Revision of Hypothesis Testing

Extra

Question 1

- A group of retailers is considering spending $150 million on a new shopping complex in the Melbourne suburb of Blue Bay. For the complex to be profitable, however, the mean weekly expenditure on a wide range of consumer products, for households within 5km, must exceed $55 per week
  - Variable type: Numerical variable
  - Key word(s): 'exceed'
  - This points to: H_1
  - Test type: H_1 must be an Upper-tail test
  - Null hypothesis: $H_0: \mu \leq \$55$
  - Alternative hypothesis: $H_1: \mu > \$55$
  - Level of Alpha to use: 5%

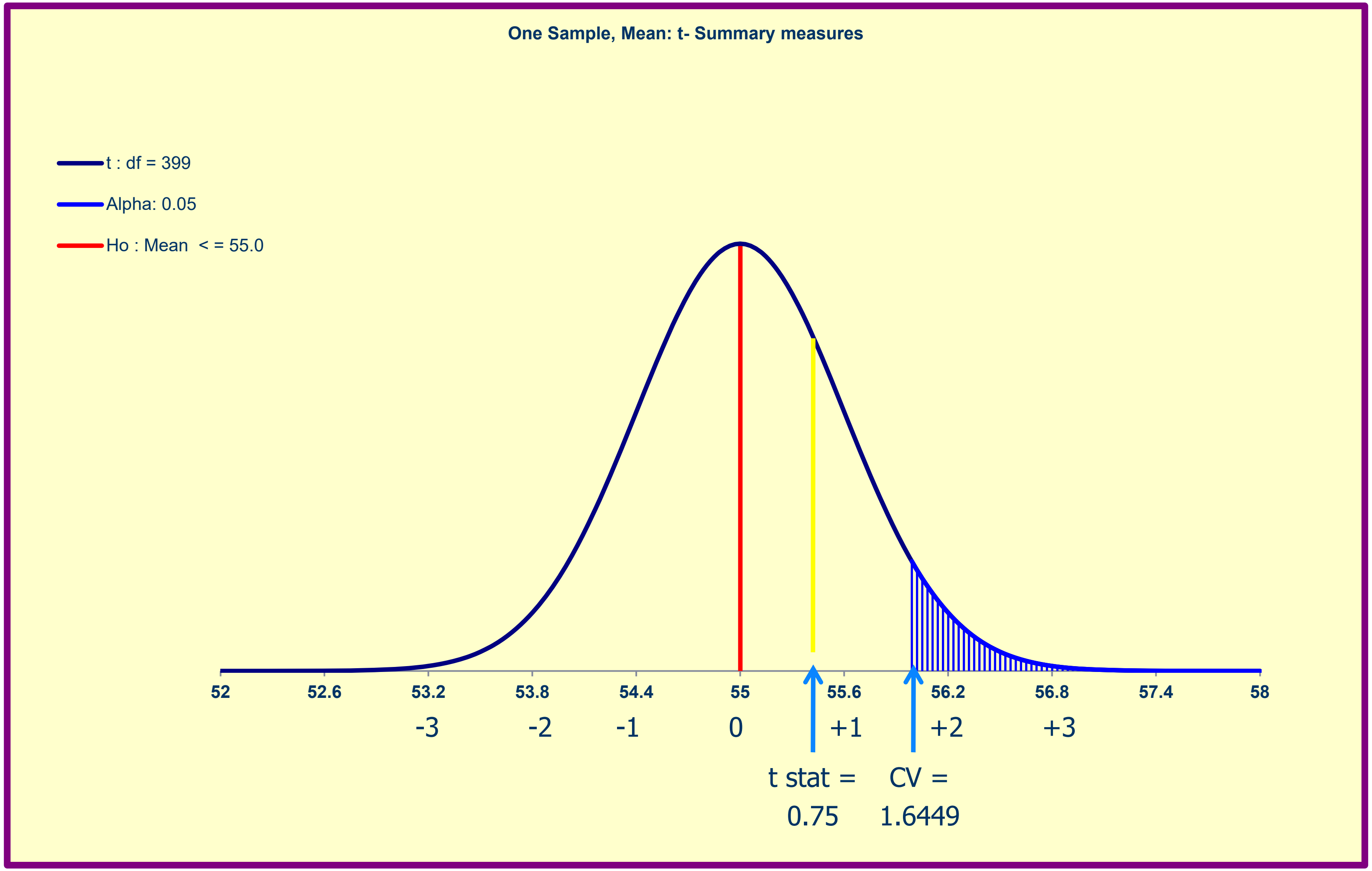
Question 1: Sample results

- A sample of 400 households yielded a sample mean of \$55.45 and standard deviation of \$12. What conclusion would you draw?

Sample Size (n):	= 400
Sample Mean ($\bar{x}$):	= 55.45
Sample SD (s):	= 12
CV of t:	= 1.6449 (df 399; tail area 0.05)
Standard Error ($S_{\bar{x}}$):	$= \frac{s}{\sqrt{n}} = \frac{12}{\sqrt{400}} = 0.6$
t-statistic (t):	$= \frac{\bar{x} - \mu}{s_{\bar{x}}} = \frac{55.45 - 55}{0.6} = 0.75$

- Conclusion: t stat is less than CV, therefore we Do Not Reject H_0
- Outcome: Complex not profitable

Question 1 Plot



Question 2

- A manufacturer has a machine that produces 10% defectives and this has proven very costly in the past. He has to decide whether to spend $400,000 on a new machine whose distributor claims will produce far fewer than 10% defectives and could save him money in the long run
  - Variable type: Categorical variable
  - Key word(s): 'fewer'
  - This points to: H_1
  - Test type: H_1 must be an Lower-tail test
  - Null hypothesis: H_0: $\pi \geq 10\%$
  - Alternative hypothesis: H_1: $\pi < 10\%$
  - Level of Alpha to use: 5%

