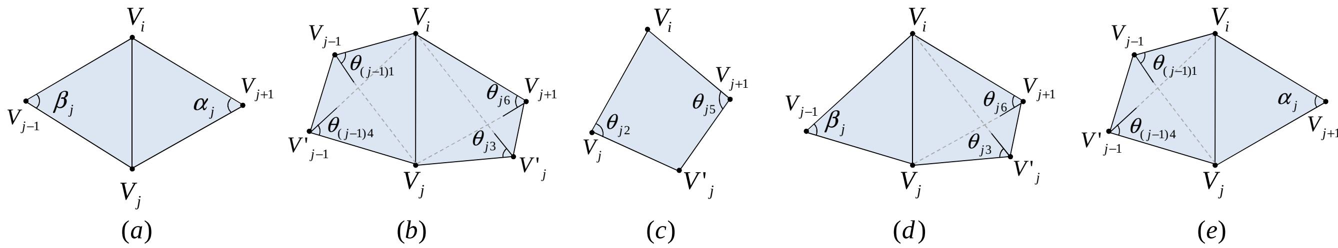

Figure 3: *The 5 basic triangle-quad cases with common vertex V_i and the relationship with V_j and V'_j. (a) Two triangles [Desbrun 1999]. (b) (c) Two quads and one quad [Xiong 2011]. (d) (e) Triangles and quads (TQLBO).*

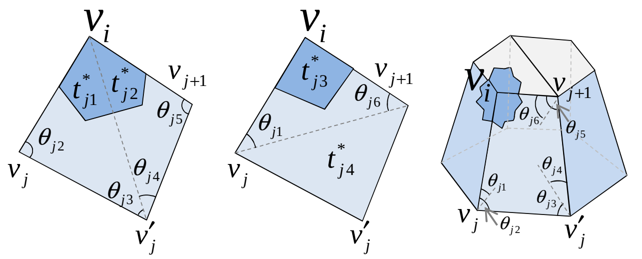

Figure 4: $t^*_{j1} \equiv \triangle\ v_i v_j v'_j$, $t^*_{j2} \equiv \triangle\ v_i v'_j v_{j+1}$, $t^*_{j3} \equiv \triangle\ v_i v_j v_{j+1}$ *Triangulations of the quad with common vertex v_i proposed by [Xiong 2011] to define Mean LBO.*

Where $q_1, q_2, ..., q_i, ..., q_m \in Q_{N_1(v_i)}$ and $t_1, t_2, ..., t_k, ..., t_r \in T_{N_1(v_i)}$.

$$A(v_i) = \frac{1}{2}\sum_{j=1}^{m}\left[A\left(t^*_{j1}\right) + A\left(t^*_{j2}\right) + A\left(t^*_{j3}\right)\right] + \sum_{k=1}^{r} A(t_k) \quad (5)$$

Applying the gradient operator to (5).

$$\nabla A(v_i) = \frac{1}{2}\sum_{j=1}^{m}\left[\nabla A\left(t^*_{j1}\right) + \nabla A\left(t^*_{j2}\right) + \nabla A\left(t^*_{j3}\right)\right] + \sum_{k=1}^{r} \nabla A(t_k) \quad (6)$$

According to (2), we have.

$$\begin{aligned}
\nabla A\left(t^*_{j1}\right) &= \tfrac{\cot\theta_{j3}\left(v_j - v_i\right) + \cot\theta_{j2}\left(v'_j - v_i\right)}{2} \\
\nabla A\left(t^*_{j2}\right) &= \tfrac{\cot\theta_{j5}\left(v'_j - v_i\right) + \cot\theta_{j4}\left(v_{j+1} - v_i\right)}{2} \\
\nabla A\left(t^*_{j3}\right) &= \tfrac{\cot\theta_{j6}\left(v_j - v_i\right) + \cot\theta_{j1}\left(v_{j+1} - v_i\right)}{2} \\
\nabla A(t_k) &= \tfrac{\cot\alpha_k(v_k - v_i) + \cot\beta_{k+1}\left(v_{k+1} - v_i\right)}{2}
\end{aligned}$$

All triangles and quads configurations of the 1-neighborhood faces adjacent to v_i can be simplified in 5 simple cases how show in figure 3.

