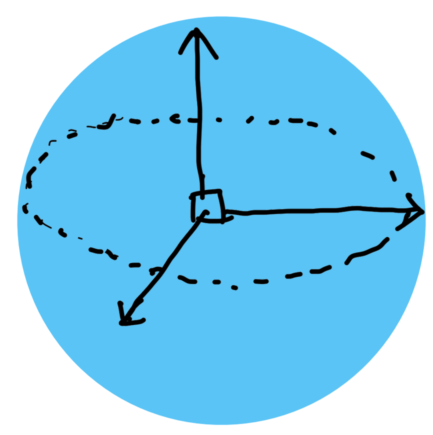
WEEK

Advanced Fluid Mechanics Week 3

Principal Stresses & finite deformation: Coordinate invariant

CI @ Tensor

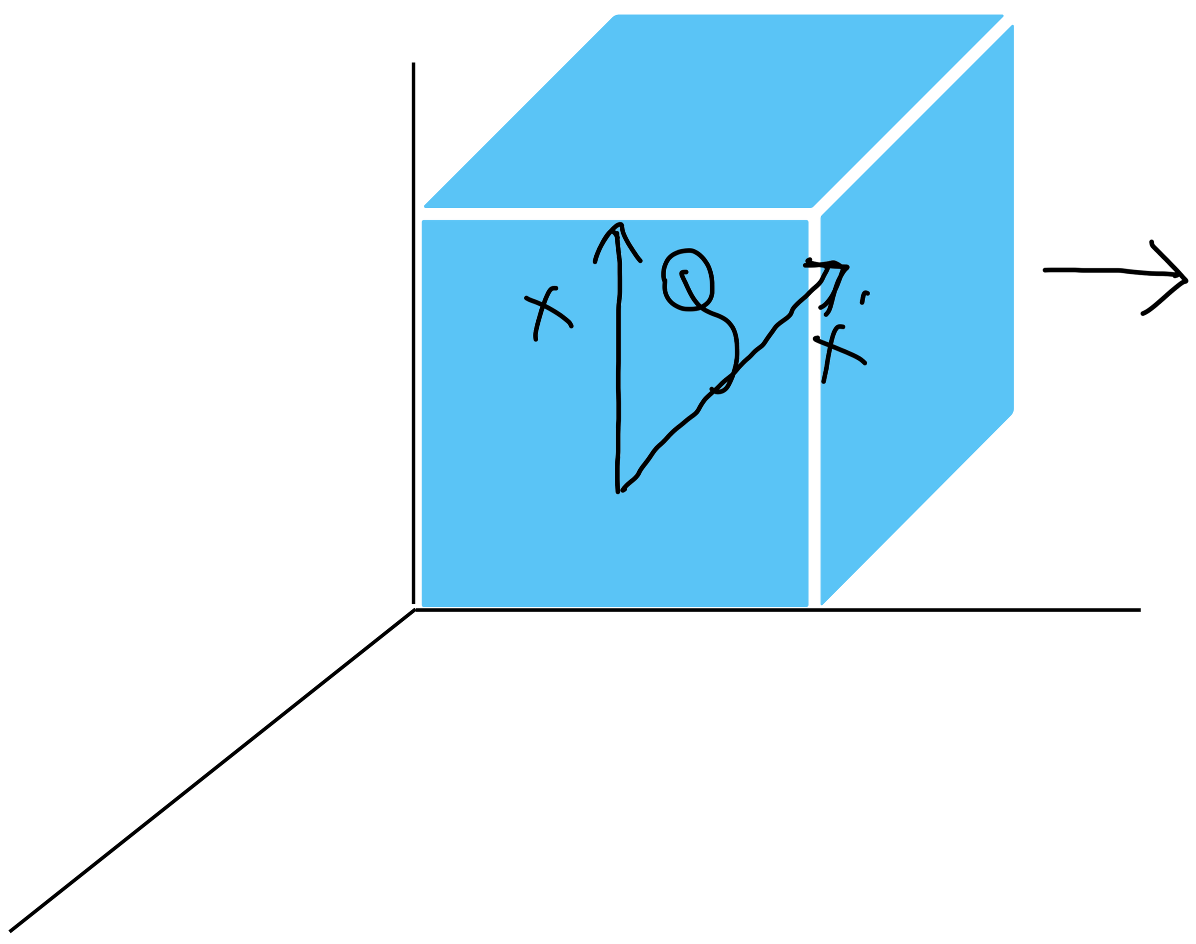


$$\begin{bmatrix} \tau_{xx} & 0 & 0 \\ 0 & \tau_{yy} & 0 \\ 0 & 0 & \tau_{zz} \end{bmatrix}$$

