

# Adapted Laplacian Operator For Hybrid Quad/Triangle Meshes

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# Adapted Laplacian Operator For Hybrid Quad/Triangle Meshes

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## Dedication

Su uso es opcional y cada autor podrá determinar la distribución del texto en la página, se sugiere esta presentación. En ella el autor dedica su trabajo en forma especial a personas y/o entidades.

Por ejemplo:

A mis padres  
o

La preocupación por el hombre y su destino siempre debe ser el interés primordial de todo esfuerzo técnico. Nunca olvides esto entre tus diagramas y ecuaciones.

Albert Einstein

# Acknowledgement

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## Abstract

This thesis proposes a novel modeling method for a hybrid quad/triangle mesh that allows to set a family of possible shapes by controlling a single parameter, the global curvature. The method uses an original extension of the Laplace Beltrami operator that efficiently estimates a curvature parameter which is used to define an inflated shape after a particular operation performed in certain mesh points. Along with the method, this work presents new applications in sculpting and modeling, with subdivision of surfaces and weight vertex groups. A series of graphics examples demonstrates the quality, predictability and flexibility of the method in a real production environment with software Blender.

## Resumen

Es el mismo resumen pero traducido al inglés. Se debe usar una extensión máxima de 12 renglones. Al final del Abstract se deben traducir las anteriores palabras claves tomadas del texto (máximo 3 y máximo 7 palabras), llamadas keywords. Es posible incluir el resumen en otro idioma diferente al español o al inglés, si se considera como importante dentro del tema tratado en la investigación, por ejemplo: un trabajo dedicado a problemas lingüísticos del mandarín seguramente estará mejor con un resumen en mandarín.

**Keywords:** laplacian smooth; curvature; sculpting; subdivision surface

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# 1 Introduction

Over the last years several, modeling techniques able to generate a variety of realistic shapes, have been developed [3]. Editing techniques have evolved from affine transformations to advanced tools like sculpting [6, 10, 19], editing, creation from sketches [12, 11], and complex interpolation techniques [17, 20]. Catmull-Clark based methods however require to interact with a minimum number of control points for any operation to be efficient, or in other words, a unicity condition is introduced by demanding a smooth surface after any of these shape operations. Hence, traditional modeling methods for subdividing surfaces from coarse geometry have become widely popular [4, 18]. These works have generalized a uniform B-cubic spline knot insertion to meshes, some of them adding some type of control, for instance with the use of creases to produce sharp edges [7], or the modification of some vertex weights to locally control the zone of influence [1]. Nevertheless these methods are difficult to deal with since they require a large number of parameters and a very tedious customization. Instead, the presented method requires a single parameter that controls the global curvature, which is used to maintain realistic shapes, creating a family of different versions of the same object and therefore preserving the detail of the original model and a realistic appearance.

Interest in meshes composed of triangles and quads has lately increased because of the flexibility of modeling tools such as Blender 3D [2]. Nowadays, many artists use a manual connection of a couple of vertices to perform animation processes and interpolation [14]. It is then of paramount importance to develop operators that easily interact with such meshes, eliminating the need of preprocessing the mesh to convert it to triangles. The shape inflation and shape exaggeration can thus be used as such brush in the sculpting process, when inflating a shape since current brushes end up by losing detail when moving vertices [19]. In contrast, the presented method inflates a mesh by moving the vertices towards the reverse curvature direction, conserving the shape and sharp features of the model.

**Contributions** This work presents an extension of the Laplace Beltrami operator for hybrid quad/triangle meshes, representing a larger mesh spectrum from what has been presented so far. The method eliminates the need of preprocessing and allows preservation of the original topology. Likewise, along with this operator, it is proposed a method to generate a family of parameterized shapes, in a robust and predictable way. This method enables customization of the smoothness and curvature, obtained during the subdivision surfaces

process. Finally, it is proposed a new brush for inflating the silhouette mesh features in modeling and sculpting.

This work is organized as follows: Section \ref{sub:1.1-Related-work} presents works related to the Laplacian mesh processing, digital sculpting, and offsetting methods for polygonal meshes; In section \ref{sec:Laplacian-Smooth} , it is described the theoretical framework of the Laplacian operator for polygon meshes; In section \ref{sec:Proposed-Method}, it is presented the method for shape inflation and applications of subdivision of surfaces and sculpting; finally some Laplacian operator results, to hybrid quad/triangle meshes are graphically shown as well as results of the shape inflation applications in sculpting, subdivision and modeling.



