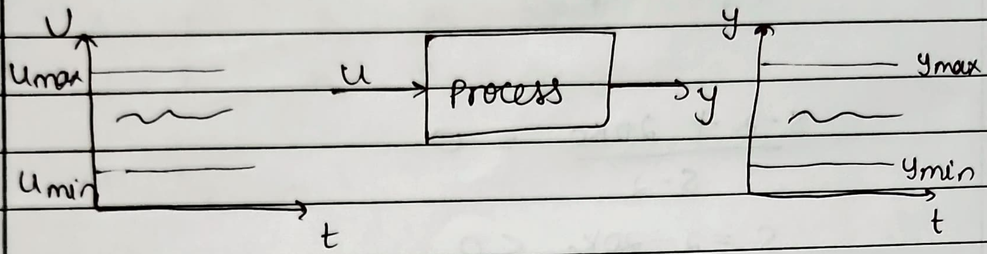
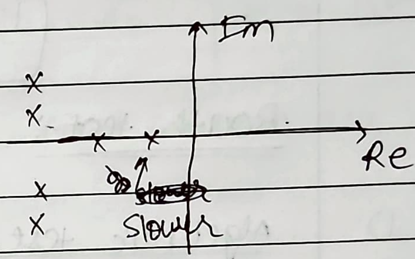
Stability

A dynamic system is said to be stable if for any change in bounded input it provides bounded output.



$G(S) = \frac{Q(S)}{P(S)}$ $\quad P(S) = 0$ {poles}



open-loop, $G_p = \frac{K_p}{\tau_p s + 1} = 0$, $s = \frac{-1}{\tau_p}$

closed Loop:

$$\bar{y} = \frac{G_c G_p G_f}{1 + G_c G_p G_f G_m} y_{sp} + \frac{G_d}{1 + G_c G_p G_f G_m} \bar{d}$$

$1 + G_p G_c G_f G_m = 0$ → characteristic equation

$s = ?$

Ex $\quad \bar{y} = \frac{5}{s-2} \bar{m} + \frac{10}{s-2} \bar{d}$ $\qquad G_m = G_f = 2$

For what value of k_c (P-controller) we can have stable close loop response?

$G_p = \frac{5}{s-2}$, $G_d = \frac{10}{s-2}$

$s - 2 = 0 \Rightarrow s = 2$, $s > 0$ unstable

_/_/_

using
P-only controller:

CE: $1 + G_p G_c G_f G_m = 0$

$$1 + \left(\frac{5}{s-2}\right)(2)\,K_c\,(2) = 0$$

$$\frac{s - 2 + 20K_c}{s-2} = 0$$

$$s = 2 - 20K_c < 0$$

$$\boxed{K_c > \frac{1}{10}}$$

Routh test:

i) Algebraic test
ii) how many roots (poles) are present in the right half plane, without knowing those poles
iii) Applicable to polynomial form of characteristic equation

$$G_p = \frac{K_p}{T_p s + 1} e^{-t_d s}$$

$$\frac{1 - \frac{t_d s}{2}}{1 + \frac{t_d s}{2}} \text{ —— approximation}$$

Steps:

① $1 + G_c G_p G_f G_m = 0$

$$a_0 s^n + a_1 s^{n-1} \text{ —— } a_n = 0$$

② a_0 should be +ve

③ a_0, a_1 —— a_n if ⓐ $a_1 < 0$ { unstable, no further analysis required }
↳ atleast one pole must be present in right plane.

ⓑ if all the cofficient are +ve { we can't predict stability } further analysis required.

All coffficient are +ve

Form Routh array

Row					
1	a_0	a_2	a_4	a_6	
2	a_1	a_3	a_5	a_7	----
3	A_1	A_2	A_3	A_4	
4	B_1	B_2	B_3	B_4	
⋮					
n+1	Z_1	--	----		

$A_1 = \dfrac{a_1 a_2 - a_0 a_3}{a_1}$ | $A_3 = \dfrac{a_1 a_6 - a_0 a_7}{a_1}$ | $B_1 = \dfrac{A_1 a_3 - a_1 A_2}{A_1}$

If all the elements in the first column are > 0 then the system is stable.

⑥ No. of poles in Right half plane = No. of sign changes

Example $s^3 + 2s^2 + (2+K_c)s + \dfrac{K_c}{\tau_i} = 0 \longrightarrow CE$, $K_c > 0$

$a_0 = 1,\ a_1 = 2,\ a_2 = 2 + K_c;\ a_3 = \dfrac{K_c}{\tau_i}$

Row				
1	1	$2+K_c$	0	$A_2 = 0$
2	2	$\dfrac{K_c}{\tau_i}$	0	$A_1 = \dfrac{\left(-\dfrac{K_c}{\tau_i} + 4 + 2K_c\right)}{2}$
3	A_1	A_2		$B_1 = \dfrac{A_1 \frac{K_c}{\tau_i} - 2A_2}{}$
4	B_1	B_2		$B_1 = \left(\dfrac{K_c}{\tau_i}\right) > 0$

$-\dfrac{K_c}{\tau_i} + 4 + 2K_c > 0$

~~$K_c\left(\frac{1}{\tau_i} - 2\right) > 4$~~

~~Assuming $\tau_i = 0.1$~~

$\boxed{0 < K_c < 0.5}$ stable

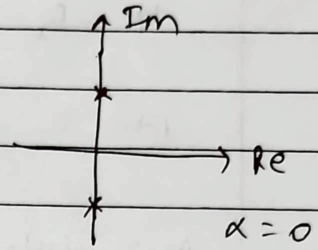
if $K_c > 0.5$ unstable

process with controller → closed loop
" without " → open "

$$\frac{4 + 2k_c - 10k_c}{2} = 0$$

$$k_c = 0.5$$

$k_c < 0.5$ All element are +ve (stable)
$k_c > 0.5$ unstable
$k_c = 0.5$ Marginally stable



$\alpha = 0$

$$2s^2 + \frac{k_c}{\tau_i} = 0$$

$$s = \pm i\sqrt{\frac{k_c}{2\tau_i}}$$

Root locus method
1) Graphical method
   ⓐ poles of CE
ii) How many poles → RHP

Ex $1 + G_c G_p G_m G_F = 0 \Rightarrow 1 + G_{OL} = 0$ (G_{OL} → $G_{open\ loop}$)

$G_p = \dfrac{0.25}{(s+1)(2s+1)}$ $\quad G_m = G_F = 2$ $\quad G_c = $ P-only $= k_c$

$$1 + \frac{k_c}{(s+1)(2s+1)} = 0 \quad ; \quad G_{OL} = \frac{k_c}{(s+1)(2s+1)}$$

poles: $-1, -\frac{1}{2}$

$$2s^2 + 3s + (1 + k_c) = 0$$

$$= \frac{-3 \pm \sqrt{9 - 8 - 8k_c}}{4}$$

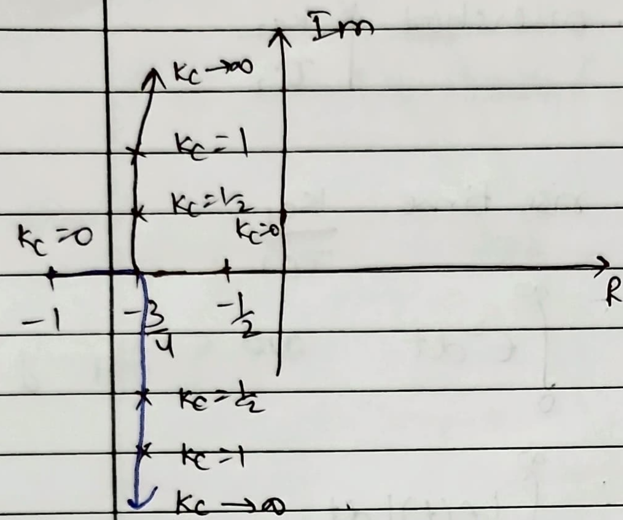
$r_1 = \dfrac{-3}{4} + \dfrac{\sqrt{1-8k_c}}{4}$, $r_2 = \dfrac{-3}{4} - \dfrac{\sqrt{1-8k_c}}{4}$

k_c : $0 \to \infty$

k_c	r_1	r_2
0	$-\frac{1}{2}$	-1

—/—/—

K_c	r_1	r_2
$1/16$	$-2.3/4$	$-3.7/4$
$1/8$	$-3/4$	$-3/4$
$\frac{1}{2}$	$\frac{-3}{4} + j\frac{\sqrt{3}}{4}$	$-\frac{3}{4} - j\frac{\sqrt{3}}{4}$
1	$\frac{-3}{4} + j\frac{\sqrt{7}}{4}$	$\frac{-3}{4} - j\frac{\sqrt{7}}{4}$



<u>Remarks:</u> ① stable

② No. of Root Loci (branches) = 2 = (No. of poles of G_{OL})

③ Root Loci originate from the poles of G_{OL}

If $K_c \uparrow$, poles moves away from Real axis. ∴ response becomes more oscillatory but never unstable

Q What will happen if we consider PI

Q This is tunning method or stability anylysis method?

