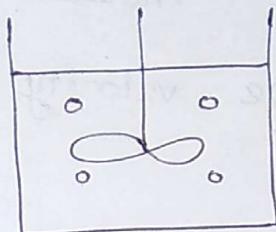
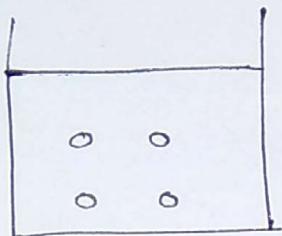


1) Why mass transfer occurs?

→ Principles, mechanisms, driving force

2) Separation process.

3) Transport process



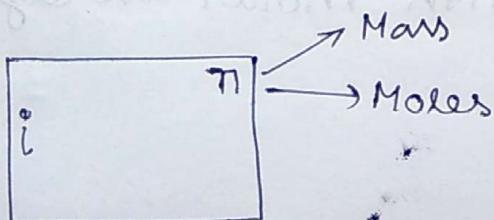
- Domain is discretized (Diffusion occurs in small areas)
- Localized diffusion/

Eddy diffusion

- molecular diffusion
- occurs in entire domain
- on imposing external velocity, molecular diffusion becomes Eddy diffusion

- path in between consecutive collisions is shorter compared to molecular diffusion
- convective mass transfer

In Isothermal, Isobaric condition, concentration difference is major driving force in diffusion



⇒  $\rho_i = \text{mass concentration of } i^{\text{th}} \text{ component} = \frac{\text{Mass}}{\text{vol.}}$

$c_i = \frac{\text{mole}}{\text{volume}} = \text{mole concentration of } i^{\text{th}} \text{ component}$

$w_i = \frac{\rho_i}{\sum \rho_i} = \text{mass fraction of } i^{\text{th}} \text{ component}$

$$x_i, y_i = \frac{c_i}{\sum c_i} = \text{mole fraction}$$

$$\sum w_i = 1, \quad \sum x_i, \sum y_i = 1$$

$$c_i \Leftrightarrow P_i, P, \quad y_i \Leftrightarrow P_i, P$$

$u_i$  = velocity of  $i^{\text{th}}$  molecule

$u$  = mass average velocity

$$= \frac{\sum p_i u_i}{\sum p_i}$$

$$U = \text{molar average velocity} = \frac{\sum c_i u_i}{\sum c_i}$$

$$\text{Flux} = \frac{\text{Rate}}{\text{area}}$$

↖      ↘  
Mass      Mole

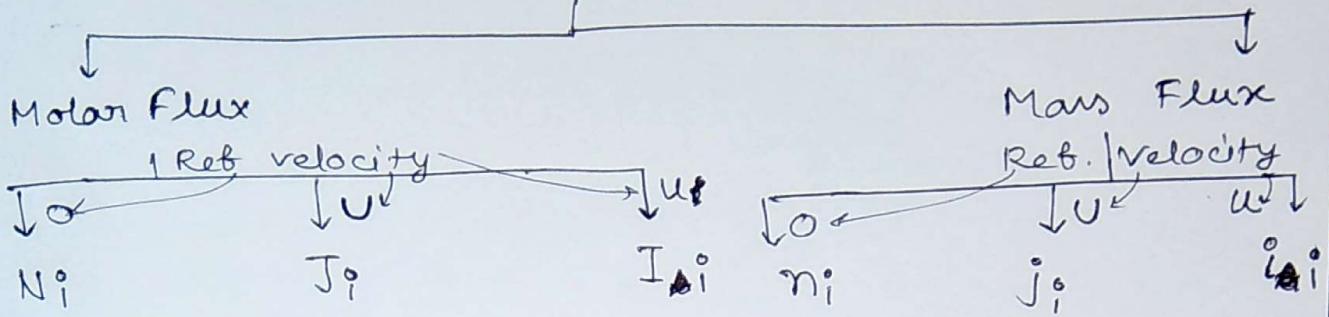
Reference  $\rightarrow$  stationary

$\rightarrow$  moving with mass average velocity  $u$

$\rightarrow$  moving with molar average velocity  $U$



Flux related  
to mole/mass



Total flux of  $i^{\text{th}}$  component

$$\begin{cases} N_i = c_i (u_i - 0) \\ J_i = c_i (u_i - U) \\ I_{\cancel{i}} = c_i (u_i - U) \end{cases} \quad \begin{cases} n_i = \rho_i (u_i - 0) \\ j_i = \rho_i (u_i - U) \\ i_i = \rho (u_i - U) \end{cases}$$

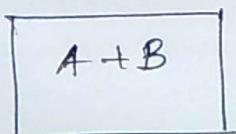
$$J_i = c_i (u_i - U) = c_i u_i - c_i \left( \frac{\sum c_i u_i}{\sum c_i} \right)$$

$$J_i = N_i - \left( \frac{c_i}{C} \left[ \sum N_i \right] \right)$$

→ convective flux

→ Total flux of all component

$J_i \Rightarrow$  diffusive flux



$$U = \frac{c_A u_A + c_B u_B}{c = (c_A + c_B)}$$



$$U = \frac{P_A u_A + P_B u_B}{P}$$

$$N_A = c_A u_A$$

$$J_A = N_A - \frac{c_A}{C} (N_A + N_B)$$

SN

1) Molecular Diffusion

2) Driving force: chemical potential

3) i)  $p_i, c_i, \omega_i, x, y$

ii)  $u_i^o, u, v$

iii)  ~~$N_i, J_i, I_i$~~

$(n_i, j_i, l_i)$

iv)  $J_i^o = N_i^o - \frac{C_i^o}{C} (\sum N_i^o)$

$$J_A^o = N_A - \frac{C_A}{C} (\sum N_A^o) = N_A - \frac{C_A}{C} (N_A + N_B)$$

Diffusion in homogeneous binary system

in single phase

Molecular Diffusion

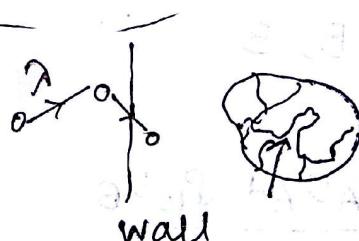
Pore/Nano  
channel  
diffusion

Eddy Diffusion  
(Convective mass transfer)

(in case of narrow channel, catalyst)

Surface Diffusion / Knudsen Diffusion

movement along  
surface



Molecular Diffusion : Single phase homogenous  
binary system

Fick's Law

flux  $\propto$  concentration gradient

$$J_A = -(D_{AB}) \frac{dc_A}{dx}$$

$$J_B = -(D_{BA}) \frac{dc_B}{dx}$$

$$D_{AB} = \frac{J_A}{\frac{dc_A}{dx}} = \frac{\text{mol/m}^2 \cdot \text{s}}{\frac{\text{mol/m}^3}{m}} = \frac{\text{m}^2}{\text{s}}$$

$$N_A = - \frac{dc_A}{dx} + \frac{c_A}{c_B} (N_A + N_B)$$

$$\cancel{N_A = - \frac{dC_A}{dx} + \frac{C_A}{c} (N_A + N_B)}$$

✓  $N_A = - D_{AB} \frac{dC_A}{dx} + \frac{C_A}{c} (N_A + N_B)$

24/07/2017

$$N_A = - D_{AB} \frac{dC_A}{dx} + \frac{C_A}{c} (N_A + N_B)$$

$$+ N_B = - D_{BA} \frac{dC_B}{dx} + \frac{C_B}{c} (N_A + N_B)$$

$$(N_A + N_B) = - (D_{AB} \frac{dC_A}{dx} + D_{BA} \frac{dC_B}{dx}) + \left( \frac{C_A + C_B}{c} \right) (N_A + N_B)$$

$$0 = D_{AB} \frac{dC_A}{dx} + D_{BA} \frac{dC_B}{dx}$$

Binary closed sol<sup>n</sup>  $C = C_A + C_B$

[const. conc.]

$$\cancel{\frac{dC}{dx}^0 = \frac{dC_A}{dx} + \frac{dC_B}{dx}}$$

$$\frac{dC_A}{dx} = - \frac{dC_B}{dx}$$

$$D_{AB} \frac{dC_A}{dx} = D_{BA} \frac{dC_A}{dx} \Rightarrow D_{AB} = D_{BA}$$

$$\mu_A = \mu_A^\circ + RT \ln a_A$$

$$= \mu_A^\circ + RT \ln (\gamma_A c_A)$$

$$\frac{d\mu_A}{dx} = 0 + \frac{RT}{x_A c_A} \cdot x_A \frac{dc_A}{dx} = \frac{RT}{c_A}$$

$$\frac{dc_A}{dx} = \frac{c_A}{RT} \frac{d\mu_A}{dx}$$

$$\frac{\partial b}{\partial x}(a=0) = \beta T$$

$$J_A = -D_{AB} \frac{dc_A}{dx} = -\frac{D_{AB} c_A}{RT} \frac{d\mu_A}{dx}$$

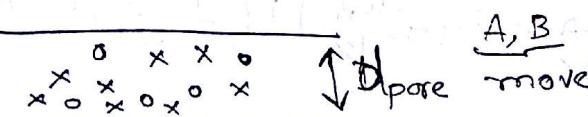
### Surface diffusion

$$J_A = -(\underline{D}_S) \frac{dc_A}{dx}$$

Diffusivity in surface



### Knudsen diffusion



$d_{pore} < \lambda \Leftarrow$  condition for Knudsen diffusion

mean free path

$$K_n = \text{Knudsen Number} = \frac{\lambda}{d_{pore}}$$

$K_n > 1 \Rightarrow$  Pore diffusion / Knudsen diffusion occurs

# Sometimes, it is hard to segregate molecular & porous diffusion.

$$J_A = - \underbrace{D_{KA}}_{\tau} \epsilon \frac{dc_A}{dx}$$

Knudsen diffusivity of A

porous volume = total volume  $\times$  porosity

$$\epsilon \rightarrow \underset{\text{correction factor}}{\overset{\text{porosity}}{\text{}}} \quad V_{\text{eff}} = \epsilon V_T \Rightarrow \epsilon = \frac{V_{\text{eff}}}{V_T}$$

$$\tau \rightarrow \text{tortuosity} = \frac{\text{tortuous length}}{\cancel{\text{actual length}}}$$

$$J_A = - \frac{D_{KA}}{\tau} \epsilon \frac{dc_A}{dx}$$

$\Rightarrow$  In presence of only Knudsen diffusion

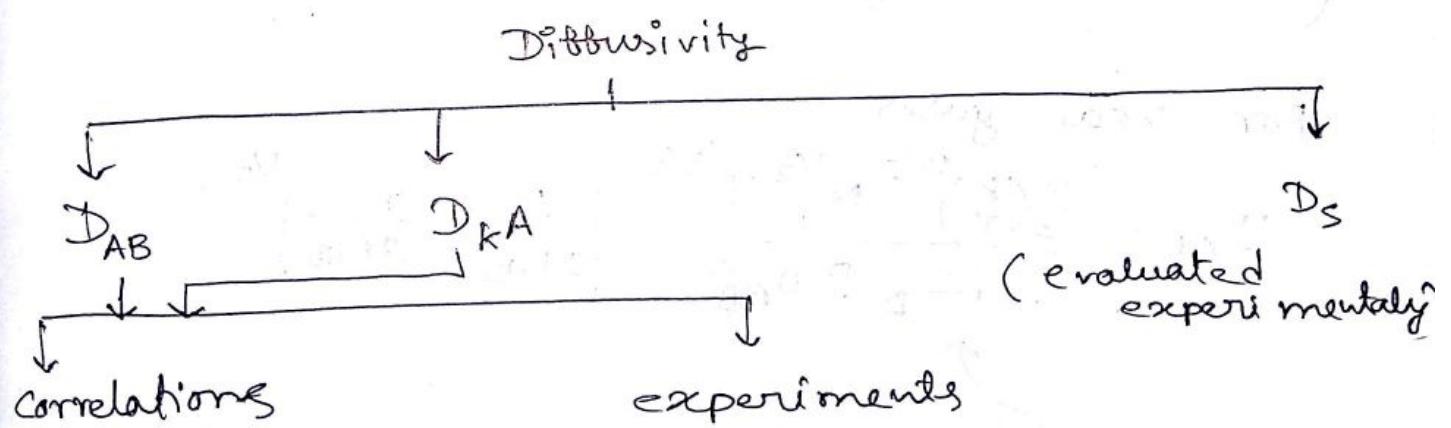
For both molecular & Knudsen diffusion are present then

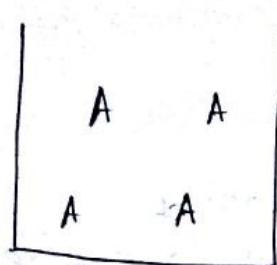
$$\frac{1}{D} = \frac{1}{D_{KA}} + \frac{1}{D_{AB}}$$

↑

Effective

26/07/17  
SN





gas phase

$$D_{AA} = \frac{1}{3} \lambda u$$

mean free path, m  
average molecular velocity

$$\lambda = \frac{kT}{\sqrt{2\pi\sigma_A^2 P}}$$

Boltzmann const.

$$u = \sqrt{\frac{8kNT}{\pi M_A}}$$

Diameter of the molecule

$$D_{AA} = \frac{2}{3} \left( \frac{k}{\pi} \right)^{3/2} N^{1/2} T^{3/2} \left[ \frac{1}{M_A} \right]^{1/2}$$

$$\frac{}{P \sigma_A^2}$$

$$D_{AB} = \frac{\frac{2}{3} \left( \frac{k}{\pi} \right)^{3/2} N^{1/2} T^{3/2}}{P \sigma_{AB}^2} \left[ \frac{1}{2M_A} + \frac{1}{2M_B} \right]^{1/2}$$

$$\frac{(\sigma_A + \sigma_B)}{2}$$

- i) Simplest of correlations
- ii) ideal binary mixture  
→ no intermolecular interaction
- iii) Non polar system

For real gases

$$D_{AB} = \frac{\frac{2}{3} \left( \frac{k}{\pi} \right)^{3/2} N^{1/2} T^{3/2}}{\sqrt{2} D P \sigma_{AB}^2} \left[ \frac{1}{2M_A} + \frac{1}{2M_B} \right]^{1/2}$$

↑  
Leonard Jones parameters

$$D = \frac{1.8 \times 10^{-7} T^{3/2}}{P \sigma_{AB}^2 - \rho_D} \left[ \frac{1}{2M_A} + \frac{1}{2M_B} \right]^{1/2}$$

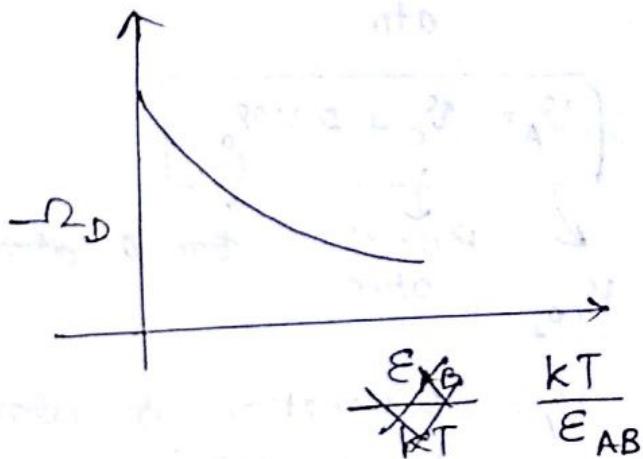
in  $\frac{m^2}{s}$

$$\rho_D = f\left(\frac{kT}{\epsilon_{AB}}\right)$$

$$\epsilon_{AB} = (\epsilon_A \epsilon_B)^{1/2}$$

Tabulated data	
$\frac{\epsilon_{AB}}{kT}$	$\rho_D$
—	—
—	—
—	—

or, graphical data



$\rho_D, \sigma_A, \sigma_B$

$$\rho_D = f\left(\frac{kT}{\epsilon_{AB}}\right)$$

$$\sigma = 1.18 V_b^{1/3}$$

↑ Molar volume at normal  
boiling point.

$$= 0.841 V_c^{1/3}$$

↓ critical volume

$$= 2.44 \left( \frac{T_c}{P_c} \right)^{1/3}$$

$$\epsilon_A/(k) = 0.77 T_C$$

Boltzmann const.

$$D_{AB} = \frac{10^{-3} \cdot T^{1.75} \left[ \frac{1}{M_A} + \frac{1}{M_B} \right]^{1/2}}{P \left[ (\sum v_A)_A^{1/3} + (\sum v_B)_B^{1/3} \right]^2}$$

$\downarrow$   
 $\frac{\text{cm}^2}{\text{s}}$

$\downarrow$   
atm

$$v_A = v_c + 2 \times v_o$$

$\downarrow$  for 'c' atom  
 $\downarrow$  for 'o' atom

$v_{CO_2}$

$v_A$  = summation of atomic volume of components

$$D_{AB} = \frac{10^{-3} T^{1.75} \left[ \frac{1}{M_A} + \frac{1}{M_B} \right]^{1/2}}{P [v_A^{1/3} + v_B^{1/3}]^2}$$

$\downarrow$

Depends on  $T$ ,  $P$ , nature of components

$$\frac{(D_{AB})_{T_1}}{(D_{AB})_{T_2}} = \frac{T_1^{3/2} P_2}{T_2^{3/2} P_1} \frac{(\rho_D)_2}{(\rho_D)_1}$$

from  
Hanschfelder  
eqn

## Diffusivity (Molecular)

27/07/17  
SN

→ Gas phase

→ liquid phase → Hydrodynamic Theory  
→ Eyring Theory

### Hydrodynamic Theory

$$\frac{D_{AB} \mu_B}{T} = \text{const.} \quad \left. \begin{array}{l} \text{Viscosity of sol}^n \\ \text{applicable for dilute solution} \end{array} \right\}$$

system temperature

### Eyring Theory

$$\frac{D_{AB} \mu_B}{T} = f(v_A) \quad \left. \begin{array}{l} \text{Molar} \\ \text{molecular volume of sol}^n \end{array} \right\}$$

Applicable for concentrated sol<sup>n</sup> also

Wilke - Chang  $\Rightarrow$  Majorly applicable for dil. sol<sup>n</sup>

Association factor

$$\left[ \frac{D_{AB} \mu_B}{T} \right]_{\text{cm}^2/\text{s}} = \frac{7.4 \times 10^{-8} (\phi_B M_B)^{1/2}}{V_A^{0.6}} \quad \text{in c.g.s.}$$

Solute viscosity for conc. sol<sup>n</sup>

$$= \frac{1.73 \times 10^{-16} (\phi_B M_B)^{1/2}}{V_A^{0.6}} \quad \text{in S.I.}$$

## Haydak - Laudie

$$D_{AB}^{\circ} = 13.26 \times 10^{-5} \bar{M}_B^{-1.14} V_A^{-0.589}$$

$$D_{BA}^{\circ} = 13.26 \times 10^{-5} \bar{M}_A^{-1.14} V_B^{-0.589}$$

$$D_{AB} \mu = (D_{AB}^{\circ} \bar{M}_B)^{X_B} (D_{BA}^{\circ} \bar{M}_A)^{X_A} \quad \left\{ \begin{array}{l} \text{for conc.} \\ \text{soft} \end{array} \right.$$

For more than 2 component system in gas phase

$$A, B, C, D, \dots \rightarrow y_A, y_B, y_C, y_D, \dots, y_n$$

$$D_{AB}, D_{AC}, D_{AD}, \dots$$

$$\frac{1}{D_{Am}} = \frac{y'_B}{D_{AB}} + \frac{y'_C}{D_{AC}} + \dots \quad \left\{ \begin{array}{l} y'_i = \frac{y_i^{\circ}}{\sum y_i^{\circ}} \\ i \neq A \end{array} \right.$$

## Knudsen Diffusivity

$$D_{AA} = \frac{1}{3} \cancel{\chi} u \quad \text{for ordinary gas phase}$$

⊕  $\cancel{\chi} = d_{\text{pore}}$  for Knudsen diffusion

$$D_{AA} = \frac{1}{3} d_{\text{pore}} \sqrt{\frac{8kNT}{\pi M_A}}$$

$$D_{KA} = 4850 d_{\text{pore}} \sqrt{\frac{T}{M_A}}$$

$$\frac{1}{D_{Ae}} = \frac{1 - \alpha \gamma_A}{D_{AB}} + \frac{1}{D_{KA}}$$

$$\left( \alpha = 1 + \frac{N_B}{N_A} \right)$$

effective diffusivity  
when both molecular  
and Knudsen  
diffusion are present

$\downarrow$   
 effective diffusivity  
 when both molecular  
 and Knudsen  
 diffusion are present

$$\left( \alpha = 1 + \frac{N_B}{N_A} \right)$$

31/07/2017

$D_{AB}|_{\text{gas}}$

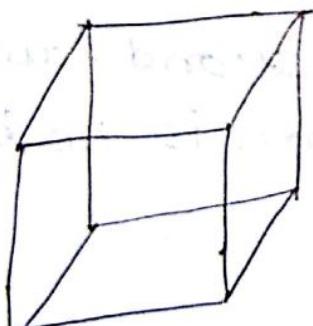
$D_{AB}|_{\text{liquid}} \rightarrow \text{dil}$

$\rightarrow \text{concentrated}$

$D_{KA}$

$D_{AE}$  or  $D_{\text{Effective}}$

$D_{AM} \rightarrow \text{for multi component mixture}$



Rate of  
 (moles in - moles out) + moles  
 generated

= Rate of mole accumulation

(0, 0, 0)

$(\sigma^x, \sigma^y, \sigma^z)$

$$[N_A|_x - N_A|_{x+\sigma^x}] \sigma^y \sigma^z + [N_A|_y - N_A|_{y+\sigma^y}] \sigma^x \sigma^z$$

$$+ [N_A|_z - N_A|_{z+\sigma^z}] \sigma^x \sigma^y + R_A \sigma^x \sigma^y \sigma^z = \frac{d(C\sigma^x \sigma^y \sigma^z)}{dt}$$

$$-\left(\frac{\partial N_{A_x}}{\partial x} + \frac{\partial N_{A_y}}{\partial y} + \frac{\partial N_{A_z}}{\partial z}\right) + R_A = -\frac{dc_A}{dt} = \frac{1}{M_A} \frac{dp^A}{dt}$$

for steady state, binary, non-reacting system  
and constant area

$$-\frac{dN_A}{dx} + R_A = \frac{dc_A}{dt}$$

$$\frac{dN_A}{dx} = 0$$

$$J_A = c_A(u_A - v)$$

for  $v=0$ , there is no bulk convective transfer

$$J_A = N_A - \left( \frac{c_A}{C} (N_A + N_B) \right)$$

$$\frac{dJ_A}{dx} = 0 \Rightarrow \frac{d}{dx} \left( -D_{AB} \frac{d(c_A)}{dx} \right) = 0$$

$$\Rightarrow \boxed{\frac{d^2 c_A}{dx^2} = 0}$$

Ficks second law  
when there is no bulk flow

Steady state

Binary

homogeneous single phase

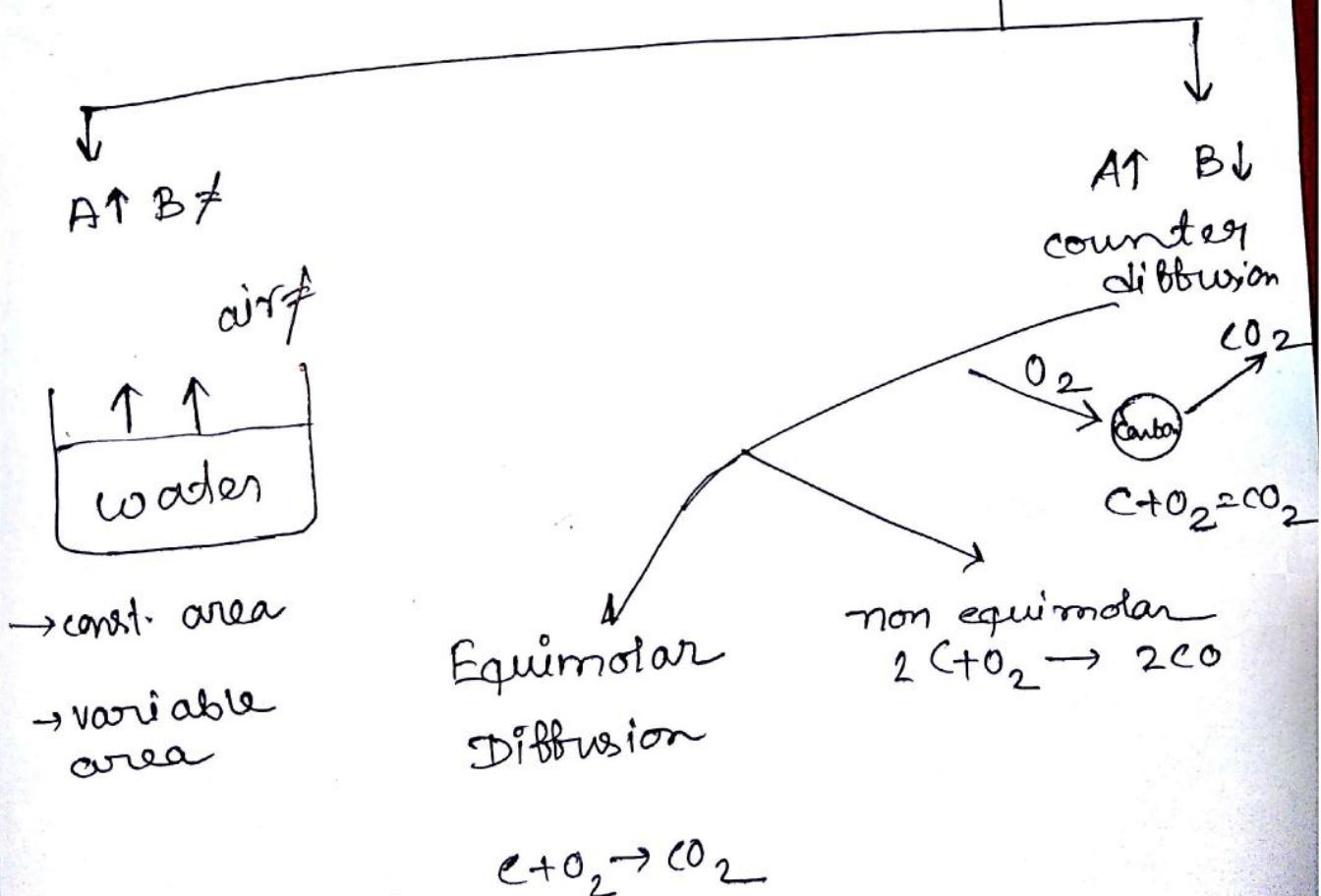
non reactive system

Molecular diffusion, 1 dimensional

① Fick's Law

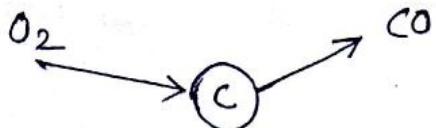
$$N_A = -D_{AB} \frac{dc_A}{dx} + \frac{c_A}{C} (N_A + N_B)$$

$$\frac{dN_A}{dx} = 0$$



02/08/17

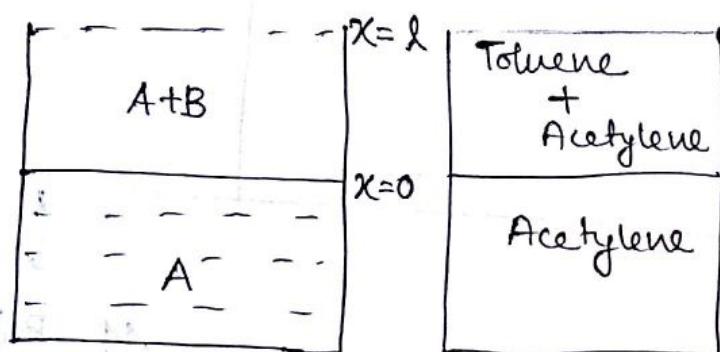
SN



$$N_A = -8 N_B$$

### Case 1

$$A \uparrow, B \neq$$



$$N_A = -D_{AB} \frac{dc_A}{dx} + \frac{c_A}{c} (N_A + N_B)$$

$$\frac{d(N_A)}{dx} = 0 \Rightarrow N_A \neq f(x)$$

$$\text{at } x=0 \quad c_A = c_{A_1}$$

$$x=1 \quad c_A = c_{A_2}$$

$$N_B = 0$$

$$N_A = -D_{AB} \frac{dc_A}{dx} + \frac{c_{A_1}}{c}$$

$$N_A \left( \frac{c_A}{c} - 1 \right) = D_{AB} \frac{dc_A}{dx}$$

$$\frac{N_A D_{AB} dx}{D_{AB} C} = \frac{dC_A}{C_A - C}$$

$$N_A \int_0^l dz = -D_{AB} \cdot C \int_{C_{A1}}^{C_{A2}} \frac{dC_A}{C - C_A}$$

$$N_A = \frac{D_{AB} \cdot C}{l} \ln \left( \frac{C - C_{A2}}{C - C_{A1}} \right)$$

$$= \frac{D_{AB} \cdot C}{l(C_B)_{em}} (C_{A1} - C_{A2})$$

Mass transfer coefficient

$$C = C_{B1} + C_{A1} = C_{B2} + C_{A2}$$

$$(C_B)_{em} = \frac{C_{B2} - C_{B1}}{\ln \left( \frac{C_{B2}}{C_{B1}} \right)}$$

Mass flux = Mass transfer coefficient  
x Δ (driving force)

in reality

$N_A \rightarrow \text{const}$

$l \rightarrow l \gg \Delta z$

$$N_A = \frac{D_A C}{(C - C_A)} \frac{dC_A}{dx} \quad \frac{d(N_A)}{dx} = 0$$

$$\frac{d}{dx} \left( \frac{D_A \cdot C}{(C - C_A)} \frac{dC_A}{dx} \right) = 0$$

$$C_{A1} = C_{A1}, \quad C_{A2} = C_{A2}$$

$$\frac{dC_A}{dx(C-C_A)} = C_1$$

$$\ln(C-C_A) = C_1 x + C_2$$

$$\frac{C-C_A}{C-C_{A1}} = \left[ \frac{C-C_{A2}}{C-C_{A1}} \right]^{x/l}$$

$$N_A = \frac{D_{AB} \cdot C}{l(Y_B)_{em}} \quad Y_A = \frac{C_{A1}}{C}$$

$$Y_B = \frac{C_{A2}}{C}$$

For gas

$$\textcircled{1} \quad N_A = \frac{D_{AB} C}{l} \left[ \frac{Y_{A1} - Y_{A2}}{(Y_B)_{em}} \right], \quad (Y_B)_{em} = \frac{Y_{B2} - Y_{B1}}{\ln(\frac{Y_{B2}}{Y_{B1}})}$$

$$\textcircled{2} \quad N_A = \frac{D_{AB} P}{RT l} \left[ \frac{P_{A1} - P_{A2}}{(P_B)_{em}} \right], \quad (P_B)_{em} = \frac{P_{B2} - P_{B1}}{\ln(\frac{P_{B2}}{P_{B1}})}$$

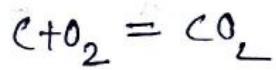
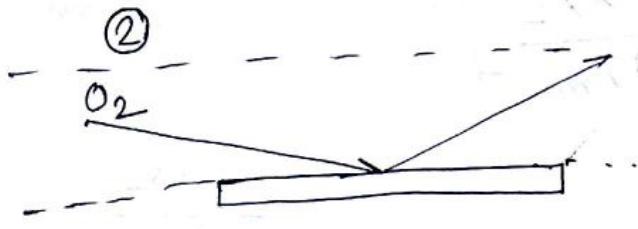
For liquid

$$N_A = \frac{D_{AB} C}{l} \left[ \frac{\chi_{A1} - \chi_{A2}}{(\chi_B)_{em}} \right], \quad (\chi_B)_{em} = \frac{\chi_{B2} - \chi_{B1}}{\ln(\frac{\chi_{B2}}{\chi_{B1}})}$$

$$C = \frac{P}{RT} = \frac{P}{(M_{\text{average}})}$$

For gas

For liquid



$$N_A = N_B$$

①

$$N_A = - D_{AB} \frac{dC_A}{dx} + \frac{C_A}{C} (N_A + N_B)$$

$$N_A = - D_{AB} \frac{dC_A}{dx}$$

$$\frac{d(N_A)}{dx} = 0 \Rightarrow N_A \neq f(x)$$

$$C_A = C_{A1}, \quad C_A = C_{A2}$$

$$N_A = - D_{AB} \frac{(C_{A1} - C_{A2})}{l}$$

$$\frac{d}{dx}(N_A) = 0$$

$$\frac{d}{dx} \left( - D_{AB} \frac{dC_A}{dx} \right) = 0$$

$$- D_{AB} \frac{dC_A}{dx} = C_1$$

$$\therefore C_A = - \frac{C_1}{D_{AB}} x + C_2$$

$$N_A = \frac{D_{AB} \cdot C}{l} \left[ \frac{x_{A_1} - x_{A_2}}{(x_B)_{\text{em}}} \right]$$

03/08/17  
SN

$$N_A = -s N_B$$

for  $s \neq 1$

### Variable Area

1.  $A \uparrow, B \neq$

a) spherical geometry

b) cylindrical geometry

2.  $A \downarrow, B \uparrow, s = 1$

a) Spherical geometry

b) Tapered tube

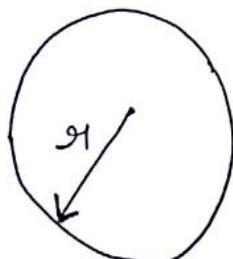
### Spherical Geometry ( $A \uparrow, B \neq$ )

at,

$$t = t, R = R$$

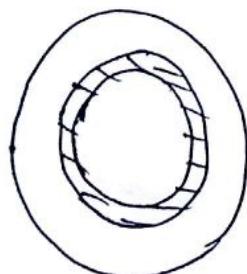
$$t = 0, R = R_{SO}$$

$$t = t_f, R = R_{sf}$$



$$t=0, r=r, C_A = C_{AS}$$

$$r=\infty, C_A = C_{A,\infty}$$



From shell balance

$$N_A \cdot A = \text{const.}$$

$$N_A \cdot 4\pi r^2 = \text{const.}$$

= w moles

$$N_A = - \frac{D_{AB} \cdot C}{(C - C_A)} \frac{d(C_A)}{d\tau_1}$$

$$N_B = 0$$

$C \equiv$  conc. in gas phase

$$\int N_A d\tau_1 = - D_{AB} \cdot C \int \frac{d(C_A)}{C - C_A}$$

~~$\frac{N_A \cdot 4\pi r^2}{4\pi R^2}$~~

$$\int_{\infty}^{r_1} \frac{W}{4\pi R^2} d\tau_1 = - D_{AB} \cdot C \int_{\infty}^{r_1} \frac{d(C_A)}{C - C_A}$$

~~$C_{A,\infty}$~~

~~$C_{A,S}$~~

$W = \text{area} \times \frac{\text{mole}}{\text{flux}}$

$$\left[ - \frac{W}{4\pi R^2} \right]_{\infty}^{r_1} = + D_{AB} \cdot C \left[ \ln \left( \frac{C - C_A}{C_{A,S}} \right) \right]_{C_{A,S}}^{C_{A,\infty}}$$

$$\frac{W}{4\pi R^2} = + D_{AB} \cdot C \ln \left[ \frac{C - C_{A,\infty}}{C - C_{A,S}} \right]$$

$$\frac{W}{4\pi D_{AB} C \cdot R^2} = \ln \left( \frac{C}{C - C_{A,S}} \right)$$

$$C - C_{A,S} = C \cdot e^{-4\pi D_{AB} C \cdot R^2}$$

$$C_{A,S} = C \left( 1 - e^{-4\pi D_{AB} C \cdot R^2} \right)$$

$$W = 4\pi D_{AB} C \left[ \ln \left( \frac{C - C_{A,\infty}}{C - C_{A,S}} \right) \right] \cdot R^2$$

Q: Q and S are two different  
processes between which  
there is no exchange

$$\frac{d}{dt} \left[ \frac{4}{3} \pi r^3 \left( \frac{P_A}{M_A} \right) \right] = -W$$

Gas phase conc.

$$= -4\pi D_{AB} C \cdot r \ln \left( \frac{C - C_{A,\infty}}{C - C_{A,S}} \right) = 4\pi \left( \frac{P_A}{M_A} \right)_n$$

$$4\pi \left( \frac{P_A}{M_A} \right) \int_{r_{so}}^{r_{sf}} dr = -4\pi D_{AB} \cdot C \ln \left( \frac{C - C_{A,\infty}}{C - C_{A,S}} \right) \int_{0}^{t_f} dt$$

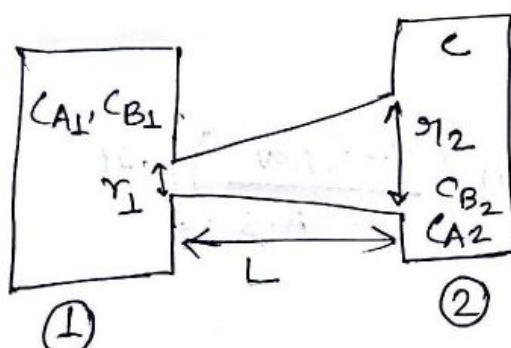
$$r_{so}^2 - r_{sf}^2 = 2D_{AB} \cdot C \left( \frac{M_A}{P_A} \right) \ln \left( \frac{C - C_{A,\infty}}{C - C_{A,S}} \right) \cdot t_f$$

for 2(a) counter  
Equimolar Diffusion

$$N_A = -D_{AB} \frac{dc_A}{dy}$$

$$\frac{r_p^2}{r_{so}^2} - \frac{r_{sf}^2}{r_{so}^2} = 2 DAB \left( \frac{U_A}{P} \right) (C_{A\infty} - C_{A,S}) t_f$$

## 2(b) Tapered pipe



$c_{A1} > c_{A2} \Rightarrow$  A diffuse from ① to ②  
- maintained at same  
temp. & pressure

$$N_A \pi r^2 = W$$

$$N_A = -D_{AB} \frac{dc_A}{dx}$$

Important

$$\frac{W}{\pi} \frac{dx}{r^2} = - D_{AB} dc_A$$

$$c_1 = c_{1L} + \frac{x}{L} (c_{12} - c_{11})$$

$$\frac{W}{\pi} \int_0^L \frac{dx}{[c_{1L} + \frac{x}{L} (c_{12} - c_{11})]^2} = - D_{AB} \int_{c_{1L}}^{c_{A2}} dc_A$$

$$W = \pi D_{AB} \frac{r_1 r_2}{C} (c_{A1} - c_{A2})$$

$$N_A \pi r^2 = W$$

$$N_A = -D_{AB} \frac{dc_A}{dx}$$

Important

$$\frac{W}{\pi} \frac{dx}{r^2} = -D_{AB} dc_A$$

$$\sigma_1 = \sigma_{1\perp} + \frac{x}{L} (\sigma_{12} - \sigma_{11})$$

$$\frac{W}{\pi} \int_0^L \frac{dx}{[\sigma_{1\perp} + \frac{x}{L} (\sigma_{12} - \sigma_{11})]^2} = D_{AB} \int_{c_{A1}}^{c_{A2}} dc_A$$

$$W = \pi D_{AB} \frac{r_1 r_2}{c} (\sigma_{12} - \sigma_{11})$$

07/08/17

### Experimental measurement

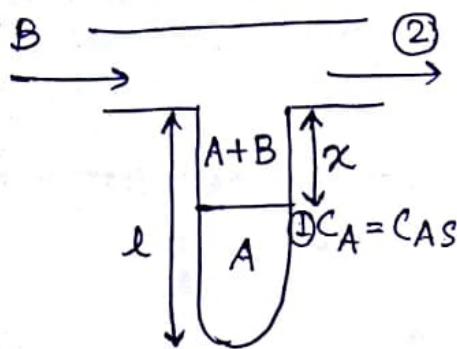
Gas

1) Stefan tube / Arnold cell

2) Twin bulb

1) Diaphragm cell

## Stefan tube



$B$  is not soluble in  $A$

+ Flow rate of  $B$  is large

$A \uparrow, B \neq$

$$N_B = 0$$

$$\begin{array}{ll} t=0, x=x_1 & C_{A1} = C_{AS} = C_{A1} \\ t=t_f, x=x_f & C_{A2} = 0 = C_{A2} \end{array}$$

$$N_A = \frac{D_{AB}C}{C_{B\text{em}}x} (C_{A1} - C_{A2})$$

$a$  = cross sectional area of vertical tube

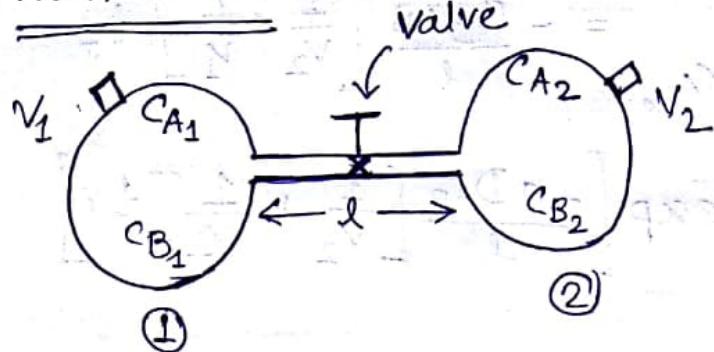
$$\frac{d}{dt} \left( a(l-x) \frac{P_A}{M_A} \right) = -N_A \cdot a$$

$$\int_{x_1}^{x_f} \frac{P_A}{M_A} dx = \frac{D_{AB}C}{C_{B\text{em}}} (C_{A1} - C_{A2}) \int_0^{t_f} dt$$

~~$$\frac{P_A}{2M_A} (x_f^2 - x_1^2) = \frac{D_{AB}C}{C_{B\text{em}}} (C_{A1} - C_{A2}) t_f$$~~

$$\frac{P_A}{2M_A} (x_f^2 - x_1^2) = \frac{D_{AB}C (C_{A1} - C_{A2}) t_f}{C_{B\text{em}}}$$

## Twin Bulb



$t, C_{A_1}, C_{A_2}$

$$t=0, C_{A_1} = C_{A_10}, \boxed{C_{A_2} = C_{A_20}}$$

$$t=t_f, C_{A_1} = C_{A_{f1}}, C_{A_2} = C_{A_{f2}}$$

$$N_A = \frac{D_{AB}}{l} (C_{A_1} - C_{A_2}) = -N_B$$

$$\frac{d}{dt}(V_1 C_{A_1}) = -N_A \cdot a$$

$$\frac{dC_{A_1}}{dt} = -\frac{a}{V_1} \frac{D_{AB}}{l} (C_{A_1} - C_{A_2}) \quad \dots \textcircled{1}$$

$$\frac{d}{dt}(V_2 C_{A_2}) = N_A \cdot a ; [N_A = -N_B]$$

$$\frac{dC_{A_2}}{dt} = \frac{a}{V_2} \frac{D_{AB}}{l} (C_{A_1} - C_{A_2}) \quad \dots \textcircled{2}$$

$$\frac{dC_{A_1}}{dt} + \frac{dC_{A_2}}{dt} = -\frac{a D_{AB}}{l} (C_{A_1} - C_{A_2}) \left[ \frac{1}{V_2} + \frac{1}{V_1} \right]$$

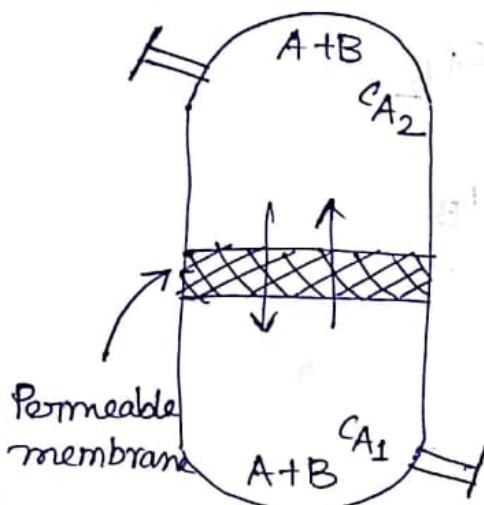
$$\int \frac{d(C_{A_1} - C_{A_2})}{d(C_{A_1} - C_{A_2})} = -\frac{a D_{AB}}{l} \left[ \frac{1}{V_2} + \frac{1}{V_1} \right] dt$$

Equimolar counter diffusion as both of the bulbs are maintained at same pressure

$$\left[ \ln \left( \frac{C_{A_1} - C_{A_2}}{C_{A_{10}} - C_{A_{20}}} \right) \right]_{C_{A_{10}}, C_{A_{20}}}^{C_{A_1}, C_{A_2}} = - \frac{\alpha D_{AB}}{l} \left[ \frac{1}{V_2} + \frac{1}{V_1} \right] t_f$$

$$\frac{C_{A_1} - C_{A_2}}{C_{A_{10}} - C_{A_{20}}} = \exp \left[ - \frac{\alpha D_{AB}}{l} \left[ \frac{1}{V_2} + \frac{1}{V_1} \right] t_f \right]$$

### Diaphragm Cell



$$t=0, C_{A_1} = C_{A_{10}}, C_{A_2} = C_{A_{20}}$$

A↑ B↓

$$t=t_f, C_{A_1}, C_{A_2}$$

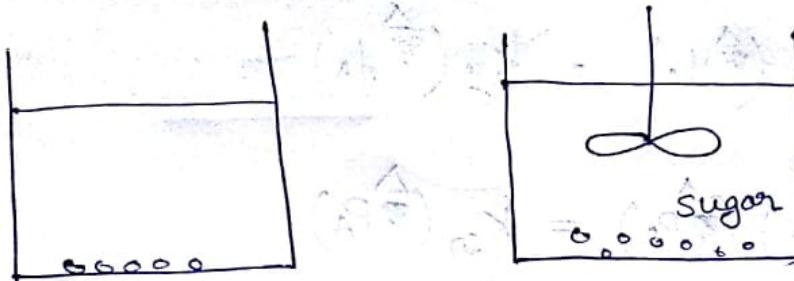
$$\int \frac{d(C_{A_1} - C_{A_2})}{(C_{A_1} - C_{A_2})} = - \frac{D_{AB} \alpha \epsilon}{l \tau} \left[ \frac{1}{V_2} + \frac{1}{V_1} \right] t_f$$

$$C_{A_{10}} - C_{A_{20}} = - D_{AB} K_{cell} t_f$$

$$K_{cell} = \frac{D_{AB} \alpha \epsilon}{l \tau} \left[ \frac{1}{V_2} + \frac{1}{V_1} \right]$$

~~09/08/17~~

09/08/17



Convective mass transfer

Sugar

i) Molecular diffusion

ii) Fick's Law

i) Eddy diffusion

ii) Fick's law is not applicable

iii) Additional effect of flow/velocity needs to be accounted

Mass flux  $\propto \Delta \nabla$  (driving force)

Mass flux = (constant) ( $\nabla \Delta$  (driving force))  
M.T.C

$$N_A = k_c (\nabla c_A) \\ = k_y (\nabla y_A) \\ = k_o (\nabla P_A)$$

gases

$$N_A = k_L (\nabla c_A) \\ = k_x (\nabla x_A)$$

liquid

$$N_A = K_c (\frac{\Delta}{\Delta} C) = \frac{K_c \cdot C \Delta \gamma_A}{C}$$

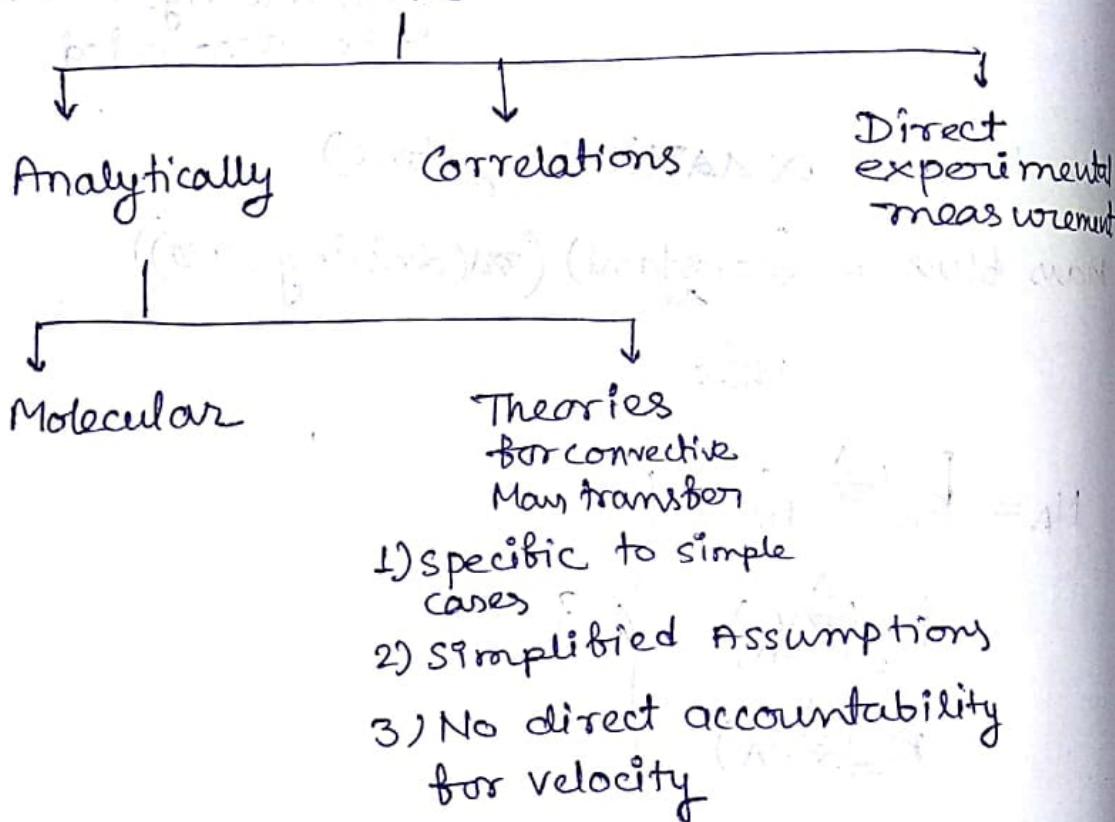
$$= (k_c c) (\Delta \gamma_A) = k_y (\Delta \gamma_A)$$

$$= \left( \frac{k_c}{RT} \right) (\Delta P_A) = k_{ca} (\Delta P_A)$$

$\frac{1}{MTC}$  = Mass transfer resistance

↳ Transfer resistances

↳ Shows all about MTC



### Theories

- i) Filum theory
- ii) Penetration theory
- iii) Surface Renewal theory
- iv) Boundary layer theory

## Gas phase

situation	$k_c$	$k_y$	$k_g$
1) Vertical tube	$k_c = k_g$	$k_y = k_g$	$k_g = k_g$
2) Horizontal tube	$k_c = k_g$	$k_y = k_g$	$k_g = k_g$
3) Vertical tray	$k_c = k_g$	$k_y = k_g$	$k_g = k_g$
4) Horizontal tray	$k_c = k_g$	$k_y = k_g$	$k_g = k_g$

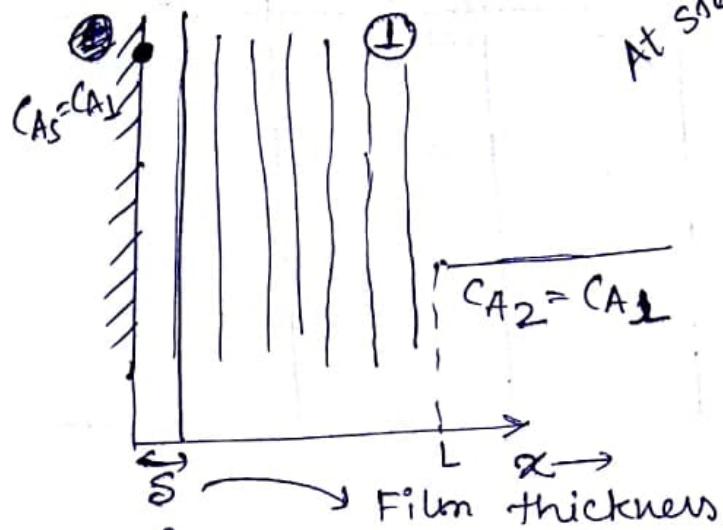
## Liquid phase

## Variable Geometry

	situation	$k_{LA}$	$k_{yA}$	$k_{gA}$
1)	Cylindrical	$\propto \frac{1}{R}$	$\propto \frac{1}{R}$	$\propto \frac{1}{R}$
2)	Spherical	$\propto \frac{1}{R^2}$	$\propto \frac{1}{R^2}$	$\propto \frac{1}{R^2}$

In case of variable geometry  $k$  changes with area.

