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qtn 1

1 (a) $x_c(t) = \sin(200\pi t) + \cos(400\pi t) \quad -\infty < t < \infty$

$x[n] = \sin\left(\frac{\pi n}{5}\right) + \cos\left(\frac{2\pi n}{5}\right) \quad -\infty < n < \infty$

Therefore $200\pi n T = \frac{\pi n}{5}$. And also $400\pi n T = \frac{2\pi n}{5}$

$T = \frac{1}{1000} s \qquad T = \frac{1}{1000} s$.

$f_s = \frac{1}{T} = 1000 Hz$

(i) The choice of T is not unique because the sampling period is periodic implying that there are other frequency that can be used for sampling

1 (b) $\Delta = 0.1$

using $\Delta = \frac{X_{max}}{2^B} = \frac{2A}{2^B}$

$2^B = \frac{2 \times 1}{0.1}$.

$2^B = 20$

$\log 2^B = \log 20$

$B = \frac{\log 20}{\log 2} = 4.3219$ bits

$SQNR = 1.76 + 6.02B$

$= 1.76 + 6.02 \times 4.3219$

$SQNR = 27.778 dB$.

1 b (ii) $\Delta = \frac{X_{max}}{2^B} = \frac{2A}{2^B}$

$2^B = \frac{2}{0.1}$

$\log 2^B = \log 20$

$B = 4.3219$

$B \approx 5$ bits

1(b) (ii)

$y[n] = -2x[n] + 4x[n-1] - 2x[n-2]$

Impulse response $h[n] = -2\delta[n] + 4\delta[n-1] - 2\delta[n-2]$

frequency response $H(e^{j\omega}) = -2 + 4e^{-j\omega} - 2e^{-j2\omega}$

$= -4e^{-j\omega}\left(\frac{e^{j\omega}}{2} + \frac{e^{-j\omega}}{2} - 1\right)$

$= -4e^{-j\omega}(\cos\omega - 1]$.

but $\cos\omega = 1 - 2\sin^2\omega/2$.

$= -4e^{-j\omega}[1 - 2\sin^2\omega/2 - 1]$

$= 8e^{-j\omega}\sin^2\omega/2$

$= 8\sin^2(\omega/2)\, e^{-j\omega}$

2. Qtn 2

$x[n] = \delta[n] - 3\delta[n-1] + 2\delta[n-3]$

$h[n] = 2\delta[n] + \delta[n-1]$

(i) $X[k] = \sum_{n=0}^{4-1} x[n] e^{-\frac{j2\pi nk}{4}}$

$X[k] = \sum_{n=0}^{3} x[n] e^{-\frac{j\pi kn}{2}}$

$X[k] = \sum_{n=0}^{3} x[n] W_4^{kn}$ where $W_4 = e^{-j\pi/2} = -i$

$X[k] = \sum_{n=0}^{3} [1, -3, 0, 2] W_4^{kn}$

$X[k] = 1 - 3W_4^{k} + 2W_4^{3k}$

$k=0$

$X[0] = 1 - 3(-i)^0 + 2(-i)^{3(0)}$

$X[0] = 0$

$k=1$

$X[1] = 1 - 3(-i)^1 + 2(-i)^{3(1)}$

$X[1] = 1 + 5j$

$k=2$

$X[2] = 1 - 3(-i)^2 + 2(-i)^{3(2)}$

$X[2] = 2$.

for $k=3$

$X[3] = 1 - 3(-i)^3 + 2(-i)^{3(3)}$

$X[3] = 1 - 5j$

$X[k] = [0, 1+5j, 2, 1-5j]$

$$H[k] = \sum_{n=0}^{3} h(n) W_4^{kn}$$

$$= \sum_{n=0}^{3} [2, 1] W_4^{kn}$$

$$= 2 + W_4^{k}$$

For $k=0$

$H[0] = 2 + (-j)^0$

$= 3$

For $= 1$

$H[1] = 2 + (-j)^1$

$= 2 - j$

$k = 2$

$H[2] = 2 + (-j)^2$

$H[2] = 1$

For $k = 3$

$H[3] = 2 + (-j)^3$

$H[3] = 2 + j$

$H[k] = [3, 2-j, 1, 2+j]$

(ii) $x(n) = [1, -3, 0, 2]$

$h(n) = [2, 1]$

h(n) \ x(n)	1	-3	0	2
2	2	-6	0	4
1	1	-3	0	2

$y(n) = [2, -5, -3, 4, 2]$

(iii) $y(n) = x(n) \circledast h(n) = \sum_{k=0}^{3} x(k)\, h[\langle n-k \rangle_4]$ $\quad n = 0 \leq n \leq 3$.

