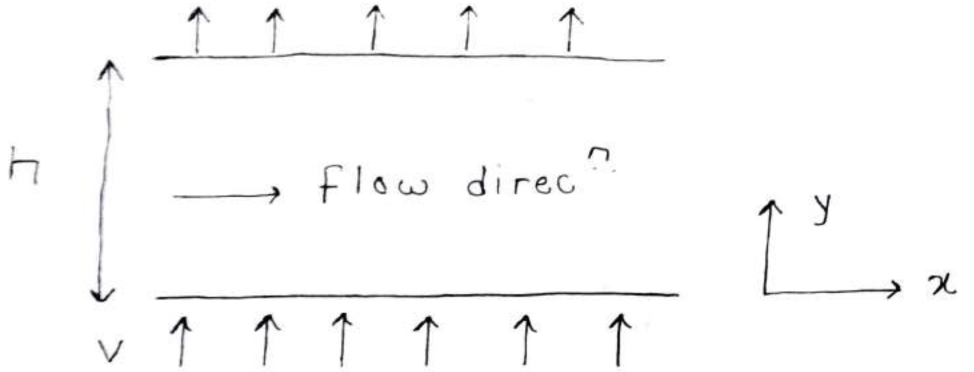
| Momentum | Heat | Mass |
|---|---|---|
| $\nabla v$ | $\nabla T$ | $\nabla C$ |
| $\tau_{yx} = -\mu \frac{dv}{dy}$ | $q = -k \frac{dT}{dy}$ | $N_A = -D_{AB} \frac{dC}{dy}$ |
| Tensor (grad. of vector) | Vector | Vector |

→ High viscosity oil spreads on solid surface. No slip condition not applicable.

- Fundamental Engg. Science
- Applications – Traditional ($M^2$, Heat, Mass)
  – New (Bio, Micro, Nano)
- Similarities / Dissimilarities – ($M^2$, Mass, Heat)
- Physical Insight
- Conservation / Flux
- Boundary Layers
- Similarity parameters
- Simultaneous heat, mass and momentum transfer
- Design eq$^n$ / Numerical sol$^n$

3/01/19

Q1




. Constant pressure gradient
. Two plates are porous and stationary
. Steady, fully developed flow.
. No body forces.

Solⁿ:

x component of the N-S eqⁿ

$$\rho\left(\frac{\partial v_x}{\partial t} + v_x\frac{\partial v_x}{\partial x} + v_y\frac{\partial v_x}{\partial y} + v_z\frac{\partial v_x}{\partial z}\right) = -\frac{\partial P}{\partial x}$$

$$+ \mu\left[\frac{\partial^2 v_x}{\partial x^2} + \frac{\partial^2 v_x}{\partial y^2} + \frac{\partial^2 v_x}{\partial z^2}\right] + \rho g$$

$$\rho v_y \frac{\partial v_x}{\partial y} = -\frac{\partial P}{\partial x} + \mu\frac{\partial^2 v_x}{\partial y^2}$$

$$\rho v_y \frac{d v_x}{d y} = -\underbrace{\frac{\partial P}{\partial x}}_{A} + \mu\frac{\partial^2 v_x}{\partial y^2}$$

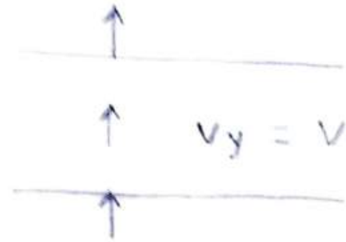
$\left\{ \frac{\partial^2 v_x}{\partial z^2} = 0 \right.$ only if the z dimension is very large compared to y $\}$

$v_y$ can be obtained from eqⁿ of continuity.

$$\underbrace{\cancel{\frac{\partial v_x}{\partial x}}}_{0} + \frac{\partial v_y}{\partial y} + \underbrace{\cancel{\frac{\partial v_z}{\partial z}}}_{0} = 0$$

B dc/dy
rface
not
entum

$$\Rightarrow \frac{\partial v_y}{\partial y} = 0$$




$$\Rightarrow v_y = \text{constant}$$

$$\Rightarrow v_y\big|_{y=0} = V$$

Governing eq$^n$

$$V \frac{dv_x}{dy} = A + \mu \frac{d^2 v_x}{dy^2}$$

Boundary conditions :-

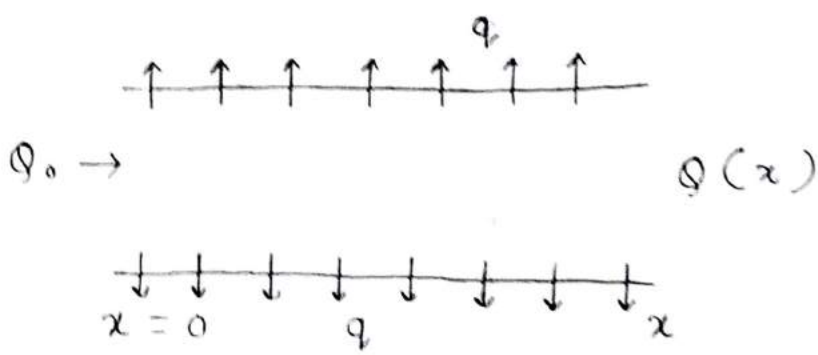
No slip $\Rightarrow$ $v_x = 0$ @ $y = 0$

$v_x = 0$ @ $y = h$

Sol$^n$ :-

$$u_x = \frac{h}{\rho V}\left(\frac{\partial p}{\partial x}\right)\left[\frac{1 - \exp\left(\frac{\rho V y}{\mu}\right)}{1 - \exp\left(\frac{\rho V h}{\mu}\right)} - \frac{y}{h}\right]$$

2.




$h << L$

* Pressure gradient is constant only if cross-sectional area and $\rho$ are constant and their is no suction

- Pressure decreases as a function of x
- Flow is dominated by viscous effects and is steady
- Length is L, Depth is W and

separation is $h$ $q$ has the unit m/s

a) Find the pressure distribution, $P(x)$ in the flow channel if $q = 0$

$x$ component of the NS eq$^n$ gets simplified to

$$\mu \frac{d^2 v_x}{dy^2} - \frac{dP}{dx} = 0$$

{ steady state, fully-developed, body forces neglected }

The is no injection and suction

$$\therefore \frac{dP}{dx} = \text{constant} = A$$

$$\mu \frac{d^2 v_x}{dy^2} = A$$

$$\mu \frac{d v_x}{dy} = A y + c_1$$

$$\mu v_x = A \frac{y^2}{2} + c_1 y + c_2$$

Boundary conditions :-

$y = 0$ $\quad v_x = 0$ $\Big\}$ No slip
$y = h$ $\quad v_x = 0$

$$\therefore v_x = \frac{1}{2\mu} A y^2 \left[ 1 - \frac{h}{y} \right]$$

$$= \frac{1}{2\mu} \left( \frac{\partial P}{\partial x} \right) y^2 \left[ 1 - \frac{h}{y} \right]$$

$$\langle v_x \rangle = \frac{1}{h} \int_0^h v_x \, dy = - \frac{1}{12\mu} \frac{\partial P}{\partial x} h^2$$

$$\therefore Q_0 = Wh \langle v_x \rangle = - \frac{1}{12\mu} \frac{\partial P}{\partial x} h^3 W.$$

$Q_0$ = Flow rate with $q = 0$.

$$\therefore \frac{dP}{dx} = -\frac{12\mu}{Wh^3} Q_0$$

Integrating and imposing the B.C that $P(x=0) = P_0$

$$\therefore P(x) = P_0 - \frac{12\mu Q_0}{Wh^3} x$$

b) Find the pressure distribution $P(x)$ in the channel for $q \neq 0$.

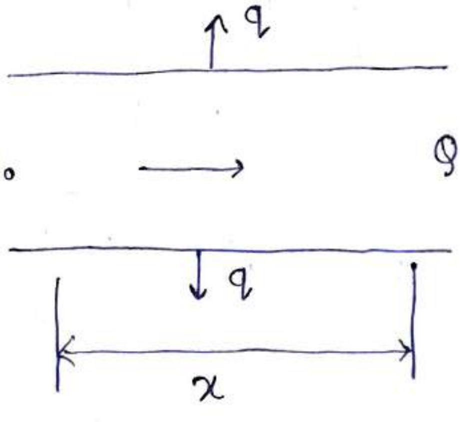
$$v_x = \frac{1}{2\mu}\left(\frac{\partial P}{\partial x}\right) y^2 \left[1 - \frac{h}{y}\right]$$

$$Q_0 = Wh\langle v_x\rangle = -\frac{1}{12\mu}\frac{dP}{dx} h^3 W$$

$$\therefore \frac{dP}{dx} = -\frac{12\mu}{Wh^3} Q_0 \longrightarrow \text{will change with } x.$$

$Q(x)$ needs to be found out.

Applying conservation of mass




$$Q(x) = Q_0 - 2qWx$$

We have $\dfrac{dP}{dx} = -\dfrac{12\mu}{Wh^3} Q(x)$

$$\Rightarrow \quad \frac{dP}{dx} = -\frac{12\mu\, Q(x)}{Wh^3} = \frac{-12\mu}{Wh^3}\left(Q_0 - 2qWx\right)$$

$$\Downarrow \quad P = P_0 \quad \text{at} \quad x = 0$$

$$P(x) = P_0 - \frac{12\mu x}{Wh^3}\left[Q_0 - \frac{\cancel{2}qWx}{\cancel{2}}\right]$$

c) Given that $P(x = L) = P_e$, find $q$.

d) Find the criterion necessary for viscous terms to dominate

$$\frac{\rho\left(\overbrace{v_x \frac{\partial v_x}{\partial x}}^{\mathrm{I}} + \overbrace{v_y \frac{\partial v_x}{\partial y}}^{\mathrm{II}}\right)}{\mu\left(\frac{\partial^2 v_x}{\partial x^2} + \frac{\partial^2 v_x}{\partial y^2}\right)} << 1 \qquad - (1)$$

$$\frac{\partial^2 v_x}{\partial x^2} << \frac{\partial^2 v_x}{\partial y^2} \Rightarrow \frac{\partial^2 v_x}{\partial x^2} \approx 0$$

$$\mu \frac{\partial^2 v_x}{\partial y^2} \approx \mu \frac{v_x}{h^2}$$

$$\frac{\partial v_x}{\partial x} \sim \frac{\partial}{\partial x}\left(Q/A\right) = \frac{d}{dx}\left(Q/A\right)$$

$$= \frac{1}{Wh}\frac{d}{dx}\left(Q_0 - 2qWx\right) = \frac{-2q}{h}$$

Ignoring the sign

$$\frac{\partial v_x}{\partial x} = \frac{2q}{h} \sim \frac{q}{h}$$

Term I becomes $v_x \frac{q}{h}$

Term II :

$v_y \approx q$ $\qquad\qquad \dfrac{\partial v_x}{\partial y} \approx \dfrac{v_x}{h}$

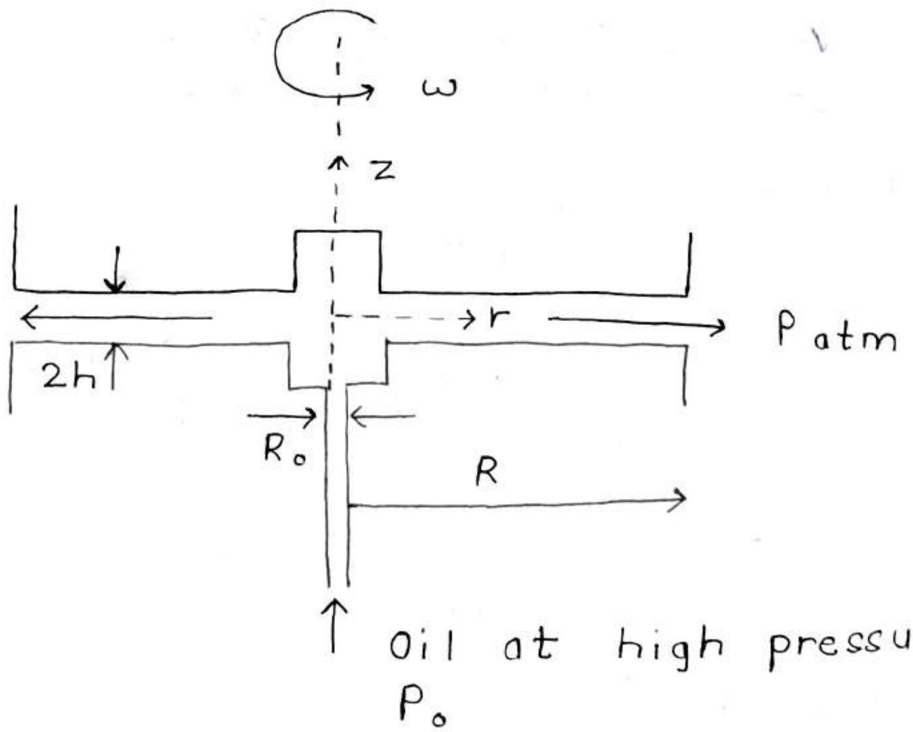
Eq$^n$ ① becomes

$$\frac{\rho\left(v_x \frac{q}{h} + q \frac{v_x}{h}\right)}{\mu v_x / h^2} << 1$$

$$\frac{q \rho h}{\mu} << 1$$ $q$ is the suction velocity

For our previous analysis to be valid, the reynolds no. based on suction velocity should be less than 1.

Q.




- Upper plate is rotating at an angular velocity $\omega$.
- Bottom plate is stationary.
- Pressure varies from $R_0$ to $R$, $P = P_0$ upto $R_0$. Convective and body force terms can be neglected.

Sol$^n$:

Velocity has two components $v_r$ and $v_\theta$. $v_z$ can be neglected as no significant motion in z direction.

i) Start with the continuity eq$^n$ to get the functional form of $v_r$.

$$\frac{1}{r}\frac{\partial}{\partial r}(\rho r v_r) + \underbrace{\cancel{\frac{\partial \rho}{\partial t}}}_{0} + \frac{1}{r}\frac{\partial}{\partial \theta}\underbrace{(\rho \cancel{v_\theta})}_{0} + \frac{\partial}{\partial z}(\rho v_z) = 0$$

$$\frac{1}{r}\frac{\partial}{\partial r}(\rho r v_r) = 0$$

$$\Rightarrow r v_r = \text{constant}$$

$$\Rightarrow v_r = \frac{c}{r}$$

$$c = f(z)$$

$$\Rightarrow v_r = \frac{f(z)}{r} \Rightarrow v_r = v_r(r, z)$$

ii) Show that $v_\theta = \frac{\omega r}{2}\left(1+\frac{z}{h}\right)$ satisfies the $\theta$ component of NS eq$^n$.

L.H.S of the $\theta$ component of NS eq$^n$ is zero as convective terms are neglected.

$$\frac{\partial p}{\partial \theta} = 0 \quad \{\text{angular symmetry}\}$$

$$\frac{1}{r^2}\frac{\partial^2 v_\theta}{\partial \theta^2} = 0 \quad \{\quad " \quad\}$$

$$\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(r v_\theta)\right) + \frac{\partial^2 v_\theta}{\partial z^2} = 0$$

iii) Write the r component of N.S. eq$^n$ to show that $r\frac{\partial P}{\partial r}$ is a constant.

L.H.S. = 0

$$\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(r v_r)\right) = 0$$

as $r v_r$ is a f$^n$ of z.

$$-\frac{\partial P}{\partial r} = -\frac{\partial^2 v_r}{\partial z^2}\mu$$

or $$-\frac{\partial P}{\partial r} + \mu\frac{\partial^2 v_r}{\partial z^2} = 0$$

$$r\frac{\partial P}{\partial r} = r\mu\frac{\partial^2 v_r}{\partial z^2} = \mu\frac{\partial^2 (\mu v_r)}{\partial z^2}$$

$$\underbrace{r\frac{\partial P}{\partial r}}_{f(r)} = \underbrace{\mu\frac{\partial^2 (r v_r)}{\partial z^2}}_{f(z)} = K$$

$P \neq f(\theta)$

$P \neq f(z)$

$$r\frac{\partial P}{\partial r} = K \Rightarrow \frac{dP}{dr} = \frac{K}{r}$$

$$P = K \ln r + c$$

At $r = R$ , $P = P_{atm}$

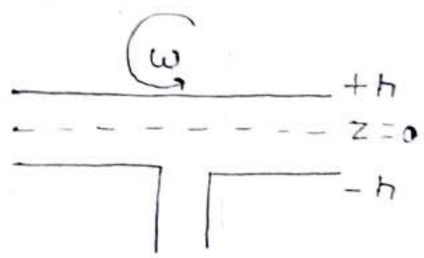
$\therefore$ $P - P_{atm} = k \ln \frac{r}{R}$

$$\Rightarrow K = \frac{P_o - P_{atm}}{\ln (r_o/R)} \quad \begin{Bmatrix} r = r_o \\ P = P_o \end{Bmatrix}$$

Find the volume flowrate (leakage rate) and the ~~axial~~ vertical load that can be supported.

$$\frac{\partial^2 (r v_r)}{\partial z^2} = \frac{K}{\mu}$$

$$r v_r = \frac{K z^2}{2\mu} + C_1 z + C_2$$




B.C: @ $z = \pm h$ $\quad v_r = 0$

$$v_r = \frac{K}{2 \mu r} (z^2 - h^2)$$

$$Q = \int_{-h}^{h} v_r \, 2\pi r \, dz = \int_{-h}^{h} \frac{K}{\cancel{2}\mu} \cdot \cancel{2}\pi (z^2 - h^2) \, dz$$

$$\Rightarrow Q = \frac{\pi K}{\mu} \left[ \frac{z^3}{3} - z h^2 \right]_{-h}^{h}$$

$$= \frac{\pi K}{\mu} \left[ \frac{h^3}{3} - h^3 - \left\{ -\frac{h^3}{3} + h^3 \right\} \right]$$

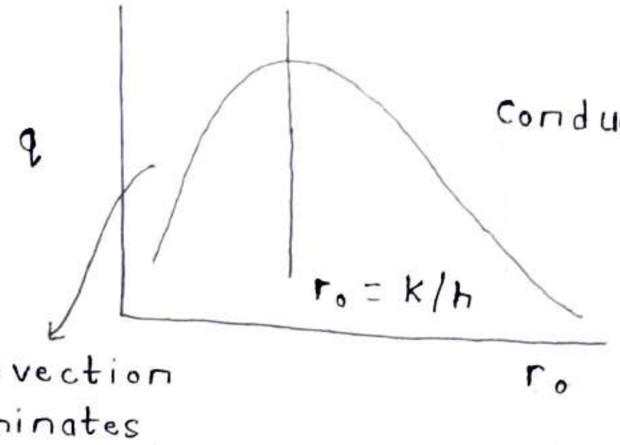
$$= -\frac{\pi K}{\mu} \frac{4}{3} h^3$$

$$Q = \frac{P_o - P_{atm}}{\ln R/R_o} \frac{4}{3} \frac{\pi h^3}{\mu}$$

$$\text{Load} = \pi R_0^2 (P_0 - P_{atm}) + \int_{R_0}^{R} (P - P_{atm}) \ldots$$

$$= \frac{\pi (P_0 - P_{atm})}{2 \ln R/R_0} (R^2 - R_0^2)$$

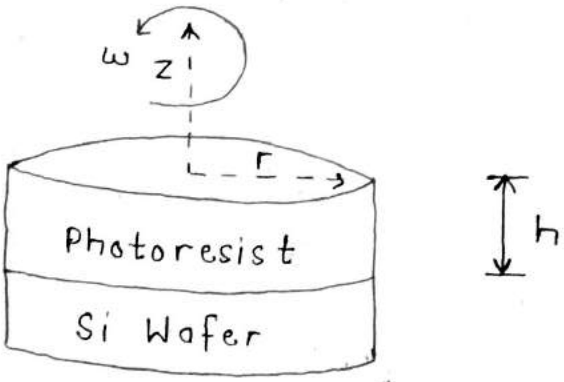
* Critical thickness of insulation not applicable for planar surface.




