Number 1

a) Part a. ; Information is lost when data is digitized because of ;

- Sampling limitations ; ADC samples a continuous analog signal at discrete time intervals and if the sampling rate is too low (below Nyquist rate), there is signal loss
- Quantization error; ADC process rounds analog values to the nearest digital level causing a small difference between the original and digitized signal.

To perfectly reconstruct the signal after ADC;

- Use a high sampling rate (above Nyquist rate)
- Use interpolation such as sinc interpolation and oversampling techniques.

b) Aliasing effect:

- Aliasing occurs when a continuous analog signal is sampled at a rate lower than the Nyquist rate ( less than twice the highest frequency of the sig
- This causes frequency components to overlap leading to distortion where high-frequency signals appear as lower frequencies in the digitized version

To reduce the aliasing effect;

- Increase the sampling rate to be atleast twice the highest frequen
- Use an anti-aliasing filter e.g a low pass filter before sampling to remo high-frequency components that could cause aliasing.

c) $x_c(t) = 2\cos(20\pi t)$ , $-\infty < t < \infty$

$x[n] = 2\cos\left(\frac{\pi n}{5}\right)$ , $\infty < n < \infty$

i) Sampling period , T

$x[n] = x_c(nT)$

$2\cos\left(\frac{\pi n}{5}\right) = 2\cos(20\pi t)$

$2\cos\left(\frac{\pi n}{5}\right) = 2\cos(20\pi nT)$

$\frac{\pi n}{5} = 20\pi nT$

$T = \frac{1}{5(20)}$

$T = \frac{1}{100}$ s

c) Due to periodicity, there should be other sampling periods such as $T = 11/100$, $T = 21/100$, $T = 31/100$ and so on thus the choice of T is not unique.

example: $\cos\left(2\pi n + \frac{\pi n}{5}\right) = 2\cos\left(n\left(2\pi + \frac{\pi}{5}\right)\right)$

$$= 2\cos\left(n\frac{11\pi}{5}\right)$$

$$\frac{n 11\pi}{5} = 20\pi n T$$

$$T = \frac{11}{100} s$$

d) $\Delta = 0.1$

Uniform distribution

Recall: $x_c(t) = 2\cos(20\pi t)$

Range = $X_{max} - X_{min}$

X max occurs when cos is max at 1; $X_{max} = 2$

X min occurs when cos in min at -1; $X_{min} = -2$

Range = $2 - -2 = 4$

$SQNR = 10\log_{10}\left(\frac{P_x}{P_n}\right)$ where $P_x$ = Signal power

$P_n$ = Noise power

$$P_x = \frac{1}{T_p}\int_0^{T_p} (2)^2\cos^2(20\pi t)\,dt$$

$$= \frac{4}{T_p}\int_0^{T_p}\frac{1}{2}\left(1 + \cos(40\pi t)\right)dt$$

$$= \frac{2}{T_p}\left[t + \frac{\sin(40\pi t)}{40\pi}\right]_0^{T_p}$$

$$= \frac{2}{T_p}\left[\left(T_p + \frac{\sin 40\pi T_p}{40\pi}\right) - \left(0 + \sin(0)\right)\right]$$

For any value of $T_p$; $\sin 40\pi T_p$ yields a zero

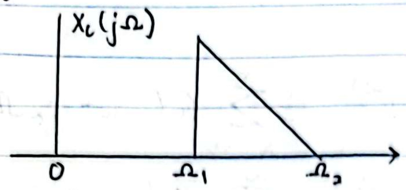
$$P_x = \frac{2}{T_p}(T_p) = 2W$$

$$P_n = \frac{\Delta^2}{12} = \frac{(0.1)^2}{12}$$

$$SQNR = 10\log_{10}\left(2 \div \left(\frac{(0.1)^2}{12}\right)\right)$$

$$= 33.802\ dB$$

Part e.




