

Analysis of Dartboard Problem using Hoeffding's Inequality

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1 Hoeffding's Inequality in terms of δ

$$Pr\left(\left|\frac{1}{n} \sum_{i=1}^n -\frac{\pi}{4}\right| \geq \frac{t}{n}\right) \leq 2e^{\frac{-2t^2}{n}}$$

$$\delta = \frac{t}{n}$$

$$t = \delta n$$

$$Pr\left(\left|\frac{1}{n} \sum_{i=1}^n -\frac{\pi}{4}\right| \geq \delta\right) \leq 2e^{-2n\delta^2}$$

2 Number of steps required for a success rate of $\frac{2}{3}$ in terms of δ

$$Pr\left(\left|\frac{1}{n} \sum_{i=1}^n -\frac{\pi}{4}\right| \geq \delta\right) \leq 2e^{-2n\delta^2}$$

$$Pr\left(\left|\frac{1}{n} \sum_{i=1}^n -\frac{\pi}{4}\right| < \delta\right) \geq 1 - 2e^{-2n\delta^2}$$

$$1 - 2e^{-2n\delta^2} = \frac{2}{3}$$

$$-2e^{-2n\delta^2} = -\frac{1}{3}$$

$$e^{-2n\delta^2} = \frac{1}{6}$$

$$-2n\delta^2 = \ln \frac{1}{6}$$

$$n = \frac{\ln 6}{2\delta^2}$$

3 Number of steps required for a success probability of at least $p_{success}$ in terms of δ

$$Pr\left(\left|\frac{1}{n} \sum_{i=1}^n -\frac{\pi}{4}\right| \geq \delta\right) \leq 2e^{-2n\delta^2}$$

$$p_{failure} = 1 - p_{success}$$

$$2e^{-2n\delta^2} = p_{failure}$$

$$e^{-2n\delta^2} = \frac{p_{failure}}{2}$$

$$-2n\delta^2 = \ln\left(\frac{p_{failure}}{2}\right)$$

$$n = \frac{-\ln\left(\frac{p_{failure}}{2}\right)}{2\delta^2}$$

$$n = \frac{\ln\left(\frac{2}{p_{failure}}\right)}{2\delta^2}$$