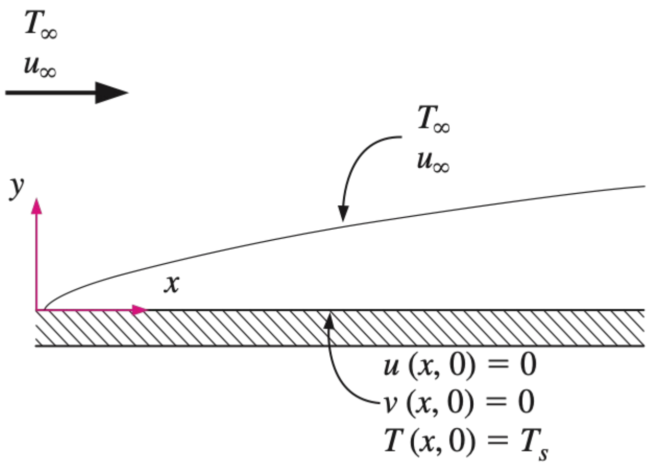
HEAT TRANSFER

[CH21204]

January 25, 2023



Continuity: $$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

Momentum: $$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \nu\frac{\partial^2 u}{\partial y^2}$$

Energy: $$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2}$$

At $x = 0$: $u(0, y) = u_\infty$, $T(0, y) = T_\infty$

At $y = 0$: $u(x, 0) = 0$, $v(x, 0) = 0$, $T(x, 0) = T_s$

As $y \to \infty$: $u(x, \infty) = u_\infty$, $T(x, \infty) = T_\infty$

$$\delta = \frac{5.0}{\sqrt{u_\infty/\nu x}} = \frac{5.0x}{\sqrt{\mathrm{Re}_x}}$$

$$C_{f,x} = \frac{\tau_w}{\rho \mathcal{V}^2/2} = \frac{\tau_w}{\rho u_\infty^2/2} = 0.664\ \mathrm{Re}_x^{-1/2}$$

$$\mathrm{Nu}_x = \frac{h_x x}{k} = 0.332\ \mathrm{Pr}^{1/3}\mathrm{Re}_x^{1/2} \qquad \mathrm{Pr} > 0.6 \qquad T_f = (T_s + T_\infty)/2.$$

$$\delta_t = \frac{\delta}{\mathrm{Pr}^{1/3}} = \frac{5.0x}{\mathrm{Pr}^{1/3}\sqrt{\mathrm{Re}_x}}$$

laminar flow over an isothermal flat plate

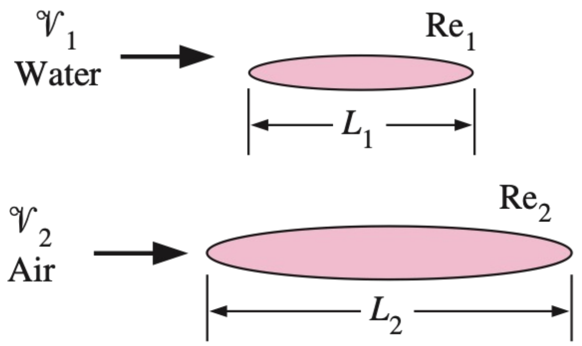
$$x^* = \frac{x}{L}, \quad y^* = \frac{y}{L}, \quad u^* = \frac{u}{\mathcal{V}}, \quad v^* = \frac{v}{\mathcal{V}}, \quad P^* = \frac{P}{\rho \mathcal{V}^2}, \quad \text{and} \quad T^* = \frac{T - T_s}{T_\infty - T_s}$$

Continuity: $$\frac{\partial u^*}{\partial x^*} + \frac{\partial v^*}{\partial y^*} = 0$$

Momentum: $$u^* \frac{\partial u^*}{\partial x^*} + v^* \frac{\partial u^*}{\partial y^*} = \frac{1}{\mathrm{Re}_L} \frac{\partial^2 u^*}{\partial y^{*2}} - \frac{dP^*}{dx^*}$$

Energy: $$u^* \frac{\partial T^*}{\partial x^*} + v^* \frac{\partial T^*}{\partial y^*} = \frac{1}{\mathrm{Re}_L \mathrm{Pr}} \frac{\partial^2 T^*}{\partial y^{*2}}$$

$$u^*(0, y^*) = 1, \quad u^*(x^*, 0) = 0, \quad u^*(x^*, \infty) = 1, \quad v^*(x^*, 0) = 0,$$
$$T^*(0, y^*) = 1, \quad T^*(x^*, 0) = 0, \quad T^*(x^*, \infty) = 1$$



Parameters before nondimensionalizing

$L, \mathcal{V}, T_\infty, T_s, \nu, \alpha$

Parameters after nondimensionalizing:

Re, Pr

If $\mathrm{Re}_1 = \mathrm{Re}_2$, then $C_{f1} = C_{f2}$

Local Nusselt number:

$$\mathrm{Nu}_x = \text{function}\,(x^*, \mathrm{Re}_L, \mathrm{Pr})$$

Average Nusselt number:

$$\mathrm{Nu} = \text{function}\,(\mathrm{Re}_L, \mathrm{Pr})$$

A common form of Nusselt number:

$$Nu = C\,\mathrm{Re}_L^m \mathrm{Pr}^n$$

$$C_f = f_4(\mathrm{Re}_L)$$

Momentum: Reynolds analogy

Heat: Chilton–Colburn analogy

$$u^*\frac{\partial u^*}{\partial x^*} + v^*\frac{\partial u^*}{\partial y^*} = \frac{1}{\mathrm{Re}_L}\frac{\partial^2 u^*}{\partial y^{*2}} - \frac{dP^*}{dx^*}$$