Question 2: Sample results

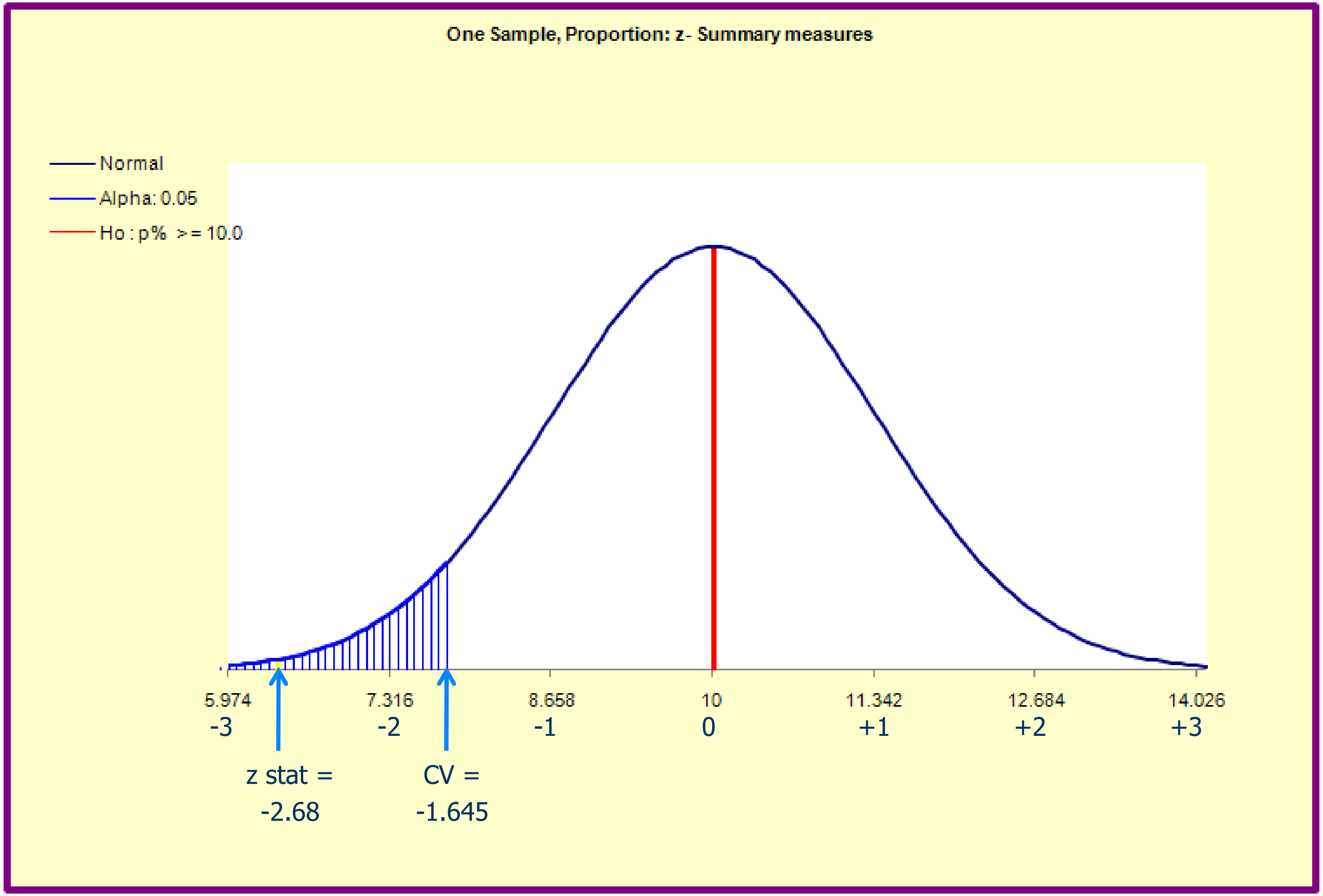
- A batch of 500 items produced on the new machine contained 32 defectives. What conclusion would you draw?

Sample Size (n):	= 500
Sample Proportion (p):	= 32/500 = 6.4%
CV of z:	= -1.645 (0.05 in the table)
Standard Error (σ_p):	$= \sqrt{\frac{\pi(1-\pi)}{n}} = \sqrt{\frac{0.1(0.9)}{500}} = 0.0134 \text{ or } 1.34\%$
z-statistic (z):	$= \frac{p-\pi}{\sigma_p} = \frac{0.064-0.1}{0.0134} = -2.68$

- Conclusion: z stat is less than CV, therefore we Reject H_0

- Outcome: Purchase new machine

Question 2 Plot



Question 3

- In 2010 a detailed market research survey showed that 45% of consumers were aware of Brand X Headache pill. In 2016, the manufacturers of the product wish to see if the same awareness level still applies

  - Variable type: Categorical variable
  - Key word(s): 'same awareness level still applies'
  - This points to: H_o
  - Test type: H_1 must be a Two-tail test
  - Null hypothesis: H_o: $\pi = 45\%$
  - Alternative hypothesis: H_1: $\pi \neq 45\%$
  - Level of Alpha to use: 5%

