TRANSPORT PHENOMENA

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TRANSPORT PHENOMENA

Basic Overview

Over the last few decades, the subject has revolutionized the way Chemical Engineering science is taught. This course deals with the unified treatment of the different transport processes, ubiquitous in industry as well as in nature. Momentum, heat and mass transfer are taught together due to the underlying similarities of the mathematics and molecular mechanisms describing such processes.

Books

1. **Transport Phenomena by Bird, Stewart and Lightfoot (Wiley)**
2. Transport Phenomena Fundamentals by J. L. Plawsky (CRC Press)
3. Heat and Mass Transfer – A Transport Phenomena Approach by K. S. Gandhi (New Age)
4. **Fluid Mechanics by R. W. Fox, P. J. Pritchard and A. T. McDonald (Wiley)**
5. **Introduction to Heat and Mass Transfer by F. P. Incropera & D. P. DeWitt (Wiley)**

Grading: 2-3 class tests for TA, relative grading

Course Plan

Formulation and solution of momentum transfer in laminar flow.

Navier-Stokes equation and its applications

Boundary Layer concepts, boundary layer thicknesses (disturbance, displacement and momentum), Blasius solution for flow over a flat plate

Use of momentum integral equation, turbulent boundary layers, fluid flow about immersed bodies, drag

Formulation and solution of heat transfer in laminar flow

Development and use of energy equation

Transient conduction - lumped capacitance, analytical solutions and other methods.

Formulation and solution of mass transfer in laminar flow. Development and use of species balance equation

Introduction to convective flow, natural convection, relevant examples from heat and mass transfer

Mathematical treatment of the similarities between heat, mass and momentum transfer, similarity parameters, and relevant analogies.

Solution of coupled heat, mass and momentum transfer problems based on analogy.

Phenomenological Relations

Newton's law, Fourier Law, Fick's Law

Governing Equations

NS Equation, Energy Equation, Species Balance Equation

Similarity Parameters

Re Pr Sc

Engineering Parameters and Analogy

f Nu Sh

f = f (Re) **Nu=f (???)** **Sh = f (???)**

TRANSPORT PHENOMENA

Application of Navier Stokes Equations

Continuity Equation

$$\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z} = 0$$

Equation of Motion – Navier Stokes equation

$$\rho D\bar{v} / Dt = -\nabla p - [\nabla . \bar{\tau}] + \rho g$$

Cartesian Coordinate – z component

$$\rho \left(\frac{\partial\ v_z}{\partial t} + v_x \frac{\partial\ v_z}{\partial x} + v_y \frac{\partial\ v_z}{\partial y} + v_z \frac{\partial\ v_z}{\partial z} \right) = -\frac{\partial\ P}{\partial z} + \mu \left[\frac{\partial^2\ v_z}{\partial x^2} + \frac{\partial^2\ v_z}{\partial y^2} + \frac{\partial^2\ v_z}{\partial z^2} \right] + \rho\ g_z$$

Relevant Boundary Conditions

No slip at the liquid-solid interface

No shear at the liquid-vapor interface

The Equation of Continuity

$$[\partial\rho/\partial t + (\nabla \cdot \rho\mathbf{v}) = 0]$$

Cartesian coordinates (x, y, z):	
$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x}(\rho v_x) + \frac{\partial}{\partial y}(\rho v_y) + \frac{\partial}{\partial z}(\rho v_z) = 0$$	(B.4-1)
Cylindrical coordinates (r, θ, z):	
$$\frac{\partial \rho}{\partial t} + \frac{1}{r}\frac{\partial}{\partial r}(\rho r v_r) + \frac{1}{r}\frac{\partial}{\partial \theta}(\rho v_\theta) + \frac{\partial}{\partial z}(\rho v_z) = 0$$	(B.4-2)
Spherical coordinates (r, θ, ϕ):	
$$\frac{\partial \rho}{\partial t} + \frac{1}{r^2}\frac{\partial}{\partial r}(\rho r^2 v_r) + \frac{1}{r \sin\theta}\frac{\partial}{\partial \theta}(\rho v_\theta \sin\theta) + \frac{1}{r \sin\theta}\frac{\partial}{\partial \phi}(\rho v_\phi) = 0$$	(B.4-3)

[a] When the fluid is assumed to have constant mass density ρ, the equation simplifies to $(\nabla \cdot \mathbf{v}) = 0$.

