V-RT

Smallpdf Tools

1. A plane surface, 25 cm wide, has its temperature maintained at 80 C. Atmospheric air, at 25C; flows parallel to the surface with a velocity of 2.8 m/s. Determine the following for a 1-m long plate:
(iii) The total drag force exerted on the plate by the air flow (in mN).
(iv) The total heat transfer rate from the plate to the air stream (in W).
For Air at the average temperature the properties can be taken as: $\rho = 1.087\ Kg/m^3$, kinematic viscosity = $1.807 \times 10^{-5}\ m^2/s$, $C_p = 1.008\ kJ/(Kg\ .\ K)$, $Pr = 0.702$, $k = 2.816\ W/(m\ .\ K)$ 5+5 = 10

i) The expression for the friction coefficient can be chosen based on the Re number.

Here $Re = \frac{VL}{\nu} = \frac{2.8\ m/s \times 1\ m}{1.807 \times 10^{-5}\ m^2/s} = 1.5 \times 10^5$, therefore Laminar. (1)

$\therefore C_{f_L} = 1.328\ Re_L^{-1/2}$ (1)

$C_{f_L} = 1.328\,(1.5 \times 10^5)^{-1/2} = 3.374 \times 10^{-3}$ (1)

The drag force $F_D = C_{f_L} A \frac{\rho U^2}{2} = 3.37 \times 10^{-3} \times (0.25 \times 1) \times \frac{1.087 \times (2.8)^2}{2}$ (1)

$F_D = 3.594 \times 10^{-3}\ N.$ (1)

ii) Using analogy

$St = \frac{C_{f_L}}{2} Pr^{-2/3}.$ (1)

$St = \frac{3.374 \times 10^{-3}}{2} \times (0.702)^{-2/3}.$

$St = 2.136 \times 10^{-3}.$ (1)

$\frac{h}{\rho C_p V} = 2.136 \times 10^{-3}.$

$h = (2.136 \times 10^{-3})(1.087)(1.008)(2.8)$

$h = 6.55\ kW/m^2 K$ (1)

Therefore, the total heat transfer rate (q)

$q = hA\Delta T = 6.55 \times (1 \times 0.25) \times (80-25)$

$q = 90\ kW.$ (2)

2 (a) A crude approximation for the x component of velocity in a laminar boundary layer is a linear variation from $v_x = 0$ at the surface to the freestream velocity, U, at the boundary layer edge ($y = \delta$). The equation for the profile is given below (where C is a constant). Evaluate the maximum value of the ratio v_y / v_x at a location $x = 0.5$ m and $\delta = 5$ mm.

$$v_x = C\,U \frac{y}{x^{1/2}} \qquad 3$$

(b) Starting with the governing equation for momentum boundary layer evaluate the value of $\partial^3 u / \partial y^3$ at $y = 0$ for an incompressible laminar boundary layer on a flat plate with zero-pressure gradient. 2

(c) A flat plate, sides a, b in length, is towed through a fluid so that the boundary layer is entirely laminar. Find the ratio of towing speeds so that the drag force remains constant regardless of whether a or b is in the flow direction. U_a is the freestream velocity if side a is in the flow direction and U_b if b is in the flow direction. 5

3+2+5=10

$v_x = CU \frac{y}{x^{1/2}}.$

Cont. eq^n $\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} = 0.$

$\frac{\partial v_x}{\partial x} = -\frac{1}{2} \frac{CUy}{x^{3/2}}$

$\therefore \frac{\partial v_y}{\partial y} = \frac{1}{2} \frac{CU}{x^{3/2}} y$ (1)

$v_y = \frac{1}{4} \frac{CU}{x^{3/2}} y^2 + C_1$

B.C. At $y = 0$, $v_y = 0 \Rightarrow C_1 = 0.$

$\therefore \frac{v_y}{v_x} = \frac{1}{4} \frac{CU}{x^{3/2}} y^2 \cdot \frac{x^{1/2}}{CUy} = \frac{y}{4x}.$ (1)

Max. value of y at any x is $y = \delta$ (the B.L. thickness)

