HEAT TRANSFER

[CH21204]

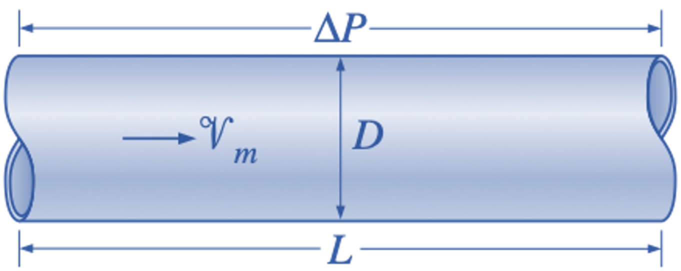
March 16, 2023

LAMINAR FLOW IN TUBES

$$\mathcal{V}(r) = 2\mathcal{V}_m\left(1 - \frac{r^2}{R^2}\right)$$

mean velocity is half of the maximum velocity

$$\Delta P = \frac{8\mu L \mathcal{V}_m}{R^2} = \frac{32\mu L \mathcal{V}_m}{D^2}$$



$$\boxed{f = \frac{64\mu}{\rho D \mathcal{V}_m} = \frac{64}{\text{Re}}}$$

Pressure drop: $\Delta P = f \dfrac{L}{D} \dfrac{\rho \mathcal{V}_m^2}{2}$

Darcy friction factor

$$C_f = \tau_s(\rho \mathcal{V}_m^2/2) = f/4$$

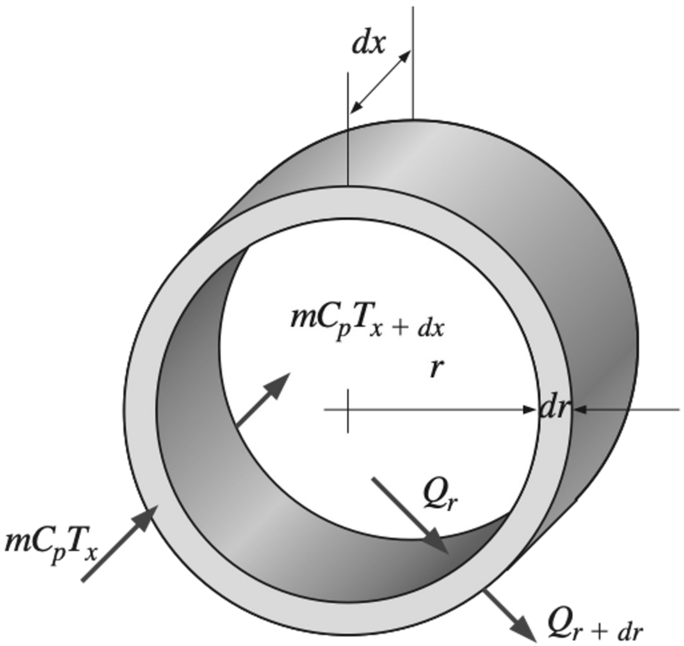
friction factor is a function of the Reynolds number only and is independent of the roughness of the tube surface

$$\dot{W}_{\text{pump}} = \dot{V}\Delta P$$

$$\dot{V} = \mathcal{V}_{\text{ave}} A_c = \frac{\Delta P R^2}{8\mu L}\pi R^2 = \frac{\pi R^4 \Delta P}{8\mu L} = \frac{\pi D^4 \Delta P}{128\mu L}$$

Poiseuille's Law

Hagen–Poiseuille flow



$$\dot{m}C_pT_x - \dot{m}C_pT_{x+dx} + \dot{Q}_r - \dot{Q}_{r+dr} = 0$$

$$\dot{m} = \rho \mathcal{V} A_c = \rho \mathcal{V}(2\pi r dr)$$

$$\rho C_p \mathcal{V} \frac{T_{x+dx} - T_x}{dx} = -\frac{1}{2\pi r dx} \frac{\dot{Q}_{r+dr} - \dot{Q}_r}{dr}$$

$$\mathcal{V} \frac{\partial T}{\partial x} = -\frac{1}{2\rho C_p \pi r dx} \frac{\partial \dot{Q}}{\partial r}$$

