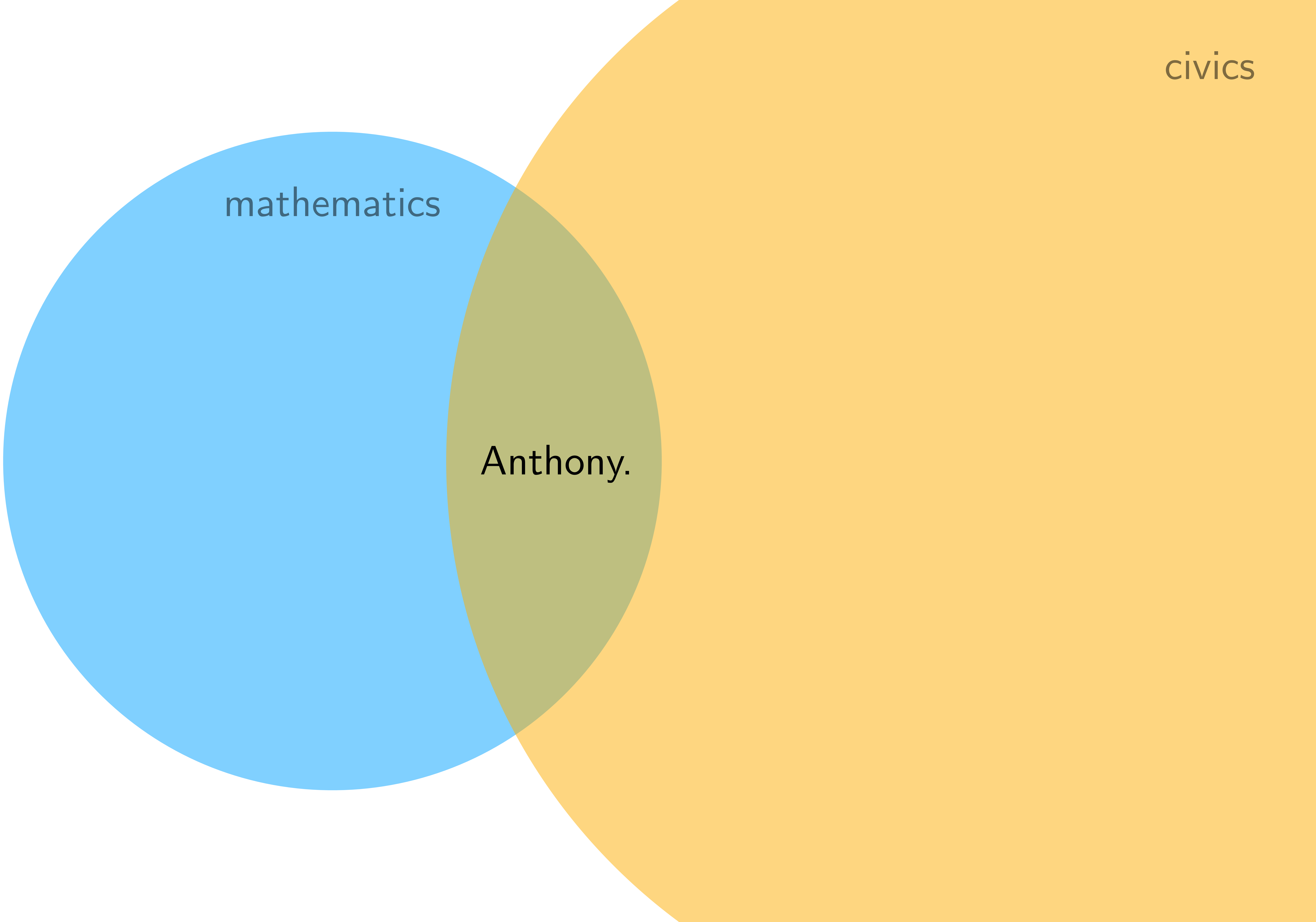
mathematics and redistricting

what we know, what we don't, and where we're going

hi! I’m Anthony.


civics
mathematics
Anthony.

the end!

what's in this talk

what's in this talk

1. a redistricting primer

what's in this talk

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2. some mathematical history

what's in this talk

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2. some mathematical history
3. contemporary research

what’s in this talk

1. a redistricting primer
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3. contemporary research
4. things that are hard

what’s in this talk

1. a redistricting primer
2. some mathematical history
3. contemporary research
4. things that are hard
5. where we’re *hopefully* going

redistricting

redistricting

the practice of dividing a jurisdiction into electoral districts.

gerrymandering

gerrymandering

the practice of redistricting to favor *or disfavor* a specific group

1 The United States Census

everyone* gets counted.

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2 Redistricting statutes

the rules of the game.

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3 Drawing the maps

where the sausage gets made?

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4 Challenging the maps

someone's angry...

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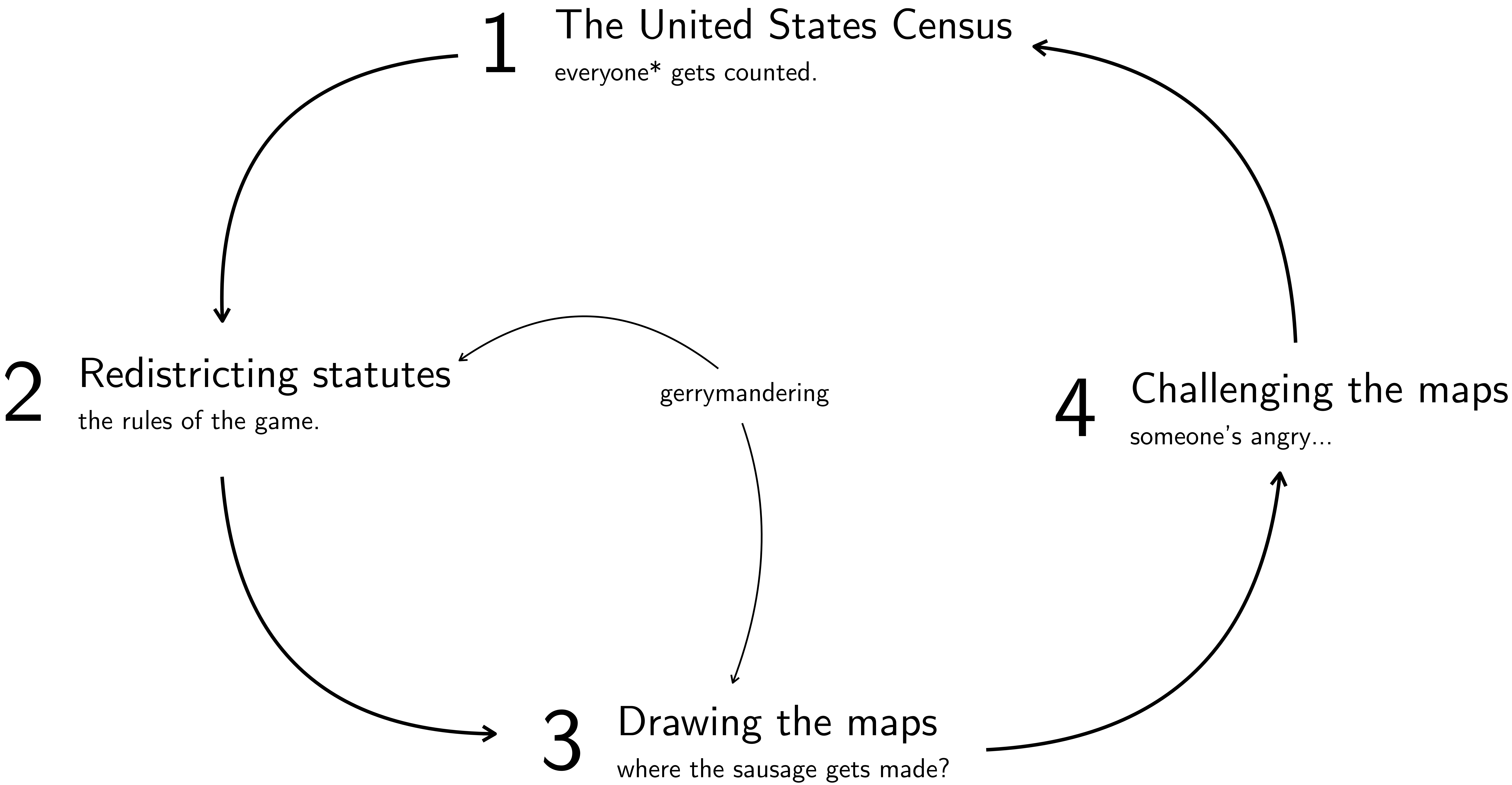
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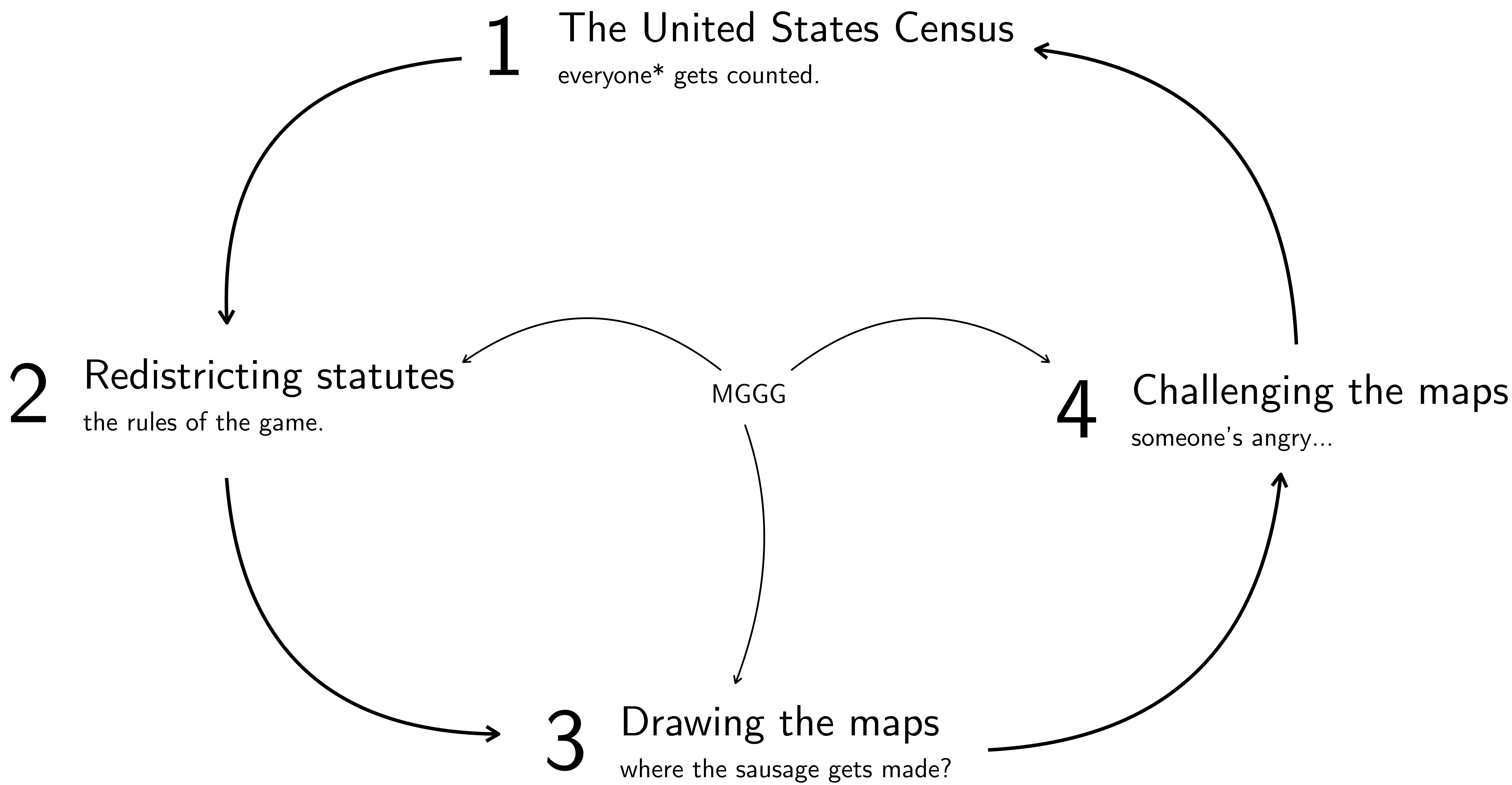
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MGGG
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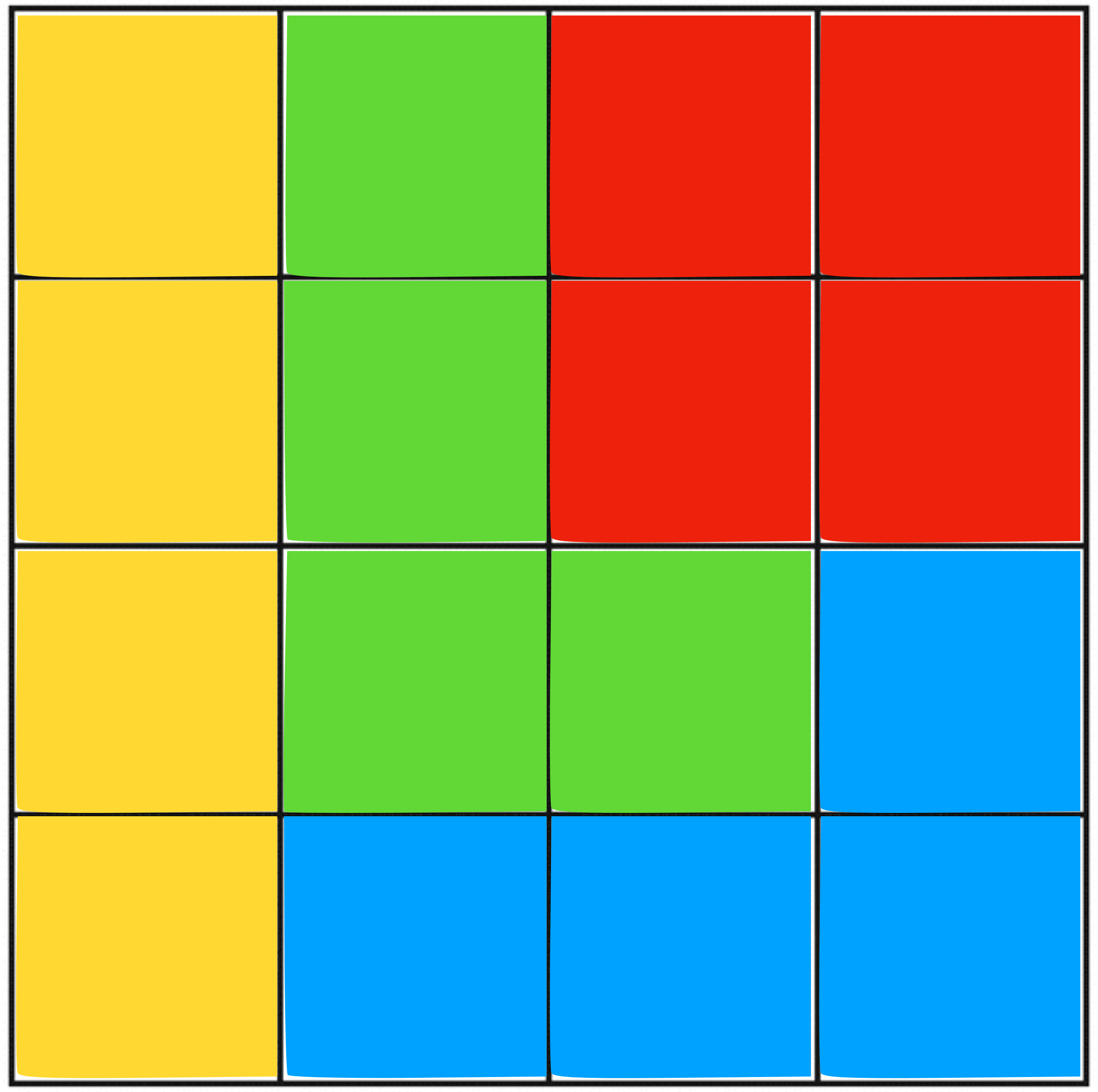
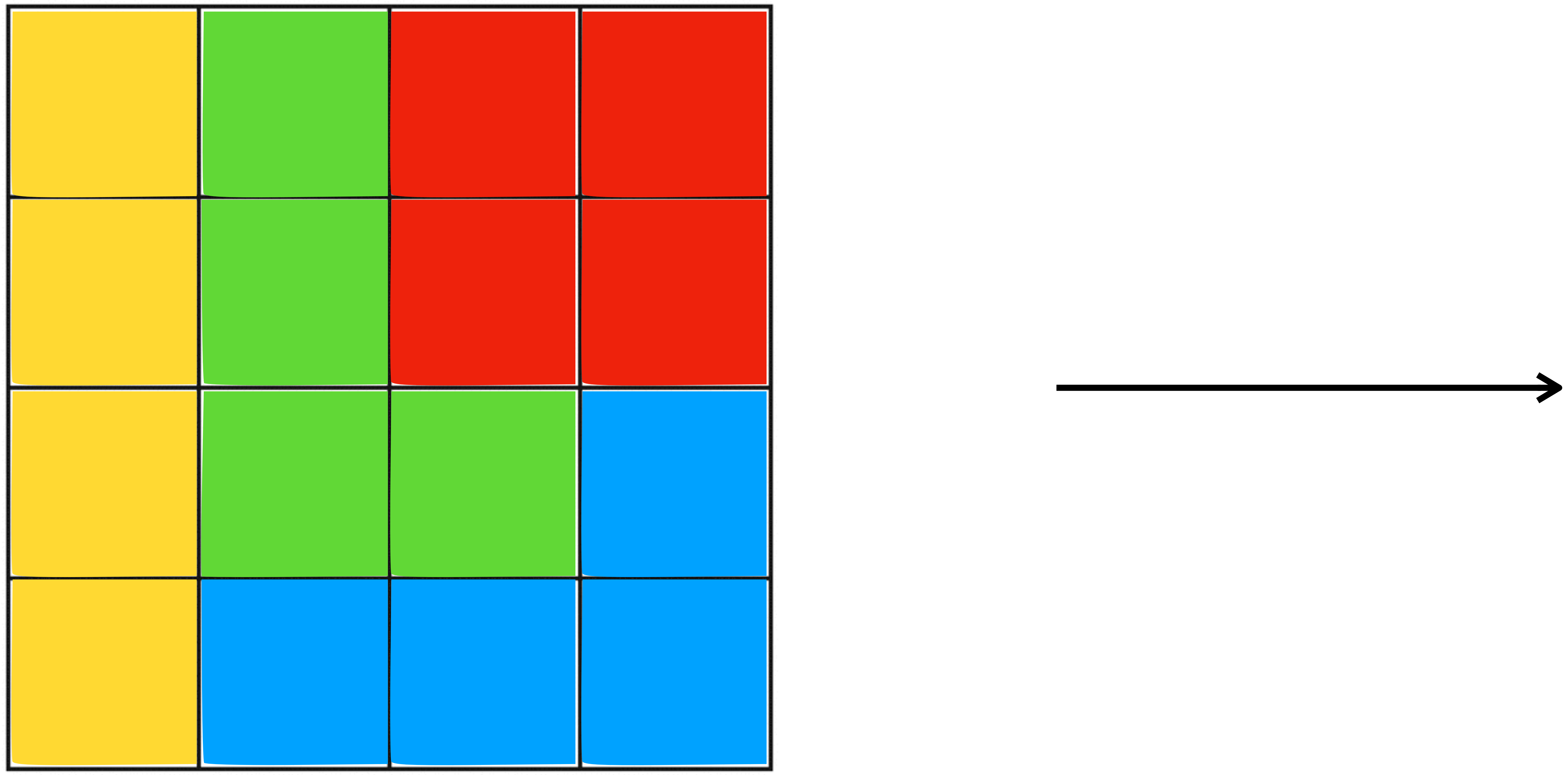
2. some mathematical history

