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DIGITAL SIGNAL PROCESSING/ ~~[illegible]~~

ASSIGNMENT 1

Q1 Consider the following sequences and their ...

a) $x[n] = 2\delta[n+2] - \delta[n-4]$ $-5 \le n \le 5$

b) $x[n] = n(2u[n] - u[n-10]) + 10e^{-0.3(n-10)}(u[n-10] - u[n-20]), 0 \le n \le 20$

Write Matlab routines to illustrate these sequences in time (n)

Plot Descrete sequences

```
% Define the range for sequence (a)
    n_a = -5:5;
    x_a = 2*(n_a == -2) - (n_a == 4);

% plot sequence (a)
    figure;
    stem(n_a, x_a, 'filled');
    grid on;
    title('sequence(a): x[n] = 2\delta[n+2] - \delta[n-4]');
    xlabel('n');
    ylabel('x[n]');

% define the range for sequence(b)
%     n_b = 0:20;

% Compute sequence (b)
    u_n = (n_b >= 0); % unit step u[n]
    u_n_10 = (n_b >= 20); % unit step u[n-20]
x_b = n_b .*(2*u_n - u_n_10 + 10* exp(-0.3*(n_b-10))*
                (u_n_10 - u_n_20);

% plot sequence (b)
figure;
stem(n_b, x_b, 'filled');
grid on;
title('sequence (b)');
xlabel('n');
ylabel('x[n]');
```

1.2 Determine whether the following systems are i) stable, ii) causal iii) linear, iv) time-invariant and memoryless

a) $T(x[n]) = e^{x[n]}$

b) $T(x[n]) = ax[n] + a$

c $T(x[n]) = x[n] + 3u(n+1)$

Soln

a) i) stability testing

$$T(x(n)) = e^{x[n]}$$

If $|x[n]| \leq M$ then

$$y[n] = e^{x[n]}$$

The function e^x is not bounded for all $x[n]$ is very large (e.g $x[n] \to \infty$)

Thus $T(x[n]) \to \infty$ which is unbounded. The system is not stable.

ii) Causuality Testing

$$T(x[n]) = e^{x(n)} \qquad n = 0, 1, -1$$

$n = 0$ when $n = 1$

$$T(x[n]) = e^{x(0)} = e^0 = 1 \qquad T(x[n]) = e^{x(1)} = e^x$$

$n = -1$

$$T(x[n]) = e^{x(-1)} = e^{-x}$$

Since $x[n]$ ~~only~~ is dependent on the present inputs of n, the system is causual

iii) $T_1[x_1[n]] = e^{x_1[n]}$

$T_2[x_2[n]] = e^{x_2[n]}$

$$T[ax_1[n] + bx_2[n]] = e^{ax_1[n] + bx[n]}$$

$$aT_1[x[n]] = e^{aX_1[n]} \quad \text{—— (i)}$$

$$bT_2[x_2[n]] = e^{bX_2[n]} \quad \text{—— (ii)}$$

Combine ~~and~~ (i) & (ii)

$$aT_1(x_1[n]) + bT_2(x_2[n]) = e^{ax_1[n]} + e^{bX_2[n]}$$

Since

$$T(aT_1(x_1[n]) + bT_2(x_2[n])) \neq aT_1(x_1[n]) + bT_2(x_2[n])$$

therefore the system is non - linear

iv) for Time - Invariant system.

~~T(x[n-n_0~~

$$T(x[n-n_0]) = y[n-n_0]$$

$$y[n] = e^{x[n]}$$

$$y[n-n_0] = e^{x[n-n_0]}$$

$$T\{x[n]\} = e^{x[n]}$$

$$T[x[n-n_0]] = e^{x[n-n_0]}$$

Since $T(x[n-n_0]) = y[n-n_0]$, then the system is Time - Invariant

(v) System is memoryless since it only depends on the present inputs of the time, n at Instant

$$T\{x[n]\} = e^{x[n]}$$

$n = -1, 0, 1$

$$T\{x[n]\} = e^{x[-1]} = e^{-x}$$

$n = 0$

$$T\{x[n]\} = e^{x(0)} = 1$$

$n = 1$ $\quad T(x[n]) = e^{x(1)} = e^{x}$

(b)

