#### Assignment MT1-2023-3

#### Section 1

# (Based on lecture notes)

- **1.1** Develop an expression for
  - (a) Drying time in the case of drying of a Spherical water droplet
  - (b) Sublimation time of a Cylindrical naphthalene rod
  - (c) Molar transfer rate from a tank 1 to tank 2 connected through a tapered tube
- **1.2** Describe experimental method for measuring the binary gas phase diffusivity using Stefan tube/Arnold diffusion cell
- **1.3** Describe experimental method for measuring the binary gas phase diffusivity using twin bulb
- **1.4** What are the ways to obtain mass transfer coefficient?
- **1.5** Derive an expression for mass transfer coefficient using following theories and discuss how the flow of fluid is accounted for them.
  - a) Film theory
  - **b**) Penetration theory
  - c) Surface renewal theory
  - **d**) Boundary layer theory
- **1.6** Write the dimensionless numbers related to heat and mass transfer along with the physical significance of them
- **1.7** Explain Reynolds's analogy and Coulburn analogy.
- **1.8** Provide the expressions for different type of Mass Transfer Coefficient for the following cases.
  - a) A is diffusing through non-diffusing B.
  - **b)** Equimolar counter diffusion.

#### Section 2

(Based on the work out problems in the text book)

From book: Principle of Mass Transfer by B.K.Dutta

#### Ex.2.18 Page no.56

**EXAMPLE 2.18** (Diffusion through a converging-diverging region) Two 'large' vessels containing mixtures of  $O_2(A)$  and  $CO_2(B)$  are connected by a tube having converging and diverging sections as shown in Figure 2.18. The partial pressure of oxygen in vessel 1 is  $p_{A1} = 0.3$  bar and that in vessel 2 is  $p_{A2} = 0.1$  bar. The total pressure is 1.013 bar in both the vessels and the temperature is  $20^{\circ}$ C. Calculate the rate of diffusion of oxygen through the tube at steady state. What would be the partial pressure of carbon dioxide and its gradient at the neck (i.e. at the point of the smallest cross-section) of the connecting tube? Given:  $D_{AB} = 0.153$  cm<sup>2</sup>/s;  $r_1 = 5$  cm;  $r_2 = 10$  cm; throat radius, r = 2.5 cm; length of the two sections,  $l_1 = 12$  cm,  $l_2 = 20$  cm.

#### Ex.2.17 Page no.55

**EXAMPLE 2.17** (Diffusion through a tapered region) Test tubes of tapered shape are sometimes used in the laboratories. Consider a 15 cm tall tapered test tube, open at the top. Half of the tube (to a depth of 7.5 cm) is full of ethyl acetate (A) at 25°C. The diameter of the tube at the top is 20 mm and that at the bottom is 12 mm. Calculate the rate of evaporation loss of the ester at the beginning. And also calculate the time of fall of the level by 2 cm. The ambient temperature is 25°C and the pressure is 1.013 bar. The data for the ester are given as follows: mol. wt. = 88; density = 900 kg/m³; vapour pressure at 25°C = 0.1264 bar; diffusivity in air =  $8.66 \times 10^{-6}$  m²/s.

## Ex.2.14 page no.51

**EXAMPLE 2.14** (*Diffusion from a sphere*) Calculate the time required for the sublimation of 3 g of naphthalene from a naphthalene ball of mass 4 g kept suspended in a large volume of stagnant air at  $45^{\circ}$ C and 1.013 bar pressure. Diffusivity of naphthalene in air under the given conditions is  $6.92 \times 10^{-6}$  m<sup>2</sup>/s, its density is  $1140 \text{ kg/m}^3$ , and its sublimation pressure at  $45^{\circ}$ C is 0.8654 mm Hg.

