CS 577 Homework 8

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Question One: 8.5

We seek to prove that the Hitting Set problem is NP-Complete. More specifically, our problem states that, given a set, $A = a_1, a_2, \ldots, a_n$, a collection of subsets, B_1, B_2, \ldots, B_m , and a number k, is there a hitting set, H, so that $|H| \leq k$?

First, we will verify that the Hitting Set problem is in NP. Given a proposed set, H, in an instance of the problem, we can in polynomial time check if there are no more than k elements in H as well as check that each B_i has at least one element that is also in H, therefore it is in NP.

Now, we must pick another problem, Y, that is known to be NP-Complete and show that $Y \leq_p$ Hitting Set. We choose the Vertex Cover Problem. Assume we are given a graph, G = (V, E). We call our set A for the Hitting Set Problem the set of all vertices in G, so A = V. Then, we determine our subsets, B_i as follows. For every edge e = (u, v) in E, we create a new B_i that just contains u and v. With this construction, then there is a hitting set of size k or less if and only if there exists a vertex cover of size k or less. To prove this, first assume we have found a vertex cover D of G of size k or less. Then, for each edge e = (u, v), $e \in E$, we have at least one, $u \in D$ or $v \in D$. But this also means that the corresponding B_i is also hit, as our corresponding hitting set must either have u or v, Thus D is also a hitting set. Now, suppose we have a hitting set of size k or less, H. Then for every edge, e, e has at least one edge in the hitting set H because H contains one of the nodes in the edge. Thus H is a vertex cover. Thus, $VertexCover \leq_p HittingSet$, and the Hitting Set problem is NP-Complete