

SIT787: Mathematics for AI

Practical Week 2

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1. For these vectors

$$\mathbf{u} = \begin{bmatrix} 0 \\ 4 \\ -1 \\ 2 \end{bmatrix} \text{ and } \mathbf{v} = \begin{bmatrix} 1 \\ 0 \\ 3 \\ -1 \end{bmatrix}$$

- Find $\mathbf{u} + \mathbf{v}$, $\mathbf{u} - \mathbf{v}$, $2\mathbf{u} + 3\mathbf{v}$
- Find the cosine between these two vectors and their lengths
- Find the distance between them.

2. Are these vectors linearly independent?

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$$\mathbf{u} = \begin{bmatrix} 0 \\ 4 \end{bmatrix} \text{ and } \mathbf{v} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

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$$\mathbf{u} = \begin{bmatrix} 4 \\ 2 \\ -6 \end{bmatrix} \text{ and } \mathbf{v} = \begin{bmatrix} 10 \\ 5 \\ -15 \end{bmatrix}$$

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$$\mathbf{u} = \begin{bmatrix} 1 \\ 2 \end{bmatrix} \text{ and } \mathbf{v} = \begin{bmatrix} 3 \\ 6 \end{bmatrix}$$

3. For these vectors

$$\mathbf{u} = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} \text{ and } \mathbf{v} = \begin{bmatrix} 0 \\ -1 \\ 1 \end{bmatrix}$$

- Find the projection of \mathbf{v} over \mathbf{u} : $\mathbf{a}_1 = \text{proj}_{\mathbf{u}}^{\mathbf{v}}$
- Find $\mathbf{a}_2 = \mathbf{v} - \mathbf{a}_1$ using the definition.
- Are \mathbf{a}_1 and \mathbf{a}_2 perpendicular (orthogonal)?
- The formulas

$$\mathbf{a}_1 = \left(\frac{\mathbf{u} \cdot \mathbf{v}}{\mathbf{u} \cdot \mathbf{u}} \right) \mathbf{u}$$

$$\mathbf{a}_2 = \mathbf{v} - \mathbf{a}_1 = \mathbf{v} - \left(\frac{\mathbf{u} \cdot \mathbf{v}}{\mathbf{u} \cdot \mathbf{u}} \right) \mathbf{u}$$

4. which two vectors are more similar considering both the distance and cosine of an angle between them?

$$\mathbf{u} = \begin{bmatrix} 0 \\ 4 \\ -1 \\ 2 \end{bmatrix}, \mathbf{v} = \begin{bmatrix} 1 \\ 0 \\ 3 \\ -1 \end{bmatrix} \text{ and } \mathbf{w} = \begin{bmatrix} 1 \\ 0 \\ 1 \\ 1 \end{bmatrix}$$

5. Find all the norms for these vectors:

$$\mathbf{u} = \begin{bmatrix} 0 \\ 4 \\ -1 \\ 2 \end{bmatrix}, \mathbf{v} = \begin{bmatrix} 1 \\ 0 \\ 3 \\ -1 \end{bmatrix} \text{ and } \mathbf{w} = \begin{bmatrix} 1 \\ 0 \\ 1 \\ 1 \end{bmatrix}$$

6. Which of the angles (if any) of triangle $\triangle ABC$, with $A = (1, -2, 0)$, $B = (2, 1, -2)$ and $C = (6, -1, -3)$ is a right angle?