$\tau_{xx} = \tau_{yy} = \tau_{zz} = k$

Condition for hydrostatic pressure

Deformation → Strain



$\boxed{X = X'}$ Deformation Tensor

$X = F X$

Pushes X to X'

Nature of DT

$$\begin{bmatrix} \sigma_1 & 0 & 0 \\ 0 & \sigma_2 & 0 \\ 0 & 0 & \sigma_3 \end{bmatrix}$$

$\hat{n}\sigma$

$\hat{n}T$

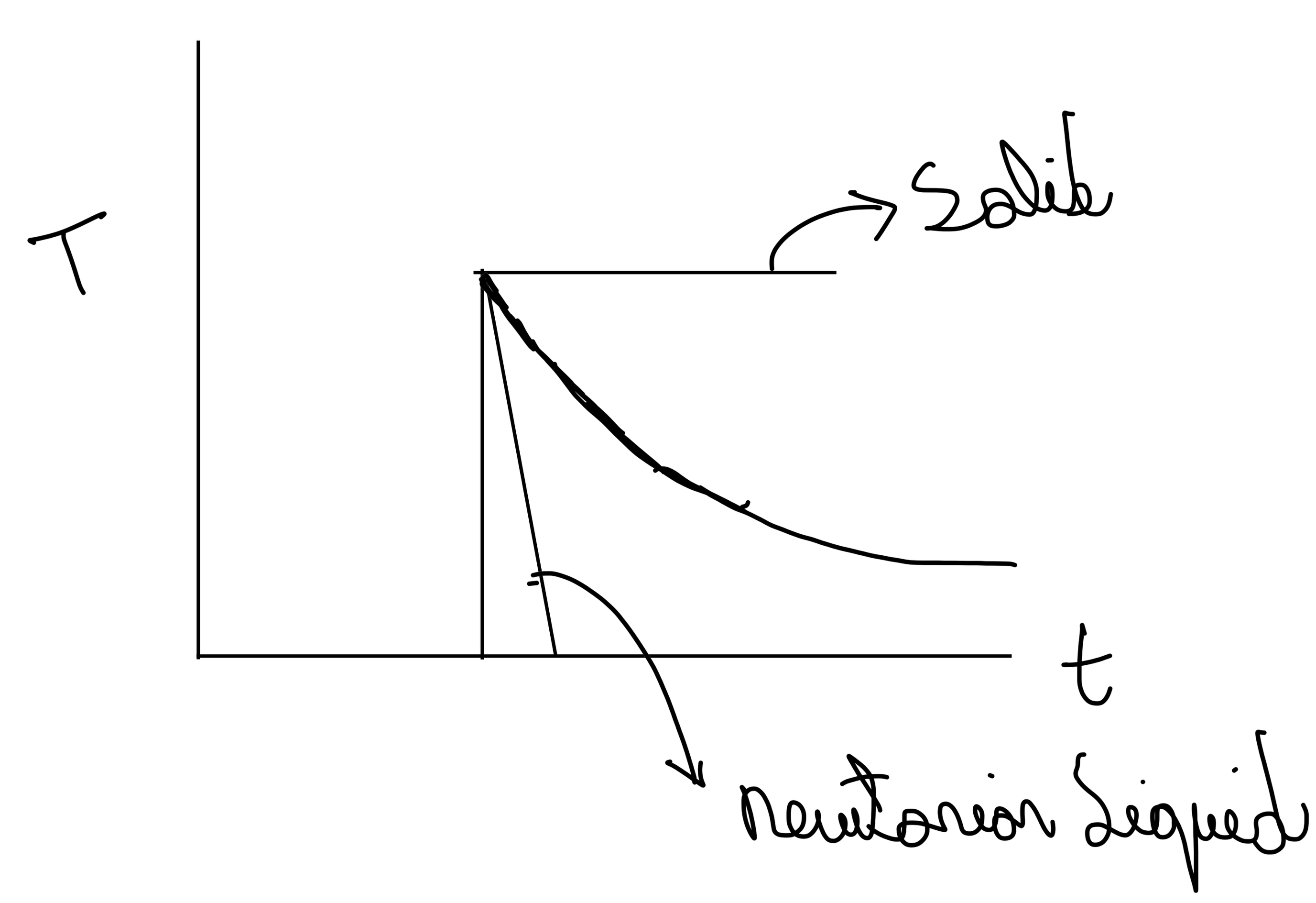
$\hat{n}[\sigma - TI] = 0$

$$\begin{bmatrix} T_{11}-\sigma & T_{12} & T_{13} \\ T_{21} & T_{22}-\sigma & T_{23} \\ T_{31} & T_{32} & T_{33}-\sigma \end{bmatrix}$$

$$\sigma^3 + (\ )\sigma^2 + (\ )\sigma = 0$$

3 coordinate invariant

Elastic Solid (Complex Fluid) Newtonian Fluid



VISCOELASTICITY

A viscoelastic material one which shows a combination of viscous and elastic effects.

