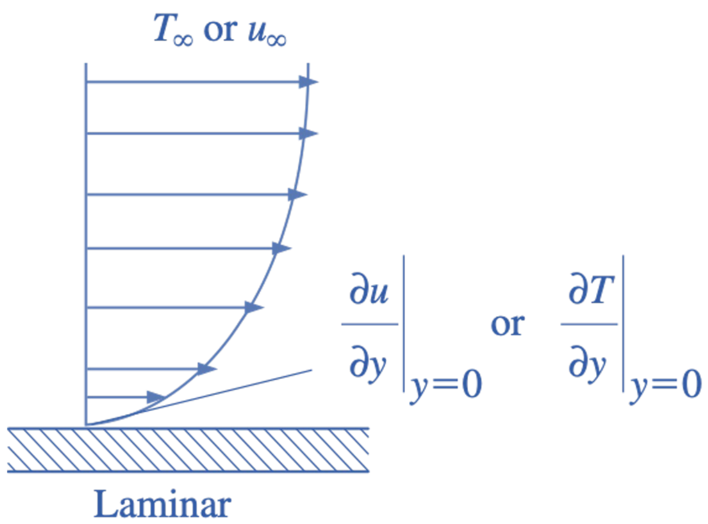
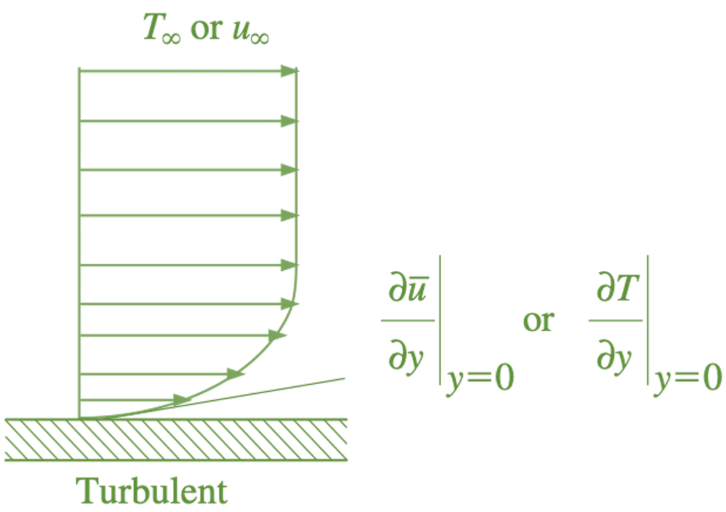
HEAT TRANSFER

[CH21204]

January 20, 2023

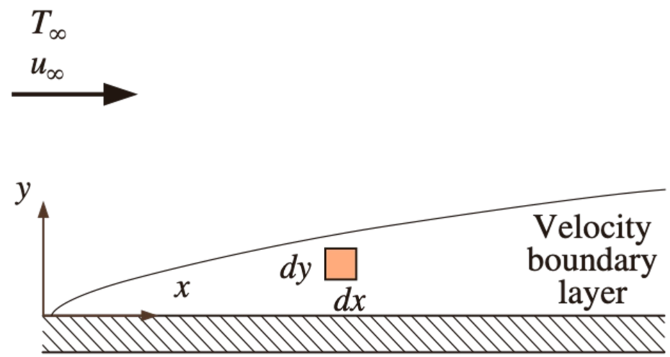




The velocity and temperature gradients at the wall, and thus the wall shear stress and heat transfer rate, are much larger for turbulent flow than they are for laminar flow.