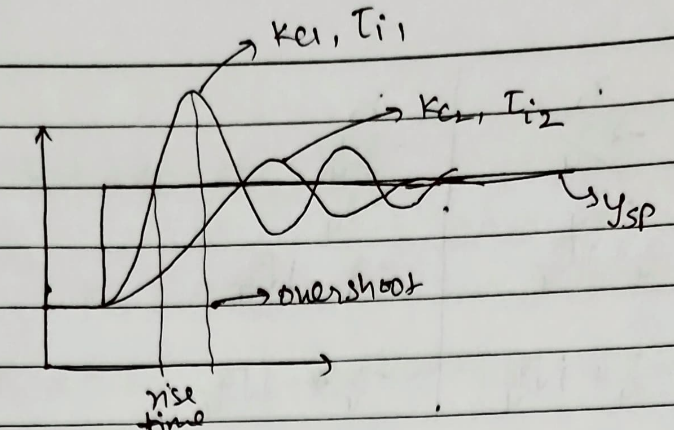
tunning → Directly you get tunning parameter value (K_c)

stability → you get a Range of K_c

Ans stability Analysis method

④ Routh test is also stability Analysis method.

<u>Time integral performance criterion:</u>



criterion 1 : min. overshoot | K_{c2}, τ_{i2}

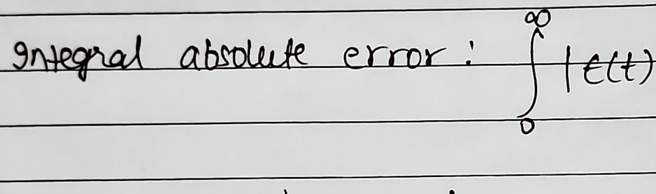
Criterion 2 : min rise time $\frac{K_{c1}}{\tau_{i1}}$

⇒ Integral square error : $\int_0^\infty \epsilon^2 dt$ and $\epsilon = y_{sp} - y$

⇒ Integral absolute error : $\int_0^\infty |\epsilon(t)| dt$

	ϵ	ϵ^2	Criterion	
2	large	larger	ISE	T, P, h
0.1	small	smaller	IAE	composition

Integral of time weighted Absolute error : $\int_0^\infty t|\epsilon(t)|dt$



Ex: $G_p = \frac{2}{s+1}$, $G_d = \frac{1}{s+1}$, $G_c = PI$, $G_f = G_m = 1$

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① closed loop transfer function:

$$\bar{y} = \frac{G_c G_p G_f}{1 + G_c G_p G_f G_m} \bar{y}_{sp} + \frac{G_d}{1 + G_c G_p G_f G_m} \bar{d}$$

$$\bar{y} = \frac{\left(\frac{2}{s+1}\right) K_c \left(1 + \frac{1}{\tau_i s}\right)(1)}{1 + \left(\frac{K_c + 1}{\tau_i s}\right)\left(\frac{2}{s+1}\right)} \bar{y}_{sp} + \frac{\left(\frac{1}{s+1}\right)}{1 + \left(\frac{K_c + 1}{\tau_i s}\right)\left(\frac{1}{s+1}\right)} \bar{d}$$

$$\bar{y} = \frac{2K_c}{\tau_i s^2 + \tau_i s + 2K_c} \bar{y}_{sp} + \frac{\tau_i s}{\tau_i s^2 + \tau_i s + K_c} \bar{d}$$

$$= \frac{1}{\frac{\tau_i s^2}{2K_c} + \frac{\tau_i s}{2K_c} + 1} \bar{y}_{sp} + \frac{\frac{\tau_i s}{K_c}}{\frac{\tau_i s^2}{K_c} + \frac{\tau_i s}{K_c} + 1} \bar{d}$$

$$\bar{y} = \frac{\frac{2}{s+1} \times K_c \left(1 + \frac{1}{\tau_i s}\right)}{1 + \frac{2}{s+1} K_c \left(1 + \frac{1}{\tau_i s}\right) + 1} + \frac{\frac{1}{s+1}}{1 + \frac{2}{s+1} K_c \left(1 + \frac{1}{\tau_i s}\right)}$$

$$= \frac{2K_c\left(1 + \frac{1}{\tau_i s}\right)}{s + 1 + 2K_c\left(1 + \frac{1}{\tau_i s}\right)} \bar{y}_{sp} + \frac{1}{s + 1 + 2K_c\left(1 + \frac{1}{\tau_i s}\right)} \bar{d}$$

$$= \frac{2K_c(\tau_i s + 1)}{\quad}$$

$$\bar{y} = \frac{\tau_i s + 1}{\tau^2 s^2 + 2\zeta \tau s + 1} \bar{y}_{sp} + \frac{(\tau_i / 2K_c) s}{\tau^2 s^2 + 2\zeta \tau s + 1} \bar{d}$$

$$\boxed{\tau = \sqrt{\frac{\tau_i}{2K_c}}} \qquad \boxed{\zeta = \frac{1}{2}\sqrt{\frac{\tau_i}{2K_c}}\,(1 + 2K_c)}$$

__/__/__

(2) <u>Servo case</u>:

$$\bar{y} = \frac{\tau_I s + 1}{\tau^2 s^2 + 2\zeta\tau s + 1} \bar{y}_{sp}$$

(3) $y_{sp} = \frac{1}{s}$

$$\bar{y} = \left(\frac{\tau_I s + 1}{\tau^2 s^2 + 2\zeta\tau s + 1}\right) \frac{1}{s}$$

(4) $y(t) = ?$

(5) $$ISE = \int_0^\infty [y_{sp} - y(t)]^2 dt$$

(6) $\frac{d(ISE)}{d\tau} = 0$ $\quad \tau^* \longrightarrow$ optimum value $\quad \tau^* = \sqrt{\frac{\tau_i}{2K_c}}$ — (I)

$\frac{d(ISE)}{d\zeta} = 0$ $\quad \zeta^* \longrightarrow$ optimum value $\quad \zeta^* = \frac{1}{2}\sqrt{\frac{\tau_i}{2K_c}}(1+2K_c)$

(II)

Using (I) and (II) find K_c and τ_i

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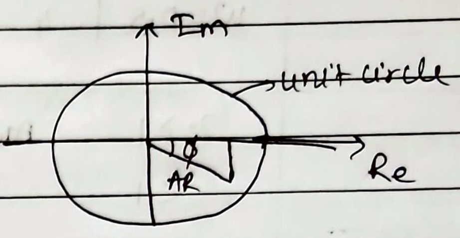
Nyquist / polar plot

$G(s) = G(j\omega) = Re + j\,Im$

$AR = \sqrt{Re^2 + Im^2}$

$\phi = \tan^{-1}\left(\frac{Im}{Re}\right)$



Ex $G(s) = \frac{K}{Ts+1}$ 1st order

$AR = \frac{K}{\sqrt{1+T^2\omega^2}}$

$\phi = -\tan^{-1}(T\omega)$

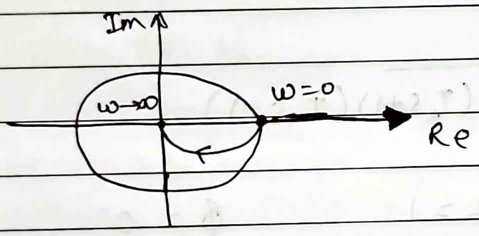
(i) $\omega = 0$ $AR = 1$ $\phi = 0$

(ii) $\omega \to \infty$ $AR = 0$ $\phi = -90^\circ$

$0 < \omega < \infty$

$1 > AR > 0$

$0 > \phi > -90^\circ$



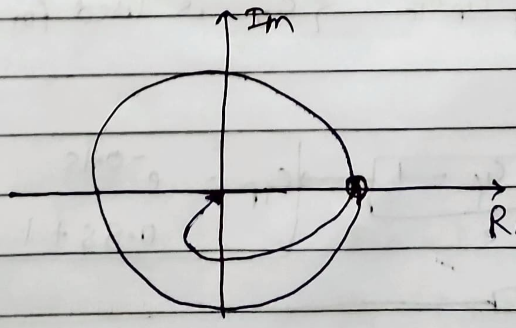
Ex $G(s) = \frac{K}{T^2s^2 + 2\zeta Ts + 1}$ $K = T = 1$