GerryChain

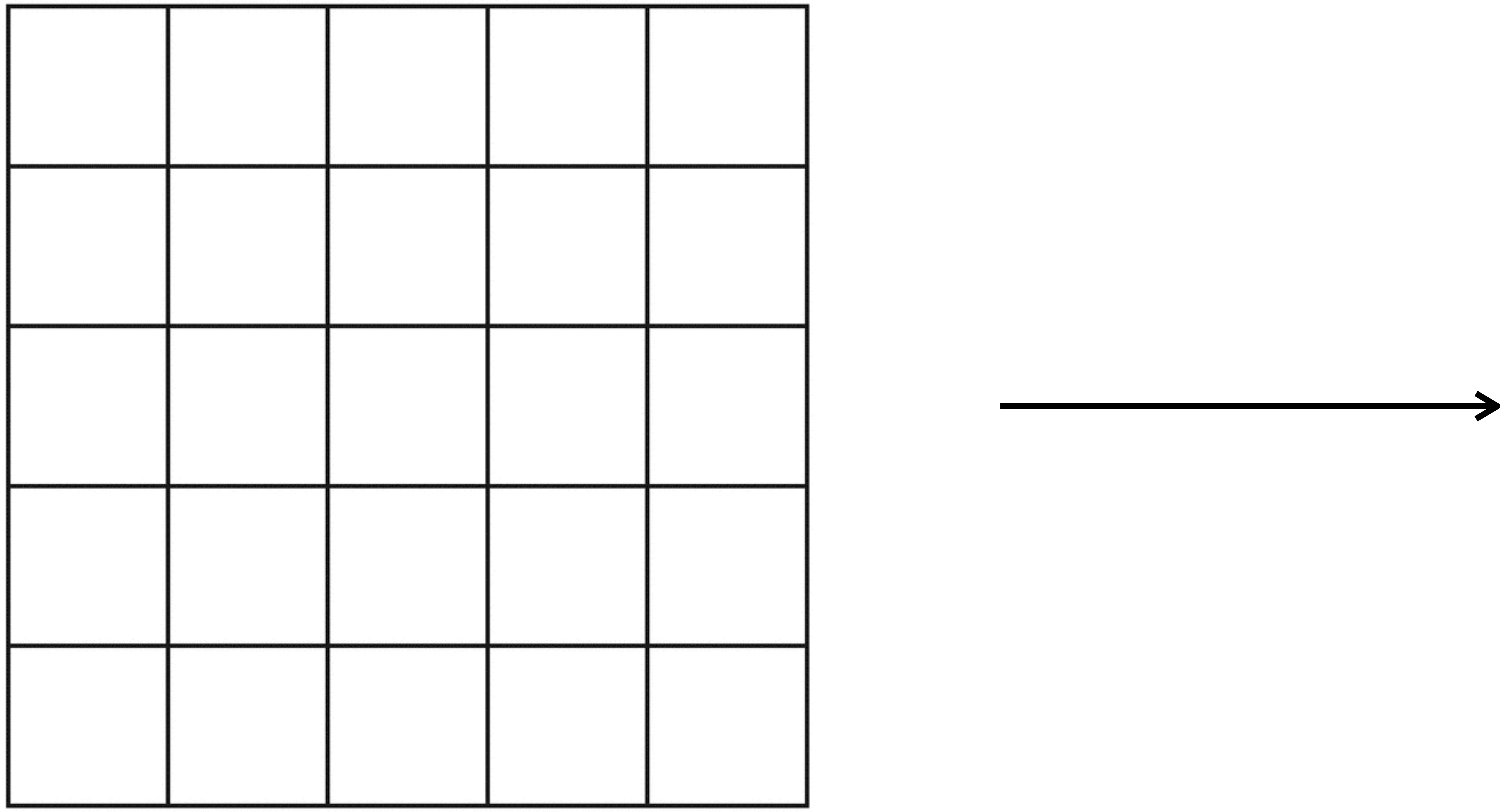

COLUMBIA
UNIVERSITY

GerryChain

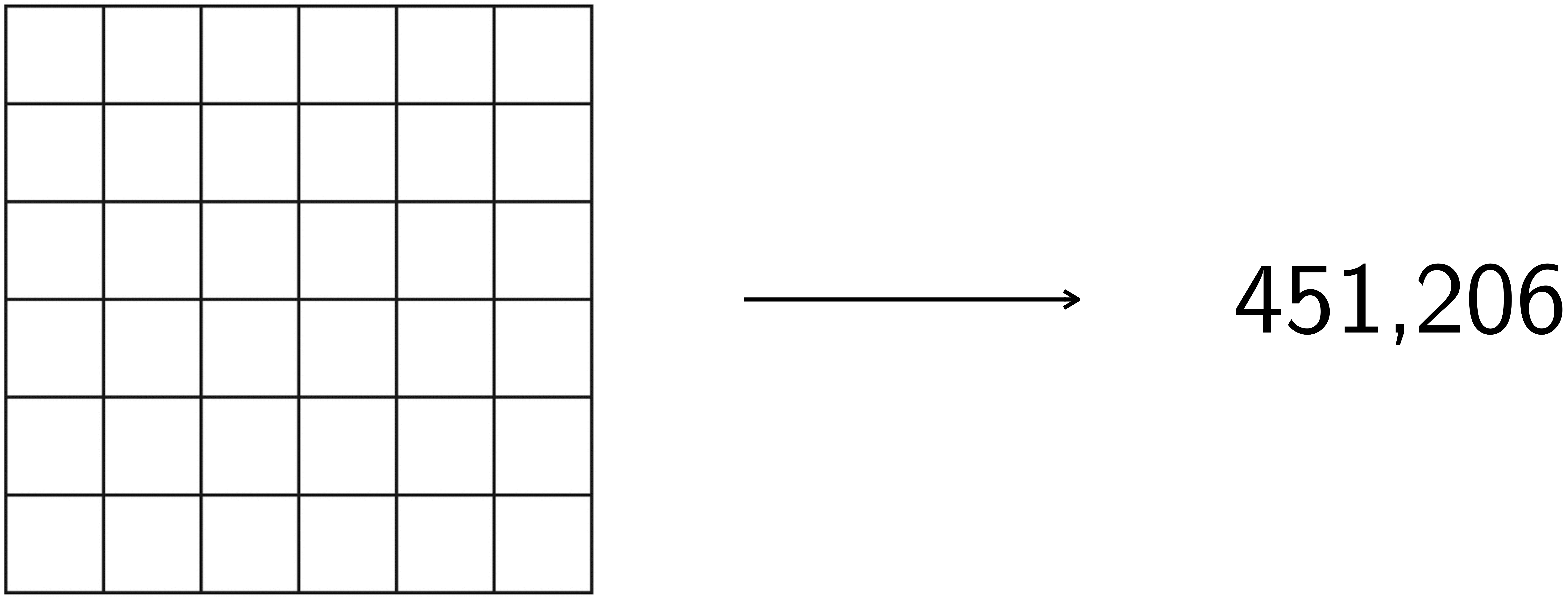


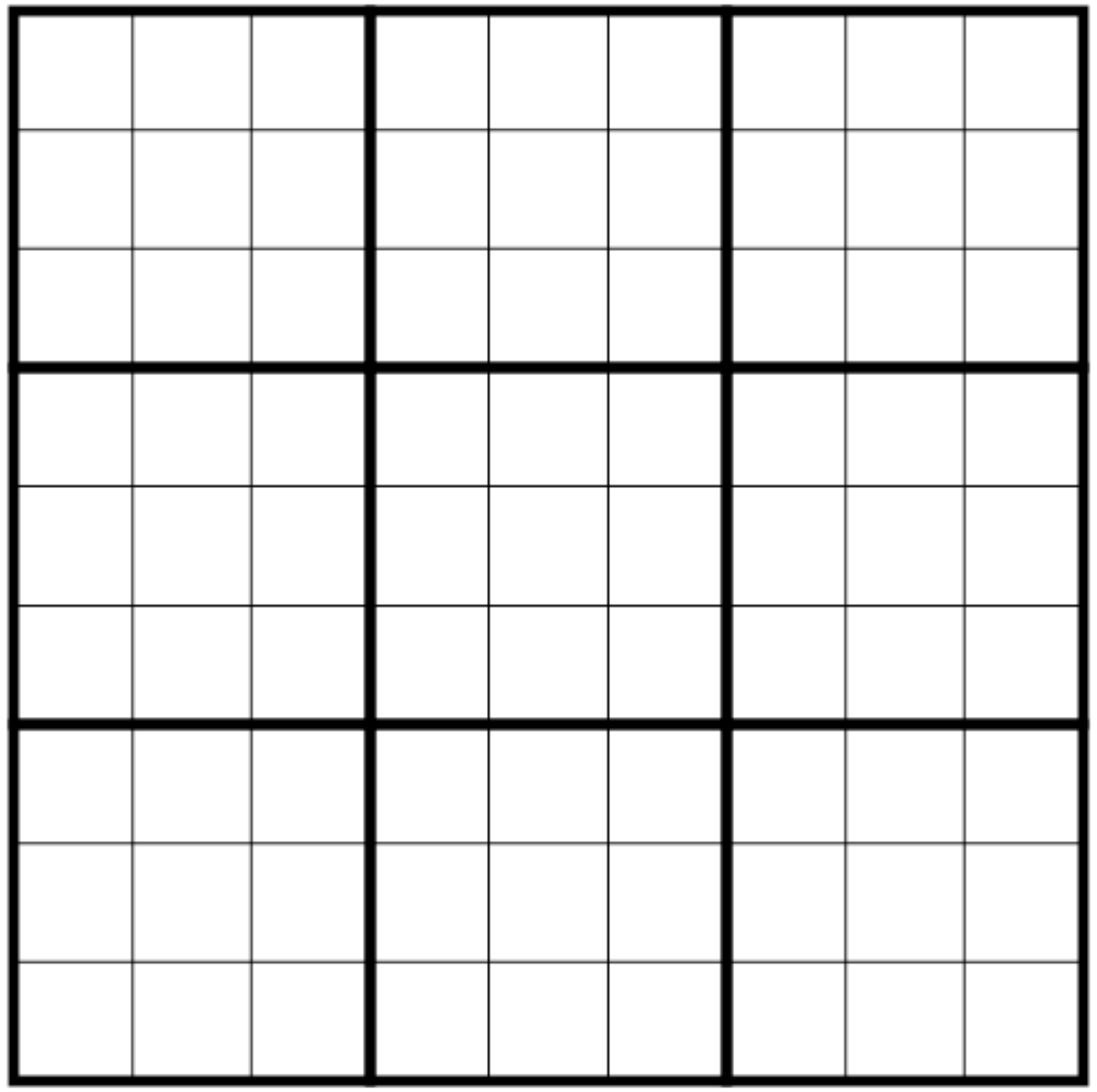


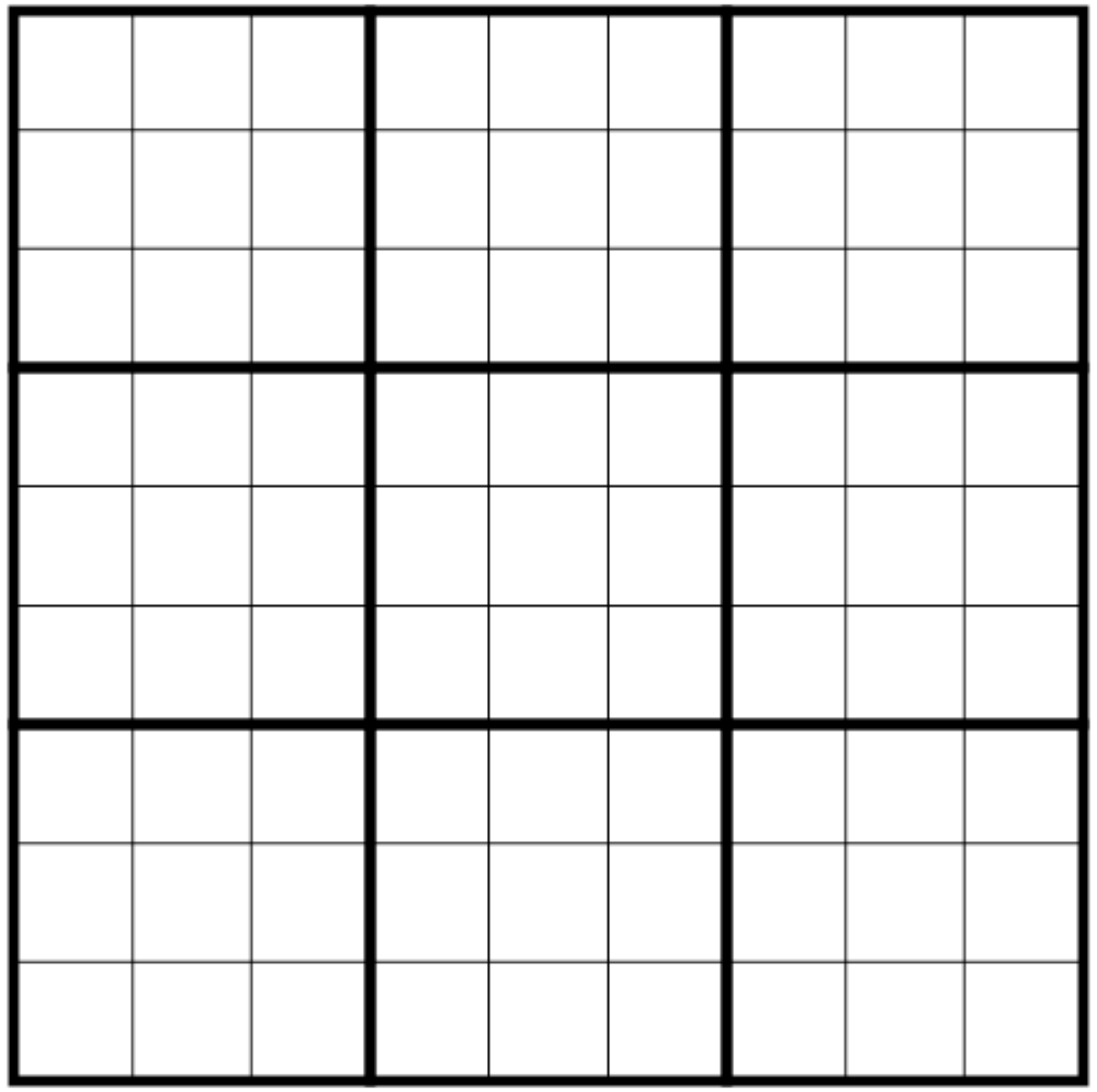
117



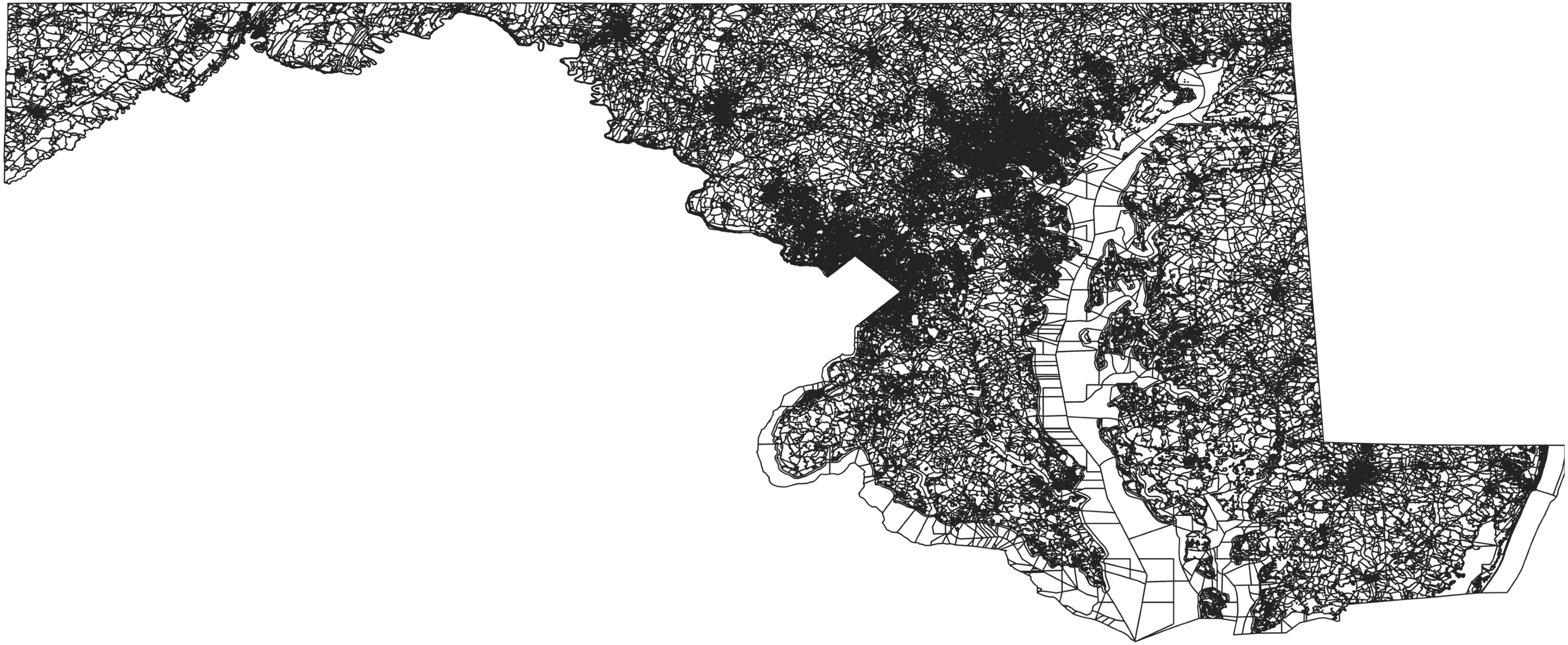
4,006

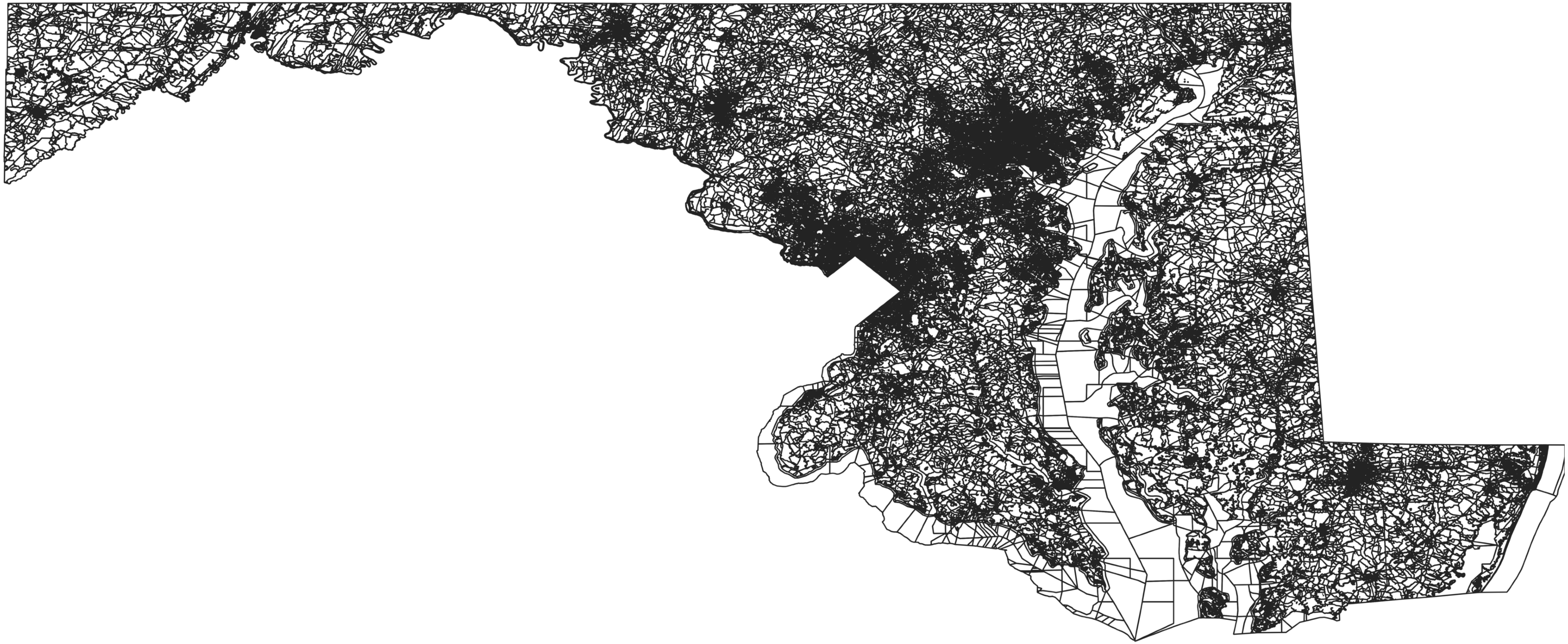

451,206





$\longrightarrow$ 706,152,947,468,301





there are 145,247 individual in the state of Maryland.