Adding insulation;

Conduction ↓ Convection ↑

- Predicting film thickness during spin coating




- Angular symmetry
- Constant $\omega$
- No pressure gradient
- No body force
- $\because$ film is thin, it can be assumed to be a rigid body, i.e., $v_\theta \neq f(z)$

Solution :

Continuity eq$^n$ :

$$\frac{1}{r}\frac{\partial}{\partial r}(r v_r) + \frac{\partial v_z}{\partial z} = 0$$

Momentum eq$^n$ in r direction gets reduced to -

$$\rho\left(v_r \frac{\partial v_r}{\partial r} - \frac{v_\theta^2}{r} + v_z \frac{\partial v_r}{\partial z}\right) = \mu\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(r v_r)\right) + \frac{\partial^2 v_r}{\partial z^2}\right]$$

As the film gets thinner $\frac{\partial v_r}{\partial r}$ becomes smaller and can be neglected.

$$\Rightarrow v_r \frac{\partial v_r}{\partial r} \sim 0$$

$v_z$ is very small and can be neglected when compared to $v_\theta$

After simplification,

$$\mu \frac{\partial^2 v_r}{\partial z^2} + \rho \frac{v_\theta^2}{r} = 0$$

$$\Rightarrow \frac{\partial^2 v_r}{\partial z^2} = -\frac{\rho}{\mu}\frac{v_\theta^2}{r}$$

$$\Rightarrow \frac{\partial v_r}{\partial z} = -\frac{\rho}{\mu}\frac{v_\theta^2}{r} z + c_1$$

$$\Rightarrow v_r = -\frac{\rho}{2\mu}\frac{v_\theta^2}{r} z^2 + c_1 z + c_2$$

@ $z = 0$, $c_2 = 0$

$$@ \quad z = h \, , \quad \frac{\partial v_r}{\partial z} = 0$$

$$\Rightarrow c_1 = \frac{\rho}{\mu} \frac{v_\theta^2}{r} h$$

$$\Rightarrow v_r = -\frac{\rho}{2\mu} \frac{v_\theta^2}{r} z^2 + \frac{\rho}{\mu} \frac{v_\theta^2}{r} hz$$

$$v_r = \frac{\omega^2 r}{\nu} \left( hz - \frac{z^2}{2} \right) \quad ; \quad v_\theta = \omega r$$

$v_z$ can be obtained from $eq^n$ of continuity.

$$\frac{1}{r} \frac{\partial}{\partial r} ( r v_r ) + \frac{\partial v_z}{\partial z} = 0$$

$$\frac{1}{r} \frac{\partial}{\partial r} \left\{ \frac{\omega^2 r^2}{\nu} \left( hz - \frac{z^2}{2} \right) \right\} + \frac{\partial v_z}{\partial z} = 0$$

$$\frac{1}{\cancel{r}} \times \frac{2 \cancel{r} \omega^2}{\nu} \left( hz - \frac{z^2}{2} \right) + \frac{\partial v_z}{\partial z} = 0$$

$$\frac{\partial v_z}{\partial z} = \frac{2\omega^2}{\nu} \left( \frac{z^2}{2} - hz \right)$$

$$v_z = \frac{2\omega^2}{\nu} \left( \frac{z^3}{6} - \frac{hz^2}{2} \right) + c$$

$$@ \quad z = 0 \, , \quad v_z = 0 \quad ; \quad c = 0$$

$$\therefore v_z = \frac{2\omega^2}{\nu} \left( \frac{z^3}{6} - \frac{hz^2}{2} \right)$$

$$v_z = \frac{\omega^2}{\nu}\left(\frac{z^3 - 3hz^2}{3}\right)$$

$$\frac{dz}{dt} = \frac{\omega^2}{\nu}\left(\frac{z^3 - 3hz^2}{3}\right)$$

$$\int_h^z \frac{dz}{z^3 - 3hz^2} = \int_0^t \frac{\omega^2}{3\nu}\, dt$$

$$\frac{dh}{dt} = \langle v_z \rangle \longrightarrow \text{Thickness avg.}$$

$$\langle v_z \rangle = \frac{1}{h}\int v_z\, dz$$

$$= \frac{1}{h}\int_0^h \left(\frac{z^3}{3} - hz^2\right) dz \cdot \frac{\omega^2}{\nu}$$

$$= \frac{1}{h}\left[\frac{z^4}{12} - \frac{hz^3}{3}\right]_0^h \cdot \frac{\omega^2}{\nu}$$

$$= \frac{1}{h}\left[\frac{h^4}{12} - \frac{h^4}{3}\right] = \frac{1}{h}\left[\frac{h^4 - 4h^4}{12}\right]$$

$$\langle v_z \rangle = -\frac{h^3}{4}\frac{\omega^2}{\nu}$$

$$\frac{dh}{dt} = -\frac{\omega^2}{4\nu} h^3$$

$$\int_{h_0}^h \frac{dh}{h^3} = \int_0^t -\frac{\omega^2}{4\nu}\, dt$$

$$-\frac{1}{2h^2}\Bigg|_{h_0}^{h} = -\frac{\omega^2}{4\nu} t$$

$$-\frac{1}{2h^2} + \frac{1}{2h_0^2} = -\frac{\omega^2 t}{4\nu}$$

$$\Rightarrow \frac{1}{h_0^2} + \frac{\omega^2 t}{2\nu} = \frac{1}{h^2}$$

$$h = \left(\frac{1}{h_0^2} + \frac{\omega^2 t}{2\nu}\right)^{-1/2}$$

Q. Two parallel, plane circular discs of radius R lie one above the other a small distance apart. The space b/w them is filled with a fluid. The upper disc approaches the bottom at constant velocity V. At r = R, $p = p_0$. $p_r \neq f(z)$

Sol$^n$ Assumptions :-

I) R is very large compared to h

II) Leakage rate is small (small $v_R$)

} $Re = \frac{h v_R \rho}{\mu}$ is small
S.S

III) $\frac{\partial P}{\partial z} \approx 0$

IV) $v_z$ is very small compared to $v_r$

V) $\frac{\partial v_z}{\partial z}$ may not be small (as z is very small)

vi) $\frac{\partial}{\partial z}\left(\frac{\partial v_z}{\partial z}\right)$ can be appreciable.

vii) $\frac{\partial v_r}{\partial r}$ is small compared to $\frac{\partial v_r}{\partial z}$.

viii) $v_\theta = 0$, no $\theta$ dependence of $v_r$, $v_z$

Eq$^n$ of continuity :-

$$\cancel{\frac{\partial \rho}{\partial t}}^{\,0} + \frac{1}{r}\frac{\partial}{\partial r}(\rho r v_r) + \frac{1}{r}\frac{\partial}{\partial \theta}(\rho \cancel{v_\theta}^{\,0}) + \frac{\partial}{\partial z}(\rho v_z) = 0$$

$$\Rightarrow \frac{1}{r}\frac{\partial}{\partial r}(r v_r) + \frac{\partial v_z}{\partial z} = 0 \quad \text{— (A)}$$

Eq$^n$ of motion in r direction —

$\underbrace{v_r}_{\text{appreciable}} \underbrace{\frac{\partial v_r}{\partial r}}_{\text{small}} \rightarrow \therefore$ neglected

$\underbrace{v_z}_{\text{small}} \underbrace{\frac{\partial v_r}{\partial z}}_{\text{appreciable}} \rightarrow \therefore$ neglected

$\frac{\partial^2 v_r}{\partial z^2}$ not neglected.

$$0 = -\frac{1}{\rho}\frac{\partial P}{\partial r} + \nu\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(r v_r)\right) + \frac{\partial^2 v_r}{\partial z^2}\right]$$

$$\Rightarrow 0 = -\frac{1}{\rho}\frac{dP}{dr} + \nu\left[\frac{1}{r}\cancel{\frac{\partial v_r}{\partial r}} - \cancel{\frac{v_r}{r^2}} + \cancel{\frac{\partial^2 v_r}{\partial r^2}} + \frac{\partial^2 v_r}{\partial z^2}\right]$$

$$\Rightarrow -\frac{1}{\rho}\frac{dP}{dr} + \nu\frac{d^2 v_r}{dz^2} = 0 \qquad - \text{(B)}$$

B.C. @ $z = 0$, $v_r = 0$
$z = h$, $v_r = 0$

$$v_r = \frac{1}{2\mu}\frac{dP}{dr}(z-h)z \qquad - \text{(C)}$$

Substituting (C) in (A) and performing the necessary steps

$z = 0$ ; $v_r = v_z = 0$
$z = h$ ; $v_r = 0$, $v_z = U$

$$U = -\frac{h^3}{12\mu}\frac{1}{r}\frac{d}{dr}\left(r\frac{dP}{dr}\right)$$

Integrating w.r.t. r

$$-\frac{12\mu U}{h^3}\frac{r^2}{2} = r\frac{dP}{dr} + c_1$$

$$\Rightarrow P - P_0 = \frac{3\mu U}{h^3}(R^2 - r^2)$$

$$\text{Force} = \int_0^R 2\pi r\,dr\,(P - P_0) \Rightarrow \frac{3\pi\mu U R^4}{2h^3}$$

17.1.19

$$\rho C_p \frac{\partial T}{\partial t} = k \left( \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} \right) + \dot{q}$$

Adiabatic Surface

$$\frac{dT}{dx} = 0$$

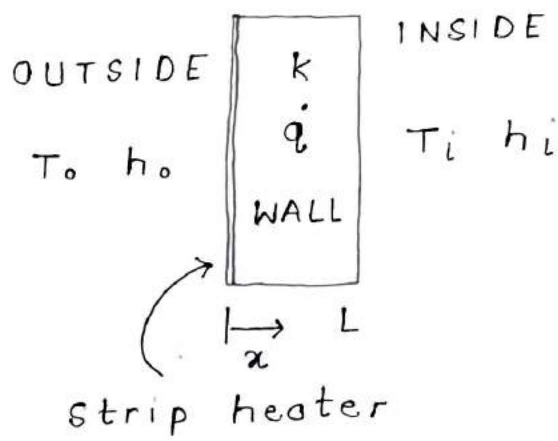
Insulated surface

$$-k \frac{dT}{dx}\bigg|_{x=L} = 0$$

$$\frac{dT}{dx} = 0$$

Mathematically, AS and IS give rise to same boundary condⁿ.

Q.




$T_i = 50°C$, $h_i = 20\ W/m^2K$

$L = 200\ mm$, $k = 4\ W/mK$

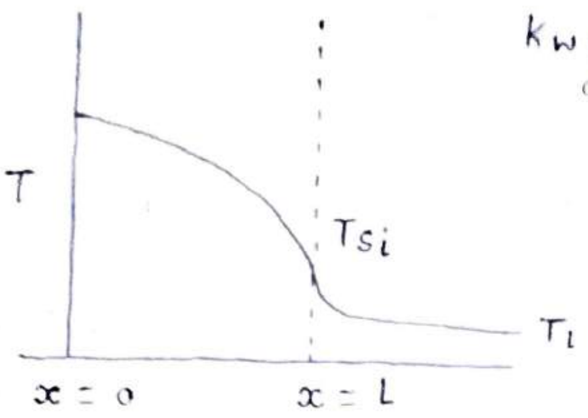
$\dot{q} = 1000\ W/m^3$, $T_o = 25°C$

$h_o = 5\ W/m^2K$.

1. If no heat is lost to the outside, Draw the T-profile.

W | Air

$$k_W \frac{dT}{dx}\Big|_{0^-} = k_A \frac{dT}{dx}\Big|$$




ii) Find T(0) and T(L) for condition 1.

$$\frac{d^2T}{dx^2} + \frac{\dot{q}}{K} = 0$$

$$\frac{d^2T}{dx^2} = -\frac{\dot{q}}{K}$$

$$\frac{dT}{dx} = -\frac{\dot{q}}{K}x + c_1$$

$$T(x) = -\frac{\dot{q}}{2K}x^2 + c_1 x + c_2$$

@ $x = 0$, $\frac{dT}{dx} = 0 \Rightarrow c_1 = 0$

$$-K\frac{dT}{dx}\Big|_{x=L} = h_i(T(L) - T_\infty)$$

$$+\frac{\dot{q}}{K}L = h_i\left(-\frac{\dot{q}}{2K}L^2 + c_2 - T_\infty\right)$$

$$c_2 = +\frac{\dot{q}}{h_i K}L + \frac{\dot{q}}{2K}L^2 + T_\infty$$

$$c_2 = 65\,^\circ C$$

$$T(x) = -125 x^2 + 65$$

$$T(L) = -125 L^2 + 65$$

$$= -125 \times 0.2^2 + 65$$

$$T(L) = 60°C \qquad T(0) = 65°C$$

OR

$$\dot{q} \cancel{A} L = h_i \cancel{A} (T_L - T_i)$$

$$T_L = \frac{\dot{q} L}{h_i} + T_i = \frac{1000 \times 0.4}{20 \times 2} + 50$$

$$T_L = 60°C$$

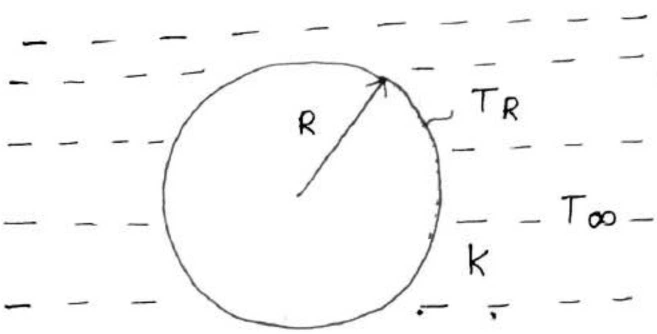
Amount of heat supplied by the strip heater for the given condition :-

$$q_{ST} = h_o (T_o - T_\infty)$$

$$= 5 (65 - 25)$$

$$= 200 \ W/m^2$$

Heated Sphere in a large, motionless body of fluid, Obtain $T_{FLUID}(r)$.




$$\frac{1}{r^2} \frac{d}{dr} \left( r^2 \frac{dT}{dr} \right) = 0$$

$$\frac{d}{dr} \left( r^2 \frac{dT}{dr} \right) = 0$$

$$r^2 \frac{dT}{dr} = C_1$$

$$\frac{dT}{dr} = \frac{C_1}{r^2}$$

$$T(r) = -\frac{C_1}{r} + C_2$$

B.C. @ $r = R \quad T = T_R$

$r \to \infty \quad T = T_\infty$

$$\Rightarrow \quad \frac{T(r) - T_\infty}{T_R - T_\infty} = \frac{R}{r}$$

Use this to obtain and expression for Nu

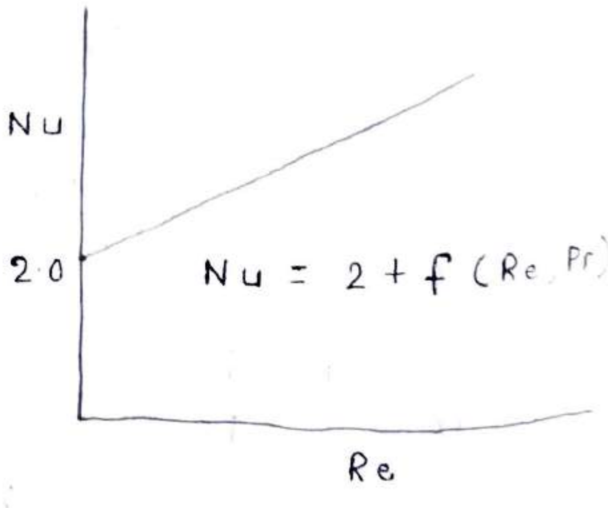
$$\frac{dT}{dr} = -\frac{R}{r^2}(T_R - T_\infty)$$

$$-kA \left.\frac{dT}{dr}\right|_{r=R} = hA(T_R - T_\infty)$$

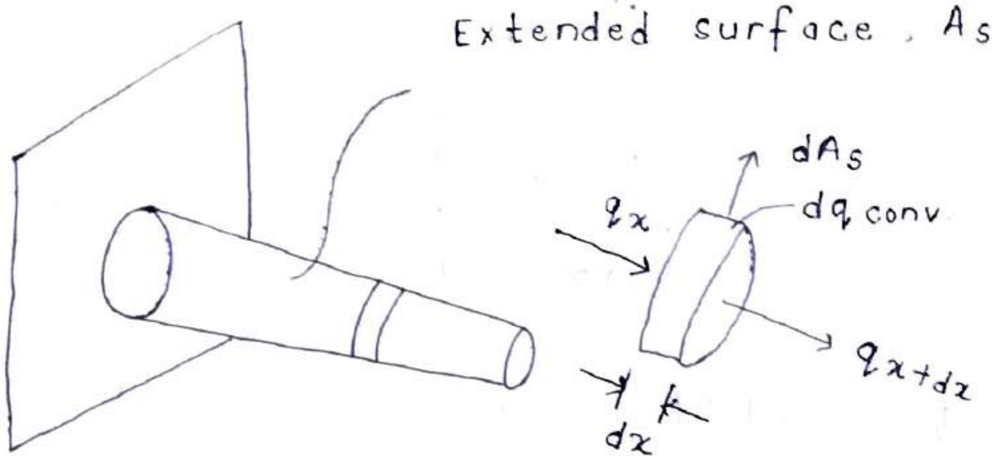
$$K \cancel{A} \frac{\cancel{R}}{R^{\cancel{2}}} (\cancel{T_R - T_\infty}) = h \cancel{A} (\cancel{T_R - T_\infty})$$

$$\frac{hR}{K} = 1$$

$$\frac{hD}{K} = 2$$



# General Conduction Analysis



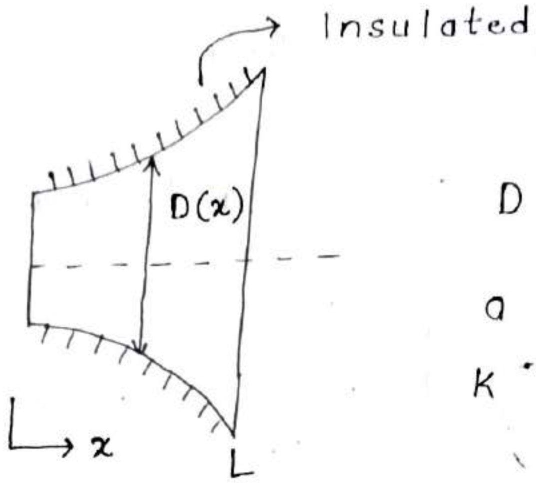

$$q_x = q_{x+dx} + dq_{conv}$$

$$q_x = -kA_c \frac{dT}{dx} \quad , \quad dq_{conv} = h\, dA_s (T - T_\infty)$$

$$\frac{d}{dx}\left(A_c \frac{dT}{dx}\right) - \frac{h}{k} \frac{dA_s}{dx} (T - T_\infty) = 0$$

$$\frac{d^2T}{dx^2} + \left(\frac{1}{A_c} \frac{dA_c}{dx}\right) \frac{dT}{dx} - \left(\frac{1}{A_c} \frac{A_s}{k} \frac{dA_s}{dx}\right) (T - T_\infty) = 0$$

Q.



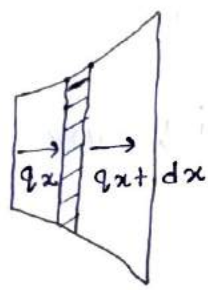

$D = a e^{1.8 x/L}$

$a = 0.8 m$, $L = 1$

$K = 8 \ W/mK$

$\dot{q} = 1989 \ W/m^3$ $\quad T_0 = 300°C$,

$Q_x |_{x=0} = 500 \ W$ $\quad$ Find $T_L$ and $Q_x|_L$




In + generation = out

$$- k \cancel{\frac{\pi a^2}{4}} e^{2 \times 1.8 x/L} \frac{dT}{dx} + 1989 \cdot \frac{\pi a^2}{4} e^{2 \times 1.8 x/L} \cdot dx$$

$$= \cancel{q_x} + \frac{d q_x}{dx} \cdot dx$$

$$\Rightarrow \frac{d q_x}{dx} = 1989 \frac{\pi a^2}{4} e^{2 \times 1.8 x/L}$$

$$q_x = 1989 \frac{\pi a^2}{4} \times \frac{L}{3.6} e^{3.6 x/L} + c_1$$

$c_1 \approx 0$ $\quad$ { In comparison to the other terms }

$$\therefore q_x = 1989 \frac{\pi a^2}{4} \frac{L}{3.6} e^{3.6 x/L}$$

$$q_x = -KA\frac{dT}{dx}$$

$$\dot{q}\,\frac{\pi a^2}{4}\,\frac{L}{3.6}\,e^{3.6x/L} = -K\,\frac{\pi a^2}{4}\,e^{3.6x/L}\,\frac{dT}{dx}\cdot\frac{L}{3.6}$$

$$\Rightarrow \quad -\frac{dT}{dx} = \frac{\dot{q}}{2K}$$

$$T = -\frac{\dot{q}}{2K}x + C_1$$

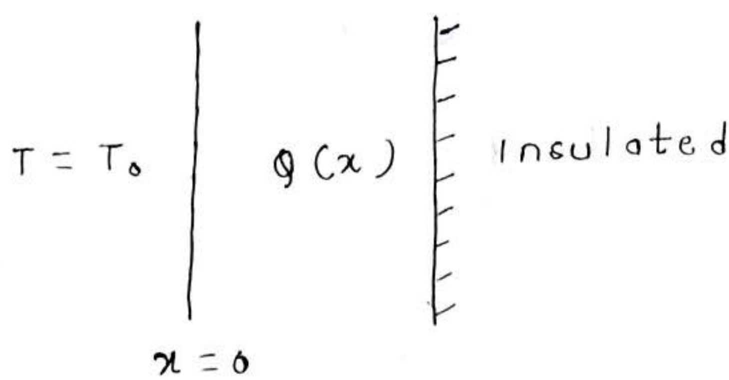
$$T|_0 = 300 = C_1$$

$$T(x) = -\frac{\dot{q}}{2K}x + 300$$

$$T|_{x=L} = 76.2\,^\circ C$$

$$q|_{x=L} = 18295\ W$$

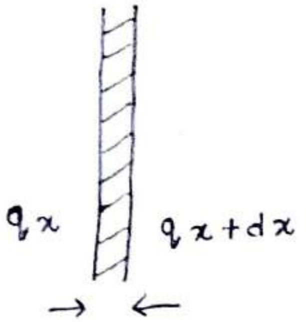
Q. $\quad q(x) = q_0\left(1 - \frac{x}{L}\right)$ {heat generation}




a) Derive the DE for temp. profile.

b) BC ?

c) Solve DE to find temp. profile.




