

# Laplacian Subdivision Surfaces

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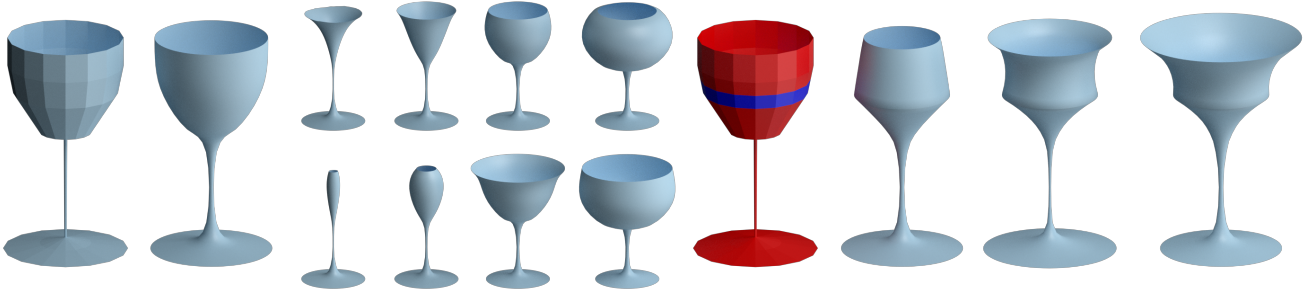


Figure 1: Laplacian subdivision surface with weight vertex group

## Abstract

This paper proposes a novel method for modelling polygonal mesh using subdivision surface and laplacian smoothing. This method use laplacian smooth for modelling global curvature in the model, to permit most flexible, robust and predictable results.

This method can correct traditional problems in extraordinary vertices present at catmull-clark subdivision method. The convergent rate of the laplacian smooth can be controlled by adjusting the weight in lambda parameter.

This proposal contains NN novel features

ESTE ABSTRACT NO SIRVE

(we provide a series of examples to graphically and numerically demonstrate the quality of our results.) debe ser cambiado copiado de desbrun 99

**CR Categories:** I.3.5 [Computer Graphics]: Computational Geometry and Object Modeling —Modeling packages

I.3.5 [Computer Graphics]: Computational Geometry and Object Modeling —Curve, surface, solid, and object representations

**Keywords:** laplacian smooth, subdivision surface

## 1 Introduction

Mencionar la gran importancia que tiene trabajar con mallas de triángulos y polígonos, ya no se necesitara preprocesar la malla para poder aplicar alguna tecnica de procesamiento que involucre el operador laplace beltrami. en el mundo real los dise;adores crean modelos con triángulos y cuadrados.

Mesh modelling from a coarse geometry is an important step in computer graphics process such as design, art and other fields.

Nuestro metodos propone tre cosas muy novedosas:

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Operador Laplace beltrami para mallas de topologia arbitraria formadas por triángulos y cuadrados, para cualquier tipo de procesamiento en geometria diferencial

Permite generar una familia de formas parametrizadas.

Controlar el nivel de suavizado y curvatura al subdividir mallas de poligonos

Enhanced brush for sculpting modelling

### 1.1 Previous work

### 1.2 Overview of our method

Nuestro metodo usa el un bosquejo de maya le aplica una subdivision cualquiera y esa subdivision la modifica a lo largo de su curvatura de flujo usando un operador laplaciano para triángulos y cuadrados

## 2 Theoretical Review of Related Work

### 2.1 Subdivision Surface

Subdivision is an iterated transformation [Warren and Weimer 2001]. Let  $F$  be a function (subdivision transformation) that maps one geometry  $M_i$  into another similar geometry with same topology  $M_{i+1}$ .

$$M_{i+1} = F(M_i) \quad (1)$$

The Catmull-Clark subdivision transformation is used to smooth a surface as the limit of sequence of subdivision steps[Stam 1998]. This method do a recursive subdivision transformation that refines the model into a linear interpolationthat is a approximate smooth surface. The process of Catmull-Clark is govern o properties of B-spline curve from multivariate spline theory[Loop 1987].

