

DROMA KIZITO

## DIGITAL SIGNAL PROCESSING / ~~PROBLEMS~~

## ASSIGNMENT 1

(a) ~~2000~~ ~~1999~~ ~~1998~~ ~~1997~~ ~~1996~~ ~~1995~~ ~~1994~~ ~~1993~~ ~~1992~~ ~~1991~~ ~~1990~~ ~~1989~~ ~~1988~~ ~~1987~~ ~~1986~~ ~~1985~~ ~~1984~~ ~~1983~~ ~~1982~~ ~~1981~~ ~~1980~~ ~~1979~~ ~~1978~~ ~~1977~~ ~~1976~~ ~~1975~~ ~~1974~~ ~~1973~~ ~~1972~~ ~~1971~~ ~~1970~~ ~~1969~~ ~~1968~~ ~~1967~~ ~~1966~~ ~~1965~~ ~~1964~~ ~~1963~~ ~~1962~~ ~~1961~~ ~~1960~~ ~~1959~~ ~~1958~~ ~~1957~~ ~~1956~~ ~~1955~~ ~~1954~~ ~~1953~~ ~~1952~~ ~~1951~~ ~~1950~~ ~~1949~~ ~~1948~~ ~~1947~~ ~~1946~~ ~~1945~~ ~~1944~~ ~~1943~~ ~~1942~~ ~~1941~~ ~~1940~~ ~~1939~~ ~~1938~~ ~~1937~~ ~~1936~~ ~~1935~~ ~~1934~~ ~~1933~~ ~~1932~~ ~~1931~~ ~~1930~~ ~~1929~~ ~~1928~~ ~~1927~~ ~~1926~~ ~~1925~~ ~~1924~~ ~~1923~~ ~~1922~~ ~~1921~~ ~~1920~~ ~~1919~~ ~~1918~~ ~~1917~~ ~~1916~~ ~~1915~~ ~~1914~~ ~~1913~~ ~~1912~~ ~~1911~~ ~~1910~~ ~~1909~~ ~~1908~~ ~~1907~~ ~~1906~~ ~~1905~~ ~~1904~~ ~~1903~~ ~~1902~~ ~~1901~~ ~~1900~~ ~~1899~~ ~~1898~~ ~~1897~~ ~~1896~~ ~~1895~~ ~~1894~~ ~~1893~~ ~~1892~~ ~~1891~~ 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$$k^2 P^2 + -P = 0$$

$$(x^2 + 2x + 1) \cdot (x^2 - 2x + 1) = (x^2 + 1)^2 = 17 \cdot 1$$

(2)  $\Delta$   $\mu_{\text{eff}}$   $\approx$   $10^{-4}$  eV

卷之三

1. *Brachypodium sylvaticum* (L.) Beauvois

1. *Castanea*

1100'  $\times$  7' thick

(3) 20' suspended from 11' 3" and 11' 2"

305:0 ~~and~~ in

(3) ~~unseen~~ ~~seen~~ ~~seen~~ A

Longer  $\beta$ -decay times ( $\beta^+$  losses  $\approx 5$ ) are 10

Experiments with  $(2 \times 2 \times 2) \times (2 \times 2 \times 2)$

$$(\text{The expression} + 91.56)(1 - 0.47) = 2.38 - 0.38$$

$$f(x_1, x_2, \dots, x_n) = \frac{1}{n} \sum_{i=1}^n g_i(x_i)$$

(ج) النحو الصرف الإعراب

160000

1. *Chlorophytum comosum* (L.) Willd. (Fig. 1)

卷之三

Q1 Consider the following sequences and their initial values

$$q) x[n] = 2\delta[n+2] + \delta[n-4] \quad \text{for } n \leq 5$$

$$b) x[n] = n(2u[n] - u[n-10]) + 10 \cdot e^{-0.3(n-10)} (u[n-10] - u[n-20]) \quad \text{for } n \geq 0$$

Write Matlab routines to illustrate these sequences in time (a)