$\Delta\Omega = \Omega_2 - \Omega_1$

$\Omega_2 = 930MHz$  $\Omega_1 = 900MHz$

$x[n] = x_c(nT)$

$\Omega_c =$ cut off frequency

i) Smallest frequency required to avoid aliasing assuming knowledge of the carrier frequency.

$\Omega_2 = 930MHz$  $\Omega_1 = 900MHz$  $\Delta\Omega = \Omega_2 - \Omega_1$

$= 930 - 900$

$\Delta\Omega = 30MHz$

$\Omega_c = \frac{\Omega_1 + \Omega_2}{2} = \frac{930 + 900}{2} = 915MHz$

To avoid aliasing; $\Omega_s \geq 2\Omega_0$

We modulate $X_c(t)$ with the exponential complex signal with $\Omega_c = \frac{\Omega_1 + \Omega_2}{2}$

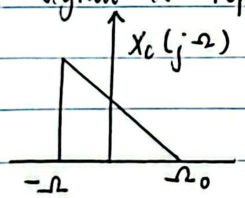
$$X_c(t)e^{j\Omega_c t} \xleftrightarrow{FT} X_c(\Omega - \Omega_c)$$

The demodulated signal is given by:

$y_c(t) = X_c(t)e^{-j\Omega_c t}$

$y_c(t) = X_c(t)e^{-j\frac{(\Omega_1 + \Omega_2)}{2}t}$

Resulting signal is represented by;




where $\Omega_0 = \frac{\Omega_2 - \Omega_1}{2}$

$= 30/2$

$\Omega_0 = 15MHz$

Recall $\Omega_s \geq 2\Omega_0$

$\Omega_s \geq 2(15)$

$\Omega_s \geq 30MHz$

ii)

a) $X(e^{j\omega}) = X(j\Omega)\Big|_{\Omega = \frac{\omega}{T_0}}$

$X(e^{j\omega})$ is periodic with $X(j\Omega) = \frac{1}{T}\sum_{K=-\infty}^{\infty} X_c(j\Omega - jK\Omega_s)$

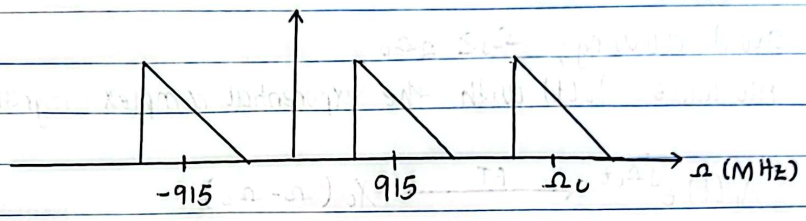
$X(j\Omega) = \frac{1}{T}\sum_{K=-\infty}^{\infty} X_c(j\Omega - jK\Omega_s)$

where $\Omega_s = 2\pi f_s$

$= \frac{2\pi}{T}$

since $\Omega_s = 30MHz$

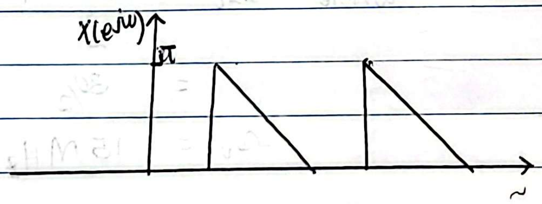
$T = \frac{2\pi}{\Omega_s} = \frac{2\pi}{30\times10^6}\ S$




$\Omega_c = \frac{2\Omega_s + 1830}{2}$ where $\Omega_s = 30MHz$

From $W = \Omega_c T_s$

$W = 915\times10^6 \times \frac{2\pi}{30\times10^6} = 61\pi$



Number 2.

a) i) A system is causal if for every choice of $n_0$, the output sequence value at the index $n = n_0$ depends only on the input sequence values for $n \leq n_0$ yet a system is non-causal if for any choice of $n_0$, the output ~~sequence~~ depends on the input sequence values of $n > n_0$

Example : Given $y[n] = x[n-n_d]$ $-\infty < n < \infty$.

