Distillation : degrees of freedom

$$f = V - E$$

No.s of eq^n	origin
$N+1$	$y = \dfrac{\alpha x}{1+(\alpha-1)x}$
N	$L = L_o + \dfrac{m - m_o}{\beta}$
$N+2$	$\dfrac{dm}{dt}$
$N+2$	$\dfrac{dx}{dt}$
$4N+5$	

N = total no.s of plates/trays

No. of variables	Type
$N+1$	y
N	L
$N+2$	m
$N+2$	x
6	$F\ Z\ D\ R\ B\ V_B$
$4N+11$	

$f = 6$

Measure F, Z

$f = 6 - 2 = 4$

CV	MV
x_D	R
x_B	V_B
m_D	D
m_B	B

①

chanser, pum

ontr, tank e

products

ell-being

continued

lopment.

s

How?

produce.

ts.

99.5 mol %

ow?

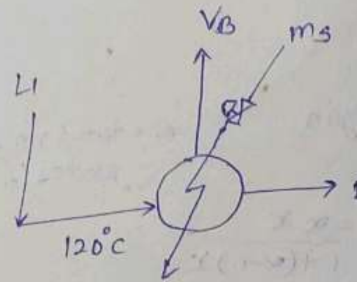
ate laws

t)

here

river

ow?



Q_R = reboiler duty
$= m_s \lambda_s$
$= V_B \lambda_B$

{ dynamics of reboiler is neglected }

(no accumulation.)

Laplace Transform :

$$L[f(t)] = \bar{f}(s) = \int_0^\infty f(t) e^{-st} dt$$

$f(t)$	$\bar{f}(s)$
1	$\frac{1}{s}$
t^n	$\frac{n!}{s^{n+1}}$
$e^{-at}\sinh\omega t$	$\frac{\omega}{(s+a)^2-\omega^2}$

$f(t)$	$\bar{f}(s)$
$t^n f(t)$	$(-1)^n \frac{d^n F(s)}{ds^n}$
$\frac{f(t)}{t^n}$	$\underbrace{\int_s^\infty \int_s^\infty \cdots \int_s^\infty}_{n} F(s)\, ds \cdots ds$

$$L\left[\int_0^t f(t)\, dt\right] = \frac{1}{s} \bar{f}(s)$$

$$L\left[\frac{df}{dt}\right] = s\bar{f}(s) - f(0)$$

$$L\left[\frac{d^n f}{dt^n}\right] = s^n \bar{f}(s) - s^{n-1} f(0) - s^{n-2} f'(0) \cdots - f^{n-1}(0)$$

f = devi
procee
f(t=0
final va
lim f(
$t\to\infty$
Initial
lim f(t
$t\to 0$
Input / fo
(i) step inp
f(t) = {
$\bar{f}(s) = A$
if A=1 ⇒

f = deviation variable $= f_t - f_{ss}$

process is initially at ss

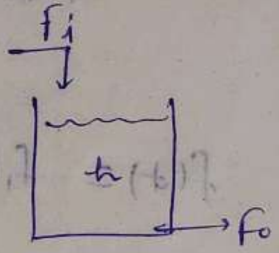
$f(t=0) = 0$

final value theorem

$$\lim_{t \to \infty} f(t) = \lim_{s \to 0} s\bar{f}(s)$$

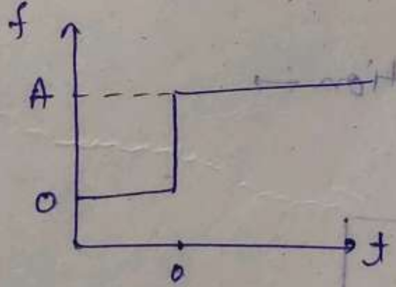
Initial value theorem

$$\lim_{t \to 0} f(t) = \lim_{s \to \infty} s\bar{f}(s)$$



Input / forcing function :-

(i) step input :-



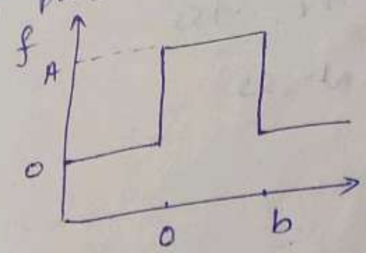
f = deviation vari

$$f(t) = \begin{cases} 0 & t<0 \\ A & t>0 \end{cases}$$

$\bar{f}(s) = \frac{A}{s}$

if $A=1 \Rightarrow$ unit step change $\bar{f}(s) = \frac{1}{s}$

(2.) pulse function:

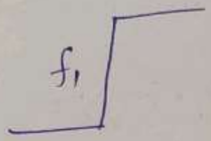


$$f(t) = \begin{cases} A & b > t > 0 \\ 0 & t > b \end{cases}$$

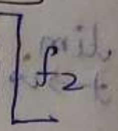
$$f(t) = A\left[u(t-0) - u(t-b)\right]$$

$$\bar{F}(s) = A\left(\frac{1}{s}e^{-0s} - \frac{1}{s}e^{-bs}\right)$$

$$\boxed{\bar{F}(s) = \frac{A}{s}\left(1 - e^{-bs}\right)}$$



$$\bar{F}(s) = \frac{\left(1 - e^{-As}\right)}{As}$$

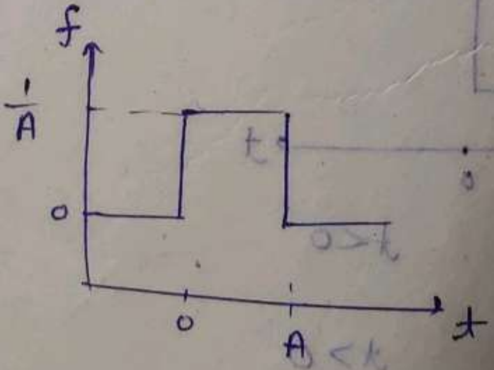


$$f_1(t) = \begin{cases} 0 & t < 0 \\ A & t > 0 \end{cases}$$

$$f_2(t) = \begin{cases} 0 & t < b \\ A & t > b \end{cases} = -f_1(t-b)$$

$$f(t) = f_1(t) - f_1(t-b)$$

unit pulse function :-



$$\bar{f}(s) = \frac{A}{s}\left[1 - e^{-bs}\right]$$

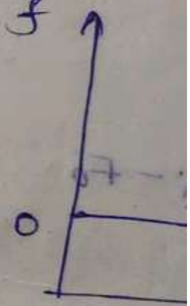
unit pulse :- $\bar{f}(s) = \frac{1 - e^{-As}}{As} = \bar{S}_A(A)$

(3.) Un

f

$L\left[\lim_{A \to 0}\right.$

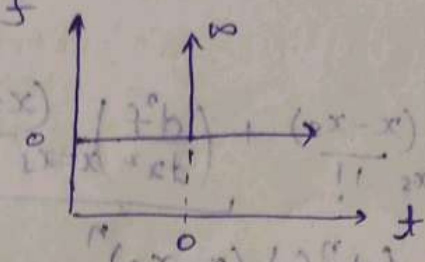
(4.) Ramp



$f(t) =$

$\bar{f}(s) =$

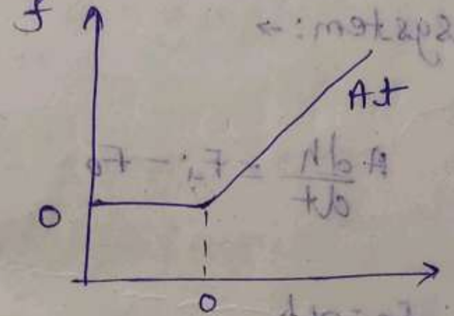
(3) Unit impulse :-



dirac fuction $\delta(t)$

$$L\left[\lim_{A\to 0} \delta_A(A)\right] = L\left[\delta(t)\right] = 1$$

(4) Ramp fuction :-



$$f(t) = \begin{cases} 0 & t<0 \\ At & t>0 \end{cases}$$

$$\bar{f}(s) = \frac{A}{s^2}$$

Nonlinear systems

$$\frac{dx}{dt} = f(x)$$

$$f(x) = f(x_s) + \left(\frac{df}{dx}\right)_{x=x_s} \frac{(x-x_s)}{1!} + \left(\frac{d^2f}{dx^2}\right)_{x=x_s} \frac{(x-x_s)^2}{2!}$$

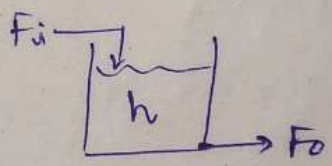
$$+ \cdots + \left(\frac{d^nf}{dx^n}\right)_{x=x_s} \frac{(x-x_s)^n}{n!} + \cdots$$

-- Linearized form

$$\frac{dx}{dt} = f(x) = f(x_s) + \left(\frac{df}{dx}\right)_{x=x_s} (x-x_s)$$

{error = truncation error}

eg. liquid tank system: →



$$A\frac{dh}{dt} = F_i - F_o$$

case 1: $F_o \propto h$ $\quad$ $F_o = \alpha h$

$$A\frac{dh}{dt} + \alpha h = F_i \quad \text{--- linear}$$

case 2: $F_o \propto \sqrt{h}$ $\quad$ $F_o = \beta\sqrt{h}$

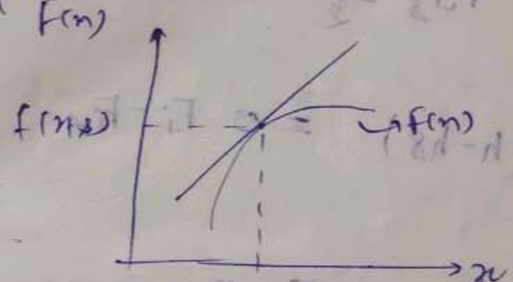
$$A\frac{dh}{dt} + \beta\sqrt{h} = F_i \quad \text{--- non linear}$$

$$\frac{dh}{dt} = \frac{F_i - \beta\sqrt{h}}{A} = f(h)$$

$$\Rightarrow A\frac{dh}{dt} + \beta\sqrt{h} = F_i$$

$$A\frac{dh}{dt} + \frac{\beta}{2\sqrt{h_s}}\, h = F_i - \frac{\beta}{2}\sqrt{h_s}$$

{At steady state all the variable = cost}



$$\frac{dx}{dt} = f(x_s) + \frac{df}{dx}\Big|_{x=x_s}(x-x_s) \quad \text{—— (2)}$$

⇐ Linearized model

at steady become

$$\frac{dx_s}{dt} = f(x_s) \quad \text{—— (3)}$$

⇐ steady state model

(2) − (3) →

$$\frac{d}{dt}(x-x_s) = \frac{df}{dx}\Big|_{x=x_s}(x-x_s)$$

$x - x_s = x'$ {deviation variable}

$$\Rightarrow \frac{dx'}{dt} = \frac{df}{dx}\Big|_{x=x_s} x'$$

$$A\frac{dh}{dt} + \frac{\beta}{2\sqrt{h_s}} h = F_i - \frac{\beta}{2}\sqrt{h_s}$$

steady state:-

$$A\frac{dh_s}{dt} + \frac{\beta}{2}\sqrt{h_s} = F_{is} - \frac{\beta}{2}\sqrt{h_s}$$

$$\Rightarrow A\frac{d}{dt}(h-h_s) + \frac{\beta}{2\sqrt{h_s}}(h-h_s) = F_i - F_{is}$$

$$\Rightarrow A\frac{d}{dt}h' + \frac{\beta}{2\sqrt{h_s}}h' = F_i - F_{is}$$

Multivariable system:-

$$\left.\begin{aligned}\frac{dx_1}{dt} &= f_1(x_1, x_2)\\ \frac{dx_2}{dt} &= f_2(x_1, x_2)\end{aligned}\right\} \text{non linear system} \quad \text{— ③}$$

$$f_1(x_1, x_2) = f_1(x_{1s}, x_{2s}) + \left.\frac{\partial f_1}{\partial x_1}\right|_{x_1 = x_{1s}}(x_1 - x_{1s})$$

$$+ \left.\frac{\partial f_1}{\partial x_2}\right|_{x_2 = x_{2s}}(x_2 - x_{2s}) + \left.\frac{\partial^2 f_1}{\partial x_1^2}\right|_{x_1 = x_{1s}} \frac{(x_1 - x_{1s})^2}{2!}$$

$$+ \left.\frac{\partial^2 f_2}{\partial x_2^2}\right|_{x_2 = x_{2s}} \frac{(x_2 - x_{2s})^2}{2!}$$

$$+ \left.\frac{\partial^2 f_1}{\partial x_1 \partial x_2}\right|_{x_s} (x_1 - x_{1s})(x_2 - x_{2s}) \ldots$$

