Problems on Analogy

Chilton Coulburn Analogy (Modified Reynold's Analogy)

$$\frac{C_f}{2} = St\,.\,\mathrm{Pr}^{2/3} = j_h = St_m\,.\,Sc^{2/3} = j_m \qquad \begin{array}{l} 0.6 < \mathrm{Pr} < 60 \\ 0.6 < Sc < 300 \end{array}$$

	Momentum	Heat	Mass
Laminar	$C_{f,x} = 0.664\,\mathrm{Re}_x^{-1/2}$ $\mathrm{Re}_x < 5\times10^5$	$Nu_x = 0.332\,\mathrm{Re}_x^{1/2}\,\mathrm{Pr}^{1/3}$ $\overline{Nu}_L = 0.664\,\mathrm{Re}_L^{1/2}\,\mathrm{Pr}^{1/3}$ $\mathrm{Re}_x < 5\times10^5,\ 0.6 < \mathrm{Pr} < 50$	$Sh_x = 0.332\,\mathrm{Re}_x^{1/2}\,Sc^{1/3}$ $\overline{Sh}_L = 0.664\,\mathrm{Re}_L^{1/2}\,Sc^{1/3}$ $\mathrm{Re}_x < 5\times10^5,\ 0.6 < Sc < 300$
Mixed/ Turbulent	$C_{f,L} = \frac{0.074}{\mathrm{Re}_L^{1/5}} - \frac{1742}{\mathrm{Re}_L}$ $5\times10^5 < \mathrm{Re}_L < 10^8$ $\mathrm{Re}_{x,c} = 5\times10^5$	$\overline{Nu}_L = \left(0.037\,\mathrm{Re}_L^{4/5} - 871\right)\mathrm{Pr}^{1/3}$ $0.6 \le \mathrm{Pr} \le 50, \mathrm{Re}_{x,c} = 5\times10^5$ $5\times10^5 < \mathrm{Re}_L < 10^8$	$\overline{Sh}_L = \left(0.037\,\mathrm{Re}_L^{4/5} - 871\right)Sc^{1/3}$ $0.6 \le Sc \le 300, \mathrm{Re}_{x,c} = 5\times10^5$ $5\times10^5 < \mathrm{Re}_L < 10^8$

Analogy

Salient Features

- Unified treatment of momentum, heat and mass transfer, based on fundamental governing equations
- Valid for laminar as well as turbulent flow conditions
- Relations are interchangeable, valid over a large range of Prandtl and Schmidt number, encompassing most of the fluids and scenarios
- A powerful tool

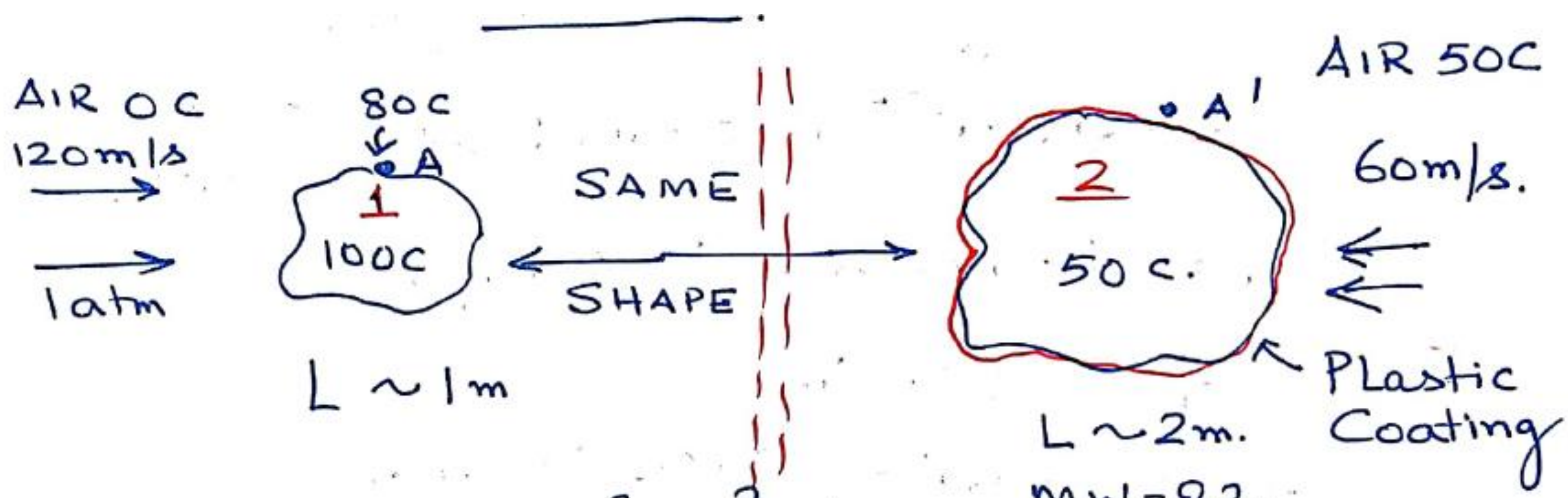
Problem on Analogy

An irregularly shaped object, 1 m long, maintained a constant temperature of 100 ^oC is suspended in an airstream having a free-stream temperature of 0 ^oC, a pressure of 1 atm, and a velocity of 120 m/s. The air temperature measured at a point near the object in the airstream is 80 ^oC. A second object having the same shape is 2 m long and is suspended in an airstream in the same manner. Both the air and the object are at 50 ^oC, total pressure equal to 1 atm and air free stream velocity is 60 m/s. A plastic coating on the surface of the object is being dried by the process. The molecular weight of the vapor is 82 and the saturation pressure at 50 ^oC for the plastic material is 0.0323 atm.

For the second object, at a location corresponding to the point of measurement of the first object, determine the vapor concentration and partial pressure.

If the average heat flux, $q^{//}$, is 2000 W/m^2 for the first object, determine the average mass flux $n_A^{//}$ (Kg/s.m^2) for the second object.

