INDIAN INSTITUTE OF TECHNOLOGY KHARAGPUR

Mid-Spring Semester Examination 2022-23

Date of Examination: ________________ **Session: (FN/AN)** ________ **Duration: 2 hrs.**

Full Marks: 30

Subject No.: CH30012 **Subject: TRANSPORT PHENOMENA**

Department/Center/School: Chemical Engineering

Specific charts, graph paper, log book etc., required

Special Instructions (if any): **NO**

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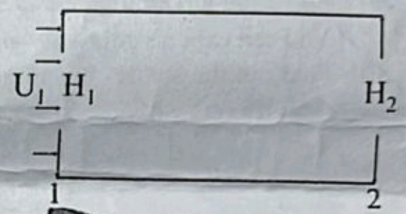
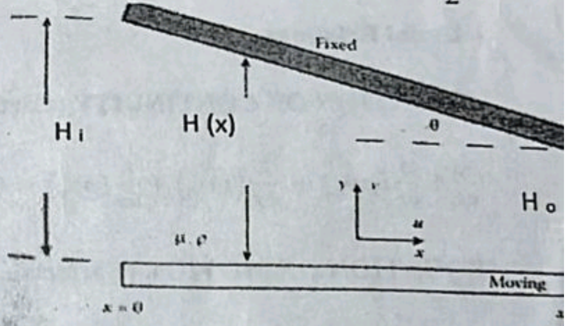
1. Lubrication flows are characterized by incompressible fluids to thin gaps e.g., the layer of water between the ice skate and the ice, the oil that lubricates the moving parts of an internal combustion engine etc. The fluid flows at relatively small velocities and so the inertial terms in the Navier-Stokes equations are insignificant compared to the diffusive terms. Since the gaps are so thin, lubrication flows can be treated as two-dimensional. Finally, if the gap is very thin, there is little effect of gravity.



Consider the lubrication flow occurring between the fixed and moving components of a bearing as shown in the figure (H_i and H_o are the gaps at $x = 0$ and $x = L$) in presence of a pressure gradient with one of the plates moving with a constant velocity U_o. Assume that there is a constant volumetric flow rate V of the lubricant in the system as a result of this motion of the plate and the applied pressure gradient.

(i) Solve the governing equations with appropriate boundary conditions to obtain the x-component of the velocity, u, as a function of the pressure gradient and H(x).

Use relevant conditions to obtain expressions for ii) dP/dx and the local pressure P(x) in terms of H(x), U_o and V, (iii) the volumetric flow rate V and (iv) the load the bearing can support in terms of the system parameters (H_i, H_o, θ, U_o) and μ.



4+4+3+3 =14

2. A laboratory wind tunnel has a flexible upper wall that can be adjusted to compensate for boundary-layer growth, giving zero pressure gradient along the test section. The wall boundary layers at both sections 1 and 2 are well represented by the 1/7-power-velocity profile. At the inlet the tunnel cross section is square, with height H_1 and width W_1, each equal to 305 mm. With freestream speed $U_1 = 26.5$ m/s, measurements show that $\delta_1 = 12.2$ mm and downstream $\delta_2 = 16.6$ mm.

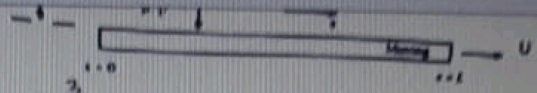
(i) Calculate the height of the tunnel walls at section 2.

(ii) Determine the equivalent length of a flat plate that would produce the inlet boundary layer thickness.

(iii) Estimate the streamwise distance between sections 1 and 2 in the tunnel. Assume standard air (kinematic viscosity = 1.45×10^{-5} m^2/s). 4+3+2=9

3. Consider an unsteady state momentum transfer event in semi-infinite domain (in +y direction), wherein a flat plat of infinite length (in $\pm$ x direction) is kept at $y = 0$, and the domain $0 < y$ contains a Newtonian liquid at rest, as shown in figure on next page. At time $t = 0$, a constant stress τ_0, has been imposed on flat plat in $+x$ direction, which is maintained for $t \geq 0$. Assume 1D momentum transport in $+y$ direction. The constitutive relation for

local pressure P(x) in terms of H(x), U_o and V, (iii) the volumetric flow rate V and (iv) the load the bearing can support in terms of the system parameters (H_i, H_o, θ, U_o) and μ.



5 ~~4+4~~+3+3=14

i) As the flow is in the lubrication regime, the inertial terms are negligible and the momentum equations become

$$\frac{\partial P}{\partial x} = \mu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right)$$

$$\frac{\partial P}{\partial y} = \mu\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right)$$

The gap betⁿ the plates is thin, thus gravity can be neglected. 1

Since $v_y = 0$ at both plates and the gap is very thin, $u >> v$ $\therefore \frac{\partial p}{\partial y} = 0$. 1

The thin gap approximation also means

$$\frac{\partial u}{\partial y} >> \frac{\partial u}{\partial x} \quad 1$$

$$\therefore \frac{\partial p}{\partial x} = \mu \frac{d^2 u}{dy^2}$$

$$\Rightarrow u = \frac{1}{2}\left(\frac{dp}{dx}\right)y^2 + C_0 y + C_1$$

At $y = 0$ $u = U_0$ 1

$y = H(x)$ $u = 0$.

$$\therefore u = U_0 + \left(\frac{1}{2\mu}\right)\frac{dp}{dx}y^2 - \left(\frac{U_0}{H(x)}\right)y - \left(\frac{H(x)}{2\mu}\right)\frac{dp}{dx}y$$

ii) H(x) may be obtained from geometry and $\frac{dp}{dx}$ from the given constant volumetric flow rate q, the Liquid.