In many subdivision surfaces methods catmull clark loop so on. the smoothness of the model is autmaically guaranteed[DeRose et al. 1998].

Subdivision surfaces with catmull clark is continuos except at a extraordinary points[Loop 1987], but with our method can correct this problem

## 2.2 Laplacian smooth

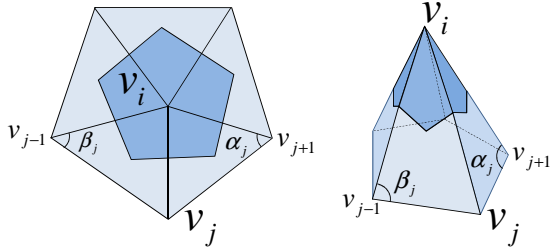
The Laplacian Smooth techniques allows you to reduce noise on a mesh's surface with minimal changes on its shape. Computer graphics objects which have been reconstructed from real world, contain undesirable noise. A laplacian smoothing removes undesirable noise while still preserves desirable geometry as well as the shape of the original model.

The functional used in many laplacian smoothing approach to constrain energy minimization is based on a total curvature of a surface  $S$ .

$$E(S) = \int_S \kappa_1^2 + \kappa_2^2 dS \quad (2)$$

Where  $\kappa_1$  and  $\kappa_2$  are the two principal curvatures of the surface  $S$ .

### 2.2.1 Gradient of Voronoi Area



**Figure 2:** Area of Voronoi region around  $v_i$  in dark blue.  $v_j$  1-ring neighbors around  $v_i$ .  $\alpha_j$  and  $\beta_j$  opposite angles to edge  $v_j - v_i$ .

Consider a surface  $S$  compound by a set of triangles around vertex  $v_i$ . We can define the *Voronoi region* of  $v_i$  as show in figure 2, The change in area produced by move  $v_i$  is named gradient of *Voronoi region* [Pinkall et al. 1993; Desbrun et al. 1999; Meyer et al. 2003].

$$\nabla A = \frac{1}{2} \sum_j (\cot \alpha_j + \cot \beta_j) (v_i - v_j) \quad (3)$$

If we normalize this gradient in equation (3) by the total area in 1-ring around  $v_i$ , we have the *discrete mean curvature normal* of a surface  $S$  as shown in equation (4).

$$2\kappa \mathbf{n} = \frac{\nabla A}{A} \quad (4)$$

### 2.2.2 Laplace Beltrami Operator

The *Laplace Beltrami operator* LBO denoted  $\Delta_g$  is used for measures mean curvature normal of the Surface  $S$  [Pinkall et al. 1993].

$$\Delta_g S = 2\kappa \mathbf{n} \quad (5)$$

The LBO has desirable features, one feature of the LBO is in direction of surface area minimization, allowing us to minimize energy using it on a total curvature of a surface  $S$  equation (2).

## 3 Proposed Method

Our method simplifies the design of irregular polygon meshes, generating a parameterized family of shapes using a set of vertices representing a coarse sketch of the desired model. Our method is iterative and converges towards a continuous and smooth version of the original model.

Unlike other methods, our method allows use arbitrary topologies types of representation as triangles and quads, exploiting the basic geometrical relationships facilitating and ensuring convergence of the algorithm and similar shapes consistent with the original shape against the other methods.

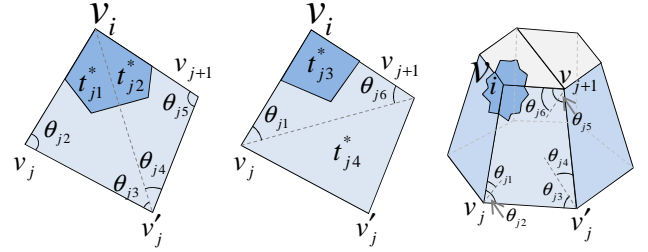
Our method allows the use of soft constraints weighting the effect of smoothing at each vertex based on a normalized weight, the weights are assigned to the control vertices of the original mesh or. The weights of the new vertices resulting from the iterations are calculated by interpolation, allowing to modify the behavior of the method on exact regions of the original model.