$\frac{dx_1}{dt} \Rightarrow F_1(x_1,$

$\frac{d}{dt}x_{1s} =$

③ − ④

$\Rightarrow \frac{d}{dt}x_1'$

similar fo

$\frac{d}{dt}x_2' =$

$\frac{dx_1'}{dt} = a_{11}$

$\frac{dx_2'}{dt} = a_{21}$

$a_{ij} = \left(\frac{\partial f_i}{\partial x_j}\right.$

$$\frac{dx_1}{dt} \Rightarrow f_1(x_1,x_2) = f_1(x_{1s},x_{2s}) + \frac{\partial f_1}{\partial x_1}\Big|_{x_{1s}}(x_1-x_{1s}) + \frac{\partial f_1}{\partial x_2}\Big|_{x_{2s}}(x_2-x_{2s}) \quad \text{--- (3)}$$

Linearized dynamic model

$$\frac{dx_{1s}}{dt} = f_1(x_{1s},x_{2s}) \quad \text{--- (4)}$$

L. static model

(3) − (4)

$$\Rightarrow \frac{dx_1'}{dt} = \frac{\partial f_1}{\partial x_1}\Big|_{x_1=x_{1s}} x_1' + \frac{\partial f_1}{\partial x_2}\Big|_{x_2=x_{2s}} x_2'$$

const coeff

similar for (B)

$$\frac{dx_2'}{dt} = \frac{\partial f_2}{\partial x_1}\Big|_{x_{1s}} x_1' + \frac{\partial f_2}{\partial x_2}\Big|_{x_{2s}} x_2'$$

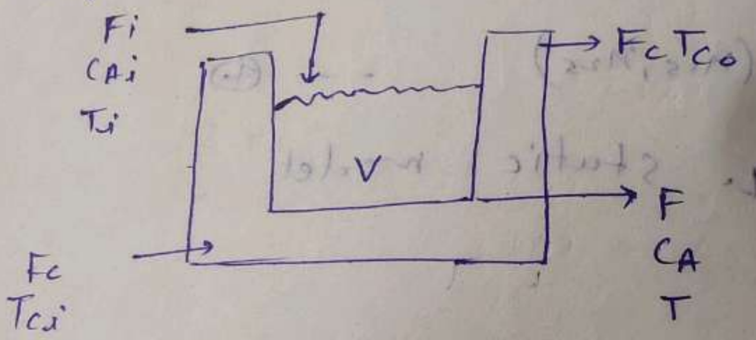
$$\frac{dx_1'}{dt} = a_{11}x_1' + a_{12}x_2'$$

$$\frac{dx_2'}{dt} = a_{21}x_1' + a_{22}x_2'$$

$$\left\{ a_{ij} = \left(\frac{\partial f_i}{\partial x_j}\right)_{xs} \right\}$$

$$\begin{bmatrix} \dot{x}_1' \\ \dot{x}_2' \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} x_1' \\ x_2' \end{bmatrix}$$

e.g. CSTR :→



$$\frac{dC_A}{dt} = \frac{1}{\tau}\left[C_{Ai} - C_A\right] - K_0 C_A e^{-E/RT}$$

τ = residence time

$$\frac{dT}{dt} = \frac{1}{\tau}(T_i - T) - \frac{Q}{V\rho C_p} + \frac{(-\Delta H) K_0 C_A e^{-E/RT}}{\rho C_p}$$

$$\left\{ Q = UA(T - T_c) \right\}$$

$$C_A e^{-E/RT} =$$

$$\begin{bmatrix} \dot{C}_A \\ \dot{T} \end{bmatrix} = \Big[\; e^{-E}$$

* Transfer function (TF)

{in time domain}

Input $f(t) \rightarrow$ [Process] $\rightarrow$ output $y(t)$

SISO $\equiv$ (single i/p single o/p) :-

in Laplace domain

Input $\bar{f}(s) \rightarrow \boxed{G(s)} \rightarrow$ output $\bar{y}(s)$

$$G(s) = TF = \frac{\bar{y}(s)}{\bar{f}(s)}$$

Linearized / linear process :-

$$a_n \frac{d^n y}{dt^n} + a_{n-1} \frac{d^{n-1} y}{dt^{n-1}} + \cdots + a_1 \frac{dy}{dt} + a_0 y = b f(t)$$

$y, f \equiv$ deviation variable

ss at time $t=0$

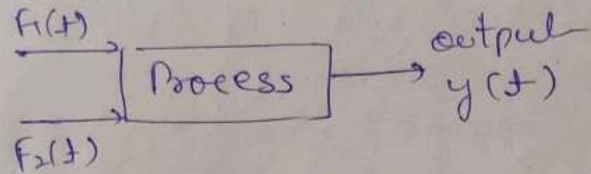
after L.T.

$$a_n s^n \bar{y}(s) + a_{n-1} s^{n-1} \bar{y}(s) + \cdots + a_1 s \bar{y}(s) + a_0 \bar{y}(s) = b \bar{f}(s)$$

$$\Rightarrow \frac{\bar{y}(s)}{\bar{f}(s)} \cdot \left[a_n s^n + a_{n-1} s^{n-1} + \cdots + a_1 s + a_0\right] = b$$

$$\Rightarrow \frac{\bar{y}(s)}{\bar{f}(s)} = \frac{b}{a_n s^n + \cdots + a_0}$$

* MISO (Multi - - -) :→



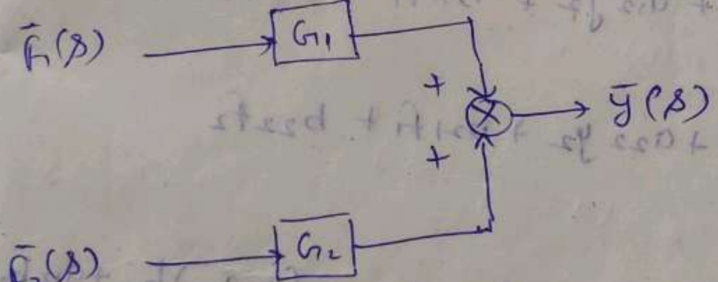
$$a_n \frac{d^n}{dt^n} y + a_{n-1} \frac{d^{n-1} y}{dt^{n-1}} + \cdots + a_0 y = b_1 f_1(t) + b_2 f_2(t)$$

L.T.

$$\Rightarrow \left[a_n s^n + a_{n-1} s^{n-1} + \cdots + a_0\right] \bar{y}(s) = b_1 \bar{f}_1(s) + b_2 \bar{f}_2(s)$$

$$\bar{y}(s) = \frac{b_1}{\left[a_n s^n + \cdots + a_0\right]} \bar{f}_1(s) + \frac{b_2}{\left[a_n s^n + \cdots + a_0\right]} \bar{f}_2(s)$$

$$\bar{y}(s) = G_1 \bar{f}_1(s) + G_2 \bar{F}_2(s)$$



*MIMO :-

$$\frac{dy_1}{dt} = a_{11} y_1 + a_{12} y_2 + b_{11} f_1 + b_{12} f_2$$

$$\frac{dy_2}{dt} = a_{21} y_1 + a_{22} y_2 + b_{21} f_1 + b_{22} f_2$$

$$\bar{y}_1(s) = \frac{(s-a_{22}) b_{11} + a_{12} b_{21}}{P(s)} \bar{F}_1(s) + \frac{(s-a_{22}) b_{12} + a_{12} b_{22}}{P(s)} \bar{F}_2(s)$$

$$\bar{y}_2(s) = \frac{(s-a_{11}) b_{21} + a_{21} b_{11}}{P(s)} \bar{F}_1(s) + \frac{(s-a_{11}) b_{22} + a_{21} b_{12}}{P(s)} \bar{F}_2(s)$$

$$P(s) = s^2 - (a_{11} + a_{22}) s - (a_{12} a_{21} - a_{11} a_{22})$$

$\bar{y}_1(s) =$

$\bar{y}_2(s) =$

$\bar{F}_1$

$\bar{F}_2$

CSTR: M

F_i

C_{Ai}

T_i

F_c

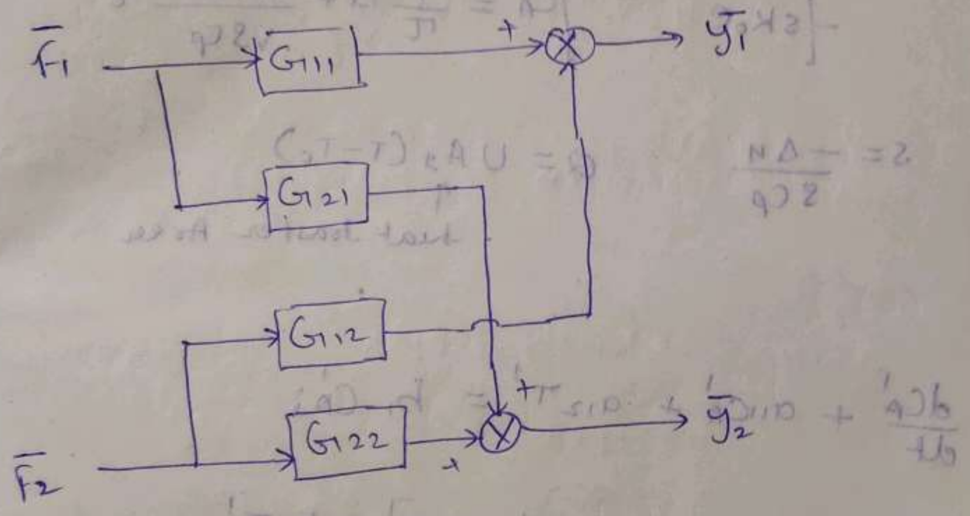
T_{ci}

$$\frac{dC_A'}{dt} + \left[\frac{1}{\tau} + \right.$$

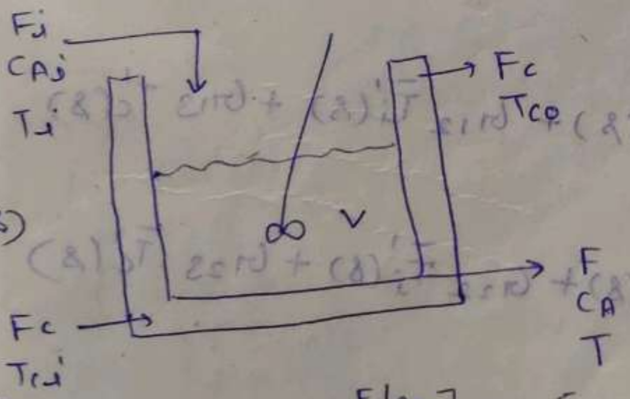
$$\bar{y}_1(s) = G_{11}\bar{F}_1(s) + G_{12}\bar{F}_2(s)$$

G_{ij} — i: output, j: input

$$\bar{y}_2(s) = G_{21}\bar{F}_1(s) + G_{22}\bar{F}_2(s)$$



CSTR: MIMO :-



$$\frac{dC_A'}{dt} + \left[\frac{1}{\tau} + K_0 e^{-E/RT_0}\right] C_A' + \left[\frac{K_0 E}{R T_0^2} e^{-E/RT_0} C_{A0}\right] T' = \frac{1}{\tau} C_{Ai}'$$

$$\frac{dT'}{dt} + \left[\frac{1}{\tau} - \frac{sk_0E}{RT_0^2} e^{-E/RT_0} C_{A0} + \frac{UA_t}{V\rho C_p}\right] T'$$

$$-\left[sk_0 e^{-E/RT_0}\right] C_A' = \frac{1}{\tau} T_i' + \frac{UA_t}{V\rho C_p} T_c'$$

$$s = \frac{-\Delta H}{\rho C_p} \qquad Q = UA_t (T - T_c)$$

heat transfer Area

$$\frac{dC_A'}{dt} + a_{11}C_A' + a_{12}T' = b_1 C_{Ai}'$$

$$\frac{dT'}{dt} = a_{21}C_A' + a_{22}T' = b_1 T_i' + b_2 T_c'$$

$$T'(0) = 0 \qquad C_A'(0) = 0$$

$$\bar{C}_A'(s) = G_{11}\bar{C}_{Ai}'(s) + G_{12}\bar{T}_i'(s) + G_{13}T_c'(s)$$

$$\bar{T}'(s) = G_{21}\bar{C}_{Ai}'(s) + G_{22}\bar{T}_i'(s) + G_{23}T_c'(s)$$

Zeros an

$$\frac{\bar{y}(s)}{\bar{F}(s)} =$$

$Q(s) = 0$
$s = ?$

$P(s) = 0$
$s = ?$

e.g. $G(s) =$

$$\frac{s-1}{s^2-3s+2}$$

First Order Process

→ A system, whose output is y is modeled by first order differential eqn.