Question 3: Sample results

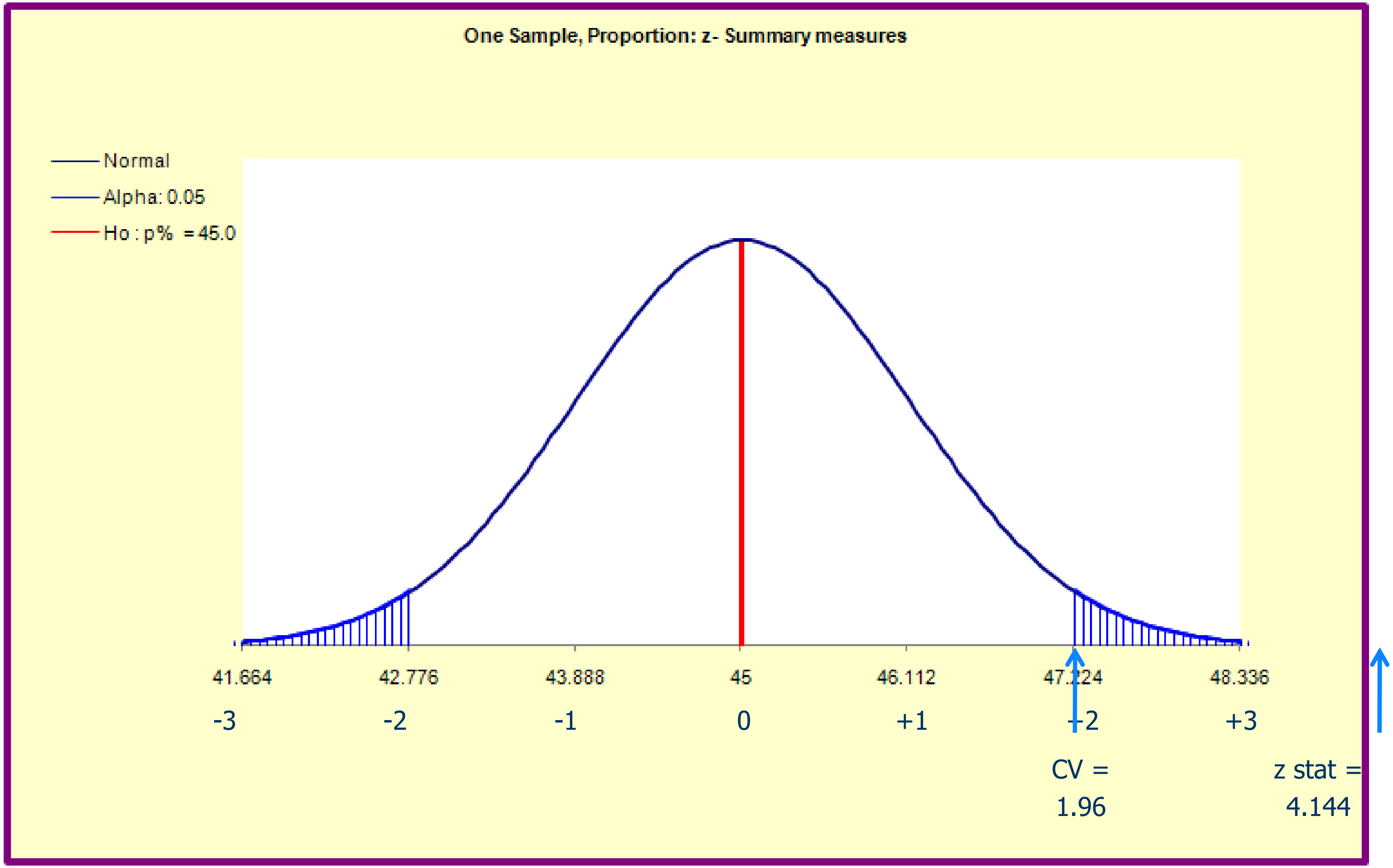
- Of 2000 consumers interviewed in 2006, 992 were aware of the product. What conclusion would you draw?

Sample Size (n):	= 2000
Sample Proportion (p):	= 992/2000 = 49.6%
CV of z:	= +/-1.96 (0.025 in the table)
Standard Error (σ_p):	$= \sqrt{\frac{\pi(1-\pi)}{n}} = \sqrt{\frac{0.45(0.55)}{2000}} = 0.0111 \text{ or } 1.11\%$
z-statistic (z):	$= \frac{p-\pi}{\sigma_p} = \frac{0.496-0.45}{0.0111} = 4.144$

