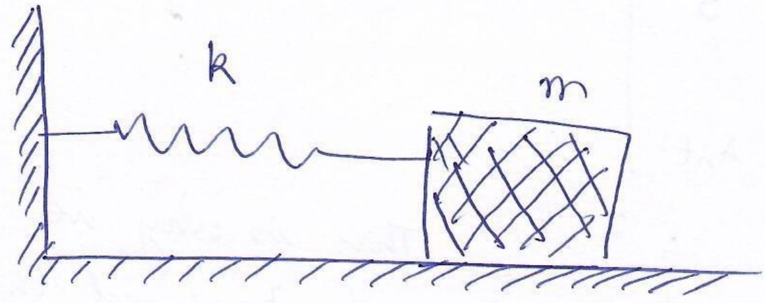
Forced Spring Mass system

Week 3

Revision

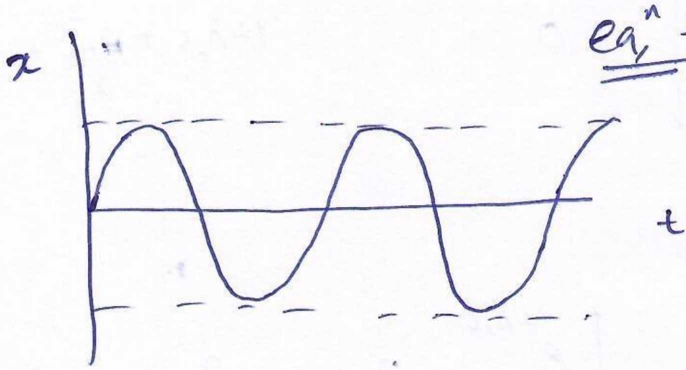


$$m\frac{d^2x}{dt^2} + kx = 0 \quad (1) \quad \text{- free undamped system}$$

$$m\frac{d^2x}{dt^2} + c\frac{dx}{dt} + kx = 0 \quad (2) \quad \text{- free vibration with damping}$$

autonomous

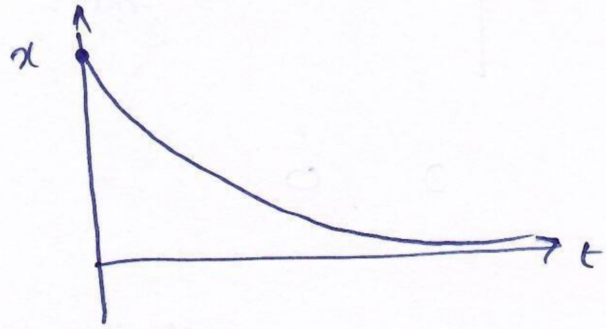
eq^n 1



eq^n (2)

overdamped system

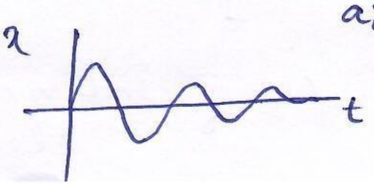
form of solution → $x(t) = C_1 e^{\lambda_1 t} + C_2 e^{\lambda_2 t}$



Underdamped system

$$x(t) = e^{-at}(C_1 \sin bt + C_2 \cos bt)$$

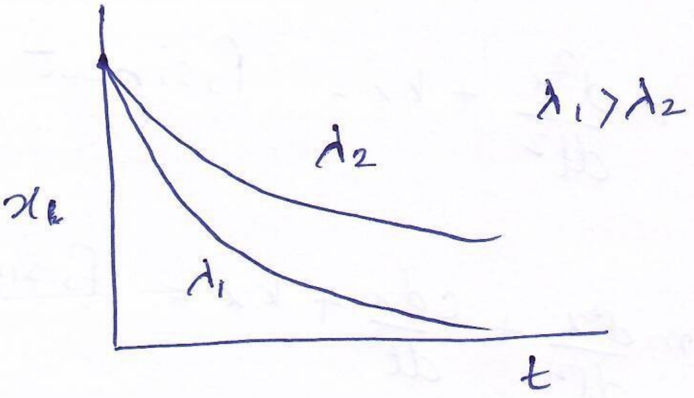
$a > 0$



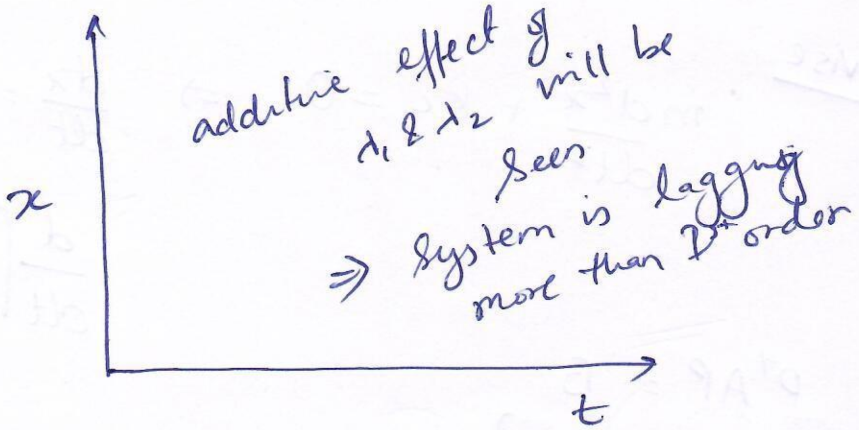
$a < 0$

In case of overdamped

In case of 1^{st} order



In case of second order



critical damping

$$x(t) = C_1 e^{-\lambda_1 t} + C_2 e^{-\lambda_2 t}$$

$$\frac{d\underline{x}}{dt} = \underline{\underline{A}}\,\underline{x}$$

$\hookrightarrow$ eigen values. (λ_1, λ_2)

if $\lambda_1 = \lambda_2$

$$\underbrace{x(t) = C_1 e^{-\lambda_1 t} + C_2 \mathbf{t} e^{-\lambda_1 t}}_{\text{critical damping}} \rightarrow \underline{\underline{\text{Prove}}}$$

for Forced system — non autonomous.

