PDC AKJ part

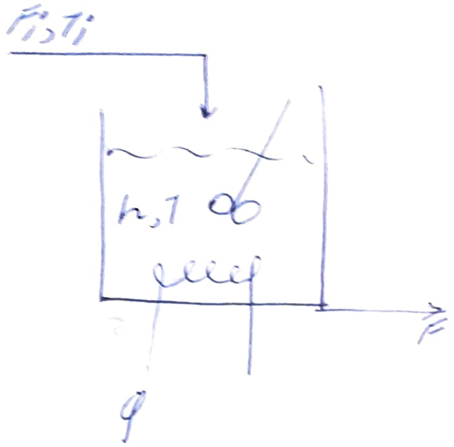
Transform calculus basis:

- $F(s) = \int_0^\infty e^{-st} f(t)\,dt = L[f(t)]$
- $L(1) = 1/s$
- $L(e^{at}) = \frac{1}{s-a}$
- $L(\sin(at)) = \frac{a}{s^2+a^2}$
- $L(\cos at) = \frac{s}{s^2+a^2}$
- $L(t f(t)) = -\frac{d}{ds} F(s)$
- $L\left(\frac{1}{t} f(t)\right) = \int_s^\infty F(s)\,ds$
- $L(f'(t)) = sF(s) - f(0)$
- $L\left(\int_0^t f(t)\,dt\right) = \frac{1}{s} F(s)$
- $L(e^{at} f(t)) = F(s-a)$ --- shifting property
- $L(f(at)) = \frac{1}{a} F(s/a)$ --- scaling property
- $L(t^n) = \frac{\Gamma(n+1)}{s^{n+1}} = \frac{n!}{s^{n+1}}$

→ variables

- Input
  - manipulated variables
  - Disturbance/load variables
    - measured LV
    - unmeasured LV
- Output
  - Unmeasured
  - measured
    - CV/Primary measurement
    - secondary measurement

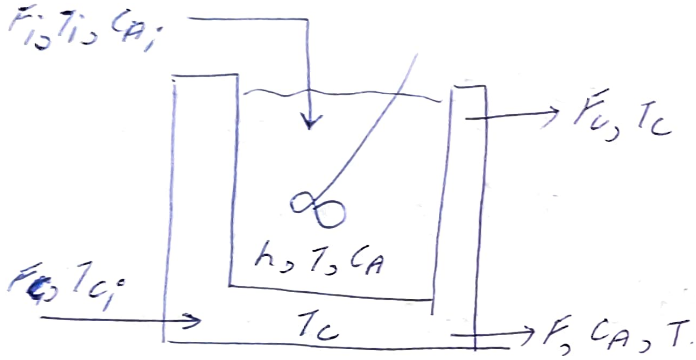
ex-1 Heating tank



Control variables : h, T

Load variables : F_i, T_i

ex-2



$A \longrightarrow$ product

1^{st} order, exothermic

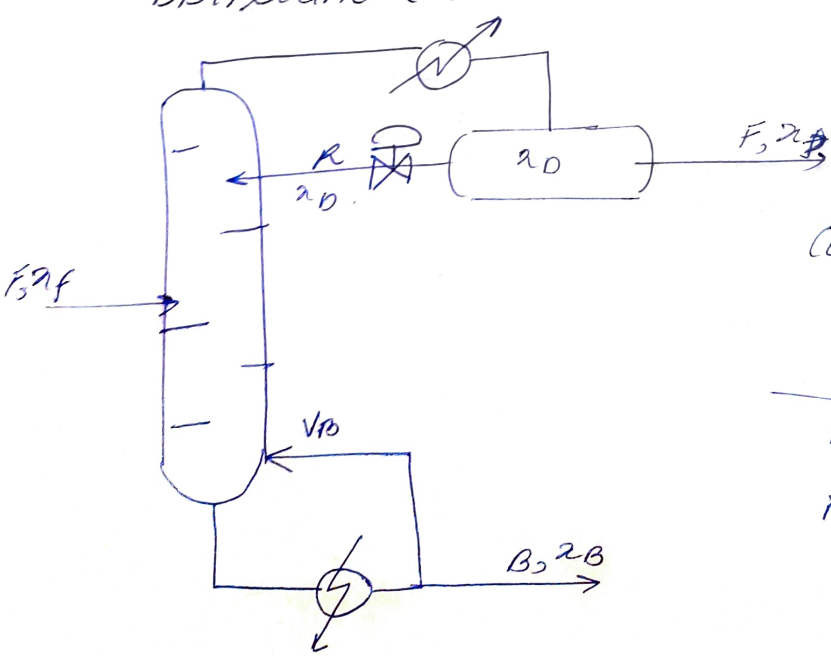
control objective : $T = T_{sp}$

CV	MV	LV
T	F_c	T_i
		C_{Ai}

input $\Rightarrow F_i, T_i, C_{Ai}, F_c, T_{ci}$

output $\Rightarrow$ ~~F_{ci}~~ T_c, F, C_A, T, h

ex-3 Distillation.

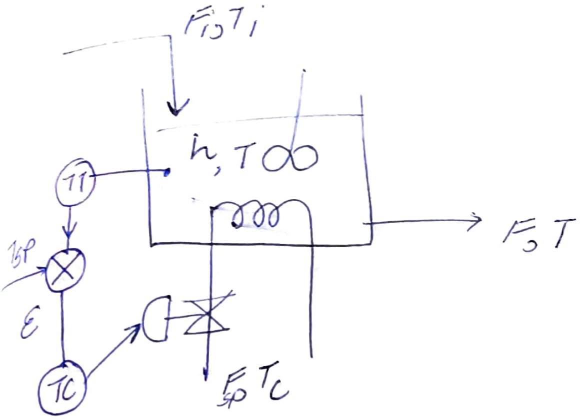


Control obj : $x_D = x_D^{sp}$

$x_B = x_B^{sp}$

CV	MV
m_D	D
m_B	B

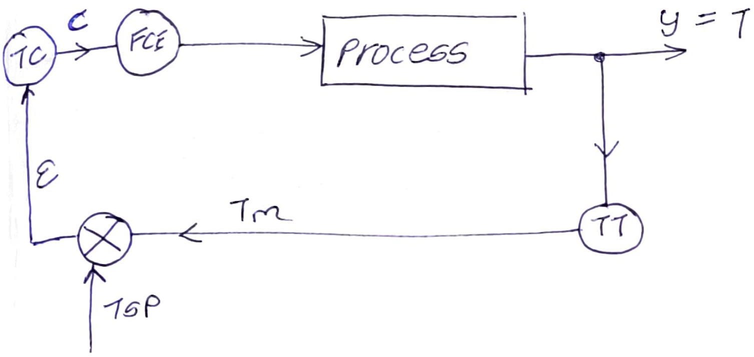
• Introduction to Feedback controllers:

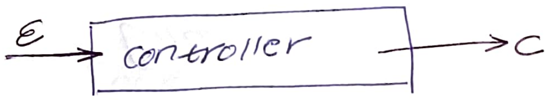


control obj: $T = T_{sp}$

CV	MV
T	F_{st}.