Viscous term → energy dissipation

elastic term → energy storage

$$\boxed{\sigma = \eta \frac{d\varepsilon}{dt}}$$

σ → stress

ε → strain

η → viscosity

Ideal elastic material

$$\sigma = E\varepsilon$$

E → Young's modulus

MODELING VISCOELASTICITY

1. Generic models
2. Empirical model
3. Differential model
4. Integral "
5. ML "

$$T = \tau_y^{0.5} + \rho t^{1/2}$$

BLOOD

$$dT = dG\,\gamma$$

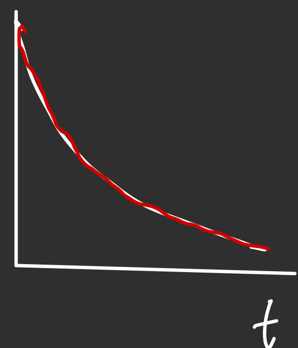
$$-\frac{dG}{dt} = M(t)$$

$$\int_0^{\tau} dT = dG\,\gamma$$

$$G_t = G_0 \exp(-t/\tau)$$

$$\frac{G_t}{G_0} = \exp(-t/\tau)$$

$\frac{G_t}{G_0}$



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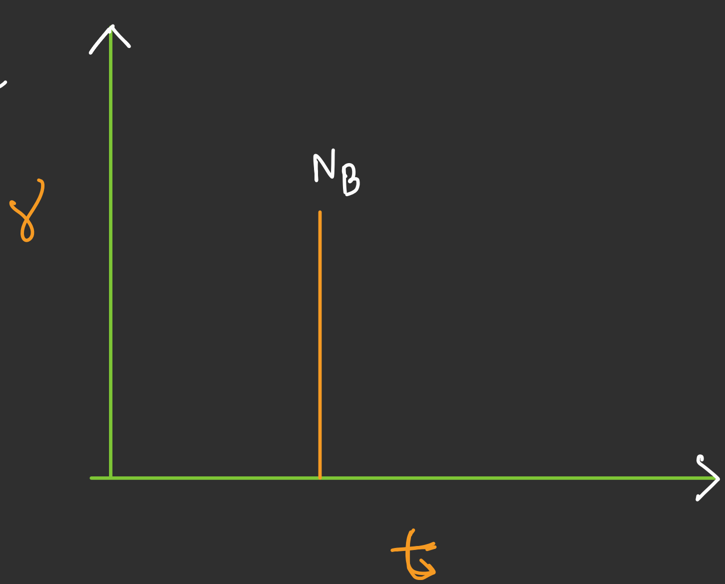
WEEK 5