$$a_1 \frac{dy}{dt} + a_0 y = b f(t)$$

Case 1: $a_0 \neq 0$

$$\frac{a_1}{a_0}\frac{dy}{dt} + y = \frac{b}{a_0} f(t)$$

$\tau_p = \frac{a_1}{a_0}$ = time const

$K_p = \frac{b}{a_0} \equiv$ static gain / steady state gain.

$$\tau_p \frac{dy}{dt} + y = K_p f$$

$$\frac{y}{f} = \frac{\Delta\, o/p}{\Delta\, i/p}$$

$$\tau_p s \bar{y}(s) + \bar{y}(s) = K_p \bar{f}(s)$$

$$\Rightarrow \frac{\bar{y}(s)}{\bar{f}(s)} = \frac{K_p}{\tau_p s + 1} = G(s)$$

(TF) of 1st order

Case 2: $a_0 = 0$

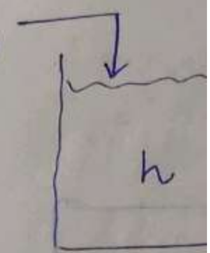
$$a\frac{dy}{dt} = b f \Rightarrow \frac{dy}{dt} = \frac{b}{a} f = K_p' f$$

$$\Rightarrow s\bar{y}(s) = K_p' \bar{f}(s)$$

$$\Rightarrow G(s) = \frac{\bar{y}(s)}{\bar{f}(s)} = \frac{K_p'}{s}$$

--- pure integr... pure capaciti...

Ex-1



$\Rightarrow A\frac{dh}{dt}$ ...

→ deviati...

$\Rightarrow A\frac{dh'}{dt}$ ...

$\Rightarrow AR$ ...

$\Rightarrow G($ ...

$\tau_p = AR$

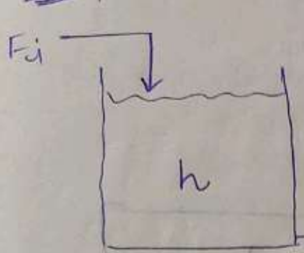
storage capacitance

Ex-2

Tst

$Q = UA_{ht}(T$ ...

Ex-1



$F_o = \frac{h}{R}$

$A\frac{dh}{dt} = F_i - F_o$

{Flow = $\frac{\text{driving Force}}{\text{resistance}}$}

$\Rightarrow A\frac{dh}{dt} + \frac{h}{R} = F_i$

$\longrightarrow$ deviation variable

$\Rightarrow A\frac{dh'}{dt} + \frac{h'}{R} = F_i'$

$\Rightarrow AR\frac{dh'}{dt} + h' = F_i' R$

$\Rightarrow G(s) = \frac{K_p}{\tau_p s + 1}$

$\tau_p = AR$

$K_p = R$

unit = time always

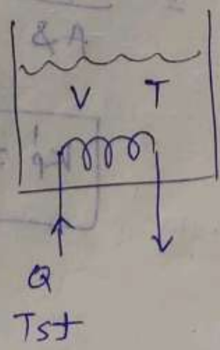
K_p unit is not fix $\rightarrow$ depends on system.

$\tau_p = A R$

A → storage capacitance

R → Resistance to mass Flow

Ex-2



Energy in − E out = Energy accu

$\Rightarrow Q = \rho V C_p \frac{dT}{dt}$

$\Rightarrow UA_{st}(T_{st} - T) = \rho V C_p \frac{dT}{dt}$

$Q = UA_{st}(T_{st} - T)$ $\Rightarrow T_{st} - T = \frac{\rho V C_p}{UA_{st}}\frac{dT}{dt}$

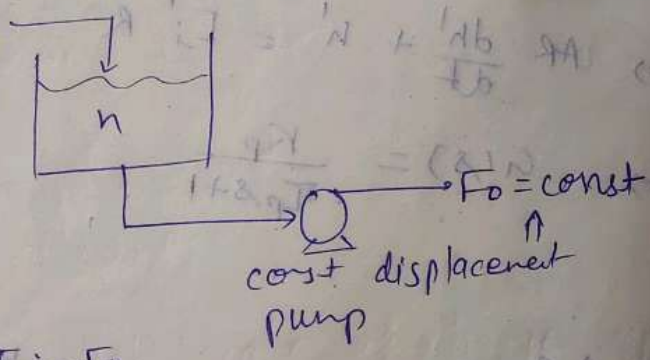
$$\frac{\rho V C_p}{U A_{st}} \frac{dT'}{dt} + T' = T_{st}'$$

τ_p $\qquad K_p = 1$

$$G(s) = \frac{K_p}{\tau_p s + 1} \qquad \boxed{G(s) = \frac{1}{\frac{\rho V C_p}{U A_{st}} s + 1}}$$

$$\tau_p = (V\rho C_p) \times \frac{1}{U A_{st}}$$

Ex-3



$$A \frac{dh}{dt} = F_i - F_o$$

$$\Rightarrow A \frac{dh'}{dt} = F_i'$$ ← deviation variable

$$\Rightarrow A s \, \bar{h}'(s) = \bar{F}_i'(s)$$

$$G(s) = \frac{\bar{h}'(s)}{\bar{F}_i'(s)} = \frac{1}{A(s)} = \frac{1}{As} = \frac{K_p'}{s}$$

$$\boxed{K_p' = \frac{1}{A}}$$

$\bar{h}'(s) = \frac{K_p}{s}$

for simplicity

$h(s) = \frac{K_p'}{s}$

$= \frac{K}{s}$

$h(t) = K$

Ex ⇒ $G(s) =$

$\bar{F}(s) = \frac{1}{s}$

$\bar{y}(s) = \frac{K_p}{\tau_p s +}$

$\frac{K_p}{\tau_p}\left(s + \frac{1}{\tau}\right.$

$\frac{K_p}{\tau_p} \int_0^t e^{-}$

if change = A

$\bar{h}'(s) = \frac{K_p'}{s} \bar{F}_i'(s)$

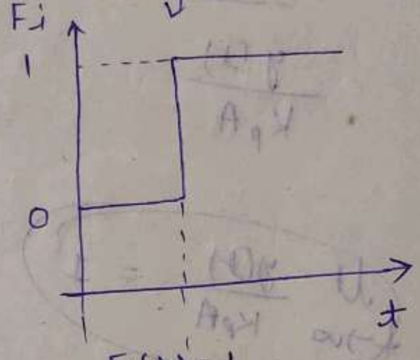
for Simplicity $\bar{h}' \rightarrow h$

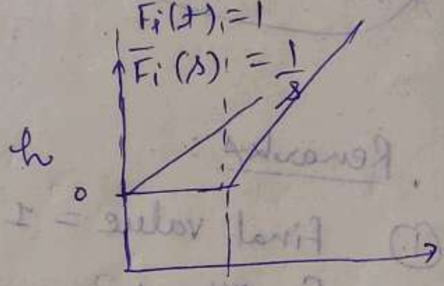
$h(s) = \frac{K_p'}{s} F_i(s)$

$= \frac{K_p'}{s^2}$

$h(t) = K_p' t$

change in i/p



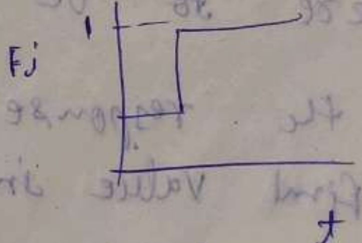


o/p = ramp fun^{t_c}

Ex ⇒ $G(s) = \frac{\bar{y}(s)}{\bar{F}(s)} = \frac{K_p}{\tau_p s + 1}$

unit step

$\bar{F}(s) = \frac{1}{s}$



$\bar{y}(s) = \frac{K_p}{\tau_p s + 1} \cdot \frac{1}{s}$

$\frac{K_p}{\tau_p} \frac{1}{(s + \frac{1}{\tau_p})} \cdot \frac{1}{s}$

$F_i = 1$

$\bar{F}(s) = \frac{1}{s}$

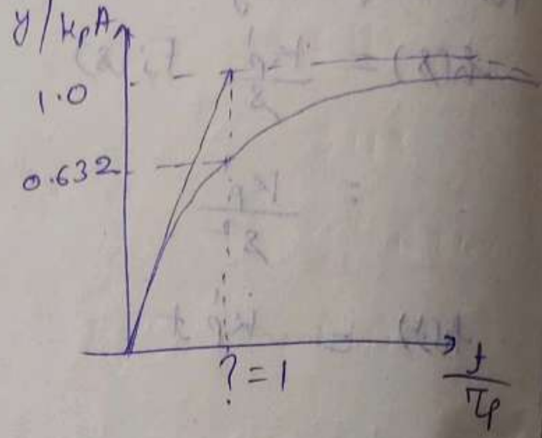
$\frac{K_p}{\tau_p} \int_0^t e^{-\frac{t}{\tau_p}} dt = -K_p \left[e^{-\frac{t}{\tau_p}} \right]_0^t$

if change = A

$\hat{y}(t) = K_p \left[1 - e^{-\frac{t}{\tau_p}} \right]$

$$\Rightarrow \boxed{y(t) = K_p A \left(1 - e^{-t/\tau_p}\right)}$$

$$\frac{y(t)}{K_p A}$$



$$\lim_{t \to \infty} \frac{y(t)}{K_p A} = 1$$

Remarks:

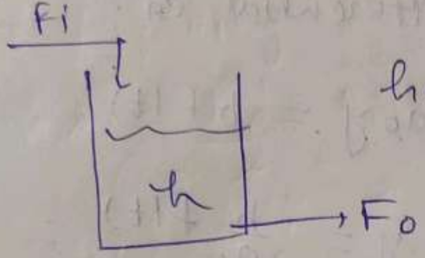
1. final value = 1
2. $$\left[\frac{d(y/K_pA)}{d(t/\tau_p)}\right]_{t=0} = 1$$

if the initial rate of change $y(t)$ were to be maintained, the response would reach its final value in 1 time const $(t = \tau_p)$

3.

t/τ_p	1	2	3	4
$\frac{y}{AK_p}$	0.632	0.865	0.950	0.982

(4.) self regulating



$h_{ss_1} = 10\,m$

$F_i \uparrow \; h \uparrow \Rightarrow h \Rightarrow$ new ss

$h_{ss2} > h_{ss1}$

A system whose output y is modeled by 2nd order differential eq^n.