Monte Carlo Markov chain

MCMC

a. large

a. large
b. contains districting plans that are “valid”

Markov chain

Markov chain

a *random process* equipped with the *memoryless property*.

$$(R_0, R_1, R_2, \ldots, R_n, \ldots)$$

draws from a random variable...

$$(R_0, R_1, R_2, \ldots, R_n, \ldots)$$

draws from a random variable...

$$(R_0, R_1, R_2, \ldots, R_n, \ldots)$$

indexed by some set

$$R_k = i_k$$

$$R_k = i_k \longrightarrow R_{k+1} = i_{k+1}$$

$$R_k = i_k \longrightarrow R_{k+1} = i_{k+1}$$

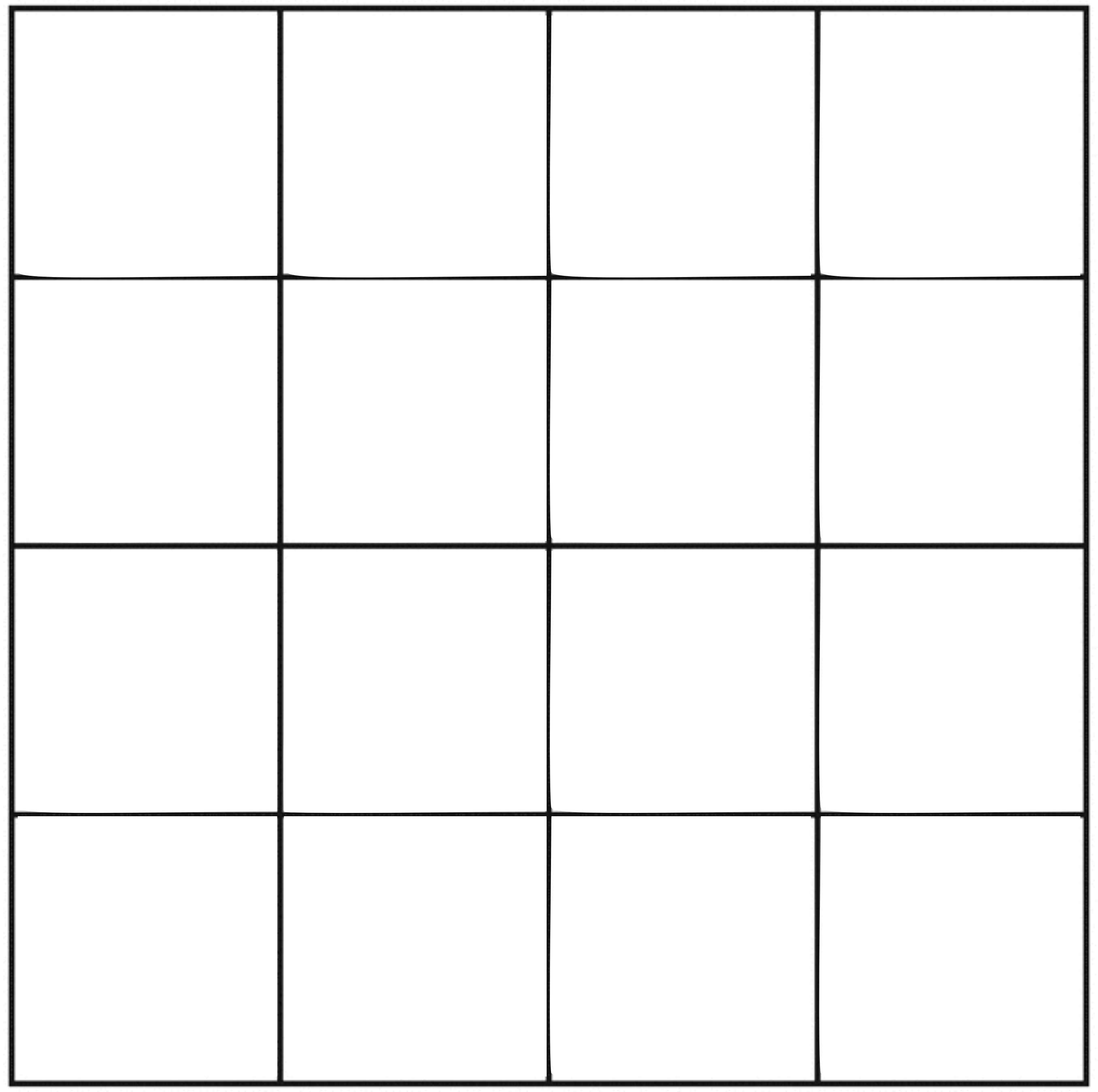
$$\mathbf{P}\left(R_{k+1} = i_{k+1} \mid R_k = i_k, R_{k-1} = i_{k-1}, \ldots\right)$$

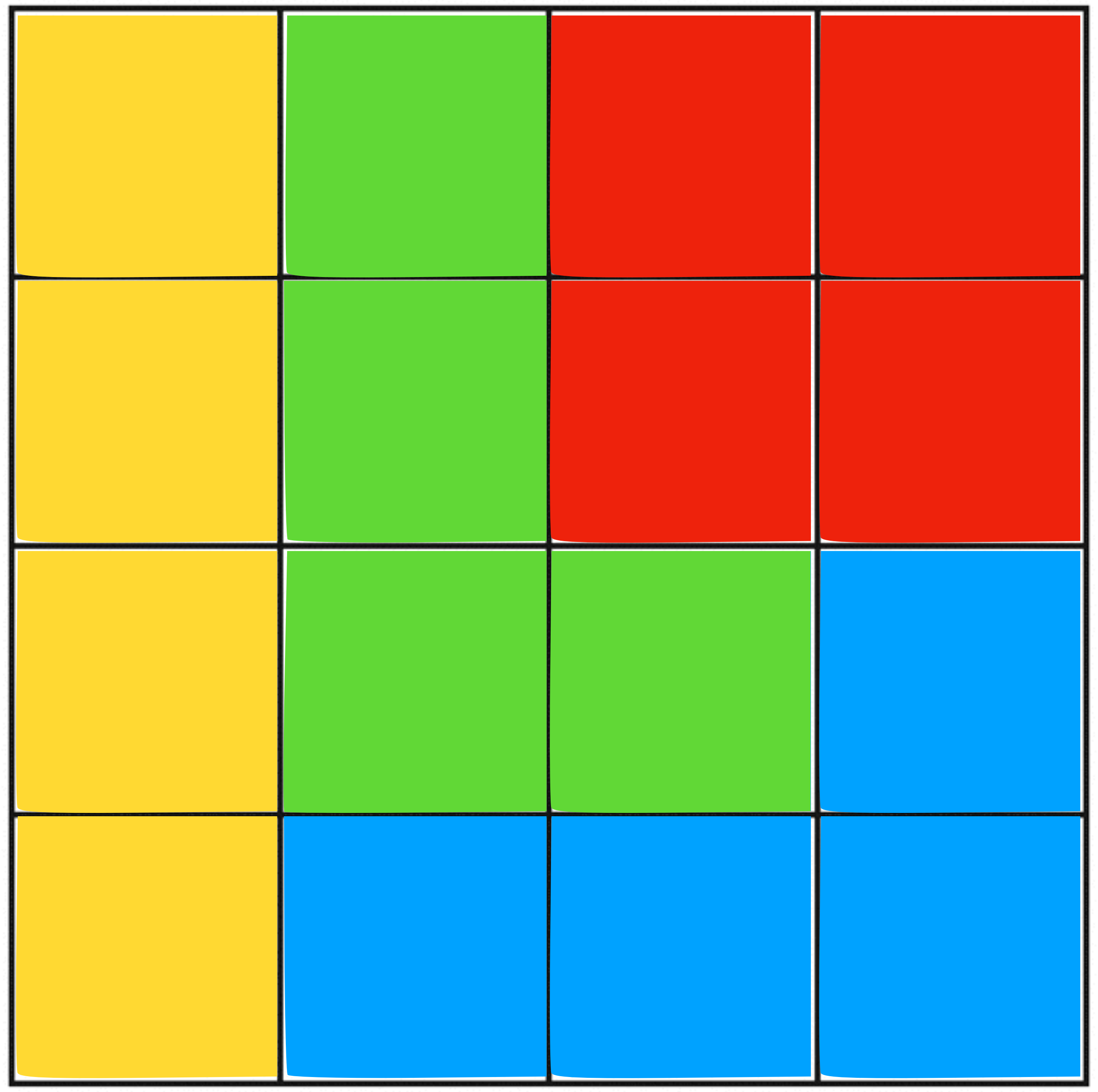
$$R_k = i_k \longrightarrow R_{k+1} = i_{k+1}$$

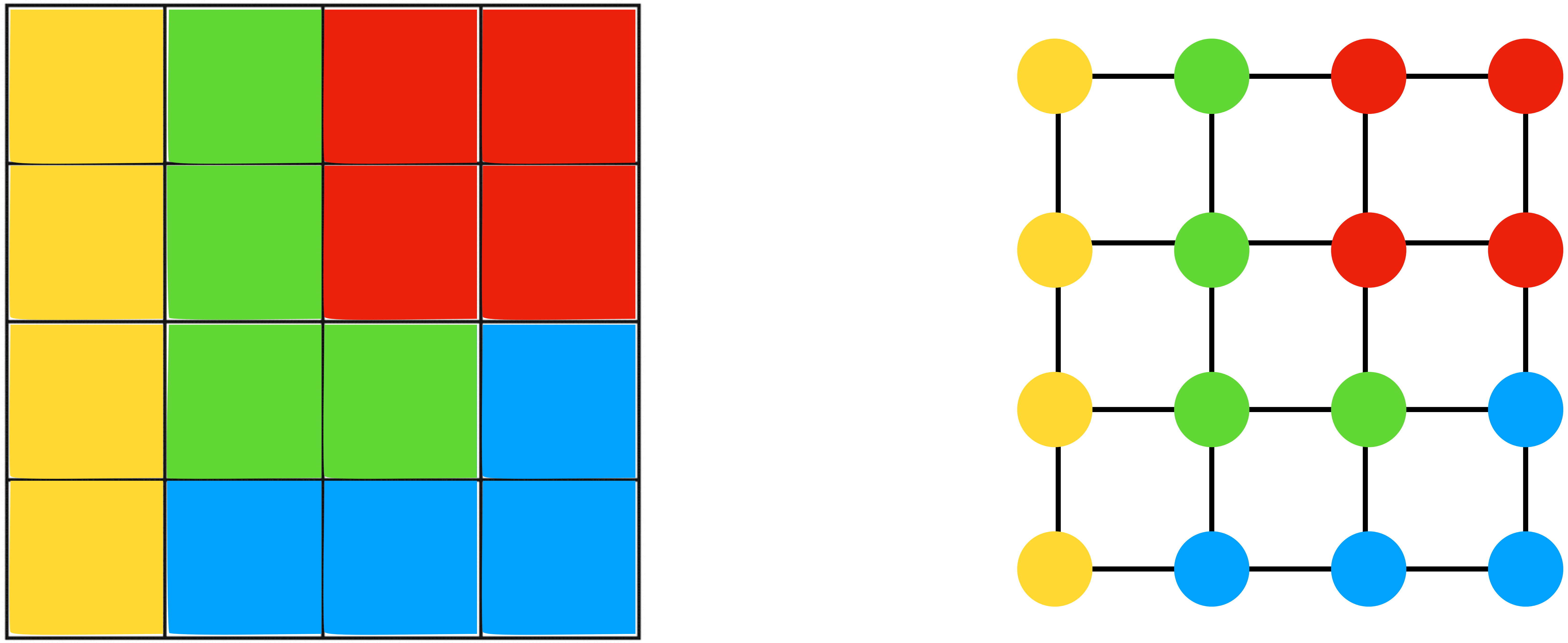
$$\mathbf{P}\left(R_{k+1} = i_{k+1} \mid R_k = i_k, R_{k-1} = i_{k-1}, \ldots\right)$$

$$= \mathbf{P}\left(R_{k+1} = i_{k+1} \mid R_k = i_k\right)$$

this is the concept foundational to GerryChain.