$$m\frac{d^2x}{dt^2} + kx = f_0 \sin\omega t$$

$$m\frac{d^2x}{dt^2} + c\frac{dx}{dt} + kx = f_0 \sin\omega t .$$

Revise

$$m\frac{d^2x}{dt^2} + kx = 0 \Rightarrow \frac{dx}{dt} = y \; ; \; \frac{dy}{dt} = -\frac{k}{m}x$$

$$\frac{d}{dt}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\frac{k}{m} & 0 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}$$

$$\frac{d\underline{X}}{dt} = \underline{\underline{A}}\,\underline{X}$$

↳ diagonalise $\underline{\underline{A}}$

$$\underline{\underline{P}}^{-1}\underline{\underline{A}}\,\underline{\underline{P}} = \underline{\underline{B}}$$

$$\underline{\underline{P}} = [\,\underline{v}_1 | \underline{v}_2\,]$$

$$\underline{\underline{P}}^{-1}\underline{\underline{A}}\,\underline{\underline{P}} = \underline{\underline{\Lambda}}$$

$$\underline{\underline{\Lambda}} = \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix}$$

$$\underline{P} = [\,v_1 \quad v_2\,]$$

$$P = \begin{bmatrix} i\sqrt{\frac{m}{k}} & -i\sqrt{\frac{m}{k}} \\ 1 & 1 \end{bmatrix}$$

$$P^{-1} = \frac{1}{2}\begin{bmatrix} -i\sqrt{\frac{k}{m}} & i\sqrt{\frac{k}{m}} \\ 1 & 1 \end{bmatrix}$$

$$\underline{\underline{\Lambda}} = \begin{bmatrix} -i\sqrt{\frac{k}{m}} & 0 \\ 0 & i\sqrt{\frac{k}{m}} \end{bmatrix}$$

$$\frac{d\underline{x}}{dt} = \underline{\underline{A}}\,\underline{x}$$

$$\frac{d}{dt}(P^{-1}x) = (\underline{\underline{P^{-1}AP}})\underline{\underline{P^{-1}}}\,\underline{x}$$

$$\frac{d}{dt}(P^{-1}x) = \underline{\underline{\Lambda}}(P^{-1}x)$$

$$y = P^{-1}x$$

$$\underline{\underline{\Lambda}} = \underline{\underline{P^{-1}}}\,\underline{\underline{A}}\,\underline{\underline{P}}$$

$$\frac{d}{dt}\underline{Y} = \underline{\underline{\Lambda}}\,\underline{Y}$$

$\hookrightarrow$

$$\underline{Y} = e^{\underline{\underline{\Lambda}}t}\,\underline{C} \qquad \underline{C} = \begin{bmatrix} c_1 \\ c_2 \end{bmatrix} \text{ initial condition}$$

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} =$$

Now, transform y back to x

$$\underline{y} = \underline{\underline{P^{-1}}}\,\underline{x} \Rightarrow \underline{x} = \underline{\underline{P}}\,\underline{y}$$

$$\underline{y} = \begin{bmatrix} e^{\left(-i\sqrt{\frac{k}{m}}\right)t} & 0 \\ 0 & e^{\left(i\sqrt{k/m}\right)t} \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \end{bmatrix}$$

$$\underline{x} = \begin{bmatrix} i\sqrt{\frac{m}{k}} & -i\sqrt{\frac{m}{k}} \\ 1 & 1 \end{bmatrix} \begin{bmatrix} e^{-i\sqrt{\frac{k}{m}}t} & 0 \\ 0 & e^{i\sqrt{k/m}\,t} \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \end{bmatrix}$$

$$\begin{bmatrix} x \\ \frac{dx}{dt} \end{bmatrix} = \underline{x} = \begin{bmatrix} \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \end{bmatrix}$$

So for forced.

$$m\frac{d^2x}{dt^2} + kx = f_0 \sin\omega t$$

$$m\frac{dy}{dt} + kx = f_0 \sin\omega t \quad ; \quad \frac{dx}{dt} = y$$

$$\frac{d}{dt}\begin{bmatrix} x \\ y \end{bmatrix} = \underbrace{\begin{bmatrix} 0 & 1 \\ -\frac{k}{m} & 0 \end{bmatrix}}\begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} 0 \\ f_0 \sin\omega t \end{bmatrix}$$

$$\left.\begin{array}{c} \underline{\underline{A}} \\ \lambda_1 \\ \lambda_2 \\ v_1 \\ v_2 \\ \underline{\underline{P}} \\ \underline{\underline{P^{-1}}} \\ \underline{\underline{\Lambda}} \end{array}\right\} \text{Same as autonomous.}$$

$$\frac{d\underline{x}}{dt} = \underline{\underline{A}}\,\underline{x} + g(t) \quad \text{(1)}$$

$$\frac{d}{dt}(P^{-1}x) = (P^{-1}A\,P)P^{-1}x + P^{-1}g(t)$$

$$\frac{d}{dt}(P^{-1}x) = \underline{\underline{\Lambda}}(P^{-1}x) + P^{-1}g(t)$$

let $\underline{\underline{P}}^{-1}\underline{x} = \underline{y}$

$\underline{\underline{P}}^{-1}\underline{g}(t) = \underline{b}$

$$\frac{d}{dt}\underline{y} = \underline{\underline{\Lambda}}\,y + \underline{b} \quad \text{(2)}$$

$$\frac{d}{dt}\underline{y} - \underline{\underline{\Lambda}}\,y = b$$

If $= e^{-\underline{\underline{\Lambda}}t}$

multiply both sides & solve for y

But $b = ?!$

$$b = P^{-1} g$$

$$b = \frac{1}{2}\begin{bmatrix} -i\sqrt{\frac{k}{m}} & i\sqrt{\frac{k}{m}} \\ 1 & 1 \end{bmatrix}\begin{bmatrix} 0 \\ f_0 \sin\omega t \end{bmatrix}$$

Put b in eq^n 2

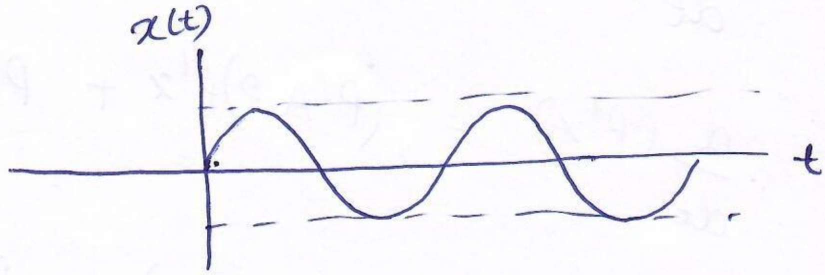
solve for $\underline{y}$

then solve for $\underline{x}$

$$\underline{x} = \underline{P}\,y$$

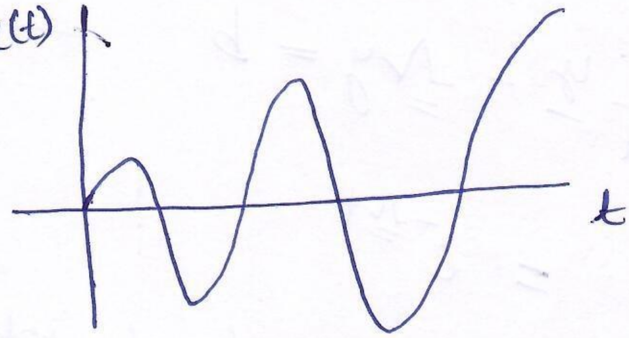
Example

1. $\frac{d^2x}{dt^2} + x = 0 \Rightarrow x(t) = C_1 \cos t + C_2 \sin t$



2. $\frac{d^2x}{dt^2} + x = \sin t \Rightarrow x(t) = C_1 \cos t + C_2 \sin t - \frac{1}{2} t \cos t$.

oscillations increase with time because effect of autonomous part is augmented by non-autonomous.