Plot Discrete sequences

% Define the range for sequence (a)

$$n_a = -5:5;$$

$$x_a = 2^{\text{sign}(n_a + 2)} - (n_a == 4);$$

% plot sequence (a)

figure;

stem(n\_a, x\_a, 'filled');

grid on;

title('sequence(a); x[n] = 2\delta[n+2] + \delta[n-4]');

xlabel('n');

ylabel('x[n]');

% define the range for sequence (b)

$$n_b = 0:20;$$

% Compute sequence (b)

$$u_n = (n_b \geq 0); \quad \text{// Unit step } u[n]$$

$$u_{n-10} = (n_b \geq 10); \quad \text{// Unit step } u[n-10]$$

$$x_b = n_b \cdot \left( 2^{\text{sign}(n_b - 10)} + 10 \cdot e^{-0.3(n_b - 10)} \right) \cdot (u_{n-10} - u_{n-20});$$

% plot sequence (b)

figure;

stem(n\_b, x\_b, 'filled');

grid on;

title('sequence(b)');

xlabel('n');

ylabel('x[n]');

1.2 Determine whether the following systems are i) stable, ii) causal  
iii) linear, iv) time-invariant and memoryless.

a)  $T[x[n]] = e^{x[n]}$

b)  $T[x[n]] = 9x[n] + a$

c)  $T[x[n]] = x[n] + 3x[n+1]$

Soln

a) i) Stability Test

$T[x[n]] = e^{x[n]}$

If  $|x[n]| \leq M$  then  $x[n]$

$y[n] = e^{x[n]}$

The function  $e^x$  is not bounded for all  $x$  (as  $x \rightarrow \infty$ )

Now  $\Rightarrow T[x(n)] \Rightarrow$  whether unbounded. The system is not stable.

ii) Causality Test

$T[x(n)] = e^{x(n)}$   $n = 0, 1, -1$

$n = 0$  when  $n = 0$   $x(0)$  when  $n = 1$   $x(1)$  when  $n = -1$   $x(-1)$

$T[x[n]] = e^{x(0)}$   $T[x[n]] = e^{x(1)}$   $T[x[n]] = e^{x(-1)}$

$\in \mathbb{C} = \mathbb{C}$

$n = -1$

$x(-1) = c$

Since  $x[n]$  only is dependent on the present input of  $n$ , the system is causal

iii)  $T_1[x_1[n]] = e^{a x_1[n]}$

$T_2[x_2[n]] = e^{b x_2[n]}$

$T[a x_1[n] + b x_2[n]] = e^{a x_1[n]} + e^{b x_2[n]}$

$a T_1[x_1[n]] = e^{a x_1[n]} \quad \text{--- (i)}$

$b T_2[x_2[n]] = e^{b x_2[n]} \quad \text{--- (ii)}$

Combine (i) & (ii)

$a T_1[x_1[n]] + b T_2[x_2[n]] = e^{a x_1[n]} + e^{b x_2[n]}$

since

$$T(aT_1(x_1[n]) + bT_2(x_2[n])) \neq aT_1(x_1[n]) + bT_2(x_2[n])$$

Therefore the system is non-linear. (iv)

iv) for Time-Invariant System.

$$T(f[n-n_0])$$

$$T(x[n-n_0]) = y[n-n_0]$$

$$y[n] = e^{x[n]}$$

$$y[n-n_0] = e^{x[n-n_0]}$$

$$x[n-n_0] = f[n-n_0]$$

$$T\{x[n]\} = e^{f[n]}$$

$$T\{x[n-n_0]\} = e^{f[n-n_0]}$$

since  $T\{x[n-n_0]\} = y[n-n_0]$ , then the system

is Time-Invariant

(v) system is memoryless since it only depends on the present input of the time,  $n$  at instant

