SIT787: Mathematics for AI
Practical Week 1

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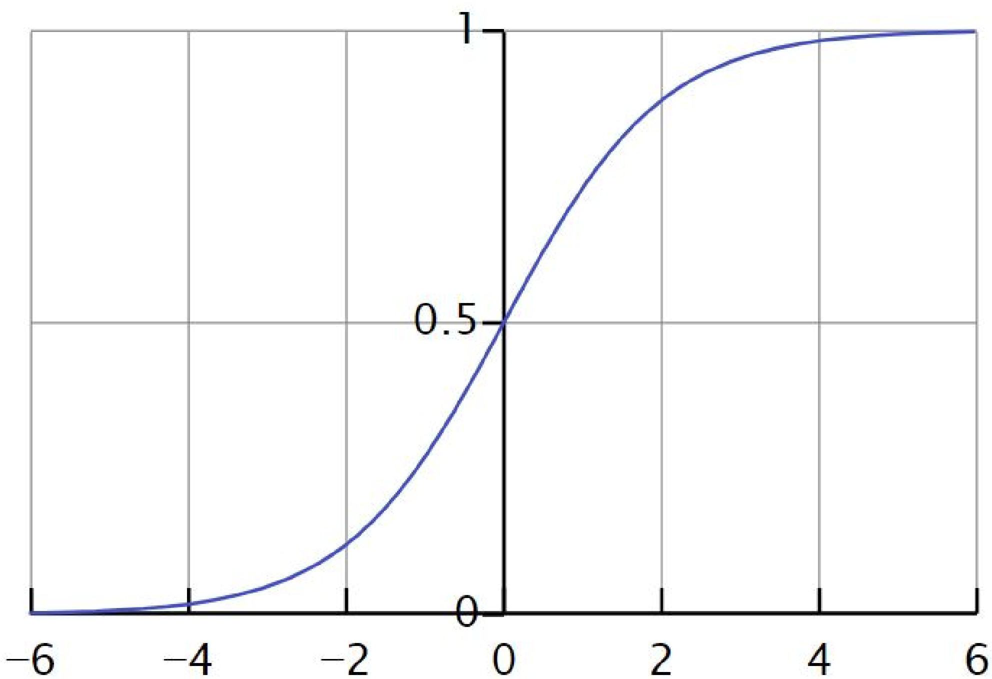
1. Consider this functions

$$f(x) = \frac{1}{1+e^{-x}}$$

- show that it can be represented as $f(x) = \frac{e^x}{1+e^x}$
- $f(0) = \frac{1}{2}$
- $f'(x) = f(x)(1 - f(x))$. [Hint: $\frac{d}{dx}(e^{kx}) = ke^{kx}$]
- A plot of the function is given as follows



  - what is the behaviour of $f(x)$ when x gets too large?
  - what is the behaviour of $f(x)$ when x gets too large from the negative side?
  - What are the domain and range of this function?

2. Consider the step function

$$f(x) = \begin{cases} 0, & \text{if } x < 0 \\ 1, & \text{if } x \geq 0 \end{cases}$$

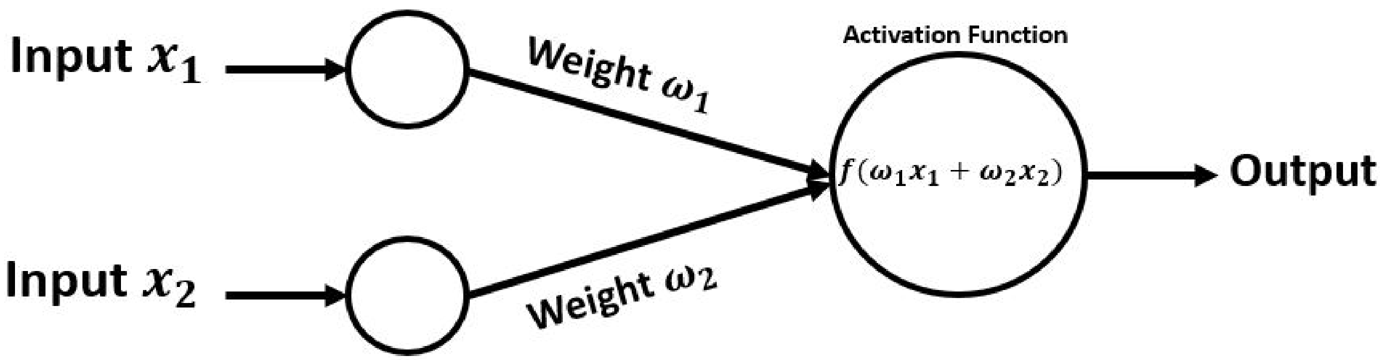
- Draw a plot of this function.

3. swish function

$$f(x) = \frac{x}{1 + e^{-x}}$$

- Considering $\sigma(x) = \frac{1}{1+e^{-x}}$, show that $f(x) = x\sigma(x)$.
- show that $f'(x) = f(x) + \sigma(x)(1 - f(x))$

4. The concept of perceptron:The functions we considered so far are types are activation functions, which are used in designing neural networks. The simplest possible neural network is a perceptron. This is called a single-layer Perceptron.



Classifying using Perceptron:

- input: $x_1, x_2, x_3, \ldots, x_n$
- weights: $\omega_1, \omega_2, \omega_3, \ldots, \omega_n$
- using a step function as an activation function

$$\text{output} = \begin{cases} 1 & \text{if } \sum_{j=1}^{n} \omega_j x_j > \text{threshold} \\ 0 & \text{if } \sum_{j=1}^{n} \omega_j x_j \leq \text{threshold} \end{cases}$$

Consider after training your model (means finding the weights) you know that you weights are $\omega_1 = 2$ and $\omega_2 = 5$, and the threshold is 4. Classify these new observation whether they belong to class 0 or class 1.

- $x_1 = 3, x_2 = -1$
- $x_1 = -2, x_2 = 7$
- $x_1 = 0, x_2 = 0$
- $x_1 = 9, x_2 = 120$