# Heat, Mass and Momentum Transfer Analogy

Introduction to Heat and Mass Transfer
Incropera and Dewitt

Continuity equation and x-momentum equation (B.L.)

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(I)}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=-\frac{1}{\rho}\frac{\partial p}{\partial x}+\gamma\frac{\partial^2 u}{\partial y^2} \quad \ldots\ldots \text{(II)} \qquad \frac{\mu}{\rho}$$

Energy equation:

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\frac{\partial^2 T}{\partial y^2}+\cancel{\dot{q}}+\cancel{\mu\phi} \quad \ldots\ldots \text{(III)} \qquad \frac{\kappa}{\rho C_P}$$

Concentration BL:

$$\underbrace{u\frac{\partial C_A}{\partial x}+v\frac{\partial C_A}{\partial y}}_{\text{Advection}}=\underbrace{D_{AB}\frac{\partial^2 C_A}{\partial y^2}+\cancel{\dot{R}_A}}_{\text{Diffusion}} \quad \ldots\ldots \text{(IV)}$$

# THE CONVECTION TRANSFER EQUATIONS

2-D Steady flow

$$\frac{v_x}{U} \quad \frac{T_s - T}{T_s - T_\infty} \quad \frac{C_{AS} - C_A}{C_{AS} - C_{A\infty}}$$

Approximations and special considerations:

Incompressible, Constant properties, Negligible body forces, non-reacting $\dot{N}_A = 0$, No energy generation $\dot{q} = 0$ , negligible viscous dissipation $\phi = 0$

B.L. approximations:

$$\left.\begin{array}{l} u \gg v \\ \dfrac{\partial u}{\partial y} \gg \dfrac{\partial u}{\partial x}, \dfrac{\partial v}{\partial x}, \dfrac{\partial v}{\partial y} \end{array}\right\} \text{Velocity B.L.}$$

$\dfrac{\partial T}{\partial y} \gg \dfrac{\partial T}{\partial x}$ ……. Thermal B.L.

$\dfrac{\partial C_A}{\partial y} \gg \dfrac{\partial C_A}{\partial x}$……. Conc. B.L.

# BOUNDARY LAYER SIMILARITY – THE NORMALIZED CONVECTION TRANSFER EQUATIONS

## Non-dimensionalizing parameters

$$x^* = \frac{x}{L} \qquad y^* = \frac{y}{L} \qquad u^* = \frac{u}{V} \qquad v^* = \frac{v}{V} \qquad T^* = \frac{T - T_s}{T_\infty - T_s} \qquad C_A{}^* = \frac{C_A - C_{AS}}{C_{A\infty} - C_{AS}}$$

| Conservation equations | Boundary conditions: Wall | Boundary conditions: Free stream | Similarity parameter |
|---|---|---|---|
| $u^*\frac{\partial u^*}{\partial x^*}+v^*\frac{\partial u^*}{\partial y^*}=-\frac{dp^*}{dx^*}+\underbrace{\frac{\nu}{VL}}_{\frac{1}{\mathrm{Re}}}\frac{\partial^2 u^*}{\partial y^{*2}}$ | $u^*(x^*,0)=0$, $v^*(x^*,0)=0$ | $u^*(x^*,\infty)=\frac{u_\infty(x^*)}{V}$ | $\mathrm{Re}_L$ |
| $u^*\frac{\partial T^*}{\partial x^*}+v^*\frac{\partial T^*}{\partial y^*}=\underbrace{\frac{\alpha}{VL}}_{\frac{1}{\mathrm{Re.Pr}}}\frac{\partial^2 T^*}{\partial y^{*2}}$ | $T^*(x^*,0)=0$ | $T^*(x^*,\infty)=1$ | $\mathrm{Re}_L,\mathrm{Pr}$ |
| $u^*\frac{\partial C_A{}^*}{\partial x^*}+v^*\frac{\partial C_A{}^*}{\partial y^*}=\underbrace{\frac{D_{AB}}{VL}}_{\frac{1}{\mathrm{Re}.Sc}}\frac{\partial^2 C_A{}^*}{\partial y^{*2}}$ | $C_A{}^*(x^*,0)=0$ | $C_A{}^*(x^*,\infty)=1$ | $\mathrm{Re}_L, Sc$ |

