MIS770 Foundation Skills in Business Analysis

DEPARTMENT OF INFORMATION SYSTEMS AND BUSINESS ANALYTICS
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Tutorial Topic 7 (Solutions)

Hypothesis Tests

Introduction

In this topic we will be looking at the purpose of a Hypothesis Test and how to create and interpret them.

Hypothesis testing follows on from Confidence Intervals and together they are probably the two most useful tools used by statisticians. They are both widely used in decision making and research and in some instances, I can use either or both, it just depends on the situation that I'm actually investigating.

I use a confidence interval when I have absolutely no idea about the value of the population parameter that I'm investigating. But I'm going to use a hypothesis test when I do have some idea about the value of the population parameter. In other words, if I have some type of prior knowledge or prior experience, or if I'm testing a standard or a claim, then I would use a Hypothesis test.

Therefore, the aims of this tutorial are to:

- identify the basic principles of hypothesis testing
- explain the assumptions of each hypothesis-testing procedure, how to evaluate them and the consequences if they are seriously violated
- use hypothesis testing to test a mean or proportion
- recognise the pitfalls involved in hypothesis testing
- identify the ethical issues involved in hypothesis testing

Textbook Questions/Answers/Readings

9.15 The process of deciding which drugs are effective and which may in fact do harm falls in the United States to the Food and Drug Administration (FDA). In April 2017 it wrote warning letters to 14 US companies who were selling products that fraudulently claim to prevent, treat or cure cancer. It is illegal to sell such products without first demonstrating to the FDA that they are safe and effective for their stated purpose. Consider a null hypothesis that a new, unapproved drug is unsafe for a specific cancer and an alternative hypothesis that it is safe.

a. Explain Type I and Type II errors in this context.

Type I error is the mistake of approving an unsafe drug. A Type II error is not approving a safe drug.

b. Which type of error do you consider it is more important to avoid?

Both errors are serious but approving an unsafe drug may have more immediate consequences resulting in deaths so should be avoided.

c. How would it be possible to lower the risk of both Type I and Type II errors?

To lower both Type I and Type II errors, select a smaller value for α and increase the sample size.

Reading: Berenson Ch 8, Section 9.1

9.29 The quality-control manager at a factory that manufactures memory cards for digital cameras needs to determine whether a large shipment of UHS cards has a mean write speed equal to 90 MB per second. The population standard deviation is 8 MB/s. A random sample of 64 cards indicates a sample mean write speed of 87.5 MB/s.

a. At the 0.05 level of significance, is there sufficient evidence that the mean write speed is different from 90 MB/s?

$\boxed{H_0 : \mu = 90 \text{ MB/s}}$

$\boxed{H_1 : \mu \neq 90 \text{ MB/s}}$

Decision rule: Reject H_0 if $\boxed{Z < -1.96 \; or \; Z > +1.96.}$

Test statistic: $\boxed{Z = \dfrac{\bar{X} - \mu}{\sigma / \sqrt{n}} = \dfrac{87.5 - 90}{8 / \sqrt{64}} = -2.50}$

Decision: Since Z = -2.5 is less than the critical bound of -1.96, reject H_0.
There is enough evidence to conclude that the mean data transfer rate differs from 90 MB/s at the 5% level of significance.

b. Calculate the p-value and interpret its meaning.

p-value = 2(0.0062) = 0.0124.

Interpretation: The probability of getting a sample of 64 cards that will yield a mean transfer rate that is farther away from the hypothesised population mean than this sample is 0.0124.

c. Construct a 95% confidence interval estimate of the population mean write speed of the memory cards.

$$\bar{X} \pm Z.\frac{\sigma}{\sqrt{n}} = 87.5 \pm 1.96.\frac{8}{\sqrt{64}} = 85.54 \le \mu \le 89.46$$

d. Compare the results of (a) and (c). What conclusions do you reach?

The results are the same. The confidence interval formed does not include the hypothesised value of 90 MB/s.