$\varepsilon = T_{sp} - T$





$\varepsilon = y_{sp} - y$

$c' = c - c_{ss}$ (excess variable)

→ P-only controller:

$c' = K_c \varepsilon(t)$

$K_c = \dfrac{c'}{\varepsilon} = \dfrac{\text{output}}{\text{input}} \Rightarrow$ proportional gain

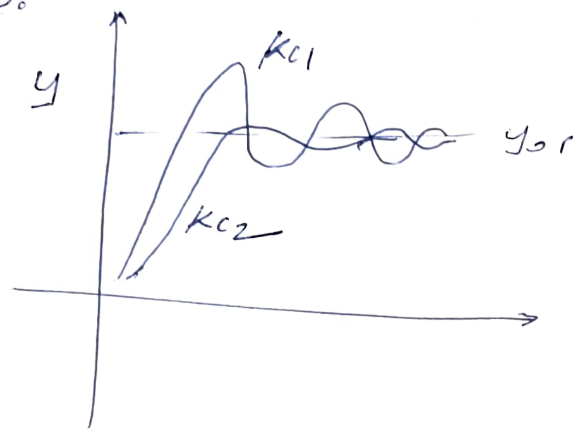
$\therefore \; c = c_s + K_c e$ ---- e or ε is error

Proportional band (PB) $= \dfrac{100}{K_c}$

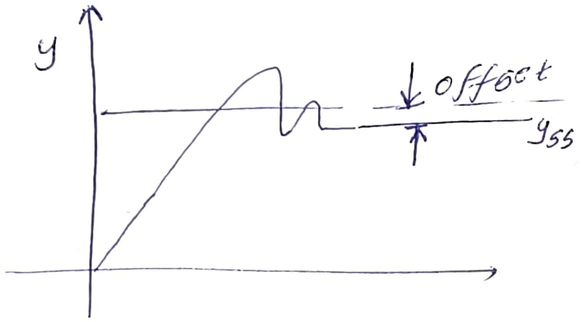
$\bar{c}'(s) = \int_0^{\infty} e^{-st} f(t)\,dt = \cancel{\;} + K_c \bar{e}(s)$

$$\boxed{K_c = \frac{\bar{c}'(s)}{\bar{e}(s)}}$$ --- Transfer fn of controller.

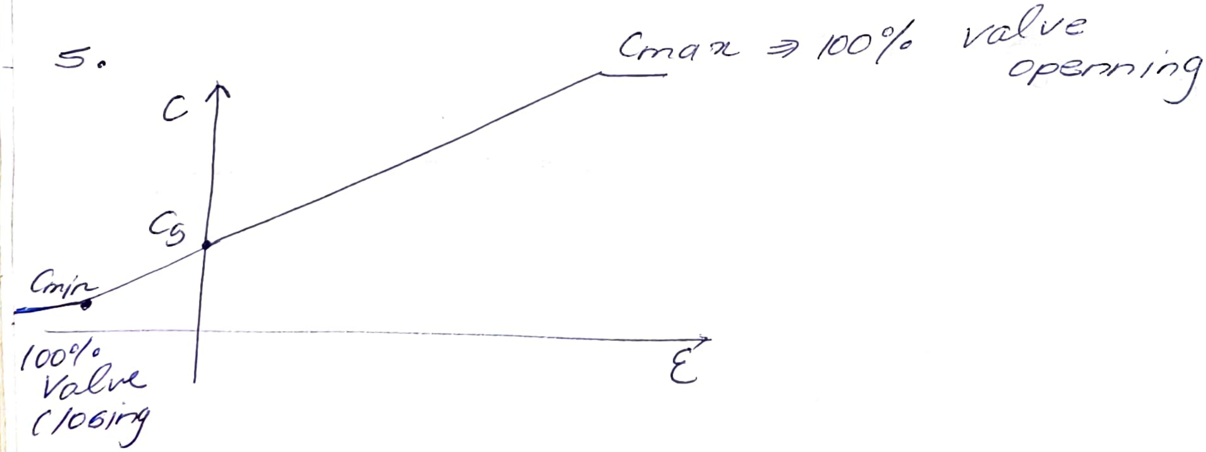
Remarks:

1. $K_c \uparrow$ more sensitive to error signal
2. K_c determines speed of controller
3. 

increasing K_c gives a faster response but at the ~~case~~ cost of more oscillations.

4. 

offset is not eliminated.

5. 

6. If K_c is very large then it acts like an on/off controller.

→ PI-controller:

$$c = c_s + \underbrace{\frac{K_c}{\tau_i}\int e\,dt}_{\text{Integral}} + \underbrace{K_c e}_{\text{Proportional}}$$

$\tau_i \Rightarrow$ integral time/reset time

$1/\tau_i \Rightarrow$ reset rate

$$c' = \frac{K_c}{\tau_i}\int e\,dt + K_c e$$

$$\bar{c}'(s) = K_c\left[\frac{1}{\tau_i}\times\frac{1}{s}\bar{e}(s) + \bar{e}(s)\right]$$

$$\bar{c}'(s) = K_c\bar{e}(s)\left(1 + \frac{1}{\tau_i s}\right)$$

$G(s)$ transfer fn $= \dfrac{\bar{c}'(s)}{\bar{e}(s)} = K_c\left(1 + \dfrac{1}{\tau_i s}\right)$

Remarks:

1. K_c decides the speed of response
2. ellimina tes offset
3. $\tau_i \rightarrow$ decreases, offset $\rightarrow 0$
4. τ_i is the time required by a controller to repeat its ~~propartial~~ proportional action change in its output

$\rightarrow$ 10/01/2025

5. 1st order process + PI controller $\Rightarrow$ 2nd order system

ex-

$$G_p = \underbrace{\frac{K_p}{\tau_p s + 1}}_{\text{open loop}} \xrightarrow{+\text{ controller}} \text{closed loop} \qquad G_c = K_c\left[1 + \frac{1}{\tau_i s}\right].$$

$$CLTF \Rightarrow \frac{G_c G_p G_f}{1 + G_c G_p G_f G_m} \qquad \bar{y}_{sp}(s)$$

simplicity: $G_f = G_m = 1$

$$CLTF = \frac{G_c G_p}{1 + G_c G_p}$$

6. If order of process increases then speed of response decreases. (slugishness)

Basis:

$$\frac{dy}{dt} = f(y, u, d) \;\}\; \text{process representatn}$$

$$\frac{y_{k+1} - y_k}{\Delta t} = f[y_k, u_k, d_k]$$

$$y_{k+1} = f[y_k, u_k, d_k]$$

$u_k \longrightarrow y_{k+1}$ } Δt control action at k^{th} time will be realised at $(k+1)^{th}$ time by the sys.

↑ sampling period

$\therefore$

$$\frac{d^2y}{dt^2} = f(y, u, d)$$

$$y_{k+2} = f[y_k, u_k, d_k, y_{k+1}]$$

$u_k \longrightarrow y_{k+2}$ } $2\Delta t$

• PID controller:

derivative time

$$c(t) = c_s + \underbrace{k_c e}_{P} + \underbrace{\frac{k_c}{\tau_i}\int e\,dt}_{I} + \underbrace{k_c \tau_D \frac{de}{dt}}_{D}$$

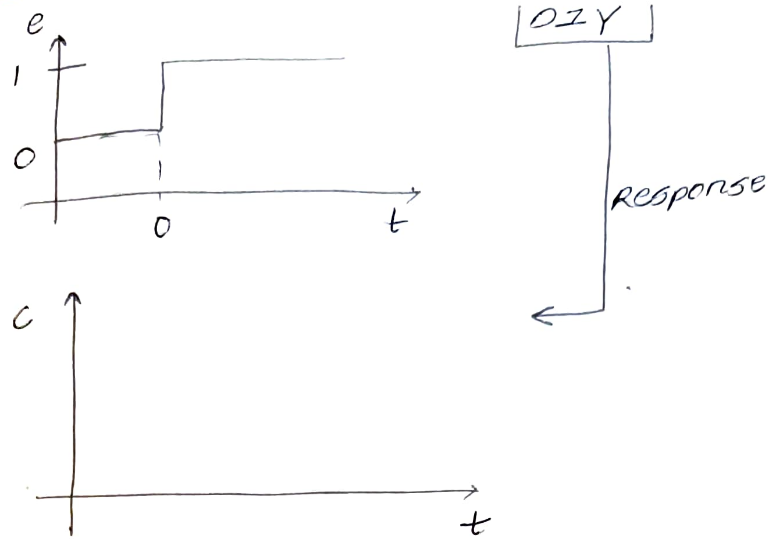
Remarks:

1. $e = const$ $\quad D = 0$ (no-derivative actn)
2. For a noisy response with almost '0' error, derivative term takes aggressive control action though it is not needed.

