CS122A: Intermediate Embedded and Real Time Operating Systems

Jeffrey McDaniel

University of California, Riverside

Error Detection

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- Additional bits are added to the end of the data to detect when errors occur

Error Detection and Correction Schemes

- Redundancy checks

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  - Parity

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  - Checksum

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- Gray code
- Forward Error Correction

Parity Bits

7 bits of data	(Count of 1 bits)	8 bits including parity	
		even	odd
000 0000	0	000 0000**0**	000 0000**1**
101 0001	3	101 0001**1**	101 0001**0**
110 1001	4	110 1001**0**	110 1001**1**
111 1111	7	111 1111**1**	111 1111**0**

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- Indicates if the number of bits with a 1 is even or odd
- Used as the simplest form of error detection
- Single errors are identified, but the error cannot be recovered without re-sending data

Parity Bits - Successful Transmission

Type of bit parity	Successful transmission scenario
Even Parity	
Odd Parity	

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Even Parity	A wants to transmit: 1001 A computes parity bit: $(1 + 0 + 0 + 1)\%2 = 0$ A adds parity bit and sends: 10010
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Even Parity	A wants to transmit: 1001 A computes parity bit: $(1 \wedge 0 \wedge 0 \wedge 1) = 0$ A adds parity bit and sends: 10010 **...TRANSMISSION ERROR...** B receives: 1**1**010
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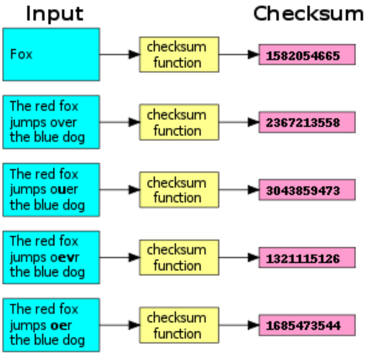
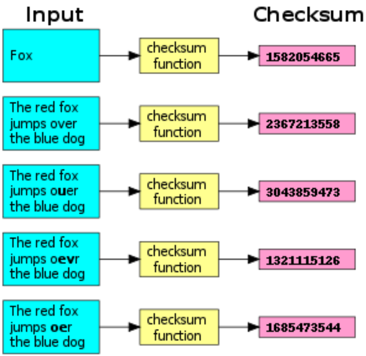
Checksum



- **Checksum function** gives checksum from data input

Checksum



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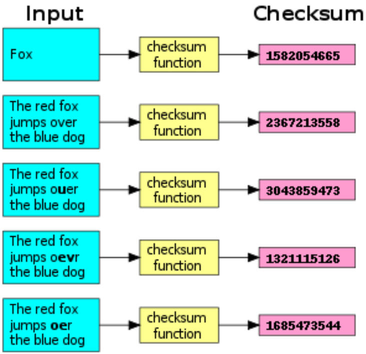

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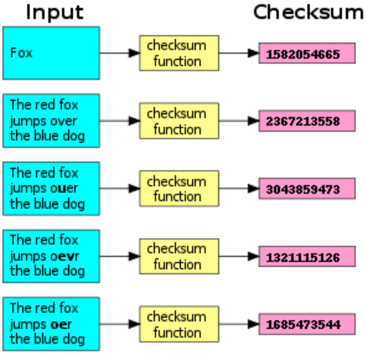

- **Checksum function** gives checksum from data input
- Produces significantly different data even for small changes
- Parity bits are simple checksum functions
- Some error-correcting codes are based on checksums

Checksum Algorithms

- Parity byte or word

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  - Break data into words

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  - Check if resulting word has *n* zeros

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- Modular sum

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  - Add all "words" as unsigned binary numbers

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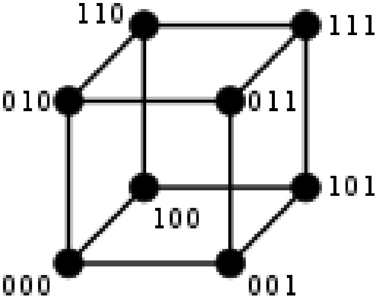
  - Above algorithms fail to detect common errors which affect many bits
  - Position dependent algorithms look not just at values, but position as well
  - Generally increases the cost of computing the checksum

Hamming Distance



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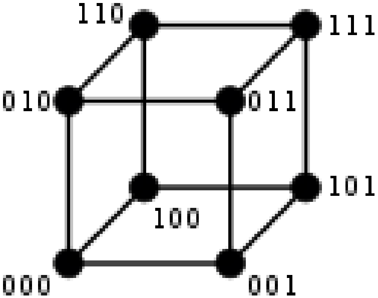