2nd order

$AR = \frac{K}{\sqrt{(1-T^2\omega^2)^2 + (2\zeta T\omega)^2}}$, $\phi = \tan^{-1}\left(\frac{-2\zeta T\omega}{1-T^2\omega^2}\right)$

i) $\omega = 0$ $AR = 1$ $\phi = 0$

ii) $\omega \to \infty$ $AR = 0$ $\phi = -180^\circ$

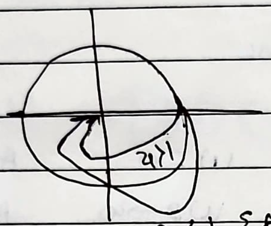


$\omega = 0.5$	$\zeta = 2$

$\omega = 0.5$	$\zeta = 2$	$AR = 0.47$
$\omega = 0.5$	$\zeta = 1$	$AR = 1.32$
$\omega = 0.5$	$\zeta = 0.8$	$AR =$

$\zeta > 1$ $AR \le 1$

< 1 > 1 few cases



$\zeta < 1$ {for some values of ω)

Ex: $G(s) = \dfrac{K}{(\tau_1 s+1)(\tau_2 s+1)(\tau_3 s+1)}$

(i) $\omega = 0$ $AR = 1$ $\phi = 0°$

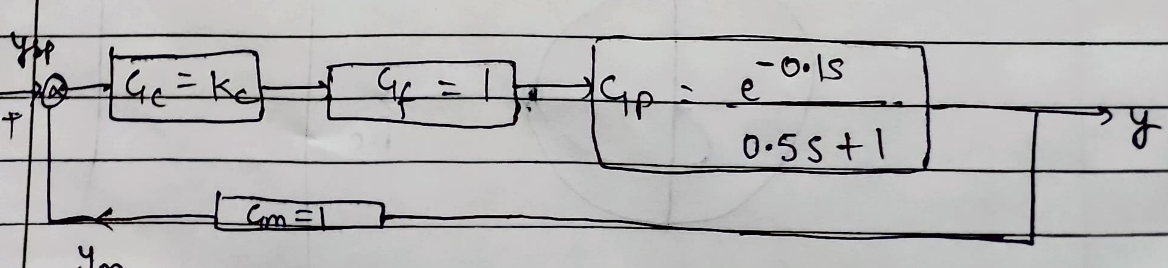
(ii) $\omega \to \infty$ $AR = 0$ $\phi = -270°$

we will use the frequency Analysis in:
1. stability
2. controller tunning

<u>Bode stability analysis</u>:

$G_{OL} = G_m G_f G_p G_c$ { G_{OL} is used for bode stability analysis }

Ex



$$G_{OL} = \frac{K_c e^{-0.1s}}{0.5s+1}$$

$$G_{OL} = \left(\frac{K_c}{0.5s+1}\right)\left(e^{-0.1s}\right)$$

$AR = AR_1 \, AR_2$

$\phi = \phi_1 + \phi_2$

$\phi_1 = -\tan^{-1}(0.5\omega)$

$\phi_2 = -0.1\omega \times \frac{180}{\pi}$ $\quad \underbrace{e^{-0.1(i\omega)}}_{AR\, e^{-0.1\omega j}}$

$$\phi = -\tan^{-1}(0.5\omega) - 0.1\omega \times \frac{180}{\pi}$$

\# The ω at which $\phi = -180$ is called as crossover frequency.

$$-180 = -\tan^{-1}(0.5\omega_{co}) - 0.1\,\omega_{co} \times \frac{180}{\pi}$$

$$\omega_{co} = 17 \frac{rad}{time}$$

$AR = AR_1 \, AR_2$

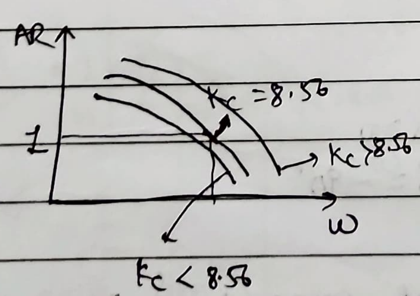
$AR_1 = \frac{K_c}{\sqrt{1+(0.5\omega)^2}} = \frac{K}{\sqrt{1+0.5^2 \times 17^2}}$

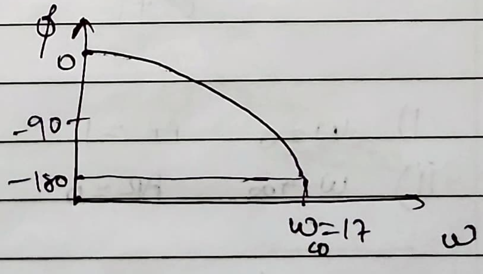
$AR_2 = 1$

$AR = 0.12\, K_c$

If $AR = 1$, $K_c = \cancel{8.33}\ 8.56$

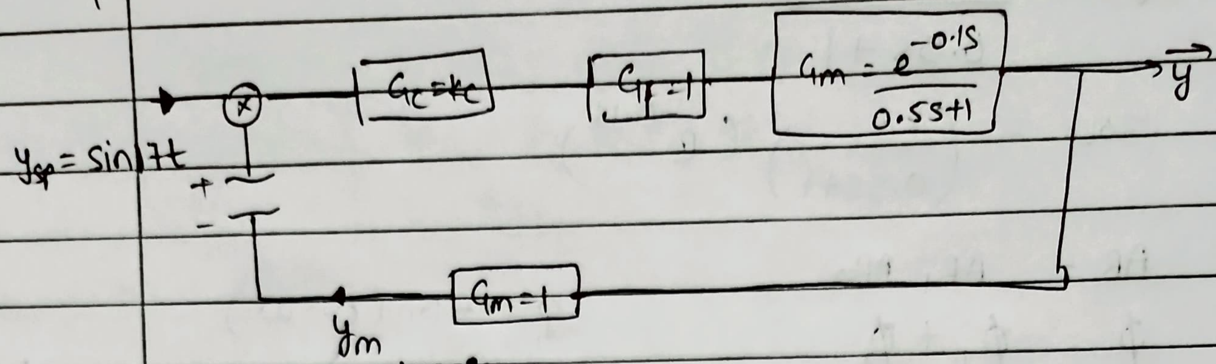
$AR > 1$	$K_c > 8.56$
$AR = 1$	$K_c = 8.56$
$AR < 1$	$K_c < 8.56$





\# considerations:

1) $AR = 1$, $\phi = -180°$ | ii) $A_{in} = 1$, $y_{sp} = A_{in}\sin\omega t$ | iii) open-loop



$y_m = -\sin 17t$

$A \sin \omega t \longrightarrow$ Process $\longrightarrow A_0 \sin(\omega t + \phi)$

$\sin 17t$ $\qquad$ $\sin(17t - \pi) \Rightarrow -\sin 17t$

(1) close the loop

$y_{sp} = 0$

AR	Input	Output	
1	$\sin 17t$	$-\sin 17t$	marginaly stable
1.2	$\sin 17t$	$-1.2 \sin 17t$	
[illegible]	$1.2 \sin 17t$	$-1.44 \sin 17t$	unstable

If feedback system is unstable, when the amplitude ratio of the corresponding open-loop transfer f^n is larger than 1 at cross over frequency

ex $G(s) = \frac{K_c}{\tau s + 1}$

i) $\omega = 0$ $\quad AR = 1$ $\quad \phi = 0$
ii) $\omega \to \infty$ $\quad AR = 0$ $\quad \phi = -90°$