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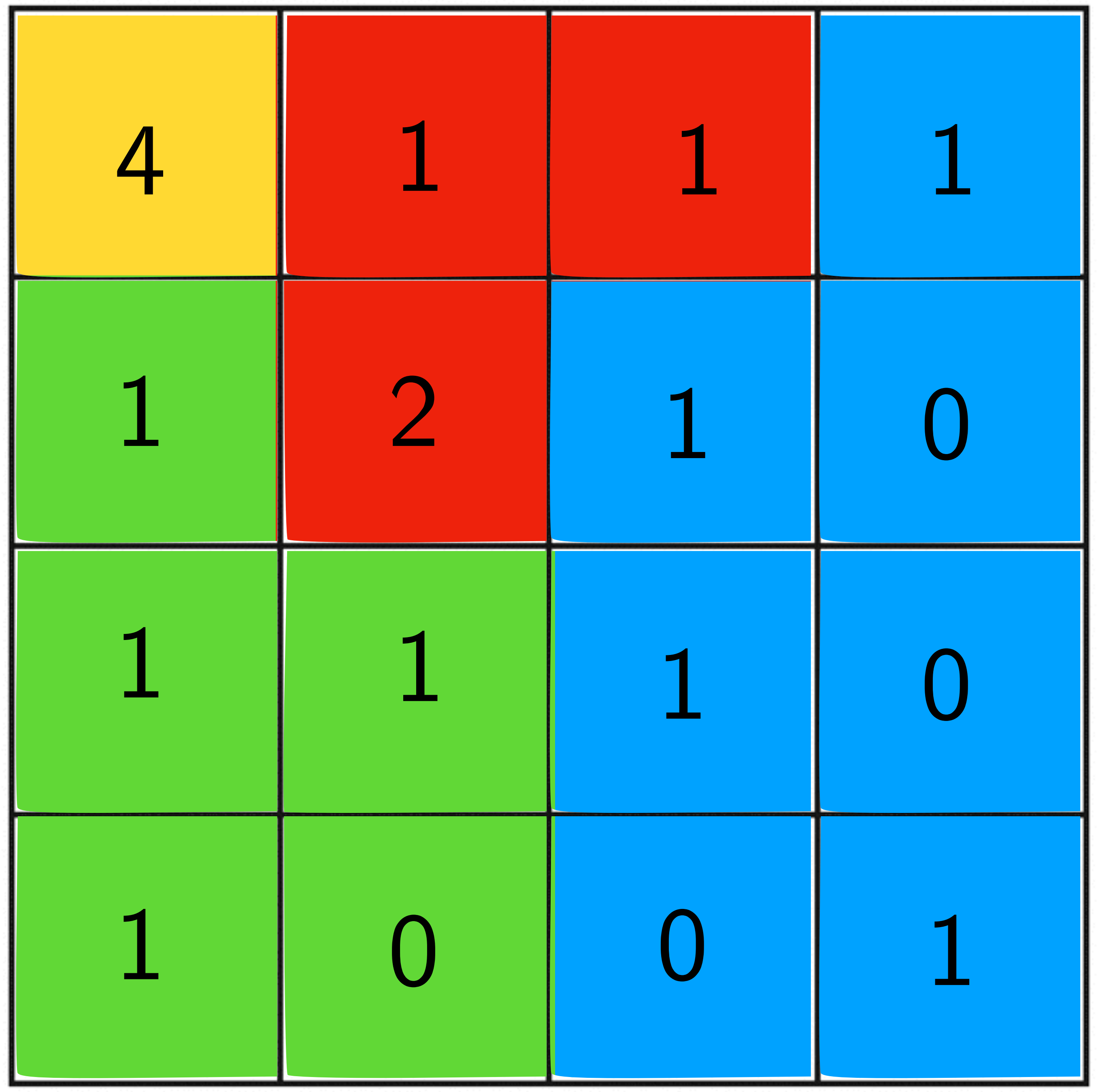
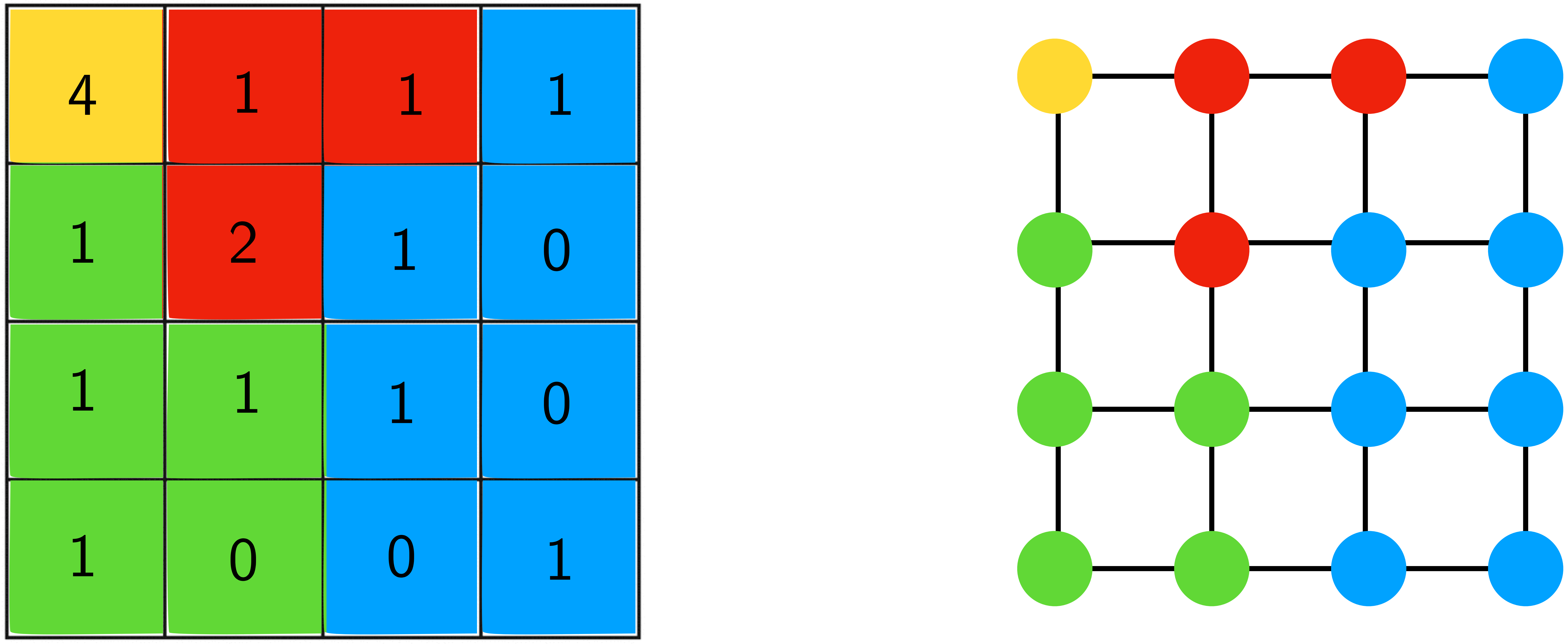
1	2	1	0
1	1	1	0
1	0	0	1


