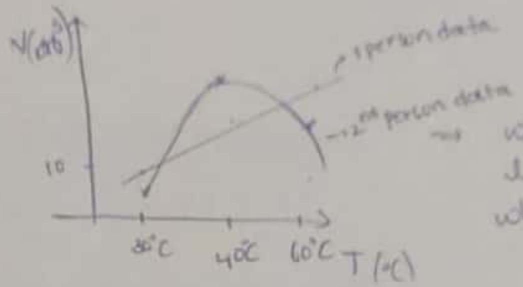
Assume $L_{n_0} \Rightarrow$ as reflux flow rate

* $V_B = \frac{Q_R}{\lambda_B}$ → reflux flow rate

7/11/23

we mainly Algebraic – Differential Equations (Next week)

Lab → we generate data
⇓
Eg measuring volume w.r.t temp



→ why engineers don't like this plot? what's wrong with this?

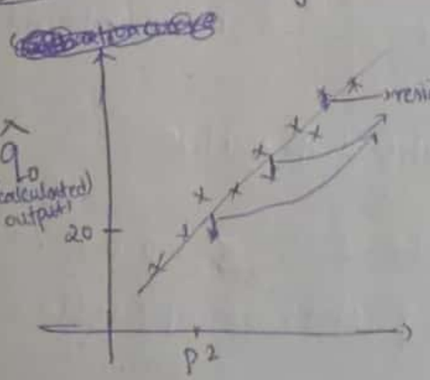
→ we are not taking all the points (we do not take experimental error)

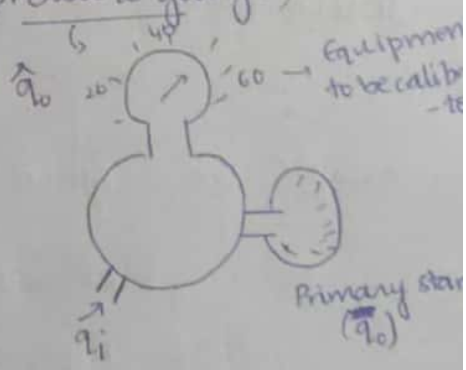
parallax error
human error

like this we can eliminate so many error, but still there will be some error which cannot be eliminated
↳ Random variable

<u>Calibration</u> : Primary standard : very accurate measurement

<u>Pressure</u> (Primary standard ⇒ <u>pressure gauge</u>)





q_i (input)

we will ~~draw~~ fit a straight line using least square error meth

$\hat{q}_o = m q_i + b$

least square error — Error : $L = \sum (q_o - \hat{q}_o)^2$ — actual

minimising the error

$\frac{dL}{dm} = 0 \qquad \frac{dL}{db} = 0$

$$\boxed{m = \frac{N \sum q_i q_o - \sum q_i \sum q_o}{\left[N \sum q_i^2 - (\sum q_i)^2\right]}}$$

$$\boxed{b = \frac{\sum q_o \sum q_i^2 - (\sum q_i q_o)(\sum q_i)}{\left[N \sum q_i^2 - (\sum q_i)^2\right]}}$$

N = Total no. of data points

How to give reliability that m, b are accurate?

m & b are obtained by q_i, q_o

q_o → random variable

q_i → not so random

m & b are fn of random variables

for random variables, we can specify probability distribution

m, b } range should be mentioned

samples ⇒ mean, standard deviation

Ex: If ~~an avg~~ m = 12.3, S_m = 0.05 (standard error), what we say, assuming this sample follow normal distribution ⇒

3 sigma

m = 12.3 ± 0.15

99% above ...

$$Y = X + C$$

$Y \to$ fn of random variable X

$E(Y) \to$ expected value of Y

$V(Y) \to$ variance of Y

$$E(Y) = E(X) + E(C)$$

$$V(Y) = V(X) + V(C)$$

$V(C) = 0$
const

variance within

$$S_m^2 = \frac{N(S_{q_o})^2}{N\sum q_i^2 - (\sum q_i)^2}$$

$$S_b^2 = \frac{(S_{q_o})^2 \sum q_i^2}{N\sum q_i^2 - (\sum q_i)^2}$$

$$S_{q_o}^2 = \frac{1}{(N-2)} \sum \left[(mq_i + b) - q_o\right]^2$$

Residual
$((mq_i + b) - q_o)$

instrument gave reading $\Rightarrow$ read reading from calibration curve

$\Downarrow$

Check what is the uncertainity ?