$\therefore \left.\frac{v_y}{v_x}\right|_{max} = \frac{\delta}{4x}.$ (1)

given for $x = 0.5\ m$, $\delta = 5 \times 10^{-3}\ m.$

$\therefore \left.\frac{v_y}{v_x}\right|_{max} = \frac{5 \times 10^{-3}}{4 \times 0.5} = 2.5 \times 10^{-3}.$

3 A new spray painting system is being evaluated for the auto industry. The paint is delivered via an atomizer that produces 10 mm particles and propels them toward the surface at avelocity of 10 m/s. The particles are a dilute suspension of pigment agents in a solvent and for modeling purposes can be assumed to be pure solvent. To form a good coating the particles must arrive at the surface with 75% by volume of the solvent remaining. The freestream concentration of the solvent is effectively zero and the solvent has a specific gravity of 0.85 and a molecular weight of 100. The whole system operates at atmospheric pressure and at the temperature of deposition the vapor pressure of the solvent is 250 mm Hg. The diffusivity of the solvent in air was measured to be 1x10-9 m^2/s and the Schmidt number for the solvent is 500. For the specified conditions the relevant mass transfer correlation can be approximated as -

$$\overline{Sh}_d = 2+\left(0.4\,Re_d^{1/2}+0.06\,Re_d^{2/3}\right)Sc^{0.4}$$

How far away from the surface can the painting nozzle be located? 10

[The calculations were done taking $d = 10\ \mu m$. The bracketed quantities denote the results corresponding to $d = 10$ mm]

Mass balance about a single particle.

IN − OUT + GEN = Acc.

$$0 - \dot{M}_{out} + 0 = \frac{dM}{dt}$$

$$-\bar{h}_m A_s (C_{As} - C_{A\infty}) = \frac{d}{dt}(C_{Ad} V)$$

2 $$-\bar{h}_m (4\pi r^2)(C_{As} - C_{A\infty}) = C_{Ad}\frac{d}{dt}\left(\frac{4}{3}\pi r^3\right)$$

$$-\bar{h}_m\left(\frac{C_{As} - C_{A\infty}}{C_{Ad}}\right) = \frac{dr}{dt}$$

Use velocity of the drop to translate from time to space

1 $$-\bar{h}_m\left(\frac{C_{As} - C_{A\infty}}{C_{Ad}}\right) = \frac{dr}{dt} = \frac{dr}{dx}\cdot\frac{dx}{dt} = \bar{V}_x\frac{dr}{dx}$$

75% of the solvent must be remaining

$$\therefore V_{final} = 0.75\,V_{initial}$$

1 $$r_{final} = \left(0.75\,r^3_{initial}\right)^{1/3} = 0.91\,r_{initial}.$$

Remaining unknown is the mass transfer coeff.

$$\bar{Sh} = 2 + \left(0.4\,Re_d^{1/2} + 0.06\,Re_d^{2/3}\right)Sc^{0.4}$$

$$Re_d = \frac{V_\infty d}{\nu} = \frac{V_\infty d}{Sc\,D_{ab}} = \frac{10\times 1\times10^{-6}}{500\times10^{-9}} = 20\ (2\times10^5)$$

$$\therefore \bar{Sh} = 2+(0.4\times 20^{1/2} + 0.06\times 20^{2/3})(500)^{0.4} = 4.23\ (4616$$

2 $$\therefore \bar{h}_m = \frac{\bar{Sh}\cdot D_{ab}}{d} = \frac{4.23\times(1\times10^{-9})}{1\times10^{-6}} = 4.23\times10^{-3}\ m/s\ (4.61\times10^{-4})$$

The rate of decay of the drop with distance from the nozzle can be calculated

$$\frac{dr}{dx} = -\frac{\bar{h}_m}{\bar{V}_x}\left(\frac{C_{As} - C_{A\infty}}{C_{Ad}}\right)$$

$$\therefore r_{final} - r_{initial} = -\frac{\bar{h}_m}{\bar{V}_x}\left(\frac{C_{As} - C_{A\infty}}{C_{Ad}}\right)(x_{final} - x_{initial})$$

1

Concentrations to be found using vap. pr., sp. gravity & mol. wt & ideal gas law.

$$C_{Ad} = \frac{0.85\times1000\ kg/m^3}{100\ kg/kmol} = 8.5\ \frac{kmol}{kg}$$

$$C_{As} = \left(\frac{P_{vap}}{P}\right)\left(\frac{P}{RT}\right) = \frac{250}{760}\times\frac{1.01\times10^5}{8314\times298}$$

1 $$= 1.34\times10^{-2}\ \frac{kmol}{m^3}$$

$x_{initial} = 0$, $r_{final} = 0.91\,r_{initial} = 0.91\ \mu m$.