### 3.1 Laplace Beltrami operator over triangular and quadrilateral meshes TQLBO

Given a mesh  $M = (V, Q, T)$ , with vertices  $V$ , quads  $Q$ , triangles  $T$ .

The area of 1-ring neighborhood ( $N_1$ ) with shared face to vertex  $v_i$  in  $M$  is.

$$A(v_i) = A(Q_{N_1(v_i)}) + A(T_{N_1(v_i)}).$$



**Figure 4:**  $t_{j1}^* \equiv \Delta v_i v_j v_{j-1}$ ,  $t_{j2}^* \equiv \Delta v_i v_{j-1} v_{j+1}$ ,  $t_{j3}^* \equiv \Delta v_i v_j v_{j+1}$  Triangulations of the quad with common vertex  $v_i$  proposed by [Xiong 2011] to define Mean LBO.

Applying the mean average area according to [Xiong et al. 2011] of all possible triangulations for each quad to  $A(Q_{N_1(v_i)})$  as show in figure 4.

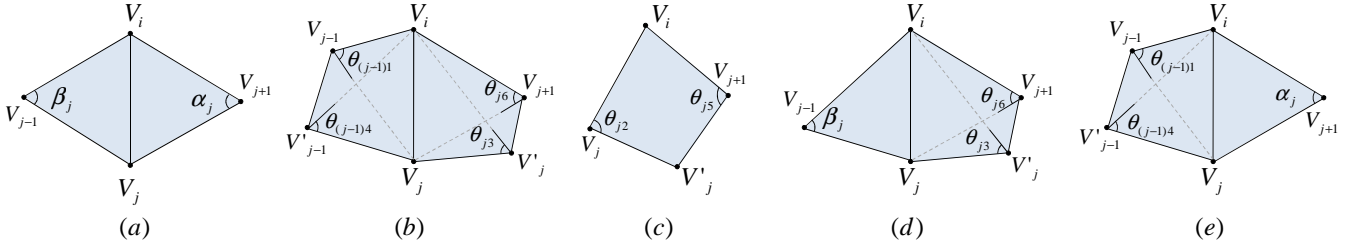
$$A(v_i) = \frac{1}{2^m} \sum_{j=1}^m 2^{m-1} A(q_j) + \sum_{k=1}^r A(t_k)$$

Where  $q_1, q_2, \dots, q_i, \dots, q_m \in Q_{N_1(v_i)}$  and  $t_1, t_2, \dots, t_k, \dots, t_r \in T_{N_1(v_i)}$ .

$$A(v_i) = \frac{1}{2} \sum_{j=1}^m [A(t_{j1}^*) + A(t_{j2}^*) + A(t_{j3}^*)] + \sum_{k=1}^r A(t_k) \quad (6)$$

Applying the gradient operator to (6).

$$\nabla A(v_i) = \frac{1}{2} \sum_{j=1}^m [\nabla A(t_{j1}^*) + \nabla A(t_{j2}^*) + \nabla A(t_{j3}^*)] + \sum_{k=1}^r \nabla A(t_k) \quad (7)$$



**Figure 3:** The 5 basic triangle-quad cases with common vertex  $V_i$  and the relationship with  $V_j$  and  $V'_j$ . (a) Two triangles [Desbrun 1999]. (b) (c) Two quads and one quad [Xiong 2011]. (d) (e) Triangles and quads (TQLBO).

According to (3), we have.

$$\begin{aligned}\nabla A(t_{j1}^*) &= \frac{\cot \theta_{j3}(v_j - v_i) + \cot \theta_{j2}(v'_j - v_i)}{2} \\ \nabla A(t_{j2}^*) &= \frac{\cot \theta_{j5}(v'_j - v_i) + \cot \theta_{j4}(v_{j+1} - v_i)}{2} \\ \nabla A(t_{j3}^*) &= \frac{\cot \theta_{j6}(v_j - v_i) + \cot \theta_{j1}(v_{j+1} - v_i)}{2} \\ \nabla A(t_k) &= \frac{\cot \alpha_k(v_k - v_i) + \cot \beta_{k+1}(v_{k+1} - v_i)}{2}\end{aligned}$$

All triangles and quads configurations of the 1-neighborhood faces adjacent to  $v_i$  can be simplified in 5 simple cases how show in figure 3.