- Conclusion: z stat is greater than CV, therefore we Reject H_0

- Outcome: Awareness has changed

Question 3 Plot



Question 4

- A special machine is used to fill coffee jars and has been set so that the amount of coffee per jar is normally distributed with mean of 152 grams. The quality control officer has to decide when the machine is playing up. If she stops operations to fix the machine, valuable production time is lost; but if she allows operations to continue, the machine could be producing a large percentage of under-weight or over-weight jars. She takes a sample of coffee jars every hour to keep a check on the machine
  - Variable type: Numerical variable
  - Key word(s): 'underweight or overweight'
  - This points to: H_1
  - Test type: H_1 must be a Two-tail test
  - Null hypothesis: $H_0: \mu = 152g$
  - Alternative hypothesis: $H_1: \mu \neq 152g$
  - Level of Alpha to use: 5%

Question 4: Sample results

- A simple random sample of 9 items taken at 11 o'clock had a mean of 151 grams, and standard deviation of 1 gram. What conclusion would you draw at 11 o'clock?

Sample Size (n): = 9

Sample Mean ($\bar{x}$): = 151g

Sample SD (s): = 1

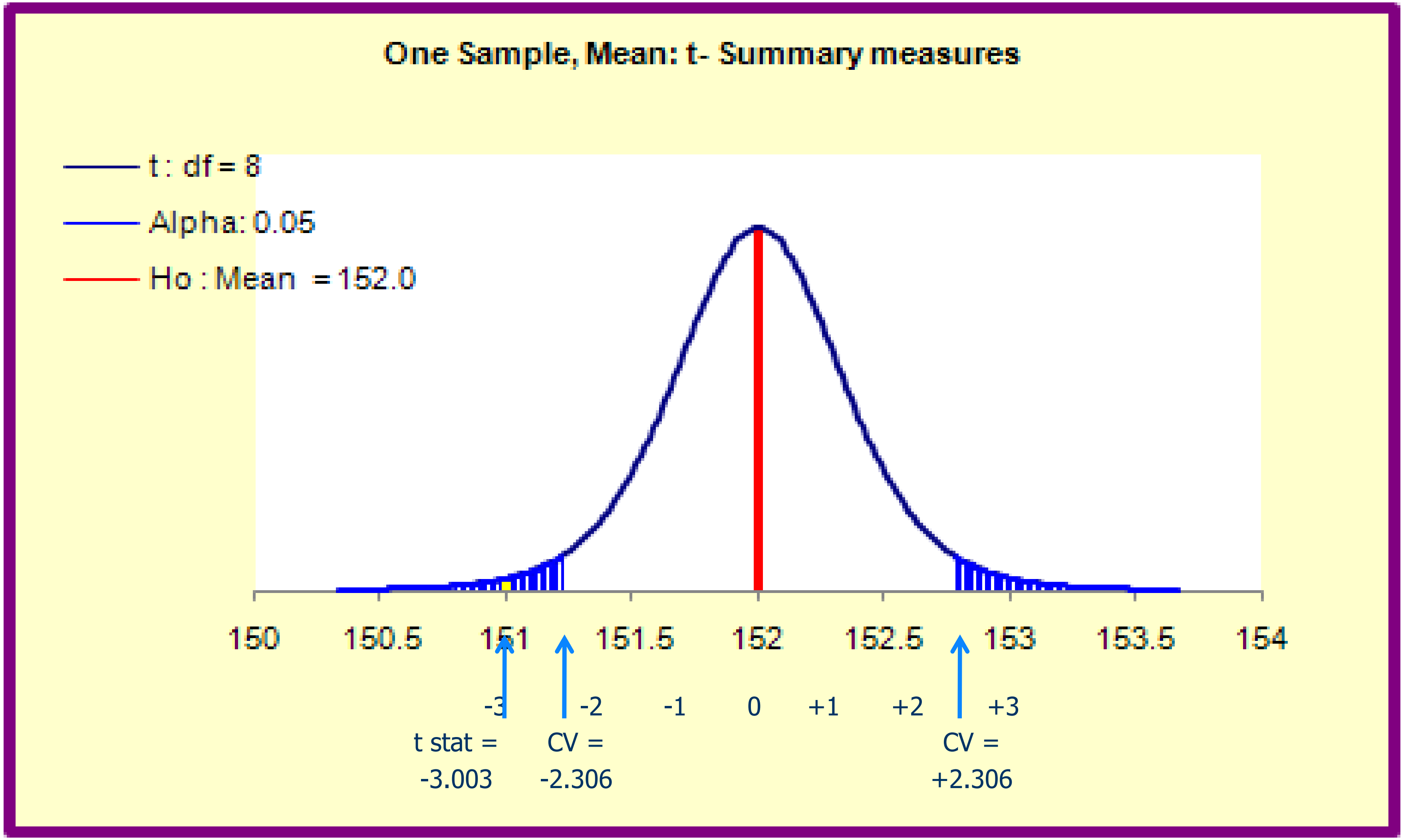
CV of t: = ±2.306 (df 8; tail area 0.025)