2nd order System

$$a_2 \frac{d^2y}{dt^2} + a_1 \frac{dy}{dt} + a_0 y = b f(t)$$

$$\frac{a_2}{a_0} \frac{d^2y}{dt^2} + \frac{a_1}{a_0} \frac{dy}{dt} + y = \frac{b}{a_0} f(t) \qquad a_0 \neq 0$$

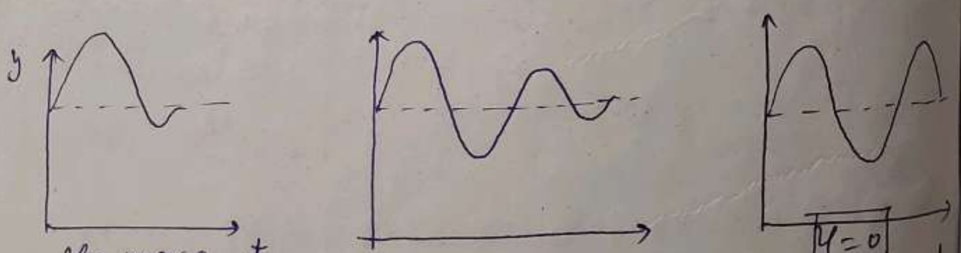
$$\Rightarrow \tau^2 \frac{d^2y}{dt^2} + 2\psi\tau \frac{dy}{dt} + y = K_p f(t)$$

$$\tau^2 = \frac{a_2}{a_0} \qquad 2\psi\tau = \frac{a_1}{a_0} \qquad K_p = \frac{b}{a_0}$$

τ = natural period

ψ = damping factor

K_p = static gain



ψ = more oscillation less

undamped

ψ is measure of amount of damping.

multicapability processes

2nd-order System :→

(i) 1st-order + 1st-order ≡ 2nd order

(ii) 1st-order + controller ≡ "

*(iii)✗ inherent second order → not controllable

$$G(s) = \frac{\bar{y}(s)}{\bar{f}(s)}$$

$$\Rightarrow \tau^2 s^2 \bar{y}(s) + 2\psi\tau s \bar{y}(s) + \bar{y}(s) = K_p \bar{f}(s)$$

$$\Rightarrow \frac{\bar{y}(s)}{\bar{f}(s)} = \frac{K_p}{\tau^2 s^2 + 2\psi\tau s + 1}$$

{TF only if deviation variable y, f}

Pole: $\tau^2 s^2 + 2\psi\tau s + 1 = 0$

$$\frac{-2\psi\tau \pm \sqrt{4\psi^2\tau^2\cancel{s^2} - 4\tau^2}}{2\tau^2}$$

$$\frac{-2\psi\tau \pm 2\tau\sqrt{\psi^2\cancel{s}-1}}{2\tau^2}$$

$$\Rightarrow \frac{-\psi \pm \sqrt{\psi^2\cancel{s}-1}}{\tau}$$

$$P_2 = \frac{-\psi + \sqrt{\psi^2\cancel{s}-1}}{\tau} \qquad P_1 = \frac{-\psi - \sqrt{\psi^2-1}}{\tau}$$

Case 1: $\psi > 1$ two distinct real poles

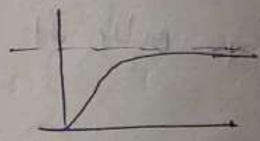
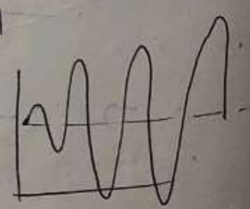
→ This kind of system is called overdamped system.

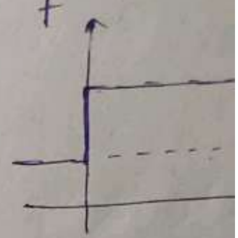
Case 2: $\psi = 1$ one real

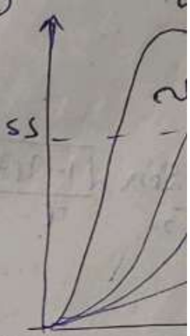
critically damped system

case 3 : $0 < \psi < 1$ conjugate poles

$$p_1 = -\frac{\psi}{\tau} + i\,\frac{\sqrt{1-\psi^2}}{\tau} \qquad p_2 = -\frac{\psi}{\tau} - i\,\frac{\sqrt{1-\psi^2}}{\tau}$$

ψ	nature	plot
$\psi \geq 1$	overdamped stable & non osc.	
$0 < \psi < 1$	underdamped stable & osc.	
$\psi = 0$	Undamped [marginally stable] in CH	
$-1 < \psi < 0$		
$\psi \leq -1$		





Repons

$G_p(s) =$

$\uparrow$

$p \equiv$ process

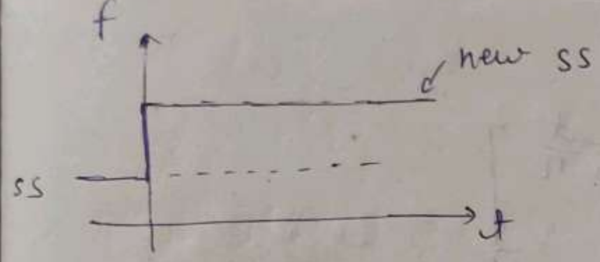
$f(t) = A$

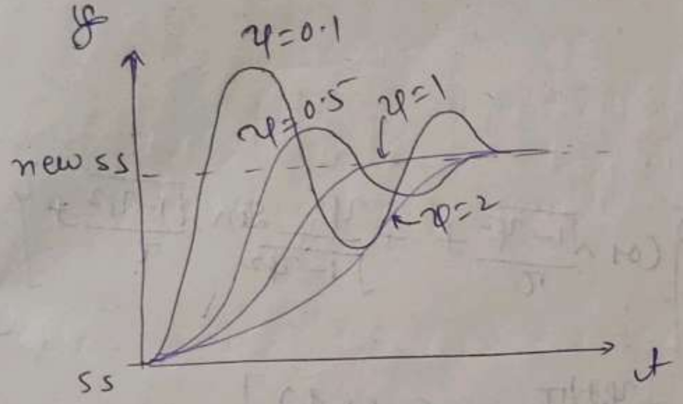
$\bar{f}(s) = \frac{A}{s}$

$y(t) = ?$

overdamped $\Rightarrow$

$y(t) = K_p A\,[$





Reponse against input Change

$$G_p(s) = \frac{K_p}{\tau^2 s^2 + 2\zeta\tau s + 1} = \frac{\bar{y}(s)}{\bar{f}(s)}$$

p ≡ process

$$\bar{y}(s) = \frac{K_p}{\tau^2 s^2 + 2\zeta\tau s + 1}\,\bar{f}(s)$$

$f(t) = A$

$\bar{f}(s) = \frac{A}{s}$

$$\bar{y}(s) = \frac{K_p}{\tau^2 s^2 + 2\zeta\tau s + 1}\,\frac{A}{s}$$

$y(t) = ?$

overdaped :-

$$y(t) = K_p A\left[1 - e^{-\frac{\zeta t}{\tau}}\left\{\cosh\frac{\sqrt{\zeta^2-1}}{\tau}t + \frac{\zeta}{\sqrt{\zeta^2-1}}\sinh\frac{\sqrt{\zeta^2-1}}{\tau}t\right\}\right]$$

$$\sinh\frac{\sqrt{\zeta^2-1}}{\tau}\,t$$

Critically damped :-

$$y(t) = K_p A\left[1 - \left(1 + \frac{t}{\tau}\right) e^{-\frac{t}{\tau}}\right]$$

Underdamped :-

$$y(t) = K_p A\left[1 - e^{-\psi t/\tau}\left\{\cos \frac{\sqrt{1-\psi^2}}{\tau} t + \frac{\psi}{\sqrt{1-\psi^2}} \sin \frac{\sqrt{1-\psi^2}}{\tau} t\right\}\right]$$

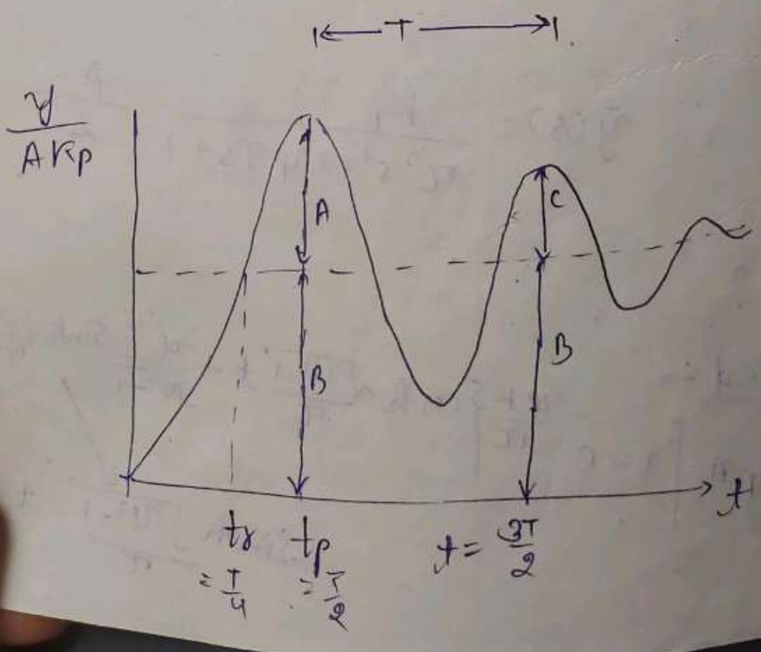
$$= K_p A\left[1 - \frac{1}{\sqrt{1-\psi^2}} e^{-\psi t/\tau} \sin(\omega t + \phi)\right]$$

$\boxed{\omega T = 2\pi}$

ω = radian frequency

rad / time $= \dfrac{\sqrt{1-\psi^2}}{\tau}$

$\phi = \tan^{-1}\left(\dfrac{\sqrt{1-\psi^2}}{\psi}\right)$ = phase angle



t_r = rise
this is
to reach

$\frac{y}{AK_p} = 1$

$t = t_0 = \frac{1}{\omega}$

t_p = peak tim
this is the
reach firs

$\frac{d}{dt}(y/AK_p)$

$A + B = \frac{y}{AK_p}$

$\frac{A}{B}$ = oversh

$C + B = \frac{y}{AK_p}$

$\frac{C}{A}$ = decay

t_r = rise time

this is the time required for $\frac{y}{AK_p}$ to reach new/final ss first time.

$$\frac{y}{AK_p} = 1 \qquad \lim_{t \to \infty} \frac{y}{AK_p} = 1$$

final value

$$t = t_r = \frac{1}{\omega}(\pi - \phi)$$

t_p = peak time

this is the time required for $\frac{y}{AK_p}$ to reach first time the maxm. value

$$\frac{d}{dt}\left(y/AK_p\right) = 0$$

$$A + B = \left.\frac{y}{AK_p}\right|_{t = t_p} \qquad B = 1$$

$$\frac{A}{B} = \text{overshoot} = \exp\left(-\frac{\pi \psi}{\sqrt{1-\psi^2}}\right)$$

$$C + B = \left.\frac{y}{AK_p}\right|_{t = \frac{3T}{2}}$$

$$\frac{C}{A} = \text{decay ratio} = (\text{overshoot})^2$$

$$= \exp\left(\frac{-2\pi\psi}{\sqrt{1-\psi^2}}\right)$$

$$G(s) = \frac{K_p}{\tau^2 s^2 + 2\psi\tau s + 1}$$

$$\omega = \frac{\sqrt{1-\psi^2}}{\tau} \quad \text{rad/time}$$

$$\omega T = 2\pi$$

$\psi = 0$ undamped sustained oscillation

$$G(s) = \frac{K_p}{\tau^2 s^2 + 1} = \frac{K_p/\tau^2}{(s - i/\tau)(s + i/\tau)}$$

poles = conjugate
real part = 0

$$\omega = \frac{\sqrt{1-\psi^2}}{\tau} \Rightarrow \omega_n = \frac{1}{\tau}$$

$$T = \frac{2\pi}{\omega} = 2\pi\tau$$
$$= 2\pi f$$

Problem :- $$\frac{\bar{y}(s)}{\bar{F}(s)} = \frac{4}{s^2 + 1.6s + 4}$$

$$= \frac{1}{\frac{1}{4}s^2 + 0.4s + 1}$$

(i) $\tau^2 = \frac{1}{4}$ $\quad$ $\tau\psi = 0.2$

$\tau = \frac{1}{2} = 0.5$ $\quad$ $\psi = 0.4$

(ii) step change $\bar{F} = \frac{10}{s}$

$\bar{y}(s) =$

Final val

$\lim_{s \to 0} s\bar{F}(s)$

η 10

overshoo

(iii) rise t

$t_r =$

$\Phi =$ ta

$t_r =$

$\bar{y}(s) = \frac{4}{s^2 + 1.6s + 4} \cdot \frac{10}{s}$ ①

Final value theorem:-

$\lim_{s \to 0} s\bar{F}(s) = \lim_{t \to \infty} f(t)$ ultimate response

$\downarrow$

η 10

overshoot $= \exp\left(\frac{-\pi\xi}{\sqrt{1-\xi^2}}\right)$

$= \exp\left(\frac{-\pi \times 0.4}{\sqrt{1-0.16}}\right)$

$= 0.25$

(iii) rise time t_r =? — ASK?