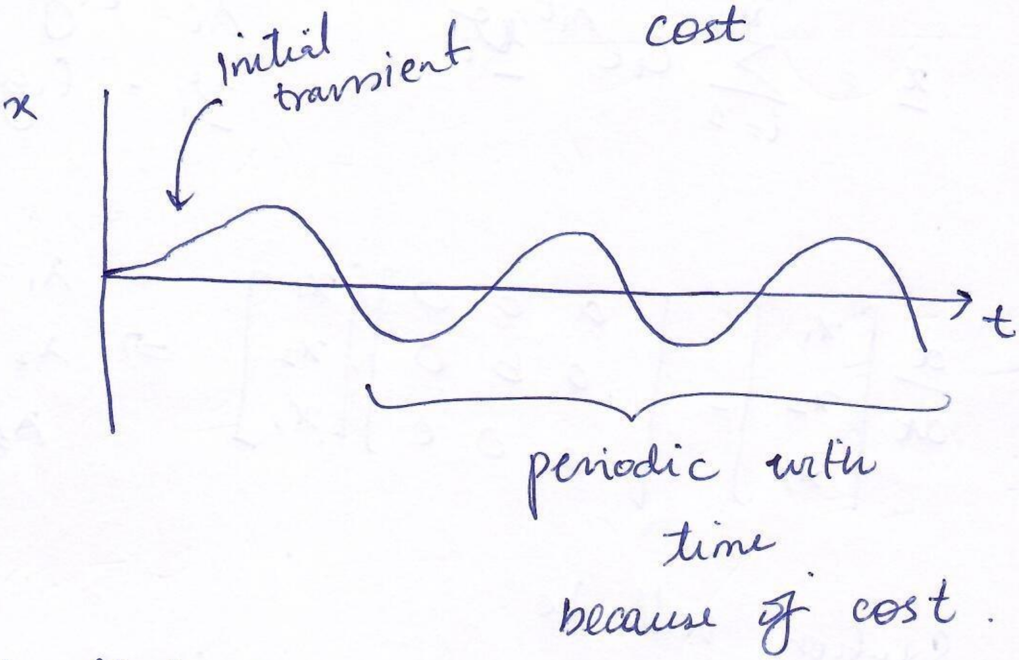


external force is increasing amplitude with time.

3. $\frac{d^2x}{dt^2} + \frac{dx}{dt} + x = 0 \Rightarrow x(t) = C_1 e^{-t/2} \sin \sqrt{3}/2\, t + C_2 e^{-t/2} \cos \frac{\sqrt{3}}{2} t$

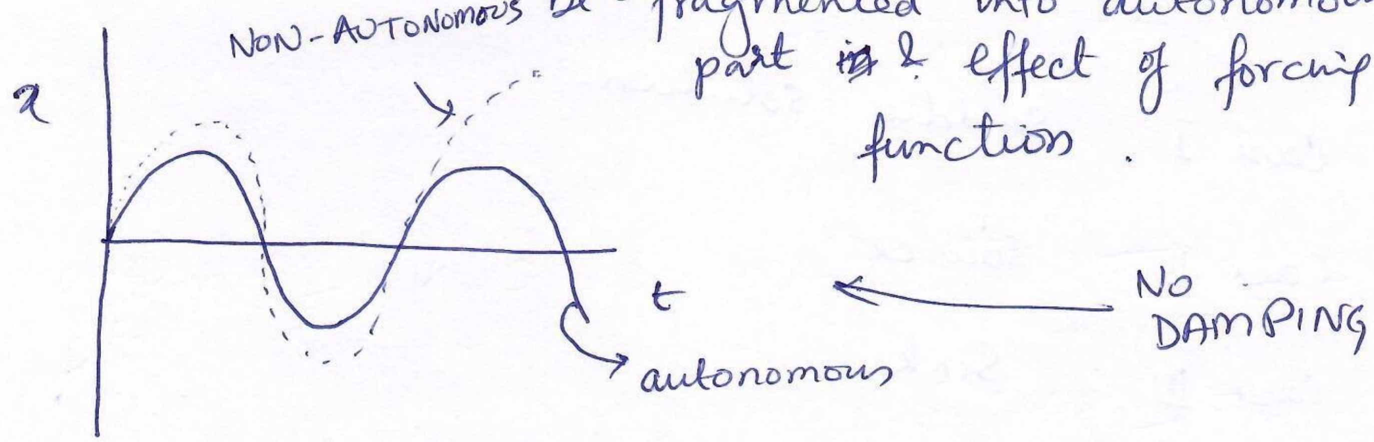
Damping

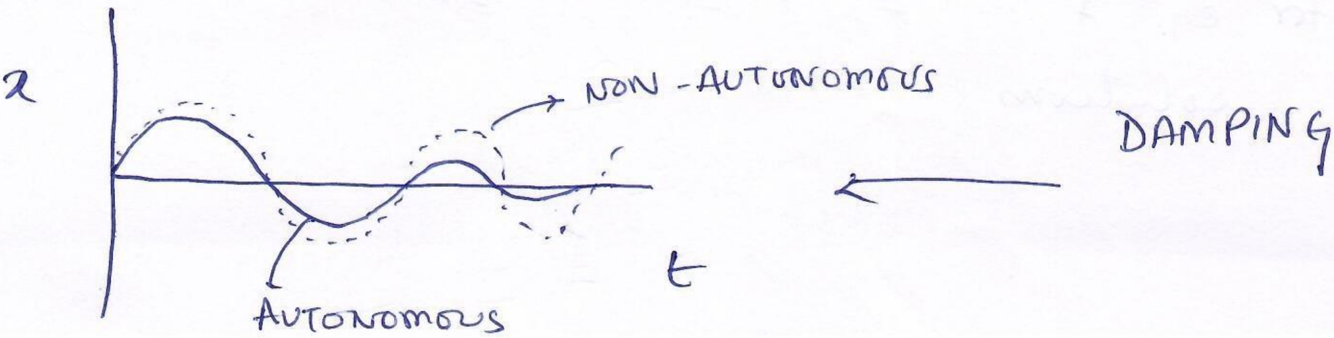
4. $\dfrac{d^2x}{dt^2} + \dfrac{dx}{dt} + x = \sin t \Rightarrow x(t) = c_1 e^{-t/2} \sin \dfrac{\sqrt{3}}{2}t + c_2 e^{-t/2} \cos \dfrac{\sqrt{3}}{2}t - \cos t$



effect of first 2 terms will decrease on time

SUMMARY → every non-autonomous system can be fragmented into autonomous part & effect of forcing function.