4 1 1 1
1 2 1 0
1 1 1 0
1 0 0 1


4 1 1 1
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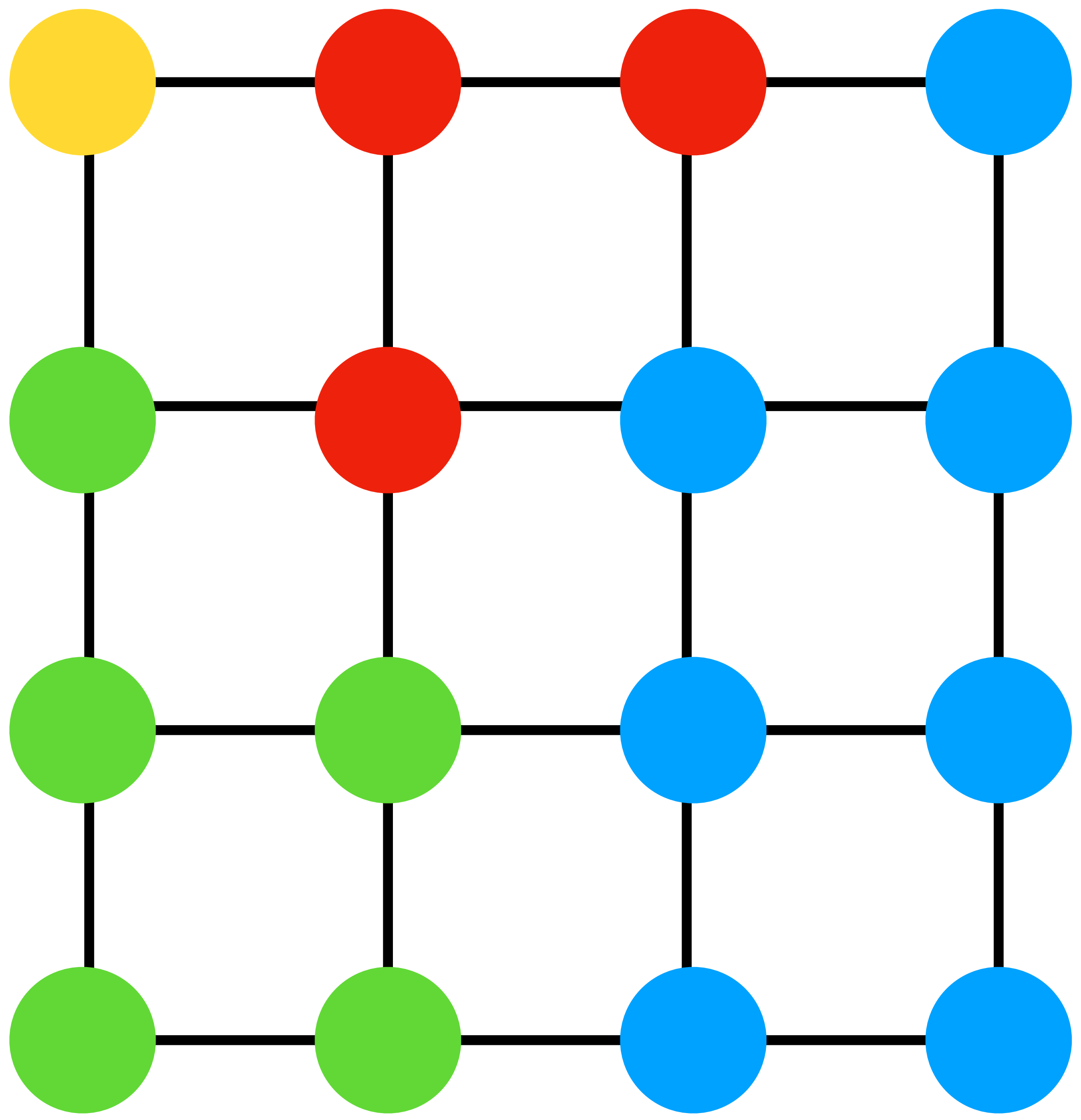
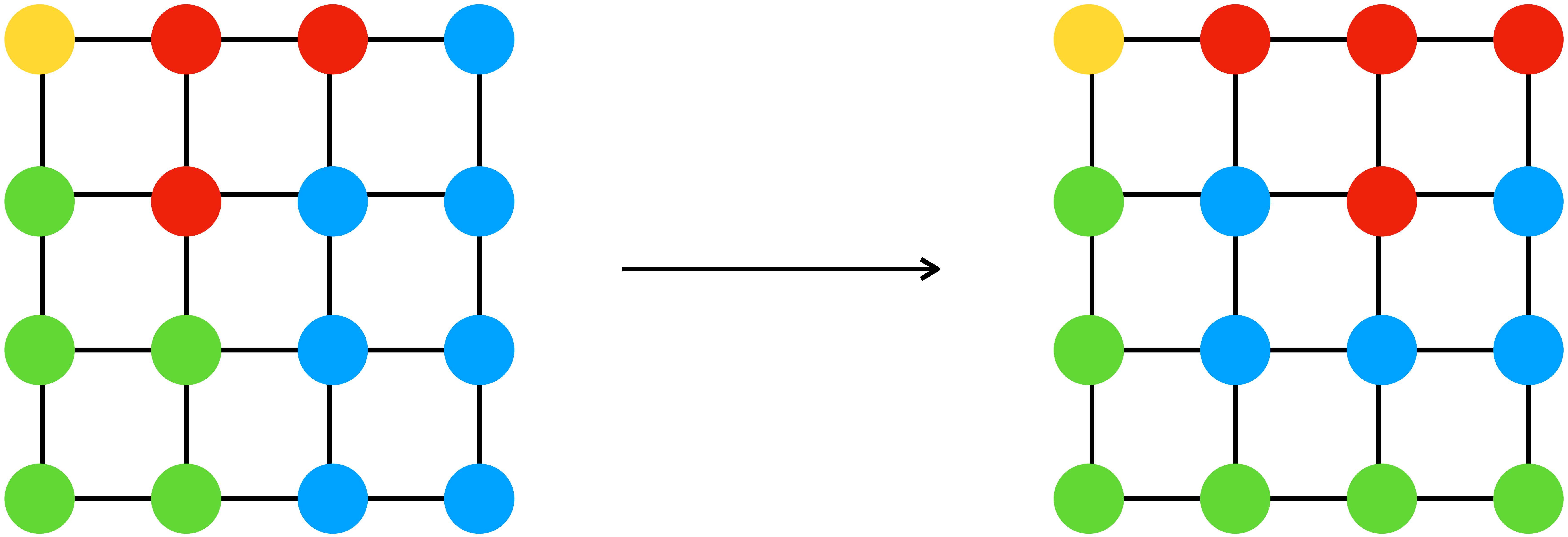