# FUNCTIONAL FORM OF THE SOLUTIONS

$$u^* = f_1\left(x^*, y^*, \mathrm{Re}_L, \frac{dp^*}{dx^*}\right)$$

$$x^* = \frac{x}{L} \qquad u^* = \frac{u}{V}$$

$$y^* = \frac{y}{L} \qquad v^* = \frac{v}{V}$$

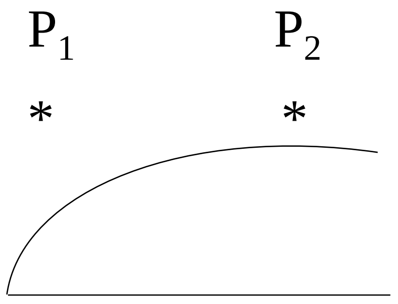
Shear stress at the surf. $y^* = 0$

$$\tau_s = \mu \frac{\partial u}{\partial y}\bigg]_{y=0} = \frac{\mu V}{L}\frac{\partial u^*}{\partial y^*}\bigg]_{y^*=0}$$

$$C_f = \frac{\tau_s}{\frac{1}{2}\rho V^2} = \frac{2}{\mathrm{Re}_L}\frac{\partial u^*}{\partial y^*}\bigg]_{y^*=0}$$

$$C_f \frac{\mathrm{Re}_L}{2} = f_2\left(x^*, \mathrm{Re}_L\right)$$

For a prescribed geometry




$$P \neq P(y)$$

$$\frac{P_2 - P_1}{\Delta x}$$

$$T^* = f_3\left(x^*, y^*, \mathrm{Re}_L, \mathrm{Pr}, \frac{dp^*}{dx^*}\right)$$

$$q_s = -k_f \left.\frac{\partial T}{\partial y}\right]_{y=0} \Rightarrow h = \frac{-k_f \left.\frac{\partial T}{\partial y}\right]_{y=0}}{T_S - T_\infty}$$

Convective heat transfer coefficient

$$h = -\frac{k_f}{L}\frac{\left(T_\infty - T_S\right)}{\left(T_S - T_\infty\right)}\left.\frac{\partial T^*}{\partial y^*}\right]_{y^*=0}$$

$$Nu = \frac{hL}{k_f} = \left.\frac{\partial T^*}{\partial y^*}\right]_{y^*=0}$$

$$Nu = f_4\left(x^*, \mathrm{Re}, \mathrm{Pr}\right)$$ For a prescribed geometry

Average Nu number $$\overline{Nu} = \frac{\bar{h}L}{\kappa_f} = f_5\left(\mathrm{Re}, \mathrm{Pr}\right)$$

$$C_A^{\ *} = f_6\left(x^*, y^*, \mathrm{Re}_L, Sc, \frac{dp^*}{dx^*}\right)$$

$$N_A^{''} = -D_{AB}\left.\frac{\partial C_A}{\partial y}\right]_{y=0} = h_m\left(C_{AS} - C_{A\infty}\right)$$ Convective mass transfer coefficient

$$h_m = \frac{-D_{AB}\left.\frac{\partial C_A}{\partial y}\right]_{y=0}}{C_{AS} - C_{A\infty}}$$

$$h_m = \frac{D_{AB}}{L}\left.\frac{\partial C_A^{\ *}}{\partial y^*}\right]_{y^*=0}$$

$$\frac{h_m}{L} = \text{Sherwood No} = \left.\frac{\partial C_A^{\ *}}{\partial y^*}\right]_{y^*=0}$$