## Film Theory



At steady state

$$\frac{d^2 C_A}{dx^2} = 0$$

$$C_{A1}|_{x=0} = C_{A1} = C_{AS}$$

$$C_{A1}|_{x=\delta} = C_{A2} = C_{Ae}$$

All the mass transfer takes place within this film thickness

$$\frac{dC_A}{dx} = k_L$$

$$dC_A = k_L x + K_2$$

$$C_{A1} = 0 + K_2 = C_{AS}$$

Neglect bulk flow

$$(N_A + N_B) = 0$$

$$C_{Ae} = k_L l + C_{AS}$$

$$k_L = \frac{C_{A,e} - C_{A,S}}{l}$$

$$C_A = \left( \frac{C_{A,e} - C_{A,S}}{\delta} \right) x + C_{A,S}$$

$$\frac{dC_A}{dx} = \frac{(C_{A,e} - C_{A,S})}{\delta}$$

$$N_A = -\frac{D_{AB}}{8} (C_{A,2} - C_{A,1}) \quad \text{where } \frac{dC}{dt} = \frac{D_{AB}}{8} \frac{(C_{A,2} - C_{A,1})}{L}$$

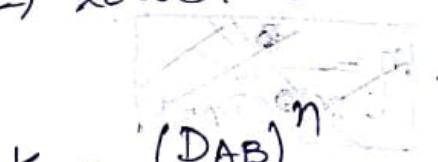
$\downarrow$   
MTC

$\delta \equiv$  hypothetical parameter

higher falling rate

$\Rightarrow$  higher M.T rate

$\Rightarrow$  lower  $\delta$

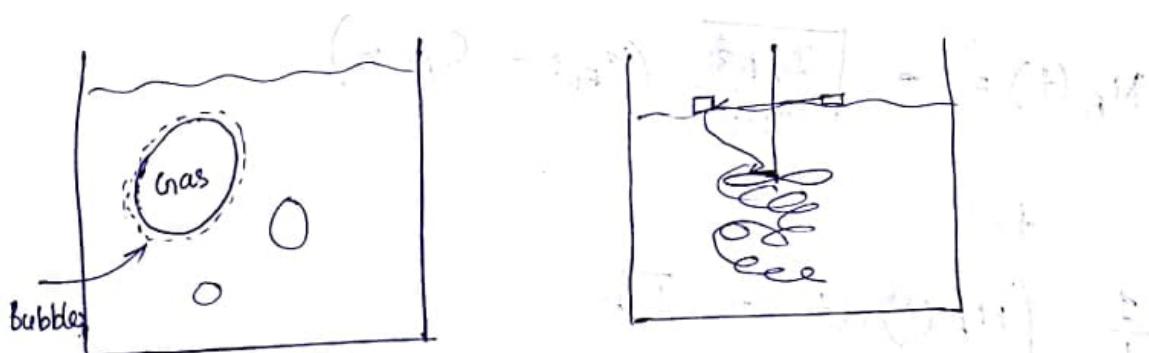


$$K_C = (D_{AB})^n$$

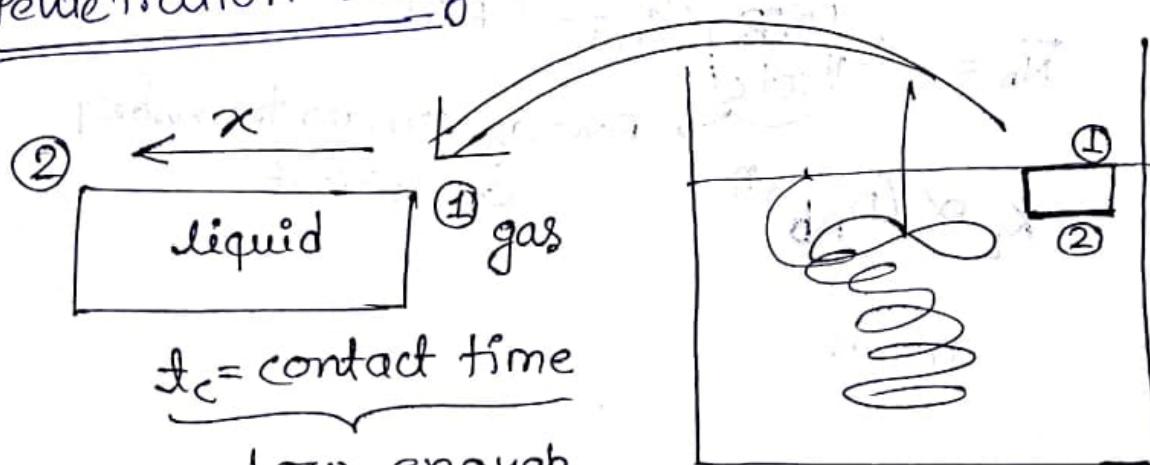
$$n < 1$$

$$\text{M.T. rate} = \frac{(A_2 - A_1)}{(A_2 - A_1)}$$

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### Penetration Theory



$$t_c = \text{contact time}$$

Low enough

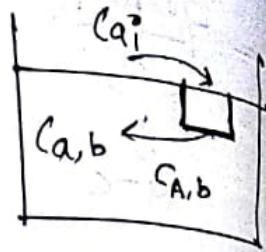
so that diffusing element component never reaches other end of the element

$$-D_{AB} \frac{d^2 C_A}{dx^2} = \frac{\delta C_A}{\delta t}$$

$$t=0, \quad C_A(x) = C_{A,b}$$

$$t > 0, \quad C_A(0) = C_{A,i}$$

$$C_A(\infty) = C_{A,b}$$



$$\frac{C_A - C_{A,b}}{C_{A,i} - C_{A,b}} = 1 - \operatorname{erf} \eta$$

$$\eta = \frac{x}{2\sqrt{D_{AB}t}}$$

$$\eta = \frac{x}{2\sqrt{D_{AB}t}}$$

$$N_A(t) = - \sqrt{\frac{D_{AB}}{\pi t}} (C_{A,i} - C_{A,b})$$

$t_c$

$$\frac{1}{t_c} \int_0^{t_c} N_A(t) dt = \bar{N}_A$$

$$\bar{N}_A = 2 \cdot \left( \frac{D_{AB}}{\pi t_c} \right) (C_{A,i} - C_{A,b})$$

Average mass transfer coefficient

$$k_L \propto (D_{AB})^\eta$$

$$\eta = 0.5$$

Film theory

$$\text{Vel} \uparrow \quad N_A \uparrow, k_e \uparrow, \delta \downarrow$$

Penetration theory

$$\text{Vel} \uparrow \quad N_A \uparrow, k_e \uparrow, t_c \downarrow$$

Surface Renewal Theory:

$$N_A = \sqrt{D_{A,B} S} (C_{A,i} - C_{A,b})$$

$S$  = surface renewal ~~theory~~ rate

$$\text{Vel} \uparrow, N_A \uparrow, k_e \uparrow, S \uparrow$$

# Does not account for velocity profile (bulk motion)

$$J_A = -D_{AB} \frac{dC_A}{dx}$$

$$q'' = -\frac{k}{\rho c_p} \frac{dT_p(C_p)}{dx}$$

$$\tau = -\frac{\mu}{\rho} \frac{d(V_p)}{dx}$$

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$$J_A = -D_{AB} \frac{dc_A}{dx}$$

→ Mass diffusivity

$$q = -\left(\frac{k}{\rho c_p}\right) \frac{d(p c_p T)}{dx}$$

→  $\frac{1}{\text{Thermal diffusivity}(\alpha)}$

$$\tau = -\left(\frac{\mu}{\rho}\right) \frac{dv}{dx}$$

→ Kinematic viscosity  
or  
Momentum diffusivity  
( $\nu$ )

$$D_{AB} \Leftrightarrow \frac{1}{\alpha} \Leftrightarrow \nu$$

$$Nu = \frac{\text{Heat transfer due to convection}}{\text{Heat transfer due to conduction}}$$

$$= \frac{h A \Delta T}{K \cdot A \frac{\Delta T}{L}} = \frac{h L}{K}$$

$Sh \equiv$  Sherwood Number

$$= \frac{\text{Mass transfer due to convective mass transfer}}{\text{molecular diffusion}}$$

Ignoring bulk flow  $N_A + N_B = 0$

$$Sh = \frac{\frac{K_c A \Delta C_A}{D_{AB} A \Delta C_A}}{\frac{l}{l}} = \frac{K_c l}{D_{AB}} = \frac{K_L l}{D_{AB}}$$

For  $N_A + N_B \neq 0$

$$Sh = \frac{K_c l}{D_{AB}} \left[ \frac{C}{(C_B)_{\text{em}}} \right]$$

$$Re = \frac{\rho v D}{\mu} = \frac{G D}{\mu}$$

$$Pr = \frac{\gamma}{\alpha} \Leftrightarrow Sc \equiv \text{Schmidt Number}$$

$$= \frac{\gamma}{D_{AB}}$$

$$\left. \begin{array}{l} Pe_H = Re \cdot Pr \\ St_H = \frac{Nu}{Pe} \end{array} \right\} \begin{array}{l} Pe_M = Re \cdot Sc \\ St_M = \frac{Sh}{Pe_M} \end{array} \right\} \text{For mass transfer}$$

Boundary layer thickness depends on Schmidt & Prandtl number.

$$\frac{k_L \cdot x}{D_{AB}} = 0.332 \left( \frac{x \nu_{AB} P}{\mu} \right)^{1/2} \left( \frac{\gamma}{D_{AB}} \right)^{1/3}$$

$$Sh_x = 0.332 (Re_x)^{1/2} \left( \frac{Sc}{Pe} \right)^{1/3}$$

$$Sh_x \propto Re^n$$

$$\propto Sc^m$$

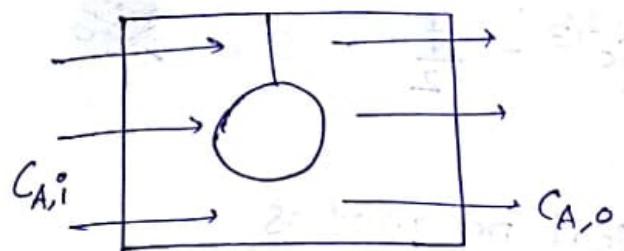
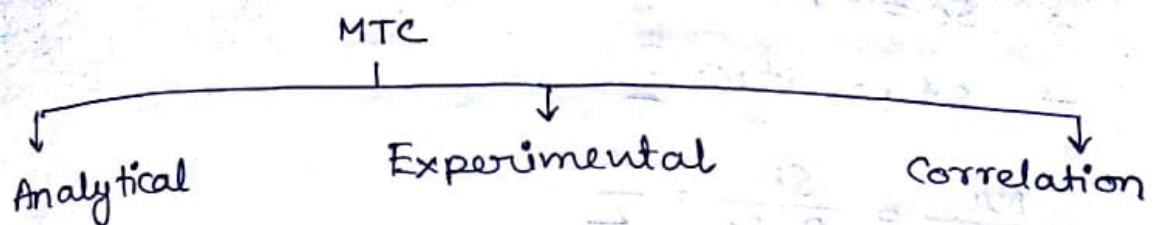
$$\frac{\int_0^L Sh_x dx}{\int_0^L x dx} = \overline{Sh}_L = 0.664 (Re_L)^{1/2} (Sc)^{1/3} = \frac{k_L \cdot L}{D_{AB}}$$

Average Sherwood number

$K_L \propto D_{AB}$	film theory
$\propto D_{AB}^{1/2}$	Penetration theory
$\propto D_{AB}^{1/2}$	Surface Renewal
$\propto D_{AB}^{2/3}$	Boundary Layer theory

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$$N_A = (MTC) D \cdot F$$



$$W = k_c \Delta C_A$$

$$k_e = f(S_h, Re, Pr)$$

Boundary Layer theory

$$S_h = f(Re, Sc)$$

### Analogous correlations

#### ① Heat, mass & momentum

### Correlations

→ analogy: Reynold's Analogy  
 Applicable for  
 → Laminar flow  
 →  $Sc = Pr \approx 1$   
 →  $(St)_H = (St)_M = f/2$

→ Experimental data

## Analogy

→ Laminar flow

$$\rightarrow Sc = Pr \approx 1$$

$$\rightarrow St_H = St_M = \frac{f}{2}$$

$$\frac{Nu}{Re \cdot Pr} = \frac{Sh}{Re \cdot Sc} = \frac{f}{2}$$

Coulbourn - Chilton : Correction for turbulent flow

$$St_H \cancel{Pr^{2/3}} = St_M \cancel{Sc^{2/3}} = \frac{f}{2} \quad \cancel{Sc^{2/3}} \quad \cancel{Pr^{2/3}}$$

1) Match the situation

2) Match process critical parameters

Sc, Re

3) Find suitable correlations

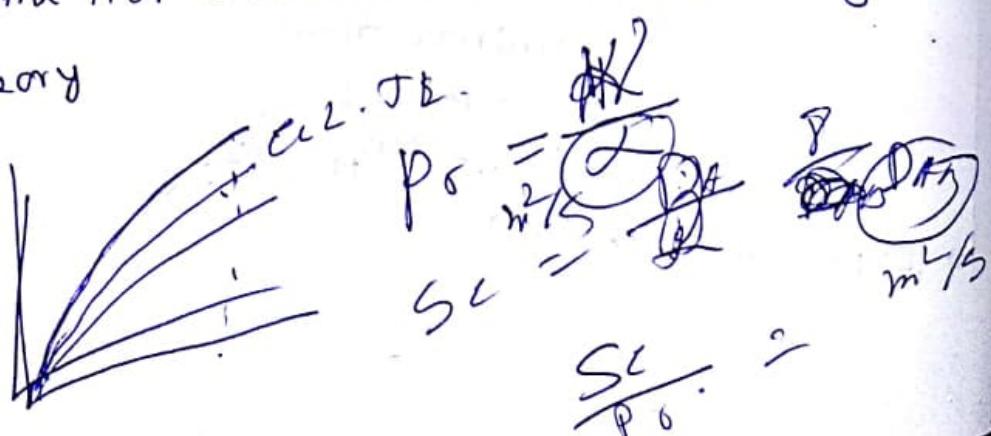
4) If no correlation is available, look for analogy.

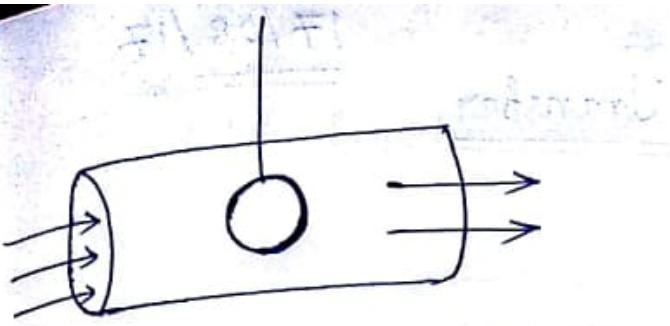
5) Use analogous correlation from heat transfer

e.g.  $Nu = 2 + 0.3 Re^{0.5} Pr^{0.33} \cancel{Re} \cancel{Pr}$

$$Sh = 2 + 0.3 Re^{0.5} Sc^{0.33}$$

6) Still not available, use boundary layer theory





from  $W \Rightarrow N_A$

	$W$	$\Delta C$
$D_i$		
$M$		
$P$		
$V$		
$D_{AB}$		

$\theta = A$

$\theta = D_{AB}$

straight and  $\theta = D_{AB}$   
straight and  $\theta = D_{AB}$   
but  $\theta = D_{AB}$  is not straight  
straight and  $\theta = D_{AB}$

length  $\rightarrow$  length  $\rightarrow$   
length  $\rightarrow$  length  $\rightarrow$

$\theta = D_{AB}$

$\theta = D_{AB}$

straight and  $\theta = D_{AB}$

$\theta = D_{AB}$

$(\sin \theta) + (\cos \theta)$

$(\sin \theta) + (\cos \theta)$

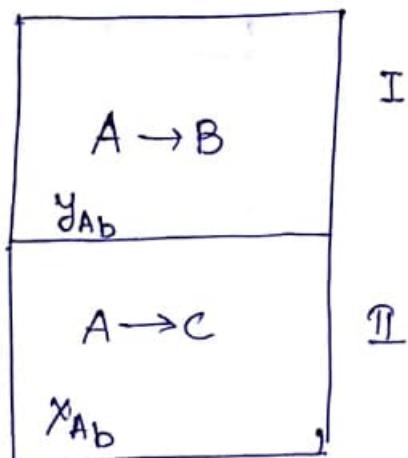
length  $\rightarrow$  length  $\rightarrow$  length  $\rightarrow$

length  $\rightarrow$  length  $\rightarrow$

length  $\rightarrow$  length  $\rightarrow$

## Inter Phase Mass Transfer

- Binary system
- Non reactive
- Two phases - Gas liq.



- 1) Is there any transfer of A? for  $\mu_1 \neq \mu_2$
- 2) From which phase to which? from higher   
 lower chemical potential
- 3) How long?  
until equilibrium reached

$$y_{Ab} > f(x_{Ab})$$

$$y_{Ab} < f(x_{Ab})$$

Mass transfer between <sup>two</sup> phases occur when

$$1) y_{AB} \neq y_A^*$$

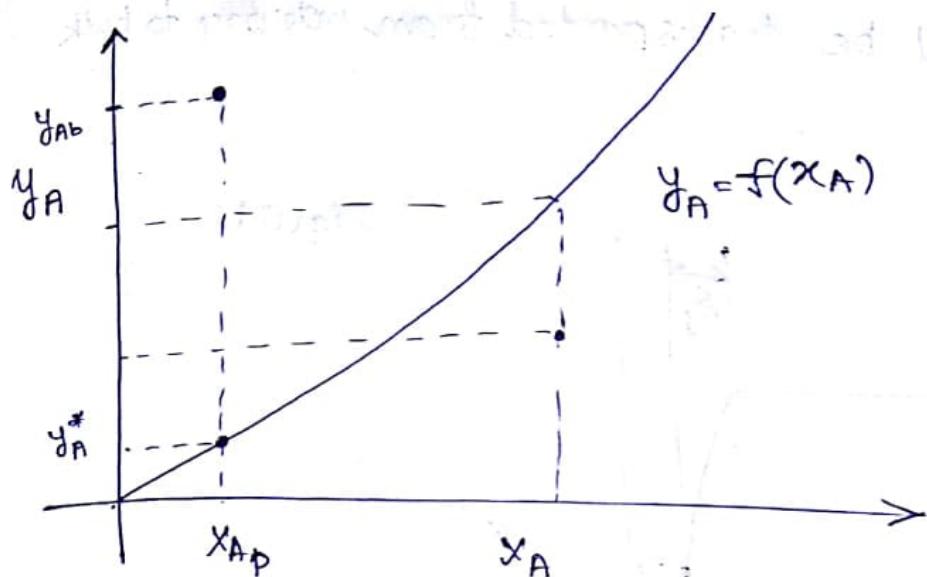
$$y_A^* = f(x_{Ab})$$

$$2) y_{AB} > f(x_{AB})$$

Mass transfer occurs from gas phase to liq phase  
⇒ Absorption

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$y_{AB} < f(x_{AB}) \Rightarrow$  stripping  
from liquid phase to gas phase



$$\mu_1 \neq \mu_2$$

higher  
chemical  
potential

$$y_A = f(x_A)$$

$$P_A = x_A P_A^S$$

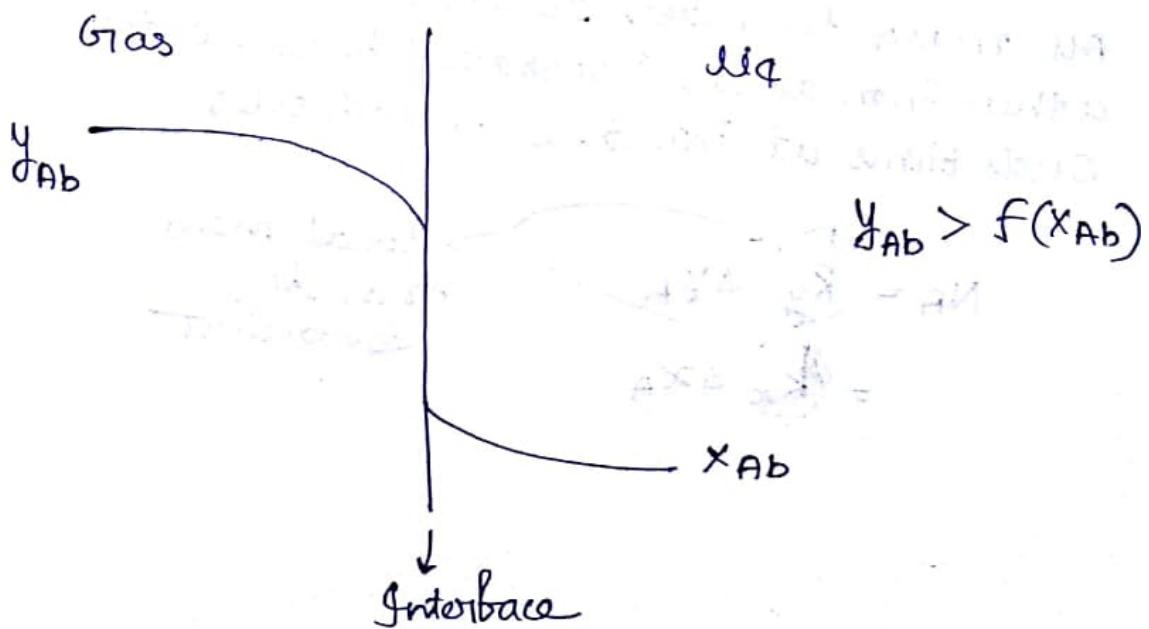
$$y_A = k_H x_A$$

Raoult's Law

Henry's Law

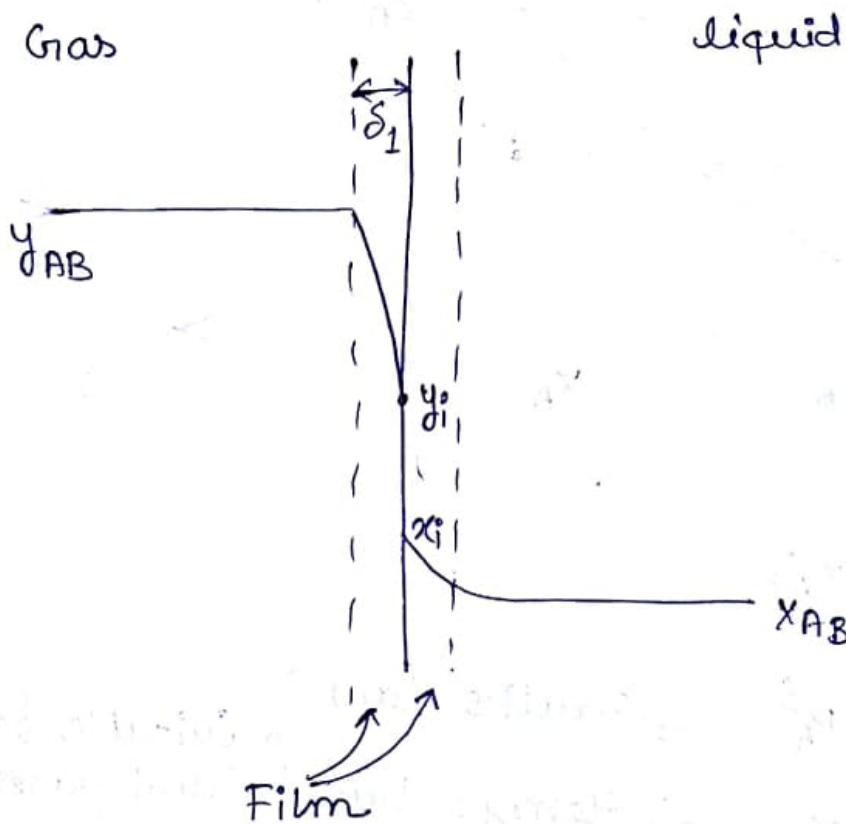
Suitable for  
ideal gases

when



gas to liquid  
phase

- 1) Transport of A from bulk to interface
- 2) At interface equilibrium will be established
- 3) A will be transported from interface to bulk



All mass transfer resistance exists in a thin film at the interface. There are two such films at interface on both sides.

$$\begin{aligned}
 N_A &= (R_y) \Delta y_A \\
 &= (R_x) \Delta x_A
 \end{aligned}
 \quad \text{Local mass transfer coefficient}$$

face

to bulk

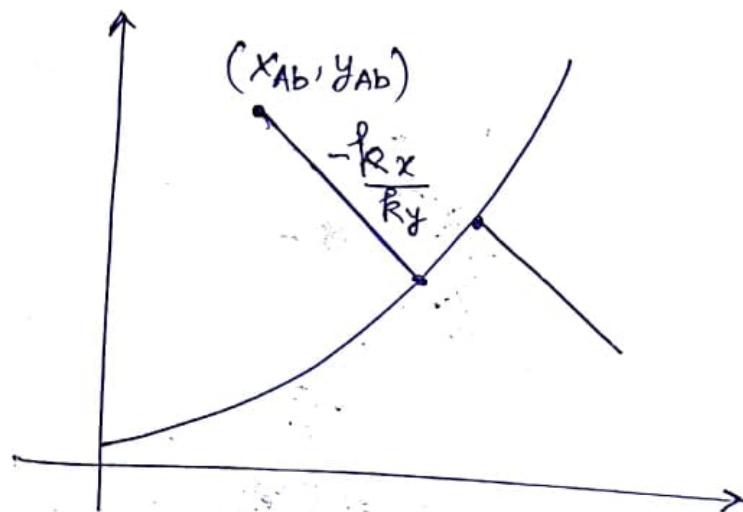
Absorption

$$N_A = k_y (y_{Ab} - y_i)$$

$$N_T = k_x (x_{Abi} - x_{Ab})$$

$$k_y (y_{Ab} - y_i) = k_x (x_{Ab} - x_i)$$

$$-\frac{k_x}{k_y} = \frac{y_{Ab} - y_i}{x_{Ab} - x_i}$$



Stripping

$$N_A = k_y (y_i - y_{Ab})$$

$$N_T = k_x (x_{Ab} - x_i)$$

$$-\frac{k_x y_i}{k_y x} = \frac{x_{Ab} - x_i}{y_{Ab} - y_i}$$

$$\frac{N_A}{K_x} = \frac{N_A}{R_x} + \frac{N_A}{m_2 K_y}$$

$$\frac{1}{K_x} = \frac{1}{R_x} + \frac{1}{m_2 K_y}$$

23/08/17

For Henry's law  $y = Hx$

$$\Rightarrow m_1 = m_2$$

$\frac{1}{K_y}$  = total M.T resistance based on overall gas phase  
driving force

$\frac{1}{K_x}$  = total M.T resistance based on liquid phase  
driving force.

$$\frac{1}{K_y} = \frac{1}{R_y} + \frac{m_1}{R_x}$$

Liq phase mass t.r.  
based on ~~gas~~ gas phase  
driving force

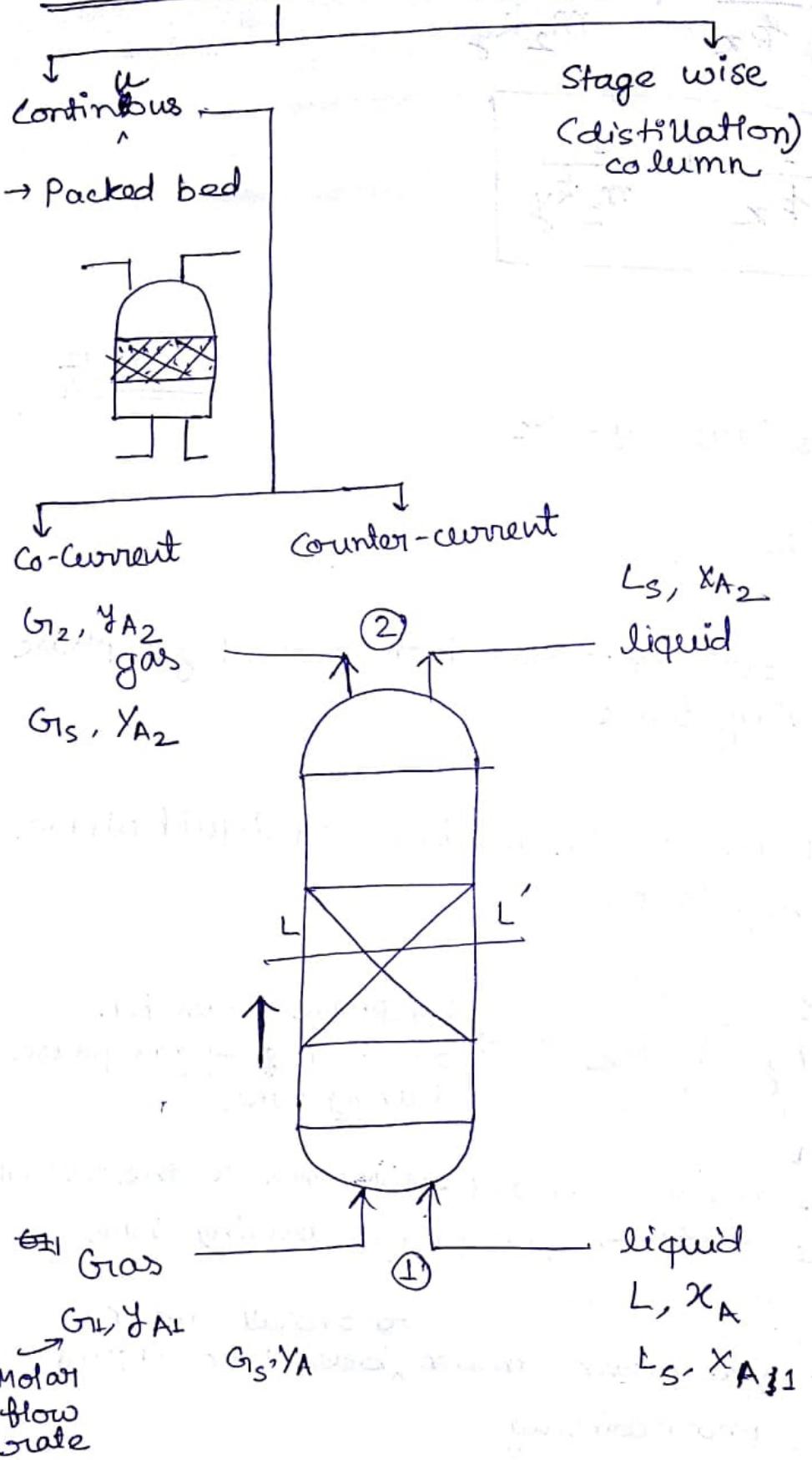
Gas phase mass transfer resistance to the overall  
resistance based on gas phase driving force.

$$\frac{1}{m_2 R_y} = \text{gas phase m.t.r. based on liquid  
phase driving}$$

to overall m.t.r.

$$\frac{1}{R_x} = \text{liq phase } \underline{\quad} " \quad$$

## Mass Transfer Equipments



Flow rate

Solute free basis

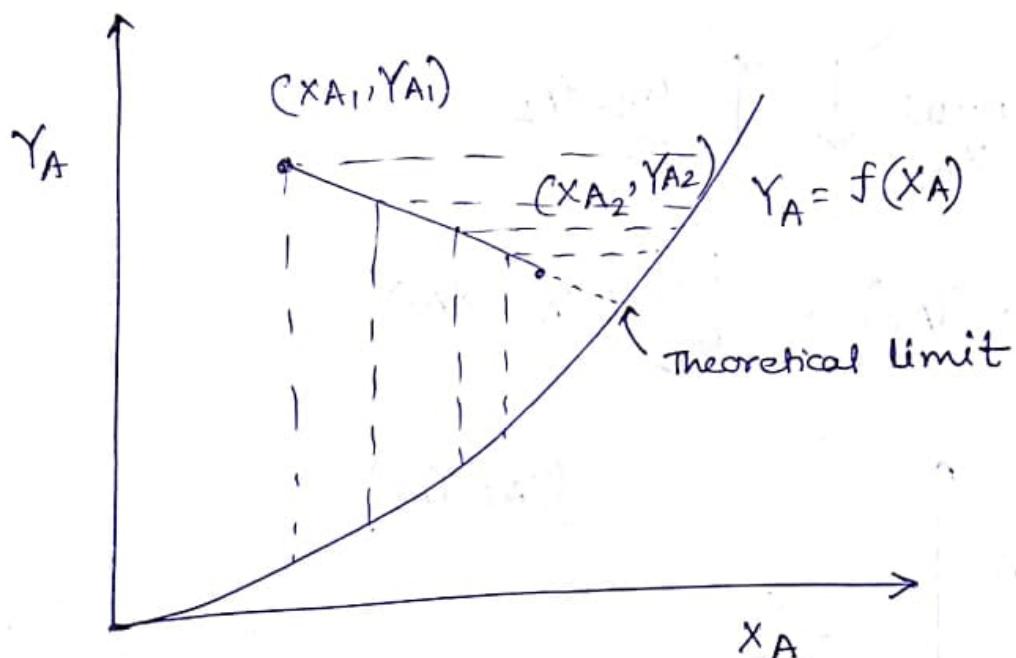
$$Y_A = \frac{y_A}{1 - y_A}$$

$$X_A = \frac{x_A}{1 - x_A}$$

$$y_A = f(x_A)$$

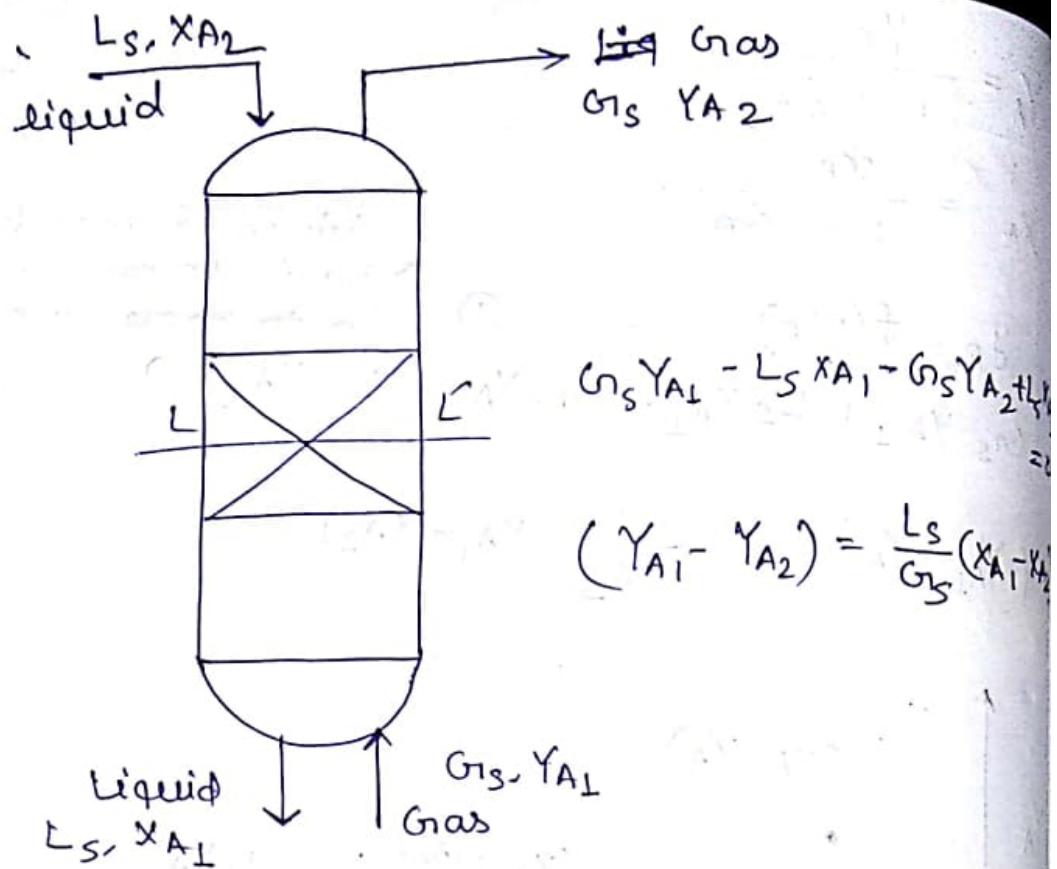
$$G_{1S} Y_{A1} + L_S X_{A1} - G_{1S} Y_{A2} - L_S X_{A2} = 0 \quad \text{--- (2)}$$

$$(Y_{A1} - Y_{A2}) = -\frac{L_S}{G_{1S}} (X_{A1} - X_{A2})$$



$$G_{1S} Y_{A1} + L_S X_{A1} - G_{1S} Y_A - L_S X_A = 0$$

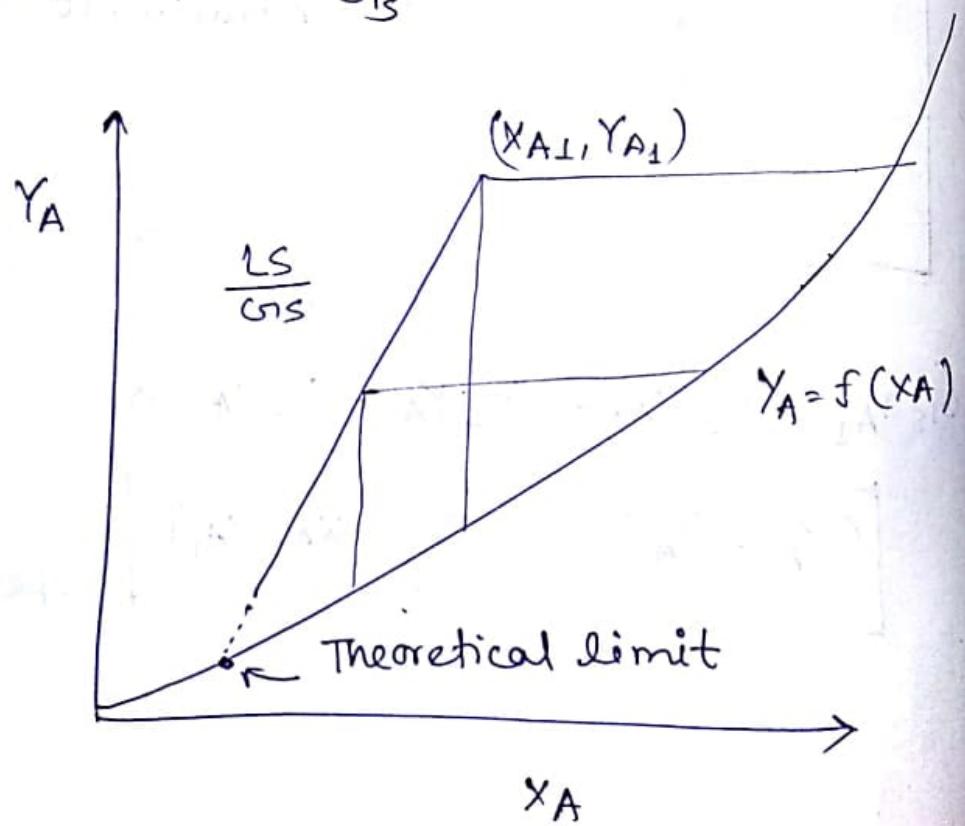
$$\boxed{(Y_A - Y_{A1}) = -\frac{L_S}{G_{1S}} (X_A - X_{A1})} \Rightarrow \text{operating eqn}$$