The Equation of Motion for a Newtonian Fluid with constant μ, ρ

The Navier Stokes Equation

Cartesian coordinates (x, y, z):

$$\rho\left(\frac{\partial v_x}{\partial t} + v_x \frac{\partial v_x}{\partial x} + v_y \frac{\partial v_x}{\partial y} + v_z \frac{\partial v_x}{\partial z}\right) = -\frac{\partial p}{\partial x} + \mu\left[\frac{\partial^2 v_x}{\partial x^2} + \frac{\partial^2 v_x}{\partial y^2} + \frac{\partial^2 v_x}{\partial z^2}\right] + \rho g_x$$

$$\rho\left(\frac{\partial v_y}{\partial t} + v_x \frac{\partial v_y}{\partial x} + v_y \frac{\partial v_y}{\partial y} + v_z \frac{\partial v_y}{\partial z}\right) = -\frac{\partial p}{\partial y} + \mu\left[\frac{\partial^2 v_y}{\partial x^2} + \frac{\partial^2 v_y}{\partial y^2} + \frac{\partial^2 v_y}{\partial z^2}\right] + \rho g_y$$

$$\rho\left(\frac{\partial v_z}{\partial t} + v_x \frac{\partial v_z}{\partial x} + v_y \frac{\partial v_z}{\partial y} + v_z \frac{\partial v_z}{\partial z}\right) = -\frac{\partial p}{\partial z} + \mu\left[\frac{\partial^2 v_z}{\partial x^2} + \frac{\partial^2 v_z}{\partial y^2} + \frac{\partial^2 v_z}{\partial z^2}\right] + \rho g_z$$

Cylindrical coordinates (r, θ, z):

$$\rho\left(\frac{\partial v_r}{\partial t} + v_r \frac{\partial v_r}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_r}{\partial \theta} + v_z \frac{\partial v_r}{\partial z} - \frac{v_\theta^2}{r}\right) = -\frac{\partial p}{\partial r} + \mu\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(r v_r)\right) + \frac{1}{r^2}\frac{\partial^2 v_r}{\partial \theta^2} + \frac{\partial^2 v_r}{\partial z^2} - \frac{2}{r^2}\frac{\partial v_\theta}{\partial \theta}\right] + \rho g_r$$

$$\rho\left(\frac{\partial v_\theta}{\partial t} + v_r \frac{\partial v_\theta}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_\theta}{\partial \theta} + v_z \frac{\partial v_\theta}{\partial z} + \frac{v_r v_\theta}{r}\right) = -\frac{1}{r}\frac{\partial p}{\partial \theta} + \mu\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(r v_\theta)\right) + \frac{1}{r^2}\frac{\partial^2 v_\theta}{\partial \theta^2} + \frac{\partial^2 v_\theta}{\partial z^2} + \frac{2}{r^2}\frac{\partial v_r}{\partial \theta}\right] + \rho g_\theta$$

$$\rho\left(\frac{\partial v_z}{\partial t} + v_r \frac{\partial v_z}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_z}{\partial \theta} + v_z \frac{\partial v_z}{\partial z}\right) = -\frac{\partial p}{\partial z} + \mu\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r \frac{\partial v_z}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 v_z}{\partial \theta^2} + \frac{\partial^2 v_z}{\partial z^2}\right] + \rho g_z$$

Spherical coordinates (r, θ, ϕ):

$$\rho\left(\frac{\partial v_r}{\partial t} + v_r \frac{\partial v_r}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_r}{\partial \theta} + \frac{v_\phi}{r \sin\theta}\frac{\partial v_r}{\partial \phi} - \frac{v_\theta^2 + v_\phi^2}{r}\right) = -\frac{\partial p}{\partial r}$$
$$+ \mu\left[\frac{1}{r^2}\frac{\partial^2}{\partial r^2}(r^2 v_r) + \frac{1}{r^2 \sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta \frac{\partial v_r}{\partial \theta}\right) + \frac{1}{r^2 \sin^2\theta}\frac{\partial^2 v_r}{\partial \phi^2}\right] + \rho g_r$$