3 order system

$$\frac{d\underline{x}}{dt} = \underline{\underline{A}}\,\underline{x}$$

$\underline{x} = N \times 1$
$\underline{\underline{A}} = N \times N$

$$\underline{x} = \sum_{i=1}^{N} C_i e^{\lambda_i t} \underline{v}_i$$

λ_i = Eigen Value
$\underline{v}_i$ = Eigen vectors

$$\frac{d}{dt}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \Rightarrow \begin{matrix} \lambda_1 = a & \underline{v}_1 = [1\ 0\ 0]' \\ \lambda_2 = b & \underline{v}_2 = [0\ 1\ 0]' \\ \lambda_3 = c & \underline{v}_3 = [0\ 0\ 1]' \end{matrix}$$

Solution will be

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = C_1 e^{at}\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} + C_2 e^{bt}\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} + C_3 e^{ct}\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \quad \text{--- (1)}$$

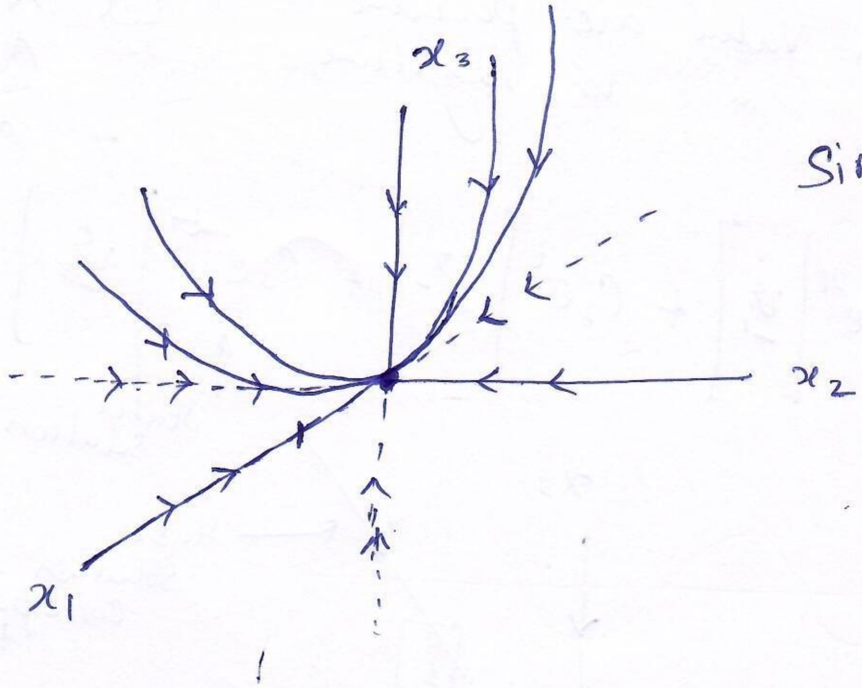
$$a, b, c \in \mathbb{R}$$

Case I : Saddle solution $a>0$, $b<0$

Case II : Source $a>0$, $b>0$

Case III : Sink $a<0$, $b<0$

for eq^n 1 ; $\underline{v}_1$; $\underline{v}_2$ & $\underline{v}_3$ all are solutions



SINK SOLUTION

all 3 axis x_1, x_2, x_3 are solution.

at equilibrium.

$$\begin{bmatrix} x_{1e} = 0 \\ x_{2e} = 0 \\ x_{3e} = 0 \end{bmatrix} \text{ — solution}$$

if $a < 0$; $b < 0$, $c < 0$ ← Sink solution

$\lim_{t\to\infty} c_1 e^{at} \underline{v}_1 \to 0$; $\lim_{t\to\infty} c_1 e^{bt} \underline{v}_2 \to 0$; $\lim_{t\to\infty} c_2 e^{ct} v_3 \to 0$

dominant Eigen value will decide direction which will act as tangent

If $a > b > c$ (all are negative)

x_1 will be tangent.