$$T\{f[n]\} = e^{f[n]}$$

$$f[n] = f[n-1] + x[n] \text{ (non-additive)}$$

$$T\{f[n]\} = e^{f[-1] + x[n]}$$

$$n=0$$

$$T\{f[n]\} = e^{f(0)} + x(0)$$

$$n=1$$

$$T\{f[n]\} = e^{f(1)} + x(1)$$

$$T\{f[n]\} = e^{f(0) + x(0)} + e^{f(1) + x(1)}$$

$$n=2$$

$$T\{f[n]\} = e^{f(2)} + x(2)$$

$$n=3$$

$$T\{f[n]\} = e^{f(3)} + x(3)$$

$$n=4$$

$$T\{f[n]\} = e^{f(4)} + x(4)$$

$$n=5$$

$$T\{f[n]\} = e^{f(5)} + x(5)$$

b)

$$T[x[n]] = a x[n] + b$$

$$i) x[n] =$$

but  $x[n] =$  ~~not bounded and so the system is not stable~~

A system is stable if every bounded input produces a bounded output

$$\text{If } |x[n]| \leq m, \text{ then } |y[n]| = |ax[n] + b| \leq |a|m + |b|$$

which is also a bounded system

Therefore the system is stable.

ii) When the values of  $n = 0, 1, 2, \dots$  are plugged in to the system equation, the system

$T[x[n]]$  will depend only on those inputs - not the future values - This shows that the system is stable.

iv) Time-Invariant Testing

Checking if the shifting of the input results in the same shift in the output

$$\text{If } x[n] = x[n - n_0]$$

$$\text{Then } y[n] = a x[n - n_0] + b$$

This is the same as shifting ~~input~~  $y[n]$  by ~~input~~  $n_0$

The system is Time Invariant

v) The output  $n$  depends on  $x[n]$  and not on past or future values

The system is memory less

vi) Linearity Testing

$$T_1(x_1[n]) = c_1 x_1[n] + b$$

$$T_2(x_2[n]) = c_2 x_2[n] + b$$

$$T_1(x_1[n]) + T_2(x_2[n]) = c_1 x_1[n] + c_2 x_2[n] + b$$

$$a(c_1 x_1[n] + c_2 x_2[n]) + b \neq c_1 a x_1[n] + c_2 a x_2[n] + b$$

The system is non-linear since

the superposition is not satisfied

c)

$$T(x[n]) = x[n] + 3u[n+1]$$

where  $u[n]$  is the ~~step~~ unit step function, defined as

$$u[n] = \begin{cases} 1 & n \geq 0 \\ 0 & n < 0 \end{cases}$$

$$y[n] = 1x[n] + 3u[n+1]$$

i) Closely Testing

A system is stable if every bounded input produces a bounded output

If  $|x[n]| \leq M$  for all  $n$ , then

$$|y[n]| = |x[n] + 3u[n+1]|$$

since  $u[n+1]$  is either 0 or 1 then the term

$3u[n+1]$  is either 0 or 3

If  $|y[n]| \leq M$  then

then  $|y[n]| \leq M + 3$  and still bounded

The system is stable

ii) Causality Testing

$$y[n] = x[n] + 3u[n+1]$$

where 0, 1, -1 is used for  $n$  values

The term  $x[n]$  depends on the present input

The term  $3u[n+1]$  depends on  $n+1$  which is not dependent on future values of  $x[n]$ , only on a fixed function  $u[n+1]$

The system is causal.

iv) Time-Invariance Testing

Suppose input is shifted by  $No$ , then

$$y[n] = x[n]$$

$$y[n] = x[n] \neq x[n - No]$$

$$x'[n] = x[n - n_0]$$

Item system output  $y'[n] = f(x[n], x[n-1])$

$$y'[n] = x[n] + 3u[n+1]$$

$$x[n-n_0] + 3u[n+1]$$

$$\text{Therefore output } y[n] = x[n] + 3u[n+1]$$

$$T(x[n]) = x[n] + 3u[n+1]$$

The term  $3u[n+1]$  does not shift according to  $n_0$ .  
Hence the system is not time-invariant (Time Invariant)