MEASUREMENT METHODS

Transducer



Methods

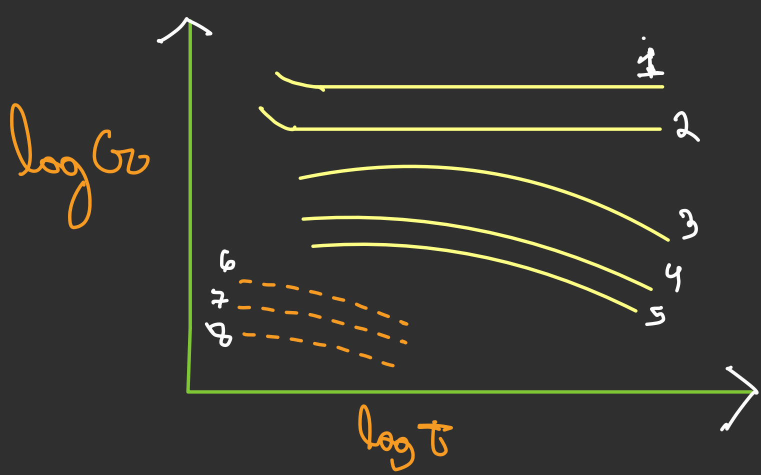
① Stress Relaxation

$$G(t) = \frac{\tau(t)}{\gamma_0}$$

Examples

Glass → 3 – 4

Memory foam → 5 – 6



CREEP

A constant load is applied and the resulting strain is measured

$\varepsilon_1 \rightarrow$ immediate elastic deformation

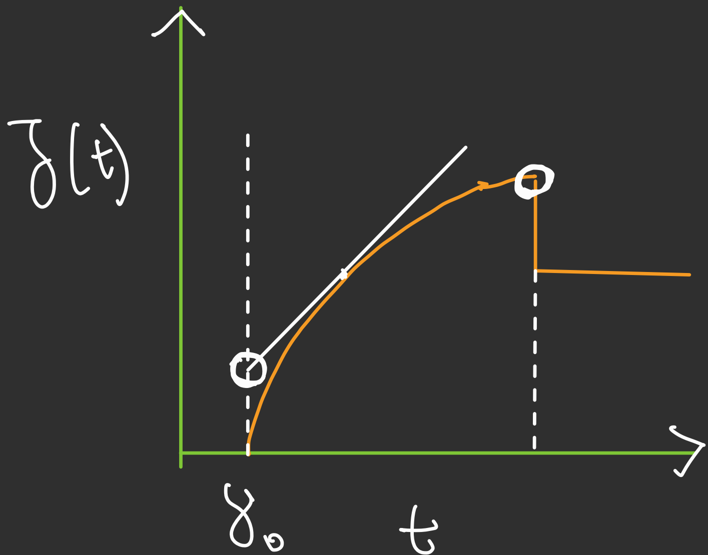
$\varepsilon_2 \rightarrow$ delayed elastic deformation

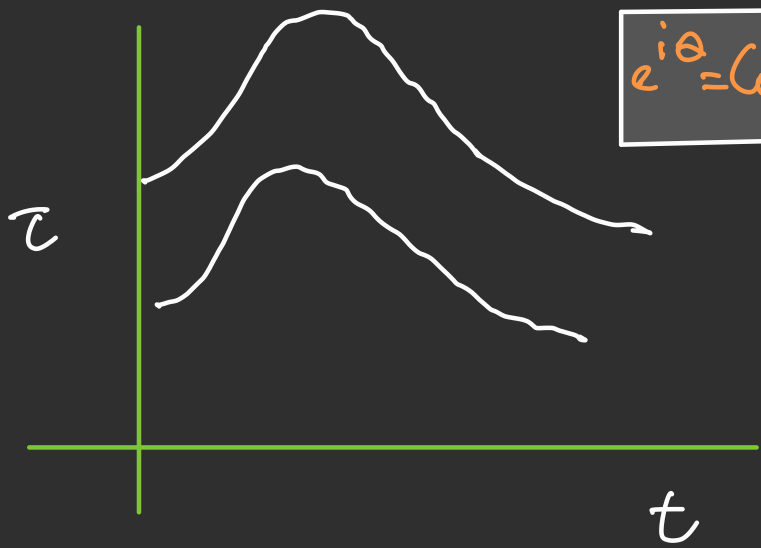
$\varepsilon_3 \rightarrow$ Newtonian flow (i.e permanent deformation)