SOURCE SOLUTION WILL BE EXACTLY OPPOSITE

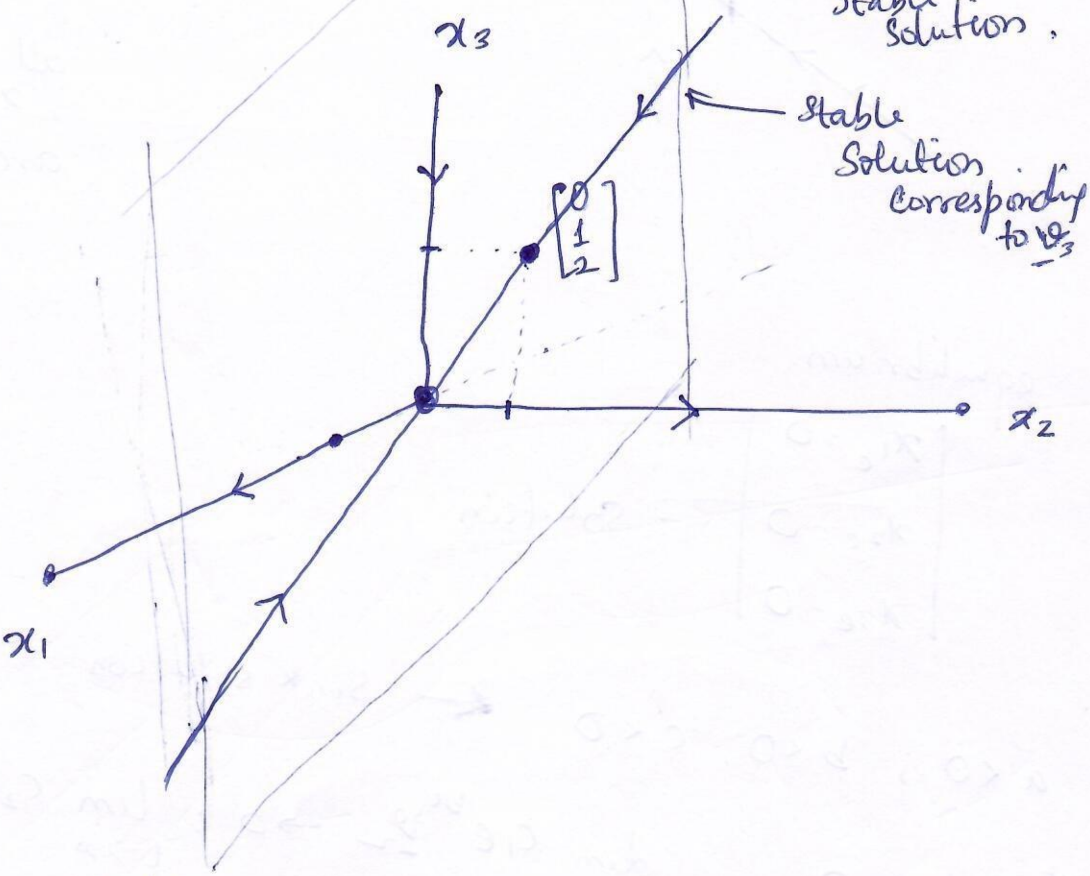
$a > 0$; $b > 0$; $c > 0$

Case 2 Eigen Value are positive
1 " " is negative. Let $\lambda_1 = 2$
$\lambda_2 = 1$
$\lambda_3 = -1$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = C_1 e^{2t}\begin{bmatrix} \underline{v}_1 \end{bmatrix} + C_2 e^{t}\begin{bmatrix} \underline{v}_2 \end{bmatrix} + C_3 e^{-t}\begin{bmatrix} \underline{v}_3 \end{bmatrix}$$

Stable Solution.

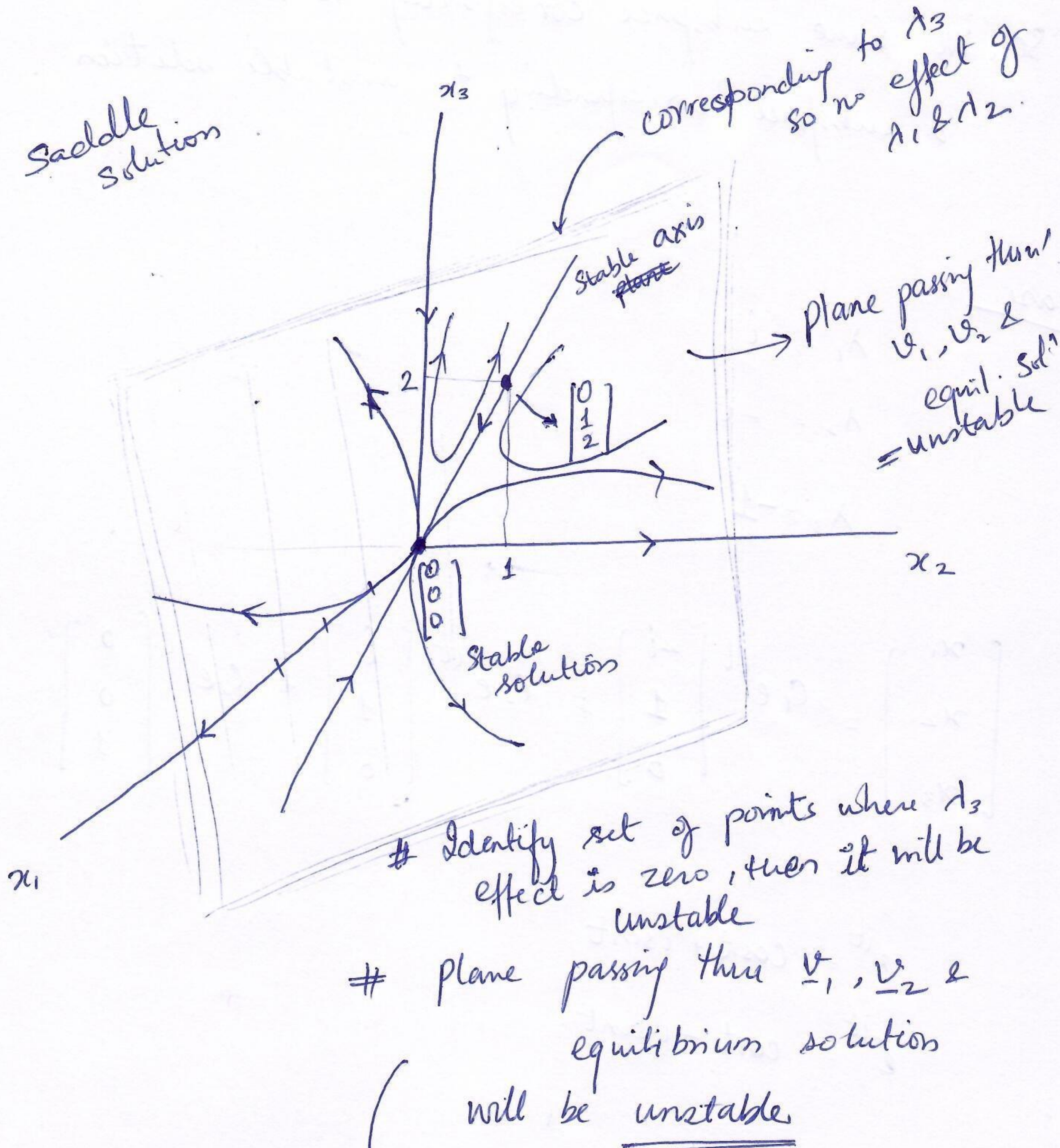
$\underline{v}_1 = \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}$ $\underline{v}_2 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ $\underline{v}_3 = \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix}$



equilibrium solution = $\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = C_1 e^{2t}\begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix} + C_2 e^{t}\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} + C_3 e^{-t}\begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix}$$

Saddle solution



Identify set of points where λ_3 effect is zero, then it will be unstable

Plane passing thru $\underline{v}_1$, $\underline{v}_2$ & equilibrium solution will be $\underline{\underline{\text{unstable}}}$

as $x_3 = 0$; all the points on this plane will cause instability.

Any point on this plane is unstable

So, we have subspace corresponding to stable solution & a subspace corresponding to unstable solution.