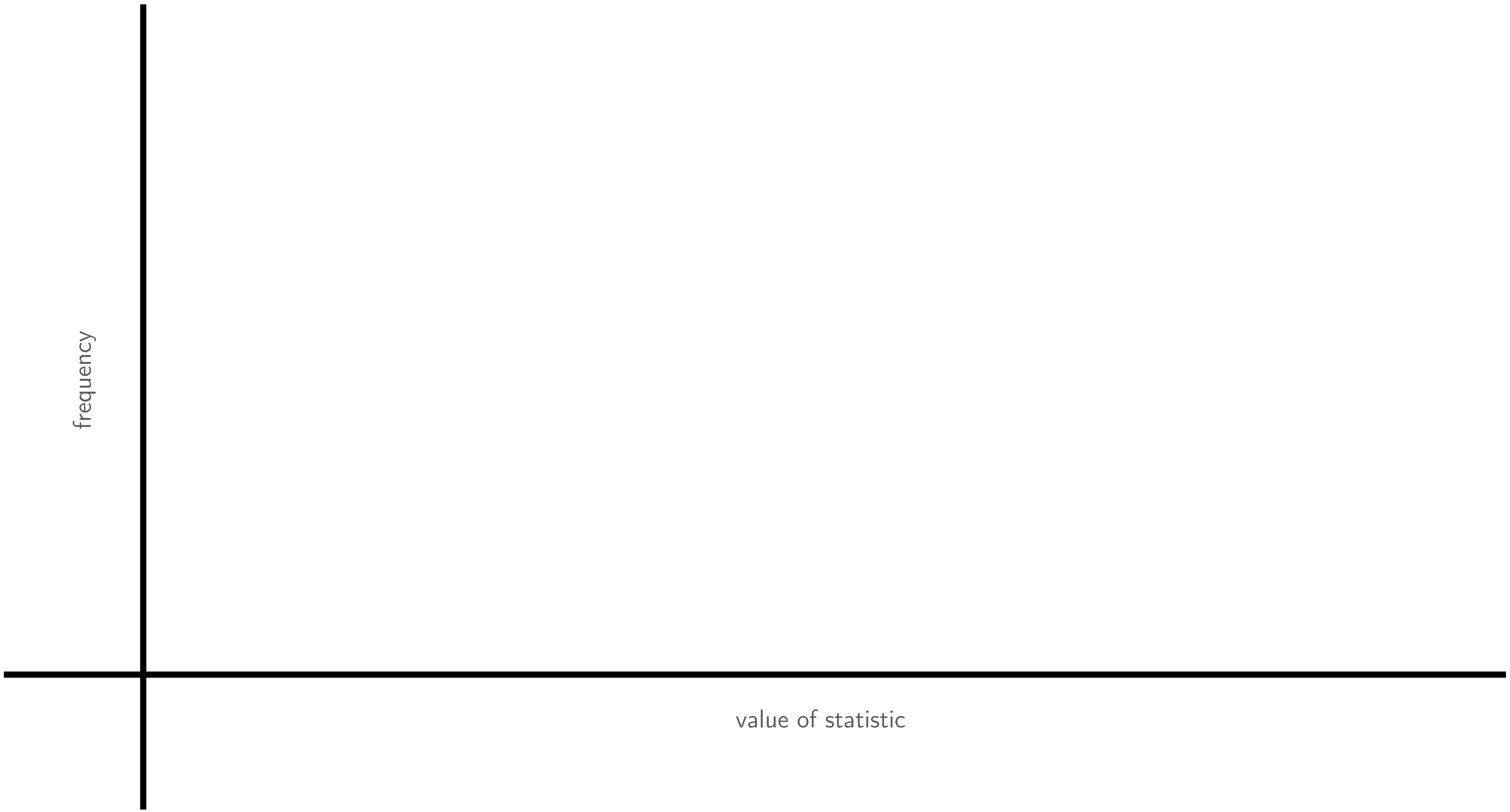
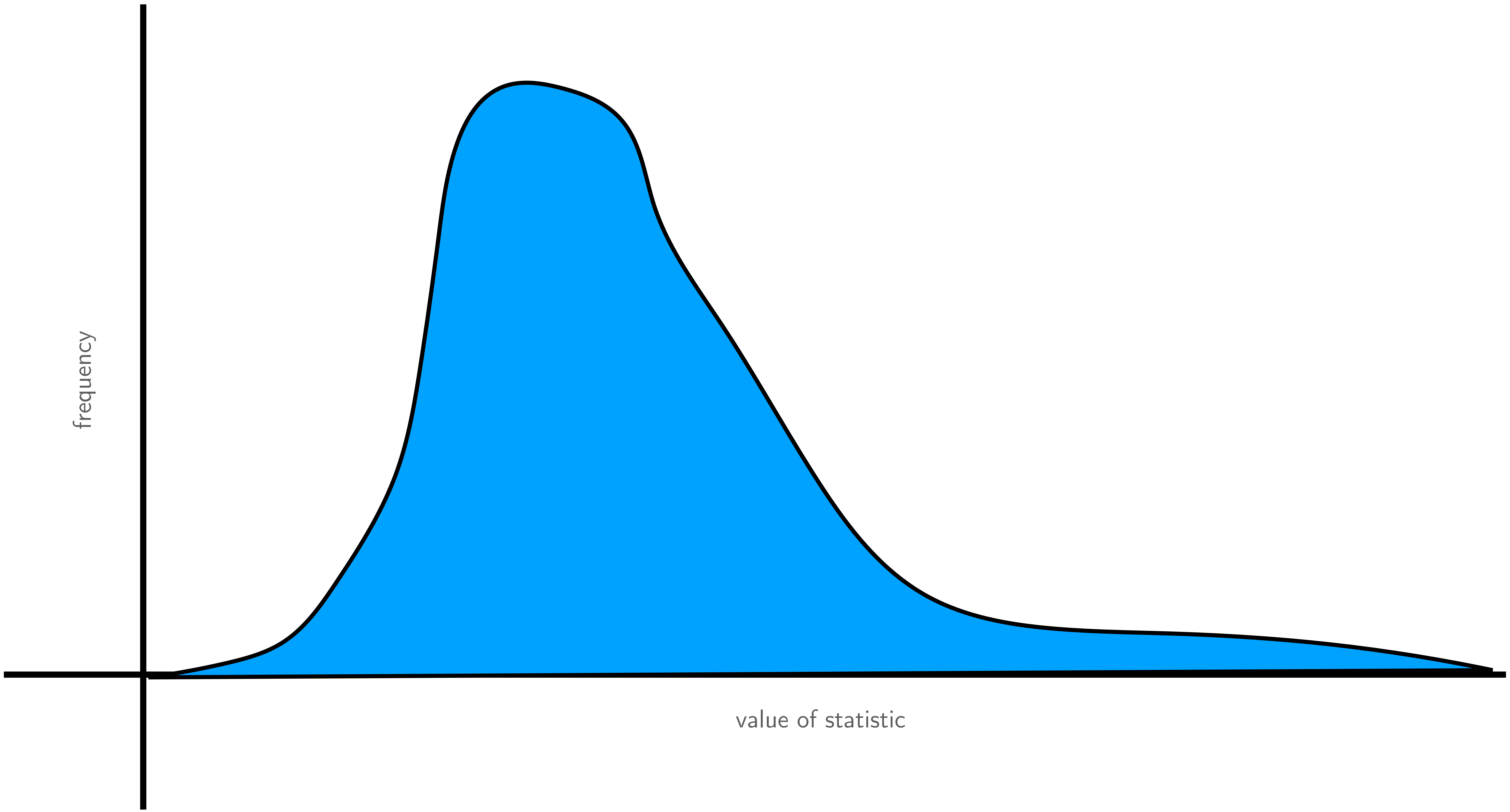
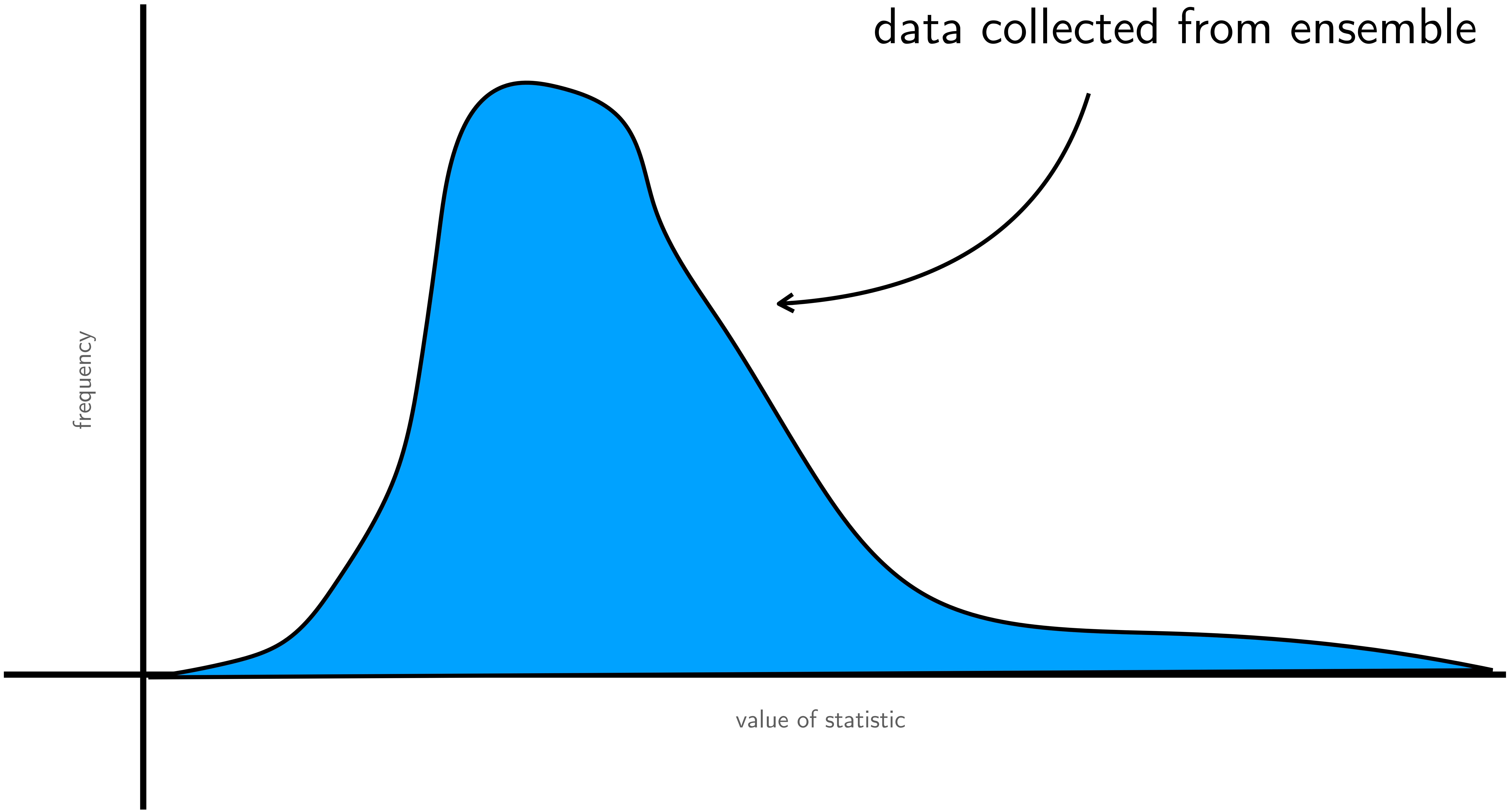
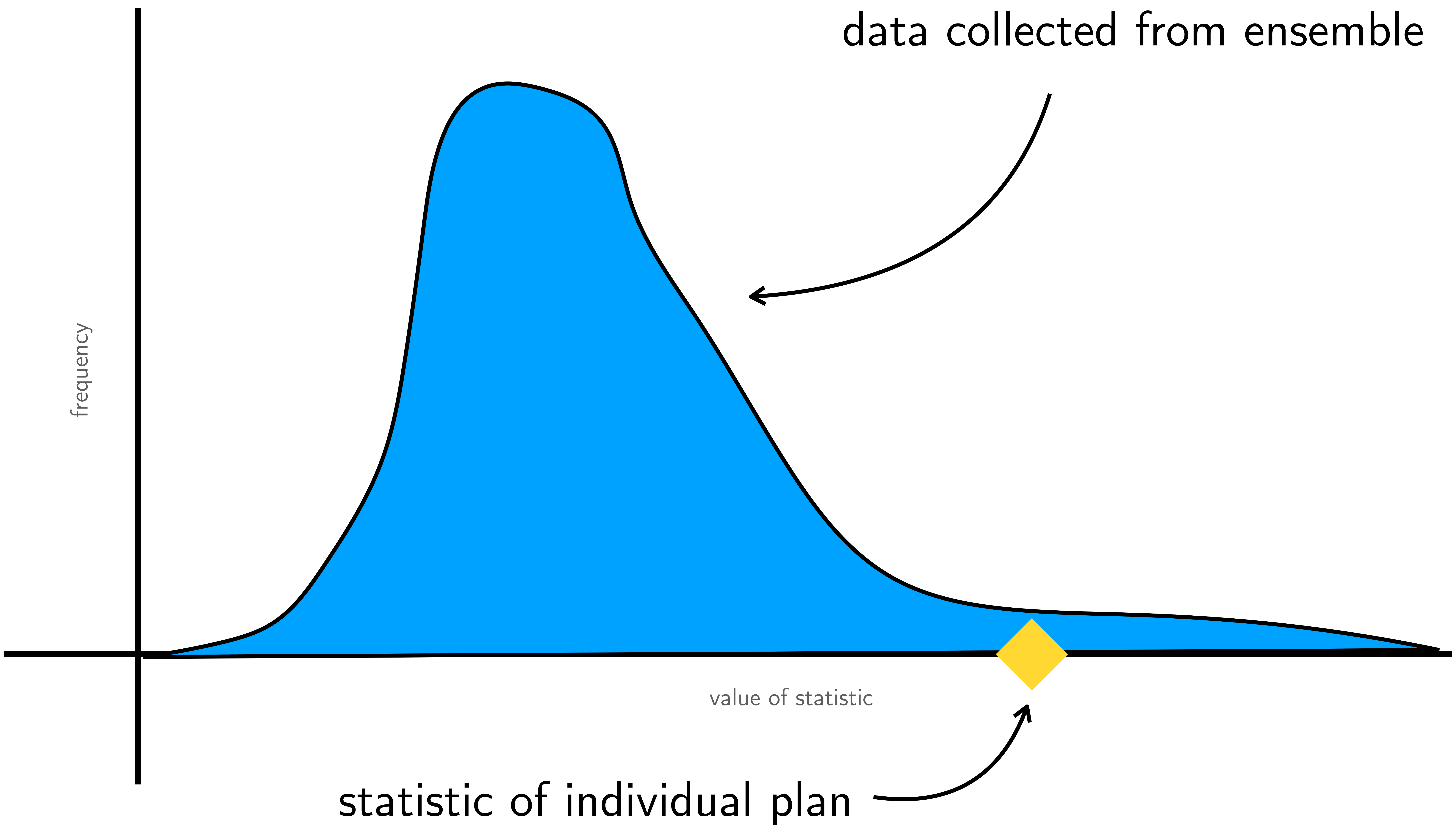