Standard Error ($S_{\bar{x}}$): $= \frac{s}{\sqrt{n}} = \frac{1}{\sqrt{9}} = 0.333$

t-statistic (t): $= \frac{\bar{x} - \mu}{s_{\bar{x}}} = \frac{151 - 152}{0.333} = -3.003$

- Conclusion: t stat is lower than CV, therefore we: Reject H_0
- Outcome: Coffee jars are underweight

Question 4 Plot



Question 5

- A company is about to launch a new product on the market. A company official claimed that for it to be successful more than 20% of households would want to own such a product. We wish to test consumer reaction to potential take-up

  - Variable type: Categorical variable
  - Key word(s): ‘more than’
  - This points to: H_1
  - Test type: H_1 must be an Upper-tail test
  - Null hypothesis: H_o: $\pi \leq 20\%$
  - Alternative hypothesis: H_1: $\pi > 20\%$
  - Level of Alpha to use: 5%

Question 5 Sample results

- Subsequently, a market research study of 400 households selected at random showed that 26% of households in the sample would buy the product. Does this result indicate the product could be successful?

Sample Size (n): = 400

Sample Proportion (p): = 26%

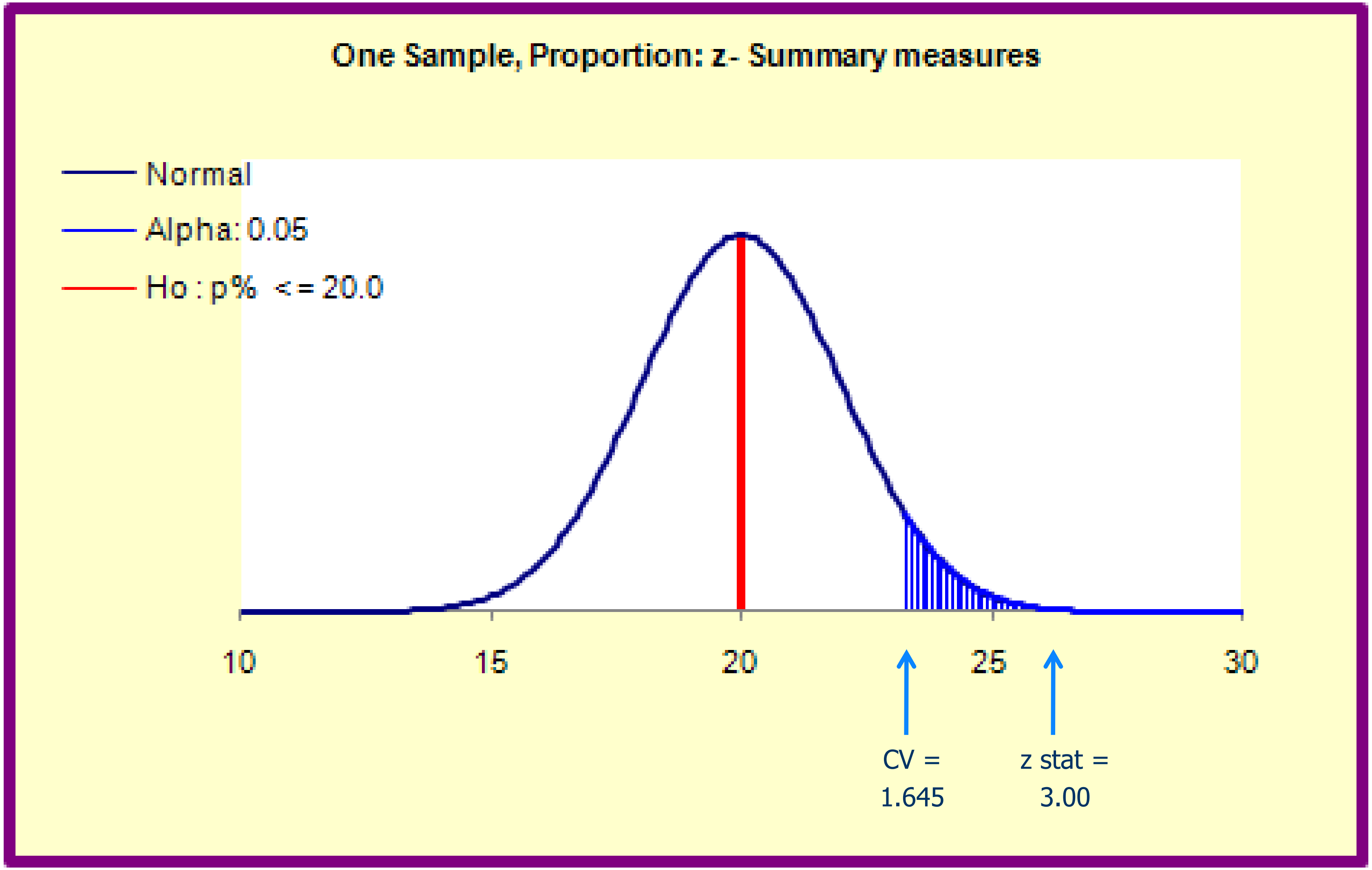
CV of z: = 1.645 (0.05 in the table)

Standard Error (σ_p): $= \sqrt{\frac{\pi(1-\pi)}{n}} = \sqrt{\frac{0.2(0.8)}{400}} = 0.02 \text{ or } 2\%$