Creep → strain a material and watch it for a long time

$$\boxed{J(t) = \frac{\gamma(t)}{\tau_0}}$$

Is $G(t) = \frac{1}{J(t)}$?





$$e^{i\theta} = \cos\theta + i\sin\theta$$

$$\gamma = \gamma_0 \sin(\omega t + \delta) \quad \tau = \tau_0 \sin\omega t$$

What is the real and imaginary part of viscoelasticity?

Real - viscous dissipation

imaginary - elastic part.

spectrum of viscoelasticity

Q $$G(s) = \int_0^\infty \frac{H(\lambda)}{\lambda} e^{-s/\lambda}\, d\lambda \qquad \eta' = \frac{\tau_0''}{\gamma_0}, \quad \eta'' = \frac{\tau_0'}{\gamma_0}$$

Prove,

$$\eta''(\omega) = \int_0^\infty G(s)\cos\omega\rho\, dt = \int_0^\infty \frac{H(\lambda)\, d\lambda}{1+(\lambda\omega)^2}$$

The relation to prove n' – real n'' – imaginary

$$n''(\omega) = \int_0^\infty G(s)\cos(\omega s)\,ds = \int_0^\infty \frac{H(\lambda)}{1+(\lambda\omega)^2}\,d\lambda$$

$$G(s) = \int_0^\infty \frac{H(\lambda)}{\lambda} e^{-s/\lambda}\,d\lambda$$

$$n''(\omega) = \int_0^\infty G(s)\cos(\omega s)\,ds$$

$$n''(\omega) = \int_0^\infty \left(\int_0^\infty \frac{H(\lambda)}{\lambda} e^{-s/\lambda}\,d\lambda \right) \cos(\omega s)\,ds$$

$$n''(\omega) = \int_0^\infty \frac{H(\lambda)}{\lambda} \left(\underbrace{\int_0^\infty e^{-s/\lambda} \cos(\omega s)\,ds}_{= \frac{\lambda}{1+(\lambda\omega)^2}} \right) d\lambda$$

$$n''(\omega) = \int_0^\infty \frac{H(\lambda)}{\cancel{\lambda}} \cdot \frac{\cancel{\lambda}}{1+(\lambda\omega)^2}\, d\lambda$$

$$\boxed{n''(\omega) = \int_0^\infty \frac{H(\lambda)}{1+(\lambda\omega)^2}\, d\lambda}$$

To prove $n''(\omega) = \int_0^\infty G(s) \cos(\omega s)\, ds$

Simple viscosity

$$\eta^*(\omega) = \frac{G^*(\omega)}{i\omega}$$

FT

$$G^*(\omega) = \int_0^\infty G(s)\, e^{-i\omega s}\, ds$$

$$G^*(\omega) = \int_0^\infty G(s)\left(\cos(\omega s) - i\sin(\omega s)\right)$$

$$\overbrace{G'(s)}^{=\eta'(s)} = \int_0^\infty G(s)\cos(\omega s)\,ds$$

$$G''(s) = \int_0^\infty G(s)\sin(\omega s)\,ds$$

$$\shortparallel$$

$$\eta''(s)$$

Ref for Linear Viscosity

1. JC Maxwell, Phil. Trans 1866, 156-249
2. JC Maxwell, Phil Trans 1867, 153 249
3. F.J. Lovett

NONLINEAR VISCOELASTICITY

Measurements / Instrumentation

Capillary Tube

$$\frac{\partial v}{\partial t} = -\nabla p \cdot \nabla T + \Sigma \rho_T \varepsilon_s$$