frequency
value of statistic


frequency
value of statistic


data collected from ensemble
frequency
value of statistic



data collected from ensemble
frequency
value of statistic
statistic of individual plan

applying this technique to redistricting is fairly revolutionary.

3. contemporary research

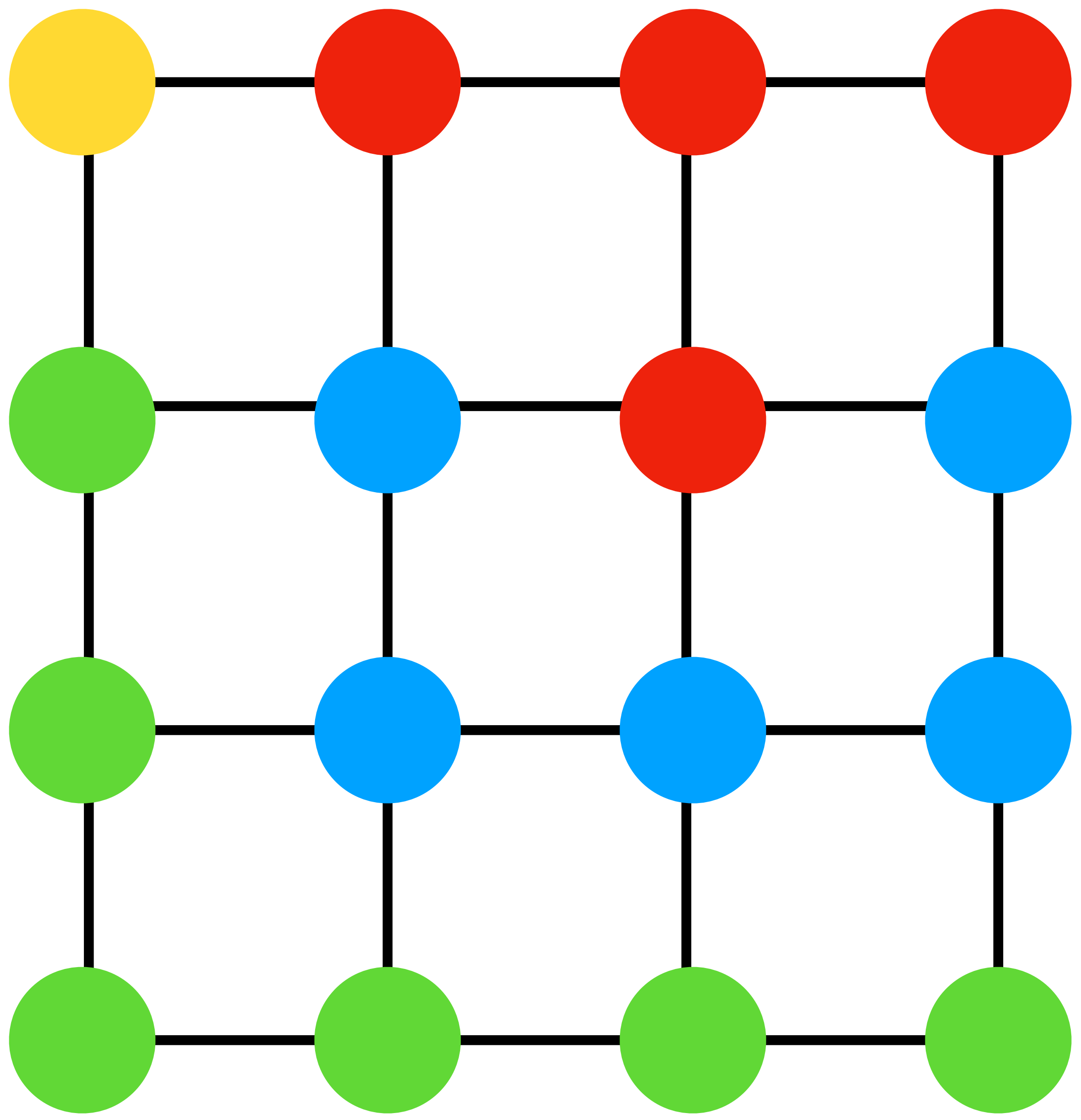
proposal and acceptance methods for Markov chains on graphs.

ReCom

ReCom

ReCombination, borrowed from molecular biology.

4	1	1	1
1	2	1	0
1	1	1	0
1	0	0	1



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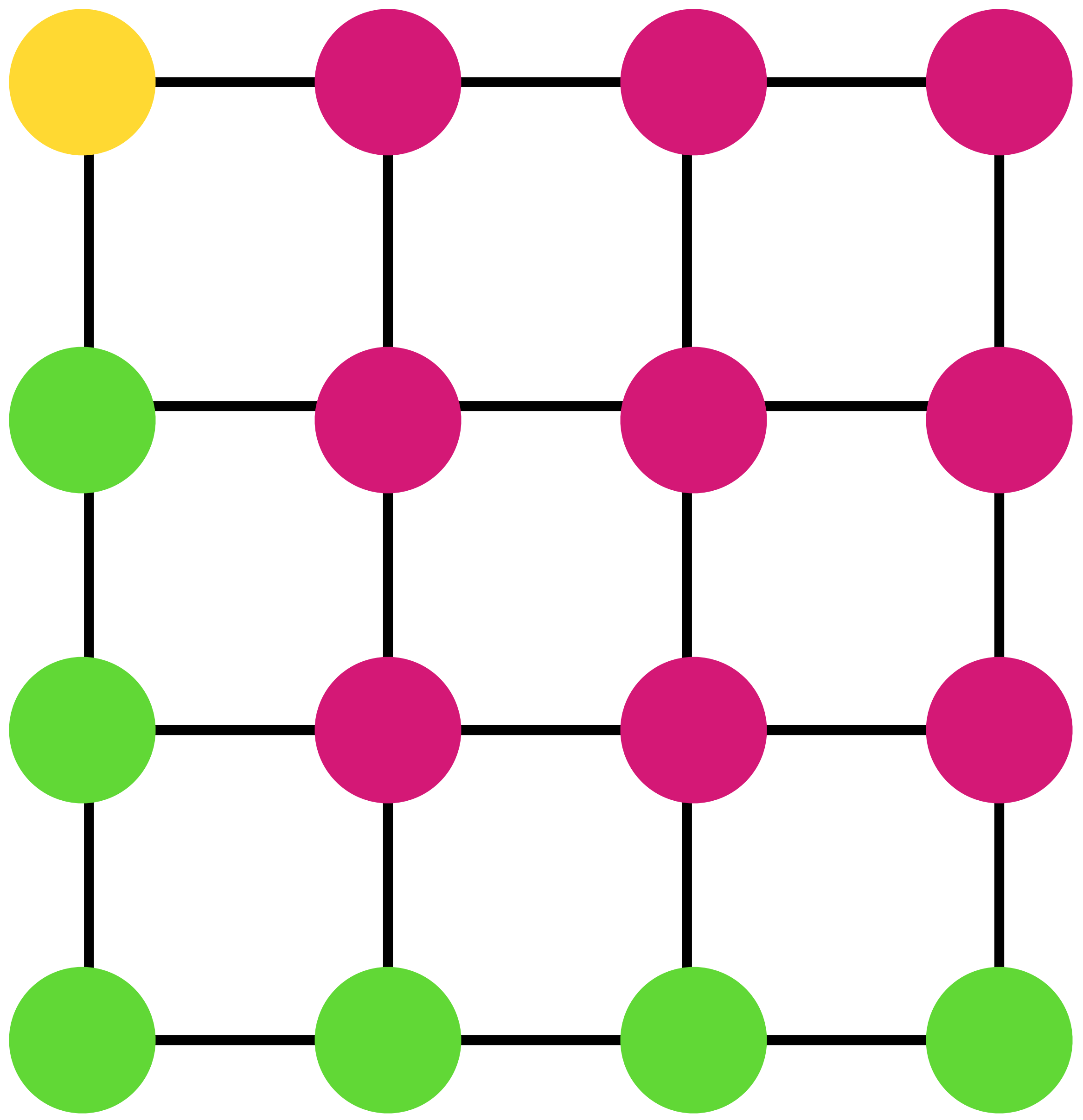

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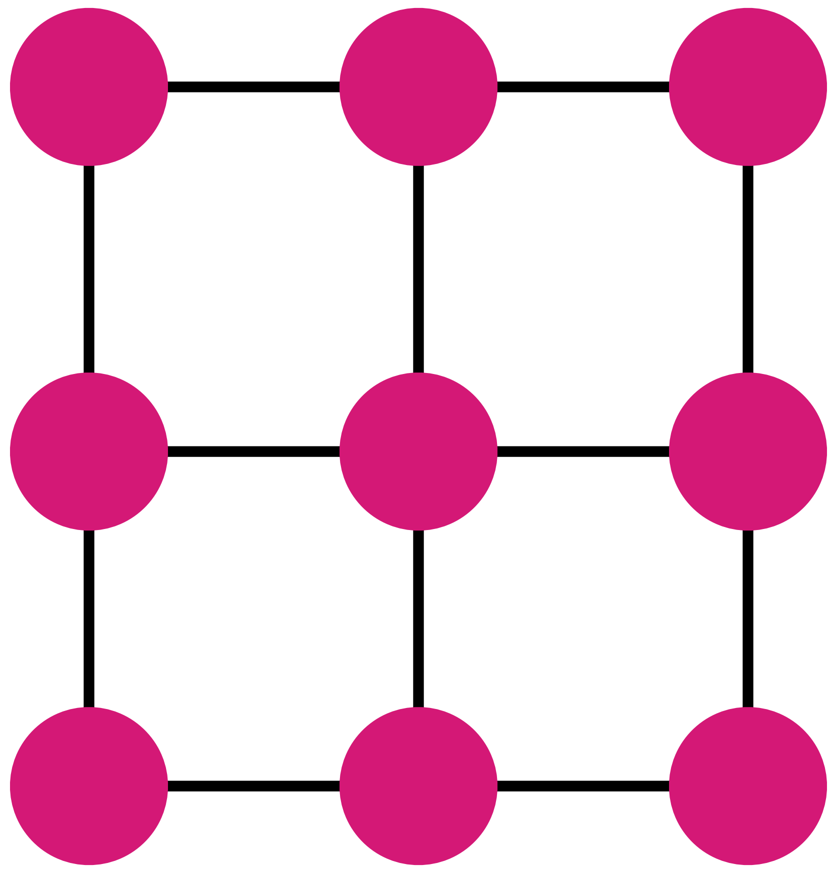