As long as 'y' is within a prescribed band, ~~there~~ it is typically due to noise and not process deviation.

3. $G_c \Rightarrow k_c \bar{e}(s) + \frac{k_c}{\tau_i s}\bar{e}(s) + k_c \tau_D [s\bar{e}(s) - \cancel{e}(0)]$

$$G_c = k_c\left[1 + \frac{k_c}{\tau_i s} + k_c \tau_D s\right]$$



- P only :- Determines speed of response.
  - cannot eliminate offset.

- PI :- Speed of response determined by k_c & τ_i.
  - eliminates offset.
  - order ↑ response becomes sluggish.

— k_c ⇒ can be increased in value till ultimate gain. above this value system becomes unstable.

5-1

$$\bar{y} = \frac{G_c G_p G_f}{1 + G_c G_p G_f G_m} \, \bar{y}_{sp}(s)$$

$$G_p(s) = \frac{k_p}{\tau_p s + 1} \qquad G_c = \frac{k_c}{\tau_i s}$$

$$G_f = G_m = 1$$

5-2

$$\bar{y}_{sp} = 1/s$$

5-3

$$\text{offset} = \text{new sp} - \text{FVT}$$

$$= \text{new sp} - \lim_{s \to 0} s\bar{y}(s)$$

$$= 0$$

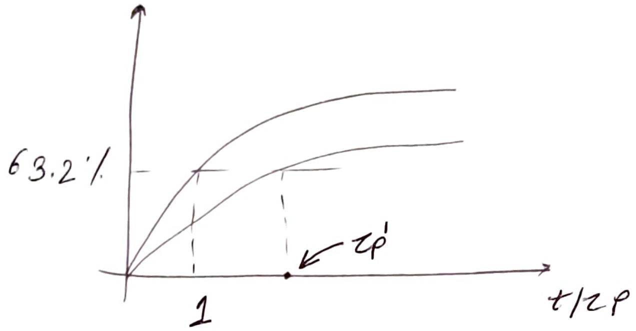
$\underbrace{\text{P+I+D}}$:- Derivative action provides stable response

Robust controller ⇒ means not very sensitive to control parameters.

- for $G_C = k_C \tau_D s$ in 5-1, 5-2, 5-3

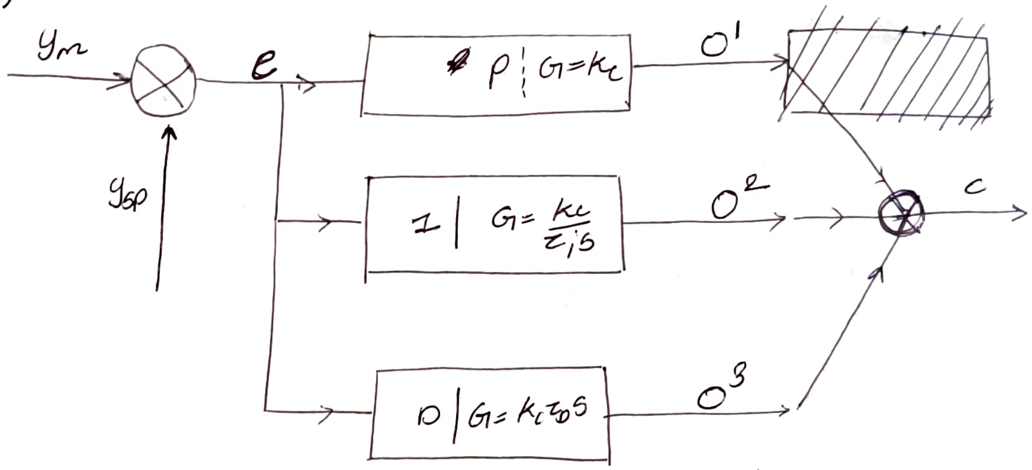
$$\bar{y}(s) = \frac{k_p'}{\tau_p' s + 1} \bar{y}_{sp}(s)$$

$$\tau_p' > \tau_p$$



→ Block diagram for PID controller

(1)



$\boxed{\text{PID (Parallel / non-interacting)}}$

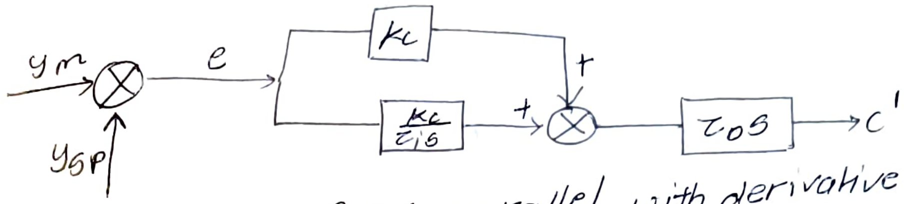
$$O^1 = \bar{\varepsilon}(s) k_c$$

$$O^2 = \bar{\varepsilon}(s) \frac{k_c}{\tau_i s}$$

$$O^3 = \bar{\varepsilon}(s) k_c \tau_D s$$

$$\cancel{G_{overall}} = \cancel{\frac{\bar{c}(s)}{\bar{\varepsilon}}}$$

(2) $\dfrac{\bar{c}'(s)}{\bar{e}(s)} = k_c\left(1+\dfrac{1}{\tau_i s}\right)\tau_D s$



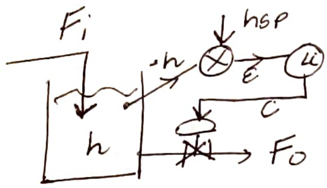
P & I parallel with derivative in series

PID (series/interacting)

- Control actions (Direct & reverse)

If input to controller increases then the output from controller must increase for direct acting controller

CV↑ ⇒ MV↑ ⇒ for direct action.

eg- 

h↑ ⇒ F_o↑

CV MV

for P-only controller,

$F_o = F_{os} + k_c(h_{sp} - h)$

** [Controller is implemented after the process reaches steady state.]

@ $e=0$ $t=0$

~~if h > hsp then~~ $\boxed{k_c \Rightarrow ve^-}$

<u>Reverse action</u> :

CV ⇒ h MV ⇒ F_i

① h↑ ⇒ F_i↓

② k_c ⇒ ve+

- D-action: $G_c(s) = k_c\tau_D s = \dfrac{\bar{c}(s)}{\bar{e}(s)}$

$\bar{c}'(s) = k_c\left(1+\dfrac{1}{\tau_i s}+\tau_D s\right)\bar{e}(s)$

for unit step change in e,

$L[t^n] =$

$$\bar{c'} = \frac{k_c}{s} + \frac{k_c}{\tau_i s^2} + k_c \tau_D$$

$$c' = k_c + L^{-1}[k_c \tau_D] + \frac{k_c}{\tau_i} t$$

$$c' = k_c + \frac{k_c}{\tau_i} t + L^{-1}[k_c \tau_D]$$

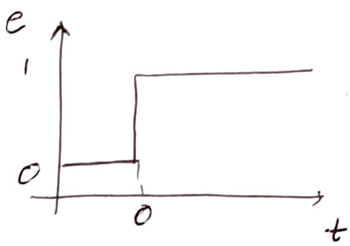
$$\boxed{c' = k_c + \frac{k_c}{\tau_i} t + k_c \tau_D \delta(t)}$$

