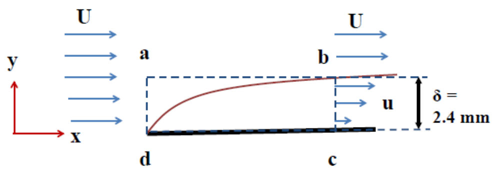
1. Air at standard conditions is flowing over a thin flat plate which is 1 m long and 0.3 m wide. The flow is uniform at the leading edge of the plate. The velocity profile in the boundary layer is assumed to be linear and U = 30 m/s. Assume that the flow conditions are independent of z and treat the flow as two dimensional. Using the control volume abcd (dc = 1m), compute the mass flow rate across surface ab. The boundary layer thickness at the end of the plate (point c in the figure) is δ = 2.4mm. Determine the magnitude and direction of the x-component of the force required to hold the plate stationary (ρ_{air} = 1.23 kg/m^3). **4+6=10**



1. At bc, $\dfrac{u}{U} = \dfrac{y}{\delta} = \eta$, U = 30 m/s

Basic equations: $0 = \cancel{\dfrac{\partial}{\partial t}\int_{CV} \rho dV} + \int_{CS} \rho \vec{V}.\vec{dA}$

$$F_{SX} + F_{BX} = \cancel{\frac{\partial}{\partial t}\int_{CV} u\rho dV} + \int_{CS} u\rho \vec{V}.\vec{dA}$$

$$\therefore 0 = \underbrace{\{-\rho U b\delta\}}_{\dot{m}_{ad}} + \int_0^{\delta} \rho U b dy + \dot{m}_{ab}, \quad \text{and } \int_0^{\delta} \rho U b dy = \rho U b\delta \int_0^1 \eta d\eta = \frac{1}{2}\rho U b\delta = \dot{m}_{bc}$$

Or $\dot{m}_{ab} = \dfrac{1}{2}\rho U b\delta = \dfrac{1}{2}\times 1.23\dfrac{kg}{m^3}\times 30\dfrac{m}{s}\times 0.3m \times 0.0024m = 0.0133\dfrac{kg}{s}$

From momentum equation

$$R_x = u_{da}\{-\rho U b\delta\} + u_{ab}\,\dot{m}_{ab} + \int_0^{\delta} \rho u^2 b dy$$

Now $\displaystyle\int_0^{\delta} \rho u^2 b dy = \rho U^2 b\delta \int_0^1 \eta^2 d\eta = \frac{1}{3}\rho U^2 b\delta$

And $u_{da} = U, u_{ab} = U$ (all of ab is outside BL)

$$R_x = -\rho U^2 b\delta + \frac{1}{2}\rho U^2 b\delta + \frac{1}{3}\rho U^2 b\delta = -\frac{1}{6}\rho U^2 b\delta$$

$$R_x = -\frac{1}{6}\times 1.23\frac{kg}{m^3}\times (30)^2\frac{m^2}{s^2}\times 0.3m\times 0.0024m = -0.133N$$

This force must be applied t the CV by the plate. Thus, to hold the plate,

$$F_x = R_x = -0.133N$$

2. The presence of CO_2 in solution is essential to the growth of aquatic plant life, with CO_2 used as a reactant in the photosynthesis. Consider a stagnant body of water in which the concentration of CO_2 (ρ_A) is everywhere zero. At time t = 0, the water is exposed to a source of CO_2, which maintains the surface (x = 0) concentration at a fixed value ρ_{Ao}. For time t > 0, CO_2 will begin to accumulate at the water-air surface and start diffusing in the water with the simultaneous utilization in the photosynthesis process, the rate of which can be expressed as the product of a homogeneous reaction rate constant, k_1''' and the local CO_2 concentration ρ_A (x, t).