## 2 Related work

Many tools have been developed for modeling, based on the Laplacian mesh processing. Thanks to the advantages of the Laplacian operator, these different tools preserve the surface geometric details when using them for different processes such as free-form deformation, fusion, morphing and other applications [17].

Offset methods for polygon meshing, based on the curvature defined by the Laplace Beltrami operator, have been developed. These methods adjust the shape offset by a constant distance, with enough precision. Nevertheless, these methods fail to conserve sufficient detail because of the smoothing, a crucial issue which depends on the offset size [21]. In volumetric approaches, in case of point-based representations, the offset boundary computation is based on the distance field and therefore when calculating such offset, the topology of the model may be different to the original [5].

[9] propose automatic feature detection and shape edition with feature inter-relationship preservation. They define salient surface features like ridges and valleys, characterized by their first and second order curvature derivatives, see [15], and angle-based threshold. Likewise, curves have been also classified as planar or non-planar, approximated by lines, circles, ellipses and other complex shapes. In such case, the user defines an initial change over several features which is propagated towards other features, based on the classified shapes and the inter-relationships between them. This method works well with objects that have sharp edges, composed of basic geometric shapes such as lines, circles or ellipses. However, the method is very limited when models are smooth since it cannot find the proper features to edit.

Digital sculpting have been traditionally approached either under a polygonal representation or a voxel grid-based method. Brushes for inflation operations only depend on the vertex normal [19]. In grid-based sculpting, some other operations have allowed to add or remove voxels since production of polygonal meshes require a processing of isosurfaces from a volume [10]. The drawback comes from the difficulty of maintaining the surface details during larger scale deformations.

### 3 Laplacian Smooth

Computer objects, reconstructed from the real world, are usually noisy. Laplacian Smooth techniques allow a proper noise reduction on the mesh surface with minimal shape changes, while still preserving a desirable geometry as well as the original shape.

Many smoothing Laplacian functionals regularize the surface energy by controlling the total surface curvature  $S$ .

$$E(S) = \int_S \kappa_1^2 + \kappa_2^2 dS \quad (3-1)$$

Where  $\kappa_1$  and  $\kappa_2$  are the two principal curvatures of the surface  $S$ .

#### 3.1 Gradient of Voronoi Area

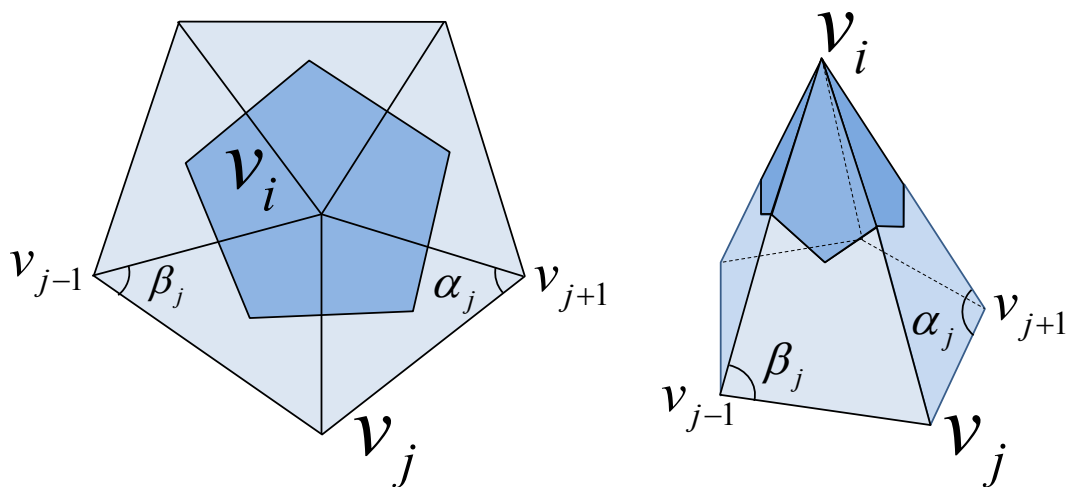


Figure 3-1: Area of the Voronoi region around  $v_i$  in dark blue.  $v_j$  belong to the first neighborhood around  $v_i$ .  $\alpha_j$  and  $\beta_j$  opposite angles to edge  $\overrightarrow{v_j - v_i}$ .

Consider a surface  $S$  composed of a set of triangles around vertex  $v_i$ . Let us define the *Voronoi Region* of  $v_i$  as show in figure **3-1**, The area change produced by the movement of  $v_i$  is called the gradient of *Voronoi region* [16, 8, 13].

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