SWENDSON-WANG

Swendson–Wang for the Potts model (Potts lattice gauge theory) in $\mathbb{Z}^d$ with $q \in$ ~~[illegible]~~ $2, \ldots$. The random-cluster model

random-cluster measure on finite graph G:

$p \in [0,1]$, $q \in (0,\infty)$, state space $\underbrace{\Omega = \{0,1\}^E}_{\text{each edge assigned a "spin"}}$

$\omega \in \Omega$, where
$\omega = (\omega(e) : e \in E)$, so
ω is a functional on E

random-cluster measure ~~$\varphi_{p,q}$~~ $\varphi_{p,q}$ given by

# of components

$$\varphi_{p,q}(\omega) = \frac{1}{Z_{RC}} \left\{ \prod_{e\in E} p^{\omega(e)} (1-p)^{1-\omega(e)} \right\} \cdot q^{k(\omega)}$$

sums up all the products and acts as a normalizing constant!

The Hamiltonian (appropriate for this context) is given by

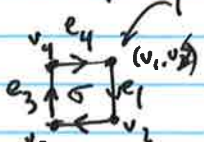
$$H(\sigma) = -\sum_{e\in E} \delta_e(\sigma), \quad \delta_e = \cancel{\delta(x,y)}\ \underbrace{\delta_{\sigma_x,\sigma_y}}_{\text{Kronecker on vertices}}$$

σ is a random variable taking values ~~uniformly~~ in $\{1,2,\ldots,q\}$

$\cancel{f \sim \sum k((\delta f)(\sigma))}$

we want to generalize this for ~~f~~ higher-order ~~homotopy~~ homology groups:

two-plaquette



we want cycles $\alpha \in C_i(X,G)$ (i.e. $\partial\alpha = 0$); $Z_i(X,G)$ is the cycle space for $C_i(X,G)$ (or just $\ker \partial_i$). boundaries of $(i+1)$-plaquettes are denoted $B_i(X,G)$, and are just $\operatorname{im} \partial_{i+1}$.

$\partial(\sigma) = e_1 + e_2 + e_3 + e_4$

$= (v_1,v_2) + (v_2,v_3) + (v_3,v_4) - (v_1,v_4)$

in the traditional Swendson-Wang algorithm:

1. we are initially given some vertex configuration (an assignment of spins to vertices, where spins are in {1, ..., q}) and want to construct an edge configuration (an assignment of spins to edges, where spins are in {0,1}) by adding an edge e between vertices with the same spin with some probability p, then assigning a spin in {0,1} to e;
2. continue doing this until we have constructed a subgraph P of G;
3. to each connected component of the graph consisting of all vertices and the edges in P (i.e. all edges with spin 1), we uniformly randomly assign all the vertices in each component a spin in {1, ..., q}.

the labeling of the vertices is the cocycle, because (when using the boundary operator) we add and subtract all the spins once, so everything is zero!

traditional:

given time t, temp β, $f_t \in C^0(X, \mathbb{F}_q)$,

1. sample a random graph $G(f)$ by:

   not ~~joining~~ adding edges between vertices with different spins

   AND

   adding edges between vertices with the same spin with probability $p = 1 - e^{-\beta H(f)}$

   sampling from $Z^0(G; \mathbb{F}_q)$

2. for each component of $G(f)$, uniformly randomly sample spins from $\{0,1\}$.
3. call the f admitted (i.e. the configuration on the edges) as f_{t+1}

cocycles on vertices

generalized

given time t, temp β, $f_t \in C^{i-1}(X; \mathbb{F}_q)$

1. sample a random i-complex P where (δf) vanishes indep. with prob. $p = 1 - e^{-\beta}$
2. select unif. random cocycle $f_{t+1} \in Z(P; \mathbb{F}_q)$

proofs of detailed balance!

working later with Grimmett (Akshin model)

what do we want to measure? → Wilson loop variable!

how does it act at the self-dual

- recreate behavior/asymptotics in $\mathbb{Z}^2_m$ $\mathbb{Z}^3_q$
- plaquettes what's the prob. that we're null-homologous?
- numerical sims in $\mathbb{Z}^4$

what's correlation btwn loop-variables

i.e. prob that they're null-homologous or homologous to each other

- independent model

prob there is a tube