DIFFERENTIAL CONVECTION EQUATIONS:

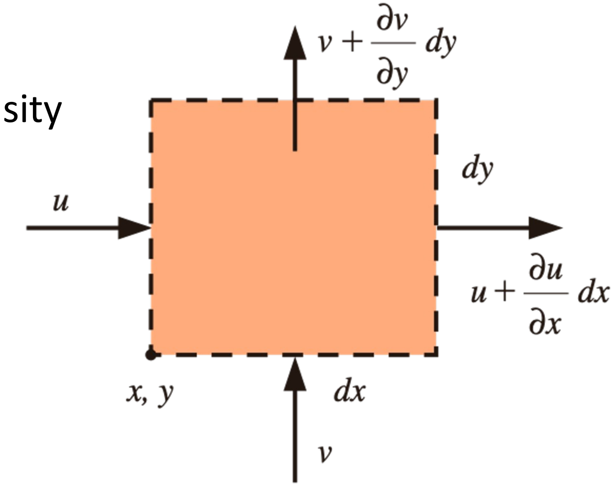
Conservation of Mass Equation:



$$\begin{pmatrix}\text{Rate of mass flow}\\ \text{into the control volume}\end{pmatrix} = \begin{pmatrix}\text{Rate of mass flow}\\ \text{out of the control volume}\end{pmatrix}$$

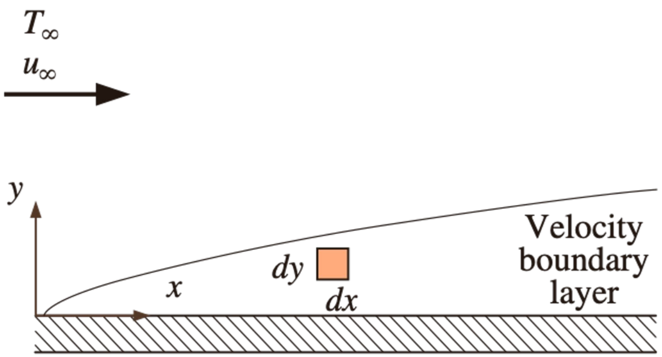
$$\boxed{\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0}$$

steady two-dimensional flow of a fluid with constant density



DIFFERENTIAL CONVECTION EQUATIONS:

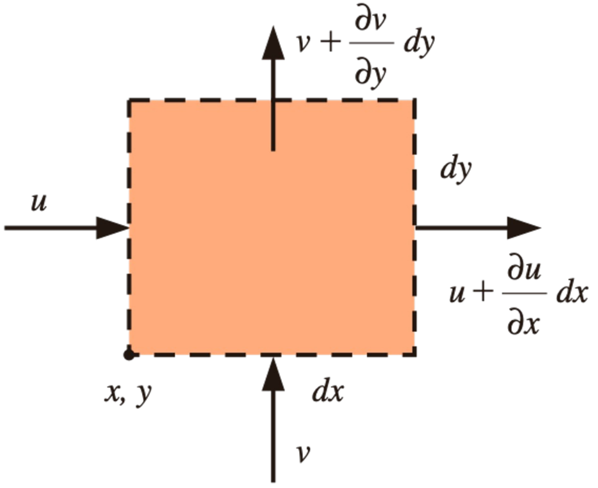
Conservation of Momentum Equation:



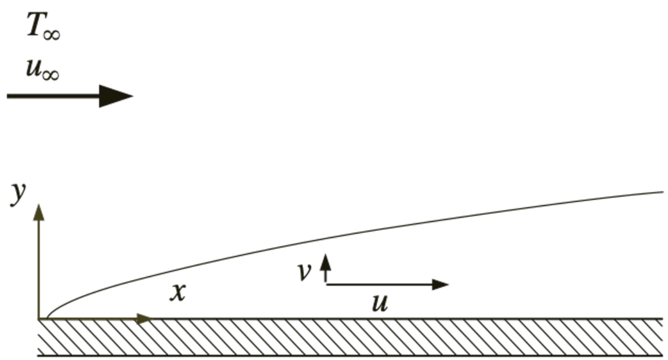
$$(\text{Mass})\begin{pmatrix}\text{Acceleration}\\ \text{in a specified direction}\end{pmatrix} = \begin{pmatrix}\text{Net force (body and surface)}\\ \text{acting in that direction}\end{pmatrix}$$

$$\rho\left(u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right) = \mu\frac{\partial^2 u}{\partial y^2} - \frac{\partial P}{\partial x}$$

x-momentum equation



Boundary layer approximations



1) Velocity components:

   $v \ll u$

2) Velocity grandients:

   $\frac{\partial v}{\partial x} \approx 0, \frac{\partial v}{\partial y} \approx 0$

   $\frac{\partial u}{\partial x} \ll \frac{\partial u}{\partial y}$

3) Temperature gradients:

   $\frac{\partial T}{\partial x} \ll \frac{\partial T}{\partial y}$

the velocity and temperature gradients normal to the surface are much greater than those along the surface.

When gravity effects and other body forces are *negligible* and the boundary layer approximations are *valid*, applying Newton's second law of motion on the volume element in the **y-direction** gives the **y-momentum equation:**

$$\frac{\partial P}{\partial y} = 0$$

$$P = P(x) \text{ and } \partial P/\partial x = dP/dx.$$

The velocity components in the free stream region of a flat plate are:

$$u = u_\infty \qquad v = 0$$

$$\partial P/\partial x = 0.$$

$$\rho C_p\left(u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}\right)=k\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}\right)$$

The net energy convected by the fluid out of the control volume =
The net energy transferred into the control volume by heat conduction

$$\rho C_p\left(u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}\right)=k\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}\right)+\mu\Phi$$

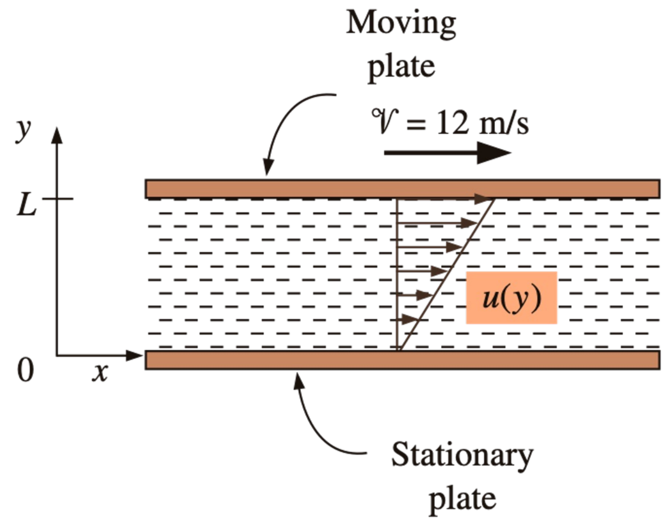
viscous dissipation function

$$\Phi=2\left[\left(\frac{\partial u}{\partial x}\right)^2+\left(\frac{\partial v}{\partial y}\right)^2\right]+\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right)^2$$

- Viscous dissipation may play a dominant role in high-speed flows, especially when the viscosity of the fluid is high (like the flow of oil in journal bearings).

- This manifests itself as a significant rise in fluid temperature due to the conversion of the kinetic energy of the fluid to thermal energy.

- Viscous dissipation is also significant for high-speed flights of aircraft.



- Obtain relations for the velocity and temperature distributions in the oil
- Determine the maximum temperature in the oil and the heat flux from oil to each plate

$$k = 0.145 \text{ W/m} \cdot \text{K} \quad \text{and} \quad \mu = 0.800 \text{ kg/m} \cdot \text{s} = 0.800 \text{ N} \cdot \text{s/m}^2$$

1. Steady operating conditions exist.
2. Oil is an incompressible substance with constant properties.
3. Body forces such as gravity are negligible.
4. The plates are large so that there is no variation in the z-direction.