$t_r = \frac{1}{\omega}(\pi - \phi)$

$\phi = \tan^{-1}\left(\frac{\sqrt{1-\xi^2}}{\xi}\right) = 60° = \frac{\pi}{3}$

$t_r = \frac{2\pi}{3\omega}$ $\omega = \frac{\sqrt{1-\xi^2}}{\tau}$

$= 1.143$ $= \frac{2\sqrt{21}}{5}$

$= 1.833$

2nd order System :→

Remarks :→

① 1st + 1st ≡ 2nd - order

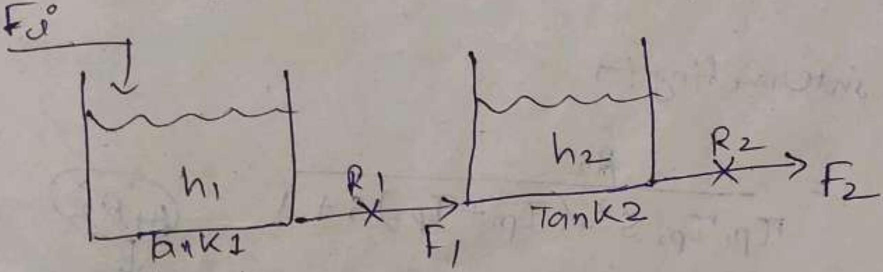
② overdamped

③ $h_2(t) =$ against unit step change in $F_i'(s)$

$$\bar{h}_2'(s) = \frac{K_{p2}}{(\tau_{p_1}s+1)(\tau_{p_2}s+1)} \cdot \bar{F}_i'(s) \longrightarrow \frac{1}{s}$$

④ S-shaped @

No. of capacities ↑ delay ↑



$$F_1 = \frac{h_1 - h_2}{R_1}$$

$$F_2 = \frac{h_2}{R_2}$$

Tank 1 : $A_1 \frac{dh_1}{dt} + \frac{h_1 - h_2}{R_1} = F_i$

$$A_1 R_1 \frac{dh_1}{dt} + (h_1 - h_2) = F_i R_1$$

Tank 2 : $A_2 \frac{dh_2}{dt} + F_2 = F_1$

$$A_2 \frac{dh_2}{dt} = \frac{h_1 - h_2}{R_1} - \frac{h_2}{R_2}$$

$$A_2 \frac{dh_2'}{dt} = \frac{h_1'}{R_1} - \left(\frac{1}{R_1} + \frac{1}{R_2}\right) h_2'$$

$$A_2 \frac{dh_2'}{dt} + \left(\frac{1}{R_1} + \frac{1}{R_2}\right) h_2' = \frac{h_1'}{R_1}$$

$$\Rightarrow A_2 R_2 \frac{dh_2'}{dt} + \left(1 + \frac{R_2}{R_1}\right) h_2' = \frac{R_2}{R_1} h_1'$$

$$G_0(s) = \frac{\bar{h}_2'(s)}{\bar{F}_i'(s)} = \frac{R_2}{\tau_{p_1}\tau_{p_2} s^2 + (\tau_{p_1} + \tau_{p_2} + A_1 R_2) s + 1}$$

--- Interacting

Non interacting :-

$$= \frac{R_2}{\tau_{p_1}\tau_{p_2} s^2 + (\tau_{p_1} + \tau_{p_2}) s + 1}$$

$A_1 R_2 \downarrow$ inter.

Remarks :

1. 1st + 1st ≡ 2nd
2. $A_1 R_2$ Interruption factor
3. $$p = \frac{-(\tau_{p_1} + \tau_{p_2} + A_1 R_2) \pm \sqrt{(\tau_{p_1} + \tau_{p_2} + A_1 R_2)^2 - 4\tau_{p_1}\tau_{p_2}}}{2\tau_{p_1}\tau_{p_2}}$$

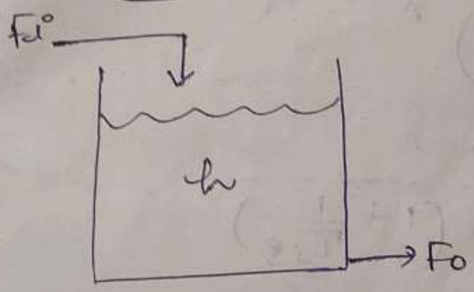
$$\underbrace{(\tau_{p_1} - \tau_{p_2})^2 + (A_1 R_2)^2 + 2(\tau_{p_1} + \tau_{p_2}) A_1 R_2}_{>0}$$

→ two distinct Real poles.

⇒ overdamped.

(4.) Interruption Increases sluggishness.

Process + Controller :→



CV	MV
h	Fo

$$A \frac{dh'}{dt} = F_i' - F_o'$$

$h' = h - h_s = h - h_{sp}$

$F_i' = F_i - F_{is}$

$F_o' = F_o - F_{os}$

Controller:

$$F_o = F_{os} + K_c h' + \frac{K_c}{\tau_i}\int h' dt$$

$\downarrow$

$\boxed{F_{os} = SS}$ ....- PI controller

case : 1 $h' = 0$ $h = h_s \leftarrow SS$

$F_o = F_{os}$

case: 2 $h' > 0$ $h > h_s$ ⇒ $F_o \uparrow$

$$F_o' = K_c h' + \frac{K_c}{\tau_i}\int h'\,dt$$

$$A\frac{dh'}{dt} + K_c h' + \frac{K_c}{\tau_i}\int h'\,dt = F_i'$$

$$\Rightarrow A s\bar{h'} + K_c\bar{h'} + \frac{K_c}{\tau_i}\frac{\bar{h'}}{s} = \bar{F_i}'$$

$$\Rightarrow \left(As + K_c\left(1+\frac{1}{\tau_i s}\right)\right)\bar{h'} = \bar{F_i}'$$

$$\Rightarrow \frac{\bar{h'}}{\bar{F_i}'} = \frac{1}{As + K_c\left(1+\frac{1}{\tau_i s}\right)}$$

$$\Rightarrow \frac{\bar{h}'(s)}{\bar{F}_i'(s)} = \frac{\tau_i s}{A\tau_i s^2 + K_c\tau_i s + K_c}$$

$$= \frac{\frac{\tau_i s}{K_c}}{\frac{A\tau_i}{K_c}s^2 + \tau_i s + 1} \rightarrow \frac{K_p s}{\tau^2 s^2 + 2\zeta\tau s + 1}$$

$$\tau_p^2 = \frac{A\tau_i}{K_c} \qquad 2\zeta\tau_p = \tau_i$$

$$\tau_p = \sqrt{\frac{A\tau_i}{K_c}} \qquad \zeta = \frac{1}{2}\sqrt{\frac{\tau_i K_c}{A}}$$

Remarks:

① 1st order process + PI ≡ 2nd order

(2.) $\sqrt{\frac{K_c \tau_i}{A}} < 2$ $\quad \zeta < 1$

$= 2$ $\quad = 1$

> 2 $\quad > 1$

(3.) K_p not includes & τ :: incl. dynamics
$\downarrow$
cost.

<u>Nth order system:→</u>

Non interacting case :→

$$\bar{f} \rightarrow \boxed{G_1} \rightarrow \boxed{G_2} \rightarrow \cdots \rightarrow \boxed{G_N} \rightarrow \bar{y}_N$$

(1.) $G_0(s) = G_1 G_2 \cdots G_N$

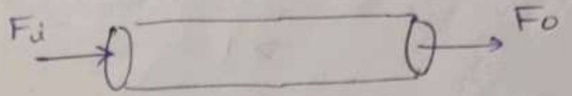
(2.) S-shaped.

(3.) order increases sluggishness.
No. of cap ↑ delay ↑

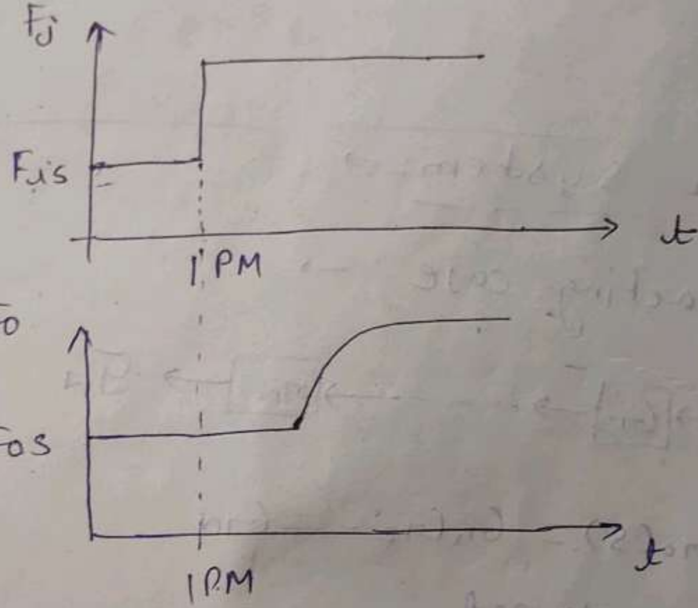
Interacting :→

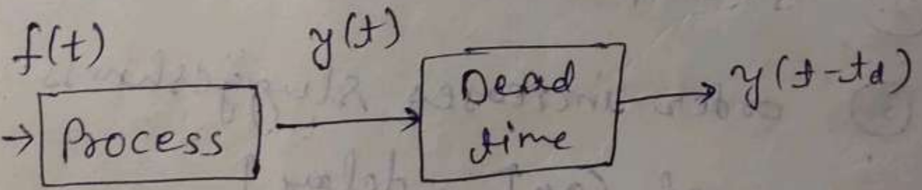
(1.) No Generlization of G_0.

Processes with Dead time :-



$F_i = F_{is} = F_{os}$ at $t = 0$





Dead time : $\dfrac{L[y(t-t_d)]}{L[y(t)]} = e^{-t_d s}$

Process : $\dfrac{L[y(t)]}{L[f(t)]} = \dfrac{K_p}{\tau_p s + 1}$

$$\frac{L[y(t-t_d)]}{L[f(t)]} = \frac{K_p e^{-t_d s}}{\tau_p s + 1} \quad \text{... FOPDT}$$

First order plus dead time

$$\frac{K_p}{\tau^2 s^2 + 2\zeta\tau s + 1} \, e^{-t_d s} \quad \text{--- SOPDT}$$

$$e^{-t_d s} = \frac{1 - \frac{t_d}{2} s}{1 + \frac{t_d}{2} s} \quad \text{Pade approximation (1st-order)}$$

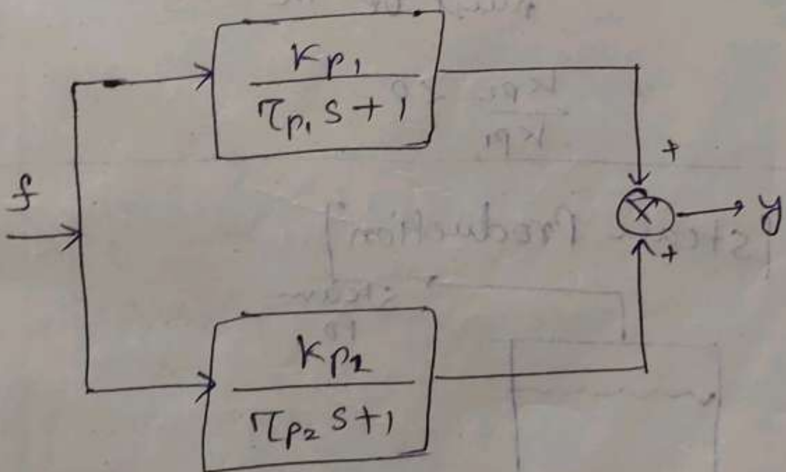
$$= \frac{t_d^2 s^2 - 6 t_d s + 12}{t_d^2 s^2 + 6 t_d s + 12} \quad \text{-- (2nd-order)}$$

2nd order pade approximation

Process with Inverse Responce.