$$\boxed{Q = \frac{\pi \, \Delta P \, r^4}{8 \eta L}}$$

Hagen - Poiseuille Eq^n

$$v_z(r) = \frac{\Delta P}{4 \eta L}\left(R^2 - r^2\right)$$

How to calculate stress strain from capillary tube //

$$\rho\left(\frac{\partial v_r}{\partial t} + v_r \frac{\partial v_r}{\partial r} + v_z \frac{\partial v_r}{\partial z} - \frac{v_\theta^2}{r}\right) = -\frac{\partial p}{\partial r} + \eta\left(\frac{\partial^2 v_r}{\partial r^2} + \frac{1}{r}\frac{\partial v_r}{\partial r} + \frac{1}{r^2}\frac{\partial^2 v_r}{\partial \theta^2} + \frac{\partial^2 v_r}{\partial z^2} - \frac{v_r}{r^2}\right)$$

$v_r = 0,\ v_\theta = 0,\ \frac{\partial v_z}{\partial t} = 0$

$$0 = -\cancel{\frac{\partial p}{\partial r}} + \nu\left(\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial v_z}{\partial r}\right)\right)$$

$\to 0$

$$\frac{\partial}{\partial r}\left(r\frac{\partial v_z}{\partial r}\right) = 0$$

$$v_z(r) = \frac{\Delta P}{4\nu L}\left(R^2 - r^2\right)$$

$$Q = \int_A v_z(r)\, dA$$

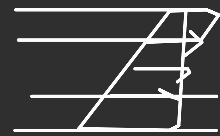
$$Q = \int_A v_z(r)\, 2\pi r\, dr$$

$$Q =$$

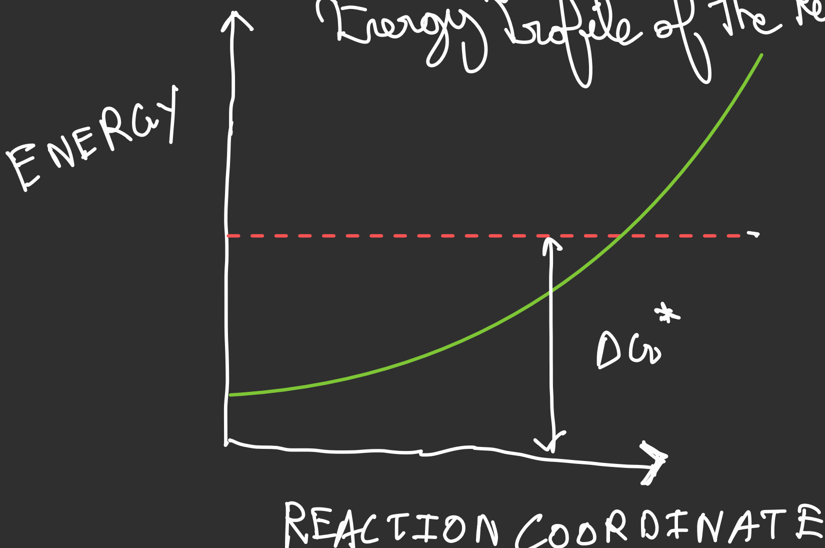
Non-Empirical Approach . Eyring.

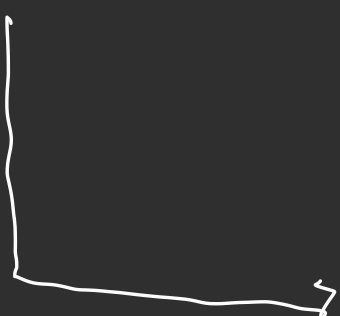
$$K = K'\underbrace{\left(\frac{K_B T}{h}\right)}_{\text{Bragg constant.}} \exp\left(-\Delta G^*/RT\right)$$

$$\tau = -\nu \frac{dv}{dy}$$



Energy Profile of the Relaxation Process





6.4.1: Eyring equation

The Eyring Equation, developed by Henry Eyring in 1935, is based on transition state theory and is used to describe the relationship between reaction rate and temperature. It is similar to the Arrhenius Equation, which also describes the temperature dependence of reaction rates. However, whereas Arrhenius Equation can be applied only to gas-phase kinetics, the Eyring Equation is useful in the study of gas, condensed, and mixed-phase reactions that have no relevance to the collision model.

