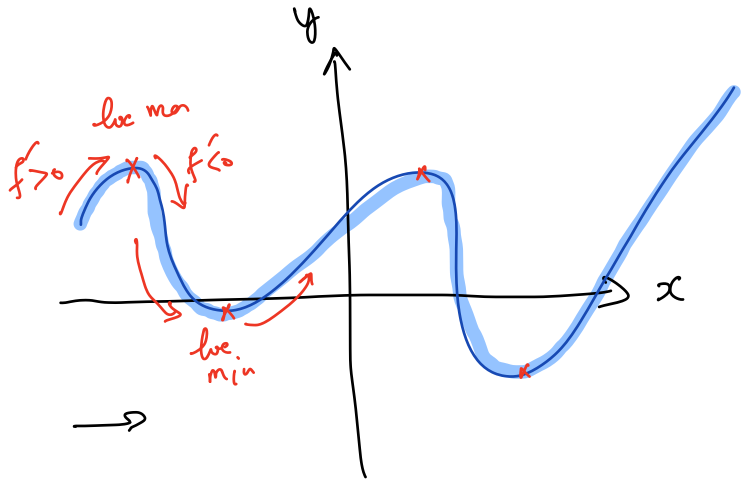
Lectures timetable optimisation

$\boxed{\text{max/min}}$ ← $y = f(x)$ extreme values { max, min }



$y = f(x)$ ① find critical points of this function

⊛ $f'(x) = 0$ roots ←

⊛ $f'(x)$ does not exist ←

② classify the critical points as max/min points

- second derivative technique

$x = c$: $f'(c) = 0$ and $f''(c) > 0$ $x = c$ is a local min

$x = d$: $f'(d) = 0$ and $f''(d) < 0$ $x = d$ is a local max

- first derivative technique

$f' > 0$ ↗ local max $x = c$ ↘ $f' < 0$

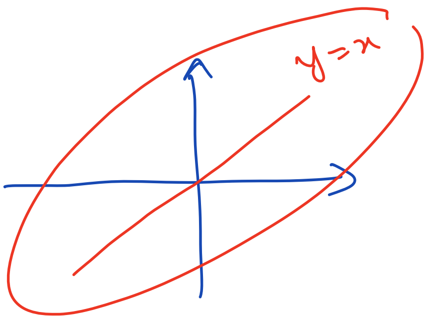
$f' < 0$ ↘ $x = c$ ↗ $f' > 0$ local min

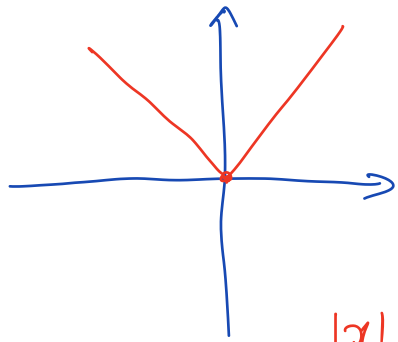
Know the sign of $f'(x)$

sign table

	$-\infty$		2		4		$+\infty$
$f'(x)$		+ ↗	0	− ↘	0	+ ↗	
			loc max		loc min		

$\boxed{\text{piecewise}}$ break points





$y = |x|$

$x = 0$

$$|x| = \begin{cases} x & x > 0 \\ -x & x < 0 \end{cases}$$

$y = |x-3|$ $\quad x - 3 \geq 0 \longrightarrow x \geq 3$

$y = |f(x)|$ $\quad f(x) \geq 0$

$$f(x) = \begin{cases} f_1(x) & x > a \\ f_2(x) & c > x \geq b \\ f_3(x) & d > x > c \end{cases}$$

$\boxed{\text{stationary points}}$ $\quad x_1 \quad x_2 \quad x_3$

$f'(x) = 0$

$f(x) = \dfrac{1}{x^2} = x^{-2}$

$f'(x)$ does not exist

$f'(x) = -2x^{-3} = \dfrac{-2}{x^3}$

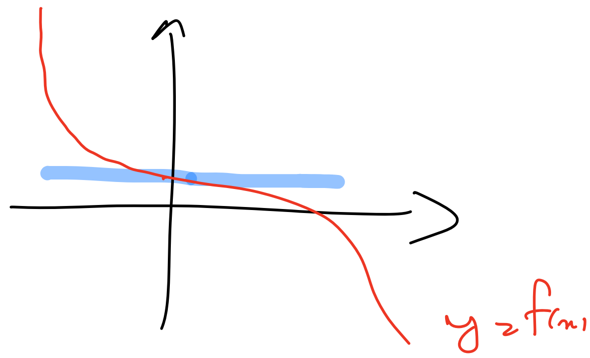
$f'(x) = 0 \longrightarrow \dfrac{-2}{x^3} = 0 \quad \times$