$T(x[n]) = ax[n] + b$

i) $x[n] =$

Lust $x[n] =$

A system is stable if every bounded input produces a bounded output

If $|x[n]| \leq m$, then $|y[n]| = |ax[n] + a| \leq |a|M + |a|$

which is also a bounded system

Therefore the system is stable.

ii) When the values of $n = 0, 1, 2, -1$ is plugged in to the system equation, the system

$T\{x[n]\}$ will depend only on those inputs – not the future values – This shows that the system is stable.

iv) Time-Invariance testing

Checking if the shifting of the input results is the same shift in the output

If $x'[n] = x[n - n_0]$

then $y'[n] = ax[n - n_0] + a$

This is the same as shifting $y[n]$ by n_0

$\therefore$ The system is a Time Invariant

vi) The output n depends on $x[n]$ and not on past or future values

The system is memory less'

iii) Linearity Testing

~~iii) Linearity Testing~~

$T_1(x_1[n]) = ax_1[n] + b$

$T_2(x_2[n]) = ax_2[n] + b$

$= T\{c_1 a x_1[n] + c_2 a x_2[n] + 2b\}$

$a(c_1 a x_1[n] + c_2 a x_2[n] + 2b) + b$

Also $c_1 T_1(x_1[n]) = c_1 a x_1[n] + b$

$c_2 T_2(x_2[n]) = c_2 a x_2[n] + b$

$c_1 T_1(x_1[n]) + c_2 T_2(x_2[n]) = c_1 a x_1[n] + c_2 a x_2[n] + b$

The system is non-linear since the superposition is not satisfied c)

$$T(x[n]) = \cancel{x[x} \; x[x] + 3u[n+1]$$

Where $u[n]$ is the ~~stable~~ step function defined as

$$u[n] = \begin{cases} 1 & n \geq 0 \\ 0 & n < 0 \end{cases}$$

i) Stability testing

A system is stable if every bounded input produces a bounded output

If $|x[n]| \leq M$ for all n, Then

$$|y[n]| = |x[n] + 3u[n+1]|$$

Since $u[n+1]$ is either 0 or 1 then the term $3u[n+1]$ is either 0 or 3

If $|y[n]| \leq M$

then $|y[n]| \leq m+3$ and still bounded

The system is stable

ii) Causality Testing

$$y[n] = x[n] + 3u[n+1]$$

When 0, 1, -1 is used as n values

the term $x[n]$ depend on the present input

The term $3u[n+1]$ depends on $n+1$ which is not dependent on future values of $x[n]$, only on a fixed function $u[n+1]$

The system is non-causal.

iii) Time-Invariance Testing

Suppose input is shifted by n_0, then

$$y'[n] = x'[n]$$

$x'[n] = x[n-n_0]$

Then system output ~~$y[n]$~~ $x'[n]$

$$y'[n] = x'[n] + 3u[n+1]$$
$$x[n-n_0] + 3u[n+1]$$

The original output $y[n] = x[n] + 3u[n+1]$

$$T(x[n]) = x[n] + 3u(n+1)$$

The term $3u[n+1]$ doe not shift according to n_0.
Hence the system is not time-Invariant (Time Invariant)

iii) Linearly Testing

$$T_1(c_1 x_1[n] + c_2 x_2[n]) = c_1 x_1[n] + c_2 x_2[n] + 3u[n+1]$$

$$c_1 T(x_1[n]) + c_2 T[x_2(n)] = c_1 x_1[n] + 3u(n+1) + c_2 x_2[n] + 3u[n+1]$$

$$T_1(c_1 x_1[n] + c_2 x_2[n]) \neq c_1 T(x_1[n]) + c_2 T(x_2[n])$$

The system is not linear

v) Memoryless Testing

$$T(x[n]) = x[n] + 3u[n+1]$$

The output n depends on $u(n+1)$, future function since the system depends on $n+1$ (a future value). It is not memoryless.

