INDIAN INSTITUTE OF TECHNOLOGY KHARAGPUR

Department of Chemical Engineering

End-semester (Autumn) Examination 2023-2024

Subject: Advanced Mathematical Techniques in Chemical Engineering (CH61015)

Remarks:

1. This question paper contains two parts: **Part A** and **Part B**. Attempt both parts.
2. Unless otherwise stated, usual mathematical notations apply.
3. Time = 3 h; maximum marks = 50; total number of printed pages = 2.

Part A: Linear algebra

1. Determine whether the following functions belong to the space of square integrable functions. Subsequently, normalise the appropriate ones.

(a) $f(x) = \frac{1}{\sqrt{x}}$ on [0,1]

(b) $g(x) = \frac{1}{x^{0.25}}$ on [0,1]

...5 marks

2. Consider the following initial value problem:

$$\frac{dx}{dt} = -2x - y$$

$$\frac{dy}{dt} = x - 2y$$

with the initial condition vector as $[1 \quad 0]^T$.

(a) Solve the above system of equations.

(b) Draw the $x - y$ phase portrait.

(c) Draw $x - t$ and $y - t$ solution projections.

...10 marks

3. Solve the following problem using Frobenius method.

$$\frac{d^2y}{dx^2} + y = 0$$

...10 marks

Part B: Differential equations

4. Find the steady state temperature distribution in a semi-circular plate of radius a insulated on both faces. The governing equation is

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2 T}{\partial \theta^2}=0$$

At $r=a$, $T=T_o$ for any θ. At $\theta=0$ and π, $T=0$. This means the temperature is maintained zero on boundary diameter. Use suitable physical boundary conditions.

...7 marks

5. Solve the following problem:

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u}{\partial r}\right)+\frac{\partial^2 u}{\partial z^2}=0$$

At $r=1, u=0$, at $z=0, u=0$, at $z=1, u=1$. Use suitable physical boundary conditions.

...8 marks

6. Solve completely using Green's function method:

$$\frac{\partial u}{\partial t}=\frac{\partial^2 u}{\partial x^2}+x \qquad (1)$$

At $t=0$, $u=1$. At $x=0$, $\dfrac{\partial u}{\partial x}=0$. At $x=1$, $\dfrac{\partial u}{\partial x}+u=0$.

...10 marks

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