In + generation = Out

$$q_x + \dot{q} A \, dx = q_{x+dx}$$

$$\cancel{q_x} + \dot{q} A \, dx = \cancel{q_x} + \frac{d q_x}{dx} dx$$

$$\dot{q} A \, \cancel{dx} = \frac{d q_x}{dx} \cancel{dx} \Rightarrow \frac{d q_x}{dx} = \dot{q} A$$

$$\frac{d}{dx}\left(-k \cancel{A} \frac{dT}{dx}\right) = \dot{q}_0 \left(1 - \frac{x}{L}\right) \cancel{A}$$

$$\frac{d^2 T}{dx^2} = \frac{\dot{q}_0}{k}\left(\frac{x}{L} - 1\right)$$

$$\frac{dT}{dx} = \frac{\dot{q}_0}{k}\left(\frac{x^2}{2L} - x\right) + c_1$$

$$0 = \frac{\dot{q}_0}{k}\left(\frac{L^{\cancel{2}}}{2\cancel{L}} - L\right) + c_1$$

$$c_1 = \frac{L}{2} \frac{\dot{q}_0}{k}$$

$$\frac{dT}{dx} = \frac{\dot{q}_0}{k}\left(\frac{x^2}{2L} - x\right) + \frac{L}{2}\frac{\dot{q}_0}{k}$$

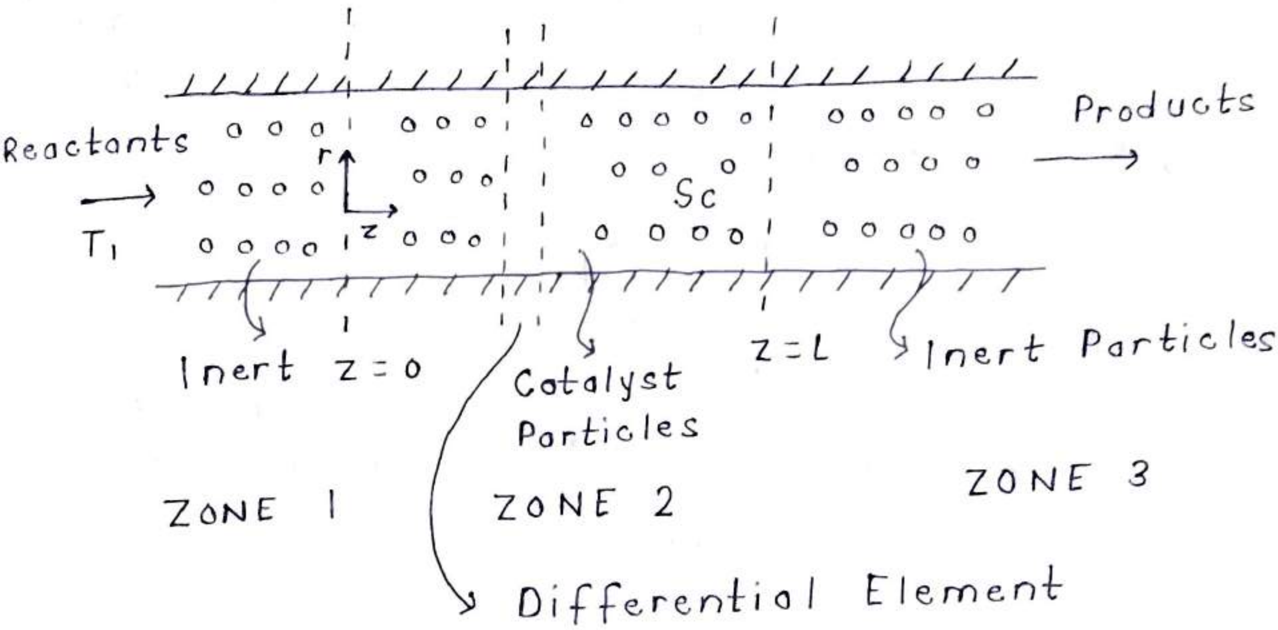
$$T(x) = \frac{\dot{q}_0}{k}\left(\frac{x^3}{6L} - \frac{x^2}{2}\right) + \frac{L}{2}\frac{\dot{q}_0}{k} x + c_2$$

$x = 0$, $T = T_0$, $c_2 = T_0$

$$T(x) = \frac{q_o}{K}\left(\frac{x^3}{6L} - \frac{x^2}{2}\right) + \frac{L}{2}\frac{q_o}{K}x + T_o$$

$$T(x) - T_o = \frac{q_o}{2K} L^2 \left(\frac{x}{L} - \frac{x^2}{L^2} + \frac{1}{3}\frac{x^3}{L^3}\right)$$

Heat Conduction with Chemical Heat Source




Volumetric Heat Generation = $S_c$

In + generation − out = 0

$$\pi R^2 q_z|_z + \pi R^2 \rho_1 v_1 C_p (T - T_o)|_z - \pi R^2 q_z|_{z+\Delta z}$$

$$-\left(\quad \checkmark \quad\right)_{z+\Delta z} + S_c \pi R^2 \Delta z = 0$$

$$\frac{dq_z}{dz} + \rho_1 v_1 C_p \frac{dT}{dz} = S_c$$

Assuming that we have finely grained catalyst particles.

For zone 1 and zone 3, $S_c = 0$

$$-k_z \frac{d^2 T}{dz^2} + \rho_1 v_1 C_p \frac{dT}{dz} = S_c$$

ZONE -

I II III

B.C.s -

$T^{I}$ $T^{II}$ $T^{III} = f(z)$

$v_1, C_p,$

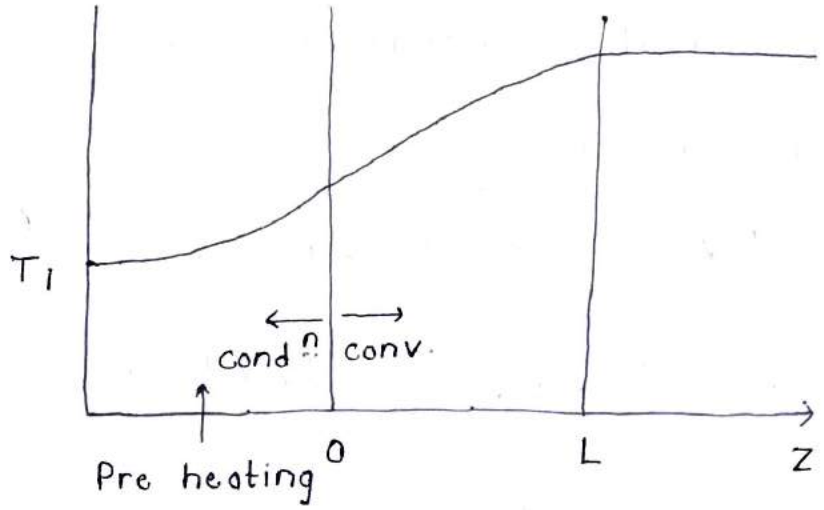
$$\theta^{I} = C_1 + C_2 e^{Bz}$$

$$\theta^{II} = C_3 e^{m_3 z} + C_4 e^{m_4 z}$$

$$\theta^{III} = C_5 + C_6 e^{Bz}$$

$\{ C_6 = 0 \}$
as $\theta \not\to \infty$
when $z \to \infty$

where, $\theta \equiv \frac{T - T^{\circ}}{T_1 - T^{\circ}}$




## Validity of LC Method :-

Biot number should be small.

Alternatives → Analytical
→ Numerical
→ Graphical

Q. Length $= L$, Diameter $= D$

$\dot{q}(x) = \dot{q}_0 \sin(x/L)$, $\dot{q}_0\ (W/m^3)$

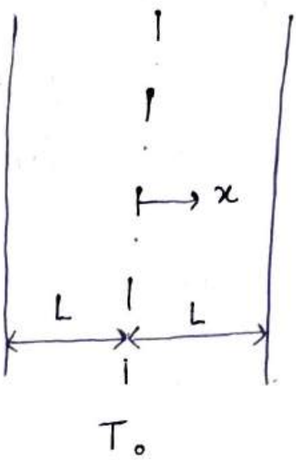
a) Obtain an expression for the local heat flux, $q''(x)$ and the total heat transfer from the fuel rod to water

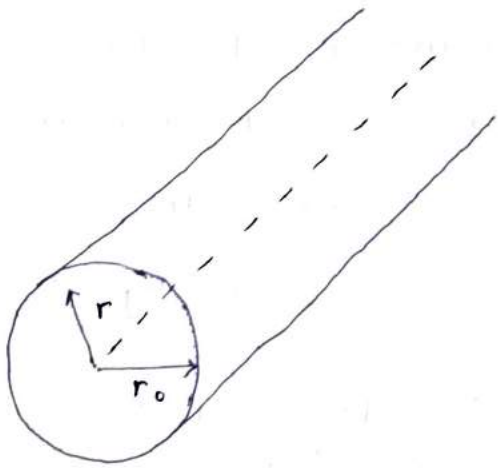
b) Find the mean temperature of the water, $T_m(x)$ as a function of $x$.

c) Find the variation of the rod temp. $T_s(x)$.

d) Develop an expression for the $x$ location of maximum $T_s$.

## HEISLER CHART



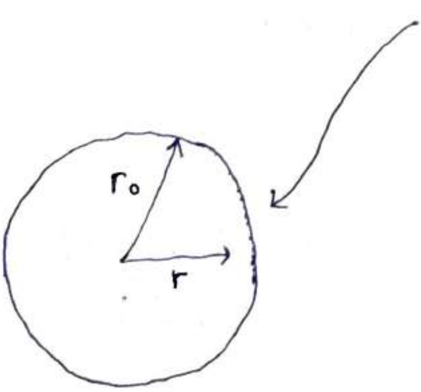



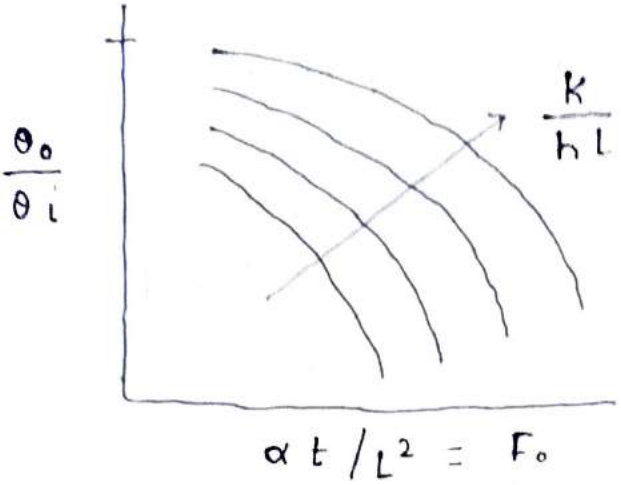

$\theta = T(x,t) - T_\infty$

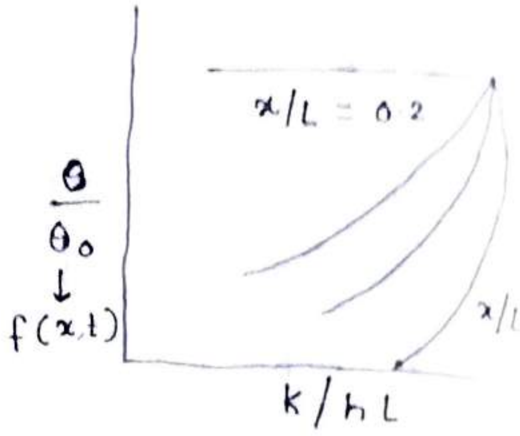
$\theta_i = T_i - T_\infty$

$T(r,t) - T_\infty$

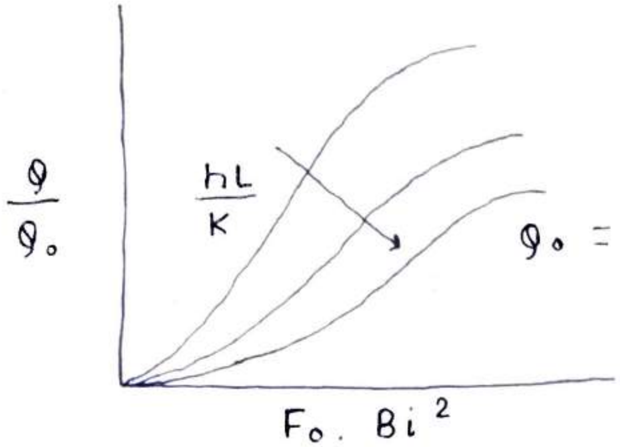
$\theta_0 = T_0 - T_\infty$






$F_o > 0.2$




$Q_0 = \rho C V (T_i - T_\infty)$

Q. Diameter of wire $= 1\,mm$ $\quad T_\infty = 25\,°C$

$R_e' = 0.01$ ohm/m $\quad$ Current $I = 100\,A$

$h = 500\ W/m^2K$ $\quad \rho_{wire} = 8000\ kg/m^3$

$C = 500\ J/kg.K$ and $k = 20\ W/mK$

i) Steady state temp. of wire

$$I^2 R_e' \cancel{L} = h \pi D \cancel{L} (T_s - T_\infty)$$

$$T_s = T_\infty + \frac{I^2 R_e'}{h \pi D}$$

$$= 88.66\,°C$$

$$Bi = \frac{hD}{k \times 4} = \frac{500 \times 0.001}{20 \times 4} = \frac{0.025}{4}$$

$$= 6.25 \times 10^{-3}$$

$\therefore$ T is not a $f^n$ of position

$$\cancel{q_{in}} - q_{out} + q_{gen} = q_{accu}$$

$$0$$

$$-h\pi D \cancel{L} (T_w - T_\infty) + I^2 R_e' \cancel{L} = \rho C_p \pi R^2 \cancel{L} \frac{\partial T_w}{\partial t}$$

ii) time required for the T to reach within 1°C of steady state temp.

$$\frac{dT_w}{dt} + \frac{h \cancel{\pi} \cancel{D} \times 4}{\rho C_p \cancel{\pi} D^{\cancel{2}}} (T_w - T_\infty) = \frac{I^2 R_e' \times 4}{\rho C_p \pi D^2}$$

$$\frac{dT_w}{dt} + \frac{4h}{\rho C_p D} T_w = \frac{4 I^2 R_e'}{\rho C_p \pi D^2} + \frac{4 h T_\infty}{\rho C_p D}$$

$$\frac{dT_w}{dt} + 0.5\, T_w = 44.33$$

$$\Rightarrow T_w \cdot e^{0.5t} = \frac{44.33}{0.5} e^{0.5t} + c$$

$$T_w \cdot e^{0.5t} = 88.66\, e^{0.5t} + c$$

i.e., $T_w = 88.66 + c\, e^{-0.5t}$

@ $t = 0$, $T_w = 25$

$\therefore$ $c = 25 - 88.66 = 63.66$

$\therefore$ $T_w = 88.66 - 63.66\, e^{-0.5t}$

Given $T_w = 87.66$

$\Rightarrow$ $87.66 = 88.66 - 63.66\, e^{-0.5t}$

$\Rightarrow$ $1/63.66 = e^{-0.5t}$

which gives ; $t = 8.307$ s

Q. Small spheres ($r_o = 1.5$ cm)
Coloured liquid has a density of
$\rho_\infty = 900\ kg/m^3$ and $T_\infty = 325$ K.

$\rho_{sphere} = \rho_o - \beta(T - T_\infty)$ $\rho_o = 910\ kg/m^3$

$\beta = 2\ kg/m^3 K$

If the polymer liquid begins its ascent at a temp. of 350 K, how long before it begins to fall back to be reheated?

$h = 20\ W/m^2K$, $C_p$ (sphere) $= 400\ J/kg\,K$

$k$ (sphere) $= 10\ W/mK$

Volume and area are constant.

Force of gravity = Force of buoyancy

$$\cancel{V}(\rho_o - \beta(T - T_\infty))\cancel{g} = \cancel{V} \times 900 \times \cancel{g}$$

$$910 - 2(T - T_\infty) = 900$$

$$5 = T - T_\infty$$

$$T = T_\infty + 5 = 325 + 5 = 330\ K$$

---

$$-h\cancel{\pi}\cancel{D^2}(T - T_\infty) = \rho C_p \frac{\cancel{4}}{3}\cancel{\pi}\frac{D^{\cancel{3}}}{\cancel{8}2}\frac{\partial T}{\partial t}$$

$$-h(T - T_\infty) = (\rho_o - \beta(T - T_\infty))C_p\frac{D}{6}\frac{\partial T}{\partial t}$$

Incorrect

$$-\frac{6h}{DC_p}(T - T_\infty) = (\rho_o - \beta(T - T_\infty))\frac{\partial T}{\partial t}$$

$$\int_{T_0}^{T} dT \left(\frac{\rho_0 - \beta(T - T_\infty)}{T - T_\infty}\right) = \int_0^t -\frac{6h}{DC_p}\, dt$$

$$\int_{T_0}^{T} \left(\frac{\rho_0}{T - T_\infty} - \beta\right) dT = -\frac{6ht}{DC_p}$$

Incorrect

$$\ln(T - T_\infty)\,\rho_0 \Big|_{T_0}^{T} - \beta(T - T_0) = -\frac{6ht}{DC_p}$$

$$\ln\left(\frac{T - T_\infty}{-T_0 + T_\infty}\right) + \beta(T_0 - T) = -\frac{6ht}{DC_p}$$

$$\ln\left(\frac{330 - 325}{350 - 325}\right) + 2(350 - 330) = -\frac{6 \times 20 t}{0.03 \times 400}$$

---

Governing equation –

$$-h(4\pi R^2)(T - T_\infty) = C_p V \frac{d}{dt}\left[T\left(\rho_0 - \beta(T - T_\infty)\right)\right]$$

$$-h(4\pi R^2)(T - T_\infty) = C_p V \frac{dT}{dt}\{\rho_0 - 2T\beta + \beta T_\infty\}$$

$$-h(\cancel{4}\cancel{\pi}\cancel{R^2})(T - T_\infty) = C_p \cdot \frac{\cancel{4}}{3}\cancel{\pi} R^{\cancel{3}} \frac{dT}{dt}\{\ ,,\ \}$$

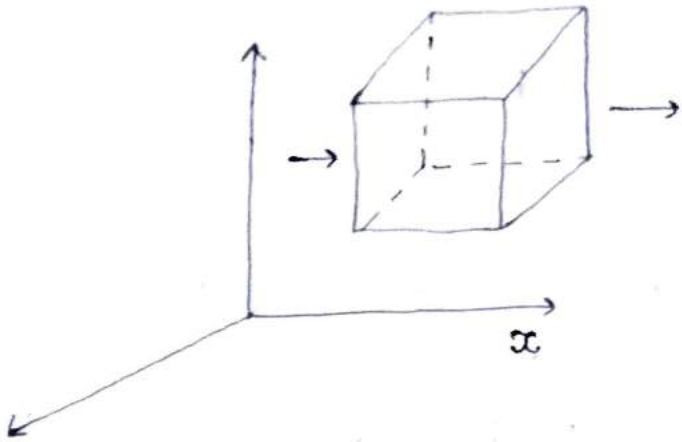
$$-20(T - 325) = \frac{dT}{dt}(1560 - 4T)$$

$$\Rightarrow \frac{dT}{dt} = \frac{10(325 - T)}{(1560 - 4T)}$$

$$\int_{350}^{330}\left(\frac{1560 - 4T}{325 - T}\right) dT = 10\int_0^t dt$$

$$\Rightarrow t = 498\,s \approx 500\,s$$

Equation of Change for a non-isothermal System -



Conservation of energy -

Rate of accu. of IE and K.E =

$$\left\{ \begin{matrix} K.E \text{ and} \\ I.E \text{ in} \end{matrix} \right\} - \left\{ \begin{matrix} K.E \text{ and} \\ I.E \text{ out} \end{matrix} \right\} + \begin{matrix} \text{viscous} \\ \text{Dissipation} \end{matrix}$$

$$\text{Accumulation} = \Delta x \Delta y \Delta z \frac{\partial}{\partial t} \left( \rho U + \frac{1}{2} \rho v^2 \right)$$

$U$ = IE / mass of fluid

$v$ = mag. of local fluid velocity

Rate of convection of IE and KE into the element -

$x$ face :

$$\Delta y \Delta z \left\{ v_x \left( \rho U + \frac{1}{2} \rho v^2 \right) \Big|_x - v_x \left( \rho U + \frac{1}{2} \rho v^2 \right) \Big|_{x+\Delta x} \right.$$

Similary for y and z face.

- Conduction :

Net Rate of energy in by conduction

$$\Delta y \Delta z \left\{ q_x|_x - q_x|_{x+\Delta x} \right\} + \Delta x \Delta z \left\{ q_y|_y - q_y|_{y+\Delta y} \right\} + \Delta x \Delta y \left\{ q_z|_z - q_z|_{z+\Delta z} \right\}$$

Work done $\begin{cases} \rightarrow \text{Against Volumetric force} \\ \rightarrow \text{Against surface forces (viscous, pressure)} \end{cases}$

Work done by gravitational force :-

$$-\rho\,(\Delta x\,\Delta y\,\Delta z)\,(v_x g_x + v_y g_y + v_z g_z)$$

Work done against pressure :-

$$\Delta y\,\Delta z\,\{P|_x - P|_{x+\Delta x}\} + \Delta x\,\Delta z\,\{P|_y - P|_{y+\Delta y}\}$$
$$+\ \Delta x\,\Delta y\,\{P|_z - P|_{z+\Delta z}\}$$

Viscous Forces :-

$$\begin{matrix} \tau_{xx}\cdot v_x & \tau_{yx}\cdot v_x & \tau_{zx}\cdot v_x \\ \tau_{xy}\cdot v_y & \tau_{yy}\cdot v_y & \tau_{zy}\cdot v_y \\ \tau_{xz}\cdot v_z & \tau_{yz}\cdot v_z & \tau_{zz}\cdot v_z \\ \downarrow & \uparrow & \uparrow \\ \text{Acting on area } \Delta y\,\Delta z & \Delta x\,\Delta z & \Delta x\,\Delta y \end{matrix}$$

Net Rate :-

$$\Delta y\,\Delta z\,\{(\tau_{xx} v_x + \tau_{xy} v_y + \tau_{xz})|_{x+\Delta x}$$
$$-\ (\tau_{xx} v_x + \ldots\ldots)|_x \quad + 2y \quad + 2z$$

$$\Rightarrow \rho C \frac{DT}{dt} = \underbrace{k\,\nabla^2 T}_{\text{Heat Conduction}} - \underbrace{T\left(\frac{\partial P}{\partial T}\right)\nabla v}_{\text{Expansion Effects}} + \underbrace{\mu\,\phi_v}_{\text{Viscous Dissipation}}$$

where $\mu\phi_v = 2\mu\left\{\left(\frac{\partial v_x}{\partial x}\right)^2 + \left(\frac{\partial v_y}{\partial y}\right)^2 + \left(\frac{\partial v_z}{\partial z}\right)^2\right\}$

$$+\ \mu\left\{\left(\frac{\partial v_x}{\partial y} + \frac{\partial v_y}{\partial x}\right)^2 + \left(\frac{\partial v_x}{\partial z} + \frac{\partial v_z}{\partial x}\right)^2 + \right.$$

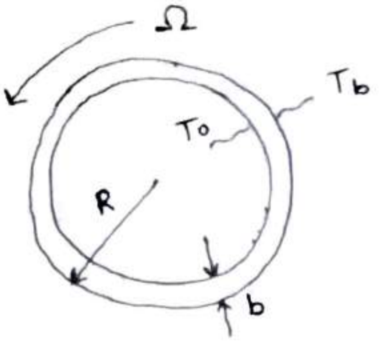
$$\left(\frac{\partial v_y}{\partial z} + \frac{\partial v_z}{\partial y}\right)^2 \Big]$$

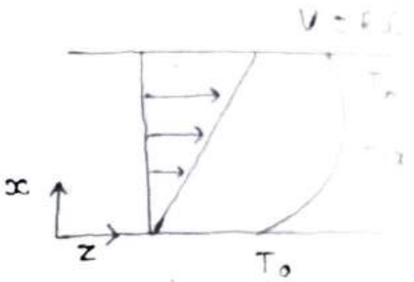
$$\frac{DT}{Dt} = \frac{\partial T}{\partial t} + v_x \frac{\partial T}{\partial x} + v_y \frac{\partial T}{\partial y} + v_z \frac{\partial T}{\partial z}$$

