INDIAN INSTITUTE OF TECHNOLOGY, KHARAGPUR
End-Autumn Semester Examination 2024-25

Date of Examination: Nov 20, 2024 Session AN Duration 3 hrs
Subject No. : CH62049 Subject Name: Microscale Transport Process
Department/Center/School: Chemical Engineering
Specific charts, graph paper, log book etc., required: No
Special Instructions (if any): Assume any data you feel are missing; Try to answer all questions of a PART in one place in the answerscript

PART-A

Q1. Multiple choice questions **[4 x 1=4 Marks]**

Consider all the statements and even if one of them is wrong then mark the entire answer as false identifying the false one with one line reason, else mark it as true.

(i) The flow in microchannels is generally regarded as straight laminar flow. This is correct for straight channels with low flow velocity and, therefore, low Re numbers. In straight channels, the flow remains laminar with straight streamlines below a Re number of 2300. Straight laminar flow changes when the fluid flows through curves, bends, or around obstacles. This flow regime in curved channel elements is often called Dean flow, where, Dean Number is = $Re\,(D/R_c)^2$.

(ii) Digital microfluidics (DMF) is implemented in one of the two following configurations – closed format, in which droplets are sandwiched between two substrates patterned with electrodes and open format in which droplets are placed atop a single substrate, housing both actuation and ground electrodes. Open DMF devices are best suited for dispensing, moving, splitting, and merging of droplets. Closed DMF devices, on the other hand, are typically not efficient for splitting and dispensing of droplets.

(iii) Disjoining pressure is the negative of the potential energy per unit volume due to the intermolecular forces and is a function of the film thickness. Disjoining describes the physical process whereby a completely spreading liquid naturally tends to disjoin a solid from the vapor by spreading an equivalent change in energy per unit area. For a completely wetting system, the surface energy decreases with the decrease in film thickness, leading to a positive disjoining pressure and a negative potential energy.

(iv) In every segment of blood vessel in the body, the flow is achieved with the least possible biological work. Two energy terms contribute to the cost of maintaining blood flow: (a) the energy required to overcome viscous drag in a fluid obeying Poiseuille's law, and (b) the energy metabolically required to maintain the volume of blood and vessel tissue involved in the flow. The optimum flow distribution and cross sections of the channels on different branching levels are influenced by - Murray's law, which states that: "If the sum of the inner radii to the power of three on each branching level is constant, the channel network will need minimal power consumption or exhibit minimal pressure loss for a given flow rate."

(v) The capillary Pressure and the suction potential for an extended meniscus act in the same direction. The thickness of an ultrathin liquid film will respond to a small change in ambient conditions.

Q2. Answer the following questions in a few sentences. **[2x2=4 marks]**

(i) In the electrowetting of sessile drops over a hydrophobic substrate, the application of voltage, causes a drop to spread progressively. Would it be possible to completely spread a droplet using electrowetting? After application of the voltage, if the power source is switched off, would the droplet return to its initial equilibrium contact angle? Explain and name the corresponding phenomenon.

(ii) Usually in any transport process, the transport of matter or energy takes places from higher to lower value of the key variable, for example heat transfer occurs due to temperature gradient from higher to lower temperature. However why does Marangoni flow take place from low to high surface tension? Justify.

Q3. A 1.5 µL droplet of 0.1 M brine solution is deposited on top of a Teflon substrate. The surface energy of bare Teflon in presence of an air ambiance is 20.5 mN/m. It is desired to increase the wettability of the droplet over the substrate by the application of a DC electric field. The solid-liquid and liquid-vapor surface energies are 50.4 mJ/m^2 and 72.8 mJ/m^2 respectively, and the dielectric breakdown voltage for a 15 µm thin Teflon layer is equal to 400 V and the relative permittivity is 2.1.