The system is causal for $n_d \geq 0$ and non-causal for $n_d < 0$

ii) - Deterministic signal is a signal whose behaviour can be exactly predicted using a mathematical expression without uncertainty. whereas a random signal is a signal that contains unpredictable variations and is best described using probabilistic models.

Example: deterministic signal: $x(t) = 5\sin(2\pi t)$

Random signal : Noise in communication systems.

iii) Periodic signal is a signal that repeats itself after a fixed interval T while an aperiodic signal is a signal that does not repeat over any fixed interval. Example: A signal is periodic if $x(t+T) = x(t)$ $\forall t$ e.g $x(t) = \cos(2\pi$

Aperiodic signal; $x(t) = e^{-t}$ which is a decaying signal that never repeats.

b) $x[n] = \delta[n] + 2\delta[n-2] - \delta[n-3]$ ; $x[n] = \{1, 0, 2, -1\}$

$h[n] = \delta[n] + 2\delta[n-1] + \delta[n-3]$ ; $h[n] = \{1, 2, 0, 1\}$

i) $X[K] = \sum_{n=0}^{N-1} x[n] e^{-j\frac{2\pi K n}{N}} = \sum_{n=0}^{3} x[n] W_4^{nK}$ where $W_4 = e^{-j2\pi/4}$

$W_4 = -i$

$X[K] = \sum_{n=0}^{3} x[n](-i)^{nK} = 1 + 2W_4^{2K} - W_4^{3K}$ $0 \leq K \leq 3$

$X[1] = 1(-i)^0 + 0(-i)^1 + 2(-i)^2 + -1(-i)^3$

$= 1 + 0 + -2 - i = -1-i$

$X[2] = 1(-i)^0 + 0(-i)^2 + 2(-i)^4 + -1(-i)^6$

$= 1 + 0 + 2 + 1 = 4$

$X[3] = 1(-i)^0 + 0(-i)^3 + 2(-i)^6 + -1(-i)^9$

$= 1 + 0 + -2 + i = -1+i$

~~$X[4] = 1(-i)^0 + 0(-i)^4 +$~~

$X[0] = 1(-i)^0 + 0(-i)^0 + 2(-i)^0 + -1(-i)^0$

$= 1 + 0 + 2 + -1 = 2$

$[X(K)] = [2, -1-i, 4, -1+i]$

$$H[K] = \sum_{n=0}^{3} [\delta[n] + 2\delta[n-1] + \delta[n-3]] W_4^{Kn} \quad 0 \le K \le 3$$

$$H[K] = 1 + 2W_4^K + W_4^{3K} \quad 0 \le K \le 3$$

$$H[K] = [4, 1-i, -2, 1+i]$$

ii) $x[n] * h[n] = \sum_{K=-\infty}^{\infty} x[n] h[n-k]$

| $x[n]$ \ $h[n]$ | 1 | 2 | 0 | 1 |
|---|---|---|---|---|
| 1 | 1 | 2 | 0 | 1 |
| 0 | 0 | 0 | 0 | 0 |
| 2 | 2 | 4 | 0 | 2 |
| -1 | -1 | -2 | 0 | -1 |

$y[n] = x[n] * h[n] = \{1, 2, 2, 4, -2, 2, -1\}$

Circular convolution $x[n] \circledast h[n]$.

$$y[n] = h[n] \circledast x[n] = \sum_{K=0}^{3} h[K]\, x[\langle n-K \rangle_4], \quad 0 \le K \le 3$$