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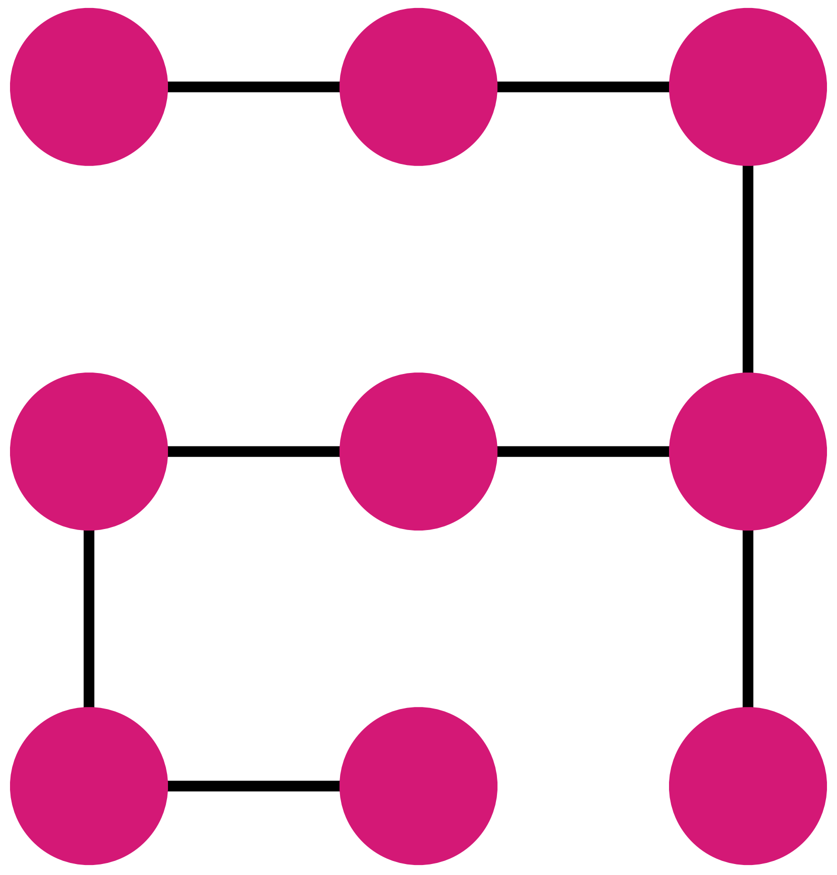
1	0	0	1



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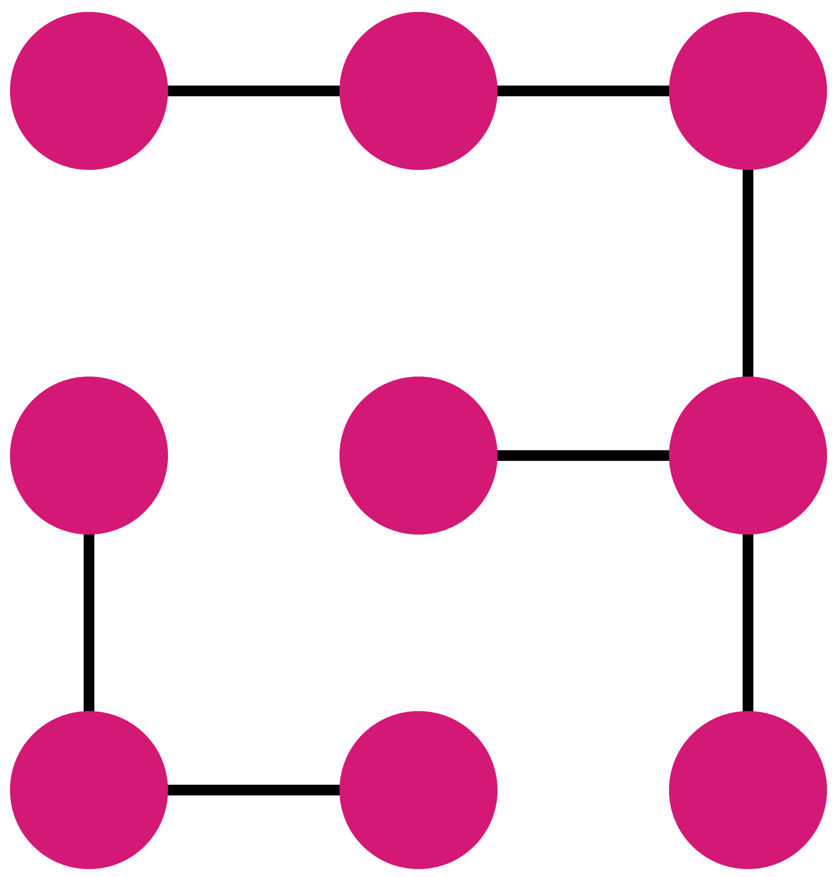

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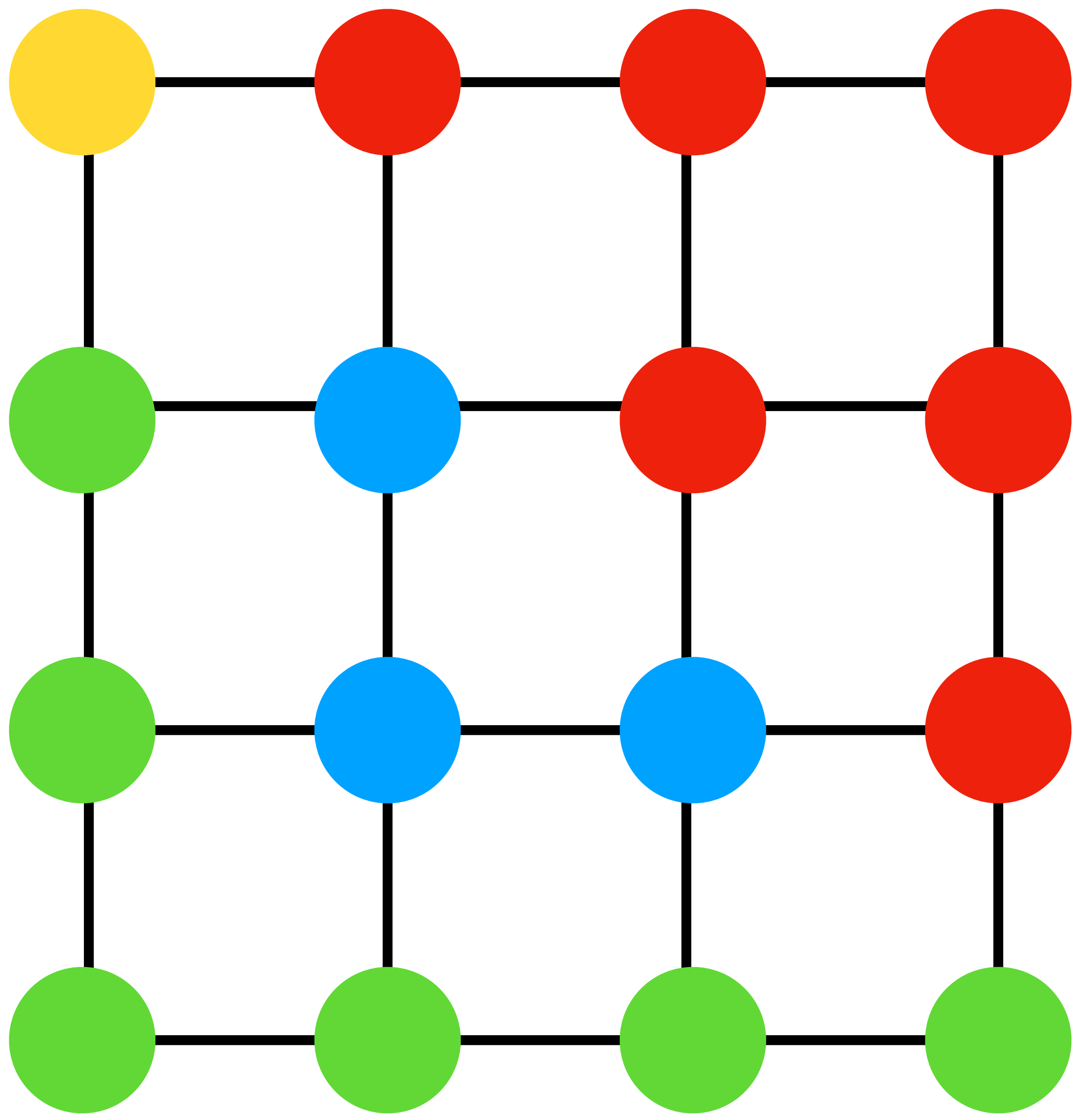

$$S : \mathscr{P} \to [0,1]$$

$$\mathbf{P}(P*) = \frac{S(P*)}{\sum_{p \in \mathscr{P}} S(p)}$$

$$\frac{\mathbf{P}(P^*)}{\mathbf{P}(P)} = \frac{\dfrac{S(P^*)}{\sum_{p \in \mathscr{P}} S(p)}}{\dfrac{S(P)}{\sum_{p \in \mathscr{P}} S(p)}} = \frac{S(P^*)}{S(P)}$$

$$\alpha = \min\left(\frac{S(P^*)}{S(P)}, 1\right)$$

$$\beta \in [0,1]$$

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$$\alpha = \min\left(\frac{S(P^*)}{S(P)}, 1\right)$$

$$\beta \in [0,1]$$

and that's it.

4. things that are hard

a. aperiodic

a. aperiodic
b. irreducible

a. aperiodic
b. irreducible
c. reversible

aperiodic

aperiodic

when a Markov chain doesn't return to any one state with regularity.

$$\mathbf{period}(P) = \gcd\left\{\text{the step numbers where we hit } P\right\}$$

irreducible

irreducible

we can get from any state to any other state.

$$\forall P, P^* \in \mathscr{P}, \exists n \quad \text{such that} \quad \mathbf{P}\left(R_{k+n} = P^* \mid R_k = P\right) > 0$$

reversible

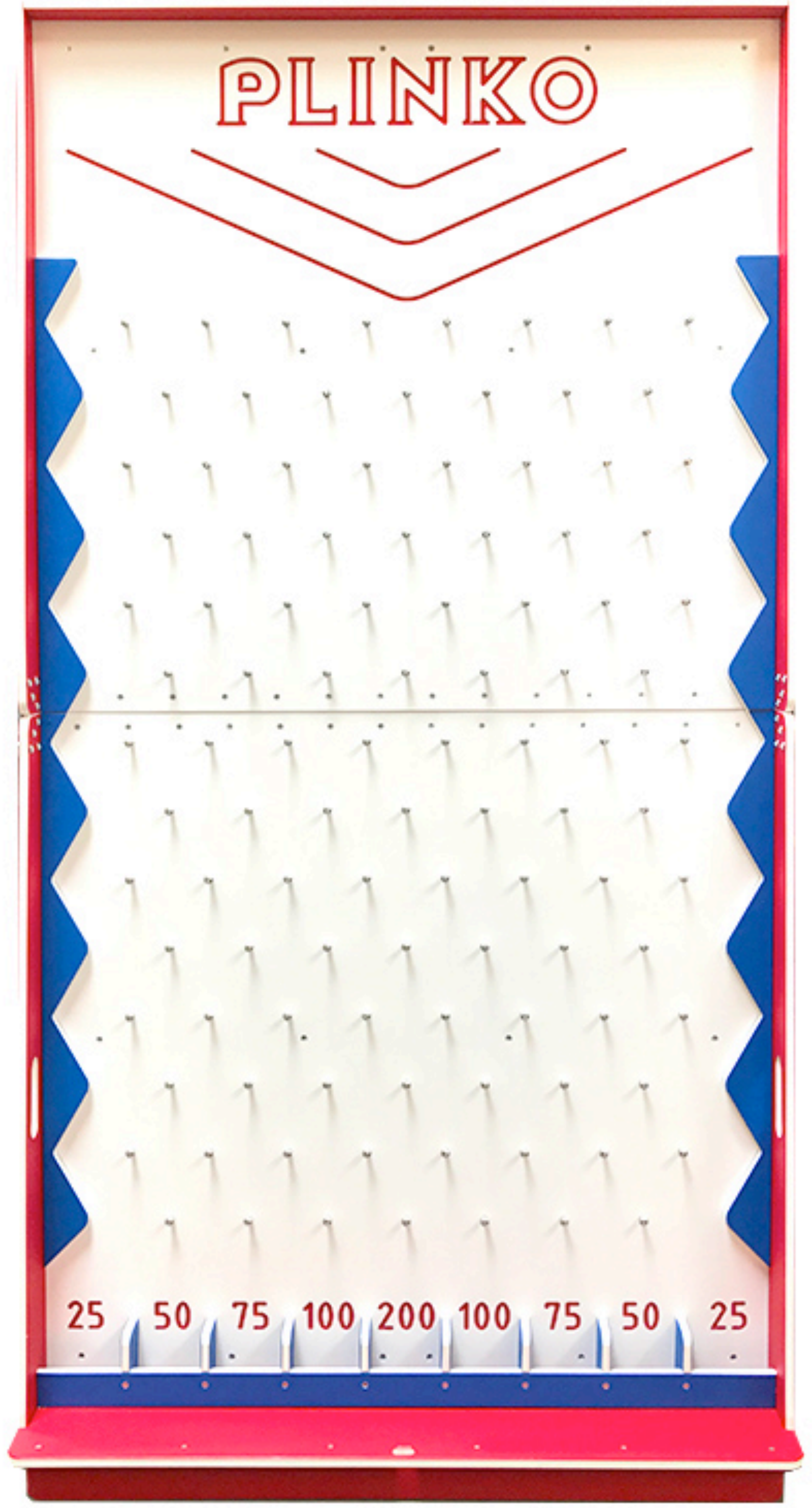
reversible

when we hit our *steady state*, the chance we move between two states is the same either way.

steady state

steady state

the limit of the distributions of our samples as our step number goes to infinity


PLINKO
25 50 75 100 200 100 75 50 25

$$\mathbf{P}(R_{m+1} = P^* \mid R_m = P) = \mathbf{P}(R_{m+1} = P \mid R_m = P^*)$$

we know our Markov chain *can* be reversible.

$$\alpha = \min\left(\frac{S(P*)}{S(P)} \cdot \frac{T_{P*\to P}}{T_{P\to P*}}, 1\right)$$

we don't know whether they're aperiodic or irreducible.

where does this leave us?

5. where we’re hopefully going

civically-minded mathematicians.

combating not-so-great ideas.

redistricting is *too human* for computers to do for us.

use your skills *for good*.

questions?

thank you!