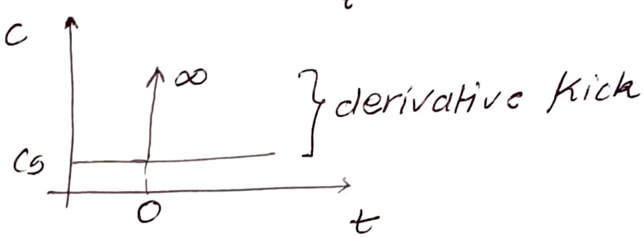
$\delta(t)$ → dirac delta fn

for just derivative terms

$\bar{c'}(s) = k_c \tau_D$

$c'(t) = k_c \tau_D \delta(t)$





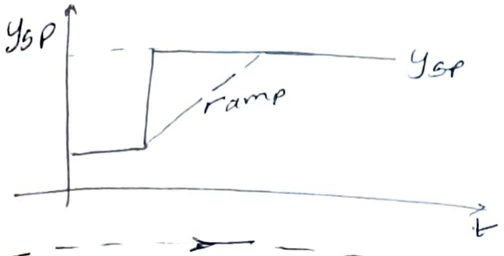
• Because of derivative kick we get ideal PID.

→ Derivative kick: (ways to address the kick)

1. $c(t) = c_s k_c e + \frac{k_c}{\tau_i} \int e\,dt + \underbrace{k_c \tau_D \frac{d}{dt}(\cancel{y_{sp}} - y)}_{}$ 0

y_{sp} zero for just derivative term.

2. y_{sp} @ ramp instead of vertical step change



$$G(s) = \frac{N(s)}{D(s)}$$

order of D > order of N ⇒ system is strictly proper

order of D = order of N ⇒ proper/semi-proper

order of D < order of N ⇒ improper (physically unrealisable)

$$G(s) = \frac{N(s)}{D(s)} = K_c \tau_D s$$ } Derivative term is improper.

To make derivative term proper,

$$G_c(s) = K_c\left[1 + \frac{1}{\tau_i s} + \tau_D s\, F(s)\right]$$ ... Real PID (Parallel/non interacting).

$$F(s) = \frac{K_f}{(\tau_F s + 1)^n}$$

τ_F = filter time constant $\qquad K_f = 1$

n = filter order.

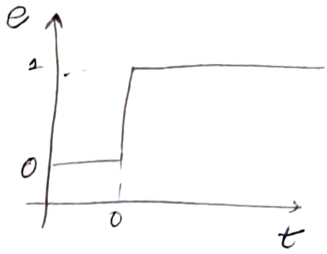
$\tau_f = \alpha \tau_D$ --- correlation

$\alpha = [0.05, 0.2]$ ... mostly $\alpha = 0.1$

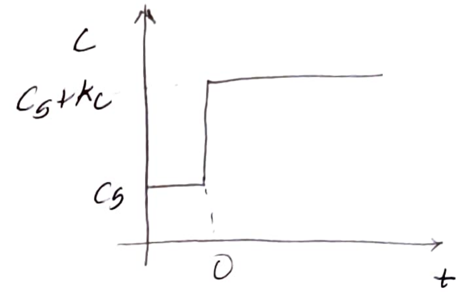
$$G_c(s) = K_c\left[1 + \frac{1}{\tau_i s}\right]\frac{\tau_D s}{(\tau_f s + 1)}$$ .... Real PID (series)

- Proportional kick.

$c(t) = c_s + k_c(y_{sp} - y)$



$c_s + k_c > c_{max}$
↑
100% openning



address kik,

1. $C(t) = c_s + k_c(\cancel{y_{sp}}^{\,0} - y)$

2. Use ramp to change sp.

- Set point weighting:

$$c = c_s + k_c(a y_{sp} - y) + \frac{k_c}{\tau_i}\int (b y_{sp} - y)\,dt + k_c \tau_D \frac{d}{dt}(c y_{sp} - y)$$

a, b, c ⇒ weights

c = 0, a < 1 } to address kicks

b = 1 } to eliminate offset

HW → Code for PID controller

$$u = u_s + k_c e + \frac{k_c}{\tau_i}\int e\,dt + k_c \tau_D \frac{de}{dt}$$

↑

$$\frac{e(t) - e(t-1)}{\Delta t}$$

$$\int e\,dt = \sum_{i=1}^{n} e(i\Delta t)\,\Delta t \quad \ldots \quad n = t/\Delta t$$

↳ Digital form of PID

3/11/25

$\Delta U(t) = U(t) - U(t-1)$

$U(t) = U_s + K_c e(t) + \cancel{\frac{K_c}{\tau_i}\int e(t)\,dt}$

$\frac{K_c \Delta t}{\tau_i} \sum e(i\Delta t)$

$+ K_D \tau_D \left[\frac{e(t) - e(t-1)}{\Delta t}\right]$

└--- form 1

$$U(t-1) = U_s + K_c e(t-1) + \frac{K_c \Delta t}{\tau_i} \sum_{i=1}^{t-\Delta t/\Delta t} e(i\Delta t) + K_D \tau_D \left[\frac{e(t-\Delta t) - e(t-\Delta t-1)}{\Delta t}\right]$$

$$\boxed{\Delta U(t) = K_c\left(e[\Delta t] - e(t-\Delta t)\right) + \frac{K_c \Delta t}{\tau_i}\left[\sum_{i=1}^{t/\Delta t} e(i\Delta t) - \sum_{i=1}^{t-\Delta t/\Delta t} e(i\Delta t)\right] + \frac{K_D \tau_D}{\Delta t}\left[e(t) - e(t-2\Delta t) - 2e(t-\Delta t)\right]}$$

$$\Delta U(t) = K_c\left(e[t] - e[t-\Delta t]\right) + \frac{K_c \Delta t}{\tau_i} e(t) + \frac{K_D \tau_D}{\Delta t}\left[e(t) + e(t-2\Delta t) - 2e(t-\Delta t)\right]$$

↳ form 2 (velocity form)

Advantage of form 2 :
1. inherent antireset windup
2. U_s not required.
3. Related to valve design?

Disadvantage : 1. if Y_{SP} becomes constant & 'I' term doesn't ~~existance~~ exist or is not used then controller becomes clueless. Bcoz $\Delta U(t)$ does not content Y_{SP} term.

