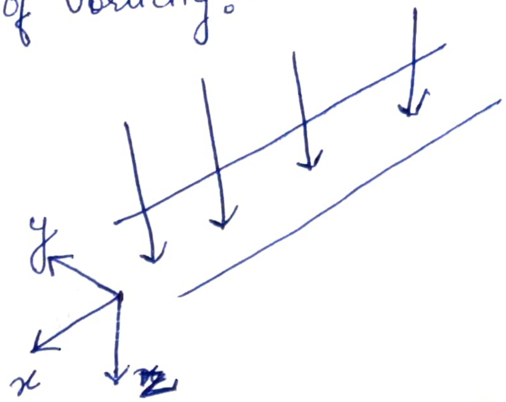
Eqⁿ of vorticity: $\frac{D\hat{\omega}}{Dt} = \nu \nabla^2 \hat{\omega} + [\hat{\omega} \cdot \nabla \hat{V}]$ — (A)

Find out the non zero components of vorticity:-



$$\vec{\omega} = \vec{\nabla} \times \vec{V}$$

$$= \begin{vmatrix} \hat{\delta}_x & \hat{\delta}_y & \hat{\delta}_z \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ \underbrace{\cancel{V_x}}_{=0} & \underbrace{\cancel{V_y}}_{=0} & V_z \end{vmatrix}$$

$$= \hat{\delta}_x \left(\frac{\partial V_z}{\partial y} \right)$$

Only non-zero velocity component is $V_z(y)$ · function of y only.

(b) Substitute the vorticity in the above vorticity eqⁿ and simplify:

$$\underbrace{\frac{\partial \hat{w}}{\partial t}}_{I} + \underbrace{\hat{V} \cdot \nabla \hat{w}}_{II} = \underbrace{\nu \nabla^2 \hat{w}}_{III} + \underbrace{[\hat{w} \cdot \nabla \hat{V}]}_{IV} \quad \text{— (B)}$$

(I) → $\frac{\partial \hat{w}}{\partial t} = 0$ steady state

W. is only funcⁿ of y

(II) $\hat{V} \cdot \nabla \hat{w} \Rightarrow \nabla \hat{w} = \sum_i \frac{\partial w_i}{\partial x_i} \hat{\delta}_i = \sum_i \frac{\partial w_x}{\partial x_i} \delta_{ix}$

$= 0$

$\Rightarrow \quad \hat{V} \cdot \nabla \hat{w} = 0$

(iv) $\hat{w} \cdot \nabla \hat{v} = \sum_i w_i \hat{\delta}_i \cdot \sum_j \sum_k \frac{\partial v_k}{\partial x_j} \hat{\delta}_j \hat{\delta}_k$

$$= \sum_i \sum_j \sum_k w_i \frac{\partial v_k}{\partial x_j} \delta_{ij} \hat{\delta}_k$$

$(i = j \Rightarrow \delta_{ij} = 1)$

$$= \sum_k w_i \frac{\partial v_k}{\partial x_i} \hat{\delta}_k$$

$$= \cancel{\sum w_x \frac{\partial v_x}{\partial x} \hat{\delta}_x +}$$

$$= \sum_k w_x \frac{\partial v_x}{\partial x} \hat{\delta}_x + w_x \frac{\partial v_y}{\partial x} \hat{\delta}_y + w_x \frac{\partial v_z}{\partial x} \hat{\delta}_z$$

$$= 0$$

v_z is only func of y & $v_x = v_y = 0$

Substituting I, II & iv in eqn B

$$\nu \nabla^2 \hat{w} = 0$$

$$\nabla^2 \hat{w} = \nabla \cdot \nabla \hat{w} = \nabla \cdot \sum_j \sum_k \frac{\partial}{\partial x_j} w_k \hat{\delta}_j \hat{\delta}_k$$

$$= \sum_i \frac{\partial}{\partial x_i} \hat{\delta}_i \cdot \sum_j \sum_k \frac{\partial}{\partial x_j} w_k \hat{\delta}_j \hat{\delta}_k$$

$(i = j \Rightarrow \delta_{ij} = 1)$

$$= \sum_i \sum_j \sum_k \frac{\partial}{\partial x_j} \frac{\partial}{\partial x_i} w_k \delta_{ij} \hat{\delta}_k$$

$$= \sum_j \sum_k \frac{\partial}{\partial x_j} \frac{\partial}{\partial x_j} w_k \hat{\delta}_k$$

$$= \sum_j \sum_k \frac{\partial^2}{\partial x_j^2} w_k \delta_k$$

$$\sum_j \frac{\partial^2}{\partial x_j^2} \omega_x \delta_x \quad \Big\}\text{ only func}^n \text{ of } y.$$

$$= \frac{\partial^2}{\partial y^2} \omega_x \delta_x$$

$$= \frac{\partial^2}{\partial y^2}\left(\frac{\partial v_z}{\partial y}\right)$$

$$= \frac{\partial^3 v_z}{\partial y^3}$$

$$\Rightarrow \boxed{\frac{\partial^3 v_z}{\partial y^3} = 0}$$