Case

$$\lambda_1 = i$$

$$\lambda_2 = -i$$

$$\lambda_3 = -1$$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = c_1 e^{it} \begin{bmatrix} -i \\ 1 \\ 0 \end{bmatrix} + c_2 e^{-it} \begin{bmatrix} i \\ 1 \\ 0 \end{bmatrix} + c_3 e^{-t} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$e^{it} = \cos t + i \sin t$$

$$e^{-it} = \cos t - i \sin t.$$

form of solution $= (Re) + i(Im)$.

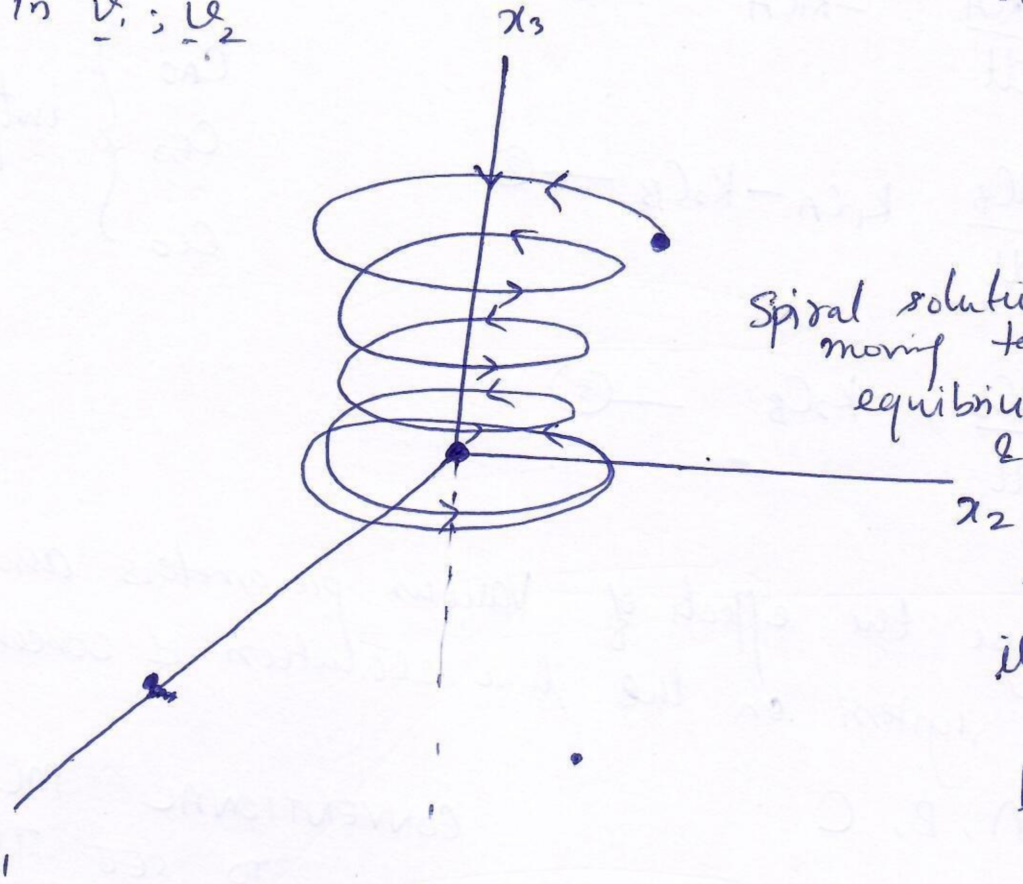
$=$ ~~[illegible]~~ .

finally . we have

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = d_1 \begin{bmatrix} \cos t \\ -\sin t \\ 0 \end{bmatrix} + d_2 \begin{bmatrix} \sin t \\ \cos t \\ 0 \end{bmatrix} + d_3 e^{-t} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

stable

as $x_3 = 0$ in $\underline{v}_1$; $\underline{v}_2$



Spiral solution moving towards equilibrium solution & on reaching $x_3 = 0$ it will become 2D. & become circular solution.

generalised Eigen vector ← Read about it.

Analysis of complex reaction systems

$$\frac{dC_A}{dt} = -K_1 C_A \quad —① \qquad A \xrightarrow{k_1} B \xrightarrow{k_2} C$$

$$\frac{dC_B}{dt} = k_1 C_A - K_2 C_B \quad —②$$

$\left.\begin{matrix} C_{A0} \\ C_{B0} \\ C_{c0} \end{matrix}\right\}$ initial conditions

$$\frac{dC_c}{dt} = k_2 C_B \quad —③$$

Q. Analyse the effects of various parameters associated with system on the time evolution of concentrations of A, B, C

CONVENTIONAL METHOD TO SEE TIME ~~VAP~~ EVOLUTION

eqⁿ ①

$$C_A = C_{A0} e^{-k_1 t}$$

eqⁿ ②.

$$\frac{dC_B}{dt} + k_2 C_B = k_1 C_{A0} e^{-k_1 t}$$

$$\hookrightarrow \frac{dx}{dt} + f(x) = g(t)$$

$$e^{k_2 t}\frac{dC_B}{dt} + k_2 e^{k_2 t} C_B = k_1 C_{A0} e^{-k_1 t} \cdot e^{k_2 t}$$

$$\frac{d}{dt}(e^{k_2 t} C_B) = k_1 C_{A0} e^{(k_2-k_1)t}$$

$$e^{k_2 t} C_B = \left(\frac{k_1}{k_2-k_1}\right) C_{A0} e^{(k_2-k_1)t} + C_1$$

at $t=0$; $C_B = C_{B0}$

$$C_{B0} = \left(\frac{k_1}{k_2-k_1}\right) C_{A0} + C_1$$

$$C_1 = C_{B0} - \left(\frac{k_1}{k_2-k_1}\right) C_{A0}$$

$$e^{k_2 t} C_B = \left(\frac{k_1}{k_2-k_1}\right) C_{A0} e^{(k_2-k_1)t} + C_{B0} - \left(\frac{k_1}{k_2-k_1}\right) C_{A0}$$

$$C_B = \left(\frac{k_1}{k_2-k_1}\right) C_{A0}\, e^{-k_2 t}\left(e^{(k_2-k_1)t} - 1\right) + e^{-k_2 t} C_{B0}$$

$$C_B = \left(\frac{k_1}{k_2-k_1}\right) C_{A0}\left(e^{-k_1 t} - e^{-k_2 t}\right) + C_{B0}\, e^{-k_2 t}$$

eqn (3)

$$\frac{dC_C}{dt} = k_2 C_B$$

$$\frac{dC_C}{dt} = \left(\frac{k_2 k_1}{k_2-k_1}\right) C_{A0}\left(e^{-k_1 t} - e^{-k_2 t}\right) + k_2 C_{B0}\, e^{-k_2 t}$$