$\phi = -180°$ is not present {Always stable}

If $\phi > -180°$ {system is always stable}

Ex $$G(s) = \frac{K_c}{\tau^2 s^2 + 2\zeta\tau s + 1}$$

i) $\omega = 0$ $\quad AR = 1 \quad \phi = 0$

ii) $\omega \to \infty$ $\quad AR = 0 \quad \phi = -180°$ $\quad$ { stable }

Ex $$G(s) = \frac{K_c e^{-t_d s}}{0.5s + 1}$$

$$\phi = \tan^{-1}(-0.5\omega) - (t_d)\,\omega \times \frac{180}{\pi}$$

$\omega = 0, \quad \phi = 0$

$\omega \to \infty, \quad \phi \to -\infty$

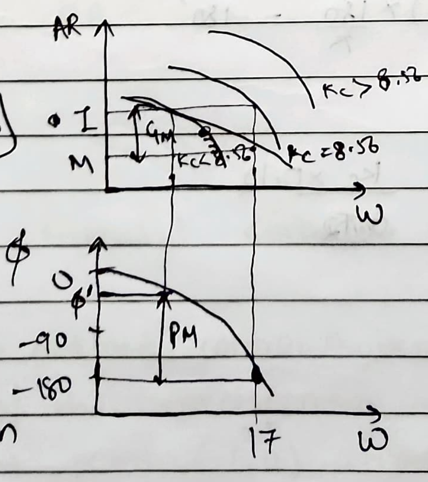
If No dead time : Instability starts from 3^{rd} order

If Dead time present: Instability starts from 1^{st} order

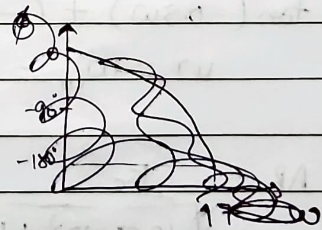
phase and gain margin:

$$G_c = K_c, \quad G_p = \frac{e^{-0.1s}}{0.5s+1}, \quad G_f = G_m = 1$$

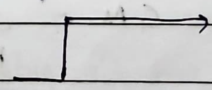
GM = Gain margin



PM = phase margin



Bode



$\phi > -180$ stable

$\phi = -180$ marginally

$\phi < -180$ unstable

$GM = \frac{1}{M}$ $\quad \log GM = \log(1) - \log$

$$\boxed{GM = \frac{1}{M}}$$

GM > 1 stable

Precisely known : GM : 1.4 - ~~1~~1.7

1.7 - 3.2

$$\boxed{\phi = -180^\circ \quad AR < 1 \quad GM > 1}$$

$$\boxed{\begin{array}{l}\phi' > -180^\circ, \quad AR = 1 \\ PM = 180 - |\phi'| > 30^\circ\end{array}}$$

Ex

$G_c = K_c \qquad G_p = \dfrac{e^{-0.1s}}{0.5s+1}$

$G_f = G_m = 1$

~~GM~~ GM = 1.7

$K_c = ?$

$$G_{OL} = \left(\frac{K_c}{0.5s+1}\right)\left(e^{-0.1s}\right)$$

$\phi = -180^\circ$

$\tan^{-1}(-0.5\omega) + (-0.1\omega) \times \dfrac{180}{\pi} = -180^\circ$

$\omega = \omega_{co} = 17$

$$AR = \frac{K_c}{\sqrt{(0.5 \times 17)^2 + 1}} = \frac{K_c \times 0.12}{\cancel{0.12}}$$

$$GM = \frac{1}{AR} = 1.7$$

$$\boxed{K_c = 4.9}$$

If ~~w~~ $t_{ds} = 0.15$

$$\phi = -180 = \tan^{-1}(-0.5\omega) + (-0.15\omega) \times \frac{180}{\pi}$$

$$\omega = \omega_{co} = 11.6$$

$$AR = \frac{K_c}{\sqrt{(0.5 \times 11.6)^2 + 1}} = 0.83 \quad \{K_c = 4.9\}$$

$$GM = \frac{1}{AR} > 1 \quad \{\text{stable}\}$$

Ex

$$G_c = K_c, \quad G_p = \frac{e^{-0.1s}}{0.5s+1}, \quad G_f = G_m = 1$$

$PM = 30°$

$K_c = ?$

$$PM = 180 - |\phi|$$

$$\phi = \pm 150° \Rightarrow -150°$$

$$G_{OL} = \left(\frac{K_c}{0.5s+1}\right)\left(e^{-0.1s}\right)$$

$$-150 = \tan^{-1}(-0.5\omega) + (-0.1\omega) \times \frac{180}{\pi}$$

$\omega = ✓$

$AR =$

Nyquist-stability criteriorn

A feedback control system is unstable if the Nyquist plot of the corresponding open loop transfer f^n encircles the point $(-1, 0)$. When ω takes a value in b/w $(-\infty, \infty)$.

Ex

$$G_{OL} = \frac{0.8\,K_c}{(5s+1)(10s+1)(15s+1)}$$

$$AR = AR_1\, AR_2\, AR_3$$

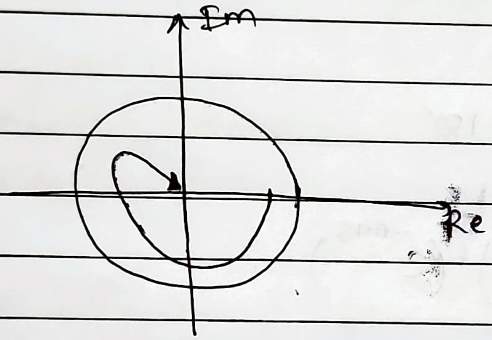
$$= \frac{(0.8K_c)}{\sqrt{1+(5\omega)^2}\sqrt{1+(10\omega)^2}\sqrt{1+(15\omega)^2}}$$

$$\phi = \tan^{-1}(-5\omega) + \tan^{-1}(-10\omega) + \tan^{-1}(-15\omega)$$

ω: $0 \rightarrow \infty$

$\omega = 0$, $AR = 0.8K_c$ $\quad \phi = 0$

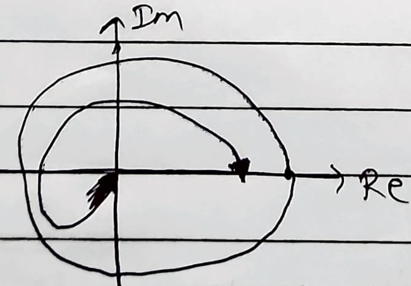
$\omega \rightarrow \infty$, $AR = 0$ $\quad \phi = -270$

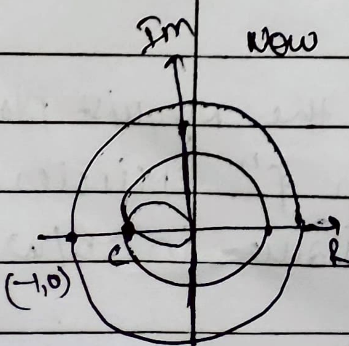


ω: $-\infty$ to 0

$\omega \rightarrow -\infty$, $AR = 0$, $\phi = 270$

$\omega = 0$, $AR = 0.8K_c$ $\quad \phi = 0$





Now $\omega(-\infty, \infty)$

$\omega = -\infty$ $\quad AR = 0$, $\quad \phi = 270$

$\omega = 0$ $\quad AR = 0.8K_c$, $\quad \phi = 0$

$\omega = \infty$ $\quad AR = 0$ $\quad \phi = -270$

$C \rightarrow$ Cross point

Remarks:

$K_c = 1$ stable { As the nyquist plot doesn't encirc $-1,0$)

$K_c = 2$ unstable

$K_c = 1.25$ Marginally stable

As
$AR = 0.8 \times 1.25$
$\Rightarrow 1$

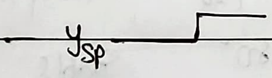
Ziegler - Nichols Tuning:

- 1942
- FRA
- online / closed-loop

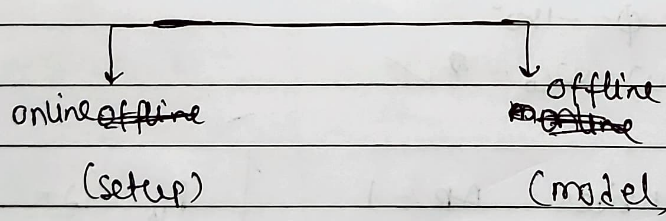
steps

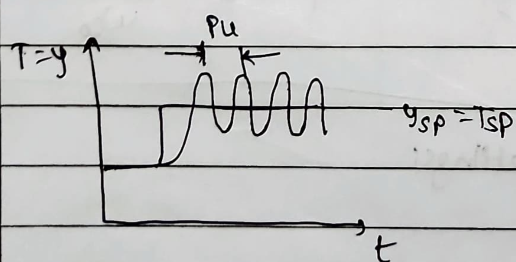
1. start up $\xrightarrow[\text{to}]{\text{bring}}$ SS

2. → P-only
   - close the loop



3.





