## HEAT TRANSFER (CH21204)

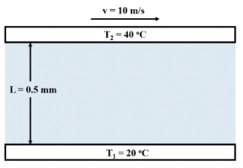
**Quiz #1 [Feb. 09, 2023] Name: Roll:**

Consider parallel flow of a liquid in between two horizontal isothermal plates having a uniform spacing of 0.5 mm. Assume that the upper plate is moving at a constant velocity of 10 m/s. The temperatures of the upper and lower plates are 40˚C and 20˚C, respectively. Estimate (a) the velocity and temperature distributions in the liquid, (b) the maximum temperature (in ˚C) and its location (in mm), and (c) the heat fluxes (in W/m²) from the liquid to each plate.

- State your assumptions (if any) clearly with justification in solving this problem.
- Assume the following liquid properties at the film temperature:

$$k = 0.15\, W/m.K \text{ and } \mu = 0.60\, kg/m.s$$

[Marks – 30]




(a) We take the x-axis to be the flow direction, and y to be the normal direction. This is parallel flow between two plates, and thus $v = 0$. Then the continuity equation reduces to:

$$\text{Continuity}: \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \longrightarrow \frac{\partial u}{\partial x} = 0 \longrightarrow u = u(y)$$

Therefore, the x-component of velocity does not change in the flow direction (i.e., the velocity profile remains unchanged). Noting that $u = u(y)$, $v = 0$, and $\partial P/\partial x = 0$ (flow is maintained by the motion of the upper plate rather than the pressure gradient), the x-momentum equation reduces to:

$$\text{x - momentum}: \rho\left(u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right) = \mu\frac{\partial^2 u}{\partial y^2} - \frac{\partial P}{\partial x} \longrightarrow \frac{d^2 u}{dy^2} = 0$$

This is a second-order ordinary differential equation, and integrating it twice gives

$$u(y) = C_1 y + C_2$$

The fluid velocities at the plate surfaces must be equal to the velocities of the plates because of the no-slip condition. Therefore, the boundary conditions are $u(0) = 0$ and $u(L) = \mathrm{v}$, and applying them gives the velocity distribution to be:

$$u(y) = \frac{y}{L}\mathrm{v}$$

Frictional heating due to viscous dissipation in this case is significant because of the high viscosity of oil and the large plate velocity. The plates are isothermal and there is no change in the flow direction, and thus the temperature depends on y only, $T = T(y)$. Also, $u = u(y)$ and $v = 0$. Then the energy equation with dissipation reduce to:

$$\text{Energy}: 0 = k\frac{\partial^2 T}{\partial y^2} + \mu\left(\frac{\partial u}{\partial y}\right)^2 \longrightarrow k\frac{\partial^2 T}{\partial y^2} = -\mu\left(\frac{\mathrm{v}}{L}\right)^2 \quad (\text{since } \partial u/\partial y = \mathrm{v}/L)$$

Now dividing both sides by $k$ and integrating twice give:

$$T(y) = -\frac{\mu}{2k}\left(\frac{y}{L}\mathrm{v}\right)^2 + C_3 y + C_4$$

Applying the boundary conditions $T(0) = T_1$ and $T(L) = T_2$ gives the temperature distribution to be:

$$T(y) = \frac{T_2 - T_1}{L}y + T_1 + \frac{\mu \mathrm{v}^2}{2k}\left(\frac{y}{L} - \frac{y^2}{L^2}\right)$$

$$T(y) = 440000y + 293.15 - 800000000y^2 \quad (\text{in K})$$

**(b)** The temperature gradient is determined by differentiating $T(y)$ with respect to $y$,

$$\frac{dT}{dy} = \frac{T_2 - T_1}{L} + \frac{\mu \mathrm{v}^2}{2kL}\left(1 - \frac{2y}{L}\right)$$

The location of maximum temperature is determined by setting $dT/dy = 0$ and solving for $y$,

$$y = L\left(k\frac{T_2 - T_1}{\mu \mathrm{v}^2} + \frac{1}{2}\right)$$

The maximum temperature is the value of temperature at this $y$, whose numeric value is:

$$y = (0.0005\ m)\left((0.15\ W/m.K)\frac{(40-20)^\circ C}{(0.6\ kg/m.s)(10\ m/s)^2} + \frac{1}{2}\right) = 0.275\ mm$$

Then

$$T_{max} = (440000 \times 0.000275) + (293.15) - (800000000 \times 0.000275^2) = 353.65\ \mathrm{K} = 80.5^\circ C$$