Density difference between the two fluids is given by: $\Delta\rho = 996 + (100 * c[M]) - 1.1544 \; kg/m^3$,

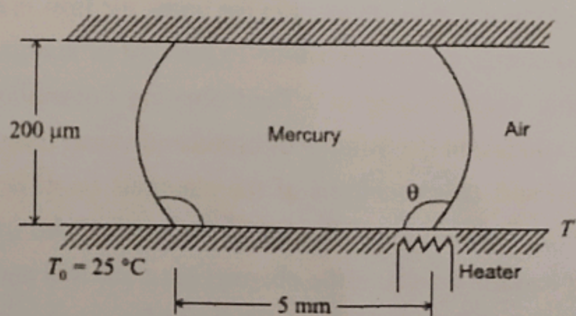
$\epsilon_0 = 8.854 \times 10^{-12} \; \frac{C^2}{N \cdot m^2}$

where $c[M]$ denotes the molar concentration of the salt in water.

i) Assume that the droplet is akin to a spherical cap throughout the process, calculate the base radius of the droplet (approx. in mm) when the actuating voltage is 370 V. **[4 marks]**

ii) Calculate the ratio of gravity Bond number to electrowetting number for the situation in problem (i). **[4 marks]**

Q4. Consider a 5mm long mercury droplet inside a microcapillary of 200 µm diameter. The temperature, T, of the contact line at the heater end is equal to 35°C, and room temperature of the other end is equal to 25°C, as depicted in the schematic below. Surface tension of mercury in air at 25°C is equal to 486.5 mJ/m². The surface tension of mercury decreases with temperature as per the following relation: $\sigma = 486.5 - 10\Delta T$, where ΔT denotes the temperature difference. The static contact angle of mercury equals 140°. Density of mercury is $13600 \; kg/m^3$. Viscosity of mercury is $1.526 \times 10^{-3} \; Pa.s$. [Hint: Consider the Poiseuille flow solution of the Navier-Stokes equation to estimate the droplet velocity, where the relationship between pressure drop and flowrate can be written as:



$$\Delta P_{Total} = \frac{8\mu L Q}{\pi R^4} \quad \textit{where } \Delta P \textit{ refers to the contribution from all posible causes}$$

Please note that the pressure drop between the two ends of the droplet involves contributions from three sources viz, capillary pressure originating due to the surface tension forces (equal to

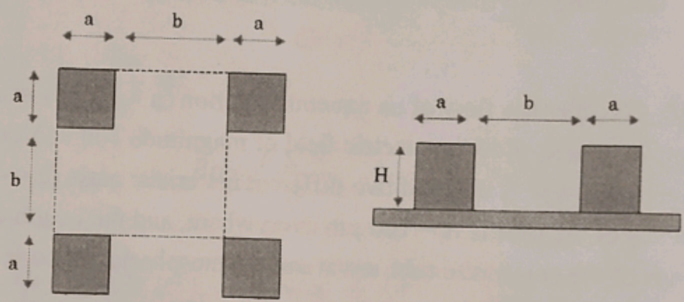
$$\Delta P_c = \frac{4\cos\theta}{d}\left[(\sigma)_A - (\sigma)_R\right]$$

, the hydrodynamic pressure due to the inclination of the channel and pressure drop due to external forces.

i. Calculate the speed of the droplet (approx. in mm/s) when the capillary is inclined at an angle of 15° from the horizontal. **[4 marks]**

ii. Calculate the ratio of droplet speeds for the two cases: I) the same situation as in problem i, and II) when the capillary is not inclined but there is an external pressure difference of 1.5×10^{-3} atm acting on the mercury droplet in the direction opposite to that of the temperature increase. **[4 marks]**

Q5. Transition from a Cassie droplet to a Wenzel droplet can be obtained by electrowetting actuation. A droplet of a certain conducting fluid shows that initially the droplet sitting on the top of the pillars in a Cassie regime, sinks down as soon as electrowetting actuation is turned on. Take the case of a lattice of square pillars along a square grid. Suppose the surface is constituted of square pillars of size a×a, height H and spacing b as shown in the Figures. Consider only one motif, as shown by the dashed line in the first figure on the left.



