Quantum Algorithms and Quantum Systems

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Quantum Computing

- Quantum computing utilizes qubits, which can be in a superposition of states (0 and 1 simultaneously), unlike classical bits.
- Operations on qubits are represented by quantum gates, which are analogous to reversible classical logic gates.
- Quantum algorithms leverage superposition and entanglement to perform parallel computations and potentially solve certain problems faster than classical algorithms.

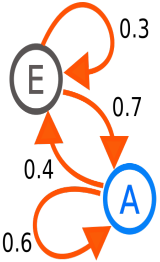
Markov Chains

Markov Chain Overview:

- Stochastic model for event sequences.
- Probability of an event depends only on the previous state.

Role in Quantum Computing:

- Understanding and Analysis
- Modeling Quantum Processes
- Probabilistic Framework



What is a Monte Carlo Method?

- Certain problems don't have known efficient deterministic solutions
- Monte Carlo methods attempt to solve these problems using stochastic processes and sampling in an efficient manner
- Since Monte Carlo methods use randomized procedures, we need to use another method, like the Chernoff Bound, to measure our probabilistic confidence in these methods' outputs

Find π using a Monte Carlo Method

Assumption

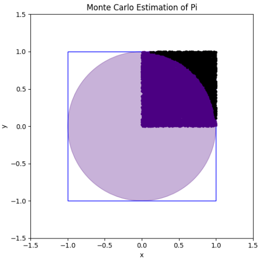
Suppose we do not know the value of π

Circle Area Formula

$A_{circle} = \pi r^2$

Find π using a Monte Carlo Method

Suppose we have a dartboard(circle inscribed in square) which we throw darts at(choose random points using continuous uniform distribution)



The percentage of hits(points inside the circle) multiplied by the area of the square should give us the area of the circle

Monte Carlo Circle Area Formula

$$A_{circle} = Pr(HIT) \times A_{square}$$

Find π using a Monte Carlo Method

Assumption

Each x_i is I.I.D. (*independently and identically distributed*)
$x_i \sim Bernoulli(p)$ where p=$\frac{A_{circle}}{A_{square}}$

$$X = \sum_{i=1}^{N} x_i$$

$$E(X) = \frac{A_{circle}}{A_{square}} N$$

$$E(X) = \frac{\pi r^2}{(2r)^2} N$$

$$E(X) = \frac{\pi}{4} N$$

Chernoff Bound

Chernoff bound: Upper bound on deviations from expected sum of independent random variables.

Multiplicative Chernoff Bound

$$Pr\left(\left|X - \frac{\pi}{4}N\right| \geq \delta \frac{\pi}{4} N\right) \leq 2^{\frac{-\delta^2 N\pi}{12}}$$

Quantum Computing Applications:

- Estimate algorithm output deviation
- Predict error accumulation probability
- Analyze and optimize success probability in amplitude amplification algorithms

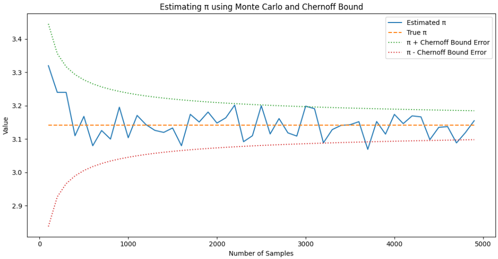
Chernoff Bounds

Chernoff Bound Importance: Precision Estimation:

- Quantifies error and uncertainty.
- Specifies the required sample size.

Significance in Monte Carlo: Optimizing Resources:

- Balances accuracy with computational resources.
- Ensures efficient Monte Carlo simulations.



Narrowing Bounds on π

Without knowing the actual value of π, we can narrow the bounds on π using a method similar to the Bisection Method

Pseudocode

while $|b - a| > \delta$
    if $\sum_{i=1}^{n} x_i > \left(\frac{a+b}{2}\right) n$
        $a \leftarrow \frac{a+b}{2}$
    else
        $b \leftarrow \frac{a+b}{2}$