**(c)** Heat flux at the plates is determined from the definition of heat flux,

$$\dot{q}_0 = -k\left.\frac{dT}{dy}\right|_{y=0} = -k\frac{T_2 - T_1}{L} - k\frac{\mu \mathrm{v}^2}{2kL}(1-0) = -k\frac{T_2 - T_1}{L} - \frac{\mu \mathrm{v}^2}{2L}$$

$$\dot{q}_0 = -(0.15)\frac{40-20}{0.0005} - \frac{0.60 \times 10^2}{2 \times 0.0005} = -6.6 \times 10^4\ \ W/m^2$$

Similarly,

$$\dot{q}_L = -k\left.\frac{dT}{dy}\right|_{y=L} = -k\frac{T_2 - T_1}{L} - k\frac{\mu \mathrm{v}^2}{2kL}(1-2) = -k\frac{T_2 - T_1}{L} + \frac{\mu \mathrm{v}^2}{2L}$$

$$\dot{q}_L = -(0.15)\frac{40-20}{0.0005} + \frac{0.60 \times 10^2}{2 \times 0.0005} = 5.4 \times 10^4 W/m^2$$

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## HEAT TRANSFER (CH21204)

**Quiz #2 [Mar. 22, 2023] Name: Roll:**

Air enters a 7-m-long section of a rectangular duct of cross section 15 cm X 20 cm at 50°C at an average velocity of 7 m/s. If the walls of the duct are maintained at 10°C, determine (a) the outlet temperature of the air, (b) the rate of heat transfer from the air, and (c) the fan power needed to overcome the pressure losses in this section of the duct.

[Marks: 10+5+5 = 20]

**Data:**

$\rho = 1.127\ \mathrm{kg/m^3}$ $\quad C_p = 1007\ \mathrm{J/kg.^\circ C}$

$k = 0.02662\ \mathrm{W/m.^\circ C}$ $\quad \mathrm{Pr} = 0.7255$

$\upsilon = 1.702 \times 10^{-5}\ \mathrm{m^2/s}$

***Properties*** We assume the bulk mean temperature for air to be 40°C since the mean temperature of air at the inlet will drop somewhat as a result of heat loss through the duct whose surface is at a lower temperature. The properties of air at this temperature and 1 atm are (Table A-15)

$\rho = 1.127\ \mathrm{kg/m^3}$ $\quad C_p = 1007\ \mathrm{J/kg.^\circ C}$

$k = 0.02662\ \mathrm{W/m.^\circ C}$ $\quad \mathrm{Pr} = 0.7255$

$\upsilon = 1.702 \times 10^{-5}\ \mathrm{m^2/s}$




***Analysis*** *(a)* The hydraulic diameter, the mean velocity of air, and the Reynolds number are

$$D_h = \frac{4A_c}{P} = \frac{4(0.15\ \mathrm{m})(0.20\ \mathrm{m})}{2[(0.15\ \mathrm{m}) + (0.20\ \mathrm{m})]} = 0.1714\ \mathrm{m}$$

$$\mathrm{Re} = \frac{\mathrm{V}_m D_h}{\upsilon} = \frac{(7\ \mathrm{m/s})(0.1714\ \mathrm{m})}{1.702 \times 10^{-5}\ \mathrm{m^2/s}} = 70{,}525$$

which is greater than 10,000. Therefore, the flow is turbulent and the entry lengths in this case are roughly

$$L_h \approx L_t \approx 10D_h = 10(0.1714\ \mathrm{m}) = 1.714\ \mathrm{m}$$

which is much shorter than the total length of the duct. Therefore, we can assume fully developed turbulent flow in the entire duct, and determine the Nusselt number from

$$Nu = \frac{hD_h}{k} = 0.023\,\mathrm{Re}^{0.8}\,\mathrm{Pr}^{0.3} = 0.023(70{,}525)^{0.8}(0.7255)^{0.3} = 158.0$$

Heat transfer coefficient is

$$h = \frac{k}{D_h}Nu = \frac{0.02662\ \mathrm{W/m.^\circ C}}{0.1714\ \mathrm{m}}(158.0) = 24.53\ \mathrm{W/m^2.^\circ C}$$

Next we determine the exit temperature of air

$$A_s = 2 \times 7[(0.15\ \mathrm{m}) + (0.20\ \mathrm{m})] = 4.9\ \mathrm{m^2}$$

$$A_c = (0.15\ \mathrm{m})(0.20\ \mathrm{m}) = 0.03\ \mathrm{m^2}$$

$$\dot{m} = \rho V A_c = (1.127\ \mathrm{kg/m^3})(7\ \mathrm{m/s})(0.03\ \mathrm{m^2}) = 0.2367\ \mathrm{kg/s}$$

$$T_e = T_s - (T_s - T_i)e^{-hA_s/(\dot{m}C_p)} = 10 - (10-50)e^{-\frac{(24.53)(4.9)}{(0.2367)(1007)}} = \mathbf{34.2^\circ C}$$