$h[n] = [1, 2, 0, 1]$ and $x[n] = [1, 0, 2, -1]$

$$y(0) = h[K]\, x[\langle -K \rangle_4] = h[0]x[0] + h[1]x[3] + h[2]x[2] + h[3]x[1]$$
$$= 1(1) + 2(-1) + 0(2) + 1(0) = -1$$

$$y(1) = h[K]\, x[\langle 1-K \rangle_4] = h[0]x[1] + h[1]x[0] + h[2]x[3] + h[3]x[2]$$
$$= 1(0) + 2(1) + 0(-1) + 1(2) = 4$$

$$y(2) = h[K]\, x[\langle 2-K \rangle_4] = h[0]x[2] + h[1]x[1] + h[2]x[0] + h[3]x[3]$$
$$= 1(2) + 2(0) + 0(1) + 1(-1) = 1$$

$$y(3) = h[K]\, x[\langle 3-K \rangle_4] = h[0]x[3] + h[1]x[2] + h[2]x[1] + h[3]x[0]$$
$$= 1(-1) + 2(2) + 0(0) + 1(1) = 4$$

$y[n] = h[n] \circledast x[n] = [-1, 4, 1, 4]$

b) ii) cont'd

Using the DFT to compute the impulse response in time.

$Y[K] = H[K] X[K]$

$H[K] = [4, 1-i, -2, 1+i]$ $\quad$ $X[K] = [2, -1-i, 4, -1+i]$

$H[K] = 1 + 2W_4^{k} + W_4^{3k}$ $\quad$ $X[K] = 1 + 2W_4^{2k} - W_4^{3k}$

$Y[K] = (1 + 2W_4^{k} + W_4^{3k})(1 + 2W_4^{2k} - W_4^{3k})$

$= 1 + 2W_4^{2k} - W_4^{3k} + 4W_4^{3k} + 2W_4^{k} - 2W_4^{4k} + W_4^{3k} - 2W_4^{5k} - W_4^{6k}$

$= 1 + 2W_4^{k} + 4W_4^{3k} - 2W_4^{4k} + 2W_4^{5k} - W_4^{6k} + 2W_4^{2k}$

$Y[K] = 1 + 2W_4^{k} + 2W_4^{2k} + 4W_4^{3k} - 2W_4^{4k} + 2W_4^{5k} - W_4^{6k}$

Using the inverse DFT of $Y[K]$

$y[n] = \{1, 2, 2, 4, -2, 2, -1\}$

Similar to linear convolution

$Y[K] = H[K] X[K]$

$= [4, 1-i, -2, 1+i] \cdot [2, -1-i, 4, -1+i]$

$= [4(2), (1-i)(-1-i), -2(4), (1+i)(-1+i)]$

$= [8, -2, -8, -2]$

$$y[n] = \frac{1}{4} \sum_{K=0}^{3} Y[K] W_4^{-nk} \quad 0 \leq n \leq 3$$

$y[0] = -1$ $\quad$ $y[1] = 4$ $\quad$ $y[2] = 1$ $\quad$ $y[3] = 4$

$y[n] = [-1, 4, 1, 4]$

Similar to circular convolution

Discussion of main difference

- The linear convolution operates on infinite length sequences whereas the circular convolution operates on finite length sequences
- DFT provides a way of obtaining the linear convolution of two sequences of finite length by obtaining the inverse DFT
- Multiplying the DFTs of two sequences is equivalent to the circular convolution of two sequences in the time-domain

Number 2c.

Length = 1500 for input $x[n]$

Length = 32 for impulse $h[n]$

DFTs and inverse DFTs of $N = 64$ to compute linear convolution with overlap-add method;

For overlap-add method; $N = L + M - 1$

$$M = N - L + 1$$

$$M = 64 - 32 + 1$$

$$M = 33$$

Number of blocks; $= \frac{1500}{33} = 45.4545$

$\cong$ 45 blocks or sections.

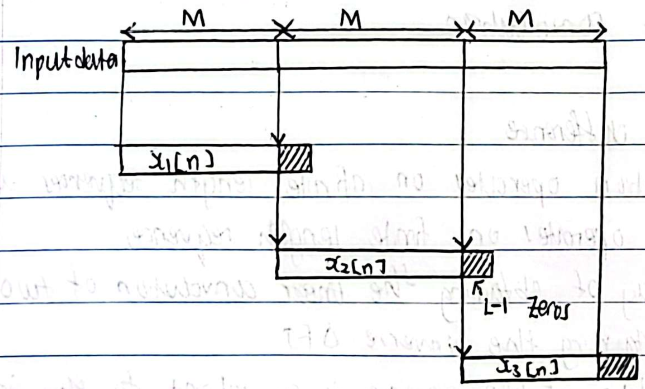
$45 \times 33 = 1485$ blocks; This leaves $(1500 - 1485) = 15$ points of data