(K_c) at which sustained osciliations are achieved is called ultimate gain

$K_u = (K_c)_{\sin}$ = ultimate gain

P_u = ultimate period

_/_/_

	K_c	τ_i	τ_D
P	$\frac{K_u}{2}$	-	-
PI	$K_u/2.2$	$P_u/1.2$	-
PID	$K_u/1.7$	$\frac{P_u}{2}$	$\frac{P_u}{8}$

For sustained osscilation $\boxed{AR = 1 \text{ and } \phi = -180^\circ}$

Bode :

$\phi = -180$ → $AR > 1$ unstable
→ $AR = 1$ M.S
→ $AR < 1$ stable

$AR = 1$ → $\phi < -180$ unstable
→ $\phi = -180^\circ$ Marginally stable
→ $\phi > -180^\circ$ Stable

consider $\phi = -180^\circ$
$\omega = \omega_{co} = \phi$ ✓

then consider $AR = 1$
$K_c = K_u =$ ✓

$P_u = \frac{2\pi}{\omega_{co}} = T$

Tyreus Luyben settings:

	K_c	τ_i	τ_D
PI	$K_u/3.2$	$2.2 P_u$	-
PID	$K_u/2.2$	$2.2 P_u$	$P_u/6.3$

__/__/__

Ex $G_{ol} = \dfrac{K_c\, e^{-0.15s}}{0.5s+1}$

$\phi = -180$

$\omega_{co} = 11.6$

$P_u = \dfrac{2\pi}{\omega_{co}} = 0.54$

$AR = 1 = \dfrac{K_c}{\sqrt{1+(11.6^2 \times 0.5^2)}}$

$1 = 0.17\,K_c$

$K_u = 5.88$

PID $K_c = \dfrac{K_u}{1.7}$, $\tau_i = \dfrac{P_u}{2} = 0.27$, $\tau_D = \dfrac{0.54}{8} = 0.0675$

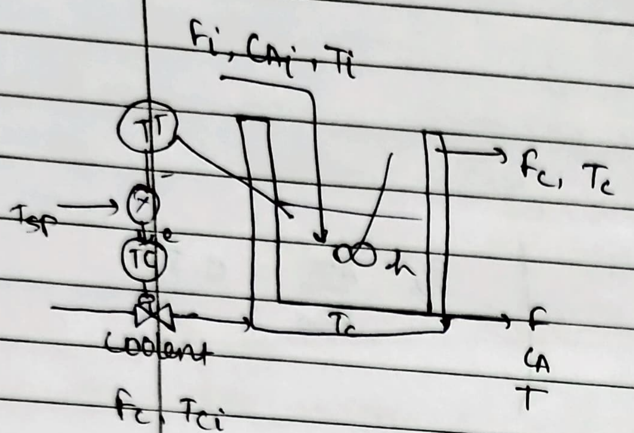
Note: we can't use ziegler-nichols for 1^{st} and 2^{nd} order

Multi-loop control:

No. of measurement	No. of Manipulated var.	controller
1	1	feed back controler (P, PI, PID)
>1	1	cascade, override, ratio
1	>1	split range

Cascade control — Dual-loop → primary / master; → secondary / slave.

Motivation:



$F_i = F =$ constant

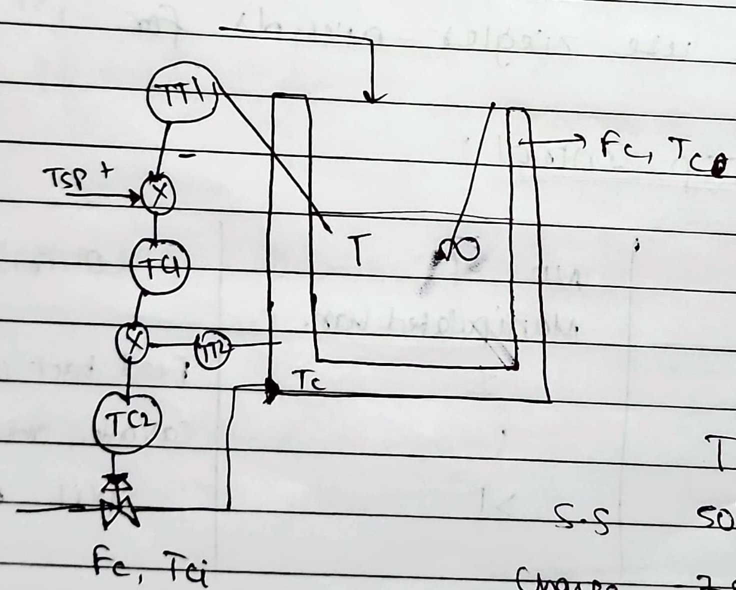
Control objective : $T = T_{sp}$

CV	MV	LV
T	F_c	T_i, T_{ci}

C_{Ai} = constant

→ T Responds much faster to changes in T_i than T_{ci}.

→ The simple FBC is more effective to reduce the effect of Distrbunce in T_i and less effective to reduce the effect in T_{ci}



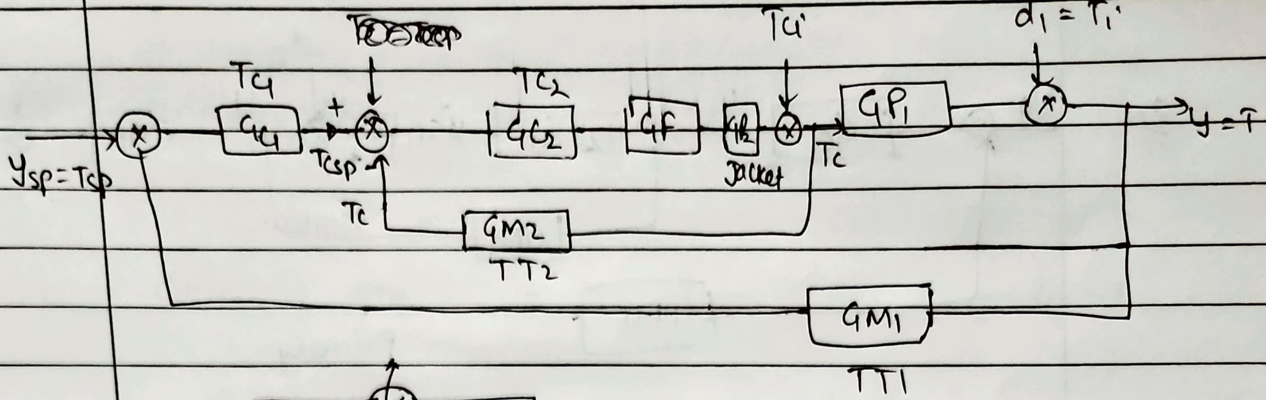
	T	T_c
S.S	50°c	20°c
Change	7.5°c	15°c = T_{csp}

