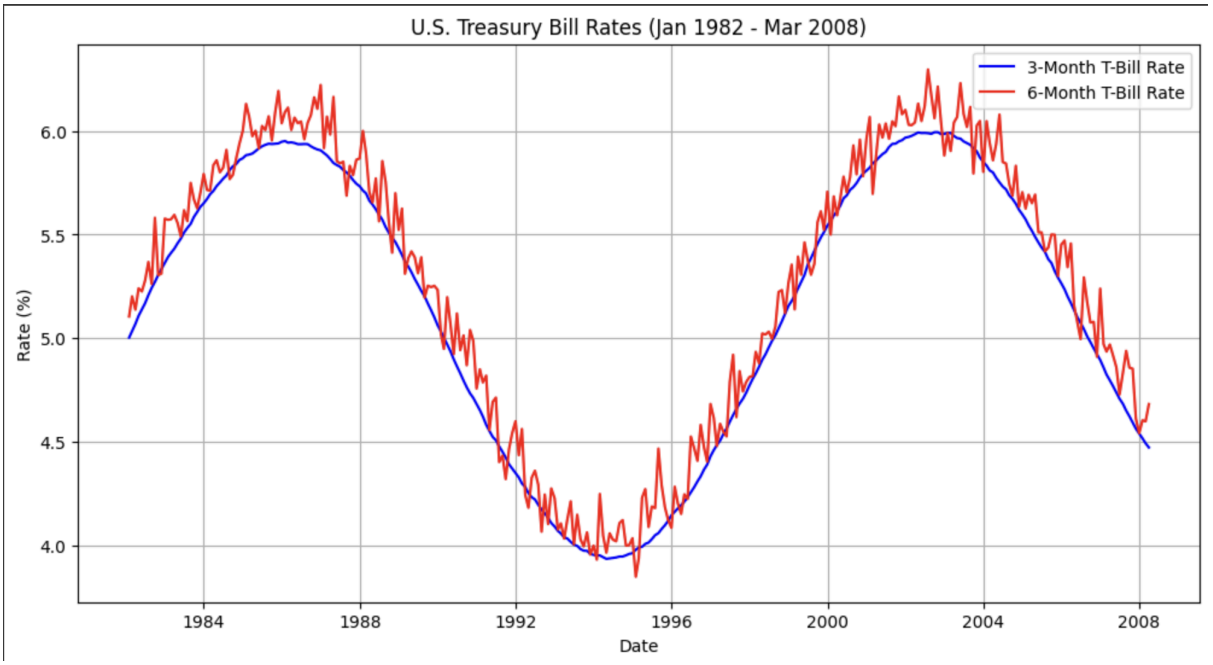
Link Collab:
https://colab.research.google.com/drive/1wru_JBJL8ETuq99wk11EbWYzdtqj85Gi?usp=sharing

Link Grok:
https://grok.com/share/c2hhcmQtMg%3D%3D_d7c56663-52c7-4e46-b8a3-9d3bc4fa9cee

Link Github:
https://github.com/aplancarte1/SERIES-DE-TIEMPO.git



Interpretations:
This analysis focuses on the relationship between the U.S. 3-month and 6-month Treasury Bill (T-Bill) rates from January 1982 to March 2008, comprising a total of 315 monthly observations. The objective is to explore their individual behavior over time, test for stationarity, examine potential cointegration, and determine the appropriate modeling strategy using time series econometrics.

To begin, both TB3M and TB6M rates were plotted on a single graph. The visual evidence shows that both series exhibit a highly similar cyclical pattern, with the 6-month rate generally staying slightly above the 3-month rate. Their movements appear tightly aligned over time, suggesting a strong comovement and the possibility of a long-run relationship between them. Peaks and troughs in both series tend to occur simultaneously, indicating responsiveness to common economic factors such as monetary policy changes and inflation expectations.

To formally assess stationarity, the Augmented Dickey-Fuller (ADF) test was applied to both series. The results confirmed that both TB3M and TB6M are stationary in levels. For the 3-month rate, the ADF statistic was -7.76 with a p-value of 0.0000, while for the 6-month rate, the statistic was -3.23 with a p-value of 0.0181. Both p-values are below the conventional 5% threshold, leading to the rejection of the null hypothesis of a unit root. This suggests that the statistical properties of both series—mean, variance, and autocovariance—remain constant over time, which is an important requirement for certain time series models.

Given the close alignment between the two series, a cointegration test was conducted using the Engle-Granger two-step approach. First, an ordinary least squares (OLS) regression was estimated with TB6M as the dependent variable and TB3M as the independent variable. The results showed an R-squared of 0.979 and a slope coefficient of approximately 1.002, indicating a nearly one-to-one relationship. In the second step, the ADF test was applied to the residuals of the regression. The residuals were found to be stationary with an ADF statistic of -16.53 and a p-value of 0.0000, providing strong evidence that the two series are cointegrated.

Economically, cointegration implies that the two interest rates, despite short-term fluctuations, are linked by a long-term equilibrium relationship. This is consistent with expectations in financial markets, where short-term instruments like 3- and 6-month T-bills are influenced by the same macroeconomic conditions and central bank policies. Cointegration suggests that deviations from equilibrium are temporary and will be corrected over time, reinforcing the idea of an efficient and interconnected short-term debt market. If the series had not been cointegrated, it would imply that the two instruments could drift apart indefinitely, which would contradict their structural similarities and shared market influences.

Finally, the question arises whether the two series should be modeled in levels or in first differences when constructing a Vector Autoregression (VAR) model with four lags. Since both series are already stationary in levels and cointegrated, there is no need for differencing. Using the series in levels preserves long-run information and allows the VAR model to capture both short-term dynamics and long-term relationships. In this context, estimating a VAR in levels is both theoretically justified and statistically appropriate.

In summary, this analysis demonstrates a robust relationship between the 3-month and 6-month T-bill rates, supported by statistical evidence of stationarity and cointegration. These findings validate the theoretical expectation that instruments with similar characteristics and maturities should move together over time, responding to the same economic forces.