+ve zero

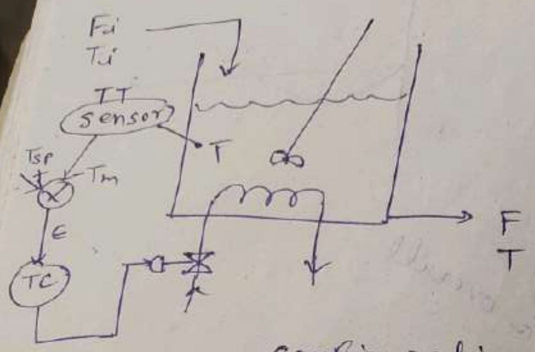
$$G(s) = \frac{Q(s)}{P(s)} \qquad \begin{array}{l} Q(s) = 0 \quad \text{zeros dynamics} \\ P(s) = 0 \quad \text{poles stability} \end{array}$$



$$\frac{y}{f} = \frac{K_{p_1}}{\tau_{p_1} s + 1} + \frac{K_{p_2}}{\tau_{p_2} s + 1} = \frac{(\tau s + 1)(K_{p_1} + K_{p_2})}{(\tau_{p_1} s + 1)(\tau_{p_2} s + 1)}$$

$$= \frac{Q(s)}{P(s)}$$

Feed back control :→



Fst

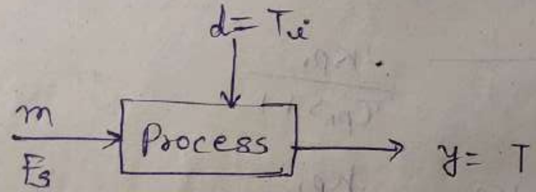
configuration

$T = T_{sp}$

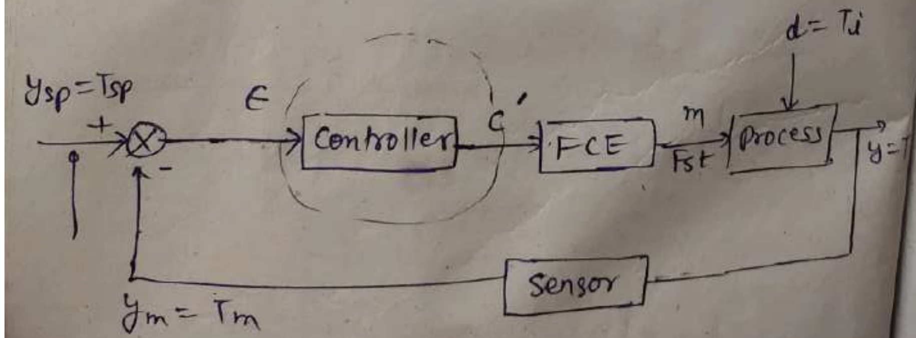
~~h = hsp~~

CV	MV	LV
T	Fst	Ti

$F_i = F$ for simplicity



$\epsilon = T_{sp} - T_m$



Block dig.

There are three individual control Action

(i) Proportional Action $\longrightarrow$ P only controller

(ii) Integral Action } PI controller

(iii) Derivative Action

P-only controller :→

$c'(t) \propto \epsilon(t)$ $\qquad$ c' = deviation variable $= c - c_s$

$c'(t) = K_c \epsilon$

$c(t) = c_s + K_c \epsilon$

K_c = proportional gain
= tunning parameter

c_s = bias signal = known

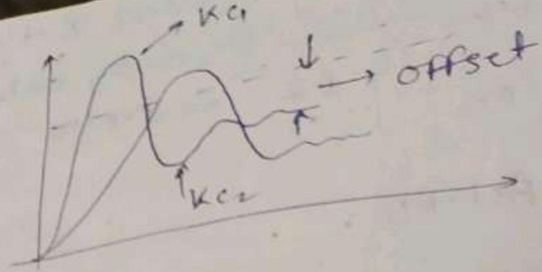
$PB = \frac{100}{K_c}$

Proportional Band

Remarks:

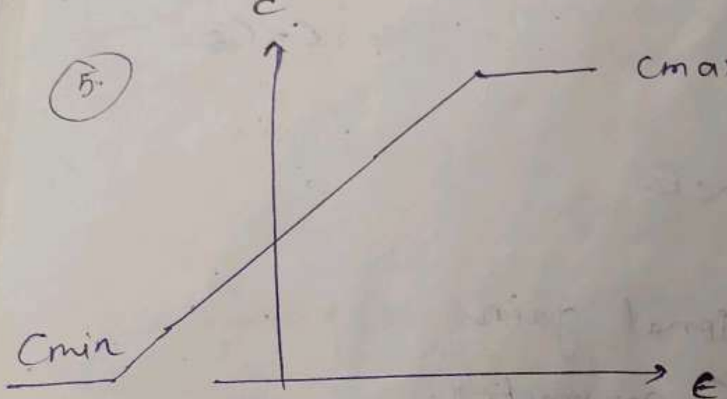
1. $K_c = \frac{c'}{\epsilon}$ = gain
2. $G_c(s) = \frac{\bar{c}'(s)}{\bar{\epsilon}(s)} = K_c$
3. with the increase of K_c value the controller becomes more sensitive to error signal.

$K_{c1} > K_{c2}$



(4) this controller can't eleminate <u>offset</u> (steady state error)

(5)

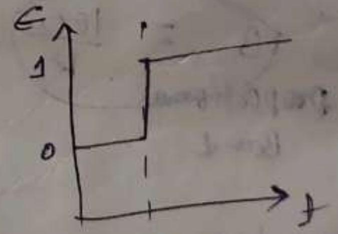


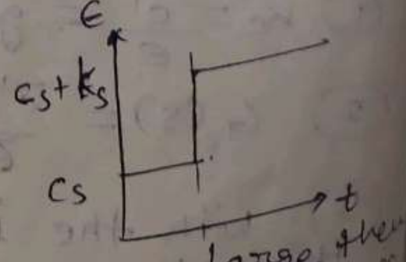
(6) $\bar{C}'(s) = K_c \bar{E}(s)$ $\quad \bar{E}(s) = \frac{1}{s}$

$\Rightarrow e'(t) = ?$

$c'(t) = K_c$

$c(t) = c_s + K_c$





(7) if Kc is very very large then p-type controller becomes on-off controller (eg. switch)

PI (Proportional Integral controller) :-

$$c(t) = c_s + \underbrace{K_c\,\epsilon(t)}_{P} + \underbrace{\frac{K_c}{\tau_i}\int \epsilon(t)\,dt}_{I}$$

$1/\tau_i$ = Reset rate

τ_i = integral time const.
Unit = time.
or (reset)

Remarks

1. $G_c(S) = K_c\left(1 + \frac{1}{\tau_i s}\right)$

2. As long as the error is present this controller keeps changing its output, to make error zero.

3. It Eliminates offset.

- 1st order + PI ≡ 2nd-order

4. sluggish response

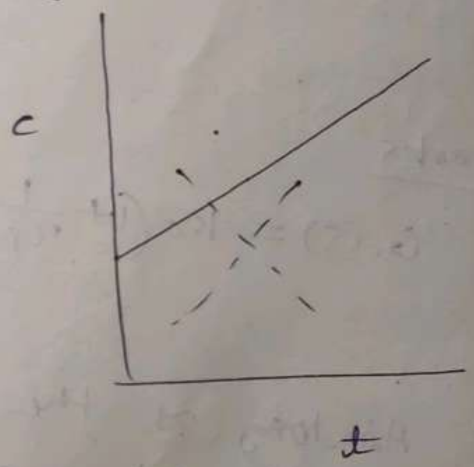
$$\bar{c}'(s) = K_c\,\bar{\epsilon}(s) + \frac{K}{\tau_i s}\,\bar{\epsilon}(s)$$

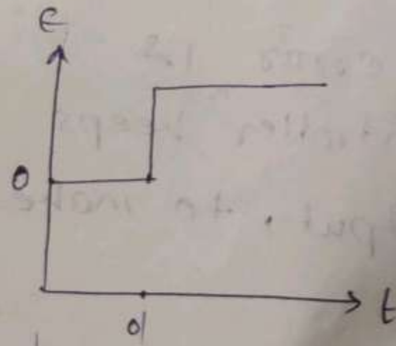
$$\bar{c}'(s) \quad K_c\left(1 + \frac{1}{\tau_i s}\right)\bar{\epsilon}(s)$$

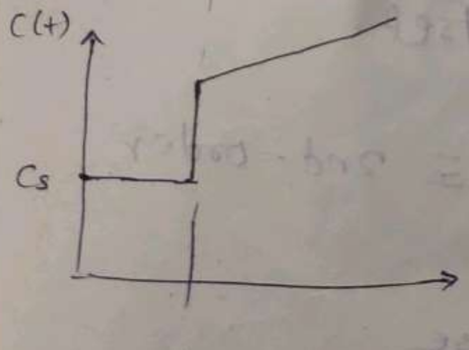
$$c'(t) = K_c\left(1 + \frac{t}{\tau_i}\right) \qquad \bar{\epsilon}(s) = \frac{1}{s}$$

$$c(t) = C_s + K_c\left[1 + \frac{t}{\tau_i}\right]$$

$c(t)$







τ_i (Res
by t
initial
output.

$t = \tau_i$

Heating T

CV : T_s

LV : T_{is}

MV : F_{st}

T_s 25

suppose F_{st} =

P-only ⇒

$t =$

$t = t$

$$\int_0^t \epsilon\, dt =$$

=

τ_i (Reset time) is the time needed by the controller to repeat its initial proportional action in the output.

$t = \tau_i \qquad c(t) = c_s + K_c + \frac{K_c \, t \to \tau_i}{\tau_i}$

Heating Tank:→

CV : $T_s = 25^\circ C$

LV : $T_{is} = 40^\circ C$

MV : $F_{sts} = 50\%$ open valve

$T_s \quad 25 \longrightarrow 20^\circ C \qquad T\downarrow$

suppose $F_{st} = 60\%$.

p-only ⇒ $\quad F_{st} = F_{sts} + K\epsilon$

$t=0 \quad 50 = 50 \quad 0$

$t=t \quad 60 = 50 \quad 10 = K_c \epsilon$

$\epsilon \neq 0$

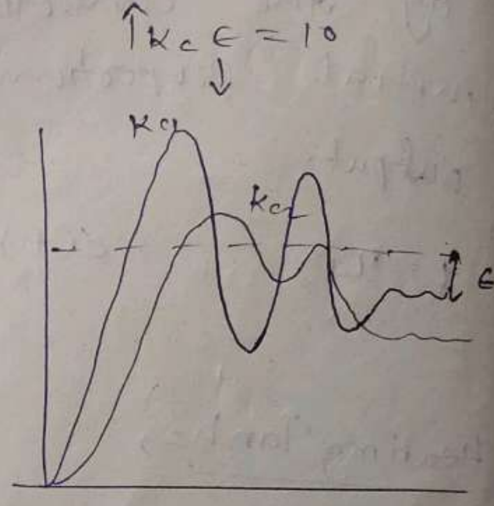
$$\int_0^t \epsilon \, dt = \sum_{i=1}^{n} \epsilon(i\Delta t)\, \Delta t$$

$$= \Delta t \left[\epsilon(\Delta t) + \epsilon(2\Delta t) + \dots + \epsilon(n\Delta t) \right]$$

$t = n\Delta t$

$F_{st} = F_{st_s} + K_c \epsilon$

$60 = 50 + 10$



$K_{c_1} > K_{c_2}$

PI:

$$F_{st} = F_{st_s} + K_c \epsilon + \frac{K_c}{\tau_i}\int \epsilon \cdot dt$$

$$60 = 50 + 0 + 10$$

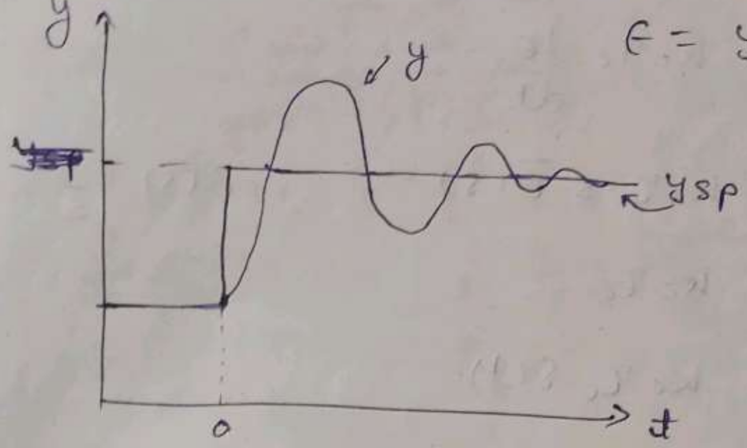
$$\int \epsilon \, dt = \sum_{i=1}^{n} \Delta t \, \epsilon(i.\Delta t) \text{ --- Rectangular}$$

$$= \Delta t \left[\epsilon(\Delta t) + \epsilon(2\Delta t) + \cdots + \epsilon(t)\right]$$

$t = n\Delta t$

time	ϵ	$\Sigma\epsilon$
0	0	0
Δt	5	5
$2\Delta t$	3	8
$\vert$		
$n\Delta t = t$	0	10

$\epsilon = y_{sp} - y$



PID controller :-

$$C = C_s + \underbrace{K_c \epsilon}_{P} + \underbrace{\frac{K_c}{\tau_i} \int \epsilon \, dt}_{I} + \underbrace{K_c \tau_D \frac{d\epsilon}{dt}}_{D}$$