Introduction

The Eyring Equation gives a more accurate calculation of rate constants and provides insight into how a reaction progresses at the molecular level. The Equation is given below:

$$k = \frac{k_B T}{h} e^{-\left(\frac{\triangle H^\ddagger}{RT}\right)} e^{\left(\frac{\triangle S^\ddagger}{R}\right)} \tag{6.4.1.1}$$

Consider a bimolecular reaction:

$$A + B \rightarrow C \tag{6.4.1.2}$$

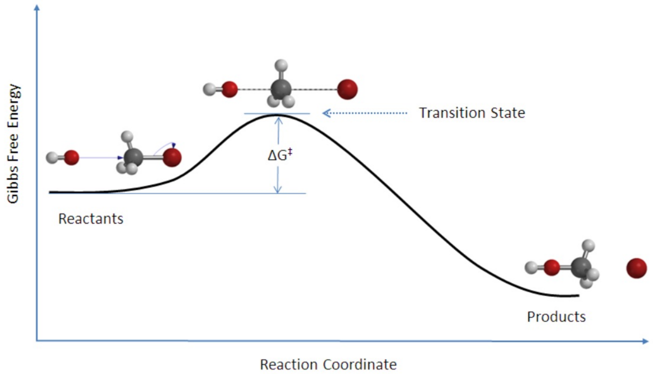
$$K = \frac{[C]}{[A][B]} \tag{6.4.1.3}$$

where K is the equilibrium constant. In the transition state model, the activated complex AB is formed:

$$A + B \rightleftharpoons AB^\ddagger \rightarrow C \tag{6.4.1.4}$$

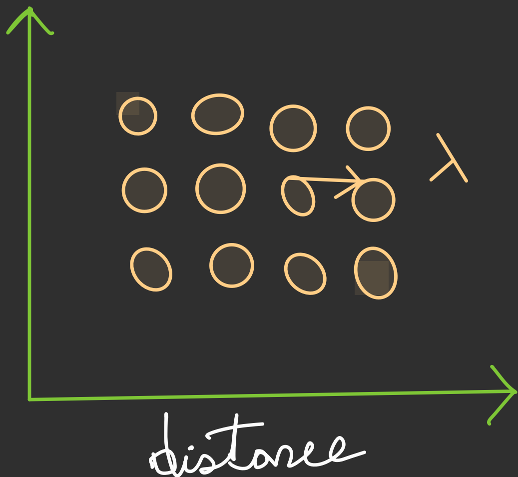
$$K^\ddagger = \frac{[AB]^\ddagger}{[A][B]} \tag{6.4.1.5}$$

There is an energy barrier, called activation energy, in the reaction pathway. A certain amount of energy is required for the reaction to occur. The transition state, $AB^\ddagger$, is formed at maximum energy. This high-energy complex represents an unstable intermediate. Once the energy barrier is overcome, the reaction is able to proceed and product formation occurs.

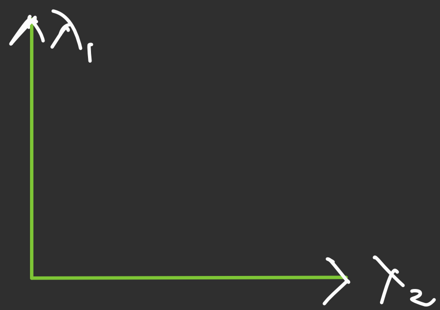


Reaction: $HO^- + CH_3Br \rightarrow [HO{-}{-}{-}CH_3{-}{-}{-}Br]^\ddagger \rightarrow CH_3OH + Br^-$