#### Ex.2.15 Page no.51

**EXAMPLE 2.15** (Diffusion from a cylinder) Sublimation of a 20 cm long naphthalene cylinder of mass 10 g occurs in a large volume of air in a room at 45°C and 1 atm pressure. The vapour diffuses through a stagnant film of air of thickness 3 mm surrounding the cylinder. The air beyond the film is well-mixed. Calculate the time required for sublimation of half of the naphthalene. Loss of mass because of sublimation from the flat ends may be neglected (as their combined area is considerably smaller than that of the cylindrical surface). The data given in Example 2.13 may be used.

#### **Ex.3.2** Page no.80

**EXAMPLE 3.2** (Calculation of the gas-phase mass transfer coefficient for an evaporating drop) The gas-phase mass transfer coefficient for the evaporation of a drop of ethyl alcohol in a stream of air at 300 K and 1.2 bar pressure is  $k_G = 2.4 \times 10^{-6} \text{ kmol/(s)} (\text{m}^2) (\text{mm Hg})$ .

- (a) Calculate the values of the mass transfer coefficient if the driving force is expressed in terms of difference in: (i) mole fraction of alcohol in the gas phase, (ii) mole ratio of alcohol, and (iii) concentration of alcohol in kmol/m<sup>3</sup>. Also calculate the coefficient, F<sub>G</sub>.
- (b) Express  $k_G$  in (i)  $lbmol/(ft^2)(s)(psi)$ , (ii)  $lbmol/(ft^2)(h)(atm)$ , and (iii)  $lbmol/(ft^2)(h)$  (inch Hg).

If the diffusivity of alcohol in air is 0.102 cm<sup>2</sup>/s at 0°C, estimate the thickness of the stagnant gas-film. Vapour pressure of alcohol = 0.0877 bar at 300 K.

#### **Ex.3.3** Page no.87

**EXAMPLE 3.3** (Mass transfer in flow past a sphere) A naphthalene ball of 1 cm diameter is suspended in a stream of air flowing at a velocity of 5 m/s at 45°C and 1 atm total pressure. Calculate the time required for its diameter to be halved. The following correlation for the Sherwood number may be used.

$$Sh = 2 + 0.6(Re)^{0.5}(Sc)^{0.33}$$

Sublimation pressure of naphthalene at  $45^{\circ}$ C is 0.8654 mm Hg. The other relevant physicochemical properties are:  $D_{AB} = 6.92 \times 10^{-6}$  m<sup>2</sup>/s; the density of naphthalene at  $45^{\circ}$ C = 1140 kg/m<sup>3</sup>;  $\rho_{air} = 1.1$  kg/m<sup>3</sup>;  $\mu_{air} = 1.92 \times 10^{-5}$  kg/m·s. Assume that the ball retains the spherical shape all through. (Is this assumption correct?)

#### **Ex.3.4** Page no.89

**EXAMPLE 3.4** (Solid dissolution in an agitated vessel) A solution of  $K_2Cr_2O_7$  is to be prepared by dissolving crystals of the solid in an agitated vessel. Twenty kilograms of the solid is charged into a vessel containing 500 kg water. The vessel is provided with a flat-bladed turbine that rotates at 120 rpm. Following data are given: density of the solid = 2690 kg/m³; average particle size = 0.6 mm; solubility of the solid in water at the given temperature = 8 wt%; average viscosity of the solution over the concentration range involved = 0.98 cP; density of the liquid = 1000 kg/m³ (assumed constant); diffusivity of potassium dichromate in water =  $1.5 \times 10^{-5}$  cm²/s. Calculate the time required for complete dissolution of the solid.

The following correlation for Sherwood number can be used to calculate the solid-liquid mass transfer coefficient.