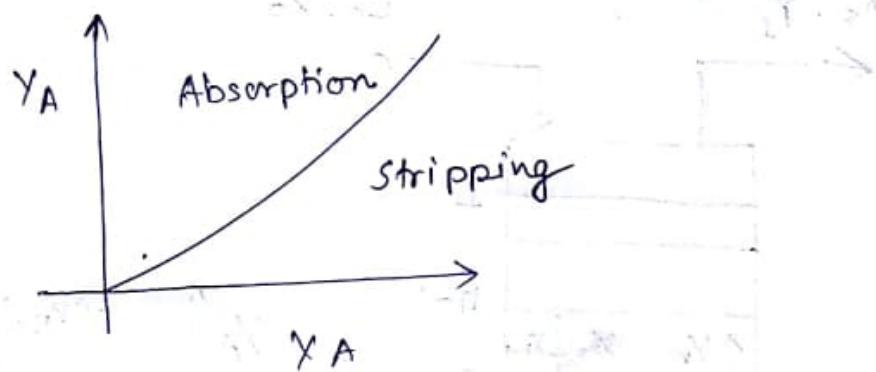


$$G_S Y_{A1} = L_S X_{A1} - G_S Y_{A2} + L_S$$

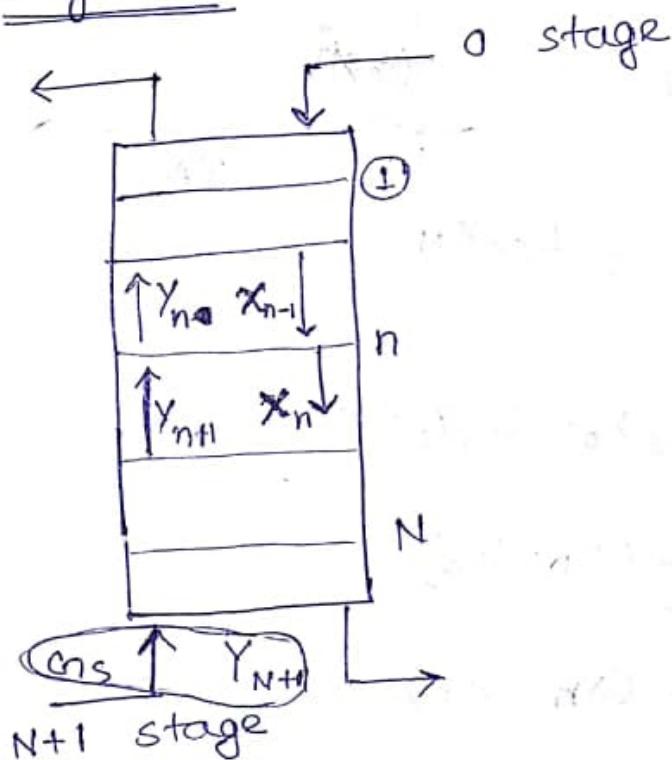
$$(Y_{A1} - Y_{A2}) = \frac{L_S}{G_S} (X_{A1} - X_{A2})$$

$$(Y_A - Y_{A1}) = \frac{L_S}{G_S} (X_A - X_{A1})$$





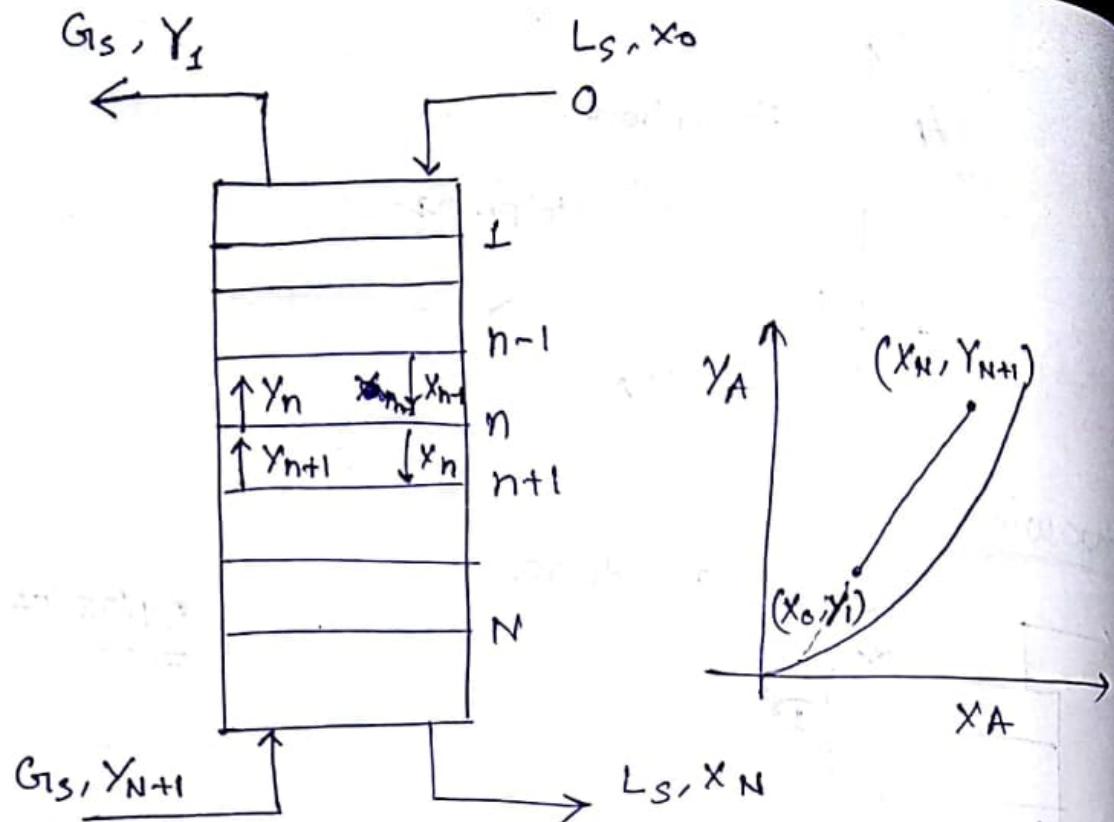
Stage wise



24/08/17

Flash distillation

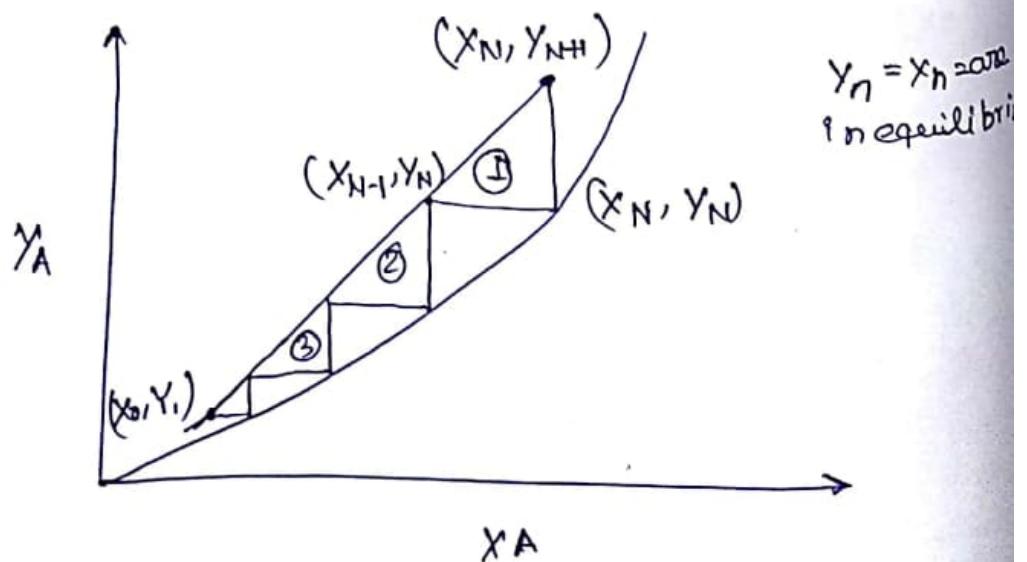
= one stage co-current distillation



$$G_s Y_{N+1} - L_s x_N + L_s x_0 - G_s Y_1 = 0$$

$$(Y_{N+1} - Y_1) = \frac{L_s}{G_s} (x_N - x_0)$$

$$(Y_{N+1} - Y_1) = \frac{L_s}{G_s} (x_n - x_0)$$



$$(Y_{n+1} - Y_1) = \frac{L_s}{G_s} (x_n - x_0) \quad \left. \begin{array}{l} \\ \\ \end{array} \right\}$$

$$Y_n = f(x_n) = \alpha x_n \quad \left. \begin{array}{l} \\ \\ \end{array} \right\}$$

$$\boxed{Y_{n+1} - \bar{A} Y_n = Y_1 - \bar{A} \alpha x_0} \quad \bar{A} = \frac{L_s}{G_s \cdot \alpha}$$

$$Y_n = \frac{\alpha x_0 - Y_1 (\bar{A})^n}{1 - \bar{A}} + \frac{Y_1 - \bar{A} \alpha x_0}{1 - \bar{A}}$$

$$Y_{N+1} = \frac{\alpha x_0 - Y_1 (\bar{A})^{N+1}}{1 - \bar{A}} + \frac{Y_1 - \bar{A} \alpha x_0}{1 - \bar{A}}$$

$$N = \frac{\log \left[ \left( \frac{(Y_{N+1} - \alpha x_0)}{Y_1 - \alpha x_0} \right) \left( 1 - \frac{1}{\bar{A}} \right) + \frac{1}{\bar{A}} \right]}{\log \bar{A}} \quad \text{for } \bar{A} \neq 1$$

for  $y_n = \frac{L_s}{G_s} x_n$

$$N = \frac{Y_{N+1} - Y_1}{Y_1 - \bar{A} \alpha x_0}$$

For stripping

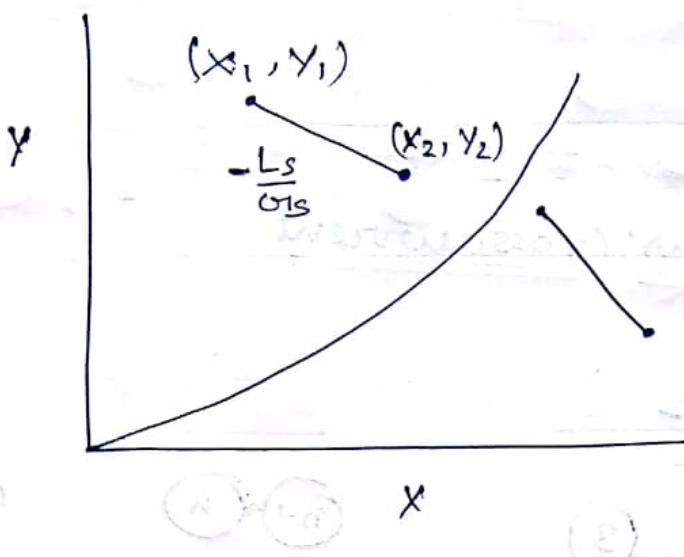
$$N = \frac{\log \left[ \left( \frac{x_0 - Y_{N+1}/\alpha}{x_N - Y_{N+1}/\alpha} \right) (1 - \bar{A}) + \bar{A} \right]}{\log (1/\bar{A})}$$

$\bar{A} \neq 1$

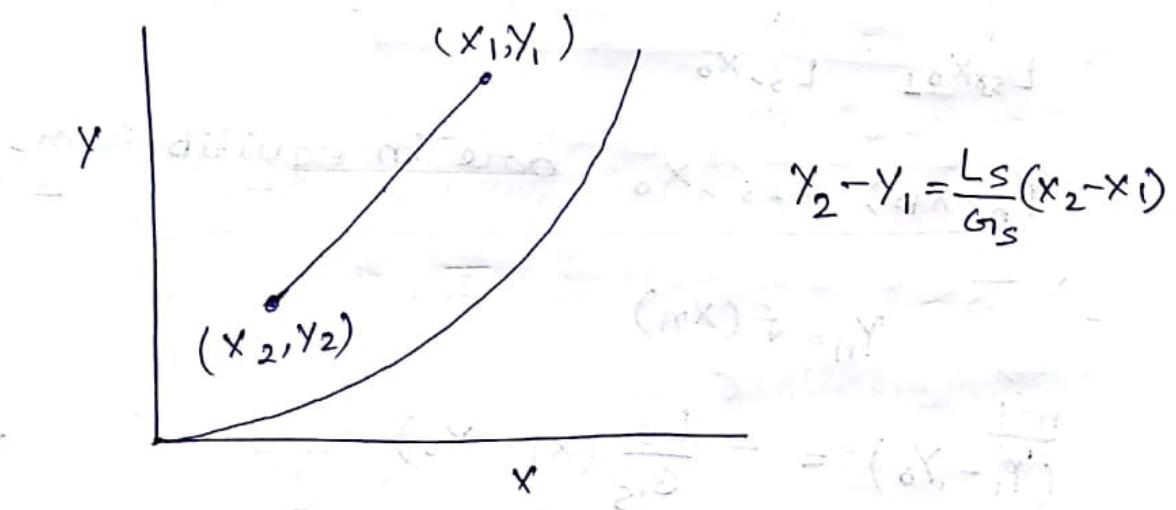
# 1. Cocurrent continuous operation

28/08/17

$$Y_2 - Y_1 = - \frac{L_s}{G_{ls}} (x_2 - x_1)$$

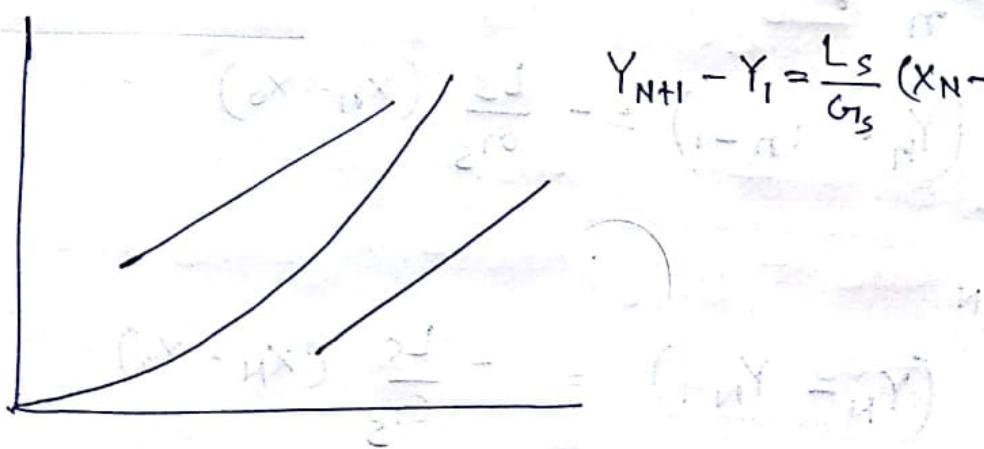


# 2. Counter-current continuous flow



# 3. Stagewise counter current

$$Y_{N+1} - Y_1 = \frac{L_s}{G_{ls}} (x_N - x_0)$$

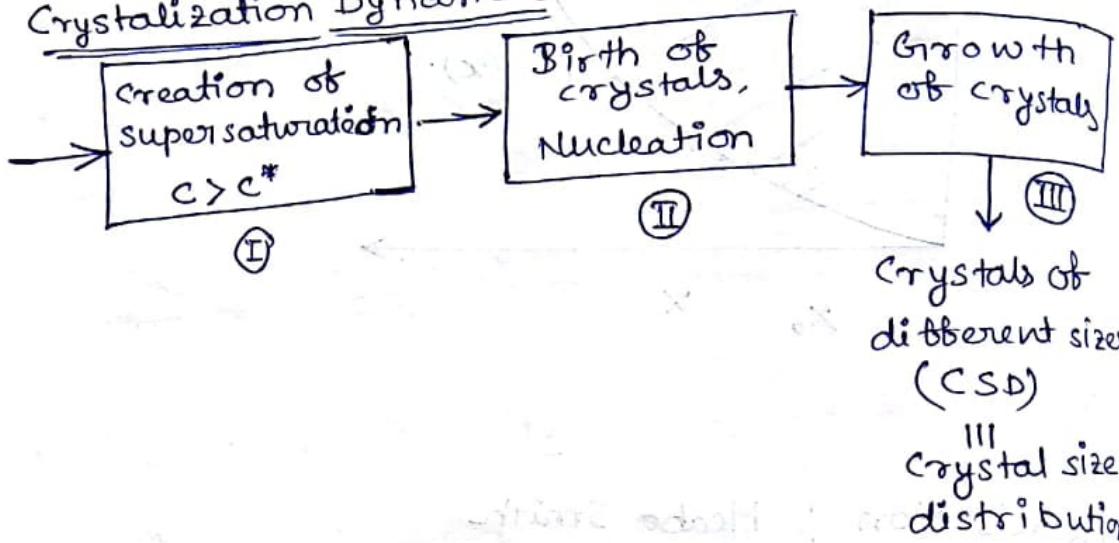


## Crystallization

30/08/17

Separation of solute in solid form  
from a solution.

### Crystallization Dynamics

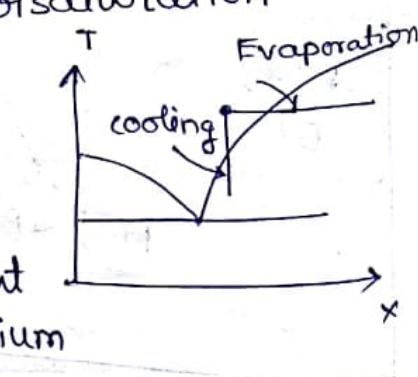


### I) Creation of supersaturation:

$$\frac{c - c^*}{c^*} = \gamma \equiv \text{Degree of supersaturation}$$

$(c - c^*) \equiv$  Extent of supersaturation

→ By cooling  
→ Evaporation



Industrial methods  
may shift the equilibrium curve.

### II) Birth of crystals / Nucleation

Rate of generation of crystals of zero size

$$= B_0$$

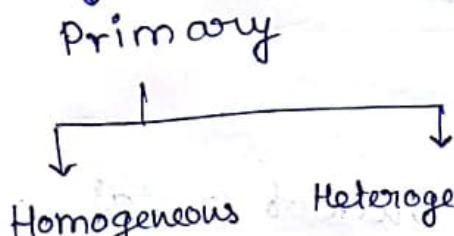
characteristic length of crystal = L

$$B_0 = \lim_{L \rightarrow 0} \frac{d(N/V)}{dt}$$

$$A_0 = \phi_a L^2$$

$$V = \phi_v L^3$$

### Nucleation



secondary  
↓  
Occurs due to storage or prolonged action of impellers

Homogeneous = No addition of fine crystals. Occurs without any external interruption

Heterogeneous = Seeds (small, fine crystals) are added to facilitate formation of crystals

For primary nucleation:

$$B_0 = A' \exp \left[ \frac{16 \pi \sigma^3 V_m^2}{3 k_B T^3 \ln(3+1)^2} \right]$$

$\nwarrow$  crystal size  
 $\uparrow$  Boltzmann const.

For heterogeneous nucleation:

$$B_0 = K_1 M_T^{-n}$$

$K_1$  } Experimentally determined  
 $n$  } determined

↓  
Mass of suspension

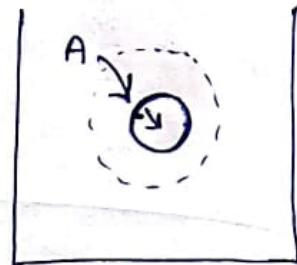
$$n = \text{order} \approx 0-5$$

$n=2$  if nothing mentioned

### III) Growth of crystals

$m_c$  = mass of crystal

$$\text{Growth rate} = \frac{dL}{dt} = G_1$$



- Transfer of solute molecules from bulk to top of surface.

$$\frac{dm_c}{dt} = k_c A_c (c - c_i)$$

- Integration of solute in the bulk of crystal.

$$\frac{dm_c}{dt} = k_p A_c (c_i - c^*)$$

$$k_L = \frac{1}{\frac{1}{k_c} + \frac{1}{k_p}}$$

$$\frac{dm_c}{dt} = k_L A_c (c - c^*) = \phi_v L^3 p_c$$

$$A_c = \phi_a L^2$$

$$\frac{d}{dt} (\phi_v L^3 p_c) = k_L A_c (c - c^*)$$

$$\frac{dm_c}{dt} \frac{dL}{dt} = k_L \left( \frac{\phi_a c^*}{3 \phi_v p_c} \right) \left( \frac{c - c^*}{c^*} \right)$$

$$\frac{dL}{dt} = k' s$$

$$G_1 = k' s$$

$$G_1 = k' s^n$$

In general,  $s$  determined experimentally

$$B_0 = k_L M_t s^n$$

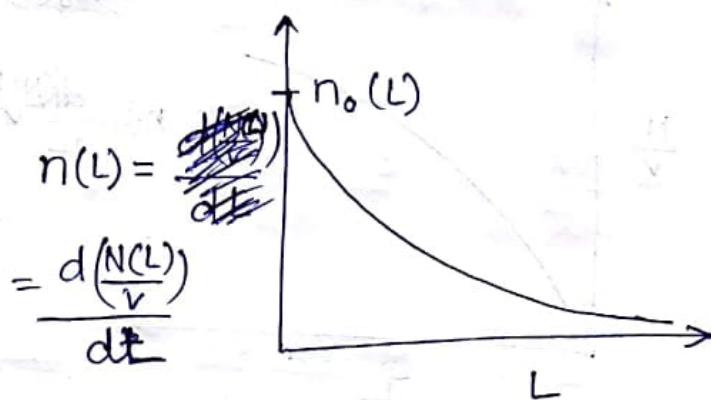
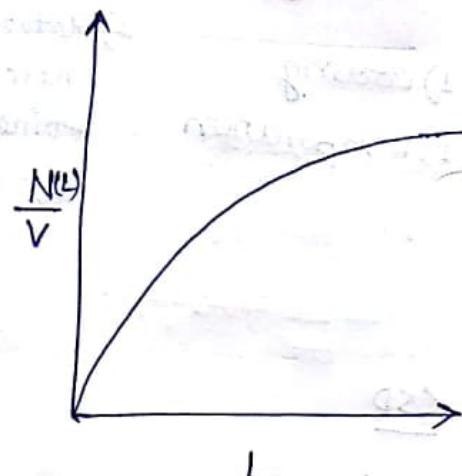
$$G = k' s^n$$

Competitive process,  
nucleation and growth of  
crystals

## CSD crystal size distribution

$$n(L) -$$

$n_0(L) \equiv$  crystal size  
distribution  
of vanishingly  
small size

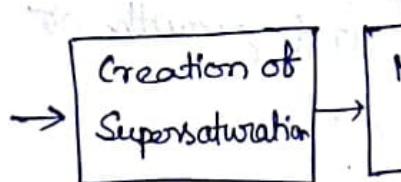


$$n_0(L) = \lim_{L \rightarrow 0} \left[ \frac{d(N(L)/V)}{dL} \right]$$

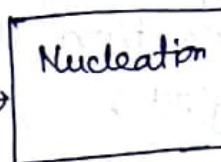
$$= \lim_{L \rightarrow 0} \frac{d(N/V)}{dt} \frac{d(L)}{dt}$$

$$n_0(L) = \frac{B_0}{G}$$

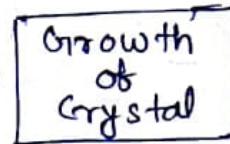
31/08/17



$$\delta = \frac{C - C^*}{C^*}$$



$$B_0 = \lim_{L \rightarrow 0} \frac{d(N(L))}{dt}$$



$$G_t = \frac{dL}{dt}$$

$$n(L) = \frac{d[N(L)]}{dL}$$

$$G_t = k_1 n$$

According to hypothesis

$$G_t = k_1 s$$

$k_1, n$  are to be determined from experiment

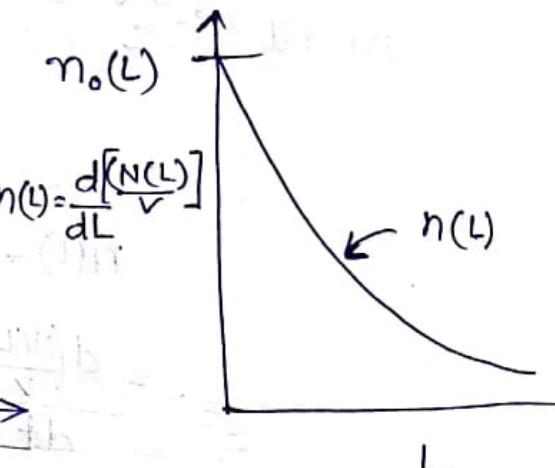
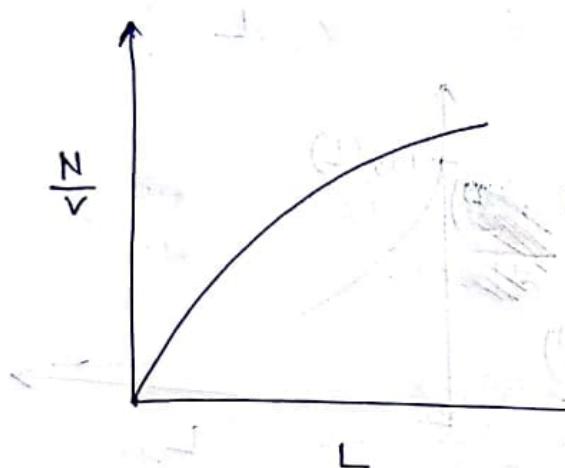
1) cooling

2) Evaporation

1) Heterogeneous nucleation  
(fine crystals)

$$B_0 = k_1 M_r s^n$$

CSD



~~avg~~

$$A_c = \int_{L_1}^{L_2} n(L) \phi_a L^2 dL$$

$$\frac{N_T}{V} = \int_{L_1}^{L_2} n(L) dL$$

$$V_c = \int_{L_1}^{L_2} n(L) \phi_v L^3 dL$$

$$M_c = V_c \rho_c = \rho_c \int_{L_1}^{L_2} n(L) \phi_v L^3 dL$$

## Crystallizer

to be created and maintained.

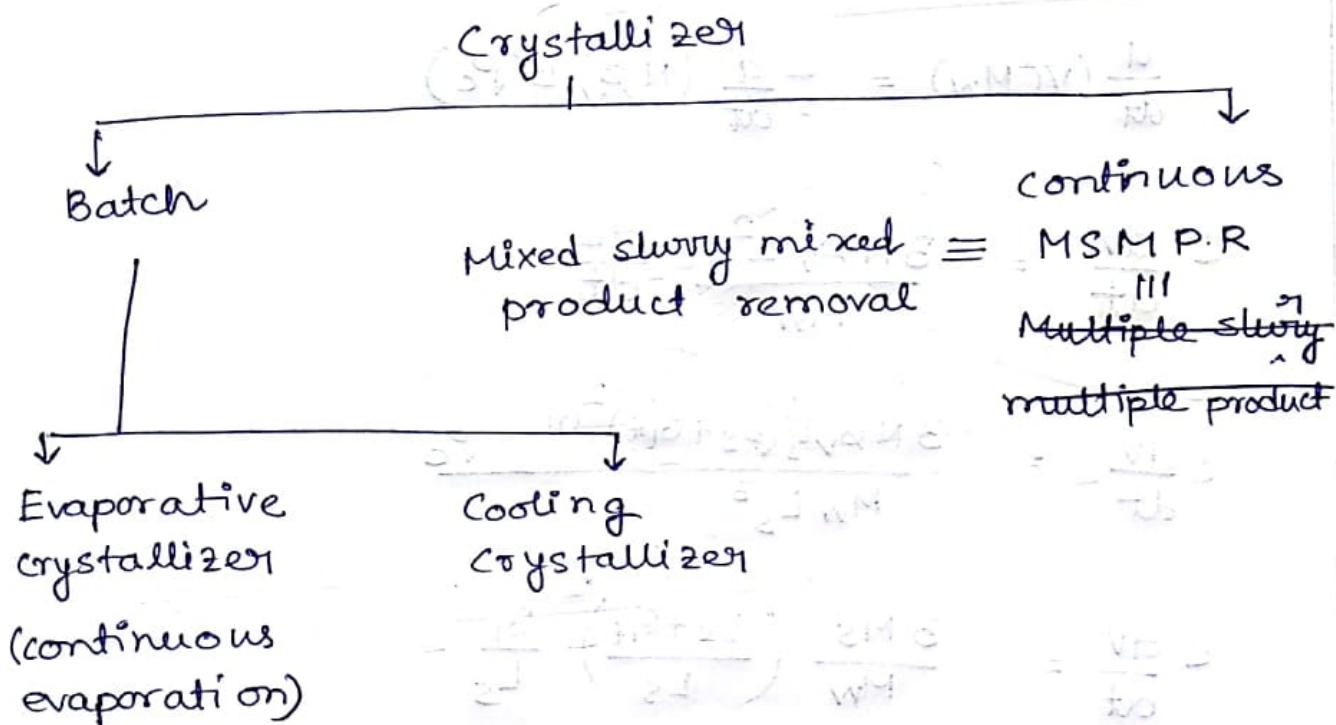
- 1) Supersaturation to be created and maintained.
- 2) Degree of supersaturation to be maintained.

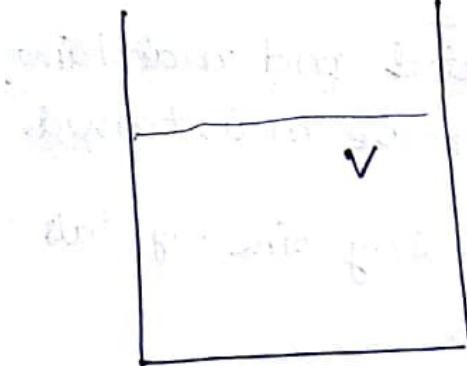
2) Nucleation  $\rightarrow$  slurry containing fine crystals

3) Time

## McCabe 'dL' Law

$$G_1 = \frac{dL}{dt} \neq f(L) = (WHSV) \frac{b}{A}$$





$$\frac{dV}{dt} = ? \Rightarrow \delta = \text{const.} = \frac{C_1}{C_2}$$

$$t=0, L=L_s, M_s = N \phi_v L_s^3$$

$$t=t, L=L, M = N \phi_v L^3$$

$$\text{at } t=t, L=G_1 t + L_s$$

$$\cancel{\frac{d}{dt}(V \cdot p)} \quad \frac{d}{dt}(V \cdot M_w) = -\frac{dM_c}{dt}$$

$$\frac{d}{dt}(V \cdot M_w) = -\frac{d}{dt}(N \phi_v L^3 P_c)$$

$$c \frac{dV}{dt} = \frac{3 N \phi_v L^2 P_c}{M_w} \frac{dL}{dt}$$

$$c \frac{dV}{dt} = \frac{3 N \phi_v L_s^3 (L_s + G_1 t)^2 G_1}{M_w L_s^3} P_c$$

$$c \frac{dV}{dt} = \frac{3 M_s}{M_w} \left( \frac{L_s + G_1 t}{L_s} \right)^2 \frac{G_1}{L_s}$$

$$L_f = L_s + G_1 t_b \leftarrow \text{Batch time}$$

04/09/17

# 1. Crystallization dynamics

## 2. CSD

## 3. Crystallizers

→ Batch

→ Continuous

i) creates supersaturation and maintains degree of supersaturation

ii) Time for crystal growth

iii) Size, size distribution

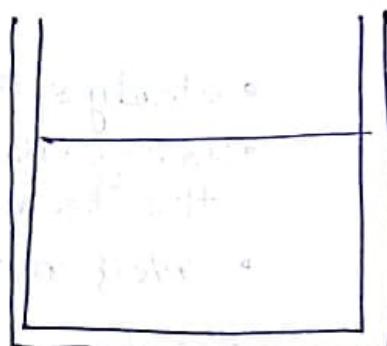
### Batch crystallizer

#### → Evaporative crystallizer

$$\frac{dV}{dt} = \text{const}, \quad L_f = L_s + Gt_b$$

#### → Cooling crystallizer

$$\frac{dT}{dt} = \text{const}, \quad L_f = L_s + Gt_b$$



$$t=0, L=L_s, M_s = N\phi_v L_s^3 p_c$$

$$t=t, L=L, M_c = N\phi_v L^3 p_c$$

$$\frac{d(M_c)}{dt} = - \frac{dM_c}{dt}$$

$$V \frac{dc}{dt} = - 3N\phi_v L^2 p_c \frac{dL}{dt}$$

$$\delta = \frac{c - c^*}{c^*}$$

→ with temp  $c^*$  varies

$$V \frac{dc}{dt} = 3N\phi_v L^2 p_c G_1 = 3N\phi_v (L_s + Gt_b)^2 p_c G_1$$

$$V \frac{dc}{dt} = 3M_S \left( \frac{L_S + G_t}{L_S} \right)^2 \frac{G_t}{L_S}$$

Now,

$$\beta = \frac{c - c^*}{c^*} = \frac{c}{c^*} - 1$$

$$c = c^* (\beta + 1) = f(T) (\beta + 1)$$

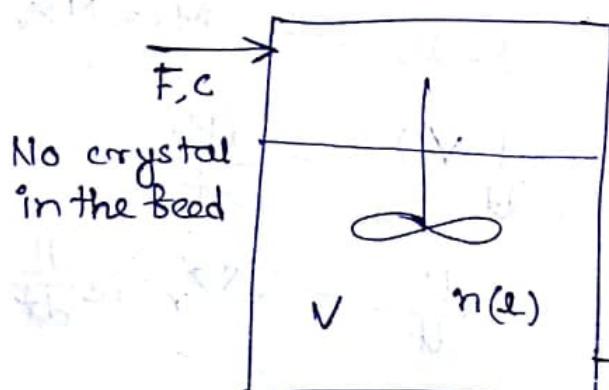
$$\frac{dc}{dT} = f'(T) (\beta + 1) \frac{dT}{dt}$$

$$V \frac{dT}{dt} = \frac{3M_S}{f'(T)(\beta + 1)} \left( \frac{L_S + G_t}{L_S} \right)^2 \frac{G_t}{L_S}$$

$G_t \rightarrow$  to be determined from kinetic data  
 $k, n$  are system dependent parameters

### Continuous Crystallizer

#### 1) Mixed slurry mixed product removal type (MSMPR)



- Steady state
- No crystals in the feed
- Well mixed
- McCabe 'dE' law is applicable

$$n(L) = \frac{d(N/v)}{dL}$$

$$n_0(L) = \lim_{L \rightarrow 0} \frac{d(N/v)}{dL}$$

No. of crystals with size  $L$  in

0

- No. of crystals with size  $L$  out

$F$  at  $n(L) \Delta L$

+ No. of crystals grown to size  $L$

$n(L) v \Delta L$

- No. of crystals grown to size  $L + \Delta L$

$n(L + \Delta L) v \Delta L$

$= 0$

$$F n(L) \Delta L = n(L) v \frac{\Delta L}{\Delta t} - n(L + \Delta L) v \frac{\Delta L}{\Delta t}$$

$$F n(L) \Delta L = n(L) v \cancel{G} - n(L + \Delta L) v G$$

$$\lim_{\Delta L \rightarrow 0} \frac{n(L) - n(L + \Delta L)}{\Delta L} = \frac{F}{v \cdot G} n(L)$$

$$-\frac{d n(L)}{d L} = \frac{F}{v \cdot G} n(L)$$

$$\frac{d n(L)}{d L} = -\frac{F}{v G} n(L) = -\frac{n(L)}{G T}$$

$$n(L)$$

$$\int \frac{d n(L)}{n(L)} = - \int \frac{1}{G T} d L$$

$$n_0(L)$$

$$\ln \left( \frac{n(L)}{n_0(L)} \right) = -\frac{L}{G T}$$

$$n(L) = n_0(L) \exp(-\frac{L}{G_1 T})$$

$$n_0(L) = \frac{B_0}{G_1}$$

$$n(L) = \frac{B_0}{G_1} \exp(-\frac{L}{G_1 T})$$

$$M_c = \int_0^L n(L) \propto L^3 dL p_c$$

06/09/17

$$n(L)$$

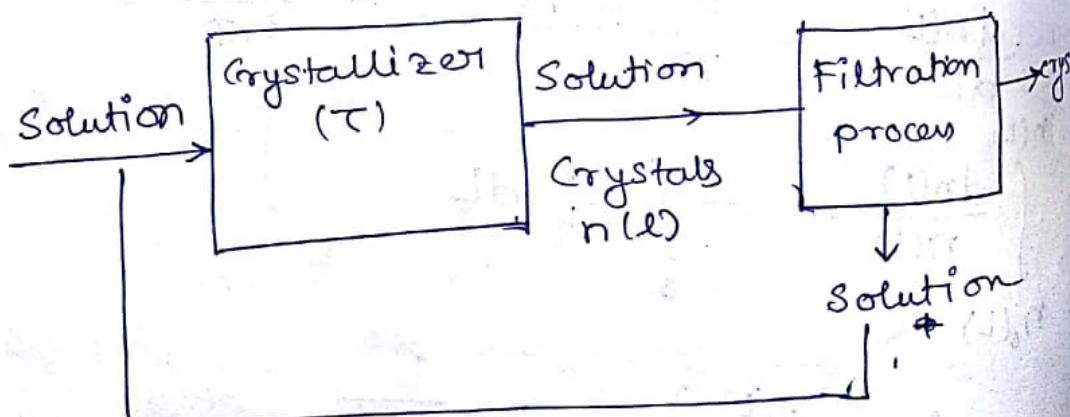
$$A c N, N/V, V_c/V, m_c/V$$

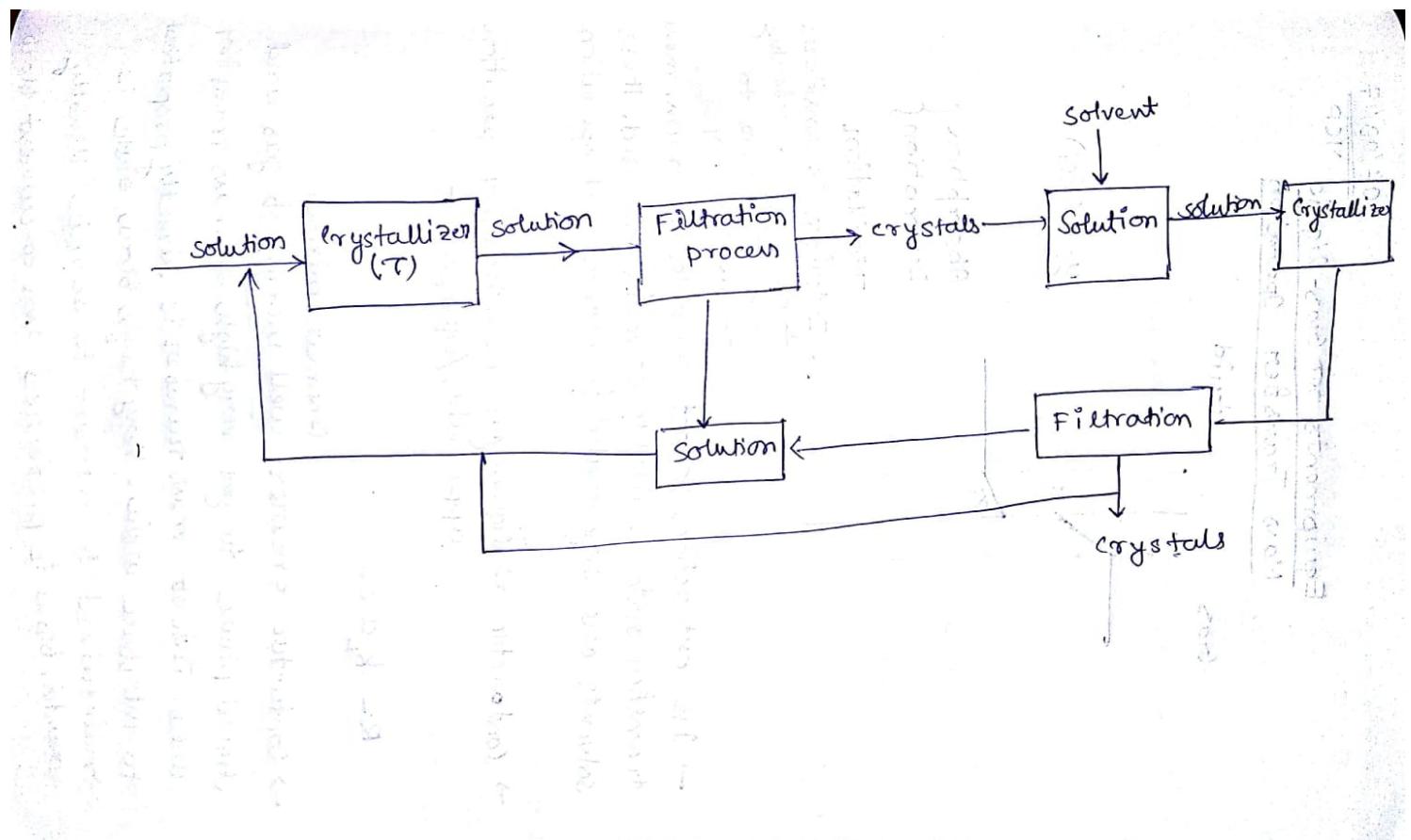
$$B_0 = k_1 S^n \quad G_1 = k_2 S^m \quad n_0(L)$$

$$\frac{dV}{dt} \Big|_t = M_S \cdot L_S \cdot G_S$$

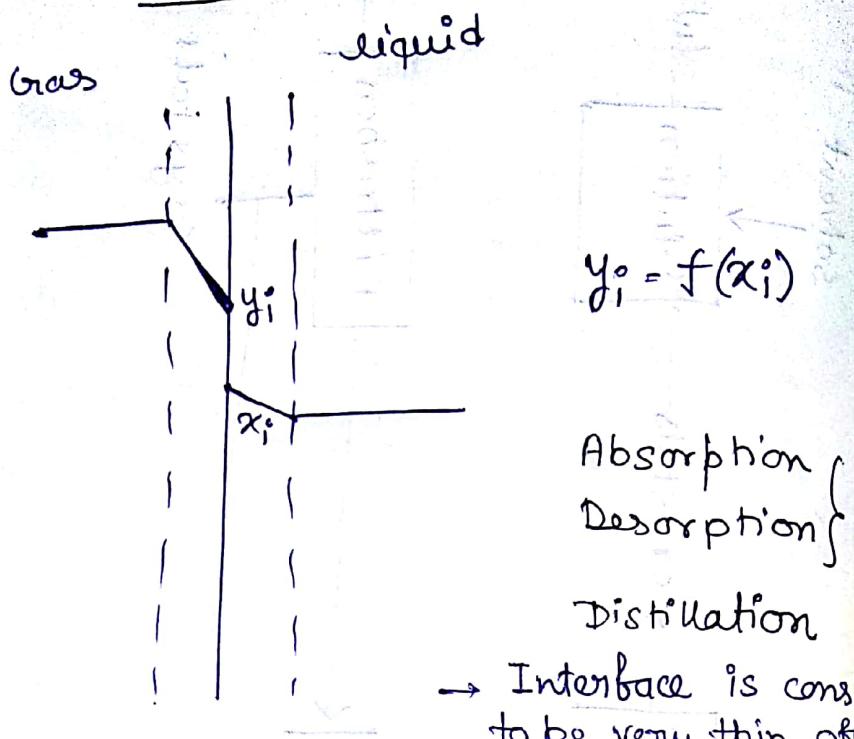
$$\frac{dT}{dt} \Big|_t = M_S \cdot L_S \cdot G_S$$

### Crystallizer





## Equipments for Gas-liquid Mass Transfer Operation



→ Interface is considered to be very thin, offering no resistance to mass transfer

- In extraction, to get suitable optimum mass transfer rate, suitable solvents are used. These solvents are regenerated for economical operation
- Contactor  $\Rightarrow$  liquid-gas mass transfer operation apparatus/equipment

$$R = k_L a \Delta c$$

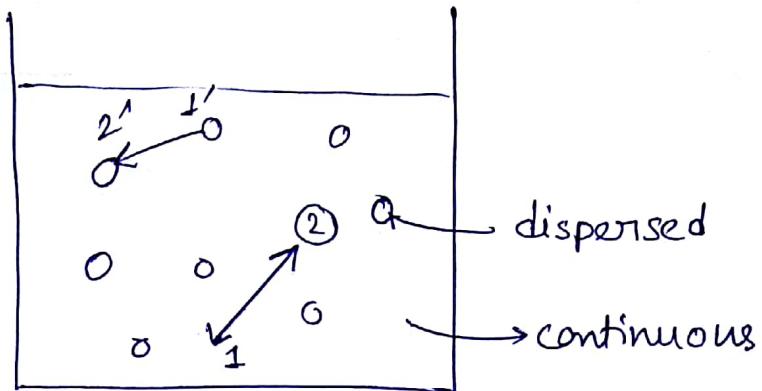
(Intimate mixing)

- Contactor ensures well mixing of gas and liquid phase to get very high of mass transfer area. Rate of mass transfer is directly proportional to interface area. Very high flow rate is maintained in interest to do this. Usually counter flow is preferred over co-current flow

## Mixing

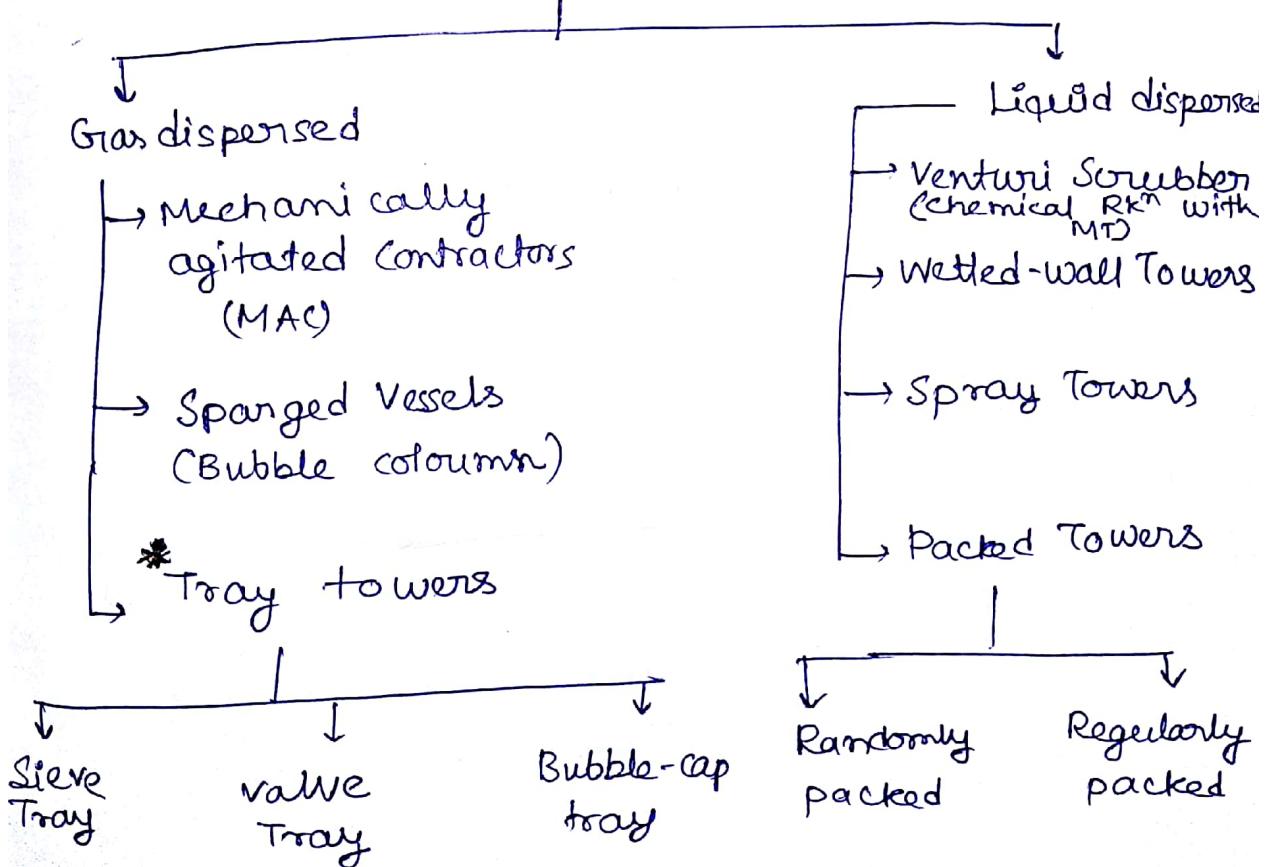
- Fine droplets of liquid in gas phase
- Bubbling the gas through liquid

One phase is dispersed in other continuous phase.



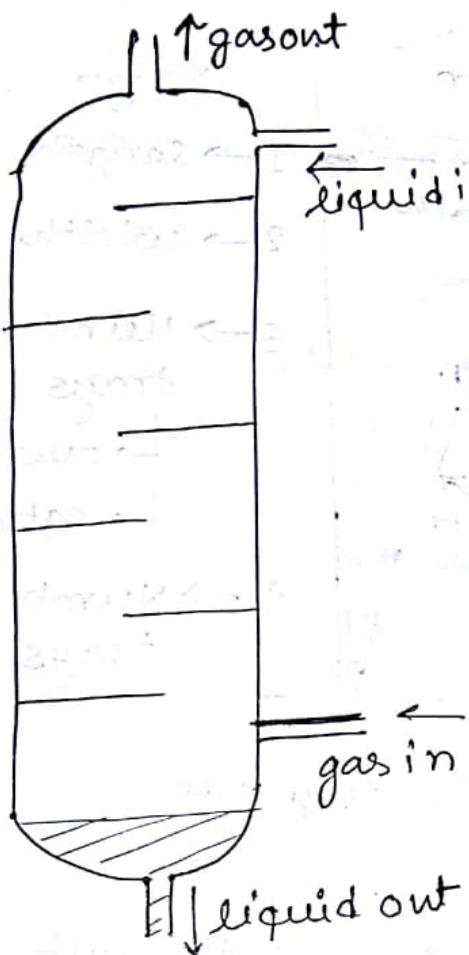
Dispersed phase is that phase for which we need to cross boundary of two phases to travel from one place to another in that phase

## Gas-liquid Contactors



## Tray Towers

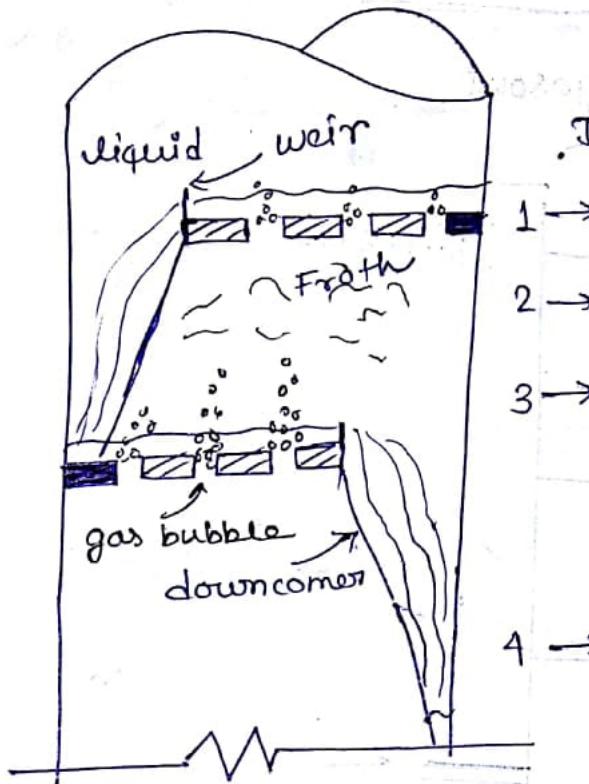
13/09/17



### Selection of Trays

	Bubble caps	valves	sieves
Relative cost	2.0	1.2	1.0
pressure drop	Highest	Intermediate	Lowest
Efficiency	Highest	Highest	lowest
vapor capacity	Lowest	Highest	Highest
Typical turndown ratio	5	1	2

Turn down ratio  $\rightarrow$  least Maximum to least feed ratio



Design

1  $\rightarrow$  length.

