Laplacian Subdivision Surface

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Outline

Laplacian smooth

The functional used in many laplacian smoothing approach to constrain energy minimization is based on a total curvature of a surface S.

$$E(S) = \int_S \kappa_1^2 + \kappa_2^2 dS \tag{1}$$

Gradient of Voronoi Area

Consider a surface S compound by a set of triangles around vertex v_i. We can define the *Voronoi region* of v_i, the change in area produced by move v_i is named gradient of *Voronoi region.*

$$\nabla A = \frac{1}{2}\sum_j \left(\cot\alpha_j + \cot\beta_j\right)\left(v_i - v_j\right) \tag{2}$$

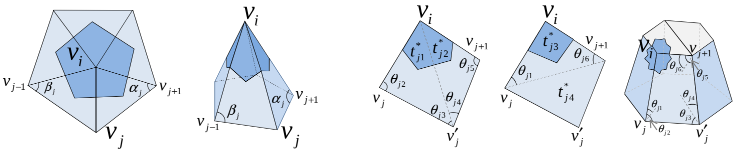


Figure: Area of Voronoi region around v_i in dark blue. v_j 1-ring neighbors around v_i. α_j and β_j opposite angles to edge $\overrightarrow{v_j - v_i}$.

Laplace Beltrami Operator

If we normalize this gradient in equation (2) by the total area in 1-ring around v_i, we have the *discrete mean curvature normal* of a surface S.

$$2\kappa\mathbf{n} = \frac{\nabla A}{A} \tag{3}$$

The LBO has desirable features, one feature of the LBO is in direction of surface area minimization, allowing us to minimize energy using it on a total curvature of a surface S equation (1).

$$\triangle_g S = 2\kappa\mathbf{n} \tag{4}$$

Normalized LBO over triangles and quads

$$L(i,j) = \begin{cases} -\dfrac{w_{ij}}{\sum\limits_{j \in N(v_i)} w_{ij}} & \text{if } j \in N(v_i) \\ \delta_{ij} & \text{otherwise} \end{cases}$$

Where $N(v_i)$ is the 1-ring neighbors with shared face to v_i, and δ_{ij} being the Kronecker delta function.

The 5 basic triangle-quad cases

$$
w_{ij} = \begin{cases}
\frac{1}{2}\left(\cot \alpha_j + \cot \beta_j\right) & \text{case } a. \\
\frac{1}{4}\left(\cot \theta_{(j-1)1} + \cot \theta_{(j-1)4} + \cot \theta_{j3} + \cot \theta_{j6}\right) & \text{case } b. \\
\frac{1}{4}\left(\cot \theta_{j2} + \cot \theta_{j5}\right) & \text{case } c. \\
\frac{1}{4}\left(\cot \theta_{j3} + \cot \theta_{j6}\right) + \frac{1}{2}\cot \beta_j & \text{case } d. \\
\frac{1}{4}\left(\cot \theta_{(j-1)1} + \cot \theta_{(j-1)4}\right) + \frac{1}{2}\cot \alpha_j & \text{case } e.
\end{cases}
$$

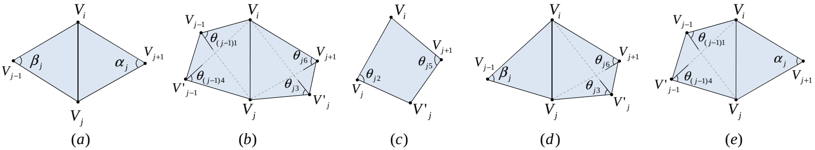
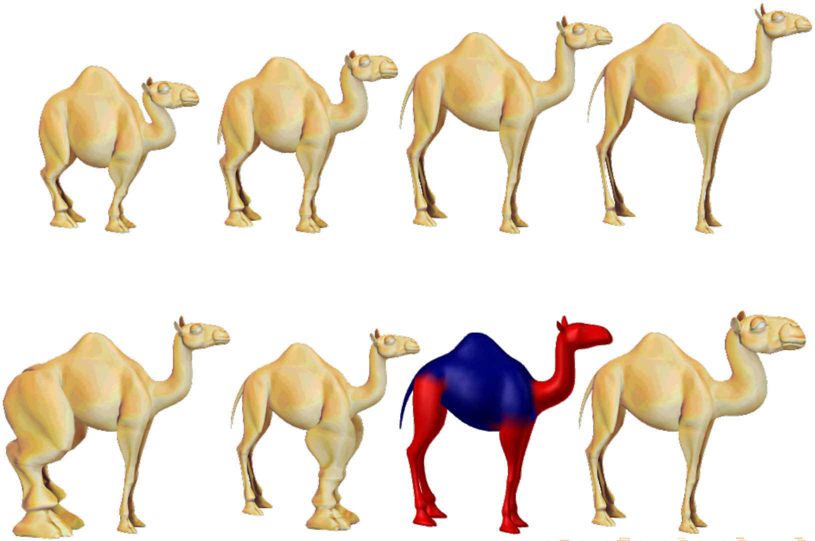


