# SIT787: Mathematics for AI
# Practical Week 3

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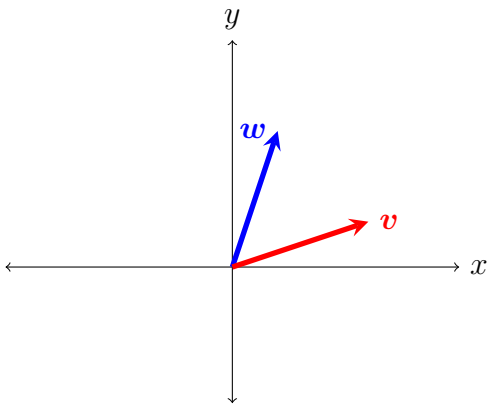
1. For $\boldsymbol{v} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$, describe all points $c\boldsymbol{v}$ with
   - $c \in \mathbb{I}$ an integer number
   - nonnegative real number $c \geq 0$
2. Find scalars $c$ and $d$ so that the linear combination $c\boldsymbol{v} + d\boldsymbol{w}$ equals $\boldsymbol{b}$:

$$\boldsymbol{v} = \begin{bmatrix} 2 \\ -1 \end{bmatrix} \qquad \boldsymbol{w} = \begin{bmatrix} -1 \\ 2 \end{bmatrix} \qquad \boldsymbol{b} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

3. draw all combinations of $c\boldsymbol{v} + d\boldsymbol{w}$
   - Restricted by $c \geq 0$ and $d \geq 0$
   - Restricted by $0 \leq c \leq 1$ and $0 \leq d \leq 1$




4. For the vectors

$$\boldsymbol{v} = \begin{bmatrix} 3 \\ 4 \end{bmatrix} \qquad \boldsymbol{w} = \begin{bmatrix} 4 \\ 3 \end{bmatrix}$$

   - Test the Schwarz inequality: $|\boldsymbol{v} \cdot \boldsymbol{w}| \leq ||\boldsymbol{v}||||\boldsymbol{w}||$
   - Test the Triangle inequality: $||\boldsymbol{v} + \boldsymbol{w}|| \leq ||\boldsymbol{v}|| + ||\boldsymbol{w}||$
   - Find the cosine of angle between them: $\cos\theta = \frac{\boldsymbol{v} \cdot \boldsymbol{w}}{||\boldsymbol{v}||||\boldsymbol{w}||}$
   - Knowing that for every angle $\theta$, $|\cos\theta| \leq 1$, can you show the Schwarz inequality?
5. Describe the vector space spanned by these three vectors and state a subspace of this vector space.

$$\boldsymbol{v} = \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix} \qquad \boldsymbol{w} = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 0 \end{bmatrix} \qquad \boldsymbol{u} = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$$

6. Show that

$$\left\{ \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 2 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 3 \end{bmatrix} \right\}$$

- are linearly independent, hence, form a basis for $\mathbb{R}^3$.
- Are the vectors mutually orthogonal?
- Use Gram-Schmidt process and orthognalise them.