Sh = 
$$\frac{k_L d_p}{D}$$
 = 2 + 0.44(Re<sub>s</sub>)<sup>0.504</sup> (Sc)<sup>0.355</sup>

where

$$Re_s = stirrer Reynolds number = \frac{N d_p^2 \rho}{\mu}$$
;  $Sc = \frac{\mu}{\rho D}$ ;  $d_p = particle diameter.$ 

#### **Ex.3.5** Page no.99

**EXAMPLE 3.5** (Gas absorption in a laminar jet) Gas absorption in a laminar liquid jet is a model experimental technique for the determination of the liquid-phase diffusivity of a soluble gas and also for studying the kinetics of gas—liquid reactions. A sketch of a laminar jet apparatus is given in Figure 3.9. A liquid jet is created by forcing the liquid through a narrow vertical tube, fitted within an enclosure. If an appropriate flow rate is maintained, the liquid comes out as a jet (that very much looks like a liquid rod or cylinder). The jet is collected in a tube of a diameter marginally greater than that of the jet and having a slightly flared opening. This liquid collection tube is oriented vertically below the jet. The solute gas, diluted with a carrier if necessary, flows through the enclosure. The rate of absorption of the gas is determined either by measuring the inlet and the outlet gas flow rates or by analyzing the concentration of the dissolved gas in the exit liquid.

In an experiment for the determination of the diffusivity of hydrogen sulphide in water, pure  $H_2S$  gas is passed through the enclosure of the jet. The following data were collected in a particular experiment at steady state: temperature = 25°C; total pressure in the jet chamber = 1.03 atm; length of the jet = 5 cm; rate of flow of water = 13.2 ml/s; absorption rate of  $H_2S$  =  $4.42 \times 10^{-4}$  g/s. The solubility of  $H_2S$  in water at 1 atm pressure and 25°C is 0.1136 kmol/ (m³)(atm). Calculate the diffusivity of  $H_2S$  in water from the above data.

## Ex.3.6 Page no.100

**EXAMPLE 3.6** (Gas absorption from bubbles) A mixture of 50% CO<sub>2</sub> and 50% N<sub>2</sub> is bubbling through water in a laboratory column at 30°C and 1 atm. The depth of water in the column is 30 cm. A single-nozzle gas distributor is used. The gas flow rate is 15 cm<sup>3</sup> per minute and the bubbles are of 1 cm diameter on the average. The bubble rise velocity is 20 cm/s. Calculate the rate of absorption of carbon dioxide. The diffusivity of CO<sub>2</sub> in water is  $2.19 \times 10^{-5}$  cm<sup>2</sup>/s. Henry's law can be used to calculate the solubility of CO<sub>2</sub> in water at the given temperature,  $p = 1860x^*$  ( $p = partial pressure of CO<sub>2</sub>, in atm; <math>x^*$  is its mole fraction in water at equilibrium).

# Ex.3.7 Page no.101

**EXAMPLE 3.7** (Gas absorption in an agitated vessel) In an experimental agitated contactor, pure carbon dioxide is being absorbed in water at 25°C and 2 atm pressure. Water is pumped into the contactor at a rate of 1 litre per minute and the carbonated water leaves the vessel continuously so that a constant volume is maintained in the contactor. The outlet water contains 2.3 g  $CO_2$  per litre. The specific interfacial area of gas-liquid contact is 80 m²/m³ of the gas-liquid dispersion; the volume of the gas-liquid dispersion is 8 litre. The liquid phase can be assumed to be well mixed. The solubility of  $CO_2$  in water can be calculated using the Henry's law. At 25°C, the Henry's law constant for  $CO_2$  is 1640 atm/mol fraction and its diffusivity in water is  $1.92 \times 10^{-9}$  m²/s. Calculate

- (a) the thickness of the liquid-film if the film theory is applicable,
- (b) the contact time between a liquid element with the gas if the penetration theory is applicable, and
- (c) the fractional surface renewal rate if the surface renewal theory is applicable.

The density of the liquid is 997 kg/m<sup>3</sup> (i.e. the same as that of water at the given temperature).

## Ex.3.8 Page no.104

**EXAMPLE 3.8** (Mass transfer in a packed bed) Water at 25°C flows through a bed of benzoic acid spheres of size  $d_p = 8$  mm at a superficial velocity of 0.022 m<sup>3</sup> per second per square metre of bed cross-section. The fractional void volume of the bed is 0.4 and the depth of the bed is 0.7 m. If the inlet water is free from benzoic acid, calculate the concentration of the acid in the effluent from the bed.