$y(0) = \sum_{k=0}^{3} x(k)\, h[\langle -k \rangle_4] = x(0)h(0) + x(1)h(3) + x(2)h(2) + x(3)h(1)$

$= 2 + 0 + 0 + 2 = 4$.

$y(1) = \sum_{k=0}^{3} x(k)\, h[\langle 1-k \rangle_4] = x(0)h(1) + x(1)h(0) + x(2)h(3) + x(3)h(2)$

$= 1 - 6 + 0 + 0 = -5$

$y(2) = \sum_{k=0}^{3} x(k)\, h[\langle 2-k \rangle_4] = x(0)h(2) + x(1)h(1) + x(2)h(0) + x(3)h(3)$

$= 0 - 3 + 0 + 0 = -3$.

$y(3) = \sum_{k=0}^{3} x(k)\, h[\langle 3-k \rangle_4] = x(0)h(3) + x(1)h(2) + x(2)h(1) + x(3)h(0)$

$= 0 + 0 + 0 + 4 = 4$.

$x(n) \circledast h(n) = [4, -5, -3, 4]$.

iv) $Y[k] = H[k]\, X[k]$

$H[k] = 2 + W_4^k$

$X[k] = [1 - 3W_4^k + 2W_4^{3k}$

$Y[k] = [2 + W_4^k][1 - 3W_4^k] + 2W_4^{3k}]$

$Y[k] = 2 - 5W_4^k - 3W_4^{2k} + 4W_4^{3k} + 2W_4^{4k}$

$y[n] = [2, -5, -3, 4, 2]$ similar to linear convolution considering $H[k] = [3, 2-j, 1, 2+j]$ and $x[n] = [0, 1+5j, 2, 1-5j]$

$Y[k] = H[k] \cdot X[k] = [3, 2-j, 1, 2+j][0, 1+5j, 2, 1-5j] = [0, 7+9j, 2, 7-9j]$

$$y[n] = \frac{1}{N} \sum_{k=0}^{N-1} Y[k] e^{\frac{j2\pi kn}{N}}$$

$$= \frac{1}{4} \sum_{k=0}^{3} Y[k] e^{\frac{j2\pi kn}{4}}$$

$$= \frac{1}{4} \sum_{k=0}^{3} Y[k] W_4^{-nk} \quad \text{where } W_4 = e^{j\pi/2} = i$$

$$y[n] = \frac{1}{4} \sum_{k=0}^{3} [0, 7+9j, 2, 7-9j] W_4^{-nk}$$

$$= \frac{1}{4} \left((7+9j) W_4^{-n} + 2 W_4^{-2n} + (7-9j) W_4^{-3k} \right) \quad 0 \le n \le 3.$$

$$y[0] = \frac{1}{4} [[7+9j](i)^0 + 2(i)^{2(0)} + (7-9j)(i)^{3(0)}] = 4$$

$$y[1] = \frac{1}{4} [(7+9j)(i)^1 + 2(i)^{2(1)} + (7-9j)(i)^{3(1)}] = -5$$

$$y[2] = \frac{1}{4} [(7+9j)(i)^2 + 2(i)^{2(2)} + (7-9j)(i)^{3(2)}] = -3$$

$$y[3] = \frac{1}{4} [(7+9j)(i)^3 + 2(i)^{2(3)} + (7-9j)(i)^{3(3)}] = 4$$

$y[n] = [4, -5, -3, 4]$ Similar to circular convolution

(v) Linear convolution operates on infinite length sequence whereas circular convolution operates on finite length sequences

- DFT provides a way of obtaining the linear convolution of two sequence of finite length by obtaining the inverse DFT

