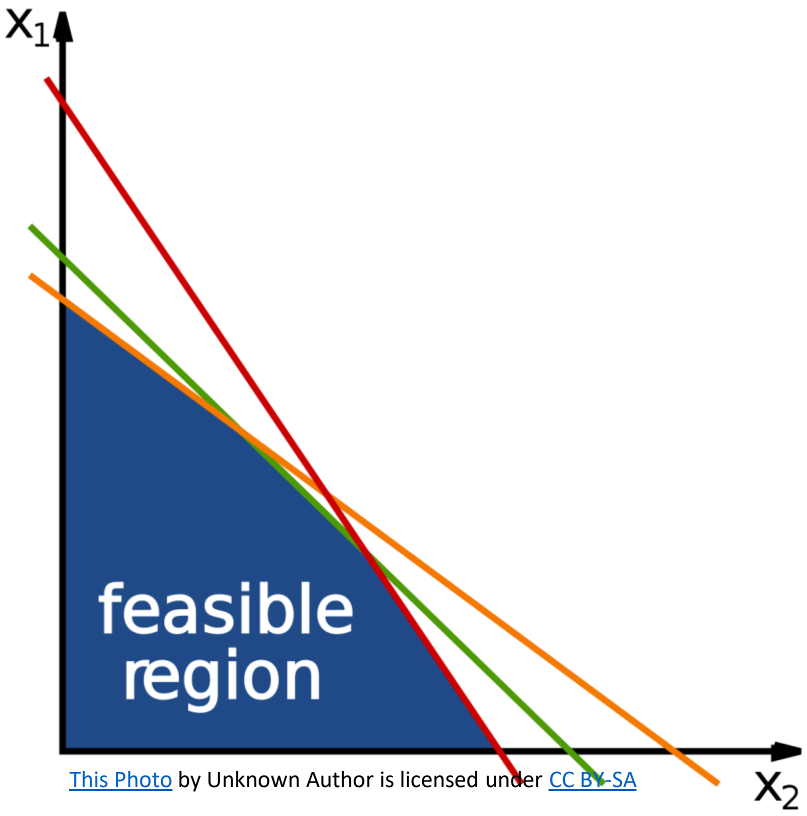

SIT718 – Introduction to Linear Programming- Week 1

Delaram Pahlevani

Introduction

- non - linear programming
- mixed integer programming
- quadratic programming
- stochastic modelling
- convex optimisation

Image Source: wikipedia

Linear Programming

Linear form:

$2x + 3y - 4z \;\geq 120$

$2x + y \leq 52$

$3y + 5z > 75$

Non-linear form

$x^2 + 7y \geq 24$

Image Source: wikipedia

Linear Programming

Linear form:

$2x + 3y - 4z \geq 120$

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Non-linear form

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Image Source: wikipedia

Elements of Linear Programming

decision variables

objective function

constraint satisfaction

Image Source: wikipedia

Toy Company

We are in charge of a toy company currently producing two types of toys: soldiers and trains:

Soldier	Train
sells for $27 and uses $10 worth of raw materials	sells for $21 and uses $9 worth of raw materials
costs $14 for labour per unit	costs $10 for labour per unit
requires 2 hours of finishing labour per unit	requires 1 hour of finishing labour per unit
requires 1 hour of carpentry labour per unit	requires 1 hour of carpentry labour per unit

Constraints
100 hours of available finishing labour every week
80 hours of available carpentry labour every week
previous data shows that at least 40 soldiers are bought each week

Image Source: wikipedia

Toy Company - Continued

	Finishing Labour	Carpentry Labour	Profit
Soldier	2	1	$27 – $10 – $14 = $3
Trains	1	1	$21 – $9 – $10 = $2
Labour Limits	100	80	

Objective function (profit) Z = Price – cost

(27s + 21t) – (10s-9t) –(14s+10t) = (27-10-14)s +(21-9-10)t = 3s+2t

Max z = 3s+2t

Image Source: wikipedia

Toy Company – Constraint Translation

Constraints	Equivalence mathematical equation
100 hours of available finishing labour every week	$2s+t \leq 100$
80 hours of available carpentry labour every week	$S+t \leq 80$
previous data shows that at least 40 soldiers are bought each week	$s \geq 40$
	$S,t \geq 0$

The solution to this LP is to build 40 soldiers and 20 trains, resulting in $160 total profit.

Assembly line

- An assembly line consists of three consecutive stations producing our toy soldiers and toy trains. The assembly times of the three workstations are listed below.

Workstation	Time to Produce Soldiers (minutes)	Time to Produce Trains (minutes)
1	3	6
2	5	5
3	4	8

Each station can operate up to 600 minutes per day, and each single toy produced must go through all three workstations.

The estimated daily maintenance times for Workstations 1, 2, 3 are 10%, 25% and 20% respectively.