$$\frac{\partial \dot{Q}}{\partial r} = \frac{\partial}{\partial r}\left(-k 2\pi r dx \frac{\partial T}{\partial r}\right) = -2\pi k dx \frac{\partial}{\partial r}\left(r \frac{\partial T}{\partial r}\right)$$

$$\alpha = k/\rho C_p$$

$$\boxed{\mathcal{V} \frac{\partial T}{\partial x} = \frac{\alpha}{r} \frac{\partial}{dr}\left(r \frac{\partial T}{\partial r}\right)}$$

rate of net energy transfer to the control volume by mass flow is equal to the net rate of heat conduction in the radial direction

Constant Surface Heat Flux:

$$\frac{\partial T}{\partial x} = \frac{dT_s}{dx} = \frac{dT_m}{dx} = \frac{\dot{q}_s p}{\dot{m} C_p} = \text{constant}$$

$$\frac{\partial T}{\partial x} = \frac{dT_s}{dx} = \frac{dT_m}{dx} = \frac{2\dot{q}_s}{\rho \mathcal{V}_m C_p R} = \text{constant}$$

$$\mathcal{V}(r) = 2\mathcal{V}_m\left(1 - \frac{r^2}{R^2}\right)$$

$$\boxed{\mathcal{V}\frac{\partial T}{\partial x} = \frac{\alpha}{r}\frac{\partial}{dr}\left(r\frac{\partial T}{\partial r}\right)}$$

$$\frac{4\dot{q}_s}{kR}\left(1 - \frac{r^2}{R^2}\right) = \frac{1}{r}\frac{d}{dr}\left(r\frac{dT}{dr}\right)$$

$$T = \frac{\dot{q}_s}{kR}\left(r^2 - \frac{r^2}{4R^2}\right) + C_1 r + C_2$$

$$\partial T/\partial x = 0 \text{ at } r = 0$$

$$T = T_s \text{ at } r = R$$

$$T = T_s - \frac{\dot{q}_s R}{k}\left(\frac{3}{4} - \frac{r^2}{R^2} + \frac{r^4}{4R^4}\right)$$

$$T_m = \frac{\int_{\dot{m}} C_p T \,\delta\dot{m}}{\dot{m} C_p} = \frac{\int_0^R C_p T(\rho \mathcal{V} 2\pi r dr)}{\rho \mathcal{V}_m (\pi R^2) C_p} = \frac{2}{\mathcal{V}_m R^2}\int_0^R T(r, x)\,\mathcal{V}(r, x)\, r dr$$

$$T_m = T_s - \frac{11}{24}\frac{\dot{q}_s R}{k}$$

$$\dot{q}_s = h(T_s - T_m)$$

$$h = \frac{24}{11}\frac{k}{R} = \frac{48}{11}\frac{k}{D} = 4.36\,\frac{k}{D}$$

$$\mathrm{Nu} = \frac{hD}{k} = 4.36$$

Circular tube, laminar ($\dot{q}_x$ = constant)

$$\mathrm{Nu} = \frac{hD}{k} = 3.66$$

Circular tube, laminar (T_s = constant)

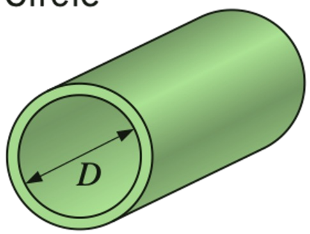
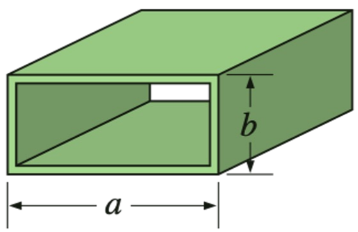
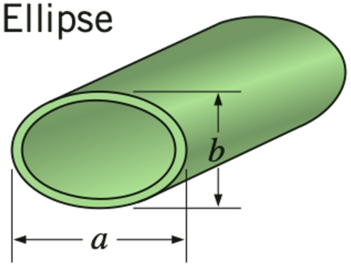
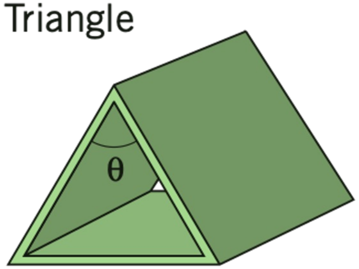
When the difference between the surface and the fluid temperatures is large, it may be necessary to account for the variation of viscosity with temperature.