Summary

Property	$T(x[n]) = e^{x[n]}$	$T(x[n]) = ax[n] + a$	$T(x[n]) = x[n] + 3u(n+1)$
stable	No ~~stable~~ ~~No~~	Yes	Yes
Causal	Yes	Yes	No ~~Yes~~ ~~No (depends on future values~~
linear	No	No	No
Time-Invariant	Yes	Yes	No (depends on future value)
Memoryless	Yes	Yes	No (depends on future value)

3) Determine the unit step response to the linear time-Invariant System. Impulse response given by
$h[n] = a^{n} u[-n], 0 < a < 1$

$$h[n] = a^{-n} u[-n] ; 0 < a < 1$$

Soln

Let $x[n] = u[n]$

$$u[n] = \begin{cases} 1, & n \geq 0 \\ 0, & n < 0 \end{cases}$$

Using the ~~Comp~~ Convolution Summary

$$y[n] = \sum_{k=-\infty}^{\infty} h[k] \, x[n-k].$$

$$y[n] = \sum_{k=-\infty}^{\infty} a^{-k} u[-k] \, x[n-k]$$

$$y[n] = \sum_{k=-\infty}^{\infty} a^{-k} u[-k] u(n-k) \quad \text{——} \circledast$$

for $n \geq 0$

$$y[n] = \sum_{k=-\infty}^{\infty} a^{-k} u[-k] \, u[-k]$$

$$y[n] = \sum_{k=-\infty}^{\infty} a^{k} \quad [\; k = -k$$

Since $h(k)$ is non zero only for $k \leq 0$ the sum simplifies follow

$$y[n] = \sum_{k=-\infty}^{\min(n,0)} a^{-k},$$

Using the sum formula for geometric series

$$\sum_{k=-\infty}^{m} r^{k} = \frac{r^{m+1}}{1-r} \quad \text{for } |r| < 1$$

for $n \leq 0$

$$y[n] = \sum_{k=-\infty}^{0} a^{-k}$$

$$= \sum_{k=-n}^{\infty} a^{+k} = \frac{a^{n}}{1-a}$$

finally

$$y[n] = \begin{cases} \frac{1}{1-a} & n \geq 0 \\ \frac{1-a^{-n}}{1-a} & n < 0 \end{cases}$$

$$= \begin{cases} \frac{a^{n}}{1-a} & n \geq 0 \\ \frac{1-a^{-n}}{1-a} & n < 0 \end{cases}$$

4. ~~Determine whether the following feature~~

4 A linear time invariant system is described by the difference equation given by

$y[n] - 5y[n-1] + 6y[n-2] = 2x[n-1]$

a) Determine the homogeneous response of the system (of $x[n]=0 \; \forall n$)

b) Determine the impulse response of the system

c) Determine the unit step response of the system

d Write matlab routines to generate the result of items b) and c) in $-20 \le n \le 120$

Soln

a)