What we want to do this time is to optimise the production plan i.e. find the numbers of each product to be produced such that the total idle time in the three workstations are minimised.

Assembly line

Workstation	Time to Produce Soldiers (minutes)	Time to Produce Trains (minutes)
1	3	6
2	5	5
3	4	8
	12	19

The maximum number of minutes available for each of the workstations are:

$$\begin{array}{ll} \text{Workstation 1} & 600 \times (1 - 10\%) = 540 \text{ minutes} \\ \text{Workstation 2} & 600 \times (1 - 25\%) = 450 \text{ minutes} \\ \text{Workstation 3} & 600 \times (1 - 20\%) = 480 \text{ minutes} \end{array}$$

Total = 540+450+180 = 1470 minutes

Objective function:

Min Z= 1470-12s – 19t

Assembly line

Workstation	Time to Produce Soldiers (minutes)	Time to Produce Trains (minutes)
1	3	6
2	5	5
3	4	8
	12	19

Objective function:

$$Z = 1470 - 12s - 19t$$

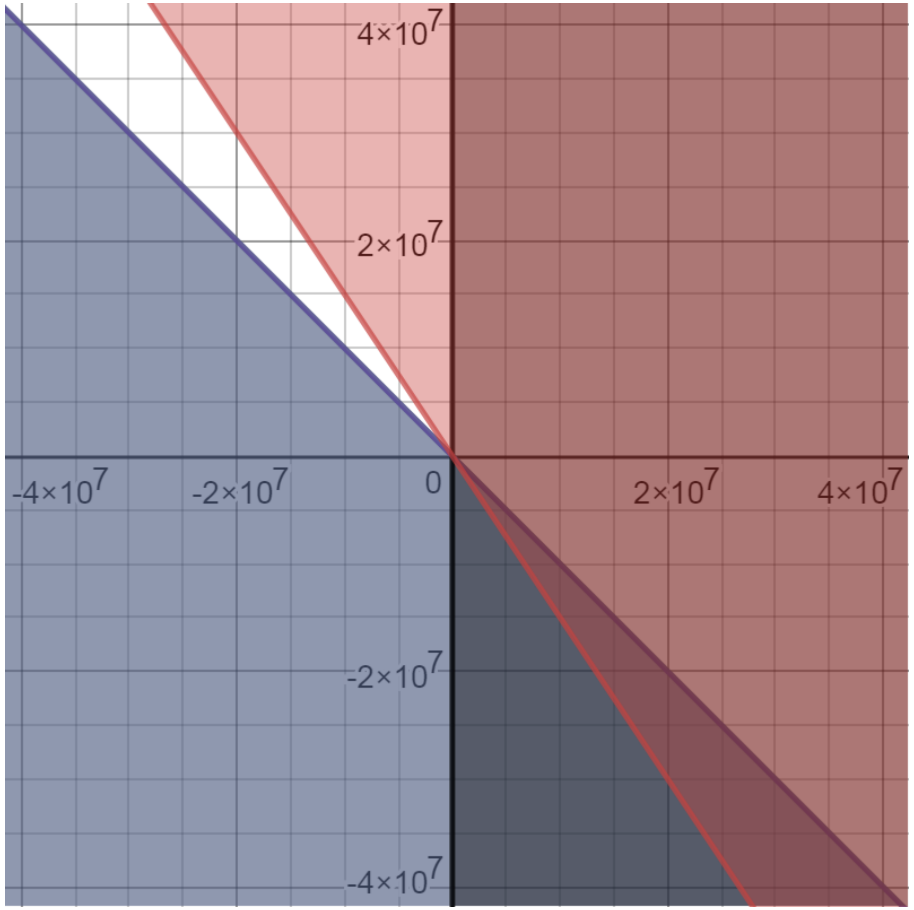
$$3s + 6t \leq 540$$
$$5s + 5t \leq 450$$
$$4s + 8t \leq 480$$
$$S, t \geq 0$$

Solving LPs with graphical Method

$$\text{Max } z = 3x + 2y = 3(40) + 2(20) = 160$$
$$2x + y \leq 100$$
$$x + y \leq 80$$
$$x \geq 40$$
$$x, y \geq 0$$

Follow these steps:

1. Draw each line only for constraints (using two points)
2. Find the area satisfies the equation
3. Find the feasible region



Sensitivity Analysis

Max $z = 3x + 2y$

$2x + 2y \leq 100$

$x + y \leq 80$

$x \geq 40$

$x, y \geq 0$

$$160 = 3x + 2y \implies y = -\frac{3}{2}x + 160$$

$$2x + y \leq 100 \implies y \leq -2x + 100$$

$$x + y \leq 80 \implies y \leq -x + 80$$

Z = cx + 2y: $y = -\frac{c}{2}x + z$

$-2 \leq -\frac{c}{2} \leq 0$

$0 \leq c \leq 4$