Then according to (4), (5), and 5 simples cases defined in figure 3 the TQLBO (Triangle-Quad LBO) of  $v_i$  is.

$$\triangle_g(v_i) = 2\kappa \mathbf{n} = \frac{\nabla A}{A} = \frac{1}{A} \sum_{v_j \in N_1(v_i)} w_{ij}(v_j - v_i) \quad (8)$$

$$w_{ij} = \begin{cases} \frac{1}{2}(\cot \alpha_j + \cot \beta_j) & \text{case a.} \\ \frac{1}{4}(\cot \theta_{(j-1)1} + \cot \theta_{(j-1)4} + \cot \theta_{j3} + \cot \theta_{j6}) & \text{case b.} \\ \frac{1}{4}(\cot \theta_{j2} + \cot \theta_{j5}) & \text{case c.} \\ \frac{1}{4}(\cot \theta_{j3} + \cot \theta_{j6}) + \frac{1}{2} \cot \beta_j & \text{case d.} \\ \frac{1}{4}(\cot \theta_{(j-1)1} + \cot \theta_{(j-1)4}) + \frac{1}{2} \cot \alpha_j & \text{case e.} \end{cases}$$

### 3.1.1 TQLBO over triangular and quadrilateral meshes

$$L(i, j) = \begin{cases} -\frac{w_{ij}}{\sum_{j \in N(v_i)} w_{ij}} & \text{if } j \in N(v_i) \\ \delta_{ij} & \text{otherwise} \end{cases}$$

Where  $N(v_i)$  is the 1-ring neighbors with shared face to  $v_i$ , and  $\delta_{ij}$  being the Kronecker delta function.

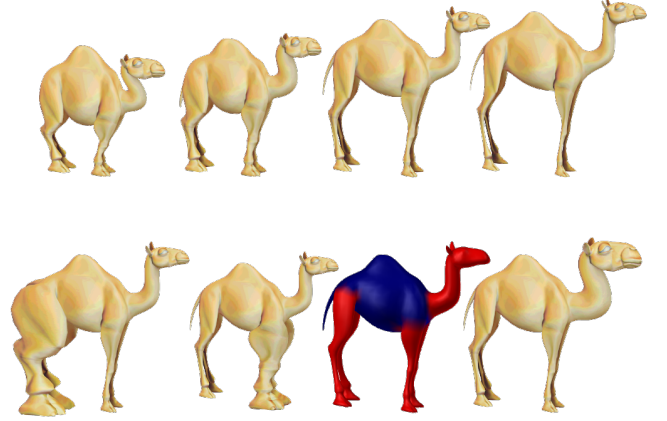
### 3.1.2 Laplacian Enhancement

El proceso de realce de los detalles de la superficie de malla aplicando el operador laplaciano que mueve los vertices a lo largo de la direccion de la curvatura normal.

realzar las caracteristicas, usando el cambio producido por la ecuacion de difusion

### 3.2 Boundaries scale-dependent

El manejo de los bordes con operador dependiente de escala



**Figure 5:** FGHFHFG

### 3.3 Anti-shrinking fairing - Volume preservation

Preservacion de volumen en el centroide funciona mejor que lo propuesto por desbrun 99

The diffusion process, induce shrinkage [Desbrun et al. 1999].

### 3.4 Weight based smooth constraints

Las familias que se generan pueden cambiar substancialmente con el ponderamiento de puntos de control especificos

## 4 Experimental Results and Applications

Se trabajo con blender software: blblbsd

se hicieron pruebas de rendimiento asdadasd

Pruebas de catmull clark vs nuestro metodo

pruebas de T Q, y nuestro metodos de Ty Q

Pruebas generando familias de objetos, que involucran y no CatCl.

### 4.1 Implementation

c y c++ blender.

## 4.2 Sparse linear system

superlu opennl

## 5 Conclusion and future work

optimizacion del metodo de solucion

aplicarlo en otras areas

## Acknowledgements

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