$$\mathrm{Nu} = 1.86\left(\frac{\mathrm{Re}\,\mathrm{Pr}\,D}{L}\right)^{1/3}\left(\frac{\mu_b}{\mu_s}\right)^{0.14}$$

Sieder and Tate

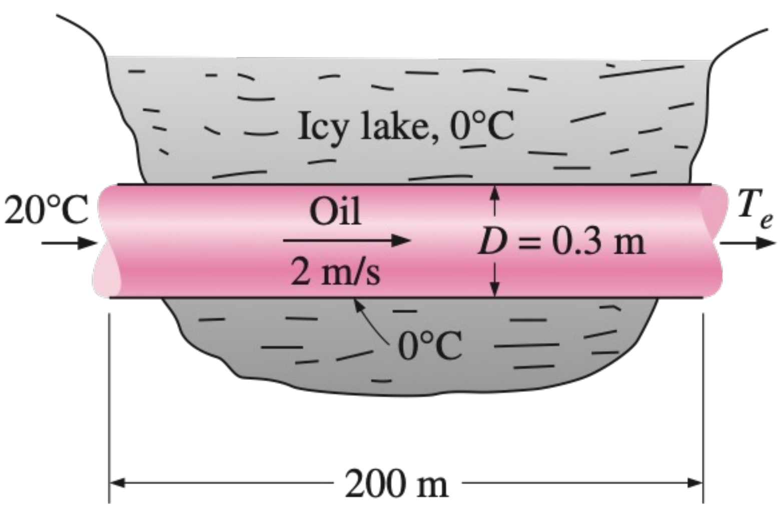
Laminar Flow in Noncircular Tubes

$D_h = 4A_c/p$, $\mathrm{Re} = V_m D_h/\nu$, and $\mathrm{Nu} = hD_h/k$

Tube Geometry	a/b or $\theta°$	*Nusselt Number* $T_s = \text{Const.}$	*Nusselt Number* $\dot{q}_s = \text{Const.}$	Friction Factor f
Circle 	—	3.66	4.36	64.00/Re
Rectangle 	a/b			
	1	2.98	3.61	56.92/Re
	2	3.39	4.12	62.20/Re
	3	3.96	4.79	68.36/Re
	4	4.44	5.33	72.92/Re
	6	5.14	6.05	78.80/Re
	8	5.60	6.49	82.32/Re
	∞	7.54	8.24	96.00/Re
Ellipse 	a/b			
	1	3.66	4.36	64.00/Re
	2	3.74	4.56	67.28/Re
	4	3.79	4.88	72.96/Re
	8	3.72	5.09	76.60/Re
	16	3.65	5.18	78.16/Re
Triangle 	θ			
	10°	1.61	2.45	50.80/Re
	30°	2.26	2.91	52.28/Re
	60°	2.47	3.11	53.32/Re
	90°	2.34	2.98	52.60/Re
	120°	2.00	2.68	50.96/Re

$$\mathrm{Nu} = 3.66 + \frac{0.065\,(D/L)\,\mathrm{Re}\,\mathrm{Pr}}{1 + 0.04[(D/L)\,\mathrm{Re}\,\mathrm{Pr}]^{2/3}}$$

$$\mathrm{Nu} = 7.54 + \frac{0.03\,(D_h/L)\,\mathrm{Re}\,\mathrm{Pr}}{1 + 0.016[(D_h/L)\,\mathrm{Re}\,\mathrm{Pr}]^{2/3}}$$



$\rho = 888\ \mathrm{kg/m^3}$ $\quad\nu = 901 \times 10^{-6}\ \mathrm{m^2/s}$

$k = 0.145\ \mathrm{W/m \cdot {}^\circ C}$ $\quad C_p = 1880\ \mathrm{J/kg \cdot {}^\circ C}$

$\mathrm{Pr} = 10{,}400$