Qn 3

(a) $y[n] - 3y[n-1] - 4y[n-2] = x[n] + 2x[n-1]$

$x[n] = 4^n u[n]$

$y[-1] = y[-2] = 0$

Homogeneous equation solution

~~let $y_h[n] = \lambda^n$~~

~~$\lambda^n - 3\lambda^{n-1} - 4\lambda^{n-2} = 0$~~

~~$\lambda = \lambda'$~~

let $y_h(n) = z^n$

$z^n - 3z^{n-1} - 4z^{n-2} = 0$

$z^{n-2}(z^2 - 3z - 4) = 0$

$z_1 = -1$ and $\lambda_2 = 4$

$y_h[n] = C_1(4)^n + C_2(-1)^n$

Particular solution

$x[n] = 4^n u[n]$

let $y_p[n] = Bn4^n$

$An4^n - 3(A(n-1)4^{n-1}) - 4(A[n-2]4^{n-2}] = 4^n + 2(4^{n-1}]$

For $n = 2$

$32A - 12A = 16 + 8$

$A = 6/5$

$y_p[n] = 1.2\, n 4^n$

$y_B[n] = C_1[4]^n + C_2(-1)^n + 1.2(n4^n]$

$y[0] - 3y(-1) - 4y(-2] = x[0] + 2x(-1]$

$y[0] = 1$

$y[0] = C_1 + C_2 + 0$

$C_1 + C_2 = 1$

For $n=1$

$$y[1]-3y[0]-4y[-1]=x(1)+2x[0]$$

$$y[1] = 3+4+2$$
$$= 9$$

$$y[1] = C_1(4)^1 + C_2(-1)^1 + 1.2(1)4^{(1)}$$

$$9 = 4C_1 - C_2 + 4.8$$

$$4C_1 - C_2 = 4.2$$

$$\therefore C_1 = \frac{26}{25}, \quad C_2 = \frac{-1}{25}$$

$$y[n] = \frac{26}{25}(4)^n - \frac{1}{25}(-1)^n + 1.2n(4)^n$$

3(b)

$$H(z) = \sum_{n=-\infty}^{\infty} h(z)z^{-n} = \sum_{n=N_1}^{N_2} a^n z^{-n} = \sum_{n-N_2=0}^{N_2} (az^{-1})^n$$

let $u=n-N_1$ when $u=0$, $u-N_1=0$

$n=u+N_1$ when $n=N_2$, $u=N_2-N_1$

$$H(z) = \sum_{u=0}^{N_2-N_1} (az^{-1})^{u+N_1} = (az^{-1})^{N_1} \sum_{u=0}^{N_2-N_1} (az^{-1})^u$$

using the sum of a GP: $S_n = \frac{a(1-r^n)}{1-r}$

$$H(z) = (az^{-1})^{N_1} \left[\frac{1-(az^{-1})^{N_2-N_1+1}}{1-az^{-1}}\right]$$

$$= \frac{(az^{-1})^{N_1} - (az^{-1})^{N_2-N_1+1}(az^{-1})^{N_1}}{1-az^{-1}}$$

$$H(z) = \frac{a^{N_1}z^{-N_1} - a^{N_2+1}z^{-(N_2+1)}}{1-az^{-1}} \qquad |z|>|a|$$

$1 - P(\cos\omega + j\sin\omega$

3 c] $H(\omega) = \dfrac{b}{(1-pe^{-j\omega})^2}$

$$H(\omega) = \frac{b}{(1-p)^2}$$

$$1 = \frac{b}{(1-p)^2}$$

$$b = (1-p)^2$$

$$|H(\omega)| = \frac{|b|}{|(1-pe^{-j\omega})^2|} = \frac{b}{\sqrt{(1-p\cos\omega)^2+(-p\sin\omega)^2} \cdot \sqrt{(1-p\cos\omega)^2(-p\sin\omega)^2}}$$

$$= \frac{b}{1+p^2\cos\omega^2 + p^2\sin\omega^2 - 2p\cos\omega}$$

$$= \frac{b}{1+p^2\underbrace{[\cos\omega^2+\sin\omega^2]}_{1} - 2p\cos\omega}$$

$$|H(\omega)| = \frac{b}{1-2p\cos\omega+p^2}$$

$$|H(\pi/4)|^2 = \frac{b^2}{[1-\sqrt{2}p+p^2]^2}$$

$$\frac{1}{2} = \frac{b^2}{[1-\sqrt{2}p+p^2]^2}$$

$$\frac{1}{\sqrt{2}} = \frac{b}{1-\sqrt{2}p+p^2}$$

$$\frac{1}{\sqrt{2}} = \frac{(1-p)^2}{1-\sqrt{2}p+p^2}$$

$$(\sqrt{2}-1)p^2 - \sqrt{2}p + (\sqrt{2}-1) = 0.$$

$p_1 = 0.32$, $p_2 = 3.09$

$b = (1-0.32)^2$ $\quad b = [1-3.09)^2$

$b = 0.4624$ $\quad b = 4.37$

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