*(b)* The logarithmic mean temperature difference and the rate of heat loss from the air are

$$\Delta T_{\ln} = \frac{T_e - T_i}{\ln\left(\frac{T_s - T_e}{T_s - T_i}\right)} = \frac{34.2 - 50}{\ln\left(\frac{10 - 34.2}{10 - 50}\right)} = 31.42^\circ\mathrm{C}$$

$$\dot{Q} = hA_s\Delta T_{\ln} = (24.53\ \mathrm{W/m^2.^\circ C})(4.9\ \mathrm{m^2})(31.42^\circ\mathrm{C}) = \mathbf{3776\ W}$$

*(c)* The friction factor, the pressure drop, and then the fan power can be determined for the case of fully developed turbulent flow to be

$$f = 0.184\,\mathrm{Re}^{-0.2} = 0.184(70{,}525)^{-0.2} = 0.01973$$

$$\Delta P = f\frac{L}{D}\frac{\rho \mathrm{V}_m^2}{2} = 0.01973\frac{(7\ \mathrm{m})}{(0.1714\ \mathrm{m})}\frac{(1.127\ \mathrm{kg/m^3})(7\ \mathrm{m/s})^2}{2} = 22.25\ \mathrm{N/m^2}$$

$$\dot{W}_{pump} = \frac{\dot{m}\Delta P}{\rho} = \frac{(0.2367\ \mathrm{kg/s})(22.25\ \mathrm{N/m^2})}{1.127\ \mathrm{kg/m^3}} = \mathbf{4.67\ W}$$

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## QUIZ #3

**April 6, 2023**

Water is to be boiled at sea level in a 30-cm-diameter mechanically polished AISI 304 stainless steel pan placed on top of a 3-kW electric burner. If 60 percent of the heat generated by the burner is transferred to the water during boiling, determine the temperature of the inner surface of the bottom of the pan. Also, determine the temperature difference between the inner and outer surfaces of the bottom of the pan if it is 6 mm thick.

Data given:

$\rho_l = 960\ \mathrm{kg/m^3}$
$\rho_v = 0.60\ \mathrm{kg/m^3}$
$\sigma = 0.06\ \mathrm{N/m}$
$Pr_l = 1.75$
$h_{fg} = 2250 \times 10^3\ \mathrm{J/kg}$
$\mu = 0.3\ \mathrm{kg.m/s}$
$C_{pl} = 4200\ \mathrm{J/kg\ C}$

$k_{steel} = 15\ \mathrm{W/m\ C}$
$C_{sf} = 0.013$
$n = 1$

The rate of heat transfer to the water and the heat flux are

$$\dot{Q} = 0.60 \times 3\ kW = 1.8\ \mathrm{kW} = 1800\ \mathrm{W}$$

$$A_s = \frac{\pi D^2}{4} = 0.07069\ m^2$$

$$\dot{q} = \frac{\dot{Q}}{A_s} \approx 25460\ \mathrm{W}/m^2$$

The temperature difference across the bottom of the pan is determined directly from the steady one-dimensional heat conduction relation to be

$$\dot{q} = k_{steel}\frac{\Delta T}{L}$$

$$\Delta T = \frac{\dot{q}L}{k_{steel}} = \frac{(25460 \times 0.006)}{15}\,^\circ\mathrm{C} = 10.184\ ^\circ\mathrm{C}$$

The Rohsenow relation which gives the nucleate boiling heat flux for a specified surface temperature can also be used to determined the surface temperature when the heat flux is given.

Assuming nucleate boiling, the temperature of the inner surface of the pan is determined from Rohsenow relation to be

$$\dot{q} = \mu_l h_{fg}\left\{\frac{g(\rho_l - \rho_v)}{\sigma}\right\}^{\frac{1}{2}}\left\{\frac{C_{p,l}(T_s - T_{sat})}{C_{sf}\ h_{fg}\ Pr_l^n}\right\}^3$$

$$25460 = (0.3) \times (2250 \times 10^3) \times \sqrt{\frac{9.8 \times (960 - 0.6)}{0.06}} \times \left\{\frac{4200 \times (T_s - 100)}{0.013 \times (2250 \times 10^3) \times 1.75}\right\}^3$$

It gives, $T_s = 100.56\ ^\circ\mathrm{C}$

Which is in the nucleate boiling range (5 to 30 °C above the surface temperature). Therefore the nucleate boiling assumption is valid.