Given: For Air (at 323 K and 1 atm) kinematic viscosity = 18.20 x 10^{-6} m^2/s, Pr = 0.71, k = 28 x 10^{-3} W/m.K, For the plastic vapor: M_A = 82 Kg/Kg mole, P_{sat} = 0.0323 atm, D_{AB} = 2.6 x 10^{-5} m^2/s



$$R = 8.205 \times 10^{-2} \ \frac{m^3 \ atm}{Kmol \ k}.$$

$$Re_1 = \frac{V_1 L_1}{\nu} = \frac{120 m/s \times 1m}{18.2 \times 10^{-6} m^2/s} = \underline{6.59 \times 10^{6}}.$$

$$Re_2 = \frac{60 m/s \times 2m}{18.2 \times 10^{-6} m^2/s} = \underline{6.59 \times 10^{-6}}.$$

$$\underline{Pr = 0.703}, \quad \underline{Sc_2} = \frac{\nu}{D_{AB}} = \frac{18.2 \times 10^{-6}}{2.6 \times 10^{-5}} = \underline{0.7}$$

Since $Re_1 = Re_2$ and $Pr_1 = Sc_2$, the dimensionless solutions of the energy and species equations must be identical.

$$T^*(x^*, y^*) = C_A^*(x^*, y^*)$$

$$\frac{T - T_s}{T_\infty - T_s} = \frac{C_A - C_{As}}{C_{A\infty} - C_{As}}.$$

$$C_{As} = \frac{p_{Asat}}{RT} = \frac{0.0323 \text{ atm}}{8.205 \times 10^{-2} \frac{m^3 \cdot atm}{Kmol \cdot K} \cdot 323 \text{ K}}$$

$$C_{As} = 1.219 \times 10^{-3} \frac{Kmol}{m^3}.$$

$$C_A = C_{As} + (C_{A\infty} - C_{As})\frac{T - T_s}{T_\infty - T_s}$$

$$= 1.219\times10^{-3} + (0 - 1.219\times10^{-3})\frac{80-100}{0-100}$$

$$\underline{C_A = 0.975\times10^{-3}}\ \text{kmol/m}^3$$

The vapor pr. is

$$\underline{p_A = C_A RT = 0.0258\ \text{atm}}$$

b) For case 1, $q'' = 2000\ \text{W/m}^2$

$$q'' = \bar{h}(T_s - T_\infty)$$

For 2 $\quad n_A'' = \bar{h}_m (C_{As} - C_{A\infty}) M_A$

From analogy $\underline{\overline{Nu}_L = \overline{Sh}_L}$ $\left(\because \begin{matrix} Pr = Sc \\ Re_1 = Re_2 \end{matrix}\right)$

$$\frac{\bar{h}}{\bar{h}_m} = \frac{L_2}{L_1} \cdot \frac{k}{D_{AB}}$$

$$\| \; n_A'' = q'' \frac{\bar{h}_m}{\bar{h}} \frac{(C_{As} - C_{A\infty})}{T_s - T_\infty} \cdot M_A \; \|$$

$$= q'' \frac{L_1 D_{AB}}{L_2 k} \frac{(C_{As} - C_{A\infty})}{(T_s - T_\infty)} M_A .$$

$$n_A'' = 2000 \frac{W}{m^2} \frac{1m \times 2.6 \times 10^{-5} m^2/s}{2m \times (28 \times 10^{-3} \frac{W}{m\,K})} \frac{(1.219 \times 10^{-3} - 0) \frac{kmol}{m^3}}{(100 - 0)K} \times 82 \frac{kg}{kmol}$$

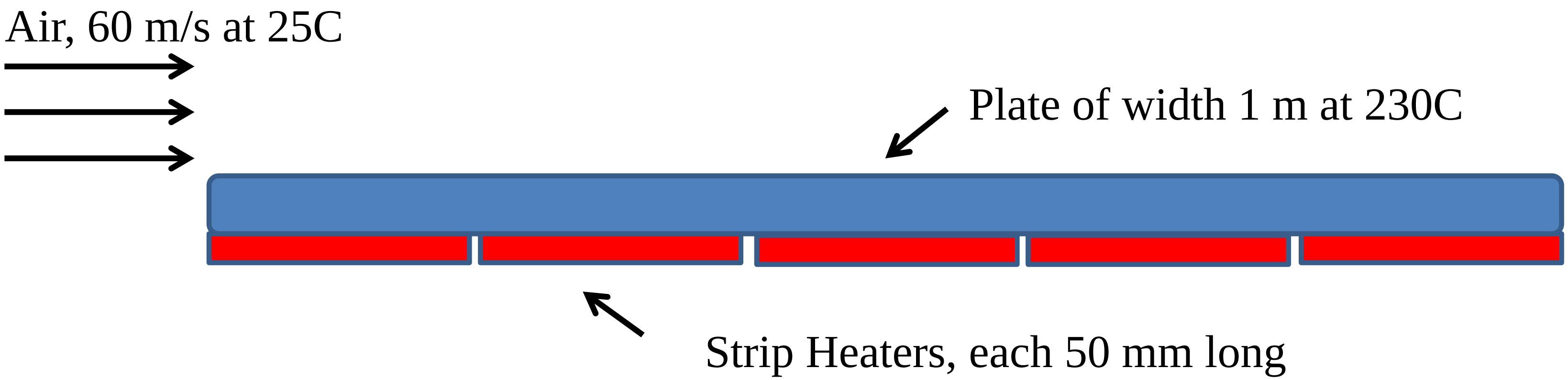
$$n_A'' = 9.28 \times 10^{-4} \; kg/m^2 \cdot s$$