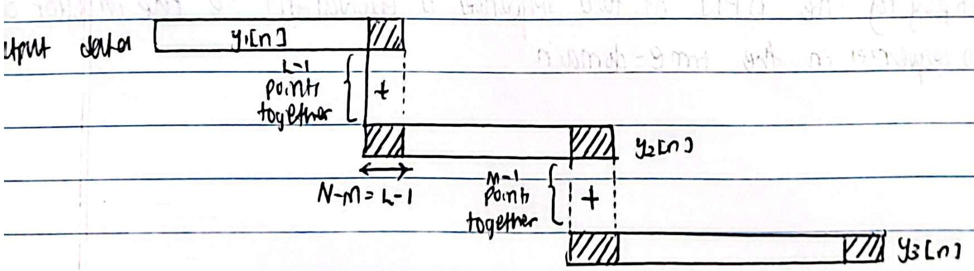
Thus the remaining data has to be zero padded to 64 and thus requires using another DFT

Thus, 46 DFTs and 46 IDFTs are required

Because $h[n]$ requires a DFT, we need 47 DFTs and 46 IDFTs to perform the convolution.

Sketch of the fast linear convolution.



2c cont'd

- The input data block is M points and the size of the DFTs and IDFT is ~~WWW~~ $N = M + L - 1$. To each data block, $L-1$ zeros are appended and the N-point DFT is computed.
- The data blocks may be represented as;

$$x_1[n] = \{x(0), x(1), \cdots, x(M-1), \underbrace{0, 0, \cdots, 0}_{L-1 \text{ zeros}}\}$$ and so on

The two-~~point~~ N-point DFTs are multiplied together to form;

$$Y_1[K] = H[K] X_1[K] \quad , \quad k = 0, 1, \cdots, N-1$$

→ The IDFT yields data blocks of length N that are free of aliasing, since the size of the DFT and IDFT is $N = M + L - 1$ and the sequences are increased to N-points by appending zeros to each block.

→ Since each data block is terminated with $L-1$ zeros, the last $L-1$ points from each output block must be overlapped and added to the ~~f~~ first $L-1$ points of the succeeding block

→ Hence this method is called overlap-add method; The overlapping and adding yields the output sequence as;

$$y[n] = \{y_1(0), y_1(1), \cdots, y_1(M-1), y_1(M) + y_2(0), y_1(M+1) + y_2(1), \\ y_1(N-1) + y_2(L-1) + y_2(L), \cdots$$

Number 3.

a) A/D , $T = 2.5ms$ D/A $T' = 1ms$

$x_a(t) = 4\cos 200\pi t + 5\sin(500\pi t)$

$x(n) = x_a(nT)$ using $T = 2.5\times10^{-3} = \frac{1}{400}$

$x(n) = 4\cos\left(\frac{200\pi n}{400}\right) + 5\sin\left(\frac{500\pi n}{400}\right)$

$x(n) = 4\cos\left(\frac{\pi n}{2}\right) + 5\sin\left(\frac{5\pi n}{4}\right)$

for $\sin\left(\frac{5\pi n}{4}\right) = \sin\left[\left(2\pi - \frac{3}{4}\pi\right)n\right]$

$= -\sin\left(\frac{3\pi}{4}n\right)$

$x(n) = 4\cos\left(\frac{\pi n}{2}\right) - 5\sin\left(\frac{3\pi}{4}n\right)$

for $T' = \frac{1}{1000} \Rightarrow y_a(t) = x\left(\frac{t}{T'}\right)$

$y_a(t) = 4\cos\left(\frac{\pi}{2}(1000t)\right) - 5\sin\left(\frac{3\pi}{4}(1000)t\right)$