### iii) Linearity Testing

$$T_1(c_1x_1[n] + c_2x_2[n]) = c_1x_1[n] + c_2x_2[n] + 3u[n+1]$$

$$c_1T(x_1[n]) + c_2T(x_2[n]) = c_1x_1[n] + 3u[n+1] + c_2x_2[n] + 3u[n+1]$$

$$T_1(c_1x_1[n] + c_2x_2[n]) \neq c_1T(x_1[n]) + c_2T(x_2[n])$$

The system is not linear

### v) Memoryless Testing

$$T(x[n]) = x[n] + 3u[n+1]$$

The output  $y[n]$  depends on  $u[n+1]$ , future function  
Since the system depends on  $n+1$  (a future value). It is  
not memoryless.

Summary

Property	$T(x[n]) = x[n]$	$T(x[n]) = ax[n] + q$	$T(x[n]) = x[n] + 3u(n+1)$
Stable	No	Yes	Yes
Causal	Yes	Yes	No (depends on future value)
Linear	No	No	No
Time-invariant & memoryless	Yes	Yes	No (depends on future value) It depends on future value

3) Define the unit step response to the linear time-invariant

System. Impulse response given by

$$h[n] = q^n u[-n], 0 < q < 1$$

$$h[n] = q^n u[-n] \cdot 0 < q < 1$$

$$\text{Let } f[n] = u[n]$$

$$f[n] = \begin{cases} 1, & n \geq 0 \\ 0, & n < 0 \end{cases}$$

$$(1 + q + q^2 + \dots + q^{n-1}) u[n] \leq 0$$

Let  $y[n]$  be the convolution sum

$$y[n] = \sum_{k=-\infty}^{\infty} h[k] x[n-k].$$

$$k = -\infty$$

$$y[n] = \sum_{k=-\infty}^{\infty} q^{-k} u[-k] x[n-k]$$

$$k = -\infty$$

$$y[n] = \sum_{k=-\infty}^{\infty} q^{-k} u[-k] u[n-k]$$

$$\text{Evaluate } k = -\infty, 0$$

$$(1 + q + q^2 + \dots + q^{n-1}) u[n] \leq 0$$

$$y[n] = \sum_{k=-\infty}^{\infty} q^{-k} u[-k] u[-k]$$

$$k = -\infty$$

$$y[n] = \sum_{k=-\infty}^{\infty} q^{-k} \left\{ \begin{array}{ll} 1, & k = -n \\ 0, & k \neq -n \end{array} \right.$$

Since  $h[k]$  is non-zero only for  $k \leq 0$ , the sum

simplifies further and we have  $y[n] = q^{-n} u[-n]$

$$y[n] = \sum_{k=-\infty}^{\infty} q^{-k}$$

$$k = -\infty$$

Using the sum formula for geometric series

$$\sum_{k=-\infty}^{\infty} r^k = \frac{1}{1-r} \quad \text{for } |r| < 1$$

for  $n \leq 0$

$$y[n] = \sum_{k=0}^0 -a$$

$k=0$

$$y[n] = \sum_{k=-n}^0 a^{n-k}$$

finally

$$(1-a)y[n] = \begin{cases} \frac{1}{1-a} & n \geq 0 \\ \frac{1-a^n}{1-a} & n < 0 \end{cases}$$

$$y[n] = \begin{cases} \frac{a^n}{1-a} & n \geq 0 \\ \frac{1-a^n}{1-a} & n < 0 \end{cases}$$

4. Determine whether the following filter

$$h[n] = (1 + \sqrt{2}) \delta[n] + \delta[n-1]$$

$$H(e^{j\omega}) = (1 + \sqrt{2}) + e^{-j\omega}$$

$$= e^{j\omega/2} + e^{-j\omega/2} + \sqrt{2}$$

$$= 2 \cos(\omega/2) + \sqrt{2}$$

$$= 2 \cos(\omega/2) + 1.414$$

¶ A linear time invariant system is described by the  
difference equation given by  
 $y[n] - 5y[n-1] + 6y[n-2] = 2x[n-1]$

