Tutorial Topic 6 (Solutions)

Confidence Intervals

Introduction

In this topic we will be looking at the purpose of a Confidence Interval and how to create and interpret them.

We calculate a Confidence Interval by putting what's called a margin of error either side of my **sample** mean average. I do this because my sample mean average is what's called a point estimate and point estimates are invariably wrong because of sampling error. Hopefully, after I construct this Confidence Interval the **true** population mean is somewhere within the interval.

Therefore, the aims of this tutorial are to:

- calculate estimates and their standard errors
- calculate and interpret Confidence Intervals for a population mean (numeric data)
- calculate and interpret Confidence Intervals for a population proportion (categorical data)
- determine the sample size necessary to develop a confidence interval for the mean or proportion

Textbook Questions/Answers/Readings

8.7 The manager of a paint supply store wants to estimate the actual amount of paint contained in 4-litre cans purchased from a nationally known manufacturer. It is known from the manufacturer's specifications that the standard deviation of the amount of paint is equal to 0.08 litres. A random sample of 50 cans is selected, and the sample mean amount of paint per 4-litre can is 3.98 litres.

a. Construct a 99% confidence interval estimate of the population mean amount of paint included in a 4-litre can.

$$\bar{X} \pm Z.\frac{\sigma}{\sqrt{n}} = 3.98 \pm 2.58.\frac{0.08}{\sqrt{50}} = 3.9508 \le \mu \le 4.0092$$

b. On the basis of your results, do you think that the manager has a right to complain to the manufacturer? Why?

Since the value of 4.0 is included in the interval, there is no reason to believe that the mean is different from 4 litres.

c. Must you assume that the population amount of paint per can is normally distributed here? Explain.

No. Since the population standard deviation is known and n = 50, from the Central Limit Theorem, we may assume that the sampling distribution of $\bar{X}$ is approximately normal.

d. Construct a 95% confidence interval estimate. How does this change your answer to (b)?

$$\bar{X} \pm Z.\frac{\sigma}{\sqrt{n}}$$

$$= 3.98 \pm 1.96.\frac{0.08}{\sqrt{50}}$$

$$= 3.9578 \le \mu \le 4.0022$$

Reading: Berenson Ch 8, Section 8.1

8.17 The energy consumption of refrigerators sold in Australia and New Zealand is checked and appliances are given a star rating to guide consumers who are about to make purchases. The consumption in kilowatts per annum is also displayed for each model on the website <www.energyrating.gov.au>. Suppose a consumer organisation wants to estimate the actual electricity usage of a model of refrigerator that has an advertised energy usage of 355 kW per annum. It tests a random sample of n = 18 fridges and finds a sample mean usage of 367 and a sample standard deviation of 30.

a. Assuming that the energy usage in the population is normally distributed, construct a 95% confidence interval estimate of the population mean energy usage for this model of refrigerator.

$$\bar{X} \pm t.\frac{S}{\sqrt{n}}$$

$$= 367 \pm 2.1098.\frac{30}{\sqrt{18}}$$

$$= 352.08 \ \leq \ \mu \leq 381.92$$

b. Do you think that the consumer organisation should accuse the manufacturer of producing fridges that do not meet the advertised energy consumption? Explain.

No, the advertised usage of 355 is within the interval.

c. Explain why an observed energy usage of 350 kW for a particular refrigerator is not unusual, even though it is outside the confidence interval developed in (a).

It is not unusual. An energy usage of 350 for a particular refrigerator is only 0.57 standard deviations below the sample mean of 367 OR 0.17 standard deviations below the population mean of 355.

Reading: Berenson Ch 8, Section 8.2

8.27 The number of older consumers in Australia is growing and they are becoming an important economic force. According to the Australian Bureau of Statistics, the proportion of the population aged 65 years and over increased from 14% in 2011 to 16% in 2016. (Australian Bureau of Statistics, Reflecting Australia- Stories from the Census, 2016, Cat. No. 2071.0, 2017). The proportion is projected to grow higher in coming years. Many older consumers feel overwhelmed when confronted with the task of selecting investments, banking services, health insurance or phone service providers. Suppose a telephone survey of 1,900 older consumers found that 27% said they felt confused when making financial decisions.

a. Construct a 95% confidence interval for the population proportion of older consumers who feel confused when making financial decisions.

$$p = \ 0.27$$

$$p \pm Z.\sqrt{\frac{p(1-p)}{n}}$$

$$= 0.27 \pm 1.96\sqrt{\frac{0.27\ (1-0.27)}{1900}}$$

$$= 0.2500 \ \leq \ \pi \ \leq 0.2899$$

b. Interpret the interval in (a).

At a 95% level of confidence, the proportion of older Australians who feel confused when making financial decisions is between 25% and 28.99%.

Reading: Berenson Ch 8, Section 8.3

8.67 The personnel manager of a large corporation wishes to study absenteeism among clerical workers at the corporation's central office during the year. A random sample of 25 clerical workers reveals the following:

- absenteeism: X = 9.7 days, S = 4.0 days
- 12 clerical workers were absent for more than 10 days

a. Construct a 95% confidence interval estimate of the mean number of absences for clerical workers last year.

$$\bar{X} \pm t.\frac{S}{\sqrt{n}}$$

$$= 9.7 \pm 2.0639.\frac{4.0}{\sqrt{25}}$$

$$= 8.049 \leq \mu \leq 11.351$$

b. Construct a 95% confidence interval estimate of the population proportion of clerical workers absent for more than 10 days last year.

$$p \pm Z.\sqrt{\frac{p(1-p)}{n}}$$

$$= 0.48 \pm 1.96\sqrt{\frac{0.48(0.52)}{25}}$$

$$= 0.284 \leq \pi \leq 0.676$$

If the personnel manager also wishes to take a survey in a branch office, answer these questions:

c. What sample size is needed to have 95% confidence in estimating the population mean to within ±1.5 days if the population standard deviation is 4.5 days?

$$n = \frac{Z^2 \times \sigma^2}{e^2}$$

$$= \frac{1.96^2 \times 4.5^2}{1.5^2}$$

$$= 34.57$$

$\boldsymbol{Rounding\ up}, use\ n = 35$

d. What sample size is needed to have 90% confidence in estimating the population proportion to within ±0.075 if no previous estimate is available?

$$n = \frac{Z^2 \times \pi(1-\pi)}{e^2}$$

$$= \frac{1.645^2 \times (0.5)(0.5)}{(0.075)^2}$$

$$= 120.268$$

$\boldsymbol{Rounding\ up}, use\ n = 121$

e. Based on (c) and (d), what sample size is needed if a single survey is being conducted?

If a single sample were to be selected for both purposes, the larger of the two sample sizes (n = 121) should be used.

Reading: Berenson Ch 8, Sections 8 to 8.4, 8.6

TEXTBOOK REFERENCE:

Basic Business Statistics: Concepts and Applications. *Berenson, M.L. Levine, D.M. Szabat, K.A. O'Brien, M. Jayne, N. Watson, J.* 5th edition. 2019. Pearson Australia Group Pty Ltd. ISBN 9781488617249. Chapter 8, sections 8 to 8.4 and 8.6