$$\rho\left(\frac{\partial v_\theta}{\partial t} + v_r \frac{\partial v_\theta}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_\theta}{\partial \theta} + \frac{v_\phi}{r \sin\theta}\frac{\partial v_\theta}{\partial \phi} + \frac{v_r v_\theta - v_\phi^2 \cot\theta}{r}\right) = -\frac{1}{r}\frac{\partial p}{\partial \theta}$$
$$+ \mu\left[\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial v_\theta}{\partial r}\right) + \frac{1}{r^2}\frac{\partial}{\partial \theta}\left(\frac{1}{\sin\theta}\frac{\partial}{\partial \theta}(v_\theta \sin\theta)\right) + \frac{1}{r^2 \sin^2\theta}\frac{\partial^2 v_\theta}{\partial \phi^2} + \frac{2}{r^2}\frac{\partial v_r}{\partial \theta} - \frac{2 \cot\theta}{r^2 \sin\theta}\frac{\partial v_\phi}{\partial \phi}\right] + \rho g_\theta$$

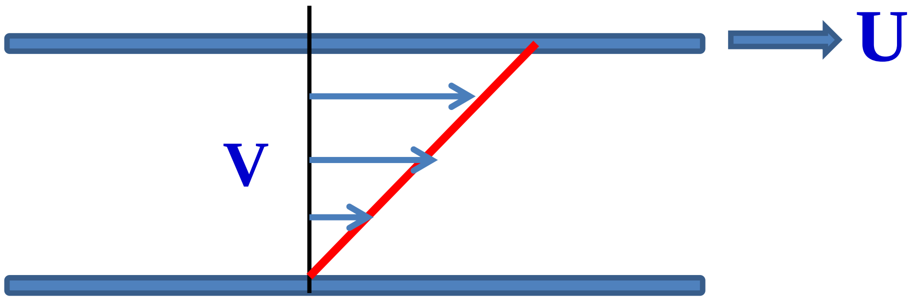
$$\rho\left(\frac{\partial v_\phi}{\partial t} + v_r \frac{\partial v_\phi}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_\phi}{\partial \theta} + \frac{v_\phi}{r \sin\theta}\frac{\partial v_\phi}{\partial \phi} + \frac{v_\phi v_r + v_\theta v_\phi \cot\theta}{r}\right) = -\frac{1}{r \sin\theta}\frac{\partial p}{\partial \phi}$$
$$+ \mu\left[\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial v_\phi}{\partial r}\right) + \frac{1}{r^2}\frac{\partial}{\partial \theta}\left(\frac{1}{\sin\theta}\frac{\partial}{\partial \theta}(v_\phi \sin\theta)\right) + \frac{1}{r^2 \sin^2\theta}\frac{\partial^2 v_\phi}{\partial \phi^2} + \frac{2}{r^2 \sin\theta}\frac{\partial v_r}{\partial \phi} + \frac{2 \cot\theta}{r^2 \sin\theta}\frac{\partial v_\theta}{\partial \phi}\right] + \rho g_\theta$$

Transport Phenomena - Momentum Transfer

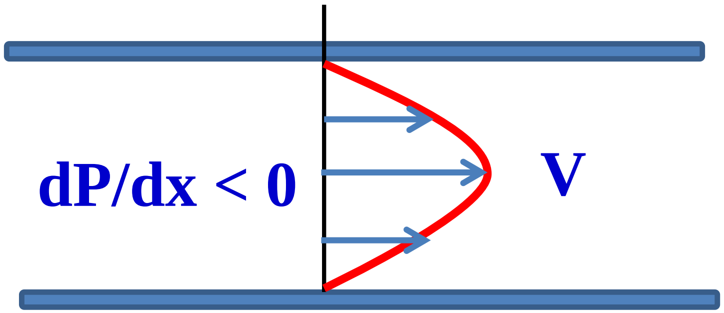
The record-write head for a computer disk storage system floats above the spinning disk on a very thin film of air (the film thickness is 0.5micron). The head location is 150mm from the disk centre line, the disk spins at 3600rpm. The record-write head is 10mm x 10mm square. Determine for standard air (density=1.23kg/m^3, viscosity=1.78x10^{-5} kg/(m.s)) in the gap between the head and the disk -
(a) the Reynold's number of the flow, (b) the viscous shear stress and (c) the power required to overcome viscous shear stress.

For such a small gap the flow can be considered as flow between parallel plates.