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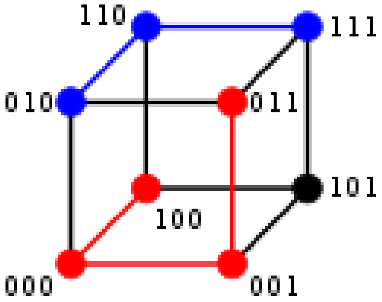

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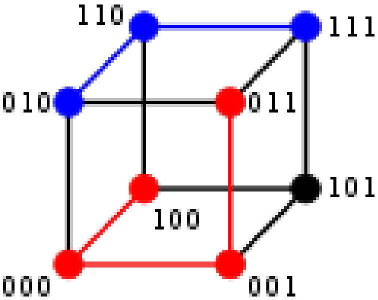

100→011 has distance 3 (red)
010→111 has distance 2 (blue)

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Gray Code

2-bit	**4-bit**
00	0000
01	0001
11	0011
10	0010
	0110
	0111
3-bit	0101
000	0100
001	1100
011	1101
010	1111
110	1110
111	1010
101	1011
100	1001
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- Named after Frank Gray of Bell Labs (1887 - 1969)
- Changes only one bit at a time
- Hamming Distance of one for adjacent values
- Gray codes are "cyclic"
- The value 7 can be changed to 0 with only one bit change

Forward Error Correction

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- Code allows limited number of errors to be detected
- Often those errors can be corrected without needing the data to be resent or recalculated
- **Systematic** codes include the data in the output without modification
- **Non-systematic** codes modify the data before output

Forward Error Correction

Triplet received	Interpreted as
000	0 (error free)
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010	0
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- Simple FEC (3,1) repetition code

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- Errors corrected by "majority vote"

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Shannon Limit

For any given degree of noise contamination
Data can be communicated nearly error-free up to a computable maximum rate

Forward Error Correction

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- *All-or-nothing tendency* becomes more pronounced closer to the **Shannon limit**
- Below the signal-to-noise ratio FEC systems don't work at all
- More advanced techniques can be used here, or hybrid approaches

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Forward Error Correction

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- Block codes
- Convolutional codes

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- Block codes
  - Work on fixed-size blocks
  - Can be hard-decoded in polynomial time to block length
  - Hard-decision algorithm, makes a decision for each bit
- Convolutional codes

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  - Work on arbitrary length streams
  - Soft-decoded, processes discretized analog signals
  - Higher error-correction performance

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- Classical block codes include Reed-Solomon coding (MLC NAND) and Hamming codes (SLC NAND)

Forward Error Correction

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- Classical block codes include Reed-Solomon coding (MLC NAND) and Hamming codes (SLC NAND)
- Viterbi algorithm commonly used for Convolutional codes

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    - Parity bit 4 covers all bit positions which have the third least significant bit set: 4–7, 12–15

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   - Parity bit 4 covers all bit positions which have the third least significant bit set: 4–7, 12–15
   - Parity bit 8 covers all bit positions which have the fourth least significant bit set: 8–15, 24–31, etc.

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1. Number the bits starting from 1: bit 1,2,3,4, etc.
2. Write the bit numbers in binary: 1, 10, 11, 100, 101, etc.
3. All bit positions which are powers of two are parity bits: 1, 2, 4, 8,etc.
4. All other bit positions are data bits
5. Each data bit is included in a unique set of 2 or more parity bits
    - Parity bit 1 covers positions with the least significant bit set: 1, 3, 5, 7, etc.
    - Parity bit 2 covers positions with the second least significant bit set: 2, 3, 6, 7, 10, 11,etc.
    - Parity bit 4 covers all bit positions which have the third least significant bit set: 4–7, 12–15
    - Parity bit 8 covers all bit positions which have the fourth least significant bit set: 8–15, 24–31, etc.
    - In general each parity bit covers all bits where the bitwise AND of the parity position and the bit position is non-zero

Hamming Code

Bit position		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	
Encoded data bits		**p1**	**p2**	**d1**	**p4**	**d2**	**d3**	**d4**	**p8**	**d5**	**d6**	**d7**	**d8**	**d9**	**d10**	**d11**	**p16**	**d12**	**d13**	**d14**	**d15**	
Parity bit coverage	**p1**	X		X		X		X		X		X		X		X		X		X		
	p2		X	X			X	X			X	X			X	X			X	X		...
	p4				X	X	X	X					X	X	X	X					X	
	p8								X	X	X	X	X	X	X	X						
	p16																X	X	X	X	X	

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