Ca_bot Inverse Kinematics

Antoine Pirrone
antoine.pirrone@gmail.Com

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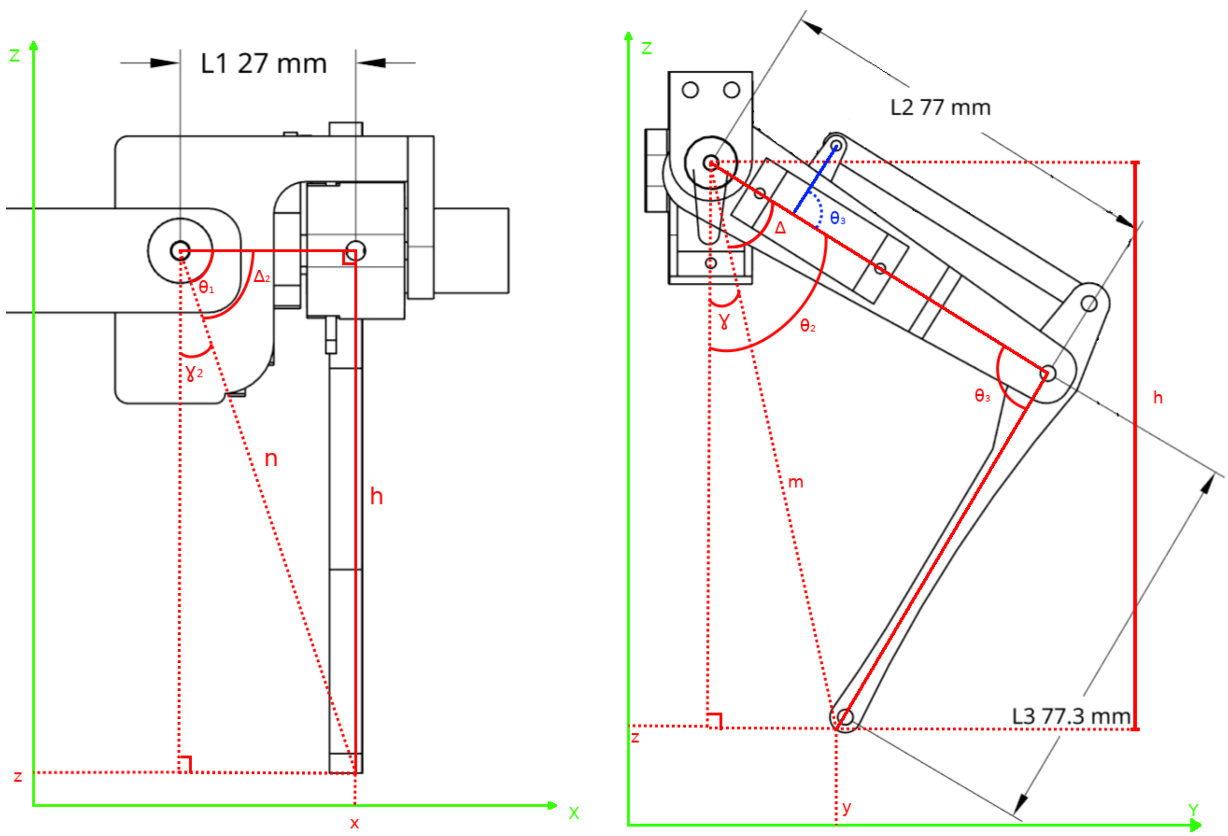

Figure 1: We use trigonometry to compute the inverse kinematics of the leg

We want to find the values of the angles θ_1, θ_2 and θ_3 given the x, y and z position of the end of the leg (see Figure **??**).

For θ_1, we first need to compute the angles γ_2 and Δ_2. Using the Pythagorean theorem, we have :

$n = \sqrt{z^2 + x^2}$
$h = \sqrt{n^2 - L1^2}$

Which allow us to compute γ_2 :

$\gamma_2 = \arcsin(\frac{x}{n})$

And Δ_2 using Al-Kashi's theorem :

$\Delta_2 = \arccos(\frac{-h^2+n^2+L1^2}{2 \cdot n \cdot L1})$

Then,

$\boxed{\theta_1 = \gamma_2 + \Delta_2}$

Same process for θ_2, this time we need to compute γ and Δ. Using the Pythagorean theorem, we have :

$m = \sqrt{y^2 + h^2}$

Thus,

$\gamma = \arcsin(\frac{y}{m})$

And using Al-Kashi's theorem again :

$\Delta = \arccos(\frac{-L3^2+m^2+L2^2}{2 \cdot m \cdot L2})$

Then,

$\boxed{\theta_2 = \gamma + \Delta}$

Eventually, we find θ_3 using Al-Kashi's theorem again :

$$\boxed{\theta_3 = \arccos(\frac{-m^2 + L2^2 + L3^2}{2 \cdot L2 \cdot L3})}$$

Note : The blue θ_3 on Figure **??** is the actual angle being actuated, but as we have a parallel mechanism here, it is actually the same value as the red one.