Calculate the ratio of the cosine of contact angles before and after electrowetting actuation in terms of geometric parameters of the pillars $\left[\frac{\cos\theta_{Before}}{\cos\theta_{After}}\right]$. Assume the contact angle with the layer of air (droplet sitting on the top of the pillars) be 180° and θ is the equilibrium contact angle with the substrate material. **[4 marks]**

PART-B

Q6. Write (in one sentence only) what major forces and underlying physical phenomena are controlling each of the following steps of an electrohydrodynamic process and how

i) release of liquid string from the needle tip
ii) breaking down of the liquid string into uniform droplets
iii) breaking down of the liquid string into highly non-uniform droplets
iv) acceleration / deceleration / terminal velocity of the droplet
v) further reduction of droplet size
vi) formation of beads in the fiber, formed from the liquid string
vii) formation of kinks in the fiber, formed from the liquid string

In an electrohydrodynamic atomization process from a capillary tip of radius 200 μm, placed at a distance of 1 cm from the ground electrode, estimate the time scale of bulk charge relaxation of double layer, hydrodynamic timescale, and the viscous timescale, and infer from these timescales whether a cone develops in this case. Also, suggest a scenario with these timescales where the cone does not develop. The flow rate is 1 μL/s over a length scale of 1 mm, fluid viscosity is 0.001 Pa.s, fluid density is 1gm/cc, ionic diffusivity is 10^{-9} m²/s, Debye length is 0.01 μm, interfacial tension as 10^{-2} N/m, and $\varepsilon_0\varepsilon_l \approx 10^{-10}$ coulomb²/(Joule-metre).

When AC voltage is applied to implement the atomization, what should be the upper limit of frequency, based on above information, such that a quasi-steady Taylor cone is produced in each half cycle? What happens to the Taylor cone, if the supply frequency exceeds the upper limit? What would be the magnitude of threshold voltage? **7×1+3+2= 12 Marks**

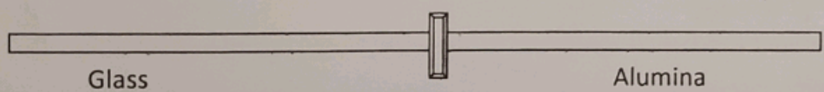
The voltage in volts is equal to the energy in joules, divided by the charge in coulombs.

Q7. A step change in tracer concentration from 0 weight% to 1 weight% (C_0) is implemented at the inlet of a microchannel, while conducting flow at constant rate of 10^{-9} m^3s^{-1}. The channel is of length 1m, and of circular cross-section with radius of 10^{-4} m. The diffusivity of the tracer in bulk is 10^{-9} m^2s^{-1}.

i) Calculate the time after which the concentration at the outlet will be 0.5 weight%.
ii) If the time, mentioned in Point (i) above is t, then tabulate the concentration at the outlet for following two time instances: 0.5 t, and 1.5 t. **1+3= 4 Marks**

Q8. Consider the flow of an aqueous solution ($\mu = 10^{-3}$ Pa-s; $\varepsilon/\varepsilon_0 = 78$) through a long narrow capillary tube under an electric field of magnitude 100 volt/cm, applied in the direction from left to right. The tube is made of two different materials: glass (ξ = -70 mV) and alumina (ξ=+40 mV). The radius of the tube is R = 100 μm everywhere, and the length of each segment is L = 1 cm. If the both ends of the composite tube are at same atmospheric pressure,

(i) What would be the pressure at the junction of two tube pieces (i.e., axially at the midpoint of the composite tube)?
(ii) Derive velocity profile u(r) in each of the two sections.
(iii) What would be the flowrate in the tube?



Assume that the EDL layer is thin, and electroosmotic velocity is uniform over the cross-section, and electrokinetic and poiseuille velocities to be simply additive. Permittivity of vacuum is 8.854×10^{-12} Joule $Volt^{-2}$ m^{-1}.

Hint: Mass conservation has to hold for the net (electro-osmotic + poiseuille) flow in two segments.

2+2+2=6 Marks

Useful Equations

$$\frac{\Delta P}{L} = \frac{8v\mu}{r^2}; \qquad u_{eof} = \epsilon\xi E/\mu; \ \frac{C}{C_0} = \frac{1}{2}\left[1 \pm erf\left(\frac{x-\bar{u}t}{\sqrt{4E_z t}}\right)\right]; \qquad C = \frac{M/A}{\sqrt{4\pi E_z t}} e^{-\frac{(Z-\bar{u}t)^2}{4E_z t}}$$

