Tutorial Topic 4 (Solutions)

Probability and Discrete Probability Distributions

Introduction

In this topic we will be looking at Probability and Discrete Probability Distributions.

In the element **Probability** we investigate elementary probability and how to calculate the probability, or chance, of an event occurring. In order to do this, we first need to understand how to distinguish between mutually exclusive, dependent and independent events. We also look at calculating conditional probabilities and understand the general addition law for probabilities. We also look at Venn diagrams and decision trees.

In the second element of this topic we'll be looking at **Discrete Probability Distributions**. Discrete Probability Distributions arise out of counting, one two three in other words, whole numbers, hence the word Discrete. For example, it could be the number of people in a queue – I can't have 3.5 people in a queue – it has to be a whole number.

Therefore, the aims of this tutorial are to:

- recognise basic probability concepts
- calculate probabilities of simple, marginal and joint events
- calculate conditional probabilities and determine whether events are independent or not
- revise probabilities using Bayes' theorem
- use counting rules to calculate the number of possible outcomes
- recognise and apply the properties of a probability distribution
- calculate average return, and measure risk associated with various investment proposals
- identify situations that can be modelled by a Binomial distribution, and calculate binomial probabilities
- identify situations that can be modelled by a Poisson distribution, and calculate Poisson probabilities

Textbook Questions/Answers/Readings

4.19 A study was done to determine the efficacy of three different headache tablets – A, B and C. One thousand study participants used all three tablets (at different times) over the period of the study with the following results:

750	reported relief from tablet A
675	reported relief from tablet B
631	reported relief from tablet C
504	reported relief from both tablets A and B
453	reported relief from both tablets A and C
350	reported relief from both tablets B and C
236	reported relief from all three tablets

a. If a study participant is selected at random, what is the probability that they

I. reported relief from tablet A?

$$\boxed{P(A) = \frac{750}{1000} = 0.75}$$

II. reported relief from tablet B?

$$\boxed{P(B) = \frac{675}{1000} = 0.675}$$

III. reported relief from tablet A and tablet B?

$$\boxed{P(A \text{ and } B) = \frac{504}{1000} = 0.504}$$

IV. reported relief from tablet A or tablet B?

$$\boxed{\begin{aligned} P(A \text{ or } B) &= P(A) + P(B) - P(A \text{ and } B) \\ &= \frac{750}{1000} + \frac{675}{1000} - \frac{504}{1000} \\ &= \frac{921}{1000} \\ &= 0.921 \end{aligned}}$$

V. did not report relief from tablet C?

$$\boxed{P(C') = \frac{1000 - 631}{1000} = \frac{369}{1000} = 0.369}$$

b. What is the probability that, if a participant reported relief from tablet A, they also reported relief from tablet B?

$$\boxed{P(B \mid A) = \frac{P(A \text{ and } B)}{P(A)} = \frac{0.504}{0.75} = 0.672}$$

c. What is the probability that, if a participant reported relief from tablet B, they also reported relief from tablet A?

$$\boxed{P(A \mid B) = \frac{P(A \text{ and } B)}{P(B)} = \frac{0.504}{0.675} = 0.7466\ldots}$$

d. Are the events ‘report relief from tablet A’ and ‘report relief from tablet B’ statistically independent? Explain.

Not statistically independent, $\boxed{P(A \mid B) <> P(A)}$

Reading: Berenson Ch 4, Section 4.2

5.3 Using the company records for the past 500 working days, the manager of Konig Motors has summarised the number of cars sold per day in the following table:

Number of cars sold per day	Frequency of occurrence
0	40
1	100
2	142
3	66
4	36
5	30
6	26
7	20
8	16
9	14
10	8
11	2
Total	**500**

a. Form the probability distribution for the number of cars sold per day.

Probability distribution is given below:

X	n	$P(X)$	$XP(X)$	$X^2P(X)$
0	40	0.080	0.000	0.000
1	100	0.200	0.200	0.200
2	142	0.284	0.568	1.136
3	66	0.132	0.396	1.188
4	36	0.072	0.288	1.152
5	30	0.060	0.300	1.500
6	26	0.052	0.312	1.872
7	20	0.040	0.280	1.960
8	16	0.032	0.256	2.048
9	14	0.028	0.252	2.268
10	8	0.016	0.160	1.600
11	2	0.004	0.044	0.484
		Sum	3.056	15.408

b. Calculate the mean or expected number of cars sold per day.

Mean: $\boxed{\mu = \sum XP(X) = 3.056}$

c. Calculate the standard deviation.

Variance $\boxed{\sigma^2 = \sum X^2P(X) - \mu^2 = 15.408 - 3.056^2 = 6.068864}$

Standard deviation $\boxed{\sigma = \sqrt{6.068864} = 2.4635...}$

Reading: Berenson Ch 5, Section 5.1

5.23 When a customer places an order with Rudy's On-Line Office Supplies, a computerised accounting information system (AIS) automatically checks to see whether the customer has exceeded their credit limit. Past records indicate that the probability of customers exceeding their credit limit is 0.05. Suppose that, in a given half hour, 20 customers place orders. Assume that the number of customers that the AIS detects as having exceeded their credit limit is distributed as a binomial random variable.

a. What are the mean and standard deviation of the number of customers exceeding their credit limits?

Mean = 1 Standard deviation = 0.9747

b. What is the probability that no customer will exceed their limit?

$P(X = 0) = 0.3585$

c. What is the probability that one customer will exceed their limit?

$P(X = 1) = 0.3774$

d. What is the probability that two or more customers will exceed their limits?

$P(X \geq 2) = 0.2642$

Reading: Berenson Ch 5, Section 5.3

5.31 Based on past experience, it is assumed that the number of flaws per metre in rolls of grade 2 paper follow a Poisson distribution with a mean of one flaw per 5 metres of paper. What is the probability that in a:

a. 1-metre roll there will be at least two flaws?

$\lambda = 0.2$ $P(X \geq 2) = 1 - P(X = 0) + P(X = 1) = 1 - (0.8187 + 0.1637) = 0.0176$

b. 10-metre roll there will be at least one flaw?

If there are 0.2 flaws per metre on average, then there are on average $\lambda = 10 \times 0.2 = 2$ flaws in a 10-metre roll.

$P(X \geq 1) = 1 - P(X = 0) = 1 - 0.1353 = 0.8647$

c. 50-metre roll there will be between five and 15 (inclusive) flaws?

$\lambda = 50 \times 0.2 = 10$ flaws on average in a 15-metre roll.

$P(5 \leq X \leq 15) = 0.9220$

Reading: Berenson Ch 5, Section 5.4

TEXTBOOK REFERENCE:

Basic Business Statistics: Concepts and Applications. *Berenson, M.L. Levine, D.M. Szabat, K.A. O'Brien, M. Jayne, N. Watson, J.* 5th edition. 2019. Pearson Australia Group Pty Ltd. ISBN 9781488617249. Chapter 4, sections 4 to 4.5 and Chapter 5, sections 5 to 5.1 and 5.3 to 5.4