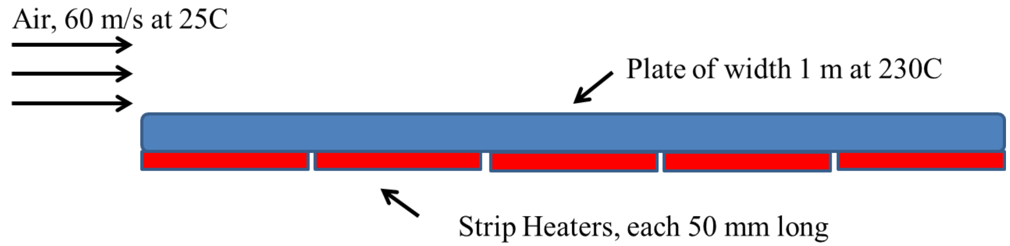
$$= 1.132 \times 10^{-5} \; \frac{kg\ moles}{m^2 s} .$$

A flat plate of 4m length, coated with a volatile substance (A) is exposed to dry, atmospheric air in parallel flow with T = 20 ^{o}C, U = 8 m/s. The plate is maintained at a constant temperature of 134 ^{o}C, by electrical heating element, and the substance evaporates from the surface. The plate has a width of 0.25 m and is well insulated at the bottom. The molecular weight of A is 150 kg/kg mol, the latent heat of vaporization of A is 5.44×10^{6} J/kg and the diffusion coefficient of A in air (B) is $D_{AB} = 7.75 \times 10^{-7}$ m^2/s. If the saturated vapor pressure of A is 0.12 atm at 134^{o}C, what is the electrical power required to maintain steady-state conditions? Neglect radiative losses. For air: kinematic viscosity = 20.92×10^{-6} m^2/s, k = 0.03W/m.K, Pr = 0.7, R = 8.205×10^{-2} m^3. atm / (Kmol . K)

Problem on Analogy

A flat plate of width 1 m is maintained at a uniform surface temperature of T_s = 230 ^oC by using independently controlled, electrical strip heaters, each of which is 50 mm long. Each heater is insulated from its neighbors, as well as on its back side. If atmospheric air at 25 ^oC flows over the plate at a velocity of 60 m/s, at what heater is the electrical input a maximum (consider and calculate for all logical possibilities)? What is the value of this input? For air : γ = 26.41 x 10 $^{-6}$ m ^{2}/s, k = 0.0338 W/m . K, Pr = 0.69.





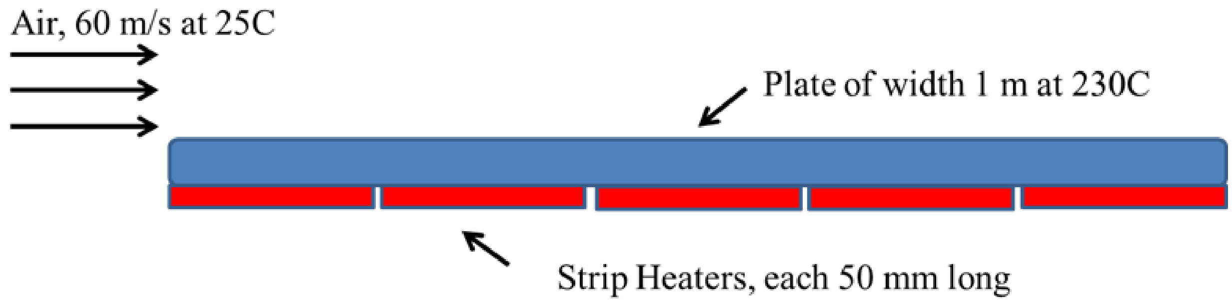
First find the point of B.L. Transition

Re_1, based on the length of the first heater

$$Re_1 = \frac{U_\infty L_1}{\nu} = \frac{60 \times 0.05}{26.41 \times 10^{-6}} = 1.14 \times 10^5$$

If $Re_{x,c} = 5 \times 10^5$, transition will occur on the 5th heater

Max. electrical power is needed for the heater with largest $\bar{h}$.



Possibilities

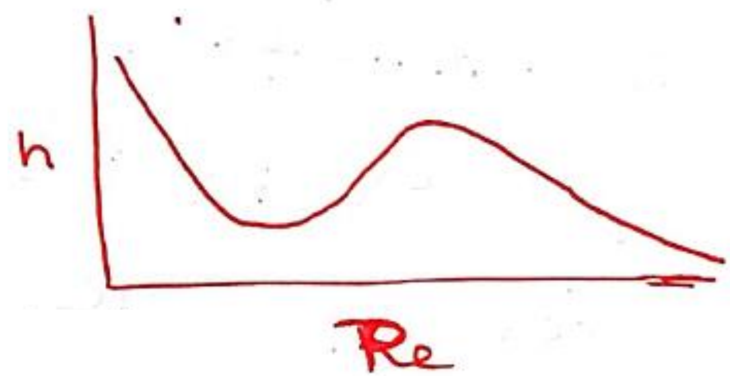
1) Heater 1 → Largest, local, laminar convect. coeff.

   " Turbulent " "

2) " 5 → "

3) " 6 → Since turbulent conditions exist over the entire heater.

$$\boxed{q_{conv} = q_{elec}}$$



For 1st heater

$$q_{conv,1} = \overline{h}_1 L_1 W (T_s - T_\infty)$$

$$\overline{Nu}_1 = 0.664 Re^{1/2} Pr^{1/3} = 0.664 (1.14 \times 10^5)^{1/2} (0.69)^{1/3}$$

$$= 198.$$

$$\overline{h}_1 = \frac{\overline{Nu}_1 k}{L_1} = \frac{198 \times 0.0338}{0.05} = 134 \ W/m^2 K$$

$$\therefore \ q_{conv,1} = 134 \times (0.05 \times 1)(230 - 25) = 1370 \ W$$

For 5th heater

$$q_{conv,5} = \bar{h}_{1-5} L_5 W (T_s - T_\infty) - \bar{h}_{1-4} L_4 W (T_s - T_W)$$

$$= \left(\bar{h}_{1-5} L_5 - \bar{h}_{1-4} L_4 \right) W (T_s - T_\infty)$$

$\bar{h}_{1-4} \rightarrow \overline{Nu}_4 = 0.664\, Re_4^{1/2} Pr^{1/3}$ (Heaters 1–4 are under Laminar condition)

$$\overline{Nu}_4 = 0.664 \left(4.56 \times 10^5\right)^{1/2} (0.69)^{1/2} = 396$$

$\bar{h}_{1-4} = 67\ W/m^2K$ $(L = 0.2m)$

$\bar{h}_{1-5} \rightarrow \overline{Nu}_5 = \left(0.037\, Re_5^{4/5} - 871\right) Pr^{1/3}$. (Transition takes place on 5th heater – Mixed flow correlation)

$Re_5 = 5.7 \times 10^5$, $L = 5L_1$.