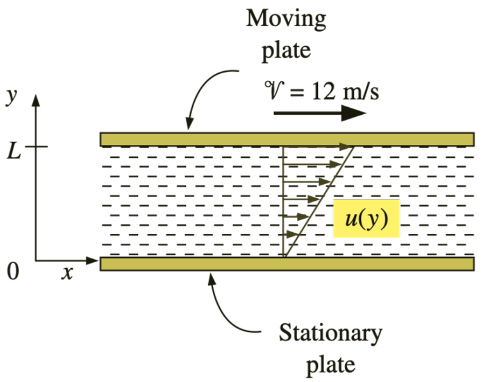
This is parallel flow between two plates, and thus v = 0.

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \rightarrow \frac{\partial u}{\partial x} = 0 \quad \rightarrow u = u(y)$$

the x-component of velocity does not change in the flow direction
(i.e., the velocity profile remains unchanged)

$$\partial P/\partial x = 0$$

$$x\text{-momentum: } \rho\left(u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right) = \mu\frac{\partial^2 u}{\partial y^2} - \frac{\partial P}{\partial x} \qquad \rightarrow \qquad \frac{d^2 u}{dy^2} = 0$$



$$u(y) = C_1 y + C_2$$

$$u(0) = 0 \text{ and } u(L) = \mathcal{V}$$

$$u(y) = \frac{y}{L}\mathcal{V}$$

Frictional heating due to viscous dissipation in this case is significant because of the high viscosity of oil and the large plate velocity.

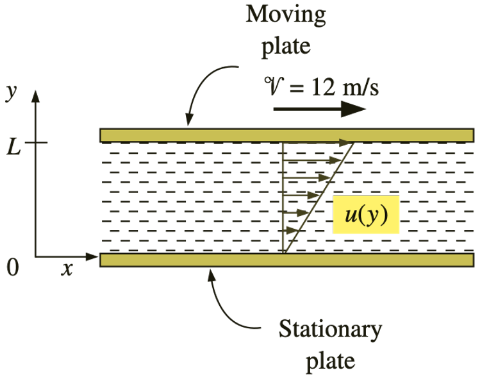
$$T = T(y).$$

$$\textit{Energy:} \qquad 0 = k\frac{\partial^2 T}{\partial y^2} + \mu\left(\frac{\partial u}{\partial y}\right)^2 \quad \rightarrow \quad k\frac{d^2T}{dy^2} = -\mu\left(\frac{\mathcal{V}}{L}\right)^2$$

$$\partial u/\partial y = \mathcal{V}/L$$

$$T(y) = -\frac{\mu}{2k}\left(\frac{y}{L}\mathcal{V}\right)^2 + C_3 y + C_4$$

$$T(0) = T_0 \text{ and } T(L) = T_0$$



$$T(y) = T_0 + \frac{\mu\mathcal{V}^2}{2k}\left(\frac{y}{L} - \frac{y^2}{L^2}\right)$$

$$\frac{dT}{dy} = \frac{\mu \mathcal{V}^2}{2kL}\left(1 - 2\frac{y}{L}\right)$$

$$\frac{dT}{dy} = \frac{\mu \mathcal{V}^2}{2kL}\left(1 - 2\frac{y}{L}\right) = 0 \qquad \rightarrow \qquad y = \frac{L}{2}$$

$$T_{\max} = T\left(\frac{L}{2}\right) = T_0 + \frac{\mu \mathcal{V}^2}{2k}\left(\frac{L/2}{L} - \frac{(L/2)^2}{L^2}\right) = T_0 + \frac{\mu \mathcal{V}^2}{8k}$$

$$\dot{q}_0 = -k\left.\frac{dT}{dy}\right|_{y=0} = -k\frac{\mu \mathcal{V}^2}{2kL}(1 - 0) = -\frac{\mu \mathcal{V}^2}{2L}$$

$$\dot{q}_L = -k\left.\frac{dT}{dy}\right|_{y=L} = -k\frac{\mu \mathcal{V}^2}{2kL}(1 - 2) = \frac{\mu \mathcal{V}^2}{2L}$$