*** PID : Linear controller, whereas chemical processes are non-linear.

Non-linear PID controller : $K_c = K_{c0}[1 + a|e|]$

P-action $\propto e^2$ or $e|e|$

e	Linear $U \propto e$	Non-linear $U \propto e^2$
small (<1)	small	smaller
0.2	$\propto 0.2$	$\propto 0.04$
>1	Large	Larger
e = 2	$\propto 2$	$\propto 4$

• non-linear controller has better flexibility

• Gain scheduling PID: K_c is not fixed

$K_c(\alpha)$ is a fn of scheduling variable (α)

This controller is used when K_p is variable..... $K_p(\alpha)$

Usteady state processes have variable gain

$K_c K_p$ = constant

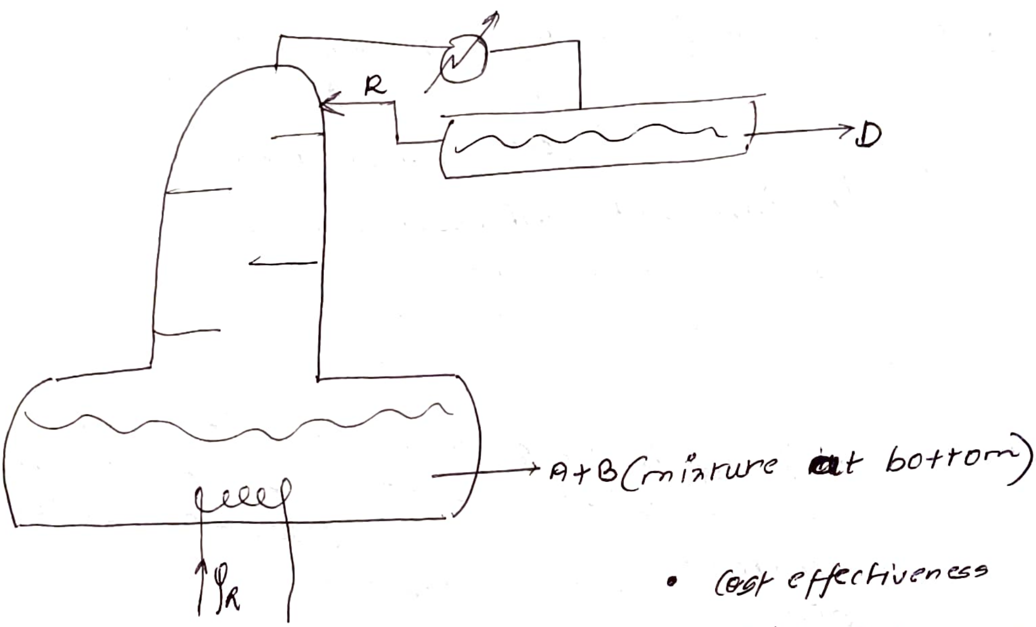
Stability margin = constant } How?

selection of α : 1. slow variable ($\alpha = y$)

(y) controll variable is slower than MV

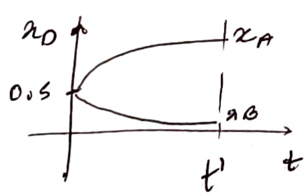
2. Measurable quantity

3. K_p is strong fn of α.



Total reflux @ begining

D = 0



• Cost effectiveness

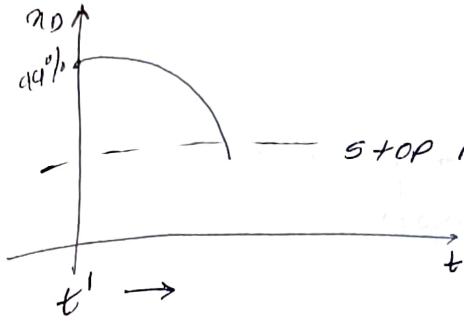
1. High P (to use cooling water instead of refrigerant)

How to create high P?

→ dont start condenser

• For batch distillation we can attain S.S. eventhough it is a batch process @ total reflux condition.

• Partial reflux condition



stop the process at whatever purity is wanted

x_D can be maintained at 99% with the help of a controler. Controller increases R to maintain purity

$$K_P = \frac{\Delta x_D}{\Delta R}$$

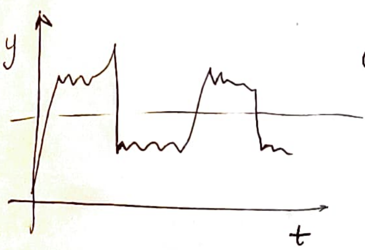
$$K_c(\alpha) = K_{c0}\frac{(1-x_{D0})}{1-x_D} \quad ; \quad x_D > x_{D0} \text{ (SS value)}$$

$$= K_{c0} \quad ; \quad x_D \le x_{D0}$$

$$K_P K_c = K_{c0} K_{P0}$$

$$K_P = \frac{\cancel{K_{c0}} K_{P0} (1-x_D)}{\cancel{K_{c0}} (1-x_{D0})}$$

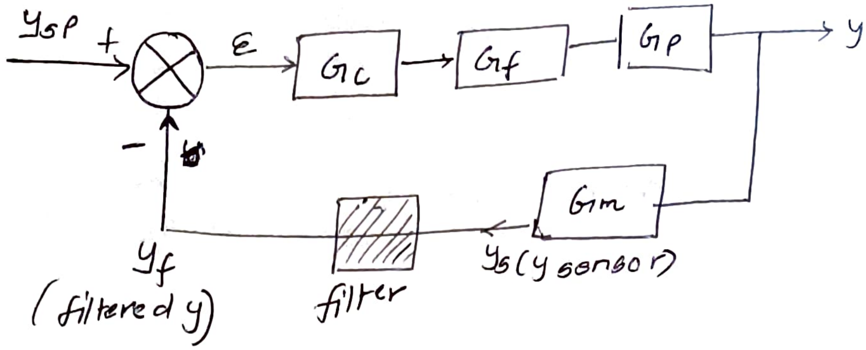
→ Data Filtering

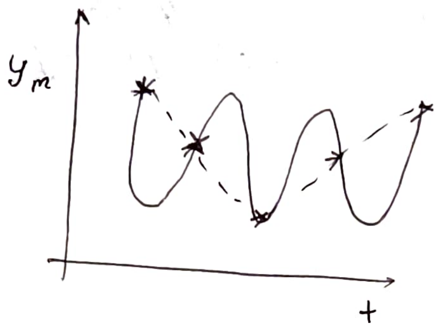


(removing noise)

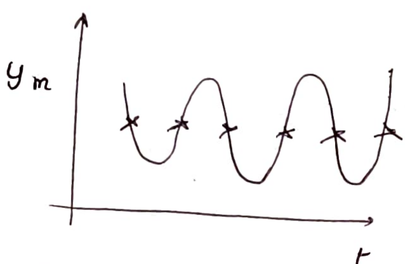
Noise sources : 1. Process itself
2. Measuring instrument
3. Electrical equipment

• To avoid 3 we use shielding or grounding.
• ex of 1 is turbulence.
[imperfect mixing, non-uniform multiphase flow]





4/3 samples per cycle



2 samples per cycle

~~eb~~ -

Aliasing

• 1st order transfer function for filter:

$$\frac{\bar{y}}{\bar{x}} = G_f = \frac{1}{\tau_f s+1} = \frac{1}{\tau_f} \times \frac{1}{(s+1/\tau_f)}$$

$$\boxed{\cancel{\frac{y}{x} = \frac{1}{\tau_f} e^{-s t/\tau_f}}}$$

$$\tau_f \bar{y} s + \bar{y} = \bar{x}$$

$$\cancel{\tau_f}\,[y] + \tau_f\left[\frac{dy}{dt} + y(0)\right] = x.$$

$$\tau_f \frac{dy}{dt} + \tau_f y(0) + y = x \quad \text{— (1)}$$

$$\tau_f\left[\frac{y(t)-y(t-\Delta t)}{\Delta t}\right] + \tau_f\, y(0) + y(t) = x(t)$$