qu	obs	q_i	q_o	$q_i q_o$	q_i^2	Residual	Residual^2
1	1	0.19	3.8	0.722	0.0361	-0.90	0.81
1	2	0.40	10.4	4.16	0.16	1.85	3.4225
1	3	0.63	15.1	9.513	0.3969	2.34	5.4756
1	4	0.60	9.3	5.58	0.36	-2.91	8.4681
1	5	0.78	14.6	11.388	0.6084	-0.90	0.81
1	6	1.05	20.9	21.94	1.1025	0.47	0.2209
1	7	1.74	31.3	54.462	3.0276	-1.75	3.0625
1	8	1.62	32.7	52.974	2.6244	1.84	3.3856
observation one value		7.01	138.1	160.739	8.3159		25.655

Calculate m b, and then residual

9/11/23 For previous problem

$\hat{q}_o = m q_i + b$

$m = 18.28$, $S_m = 1.4034$ } solve it

$b = 1.24$, $S_b = 1.3137$

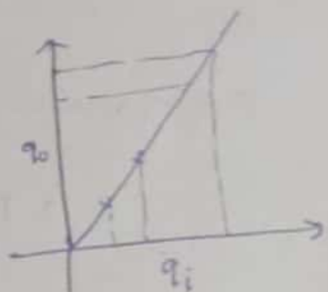
$\sqrt{S_m^2 = \frac{8 (S_{q_o})^2}{8(7.293) - (7.01)^2}} = 15.8915$

$S_{q_o}^2 = \frac{1}{(8-2)} (25.6552) = 4.276$

$\boxed{S_m = 3.986}$

$m = \frac{8(160.739) - (7.01 \times 138.1)}{8(7.293) - (7.01)^2}$

= [illegible]



$m = 18.28 \pm 4.2$

$b = 1.24 \pm 3.9$

if $q_o = 24.61$ then $q_i = 1.2781$, how uncertain the value of q_i is?

$\pm 3(S_{q_i}) \Rightarrow 1.2781 \pm 3(0.113)$

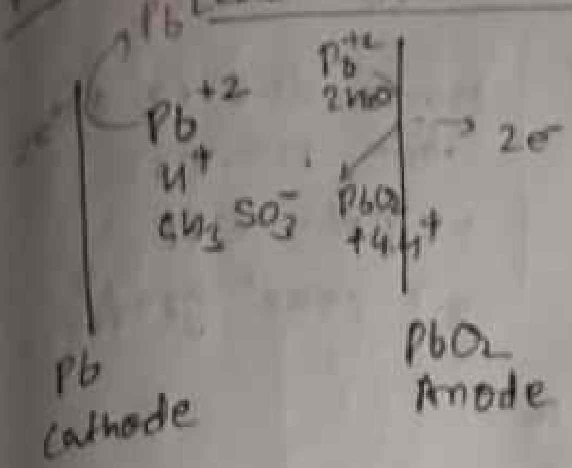
$\boxed{S_{q_i} = \sqrt{\frac{S_{q_o}^2}{m^2}} = 0.113}$

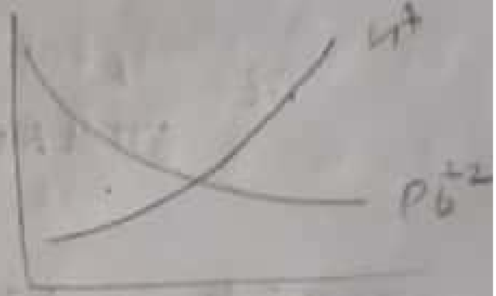
$\boxed{\pm 3(\text{sigma})}$ ⇒ from here trying to estimate speed, that we do by relative diff of random

standard deviation

Flux = Velocity × Density

Lead acid Battery :

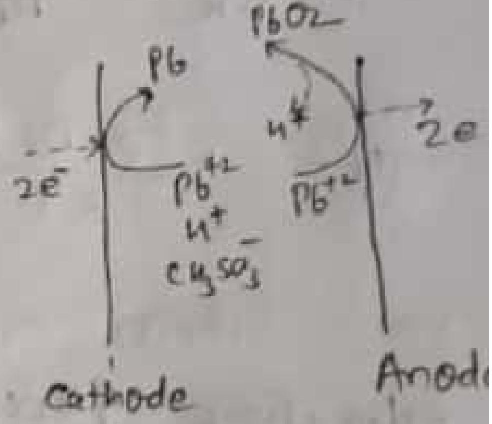




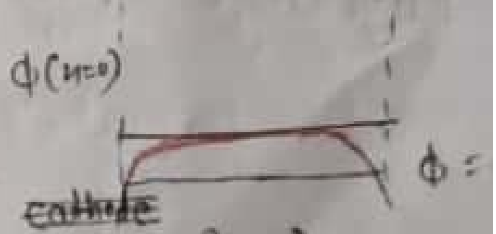
10/11/23

Develop model equations.