$$u^*\frac{\partial T^*}{\partial x^*} + v^*\frac{\partial T^*}{\partial y^*} = \frac{1}{\mathrm{Re}_L\,\mathrm{Pr}}\frac{\partial^2 T^*}{\partial y^{*2}}$$

$$u^*\frac{\partial u^*}{\partial x^*} + v^*\frac{\partial u^*}{\partial y^*} = \frac{1}{\mathrm{Re}_L}\frac{\partial^2 u^*}{\partial y^{*2}}$$

$$u^*\frac{\partial T^*}{\partial x^*} + v^*\frac{\partial T^*}{\partial y^*} = \frac{1}{\mathrm{Re}_L}\frac{\partial^2 T^*}{\partial y^{*2}}$$

$$\left.\frac{\partial u^*}{\partial y^*}\right|_{y^*=0} = \left.\frac{\partial T^*}{\partial y^*}\right|_{y^*=0}$$

$$\tau_s = \mu \frac{\partial u}{\partial y}\bigg|_{y=0} = \frac{\mu \mathcal{V}}{L} \frac{\partial u^*}{\partial y^*}\bigg|_{y^*=0} = \frac{\mu \mathcal{V}}{L} f_2(x^*, \mathrm{Re}_L)$$

$$C_{f,x} = \frac{\tau_s}{\rho \mathcal{V}^2/2} = \frac{\mu \mathcal{V}/L}{\rho \mathcal{V}^2/2} f_2(x^*, \mathrm{Re}_L) = \frac{2}{\mathrm{Re}_L} f_2(x^*, \mathrm{Re}_l) = f_3(x^*, \mathrm{Re}_L)$$

$$\mathrm{Nu}_x = \frac{hL}{k} = \frac{\partial T^*}{\partial y^*}\bigg|_{y^*=0} = g_2(x^*, \mathrm{Re}_L, \mathrm{Pr})$$

$$C_{f,x} \frac{\mathrm{Re}_L}{2} = \mathrm{Nu}_x \qquad (\mathrm{Pr} = 1)$$

Profiles:	$u^* = T$
Gradients:	$\frac{\partial u^*}{\partial y^*}\Big\vert_{y^*=0} = \frac{\partial T^*}{\partial y^*}\Big\vert_{y^*=0}$
Analogy:	$C_{f,x} \frac{\mathrm{Re}_L}{2} = \mathrm{Nu}_x$

$$\frac{C_{f,x}}{2} = \mathrm{St}_x \qquad (\mathrm{Pr} = 1)$$

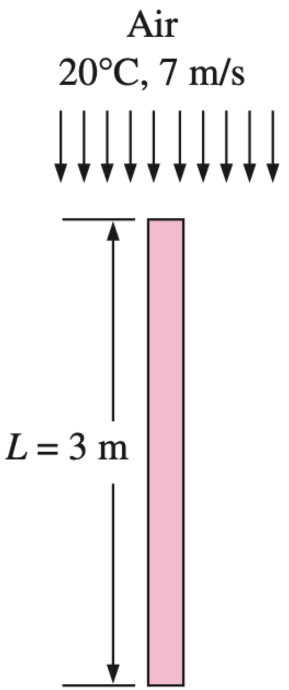
$$\mathrm{St} = \frac{h}{\rho C_p \mathcal{V}} = \frac{\mathrm{Nu}}{\mathrm{Re}_L \mathrm{Pr}}$$

$$C_{f,x} = 0.664\,\mathrm{Re}_x^{-1/2} \qquad \text{and} \qquad \mathrm{Nu}_x = 0.332\,\mathrm{Pr}^{1/3}\,\mathrm{Re}_x^{1/2}$$

Modified Reynolds analogy or Chilton–Colburn analogy:

$$C_{f,x}\frac{Re_L}{2} = \mathrm{Nu}_x\,\mathrm{Pr}^{-1/3} \qquad \text{or} \qquad \frac{C_{f,x}}{2} = \frac{h_x}{\rho C_p \mathcal{V}}\mathrm{Pr}^{2/3} \equiv j_H \qquad 0.6 < \mathrm{Pr} < 60.$$

Colburn j-factor



A 2-m 3-m flat plate is suspended in a room, and is subjected to air flow parallel to its surfaces along its 3-m-long side. The free stream temperature and velocity of air are 20˚C and 7 m/s. The total drag force acting on the plate is measured to be 0.86 N.

Calculate the average convection heat transfer coefficient.

$$\rho = 1.204\ \text{kg/m}^3, \quad C_p = 1.007\ \text{kJ/kg}\cdot\text{K}, \quad \text{Pr} = 0.7309$$

$$F_f = C_f A_s \frac{\rho \mathcal{V}^2}{2}$$

$$A_s = 2WL = 2(2\ \text{m})(3\ \text{m}) = 12\ \text{m}^2$$

$$C_f = \frac{F_f}{\rho A_s \mathcal{V}^2/2} = \frac{0.86\ \text{N}}{(1.204\ \text{kg/m}^3)(12\ \text{m}^2)(7\ \text{m/s})^2/2}\left(\frac{1\text{kg}\cdot\text{m/s}^2}{1\ \text{N}}\right) = 0.00243$$

$$h = \frac{C_f}{2}\frac{\rho \mathcal{V} C_p}{\text{Pr}^{2/3}} = \frac{0.00243}{2}\frac{(1.204\ \text{kg/m}^3)(7\ \text{m/s})(1007\ \text{J/kg}\cdot{}^\circ\text{C})}{0.7309^{2/3}} = 12.7\ \text{W/m}^2\cdot{}^\circ\text{C}$$