$$y[n] - 5y[n-1] + 6y[n-2] = 2x[n-1]$$

a Homogeneous response

Let $y_h[n] = Z^n$

~~$Z^n - 5Z$~~

$$Z^2 + 5Z^{n-1} + 6Z^{n-2} = 0$$

$$Z^{n-2}[Z^2 + 5Z + 6] = 0$$

$$Z^{n-2}[Z^2 - 5Z + 6] = 0$$

$$Z^2 - 5Z + 6 = 0$$

$$(Z-2)(Z-3) = 0 \quad Z = 2, 3$$

for homogeneous solution

$$y_h[n] = C_1 2^n + C_2 3^n$$

C_1 & C_2 are constants

b) Impulse response $H(z) = \dfrac{Y(z)}{X(z)}$

$y[n] - 5y[n-1] + 6y[n-2] = 2x[n-1]$

Taking Z-Transform on both sides

$Y(z) - 5z^{-1}Y(z) + 6z^{-2}Y(z) = 2z^{-1}X(z)$

$Y(z)\{1 - 5z^{-1} + 6z^{-2}\} = 2z^{-1}X(z)$

$$\frac{Y(z)}{X(z)} = H(z) = \frac{2z^{-1}}{1 - 5z^{-1} + 6z^{-2}}$$

$$H(z) = \frac{2z^{-1}}{(1-2z^{-1})(1-3z^{-1})}$$

$$H(z) = \frac{A}{1-2z^{-1}} + \frac{B}{1-3z^{-1}}$$

$$\frac{2z^{-1}}{(1-2z^{-1})(1-3z^{-1})} = \frac{A(1-3z^{-1}) + B(1-2z^{-1})}{(1-2z^{-1})(1-3z^{-1})}$$

$A + B = 0$ —— (i)

~~$-3A + B$~~

$-3A - 2B = 2$ —— (ii)

$A = -B$ —— (iii)

Solving (ii) & (iii)

$-3A + 2A = 2$

$A = -2$

$B = -A = -(-2) = 2$

$B = 2$

$$H(z) = \frac{-2}{1-2z^{-1}} + \frac{2}{1-3z^{-1}}$$

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$h[n] = IZT\{H(z)\}$

$h[n] = -2(2)^n u[n] + 2(3)^n u[n]$

$h[n] = 2(3^n - 2^n)u[n]$

impulse response

c)

Let $x[n] = u(n)$

$X(z) = U[z] = \frac{1}{1-z^{-1}}$

$$Y(z) = \frac{H(z) \cdot X(z)}{(1-2z^{-1})(1-3z^{-1})(1-z^{-1})}$$

$$Y(z) = \frac{A}{1-2z^{-1}} + \frac{B}{1-3z^{-1}} + \frac{C}{1-z^{-1}}$$

$y(z) = A(1-3z^{-1})(1-z^{-1}) + B(1-2z^{-1})(1-z^{-1}) + C(1-2z^{-1})(1-3z^{-1}) = 2z^{-1}$

Taking

$z^{-1} = 1$

$2 = C(-1)(-2)$

$C = 1$

$z^{-1} = \frac{1}{3}$

$\frac{2}{3} = B\left(\frac{1}{3}\right)\left(\frac{2}{3}\right)$

$B = 3$

Taking

$z^{-1} = \frac{1}{2}$

$1 = A\left(-\frac{1}{2}\right)\left(\frac{1}{2}\right)$

$A = -4$

$$Y(z) = \frac{-4}{1-2z^{-1}} + \frac{3}{1-3z^{-1}} + \frac{1}{1-z^{-1}}$$

Taking inverse Z-Transform on both sides

$$y[n] = -4\,[2^n]\,u[n] + 3\,[3^n]\,u[n] + u[n]$$

$$y[n] = [-4(2^n) + 3(3^n) + 1]\,u[n]$$

Unit step response of the system

d) Matlab Code.

```
clc; clear; close all;
% define range
n = -20: 120;
% define system coefficients
a = [1, -5, 6]; % coefficient of y[n]
b_impulse = [0, 2]; % coefficient for impulse response (x[n]= delta[n-1])
b_step = 2 * ones(1, 121+20); % coefficient for step response (x[n]=u[n-1])

% Compute impulse response
h = filter(b_impulse, a, (n==1)); % δ[n-1]

% Compute ~~percentage~~ step response
u = double(n>=0) % unit step function
y_step = filter (b_step, a, u);

% plot impulse response
figure;
stem (n, h, 'filled');
xlabel ('n'); ylabel ('h[n]');
title('Impulse Response');
grid on;
```

% plot step response.

```
figure;
stem (n, y_step, 'filled');
xlabel ('n'); ylabel ('y_{step} [n]');
title ('step response');
grid on;
```

5 Consider the linear time invariant system described by the difference equation given by

$$y[n] = -2x[n] + 4x[n-1] - 2x[n-2]$$

a) Determine the Impulse response of the System.

b) Determine the frequency response of the system ~~Expressed~~ Express your answer in the form

$$H(e^{j\omega}) = A(e^{j\omega})e^{-j\omega n d}$$

c) Sketch figure with the magnitude $|X(e^{j\omega})|$ and the phase $\angle H(e^{j\omega})$ response.