2  $\rightarrow$  width

3  $\rightarrow$  Number of ideal trays

$\rightarrow$  operating line

$\rightarrow$  eqbm. relationship

4  $\rightarrow$  Number of real trays

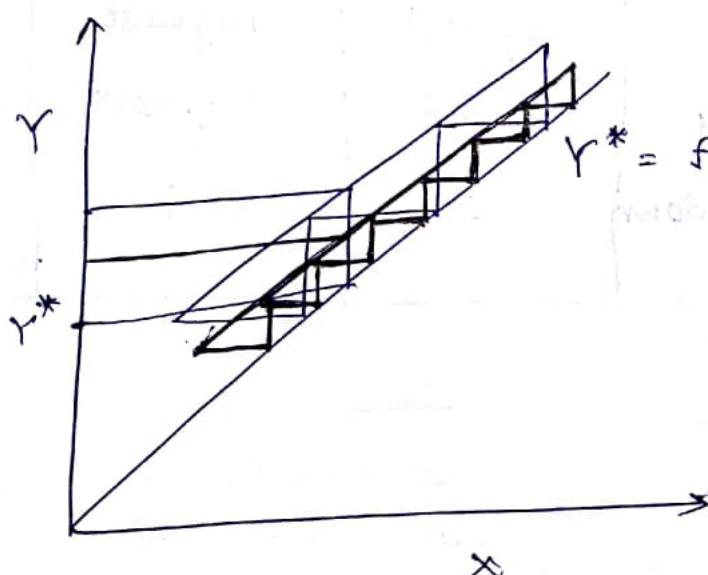
Gas-liquid sieve tray tower

Difficulty level

$\rightarrow$  Driving force is less  $\Rightarrow$  difficult

$\rightarrow$  Driving force is more  $\Rightarrow$  easier

Lesser driving force  $\Rightarrow$  Lesser separation between equilibrium line & operating line



Real trays  
can not deliver  
at eqbm at each  
stage

1) number of real trays

$$= \frac{\text{number of ideal trays}}{\text{fractional efficiency}}$$

5) Tray spacing 6" - 36"

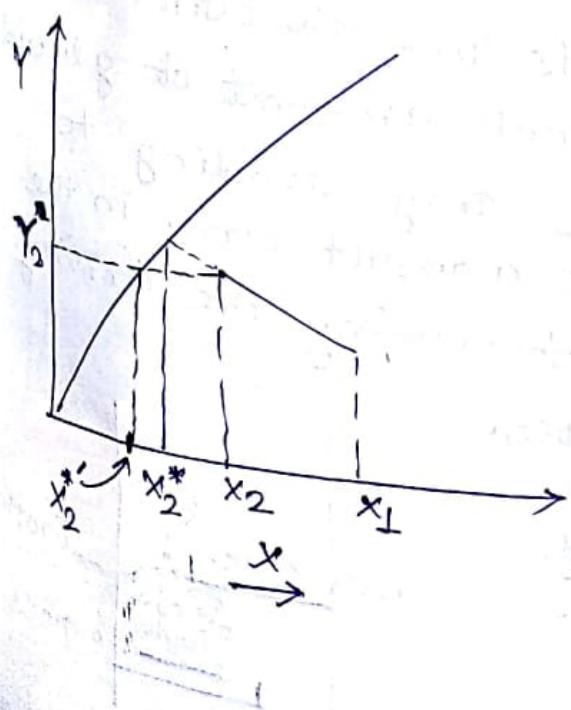
depends on diameter length, diameter  
usually 24" spacing is required

6) Length =  $\{(\text{number of total real trays} - 1) \times$   
Spacing  $\{+ \} 15 \text{ to } 20\%$

7) Diameter  $\Rightarrow$  depends on volume of feed to be  
processed

Efficiency of such towers  $\approx 50\text{-}60\%$

Efficiency:



$$E_L = \frac{x_1 - x_2}{x_1 - x_2^*}$$

$$E_{ML} = \frac{x_1 - x_2}{x_1 - x_2^*}$$

Murphree Efficiency

Efficiency is increased with

- Mass Transfer rate higher
- Large contact time
- Higher surface area
- Higher mass transfer coefficient with higher turbulence

#### Large contact time

- Higher depth of liquid pool in tray

#### Higher surface area

- Smaller bubbles by high gas flow velocity
- Higher velocity increases turbulence in the liquid pool

#### Operating Difficulty

- Heavy agitation results in froth formation. Bubbles take away small amount of liquid from lower to higher tray, leading to liquid entrainment. As a result conc. in the upper tray increases, decreasing the driving force for mass transfer.
- For high flow velocity, pressure drop is ~~low~~ high. Thus pressure at bottom is very high at low at top. Operational cost increases

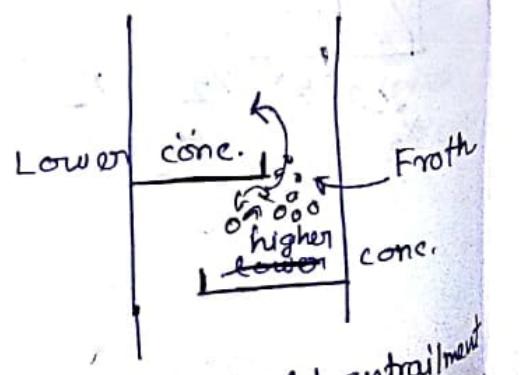
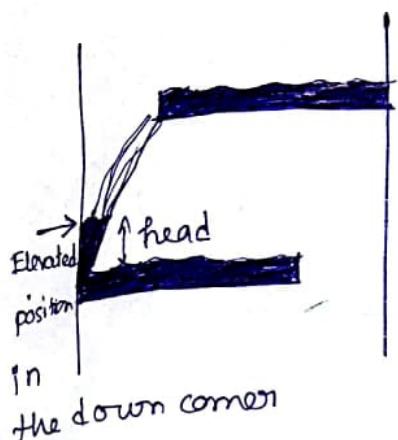


Fig: Liquid entrainment

- Hydrostatic head is required for the liquid to flow from lower to higher pressure zone in the down comer.

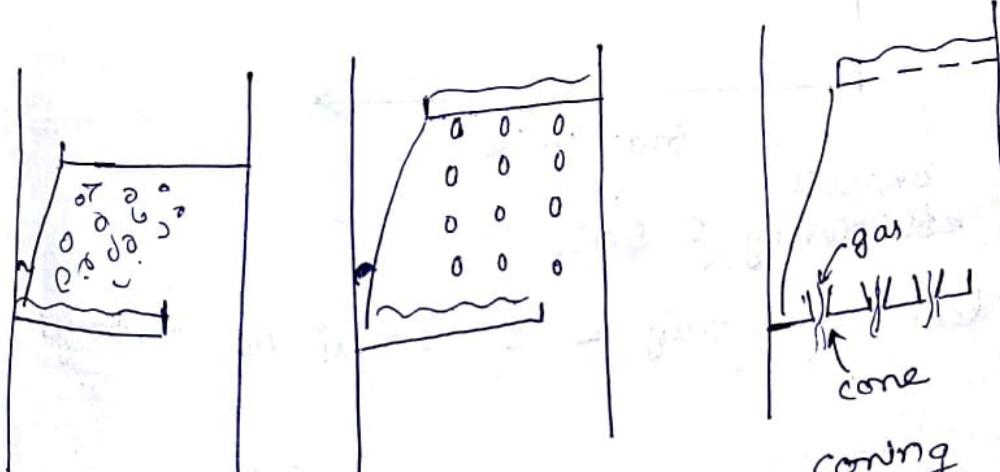


- A situation may arise when the required head reaches the upper tray leading to flooding of tower.

Priming: Excessive entrainment in forming liquid leads to high pressure drop

Dumping: Due to low flow velocity, liquid will start weeping. At further low velocity the liquid do not go to down corner and come down directly through holes

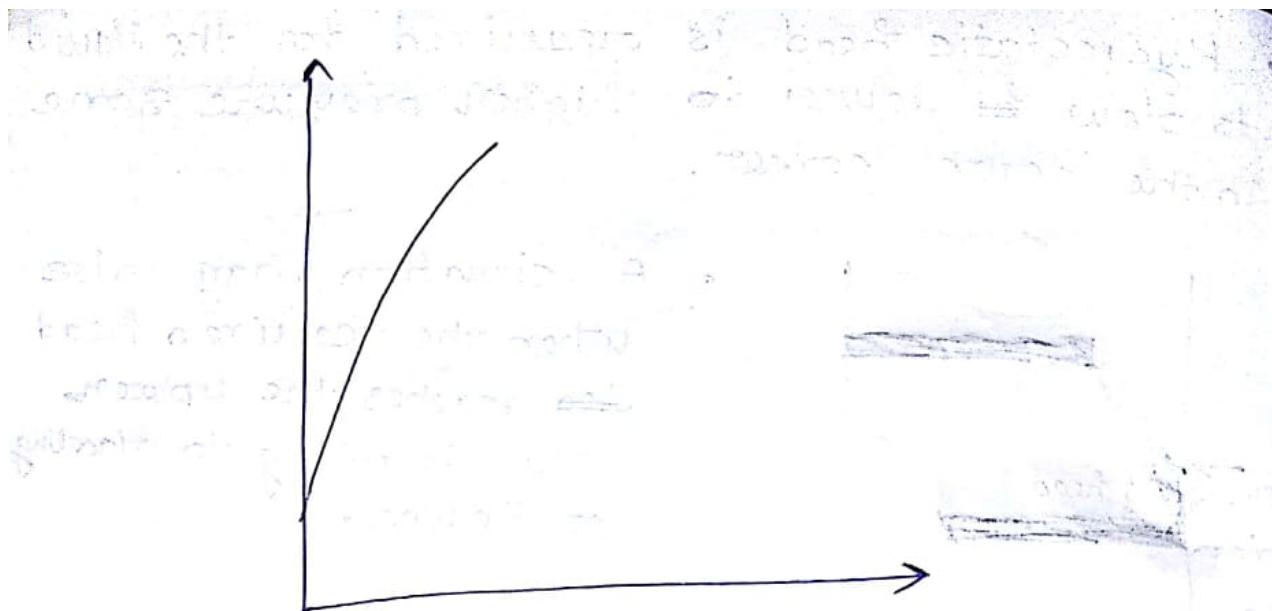
Coupling: High velocity of gas, low liquid flow rate



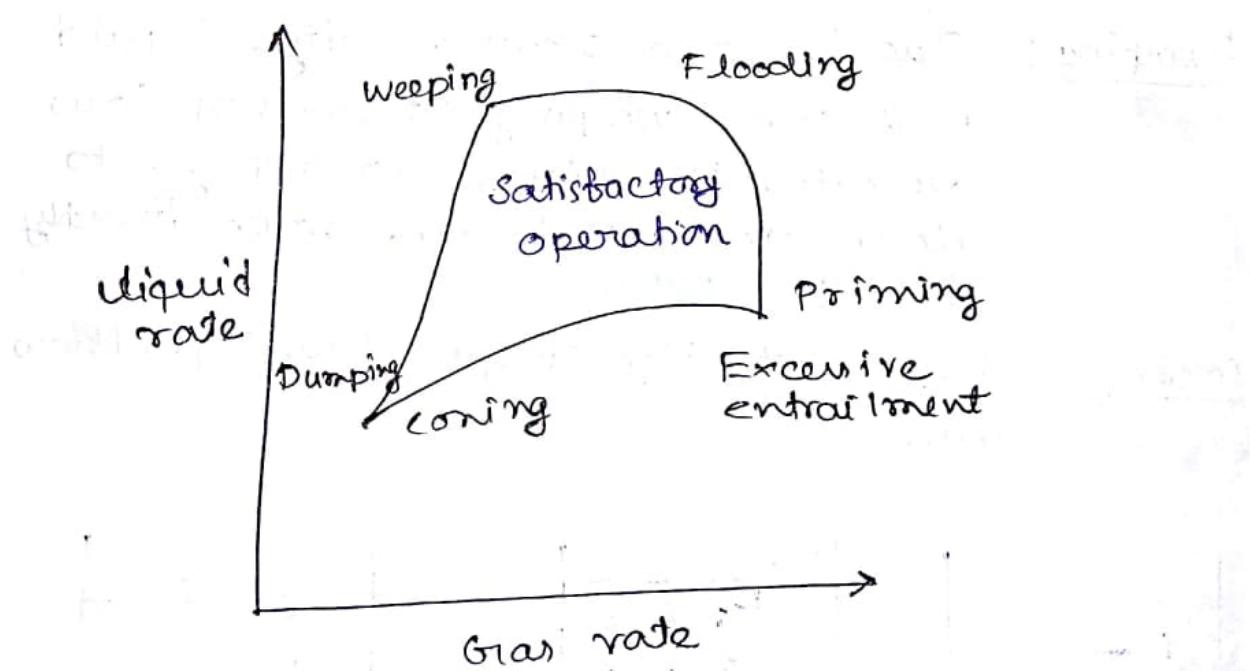
Priming

Dumping

coupling



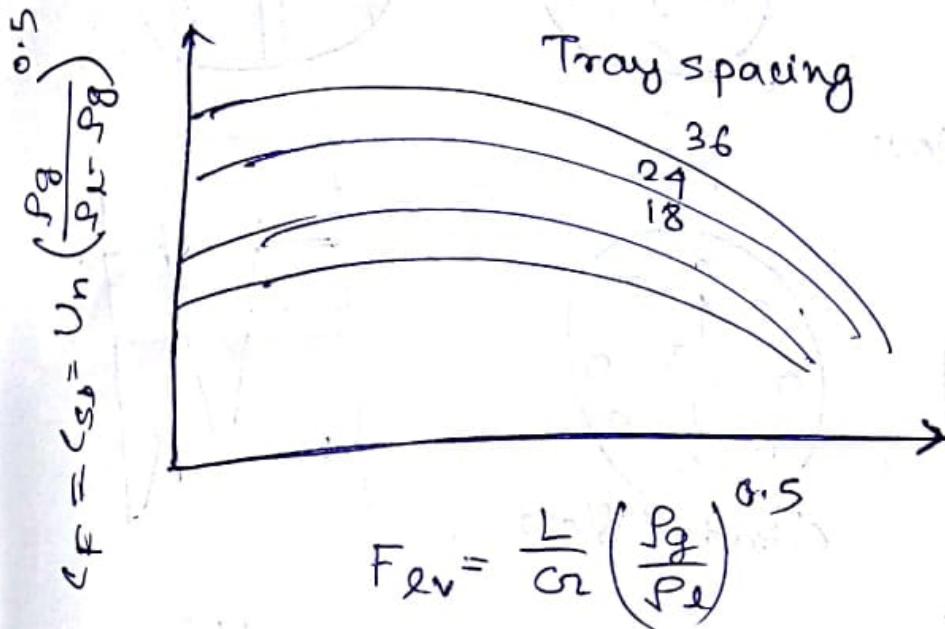
Graph of liquid rate vs. gas rate



overall efficiency  $\approx 50\%$

Real no. of trays =  $2 \times$  Ideal no. of trays

Correlation of flooding velocity in bubble-cap column and perforated column by Fair and Matthews



$$F_{cv} = \frac{L}{c_2} \left( \frac{P_g - P_e}{P_g - P_2} \right)^{0.5}$$

Actual velocity < flooding velocity

For nonboiling liquid

Actual velocity  $\approx$  85% of flooding velocity

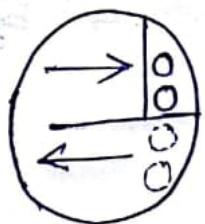
For boiling liquid

Actual velocity  $\lesssim$  75% of flooding velocity

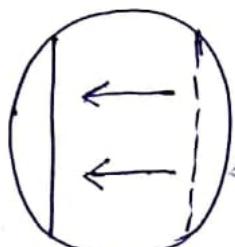
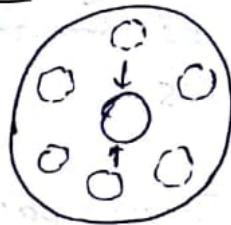
Gas flow rate  $\Rightarrow$  Area required

design area  $\approx$  1.2  $\xrightarrow{\text{calculated area}}$

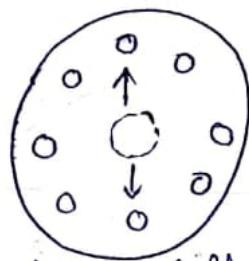
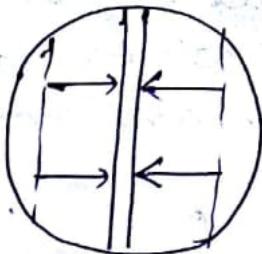
## Tray Arrangements



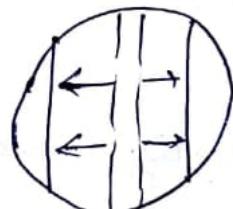
a) Reversible flow



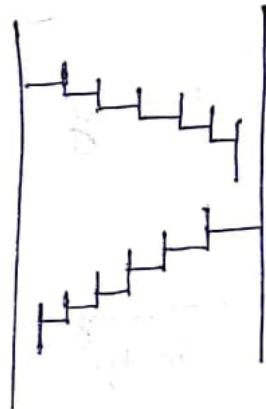
b) Cross flow



c) Radial flow



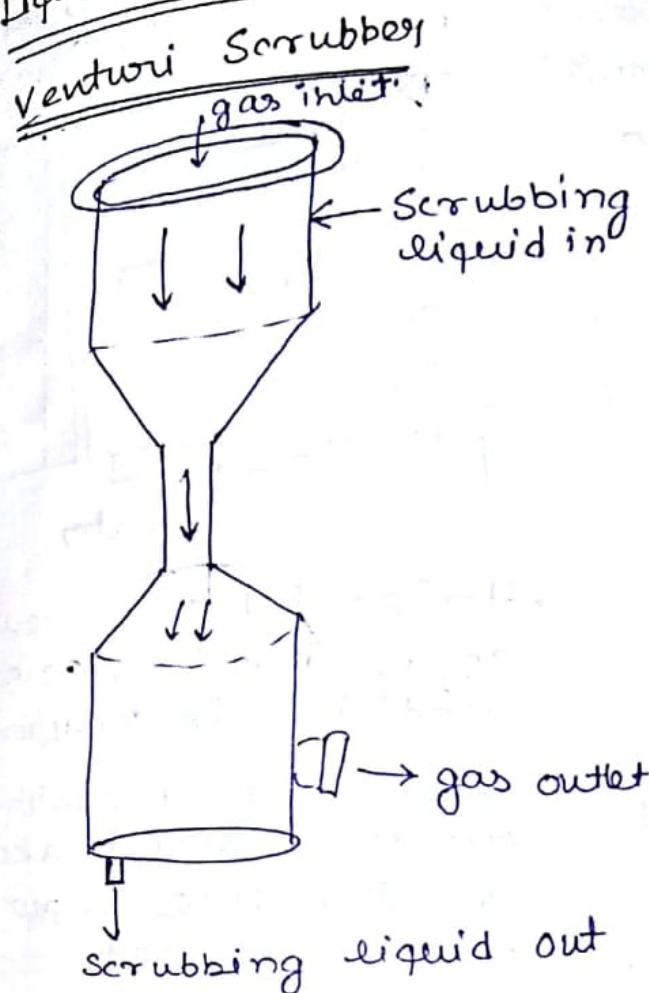
d) Split flow



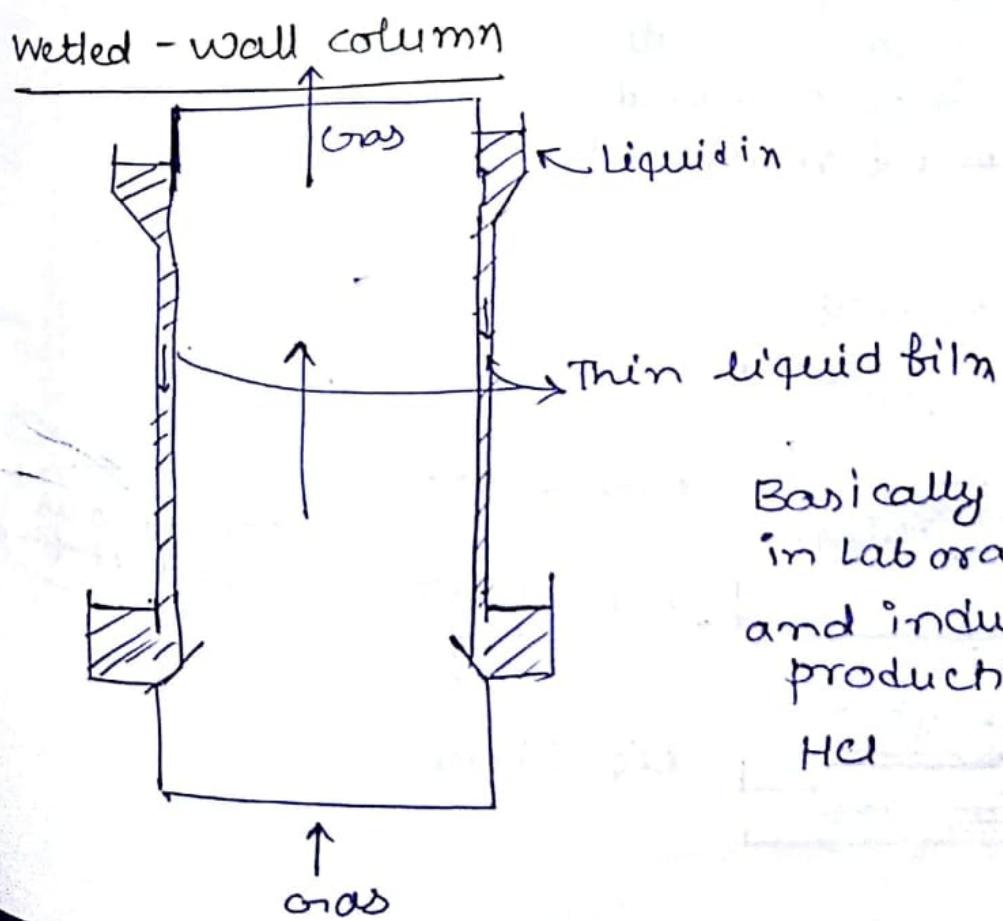
e) Cascade trays

Cascade trays are used to maintain almost equal liquid level. In a single tray, the height of liquid varies resulting in non uniform gas distribution.

## Liquid Dispersed type gas-liquid contactor

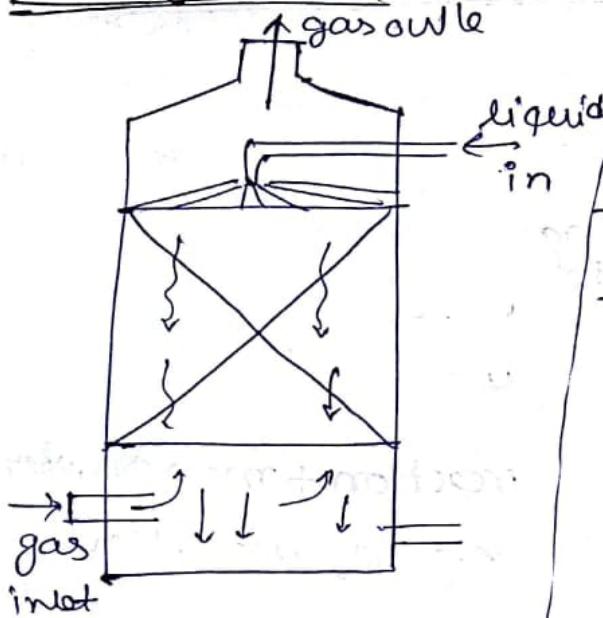


reaction + mass transfer  
e.g.  $\text{SO}_2$  adsorption

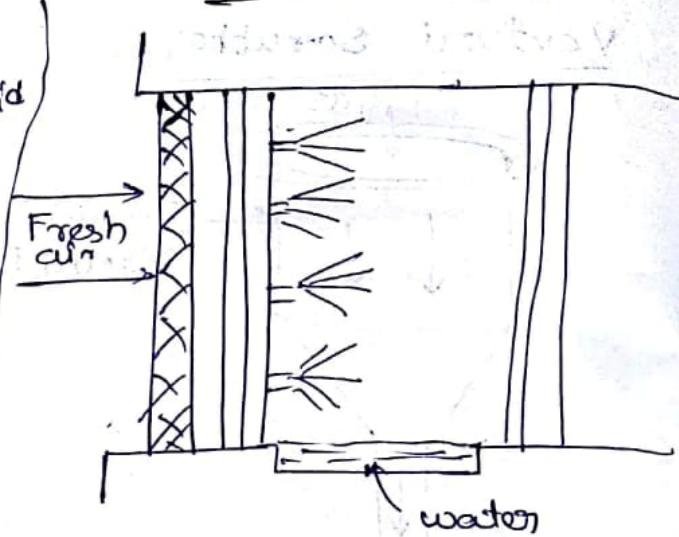


Basically used  
in laboratory  
and industrial  
production of  
 $\text{HCl}$

## Spray Tower



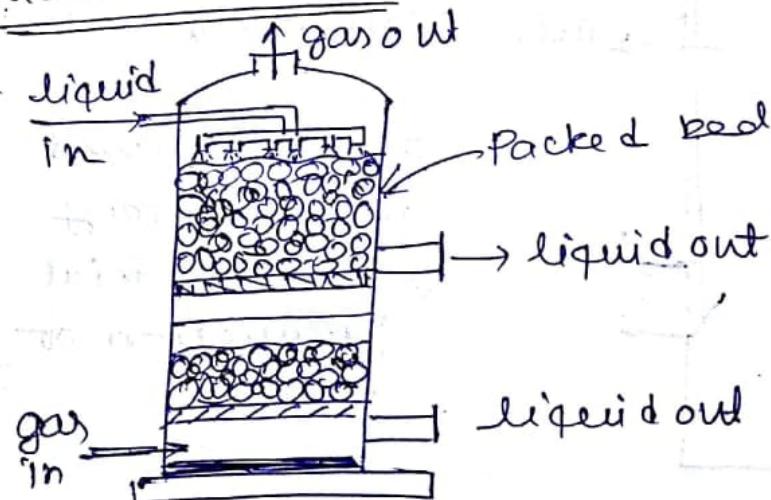
## Spray chamber



- For large diameter tower there is batch mixing, reducing counter-current benefit
- For very low diameter, the liquid drops will come very fast to wall and the flow downward in ~~dropwise~~ film wise manner

- Horizontal co-current spray chambers are used in air conditioning
- Co-current or counter-current effect makes no difference as pure liquid is used

## Packed towers



continuous contactor

## Packing Material

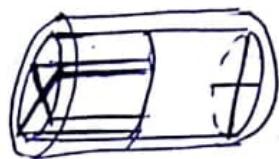
Rasching ring



Lewing Ring



Cross partition ring



Pall ring

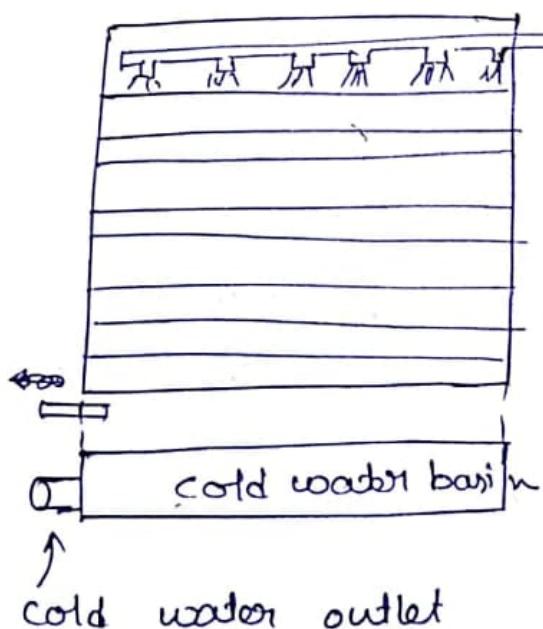


Saddles.

Tellerettes

- Metal
- Plastic

## Cooling Towers

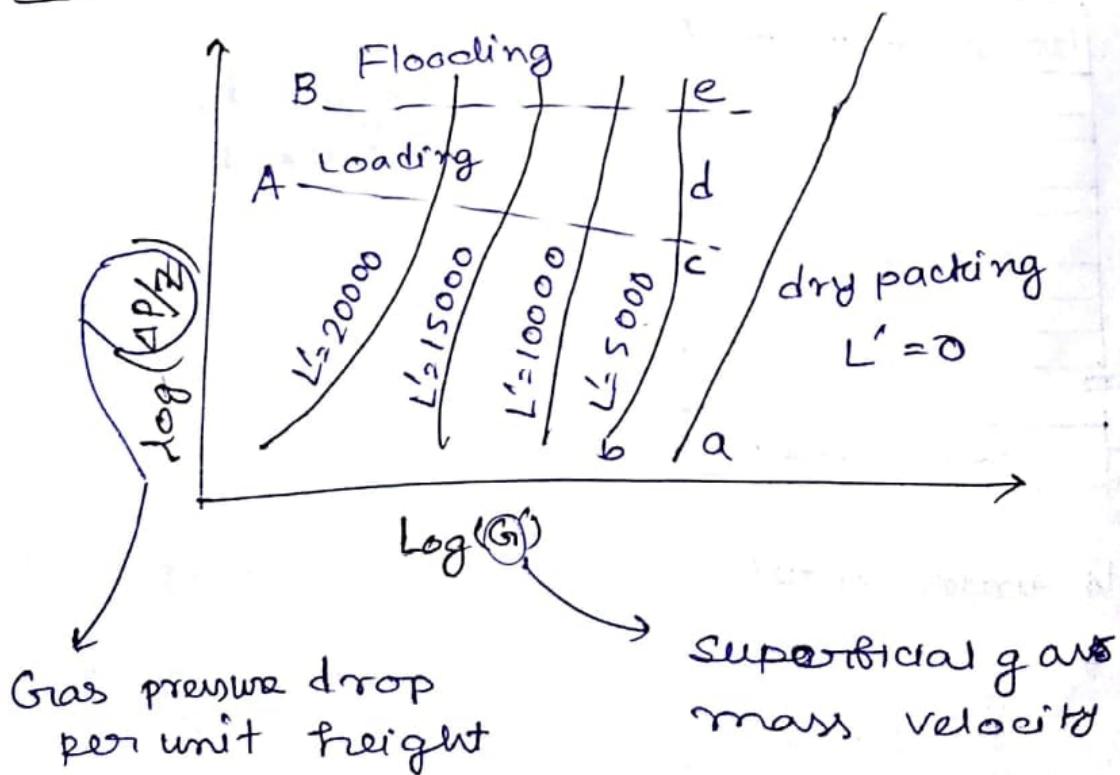


Regularly packed

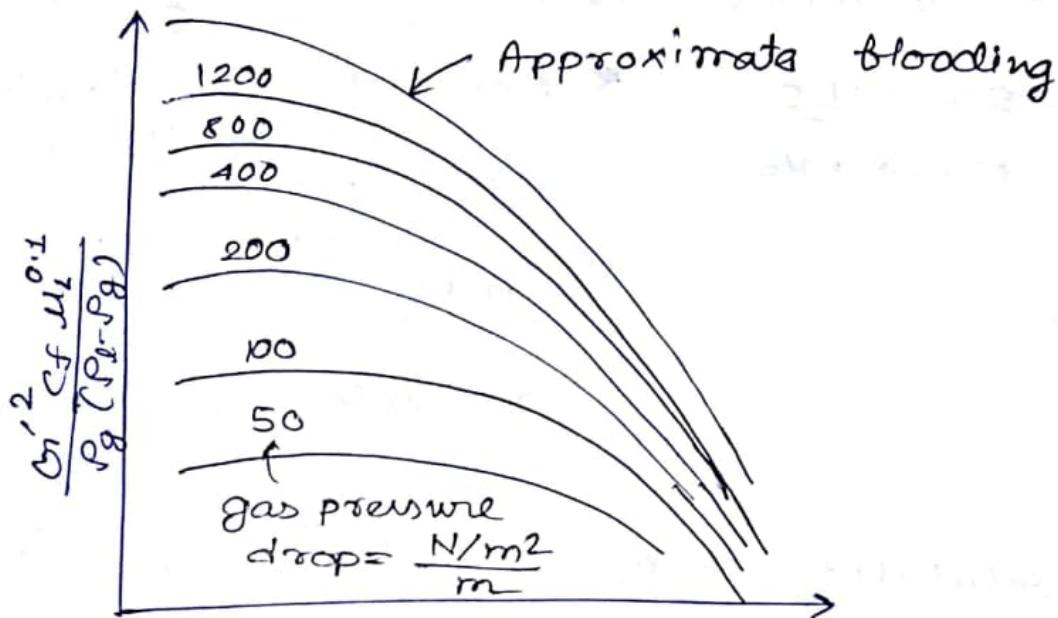
## Tower packing characteristics

- Provide large interfacial surface between liquid and gas. The surface of packing per unit volume ( $a_p$ ) should be zero
- Void volume, or fraction should be large to allow flow of gas & liquid without enough high pressure drop
- chemically inert to the fluids to be processed
- Having structural strength to permit easy handling and installation
- Represent low cost

## Loading and flooding in packed towers



liquid hold up beyond ~~point~~ line A increases leading to higher pressure drop. When there is 100% flooding, gas becomes dispersed phase instead of liquid



$$\frac{L'}{G'} \left( \frac{\rho_A}{\rho_L - \rho_A} \right)^{1/2}$$

Plot: Pressure Drop and flooding in random-packed towers

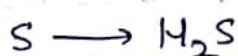
Tray tower vs. Packed tower.

- ~~small~~ Packed towers require smaller pressure drop
- Packed towers provide substantially smaller liquid hold up
- low  $L/G$  best suited in tray towers
- higher  $L/G$  " " " " packed "
- Tray towers and suitable for cooling
- Tray towers suitable for side streams
- Foaming suitable for Packed
- Corrosion - packed tower
- Cleaning - tray towers

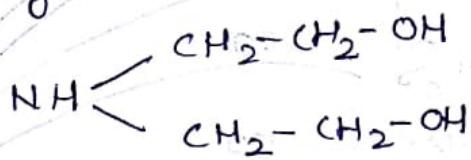
## Gas Absorption

### & Desorption or stripping

- \* Hydrotreating is used to remove sulphur from crude diesel fraction by reacting with hydrogen.

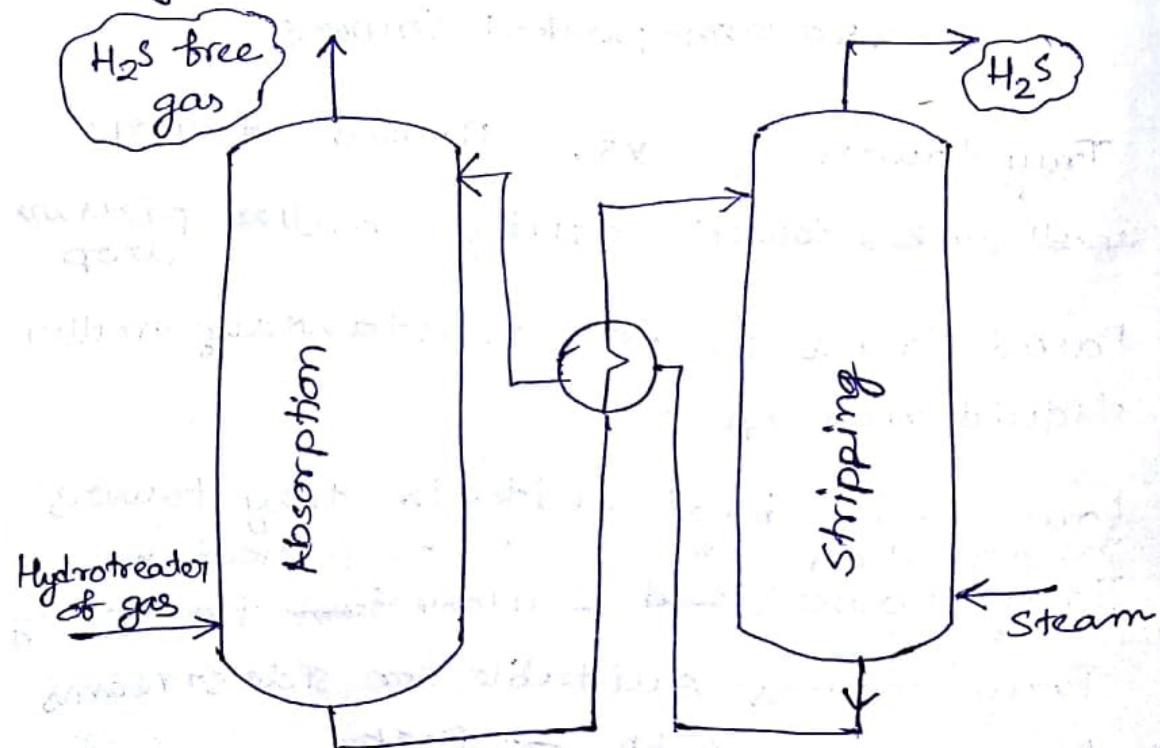


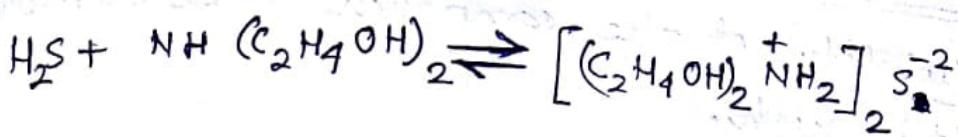
- \* Aqueous alkanolamine solvents are used, e.g. DEA



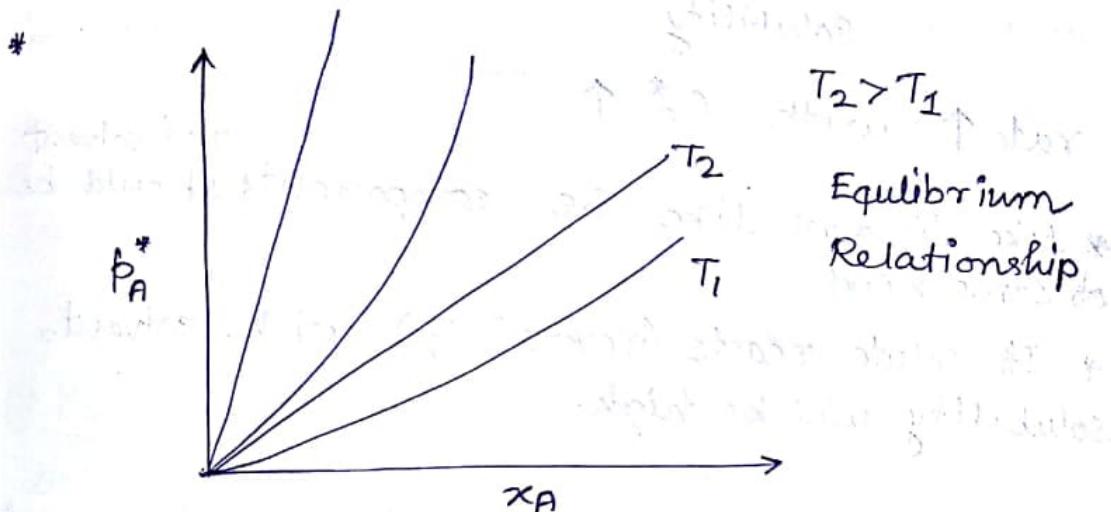
for selective removal of  $H_2S$

- \* Solubility of  $H_2S$  in DEA is very high
- \* Desorption is required to regenerate the solvent
- \* They → (Absorption as well as desorption)
- \* They are performed side by side in two columns





\* Further the phases are away from equilibrium higher will be the mass transfer rate.



\* gas adsorption is accompanied with generation of heat. With increasing temperature, solubility decreases

\* For ideal soln and ideal gas system

Raoult's law is used  $P_A^* = P_A x_A$

\* For dilute solution, Henry's Law is used

$$y_A^* = \frac{P_A}{P_t} = m_A x_A$$

## Properties of Absorbent solvent

1) Solubility of the component to be removed should be high.

$$\text{Rate} = k_L a (c_A^* - c_{A,B})$$

↓  
Solubility

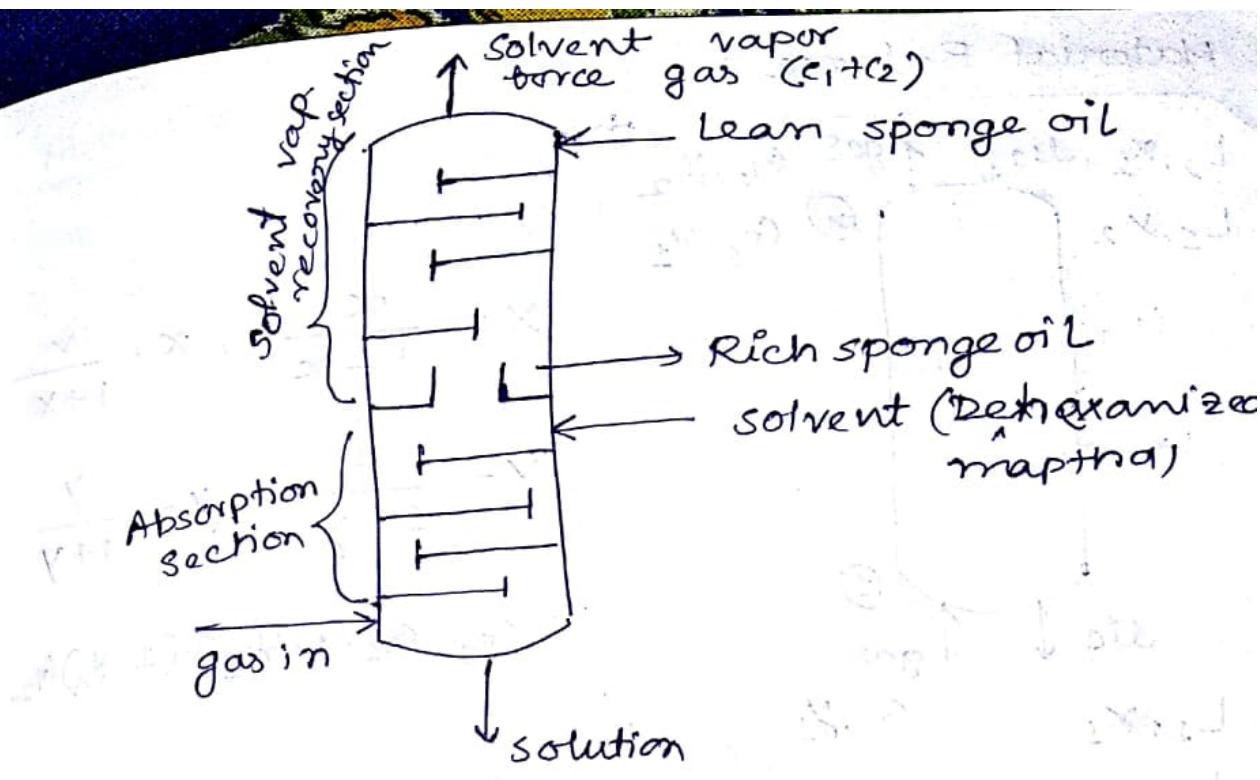
rate  $\uparrow$  with  $c_A^* \uparrow$

\* like dissolves like i.e. components should be of same kind

\* If solute reacts (reversibly) with solvent, solubility will be high.

2) Volatility of solvent should be low to prevent solvent loss. The solvent should not be perfectly non-volatile as that will result in higher boiling point leading to low solubility,

Gas processing unit: Solvent should be of higher volatility, e.g. Dehexanized naptha. In this case loss of solvent due to evaporation is high. There is a need of solvent recovery section in the section



③ Corrosiveness

Solvent should not be corrosive.