Energy Equation (in all coordinate system)

$$\underbrace{\rho C_p \left(\frac{DT}{Dt}\right)}_{\text{Convective}} = \underbrace{k \nabla^2 T}_{\text{Conductive}} + \underbrace{\mu \phi_v}_{\substack{\text{Viscous} \\ \text{Dissipation}}} + \underbrace{\dot{Q}}_{\substack{\text{Heat} \\ \text{generation}}}$$

Q




Thin gap parallel plate approximation
no pressure gradient

$$V_z = R\Omega \frac{x}{b}$$

Energy Eq$^n$ :-

$$\rho C_p \left(\cancel{\frac{\partial T}{\partial t}}_0 + \cancel{v_x}_0 \frac{\partial T}{\partial x} + \cancel{v_y}_0 \frac{\partial T}{\partial y} + v_z \cancel{\frac{\partial T}{\partial z}}_0\right) =$$

$$k\left(\frac{\partial^2 T}{\partial x^2} + \cancel{\frac{\partial^2 T}{\partial y^2}}_0 + \cancel{\frac{\partial^2 T}{\partial z^2}}_0\right) + 2\mu$$

$$\left\{\left(\cancel{\frac{\partial v_x}{\partial x}}_0\right)^2 + \left(\cancel{\frac{\partial v_y}{\partial y}}_0\right)^2 + \left(\cancel{\frac{\partial v_z}{\partial z}}_0\right)^2\right\} + \mu$$

$$\left\{\left(\cancel{\frac{\partial v_x}{\partial y}}_0 + \cancel{\frac{\partial v_y}{\partial x}}_0\right)^2 + \left(\cancel{\frac{\partial v_x}{\partial z}}_0 + \frac{\partial v_z}{\partial x}\right)^2 +\right.$$

$$\left.\left(\cancel{\frac{\partial v_y}{\partial z}}_0 + \cancel{\frac{\partial v_z}{\partial y}}_0\right)^2\right\} + \cancel{\dot{Q}}_0 = 0$$

$$\Rightarrow \quad k \frac{\partial^2 T}{\partial x^2} + \mu \left( \frac{d v_z}{d x} \right)^2 = 0$$

$$v_z = R\Omega \frac{x}{b} \quad \Rightarrow \quad \frac{d v_z}{d x} = \frac{R\Omega}{b}$$

$$\Rightarrow \quad K \frac{d^2 T}{d x^2} + \mu \frac{R^2 \Omega^2}{b^2} = 0$$

$$\frac{d^2 T}{d x^2} = - \frac{\mu}{K} \frac{R^2 \Omega^2}{b^2}$$

$$\frac{d T}{d x} = - \frac{\mu}{K} \frac{R^2 \Omega^2}{b^2} x + c_1$$

$$T(x) = - \frac{\mu}{K} \frac{R^2 \Omega^2}{2 b^2} x^2 + c_1 x + c_2$$

@ $x = 0$ $\quad T = T_0 \quad \Rightarrow \quad c_2 = T_0$

@ $x = b$ $\quad T = T_b$

$$T_b = - \frac{\mu}{K} \frac{R^2 \Omega^2}{2 \cancel{b^2}} \times \cancel{b^2} + c_1 b + T_0$$

$$\Rightarrow \quad c_1 = \frac{T_b}{b} + \frac{\mu}{K} \frac{R^2 \Omega^2}{2 b} - \frac{T_0}{b}$$

$$T(x) = - \frac{\mu}{K} \frac{R^2 \Omega^2}{2 b^2} x^2 + \left( \frac{T_b - T_0}{b} \right) x + \frac{\mu}{K} \frac{R^2 \Omega^2}{2 b} x + T_0$$

$$\frac{T - T_0}{T_b - T_0} = \left( \frac{x}{b} \right) + \frac{1}{2} \frac{\mu R^2 \Omega^2}{K (T_b - T_0)} \left[ \left( \frac{x}{b} \right) - \left( \frac{x}{b} \right)^2 \right]$$

$\dfrac{\mu v^2}{k(T_b - T_o)}$ gives $\dfrac{\text{Viscous losses}}{\text{Conductive HT}}$

↳ Brinkmann Number

$$\frac{1}{T_b - T_o} \frac{dT}{dx} = \frac{1}{b} + \frac{1}{2} \frac{\mu v^2}{k(T_b - T_o)} \times \left[ \frac{1}{b} - 2\frac{x}{b} \right]$$

For maxima,

$$0 = \frac{1}{b} + \frac{\alpha}{b} - \frac{2\alpha x}{b^2}$$

$$\frac{2\alpha x}{b^{\cancel{2}}} = \frac{(\alpha+1)}{\cancel{b}}$$

$$x = \left(\frac{\alpha+1}{2\alpha}\right) b$$

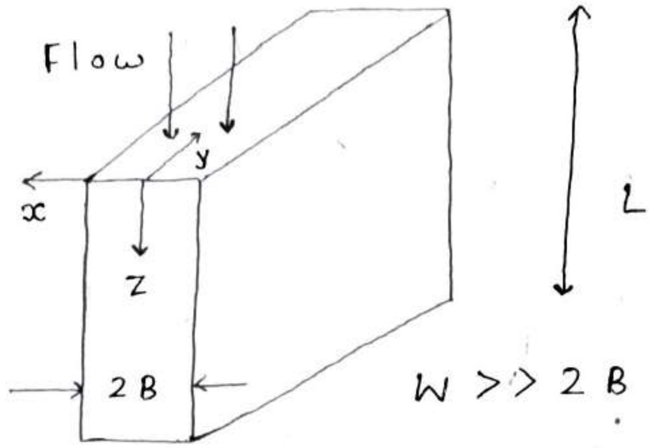
$$0 < \frac{\alpha+1}{2\alpha} < 1 \Rightarrow 0 < \alpha + 1 < 2\alpha$$

$1 < \alpha$ where $\alpha$ is $\dfrac{\text{Brinkmann}}{2}$

$$\frac{Br}{2} > 1 \Rightarrow Br > 2$$

$\therefore$ Brinkmann Number $> 2$

Q.




Find $T = f(x)$ Fully developed flow

Energy Eq$^n$ gives :-

$$0 = k \frac{d^2 T}{dx^2} + \mu \left( \frac{\partial v_z}{\partial x} \right)^2 \quad - (1)$$

$T = T_0$ at $x = \pm B$

$v_z = f(x) \longrightarrow$ N.S.

$$v_z = \frac{(P_0 - P_L) B^2}{2 \mu L} \left[ 1 - \left( \frac{x}{B} \right)^2 \right]$$

$$\frac{\partial v_z}{\partial x} = \frac{(P_0 - P_L) \cancel{B^2}}{\cancel{2} \mu L} \times \left( - \frac{\cancel{2} x}{\cancel{B^2}} \right)$$

$$\frac{\partial v_z}{\partial x} = - x \left( \frac{P_0 - P_L}{\mu L} \right) \quad - (2)$$

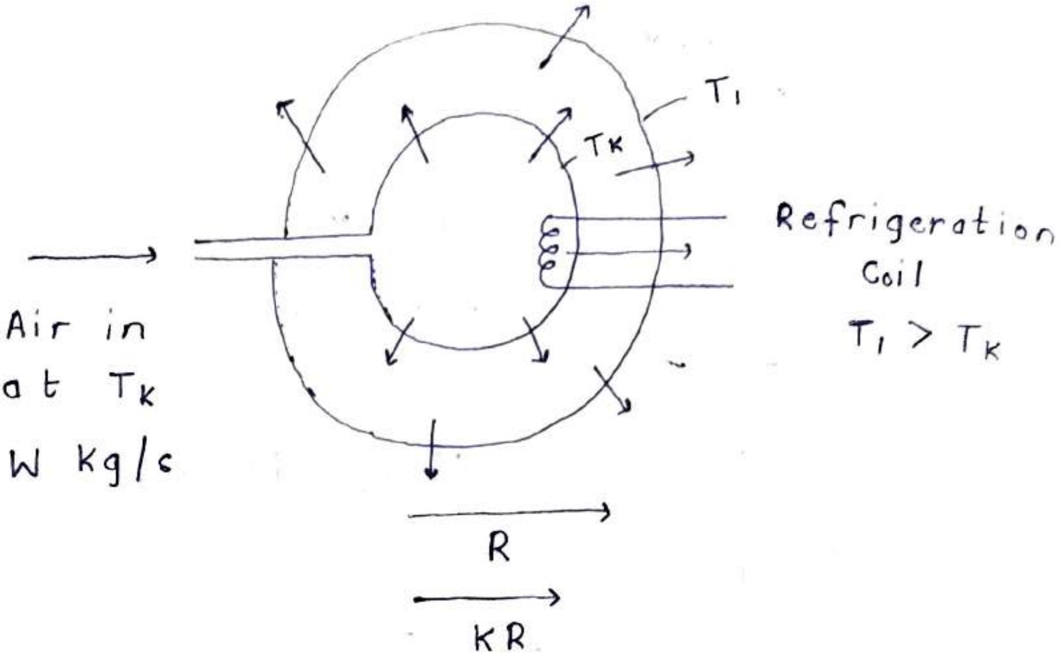
Substituting (2) in (1) gives

$$\frac{d^2 T}{dx^2} = - \frac{\mu}{k} \left( \frac{P_0 - P_L}{\mu L} \right)^2 x^2$$

which on solving and putting the B.C.s give

$$T(x) = T_0 + \frac{\mu}{k} \left( \frac{P_0 - P_L}{\mu L} \right)^2 \frac{B^4}{12} \left( 1 - \frac{x^4}{B^4} \right)$$

Q. Transpiration Cooling




From continuity ;

$$4\pi r^2 v_r \rho = \text{constant} = W$$

$$\Rightarrow v_r = \frac{W}{4\pi r^2 \rho} \qquad \{ r\uparrow \quad v_r \downarrow \}$$

Energy eq$^n$ in spherical coordinates get reduced to

$$\rho C_p v_r \frac{\partial T}{\partial r} = \frac{k}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial T}{\partial r}\right)$$

- No viscous dissipation as flow is slow
- No heat generation.

Substituting $v_r$ –

$$\cancel{\rho} C_p \frac{W}{4\pi \cancel{r^2} \cancel{\rho}} \frac{dT}{dr} = \frac{k}{\cancel{r^2}} \frac{d}{dr}\left(r^2 \frac{dT}{dr}\right)$$

$$\frac{dT}{dr} = \frac{4\pi k}{W C_p} \frac{d}{dr}\left(r^2 \frac{dT}{dr}\right)$$

$$\frac{dT}{dr} = \frac{4\pi k}{W C_p} \frac{d}{dr}\left(r^2 \frac{dT}{dr}\right)$$

$$\frac{T - T_1}{T_K - T_1} = \frac{e^{-R_o/r} - e^{-R_o/R}}{e^{-R_o/KR} - e^{-R_o/R}}$$ { with transpira$^n$

where, $R_o = \frac{W C_p}{4\pi k}$

For small $R_o$.

$$\Rightarrow \frac{T - T_1}{T_K - T_1} = \frac{\frac{1}{r} - \frac{1}{R}}{\frac{1}{KR} - \frac{1}{R}}$$ → no transpiration

For very small value of W and R very large, Nu = 2 can be obtained from the above temp. profile.

For stagnant fluid, equality of conduction and convective flux holds.

$$-4\pi (KR)^2 \frac{dT}{dr}\Big|_{r=R} k = h (4\pi K^2 R^2)(T_K - T_1)$$

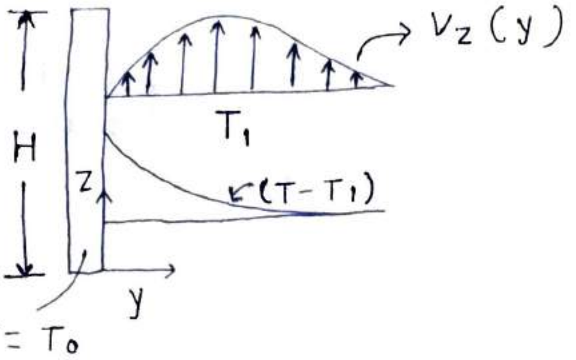
Nu = 2 gives the minimum possible value of h possible for a sphere.

$$Q = -4\pi (K^2 R^2)\, q''_r \Big|_{r=KR}$$

$$= 4\pi k (K^2 R^2) \frac{dT}{dr}\Big|_{r=KR}$$

$$\text{Efficiency } (\epsilon) = \frac{-Q_o + Q}{Q_o}$$ { $Q_o$ = w/o transpiration

# Free Convection from a Vertical Plate




$H >> \delta_t$

@ $y = 0$ $\quad v_z = 0$

as $y \to \infty$ $\quad v_z = 0$

$v_y = 0$

$z \to -\infty$ $\quad v_z = 0$

Continuity gives ;

$$\frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z} = 0$$

z Component of NS $eq^n$ :-

$$\rho \left( v_y \frac{\partial v_z}{\partial y} + v_z \frac{\partial v_z}{\partial z} \right) = \mu \left( \frac{\partial^2 v_z}{\partial y^2} + \frac{\partial^2 v_z}{\partial z^2} \right) + \rho g \beta (T - T_1)$$

$$\beta \equiv \frac{1}{V} \frac{\Delta V}{\Delta T} \Rightarrow \Delta V = \beta V \Delta T$$

Buoyant force $= \rho g \Delta V = \rho g \beta V \Delta T$

Energy Equation :-

$$\rho C_p \left( v_y \frac{\partial}{\partial y} (T - T_1) + v_z \frac{\partial}{\partial z} (T - T_1) \right)$$

$$= k \left( \frac{\partial^2}{\partial y^2} (T - T_1) + \frac{\partial^2}{\partial z^2} (T - T_1) \right)$$

$v_z >> v_y$

$\frac{\partial T}{\partial y} >> \frac{\partial T}{\partial z}$

→ No heat generation

→ No viscous loss (slow flow)

↳ Because length scale in y is $\delta_t$ ( thermal boundary layer) which is much greater than h.

z dir : Convection prevalent
y dir : Conduction ,,

Non - dimensionalising the governing eq$^n$:

$$\theta = \frac{T - T_1}{T_0 - T_1} \quad , \text{ temp}$$

$$\phi_z = \left( \frac{\mu}{B \alpha H} \right)^{1/2} y \longrightarrow \text{Vertical velo.}$$

$$\phi_y = \left( \frac{\mu H}{\alpha^3 B} \right)^{1/4} v_y \longrightarrow \text{Horizontal velo.}$$

$$\alpha = \frac{k}{\rho C_p} \quad ; \quad B = \rho g \beta (T_0 - T_1)$$

$$\xi = z/H \quad , \quad \eta = \left( \frac{\beta}{\alpha \mu H} \right)^{1/4} y$$

$$\frac{\partial \phi_y}{\partial \eta} + \frac{\partial \phi_z}{\partial \xi} = 0 \quad \{ \text{continuity eq}^n \}$$

$$\frac{1}{Pr} \left( \phi_y \frac{\partial \phi_z}{\partial \eta} + \phi_z \frac{\partial \phi_z}{\partial \xi} \right) = \frac{\partial^2 \phi_z}{\partial \eta^2} + \theta$$

(Momentum eq$^n$)

$$\phi_y \frac{\partial \theta}{\partial \eta} + \phi_z \frac{\partial \theta}{\partial \xi} = \frac{\partial^2 \theta}{\partial \eta^2} \quad (\text{Energy eq}^n)$$

For very small values of velocity L.H.S. of eq$^n$ 2 can be neglected.

From engineering point pof view, heat lost by plate is important.

$$\left.\frac{\partial \theta}{\partial \eta}\right|_{\eta=0} \rightarrow \text{important}$$

is a $f^n$ of $z$

$$q''_{avg} = \frac{1}{H}\int_0^H -k \left.\frac{\partial T}{\partial y}\right|_{y=0} dz = q''$$

$$q''_{avg} = k(T_0 - T_1)\left(\frac{B}{\mu\alpha H}\right)^{1/4} \int_0^1 -\left.\frac{\partial \theta}{\partial \eta}\right|_{\eta=0}$$

$$\frac{B}{\mu\alpha H} = \frac{\rho \beta g (T_0 - T_1) \rho C_p}{\mu k H}$$

$$= \frac{\rho^2 \beta g H^3 \Delta T}{\mu^2} \cdot \frac{C_p \mu}{k} \cdot \frac{1}{H^4}$$

$$= Gr \; Pr \; \frac{1}{H^4} \quad \{\text{Groshoff Number}\}$$

$$\left(\frac{B}{\mu\alpha H}\right)^{1/4} = \frac{1}{H}(Gr\,Pr)^{1/4}$$

$$\Rightarrow \theta = f(\eta, \xi, Pr)$$

$$\Rightarrow \left.\frac{\partial \theta}{\partial \eta}\right|_{\eta=0} = f(\xi, Pr)$$

$$\Rightarrow \int_0^1 \left.\left(\frac{\partial \theta}{\partial \eta}\right)\right|_{\eta=0} d\xi = f(Pr) = c \text{ (Let)}$$

This process is characterised by weak dependence on $Pr$

Experimentally it is found

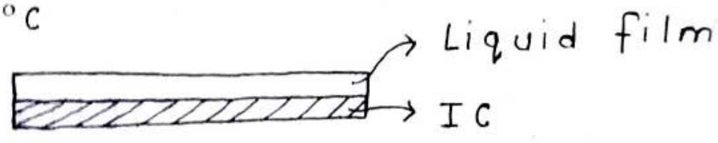
| Pr | 10 | $10^2$ | $10^3$ | 0.1 |
|---|---|---|---|---|
| C | 0.612 | 0.652 | 0.653 | 0.52 |

$$q''_{avg} = k\,(\Delta T)\,\frac{1}{H}\,(Gr\,Pr)^{1/4}\,f(Pr)$$

$$\simeq k\,(\Delta T)\,\frac{1}{H}\,(Gr.\,Pr)^{1/4}\,(0.6\ \ )$$

Q. Atmosphere

$T_\infty = 35°C$




Relative humidity = 50 %

$T_\infty = 35°C$

$k_c = 2.5 \times 10^{-4}\ mol/Pa.s.m^2$

$q''$ from the chip $= 10\ W/cm^2$

a) If the heat of vaporisation of the liquid is 35 kJ/mol, what is the flux of vapor from the surface of the liquid at steady state ?

$\dot{m} \times 35 \times 10^3 = 10 \times 10^4$

$\dot{m} = 2.857\ mol/m^2 s$

b) The vapor pressure of the liquid can be represented by the following eqn :-

$$\log_{10} P^{sat}\,(mm\ Hg) = 7.96681 - \frac{1668.21}{T + 228.0}$$

What is the temperature of the

surface of the liquid ?

$$R.H = \frac{p_A}{p_{sat}} \times 100 = 0.5$$

$T = 35°C$

Using Antoinne's Equation,

$p_{sat} = 42.05$ mm Hg

$p_A = 21.025$ mm Hg $= \frac{2761}{2803}$ (Pa)

$\dot{m} = k_c \Delta P = 2.5 \times 10^{-4} (P_{LS} - 2761)$

$q'' = \dot{m} h = 2.5 \times 10^{-4} (P_{LS} - 2761) h$

$2.857 = 2.5 \times 10^{-4} (P_{LS} - 2761)$

$P_{LS} = 1.42 \times 10^4$ Pa

Liquid surface will be at saturation.

$$\therefore \log_{10} \left( \frac{1.42 \times 10^4}{1.013 \times 10^5 / 760} \right) = 7.96681 - \frac{1668.21}{T + 228}$$

$\therefore T = 53°C$

c) What is the temperature at the surface of the chip assuming a liquid film thickness of 100 μm and a (k) for the liquid of 0.5 W/mK ?

$$-k \frac{\Delta T}{L} = q''$$

$$-0.5 \frac{(-T_s + 53)}{100 \times 10^{-6}} = 10 \times 10^4$$

$\Rightarrow T_s - 53 = \frac{10}{0.}$

$\Rightarrow T_s = 73°C$

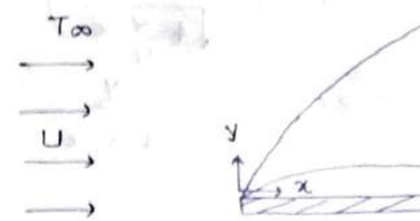
d) if heat flux is
present value, fi

$$-\frac{0.5 (53 - T_s)}{100 \times 10^{-6}} =$$

$\Rightarrow T_s - 53 = 22$

$\Rightarrow T_s = 75°C$

Flow of Liquid Metal




$$\rho C_p \left( v_x \frac{\partial T}{\partial x} + v_y \right.$$

$v_x >> v_y \quad \frac{\partial T}{\partial y}$

$$v_x \frac{\partial T}{\partial x} + v_y \frac{\partial T}{\partial y}$$

$\frac{\partial T}{\partial y}$ will now be
of boundary lay
$v_x = U$ (every

$$\frac{\partial T}{\partial x} = \frac{\alpha}{U} \frac{\partial^2 T}{\partial y^2}$$

$\Rightarrow \quad T_s - 53 = \frac{10}{0.5}$

$\Rightarrow \quad T_s = 73^\circ C$

d) if heat flux is 10% more than present value, find $T_s$.