$\bar{h}_{1-5} = 74\ W/m^2K$

$$\therefore q_{conv,5} = (74 \times 0.25 - 67 \times 0.2) \times 1\, (230 - 25)$$

$$\boxed{q_{conv,5} = 1050\ W}$$

For 6th heater

$$q_{conv6} = \left(\bar{h}_{1-6} L_6 - \bar{h}_{1-5} L_5\right) w \left(T_S - T_\infty\right)$$

For $\bar{h}_{1-6}$ $\quad Re_6 = 6.84 \times 10^5$, $L_6 = 6 L_1$

$$\overline{Nu}_6 = \left[0.037\left(6.84 \times 10^5\right)^{4/5} - 871\right](0.69)^{1/3} = 753$$

(Mixed flow on 6th)

$$\bar{h}_{1-6} = 85 \ W/m^2 K$$

$$q_{conv6} = (85 \times 0.3 - 74 \times 0.25) \times 1 \times (230 - 25)$$

$$\boxed{q_{conv,6} = 1440 W}$$

$$\therefore \quad q_{conv6} > q_{conv1} > q_{conv5}$$

Sixth plate has the largest power requirement.

Problem on Analogy

As a means of supplying fresh water to arid regions of the world, it has been advocated that the icebergs be towed from polar regions. Icebergs that are considered to be best suited for towing are those that are relatively broad and flat. Consider an iceberg that is 1 km long by 0.5 km wide and of depth D = 0.25 km. It is proposed that this iceberg be towed at 1 km/h in the direction of its length for 6000 km through water whose average temperature (over the trip) is 10 ^{o}C.

As a first approximation, the interaction of the iceberg with its surroundings may be assumed to be dominated by conditions at the bottom (1 km x 0.5 km) surface. The latent heat of fusion of ice is 3.34 x 10 ^{5} J/kg.

(a) What is the average recession rate, dD/dt at the bottom surface?
(b) What is the power required to move the iceberg at the designated speed?
(c) If towing costs amount to $ 1/kW.h of power requirement, what is the minimum cost of fresh water at the destination?

Properties:
Ice : ρ_i = 920 kg/m ^{3}, Latent heat of fusion, h_{sf} = 3.34 x 10 ^{5} J/kg,

Water : ρ = 1000 kg/m ^{3}, k = 0.58 W/m.K, Kinematic viscosity = 1.5 x 10 $^{-6}$ m ^{2}/s, Pr = 11.

Conservation of energy dictates that the rate at which heat is convected to the ice must be equal to the rate of energy used for melting

$$\bar{h}_L A_S (T_\infty - T_s) = \dot{E}_m = h_{sf} \frac{dm}{dt} = h_{sf} \frac{d}{dt}\left(\rho_{ice} A_S D\right)$$

The average recession rate is therefore,

$$\frac{dD}{dt} = \frac{\bar{h}_L \left(T_\infty - T_s\right)}{\rho_{ice} h_{sf}}$$

Here $Re_L = VL/(\mu/\rho) = 1000/3600$ m/s x 1000m x 1/ (1.5×10^{-6} m^2/s)

$Re_L = 1.85 \times 10^8$

Since $Re_L \gg Re_{x,c}$ Nusselt number can be estimated from

$$\overline{Nu_L} = \left(0.037\,\mathrm{Re}^{4/5} - 871\right)\mathrm{Pr}^{1/3} = 3.38\times10^5$$

$$\bar{h}_L = \frac{k}{L}\overline{Nu_L} = \frac{0.058}{1000}\times3.38\times10^5 = 196\,\frac{W}{m^2K}$$

Therefore, the average recession rate is

$$\frac{dD}{dt} = \frac{196\times(10-0)}{920\times3.34\times10^5}m/s = 0.64\times10^{-5}\,m/s = 0.023\,m/hr$$

The power required to tow the iceberg is

$$P = F_D \times V = \bar{\tau}_{s,L} A_S V \quad , \qquad F_D = Drag\ force$$

$$\bar{\tau}_{s,L} = \bar{C}_{f,L}\left(\frac{\rho V^2}{2}\right), \ \bar{C}_{f,L} = 0.074\,\mathrm{Re}_L^{-1/5} = 0.074\left(1.85\times10^{8}\right)^{-1/5}$$

$$\bar{C}_{f,L} = 1.64\times10^{-3}$$

Therefore

$$P = \bar{C}_{f,L} A_S\left(\frac{\rho V^2}{2}\right) = 1.64\times10^{-3}\times5\times10^{5}\,m^2\times\left(1000\frac{Kg}{m^3}\times\frac{1000}{3600}\frac{m}{s}\right)^2\times\frac{1}{2}$$

$$P = 8788\,Kg\,\frac{m^2}{s^2} = 8.8\,KW$$

The minimum cost of fresh water is determined by the transportation cost, C_t , and the volume of the water arriving at the destination, V_s. Hence, the minimum cost $= C_t / V_s$

$$\frac{C_t}{V_s} = \frac{P \times (Towing\,Time) \times (Towing\,Rate)}{\left(D - \frac{dD}{dt} \times Towing\,Time\right) A_S} = \frac{8.8\,KW \left(\frac{6000\,Km}{1Km/hr}\right) \$1/KWhr}{\left(250 - 0.023\frac{m}{hr} \times \frac{6000\,Km}{1Km/hr}\right) 5\times10^5 m^2}$$