$$C_c = \left(\frac{k_1 k_2}{k_2 - k_1}\right) C_{A0} \left(\frac{-1}{k_1}\right) e^{-k_1 t} - \left(\frac{k_1 k_2}{k_2 - k_1}\right) C_{A0} \left(\frac{-1}{k_2}\right) e^{-k_2 t} - C_{B0} e^{-k_2 t} + C_2$$

$$C_c = \left(\frac{-k_2}{k_2 - k_1}\right) C_{A0} e^{-k_1 t} + \left(\frac{k_1}{k_2 - k_1}\right) C_{A0} e^{-k_2 t} - C_{B0} e^{-k_2 t} + C_2$$

at $t = 0$ $\quad C_c = C_{C0}$

$$C_{C0} = \left(\frac{-k_2}{k_2 - 1}\right) C_{A0} + \left(\frac{k_1}{k_2 - k_1}\right) C_{A0} - C_{B0} + C_2$$

$$C_2 = C_{A0} + C_{B0} + C_{C0}$$

$$C_c = \left(\frac{-k_2}{k_2 - k_1}\right) C_{A0} e^{-k_1 t} + \left(\frac{k_1}{k_2 - k_1}\right) C_{A0} e^{k_2 t} - C_{B0} e^{-k_2 t} + C_{A0} + C_{B0} + C_{C0}$$

then we see how C_A, C_B & C_c are changing with time.

Matrix Solution

$$\frac{dC_A}{dt} = k_1 C_A$$

$$\frac{dC_B}{dt} = k_1 C_A - k_2 C_B$$

$$\frac{dC_C}{dt} = k_2 C_B$$

ORDER – 3

Linear system

DYNAMICAL VARIABLE $\rightarrow \begin{bmatrix} C_A \\ C_B \\ C_C \end{bmatrix}$

Autonomous eqn

$$\frac{d}{dt}\begin{bmatrix} C_A \\ C_B \\ C_C \end{bmatrix} = \underbrace{\begin{bmatrix} -k_1 & 0 & 0 \\ k_1 & k_2 & 0 \\ 0 & k_2 & 0 \end{bmatrix}}_{\underline{\underline{A}}} \begin{bmatrix} C_A \\ C_B \\ C_C \end{bmatrix}$$

$\Rightarrow \lambda_1 = 0 \quad \underline{v}_1 = [0 \; 0 \; 1]'$

$\lambda_2 = -k_1 \quad \underline{v}_2 = \begin{bmatrix} \frac{k_1 - k_2}{k_2} \\ \frac{-k_1}{k_2} \\ 1 \end{bmatrix}$

$\lambda_3 = -k_2$

$\hookrightarrow \underline{v}_3 = \begin{bmatrix} 0 \\ -1 \\ 1 \end{bmatrix}$

$$\frac{d\underline{X}}{dt} = \underline{\underline{A}}\,\underline{X}$$

at equilibrium

$C_{Ae} = 0$

$C_{Be} = 0$

$C_{Ce} = ?$ let it be α

$\alpha = C_{A0} + C_{B0} + C_{C0}$

Revise

E. Value are purely imaginary = oscillatory behaviour

Complex no. with real part >0 = oscillatory behaviour with increasing amplitude

Complex no. with real part <0 = oscillatory behaviour with decreasing amplitude

Real eigen values >0 = unstable

<0 = stable system

Some E. Value >0 & Some E. Value <0 = saddle solution

Case 1. $k_1 > 0$; $k_2 > 0$ $\Rightarrow$ $\lambda_2 < 0$; $\lambda_3 < 0$

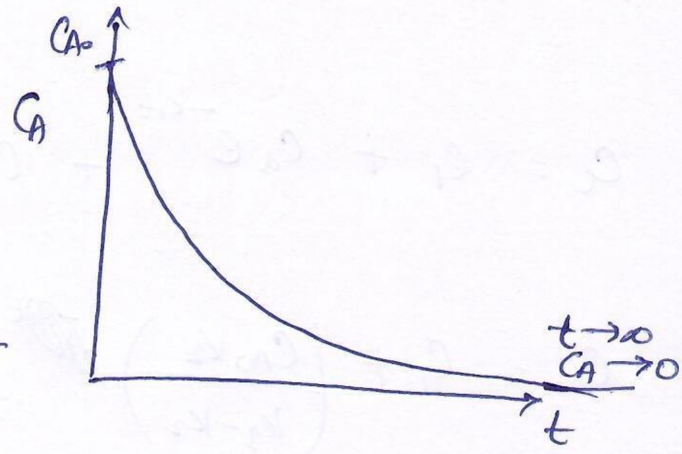
$\hookrightarrow$ Stable solution.

$\boxed{\lambda_1 = 0}$ $\leftarrow$ How will this impact the system ??

$$\begin{bmatrix} C_A \\ C_B \\ C_C \end{bmatrix} = C_1 \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} + C_2 e^{-k_1 t} \begin{bmatrix} \frac{k_1 - k_2}{k_2} \\ \frac{-k_1}{k_2} \\ 1 \end{bmatrix} + C_3 e^{-k_2 t} \begin{bmatrix} 0 \\ -1 \\ 1 \end{bmatrix}$$

$$C_A = C_2\left(\frac{k_1-k_2}{k_2}\right)e^{-k_1 t} \quad \left(\text{at } t=0 \quad C_A = C_{A0}\right)$$

$$C_{A0} = C_2\left(\frac{k_1-k_2}{k_2}\right)$$



$$C_2 = \left(\frac{C_{A0}\, k_2}{k_1-k_2}\right)$$

$$C_A = \left(\frac{k_2 C_{A0}}{k_1-k_2}\right)\left(\frac{k_1-k_2}{k_2}\right)e^{-k_1 t}$$

$$C_A = C_{A0}e^{-k_1 t}$$

$$C_B = C_2 e^{-kt}\left(\frac{-k_1}{k_2}\right) + C_3 e^{-k_2 t} \quad \text{at } t=0 \quad C_B = C_{B0}$$

$$C_{B0} = C_2\left(\frac{-k_1}{k_2}\right) - C_3$$

$$C_3 = C_2\left(\frac{-k_1}{k_2}\right) - C_{B0}$$

$$C_3 = \left(\frac{C_{A0}\, k_2}{k_1-k_2}\right)\left(\frac{-k_1}{k_2}\right) - C_{B0}$$