T_{C1} → Primary controller

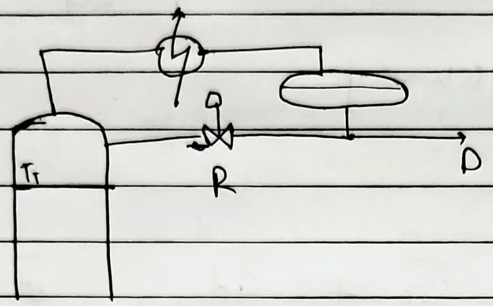
T_{C2} → secondary controller

	sensor	controller
process I (Reactor	TT1	TC1
Process II (JACKET)	TT2	TC2

_/_/__

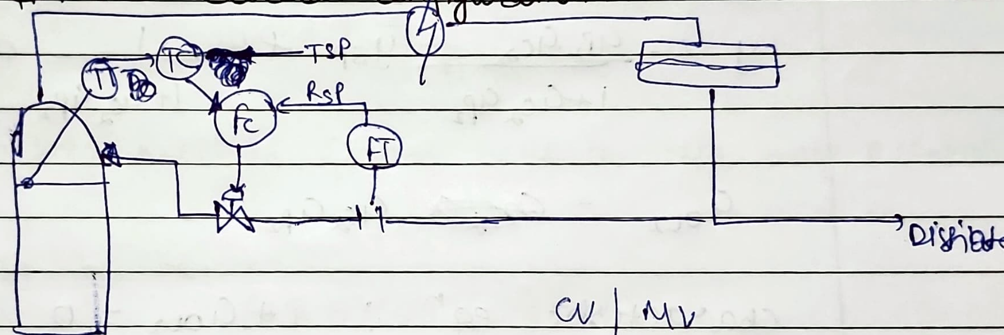


Q



CV	MV
T_T	R

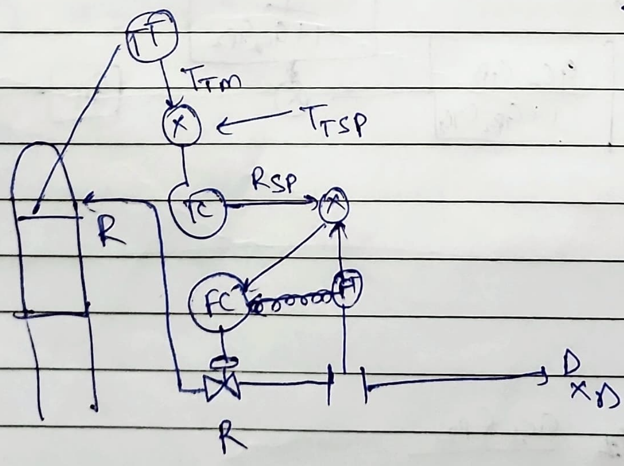
- make cascade configuration:



CV	MV
T_T	R

TC → primary controller

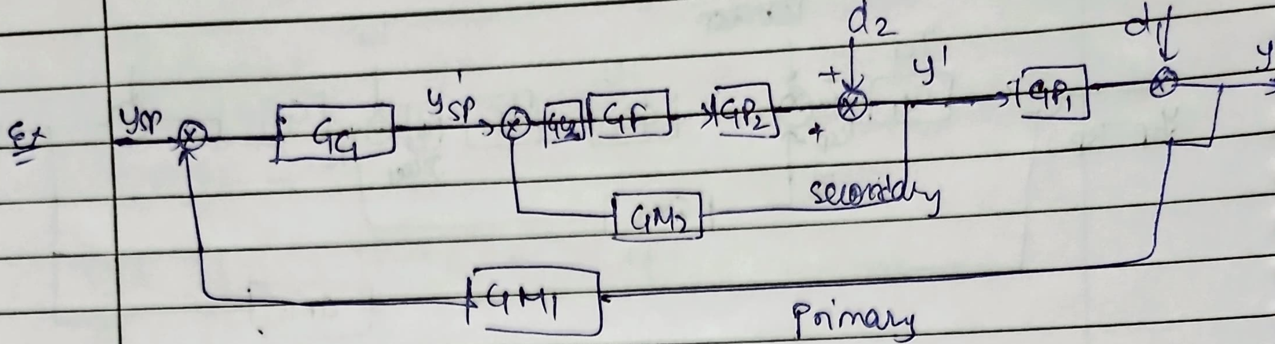
FC → secondary controller



$\left\{\begin{array}{l} \text{PI / PID} \\ \text{P / PI} \end{array}\right\}$ ← TC → Primary controller measure T

← FC → secondary controller measures flow (

Ex



$G_F = G_{M_2} = G_{M_1} = 1$ {assume}

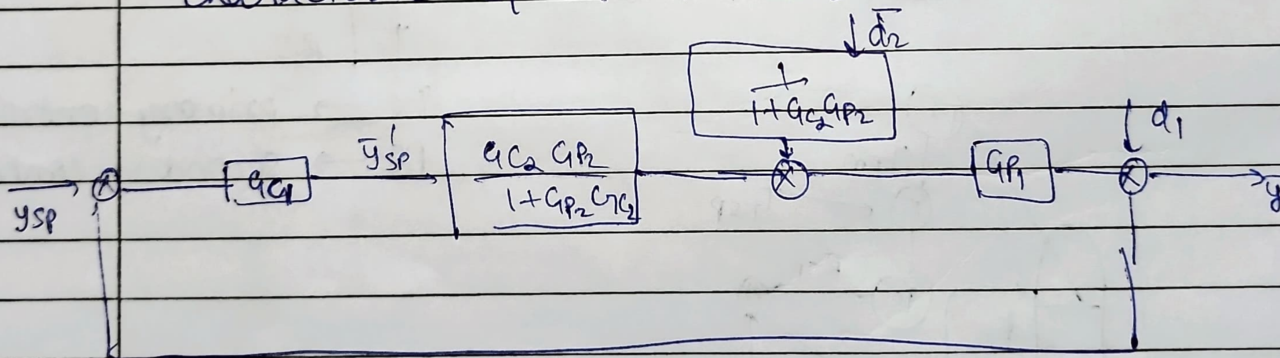
secondary loop:

$$y = \frac{G_p G_f G_c}{1 + G_p G_c G_f G_M}\,\overline{y_{sp}} + \frac{G_d}{1 + G_c G_p G_f G_m}\,\overline{d}$$

$$y = \frac{G_{P_2} G_{C_2}}{1 + G_{C_2} G_{P_2}}\,\overline{y}_{sp} + \frac{1}{1 + G_{C_2} G_{P_2}}\,\overline{d_2}$$

$$G_{OL_2} = G_{C_2} G_{P_2}$$

characteristic eq^n : $1 + G_{OL_2} = 0$



Primary loop

$$G_{OL_1} = G_{C_1} G_{P_1} \frac{G_{C_2} G_{P_2}}{1 + G_{C_2} G_{P_2}}$$

Remarks: order of G_{OL_1} > order of G_{OL_2}

primary loop (Temp) is slower secondary loop (Flow)

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Exam problem

primary loop : $G_{P_1} = \frac{1}{(0.5s+1)(s+1)}$ $G_{m_1} = 1$ PID

Secondary loop : $G_{P_2} = \frac{1}{1.5s+1}$ $G_{m_2} = G_f = 1$ P

$G_{OL_2} = G_{C_2} G_{P_2} = \frac{K_C}{1.5s+1}$; ~~w~~ $\boxed{\begin{array}{l} 0 < \omega < \infty \\ 0 > \phi > -90 \end{array}}$

$K_{C_2} = 5$ { consider }

$G_{OL_1} = \cancel{K_C} \frac{\frac{5}{1.5s+1}}{1 + \frac{5}{1.5s+1}}$ $G_{C_1} G_{P_1}$

Nichol's method { ~~we~~ have to take only P controller }

$G_{OL_1} = K_{C_1} \frac{(5/6)}{(0.5s+1)(s+1)(0.25s+1)}$

step ① $\phi = -180°$

step ② $\omega_{Co} = ✓$

③ ~~t ultimate period~~ $= \frac{2\pi}{\omega}$

④ consider AR = 1

$K_C = K_u = ✓$

⑤ Z-N tunning settings.

$K_C = ✓$ $\tau_I = ✓$

$\tau_D = ✓$

_/_/__

$$-180 = \tan^{-1}(-0.5\omega) + \tan^{-1}(-\omega) + \tan^{-1}(-0.25\omega)$$

$$-180 = \tan^{-1}\left(\frac{-0.5\omega - \omega}{1 - 0.5\omega^2}\right) + \tan^{-1}(-0.25\omega)$$

$$0.25\omega = \frac{-1.5\omega}{1 - 0.5\omega^2}$$

$$\omega_{co} = \sqrt{14} \Rightarrow 3.74$$

$$AR = \frac{5/6 \, K_c}{\sqrt{(1+0.25\omega^2)(1+\omega^2)(1+0.25\omega^2)}}$$

AR = 1

$$K_c = 11.25 \times \frac{6}{5} = 13.5$$

$$K_c = 13.5 = K_U$$

$$K_{c_1} = \frac{K_U}{1.7} = 7.94 \qquad P_u = \frac{2\pi}{\omega_{co}} = 1.67$$

$$\tau_i = \frac{P_u}{2} \qquad \tau_D = P_u/8$$

$$= 0.84 \qquad = 0.21$$

Override / constraint control:

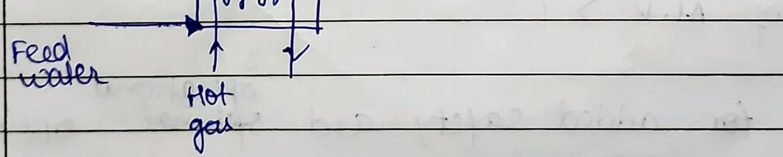
→ no. of measurement > 1
no. of MV $= 1$

No. of meast.	No MV	controller
>1	1	

During the normal operation of the plan or startup or during the shutdown condn some abnormal situation may arrive which may leads to distruction of the operationg personel and/or equipment. In such a condn, a special type of swich is used.