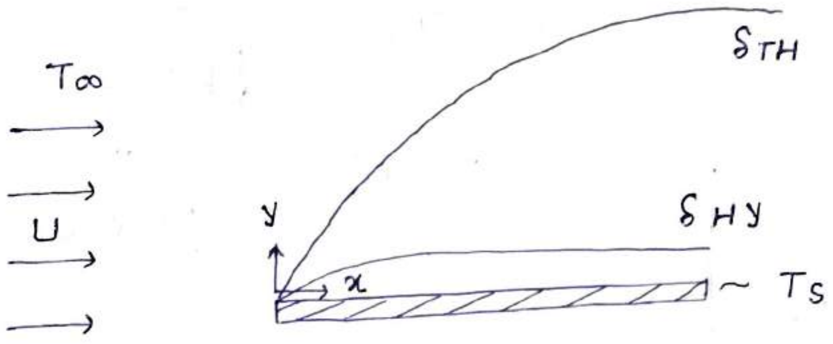
$$-\frac{0.5\,(53 - T_s)}{100 \times 10^{-6}} = 11 \times 10^4$$

$\Rightarrow \quad T_s - 53 = 22$

$\Rightarrow \quad T_s = 75^\circ C$

Flow of Liquid Metal Over a Flat Plate

$Pr << 1$




$T = f(x, y)$

$V_x = f(x, y)$

$$\rho C_p \left( V_x \frac{\partial T}{\partial x} + V_y \frac{\partial T}{\partial y} \right) = K \left( \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right)$$

$V_x >> V_y \qquad \frac{\partial T}{\partial y} >> \frac{\partial T}{\partial x} \qquad \frac{\partial^2 T}{\partial x^2}$ small

$$V_x \frac{\partial T}{\partial x} + V_y \frac{\partial T}{\partial y} = \alpha \frac{\partial^2 T}{\partial y^2}$$

$\frac{\partial T}{\partial y}$ will now be small as thickness of boundary layer will be large.

$V_x = U$ (everywhere)

$$\frac{\partial T}{\partial x} = \frac{\alpha}{U} \frac{\partial^2 T}{\partial y^2}$$

Define $T^* = \dfrac{T - T_\infty}{T_s - T_\infty}$

$$\frac{\partial T^*}{\partial x} = \frac{\alpha}{U} \frac{\partial^2 T^*}{\partial y^2}$$

B.C: i> $y = 0$ $\quad T^* = 1$

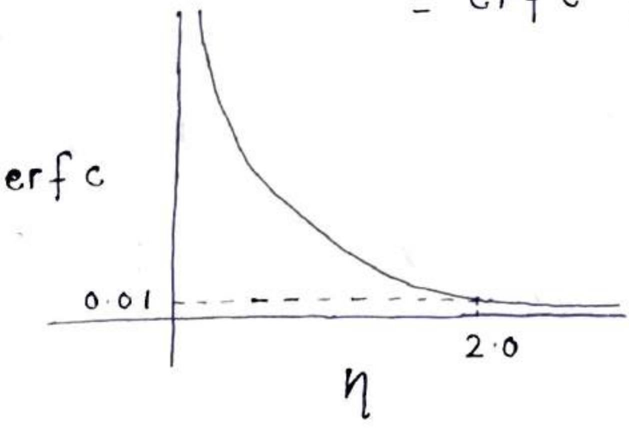
ii> $y = \infty$ $\quad T^* = 0$

iii> $x = 0$ $\quad T^* = 0$ $\quad (T = T_\infty)$

$$\eta = \frac{y}{\sqrt{4 \frac{\alpha}{U} x}}$$

$$\frac{T - T_\infty}{T_s - T_\infty} = T^* = 1 - \text{erf}\left(\frac{y}{\sqrt{4 \frac{\alpha}{U} x}}\right)$$

$$= \text{erfc}\left(\frac{y}{\sqrt{4 \frac{\alpha}{U} x}}\right)$$




When $\eta = 2$, $y = \delta_t$ (thickness of BL)

$$\frac{d}{dx}\left\{ \text{erf} \frac{x}{\sqrt{4At}} \right\}\Bigg|_{x=0} = \frac{1}{(\pi A t)^{1/2}}$$

$$\frac{dT^*}{dy}\Bigg|_{y=0} = -\sqrt{\frac{U}{\pi \alpha x}}$$

$$-k \frac{\partial T}{\partial y} \Big|_{y=0} = h ( T_s - T_\infty )$$

$$-k ( \cancel{T_s - T_\infty} ) \frac{\partial T^*}{\partial y} \Big|_{y=0} = h ( \cancel{T_s - T_\infty} )$$

$$+ k \sqrt{\frac{U}{\pi \alpha x}} = h$$

$$Nu_x = \frac{h x}{k} = \sqrt{\frac{U}{\pi \alpha x}} \cdot x = \sqrt{\frac{U x}{\pi \alpha}}$$

$$Nu_x = K' \sqrt{Pr \, Re} = K' \sqrt{Pe} = \frac{1}{\sqrt{\pi}} \sqrt{Pe}$$

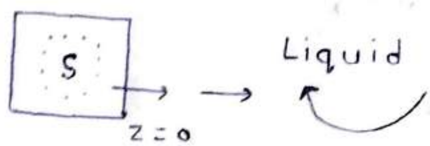
$$Nu_x = 0.565 ( Pe )^{1/2}$$

$$Nu_{,avg} = \frac{1}{L} \int_0^L 0.565 ( Pe_x )^{1/2} dx$$

28/02/19

## Mass Transfer (BSL)

$$N_{Az} = -cD_{AB}\frac{\partial x_A}{\partial z} + x_A(N_{Az} + N_{Bz})$$



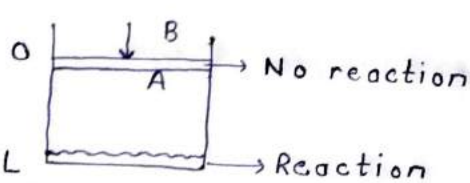

$$\Rightarrow -D_{AB}\left.\frac{dC_A}{dz}\right|_{z=0} = h_m\left(C_A|_{z=0} - C_\infty\right)$$

$\hookrightarrow$ MTC

$$\frac{hD}{k} = Nu = f(Re, Pr)$$

$$\frac{h_m D}{D_{AB}} = Sh = f(Re, Sc)$$

Mass In − Mass out + Generation = 0
↓
(Homogeneous reaction)




rate of reaction = diffusion rate of B

$$\text{Reaction} = -D_{BA}\left.\frac{dc_B}{dz}\right|_{z=0}$$

At the interface, only conduction

$k_1'''$ : homogeneous

$k_1''$ : heterogeneous

Fast reactio

$C_B = 0$

$N_{Az}|_z\, S - N$

$\lim_{\Delta z \to 0} N_{Az}$

Q.

Air
↓

$C_{Ao}$

Find :

1> $O_2$ conc

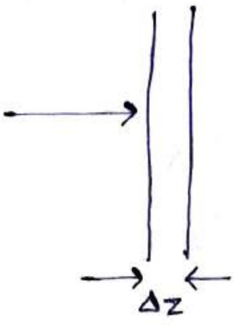
$N_A|_{x=x}\, S -$

$-\frac{dN_A}{dx} =$

$-\frac{d}{dx}(-D_{A}$

Fast reaction : No B at the interface (surface of catalyst)

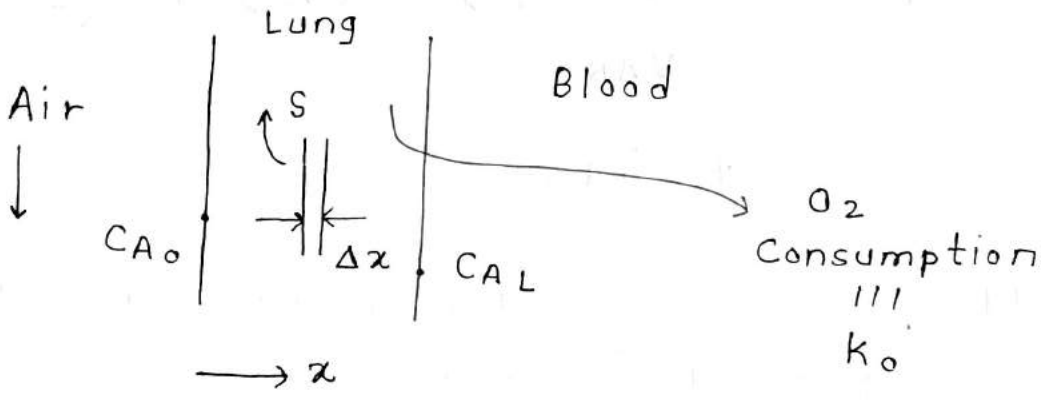
$C_B = 0$




$$N_{AZ}\Big|_{z} S - N_{AZ}\Big|_{z+\Delta z} S \pm \text{Reaction} = 0$$

$$\lim_{\Delta z \to 0} \frac{N_{Az}\big|_{z+\Delta z} - N_{Az}\big|_{z}}{\Delta z} = r_A$$

Q.




Find :

1) $O_2$ conc profile in tissue.

$$N_A\Big|_{x=x} S - N_A\Big|_{x+\Delta x} S - k_o''' S \Delta x = 0$$

$$-\frac{dN_A}{dx} = k_o'''$$

$$-\frac{d}{dx}\left(-D_{AB}\frac{dC_A}{dx}\right) = k_o'''$$

$$D_{AB}\frac{d^2C_A}{dx^2} = k_o'''$$

$$C_A = \frac{k_o'''}{D_{AB}}\frac{x^2}{2} + c_1 x + c_2$$

$x = 0$ , $C_A = C_{A_0}$

$x = L$ , $C_A = C_{A_L}$

$$c_2 = C_{A_0}$$

$$\frac{C_{A_L} - \frac{k_o'''}{D_{AB}}\frac{L^2}{2} - C_{A_0}}{L} = c_1$$

$$C_A = \frac{k_o'''}{2D_{AB}}(x^2 - Lx) + \left(\frac{C_{AL} - C_{A_0}}{L}\right)x + C_{A_0}$$

11> $O_2$ assimilation rate by the blo... per unit area

Rate to blood per unit area

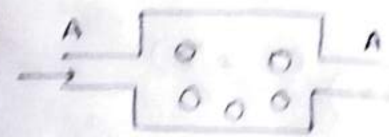
$$= -D_{AB}\left.\frac{dC_A}{dx}\right|_{x=L}$$

$$= -\frac{k_o L}{2} + \frac{D_{AB}}{L}\left[C_{A_0} - C_{A_L}\right]$$

11|03|19

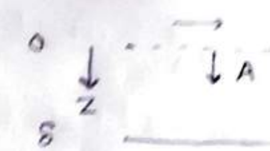
Diffusion with ...
Reaction

Catalytic reactor



$N_{Az} = -D_{AB}\frac{d...}{d...}$

If the MT b... the catalyst su... to be flat.



At the surface ... and irreversible ... Find an express... of conversion f...

$N_{Az}|_z \; \delta - N_{...}$

$\frac{d}{dz}(N_{Az}) = ...$

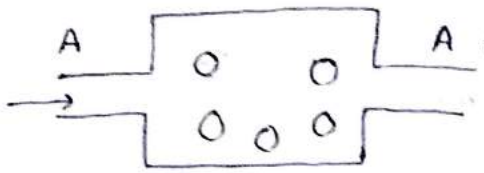
$N_{Az} = -D_{AB} ...$

$N_{A_2z} = -\frac{N_{Az}}{2}$

11/03/19

Diffusion with Heterogeneous Chemical Reaction

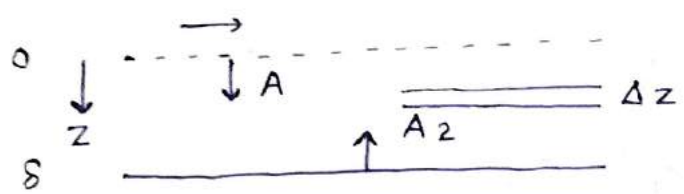
Catalytic reactor $2A \rightarrow A_2$




Catalyst surface
$2A \rightarrow A_2$

$$N_{Az} = -D_{AB}\frac{dC_A}{dz} + x_A(N_A + N_B)$$

If the MT boundary layer is very thin, the catalyst surface can be approximated to be flat.




$\delta <<$ Radius of pellet

At the surface, the reaction is inst. and irreversible.

Find an expression of the local rate of conversion from $A \rightarrow A_2$.

$$N_{Az}\big|_z S - N_{Az}\big|_{z+\Delta z} S = 0$$

$$\frac{d}{dz}(N_{Az}) = 0$$

$$N_{Az} = -D_{AB}\frac{dC_A}{dz} + x_A(N_{Az} + N_{A_2z})$$

$$N_{A_2z} = -\frac{N_{Az}}{2}$$

$$\Rightarrow N_{Az} = -\frac{C D_{AA_2}}{1 - \frac{x_A}{2}} \frac{dx_A}{dz}$$

B.C. $x_A = x_{A0}$ @ $z = 0$

$x_A = 0$ @ $z = \delta$

$$\Rightarrow \left(1 - \frac{x_A}{2}\right) = \left(1 - \frac{x_{A0}}{2}\right)^{(1 - z/\delta)}$$

$$N_{Az} = \frac{2 C D_{AA_2}}{\delta} \ln\left(\frac{1}{1 - \frac{x_{A0}}{2}}\right)$$

Rate at which A gets transformed to $A_2$ $= N_{Az} = \frac{2 C D_{AA_2}}{\delta} \ln\left(\frac{1}{1 - \frac{x_{A0}}{2}}\right)$

Q $NO + CO \rightleftharpoons \frac{1}{2} N_2 + CO_2$

$r_A = -k_1'' C_A$

One dimensional diffusion only through a thin gas film of thickness L.

$T = 500°C$ $P = 1.2$ bar

Thickness = 1 mm $k_1'' = 0.05$ m/s

$A = 200$ cm$^2$ $D_{AB} = 10^{-4}$ m$^2$/s

$x_{AL} = 0.15$

i) What is the mole fraction of NO at the catalyst surface?

ii) What is the NO removal rate for a surface area?

$$N_{NO}\Big|_{z} S - N_{NO}\Big|_{z+\Delta z} = 0$$

$$\Rightarrow \frac{dN_{NO}}{dz} = 0$$

$$N_{NO} = -D_{AB}\frac{dC_{NO}}{dz} + x_{NO}(\cancel{N_{NO} + N_{N_2}})^{\approx 0}$$

$$N_{NO} = -D_{AB}\frac{dC_{NO}}{dz} = -D_{AB}\, C\, \frac{dx_{NO}}{dz}$$

$$\Rightarrow -D_{AB}\, C\, \frac{d^2 x_{NO}}{dz^2} = 0$$

$$\Rightarrow x_{NO} = C_1 z + C_2$$

$$@\ z = 0 \qquad x_{NO} = x_{NO,S}$$

$$@\ z = \delta = L \qquad x_{NO} = x_{AL}$$

$$x_{NO} = \left(\frac{x_{NO,L} - x_{NO,S}}{L}\right) z + x_{NO,S}$$

$$N_{NO} = -D_{AB}\cancel{C}\frac{dx_{NO}}{dz} = -k_1''\cancel{C}\, x_{NO,S}$$

$$D_{AB}\left(\frac{x_{NO,L} - x_{NO,S}}{L}\right) = k_1'' x_{NO,S}$$

$$\frac{\frac{D_{AB}\, x_{NO,L}}{L}}{\left(\frac{D_{AB}}{L} + k_1''\right)} = x_{NO,S}$$

$$\Rightarrow x_{NO,S} = 0.1$$

ii) $N_{NO}|_{z=0} = -r_A$

$$-D_{AB} \frac{dx_{NO}}{dz} \cdot C = r_A$$

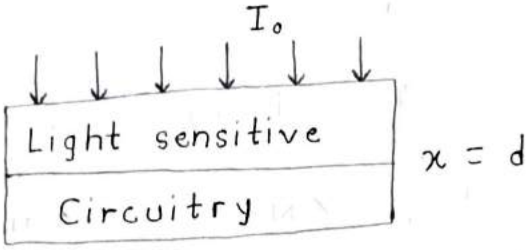
$$+D_{AB} \frac{(x_{NO,L} - x_{NO,s})}{L} C \cdot S = -r_A$$

$$r_A = -9.34 \times 10^{-2} \times 200 \times 10^{-4}$$

$$r_A = -1.87 \times 10^{-3} \text{ mol/s}$$

$\therefore$ NO removal rate $= -1.87 \times 10^{-3}$ mol/s

Q.




$I = I_o \exp(-\alpha x)$ and $M = m_o I$

M : rate of electron generation

a) Determine the concentration profile of electrons, $C_e$, in the light sensitive layer. The flow of electrons is governed by a law similar to Fick's Law. The diffusivity of electrons is governed in the light sensitive layer is $D_{es}$.

$$N_A|_z \cancel{S} - N_A|_{z+\Delta z} \cancel{S} = -m_o I \cancel{S} \Delta z$$

$$-\frac{dN_A}{dz} = -m_o I$$

$$\frac{dN_A}{dx} = m_o I_o \exp(-\alpha x)$$

From Fick's Law,

$$N_A = -D_e \frac{dC_A}{dx}$$

$$-D_e \frac{d^2 C_A}{dx^2} = m_o I_o \exp(-\alpha x)$$

$$D_e \frac{dC_A}{dx} = \frac{m_o I_o}{\alpha} \exp(-\alpha x) + c_1$$

$$D_e C_A = -\frac{m_o I_o}{\alpha^2} e^{-\alpha x} + c_1 x + c_2$$

$$x = 0, \quad C_A = 0$$

$$x = d, \quad -D_{Ac} \left.\frac{dC_A}{dx}\right|_d = \int_0^d m_o I_o e^{-\alpha x} dx$$

$$\therefore C_e = \frac{m_o I_o}{\alpha^2 D_{es}} \left[1 - e^{-\alpha x}\right] -$$

$$\frac{m_o I_o}{\alpha D_{AB}} x$$

Q Time-release drug may be assumed to be a rod of overall radius $r_o$ (m) and Length L (m)

Diffusivity = $D_{AB}$

At the inner edge of the coating ($r_i$) the composition of the drug (mole fraction) is $x_{ai}$

Mass Transfer coefficient = $K_{ac}$

$x_{a,\infty} = 0$

Total conc. of all the species within the coating is $C_t$ (constant)

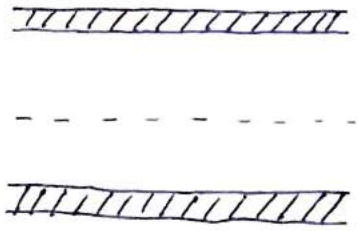
It is a diffusion only process

Calculate the rate of drug release (mg/hr). $r_i = 3$ mm, $r_o = 5$ mm,

$D_{AB} = 10^{-10}$ $m^2/s$, $K_{ac} = 0.1$ m/s

L = 5 mm, $C_t = 0.4$ kg/$m^3$,

$x_{ai} = 0.9$



$$N_A \cdot 2\pi r L \Big|_{r=r} - N_A\, 2\pi r L \Big|_{r+\Delta r} = 0$$

$$2\pi L \left( -\frac{d}{dr}(N_A r) \right) = 0$$

$$\frac{d}{dr}(r N_A) = 0$$

$$N_A = -D_{AB}\frac{dC_A}{dr} + x_A(\cancel{N_A + N_B})^{\approx 0}$$

$$= -D_{AB}\, C\, \frac{dx_A}{dr}$$

$$\frac{d}{dr}\left(-D_{AB}\cdot C\, r\frac{dx_A}{dr}\right) = 0$$

$$\frac{d}{dr}\left(r\frac{dx_A}{dr}\right) = 0$$

$$r\frac{dx_A}{dr} = c$$

$$\frac{dx_A}{dr} = \frac{c}{r}$$

$$x_A = c \ln r + d$$

$$x_A = x_{Ai} \quad \text{at } r = r_i$$

$$N_A \big|_{r=r_o} = k_{ac}\,(C_{A\,surf} - \cancel{C_{A\infty}}^{0})$$

$$-D_{AB}\, C_t \left.\frac{dx_A}{dr}\right|_{r=r_o} = k_{AC}\, C x_{As}$$

$$x_A = x_{Ai} + \frac{k_{AC}}{D_{AB}}\, r_o\, x_{Ao} \ln\frac{r_i}{r}$$

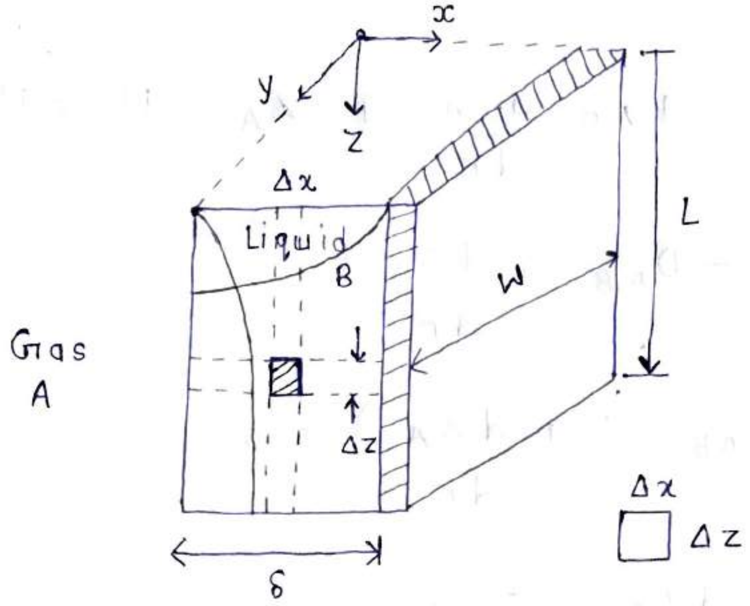
Rate of Drug Release = Area X coeff. X $\Delta C$.

$$= 2\pi r_o L\, x_{Ao}\, C_t$$

$$x_{Ao} = 3.52 \times 10^{-7}$$

Rate $= 8 \times 10^{-3}$ mg/hr

7.03.2019




Diffusion in a falling film –

Eq$^n$ of change for multicomponent system :

$$\frac{\partial \rho_A}{\partial t} + \frac{\partial n_{Ax}}{\partial x} + \frac{\partial n_{Ay}}{\partial y} + \frac{\partial n_{az}}{\partial z} = r_A$$

Vector form :–

$$\frac{\partial \rho_A}{\partial t} + (\nabla . n_A) = r_A \quad -1$$

$$\frac{\partial \rho_B}{\partial t} + (\nabla . n_B) = r_B \quad -2$$

$$v = \frac{\Sigma \rho_i v_i}{\Sigma \rho_i} \quad , \quad v^* = \frac{\Sigma c_i v_i}{\Sigma c_i}$$

$\rho = \rho_A + \rho_B \qquad c = c_A + c_B$

$n_A + n_B = n = \rho V$

A is producing B.

$r_A + r_B = 0$ (when expressed in mass terms)

may or may not be zero when

expressed in mole terms.