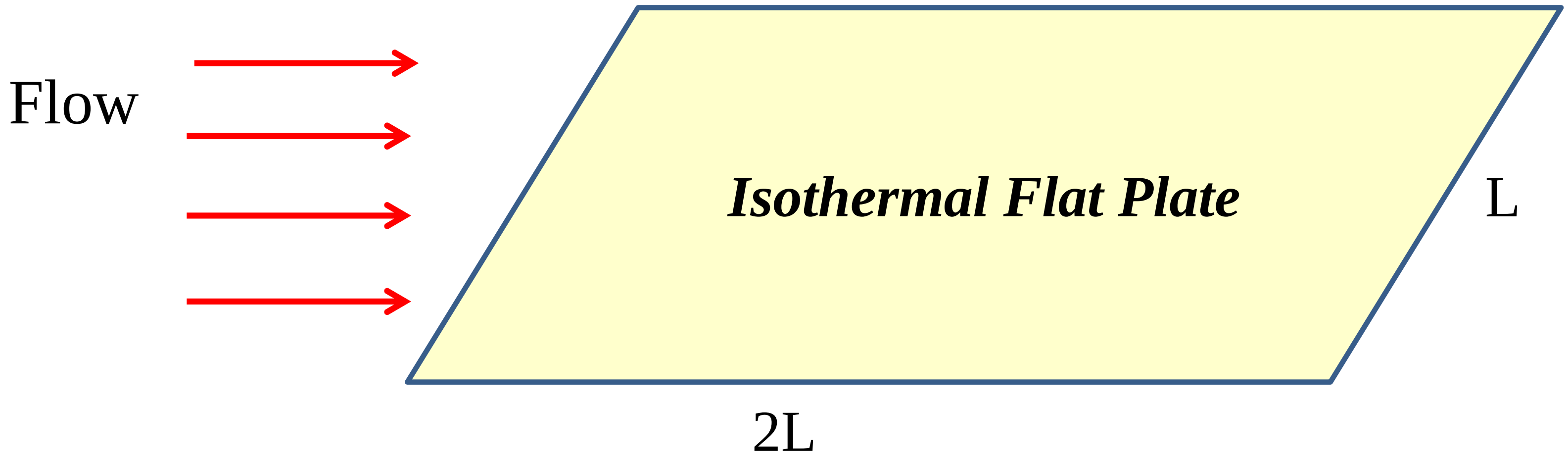
$$\frac{C_t}{V_s} = 0.0009\,\$/m^3$$

Only a rough estimate, as the effects of top an side surfaces are not considered.

Heat transfer coefficient during flow over a flat plate

Explain under what conditions the total rate of heat transfer from an isothermal flat plate of dimensions L by 2L would be the same, independent of whether parallel flow over the plate is directed along the side of length L or 2L.

With a critical Reynold's number of $5x10^5$, for what values of Re L would the total heat transfer be independent of orientation? Consider all possibilities and comment whether your result depends on the nature of the fluid.