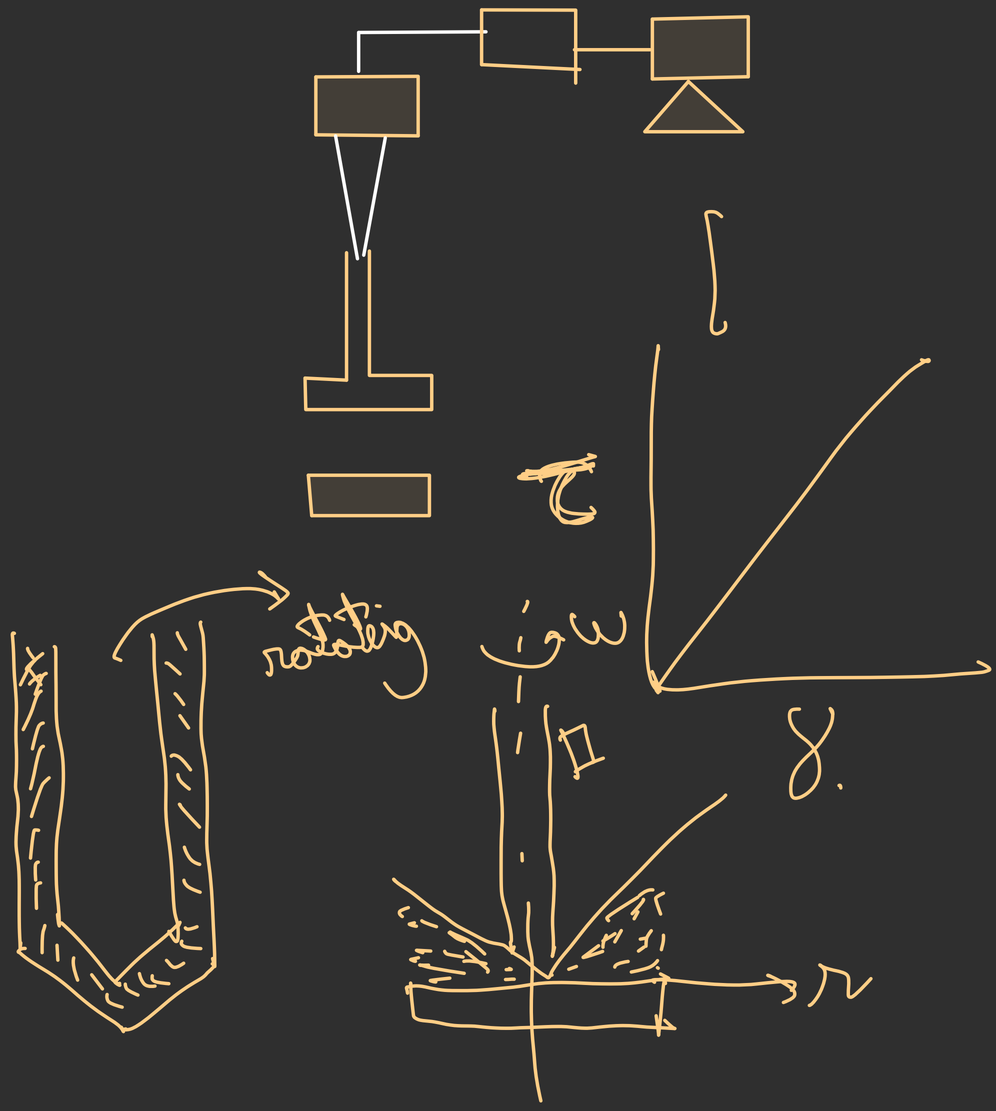
Visualization



$$U_\lambda = \lambda(k_r - k_f)$$



MODERN TECHNIQUES (RHEOMETRY)



Assumptions

1. Steady, laminar, Isothermal
2. $v_\phi(r,\theta)$ only $v_r = v_\theta = 0$
3. Cone angle $\beta < 0.1$ rad $(5-6)$
4. Spherical & liquid boundary

$$r\Big|\quad \frac{\rho v_\theta^2}{r} = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \tau_{rr}\right) - \frac{\tau_{\phi\phi} + \tau_{\theta\theta}}{r}$$

$$\theta\Big|\quad 0 = \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}\left(\tau_{\theta\theta}\sin\theta\right) - \frac{\cos\theta\,\tau_{\theta\theta}}{r}$$

$$\phi\Big|\quad 0 = \frac{1}{r}\frac{\partial}{\partial\theta}\tau_{\theta\phi} + \frac{1}{r}\cos\theta\,\tau_{\theta\phi}$$

FIND

RELEVANT SHEAR STRESS TO TORQUE