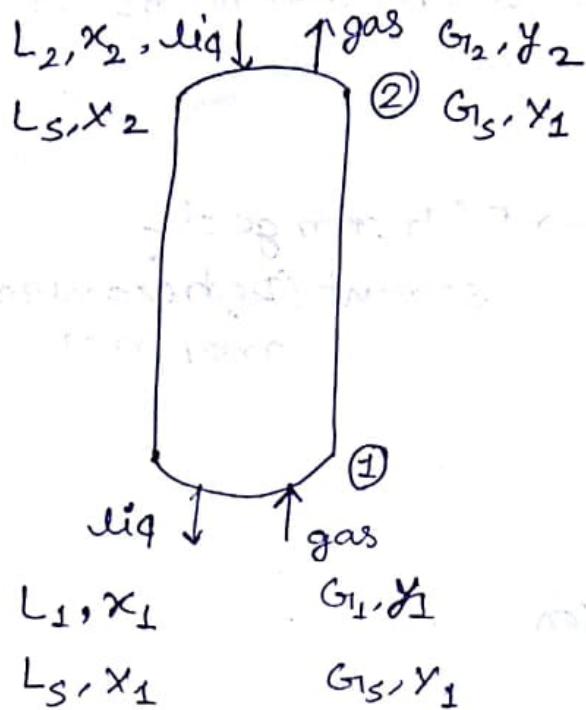
④ Viscosity of solvent should be low

⑤ Freezing point of the solvent should be low

⑥ Cost

⑦ Miscellaneous

## Material Balance



$$x = \frac{x}{1-x}; x = \frac{x}{1+x}$$

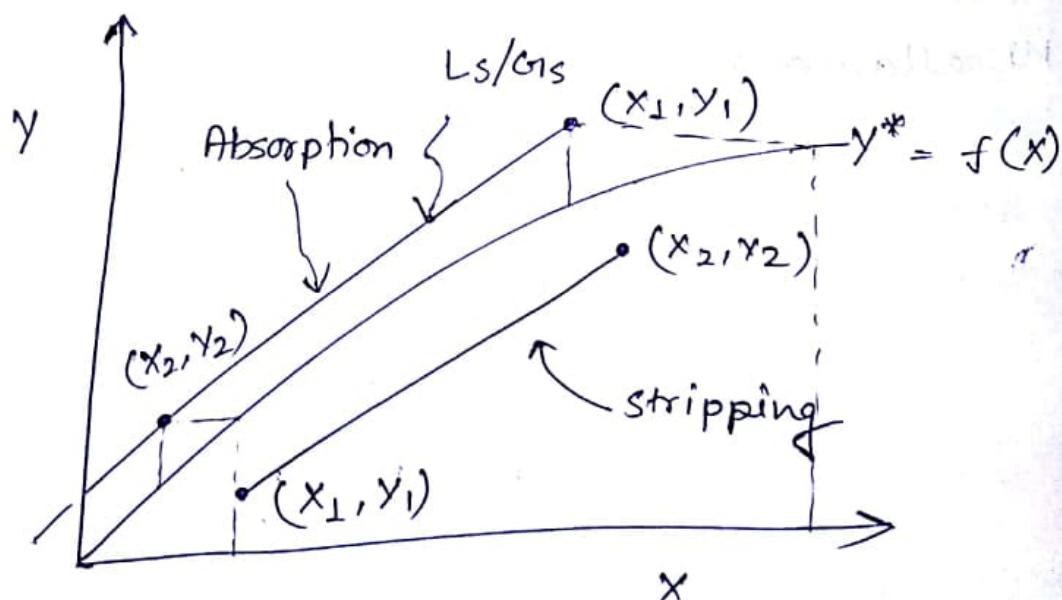
$$y = \frac{y}{1-y}; y = \frac{y}{1+y}$$

$$G_S = G_1 (1 - y_1) = (1 - y_2) G_2$$

$$G_S y_1 + L_S x_2 = G_S Y_2 + L_S X_1$$

$$G_S (y_1 - y_2) = L_S (x_1 - x_2)$$

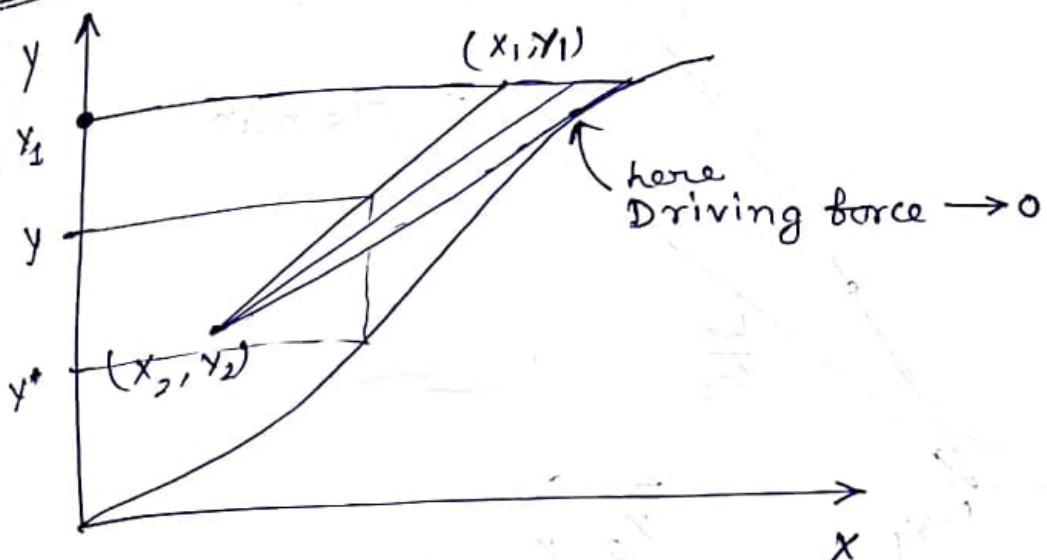
$$y_1 - y_2 = \frac{L_S}{G_S} (x_1 - x_2)$$



$$(y_1 - y) = \frac{L_S}{G_S} (x_1 - x)$$

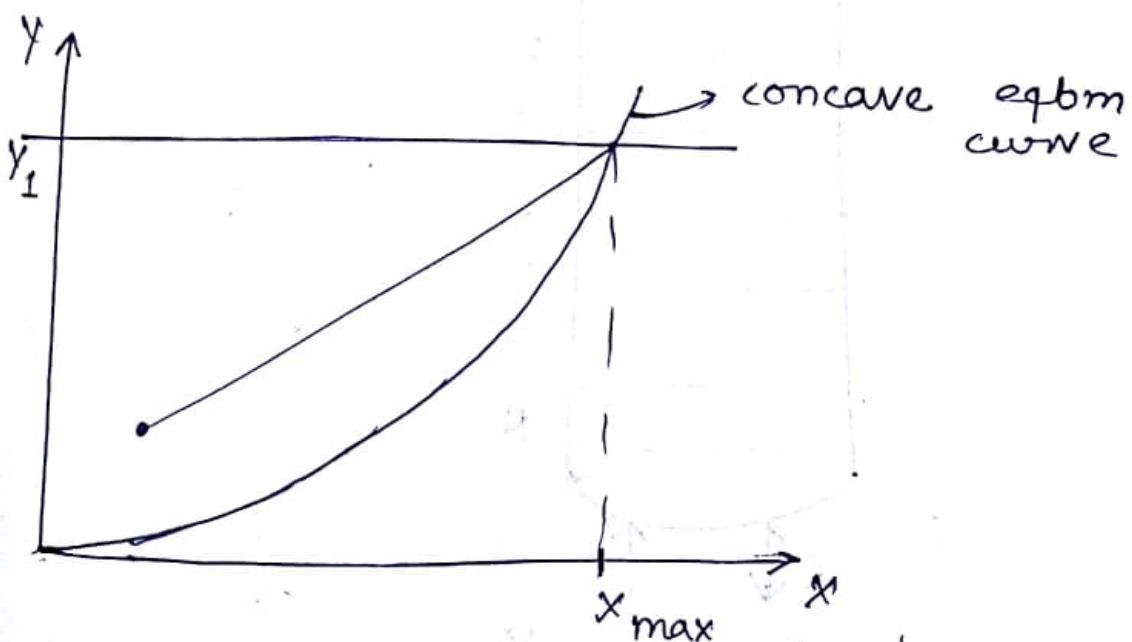
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Absorption



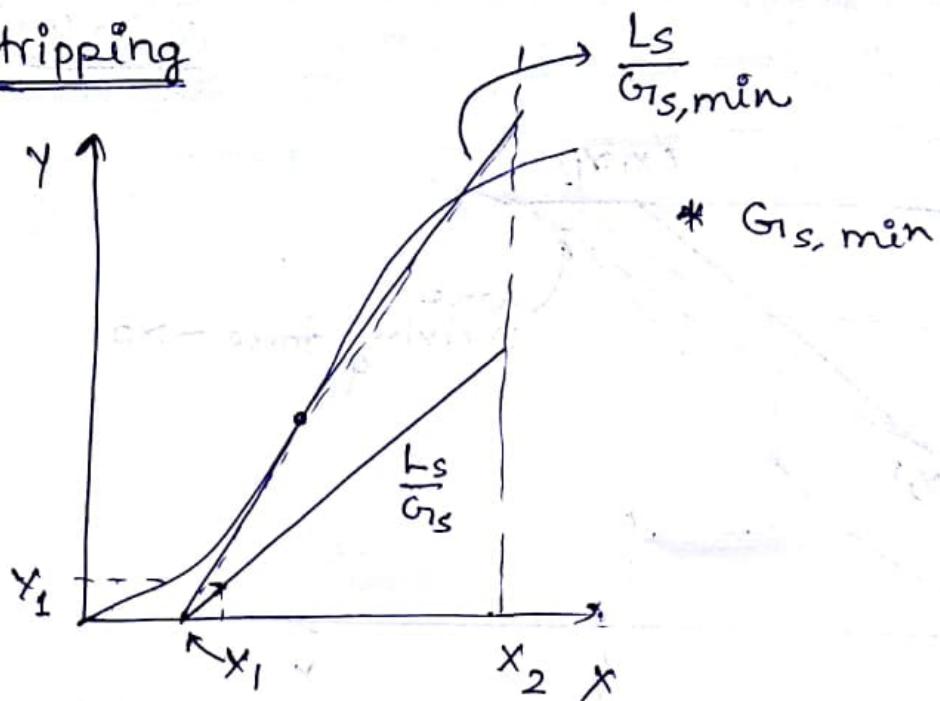
\* The liquid rate at which the operating line touches the equilibrium curve, is called minimum liquid rate

$$* L_{s, \min} \times (1.2 \text{ to } 2) = L_s \quad \text{Actual liquid rate}$$



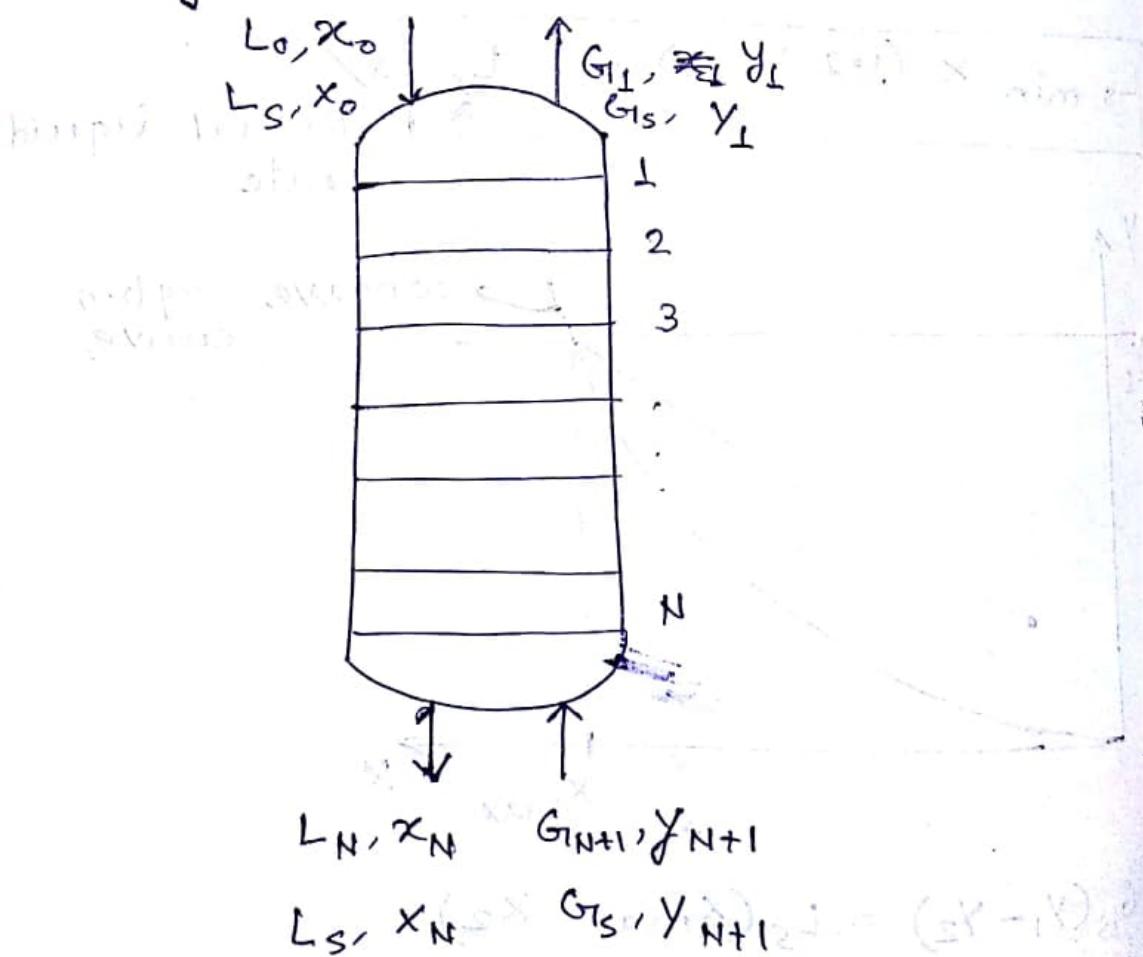
$$G_s(y_1 - y_2) = L_s(x_{\max} - x_2)$$

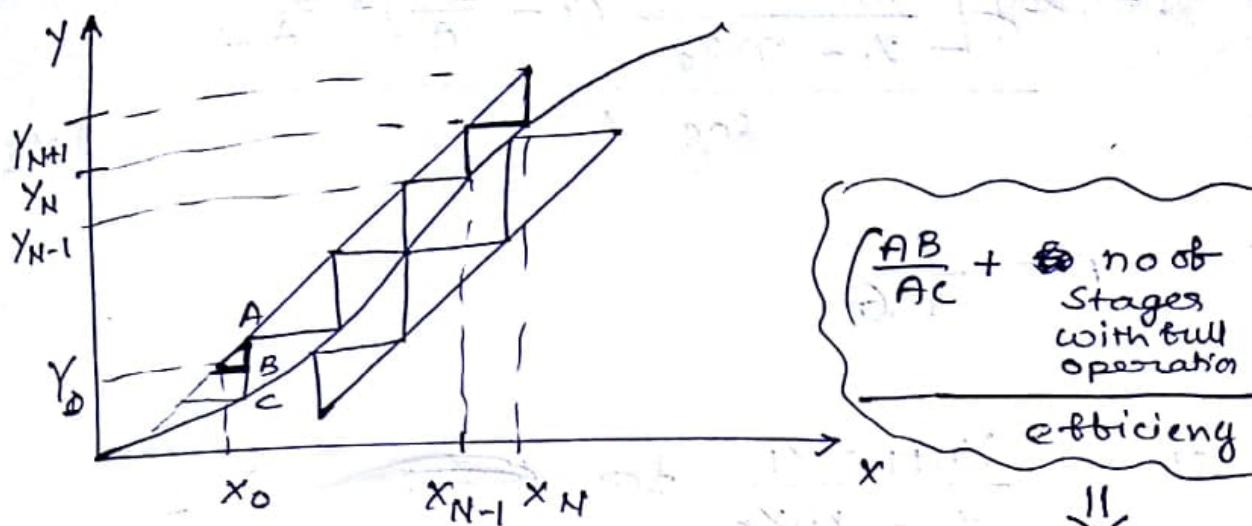
## Stripping



stripping is the reverse of absorption

\* co-current process is preferred in case of pure gas and reversible reaction or fast reaction





$\frac{AB}{AC} + \text{no of stages with full operation}$

efficiency

Roundup  
to next whole  
no-

For dilute system

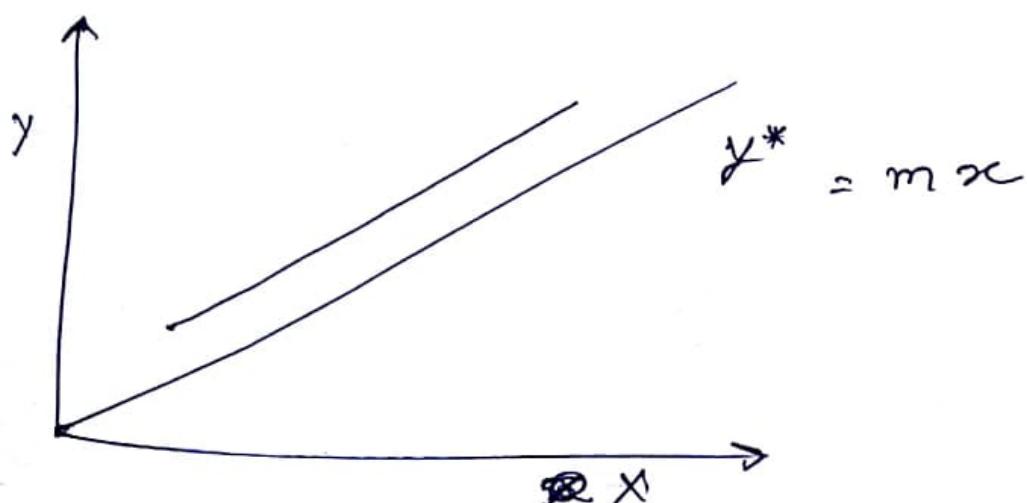
$$y = \frac{y}{1-y} \approx y$$

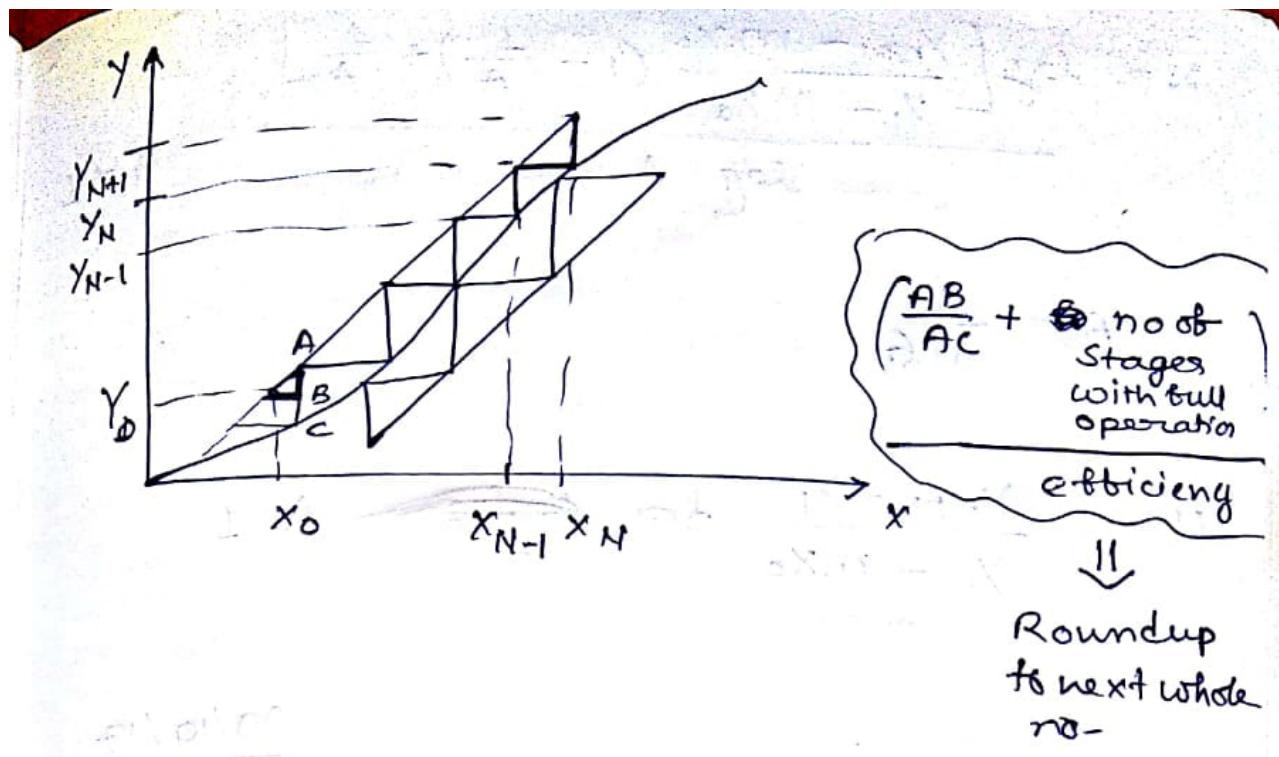
$$x = \frac{x}{1-x} \approx x$$

$$L_1 \approx L_2 = L$$

$$G_{11} \approx G_{12} = G_1$$

Use Kremser's eq<sup>n</sup>





For dilute system

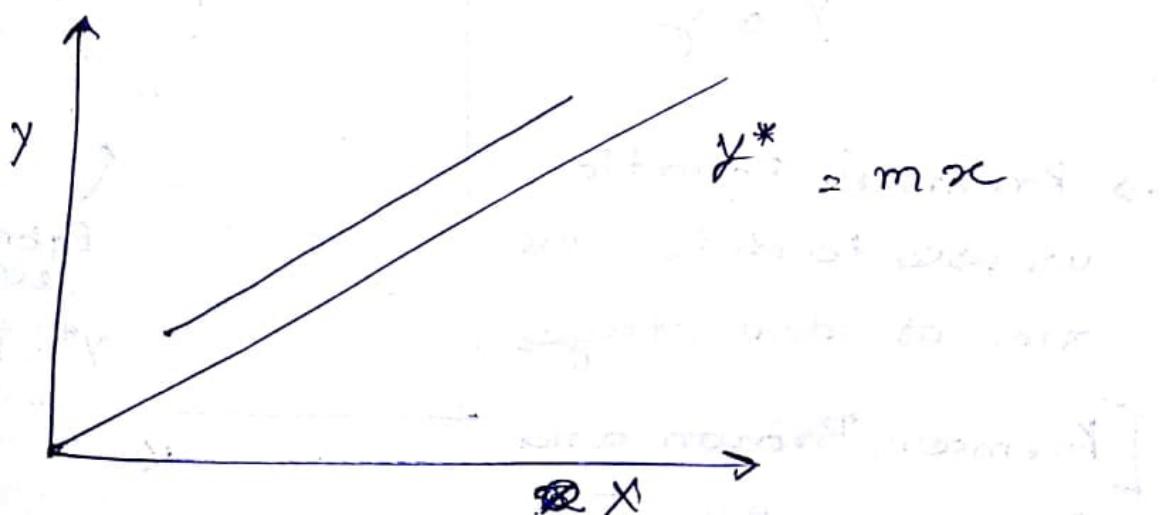
$$y = \frac{y}{1-y} \approx y$$

$$x = \frac{x}{1-x} \approx x$$

$$L_1 \approx L_2 = L$$

$$G_{11} \approx G_{12} = G_1$$

Use Kremser's eqn



$$N = \frac{\log \left[ \frac{y_{n+1} - mx_0}{y_1 - mx_0} \left( 1 - \frac{1}{A} \right) + \frac{1}{A} \right]}{\log A}$$

$$A = \frac{L}{mG}$$

$$N = \frac{y_{n+1} - y_1}{y_1 - mx_0} \text{ for } \cancel{A=1}$$

09/10/17

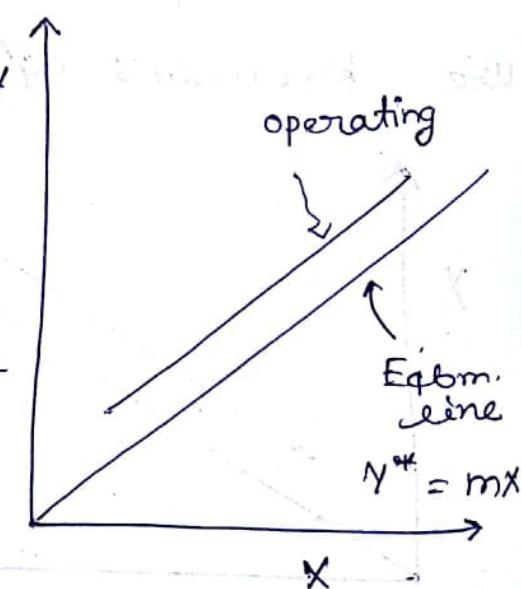
### Design of Tray Tower for Dilute Gas

- Operating line is straight line (both  $y$  vs  $x$  and  $Y$  vs.  $X$ )
- For concentrated gas operating line is st. line <sup>not</sup>
- Dilute gas will follow Henry's Law

$$y^* = mx$$

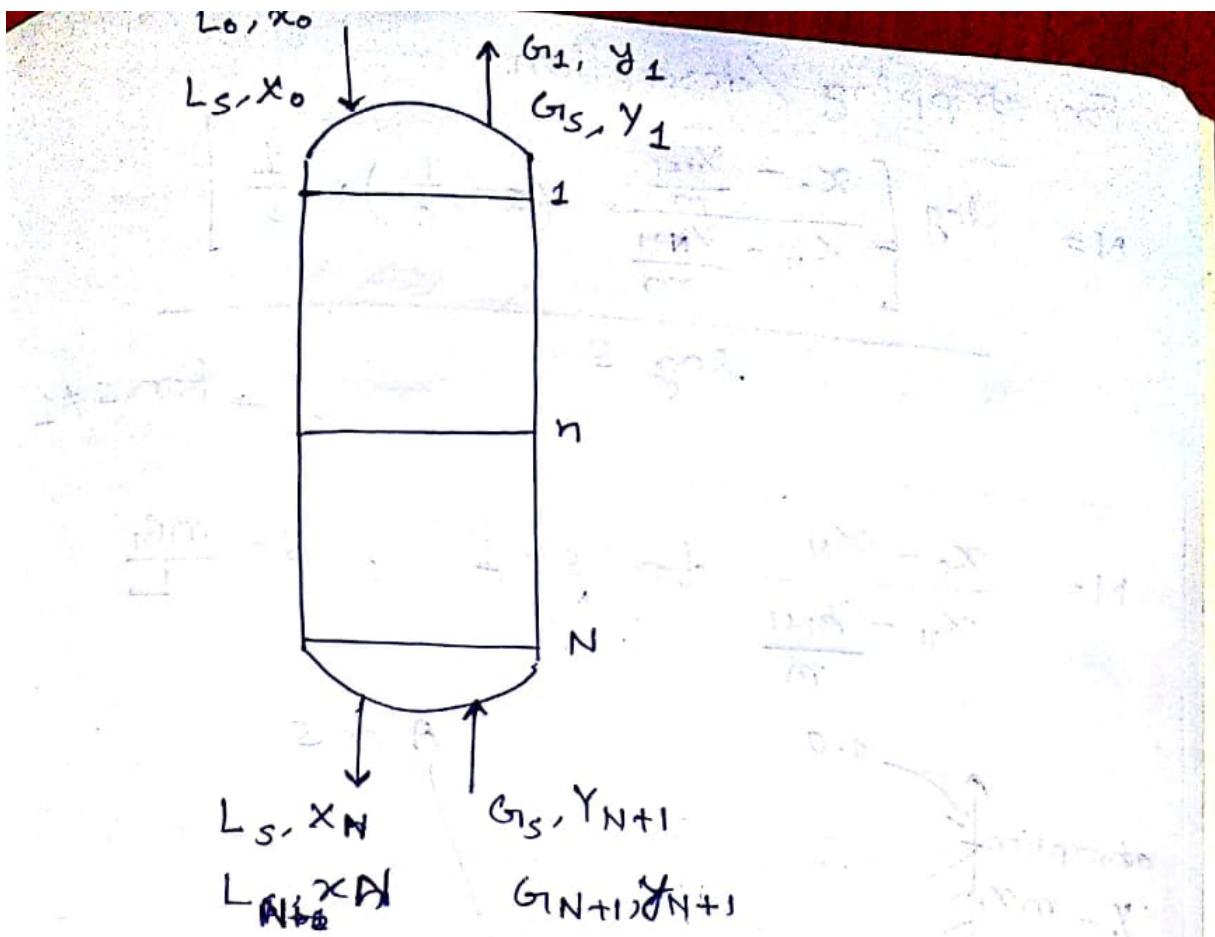
where  $x \approx x$

$$y \approx Y$$



- Kremser's Equation is used to determine no. of ideal stages

[Kremser, Brown and Sonderis Eq<sup>n</sup>]



For absorption:

$$N = \frac{\log \left[ \frac{Y_{N+1} - mX_0}{Y_1 - mX_0} \cdot \left( 1 - \frac{1}{A} \right) + \frac{1}{A} \right]}{\log A} \quad \text{for } A \neq 1$$

$$A = \frac{L}{mG_1}$$

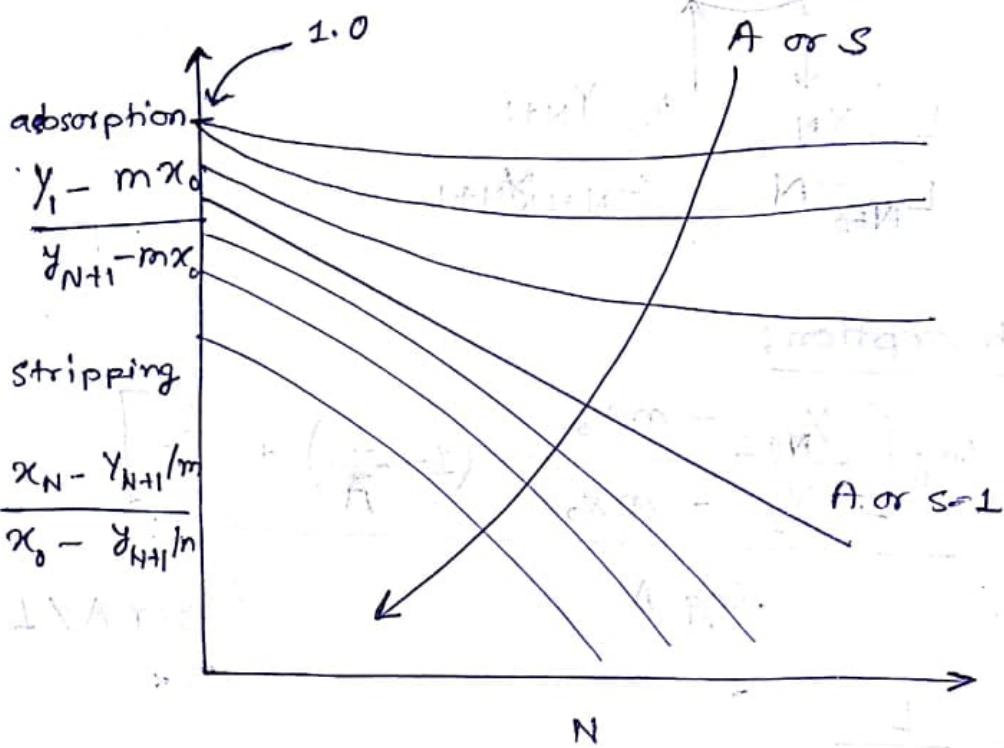
for  $A = 1$

$$N = \frac{Y_{N+1} - Y_1}{Y_1 - mX_0} \quad \text{for } A = 1$$

For stripping / desorption

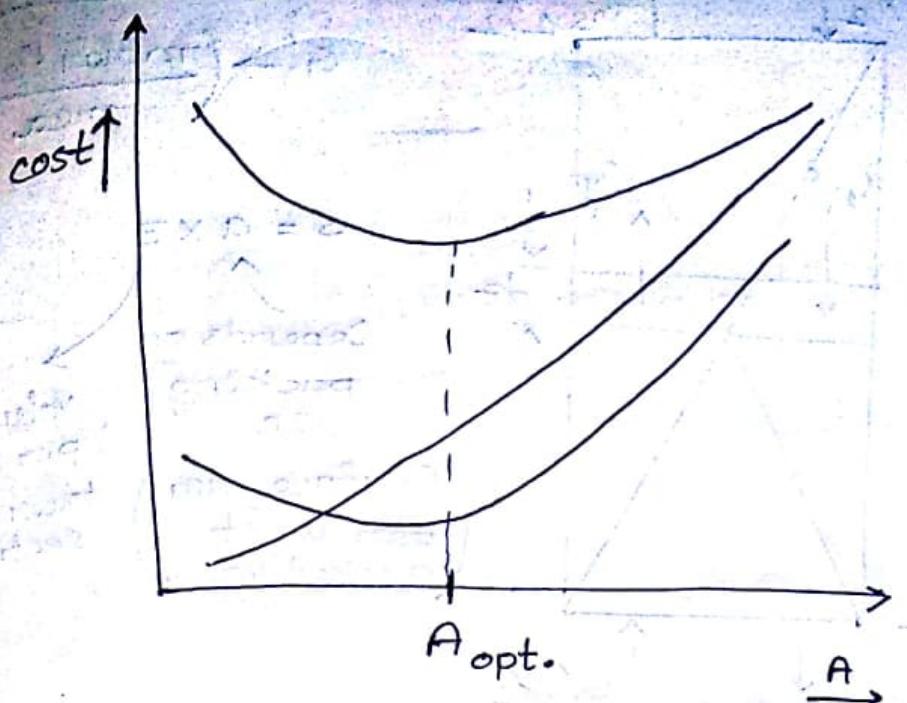
$$N = \frac{\log \left[ \frac{x_0 - \frac{y_{N+1}}{m}}{x_N - \frac{y_{N+1}}{m}} \cdot \left( 1 - \frac{1}{s} \right) + \frac{1}{s} \right]}{\log s} \quad \text{for } s \neq 1$$

$$N = \frac{x_0 - x_N}{x_N - \frac{y_{N+1}}{m}} \quad \text{for } s = 1, \quad s = \frac{mG}{L}$$



→ For  $A$  or  $s < 1$ , the upper part of operating line can be approximated as the eqbm. curve.

→ For increased value of  $A$ , solution will be dilute i.e.  $L$  will be high. So, regeneration of solvent from dilute sol<sup>n</sup> is harder and economically not feasible.



→ In general,  $A_{opt} \in (1.25, 2.0)$  ✓

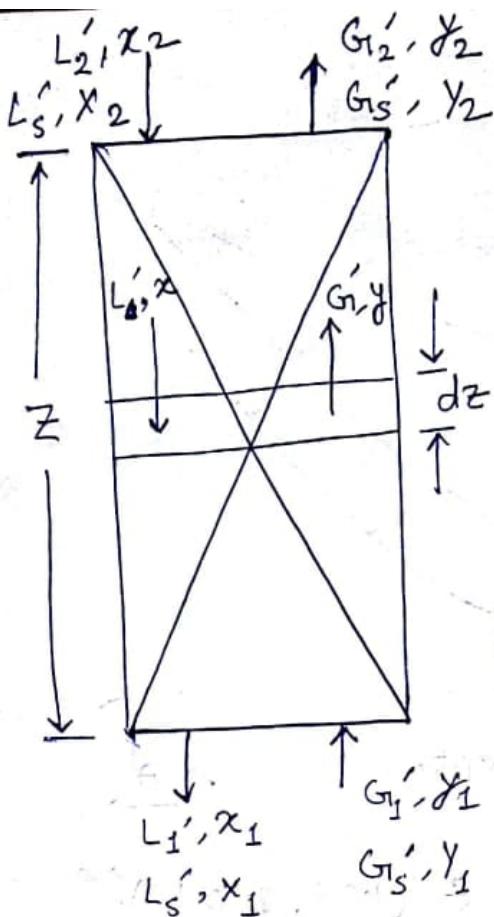
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## Design of continuous Contact Equipment

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(Packed Tower)

- In reality, there is change in concentration in trays, discontinuously
- In continuous contactor, there is continuous change in concentration of both gas and liquid.
- HETP (Height equivalent theoretical plates) method is used to ~~design~~ design a packed bed contactor
- HETP does not consider the difference between operations of stage wise and continuous contact equipment.
- Transfer Unit was introduced by Coulburn in 1939



TRANSFER UNIT

$G_i' \rightarrow$  (molar flowrate area)

$$S = a \times z$$

Depends on packing size

(Surface area per unit packed volume)

Volume per unit tower cross section

$$ds = a dz$$

$$-d(G'_y) = N_A ds = N_A a dz$$

$$\text{or, } -G'_y dy - y dG'_y = N_A a dz$$

$$\text{or, } -G'_y dy - y N_A a dz = N_A a dz$$

$$\text{or, } -G'_y dy = (1-y) N_A a dz$$

$$\text{(or, } -\frac{G'_y dy}{1-y} = N_A a dz)$$

$$-dG'_y = N_A a dz$$

$$-\cancel{d(G'_y)} = \cancel{d(G'_y)}$$

$$-d(G'_y)$$

$$= -d\left(\frac{G'_y y}{1-y}\right)$$

$$= -G'_y d\left(\frac{y}{1-y}\right)$$

$$= -\frac{G'_y}{(1-y)} \frac{dy}{(1-y)^2}$$

$$= -\frac{G'_y}{(1-y)} \frac{dy}{1-y}$$

$$= -G'_y \frac{dy}{1-y}$$

$$-\frac{G' dy}{(1-y)} = k_y \alpha (y - y_i) dz$$

$$\Rightarrow dz = -\int \frac{G' dy}{k_y \alpha (1-y)(y-y_i)}$$

0  $y_1$   $y_2$  almost constant throughout the tower

$$z = \int_{y_2}^{y_1} \frac{G' dy}{k_y \alpha (1-y)(y-y_i)}$$

$\frac{G'}{k_y \alpha y_{BM}}$   $\rightarrow$  almost constant throughout the tower

$$y_{BM} = \frac{(1-y_i) - (1-y)}{\ln(\frac{1-y_i}{1-y})}$$

$$z = \frac{H_{tg}}{k_y \alpha y_{BM}} \int_{y_2}^{y_1} \frac{dy}{(1-y)(y-y_i)} = H_{tg} \times N_{tg}$$

$$= \frac{G'}{k_y \alpha y_{BM}} \int_{y_2}^{y_1} \frac{(y-y_i)}{\left[ \ln\left(\frac{1-y_i}{1-y}\right) \right] [(1-y)(y-y_i)]} dy$$

$$= \frac{G'}{k_y \alpha y_{BM}} \int_{y_2}^{y_1} \frac{dy}{(1-y) \ln(\frac{1-y_i}{1-y})}$$

$$\text{Usually, } G_i' = \frac{(G_{i1}' + G_{i2}')}{2}$$

$$Y_{BM} = \frac{(Y_{BM1} + Y_{BM2})}{2}$$

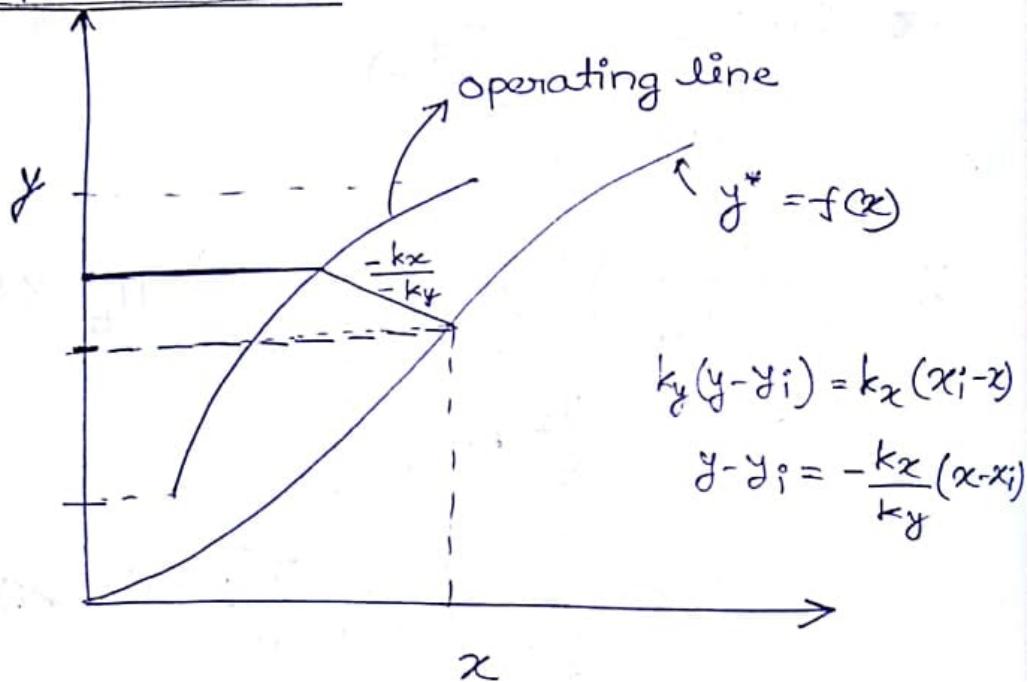
$H_{tg} \rightarrow$  Height of gas phase transfer unit

$N_{tg} \rightarrow$  Number of gas phase transfer unit

$\rightarrow$  Relative ease of mass transfer operation  
 $H_{tg}$

$N_{tg} \rightarrow$  Difficulty involved in achievement of desired degree of separation

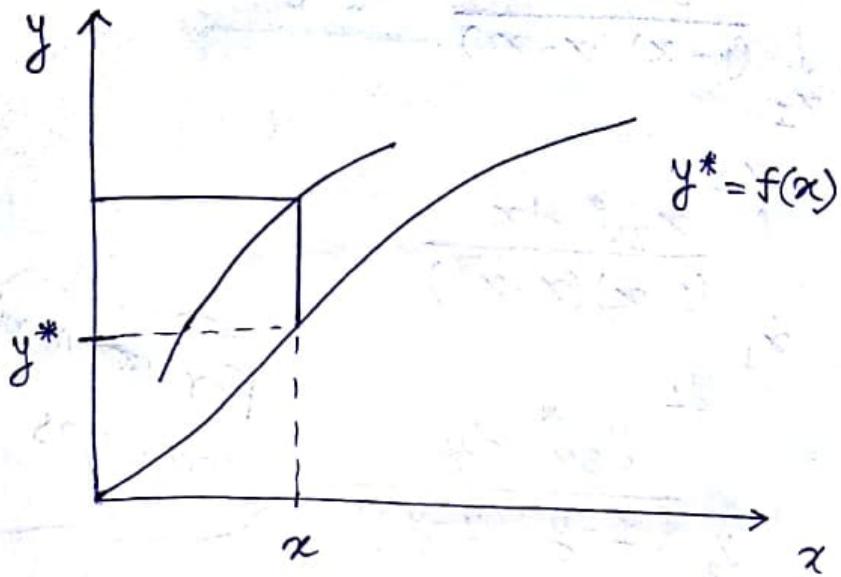
### Graphical Method



$$\rightarrow Z = \underbrace{H_{tg} \times N_{tg}}_{\text{Based on overall transfer coefficient}}$$

$$= \frac{G_i'}{K_y a Y_{BM}^*} \int_{y_2}^{y_1} \frac{Y_{BM}^* dy}{(1-y)(y-y^*)}$$

$$y_{BM}^* = \frac{(1-y^*) - (1-y)}{\ln \frac{(1-y^*)}{(1-y)}}$$



$$\bar{x} = H_{tL} \times N_{tL} = H_{t_{0L}} \times N_{t_{0L}}$$

$$H_{tL} = \frac{L'}{k_x a x_{BM}} \quad N_{tL} = \int_{x_2}^{x_1} \frac{x_{BM} dx}{(1-x)(x-x)}$$

$$H_{t_{0L}} = \frac{L'}{k_x a x_{BM}^*} \quad N_{t_{0L}} = \int_{x_2}^{x_1} \frac{x_{BM} dx}{(1-x)(x^*-x)}$$

Stripped

$$N_{tg} = \int_{y_2}^{y_1} \frac{y_{BM} dy}{(1-y)(y-y_i)}$$

$$\approx \int_{y_2}^{y_1} \frac{dy}{y-y_i} = \frac{y_1 - y_2}{(y-y_i)_{av}}$$

$$y_{BM} \approx 1$$

$$1-y \approx 1$$

$$N_{tg} = 1, \quad y_1 - y_2 = (y - y_i^*)_{av}$$

Stripper

$$N_{tL} = \int_{x_1}^{x_2} \frac{x_{BM} dx}{(1-x)(x-x_i)}$$

$$N_{t_{OL}} = \int_{x_1}^{x_2} \frac{x_{BM}^* dx}{(1-x)(x-x^*)}$$

$$N_{t_{OG}} = \int_{y_2}^{y_1} \frac{y_{BM}^* dy}{(1-y)(y-y^*)}$$

For moderately conc. gas

$$y_{BM}^* = \frac{(1-y) + (1-y^*)}{2}$$

$$y_{BM}^* = \frac{2(1-y) + (y-y^*)}{2}$$

$$N_{t_{OG}} = \int_{y_2}^{y_1} \frac{dy}{y-y^*} + \frac{1}{2} \int_{y_2}^{y_1} \frac{dy}{1-y}$$

$$= \int_{y_2}^{y_1} \frac{dy}{y-y^*} + \frac{1}{2} \ln \left( \frac{1-y_2}{1-y_1} \right)$$

For dilute Gas (system)

$$N_{tG_1} = \int_{y_2}^{y_1} \frac{dy}{y - y_i}$$

$$N_{tG_2} = \int_{y_2}^{y_1} \frac{dy}{y - y^*}$$

→ In this range of  $y_1$  to  $y_2$  eqbm. line may be considered as straight line given by

$$y^* = mx + r$$

$$y = \frac{L}{G_1} (x - x_2) + y_2$$

$$\begin{aligned} y - y^* &= \left( \frac{L}{G_1} - m \right) x + \left( y_2 - \frac{L}{G_1} x_2 - r \right) \\ &= qx + s \end{aligned}$$

$$N_{tG_1} = \int_{y_2}^{y_1} \frac{dy}{qx + s} = \frac{L}{G_1} \int_{y_2}^{y_1} \frac{dx}{qx + s}$$

$$= \frac{L}{G_1 q} \ln \left( \frac{qx_1 + s}{qx_2 + s} \right)$$

$$= \frac{L}{G_1 q} \ln \left( \frac{y_1 - y_1^*}{y_2 - y_2^*} \right)$$

$$\frac{L}{Gq} = \frac{y_1 - y_2}{x_1 - x_2} \cdot \frac{1}{\left( \frac{y_1 - y_2}{x_1 - x_2} - m \right)} \quad q = \frac{L}{G} - m$$

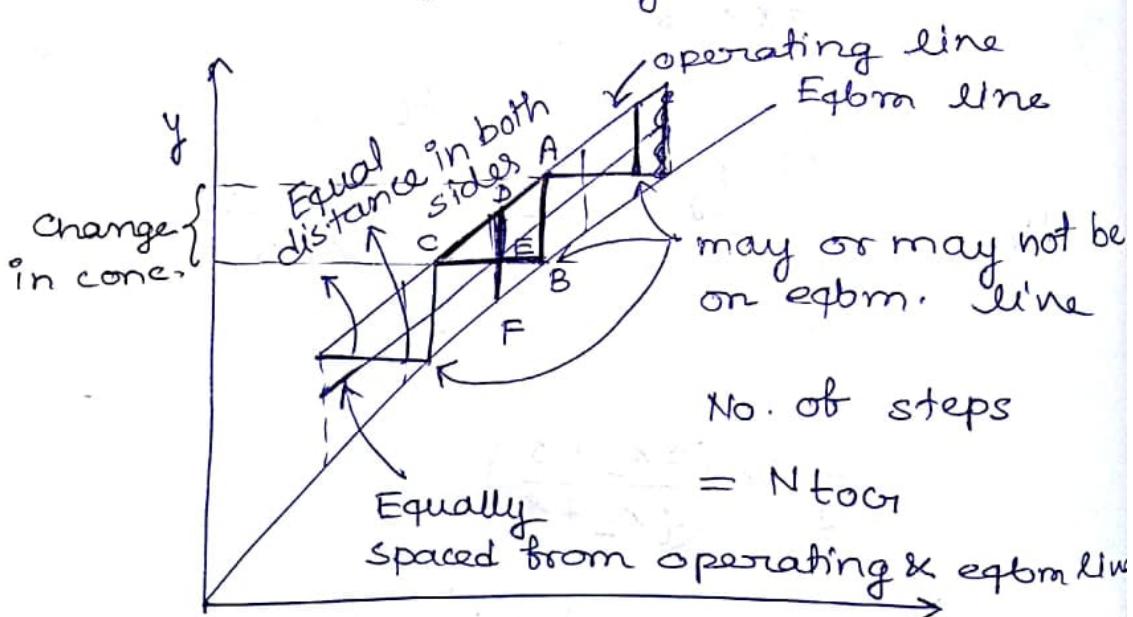
$$= \frac{y_1 - y_2}{(x_1 - x_2)(y_1 - y_2 - m(x_1 - x_2))} = \frac{y_1 - y_2}{(y_1 - mx_1 - n)(y_2 - mx_2 - n)}$$

$$= \frac{y_1 - y_2}{(y_1 - y_1^*) - (y_2 - y_2^*)}$$

$$N_{tocr} = \frac{\frac{y_1 - y_2}{(y_1 - y_1^*) - (y_2 - y_2^*)}}{\ln \left( \frac{y_1 - y_1^*}{y_2 - y_2^*} \right)} = \frac{y_1 - y_2}{(y - y^*)n}$$

$$N_{tocr} = \frac{\text{total change in cone.}}{\text{average driving force}}$$

12/10/17



$A B \equiv$  total change in concentration

$\left. \begin{array}{l} x \\ DF \end{array} \right\} \equiv$  overall driving force

$$\frac{AB}{BC} = \frac{DE}{CE}$$

$$AB = \frac{BC}{CE} \cdot DE = 2DE$$

$$AB = 2DE = DF$$

### Dilute System - Henry's Law

Eqbm. line:  $y^* = mx$

$$y = \frac{L}{G_1}(x - x_2) + y_2 : \text{operating line}$$

$$= \frac{L}{G_1} \left( \frac{y^*}{m} - x_2 \right) + y_2$$

$$y = \cancel{\frac{L}{mG_1}}^A (y^* - mx_2) + y_2$$

$$y^* = \frac{1}{A} (y - y_2) + mx_2$$

$$\checkmark y - y^* = \left(1 - \frac{1}{A}\right)y + \frac{y_2}{A} - mx_2$$

for dilute systems

$$N_{toco} = \int_{y_2}^{y_1} \frac{dy}{y - y^*}$$

$$y_{BM} \approx 1$$

$$1 - y \approx 1$$

$$= \int_{y_2}^{y_1} \frac{dy}{\left(1 - \frac{1}{A}\right)y + \frac{y_2}{A} - mx_2}$$

$$= -\frac{1}{\left(1 - \frac{1}{A}\right)} \left[ \ln \left[ \left(1 - \frac{1}{A}\right)y + \frac{y_2}{A} - mx_2 \right] \right]_{y_2}^{y_1}$$

$$\frac{1}{1 - \frac{1}{A}} \ln \left[ \frac{\left(1 - \frac{1}{A}\right) y_2 + \frac{y_2}{A} - mx_2}{\left(1 - \frac{1}{A}\right) y_2 + \frac{y_1}{A} - mx_1} \right]$$

~~$$= -\frac{1}{1 - \frac{1}{A}} \ln \left( \frac{y_2 - mx_2}{y_1 - mx_1} \right)$$~~

$$= \frac{1}{1 - \frac{1}{A}} \ln \left[ \frac{(y_1 - mx_2) - \frac{1}{A} (y_2 - mx_2) + \frac{1}{A} (y_2 - mx_1)}{y_2 - mx_2} \right]$$

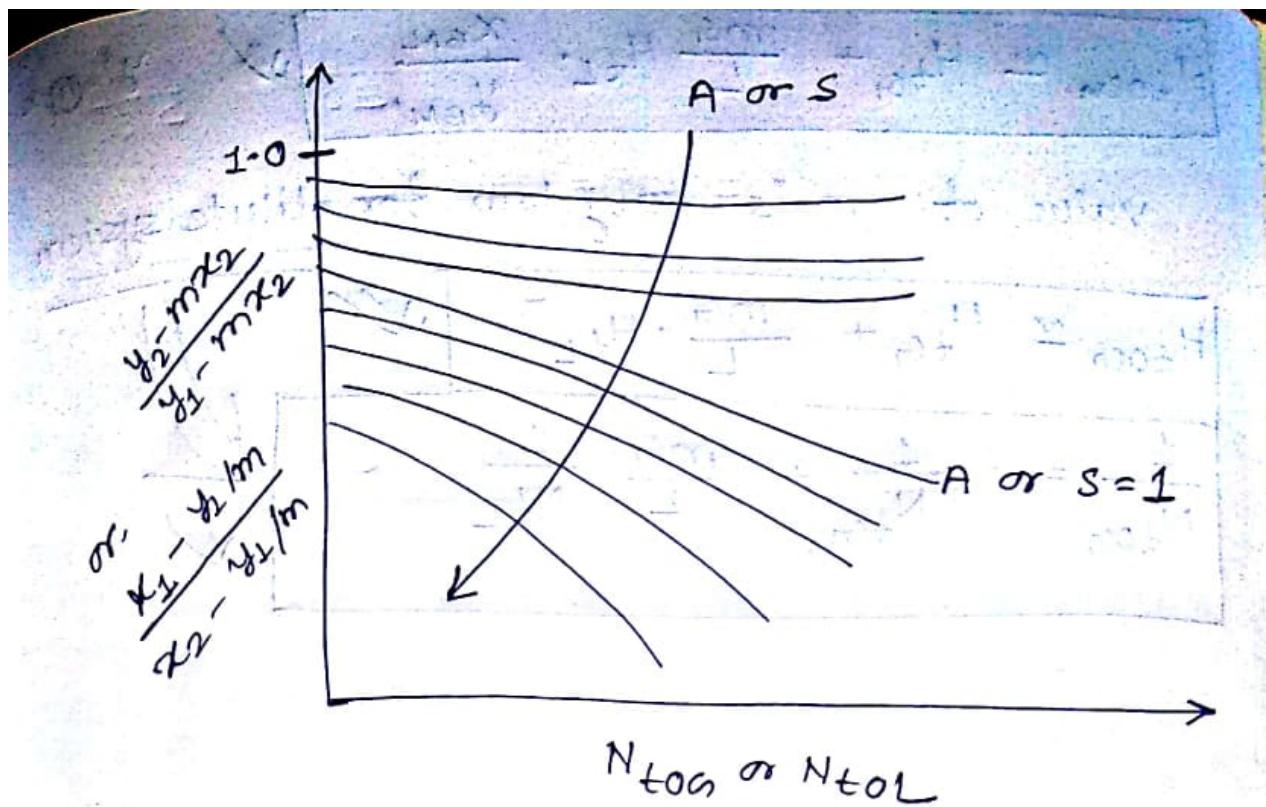
For absorption

$$N_{tow} = \frac{1}{1 - \frac{1}{A}} \ln \left[ \frac{y_1 - mx_2}{y_2 - mx_2} \left(1 - \frac{1}{A}\right) + \frac{1}{A} \right] \checkmark$$

For stripper  $\rightarrow$  only difference from Kremser Eq^n

$$N_{tow} = \frac{1}{1 - \frac{1}{S}} \ln \left[ \frac{x_2 - y_1/m}{x_1 - y_2/m} \left(1 - \frac{1}{S}\right) + \frac{1}{S} \right] \checkmark$$

$$N_{tow} = \frac{1}{1 - A} \ln \left[ \frac{x_2 - y_1/m}{x_1 - y_1/m} (1-A) + A \right] \checkmark$$



In general      A optimum  $\in (1.25, 2.0)$

$$Z = H_{tL} \times N_{tL} = H_{tG1} \times N_{tG1} = H_{tOG1} \times N_{tOG1}$$

$$H_{tOG1} = \frac{G_1'}{K_y a \frac{y^*}{y_{BM}}} = \frac{G_1'}{y_{BM}} \left( \frac{1}{K_y a} + \frac{m}{K_x a} \right)$$

slope  
 of eqbm.  
 line in  
 the operat-  
 ing  
 range

$$= \frac{G_1'}{K_y a \frac{y^*}{y_{BM}}} + \frac{m G_1'}{L'} \cdot \frac{L'}{K_x a x_{BM}} \cdot \frac{x_{BM}}{y_{BM}^*}$$

$$= H_{tG1} \cdot \frac{y_{BM}}{y_{BM}^*} + \frac{m G_1}{L} H_{tL} \frac{x_{BM}}{y_{BM}^*}$$

For dilute system

$$y - y_i \approx y - y^*$$

$\rightarrow$  most of the resistance is offered by  
gas phase

$$H_{\text{toge}} \simeq H_{tG} + \frac{mG}{L} H_{tL} \cdot \frac{x_{BM}}{y_{BM^*}} \quad \checkmark$$

value of  $m$  is very low for dilute system

$$H_{t+G_1} \approx H_{t+G_1} + \frac{mG_1}{L} \cdot H_{t+L}$$

$$\frac{1}{N_{tG}} = \frac{1}{N_{tG}} + \frac{mG}{L} \frac{1}{N_t L}$$

$$H_{tOCN} \approx H_{tG_1} + \frac{mG_1}{L} H_{tL} \cdot \frac{x_{BM}}{y_{BM^*}} \quad \checkmark \quad y^* = M_2$$

value of  $m$  is very low for dilute system

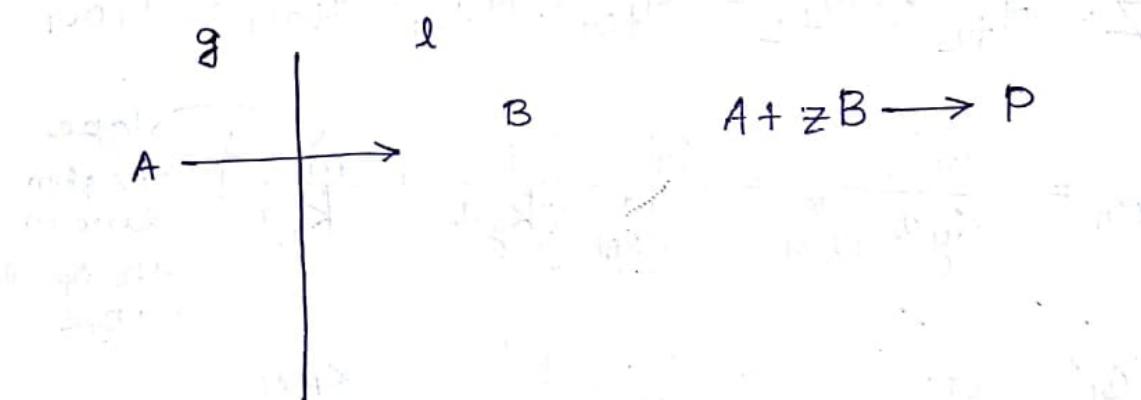
$$H_{tOCN} \approx H_{tG_1} + \frac{mG_1}{L} \cdot H_{tL} \quad \checkmark$$

$$\frac{1}{N_{tOCN}} = \frac{1}{N_{tG_1}} + \frac{mG_1}{L} \frac{1}{N_{tL}} \quad \checkmark$$

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### Absorption with Chemical Reaction

- When there are multiple processes taking place, usually the slowest step is rate determining.