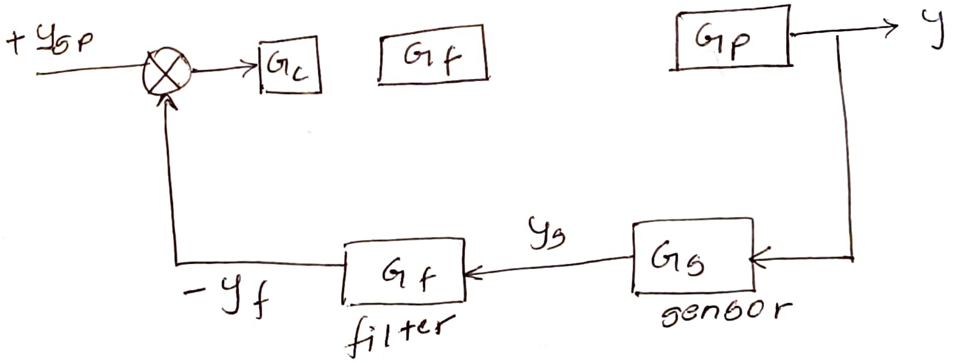
$$\frac{\tau_f\, y(t)}{\Delta t} + y(t) = x(t) - \tau_f\, y(0) + \frac{\tau_f}{\Delta t}\, y(t-\Delta t).$$

$$y(t)\left[\frac{\tau_f}{\Delta t}+1\right] = x(t) - \tau_f\, y(0) + \frac{\tau_f}{\Delta t}\, y(t-\Delta t)$$

$$y(t) = \left(\frac{\Delta t}{\tau_f+\Delta t}\right)x(t) - \frac{\overset{0}{\cancel{\tau_f\, y(0)\,\Delta t}}}{\tau_f+1} + \left(\frac{\tau_f}{\tau_f+\Delta t}\right) y(t-\Delta t)$$

$$\boxed{y_f(t) = \left[\frac{\Delta t}{\tau_f+\Delta t}\right]x(t) + \left[\frac{\tau_f}{\tau_f+\Delta t}\right] y_f(t-\Delta t)}$$ ---- $x(t)\equiv y_s(t)$ ---- (2)

1/02/25



$$G_f = \frac{K}{\tau_f s+1}$$; $K=1$ (find why?)

$$y_f(t) = f\, y_s(t) + (1-f)\, y_f(t-\Delta t)$$

$$f = \frac{\Delta t}{\Delta t+\tau_p}$$ = filter function

case1: $f=0$, we are ignoring y_s

case2: $f=1$, no filtering

$\longrightarrow$

$$f = \frac{\Delta t}{\Delta t + \tau_f}$$

$$\Delta t f + \tau_f \times f = \Delta t$$

$$\boxed{\tau_f = \frac{\Delta t(1-f)}{f}} \Rightarrow \tau_f = \Delta t\left(\frac{1}{f} - 1\right)$$

Case A: $f \uparrow$ $\quad \tau_f \downarrow$

$\Downarrow$

- $\underbrace{\text{More noise}}_{\text{disadv.}}$ $\Rightarrow$ • $\underbrace{\text{faster response}}_{\text{adv.}}$

• Case B: $f \downarrow$ $\quad \tau_f \uparrow$

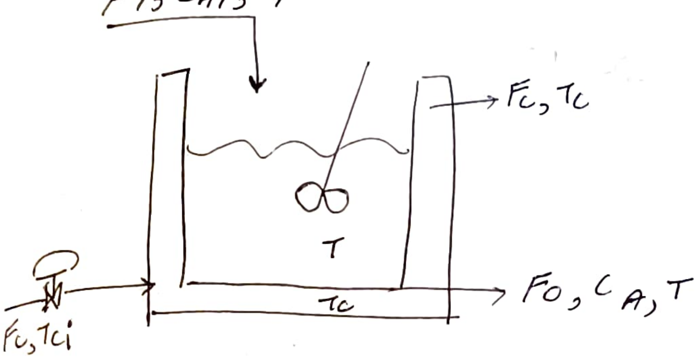
- $\underbrace{\text{Low noise}}_{\text{adv}}$ • $\underbrace{\text{slow response}}_{\text{disadv.}}$

from experience: $\tau_f = 10-12\,\Delta t$

* Enhanced single loop control

No. of measurements	No. of MV	Name of control scheme
1	1	simple FBC
>1	1	cascade, ratio, override
1	>1	split range control.

• Cascade control



$A \xrightarrow{exo} Pdt$

$F_i = F_o$ (but not constant)

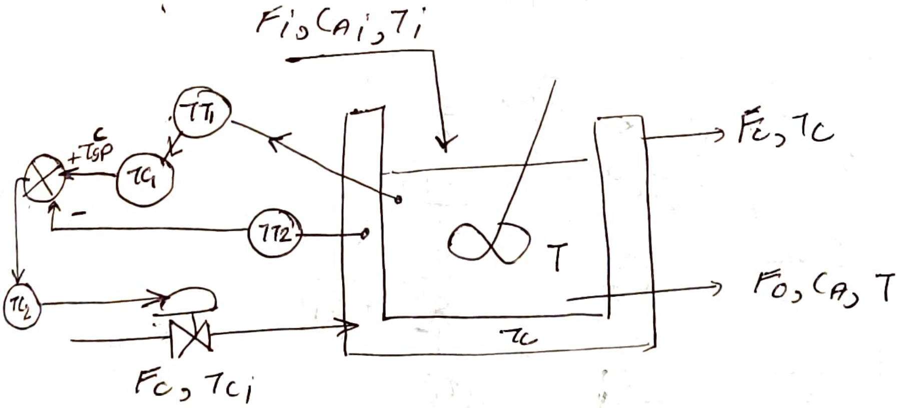
C_{Ai} = const

Objective : $T = T_{sp}$.

CV	MV
T	F_c

→ temperature (T) response is much faster to changes in T_i.

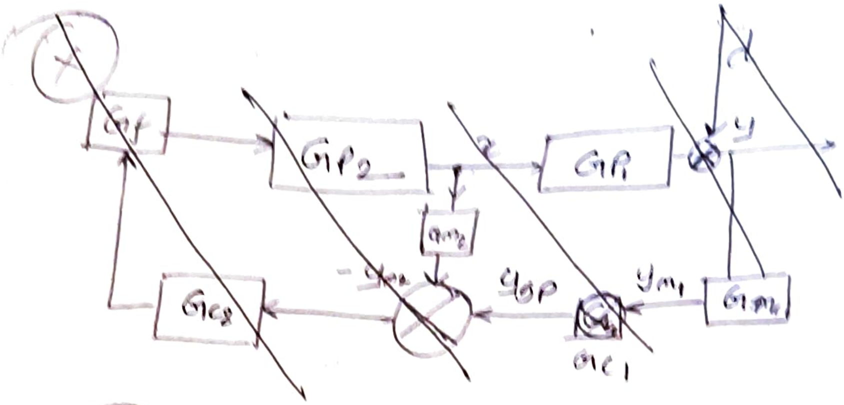
→ FBC controller is less effective to reduce the effect of disturbance in (Ti) T_{ci}