$$Sh = f_7\left(x^*, \mathrm{Re}_L, Sc\right)$$ For a prescribed geometry

$$\overline{Sh} = \frac{\overline{h_m}L}{D_{AB}} = f_8\left(\mathrm{Re}_L, Sc\right)$$

| Conservation equations | Boundary conditions | | Similarity parameter |
|---|---|---|---|
| | Wall | Free stream | |
| $u^* \frac{\partial u^*}{\partial x^*} + v^* \frac{\partial u^*}{\partial y^*} = -\frac{dp^*}{dx^*} + \underbrace{\frac{\nu}{VL}}_{\frac{1}{\mathrm{Re}}} \frac{\partial^2 u^*}{\partial y^{*2}}$ | $u^*(x^*, 0) = 0$ <br> $v^*(x^*, 0) = 0$ | $u^*(x^*, \infty) = \frac{u_\infty(x^*)}{V}$ | $\mathrm{Re}_L$ |
| $u^* \frac{\partial T^*}{\partial x^*} + v^* \frac{\partial T^*}{\partial y^*} = \underbrace{\frac{\alpha}{VL}}_{\frac{1}{\mathrm{Re}.\mathrm{Pr}}} \frac{\partial^2 T^*}{\partial y^{*2}}$ | $T^*(x^*, 0) = 0$ | $T^*(x^*, \infty) = 1$ | $\mathrm{Re}_L, \mathrm{Pr}$ |
| $u^* \frac{\partial C_A^*}{\partial x^*} + v^* \frac{\partial C_A^*}{\partial y^*} = \underbrace{\frac{D_{AB}}{VL}}_{\frac{1}{\mathrm{Re}.Sc}} \frac{\partial^2 C_A^*}{\partial y^{*2}}$ | $C_A^*(x^*, 0) = 0$ | $C_A^*(x^*, \infty) = 1$ | $\mathrm{Re}_L, Sc$ |

# THE REYNOLD'S ANALOGY

$$\frac{dp^*}{dx^*} = 0$$

$Pr = Sc = 1 \Rightarrow$ Solutions of $u^*$ $T^*$ and $C_A^*$ must be equivalent.

$$f_1 = f_3 = f_6$$

Geometric similarity

Dynamic similarity

Same is true for $C_f$ ,$Nu$ and $Sh$

$$f_2 = f_4 = f_7$$

$$\boxed{C_f \frac{Re_L}{2} = Nu = Sh}$$

$\frac{Nu}{Re.Pr}$ - Stanton no for heat transfer

$\frac{Sh}{Re.Sc}$ - Stanton no for mass transfer

$$\frac{C_f}{2} = \frac{Nu}{Re_L.Pr} = \frac{Sh}{Re_L.Sc}$$

$\boxed{\frac{C_f}{2} = St = St_m}$ ……….Reynold's analogy

$$u^* = f_1\left(x^*, y^*, \mathrm{Re}_L, \frac{dp^*}{dx^*}\right)$$

$$T^* = f_3\left(x^*, y^*, \mathrm{Re}_L, \mathrm{Pr}, \frac{dp^*}{dx^*}\right)$$

$$C_A{}^* = f_6\left(x^*, y^*, \mathrm{Re}_L, Sc, \frac{dp^*}{dx^*}\right)$$

$$C_f \frac{\mathrm{Re}_L}{2} = f_2\left(x^*, \mathrm{Re}_L\right)$$

$$Nu = f_4\left(x^*, \mathrm{Re}, \mathrm{Pr}\right)$$ For a prescribed geometry

$$Sh = f_7\left(x^*, \mathrm{Re}_L, Sc\right)$$ For a prescribed geometry

## Modified Reynold's analogy

## Chilton Colburn analogy

$$\frac{C_f}{2} = St.\Pr^{\frac{2}{3}} = j_H \qquad 0.6 < \Pr < 60$$

$$\frac{C_f}{2} = St_m.Sc^{\frac{2}{3}} = j_m \qquad 0.6 < Sc < 300$$

$$\frac{C_f}{2} = St.\Pr^{\frac{2}{3}} = St_m.Sc^{\frac{2}{3}}$$

$$\boxed{\frac{C_f}{2} = j_H = j_m}$$

$j$ =Colburn '$j$' factor

# BOUNDARY LAYER ANALOGIES

$$\mathrm{Pr} = \frac{\nu}{\alpha} = \frac{\text{Momentum diffusivity}}{\text{Thermal diffusivity}}$$

