APRIL 11^{th}

Let $\varepsilon > 0$ be given

- funding: industrial immersion, Ben's startup
  - ↪ decisions to be made for funding!
- start programming algorithms!
  - ↪ don't <u>need</u> to know how to do stuff!
  - ↪ start in 3-d for the sampling algo
  - ↪ cut a flat torus into a bunch of "cubes"
  - ↪ doing the "same thing" as in lower-D
  - ~~↪ given spins on edges, find cocycle~~
  - ↪ given spins on edges, find a 0-sum boundary, <u>then</u> pick a random cocycle
  - ↪ <u>clock model</u> is same in $\mathbb{Z}^2$, $\underline{\mathbb{Z}^3}$! cool af

□

Let $\varepsilon > 0$ be given

- talking about $\mathbb{Z}^2$ first (and $\mathbb{Z}^3$)
  - ↪ slowest thing is sampling from the null space of a matrix
  - ↪ <u>linbox</u> for sampling random sol'ns of mx null space
  - ↪ translating to C++ some of the things which go back and forth btwn matrices and graphs!
  - ↪ find graph ⟷ mx stuff from books!
- in $\mathbb{Z}^3$ at the critical temp you get ch. (?) surfaces on tori → Grimmett ~~ch. 8~~
  - ↪ duality here for surfaces @ crit. temps! gives us an exact write-downable β temp for phase trans'n in

□

APRIL 4^{th}

Let $\varepsilon > 0$ be given

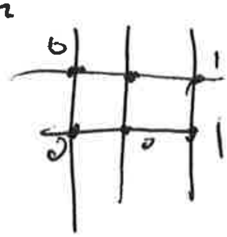
finite graph G

"spins" $\in \{0,1\}$ on the vertices of ~~G~~ G
$= \mathbb{Z}_2$

cohomology $\begin{cases} C_i(X, \mathbb{Z}_2) \\ \cancel{C^i(X, \mathbb{Z}_2} \end{cases}$

$C^i(X', \mathbb{Z}_2) = \mathrm{Hom}(C_i(X, \mathbb{Z}_2))$

in $\mathbb{Z}^2$



want nearest neighbors to have same sign!

$H = \{\#$ of pairs of nearest nbrs w/ same spin$\}$

energy

$f \in C^0(G, \mathbb{Z}_2)$ $\longrightarrow$ coboundary

$\delta f \in C^1(G, \mathbb{Z}_2)$

$\delta(f)(e) = f(\delta(e))$

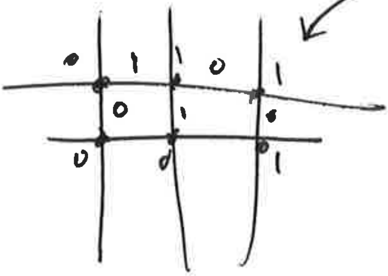
Kronecker

$$H = \sum_{e \in \text{edges}} -K(\delta f(e), 0)$$

$\square$

Let $\varepsilon > 0$ be given

all the 0-weight edges



corellat

Gibbs measures! yay!

<u>Def</u> Ising model on G w/ inverse temp β is

$$\mu_\beta(f) = \frac{1}{Z} e^{-H(f)\beta}$$

$\frac{1}{Z} e$

$$\frac{1}{e^{H(f)\cdot\beta}} \to 0$$

$\beta \to \infty$

$\square$

$\mu_\beta(f) = \frac{1}{Z} e^{-H(f)\beta}$ $\quad \mathcal{T}_\beta(x,y) = \mathbb{P}(f(x) = f(y)) - \frac{1}{2}$

Let $\varepsilon > 0$ be given

ratios!

thm. critical value of inverse temp $\beta_c > 0$ such that if $\underbrace{\beta > \beta_c}_{\text{low temp}}$,

there's a $c > 0$ s.t. ~~$\mathcal{T}_\beta(x,y) > 0$~~

$$\mathcal{T}_\beta(x,y) > c > 0 \quad \forall x, y,$$

if $\beta < \beta_c$: $\exists\, c' > 0$ s.t.

$$\mathcal{T}_\beta(x,y) \le e^{-c'\|x-y\|}$$

Glauber dynamics in the low-temp regime (i.e. $\beta > \beta_c$):

- start w/ independent spins @ $t=0$
- at each time step, flip a spin
  - $\hookrightarrow$ do rejection sampling w/ Hamiltonian

$\square$

Let $\varepsilon > 0$ be given

- NONLOCAL MC!

  ↪ want to be easy to go from graphs to spins, spins to graphs

  ↪ give adjacent component different spins, but all vert's in the same component has the same spin

$G(f)$ a random grap where eges so $\delta f = 0$ are included ind. w/ probability $p = 1 - e^{-\beta}$

Exercise: can we go $f \to G(f) \to f$ and recover the measure f by independently assinging spins to each component

$\square$

Let $\varepsilon > 0$ be given

Swenson–Wang algo:

Given $f \in C^0(G, \mathbb{Z}^2)$ (G a box in $\mathbb{Z}_2$). want to update f.

1. compute $G(f)$ by including edges so $\delta f = 0$ ind. w/ probability $p = 1 - e^{-\beta}$
2. compute components of $G(f)$
3. assign indep. elements to each component to get f_{new}

has nice properties! invariant distribution is μ_β ← algo samples this!

$\square$

Let $\varepsilon > 0$ be given

2

3

$\Big\}$ same as sampling a uniform element of

$Z^{0}(G, \mathbb{Z}_2) = N(\delta)$

want to turn this to Z'

$\square$