$$Dispersion\ coefficient = \frac{R^2\bar{u}^2}{48D}$$

$$\rho\frac{D\vec{w}}{Dt} = \rho\left(\frac{\partial}{\partial t} + \vec{w}\cdot div\right)\vec{w} = -grad\, p + \mu\nabla^2\vec{w} + \vec{k} \qquad \Delta P = \sigma_{lv}\left(\frac{1}{r_1} + \frac{1}{r_2}\right) = \sigma_{lv} K$$

$$\cos\theta_Y = \frac{\sigma_{sv} - \sigma_{sl}}{\sigma_{lv}}$$

The non-dimensional form of the z component (flow direction) of the equation of motion for flow through a duct

$$v^* \frac{\partial w^*}{\partial y^*} + w^* \frac{\partial w^*}{\partial z^*} = -\frac{\partial p}{\partial z^*} + \frac{\partial}{\partial y^*}\left[\eta^* \frac{\partial w^*}{\partial y^*}\right] + \frac{1}{\mathrm{Re}^2}\frac{\partial}{\partial z^*}\left[\eta^* \frac{\partial w^*}{\partial z^*}\right]$$

$$\rho c_p \frac{dT}{dt} = \frac{dp}{dt} + \varepsilon - div\,(k\, grad\, T) \qquad w(x=0) = \zeta\left(\frac{\partial w}{\partial x}\right)_{y=0}$$

$$\Lambda p = \left(C_f \frac{l}{d_h} + \zeta \mathrm{Re}\right)\frac{\rho v^2}{2}\frac{\mathrm{Re}}{d_h^2} \qquad \zeta \approx \frac{2-\beta}{\beta}\Lambda$$

$$v^* \frac{\partial T^*}{\partial y^*} + w^* \frac{\partial T^*}{\partial z^*} = \frac{1}{\mathrm{Pr}}\frac{\partial}{\partial y^*}\left[\lambda^* \frac{\partial T^*}{\partial y^*}\right] + \frac{1}{\mathrm{Pr}\mathrm{Re}^2}\frac{\partial}{\partial z^*}\left[\lambda^* \frac{\partial T^*}{\partial z^*}\right] + \frac{\mathrm{Ec}}{\mathrm{Re}^2}\Phi^*$$

$$Gr = \frac{g\beta s^3 (T_{w1} - T_{w2})}{\nu^2} \qquad Kn = \frac{\Lambda}{L}$$

$$Bo = \frac{\rho g R^2}{\sigma} \qquad \frac{ds}{dt} = \frac{1}{T}\left\{\frac{\varepsilon}{\rho} - \frac{1}{\rho} div\, \vec{q}\right\}$$

$$\Lambda = \frac{1}{\sqrt{2} n \sigma}$$

Le (=Sc/Pr) = α/D, ($\alpha = k/\rho c_p$) **$\mathrm{Ec}/\mathrm{Re}^2 = v^2/(c_p \Delta T\, d^2)$,** **$\mathrm{Ec} = w_r^2 / c_p \Delta T$** $\lambda = \frac{kT}{\sqrt{2}\, p\sigma}$

$$F_{Driving} = 2\gamma w \sin\left(\frac{\theta_R + \theta_L}{2}\right)(\cos\theta_R - \cos\theta_L) \qquad \sigma_{sl}^{eff}(U) = \sigma_{sl} - \frac{\varepsilon_0 \varepsilon_d}{2d} U^2$$

$$\cos\theta = \cos\theta_0 + \frac{\epsilon_0 \epsilon_r}{2d\sigma} V^2 \; ; \; \cos\theta = \cos\theta_Y + \frac{\varepsilon_0 \varepsilon_d}{2d\sigma_{lv}} U^2$$

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} \; ; \quad \cos\theta_W = r\cos\theta\,, \quad \cos\theta_C = f\cos\theta + (1-f)\cos\theta_0$$

The volume and surface area of a spherical cap can be expressed as

$$V(a,\theta) = \frac{\pi}{3}\frac{a^3}{\sin^3\theta}(2 - 3\cos\theta + \cos^3\theta)$$

Where V is the volume of the droplet, a is the base (wetted) radius; R is the radius of the entire spherical droplet and θ is the contact angle. The surface area can be expressed as

$$S(a,\theta) = \frac{2\pi a^2}{1 + \cos\theta} \qquad Ma = \frac{\Delta\gamma R}{\rho\nu\alpha}$$