C_i, ϕ



$$-z_i e \frac{\partial \phi}{\partial x} = 6\pi R_s u \mu$$



Mass bal:

$[Pb^{+2}]$

$$\frac{\partial C_i}{\partial t} = -\frac{\partial N_i}{\partial x}$$

C_i = concⁿ of i^{th} ion

N_i = net Flux of i^{th} ion

ϕ = electric patential

z_i = valency of i^{th} ion

Pb^{+2} ($z_{Pb} = 2$)

$$N_i = -D_i \frac{\partial C_i}{\partial x} + (\quad)$$

Migration flux

there is no flow ⇒ no convection

moves due to only two reasons:
1.) pot. diff
2.) concⁿ gradient

$$-z_i e \frac{\partial \phi}{\partial x} = 6\pi R_s u \mu$$

$$D_i = \frac{RT}{6\pi \mu R_s}$$

$$-z_i e \frac{\partial \phi}{\partial x} = \frac{RT}{D_i} \cdot u$$

$$u = -\frac{z_i D_i e}{RT} \frac{\partial \phi}{\partial x}$$

$$\text{Migration flux} = -\frac{z_i D_i e}{RT} \frac{\partial \phi}{\partial x} \cdot \underbrace{C_i N_A}_{\text{no. conc}^n}$$

$$= -\frac{z_i D_i F}{RT} \frac{\partial \phi}{\partial x} C_i$$

$$\frac{\partial C_i}{\partial t} = -\frac{\partial}{\partial x}\left[-D_i \frac{\partial C_i}{\partial x} - \frac{z_i D_i C_i F}{RT} \frac{\partial \phi}{\partial x}\right]$$

$i = 1, 2, 3$

Poisson Eqn

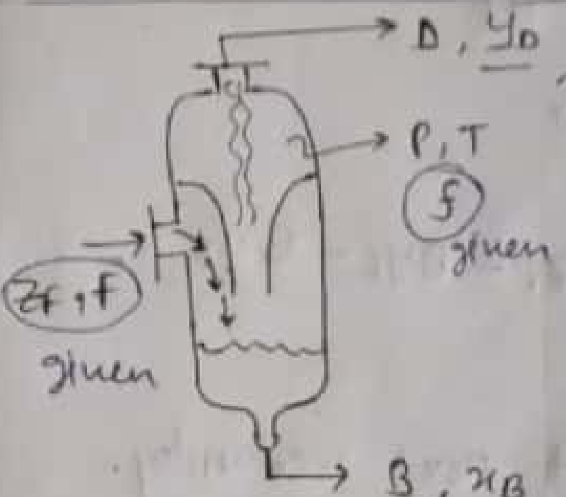
$$\nabla^2\phi = -\frac{F}{\epsilon}\sum_i z_i c_i$$

$\nabla^2\phi$ → small; $\frac{F}{\epsilon}$ → $\sim 10^{16}$

$$\boxed{\sum_i z_i c_i \cong 0}$$ electroneutrality condition.

DAE system.

Assumptions:
α = given
relative volatility = const.



steady state Mass bal. :

$$F = D + B \quad — ③$$

$$z_F F = y_D D + x_B B \quad — ①$$

↳ more volatile species bal.