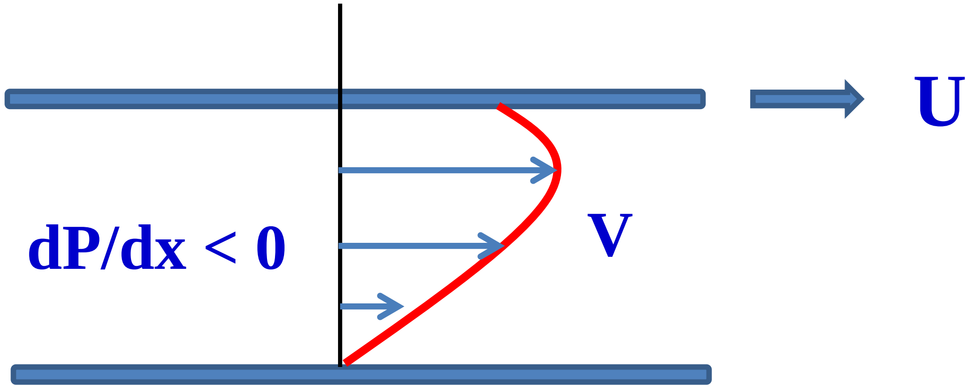
Flow between parallel plates



Couette Flow



Pressure gradient Driven Flow



Couette and Pressure gradient Driven Flow

Solution

Since the gap is too small, the situation can be approximated by the case of flow between two parallel plates with one plate moving and the other stationary with no applied pressure gradient.

Here,

$V = R\omega = 0.15$ m x 3600 rev/min x 2 π rad/rev x 1/60 min/s=56.5 m/s

$Re = (\rho\ V\ a)/\mu = (V\ a)/(\mu/\rho) = 56.5$ m/s x 0.5×10^{-6} m/(1.45 x 10^{-5} m^2 /s) = 1.95 (Laminar Flow)

For Couette flow between two parallel plates, with no applied pressure gradient, the velocity profile will be linear with no-slip boundary conditions at the bottom (stationary) and top (moving) plates.

Therefore, the shear stress, τ, can be expressed as

$\tau = \mu du/dy = \mu V/a$ (linear velocity profile)

$\tau = 1.78 \times 10^{-5}$ kg /(m.s) x 56.5 m/s x 1/(0.5x10 $^{-6}$ m) = 2.01 KN/m^2

Therefore,

Force, $F = \tau A = \tau \times (W \times L)$, Torque, $T = F \times R = \tau \times (WL) \times R$

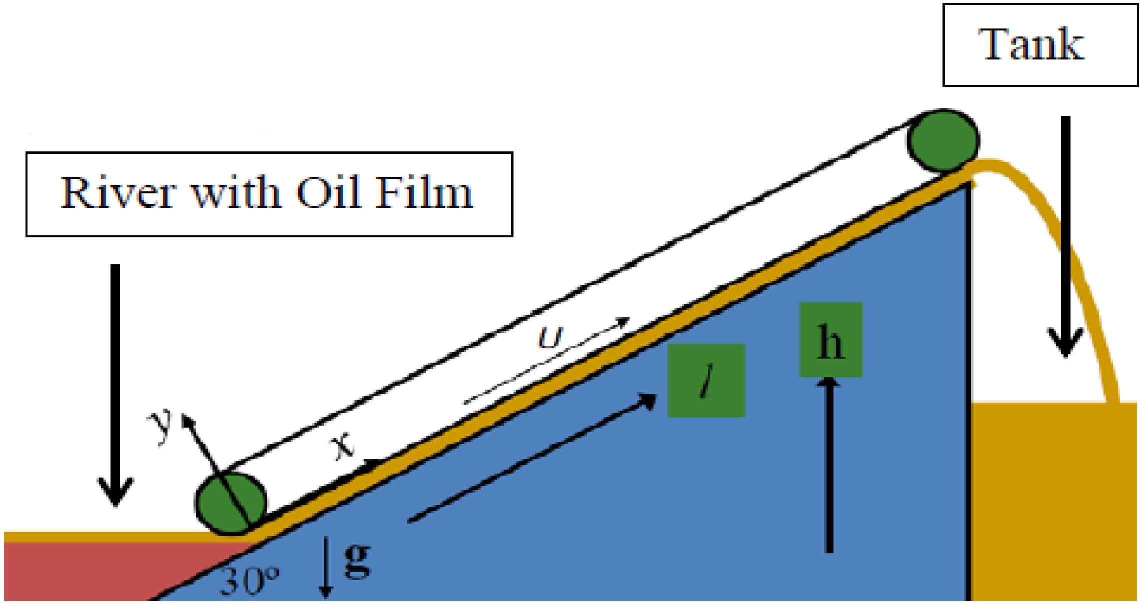
Power dissipation rate. $P = T \times \omega = \tau\, WL\, R\, \omega$