(a) Modify the species balance equation (expressed in terms of CO_2 mass concentration ρ_A) by cancelling the terms (with one-word reason) that are not relevant. What does each term in the equation represent physically?

b) For this problem, assume the case of a "deep" body of water and write appropriate boundary conditions that could be used to obtain the solution. Also assume negligible CO_2 consumption in the photosynthesis process and modify the governing equation accordingly.

c) For the condition of Part (b), evaluate the concentration distribution of CO_2 in the stagnant body of the water. **3+3+3=9**

ASSUMPTIONS: (1) One-dimensional diffusion in x, (2) Constant properties, including total density ρ, (3) Water is stagnant.

The species balance equation in mass concentration terms is

$$\left(\frac{\partial \rho_A}{\partial t} + v_x \frac{\partial \rho_A}{\partial x} + v_y \frac{\partial \rho_A}{\partial y} + v_z \frac{\partial \rho_A}{\partial z}\right) = D_{AB}\left[\frac{\partial^2 \rho_A}{\partial x^2} + \frac{\partial^2 \rho_A}{\partial y^2} + \frac{\partial^2 \rho_A}{\partial z^2}\right] + R_A$$

For the current problem –

$$D_{AB}\,\frac{\partial^2 \rho_A}{\partial x^2} - k_1 \rho_A = \frac{\partial \rho_A}{\partial t}$$

The first term on the LHS represents *net* transport of CO_2 into a differential control volume by diffusion.

The second term represents the rate of CO2 consumption due to chemical reactions.

The term on the right-hand side represents the rate of increase of CO2 storage within the control volume.

(b) For a deep body of water, appropriate boundary conditions are

$$\rho_A(0,t) = \rho_{A0}$$
$$\rho_A(\infty,t) = 0$$

and, with negligible chemical reactions, the species diffusion equation reduces to

$$\frac{\partial^2 \rho_A}{\partial x^2} = \frac{1}{D_{AB}}\frac{\partial \rho_A}{\partial t}$$

(c) With an initial condition, $\rho_A(x,0) = \rho_{A,i} = 0$, the problem is analogous to that involving heat transfer in a semi-infinite medium with constant surface temperature. By analogy, the species concentration is then

$$\frac{\rho_A(x,t) - \rho_{A,0}}{-\rho_{A,0}} = erf\left(\frac{x}{2(D_{AB}t)^{1/2}}\right)$$

$$\rho_A(x,t) = \rho_{A,0}\ erfc\left(\frac{x}{2(D_{AB}t)^{1/2}}\right)$$

3. Dry air flows at T_∞ = 300K over trays filled with water with a velocity of 15 m/s. The water surface temperature is maintained at 330 K by radiant heaters. Find (a) Evaporative flux (kg/s.m^2) at a distance of 1 m from the leading edge, and (b) The heat flux at this distance required to maintain water temperature at 330K. The following properties are known: Air: kinematic viscosity = 17.40 x 10 $^{-6}$ m^2/s, k = 0.0274 W/m.K, Pr = 0.705, Water vapour-air: D_{AB} = 0.28x10^{-4} m^2/s, Sc = 0.616, Saturated water vapour $\rho_{A,\,sat}$ = 0.1134 kg/m^3, h_{fg} = 2366 KJ/kg **3+5=8**

(a) The evaporative flux of water vapor (A) is

$$\dot{m}_{A,x} = h_{m,x}(\rho_{A,s} - \rho_{A,\infty}) = h_{m,x}(\rho_{A,sat}(T_s) - \phi_\infty \rho_{A,sat}(T_\infty))$$

Evaluate Re_x to determine the nature of the flow and then select the proper correlation.

$$\mathrm{Re}_x = \frac{u_\infty x}{\gamma} = \frac{15\frac{m}{s}\times 1m}{17.40\times10^{-6}\frac{m^2}{s}} = 8.621\times10^5$$

Hence the flow is turbulent, and invoking the heat-mass analogy

$$Sh_x = \frac{h_m x}{D_{AB}} = 0.0296\,\mathrm{Re}_x^{\frac{4}{5}}\,Sc^{\frac{1}{3}}$$

$$h_m = \frac{0.28\times10^{-4}\frac{m^2}{s}}{1m}\times 0.0296\left(8.621\times10^5\right)^{\frac{4}{5}}\left(0.616\right)^{\frac{1}{3}} = 3.952\times10^{-2}\frac{m}{s}$$