1 $$x_{final} = \frac{r_{initial} - r_{final}}{\frac{\bar{h}_m}{V_x}\left(\frac{C_{As} - C_{A\infty}}{C_{Ad}}\right)} + x_{initial}$$

$$= \frac{1\times10^{-6} - 0.91\times10^{-6}}{\frac{4.23\times10^{-3}}{10}\left(\frac{1.34\times10^{-2} - 0}{8.5}\right)}$$

1 $$x_{final} = 0.135\ m.\ (1230\ m)$$

Governing eqn for incompressible, laminar b.L on a flat plate,

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \nu\frac{\partial^2 u}{\partial y^2}$$

Differentiating w.r.t. y

$$\Rightarrow \nu\frac{\partial^3 u}{\partial y^3} = \frac{\partial u}{\partial y}\cdot\frac{\partial u}{\partial x} + u\frac{\partial^2 u}{\partial x \partial y} + \frac{\partial v}{\partial y}\cdot\frac{\partial u}{\partial y} + v\frac{\partial^2 u}{\partial y^2}$$

1

At $y=0$ $\quad u = 0 = v$ but the derivatives are not zero.

$$\Rightarrow \nu\frac{\partial^3 u}{\partial y^3}\bigg|_{y=0} = \frac{\partial u}{\partial y}\cdot\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\cdot\frac{\partial u}{\partial y}$$

$$= \frac{\partial u}{\partial y}\underbrace{\left[\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right]}_{=0 \text{ from continuity, incompressible flow}} \quad 1$$

$$\therefore \nu\frac{\partial^3 u}{\partial y^3}\bigg|_{y=0} = 0$$

(b) Show that, if a flat plate, sides a, b in length, is towed through a fluid so that the boundary layer is entirely laminar, the ratio of towing speeds so that the drag force remains constant regardless of whether a or b is in the flow direction is given by

$$\frac{U_a}{U_b} = \sqrt[3]{\frac{a}{b}}$$

where U_a is the freestream velocity if side a is in the flow direction and U_b if b is in the flow direction.

The drag force is given by

1 $\quad F_D = C_D \frac{1}{2}\rho v^2 A$ and $\underset{(\text{laminar})}{C_D} = \frac{1.328}{\sqrt{Re_L}}$ 1

$$F_{D_a} = C_{D_a}\frac{1}{2}\rho V_a^2 (a \times b)$$

$$= \frac{1.328}{\sqrt{\frac{a V_a \rho}{\mu}}}\frac{1}{2}\rho V_a^2\, ab. \quad 1$$

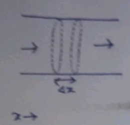
$$F_{D_b} = \frac{1.328}{\sqrt{\frac{b V_b \rho}{\mu}}}\frac{1}{2}\rho V_b^2\, ab \quad 1$$

Since $F_{D_a} = F_{D_b}$

$$V_a^{3/2} a^{1/2} b = V_b^{3/2} a b^{1/2}$$

$$\left(\frac{V_a}{V_b}\right) = \left(\frac{a}{b}\right)^{1/3} \quad 1$$

7. Air passes through a naphthalene tube, that has an inside diameter of 2.5 cm, flowing at a bulk velocity of 15 m/s. The air is at 283 K and an average pressure of 101300 Pa. Assuming that the change in pressure along the tube is negligible and that the naphthalene surface is at 283 K, determine i) the concentration profile of naphthalene as a function of the axial position in the tube, assuming the bulk velocity to be a constant and ii) the length of tube (approx. in m) that is necessary to produce a naphthalene concentration in the exiting gas stream of 4.75×10^{-4} mol/m^3. At 283 K, naphthalene has a vapor pressure of 3 Pa and a diffusivity in air of 5.4×10^{-6} m^2/s. Use $C_f = 0.0058$. 5+5=10



The first step is to identify that the concentration of napthalene, C_A, is a function of x but not of r.