- Determine the homogeneous response of the system ( $y[n] = 0$  for  $n < 0$ )
- Determine the impulse response of the system
- Determine the unit step response of the system
- Write MATLAB m-file to generate the result of item b) and c) for  $0 \leq n \leq 120$

Homogeneous response

a)

$$y[n] - 5y[n-1] + 6y[n-2] = 2x[n-1]$$

a) Homogeneous response

$$\text{Let } y_h[n] = z^n$$

$$z^n - 5z^{n-1} + 6z^{n-2} = 0$$

$$z^{n-2} [z^2 - 5z + 6] = 0$$

$$z^{n-2} [z^2 - 5z + 6] = 0 \quad \text{or} \quad z^2 - 5z + 6 = 0$$

$$z^2 - 5z + 6 = 0$$

$$(z-2)(z-3) = 0 \quad z = 2, 3$$

for homogeneous solution

$$y_h[n] = C_1 2^n + C_2 3^n$$

$C_1$  and  $C_2$  are constants

b) Impulse response  $H(z) = \frac{T(z)}{X(z)}$

$$y[n] = 5y[n-1] + 6y[n-2] = 2x[n-1]$$

Taking Z-transform on both sides

$$Y(z) - 5z^{-1}Y(z) + 6z^{-2}Y(z) = 2z^{-1}X(z)$$

$$Y(z) \{1 - 5z^{-1} + 6z^{-2}\} = 2z^{-1}X(z)$$

$$\frac{Y(z)}{X(z)} = V(z) = \frac{2z^{-1}}{1 - 5z^{-1} + 6z^{-2}}$$

$$H(z) = \frac{2z^{-1}}{(1-2z^{-1})(1-3z^{-1})}$$

$$H(z) = \frac{A}{1-2z^{-1}} + \frac{B}{1-3z^{-1}}$$

$$\frac{2z^{-1}}{(1-2z^{-1})(1-3z^{-1})} = \frac{A(1-3z^{-1})}{(1-2z^{-1})(1-3z^{-1})} + \frac{B(1-2z^{-1})}{(1-2z^{-1})(1-3z^{-1})}$$

$$A + B = 0 \quad \text{--- (i)}$$

~~$$-3A + B = 2$$~~

$$-3A + 2B = 2 \quad \text{--- (ii)}$$

$$A = -B \quad \text{--- (iii)}$$

Solving (i) & (ii)

$$\begin{array}{l|l} -3A + 2A = 2 & B = -A = -(-2) = 2 \\ A = -2 & B = 2 \end{array}$$

$$H(z) = \frac{-2}{1-2z^{-1}} + \frac{2}{1-3z^{-1}}$$

$$h[n] = 1z + \{H(z)\}$$

$$h[n] = -2(2)^n u[n] + 2(3)^n u[n]$$

$$h[n] = 2(3^n - 2^n)u[n]$$

impulse response

c)

$$H(z) = \frac{5z}{(z-2)(z-3)}$$

$$\text{let } x[n] = u[n]$$

$$x(z) = u[z] = \frac{1}{(z-2)(z-3)}$$

$$y(z) = H(z) \cdot x(z)$$

$$y(z) = \frac{A}{z-2} + \frac{B}{z-3} + \frac{C}{z-1}$$

$$y(z) = A(1-3z)(1-z) + B(1-2z)(1-z) + C(1-2z)(1-3z) = 2z$$

$$\text{take } j$$

$$\frac{1}{z} = 1$$

$$2 = C(-1)(-2)$$

$$C = 1$$

$$z^1 = \frac{1}{3}$$

$$\frac{2}{3} = B\left(\frac{1}{3}\right)\left(\frac{2}{3}\right)$$

$$B = 3$$

hence

$$z^1 = \frac{1}{2}$$

$$1 = \frac{1}{2}(-\frac{1}{2})(\frac{1}{2})$$

$$A = -\frac{4}{3}$$

$$y(z) = \frac{-4 + 3 + 1}{1 - 2z^1 - 3z^1 - 1z^1}$$

(Any inverse Z-Transform or both filter

$$y[n] = -4[2^n]u[n] + 3[3^n]u[n] + 1[n]$$

$$y[n] = [-4(2^n) + 3(3^n) + 1]u[n]$$

Unit Step regarding the system

d) Matlab Code:

clc; clear; close all;