For laminar B.L. $\dfrac{\delta}{\delta_t} = \mathrm{Pr}^n$

For gas: $\delta_t \approx \delta$ For liquid metal: $\delta_t \gg \delta$

For oil: $\delta_t \ll \delta$

$$Sc = \frac{\nu}{D_{AB}} = \frac{\text{Momentum diffusivity}}{\text{Mass diffusivity}} \qquad \frac{\delta}{\delta_c} = Sc^n$$

Lewis number Le = $\dfrac{\text{Thermal diffusivity}}{\text{Mass diffusivity}} = \dfrac{\alpha}{D_{AB}} = \dfrac{\nu}{D_{AB}} . \dfrac{\alpha}{\nu} = \dfrac{Sc}{\mathrm{Pr}}$

For simultaneous heat and mass transfer: $\dfrac{\delta_t}{\delta_c} \approx Le^n$ For most applications: $\boxed{n = \frac{1}{3}}$

# Chilton Colburn analogy (Modified Reynold's analogy)

$$St = \frac{Nu}{\mathrm{Re}.\mathrm{Pr}}$$

$$0.6 < \mathrm{Pr} < 60$$

$$\frac{C_f}{2} = St.\mathrm{Pr}^{\frac{2}{3}} = j_H = St_m.Sc^{\frac{2}{3}} = j_m$$

$$St_m = \frac{Sh}{\mathrm{Re}.Sc}$$

$$0.6 < Sc < 300$$

| | Momentum | Heat | Mass |
|---|---|---|---|
| Laminar | $C_{f,x} = 0.664\,\mathrm{Re}_x^{-1/2}$<br>$\mathrm{Re}_x < 5\times10^5$ | $Nu_x = 0.332\,\mathrm{Re}_x^{1/2}\,\mathrm{Pr}^{1/3}$<br>$\overline{Nu}_L = 0.664\,\mathrm{Re}_L^{1/2}\,\mathrm{Pr}^{1/3}$<br>$\mathrm{Re}_x < 5\times10^5, 0.6 < \mathrm{Pr} < 50$ | $Sh_x = 0.332\,\mathrm{Re}_x^{1/2}\,Sc^{1/3}$<br>$\overline{Sh}_L = 0.664\,\mathrm{Re}_L^{1/2}\,Sc^{1/3}$<br>$\mathrm{Re}_x < 5\times10^5, 0.6 < Sc < 300$ |
| Mixed / Turbulent | $C_{f,L} = \frac{0.074}{\mathrm{Re}_L^{1/5}} - \frac{1742}{\mathrm{Re}_L}$<br>$5\times10^5 < \mathrm{Re}_L < 10^8$<br>$\mathrm{Re}_{x,c} = 5\times10^5$ | $\overline{Nu}_L = \left(0.037R^{4/5} - 871\right)\mathrm{Pr}^{1/3}$<br>$0.6 \le \mathrm{Pr} \le 50, \mathrm{Re}_{x,c} = 5\times10^5$<br>$5\times10^5 < \mathrm{Re}_L < 10^8$ | $\overline{Sh}_L = \left(0.037\,\mathrm{Re}^{4/5} - 871\right)Sc^{1/3}$<br>$0.6 \le Sc \le 300, \mathrm{Re}_{x,c} = 5\times10^5$<br>$5\times10^5 < \mathrm{Re}_L < 10^8$ |