Figure: The 5 basic triangle-quad cases with common vertex V_i and the relationship with V_j and V_j'. (a) Two triangles. (b) (c) Two quads and one quad. (d) (e) Triangles and quads.

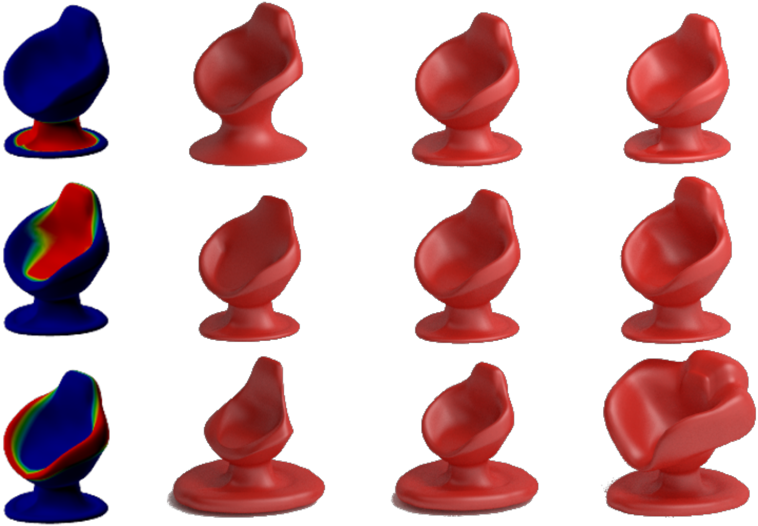
Results

Enhancing Features



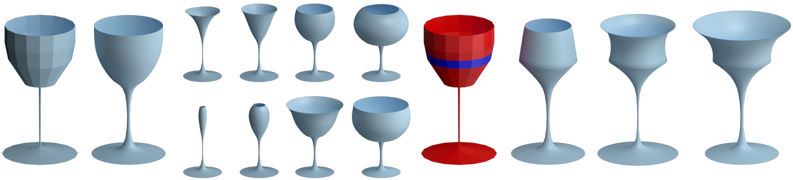
Results

Enhancing Features

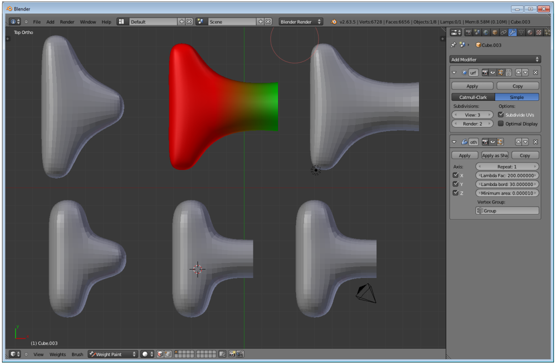


Results

Enhancing Features

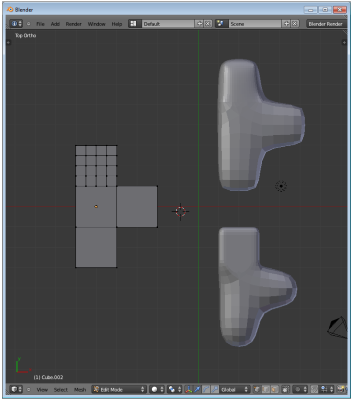


Blender



- Blender is the free open source 3D content creation suite, available for all major operating systems under the GNU General Public License..
- Use OpenNL for solve sparse system

Results


Blender
File Add Render Window Help
Default
Scene
Blender Render
Top Ortho
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View Select Mesh
Edit Mode
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