Hence the evaporative flux at x = 1m is

$$\dot{m}_{A,x} = 3.952\times10^{-2}\frac{m}{s}\left(0.1134\frac{kg}{m^3} - 0\right) = 4.48\times10^{-3}\frac{kg}{s.m^2}$$

(b) From an energy balance on the differential element at x = 1m,

$$q''_{rad} = q''_{conv} + q''_{evap} = h_x\left(T_s - T_\infty\right) + \dot{m}_{A,x}\,h_{fg}$$

To estimate h_x, invoke the heat-mass analogy using the correlation

$$\frac{Nu_x}{Sh_x} = \left(\frac{\mathrm{Pr}}{Sc}\right)^{\frac{1}{3}} \text{ or } h_x = h_{m,x}\frac{k}{D_{AB}}\left(\frac{\mathrm{Pr}}{Sc}\right)^{\frac{1}{3}};$$

$$h_x = 3.952\times10^{-2}\frac{kg}{s.m^2}\frac{0.0274\frac{W}{m.K}}{0.28\times10^{-4}\frac{m^2}{s}}\left(\frac{0.705}{0.616}\right)^{\frac{1}{3}} = 40.45\frac{W}{m^2K}$$

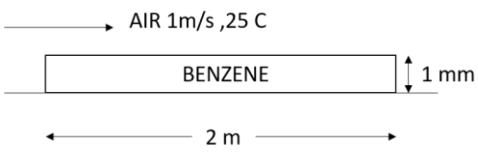
Hence, the radiant flux is

$$q''_{rad} = 40.45\frac{W}{m^2K}\left(330-300\right)K + 4.48\times10^{-3}\frac{kg}{s.m^2}\times 2366\times10^3\frac{J}{Kg}$$

$$q''_{rad} = 1214\frac{W}{m^2} + 10600\frac{W}{m^2} = 11813\frac{W}{m^2}$$

4. Benzene, a harmful chemical, has been spilled on the laboratory floor and has spread to a length of 2m. If a film 1 mm depth is formed, how long will it take for the Benzene to completely evaporate? Ventilation in the laboratory provides for airflow parallel to the surface at 1 m/s, and the benzene and air are both at 25 C. The mass densities of Benzene in the saturated vapor and liquid states are known to be 0.417 and 900 kg/m^3, respectively. For Air: kinematic viscosity = 15.7x10^{-6} m^2/s; D_{AB} = 0.88x10^{-5} m^2/s, Sc = 1.78. **9**

3. $\rho_{sat} = 0.417 \dfrac{kg}{m^3}$



$\rho_{liq} = 900 \dfrac{kg}{m^3}$

Calculate transition length x_{tr} (Lam- Turb)

Taking $\mathrm{Re}_{tr} = 5\times10^5 \quad x_{tr} = 7.85m$

Thus, the film will be under laminar flow.

Mass balance IN-OUT =ACCUM

$$-h_m A\Delta C\Delta t - A\Delta H\rho = 0$$

$$-\frac{d}{dt}(\rho AH) = \dot{m} = \overline{h_m}A\Delta C$$

$$\frac{dH}{dt} = -\frac{\overline{h_m}\Delta C}{\rho}, \text{ at } t = 0, H = h = 1mm$$

$$H = \frac{\overline{h_m}t\Delta C}{\rho_L} \Rightarrow t = \frac{H\rho_L}{h_m\Delta C}$$

Evaluate h_m

$$\mathrm{Re}_L = \frac{2\times1}{15.76\times10^{-6}} = 126903$$

$$Sh_L = 0.664\,\mathrm{Re}_L^{\frac{1}{2}}\,Sc^{\frac{1}{3}}$$

$$\overline{h_m} = 1.26\times10^{-3}$$

$$t = \frac{H\rho_L}{h_m\Delta C} = \frac{1\times10^{-3}\times900}{1.26\times10^{-3}\times0.417}s \quad t = 1711s$$