no solution

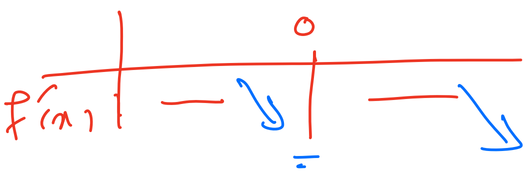
$f'(0) = \frac{-2}{0^3}$ ✗

$f'(x)$ does not exist at $x = 0$

critical points $= \{0\}$

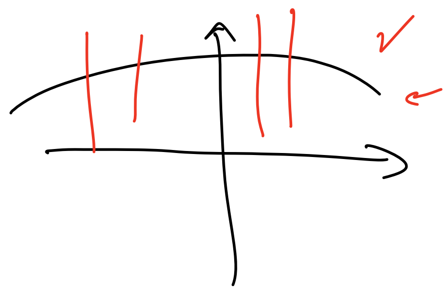


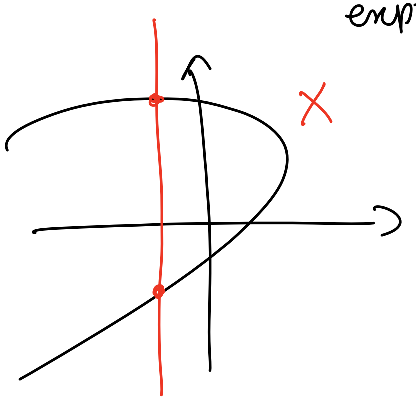
point of inflection



$f''(x) = 0$

$y = f(x)$ expression

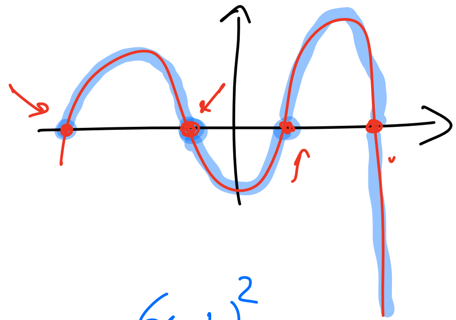




equation $\quad E(x) = H(x)$

$x = 10, 20, 37$

$E(x)$



$f(x) = y = (x-1)^2(x-\sqrt{3})(x+\sqrt{3})$

① roots of this functn

$f(x) = 0$

$$(x-1)^2(x-\sqrt{3})(x+\sqrt{3}) = 0 \rightarrow \begin{cases} (x-1)^2 = 0 \rightarrow x = 1 \\ \text{or} \\ (x-\sqrt{3}) = 0 \rightarrow x = \sqrt{3} \\ \text{or} \\ (x+\sqrt{3}) = 0 \rightarrow x = -\sqrt{3} \end{cases}$$

roots $= \{-\sqrt{3}, 1, \sqrt{3}\}$

	$-\infty$	-2	$-\sqrt{3}$	0	1	$\frac{3}{2}$	$\sqrt{3}$	2	$+\infty$
$f(x)$		$+$	0	$-$	0	$-$	0	$+$	

