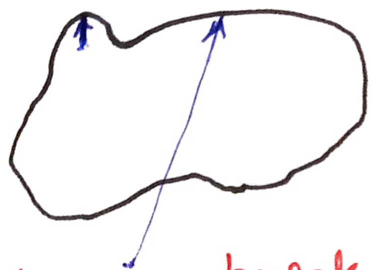
Droplet, squeezed away from spherical shape will have regions of higher and lower curvature



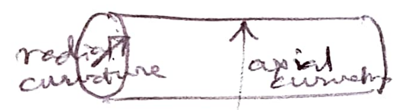
Resulting imbalance of Laplace pressure drives the liquid to a perfectly symmetric sphere.

Spontaneous break-up of liquid string into droplets.
(or gas string into bubbles)

Cylinder of gas or liquid surrounded by another immiscible fluid

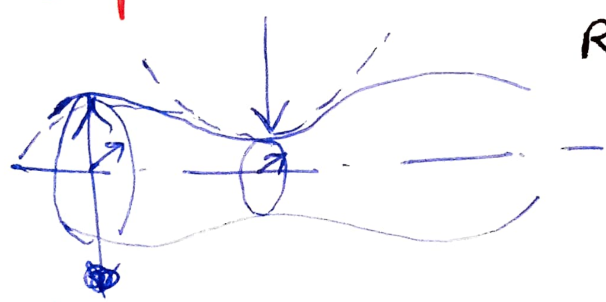
Principal radii of curvature

Axial Curvature = 0



Radial curvature defined by the radius of the cylinder

Small deformation of cylindrical shape due to thermal fluctuations.
(Sinusoidal modulations)



Radial Curvature (At the peaks) < Radial Curvature (At the trough)

Radial curvature (and corresponding Principal radius) dominates for large wavelength

$\Rightarrow$ (Laplace pressure)$_{peak}$ < (Laplace Pressure)$_{trough}$

$\Rightarrow$ Fluid driven from trough to peak
$\Rightarrow$ perturbation gets amplified.

For short wavelength, axial curvature dominates.

Axial curvature at trough is negative

(Laplace Pressure)$_{peak}$ > (Laplace Pressure)$_{trough}$

$\Rightarrow$ Fluid driven from peak to trough $\Rightarrow$ Decay of perturbation

Rayleigh Instability

Spontaneous break-up into droplets / bubbles

Perturbations with large wavelength gets amplified

Perturbation with short wavelength gets decayed.

Critical wavelength below which the fluctuations decay

$\Rightarrow$ defines the size of droplet / bubble.

Critical wavelength = circumference of the jet for a stream of inviscid liquid falling through air.

The wavelength that provides fastest growth of perturbation

$\approx$ 1.43 times the circumference

$\approx$ Characteristic diameter of droplets.

When the immiscible fluid surrounding the gas or liquid string is inside a channel, viscous effect on the surrounding continuous fluid during movement due to collapsing ~~trough~~ trough slows down the break-up

Dimensionless Number

$$\text{Bond No.} = \frac{\rho g L^2}{\sigma}$$

Bond number is used to characterise the shape of bubbles moving in a surrounding fluid

Compares gravitational force acting on the fluid with interfacial force $= \frac{(\rho L^3) g}{\left(\frac{\sigma}{L}\right) L^2}$

Bo No. << 1
$\Rightarrow$ almost spherical droplet with L as diameter

Bo No. >> 1
$\Rightarrow$ flat puddle of liquid.

$$\text{Capillary No.} = \frac{\mu v}{\sigma}$$

at low capillary number, the surface force dominates over viscous forces. So, the bubble remains stable and resist deformation.

or alternatively $\frac{\mu u d}{4 \sigma L}$

Compares viscous forces with the interfacial forces.

viscous stress $\equiv \tau \equiv \mu \frac{v}{L}$

Interfacial Stress $= \frac{\text{Force}}{\text{Area}} = \frac{\sigma L}{L^2}$

Ratio $= \frac{\mu V}{\sigma}$

For channel dimension (L), and droplet dimension (d), the ratio becomes $\frac{\mu V/L}{\left(\frac{\sigma \pi d}{\pi d^2/4}\right)} = \frac{\mu V d}{4 \sigma L}$

$$\text{Weber No.} = \frac{\rho u^2 L}{\sigma}$$

Weber number indicates whether kinetic or surface tension energy is dominant.

compares inertial and interfacial stresses

$$= \frac{(\text{mass})(\text{acceleration})}{\text{interfacial force}}$$

$$= \frac{(\rho L^3)(u^2/L)}{\sigma L}$$

Significance of Capillary Number

- (*) In shear flow, large droplets are elongated, and undergo splitting into droplets due to Rayleigh Plateau instability.
- (*) The fragmentation continues until the radius of the droplet is small enough such that the laplace pressure balances the shear stress (Ca No. $\approx$ 1).
- (*) The order of magnitude of the droplet radius can be approximated by setting Ca No. = 1 $\Rightarrow$ $d \approx 4L\left(\frac{\gamma}{\mu u}\right)$

Significance of Weber Number

- (*) A large value implies formation of elongated jet that breaks up into droplets far away from the nozzle
- (*) A small value implies dripping of droplets without formation of pronounced jet.