Then according to (3), (4), and 5 simples cases defined in figure 3 the TQLBO (Triangle-Quad LBO) of v_i is.

$$\triangle_g(v_i) = 2\kappa\mathbf{n} = \frac{\nabla A}{A} = \frac{1}{2A}\sum_{v_j \in N_1(v_i)} w_{ij}\left(v_j - v_i\right) \quad (7)$$

$$w_{ij} = \begin{cases}
(\cot\alpha_j + \cot\beta_j) & \text{case } a. \\
\frac{1}{2}\left(\cot\theta_{(j-1)1} + \cot\theta_{(j-1)4} + \cot\theta_{j3} + \cot\theta_{j6}\right) & \text{case } b. \\
(\cot\theta_{j2} + \cot\theta_{j5}) & \text{case } c. \\
\frac{1}{2}\left(\cot\theta_{j3} + \cot\theta_{j6}\right) + \cot\beta_j & \text{case } d. \\
\frac{1}{2}\left(\cot\theta_{(j-1)1} + \cot\theta_{(j-1)4}\right) + \cot\alpha_j & \text{case } e.
\end{cases}$$

3.1.1 TQLBO over triangular and quadrilateral meshes

$$L(i,j) = \begin{cases}
-\frac{w_{ij}}{\sum\limits_{j \in N(v_i)} w_{ij}} & \text{if } j \in N(v_i) \\
\delta_{ij} & \text{otherwise}
\end{cases}$$

Where $N(v_i)$ is the 1-ring neighbors with shared face to v_i, and δ_{ij} being the Kronecker delta function.

3.1.2 Laplacian Enhancement

El proceso de realce de los detalles de la superfice de malla aplicando el operador laplaciano que mueve los vertices a lo largo de la direccion de la curvatra normal.

realzar las caractersisticas, usando el cambio producido por la ecuacion de difusion

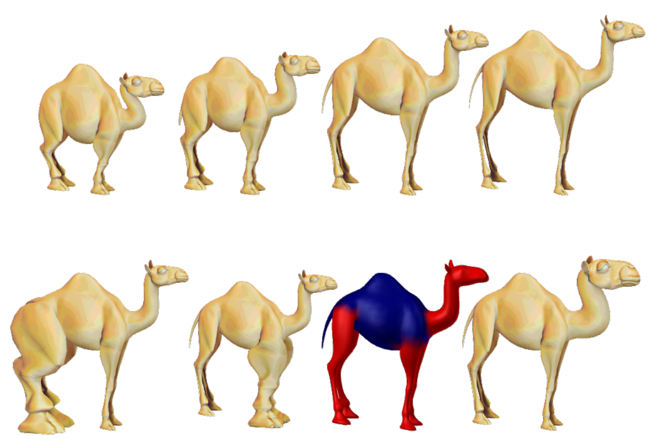

Figure 5: *FGHFHFG*

3.2 Boundaries scale-dependent

El manejo de los bordes con operador dependiente de escala

3.3 Anti-shrinking fairing - Volume preservation

Preservacion de volumen en el centroide funciona mejor que lo propuesto por desbrum 99

The diffusion process, induce shrinkage [Desbrun et al. 1999].

3.4 Weight based somooth constraints

Las familias que se generan pueden cambiar substancialmente con el ponderamiento de puntos de control especificos

3.5 Subdivision Surface

Subdivision is an iterated transformation [Warren and Weimer 2001]. Let F be a function (subdivision transformation) that maps one geometry M_i into another similar geometry with same topology M_{i+1}.

$$M_{i+1} = F(M_i) \tag{8}$$

The Catmull-Clark subdivision transformation is used to smooth a surface as the limit of sequence of subdivision steps[Stam 1998]. This method do a recursive subdivision transformation that refines the model into a linear interpolationthat is a approximate smooth surface. The process of Catmull-Clark is govern o properties of B-spline curve from multivariate spline theory[Loop 1987].

In many subdivision surfaces methods catmull clark loop so on. the smoothness of the model is autmaically guarantteed[DeRose et al. 1998].

Subdivision surfaces with catmull clark is continuos except at a extraordinary points[Loop 1987], but with our method can correct this problem

4 Experimental Results and Applications

Se trabajo con blender software blblbsd

se hicieron pruebas de rendimiento asdadasd

Pruebas de catmull clark vs nuestro metodo

pruebas de T Q, y nuestro metodos de Ty Q

Pruebas generando familias de objetos, que involucran y no CatCl.

4.1 Implementation

c y c++ blender.

4.2 Sparse linear system

superlu opennl

5 Conclusion and future work

optimizacion del metodo de solucion

aplicarlo en otras areas

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