$$C_A = f(x) \text{ only} \qquad 1$$

Making a balance of napthalene, over the control volume of length Δx and ~~idn~~ identifying sublimation of napthalene to the flowing air stream,

$$\underset{\text{IN}}{C_A \cdot V \cdot \frac{\pi D^2}{4}\Bigg|_x} - \underset{\text{OUT}}{C_A \cdot V \cdot \frac{\pi D^2}{4}\Bigg|_{x+\Delta x}} \qquad 1$$

$$+ h_m (C_{As} - C_A)\pi D = 0 \quad \text{(at steady state)}$$

as, $\Delta x \rightarrow 0$

$$\boxed{\frac{dC_A}{dx} = \frac{4}{D}\frac{h_m}{V}(C_{As} - C_A)} \quad \text{Gov. Eqn.} \qquad 1$$

let, $\theta = C_A - C_{As} \Rightarrow \frac{d\theta}{dx} = \frac{dC_A}{dx}$

$$\frac{d\theta}{dx} = -\frac{4}{D}\frac{h_m}{V}\theta$$

Integrating,

$$\int_{\theta_0}^{\theta_L} \frac{d\theta}{\theta} = -\frac{4}{D}\cdot\frac{h_m}{V}\int_0^L dx$$

BC → 1 (2)

$$\therefore \ln\left(\frac{\theta_L}{\theta_0}\right) = -\frac{4}{D}\frac{h_m}{V} L$$

or $$\ln\left[\frac{C_{AL} - C_{As}}{C_{A0} - C_{As}}\right] = -\frac{4}{D}\frac{h_m}{V}\cdot L \qquad 1$$

Now, $Re = \frac{DV}{\nu}$ Given, $C_f = 0.0058$

$$Sc = \frac{\nu}{D_{AB}}$$

Using Chilton - Coulburn analogy:

$$St \cdot Sc^{2/3} = \frac{C_f}{2} \qquad 1$$

or $$\frac{Sh}{Re \cdot Sc} \cdot Sc^{2/3} = \frac{C_f}{2}$$

or $$Sh = \frac{C_f}{2} Re\, Sc^{1/3} \qquad 1$$

or $$\frac{h_m \cdot D}{D_{AB}} = \frac{C_f}{2}\cdot\frac{DV}{\nu}\left(\frac{\nu}{D_{AB}}\right)^{1/3}$$

or $$\frac{h_m}{V} = \frac{C_f}{2}\left(\frac{D_{AB}}{\nu}\right)\left(\frac{\nu}{D_{AB}}\right)^{1/3}$$

3

or. $\frac{h_m}{V} = \frac{C_f}{2} \frac{1}{Sc} \cdot Sc^{1/3}$

or $\frac{h_m}{V} = \frac{C_f}{2} \cdot \frac{1}{Sc^{2/3}}$

$$Sc = \frac{\nu}{D_{AB}} = \left(\frac{1.415 \times 10^{-5}}{5.4 \times 10^{-6}}\right) = 2.62.$$

$$\therefore \frac{h_m}{V} = \frac{0.0058}{2} \cdot \frac{1}{(2.62)^{2/3}}$$

1

$$\frac{h_m}{V} = 1.52 \times 10^{-3}$$

Therefore, $\ln\left[\frac{C_{A_L} - C_{A_S}}{C_{A_0} - C_{A_S}}\right] = -\frac{4}{0.025} \cdot (1.52 \times 10^{-3})\, L$

Now, $C_{A_L} = 4.75 \times 10^{-4}$ mol/m^3 (given)

$$C_{A_S} = \frac{P_A}{RT}$$

$$= \frac{3 \text{ Pa}}{8.314 \text{ J K}^{-1} \text{mol}^{-1} \times 283 \text{ K}}$$

1

$$= 1.275 \times 10^{-3} \frac{\text{mol}}{\text{m}^3}$$

$$\ln\left[\frac{4.75 \times 10^{-4} - 1.275 \times 10^{-3}}{0 - 1.275 \times 10^{-3}}\right] = -\frac{4}{0.025} \cdot (1.52 \times 10^{-3})\, L$$

or $-0.4684 = -0.2432\, L$

1

or $L = \cancel{0.5192 \text{ m}}$ 1.9 m