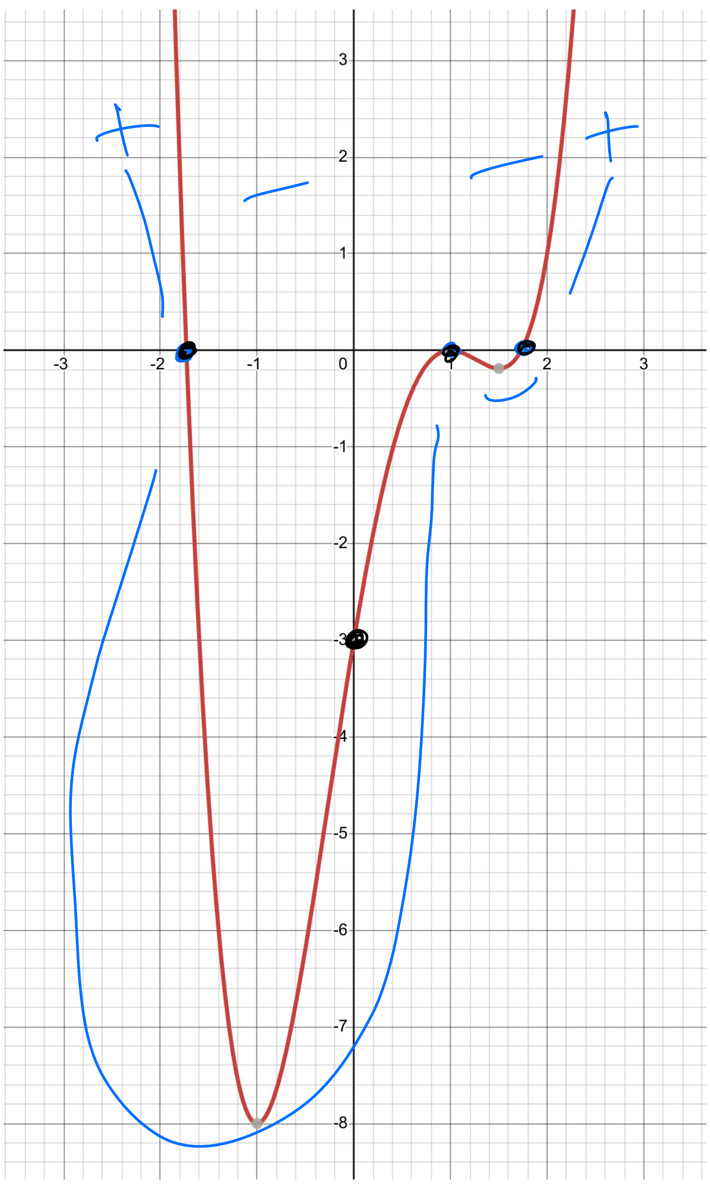
$\sqrt{3} \simeq 1.7$

$$f(0) = (0-1)^2(0-\sqrt{3})(0+\sqrt{3}) = -3 < 0$$

$$f(2) = (2-1)^2\underbrace{(2-\sqrt{3})}_{>0}\underbrace{(2+\sqrt{3})}_{>0} > 0$$

$$f(-2) = \underbrace{(-2-1)^2}_{9>0}\underbrace{(-2-\sqrt{3})}_{<0}\underbrace{(-2+\sqrt{3})}_{<0} > 0$$

$$f\left(\frac{3}{2}\right) = \underbrace{\left(\frac{3}{2}-1\right)^2}_{>0}\left(\underset{1.5}{\frac{3}{2}} - \underset{1.7}{\sqrt{3}}\right)\underbrace{\left(\frac{3}{2}+\sqrt{3}\right)}_{>0} < 0$$



$f'(x)$ + −

$y = f(x) = xe^{-x}$

intercepts $\begin{cases} x\text{-intercept} & y = 0 \\ y\text{-intercept} & x = 0 \end{cases}$

$x = 0 \rightarrow \quad y = (0)\,e^{-(0)} = 0 \qquad (0, 0) \quad$ y-intercep

$y=0 \rightarrow \underline{x}\, e^{-x} = 0 \rightarrow x = 0 \rightarrow (0,0)$

$e^{-x} > 0$

x-inter pt

critical points

$f'(x) = (1)(e^{-x}) + (x)(-e^{-x})$

$= e^{-x} - xe^{-x} = \boxed{(1-x)e^{-x}}$

$f'(x) = 0 \rightarrow (1-x)e^{-x} = 0 \rightarrow 1-x=0 \rightarrow \boxed{x=1}$

$f(1) = (1)e^{-1} = e^{-1} = \frac{1}{e}$

critical points $= \{(1, \frac{1}{e})\}$

$f''(x) = (-1)(e^{-x}) + (1-x)(-e^{-x})$

$= -e^{-x} + (x-1)e^{-x}$

$\rightarrow \quad = (-1 + x - 1)\, e^{-x} = (-2+x)e^{-x}$

at $x=1$, $\underline{f'(1) = 0}$ $\quad f''(1) = (-2+1)e^{-1} = -e^{-1} < 0$

$\rightarrow (1, \frac{1}{e})$ is a local max point.

$\frac{1}{e} \simeq 0.37$

	$-\infty$	0	1	2	$+\infty$
$f'(x) = (1-x)e^{-x}$		$+$	0	$-$	

local

max

$$f'(0) = (1-0)e^{-0} = 1 > 0$$

$$f'(2) = (1-2)e^{-2} = -e^{-2} < 0$$

$(-\infty, 1)$ $f(x)$ is increasing

$(1, +\infty)$ $f(x)$ is decreasing

$f(x)$ $(0,0)$

$(1, \frac{1}{e})$ local max

0.37