Tutorial Topic 5 (Solutions)

Continuous Distributions and Sampling Distributions

Introduction

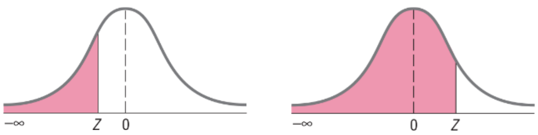
In the last topic we looked at discrete distributions, that is, distributions which involved whole numbers such as, one two three four et cetera. In this topic we will look at continuous distributions. A continuous number, in reality, has no steps or discrete values therefore we can immediately say that all decimal values are continuous. There are occasions when a whole number may also be classified as effectively continuous. We would do this when we're talking about large numbers such as salaries and house prices.

Therefore, the aims of this tutorial are to:

- calculate probabilities from the normal distribution
- determine whether a set of data is approximately normally distributed
- calculate probabilities from the uniform distribution
- calculate probabilities from the exponential distribution
- interpret the concept of the sampling distribution
- calculate probabilities related to the sample mean
- recognise the importance of the Central Limit Theorem
- calculate probabilities related to the sample proportion

Textbook Questions/Answers/Readings

6.11 A statistical analysis of 1,000 long-distance telephone calls made from the headquarters of the Bricks and Clicks Computer Corporation indicates that the length of these calls is normal with μ = 240 seconds and σ = 40 seconds. Draw normal distribution diagram, calculate Z score then use Table E.2



$$Z=\frac{X-\mu}{\sigma}$$

a. What is the probability that a call lasted less than 180 seconds?

P(X < 180) = P(Z < –1.50) = 0.0668

b. What is the probability that a particular call lasted between 180 and 300 seconds?

P(180 < X < 300) = P(– 1.50 < Z < 1.50) = 0.9332 – 0.0668 = 0.8664

c. What is the probability that a call lasted between 110 and 180 seconds?

P(110 < X < 180) = P(– 3.25 < Z < – 1.50) = 0.0668 – 0.00058 = 0.06622

d. What is the length of a particular call if only 1% of all calls are shorter?

P(X < A) = 0.01 P(Z < – 2.33) = 0.01

A = 240 – 2.33(40) = 146.80 seconds

Reading: Berenson Ch 6, Section 6.2

6.29 Customers arrive at the drive-through window of a fast-food restaurant at an average of two per minute during the lunch hour. [Use Excel file **EXPONENTIAL_PROBABILITY.XLSX**]

a. What is the probability that the next customer will arrive within 1 minute? (means up to 1 minute)

P(arrival time $\leq$ 1) = 0.8647

b. What is the probability that the next customer will arrive within 5 minutes?

P(arrival time $\leq$ 5) = 0.999955

c. During the dinner time period, the average arrival rate is one per minute. What are your answers to (a) and (b) for this period?

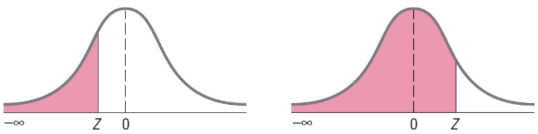
If $\lambda = 1$,

(a) P(arrival time $\leq$ 1) = 0.6321

(b) P(arrival time $\leq$ 5) = 0.9933

Reading: Berenson Ch 6, Section 6.5

7.9 A company is having a new corporate website developed. In the final testing phase the download time to open the new home page is recorded for a large number of computers in office and home settings. The mean download time for the site is 3.61 seconds. Suppose that the download times for the site are normally distributed with a standard deviation of 0.5 seconds. If you select a random sample of 30 download times: Draw normal distribution diagram, calculate Z score then use Table E.2



a. What is the probability that the sample mean download time is less than 3.75 seconds?

$P(\bar{X} < 3.75) = P(Z < 1.5336) = 0.9370$

b. What is the probability that the sample mean is between 3.70 and 3.90 seconds?

$P(3.7 < \bar{X} < 3.9) = P(0.9859 < Z < 3.1768) = 0.1604$

c. The probability is 80% that the sample mean is between which two values symmetrically distributed around the population mean?

$P(A < \bar{X} < B) = P(-1.2816 < Z < 1.2816) = 0.80$

A = 3.61 – 1.2816 (0.0913) = 3.4930; B = 3.61 + 1.2816 (0.09131) = 3.7270

d. The probability is 90% that the sample mean is less than what value?

$P(\bar{X} < A) = P(Z < 1.2816) = 0.90$; A = 3.61 + 1.2816 (0.09130) = 3.7270

Reading: Berenson Ch 7, Section 7.2

7.19 The Australian Tax Office carries out a range of verification checks and audits for the goods and services tax (GST) including Business Activity Statement integrity audits. Assume that currently no additional tax is collected for 25% of such audits. Suppose that you select a random sample of 100 audits. What is the probability that the sample will have:

The underlying distribution of the sample proportion is binomial but it can be approximated by a normal distribution if $n\pi \geq 5$ and $n(1-\pi) \geq 5$ with the resulting mean equal to **π.**

Therefore, we can use the Z formula for Proportions.

$$Z=\frac{p-\pi}{\sqrt{\frac{\pi(1-\pi)}{n}}}$$

a. between 24% and 26% of audits that collect no additional tax?

P(0.24 < p < 0.26) = P(–0.2309 < Z < 0.2309) = 0.1826

b. between 20% and 30% of audits that collect no additional tax?

P(0.20 < p < 0.30) = P(–1.1547 < Z < 1.1547) = 0.7518

c. more than 30% of audits that collect no additional tax?

P(p > 0.30) = P(Z > 1.1547) = 0.1241

Reading: Berenson Ch 7, Section 7.3

TEXTBOOK REFERENCE:

Basic Business Statistics: Concepts and Applications. *Berenson, M.L. Levine, D.M. Szabat, K.A. O'Brien, M. Jayne, N. Watson, J.* 5th edition. 2019. Pearson Australia Group Pty Ltd. ISBN 9781488617249. Chapter 6, sections 6 to 6.5 and Chapter 7, sections 7 to 7.3