① + ② gives,

$$\frac{\partial \rho}{\partial t} + (\nabla \rho V) = 0$$

$$\frac{\partial C_A}{\partial t} + (\nabla N_A) = R_A$$

$$\frac{\partial C_B}{\partial t} + (\nabla N_B) = R_B$$

$$\frac{\partial C}{\partial t} + \nabla C V^* = (R_A + R_B)$$

$$n_A - W_A (n_A + n_B) = -\rho D_{AB} \nabla \omega_A$$

↳ mass fraction

$$\frac{\partial \rho_A}{\partial t} + (\nabla \rho_A V) = (\nabla \rho D_{AB} \nabla \omega_A) + r_A$$

Special Case :-

1) CONST $\rho$ & $D_{AB}$ :-

$$\frac{\partial \rho_A}{\partial t} + \rho_A (\underbrace{\cancel{\nabla V}}_{0}) + V \nabla \rho_A = D_{AB} \nabla^2 \rho_A + r_A$$

$$\frac{D \rho_A}{D t} = D_{AB} \nabla^2 \rho_A + r_A$$

÷ by $M_A$

$$\frac{D C_A}{D t} = D_{AB} \nabla^2 C_A + R_A$$

Q.


Pure $N_2$ flows from right to left (with a velocity v), as shown in figure. Derive the concentration distribution of $O_2$ in the tube. Assume steady state is reached.

A : $N_2$ B : $O_2$

$$\frac{D\cancel{C_A}}{Dt} = D_{AB} \nabla^2 C_A + \cancel{R_A}$$

0 0

(only time component)

$$\Rightarrow v_z \frac{\partial C_A}{\partial z} = D_{AB} \frac{\partial^2 C_A}{\partial z^2}$$

W is very large.

@ $x = 0$, conc. of gas is known.

Species Balance Eqⁿ –

$$\underset{0\ (SS)}{\cancel{\frac{\partial C_A}{\partial t}}} + \underset{0}{\cancel{v_x}} \frac{\partial C_A}{\partial x} + v_y \underset{0}{\cancel{\frac{\partial C_A}{\partial y}}} + v_z \frac{\partial C_A}{\partial z} = D_{AB} \left( \frac{\partial^2 C_A}{\partial x^2} + \underset{0}{\cancel{\frac{\partial^2 C_A}{\partial y^2}}} + \frac{\partial^2 C_A}{\partial z^2} \right) + \underset{0}{\cancel{R_A}}$$

. No reaction b/w gas and liquid

. $D_{AB}$ is a constant.

$$v_z \frac{\partial C_A}{\partial z} = \left( \frac{\partial^2 C_A}{\partial x^2} + \frac{\partial^2 C_A}{\partial z^2} \right) D_{AB}$$

In z direction, convection dominates and $\frac{\partial^2 C_A}{\partial z^2}$ can be neglected.

$$v_z \frac{\partial C_A}{\partial z} = D_{AB} \frac{\partial^2 C_A}{\partial x^2}$$

$$v_z = V_{max} \left[ 1 - \frac{x^2}{\delta^2} \right]$$

$$\Rightarrow V_{max} \left[ 1 - \frac{x^2}{\delta^2} \right] \frac{\partial C_A}{\partial z} = D_{AB} \frac{\partial^2 C_A}{\partial x^2}$$

The liquid starts as pure B.

∴ @ $z = 0$, $C_A = 0$

@ $x = 0$, $C_A$ = interfacial concⁿ

If $\delta$ is very large @ $x = \delta$, $C_A = 0$

But @ $x = \delta$ $\quad \dfrac{\partial C_A}{\partial x} = 0$

if $D_{AB}$ is very small and L is small, the thickness of B.L. will be small;

$V_z = V_{max}$ throughout the concentration boundary layer

Limiting $sol^n$ :

$$V_{max} \frac{\partial C_A}{\partial z} = D_{AB} \frac{\partial^2 C_A}{\partial x^2}$$

@ $x = \infty$ $\quad C_A = 0$

$$\frac{C_A}{C_{A_0}} = 1 - \operatorname{erf} \frac{x}{\sqrt{4 D_{AB} \frac{z}{V_{max}}}}$$

Absorption of A in B ;

$$W_A = \int_0^L \int_0^W N_{Ax}\Big|_{x=0} \, dz \, dy$$

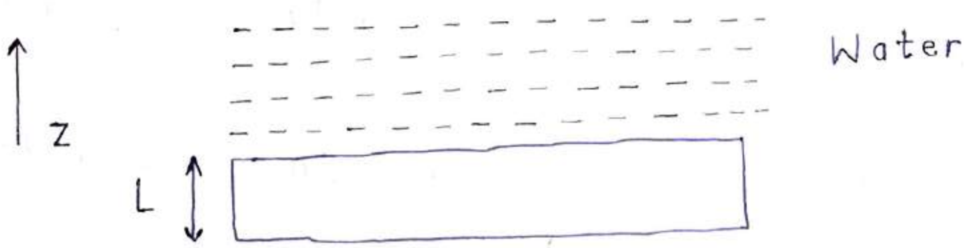
$$N_{Ax} = - D_{AB} \frac{\partial C_A}{\partial x}\Big|_{x=0}$$

$$\frac{\partial}{\partial x}\left[\operatorname{erf} \frac{x}{\sqrt{4az}}\right]\Big|_{x=0} = \frac{1}{(\pi a z)^{1/2}}$$

$$N_{Ax}(z)\Big|_{x=0} = C_{A_0}\sqrt{\frac{D_{AB}\,V_{max}}{\pi z}}$$

Q. Salt slab of thickness L. The salt dissolves in water. At interface $\rho_A = \rho_{As}$. Initially $\rho_A = 0$, how does density vary with position and time after contact? What is the surface recession rate $dL/dt$ and how does this surface recession vary with time?

If $\rho_A = 2165 \text{ kg/m}^3$ and $\rho_{As} = 380 \text{ kg/m}^3$, by much will the surface recede after 24 h? $D_{AB} = 1.2 \times 10^{-9} \text{ m}^2/\text{s}$




$$\frac{\partial \rho_A}{\partial t} = D_{AB}\frac{\partial^2 \rho_A}{\partial z^2}$$

@ $t = 0$ $\quad C_A = 0 \quad \forall z$

@ $z = 0$ $\quad \rho_A = \rho_{As} \quad \forall t$

@ $z \to \infty$ $\quad \rho_A = 0 \quad \forall t$

$$\rho_A = \rho_{A0}\left(1-\mathrm{erf}\,\frac{z}{\sqrt{4D_{AB}t}}\right)$$

$$n_A\big|_{z=0} = -D_{AB}\frac{d\rho_A}{dz}\Big|_{z=0}$$

$$n_A\big|_{z=0} = \frac{D_{AB}}{(\pi a t)^{1/2}}\cdot\rho_{A0}$$

$$\cancelto{0}{\text{In}} - \text{Out} + \cancelto{0}{\text{generation}} = \text{accumulation}$$

$$-\frac{D_{AB}\cdot\rho_{A0}}{(\pi a t)^{1/2}}\cancel{A} = \frac{d}{dt}(\rho\cancel{A}L)$$

$$-\frac{D_{AB}\cdot\rho_{A0}}{(\pi D_{AB}t)^{1/2}} = \rho\frac{dL}{dt}$$

$$\int_{L_0}^{L} dL = -\frac{D_{AB}\rho_{A0}}{\rho(\pi D_{AB})^{1/2}}\int_0^t \frac{dt}{t^{1/2}}$$

$$L-L_0 = -\sqrt{\frac{D_{AB}}{\pi}}\,\frac{\rho_{A0}}{\rho}\cdot 2t^{1/2}$$

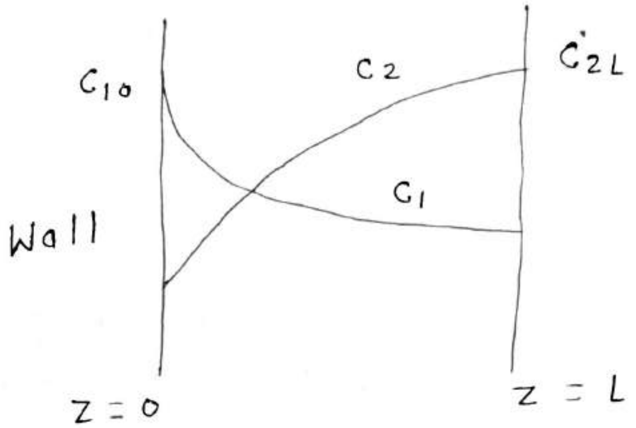
$$L = L_0 - 2\sqrt{\frac{D_{AB}}{\pi}}\,\frac{\rho_{A0}}{\rho}\,t^{1/2}$$

$$L_0 - L = 2\sqrt{\frac{D_{AB}}{\pi}}\,\frac{\rho_{A0}}{\rho}\,t^{1/2}$$

$t = 24$ h

$L_0 - L = 2.017\times10^{-3}$ m

Q. A film of stagnant liquid of thickness $L$.





2 is more in quantity

Species 1 and 2 react at a rate $R_{12} = -k C_1 C_2$. Reaction is homogeneous throughout the film.

Governing Eq$^n$ :-

$$D_{AB} \frac{d^2 C_A}{dx^2} + R_A = 0$$

$$\Rightarrow D_{AB} \frac{d^2 C_1}{dz^2} - k C_1 C_2 = 0$$

@ $z = 0$, $C_1 = C_{10}$

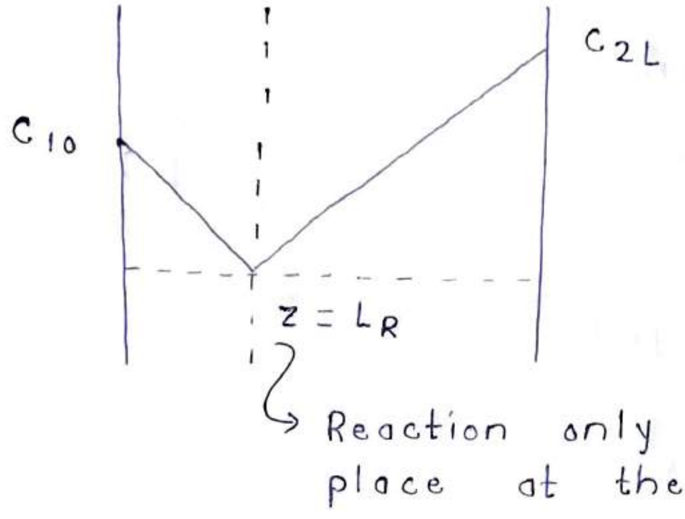
$z = L$, $C_1 = 0$ {No $C_1$ present}

$$D_{AB} \frac{d^2 C_2}{dz^2} - k C_1 C_2 = 0$$

@ $z = 0$, $\frac{dC_2}{dz} = 0$

$z = L$, $C_2 = C_{2L}$

· The reaction is very fast. The two species meet at a junction and react instantaneously.




On the right side of $z = L_R$, no $C_1$.

$$\therefore \frac{d^2 C_1}{dz^2} = 0$$

with $C_1 = C_{10}$ at $z = 0$

and $C_1 = 0$ at $z = L_R$

Similary,

$$\frac{d^2 C_2}{dz^2} = 0$$

with $C_2 = C_{2L}$ at $z = L$

and $C_2 = 0$ at $z = L_R$

$$C_1 = a_1 + b_1 z$$

$$C_2 = a_2 + b_2 z$$

$$a_1 = C_{10}, \quad b_1 = -\frac{C_{10}}{L_R}$$

$$a_2 = -\frac{C_{2L} L_R}{L - L_R}, \quad b_2 = \frac{C_{2L}}{L - L_R}$$

but $L_R$ is unknown.

Additional Physical Statement

1 mole of species 1 reacts with 1 mole of 2, at steady state

$$N_1 = -N_2$$

$$-D_1 \frac{dc_1}{dz} = D_2 \frac{dc_2}{dz}$$

$$-D_1 b_1 = D_2 b_2$$

$$\frac{C_{10} D_1}{L_R} = \frac{C_{2L} D_2}{L - L_R}$$

$$\Rightarrow \quad L_R = \frac{L}{1 + \frac{C_{2L} D_2}{C_{10} D_1}}$$

In absence of reaction;

$$\frac{dc_1}{dz} = \frac{C_{10} - 0}{L} = \frac{C_{10}}{L}$$

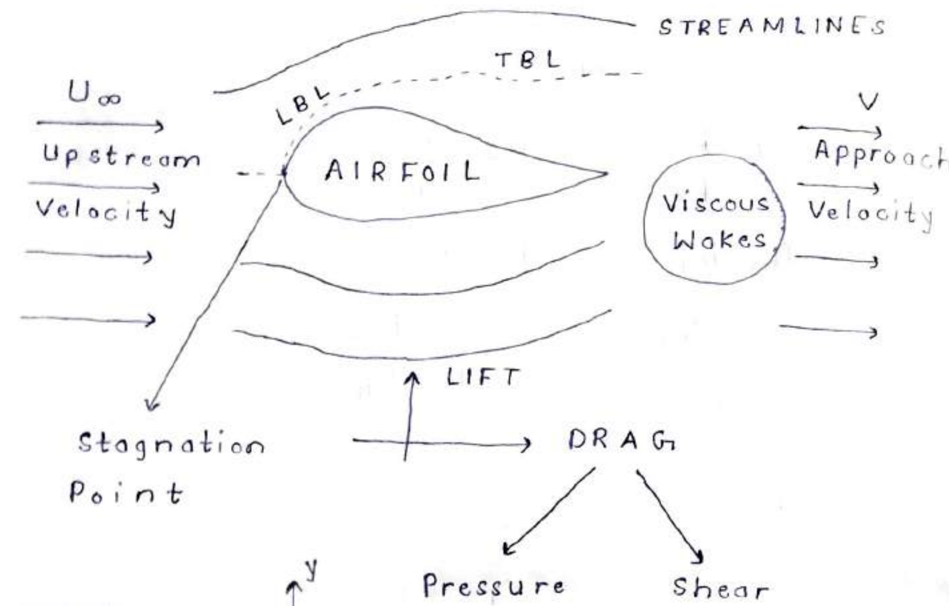
In presence of reaction,

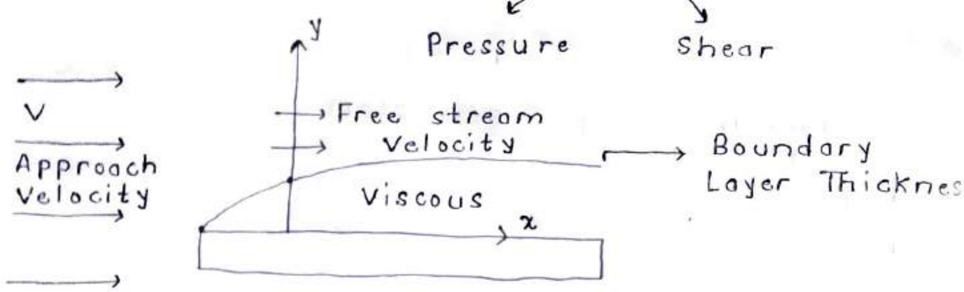
L changes to $L_R$.

$$N_1 = \frac{C_{10} D_1}{L} \left( 1 + \frac{C_{2L} D_2}{C_{10} D_1} \right)$$

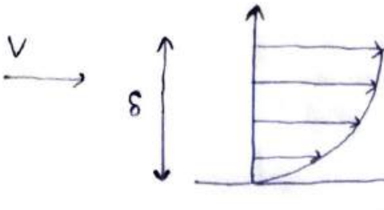
14 | 03 | 2019

## Hydrodynamic Boundary Layer :



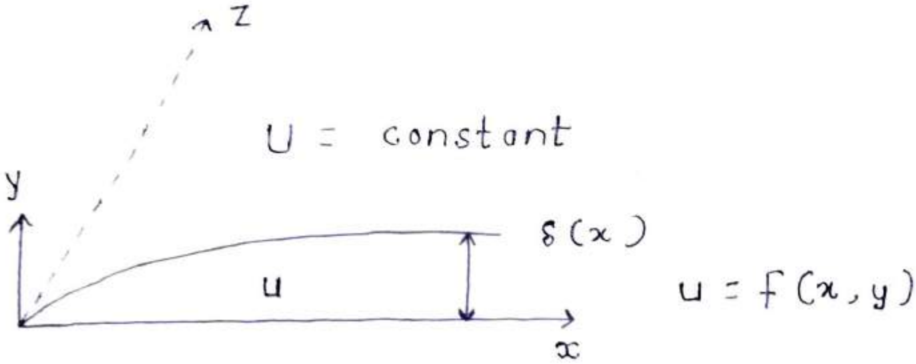




$V = U_\infty$ {for flat plate}




$V_x \approx 0.99\,V \approx 0.99\,U_\infty$

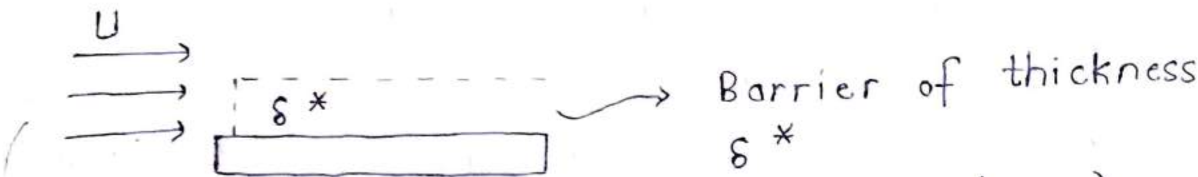

$V_x(x, y)$ in the BL




Mass flow in the BL in absence of viscosity $= \int_0^{\delta} \rho U \, dy$

In presence of viscosity $= \int_0^{\delta} \rho u \, dy$

Reduction in mass flow rate $= \rho \int_0^{\delta} (U-u) \, dy$




Inviscid flow (velocity U everywhere)

Reduction in mass flow rate in this case $= \rho U \delta^*$

If $\rho U \delta^* = \rho \int_0^{\delta} (U-u) \, dy$

then $\delta^*$ is displacement thickness

$$\delta^* = \int_0^{\infty} \left(1 - \frac{u}{U}\right) dy \approx \int_0^{\delta} \left(1 - \frac{u}{U}\right) dy$$

Length scale in case of flat plate can be $x$ or $\delta(x)$.

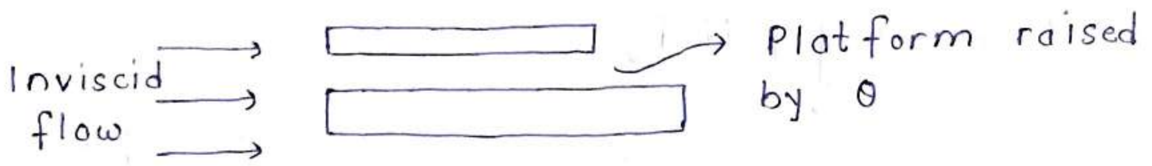
↳ For Reynolds no. calculation

For flat plate,

$$Re = \frac{\rho U x}{\mu}, \quad U: \text{free stream velocity}$$

Momentum Thickness ($\theta$):

Reduction in momentum $= \int_0^{\delta} \rho u (U-u)\, dy$




Reduction in momentum in this case

$$= \rho U^2 \theta$$

$$\rho U^2 \theta = \int_0^{\infty} \rho u (U-u)\, dy$$

$$\theta = \int_0^{\infty} \frac{u}{U}\left(1 - \frac{u}{U}\right) dy \approx \int_0^{\delta} \frac{u}{U}\left(1 - \frac{u}{U}\right) dy$$

Boundary Layer Approximations :-

Laminar flow on a flat plate -

$$v_x \frac{\partial v_x}{\partial x} + v_y \frac{\partial v_x}{\partial y} = \nu \left[ \frac{\partial^2 v_x}{\partial x^2} + \frac{\partial^2 v_x}{\partial y^2} \right]$$

$v_x >> v_y$ and $\frac{\partial v_x}{\partial y} >> \frac{\partial v_x}{\partial x}$. So, none can be neglected.

$$\frac{\partial^2 v_x}{\partial x^2} << \frac{\partial^2 v_x}{\partial y^2}$$

$$\therefore \; v_x \frac{\partial v_x}{\partial x} + v_y \frac{\partial v_x}{\partial y} = \nu \frac{\partial^2 v_x}{\partial y^2}$$

$y = 0 \qquad v_x = 0$

$y = \infty \qquad v_x = U$

$x = 0 \qquad v_x = U$

Blasius' Solution :-

$$\frac{v_x}{U} = g(\eta) \qquad \eta \sim \frac{y}{\delta(x)}$$

$$v_x = \frac{\partial \psi}{\partial y}, \quad v_y = -\frac{\partial \psi}{\partial x} \qquad \psi : \text{Stream function}$$

$\psi$ should be an exact differential.