Eg ① HSS – High selector switch
prevents to exceeds the upper limit (constraint)
② LSS – Low selector switch
prevents to exceed the lower limit.

Boiler



control obj: $P = P_{sp}$, $h > h_{min}$

LSS is used because we need to maintain the minimum height.

LSS output (C) $= \min(C_1, C_2)$

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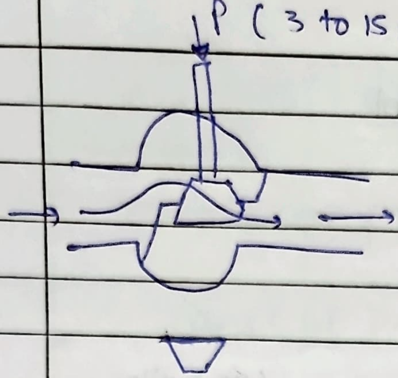


open to air value

h ⟶ $\boxed{LC}$ ⟶ C_2

If h↑ C_2↑ Direct action (value opening)

If P↑ C_1↑ Direct action

Initially consider LSS output (S) = min(C_1, C_2) = C_1

suddenly $h < h_L$ { LC ⟶ close the value to ↑ height }
$C_2 = 0$

Now LSS output(C) = C_2

So here level controler overrides the pressure controller

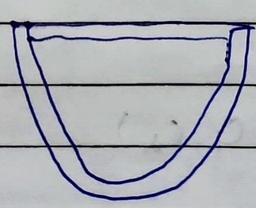
split range control

① No. of meas. = 1
No. of M.V > 1

② used for added safety and ~~optimal~~ operational optimality

③ It is not so common in chemical engineering.

Non-isothermal Batch Reactor:



Batch process is unsteady

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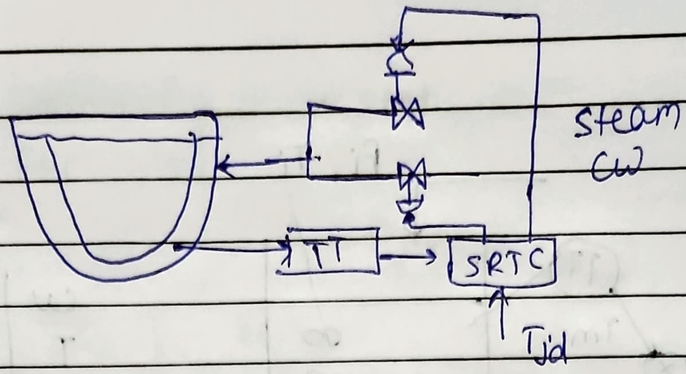
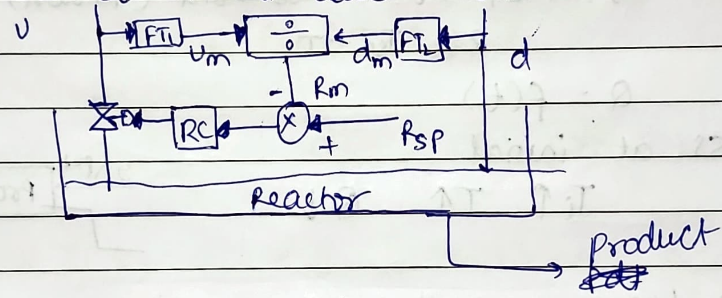


Ratio control scheme:

This controller is employed to maintain the ratio of two process streams. {typically flow rates

$R = \frac{u}{d}$ u → MV, d → Disturbance {Both are measured}

d is sometimes called wild stream



RC (Ratio controller) → P / PI / PID

sem exam

Ex (Reflux ratio in distillation)$_{min}$ = $\frac{L}{D} = \frac{R}{D}$

Examples
→ mixing of 2 streams of (colour)
→ maintain the pH of Reactor
→ fuel and air ratio of combustion channel.

Feedforward control:

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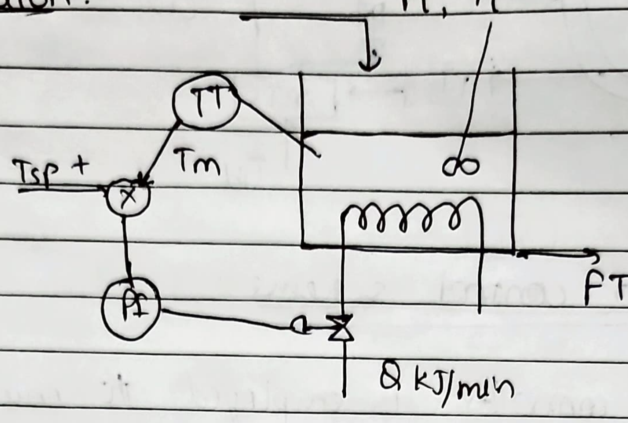
Motivation:



CV	MV
T	Q

Let us consider
Feedback controller (PI)

$$Q = Q_s + K_c e + \frac{K_c}{\tau_i}\int e\,dt$$

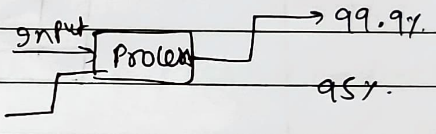
$Q_s, K_c, \tau_i \Rightarrow$ constant (known)

$e = T_{sp} - T$

$Q = f(t)$

SS at initial

$T_i \uparrow \quad T \uparrow \quad Q \downarrow$



→ feed back controller takes action after the effect of disturbance has been felt by the process.

→ perfect control can never be achieved by FBC.

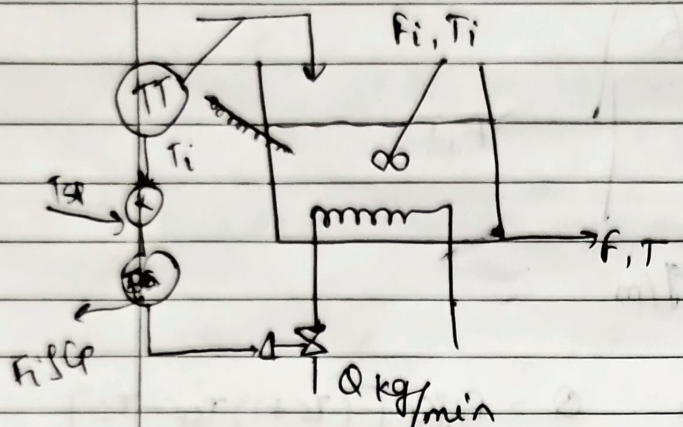
Perfect control:

→ Measure LV / disturbance } This is done in
→ control action } Feed forward controller