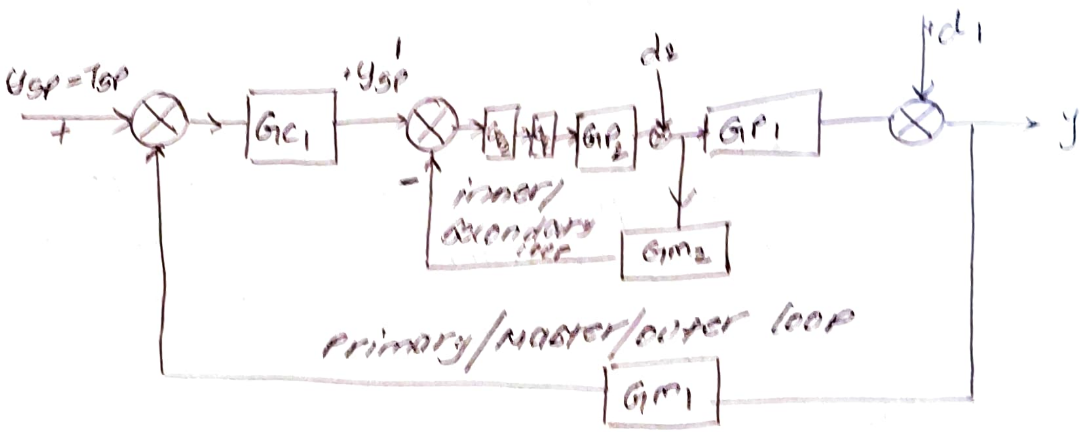


TC1 ⇒ Primary/Master controller

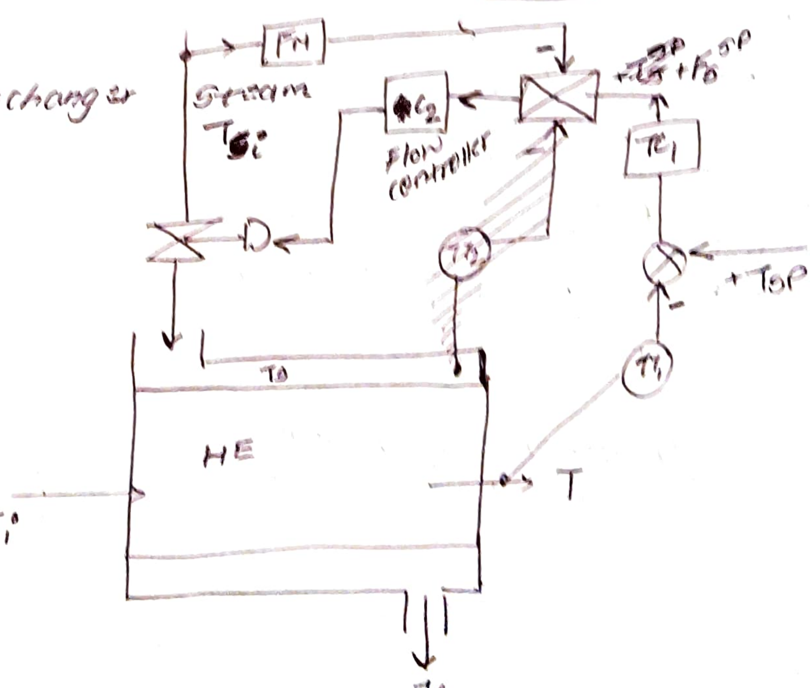
TC2 ⇒ Slave controller

Process	Sensor	Controller
1. Reactor	TT1	TC1
2. Jacket	TT2	TC2

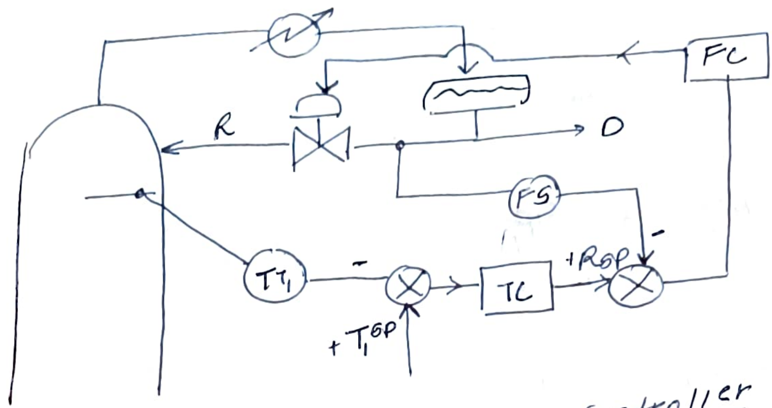




ex - Heat exchanger



Distillation [obj $T_1 = T_{1,SP}$]



Controller Choice

- Primary controller → PID
- Secondary controller → P

CV	MV
T_1/10/?D	R
T_B/?B	V_B / Q_R
m_D	D
m_B	B

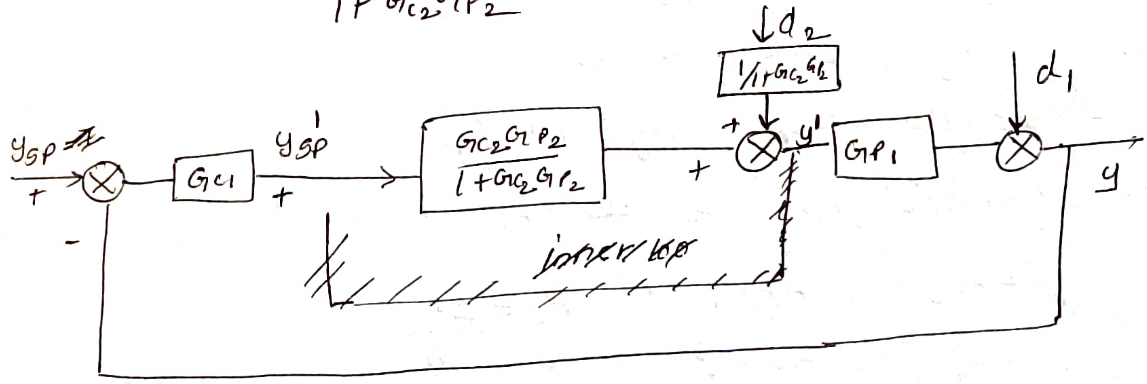
→ Secondary loop

considering $G_f, G_{m_1}, G_{m_2} = 1$

$$G_{OL2} = G_{c_2} G_{P_2}$$

CE: $1 + G_{OL2} = 0$.
$1 + G_{c_2} G_{P_2} = 0$
} for stability analysis find poles

CLTF ⇒ $\bar{y}'(s) = \frac{G_{c_2} G_{P_2}}{1 + G_{c_2} G_{P_2}} \bar{y}'_{sp}(s) + \frac{1}{G_{c_2} G_{P_2} + 1} \bar{d}_2(s)$



→ Primary loop: Slower than secondary loop

$$G_{OL1} = G_{C1} G_{P1} \times \frac{G_{C2} G_{P2}}{1 + G_{C2} G_{P2}}$$

Pb: Primary: $G_{P_1} = \frac{1}{(0.5s+1)(s+1)}$; $G_m = 1$; $G_{C_1} \equiv PI$

Secondary: $G_{P2} = \frac{1}{(1.5s+1)}$; $G_{m_2} = G_f = 1$

$G_{C_2} = K_{C2} = 5$

Tune primary controller (PI) using Z-N method
When $K_{C2} = 5$

Sol^n

$$G_{OL2} = G_{C_2} G_{P2} = \frac{5}{1.5s+1}$$

• Any value of K_{C_2} can be picked up.

$$G_{OL_1} = \frac{G_{C_1} \quad 1}{(0.5s+1)(s+1)} \times \left[\frac{5 \times 1}{(1.5s+1)\left[1 + \frac{5}{1.5s+1}\right]} \right]$$

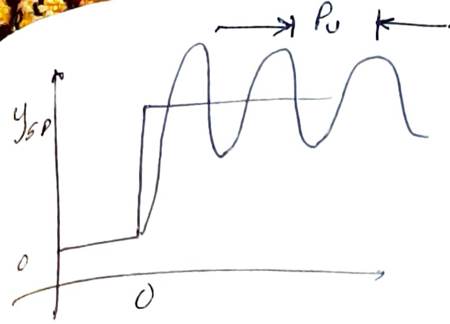
$$= \frac{G_{C_1}}{(0.5s+1)} \times \frac{1}{(s+1)} \left[\frac{5}{6 + 1.5s} \right]$$

