

$$Q = \{q_1, q_2, \dots, q_m\} | q_i = \{v_0, v_1, v_2, v_3\} \in \mathbb{R}^3, i = [1, m]$$

$$T = \{t_1, t_2, \dots, t_k\} | t_j = \{v_0, v_1, v_2\} \in \mathbb{R}^3, j = [1, k]$$

$$A(p) = A(Q) + A(T)$$

$$A(p) = \frac{1}{2^m} \sum_{1 \leq i \leq m} 2^{m-1} A(q_i) + \sum_{1 \leq j \leq k} A(t_j)$$

$$A(p) = \frac{1}{2} \sum_{1 \leq i \leq m} A(q_i) + \sum_{1 \leq j \leq k} A(t_j)$$

$$A(p) = \frac{1}{2} \sum_{1 \leq i \leq m} [A(T_{i1}) + A(T_{i2}) + A(T_{i3})] + \sum_{1 \leq j \leq k} A(t_j)$$

$$\nabla A(p) = \nabla \left(\frac{1}{2} \sum_{1 \leq i \leq m} [A(T_{i1}) + A(T_{i2}) + A(T_{i3})] + \sum_{1 \leq j \leq k} A(t_j) \right)$$

$$\nabla A(p) = \left(\frac{1}{2} \sum_{1 \leq i \leq m} [\nabla A(T_{i1}) + \nabla A(T_{i2}) + \nabla A(T_{i3})] + \sum_{1 \leq j \leq k} \nabla A(t_j) \right)$$

$$\nabla A(T_{i1}) = \frac{\cot \alpha_{i3}(p_i - p) + \cot \alpha_{i2}(p'_i - p)}{2}$$

$$\nabla A(T_{i2}) = \frac{\cot \alpha_{j5}(p'_i - p) + \cot \alpha_{i4}(p_{i+1} - p)}{2}$$

$$\nabla A(T_{i3}) = \frac{\cot \alpha_{i6}(p_i - p) + \cot \alpha_{i1}(p_{i+1} - p)}{2}$$

$$\nabla A(t_j) = \frac{(\cot \alpha + \cot \beta)(p_j - p)}{2}$$

$$\nabla A(p) = \frac{1}{2} \sum_{1 \leq i \leq m} \left[\frac{\cot \alpha_{i3}(p_i - p) + \cot \alpha_{i2}(p'_i - p)}{2} + \frac{\cot \alpha_{j5}(p'_i - p) + \cot \alpha_{i4}(p_{i+1} - p)}{2} + \frac{\cot \alpha_{i6}(p_i - p) + \cot \alpha_{i1}(p_{i+1} - p)}{2} \right]$$

$$+ \sum_{1 \leq j \leq k} \frac{(\cot \alpha + \cot \beta)(p_j - p)}{2}$$

$$\nabla A(p) = \frac{1}{2} \sum_{1 \leq i \leq m} \left[\frac{(\cot \alpha_{i3} + \cot \alpha_{i6})(p_i - p) + (\cot \alpha_{i4} + \cot \alpha_{i1})(p_{i+1} - p)}{2} + \frac{(\cot \alpha_{i2} + \cot \alpha_{j5})(p'_i - p)}{2} \right]$$

$$+ \sum_{1 \leq j \leq k} \frac{(\cot \alpha + \cot \beta)(p_j - p)}{2}$$

$$\nabla A(p) = \frac{1}{4} \sum_{1 \leq i \leq m} (\cot \alpha_{i3} + \cot \alpha_{i6})(p_i - p) + (\cot \alpha_{i4} + \cot \alpha_{i1})(p_{i+1} - p)$$

$$+ \frac{1}{4} \sum_{1 \leq i \leq m} (\cot \alpha_{i2} + \cot \alpha_{j5})(p'_i - p)$$

$$+ \sum_{1 \leq j \leq k} \frac{(\cot \alpha + \cot \beta)(p_j - p)}{2}$$

$$\nabla A(p) = \frac{1}{4} \sum_{1 \leq i \leq m} (\cot \alpha_{i3} + \cot \alpha_{i6})(p_i - p)$$

$$+ \frac{1}{4} \sum_{1 \leq i \leq m} (\cot \alpha_{i4} + \cot \alpha_{i1})(p_{i+1} - p)$$

$$+ \frac{1}{4} \sum_{1 \leq i \leq m} (\cot \alpha_{i2} + \cot \alpha_{j5})(p'_i - p)$$

$$+ \sum_{1 \leq j \leq k} \frac{(\cot \alpha + \cot \beta)(p_j - p)}{2}$$

$$\nabla A(p) = \frac{1}{4} \sum_{1 \leq i \leq m} (\cot \alpha_{i3} + \cot \alpha_{i6})(p_i - p)$$

$$\begin{aligned}
& + \frac{1}{4} \sum_{1 \leq i \leq m} (\cot \alpha_{(i-1)4} + \cot \alpha_{(i-1)1})(p_i - p) \\
& + \frac{1}{4} \sum_{1 \leq i \leq m} (\cot \alpha_{i2} + \cot \alpha_{j5})(p'_i - p) \\
& + \sum_{1 \leq j \leq k} \frac{(\cot \alpha + \cot \beta)(p_j - p)}{2}
\end{aligned}$$

$$\begin{aligned}
\nabla A(p) &= \frac{1}{4} \sum_{1 \leq i \leq m} (\cot \alpha_{i3} + \cot \alpha_{i6} + \cot \alpha_{(i-1)4} + \cot \alpha_{(i-1)1})(p_i - p) \\
& + \frac{1}{4} \sum_{1 \leq i \leq m} (\cot \alpha_{i2} + \cot \alpha_{j5})(p'_i - p) \\
& + \frac{1}{2} \sum_{1 \leq j \leq k} (\cot \alpha + \cot \beta)(p_j - p) \\
\nabla A(p) &= \sum_{1 \leq i \leq m} w_i(p_i - p) + \sum_{1 \leq i \leq m} w'_i(p'_i - p) + \sum_{1 \leq j \leq k} w_j(p_j - p)
\end{aligned}$$