Combination of variables Method :-

Near the BL -

$$v_x \sim U, \quad y \sim \delta \qquad \frac{\partial v_x}{\partial y} \approx 0$$

Governing eq$^n$ reduces to

$$U \frac{U}{x} \sim \nu \frac{U}{\delta^2}$$

$$\delta^2 \sim \frac{\nu x}{U} \Rightarrow \delta \sim \sqrt{\frac{\nu x}{U}}$$

From Blasius' solution

$$\eta = \frac{y}{\delta} = y \sqrt{\frac{U}{\nu x}}$$

Stream f$^n$ need to be non-dimensional.

$$f(\eta) = \frac{\psi}{\sqrt{\nu x U}}$$

$$v_x = \frac{\partial \psi}{\partial y} = U \frac{df}{d\eta}$$

$$v_y = -\frac{\partial \psi}{\partial x} = \frac{1}{2}\sqrt{\frac{\nu U}{x}}\left[\eta \frac{df}{d\eta} - f\right]$$

Substituting everything in the governing eq$^n$ gives,

$$2\frac{d^3 f}{d\eta^3} + f\frac{d^2 f}{d\eta^2} = 0$$

$$\eta = 0 \qquad f = \frac{df}{d\eta} = 0$$

$$\eta = \infty \qquad f' = 1$$

For $\eta = 5$, $f' = \frac{v_x}{U} = 0.99$

$$5 = \delta \sqrt{\frac{U}{\nu x}}$$

$$\boxed{\delta(x) = \frac{5x}{\sqrt{Re_x}}}$$

$$\tau_w = \mu \frac{\partial v_x}{\partial y}\bigg|_{y=0}$$

$$= \mu \frac{\partial}{\partial y}\frac{\partial \psi}{\partial y}\bigg|_{y=0}$$

$$= \mu \frac{\partial}{\partial y} U \frac{df}{d\eta}\bigg|_{\eta=0}$$

$$= \mu U \underbrace{\frac{d^2 f}{d\eta^2}}\frac{\partial \eta}{\partial y}\bigg|_{\eta=0}$$

$\hookrightarrow$ 0.332 @ $\eta = 0$

$$\tau_w = \mu U \sqrt{\frac{U}{\nu x}} \times 0.332$$

$$\tau_w = \frac{0.332 \rho U^2}{\sqrt{Re_x}}$$

$$C_f = \frac{\tau_w}{\frac{1}{2}\rho U^2} = \frac{0.664}{\sqrt{Re_x}}$$

↳ Shear stress Coefficient

$$C_D = \frac{\int_A \tau_w \, dA}{\frac{1}{2}\rho U^2 A}$$

A : projected Area

$C_D$ : Drag coefficient

$$C_D = \frac{1}{L}\int_0^L \frac{0.664}{\sqrt{Re_x}}\, dx = 2\,\overline{C}_{f,L}$$

Q. Use the numerical results of Howarth to evaluate the following quantities :-

1) $\delta^*/\delta$ evaluate at $\eta = 5$, $\eta \to \infty$

$y = 5\delta$ and $y \to \infty$

$$\delta^* = \int_0^{\delta} \left(1 - \frac{u}{U}\right) dy$$

$$u = U \frac{df}{d\eta} = U \times 0.99$$

$$\delta^* = \int_0^{\delta} (1 - 0.99)\, dy = 0.01\,\delta$$

$$\Rightarrow \frac{\delta^*}{\delta} = 0.01$$

---

$$\delta^* = \int_0^\delta \left(1 - \frac{Vx}{U}\right) \sqrt{\frac{\nu x}{U}}\, d\eta$$

$$= \sqrt{\frac{\nu x}{U}} \int_0^\delta \left(1 - \frac{df}{d\eta}\right) d\eta$$

$$= \sqrt{\nu x / U}\, \left[\eta - f\right]_0^\delta$$

$$= \sqrt{\nu x / U}\, \left[5 - 0 - (3.28329 - 0)\right]$$

$$\delta^* = \sqrt{\frac{\nu x}{U}}\, (1.717)$$

$$\delta = \frac{5.0\, x}{\sqrt{Re_x}} \Rightarrow \delta = 5.0 \sqrt{\frac{\nu x}{U}}$$

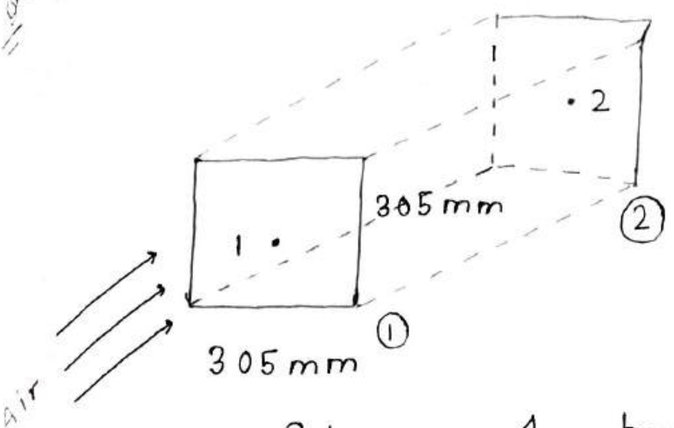
$$\Rightarrow \frac{\delta^*}{\delta} = \frac{1.717}{5} = 0.343$$

When $\eta \rightarrow \infty$

$$\delta^* = \sqrt{\frac{\nu x}{U}}\, \left[\eta - f\right]_0^\infty$$

$$= \sqrt{\frac{\nu x}{U}}\, \left[8.4 - 6.674\right]$$

Wind Tunnel




At ①

Freestream Velocity

$\rightarrow U_1 = 26\ m/s$

$\delta_1^* = 1.5\ mm$

Bounded flow 4 boundary layers.

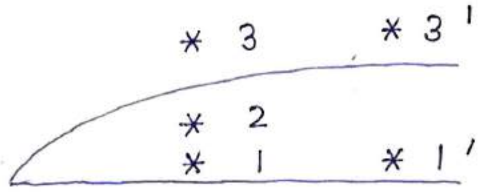
Velocity keeps on increasing from ① to ②

At ②, $U_2 = ?$ $\delta_2^* = 2.1\ mm$

Calculate the change in pressure b/w ① and ② as a fraction of the freestream Dynamic pressure.

i.e., $\dfrac{P_1 - P_2}{\frac{1}{2}\rho V_1^2}$

B.E. can be applied for the inviscid flow.




1 & 2 no press diff.

1 & 1* : press. diff.

For unbounded : 3 & 3' no pressure diff.

For bounded : 3 & 3' pressure diff.

Applying Bernoulli's Equation -

$$P_1 + \frac{1}{2}\rho V_1^2 + \rho g \cancel{z_1} = P_2 + \frac{1}{2}\rho V_2^2 + \cancel{\rho g} z_2$$

Continuity eq$^n$ –

$$A_1 V_1 = A_2 V_2$$

$$A_1 = (305 - 2\delta_1^*)^2$$

$$A_2 = (305 - 2\delta_2^*)^2$$

$$V_2 = \frac{(305 - 2\delta_1^*)^2}{(305 - 2\delta_2^*)^2} V_1$$

$$= \frac{(305 - 2 \times 1.5)^2}{(305 - 2 \times 2.1)^2} \times 26$$

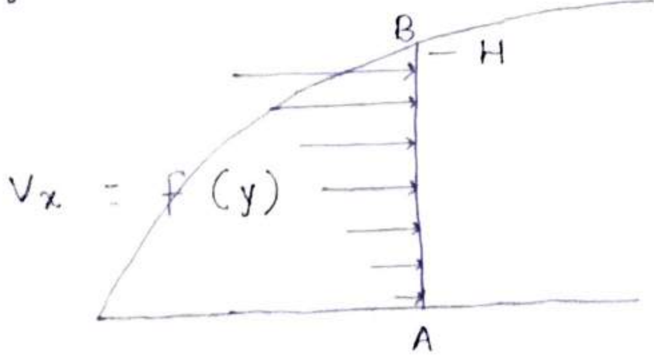
$$V_2 = 26.207 \text{ m/s}$$

## MACROSCOPIC BALANCE (Integral Approach)

$$\left.\frac{dN}{dt}\right|_{syst} = \frac{\partial}{\partial t}\int_{CV} \eta \rho \, d\forall + \int_{CS} \eta \rho \vec{V} \cdot d\vec{A} \quad -(1)$$

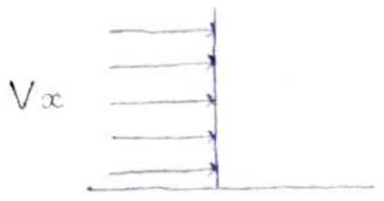
N = Extensive property

$\eta$ = Intensive property = N / Mass




$$\rho \int_0^H V_x \, dy \, dz$$

For inviscid flow :-




$$\rho V_x H Z$$

Boundary Conditions

$V_x = 0 \quad , \quad y = 0$ $\qquad \{ U_\infty$ = free-stream Velocity

$V_x = U_\infty \quad , \quad y = h$

$\frac{dV_x}{dy} = 0 \quad , \quad y = h$

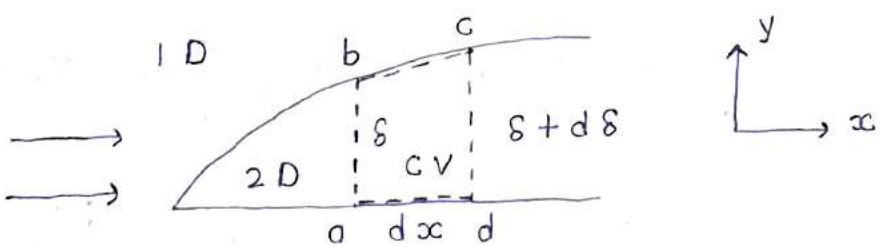
Let us assume the velocity profile to be

$$\frac{V_x}{U_\infty} = a + b\left(\frac{y}{\delta}\right) + c\left(\frac{y}{\delta}\right)^2$$

Momentum :- $N = \vec{P} \Rightarrow \eta = \vec{V}$

$$\frac{dN}{dt} = \frac{d\vec{P}}{dt} = \vec{F}$$

$$\vec{F} = \frac{\partial}{\partial t}\int_V \vec{V}\rho\, d\forall + \int_{CS} \vec{V}\rho\, \vec{V}\cdot d\vec{A}$$




$$\vec{F_s} + \vec{F_B} = \frac{\partial}{\partial t}\int_{CV} \vec{V}\rho\, d\forall + \int_{CS} \vec{V}\rho\, \vec{V}\cdot d\vec{A}$$

Continuity eq$^n$ -

$$0 = \frac{\partial}{\partial t}\int_{CV} \rho\, d\forall + \int_{CS} \rho\vec{V}\cdot d\vec{A}$$

Steady , 2D Flow

$$\int_{CS} \rho\vec{V}\cdot d\vec{A} = 0$$

$$\dot{m}_{ab} + \dot{m}_{bc} + \dot{m}_{cd} + \dot{m}_{ad} = 0$$

$$\dot{m}_{ad} = 0 \quad \{ ad \text{ lies very close to the solid} \}$$

$$\dot{m}_{bc} = -\dot{m}_{ab} - \dot{m}_{cd}$$

$$m_{x+dx} = \dot{m}_x + \left.\frac{\partial \dot{m}}{\partial x}\right|_x dx$$

$$m_{cd} = \left\{ \int_0^{\delta} \rho V_x \, dy + \frac{\partial}{\partial x}\left[ \int_0^{\delta} \rho V_x \, dy \right] dx \right\}$$

$$F_{sx} = \int_{CS} V_x \, \rho \vec{V} \cdot d\vec{A}$$

$$F_{sx} = mf_{ab} + mf_{bc} + mf_{cd}$$

$$mf_{ab} = -\left\{ \int_0^{\delta} V_x \rho V_x \, dy \right\} dz$$

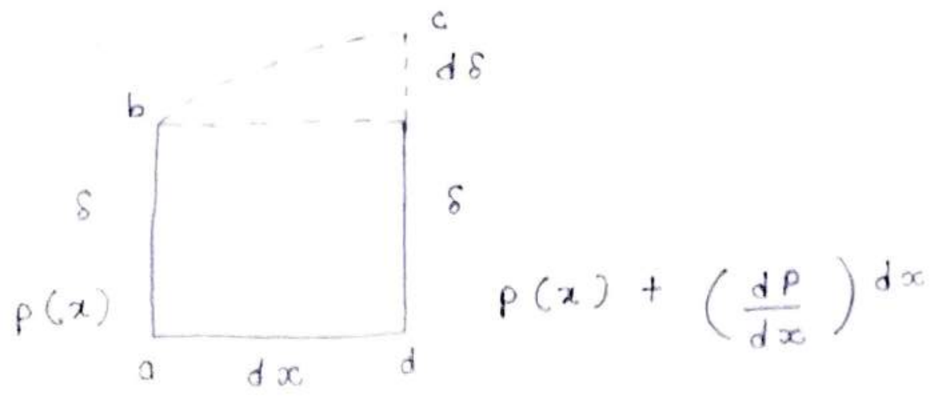
$$mf_{cd} = mf_{ab} + \frac{\partial}{\partial x}(mf_{ab})\, dx$$

$$mf_{cd} = \left\{ \int_0^{\delta} V_x \rho V_x \, dy + \frac{\partial}{\partial x}\left[ \int_0^{\delta} V_x \rho V_x \, dy \right] dx \right\}$$

$$m_{bc} = -\left\{ \frac{\partial}{\partial x}\left[ \int_0^{\delta} \rho V_x \, dy \right] dx \right\} dz$$

$$mf_{bc} = U m_{bc} = -U \left\{ \frac{\partial}{\partial x}\left[ \int_0^{\delta} \rho V_x \, dy \right] dx \right\}$$

Surface Forces on the CV



Pr on bc $= P(x) + \frac{1}{2}\left(\frac{\partial P}{\partial x}\right) dx$
(AM of Pr @ b & c)
Area on which it acts = Projected area

Momentum Integral Equation :-

$$\frac{\tau_w}{\rho} = \frac{d}{dx}\left(U^2 \theta\right) + \delta^* U \frac{dU}{dx}$$

$$\theta = \int_0^{\delta} \frac{v_x}{U}\left(1 - \frac{v_x}{U}\right) dy$$

$$\delta^* = \int_0^{\delta} \left(1 - \frac{v_x}{U}\right) dy$$

For flat plate, $\frac{dU}{dx} = 0$

$$\frac{\tau_w}{\rho} = U^2 \frac{d\theta}{dx}$$

The MI eq$^n$ doesnot assume whether the flow is laminar or turbulent.

For flat plate,

$$\frac{\tau_w}{\rho} = U^2 \frac{d}{dx} \int_0^{\delta} \frac{v_x}{U}\left(1 - \frac{v_x}{U}\right) dy$$

$$\tau_w = \rho U^2 \frac{d\delta}{dx} \underbrace{\int_0^{1} \frac{v_x}{U}\left(1 - \frac{v_x}{U}\right) d\eta}_{\text{Constant} = c}$$

$$-\mu \frac{dv_x}{dy}\Big|_{y=0} = \rho c U^2 \frac{d\delta}{dx}$$

$\underline{\underline{9}}$ $\tau_w = \rho U^2 \frac{d\delta}{dx} \left[ \int_0^1 \frac{v_x}{U} \left(1 - \frac{v_x}{U}\right) d\eta \right]$

$$\frac{v_x}{U} = a + b\eta + c\eta^2$$

$$\int_0^1 (a + b\eta + c\eta^2)(1 - a - b\eta - c\eta^2)\, d\eta$$

$$\int_0^1 \{ a - a^2 - ab\eta - ac\eta^2 + b\eta - ab\eta - b^2\eta^2 - c^2\eta^4 - bc\eta^3 + c\eta^2 - ac\eta^2 - bc\eta^3 \} d\eta$$

$$\int_0^1 ( a - a^2 - 2ab\eta - 2ac\eta^2 + b\eta - b^2\eta^2 - 2bc\eta^3 + c\eta^2 - c^2\eta^4 )\, d\eta$$

$$\left[ a\eta - a^2\eta - ab\eta^2 - \frac{2}{3}ac\eta^3 + \frac{b}{2}\eta^2 - \frac{b^3}{3}\eta^3 - \frac{bc}{2}\eta^3 + \frac{c}{3}\eta^3 - \frac{c^2}{5}\eta^5 \right]_0^1$$

$$= a - a^2 - ab - \frac{2}{3}ac + \frac{b}{2} - \frac{b^3}{3} - \frac{bc}{2} + \frac{c}{3} - \frac{c^2}{5}$$

$$a = 0, \quad b = 2, \quad c = -1$$

$$\therefore \; 1 - \frac{8}{3} + 1 - \frac{1}{3} - \frac{1}{5}$$

$$2 - 3 - \frac{1}{5} = \frac{2}{15}$$

$$\frac{V_x}{U} = 2\eta - \eta^2$$

$$+\mu \left.\frac{dV_x}{dy}\right|_{y=0} = \rho \frac{2}{15} U^2 \frac{d\delta}{dx}$$

$$+\frac{\cancel{2}\cancel{U}\mu}{\delta} = \frac{\cancel{2}}{15} \rho U^{\cancel{2}} \frac{d\delta}{dx}$$

$$+\frac{\mu}{\delta} = \frac{\rho U}{15} \frac{d\delta}{dx}$$

$$\delta\, d\delta = +\frac{15\mu}{\rho U} dx$$

$$\frac{\delta^2}{2} = \frac{15\mu}{\rho U} x$$

$$\delta(x) = \sqrt{\frac{30\mu x}{\rho U}} = \frac{\sqrt{30}\, x}{\sqrt{Re_x}}$$

$$\delta(x) = \frac{5.48\, x}{\sqrt{Re_x}}$$

Blasius Sol$^n$ $\delta = \frac{5x}{\sqrt{Re_x}}$

$$C_f = \frac{\tau_w}{\frac{1}{2}\rho U^2} = \frac{2\mu(U/\delta)}{\frac{1}{2}\rho U^2} = \frac{4\mu}{\rho U \delta}$$

$$C_f = \frac{0.73}{\sqrt{Re_x}}$$

Q. $\frac{V_x}{U} = \frac{3}{2}\left(\frac{y}{\delta}\right) - \frac{1}{2}\left(\frac{y}{\delta}\right)^3$

$\nu = 1.0 \times 10^{-6}\ m^2/s$ $L_{plate} = 0.1524\ m$

$W_p = 0.914\ m$ $U = 1.22\ m/s$

$\rho = 10^3\ kg/m^3$

$$\delta = \frac{4.64\, x}{\sqrt{Re_x}}$$

$$\tau_w = +\mu \left.\frac{dv_x}{dy}\right|_{y=0}$$

$$\tau_w = +\mu \times \frac{3}{2} \frac{U}{\delta} = \frac{3}{2} \frac{\mu U}{\delta}$$

$$\tau_w = \frac{3}{2} \mu U \times \frac{\sqrt{Re_x}}{4.64\, x}$$

$$= \frac{3}{2} \mu U \times \left(\sqrt{\frac{\nu}{U}}\right)^{-1} \times \frac{1}{4.64 \sqrt{x}}$$

$$\tau_w = \frac{1.5}{4.64} U^{3/2} (\rho \mu)^{1/2} x^{-1/2}$$

$$F = \int_0^x \tau_w \, W \, dx$$

$$F = \frac{1.5}{4.64} U^{3/2} (\rho \mu)^{1/2} W \int_0^x x^{-1/2} dx$$

$$F = \frac{3}{4.64} U^{3/2} (\rho \mu)^{1/2} W x^{1/2}$$

$$\tau_w \big|_{min} \Rightarrow x = L$$

$$F_{max} = F \big|_{x = L}$$

$$= 0.31 \text{ N}$$

# Turbulent

Power Law Eq$^n$ :-

$$\frac{\overline{V_z}}{U} = \left(\frac{y}{R}\right)^{1/7}$$

$y$ → distance from pipe wall

$R$ → Radius

Avg.

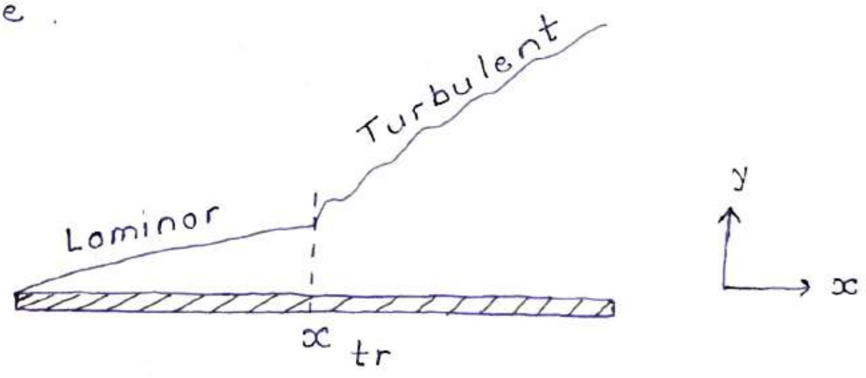
$U$ → centreline velocity

$$\frac{\overline{V}}{U} = \frac{2n^2}{(n+1)(2n+1)}$$

$n \sim 7^{-1}$ ; $\frac{\overline{V}}{U} = 0.8$

$\frac{y}{R} < 0.04$ ⟶ Infinite Velocity gradient at wall

It also doesnot give 0 slope at centreline.




$$\tau_W = -\frac{R}{2}\frac{\partial P}{\partial Z}$$

$$\frac{P_1 - P_2}{\rho} \equiv \frac{\Delta P}{\rho} = h$$

( h = HEAD LOSS FOR A HORIZONTAL PIPE )

$$h = f \frac{L}{D} \frac{\overline{V}^2}{2}$$

$$\tau_W = \frac{R}{2}\frac{\Delta P}{L} = \frac{R}{2L}\rho h$$

For a smooth pipe ; $f = \frac{0.3164}{Re^{0.25}}$

(Blasius Correlation)

$$\tau_w = \frac{R}{2L} \rho f \frac{L}{D} \frac{\bar{V}^2}{2} = \frac{1}{8} \rho f \bar{V}^2$$

$$\tau_w = 0.03325 \rho \bar{V}^2 \left( \frac{\nu}{R \bar{V}} \right)^{0.25}$$

Substituting $\bar{V} = 0.8 U$

$$0.0225 \left( \frac{\nu}{U\delta} \right)^{1/4} = \frac{d\delta}{dx} \int_0^1 \eta^{1/7} (1 - \eta^{1/7})$$

$$= \frac{7}{72} \frac{d\delta}{dx}$$

$$\rightarrow \frac{4}{5} \delta^{5/4} = 0.23 \left( \frac{\nu}{U} \right)^{1/4} x + c$$

If the flow is turbulent from the very beginning,

$x = 0 \ , \ \delta = 0 \Rightarrow c = 0$

$$\frac{\delta}{x} = 0.37 / (Re_x)^{1/5}$$

$$C_f = \frac{\tau_w}{\frac{1}{2} \rho U^2} = 0.045 \left( \frac{\nu}{U\delta} \right)^{1/4}$$

$$= \frac{0.0577}{(Re_x)^{1/5}} \qquad 5 \times 10^5 < Re_x < 10^7$$

## Fluid Flow About Immersed Bodies

$$\bar{F} = \int_{\text{body surface}} d\bar{F}_{shear} + \int_{\text{body surface}} d\bar{F}_{pressure}$$

$$\text{Drag} \quad C_D = \frac{F_D}{\frac{1}{2} \rho V^2 A} = \frac{\int_{PS} \tau_w \, dA}{\frac{1}{2} \rho V^2 A}$$

$$C_D = f(Re)$$

For Laminar Flow,

$$C_D = \frac{1.328}{\sqrt{Re_L}}$$

$$C_f = \frac{0.664}{\sqrt{Re_x}}$$

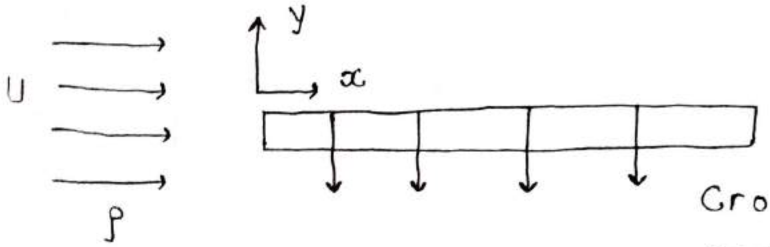
For Turbulent Flow,

$$C_f = \frac{0.0577}{(Re_x)^{1/5}} \qquad C_D = \frac{0.072}{(Re_L)^{1/5}}$$

Mixed Flow :-

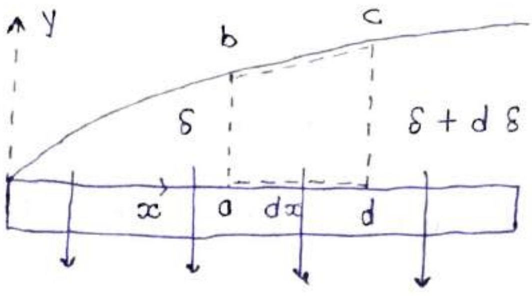
$$C_{D\,Turb} = \frac{0.074}{Re_L^{1/5}} - \frac{1740}{Re_L} \; ; \quad Re < 10^7$$

Q. Steady, Incompressible Flow :-
Find an expression for axial grad. of $\theta$, (i.e., $\frac{d\theta}{dx}$) in terms of $\tau_w$, $v_o$, $U$ and $\rho$.