Steps for ZN-method:

1. Bring process to SS.
2. Employ P only controller
3. Introduce step change in Y_{sp}
4. Ultimate gain K_U & P_U

$\phi = -180$ | $AR = 1$ $\quad P_U = \frac{2\pi}{\omega_{co}}$

$\omega_{co} = ?$ | $K_U = ?$



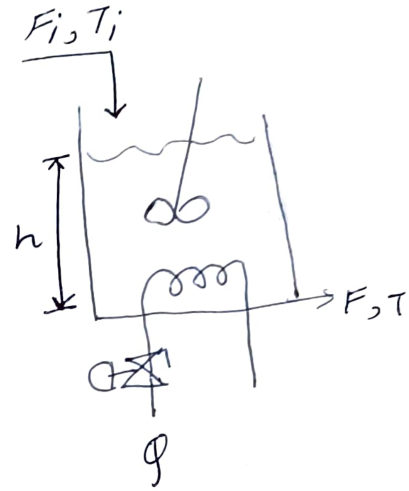
K_c at which we get sustained oscillation
$K_c = K_u$

Experimental

→ Feed forward Control scheme

Motivation :
control obj : $T = T_{SP}$

Assumption: $F_i = F$
h = constant



PI controler (FBC)

$$Q = Q_s + K_c e + \frac{K_c}{\tau_i} \int e\,dt.$$

$e = y_{SP} - y$

$Q_s, K_c, \tau_i \Rightarrow$ constant

Q changes when error signal changes i.e. T changes.

- FBC takes action only when temperature change is detected.

Situation: Process is @ SS initially.
Sudden increase in $T_i \uparrow$ ∴ $T \uparrow$ $e \downarrow$ $Q \downarrow$

- FBC takes action after the effect of disturbance has been felt by the process.

To overcome above the controller : 1. must know LV
2. Take action beforehand. FFC

$\underbrace{\text{2. Take action beforehand}}_{\text{Feed Forward Controller}}$

- FBC ~~acts~~ after the fact in a compensatory manner.
- FFC acts before hand in an anticipatory manner.

Model eqn,

in - out + gen - con = acc

$$\rho F_i T_i C_{p_i} - F T \rho C_p + \varphi = \frac{d(V\rho C_p T)}{dt}$$

const ρ, C_p & $F_i = F$

$$\varphi + F_i \rho C_p (T_i - T) = V\rho C_p \frac{dT}{dt}$$

$$\frac{\varphi}{\rho C_p V} + \frac{F_i}{V}(T_i - T) = \frac{dT}{dt}$$

$$\frac{dT}{dt} + \frac{F_i T}{V} = \frac{\varphi}{\rho C_p V} + \frac{F_i T_i}{V}$$

$$\boxed{\frac{V}{F_i}\frac{dT}{dt} + T = \frac{\varphi}{\rho C_p F_i} + T_i}$$

1. Static feed forward controller → @ SS
2. Dynamic feed forward controller

] 2 types of FFC

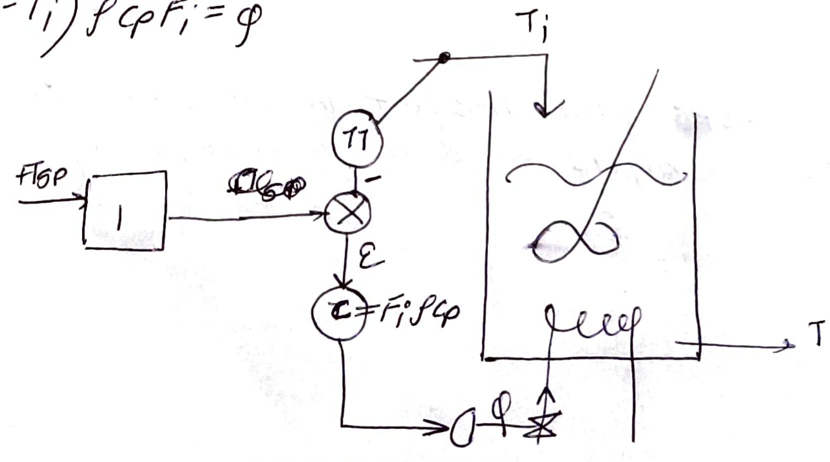
@ SS

$$T = \frac{\varphi}{\rho C_p F_i} + T_i$$

$$\boxed{(T - T_i)\rho C_p F_i = \varphi}$$ } no set point available. Controller becomes direction less

↓ Replace T by Tsp.

$$(T_{sp} - T_i)\rho C_p F_i = \varphi$$



Dynamic FFC

$$\tau \frac{dT}{dt} + T = T_i + \frac{\varphi}{F_i \rho C_p}$$

$$\tau[s\bar{T}(s)] + \bar{T} = \bar{T}_i + \frac{\bar{\varphi}}{F_i \rho C_p}$$

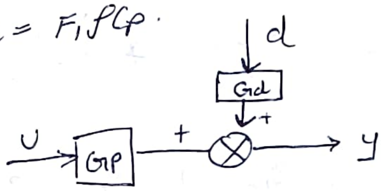
$$F_i \rho C_p\left(\tau s \bar{T}(s) + \bar{T} - \bar{T}_i\right) = \bar{\varphi}$$

$$\bar{\varphi} = F_i \rho C_p\left[\bar{T}(s)(\tau s + 1) - \bar{T}_i\right] \Rightarrow \left(\bar{T} \equiv \bar{T}_{sp}\right)$$

$\boxed{1} \xRightarrow{\text{Replace}} \boxed{\tau s + 1}$ - - - - in previous configuration.

$\bar{\varphi}(s) = [G_{sp} y_{sp} - d] G_c$ ..... general expression

$G_{sp} = \tau s + 1$ $\quad G_c = F_i \rho C_p$.



$y = G_p U + G_d d$

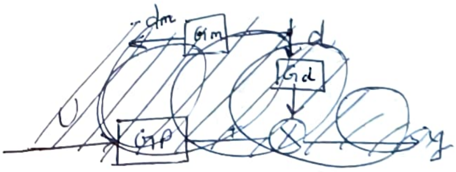
Replacing y by y_{sp}

$y_{sp} = G_p U + G_d d$

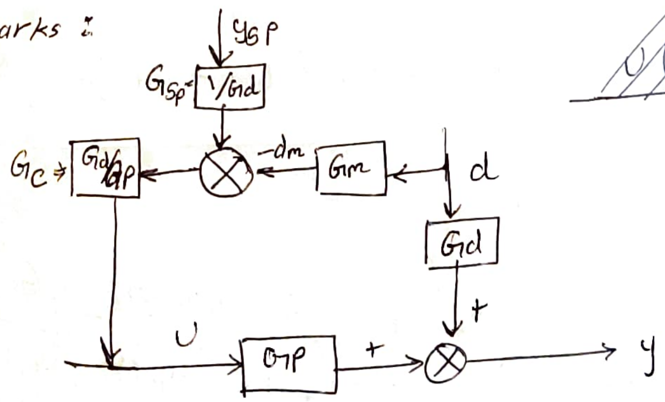
$$U = \frac{1}{G_p}\left[y_{sp} - G_d d\right]$$

$$U = \frac{G_d}{G_p}\left[\frac{1}{G_d} y_{sp} - d\right]$$

$\cancel{G_c = dd}$ $\quad G_c = \frac{G_d}{G_p}$; $G_{sp} = \frac{1}{G_d}$ } from general expressn.



Remarks :



Closed control loop