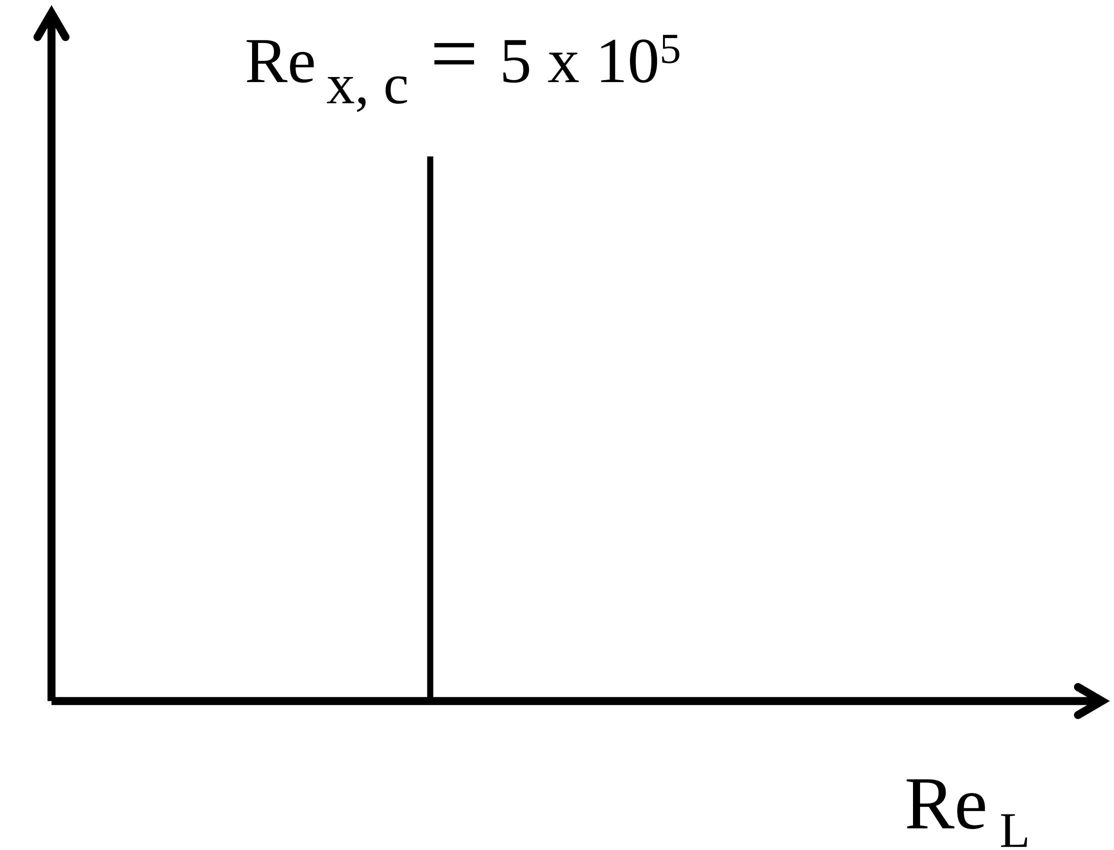


For Laminar Flow

$$\overline{h_L} \propto L^{-\frac{1}{2}} \quad Re_L < Re_{x,c}$$

For Mixed (Laminar & Turbulent Flow

$$\overline{h_L} = C_1 L^{-1/5} - C_2 L^{-1/2}$$
$$Re_L > Re_{x,c}$$



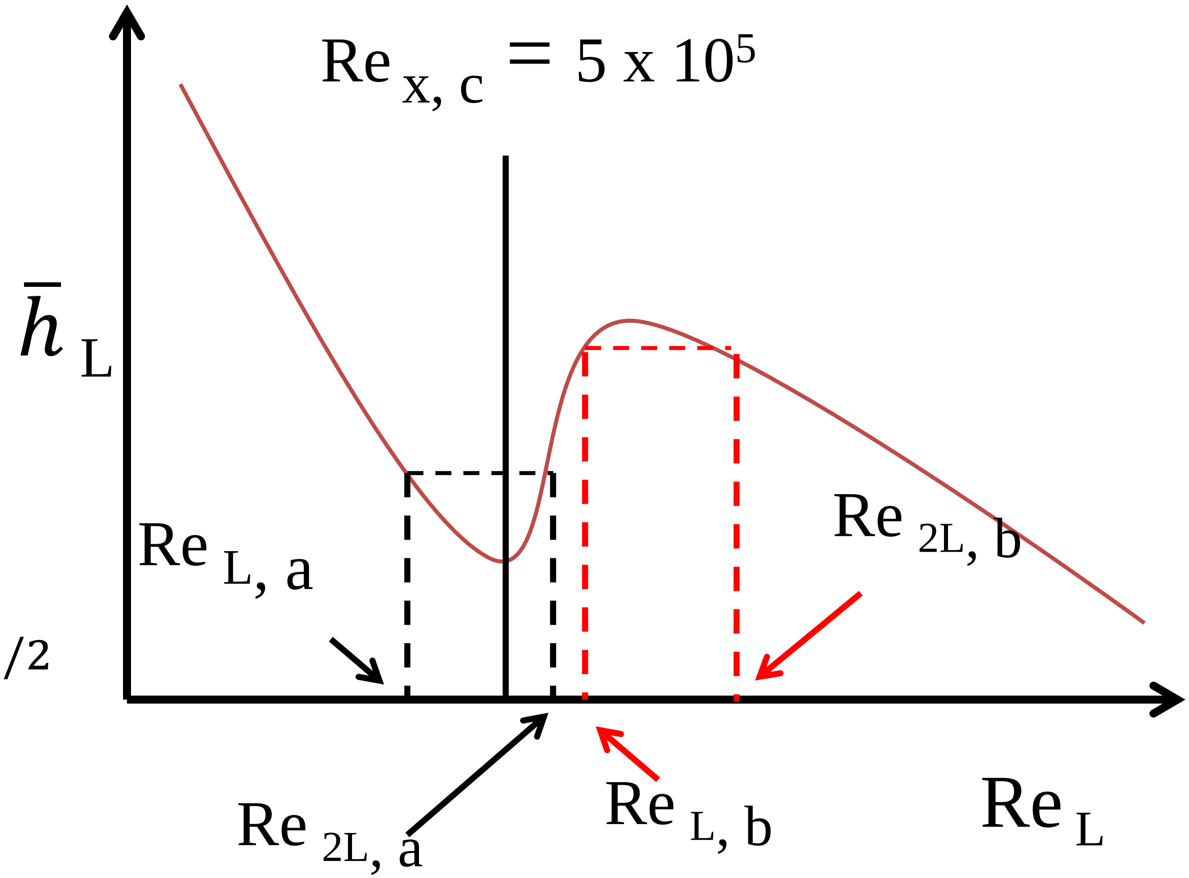
Schematic (not to scale) of the variation of heat transfer coefficient with Reynold’s No.

For Laminar Flow

$$\overline{h_L} \propto L^{-\frac{1}{2}} \; Re_L < Re_{x,c}$$

For Mixed (Laminar & Turbulent Flow

$$\overline{h_L} = C_1 L^{-1/5} - C_2 L^{-1/2}$$
$$Re_L > Re_{x,c}$$



Schematic (not to scale) of the variation of heat transfer coefficient with Reynold's No.

Two Possibilities

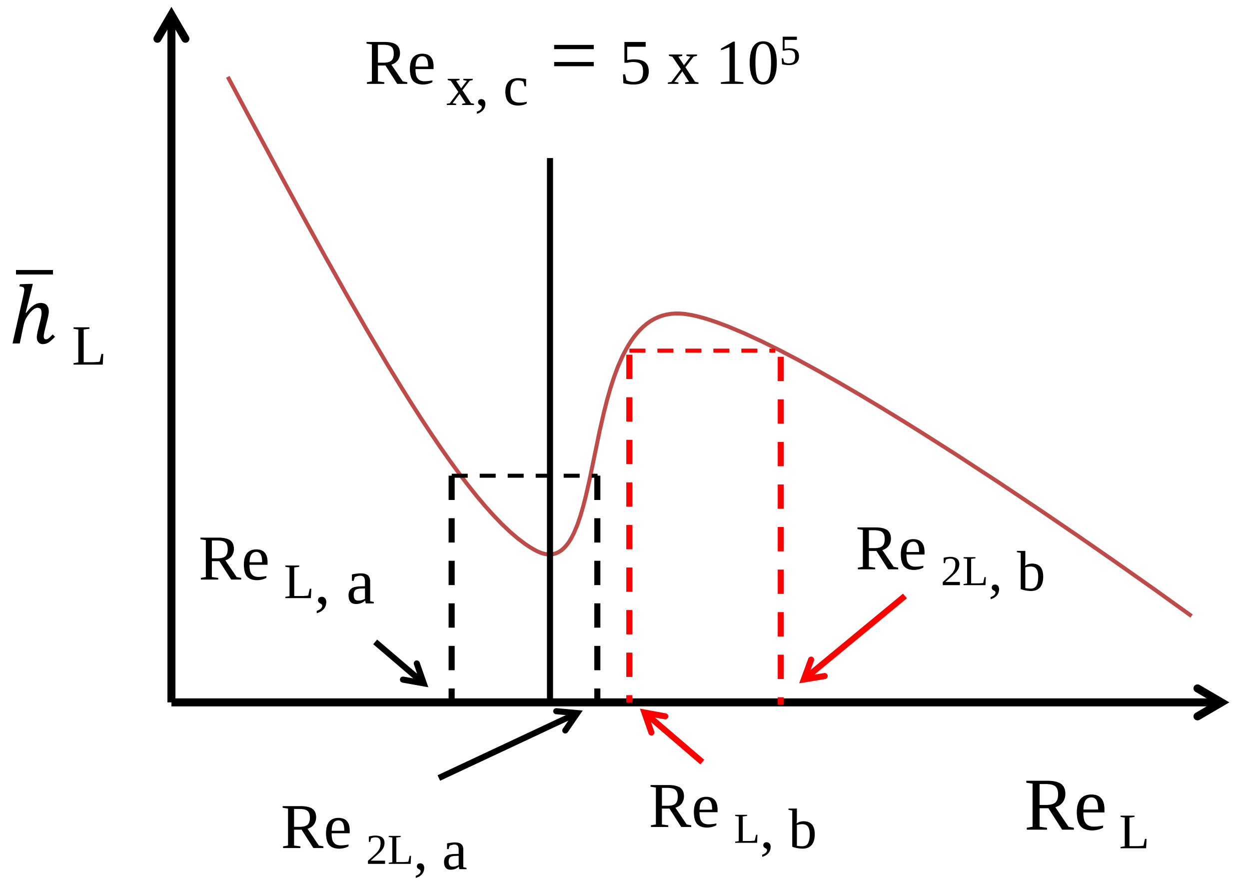
1. Laminar flow on the shorter plate, mixed flow on the larger plate
2. Mixed boundary layer on both plates