z-statistic (z): $= \frac{p-\pi}{\sigma_p} = \frac{0.26-0.2}{0.02} = 3.00$

- Conclusion: z stat is greater than CV, therefore we Reject H_0
- Outcome: Product would be successful

Question 5 Plot



Question 6

- It was claimed that the mean selling price for homes in a certain suburb was more than \$360,000. We wish to test this claim

– Variable type:	Numerical variable
– Key word(s):	'more than'
– This points to:	H_1
– Test type:	H_1 must be an Upper-tail test
– Null hypothesis:	H_0: $\mu \leq \$360{,}000$
– Alternative hypothesis:	H_1: $\mu > \$360{,}000$
– Level of Alpha to use:	5%

Question 6: Sample results

- A simple random sample of size 100 recently sold homes yielded a mean of $367,600 and a standard deviation of $20,000. Test the validity of the claim made using these sample results

Sample Size (n):	= 100
Sample Mean ($\bar{x}$):	= $367,600
Sample SD (s):	= $20,000
CV of t:	= 1.6604 (df 99; tail area 0.05)
Standard Error ($S_{\bar{x}}$):	$= \frac{s}{\sqrt{n}} = \frac{20000}{\sqrt{100}} = 2000$
t-statistic (t):	$= \frac{\bar{x} - \mu}{s_{\bar{x}}} = \frac{367600 - 360000}{2000} = 3.8$

- Conclusion: t stat is greater than CV, therefore we Reject H_0
- Outcome: Selling price is more than $360,000

Question 6 Plot