% Define range

n = -20:120;

% Define system coefficients

a = [1, -5, 6]; % Coefficients of y[n]

b\_impulse = [0, 2]; % Coefficient for Impulse response ( $x[n] = \delta[n]$ ),

b\_step = 2 \* ones(1, 121+20); % Coefficient for step response ( $x[n] = 1$ )

[n, y] = filter(b\_step, a, b\_impulse);

% Compute impulse response

h = filter(b\_impulse, a, (n=1); 0/n & [n-1])

% Compute percentage response

u = ones(n >= 0); % Unit step function

y\_step = filter(b\_step, a, u);

% Plot impulse response

figure;

stem(n, h, 'filled');

xticklabel('n'); ylabel('h[n]');

title('Impulse Response');

grid on;

1. plot step response.

figure

```
stem (n, y - step, 'filled');  
xlabel ('n'); ylabel ('y - step[n]');  
title ('Step response');  
grid on;
```

3

Consider the linear, time invariant system described by the difference equation given by

$$y[n] = -2x[n] + 4x[n-1] - 2x[n-2]$$

a) Determine the impulse response of the system.

b) Determine the frequency response of the system. Express

your answer in the form

$$H(e^{j\omega}) = A(e^{j\omega})e^{-j\omega D}$$

c) Sketch figure with the magnitude  $|H(e^{j\omega})|$  and the phase  $\angle H(e^{j\omega})$  response.

$$y[n] = -2x[n] + 4x[n-1] - 2x[n-2]$$

a) using  $y[n] = x[n] * h[n]$

$$y[n] = x[n] * \{-2\delta[n] + 4\delta[n-1] - 2\delta[n-2]\}$$

$$h[n] = -2\delta[n] + 4\delta[n-1] - 2\delta[n-2]$$

$$h[n] = \{-2, 4, -2\}$$

B

b)

$$h(n) = -2s(n) + 4s(n-1) - 2s(n-2)$$

Take 2<sup>r</sup> Transform on both sides

$$H(e^{-j\omega}) = -2 + 4e^{-j\omega} + 2e^{-j\omega}$$

$$H(e^{j\omega}) = -2 + 4e^{j\omega} - 2e^{j\omega}$$

$$= -2 + 2(2 - e^{-j\omega}) e^{j\omega}$$

$$H(e^{j\omega}) = -4e^{j\omega} \left[ \frac{e^{j\omega}}{2} - 1 + \frac{1}{2} e^{j\omega} \right]$$

$$H(e^{j\omega}) = -4e^{j\omega} \left( \frac{e^{j\omega} + e^{-j\omega}}{2} - 1 \right)$$

$$= -4e^{j\omega} \left( \underbrace{e^{j\omega} + e^{-j\omega}}_{2} - 1 \right)$$