Soln:

$$y[n] = -2x[n] + 4x[n-1] - 2x(n-2]$$

a) using $y[n] = x[n] * h[n]$

$$y[n] = x[n] * [-2\delta[n] + 4\delta[n-1] - 2\delta(n-2]]$$

$$h[n] = -2\delta(n) + 4\delta(n-1) - 2\delta(n-2)$$

$$h[n] = \{-2, 4, -2\}$$

b)

b)

$$h(n) = -2\,\delta(n) + 4\,\delta(n-1) - 2\,\delta(n-2)$$

Taking Z-Transform on both sides

$$H(e^{-j\omega}) = -2 + 4e^{-j\omega} + 2e^{-2j\omega}$$

$$H(e^{j\omega}) = -2 + 4e^{-j\omega} - 2e^{-j\omega} \cdot e^{-j\omega}$$

$$= -2 + 2(2 - e^{-j\omega})\,e^{-j\omega}$$

$$H(e^{j\omega}) = -4e^{-j\omega}\left[\frac{e^{j\omega}}{2} - 1 + \frac{1}{2}e^{-j\omega}\right]$$

$$H(e^{j\omega}) = -4e^{-j\omega}\left(\frac{e^{j\omega} + e^{-j\omega}}{2} - 1\right)$$

$$= -4e^{-j\omega}\left(\frac{e^{j\omega} + e^{-j\omega}}{2} - 1\right)$$

but $\frac{e^{j\omega} + e^{-j\omega}}{2} = \cos(\omega)$

$$H(e^{j\omega}) = -4e^{-j\omega}(\cos\omega - 1)$$

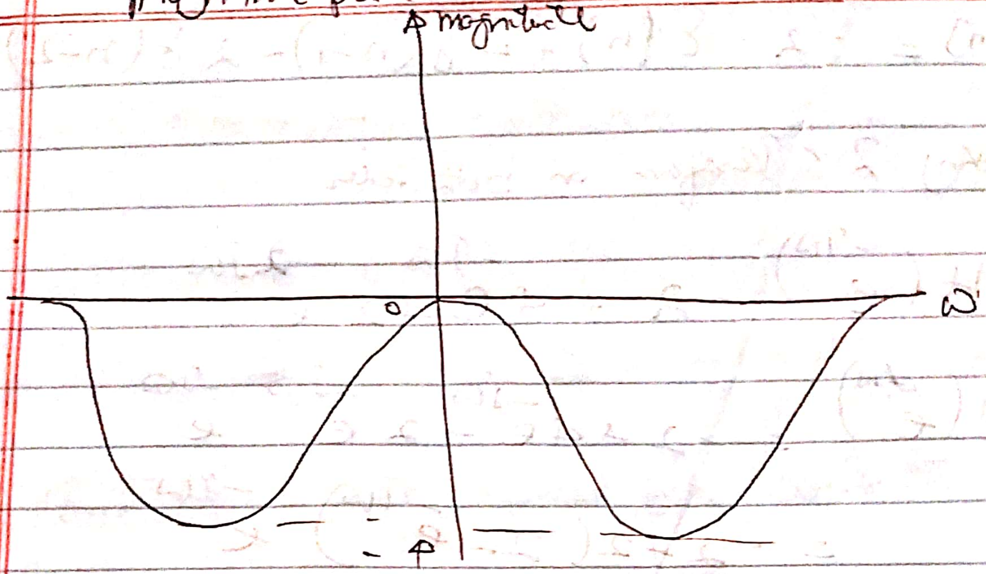
c) from

$$H(e^{j\omega}) = -4e^{-j\omega}(\cos\omega - 1)$$

$$|H(e^{j\omega})| = 4(\cos\omega - 1) \quad \cancel{\cos} - 1$$

$$\angle H(e^{j\omega}) = \omega$$

Magnitude plot



phase plot