ii)
5
iii)

$$V = \int_0^{H(x)} u\,dy = u_0 y + \frac{1}{6\mu}\frac{dp}{dx}y^3 - \left[\frac{U_0}{2H(x)} + \frac{H(x)}{4\mu}\frac{dp}{dx}\right]y^2 \Bigg|_0^{H(x)}$$ (1)

$$V = \frac{U_0}{2}H(x) - \frac{1}{12\mu}\frac{dp}{dx}H^3(x).$$ (1)

$$\therefore \frac{dp}{dx} = \left[\frac{u_0}{2}H(x) - V\right]\frac{12\mu}{H^3(x)}$$

$$\therefore p(x) = \int_0^x \left[\frac{U_0}{2}H(x) - V\right]\frac{12\mu}{H^3(x)}\,dx.$$ (1)

$H(x)$ can be expressed as (1)

$$H(x) = \frac{H_0 - H_i}{L}x + H_i$$

$$\therefore p(x) = \frac{6x\,[H(x) - H_0]\,H_i^2}{H^2(x)\,[H_i + H_0]}$$ (1)

and $$V = U_0 H_i \left[\frac{H_0}{H_i + H_0}\right].$$ (1)

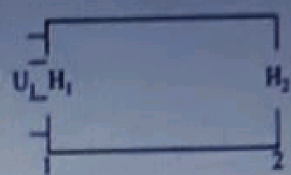
iv) The pressure distribution exerts a vertical force. The load per unit width that the bearing can support is given by the vertical component of the pressure force.

$$\text{Load} = \int_0^L p(x)\cos\theta\,dx.$$ (2)

$$\text{Load} = 6\mu U_0\left(\frac{\cos\theta}{\tan^2\theta}\right)\left[\ln\frac{H_i}{H_0} - 2\left(\frac{H_i - H_0}{H_i + H_0}\right)\right].$$ (1)

The load bearing capacity increases with increased flow rate, increases as the two surfaces become more parallel to each other and increases as the gap width decreases.

2. A laboratory wind tunnel has a flexible upper wall that can be adjusted to compensate for boundary-layer growth, giving zero pressure gradient along the test section. The wall boundary layers are well represented by the 1/7-power-velocity profile. At the inlet the tunnel cross section is square, with height H_1 and width W_1, each equal to 305 mm. With freestream speed U_1 - 26.5 m/s, measurements show that δ_1 = 12.2 mm and downstream δ_2 = 16.6 mm.



(i) Calculate the height of the tunnel walls at section 2.

(ii) Determine the equivalent length of a flat plate that would produce the inlet boundary layer thickness.

(iii) Estimate the streamwise distance between sections 1 and 2 in the tunnel. Assume standard air (kinematic viscosity = 1.45×10^{-5} m²/s). 4 + 3 + 2

1 $H_1 = 305$ mm, $\delta_1 = 12.2$ mm, $U_1 = 26.5$ m/s.

2 $\delta_2 = 16.6$ mm.

i) Using equation of continuity and concept of δ^* 1

$A_1 U_1 = A_2 U_2$, A is the effective flow area

Given $U_1 = U_2$ and zero pressure gradient flow

$A_1 = A_2$ 1

$(w-2\delta_1^*)(H_1-2\delta_1^*) = (w-2\delta_2^*)(H_2-2\delta_2^*)$

$$H_2 = \frac{(w-2\delta_1^*)(H_1-2\delta_1^*)}{(w-2\delta_2^*)} + 2\delta_2^* \quad -(i)$$

From definition of δ^*

$$\delta^* = \int_0^\delta \left(1-\frac{u}{U}\right)dy = \delta\int_0^1\left(1-\frac{u}{U}\right)d\eta \quad \left(\eta = \frac{y}{\delta}\right)$$

$$\delta^* = \delta\int_0^1 \left(1-\eta^{1/7}\right)d\eta = \delta\left[\eta - \frac{7}{8}\eta^{8/7}\right]_0^1 = \frac{\delta}{8} \quad -(ii)$$

1

Using eq^n (i)

$$H_2 = \frac{\left(305-\frac{2\times12.2}{8}\right)\left(305-\frac{2\times12.2}{8}\right)}{\left(305-\frac{2\times16.6}{8}\right)} + \frac{16.6\times2}{8}$$

1

$H_2 =$ ~~323 mm~~ 307.2 mm

(ii) For flat plate turbulent B.L. with $1/7^{th}$ power law

$$\frac{\delta}{x} = \frac{0.37}{(Re_x)^{1/5}} \Rightarrow \delta = 0.37\left(\frac{\nu}{U}\right)^{1/5} x^{4/5} \quad ①$$

$$x = \left[\frac{\delta}{0.37}\right]^{5/4}\left(\frac{U}{\nu}\right)^{1/4}$$

At ① $\delta = 12.2$ mm. ①

$$x_1 = \left[\frac{0.0122}{0.37}\right]^{5/4}\left(\frac{26.5}{1.45\times10^{-5}}\right)^{1/4}$$

$$\underline{x_1 = 0.517 \text{ m}} \quad ①$$

The length of a flat plate that will produce a B.L. thickness of $\delta_1 = 12.2$ mm.

(iii) At ② $\delta = 16.6$ mm.

$$\therefore x_2 = \left[\frac{0.0166}{0.37}\right]^{5/4}\left(\frac{26.5}{1.45\times10^{-5}}\right)^{1/4} \quad ①$$

$$x_2 = 0.759 \text{ m.}$$

$\therefore$ Approximate distance between ① and ②

$$x_2 - x_1 \quad ①$$

$$= \underline{0.242 \text{ m}}.$$