but  $\frac{e^{j\omega} + e^{-j\omega}}{2} = \cos(\omega)$

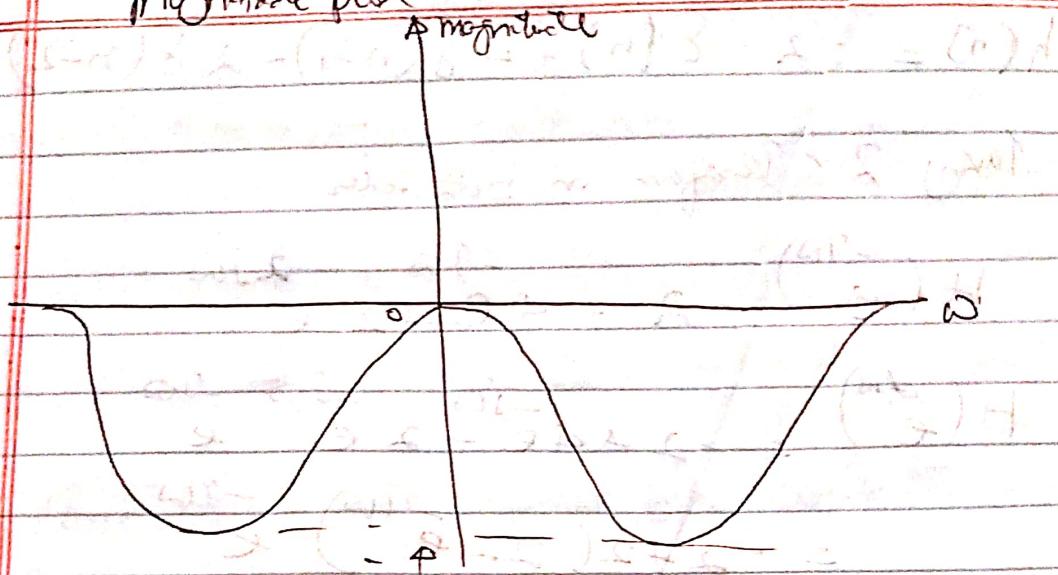
$$H(e^{j\omega}) = -4e^{j\omega} (\cos\omega - 1)$$

c)  $H(e^{j\omega}) = -4e^{-j\omega} (\cos\omega - 1)$

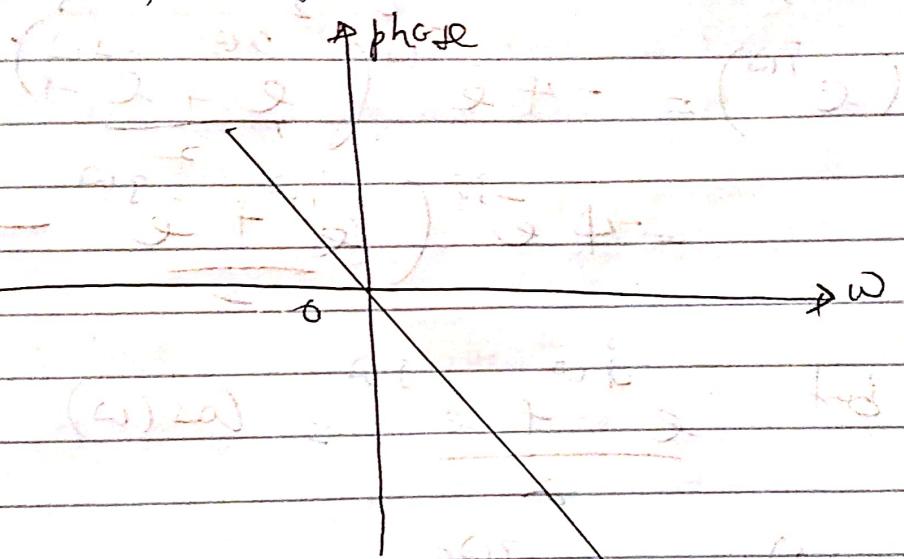
$$|H(e^{j\omega})| = 4(\cos\omega - 1)$$

$$\angle H(e^{j\omega}) = \omega$$

Magnitude plot



phase plot



$$(1 + \cos \omega) \cdot \sin \omega = \left(\frac{\pi}{2}\right) \omega$$

$$(1 + \cos \omega) \cdot \sin \omega = \left(\frac{\pi}{2}\right) \omega$$

$$(1 + \cos \omega) \cdot \sin \omega = \left(\frac{\pi}{2}\right) \omega$$

$$\cos \omega = \frac{\pi}{2} - \sin \omega$$