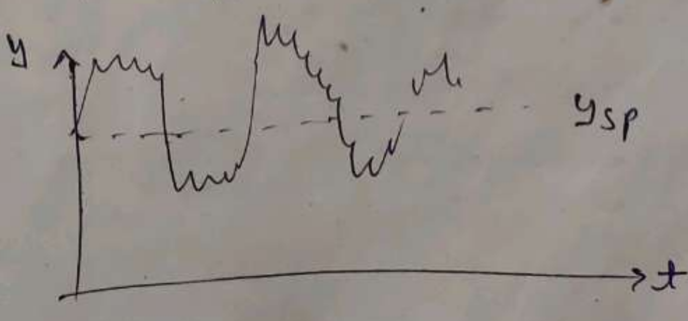
τ_D = derivative time @ turning

Remarks :-

(1) $G_c(s) = K_c \left(1 + \frac{1}{\tau_i s} + \tau_D s\right)$

(2) if $\epsilon = const$ $\quad \frac{d\epsilon}{dt} = 0$

(3) for a noisy response with almost 0 error derivative terms leads to aggressive ~~controller~~ control action although it is not needed.



$$c'(t) = K_c \tau_c \frac{d\epsilon}{dt}$$

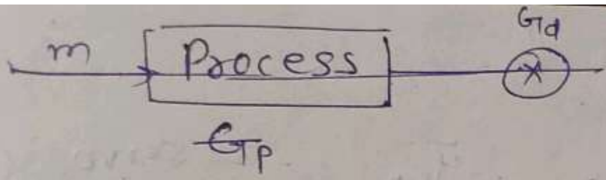
$$\bar{c}'(s) = K_c \tau_c s \, \bar{\epsilon}(s)$$

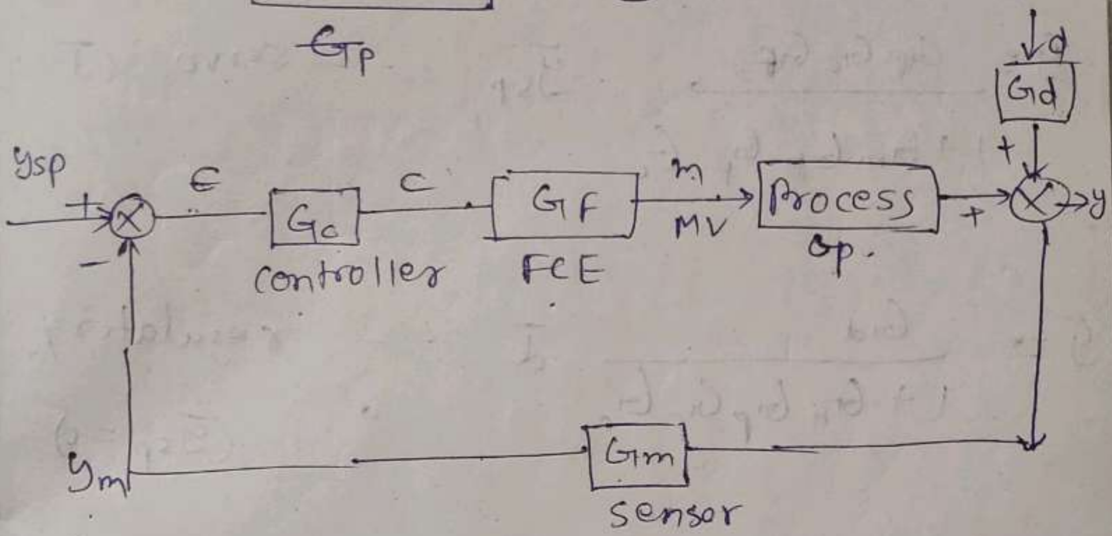
$$\epsilon(s) = \frac{1}{s}$$

$$\bar{c}'(s) = K_c \tau_c$$

$$c'(t) = K_c \tau_c \, \delta(t)$$

(4.) $G_c(s) = K_c\left(1 + \frac{1}{\tau_i s} + \tau_d s\right.$





Process : $\bar{y} = G_p \bar{m} + G_d \bar{d}$

sensor : $\bar{y}_m = G_m \bar{y}$

controller : $\bar{C} = G_c \bar{\epsilon} = G_c(\bar{y}_{sp} - \bar{y}_m)$ P, I, D

FCE : $\bar{m} = G_f \bar{C} = G_f G_c(\bar{y}_{sp} - \bar{y}_m)$

$$= G_f G_c(\bar{y}_{sp} - G_m \bar{y})$$

$$y = \underbrace{\frac{G_c G_p G_f}{1 + G_c G_p G_f G_m} \bar{y}_{sp} + \frac{G_d}{1 + G_m G_p G_f G_c} \bar{d}}$$

↳ closed loop transfer fcn. (CLTF)

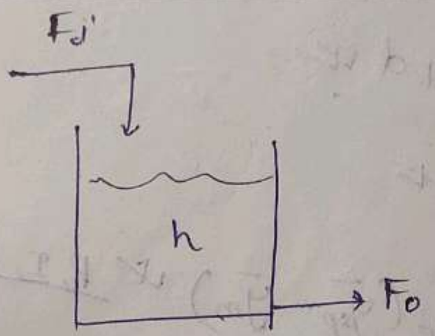
---- Open loop T. F. (OLTF)

$\cancel{G_p = \frac{k_p}{\tau_p s + 1}}$

$G_c G_f G_p G_m$

$$\bar{y} = \frac{G_p G_c G_f}{1 + G_m G_p G_f G_c} \bar{y}_{sp} \quad \text{— servo} \quad (\bar{d} = 0)$$

$$\bar{y} = \frac{G_d}{1 + G_m G_p G_f G_c} \bar{d} \quad \text{— regulatory} \quad (\bar{y}_{sp} = 0)$$



CV	MV	LV
h	Fo	Fi
y	m	d

$$A \frac{dh'}{dt} = F_i' - F_o'$$

$$\Rightarrow \quad A s \bar{h}' = \bar{F}_i' - \bar{F}_o'$$

$$\Rightarrow \quad \bar{h}' = \frac{1}{As} \bar{F}_i' - \frac{\bar{F}_o'}{As}$$

$$= G_p \bar{m} + G_d \bar{d}$$

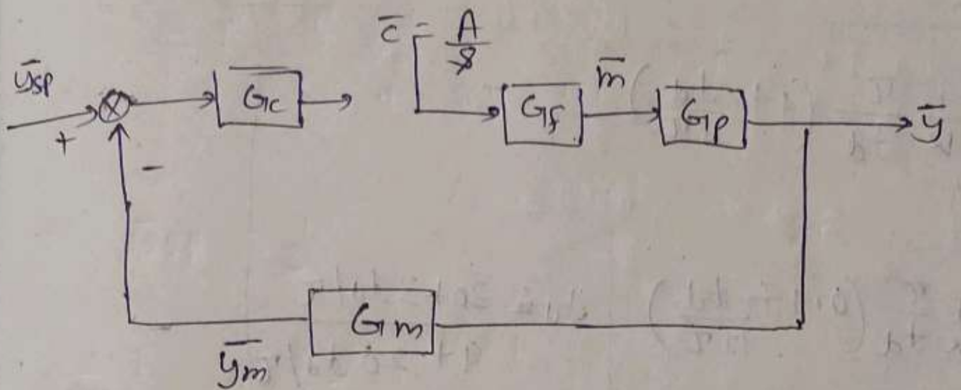
$$G_d = \frac{1}{As}$$

$$G_p = \frac{-1}{As}$$

$$G_m = \frac{K_m}{\tau_m^2 s^2 + 2 \zeta_m \tau_m s + 1}$$

$$G_f = \frac{K_f}{\tau_f s + 1}$$

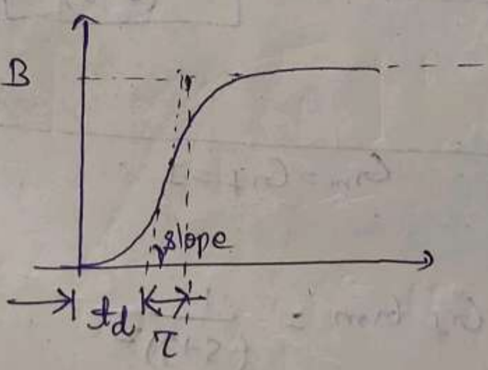
Cohen-coon Method



Block diagram

$$\frac{\bar{y}_m(s)}{\bar{c}(s)} = G_f \, G_p \, G_m = G_{PRC}$$

$$= \frac{K}{\tau s + 1} e^{-t_d s}$$



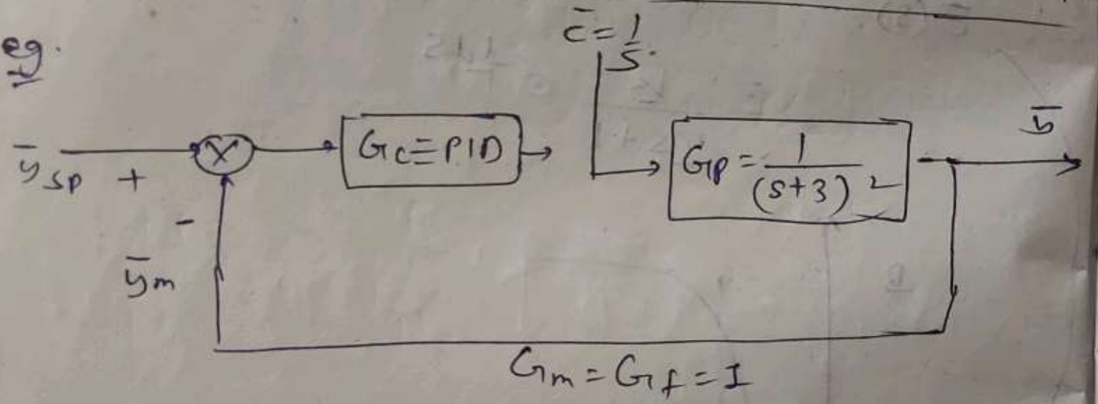
$$\lim_{t \to \infty} y_m(t) = ? = B \quad \{\text{final value Th.}\}$$

$$K = \text{gain} = \frac{\Delta out\big|_{ss}}{\Delta in\big|_{ss}} = \frac{B}{A}$$

$$\tau = \frac{B}{\text{slope}}$$

	K_c	τ_i	τ_d
P	$= \frac{1}{K}\frac{\tau}{t_d}\left(1+\frac{t_d}{3\tau}\right)$	-	-
PI	$\frac{1}{K}\frac{\tau}{t_d}\left(0.9+\frac{t_d}{12\tau}\right)$	$t_d \frac{30+3t_d/\tau}{9+20t_d/\tau}$	-
PID	$\frac{1}{K}\frac{\tau}{t_d}\left(\frac{4}{3}+\frac{t_d}{4\tau}\right)$	$t_d \frac{32+6t_d/\tau}{13+8t_d/\tau}$	$t_d \frac{4}{11+2t_d/\tau}$

eg.