Reading: Berenson Ch 8, Section 9.2

9.45 A New Zealand researcher believes that, on average, teenagers aged 16–19 living in a major city will post photographs on social network sites more than 10 times a week. Suppose she wishes to find statistical evidence to support this. Let μ represent the population mean number of times 16–19-year-old teenagers in this city post photos on social network sites.

a. State the null and alternative hypotheses.

H_0: $\mu \le 10$

A teenager will post photographs on social networking sites 10 times a week or less.

H_1: $\mu > 10$

A teenager will post photographs on social networking sites more than 10 times a week.

b. Explain in the context of the above scenario the meaning of Type I and Type II errors.

A Type I error occurs when you conclude the mean number of times that teenagers post photos on social networking sites is more than 10 when in fact the mean number is not more than 10.

A Type II error occurs when you conclude the mean number of times that teenagers post photos on social networking sites is not more than 10 when in fact the mean number is more than 10.

c. Suppose the researcher carries out a study in the city in which you live. Based on past studies, she assumes that the standard deviation of the number of times teenagers aged 16–19 post photos on social network sites is 1.6. She takes a sample of 100 teenagers aged 16–19 and finds that the mean number of times they post photos per week is 10.87. At the 0.01 level of significance, find whether there is sufficient evidence that the mean number of times a week photos are posted is greater than 10. Use the p-value approach.

Decision rule: Reject H_0 if $Z > +2.33$

Test statistic: $\boxed{Z = \frac{\bar{X} - \mu}{\sigma / \sqrt{n}} = \frac{10.87 - 10}{1.6 / \sqrt{100}} = 5.4375}$

Decision: Since Z_{calc} = 5.4375 is greater than the critical bound of +2.33, reject H_0.
There is enough evidence to conclude the population mean number of times teenagers post photos on social networking sites is more than 10 times per week, at the 1% level of significance.

d. Interpret the meaning of the p-value in (c).

When the null hypothesis is true, the probability of obtaining a sample whose mean is 10.87 times or more is virtually zero (p-value = $P(Z > 5.4375) = 0.000$).

Reading: Berenson Ch 8, Section 9.3

9.69 The QILT 2016 Graduate Outcomes Survey found that 70.9% of undergraduates had found full-time work within four months of completing their degrees (<www.qilt.edu.au>). Now imagine that a researcher carries out a follow-up study by surveying a representative sample of 1,000 recent graduates from undergraduate degree courses and finds that 68.1% have found full-time work within four months of completing their degrees.

a. At the 0.01 level of significance, can you state that the percentage of new graduates who have found full-time work within four months of completing their degrees has decreased since 2016?

$\boxed{H_0 : \pi \geq 0.709}$

$\boxed{H_1 : \pi < 0.709}$

Decision rule: If $Z < -2.33$, reject H_0.

Test statistic: $\boxed{Z = \frac{p - \pi}{\sqrt{\frac{\pi(1-\pi)}{n}}} = \frac{0.681 - 0.709}{\sqrt{\frac{0.709(0.291)}{1000}}} = -1.9493}$

Decision: Since Z_{calc} = -1.9493 is not below the critical bound of Z = -2.33, do not reject H_0.

b. At the 0.05 level of significance, can you state that the proportion of new graduates who have found full-time work within four months of completion has changed since 2016?

There is not enough evidence to show that fewer than 70.9% of new graduates are finding work with four months of completing their degrees , at the 5% level of significance. .

c. What conditions needed to be met in order to answer parts (a) and (b)?

The sample used in (a) and (b) needs to be random. As the sample size is large, the conditions that $n\pi > 5$ and $n(1-\pi) > 5$ should have been met.

Reading: Berenson Ch 8, Section 9.5

TEXTBOOK REFERENCE:

Basic Business Statistics: Concepts and Applications. *Berenson, M.L. Levine, D.M. Szabat, K.A. O'Brien, M. Jayne, N. Watson, J.* 5th edition. 2019. Pearson Australia Group Pty Ltd. ISBN 9781488617249. Chapter 9, sections 9 to 9.5 and 9.7.