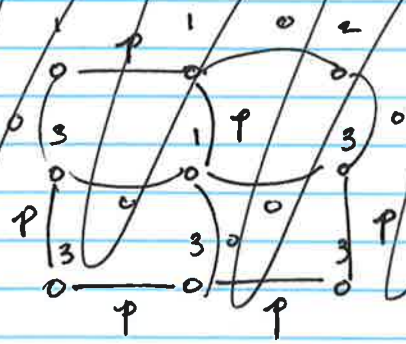
May 24th, 2023

~~Generalizing Swendsen-Wang (again)~~



~~sampling a random graph G by adding edges btwn vertices w/ same spin with prob. $p = 1-e^{-\beta}$; re-sample spins on particles uniformly for each component~~

~~cocycle in $Z^1(G; \mathbb{F}_q)$, so... vanishes on boundaries (i.e. for~~

Meeting w/ Paul! ~~Paul~~

doing 3d first, then 4D

~~Paul is content~~

using duality for Ising!

- making sure to get papers and reading the literature!
- benchmarks for simulations
- S-T conjecture (in Dropbox) → Caselle 1995
- getting read up on the physics stuff!

Grimmett: does talk about preferred spin?

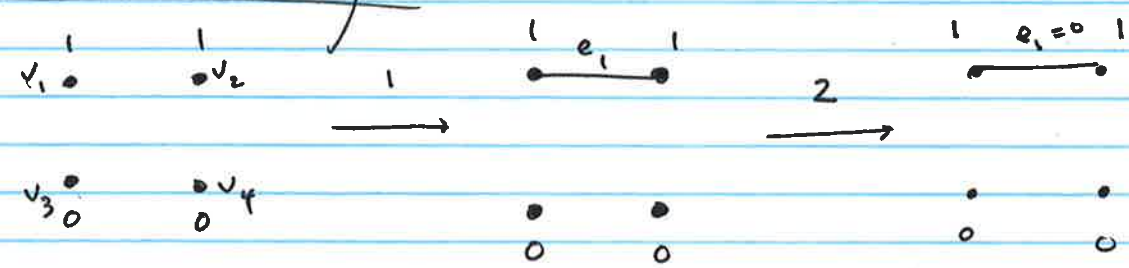
- can we interpret phase transitions in terms of "Polyakov variables" (i.e. same curves but on the torus)?
- what to start with:
  - → Wegner introduced Ising lattice gauge theory (motivation)
- mass gap correlation and criticality of Wilson loop variables measure first ✓
  - → Kreutz has offered to look at stuff! "expert" on comp'nal lattice gauge theory
- keep in mind that the time and space direction are the same (~~"space"~~ (modeling 3-d space in 1-d time)
- what's the plan:

invaded-cluster algorithm?

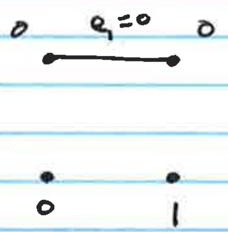
  - → programming things ASAP!
  - → try to find behavior at criticality of the random vars we want in existing papers!
  - → look at literature!
  - → comparing "single-flip" Swendson-Wang vs. "big-hop" Swendson-Wang!
- in (2+1)-dim version, different behavior for plaquettes in the plane? Caselle 1993, but maybe there isn't a ~~giant~~ difference! useful bc it defines critical exponents!

Swendson-Wang:

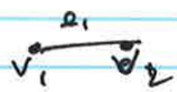
$P = \{e_1\}$



$\sigma = \{v_1 = 0,\ v_2 = 0,\ v_3 = 0,\ v_4 = 1\}$ this is f!



~~(δf)~~



let $\sigma = e_1$

δ

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suppose we have some σ_t, a "configuration" (spins) on the vertices. we want to compute σ_{t+1}:

1. for all $e \in E$, set the weight of e to 0 if $\sigma_t(u) \neq \sigma_t(v)$, and
$$\sigma_t(u) = \sigma_t(v) \begin{cases} 1 & \text{prob. } p \\ 0 & \text{prob } 1-p \end{cases}$$
2. re-sample ~~weights of~~ spins on each component

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recall that, on a graph into $\mathbb{Z}_q$, we have, for $e = (u, v)$,

$$\partial(e) = u - v$$

and for $f : C_0(G) \to \mathbb{Z}_q$, $f(u) = f(v)$, so

$$\partial(e) = p - p = 0$$

so f a cocycle.