→ Feed back controller acts after the fact in a compensatory manner

→ FFC acts beforehand in a anticipitary manner

FFC → Static
FFC → Dynamic



$$\frac{d}{dt}(\rho V C_p T) = \rho F_i C_p T_i - \rho F C_p T + Q$$

$$\frac{dT}{dt} = \frac{F_i}{V}(T_i - T) + \frac{Q}{V\rho C_p}$$

Static FFC: $\frac{dT}{dt} = 0 \quad \therefore \quad \frac{F_i}{V}(T_i - T) + \frac{Q}{V\rho C_p} = 0$

$$Q = F_i \rho C_p (T_{sp} - T_i)$$

Dynamic model: $\frac{dT}{dt} = \frac{F_i}{V}(T_i - T) + \frac{Q}{V\rho C_p}$

$$s\,T(s) = -\frac{F_i}{V}(T(s)) + \frac{Q(s)}{V\rho C_p}$$

$$\frac{Q(s)}{V\rho C_p} = \left(\frac{F_i}{V} + s\right) T(s)$$

$$Q(s) = \left(\frac{F_i}{V} + s\right) \rho V C_p\, T(s)$$

$$\frac{Q(s)}{T(s)} =$$

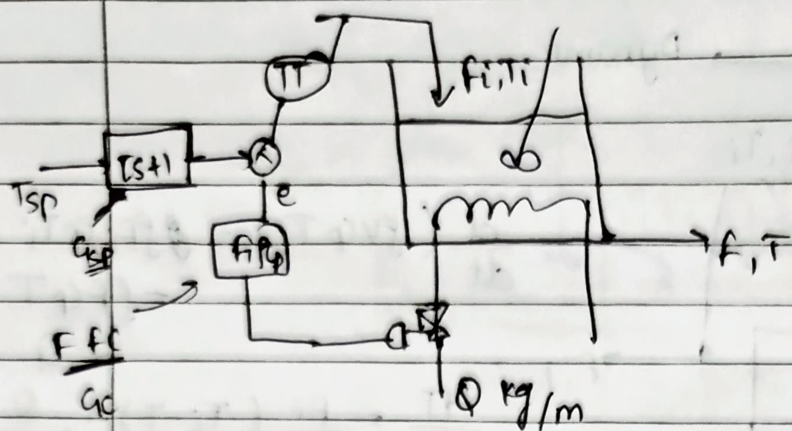
X

$$\tau \frac{dT'}{dt} + T' = T_i' + \frac{Q'}{F_i \rho C_p} \qquad \tau = \frac{V}{F_i}$$

$$\tau s(T(s)) + T(s) = T_i(s) + \frac{Q(s)}{F_i \rho C_p}$$

$$\bar{Q}'(s) = F_i \rho C_p \left[\bar{T}'(s)(\tau s + 1) - \bar{T}_i'(s) \right]$$

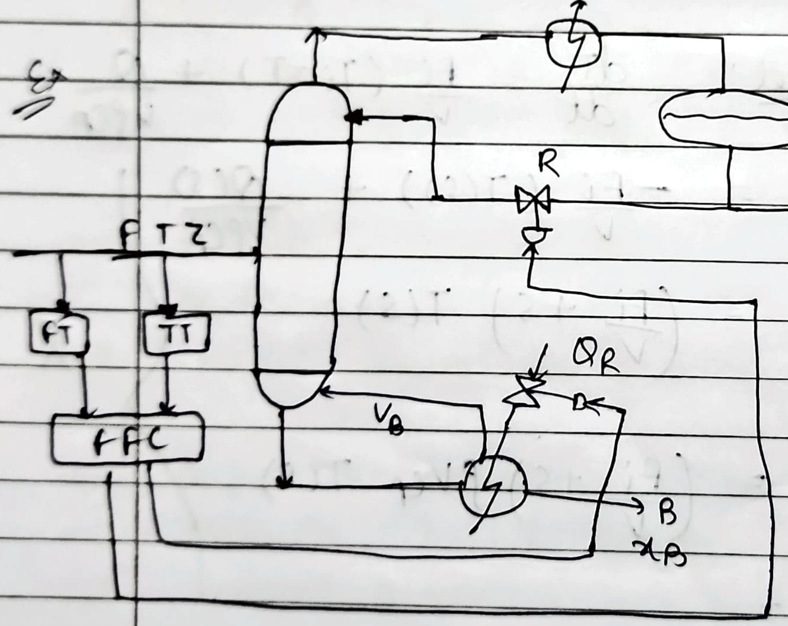
$$\bar{Q}'(s) = F_i \rho C_p \left[(\tau s+1) T_{sp}' - T' \right]$$



Energy balance : $Q = F_i \rho C_p \left[(\tau s+1) T_{sp} - T_i \right]$

General form of Manupulated variable : $m = G_c \left[G_{sp} y_{sp} - d \right]$

Ex



CV	MV
x_D	R
x_B	Q_B (or V_B)

Generalized FFC :

$$y_{sp} = G_p m + G_d d$$

$$\frac{y_{sp}}{G_d} = \frac{G_p}{G_d} m + d$$

$$\left(\frac{y_{sp}}{G_d} - d\right)\frac{G_d}{G_p} = m$$

$$m = \frac{G_d}{G_p}\left(\frac{y_{sp}}{G_d} - d\right)$$

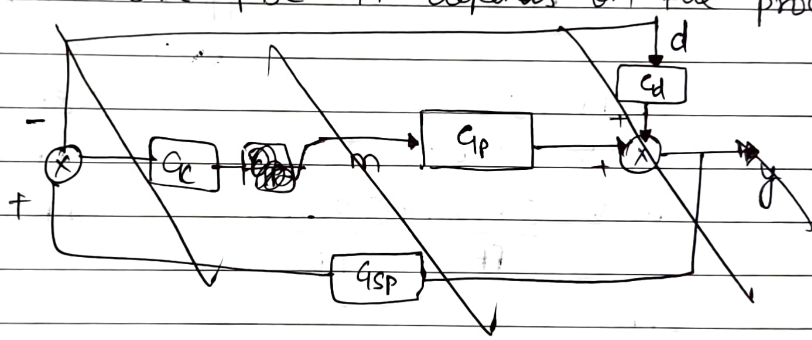
$$G_c = \frac{G_d}{G_p}, \quad G_{sp} = \frac{1}{G_d}$$

Remarks:

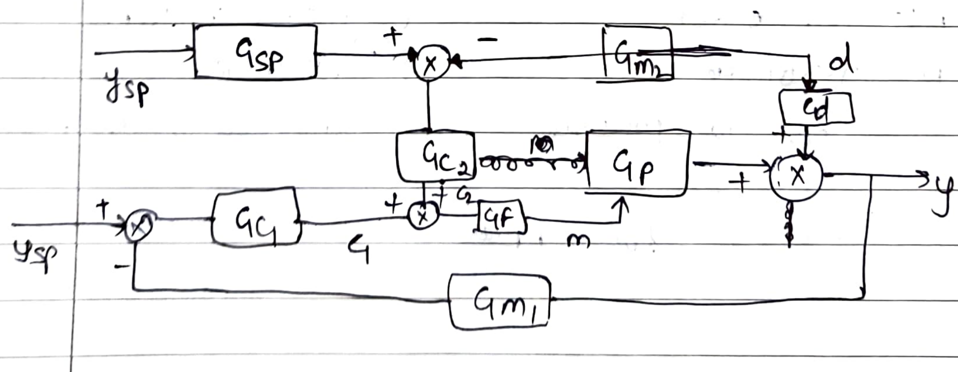
① $G_c = \dfrac{G_d}{G_p}$ —— FFC

$c = c_s + K_c \epsilon$ —— FBC

② It is not like FBC it depends on the process



③ Block diagram: (FF + FB) controller



$y = G_p m + G_d d$ —— open-loop

$m = G_f(c_1 + c_2)$

_/_/_

$$C_1 = G_{C_1}(y_{sp} - G_{m_1} y)$$

$$C_2 = G_{C_2}(G_{sp} y_{sp} - G_{m_2} d)$$

So, $m = G_f[G_{C_1}(y_{sp} - G_{m_1} y) + G_{C_2}(G_{sp} y_{sp} - G_{m_2} d)]$

$$y = \left[\frac{G_p G_f G_{C_1} + G_p G_f G_{C_2} G_{sp}}{1 + G_p G_f G_{C_1} G_{m_1}}\right] y_{sp} + \left[\frac{G_d - G_p G_f G_{C_2} G_{m_2}}{1 + G_p G_f G_{C_1} G_{m_1}}\right] d$$

CE: $1 + G_p G_f G_{C_1} G_{m_1} = 0$ — (FF + FB)

CE ⇒ same for FFC

So ∴ FFC doesn't affect stability.

Process		controller
1 LV	measured	FF
2 LV	1 meas. + 1 unmeas. FF / FB	FF + FB

FFC

Advantages	Disadvantages
1. Acts beforehand	1. Model required
2. No instability	2. model Parameter estimation
3. Good for slow processes	3. unmeasured dist. don't measure

⇒ vice versa for FBC