$= 4\cos(500\pi t) - 5\sin(750\pi t)$

b) $x(n) = \left(\frac{1}{2}\right)^n u(n) - \frac{1}{4}\left(\frac{1}{2}\right)^{n-1} u(n-1)$ ; $y(n) = \left(\frac{1}{3}\right)^n u[n]$

using $Z[\alpha^n u[n]] = \frac{1}{1-z^{-1}}$ , $|z| > |\alpha|$

$y[n] = \left(\frac{1}{3}\right)^n u[n]$

$Y(z) = \frac{1}{1-\frac{1}{3}z^{-1}}$ ROC $|z| > 1/3$

$X(z) = \frac{1}{1-\frac{1}{2}z^{-1}} - \frac{1}{4}\frac{z^{-1}}{(1-\frac{1}{2}z^{-1})}$ , $|z| > \frac{1}{2}$

$Y(z) = \frac{z}{z-\frac{1}{3}}$ $X(z) = \frac{z}{z-\frac{1}{2}} - \frac{1}{4}\left(\frac{1}{z-\frac{1}{2}}\right)$

$= \frac{z - 1/4}{z - 1/2}$

$H(z) = \frac{Y(z)}{X(z)} = \frac{z}{z-1/3} \cdot \frac{z-1/2}{z-1/4}$

$H(z) = \frac{z(z-1/2)}{(z-1/3)(z-1/4)}$

3b cont'd

$$\frac{H(z)}{z} = \frac{z-1/2}{(z-1/3)(z-1/4)}$$

$$\frac{z-1/2}{(z-1/3)(z-1/4)} = \frac{A}{z-1/3} + \frac{B}{z-1/4}$$

$$z-1/2 = A(z-\tfrac{1}{4}) + B(z-1/3)$$

when $z = 1/4$   when $z = 1/3$

$-1/4 = -1/12 B$   $-1/6 = 1/12 A$

$B = 3$   $A = -2$

$$H(z) = \frac{-2z}{z-1/3} + \frac{3z}{z-1/4}$$

$$h[n] = z^{-1}[H(z)]$$
$$= -2\left(\frac{1}{3}\right)^n u[n] + 3\left(\frac{1}{4}\right)^n u[n]$$
$$= \left(3\left(\frac{1}{4}\right)^n - 2\left(1/3\right)^n\right) u[n]$$

3b(ii)

$$H(z) = \frac{z(z-1/2)}{(z-1/3)(z-1/4)}$$
$$= \frac{z^2 - z/2}{z^2 - z/4 - z/3 + 1/12}$$

$$\frac{Y(z)}{X(z)} = \frac{z^2(1-\frac{1}{2}z^{-1})}{z^2(1-\frac{1}{4}z^{-1}-\frac{1}{3}z^{-1}+\frac{1}{12}z^{-2})}$$

$$Y(z)[1-\tfrac{1}{4}z^{-1} - \tfrac{1}{3}z^{-1} + \tfrac{1}{12}z^{-2}] = X[z](1-\tfrac{1}{2}z^{-1})$$

$$z^{-1}\{Y(z) - \tfrac{1}{4}z^{-1}Y(z) - \tfrac{1}{3}z^{-1}Y(z) + \tfrac{1}{12}z^{-2}Y(z)\} = z^{-1}\{X(z) - \tfrac{1}{2}z^{-1}X(z)\}$$

$$y[n] - \frac{1}{4}y(n-1) - \frac{1}{3}y(n-1) + \frac{1}{12}y(n-2) = x[n] - \frac{1}{2}x[n-1]$$

$$y[n] - \frac{7}{12}y[n-1] + \frac{1}{12}y[n-2] = x[n] - \frac{1}{2}x[n-1]$$

(iii) The system is stable because all poles lie inside the unit circle.

(3c) Advantages:

- Higher accuracy and noise immunity
- DSP allows for easy modification of signal characteristics

Disadvantage:

- Increased processing delay

CamScanner