$P = 2.01\times10^{-3} \times 0.01\times 0.01 \times 0.15 \times 3600 \times 2\pi \times 1/60 = 11.4$ W

Thus the power required to overcome viscous shear stress is 11.4 W

Transport Phenomena - Momentum Transfer
Problem 2

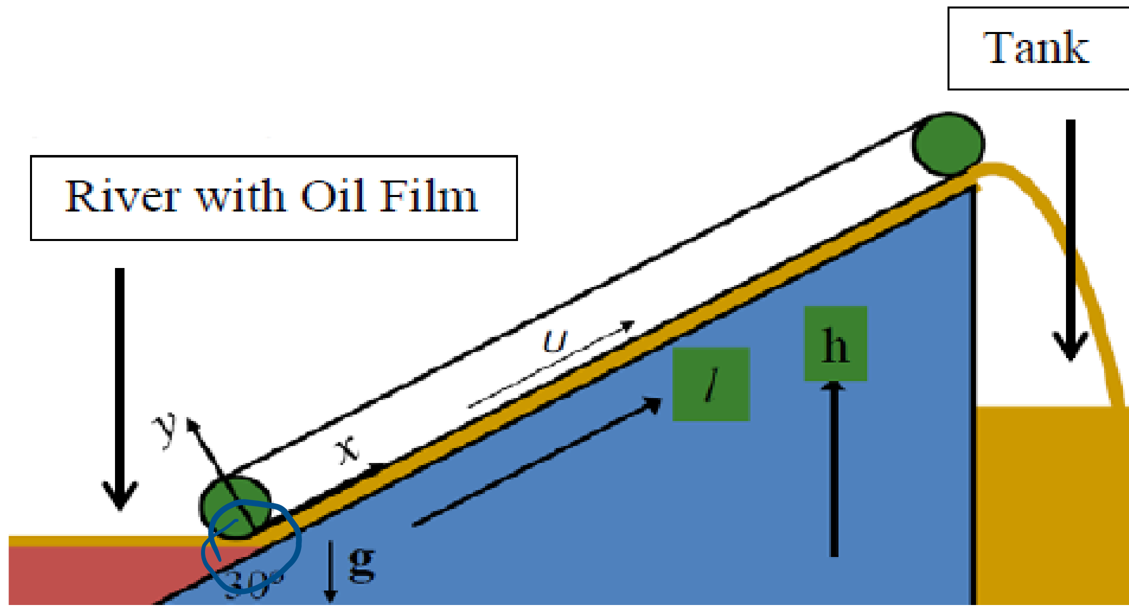
An oil skimmer uses a 5 m wide x 6 m long moving belt above a fixed platform (θ = 30º) to skim oil off of rivers (T = 10ºC). The belt travels at 3 m/s. The distance between the belt and the fixed platform is 2 mm.



The belt discharges into an open tank on the ship. The fluid is actually a mixture of oil and water. To simplify the analysis, assume crude oil dominates. Find the discharge of oil into the tank on the ship, the force acting on the belt and the power required (kW) to move the belt. For oil: $\rho = 860\ kg/m^3$, viscosity, $\mu = 1x10^{-2} N.s/m^2$

The x-component of the equation of motion is given as

$$\rho\left(\cancel{\frac{\partial u}{\partial t}} + u\cancel{\frac{\partial u}{\partial x}} + \cancel{v}\frac{\partial v}{\partial y} + \cancel{w}\frac{\partial w}{\partial z}\right) = -\cancel{\frac{\partial P}{\partial x}} + \mu\left[\cancel{\frac{\partial^2 u}{\partial x^2}} + \frac{\partial^2 u}{\partial y^2} + \cancel{\frac{\partial^2 u}{\partial z^2}}\right] + \rho\, g_x$$