- Depending on nature of reaction, either absorption or reaction will be dominating the entire process.
- Contactors are used to increase mass transfer rate to maximum.
- Even after that, mass transfer rate can be increased if reaction takes place.

Heterogeneous Reaction  
Vol. 2 by Sharma

Absorption with chemical reaction

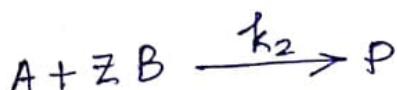
→ Regime 1 : Very slow reaction

→ Regime 2 : Slow reaction

→ Regime 3 : Fast reaction

→ Regime 4 : Instantaneous reaction

- B is in large excess  $\Rightarrow c_B \approx c_{B_0} = \text{const.}$



$$r_A = l k_2 c_A c_B$$

$$r_A = l (k_2 c_{B_0}) c_A$$

$$r_A = l k_1 c_A$$

,  $l = \frac{\text{volume fraction of liquid}}{\text{volume of liquid}}$

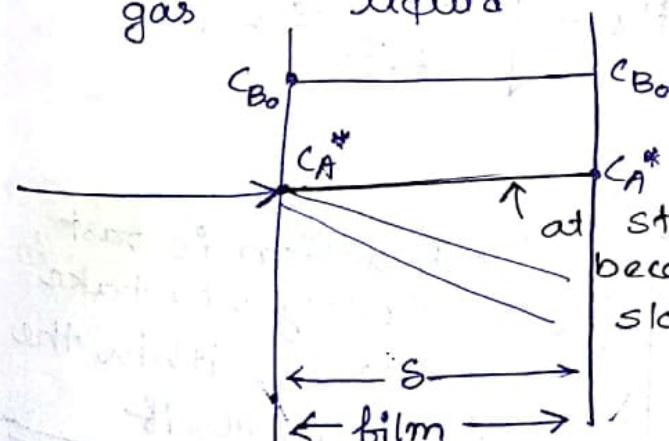
$= \frac{\text{volume of liquid}}{\text{Total volume}}$

Regime 1:

$$R_A = l k_2 c_{B_0} c_A^*$$

,  $R_A = \frac{\text{rate of absorption}}{\text{Mass Transfer}}$

gas      liquid



steady state, liquid conc. becomes const  $\approx c_A^*$  due to slow reaction rate

Condition:

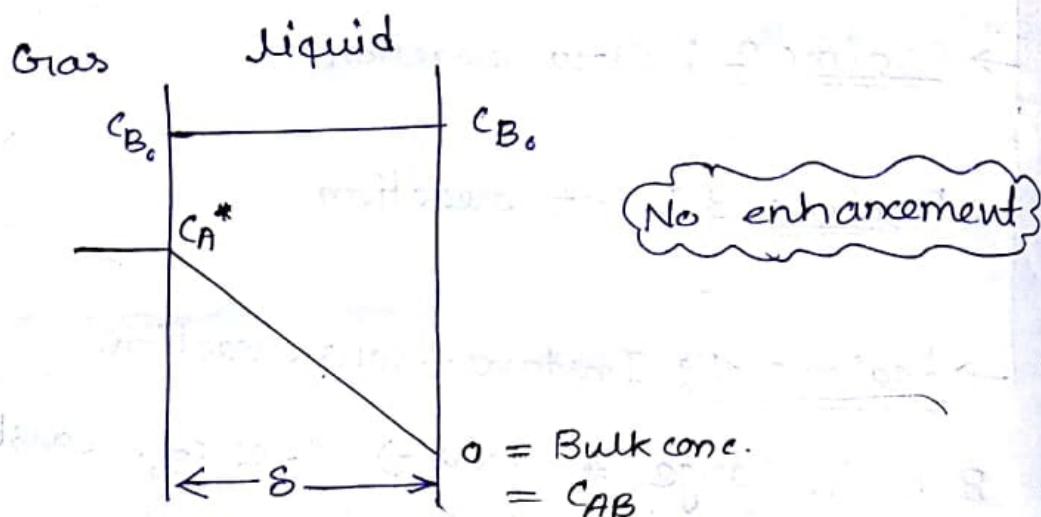
$$K_a c_A^* \gg l k_2 c_{B_0} c_A^*$$

$$K_a \gg l k_2 c_{B_0}$$

No enhancement in rate of absorption is possible

## Regime 2

- $R_A = k_L a c_A^*$



condition:  $k_L a \ll k_2 c_{B0}$

- Reaction is not fast enough to take place within the film. But reaction occurs beyond the film.

condition for above situation:

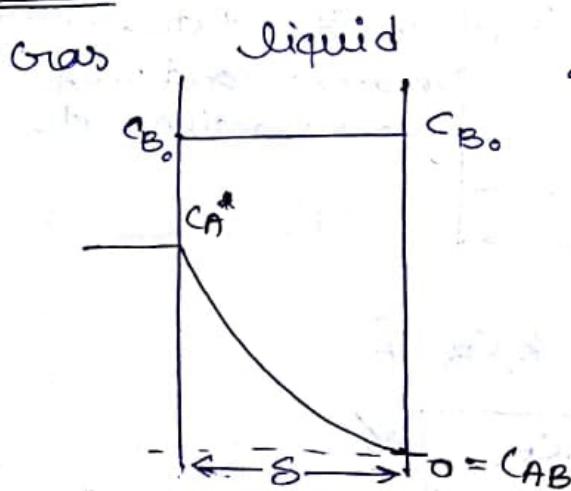
$$\sqrt{M} < 1$$

$$\frac{\sqrt{D_A k_2 c_{B0}}}{k_L} < 1$$

$$M = \frac{D_A k_2 c_{B0}}{k_L^2}$$

Hatta number,  $Ha = \sqrt{\frac{D_A k_2 c_{B0}}{k_L^2}} = \sqrt{M}$

## Regime 3



- Reaction is fast enough to take place within the film itself

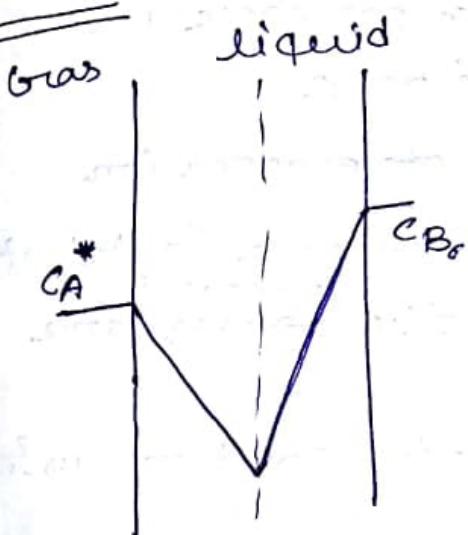
✓

- $R_A = k_L a c_A^* \frac{\sqrt{M}}{\tanh \sqrt{M}}$
- $\sqrt{M} \gg 3$

- Mars transfer rate is enhanced over pure mass transfer.

$$\phi = \frac{\sqrt{M}}{\tanh \sqrt{M}} = \text{enhancement rate} \quad \checkmark$$

Regime 4



- There is no co-existence of species A and B in the liquid phase (both film & bulk)
- Reaction is instantaneous

- $R_A = k_L a c_A^* \left( 1 + \frac{D_B c_{B0}}{D_A c_A^*} \right) \xrightarrow{q}$

$$R_A = k_L a c_A^* (1+q) \quad \checkmark$$

$$\sqrt{M} \gg (1+q) \quad \checkmark$$

$$\phi = 1+q \quad \checkmark$$

\* Phase transfer catalyst facilitates transfer of reactant from aqueous phase to organic phase

!

~~Study of two immiscible solvents~~

## Distillation

23/10/17

- All the components are present in both the phases (gas and liquid) [distinguishing characteristic from evaporation]
- Depending on their volatility (B.P) components are concentrated in either of the phases.
- More volatile component in gas phase and less volatile component in liquid phase
- No mass transfer in case of evaporation
- Equimolar counter diffusion takes place if latent heat of vaporisation of both the phases are equal
- In evaporation there is no transport of mass from one phase to other.

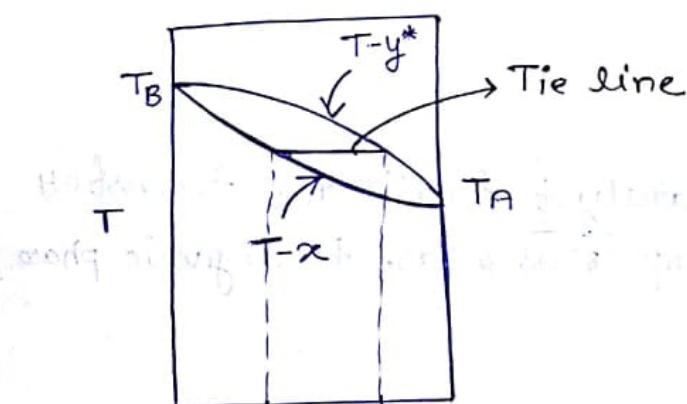
## Vapor-liquid Equilibria (VLE)

A + B

$$f = C \bar{P} + 2$$

$$= 2 \bar{P} + 2$$

Const. Pressure : T-x, y diagram :



$$x_A = 0$$

$$x_B = 1$$

$$x, y^*$$

$$x_A = 1$$

$$x_B = 0$$

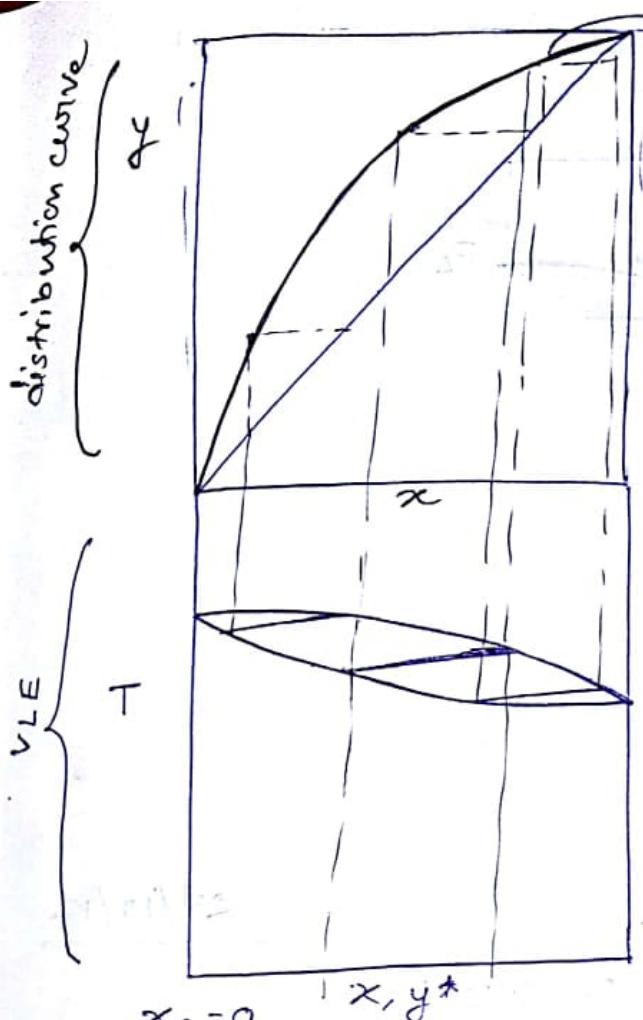
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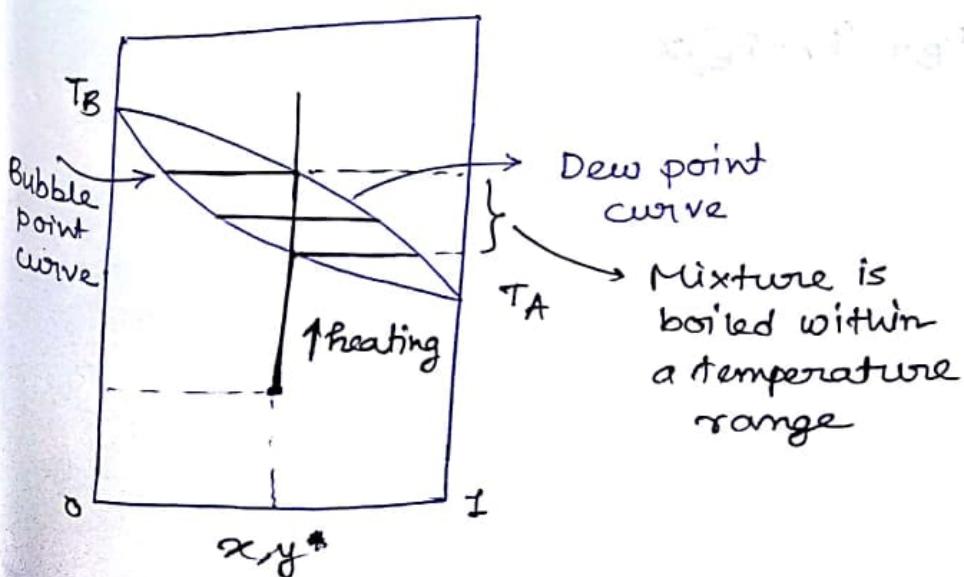
A?  
CFP + 2  
2 + 2 + 2



$$x_B = 1$$

$$\alpha_{AB} = \frac{y^*/(1-y^*)}{x/(1-x)} = \frac{y^*(1-x)}{x(1-y^*)}$$

$\alpha_{AB} > 1$  for distillation to occur.

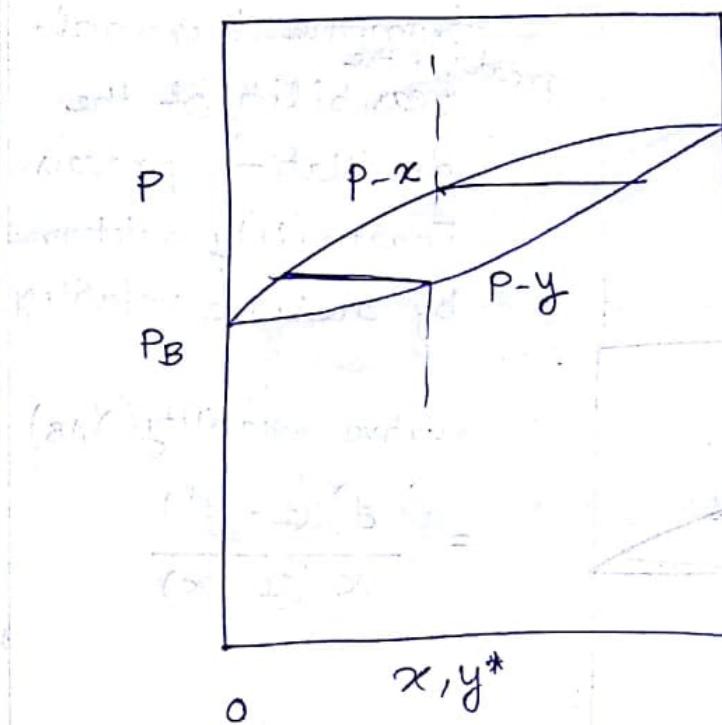


Further away the distribution diagram from the diagonal, greater is the feasibility of the distillation process. Feasibility is determined by relative volatility

Relative volatility ( $\alpha_{AB}$ )

$$= \frac{y^*/(1-y^*)}{x/(1-x)}$$

Const T P-xy VLE



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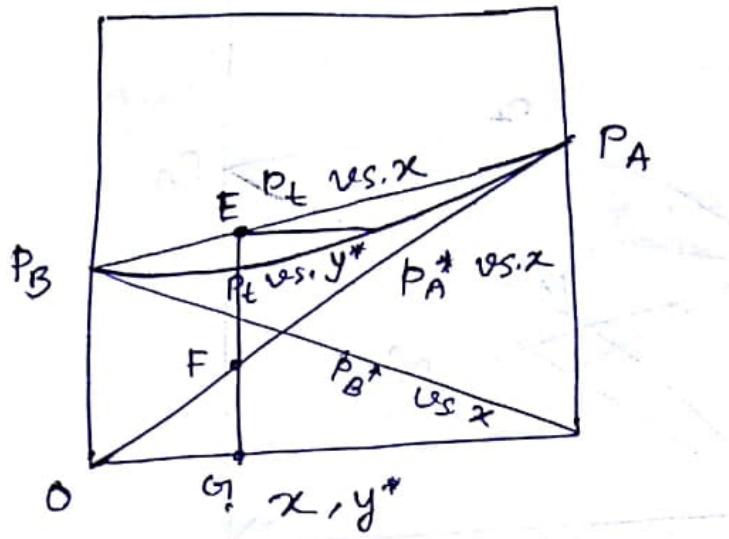
Ideal solution: obeys Raoult's Law

$$P_A^* = P_A x$$

$$P_B^* = P_B (1-x)$$

$$P_t = P_A^* + P_B^* = P_A x + P_B (1-x)$$

$$= P_B + (P_A - P_B)x$$



$$y^* = \frac{P_A^*}{P_t} = \frac{FG}{EG}$$

$$y^* = \frac{P_A^*}{P_t} = \frac{P_A x}{P_t}$$

$$1 - y^* = \frac{P_B^*}{P_t} = \frac{P_A(1-x)}{P_t}$$

$$\frac{y^*}{1-y^*} = \frac{P_A x}{P_B(1-x)}$$

$$\frac{y^*/(1-y^*)}{x/(1-x)} = \frac{P_A}{P_B}$$

$$\alpha_{AB} = \frac{P_A}{P_B}$$

$$y^* = \frac{\alpha x}{1+(\alpha-1)x}$$

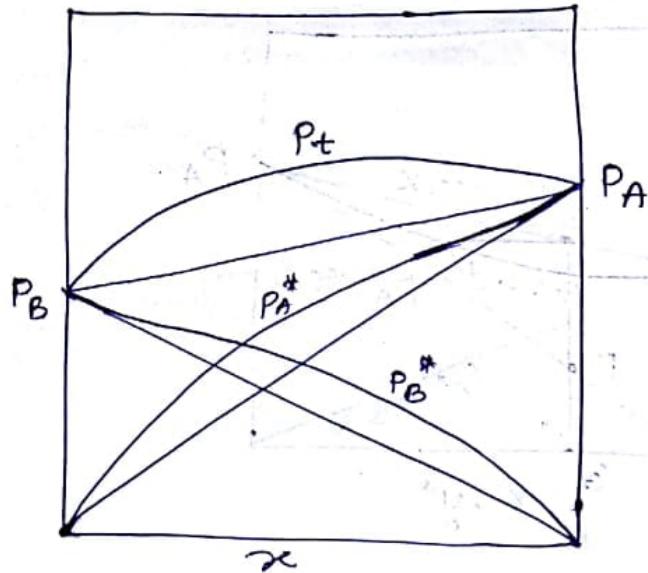
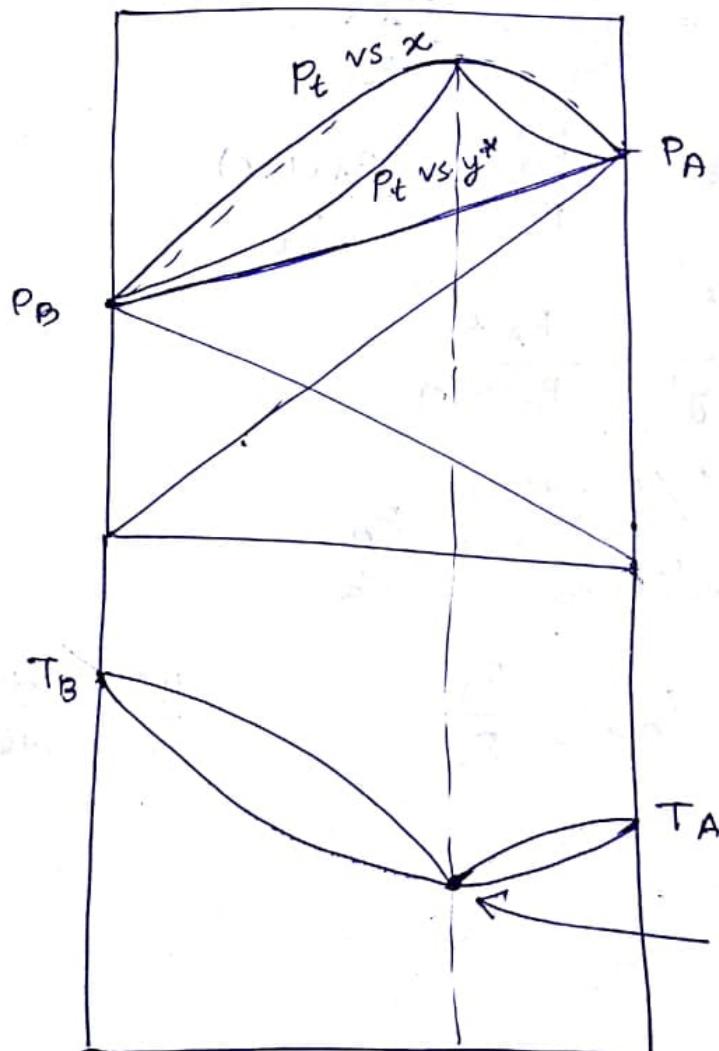


fig : +ve deviation from ideality



Minimum  
boiling  
azeotropes  
(constant boiling  
mixture)

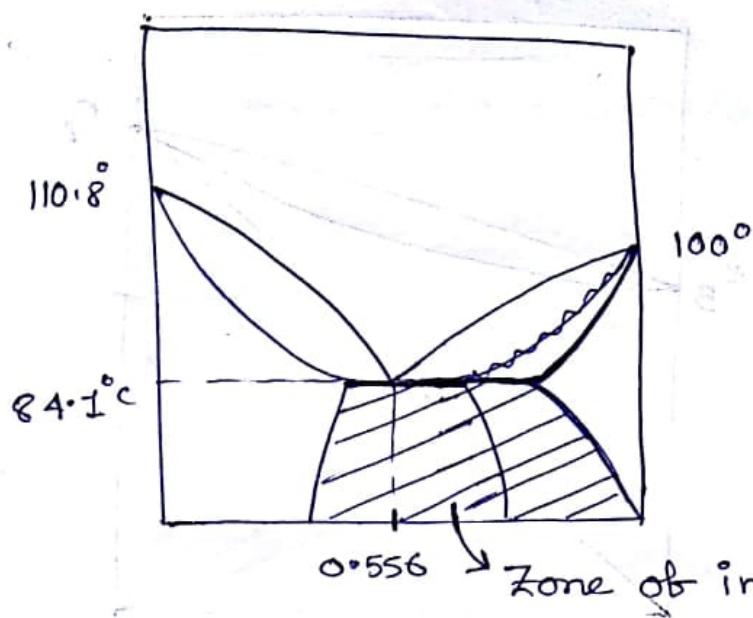


fig: Extreme deviation from ideality  
(Heteroazeotropes)

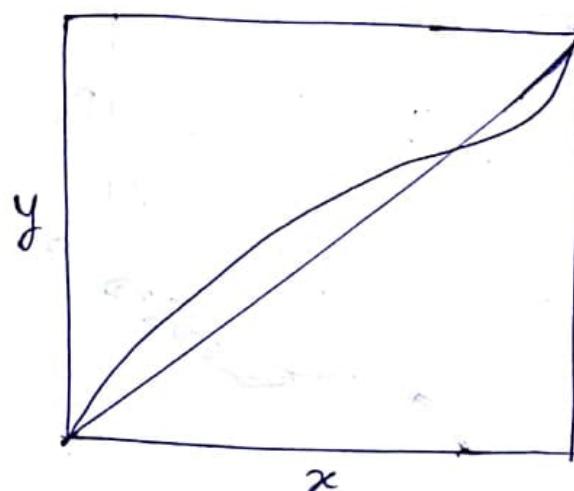
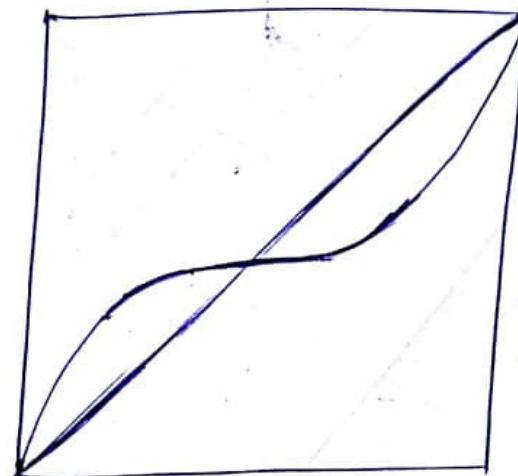


fig: for normal azeotropes

Dean & Stark  
Apparatus



Toluene + water  
110.8°C      111°C  
↓  
~~84.1°C~~  
44.4 mole%  
Toluene

fig: for heteroazeotropes

of King  
2)

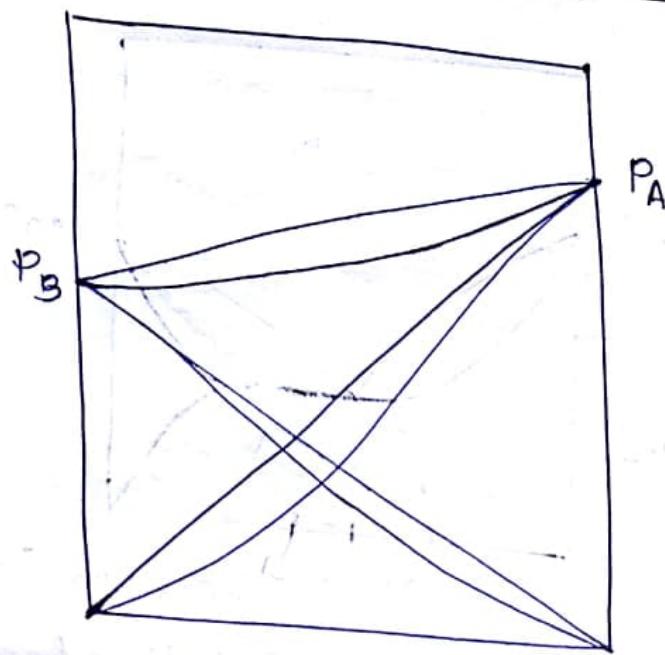
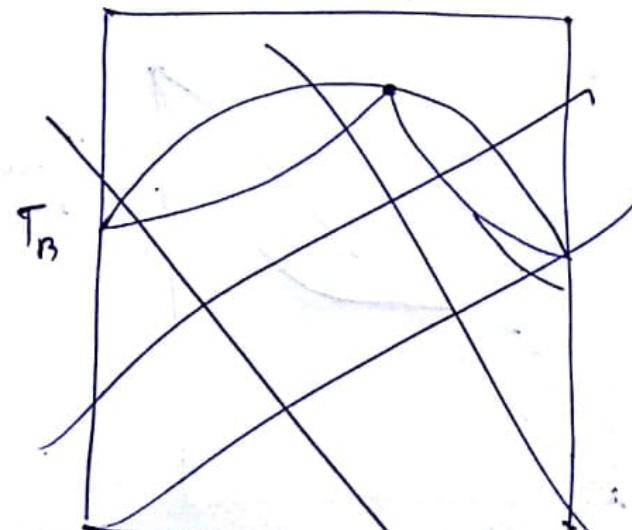
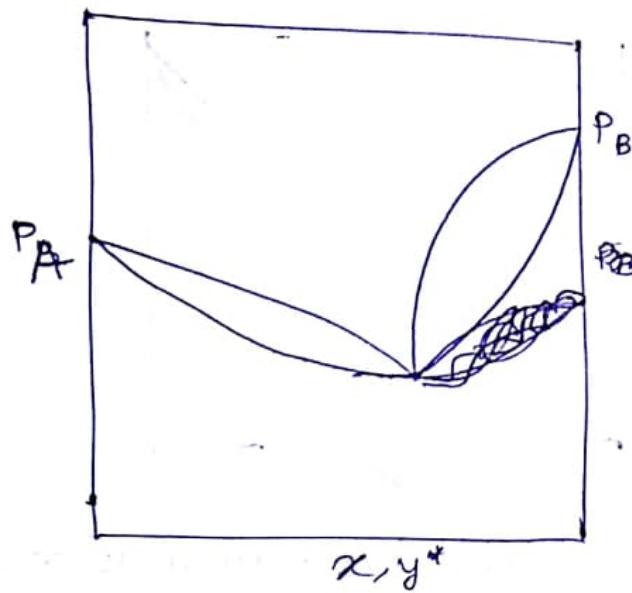


fig: Negative Deviation



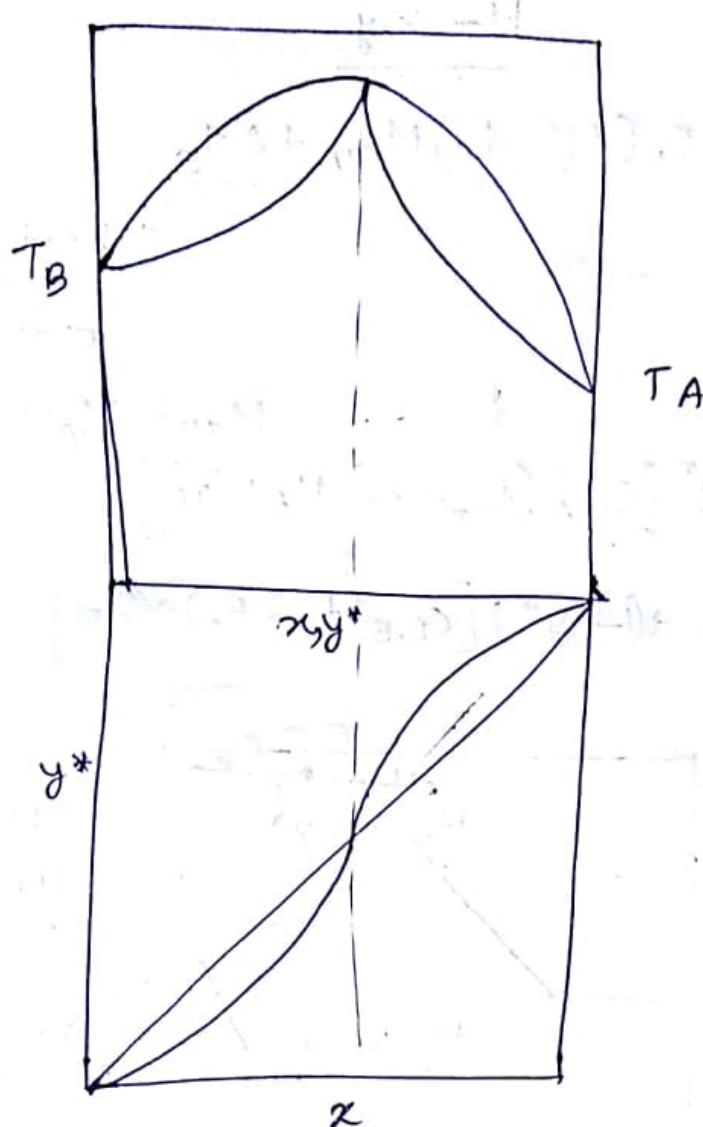
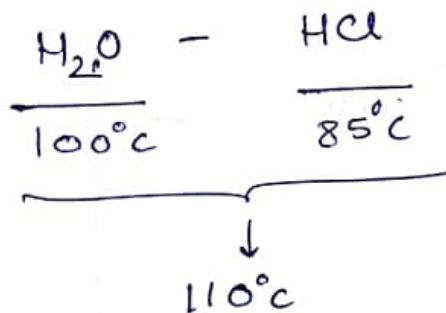


fig: Maximum boiling azeotrope

e.g.  
1)



$110^\circ\text{C}$

11.1 mole% HCl

2) Chloroform- acetone acetic acid

H - x, y

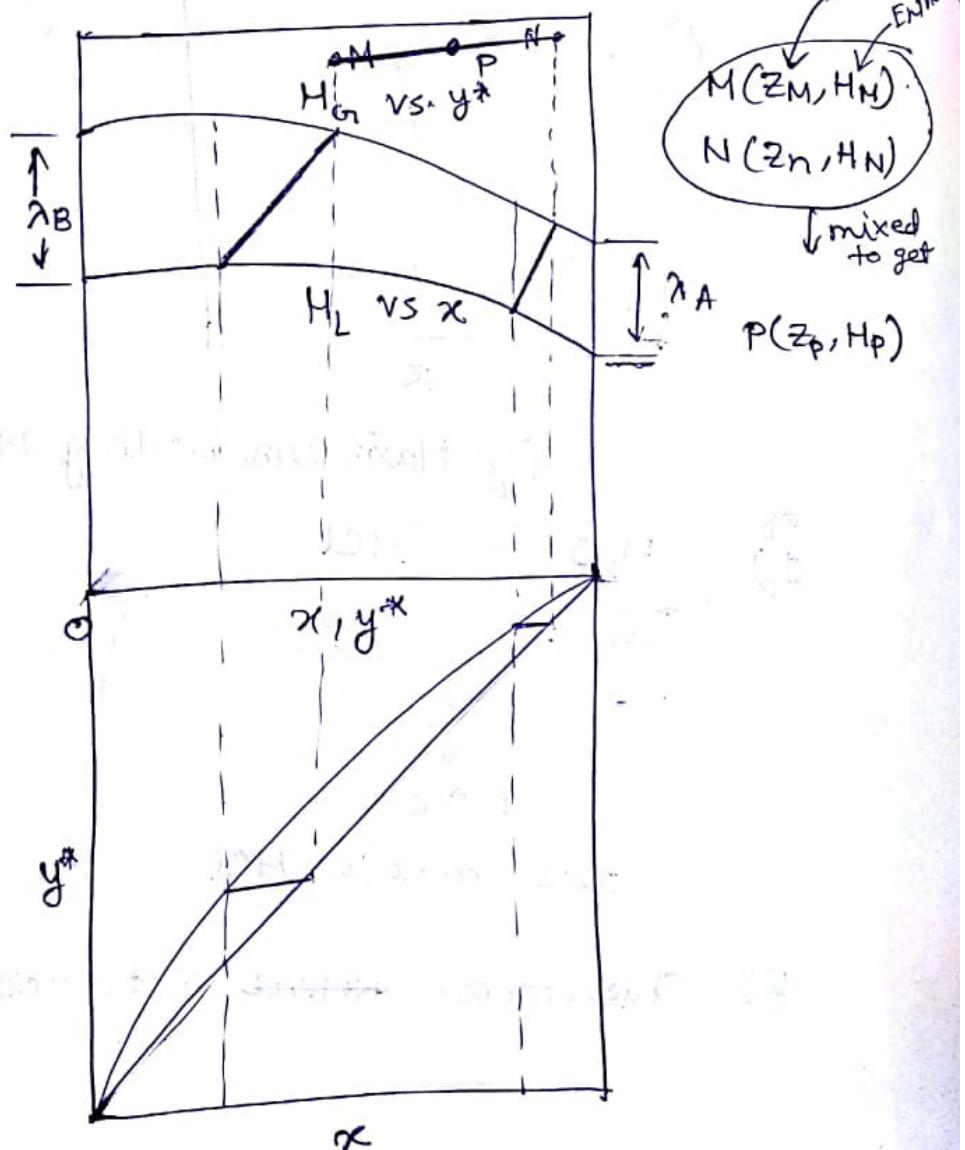
$$H_L = c_L(t_L - t_0) M_{av.} + \Delta H_s$$

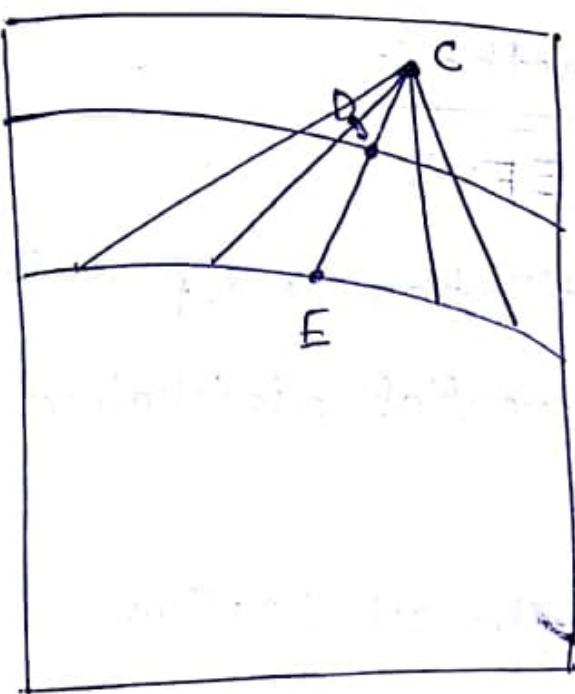
heat of soln  
or  
heat of mixing

$$M_{av.} = M_A x + M_B (1-x)$$

$$H_{G1} = y^* [c_{L,A} (t_{G1} - t_0) M_A + \lambda_A]$$

$$+ (1-y^*) [c_{L,B} (t_{G1} - t_0) + \lambda_B]$$





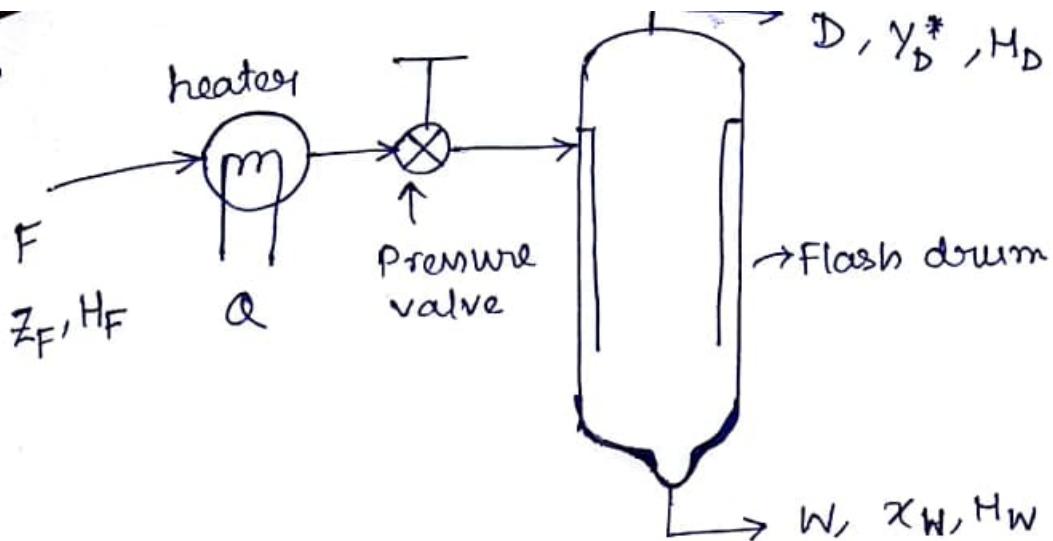
26/10/17

- Simple distillation method
  - Single stage
  - less rectification/purification
- Complex method
  - multiple stage
  - Reflux stream
  - Continuous distillation
  - Better purification

- Simple distillation methods

- 1) Flash distillation (EFV)
  - ✓ or Equilibrium flash distillation or vaporization
- 2) Simple batch or differential distillation
- 3) Steam Distillation

- Efv used for crude characterisation
  - ↓ curve
  - Volume distilled vs. Temperature (analogous to ASTM distillation of petroleum products)
- Differential distillation is carried out in such a manner that liquid & vapor formed are maintained at equilibrium
- Steam distillation is used for pharmaceuticals & fragrances.

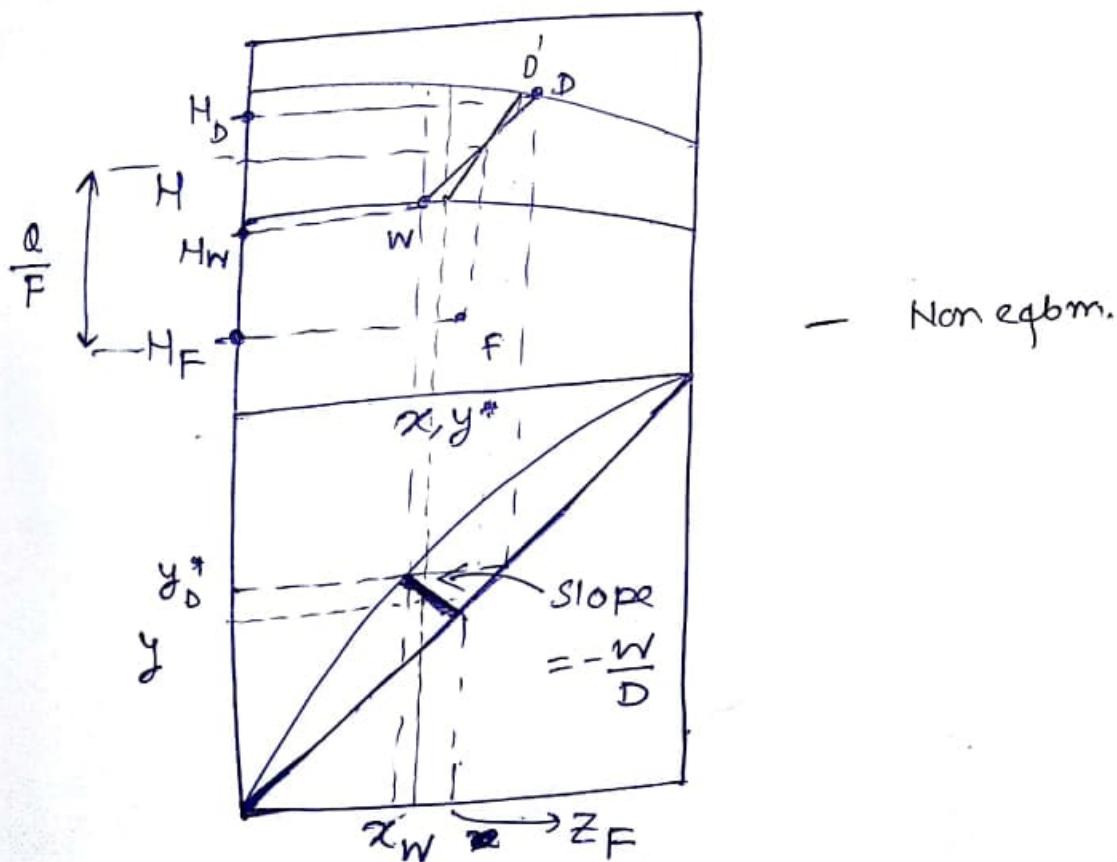


Feed enters the flash drum tangentially.