The following data are available: diffusivity of benzoic acid in water =  $10^{-5}$  cm<sup>2</sup>/s; kinematic viscosity of the liquid = 0.95 centistoke; solubility of benzoic acid in water at the given temperature = 3.01 kg/m<sup>3</sup>. For a packed bed of sphere, the specific surface area (i.e. the area of the spheres per unit volume of the bed) is given by:  $\bar{a} = 6(1 - \varepsilon)/d_p$ .

#### Ex.3.9 Page no.105

**EXAMPLE 3.9** (Mass transfer in a wetted-wall column) As stated in Section 3.6, one of the experimental devices for the determination of the mass transfer coefficient is a wetted-wall column. In an experiment, benzene is absorbed from a stream of nitrogen in a wetted-wall column in which the absorbent is a non-volatile mineral oil. Under the conditions, the liquid-phase mass transfer resistance can be safely neglected. The mass flow rate of the feed gas is 13,500 kg/h·m² at 30°C and 1.013 bar pressure. Mole fractions of benzene in the inlet and outlet gases are 0.02 and 0.0052 respectively. The interfacial concentration of benzene on the gas side is negligibly small.

Calculate the gas-phase mass transfer coefficient.

Can you predict the mass transfer coefficient for the absorption of ammonia in a dilute solution of H<sub>2</sub>SO<sub>4</sub> in the same wetted-wall column if the flow rate of air is 4.51 kg/m<sup>2</sup>·s at 25°C and 1.013 bar, and the partial pressure of ammonia in the feed gas is 10 mm Hg?

Given: i.d. of the column = 3.5 cm; thickness of the liquid film is small; height of the wetted section = 3 m; viscosity: for nitrogen (at  $30^{\circ}$ C) = 0.018 cP, for air (at  $25^{\circ}$ C) = 0.0183 cP; diffusivity of benzene in N<sub>2</sub> at  $30^{\circ}$ C = 0.0973 cm<sup>2</sup>/s, that of NH<sub>3</sub> in air at  $25^{\circ}$ C = 0.231 cm<sup>2</sup>/s.

Since the concentration of benzene in the gas is small, the change of flow rate over the column also remains small. So calculations may be done on the basis of the average gas flow rate. It is also given that the Colburn factor  $j_D$  varies as  $Re^{-0.23}$ .

## From book: Mass Transfer Operations by Treybal

# Ex. 3.4 page no.69

What is the heat-transfer analog to this Eq.

$$\frac{c_{A,i} - \bar{c}_{A,L}}{c_{A,i} - c_{A0}} = 0.7857e^{-5.1213\eta} + 0.1001e^{-39.318\eta} + 0.03599e^{-105.64\eta} + \cdots$$

#### Section 3

# (Unsolved problems)

# (Assume atmospheric temperature and pressure, and required constants if not mentioned.)

- **3.1** At 318 K and 1 atm, a naphthalene ball of 3 mm radius is suspended in open air. The surface pressure of the naphthalene can be assumed to be at 318 K is 0.555 mm Hg. The D<sub>AB</sub> of naphthalene in air at 318 K is 6.92 X  $10^{-6}$  m<sup>2</sup>/sec. Calculate the rate of evaporation of naphthalene from the surface.
- **3.2** A 20-cm-long, cylindrical graphite (pure carbon) rod is inserted into an oxidizing atmosphere at 1145 K and 1.013 X  $10^5$  Pa pressure. The oxidizing process is limited by the diffusion of oxygen counter flow to the carbon monoxide that is formed on the cylindrical surface. Under the conditions of the combustion process, the diffusivity of oxygen in the gas mixture may be assumed to be  $1.0 \times 10^{-5}$  m<sup>2</sup>/s.
- (a) Determine the moles of CO that are produced at the surface of the rod per second at the time when the diameter of the rod is 1.0 cm and the oxygen concentration that is 1.0 cm radial distance from the rod is 40 mol%. Assume a steady state process.
- (b) What would be the composition of oxygen 1.0 cm from the center of the rod.
- **3.3** A stream of air at 200 KPa pressure and 300 K is flowing on the top surface of a thin flat sheet of solid naphthalene of length 0.2 m with a velocity of 20 m/sec. The other data are:

Mass diffusivity of naphthalene vapor in air =  $6 \times 10^{-6} \text{ m}^2/\text{sec}$ 

Kinematic viscosity of air =  $1.5 \times 10^{-5}$  m<sup>2</sup>.s<sup>-1</sup>

Concentration of naphthalene at the air-solid naphthalene interface =1 x  $10^{-5}$  kmol/m<sup>3</sup>

Calculate:

- (a) the average mass transfer coefficient over the flat plate
- (b) the rate of loss of naphthalene from the surface per unit width

Note: For heat transfer over a flat plate, convective heat transfer coefficient for laminar flow can be calculated by the equation.

$$Nu = 0.664 \text{ Re}_L^{1/2} \text{ Pr}^{1/3}$$

you may use analogy between mass and heat transfer

**3.4** A solid disc of benzoic acid 3 cm in diameter is spin at 20 rpm and 25°C. Calculate the rate of dissolution in a large volume of water. Diffusivity of benzoic acid in water is  $1.0 \times 10^{-5}$  cm<sup>2</sup>/sec, and solubility is 0.003 g/cc. The following mass transfer correlation is applicable:

Sh = 0.62 Re 
$$^{1/2}$$
 Sc  $^{1/3}$ 

Where Re=  $\frac{D^2\omega\rho}{\mu}$  and  $\omega$  is the angular speed in radians/time.

- 3.5 Air flows over a solid slab of frozen carbon dioxide (dry ice) with an exposed cross-sectional surface area of 1 x  $10^{-3}$  m<sup>2</sup>. The carbon dioxide sublimes into the 2 m/s flowing stream at a total release rate of 2.29 x  $10^{-4}$  mol/s. The air is at 293 K and 1.013 x  $10^{5}$  Pa pressure. At that temperature, the diffusivity of carbon dioxide in air is  $1.5 \times 10^{-5}$  m<sup>2</sup>/s and the kinematic viscosity of the air is  $1.55 \times 10^{-5}$  m<sup>2</sup>/s. Find the mass transfer coefficient.
- **3.6** Dittus and Boelter proposed the following equation for correlating the heattransfer coefficient for turbulent flow in a pipe

$$Nu = \frac{hD}{k} = 0.023 \, Re^{0.8} Pr^{1/3}$$

What should be the corresponding equation for the mass-transfer coefficient when the transfer is to a turbulent fluid flowing in a pipe?

- **3.7** Determine the Schmidt number for
- (a) oxygen in air at 300 K and 1.0 atm; and
- (b) oxygen in liquid water at 300 K.

At 300 K, the diffusion coefficient of oxygen in liquid water is  $1.5 \times 10^{-9} \text{ m}^2/\text{s}$ .

**3.8** For flow of a fluid at right angles to a circular cylinder, the average heat transfer coefficient (averaged around the periphery) for fluid Reynolds numbers in the range 1 to 4000 is given by

$$Nu_{av} = 0.43 + 0.532 Re^{0.5} Pr^{0.31}$$

where Nu and Re are computed with the cylinder diameter and fluid properties are taken at the mean of the cylinder and bulk-fluid temperatures.

Estimate the rate of sublimation of a cylinder of uranium hexafluoride, UF<sub>6</sub>, 6 mm diameter, exposed to an airstream flowing at a velocity of 3 m/s. The surface temperature of the solid is  $43^{\circ}$ C, at which temperature the vapor pressure of UF<sub>6</sub> is 400 mmHg (53.32 kN/m<sup>2</sup>). The bulk air is at 1 std atm pressure,  $60^{\circ}$  C.