Considering one-dimensional flow ($v = 0 = w$), no applied pressure gradient, u is a function of only y, and a steady state process, the above equation reduces to the following governing equation

$$\frac{d^2u}{dy^2} = \frac{\rho g}{\mu} \sin\theta$$

The relevant boundary conditions (no slip) are

BC 1 At $y = 0$ $u = 0$
BC 2 At $y = h$, $u = U$

Therefore,

$$\frac{du}{dy} = \frac{\rho g sin\theta}{\mu} y + A \qquad and \qquad u = \left(\frac{\rho g sin\theta}{\mu}\right) \frac{y^2}{2} + Ay + B$$

Applying the BC s ➔ B = 0 and $A = \left(\frac{\rho g sin\theta}{\mu}\right)\frac{h}{2} + \frac{u}{h}$

Thus

$$u = -\left(\frac{\rho g sin\theta}{\mu}\right)\left(\frac{hy}{2} - \frac{y^2}{2}\right) + \frac{Uy}{h}$$

The volumetric flow rate per unit width of the film is given by

$$Q = \int_0^h u dy = -\int_0^h \left[\left(\frac{\rho g sin\theta}{\mu}\right)\left(\frac{hy}{2} - \frac{y^2}{2}\right) + \frac{Uy}{h}\right] dy = -\frac{\rho g h^3}{12\mu} sin\theta + \frac{Uh}{2}$$

$$Q = -\frac{860\ x\ 9{,}81\ x\ 0.002}{12\ x\ 10^{-2}} \sin 30 + \frac{3\ x\ 0.002}{2} = 0.0027 \frac{m^2}{s}\ (per\ unit\ width)$$

For the width of 5 m, the volumetric flow rate will be equal to 0.0135 m^3/s.

Evaluate $\tau = \mu\, du/dy$ at the moving belt from the expression for velocity as

$$\frac{du}{dy} = -\left(\frac{\rho g sin\theta}{\mu}\right)\left(\frac{h}{2} - y\right) + \frac{U}{h}$$

And at the moving belt,

$$\tau = \mu\left(\frac{du}{dy}\right)_{y\ =\ h} = \left(\frac{\rho g sin\theta}{2}\right) h + \frac{\mu U}{h}$$

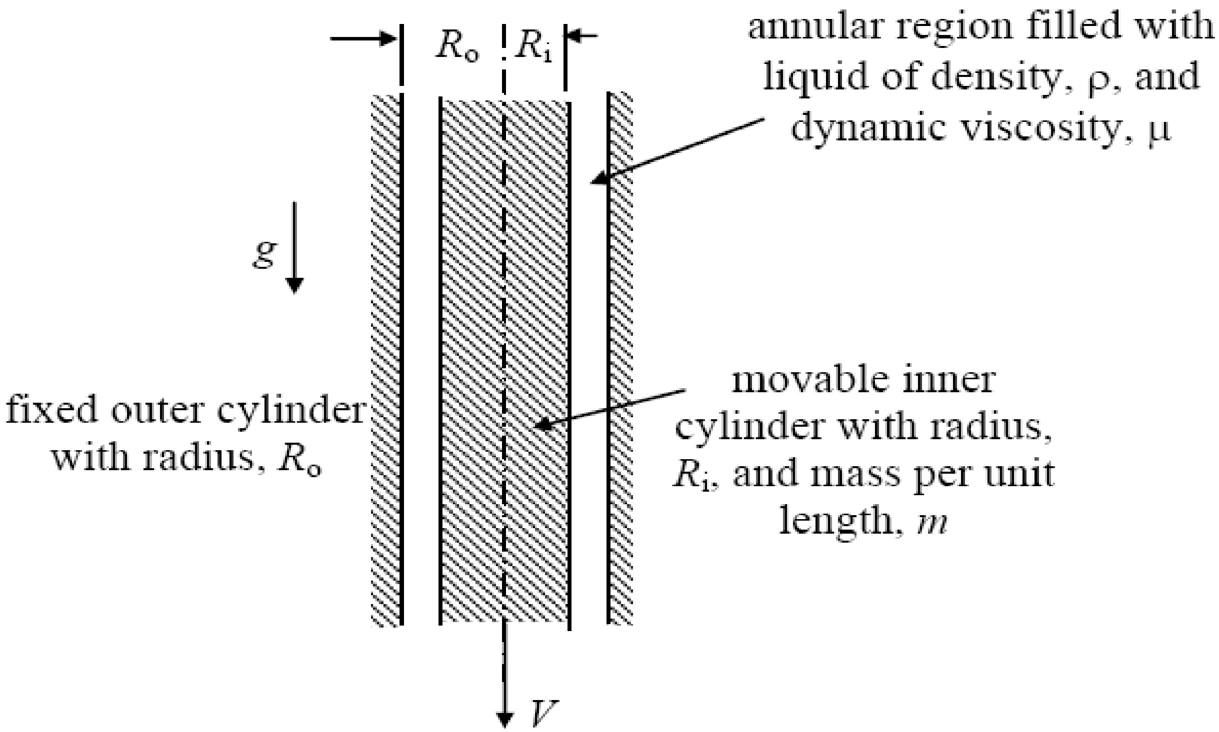
Using the values h = 0.002m, $\mu = 10^{-2}$ N.s/m^2 and U = 3 m/s