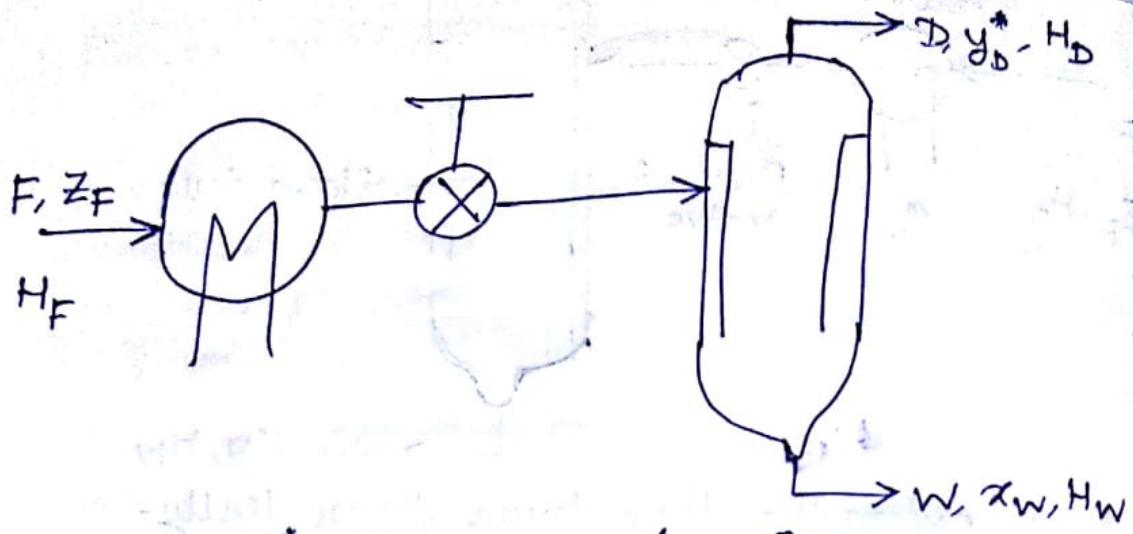
- 1)  $F = D + W$  -- overall material balance
- 2)  $FZ_F = Dy_D^* + Wx_W$  -- component balance
- 3)  $Q + FH_F = DH_D + WH_W$  - - - Energy balance  
(Enthalpy)

from 1, 2, 3

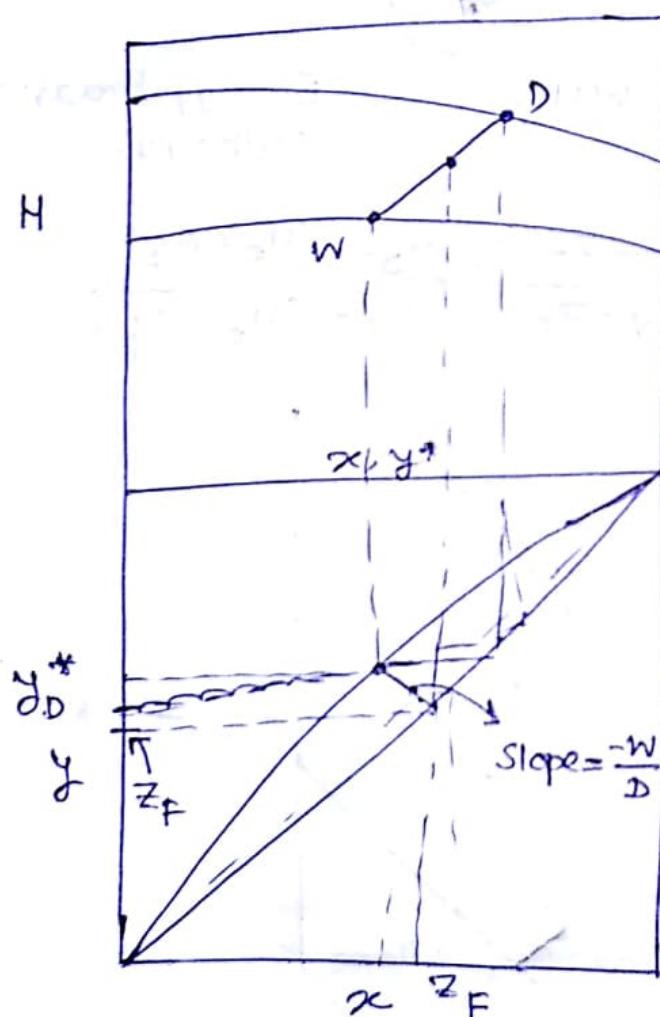
$$-\frac{W}{D} = \frac{y_D^* - Z_F}{x_W - Z_F} = \frac{H_D - (H_F + Q/F)}{H_W - (H_F + Q/F)}$$

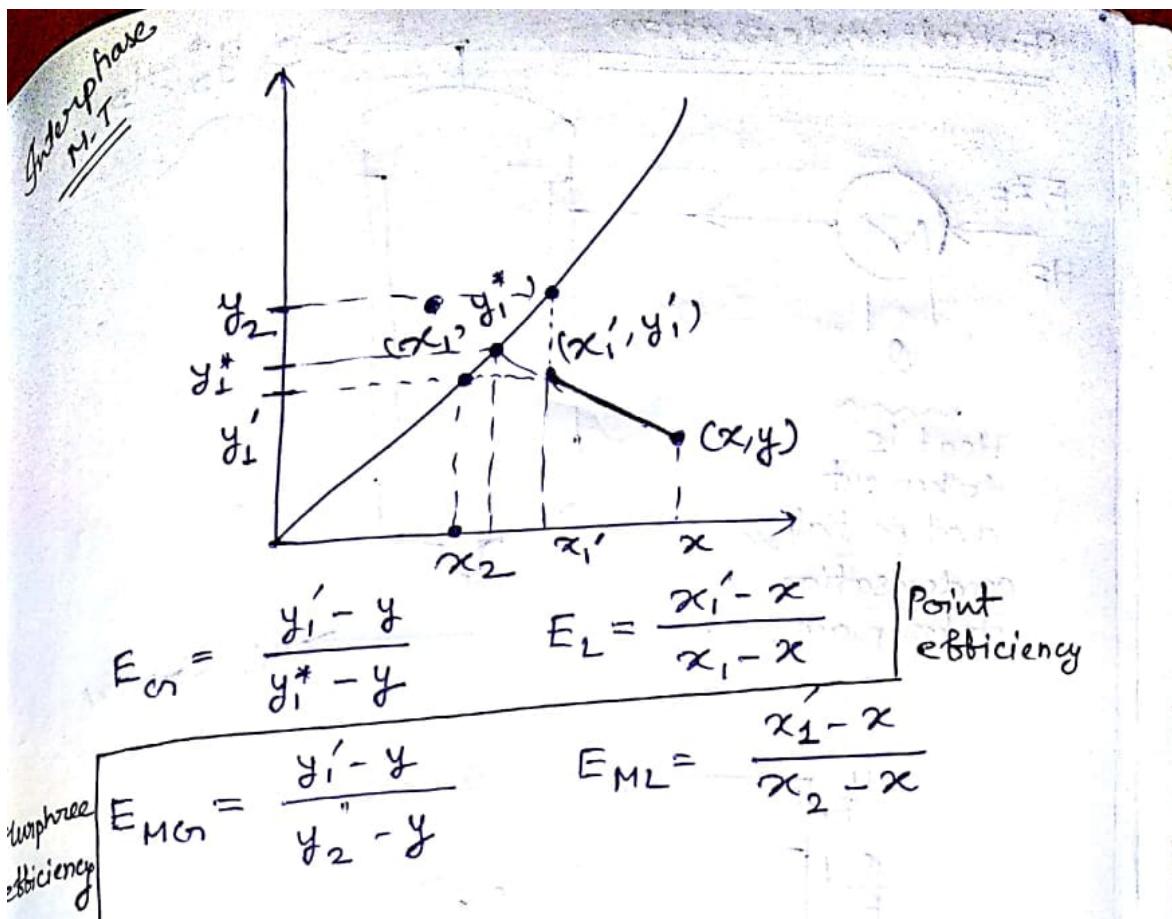


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$$-\frac{w}{D} = \frac{x_D^* - z_F}{x_w - z_F} = \frac{H_D - (H_F + \frac{\rho}{F})}{H_w - (H_F + \frac{\rho}{F})}$$



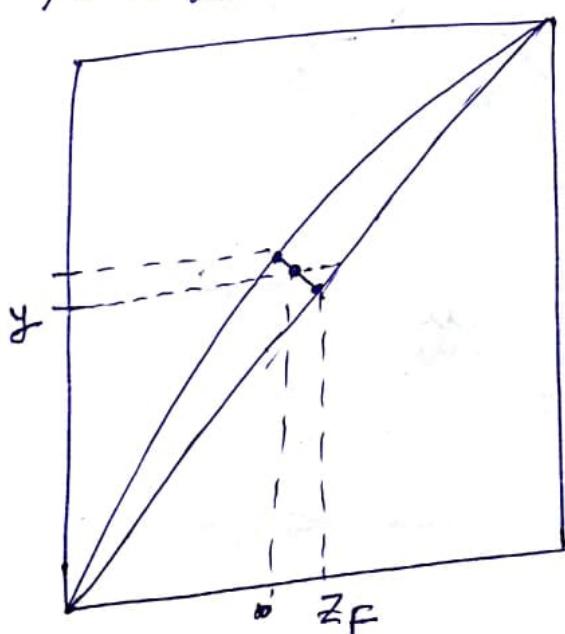


$\Rightarrow$

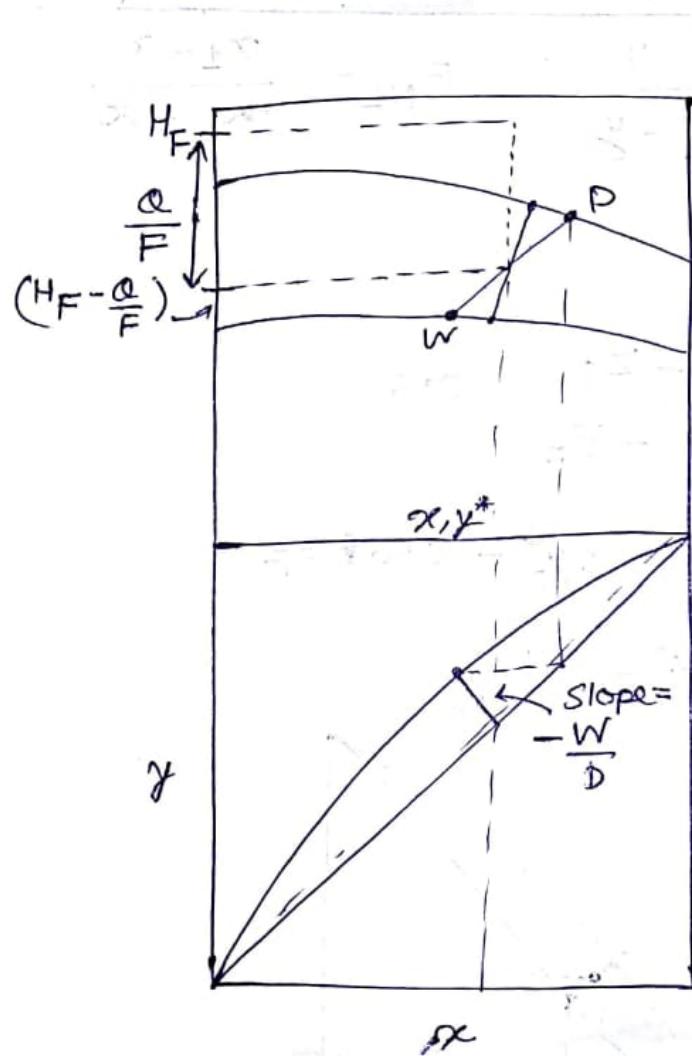
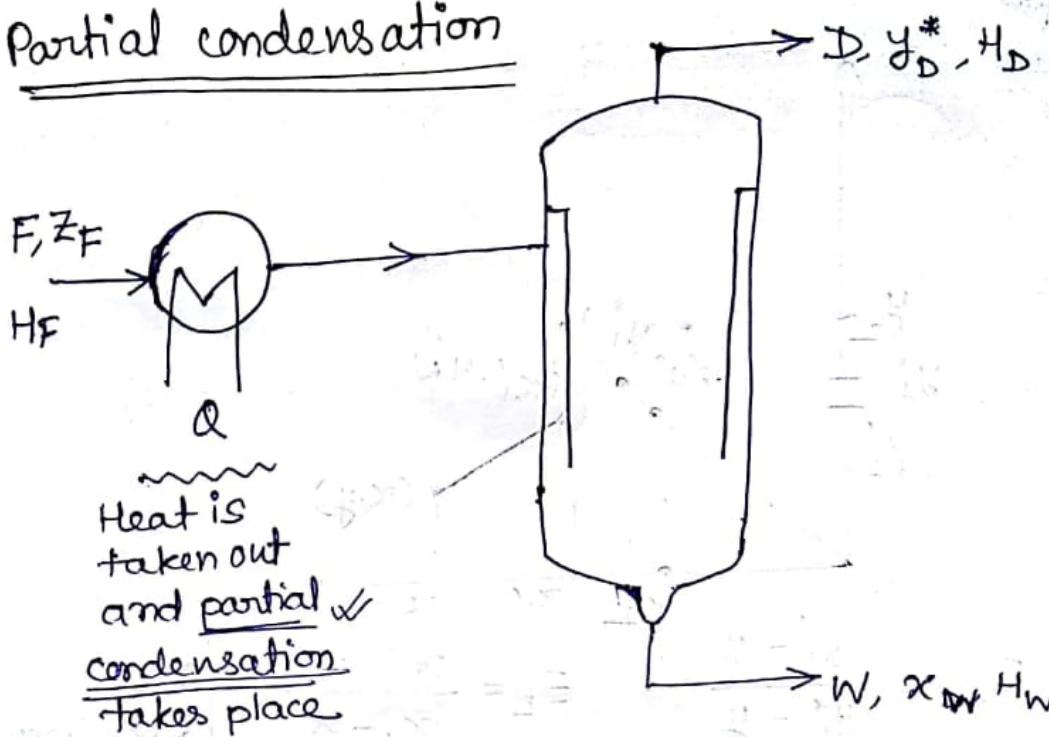
$$E_{G1} = \frac{y - z_F}{y_D^* - z_F} = 0.6$$

$$y - z_F = 0.6 y_D^* - 0.6 z_F$$

$$y = 0.6 y_D^* + 0.4 z_F$$



## Partial condensation



# Multicomponent System - Ideal solutions

Components: A, B, C, ..., N

$$p_j^* = p_j x_j$$

$$\Rightarrow \frac{p_j^*}{p_t} = \frac{p_j^\circ}{p_t} x_j = m_j x_j$$

$$\Rightarrow y_i^* = m_j x_j \Rightarrow m_j = \frac{y_i^*}{x_j}$$

For multicomponent mixture

$$y_{i,D}^* = m_j x_{j,W} \quad \dots \quad (i)$$

$$+ \frac{W}{D} = \frac{y_{i,D}^* - z_{j,F}}{z_{j,F} - x_{j,W}} \quad \dots \quad (ii)$$

$$\text{Replacing } x_{j,W} = \frac{y_{i,D}^*}{m_j}$$

$$\frac{W}{D} = \frac{y_{i,D}^* - z_{j,F}}{z_{j,F} - \frac{y_{i,D}^*}{m_j}}$$

$$\Rightarrow y_{j,D}^* \left[ \frac{W}{D m_j} - 1 \right] = -z_{j,F} \left( 1 + \frac{W}{D} \right)$$

$$\Rightarrow y_{j,D}^* = \frac{z_{j,F} (1 + W/D)}{1 + W/D x_{j,W}}$$

$$\boxed{\sum y_{i,D}^* = 1.0}$$

✓ check for correctness  
of obtained solution.

## Partial condensation of multi-component mixture

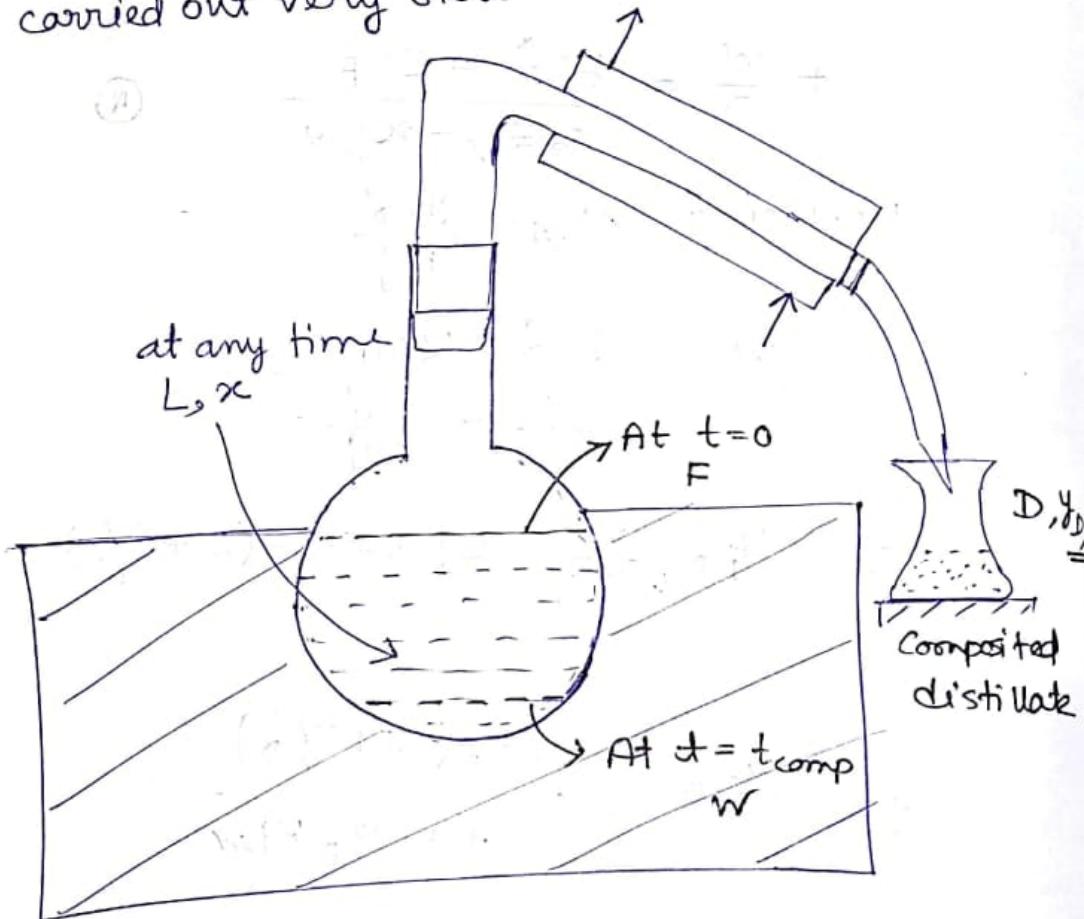
$$x_{j,w} = \frac{z_{j,F}(1 + w/D)}{m_j + w/D}$$

$$\sum x_{j,w} = 1.0$$

01/11/17

### Differential Distillation

- discomposed distillate vs. composited distillate
- carried out very slow

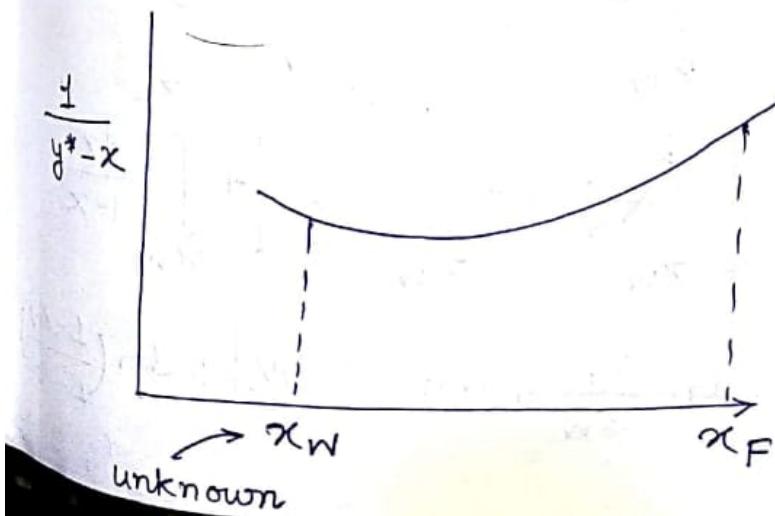


- Combination of infinite no. of flash distillation with very small amount of liquid vaporised with very small (fraction distillated) in each step.
- Differential material balance is required

Total moles	Moles of A
0	0
$\text{in}$	$y^* dD$
$\text{out}$	$dL$
Accumulation	$y^* dD - dL$
$\text{In} - \text{out}$	= accumulation
$0 - dD = dL$	①
$0 - y^* dD = d(Lx) = Ldx + xdl$	②
from ① & ②	$y^* dL = Ldx + xdl$
$(y^* - x) dL = Ldx$	
$\int_{F}^{W} \frac{dL}{L} = \int_{x_F}^{x_W} \frac{dx}{y^* - x}$	
$\ln \frac{F}{W} = \int_{x_W}^{x_F} \frac{dx}{y^* - x}$	NE Rayleigh's Eq <sup>n</sup>

composited distillate composition can be obtained from component balance of A

$$Fx_F = Dy_{D,\text{ave}} + Wx_{W,\text{ave}}$$



$x_W$  to be chosen, by trial & error

$$\ln\left(\frac{F}{D}\right) = \int_{y_F}^{y_D} \frac{dy}{y - x^*}$$

For system with constant relative volatility, graphical soln is not required although ~~graph~~ soln by trial and error is to be obtained.

$$y^* = \frac{\alpha x}{1 + (\alpha - 1)x} \quad \text{where } \alpha = \text{const.}$$

$$\ln \frac{F}{W} = \int_{x_F}^{x_W} \frac{(1 + (\alpha - 1)x)dx}{\alpha x - x(1 + (\alpha - 1)x)}$$

$$\ln \frac{F}{W} = \int_{x_F}^{x_W} \frac{(1 + (\alpha - 1)x)dx}{-(\alpha - 1)x^2 + (\alpha - 1)x}$$

$$\ln \frac{F}{W} = \int_{x_F}^{x_W} \frac{(1 + (\alpha - 1)x)dx}{x(\alpha - 1)(1 - x)}$$

✓ For  $\alpha = \text{const.}$   
i.e. rel. volatility  
is const.

$$\ln \frac{F}{W} = \frac{1}{(\alpha - 1)} \int_{x_W}^{x_F} \frac{dx}{x(1-x)} + \int_{x_W}^{x_F} \frac{dx}{x(1-x)}$$

$$\ln \frac{F}{W} = \frac{1}{\alpha - 1} \left[ \int_{x_W}^{x_F} \frac{dx}{x} + \int_{x_W}^{x_F} \frac{dx}{1-x} \right] + \int_{x_W}^{x_F} \frac{dx}{1-x}$$

$$\ln \frac{F}{W} = \frac{1}{\alpha - 1} \left[ \ln \frac{x_F}{x_W} + \ln \frac{1 - x_W}{1 - x_F} \right] + \ln \left( \frac{1 - x_W}{1 - x_F} \right)$$

$$\ln \frac{F}{W} = \frac{1}{\alpha-1} \ln \frac{x_F(1-x_W)}{x_W}$$

$$(\alpha-1) \ln \frac{F}{W} = \ln \frac{x_F}{x_W} + \ln \frac{1-x_W}{1-x_F} + (\alpha-1) \ln \frac{1-x_W}{1-x_F}$$

$$(\alpha-1) \ln \frac{F}{W} = \ln \frac{x_F}{x_W} + \alpha \ln \frac{1-x_W}{1-x_F}$$

$$\ln \left( \frac{Fx_F}{Wx_W} \right) = \alpha \ln \frac{F(1-x_F)}{W(1-x_W)}$$

$$\log \left( \frac{Fx_F}{Wx_W} \right) = \alpha \log \frac{F(1-x_F)}{W(1-x_W)}$$

✓ System with  
const. rel.  
volatility

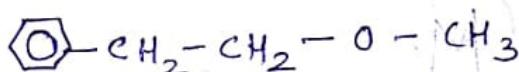
### Steam Distillation

$A+B$  → Not volatile compared to A

High boiling

A deteriorates in quality at its boiling point

Kewra water → 2-phenyl ethyl methyl ether



$A+B$

+ C (steam)

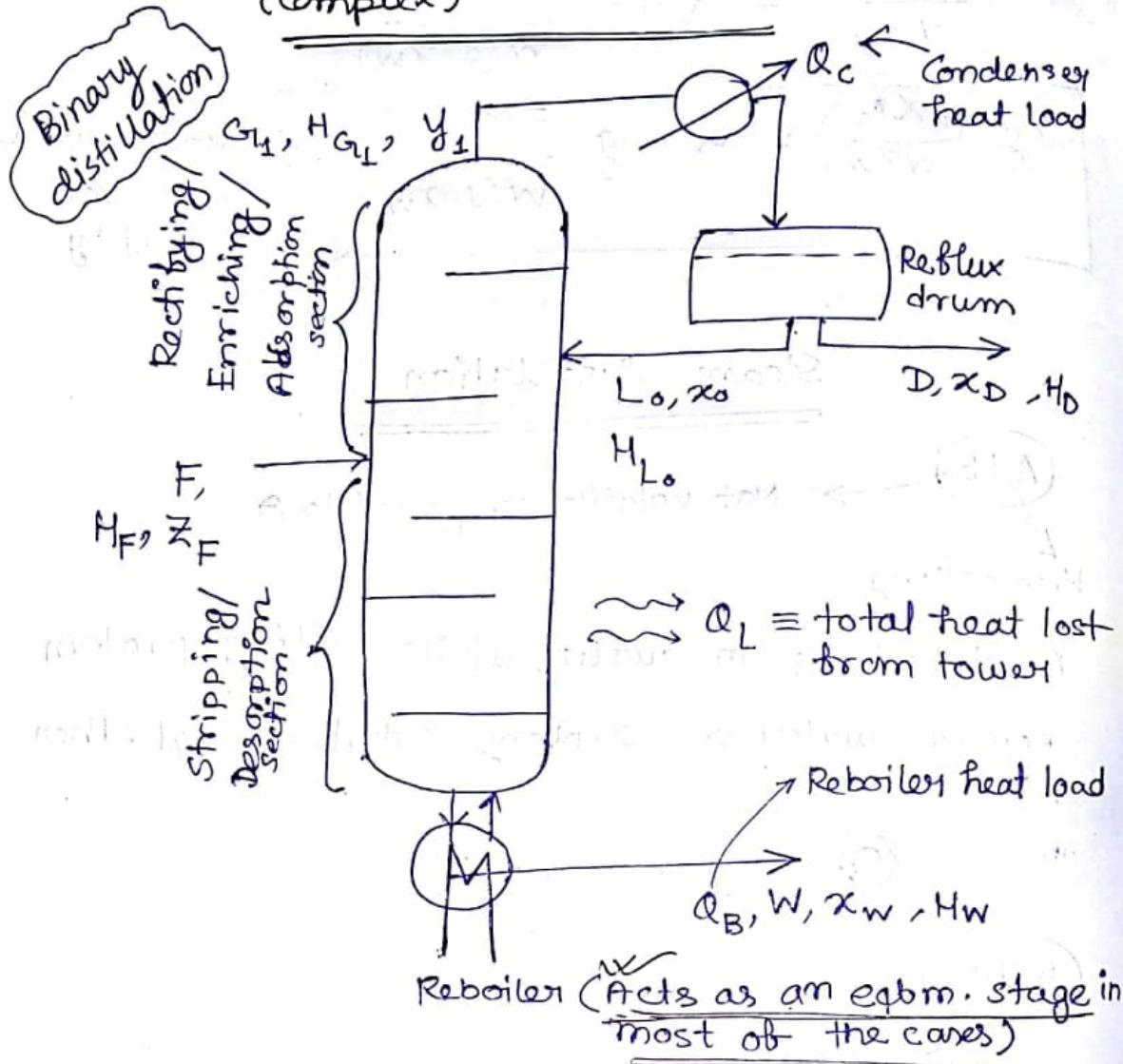
$$P_t = P_A + P_C$$

Vapor pressure of components as the components are immiscible and exert pressure independent of the other components. The mixture will boil at lower temperature compared to the boiling point of pure A. Vapor mixture is condensed, which form two separate layers. Usually the bottom

layer (<sup>extracted</sup><sub>water</sub>) is refluxed back.

$$\frac{P_A}{P_t} = y_A \text{ in vapor} \quad \frac{P_C}{P_t} = y_C \text{ in vapor}$$

### Fractional Distillation (Complex)



i.e. Total no. of trays = (obtained from stage wise construction - 1)

\* Sometimes, partial condenser need to be considered as eqbm. stage but most often not.

Find this in a more detailed book or general notes. It's a good idea to have a look at some of the books mentioned above, as they provide a more detailed explanation of the theory and application of fractional distillation.

$$G_1 H_{G_1} = Q_c + D H_D + L_o H_{L_o} \quad \dots \textcircled{1}$$

$$Q_c = G_1 H_{G_1} - D H_D - L_o H_{L_o}$$

$$G_1 = L_o + D \quad \dots \textcircled{2}$$

$$R = \frac{L_o}{D} \quad \dots \textcircled{3}$$

$= D(R+1)$  Reflux ratio

$$Q_c = D(R+1) H_{G_1} - D H_D - R D H_{L_o}$$

$$Q_c = D[(R+1) H_{G_1} - H_D - R H_{L_o}] \quad \boxed{\text{Condenser heat load}}$$

$$F H_F + Q_B = Q_c + D H_D + W H_D + Q_L$$

$$Q_B = Q_c + D H_D + W H_D + Q_L - F H_F \quad \boxed{\text{Reboiler heat load}}$$

02/11/17

\* Diameter is decided considering flooding in tower as well as the nature of liquid (foaming or non-foaming)

\* Actual fluid velocity is a fraction of the flooding velocity. The fraction is determined by nature of flow (lower for foaming, higher for non-foaming)

\* Packed towers are used for foaming liquids as gas is bubbled but not very high volume of foam is formed in a packed tower. As a result, very high ~~gas~~ gas flow rates can be employed.

1) Ponchon & Savarit Method → Detailed enthalpy composition as well as  $x$  vs  $y^*$  distribution data (e.g. data is required)  
 2) McCabe & Thiele method  
 Only equilibrium data is required

$1^{st}$  method is applicable to all systems, whereas  
 $2^{nd}$  method is applicable to \*restricted systems  
(where molar latent heat of vaporisation of components are equal), although  $2^{nd}$  method can be employed to most of the systems with modifications.

McCabe Thiele Method is based on Principle of Equimolar overflow and vaporisation.

Unless above assumption is considered, i.e. in  $1^{st}$  method the flow rates of liquid and gas change along the length of the tower. As a result, the operating line slope will change with tower length. In such cases, detailed enthalpy composition data is required.

#### Assumptions

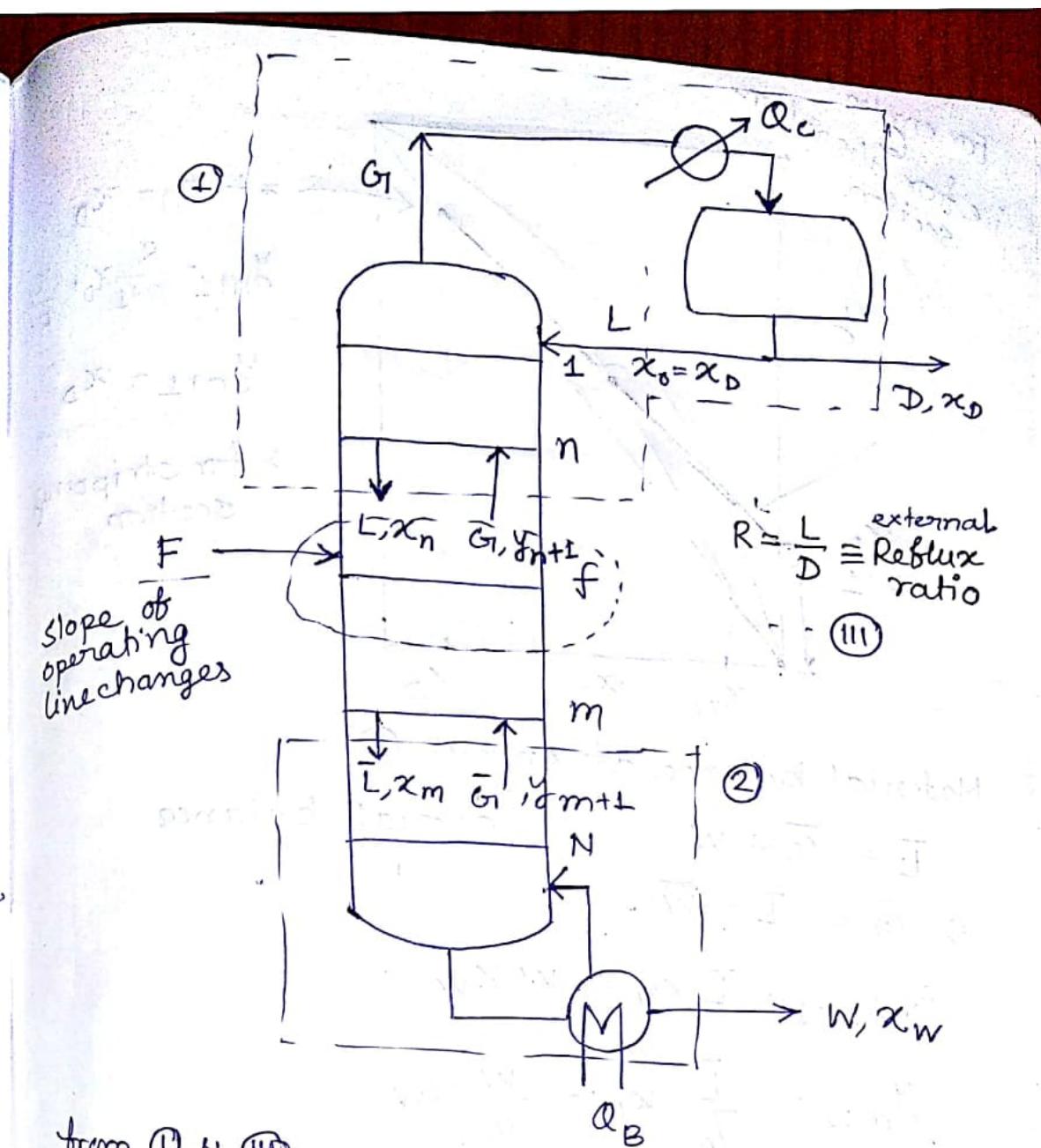
- Total condenser at  $\swarrow$
- Reflux liquid is bubble point (no subcooling)
- Gas is at saturated.  $\swarrow$
- i.e. Liquid and Gas rates are equal.

Mole balance in envelope I

$$\textcircled{I} \quad G_1 = L + D \quad \dots \quad \text{Total material balance}$$

$$\textcircled{II} \quad G_1 Y_{n+1} = L x_n + D x_D \quad \dots \quad \text{Total component balance}$$

$$Y_{n+1} = \frac{L}{G_1} x_n + \frac{D}{G_1} x_D$$



from ① & ③

$$G_1 = RD + D = D(R+1)$$

similarly from ② & ③

$$y_{n+1} = \frac{RD}{D(R+1)} x_n + \frac{D}{D(R+1)} x_D$$

$$y_{n+1} = \left( \frac{R}{R+1} \right) x_n + \left( \frac{1}{R+1} x_D \right)$$

slope                                  intercept

eq<sup>n</sup> of  
operating  
line  
for ~~st~~  
absorption  
section

For envelop  
① i.e for absorption  
section

$y_{n+1} = \frac{R}{R+1} x_D + \frac{x_0}{R+1}$

$$\text{at } x_n = x_D \\ \text{or } x_n = \frac{x_D}{R+1}$$

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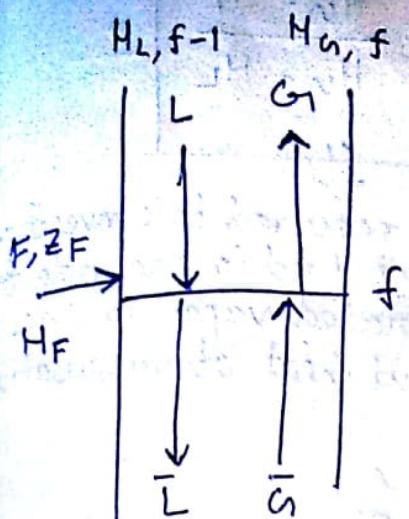
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08/11/17

Overall material balance:

$$F + L + \bar{G} = \bar{L} + \bar{G}$$

$$\bar{G} - G_1 = (\bar{L} - L) \rightleftharpoons F$$

Enthalpy balance: ①

$$FH_F + L H_{L,f-1} + \bar{G} H_{G,f+1} \\ H_{L,f} \rightleftharpoons H_{G,f+1} = \bar{L} H_{L,f} + G H_{G,f}$$

Assumption: Change in composition of the streams at the stage where feed is entering is not large.

Thus we can conclude,  $H_{L,f-1} = H_{L,f}$  and

$$H_{G,f} = H_{G,f+1}$$

$$\Rightarrow FH_F + L H_L + \bar{G} H_G = \bar{L} H_L + G H_G$$

$$(\bar{G} - G_1) H_G = (\bar{L} - L) H_L - FH_F \quad \text{--- ②}$$

Replacing  $\bar{G} - G_1 = (\bar{L} - L) + F$  from ①

$$(\bar{L} - L + F) H_G = (\bar{L} - L) H_L - FH_F$$

$$(\bar{L} - L) (H_G - H_L) = F(H_G - H_F)$$

$$\boxed{\frac{\bar{L} - L}{F} = \frac{H_G - H_F}{H_G - H_L}} = q \quad \text{--- ③}$$

$H_n$  = specific enthalpy of saturated vapor

Molar latent heat  $H_L$  = specific enthalpy of saturated liquid

$$\bar{L} - L = Fq$$

$$q = \frac{H_{G1} - H_F}{H_{G1} - H_L}$$

$$\bar{G} - G_1 = Fq - F$$

$$\boxed{\bar{G} - G_1 = F(q-1)} \quad \text{--- (V)}$$

Heat required to convert one mole of feed to one mole of saturated vapor to the latent heat of vaporisation

For stripping section

$$\bar{G}_D = \bar{L}x - w x_w$$

--- ①

For absorption section

$$G_D y = Lx + D x_D$$

--- ②

Subtracting ② from ①

$$(\bar{G} - G_1)y = (\bar{L} - L)x - (D x_D + w x_w)$$

For entire column

$$F = \cancel{G_F} D + w$$

--- ③

$$F z_F = D x_D + w x_w$$

--- ④

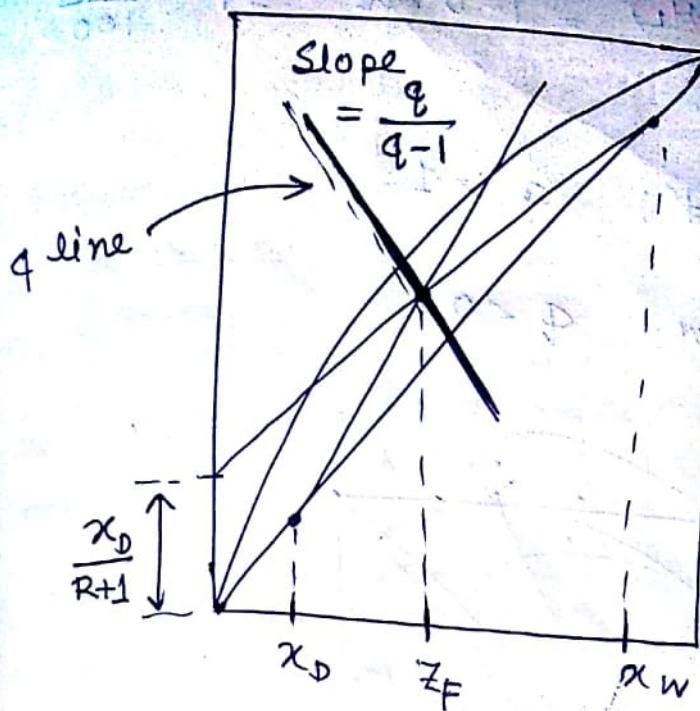
$$\Rightarrow y = \left( \frac{\bar{L} - L}{\bar{G} - G_1} \right) x - \frac{F z_F}{\bar{G} - G_1}$$

$$\boxed{y = \frac{Fq}{F(q-1)} x - \frac{F z_F}{F(q-1)}} \quad \text{--- ⑤}$$

Locus of point of intersection of top and bottom section operating line

$$\boxed{y = \frac{q x}{q-1} - \frac{z_F}{q-1}} \quad \text{--- ⑥}$$

--- ⑦

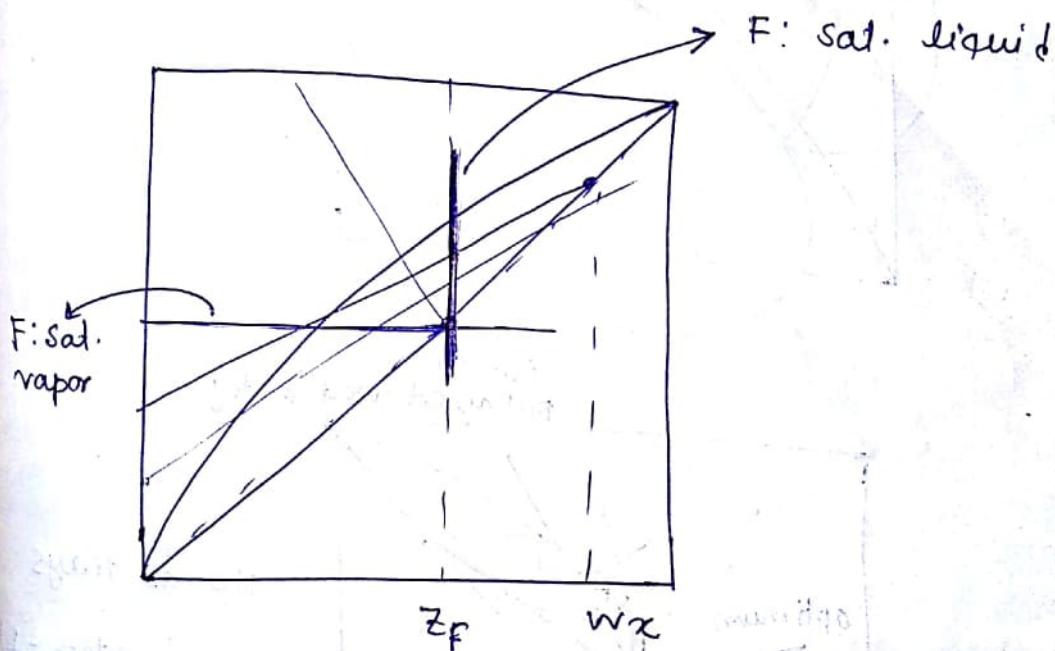


for  $x = z_w$   
from (VII)

$$y = z_w$$

i) Feed: Saturated liquid

$$q = 1 \Rightarrow \text{slope} = \infty$$



ii) Feed: Saturated liquid,  $q = 0 \Rightarrow \text{slope} = 0$

iii) Feed: liquid + vapor,  $x\% \text{ vapor}$

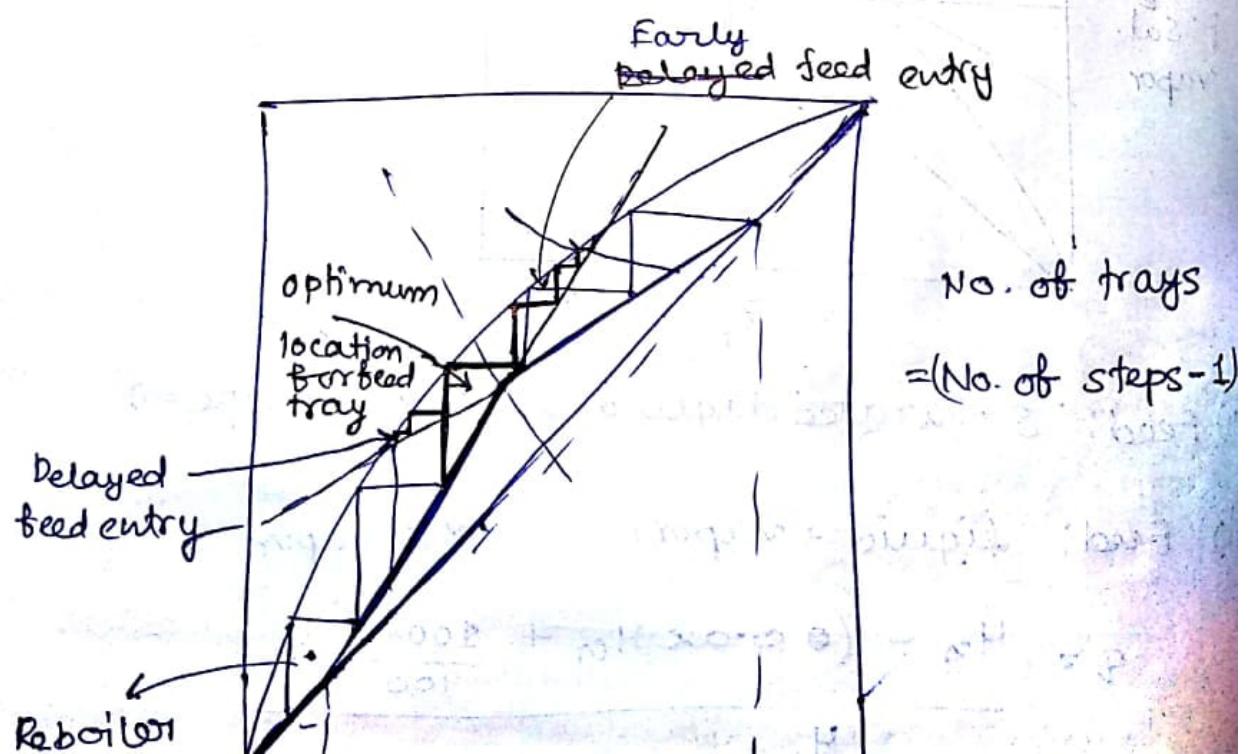
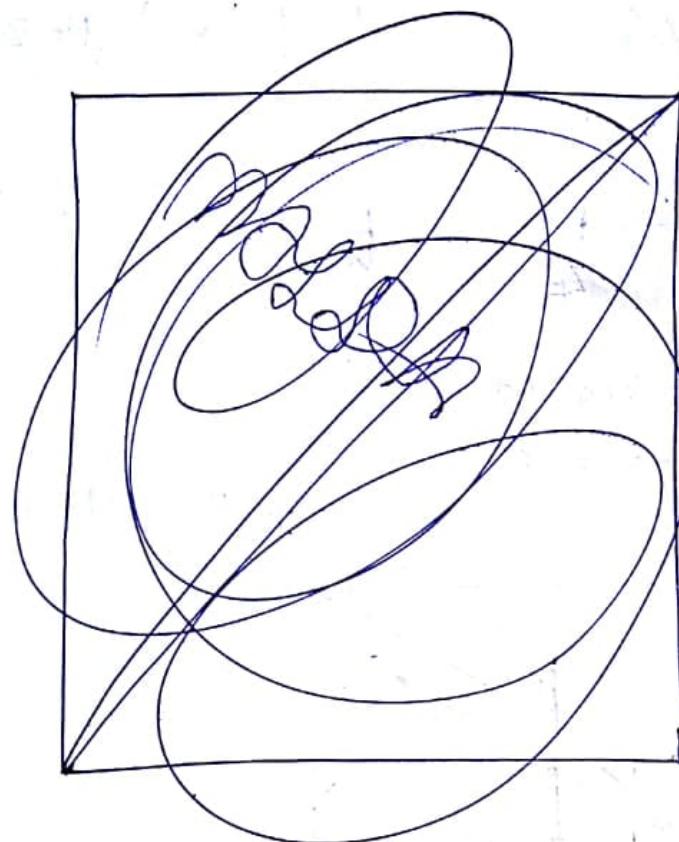
$$q = \frac{H_{G1} - (0.02x H_{G1} + \frac{100-x}{100} H_L)}{H_{G1} - H_L}$$

$$= \frac{(1-0.02x) H_{G1} - (1-0.02x) H_L}{H_{G1} - H_L}$$

$$q = \frac{(1 - 0.0x)(H_g - H_L)}{(H_g - H_L)} = 1 - 0.0x = 1 - \frac{x}{100}$$

iv) Sub cooled liquid :  $q > 1$

v) Super heated vapor :  $q < 0$

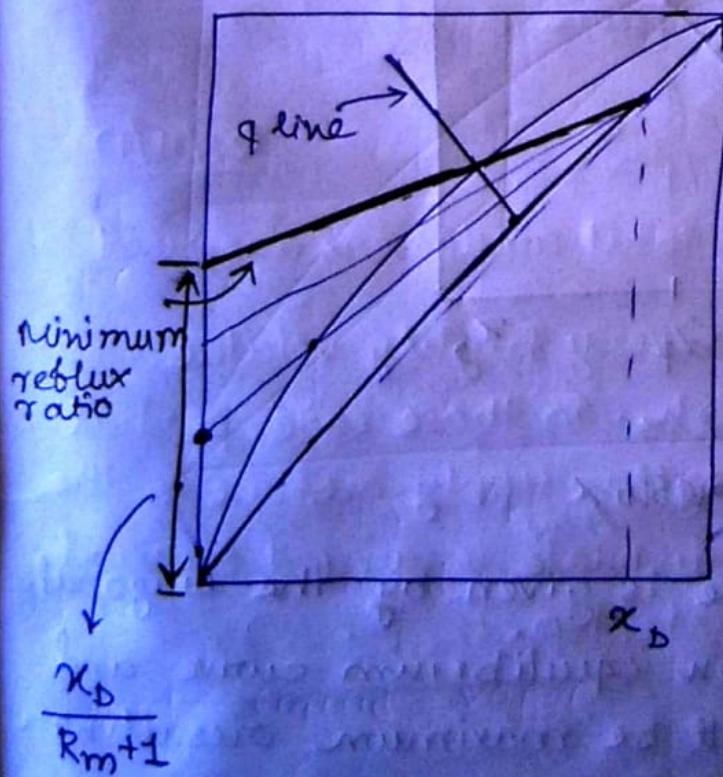


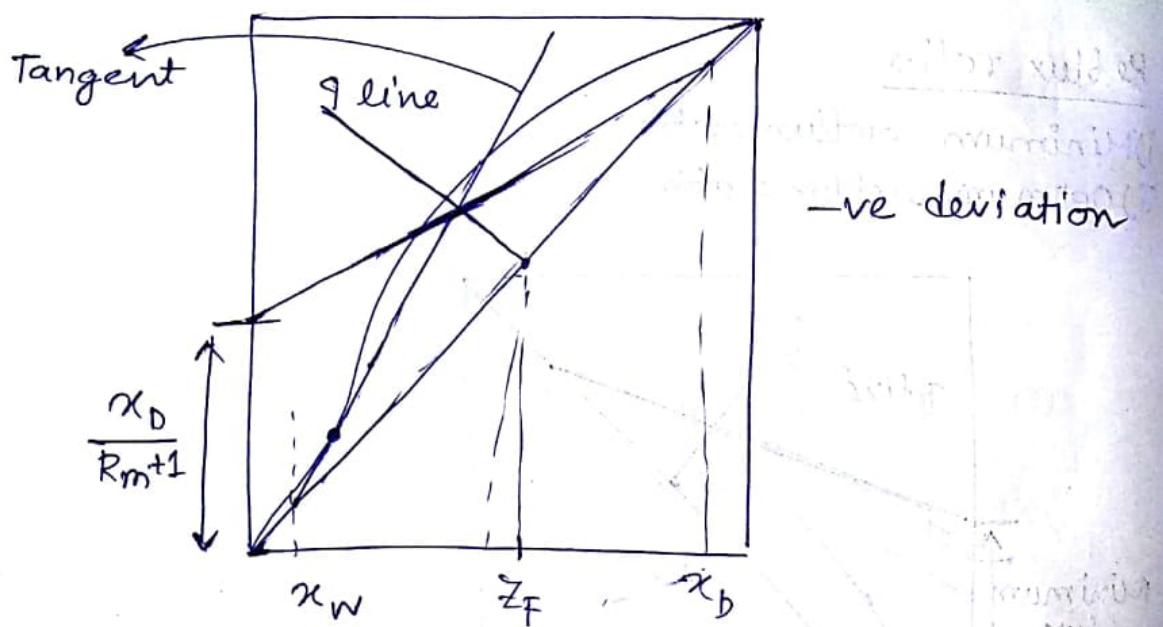
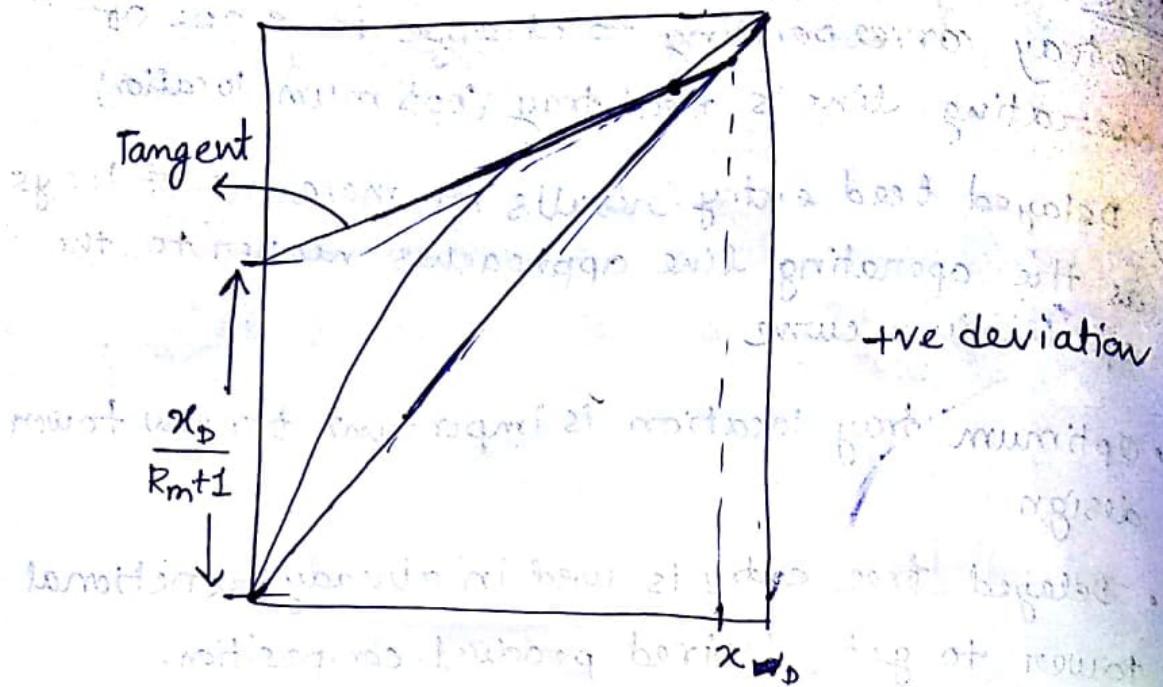
Note

- 1) The tray corresponding to change in slope of operating line is feed tray (optimum location)
- 2) Delayed feed entry results in more no. of trays as the operating line approaches nearer to the equilibrium curve
  - optimum tray location is important for new tower design
  - Delayed feed entry is used in already functional tower to get desired product composition.