$$\left.\begin{aligned} fF &= D \\ (1-f)F &= B \end{aligned}\right\}$$

$$y_D = \frac{\alpha x_B}{1+(\alpha-1)x_B}$$

$$\cancel{y_D = K_i x_B}$$

$$z_F = f y_D + (1-f) y_B$$

DAE

$$D_1 \frac{dc_1}{dx} + \frac{z_1 c_1 D_1 F}{RT} \frac{d\phi}{dx} = k_1$$

$$D_2 \frac{dc_2}{dx} + \frac{z_2 c_2 D_2 F}{RT} \frac{d\phi}{dx} = k_2$$

$$D_3 \frac{dc_3}{dx} + \frac{z_3 c_3 D_3 F}{RT} \frac{d\phi}{dx} = k_3$$

$$z_1 c_1 + z_2 c_2 + z_3 c_3 = 0$$

$$\frac{dx_1}{dt} = f_1 (x_1, x_2, x_3, t)$$

$$\frac{dx_2}{dt} = f_2 (x_1, x_2, x_3, t)$$

$$0 = f_4 (x_1, x_2, x_3, t)$$

$$D_1 \frac{dC_1}{dx} + \frac{z_1 C_1 D_1 F}{RT} \frac{d\phi}{dx} = k_1$$

$$D_2 \frac{dC_2}{dx} + \frac{z_2 C_2 D_2 F}{RT} \frac{d\phi}{dx} = k_2$$

$$D_3 \frac{dC_3}{dx} + \frac{z_3 C_3 D_3 F}{RT} \frac{d\phi}{dx} = k_3$$

$$z_1 C_1 + z_2 C_2 + z_3 C_3 = 0.$$

$$D_i = \frac{RT}{6\pi\mu R_s}$$

$$-z_i e \frac{\partial\phi}{\partial x} = \frac{RT}{D_i} \times u$$

flux = velocity × density.

$$u = -\frac{z_i D_i e}{RT} \cdot \frac{\partial\phi}{\partial x}$$

Migration flux $= \frac{-z_i D_i e}{RT} \cdot \frac{\partial\phi}{\partial x} \cdot c_i$

faraday const.

$$= -\frac{z_i D_i F}{RT} \cdot \frac{\partial\phi}{\partial x} \cdot c_i$$

c_i → molar conc.

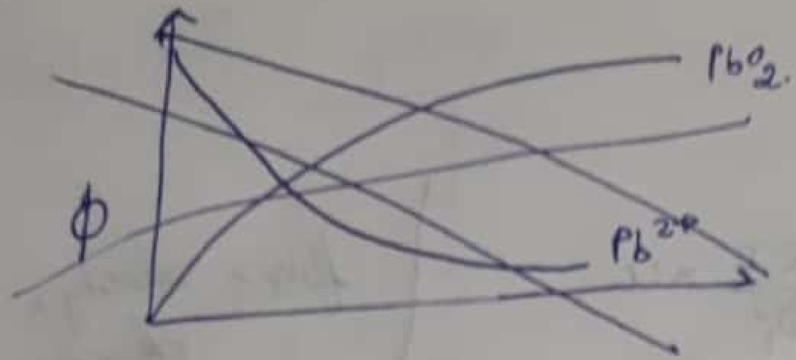
$$\frac{\partial c_i}{dt} = -\frac{\partial}{\partial x}\left[-D_i \frac{\partial c_i}{\partial x} - \frac{z_i c_i D_i F}{RT}\frac{\partial\phi}{\partial x}\right]$$

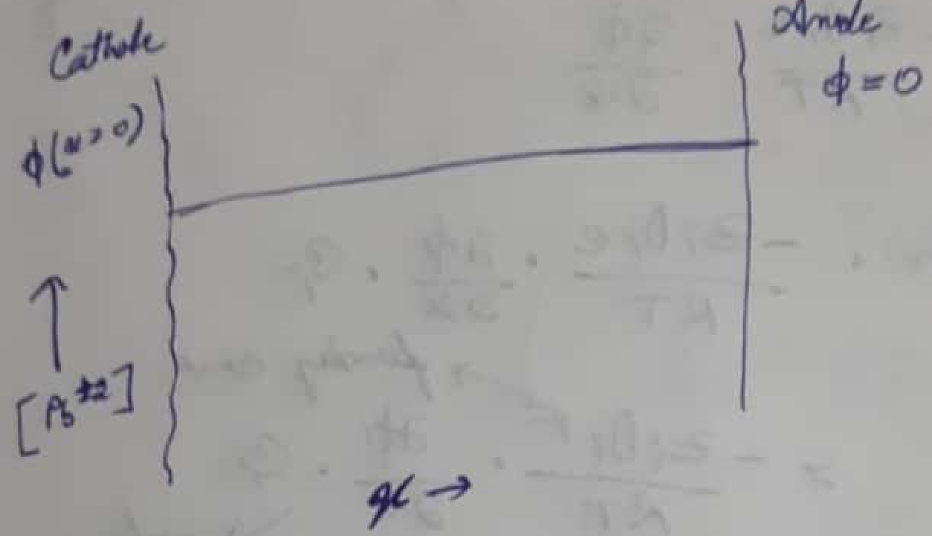
$i = 1, 2, 3$

Poisson's eqn

$$\nabla^2\phi = -\frac{F}{\epsilon}\sum z_i c_i$$

practically, $\sum z_i c_i \approx 0$ (electroneutrality of soln ↓ valid at all x)





Net flux due to :-

→ chemical potential

→ electric "

Develop Model Eq^n :-

charge of ion (ith) → electric potential. → grad.

$$\underbrace{-z_i e}_{\text{charge}} \frac{\partial \phi}{\partial x} = 6\pi \underbrace{R_s}_{\text{rad. of ion}} \times \underbrace{u}_{\text{vel. of ion}} \times \underbrace{\mu}_{\text{viscosity of medium}}$$

force on a ion

conc. of ith ion. → Net flux.

