Flow of gas through microchannel

NPTEL lec 25(from 31:10)

Knudsen No. $Kn = \frac{\lambda}{L}$

λ_{N_2} at 1 bar $= 70\ nm$ $\Rightarrow$ $Kn > 10^{-1}$ can be rarely obtained in a channel of μm characteristic dimension.

When

$Kn \leq 10^{-2}$	Continuum flow with no slip boundary condition
$10^{-2} < Kn \leq 10^{-1}$	" " " <u>slip</u> " "
$10^{-1} < Kn \leq 10$	Transition flow (Burnett equation)
$Kn > 10$	Free molecular flow

<u>Slip boundary condition</u>

(*) The sublayer at the interface, few mean free paths thick is referred as Knudsen layer.

(*) When Kn is small (< 0.1), the ~~continuum~~ average velocity and temperature of gas molecules near the wall is continuous and equal to the velocity and temperature of wall. The Knudsen layer is insignificant and the continuum solution for bulk flow can be extrapolated to the surface.

(*) When Kn is large, collision frequency is low. equilibrium between the wall and near-wall cannot be established. In that case, the tangential velocity, one mean free path away from the wall

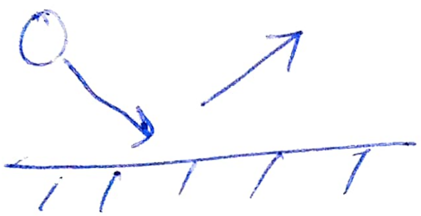
$$= \boxed{u_{wall} + \lambda \left.\frac{du}{dy}\right|_{wall}} = \text{Slip velocity under 1}^{st}\text{ order slip flow condition}$$

(Taylor series expansion ~~with~~ and ignoring higher order terms)

Additional considerations in slip flow

Interactions between gas and solid molecules.

Tangential momentum accommodation coefficient



$$\sigma_v = \frac{\tau_i - \tau_r}{\tau_i - \tau_w}$$

τ_i = tangential momentum of incoming molecules

τ_r = " " " reflected "

τ_w = " " " the wall = 0

$\Rightarrow$ A reduction in velocity after collision

$\Downarrow$

Expression by Maxwell for an isothermal wall

$$\boxed{u_{gas} - u_{wall} = \frac{2-\sigma_v}{\sigma_v}\lambda\frac{\partial u}{\partial y}}$$

$\Longleftarrow$

For mono atomic gases

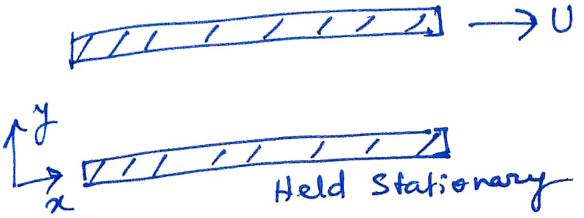
$$u_{gas} - u_{wall} = \frac{2-\sigma_v}{\sigma_v}\frac{\mu\sqrt{\pi}}{\rho\sqrt{2RT_w}}\frac{\partial u}{\partial y}$$

T_w is the wall temperature

A slip length L_s can be defined such that the slip velocity

$$= u_w + L_s\frac{du}{dy}$$

First order slip for flow between two infinite parallel plates



Steady, fully developed flow

(*) Translation invariance of the set-up along x and z direction.

$\Rightarrow$ velocity field can only depend on y

(*) Driving force points to x-direction.

$\Rightarrow$ only x-component of velocity is non-zero.

$$\Rightarrow \mu \frac{\partial^2 u}{\partial y^2} = 0$$

$$\Rightarrow u = c_1 y + c_2$$

For NO slip boundary condition

$$\frac{u}{U} = \frac{y}{h}$$

or $\bar{u} = \bar{y}$

For SLIP Boundary Condition

the governing equation remains same

B.C.s on the lower wall $(y=0, \bar{y}=0)$, $\bar{U}_W = 0$, and $\frac{\partial \bar{u}_s}{\partial n} = \frac{\partial \bar{u}_s}{\partial \bar{y}}$

On the upper wall,

n is the unit normal vector

$\equiv \frac{y}{h}$ for lower wall

and $= -\frac{y}{h}$ for upper wall

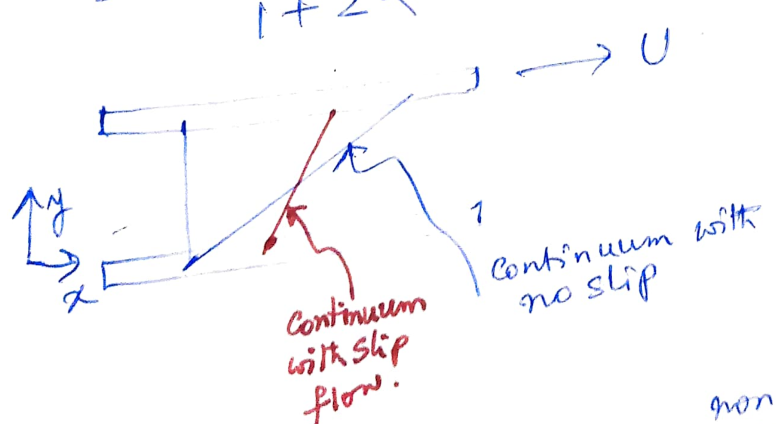
$$\bar{U}_W = 1$$

$$\frac{\partial \bar{u}_s}{\partial n} = -\frac{\partial \bar{u}_s}{\partial \bar{y}}$$

If $\alpha = \frac{2-\sigma_v}{\sigma_v} Kn$ where $Kn = \frac{\lambda}{h}$

Application of boundary condition will lead to

$$\left.\begin{aligned} c_1 &= \frac{1}{1+2\alpha} \\ c_2 &= \frac{\alpha}{1+2\alpha} \end{aligned}\right\} \quad \bar{u} = \frac{1}{1+2\alpha}\bar{y} + \frac{\alpha}{1+2\alpha}$$



The non-dimensional volume flow rate per unit depth in the z-direction ⊥ to paper

$$Q = \int_0^1 \bar{u}\, d\bar{y} = 0.5$$

This value is the same as the non-dimensional volume flow rate per depth for continuum flow solution with no-slip boundary condition.