Reflux ratio

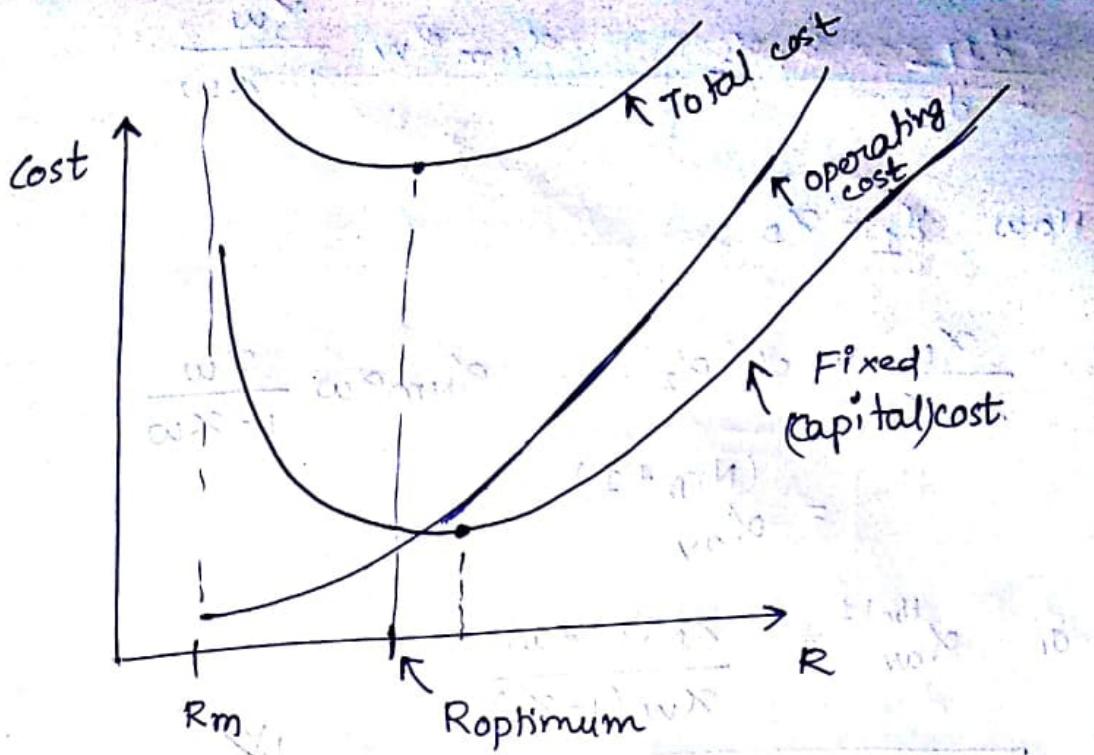
- i) Minimum reflux ratio
- ii) Optimum reflux ratio





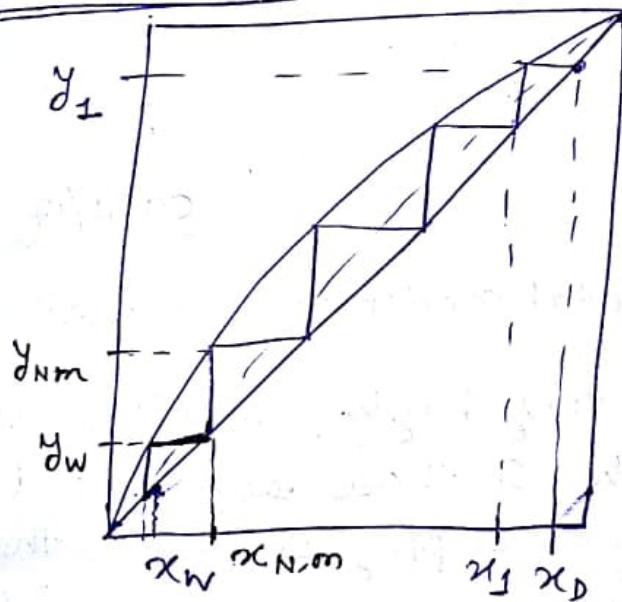
$R_m \rightarrow \infty \Rightarrow$  Total reflux  
 the operating line is given by the diagonal.  
 i.e. distance between equilibrium curve and  
 operating line will be maximum resulting in  
 minimum no. of trays

For minimum reflux ratio, no. of trays required  
 is very high ( $\rightarrow \infty$ )



Usually,  
 $R_{\text{optimum}} \in (1.2 \text{ to } 1.8) \times R_m$

For total Reflux



$$\frac{y_w}{1-y_w} = \alpha_w \frac{x_w}{1-x_w}$$

$$\frac{x_{Nm}}{1-x_{Nm}} = \alpha_w \frac{x_w}{1-x_w}$$

$$\frac{y_{Nm}}{1-y_{Nm}} = \alpha_{Nm} \frac{x_{Nm}}{1-x_{Nm}} = \alpha_{Nm} \alpha_w \frac{x_w}{1-x_w}$$

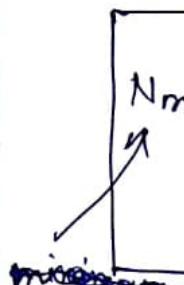
$$\frac{y_1}{1-y_1} = \alpha_1 \alpha_2 \cdots \alpha_{Nm} \alpha_w \frac{x_w}{1-x_w}$$

$$\text{Now } y_1 = x_D$$

$$\frac{x_D}{1-x_D} = \alpha_1 \alpha_2 \cdots \alpha_{Nm} \alpha_w \frac{x_w}{1-x_w}$$

$$= \frac{(Nm+1)}{\alpha_{av}}$$

$$\text{or } \alpha_{av}^{Nm+1} = \frac{x_D(1-x_w)}{x_w(1-x_D)}$$

$$N_m + 1 = \frac{\log \left( \frac{x_D(1-x_w)}{x_w(1-x_D)} \right)}{\log \alpha_{av}}$$


✓

Fenske's Eqn

no. of stages

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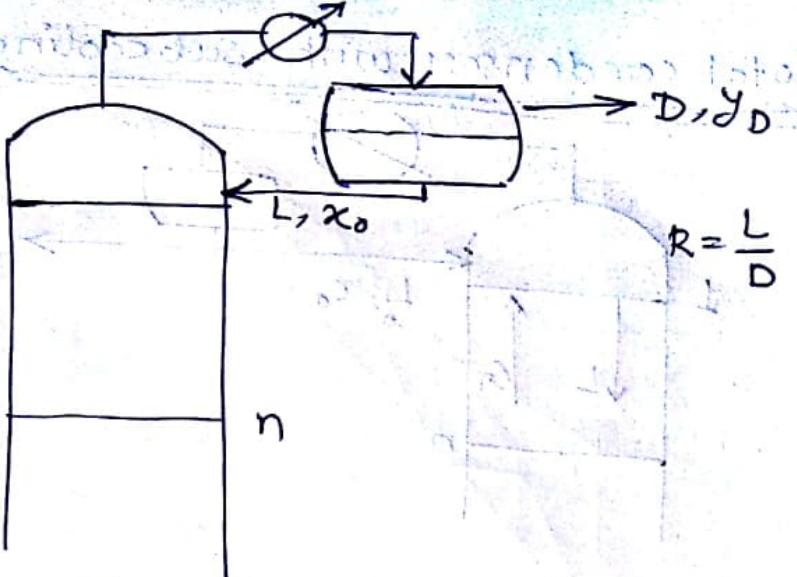
- In ideal situation, total condensers are used
- If top product is very highly volatile, condense load will be very high. In those case, partial condenser is used only to supply reflux to the tower, rest of the distillate is carried forward as vapor to the next step.

- partial condenser

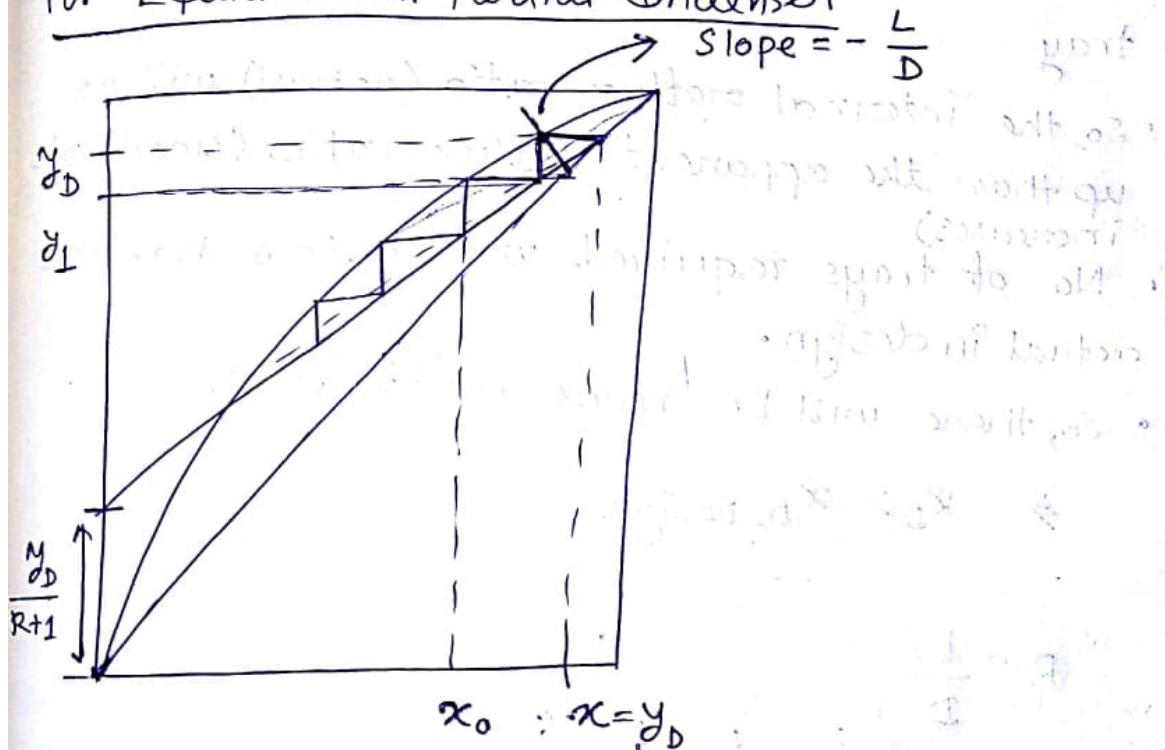
→ Equilibrium partial condenser ( $y_D \neq x_0$ )

→ Differential condenser (not feasible)

→ Rapid condenser (There is no change in composition of distillate & reflux stream)  
 $y_D = x_0$



For Equilibrium Partial condenser



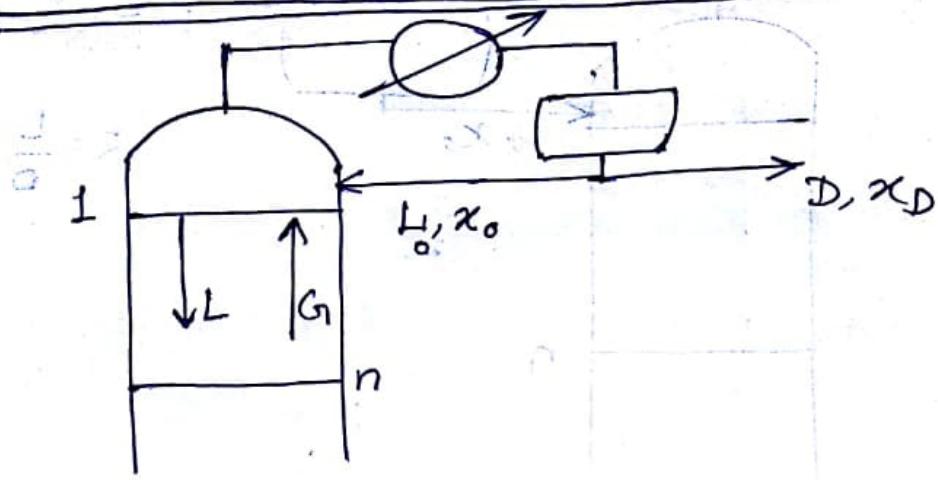
Total no. of trays

$$= (\text{Total no of steps} - 2)$$

$1 + 1$

Equilibrium Reboiler  
partial condenser

## Total condenser with subcooling (cold reflux)



- Results in condensation of vapor at the top tray
  - So, the internal reflux ratio (actual) will go up than the apparent reflux ratio. (Liquid rate increases)
  - No. of trays required, will be less than the actual in design.
  - So, there will be more purification
- $\Rightarrow x_D > x_{D, \text{design}}$

$$R = \frac{L_0}{D}$$

$$L = L_0 + \frac{L_0 C_{L0} M_{av} (t_{BP,R} - t_R)}{\lambda_{av}}$$

$$L = RD + RD \cdot C_{L0} M_{av} \frac{(t_{BP,R} - t_R)}{\lambda_{av}}$$

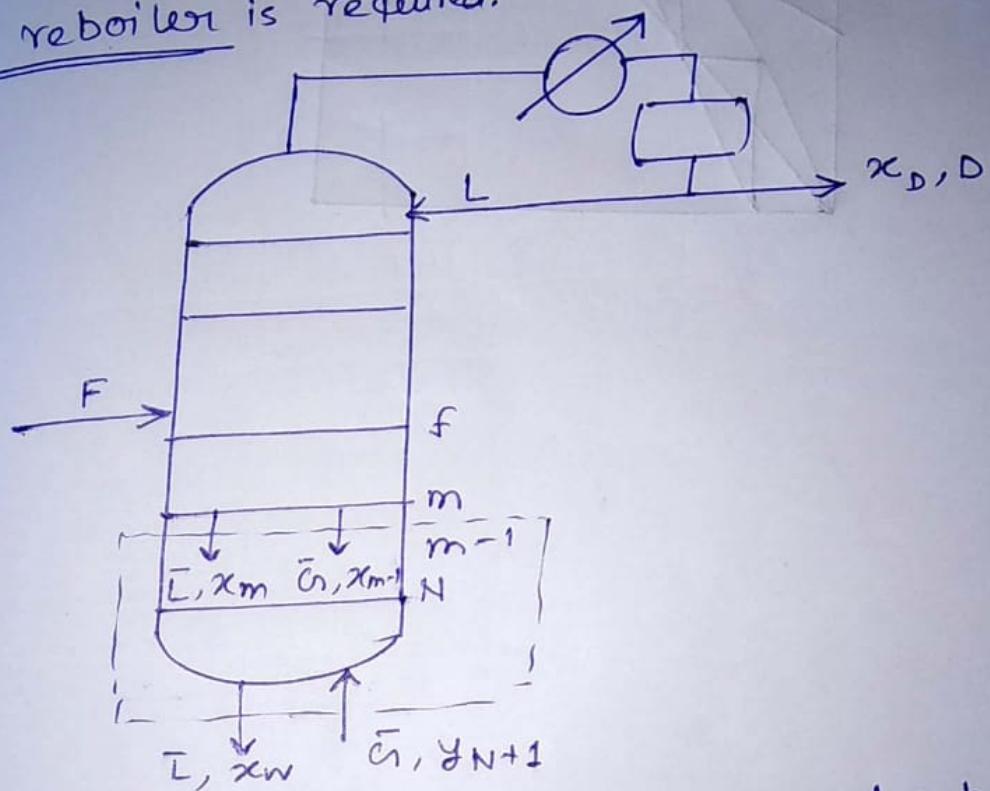
$$L = RD \left[ 1 + \frac{C_{L0} M_{av} (t_{BP,R} - t_R)}{\lambda_{av}} \right]$$

$$R' = L - R \left[ 1 + \frac{C_{L0} M_{av} (t_{BP,R} - t_R)}{\lambda_{av}} \right]$$

so operating line:

$$y_{n+1} = \frac{R'}{R'+1} x_n + \frac{x_D}{R'+1} L$$

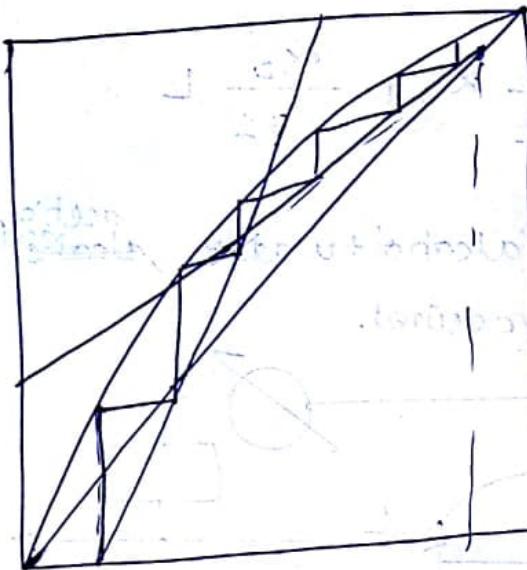
For aqueous soln (alcohol + water, ~~acetic acid + water~~)  
no. reboiler is required.



Because under product is water and reboiler is ~~using~~ producing steam, reboiler can be removed and instead utility steam stream can be fed

$$\bar{G} y_{m+1} + \bar{L} x_w = \bar{L} x_m + \bar{G} y_{N+1}$$

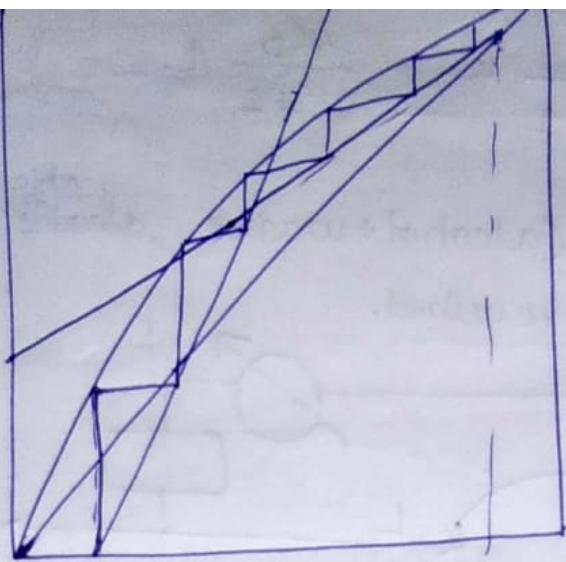
$$\frac{\bar{L}}{\bar{G}} = \frac{y_{m+1}}{x_m - x_w}$$



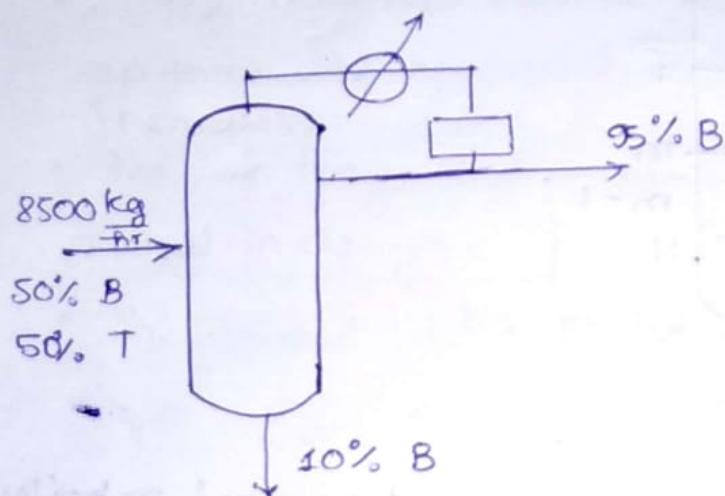
Belajar kunci jawaban dan soal matematika sekolah  
di matematika semester pertama dan semester kedua  
untuk mereka yg sdh belajar dan memahami  
bahan dan

Lihat di bawah ini dan ikuti langkah

$$\frac{1}{2} \times 10 \times 10 = 50$$



13/11/17



$$B = 48$$

$$T = 92$$

$$(M_{av})_{\text{Feed}}$$

$$= 39 + 46$$

$$= 85$$

No. of moles in feed

$$= 100 \text{ kmoles}$$

Basis: 1 hr.

$$S_0 = 0.95 D + 0.1 W$$

$$\left. \begin{array}{l} W + D = 100 \\ W = 100 - D \end{array} \right\} \quad \begin{array}{l} S_0 = 0.95 D + 0.1(100 - D) \\ S_0 = 0.95 D + 10 - 0.1D \\ 40 = 0.85 D \end{array}$$

$$D = \frac{40}{0.85} = 47.06$$

$$W = 52.94$$

$$D = 4263.53 \text{ kg/hr}$$

$$W = 52.94 (0.478 + 0.9 \times 92)$$

$$= 4796.36 \text{ kg/hr}$$

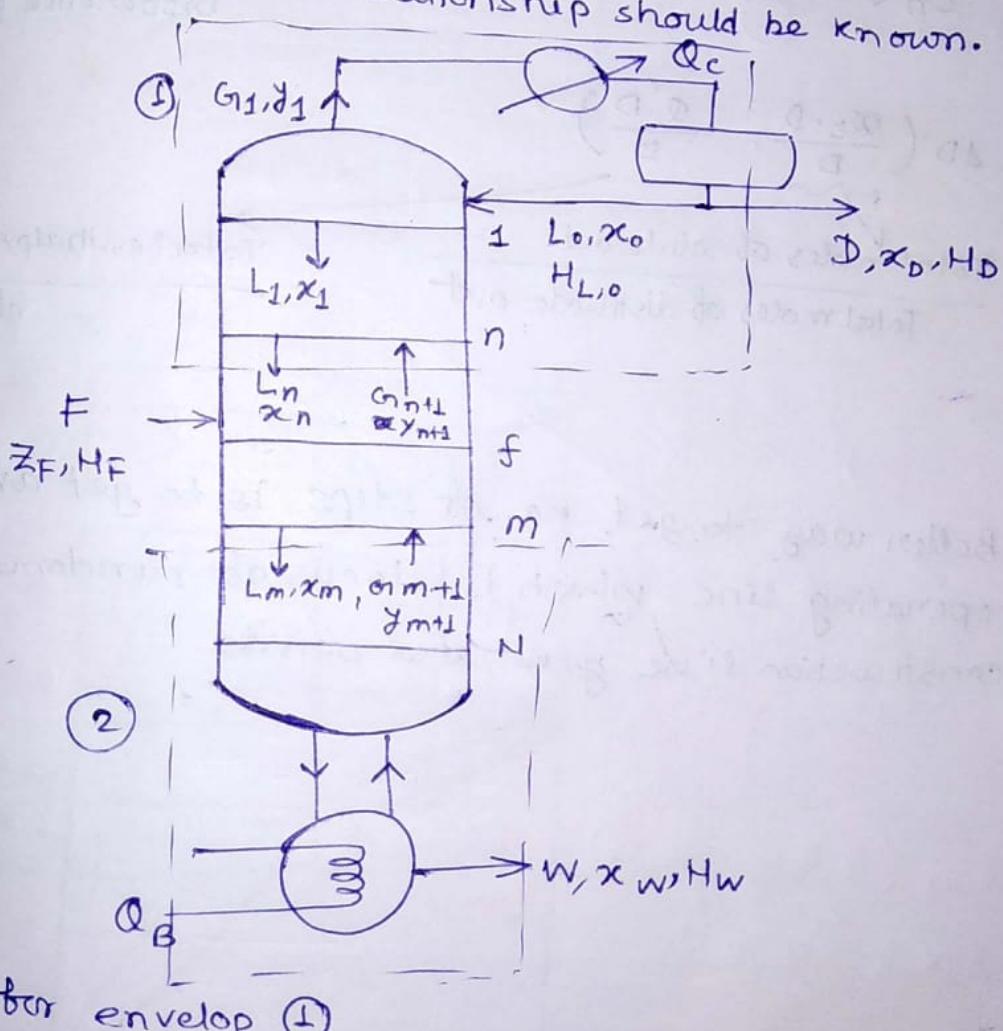
$$D = 3703.69 \text{ kg/hr.}$$

15/11/17

### Ponchon & Savarit's Method

(for determination  
of no. of ideal trays)

- Detailed enthalpy composition ( $H-x-y$ ) as well as ( $y-x$ ) curve or relationship should be known.



for envelop ①

$$G_{n+1} = L_n + D \quad \dots \text{overall component balance}$$

$$\Rightarrow G_{n+1} - L_n = D$$

$$G_{n+1} Y_{n+1} - L_n X_n = D X_D \quad \text{--- component balance}$$

$$G_{n+1} H_{O,n+1} = L_n H_{L,n} + D H_D + Q_c \quad \text{--- enthalpy balance}$$

$$G_{n+1} H_{O,n+1} - L_n H_{L,n} = D (H_D + \frac{Q_c}{D}) = D Q'$$

Eliminating D,

$$\frac{L_n}{G_{n+1}} = \frac{x_D - y_{n+1}}{x_D - x_n} = \frac{\alpha' - H_{O,n+1}}{\alpha' - H_{L,n}}$$

The line passes through

$$L_n (x_n, H_{L,n}), G_{n+1} (y_{n+1}, H_{O,n+1}), \Delta D (\alpha_D, \alpha')$$

↓  
Difference point

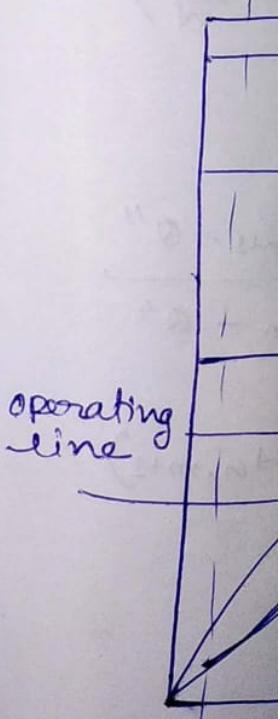
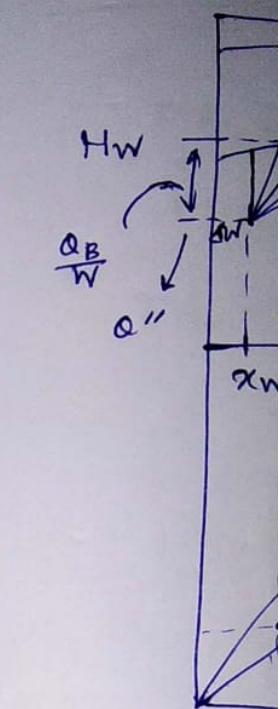
$$\Delta D \left( \frac{x_D \cdot D}{D}, \frac{\alpha' \cdot D}{D} \right)$$

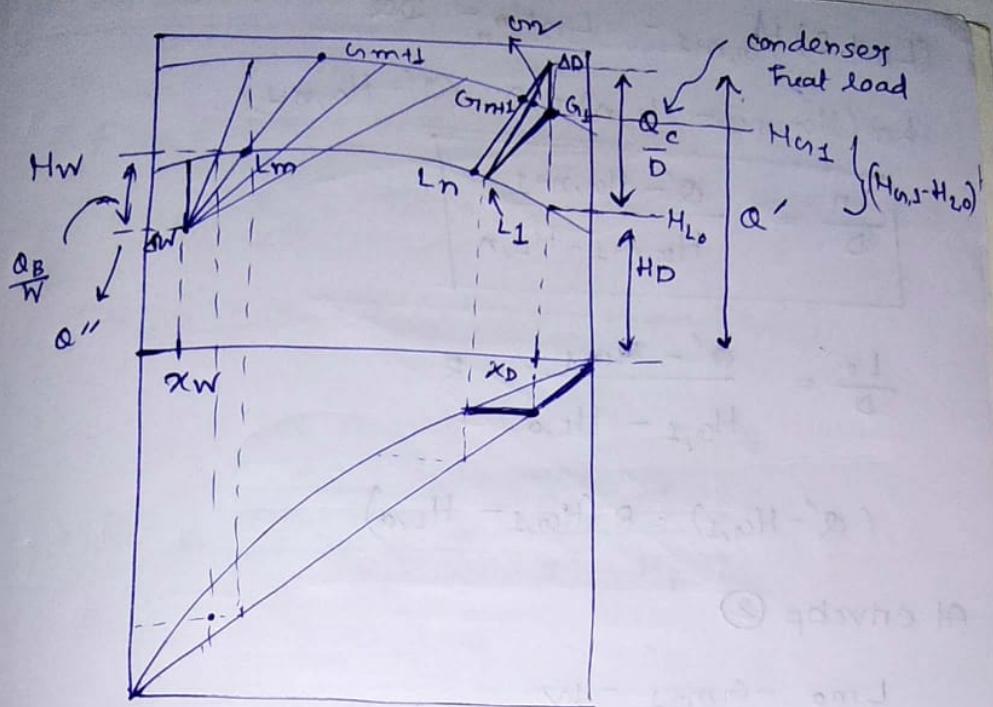
Total moles of solute out

Total moles of distillate out

Total enthalpy of solute out  
~ ~ ~ ob distillate out

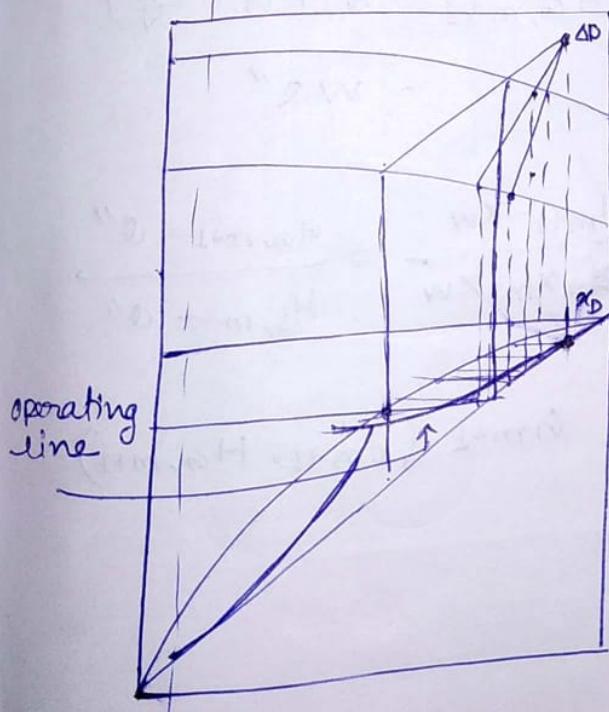
- Better way to get no. of steps is to get an operating line which is locus of random construction line generated points





2)  
point

sb solute  
out  
distillat  
out



$$(L_n + D) H_{G,n+1} - L_n H_{L,n} = D Q'$$

$$L_n (H_{G,n+1} - H_{L,n}) = D (Q' - H_{G,n+1})$$

$$\boxed{\frac{L_n}{D} = \frac{Q' - H_{G,n+1}}{H_{G,n+1} - H_{L,n}}}$$

$$\frac{L_0}{D} = \frac{Q' - H_{G,1}}{H_{G,1} - H_{L,0}} = R$$

$$(Q' - H_{G,1}) = R (H_{G,1} - H_{L,0})$$

At envelope ②

$$L_{m+1} - G_{m+1} = W$$

$$L_m x_m - G_{m+1} y_{m+1} = W x_W$$

$$\begin{aligned} H_{L,m} L_m + Q_B &= G_{m+1} H_{G,m+1} + W H_W \\ \Rightarrow L_m H_{L,m} - G_{m+1} H_{G,m+1} &= W (H_W - \frac{Q_B}{W}) \\ &= W Q'' \end{aligned}$$

$$\frac{L_m}{G_{m+1}} = \frac{y_{m+1} - x_W}{x_m - x_W} = \frac{H_{G,m+1} - Q''}{H_{L,m} - Q''}$$

$$\begin{aligned} L_m (x_m, H_{L,m}), \quad G_{m+1} (y_{m+1}, H_{G,m+1}) \\ \Delta W (x_W, Q'') \end{aligned}$$

for whole tower

$$F = D + w$$

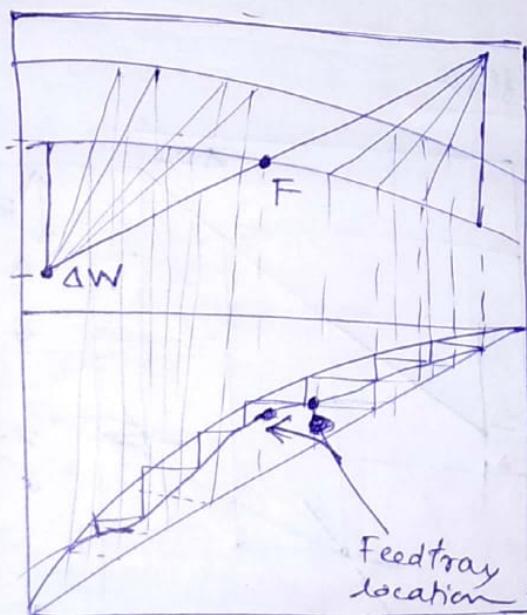
$$FZ_F = D\chi_D + w\chi_W$$

$$FH_F + Q_B = DH_D + Q_C + WH_W$$

Ignoring heat loss to the surrounding

$$FH_F = D(H_D + Q_C/D) + w(H_W - Q_B/w)$$
$$= DQ' + wQ''$$

$$\frac{D}{w} = \frac{Z_F - \chi_W}{\chi_D - Z_F} = \frac{H_F - Q''}{Q' - H_F}$$



considering  
feed to be  
liquid at  
bubble point

## Special Type of Distillation (Imp for GATE) 16/11/17

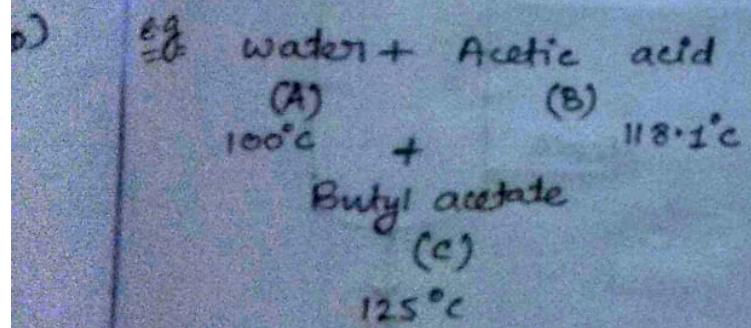
- used for separating binary mixture having very low <sup>rel.</sup> volatility or azeotropic mixture.
- For normal distillation, if relative volatility is very small then
  - tower should be longer
  - huge sized reboiler & condenser are required leading to higher cost.

### Azeotropic Distillation

(A+B) Relative volatility is very low

+C ← Component C forms low boiling azeotrope  
↓ either with A or with B.

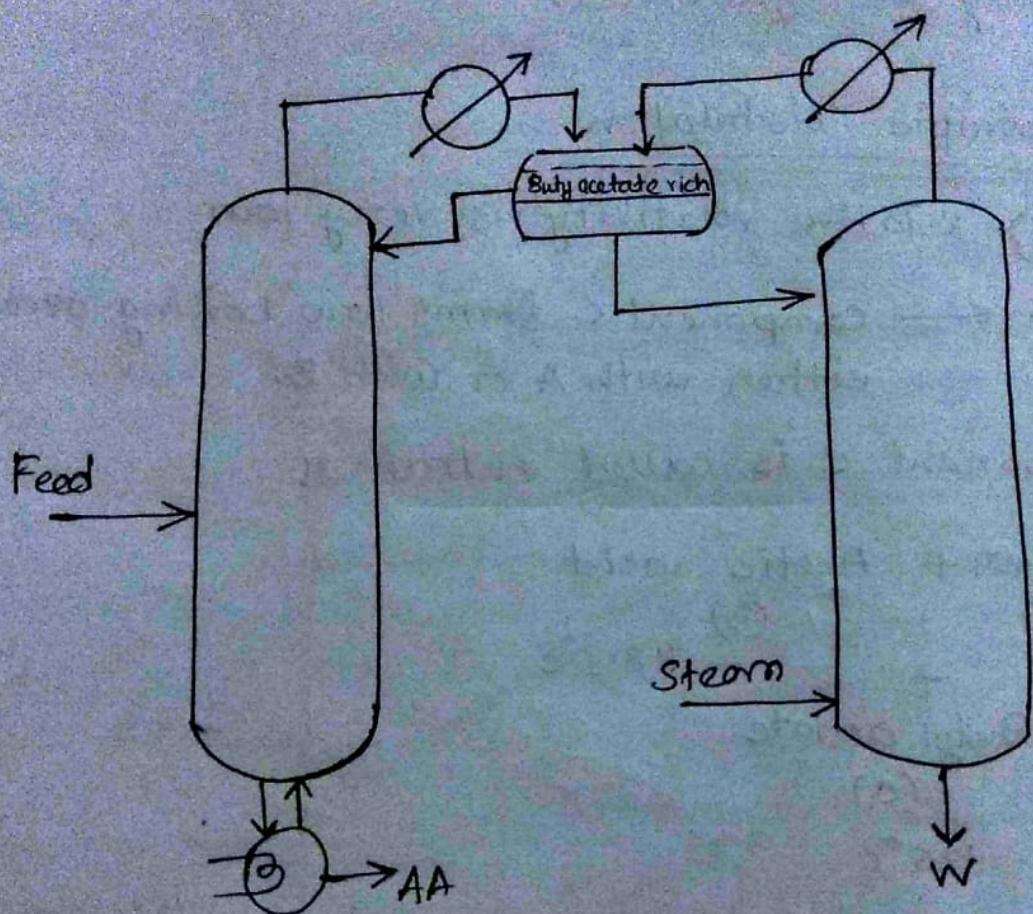
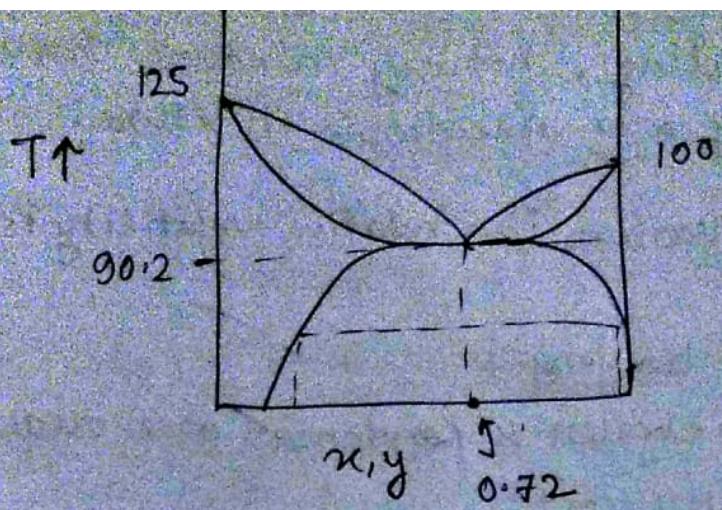
Component C is called entrainer.



Butyl acetate + water form a low boiling azeotrope

90.2°C  
72 mole% water

From top, azeotropic mixture is obtained. From bottom almost pure acetic acid is obtained. Water & Butyl acetate form hetero azeotrope which separate in components on condensation.

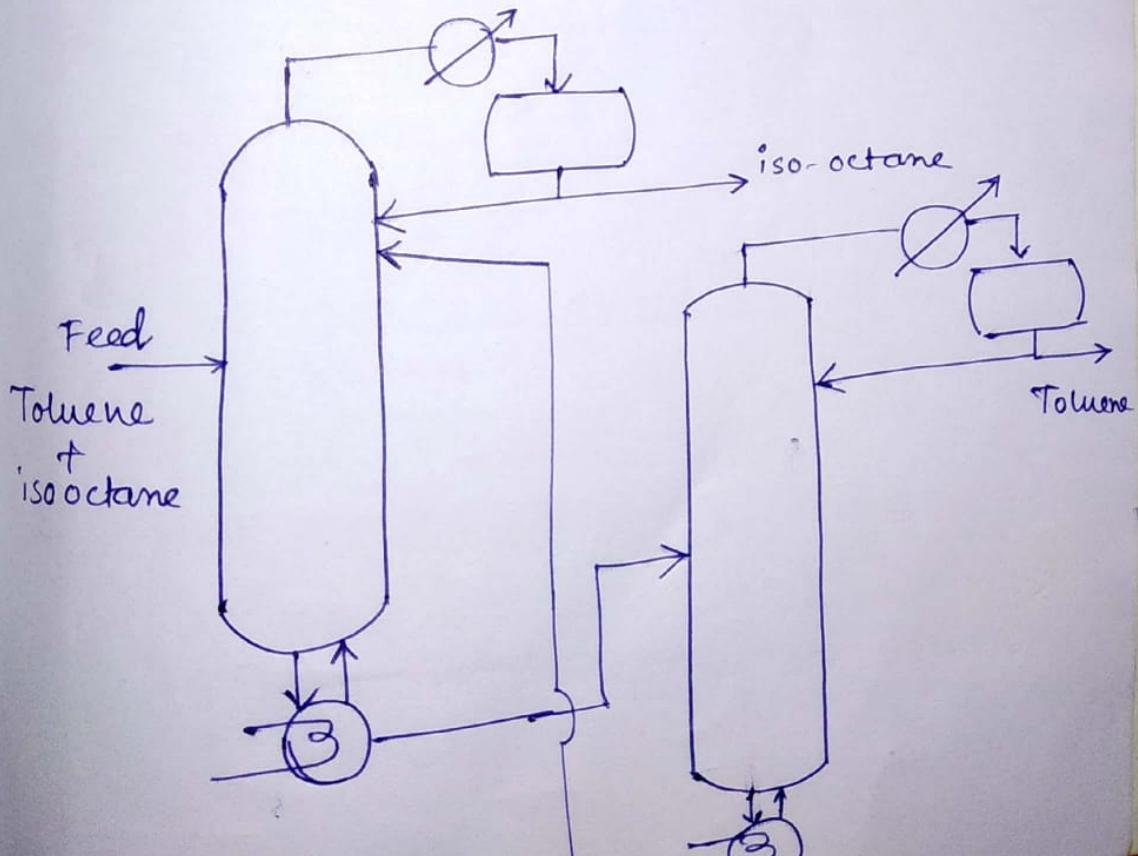
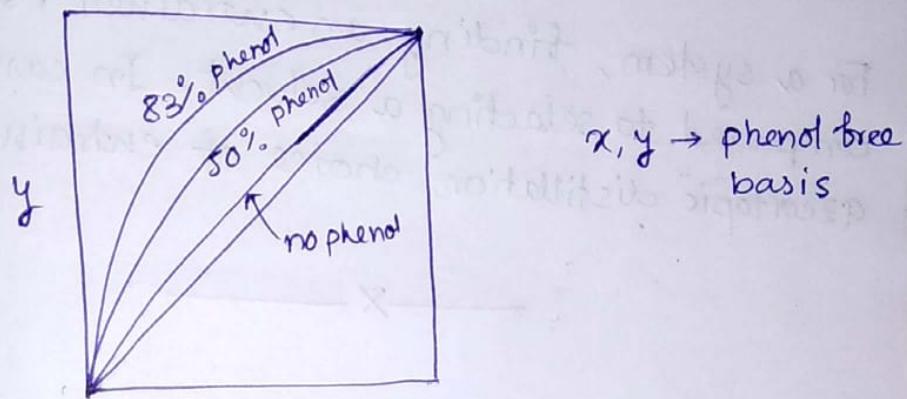


- Entrainier should be such that, it should forms heteroazeotrope with the minority component.
- Entrainier should have low latent heat of vaporisation.

## Extractive Distillation

- Relative volatility of the components is very low.  
 $(A+B) + C$   
 In solvent, changes the relative volatility of the components

Iso octane + Toluene  
 (A) (B)  
 99.3°C 110.8°C  
 +  
 Phenol  
 (C)  
 181.4°C



- 1) Selectivity: The solvent should be of high selectivity. It should be able to change the vapor liquid equilibria by great extent.
- 2) Capacity: Solubility of component should be high. Solvents capacity to dissolve the component should be high.
- 3) Volatility: Should be low.

4) viscosity  $\downarrow$ , freezing point  $\downarrow$ , non-corrosive, non-toxic

For a system, finding an entrainer is difficult compared to selecting a solvent. In case of azeotropic distillation, choice of entrainer is limited.