In both cases it is required that

$$\overline{h_L} = \overline{h_{2L}}$$

And

$$Re_{2L} = 2\,Re_L$$



Schematic (not to scale) of the variation of heat transfer coefficient with Reynold's No.

Case I

From expressions for $\overline{h_L}$ in laminar and mixed flow

$$0.664\ \frac{k}{L}\ {Re_L}^{1/2}\ Pr^{1/3} = \frac{k}{2L}\left(0.037\ {Re_{2L}}^{4/5} - 871\right)Pr^{1/3}$$

$$0.664\ {Re_L}^{1/2} = \left(0.032\ {Re_L}^{4/5} - 435\right)$$

Since $Re_L < 5 \times 10^5$ and $re_{2L} = 2\ Re_L$ the required value of Re_L may be narrowed to the range

$2.5 \times 10^5 < Re_L < 5 \times 10^5$

From a trial and error solution,

$\boldsymbol{Re_L = 3.2 \times 10^5}$

Case II For mixed flow on both plates

$$\frac{k}{L}\left(0.037\, Re_L{}^{4/5} - 871\right)Pr^{1/3} = \frac{k}{2L}\left(0.037\, Re_{2L}{}^{4/5} - 871\right)Pr^{1/3}$$

$$\left(0.037\, Re_L{}^{4/5} - 871\right) = \left(0.032\, Re_L{}^{4/5} - 871\right)$$

$$\boxed{\mathbf{Re_L \approx 1.5 \times 10^6}}$$

Observations

1. It is impossible to satisfy the requirement that $\overline{h_L} = \overline{h_{2L}}$ if $Re_L < 0.25 \times 10^5$ (Laminar flow on both plates)

2. The results are independent of the nature of the fluid

Problem on Analogy

It's a sunny winter day. In your haste to get to the shopping mall you accidentally left your heat sensitive medicine in the car with its interior dimensions as L = 2 m, Area = 2.5 m^2, Volume = 4 m^3. Solar radiation provides a heat flux of 116.5 W/m^2 through the sun-roof and windows (Area = 2.5 m^2). A breeze (v_∞ = 5 m/s, T_∞ = 25 °C) blows over the car. From your Transport Phenomena course you know that the friction factor for flow over the car obeys the following relation -

$$\frac{\overline{c_f}}{2} = 0.05\,\mathrm{Re}_l^{-0.35}$$

Assuming only air inside the car (V_{air} = 0; T(t = 0) = 25 °C), the fact that the medicine can withstand a temperature of 55 °C, and that all the net energy from the sun gets transferred to the air inside the greenhouse, and that the temperature variation of air inside the car can be neglected:

a) What is the heat transfer coefficient?

b) How fast does the temperature initially rise or fall in the greenhouse?

c) Will the medicine become ineffective before you have finished your shopping? Why?

Given: $\gamma = 15.7\text{x}10^{-6}$ m^2/s, Pr ≈ 1, k_f = 0.026 W/m K, ρ =1.18 kg/m^3, C_p=1005 J/kg K

(a) The heat transfer coefficient can be found from analogy.

As Pr ≈ 1, Reynold's analogy between heat and momentum transfer is applicable

$$\overline{Nu} = C_f \frac{\mathrm{Re}_L}{2} = 0.05\,\mathrm{Re}_L^{-0.35}\,\mathrm{Re}_L = 0.05\,\mathrm{Re}_L^{0.65} \quad \text{as} \quad \frac{\overline{c_f}}{2} = 0.05\mathrm{Re}_l^{-0.35}$$

$$\mathrm{Re}_L = \frac{2m \times 5\,m/s}{15.6 \times 10^{-6}\,m^2/s} = 6.4 \times 10^5$$

$$Nu = \frac{\bar{h}\,L}{k} = 0.05 \times \left(6.4 \times 10^5\right)^{0.65} = 297.5$$

$$\bar{h} = 3.9\,W/(m\,K)$$

(b) The temperature in the car is mentioned as uniform.