$V = -V_o$ at $y = 0$

Cross flow filtr$^n$ process.




Ans

$$\frac{d\theta}{dx} = \frac{\tau_w}{\rho U^2} - \frac{V_o}{U}$$

$$\theta = \int_0^\delta \frac{u}{U}\left(1 - \frac{u}{U}\right) dy$$

$$0 = \frac{\partial}{\partial t}\int_{CV} \rho \, d\forall + \int_{CS} \rho \vec{V} \cdot d\vec{A}$$

$$F_{Sx} + F_{Bx} = \frac{\partial}{\partial t}\int_{CV} u \rho \, d\forall + \int_{CS} u \rho \vec{V} \cdot d\vec{A}$$

$$\dot{m}_{ab} + \dot{m}_{bc} + \dot{m}_{cd} + \dot{m}_{da} = 0$$

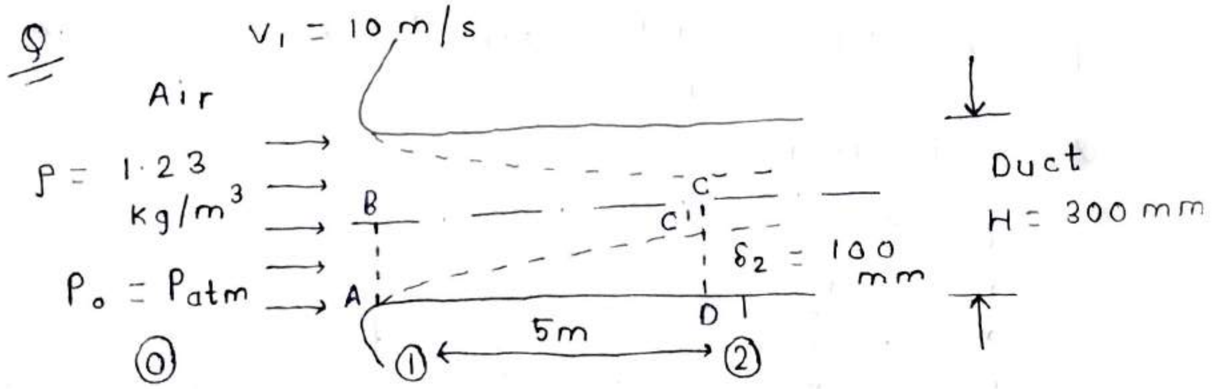
$$\dot{m}_{ab} = \left(\int_0^\delta \rho u \, dy\right) dz$$

$$\dot{m}_{cd} = \dot{m}_{ab} + \frac{d \dot{m}_{ab}}{dx} dx$$

$$= \left(\int_0^\delta \rho u \, dy + \frac{d}{dx}\left(\int_0^\delta \rho u \, dy\right) dx\right) dz$$

$$\dot{m}_{ad} = + \rho V_0 \, dx \, dz$$

Q




$$\frac{u}{U} = \left(\frac{y}{\delta}\right)^{1/7}$$

1. Find relation between $\delta^*$ & $\delta$
2. Find static gauge pressure at 2
3. Evaluate av. shear stress b/w 1 and

$$\delta^* = \int_0^{\delta} \left(1 - \frac{u}{U}\right) dy$$

$$= \int_0^{\delta} \left(1 - \left(\frac{y}{\delta}\right)^{1/7}\right) dy$$

$$= y - \frac{1}{\delta^{1/7}} \frac{y^{8/7}}{8/7} \Big|_0^{\delta}$$

$$\delta^* = \delta - \frac{7}{8} \frac{\delta^{8/7}}{\delta^{1/7}} = \frac{\delta}{8}$$

$$\frac{\delta^*}{\delta} = \frac{1}{8}$$

$$(\delta^*)$$

$$(300) \cancel{W} \; V_1 = (300 - 2 \times \frac{100}{8}) \cancel{W} \; V_2$$

$$V_2 = \frac{3 V_1}{2.75} = \frac{3 \times 10}{2.75} = \frac{30}{2.75} \text{ m/s} = 10.91 \text{ m/s}$$

---

$$\underset{0}{\cancel{P_1}} + \cancel{\rho g z_1} + \frac{1}{2} \rho V_1^2 = P_2 + \cancel{\rho g z_2} + \frac{1}{2} \rho V_2^2$$

$$P_2 = \frac{1}{2} \rho V_1^2 - \frac{1}{2} \rho V_2^2$$

$$P_2 = \frac{1}{2} \times 1.23 \left(100 - \frac{900}{2.75^2}\right)$$

$$= -1.23 \times 400$$

$$= -492$$

---

$$P_0 = P_2 + \frac{1}{2} \rho V_2^2$$

$$(P_2 - P_0) = -\frac{1}{2} \rho V_2^2$$

$$P_{2g} = -\frac{1}{2} \times 1.23 \times 10.91^2$$

$$= -\cancel{62.28} \text{ Pa}$$

$$73.068$$

Momentum entering the control volume at AB $= \rho V_1^2 \frac{H}{2} W$

$M^2$ leaving at CD $= \left( \int_0^{\delta_2} \rho u_2^2 \, dy \right) W +$

$$\rho V_2^2 \left( \frac{H}{2} - \delta_2^* \right) W$$

$$F_{sx} = \int_{cs} u \rho \vec{V} \cdot d\vec{A}$$

$$= -V_1 \left\{ \rho V_1 \frac{H}{2} W \right\} + \int_0^{\delta_2} u_2 \rho u_2 W \, dy$$

$$+ \rho V_2^2 \left( \frac{H}{2} - \delta_2^* \right) W$$

$$F_{sx} = \underbrace{(P_1 - P_2) \frac{H}{2} W}_{\text{Pressure force on CV}} - \underbrace{\tau L W}_{\text{Shear force on CV}}$$

$\tau = 0.3 \ N/m^2$ (Ans)

where, $u_2 = V_2 \left( \frac{y}{\delta} \right)^{1/7}$

**Q.** The laminar to turbulent transition on a new cricket ball (dia 7.2 cm) occurs at a $Re_D$ of about $1.4 \times 10^5$ if the flow doesn't encounter the seam. But it can be triggered by the seam, e.g., at a $Re_D$ as low as $9.5 \times 10^4$ (when it is at $30°$ to the airflow). Use the above info. to advice a seam bowler about the speed in which he has to bowl to achieve swing of the ball. A very brief (two lines) reason accompany your suggestion.

The kinematic viscosity of air is $1.5 \times 10^{-5}$ $m^2/s$.

$$Re_D = \frac{VD}{\nu}$$

$$1.4 \times 10^5 = \frac{V \times 7.2 \times 10^{-2}}{1.5 \times 10^{-5}}$$

$$V = 105 \text{ km/h}$$

$$9.5 \times 10^4 = \frac{V \times 7.2 \times 10^{-2}}{1.5 \times 10^{-5}}$$

$$V = 71.25 \text{ km/h}$$

For the ball to swing, it should encounter laminar on one side and turbulent on the other side.

So the velocity should lie in b/w the range for the ball to swing.

Mass of ball = 0.156 kg

Speed of ball = $V_o$ > upper critical speed

$C_D$ = 0.15

The ball starts to late-swing at a

Vaibhav

distance of 15 m from the bowling end Can a swing bowler plan his delivery for a late swing or whether this delivery is just a matter of chance ?

Given $\rho_{air} = 1.22\ kg/m^3$

$$C_D = \frac{F_D}{\frac{1}{2}\rho v^2 A_P}$$

$$m\frac{dv}{dt} = -F_D$$

$$m\frac{dv}{dx}\cdot\frac{dx}{dt} = -\frac{1}{2}C_D\rho v^2 A_P$$

$$m\cancel{v}\frac{dv}{dx} = -\frac{1}{2}C_D\rho v^{\cancel{2}} A_P$$

$$\int_{V_0}^{V_C}\frac{dv}{v} = -\frac{1}{2}\int_0^{15}\frac{C_D\rho A_P}{m}\,dx$$

$$\ln\left(\frac{V_C}{V_0}\right) = -\frac{C_D\rho A_P(15)}{2m}$$

$$\ln\left(\frac{V_C}{V_0}\right) = -0.03582$$

$$V_C = 105\ km/h$$

$$V_0 = 108.83\ km/h$$

A super tank
of 600,000
beam (widt
D = 25 m
(7.20 m/s)
$10^{-6}\ m^2/s$)

1. thickness
ship (

2. total s
the ship

3. power r
force

4. K.E of
a sta
vecocit
as U-

5. Minim

$u\frac{\partial u}{\partial x}$ +

$u\frac{\partial T}{\partial x}$ +

$u\frac{\partial C_A}{\partial x}$ -

A super tanker has a displacement (mass) of 600,000 metric tons and L = 300 m, beam (width) = 80 m and draft (depth), D = 25 m. The ship moves at 14 knots (7.20 m/s) through seawater ($\nu$ = 1.9 × $10^{-6}$ $m^2/s$). $\rho_s = 1.025 \times 10^3$ Find

1. thickness of BL at the end of the ship (1.7 m)
2. total skin friction drag acting on the ship (1.6 MN)
3. power required to overcome the drag force (11.4 MW)
4. K.E. of the ship b.l., considering a stationary observer, with the vecocity of the b.l., being seen as U − u. (2.73 × $10^7$ J) (water) (1.55 × $10^{10}$ J) (ship)
5. Minimum distance to stop the ship

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = -\frac{1}{\rho} \frac{\partial P}{\partial x} + \nu \frac{\partial^2 u}{\partial y^2}$$

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha \frac{\partial^2 T}{\partial y^2} + \cancel{\mu \phi} + \cancel{q}$$

$$u \frac{\partial C_A}{\partial x} + v \frac{\partial C_A}{\partial y} = D_{AB} \frac{\partial^2 C_A}{\partial y^2} + \cancel{R_A}$$

The Convection Transfer Equations

$$\frac{u_x}{u_\infty}, \quad \frac{T_s - T}{T_s - T_\infty} = T^*, \quad \frac{C_{As} - C_A}{C_{As} - C_\infty} = C_A^*$$

Approximations and Special Consideration

Incompressible flow, constant properties, negligible body forces.

Boundary layer similarity :-

$y^* = y/L \quad x^* = x/L \quad v^* = v/V$

$u^* = u/V$

Momentum Transfer -

$$u^* \frac{\partial u^*}{\partial x^*} + v^* \frac{\partial u^*}{\partial y^*} = -\frac{\partial P^*}{\partial x^*} + \frac{\nu}{VL} \frac{\partial^2 u^*}{\partial y^{*2}}$$

$$u^* \frac{\partial T^*}{\partial x^*} + v^* \frac{\partial T^*}{\partial y^*} = \frac{\alpha}{VL} \frac{\partial^2 T^*}{\partial y^{*2}}$$

V : approach velocity

BCs -

Wall

$u^*(x^*, 0) = 0$

$v^*(x^*, 0) = 0$

Freestream

$u^*(x^*, \infty) = \frac{u_\infty(}{V}$

$$u^* = f_1\left(x^*, y^*, Re_L, \frac{\partial P^*}{dx^*}\right)$$

Knowing the geometry of the system we can obtain $\frac{\partial P^*}{\partial x^*}$ as a $f^n$ of ve

Shear Stress at the surface ($y^* = 0$)

$$\tau_s = \mu \frac{\partial u}{\partial y}\Big|_{y=0} = \frac{\mu V}{L} \frac{\partial u^*}{\partial y^*}\Big|_{y^*=0}$$

$$C_f = \frac{\tau_s}{\frac{1}{2}\rho V^2}$$

$$C_f = f_2(x^*, Re_L) \Rightarrow f_2 = C_f \frac{Re_L}{2}$$

$$T^* = f_3\left(x^*, y^*, Re_L, Pr, \frac{dp^*}{dx^*}\right)$$

$$q_s = -k_f \frac{\partial T}{\partial y}\Big|_{y=0} \Rightarrow h = \frac{-k_f \frac{\partial T}{\partial y}\Big|_{y=0}}{T_s - T_\infty}$$

$$h = -\frac{k_f}{L} \frac{(T_\infty - T_s)}{(T_s - T_\infty)} \frac{\partial T^*}{\partial y^*}\Big|_{y^*=0}$$

$$Nu = \frac{hL}{k_f} = \frac{\partial T^*}{\partial y^*}\Big|_{y^*=0}$$

$$\begin{cases} Nu = f_4(x^*, Re, Pr) \\ \overline{Nu} = \frac{\bar{h} L}{k_f} = f_5(Re_L, Pr) \end{cases}$$

for a prescribed geometry

If $-\frac{\partial p^*}{\partial x^*} = 0$ and $Sc = Pr = 1$ and $u_\infty = V$; the three eq$^n$s are identical. So, the sol$^n$ of one can be used as the sol$^n$ of the other. The three eq$^n$s are dynamically similar.

Reynold's Analogy :-

$$C_f \frac{Re_L}{2} = Nu = Sh$$

$$\frac{C_f}{2} = \frac{Nu}{Re_L \, Pr} = \frac{Sh}{Re_L \, Sc}$$

$$\frac{C_f}{2} = St_H = St_m \rightarrow$$ Reynold's Analogy

Chilton Coulburn Analogy :-

$$\frac{C_f}{2} = St . Pr^{2/3} = j_H \qquad 0.6 < Pr < 60$$

$$\frac{C_f}{2} = St_m . Sc^{2/3} = j_M \qquad 0.6 < Sc < 300$$

$$\frac{C_f}{2} = j_H = j_m$$

Lewis No. $Le = \dfrac{\text{Thermal Diffusivity}}{\text{Mass Diffusivity}}$

$$= (Pr / Sc)^{-1}$$

$$= (\nu / D_{AB}) . \frac{\alpha}{\nu} = \frac{\alpha}{D_{AB}}$$

For simultaneous heat and mass transfer :-

$$\frac{\delta_t}{\delta_c} \approx Le^n$$ for most applications $n = 1/3$

Q. An irregularly shaped object, 1m, $T = 100°C$ suspended in air $T_{air} = 0°C$ $P = 1$ atm $v = 120$ m/s.

The $T_{air}$ measured at a point near the object in the airstream is 80°C.

Second object : 2m. $T_{obj} = T_{air} = 50°C$ $p = 1$ atm $v = 60$ m/s. Plastic coating being dried. MW vap = 82 $P^{sat} = 0.0323$ atm.

a) For second object, at a location corresponding to the point of measurement of the object, determine the vapor conc. and partial pressure.

b) If the avg. heat flux, $q''$, is 2000 $W/m^2$ for the first object, determine the avg. mass flux $n_A''$ (kg/s. /$m^2$) for the second object.

Given : For air (at 323 K and 1 atm)

kinematic viscosity = $18.20 \times 10^{-6}$ $m^2/s$

$Pr = 0.71$, $k = 28 \times 10^{-3}$ W/mK

$M_A = 82$ kg/kg mole, $P_{sat} = 0.0323$ atm

$D_{AB} = 2.6 \times 10^{-5}$ $m^2/s$ $R = 0.08205$

a) $Pr = 0.71$ $Sc = 0.7$

$Re_1 = Re_2$ $x^*$ is same in both cases, $y^*$ also, $\frac{dP^*}{dx^*}$ is same

$\therefore \quad T^* = C^*$

$$\frac{100-80}{100-0} = \frac{-C_A + 1.219\times10^{-3}}{1.219\times10^{-3} - 0}$$

$\Rightarrow \quad C_A = 0.9752\times10^{-3}\ kmol/m^3$

$P_A = C_A RT = 0.0258\ atm$

b) $q'' = \bar{h}\,(T_s - T_\infty)$

$n_A'' = \bar{h}_m\,(C_{As} - C_{A\infty})\,M_A$

$\overline{Nu_L} = \overline{Sh_L}$

$$\frac{\bar{h}}{\bar{h}_m} = \frac{L_2}{L_1}\ \frac{k}{D_{AB}}$$

$$\bar{h} = \frac{2000}{100-0} = 2$$

$$2\times\frac{1}{2}\times\frac{2.6\times10^{-5}}{28\times10^{-3}} = \bar{h}_m$$

$\bar{h}_m = 9.28\times10^{-4}$

$n_A'' = \bar{h}_m\,\Delta c$

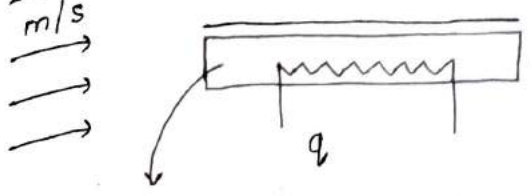
$= 9.28\times10^{-4}\,(1.219-0.9752)\times$

$\times 82 / 2$

$= 9.28\times10^{-4}\ kg/m^2s$

Q. 

$\rho_{A\infty} = 0$




$L = 1m$

$A_s = 1m^2$

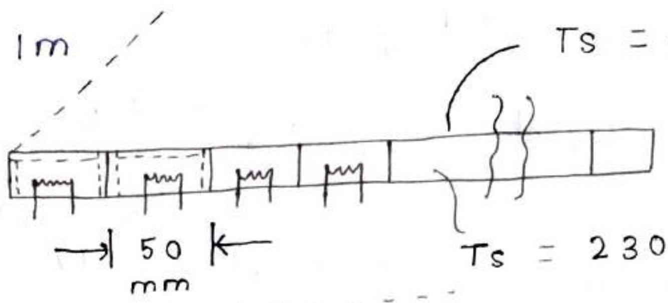
$T_s = 310 K$

$\dot{E}_{in} - \dot{E}_{out} = 0$ CONC. DIFF. $= (\rho_{A sat} - \rho_{A\infty})$

$\dot{E}_{in} = \dot{E}_{out}$ = Loss due to Convection + Loss for evaporation

Q. A flat plate of width 1m is maintained at a uniform surface temp. of $T_s = 230°C$ by using strip heaters of length 50 mm. Each heater is insulated from its neighbours as well as on its back side. Atmospheric air at 25°C flows over the plate at a velocity of 60 m/s, at what heater is the electrical input a maximum? What is the value of this input?

For air $\nu = 26.41 \times 10^{-6}\ m^2/s$, k = 0.0338 W/m, Pr = 0.69

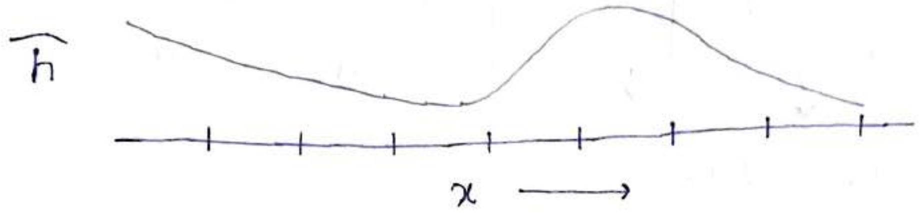


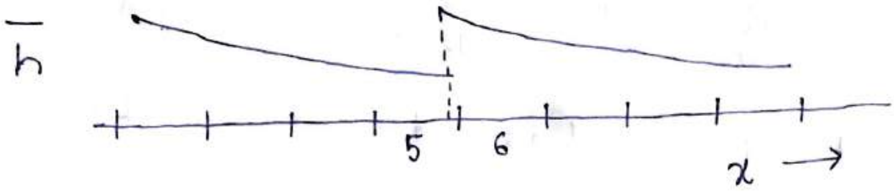

$q_1$ $q_2$

Possibility 1 : First heater

" 2 : Heater at which $Re = 5 \times 10^5$

$$Re = \frac{Ux}{\nu}$$




$$5 \times 10^5 = \frac{60 \times x}{26.41 \times 10^{-6}}$$

$$x = 0.22 \text{ m}$$

$$x = 220 \text{ cm}$$

$$\text{Heater No.} = \frac{220}{50} \approx 5$$

P3 : Heater 6 for which the flow is turbulent for the entire heater

Mixed flow ← → Laminar flow

$$Q_5 = (\bar{h}_5 A_5 - \bar{h}_4 A_4)\Delta T$$

$\bar{h}_2$ means the heat transfer coefficient upto the 2nd plate

$$A_5 = L_5 W \qquad A_6 \neq A_5$$

$$\overline{Nu}_{\text{Laminar}} = 0.664 (Re)^{1/2} Pr^{1/3}$$

$$\overline{Nu}_{\text{Mixed flow}} = (0.037 Re^{4/5} - 871) Pr^{1/3}$$

$q_{conv\,6}$ > $q_{conv\,1}$ > $q_{conv\,5}$

1440 W    1370 W    1050 W