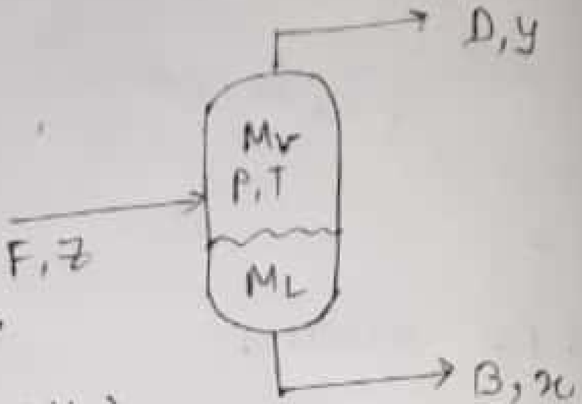
$$\frac{\partial C_i}{\partial t} = -\frac{\partial N_i}{\partial x}$$

$$N_i = \underbrace{-D_i \frac{dC_i}{\partial x}}_{\text{diffusive flux}} + \underbrace{(\quad)}_{\text{Migration flux.}}$$

12/13 Marks JC SIR

In - out + gen - consump = Accu

$$z_f F - (yD + x_B B) = \frac{d(yV + xL)}{dt}$$



Overall Mass bal:→

$$F - D - B = \frac{d}{dt}(M_V + M_L)$$

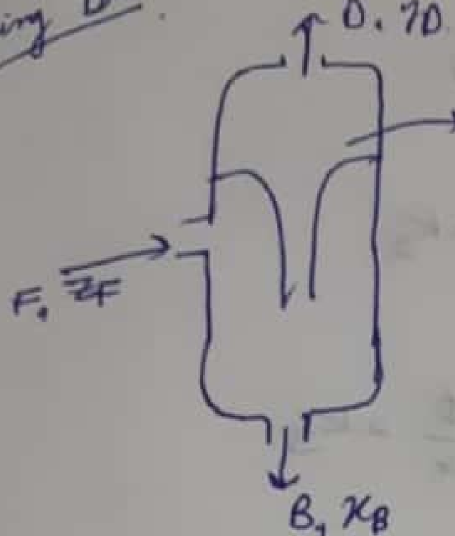
component:→

$$\frac{dM_i}{dt} = F z_i - D y_i - B x_i$$

H.W. Submit Next wednesday.

Transient mass bal. for this flash vessel. and formulate closed set of model eqn

Flashing Drum.



f → frac. of feed vaporized.

$$F z_F = D y_D + B x_B$$

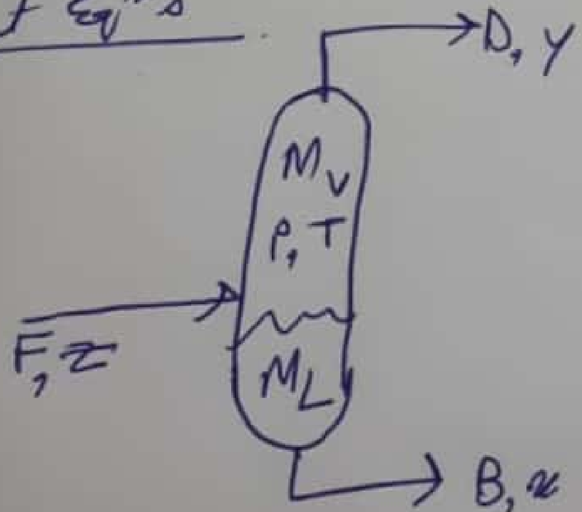
$$f = D/F$$

$$\alpha = \frac{y_i / x_i}{y_j / x_j}$$

$$z_F = f y_D + (1-f) x_B.$$

$$y_D = \frac{\alpha x_B}{1 + (\alpha - 1) x_B}$$

Transient Eq^{n}s.



$$\frac{d}{dt}\left(M_V + M_L\right) = F - D - B$$

Component Mass balance.

$$\frac{dM_i}{dt} = F z_i - D y_i - B x_i$$