

# SIT787: Mathematics for AI

## Practical Week 4

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1. Consider this matrix

$$A = \begin{bmatrix} 1 & 3 & -2 \\ 3 & 5 & 6 \\ 2 & 4 & 3 \end{bmatrix}$$

- are the columns independent?
- what is the column space of  $A$ ?
- what is the nullspace of  $A$ ?
- what is the rank of  $A$ ?
- is the matrix invertible? If so find its inverse.

2. Find the product of these two matrices using inner and outer products:

$$A = \begin{bmatrix} 1 & -4 & 0 \\ 0 & 2 & -1 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 0 \\ -1 & 1 \\ 0 & 1 \end{bmatrix}$$

3. For matrix

$$A = \begin{bmatrix} 1 & -1 \\ 0 & 2 \end{bmatrix}$$

- Find  $A^2 = AA$  and  $A^3 = AA^2$
- find  $f(A)$  for  $f(x) = x^2 - x + 2$

4. Solve these systems using Gauss Elimination:

$$\begin{cases} x + 3y - 2z = 5 \\ 3x + 5y + 6z = 7 \\ 2x + 4y + 3z = 8 \end{cases}$$

$$\begin{cases} x + z = 5 \\ 2x - y + 3z = 7 \\ 4x - 2y + 6z = 8 \end{cases}$$

5. Consider

$$A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 2 & 2 \\ 3 & 0 & 4 \end{bmatrix}$$

Find matrices  $L$  and  $U$  so that  $A = LU$

$$L = \begin{bmatrix} 1 & 0 & 0 \\ a & 1 & 0 \\ b & c & 1 \end{bmatrix} \text{ and } U = \begin{bmatrix} x & y & z \\ 0 & t & u \\ 0 & 0 & v \end{bmatrix}$$

6. Find the inverse of these matrices

$$A = \begin{bmatrix} 1 & 0 & 2 \\ 2 & -1 & 3 \\ 4 & 1 & 8 \end{bmatrix}, \text{ and } B = \begin{bmatrix} 1 & 0 & 1 \\ 2 & -1 & 3 \\ 4 & -2 & 6 \end{bmatrix}$$