$\tau \big|_{\text{at the belt}}$ = 19.21 N/m^2

Power = (τ x L x W) U = 19.21 x 6 x 5 x 3 = 1.73 KW

Transport Phenomena - Momentum Transfer
Problem 3

Consider two concentric cylinders with a Newtonian liquid of constant density, ρ, and viscosity, μ, contained between them. The outer pipe, with radius, *R*o, is fixed while the inner pipe, with radius, *R*i, and mass per unit length, *m*, falls under the action of



gravity at a constant speed. There is no pressure gradient within the flow and no swirl velocity component. Determine the vertical speed, V, of the inner cylinder as a function of the following parameters: g, R_o, R_i, m, ρ, and μ. The space between the two cylinders is **not** ‘too small’ compared to the radii of the cylinders

Solution

For the inner cylinder moving at constant velocity, the downward force is exactly balanced by the viscous force as

$$(\tau_w\ A_w)\Big|_{Inner\ Cylinder} = m\ L\ g$$

The z component of the equation of motion in cylindrical coordinate is

$$\rho\left(\cancel{\frac{\partial v_z}{\partial t}} + \cancel{v_r\frac{\partial v_z}{\partial r}} + \cancel{\frac{v_\theta}{r}\frac{\partial v_z}{\partial \theta}} + v_z\cancel{\frac{\partial v_z}{\partial z}}\right) = -\cancel{\frac{\partial p}{\partial z}} + \mu\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial v_z}{\partial r}\right) + \frac{1}{r^2}\cancel{\frac{\partial^2 v_z}{\partial \theta^2}} + \cancel{\frac{\partial^2 v_z}{\partial z^2}}\right] + \rho g_z$$

Cancelling the terms with the observations i) steady state, ii) v_z is a function of r only, not of z or θ, iii) no applied pressure gradient

The following simplified form of the NS equation can be obtained.

Governing equation

$$\frac{1}{r}\frac{d}{dr}\left(r\frac{dv_z}{dr}\right) = -\ \frac{\rho g}{\mu}$$

Upon integration (with C_1 and C_2 being the constants of integration)

$$v_z = -\ \frac{\rho g r^2}{4\mu} + C_1 \ln r + \ C_2$$

Boundary Conditions

$v_z = V$ at $r = R_i$ => $$V = -\ \frac{\rho g}{4\mu}\ Ri^2\ +\ C_1 \ln R_i +\ C_2$$

$v_z = 0$ at $r = R_o$ => $$0 = -\ \frac{\rho g}{4\mu}\ Ro^2\ +\ C_1 \ln R_o +\ C_2$$

Therefore $$V = \frac{\rho g}{4\mu}\left(Ro^2 - Ri^2\right) + C_1 \ln\frac{R_i}{R_o}$$

$$C_1 = \frac{1}{\ln\frac{R_i}{R_o}}\left[V - \frac{\rho g}{4\mu}\left(Ro^2 - Ri^2\right)\right]$$

$$\frac{dv_z}{dr} = -\frac{\rho g r}{2\mu} + \frac{C_1}{r} \qquad \tau_{rz} = \mu\frac{dv_z}{dr} = -\frac{\rho g r}{2} + \frac{C_1\mu}{r}$$

Since the force on the inner cylinder = force due to gravity

$$\tau\big|_{r=Ri} 2\pi R_i L = m\, L\, g$$

Upon substitution for the expression of the shear stress

$$V = R_i \ln\frac{R_i}{R_o}\left(\frac{\rho g R_i}{2\mu} - \frac{mg}{2\pi R_i \mu}\right) - \frac{\rho g}{4\mu}\left({R_i}^2 - {R_0}^2\right)$$