(1) $\frac{\bar{y}_m}{\bar{c}} = G_p G_f G_m = \frac{1}{(s+3)^2}$

$\bar{y}_m = \frac{1}{s(s+3)^2}$

$y_m(t) = ?$ $\quad y_m(t) = \int_0^t t\, e^{-3t}\, dt$

$= \frac{1}{9}\left[1 - (1+3t)e^{-3t}\right]$

$K \rightarrow \tau \rightarrow t_d$

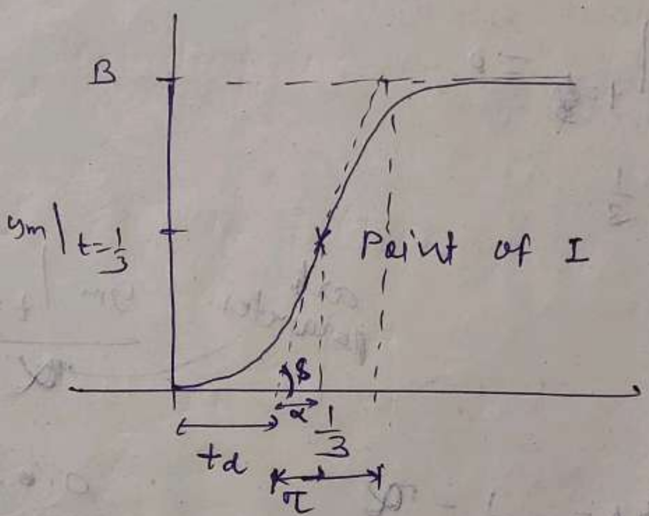
$\Rightarrow k = \frac{B}{A}$

$A = 1$

$$B = \lim_{t \to \infty} y_m(t)$$

$$= \frac{1}{9}\left[1 - 0\right] = \frac{1}{9}$$

$k = \frac{1}{9}$



$$\frac{d^2 y_m(t)}{dt^2} = 0 \quad \text{POI}$$

$$\Rightarrow \quad 0 - \frac{1}{9}\left(-3e^{-3t}(1+3t) + e^{-3t}(3) \right)$$

$$\Rightarrow \quad \frac{9t\,e^{-3t}}{9} = \underline{t\,e^{-3t}}$$

$$\frac{d}{dt}\, t e^{-3t}$$

$$e^{-3t} + t\left(-3e^{-3t}\right) = 0$$

$$\Rightarrow \quad e^{-3t}(1 - 3t) = 0$$

$$\boxed{t = \frac{1}{3}} = \cancel{td}$$

$$[y_m]_{t=\frac{1}{3}} = 0.0294$$

$$[y'_m]_{t=\frac{1}{3}} = S = 0.123$$

$$\begin{cases} y''_m \big|_{t=\$} = 0 \\ t = \frac{1}{3} \end{cases}$$

arb parameter.

$$\frac{y_m \big|_{t=\frac{1}{3}}}{\alpha} = 0.123$$

$$t_d = \frac{1}{3} - \alpha \qquad \frac{0.024}{\text{sec}} = \alpha$$

$$t_d = 0.0943 \text{ sec.}$$

$$\frac{B}{\tau} = 0.123 \Rightarrow \tau = 0.9033$$

$\because$ $G_c \equiv$ PID

K_c ✓ τ_i ✓ τ_d ✓

✓ Which tunning Method is Best?

A S

Input

output

ϕ

AR = A

ϕ = P

z = a

$|z| = \sqrt{a^2}$

arg(z) =

Ziegler - Nichols Turning :→ - 1942

- This Method is based on freq. response Analysis.
- online / closed loop turning method

steps =

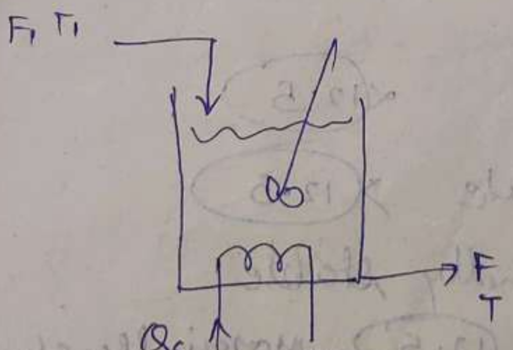
(1.) start up $\xrightarrow[\text{to}]{\text{bring}}$ SS

(2.) - p- only controller ✓ always no matter what is the controller
- close the loop
- ysp

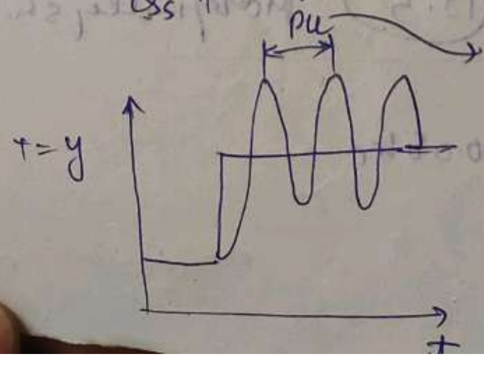
(3.)

offline
(setup)
(=no. maths mod.)

Offline
(model)
(=) no. setup)



CV	MV
T	Q_s



Pu = Ulitmate Period

ysp = Tsp

(K_c) sine wave = Ultimate gain = K_u

the value of K_c at which we get sustained oscillation

	K_c	τ_I	τ_D
P	$K_u/2$	–	–
PI	$K_u/2.2$	$P_u/1.2$	
PID	$K_u/1.7$	$P_u/2$	$P_u/8$

<u>Bode</u>

$\phi = -180$ →
- → $AR > 1$ Unst.
- → $AR = 1$ M.S.
- → $AR < 1$ St.

$AR = 1$ →
- $\phi < -180$ Unst
- $\phi = -180$ M.S
- $\phi > -180$ St.

$\phi = -180$

$\omega = \omega_{co}$ → ?

$$P_u = \frac{2\pi}{\omega_{co}} = T$$

$AR = 1$

$K_c = K_u = ?$

Tyreus - Luyben settings :→

	K_c	τ_I	τ_D
PI	$K_u/3.2$	$2.2\,P_u$	–
PID	$K_u/2.2$	$2.2\,P_u$	$P_u/6.3$

Ex: $G_{OL} = \dfrac{K_c e^{-0.15s}}{0.5s+1}$

PID controller

$K_c = K_u = ?$
$\omega_{co} = ?$

$$\frac{K_c}{0.5j\omega + 1} e^{-0.15j\omega}$$

$AR = 1$ $\qquad \phi = -180^\circ$

$$\frac{K_c}{\sqrt{1+(0.5\omega)^2}} \angle \tan^{-1} 0.5\omega$$

$$\frac{K_c}{\sqrt{1+(0.5\omega)^2}} = AR$$

$$\phi = \cancel{\tan}^{-1}(-0.15\omega) + \tan^{-1}(-0.5\omega) = -180^\circ$$

$K_c = K_u = 5.88$

$\omega = \cancel{0.7585}\ \underline{11.6}$

$\omega = 290$

PID

$\dfrac{K_u}{1.7}$ $\quad \dfrac{P_u}{2}$ $\quad \dfrac{P_u}{8}$

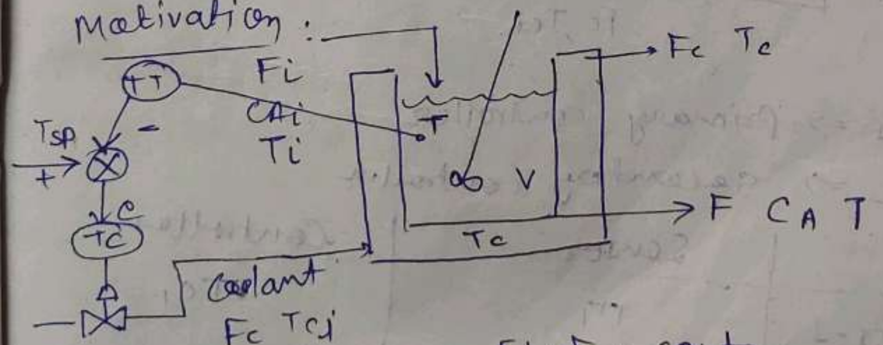
Multi Loop control

No of measurments.	No. of MV	Controller
1	1	FBC P|PI|PID
>1	1	Cascade, override, ratio
1	>1	Split range

Cascade control:

Dual loop — Primary/master, secondary/slave

Motivation:



$F_i = F = cost$

CV	MV	LV
T	F_c	T_i, T_{ci}

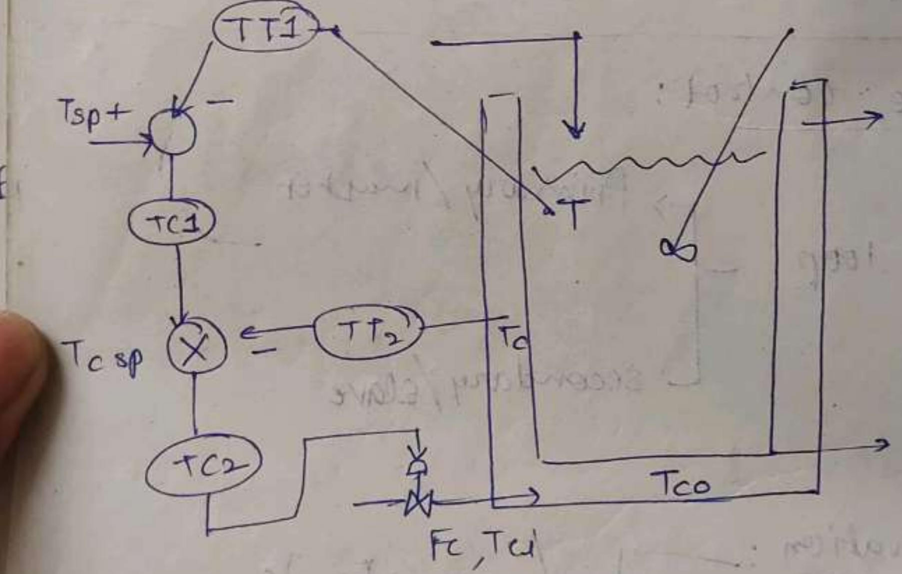
$C_{Ai} = cost$

T response much faster to changes in T_i or T_{ci} ?

↓ ∵ direct.

→ This simple FBC is more effective to reduce the disturbance in T_i than T_{ci}.

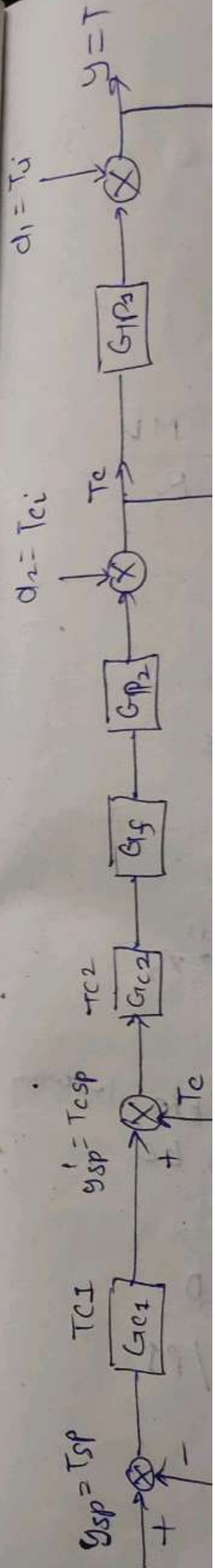
→ cascade controller can control both the disturbance effectively.

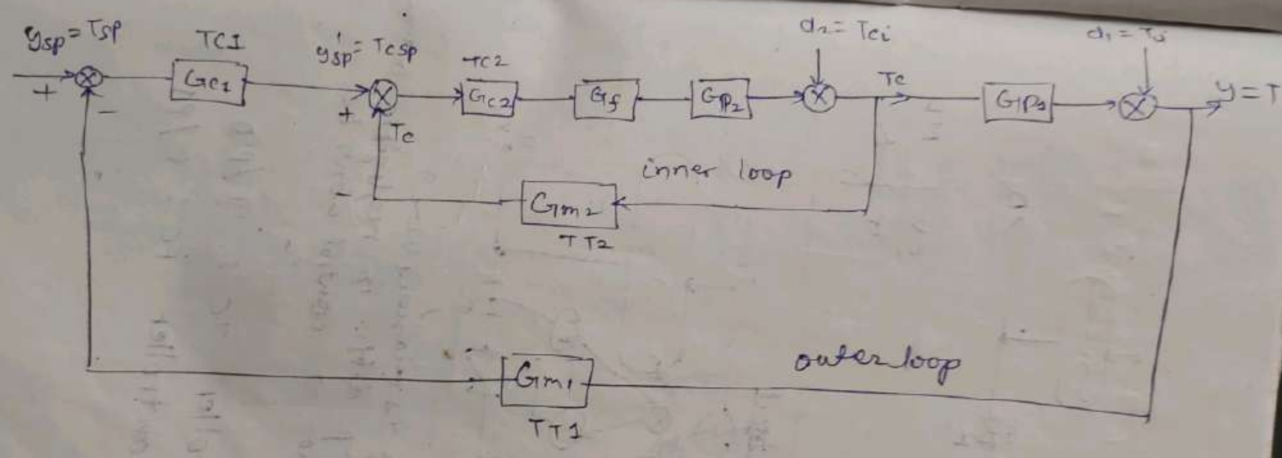


TC_1 ⇒ Primary controller

TC_2 ⇒ Secondary controller

	Sensor	Controller
Process-1 (reactor)	TT_1	TC_1
Process-2 (Jacket)	TT2	TC2





ysp = Tsp
TC1
Gc1
y'sp = Tcsp
TC2
Gc2
Gf
Gp2
d2 = Tci
Tc
Gp1
d1 = Ts
y = T
inner loop
Gm2
TT2
outer loop
Gm1
TT1