Answer Key

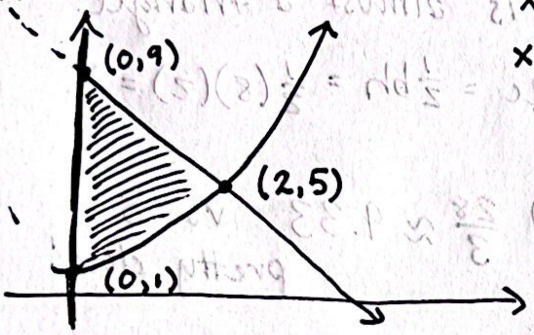
Week 1 Recitation Problems
Math 114-004-307/308

1. Graph (shading) the region bounded by the following three curves (in the first quadrant):

   (a) $y = f(x) = x^2 + 1$,

   (b) $y = g(x) = 9 - 2x$, and

   (c) the y-axis

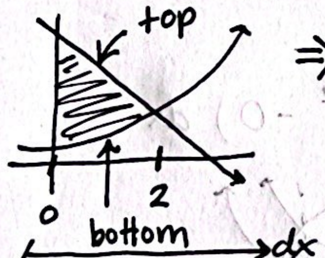


for intersections:

$$x^2+1 = 9-2x$$
$$x^2+2x+1-9=0$$
$$x^2+2x-8=0$$
$$(x+4)(x-2)=0$$
$$x=-4,\ 2$$
$$f(2)= 2^2+1=5$$
$$\Rightarrow (2,5)$$

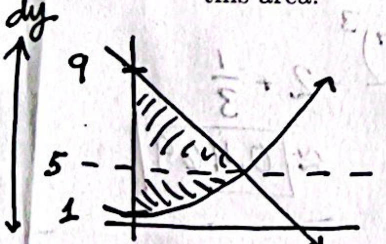
2. (i) Write one (or two) x-integral(s) giving the exact area of this region and (ii) compute explicitly this area.

Area between curves = $\int (\text{top} - \text{bottom})\, d\_$



$$\Rightarrow \int_0^2 (g(x)-f(x))\,dx \Rightarrow \int_0^2 \left((9-2x)-(x^2+1)\right) dx$$
$$\Rightarrow \int_0^2 (9-2x-x^2-1)\,dx \Rightarrow \int_0^2 (8-2x-x^2)\,dx$$
$$\Rightarrow 8x - \frac{2x^2}{2} - \frac{x^3}{3}\Big]_0^2 = 8(2)-(2)^2-\frac{(2)^3}{3}$$
$$=16-4-8/3$$
$$=\boxed{28/3}$$

3. Now (i) write one (or two) y-integral(s) for the area of this region and (ii) compute explicitly this area.



Since there are two different "tops", break the integral into pieces!

$$\int_1^5 (f(y) - (y\text{-axis}))\,dy + \int_5^9 (g(y) - (y\text{-axis}))\,dy$$

$$f(x)=x^2+1 \rightarrow y=x^2+1 \rightarrow y-1=x^2 \rightarrow x=\sqrt{y-1}$$
$$g(x)=9-2x \rightarrow y=9-2x \rightarrow 2x=9-y \rightarrow x=\frac{9}{2}-y/2$$

$$\int_1^5 (\underbrace{\sqrt{y-1}}_{(y-1)^{1/2}}-0)\,dy + \int_5^9 (9/2 - y/2 - 0)\,dy \Rightarrow \frac{(y-1)^{3/2}}{3/2}\Big]_1^5 + \frac{9}{2}y - \frac{y^2}{4}\Big]_5^9$$

$$=\left(\frac{4^{3/2}}{3/2} - \frac{0^{3/2}}{3/2}\right) + \left(\frac{9}{2}(9) - \frac{81}{4} - \frac{9}{2}(5) - \frac{25}{4}\right)$$

$$= \frac{16}{3} + \frac{12}{3} = \boxed{28/3}$$

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Answer Key

Sanity checks:

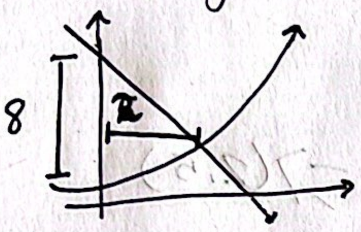
(1) Are your results in 2. and 3. the same?

(2) Can you find a way (or ways) to approximate the area without using calculus?

(3) Compare your approximation to the actual value you got for the area in (b) and (c).

1) hopefully!

2) use geometry! The region is almost a triangle.



Area of triangle $= \frac{1}{2}bh = \frac{1}{2}(8)(2) = 8$

3) $\frac{28}{3} \approx 9.33$ vs 8

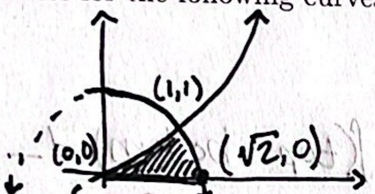
pretty close!

4. Repeat the same process for the following curves:

(a) $h(x) = x^3$

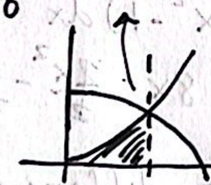
(b) $j(x) = 2 - x^2$

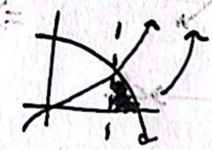
(c) the x-axis.



Hint: the intersections are at $(0, 0)$, $(1, 1)$, and $(\sqrt{2}, 0)$, and your answer will be a decimal!

x-integral: $\int_0^1 (x^3 - 0)\,dx + \int_1^{\sqrt{2}} (2 - x^2 - 0)\,dx$





$\Rightarrow \left. \frac{x^4}{4} \right]_0^1 + \left. 2x - \frac{x^3}{3} \right]_1^{\sqrt{2}} = \frac{1}{4} + 2\sqrt{2} - \frac{(\sqrt{2})^3}{3} - 2 + \frac{1}{3}$

$\approx \boxed{0.469}$

y-integral: $y = x^3 \Rightarrow x = \sqrt[3]{y} = y^{1/3}$ $\quad y = 2 - x^2 \Rightarrow x^2 = 2 - y$

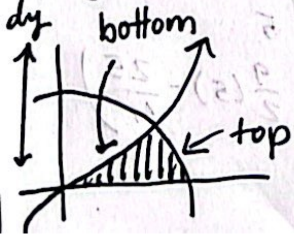
$\Rightarrow x = \sqrt{2-y}$

$\int_0^1 \left(\sqrt{2-y} - y^{1/3}\right) dy \Rightarrow \int_0^1 (2-y)^{1/2} - y^{1/3}\,dy \Rightarrow \left. -\frac{(2-y)^{3/2}}{3/2} - \frac{y^{4/3}}{4/3} \right]_0^1$

$u = 2 - y$

$du = -dy$

$= -\frac{1^{3/2}}{3/2} - \frac{1^{4/3}}{4/3} + \frac{2^{3/2}}{3/2} + 0$



$\approx \boxed{0.469}$ ✓

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