Therefore, lumped capacitance model is valid

IN - OUT + GEN = ACCUMULATION

$$q^{//} A - \bar{h} A\left(T - T_{\infty}\right) + 0 = \rho C_p V \frac{dT}{dt}$$

$$Let \quad \theta = T - T_{\infty}$$

$$\boxed{q^{//} A - \bar{h} A\theta = \rho C_p V \frac{d\theta}{dt}}$$

Governing Equation

$$At\ t = 0, T = T_{\infty} \quad \Rightarrow \quad \frac{d\theta}{dt} = \frac{q_A^{//} A}{\rho C_p V} = \frac{116.5 \times 2.5}{1.18 \times 4 \times 1005} = 0.06\, C/s$$

(c)
$$\frac{d\theta}{q_A^{//} A - \bar{h} A \theta} = \frac{dt}{\rho C_p V}$$

$$-\ln\left(q_A^{//} A - \bar{h} A \theta\right) = \frac{\bar{h} A}{\rho C_p V} t + C_1$$

At t = 0, θ = 0 ⇨ $C_1 = -\ln\left(q_A^{//}\right)$

$$t = \frac{\rho C_p V}{\bar{h} A} \ln\left[\frac{q_A^{//}}{q_A^{//} - \bar{h} A \theta}\right] \quad ; \quad \theta = T - T_\infty$$

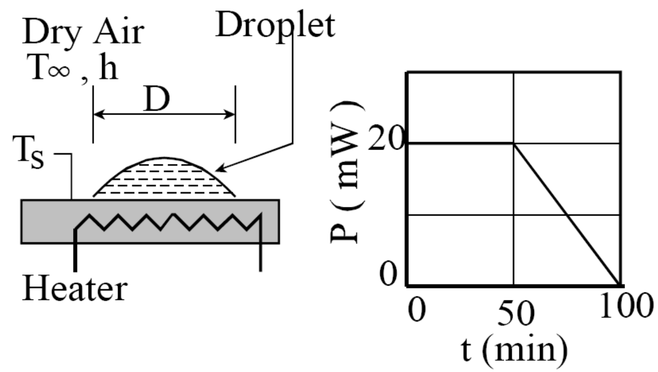
For T = 55 C, from the above equation, **t = 2782 s ≈ 46 mins**

Therefore the shopping needs to be completed within 46 minutes

Problem on Analogy

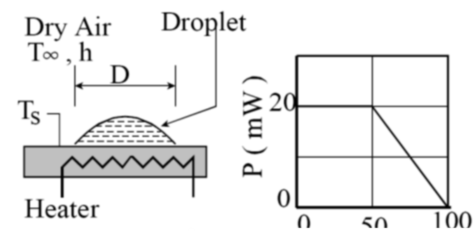
An experiment is conducted to determine the average mass transfer coefficient of a small droplet using a heater controlled to operate at a constant temperature. The power history required to completely evaporate the droplet at a temperature of 37°C is shown in the figure. It was observed that, as the droplet dried, its wetted diameter on the heater surface remained nearly constant at a value of 4 mm.

(a) Calculate the average mass transfer convection coefficient based on the wetted area during the evaporating process when the droplet, heater, and the dry ambient air are at 37 °C.



(b) How much energy will be required to evaporate the droplet if the dry ambient air temperature is 27 °C, while the droplet-heater temperature remains at 37 C?

Properties: Saturated water (37 °C): h_{fg} = 2414 kJ/kg, $\rho_{A,\,sat} = 1/v_g = 1/22.93 =$ 0.04361 kg/m^3, Air-water vapor (Ts = 37 °C, 1 atm): $D_{AB} = 0.276\times10^{-6}$ m^2 /s Air (T_{avg} = (27+37)/2=305K, 1 atm): ρ = 1.1448 kg/m^3, c_p = 1008 J/kg.K, ν = 16.39x10^{-6} m^2/s, Pr = 0.706



For the isothermal conditions (37°C), the electrical energy Q required to evaporate the droplet during the interval of time $\Delta t = t_e$ follows from the area under the P-t curve.

$$Q = \int_0^{t_e} P\,dt = \left[20\times10^{-3}\,W \times (50\times60)\,s + 0.5\times 20\times10^{-3}\,W\,(100-50)\times 60\,s\right]$$

$$Q = 90\,J$$

From an overall energy balance, the mass of the droplet is

$$Q = M h_{fg} \longrightarrow M = Q/h_{fg} = 90\,J / 2414\times10^{3}\,J/kg = 3.728\times10^{-5}\,kg$$

$$M = \dot{m}\,t_e = \bar{h}_m A_s(\rho_{As} - \rho_{A\infty})\,t_e = \bar{h}_m A_s \rho_{As}(1-\phi_\infty)\,t_e$$

$$\phi_\infty = 0$$

$$3.728\times10^{-5}\,kg = \bar{h}_m\left[\frac{\pi(0.004)^2}{4}\,m^2 \cdot 0.04361\,\frac{kg}{m^3} \times 100\times60\,s\right]$$

$$\bar{h}_m = 0.0113\,m/s.$$

b) From an overall energy balance, the energy reqd. is

$$Q = m h_{fg} + \bar{h} A_s (T_s - T_\infty)$$

$$\frac{\bar{h}}{\bar{h}_m} = \frac{k}{D_{AB} Le^n} = \rho C_p Le^{2/3}.$$

$$Sc = \frac{\nu}{D_{AB}} = \frac{16.39 \times 10^{-6} \, m^2/s}{0.276 \times 10^{-4} \, m^2/s} = 0.594, \quad Le = \frac{Sc}{Pr} = 0.841$$

$$\therefore \bar{h} = 0.0113 \frac{m}{s} \times 1.1448 \frac{kg}{m^3} \times 1008 \frac{J}{kg\,K} \times (0.841)^{2/3} = 11.62 \, W/m^2 K$$

$$\therefore Q = 3.728 \times 10^{-5} \, kg \times 2414 \, kJ/kg + 11.62 \frac{W}{m^2 K} \left(\pi \frac{(0.004m)^2}{4} \right) (37 - 27)^\circ C$$

$$Q = (90.00 + 0.00145) J = \underline{\underline{90 J}}$$

Convection heat loss is quite small compared to the evaporation heat requirement.