$$C_B = \left(\frac{C_{A0}\, k_2}{k_1-k_2}\right)e^{-k_1 t}\left(\frac{-k_1}{k_2}\right) - \left(\frac{C_{A0}\, k_2}{k_1-k_2}\right)\left(\frac{-k_1}{k_2}\right)e^{-k_2 t} + C_{B0}e^{-k_2 t}$$

$$C_B = \left(\frac{C_{A0}\, \cancel{k_2}}{k_1-k_2}\right)\left(\frac{-k_1}{\cancel{k_2}}\right)\left[e^{-k_1 t} - e^{-k_2 t}\right] + C_{B0}e^{-k_2 t}$$

$$C_B = \left(\frac{k_1 C_{A0}}{k_1 - k_2}\right)\left[e^{-k_1 t} - e^{-k_2 t}\right] + C_{B0}\, e^{-k_2 t}$$

$$C_C = C_1 + C_2 e^{-k_1 t} + C_3 e^{-k_2 t}$$

at $t=0$

$C_C = C_{C0}$

$$C_{C0} = C_1 + \left(\frac{C_{A0} k_2}{k_1 - k_2}\right)\cancel{e^{-k_1 t}} + \left(\frac{-C_{A0} k_1}{k_1 - k_2}\right)\cancel{e^{-k_2 t}} - C_{B0}\cancel{e^{-k_2 t}}$$

$$C_1 = C_{C0} - \left(\frac{C_{A0} k_2}{k_1 - k_2}\right)\cancel{e^{-k_1 t}} + \left(\frac{C_{A0} k_1}{k_1 - k_2}\right)\cancel{e^{-k_2 t}} + C_{B0}\cancel{e^{-k_2 t}}$$

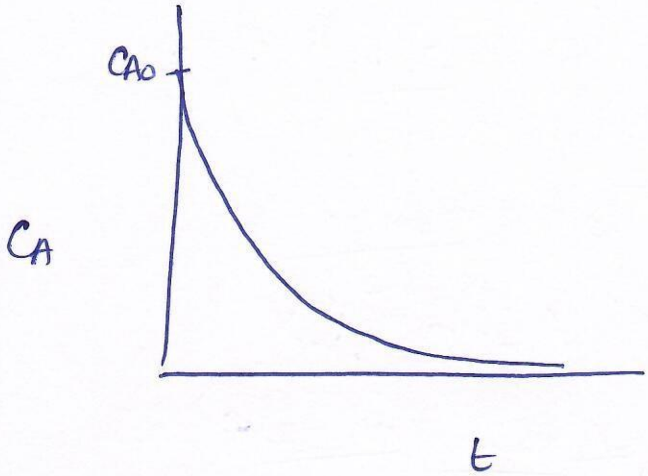
$$C_1 = C_{C0} - \left(\frac{C_{A0}}{k_1 - k_2}\right)\left(k_2 \cancel{e^{-k_1 t}} + k_1 \cancel{e^{-k_2 t}}\right) + C_{B0}\cancel{e^{-k_2 t}}$$

$$C_1 = C_{A0} + C_{B0} + C_{C0}$$

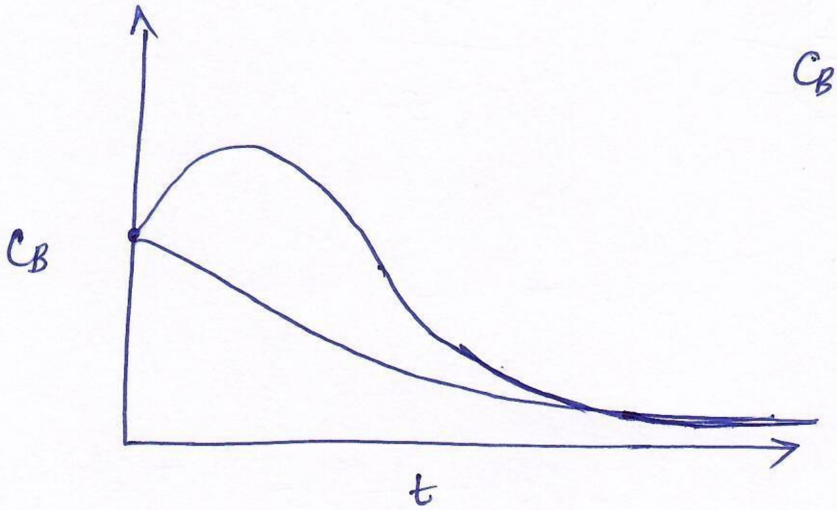
$\cancel{C_0 = C_0}$ -

$$C_C = C_{A0} + C_{B0} + C_{C0} + \left(\frac{C_{A0} k_2}{k_1 - k_2}\right) e^{-k_1 t} + \left(\frac{C_{A0} k_1}{k_2 - k_1}\right) e^{-k_2 t} - C_{B0}\, e^{-k_2 t}$$

Finally, we have



$C_A = C_{A0} e^{-k_1 t}$



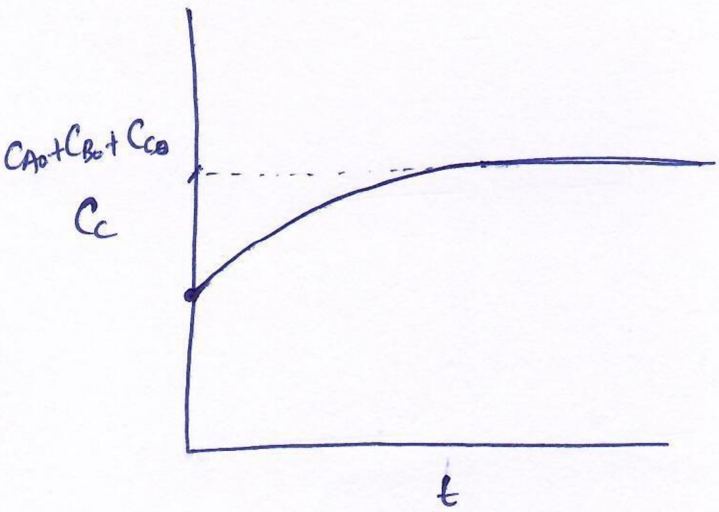
$C_B = (\quad)[e^{-ve} - e^{-ve}] +$

$(\,) e^{-ve}$

time $\to \infty$

$C_B \to 0$

depending on initial condition we can have any trajectory



$C_C = [\text{const}]_0 + (\quad) e^{-ve} +$

$(\,) e^{-ve} + (\quad) e^{-ve}$

so as $t \to \infty$

$e^{-ve} \to 0$

so effect of exp. part diminishes.

so at $t \to \infty$

$C_C \to$ constt