Heat Exchanger

(r) $A \propto \frac{1}{\Delta T_{LMTD}}$

$80 \longrightarrow 50$

$30 \longrightarrow 45$

$$\frac{A_{counter}}{A_{cocurrent}} \propto \frac{(\Delta T_{LMTD})_{cocurrent}}{(\Delta T_{LMTD})_{counter}}$$

$$= \frac{19.54}{26.8}$$

$$\therefore 1 - \frac{A_{counter}}{A_{cocurrent}} = \left[1 - \frac{(\Delta T_{LMTD})_{cocurrent}}{(\Delta T_{LMTD})_{counter}}\right] \times 100$$

$$= 0.271 \times 100$$

$$= 27.1\%$$

(2) $A = \dfrac{Q}{U_0 (\Delta T)_{LMTD}}$

$70 \longrightarrow 40$

$40 \longleftarrow 25$

$\Delta T_{LMTD} = \dfrac{30-15}{\ln(30/15)}$

$= 21.64$

$Q = \dot{m} c_P \Delta T$

$= \dfrac{1000}{3600} \times 2 \times 10^3 \times (70-40)$

$= 16.6 \text{ kW}$

$\therefore A = \dfrac{16.6 \times 10^3}{0.2 \times 10^3 \times 21.64}$

$A = 3.85 m^2$

(3) $(\dot{m}c_p)_{hot fluid} = (\dot{m}c_p)_{cold fluid}$

$\Rightarrow T_1 - T_2 = t_2 - t_1$

$90 - 40 = t_2 - 10$

$t_2 = 60c$

$90 \longrightarrow 40$

$t_2 \longrightarrow 10$

$90 \longrightarrow 40$

$60 \longleftrightarrow 10$

$\Delta T_{LMTD} = \frac{30+30}{2} = 30$

$U_i = \frac{Q}{A_i (\Delta T)_{LMTD}}$

$Q = (\dot{m}\,c_p)_{water} \Delta T = 1 \times 4.2 \times 10^3 \times 50 = 210\,kW$

$\therefore U_i = \frac{210 \times 10^3}{30 \times 3.14 \times 0.06 \times 5} = 7.427\,kW/m^2K$

1.

$k = 43$; $r_1 = 0.015$; $r_2 = 0.04$; $T_0 = 250°C$; $T_\infty = 35°C$; $h = 43$

$L = 0.025$

$L_c = 0.0255$

$r_{2c} = 0.0405$

$r_{2c}/r_1 = 2.7$

$$L_c^{3/2}(h/kA_m)^{1/2} = 0.825$$

$$\eta_f = 0.59$$

$q = (45)(2)\pi(0.0405^2 - 0.015^2)(250 - 35)(0.59) = 5.08\ W$

2.

The general solution of eq. 2–19 (b) is:

$\theta = T - T_\infty = c_1 e^{-mx} + c_2 e^{mx}$ (1)

boundary conditions:

(1) at $x = 0$ $\theta = \theta_1 = T_1 - T_\infty$

(2) at $x = L$ $\theta = \theta_2 = T_2 - T_\infty$

from:

(1) $\theta_1 = c_1 + c_2$ $c_1 = \theta_1 - c_2$

(2) $\theta_2 = c_1 e^{-mL} + c_2 e^{mL}$

$$\theta_2 = (\theta_1 - c_2)e^{-mL} + c_2 e^{mL} = \theta_1 e^{-mL} - c_2(e^{-mL} - e^{mL}$$

$$c_2 = \frac{\theta_2 - \theta_1 e^{-mL}}{e^{mL} - e^{-mL}} \quad (2)$$

$$c_1 = \theta_1 - c_2 = \theta_1 - \frac{\theta_2 - \theta_1 e^{-mL}}{e^{mL} - e^{-mL}}$$

$$c_1 = \frac{\theta_2 - \theta_1 e^{mL}}{e^{-mL} - e^{mL}} \quad (3)$$

Then eq. (1) becomes

$$\theta = \frac{\theta_2 - \theta_1 e^{mL}}{e^{-mL} - e^{mL}} e^{-mx} + \frac{\theta_2 - \theta_1 e^{-mL}}{e^{mL} - e^{-mL}} e^{mx}$$

$$\theta = \frac{e^{-mx}(\theta_2 - \theta_1 e^{mL}) + e^{mx}(\theta_1 e^{-mL} - \theta_2)}{e^{-mL} - e^{mL}}$$

Part heat lost by rod:

$$q = -kA\left.\frac{d\theta}{dx}\right|_{x=0} + kA\left.\frac{d\theta}{dx}\right|_{x=L}$$

$$\frac{d\theta}{dx} = m\left[\frac{-e^{-mx}(\theta_2 - \theta_1 e^{mL}) + e^{mx}(\theta_1 e^{-mL} - \theta_2)}{e^{-mL} - e^{mL}}\right]$$

$$q = \frac{kAm[-e^{-mL}(\theta_2 - \theta_1 e^{mL}) + e^{mL}(\theta_1 e^{-mL} - \theta_2)]}{e^{-mL} - e^{mL}}$$

$$+ \frac{kAm[-(\theta_2 - \theta_1 e^{mL}) + (\theta_1 e^{-mL} - \theta_2)]}{e^{-mL} - e^{mL}}$$

$$q = \frac{kAm[(\theta_2 - \theta_1 e^{mL})(1 - e^{-mL}) + (\theta_2 - \theta_1 e^{-mL})(1 - e^{mL})]}{e^{-mL} - e^{mL}}$$

3.

$$\frac{d^2\theta}{dx^2}-\frac{hP}{kA}\theta=0 \text{ let } m=\sqrt{\frac{hP}{kA}}$$

$T_\infty=38$ $\quad d=12.5$ mm $\quad L=30$ cm $\quad h=17$

$\theta=c_1e^{mx}+c_2e^{-mx}$ at $x=0$ $\quad \theta=200-38=162$ $\quad k=386$

$P=\pi d$ $\quad A=\frac{\pi d^2}{4}$ $\quad x=0.3$ $\quad \theta=93-38=55$

$$m=\left[\frac{(17)\pi(0.0125)(4)}{(386)\pi(0.0125)^2}\right]^{1/2}=3.754 \qquad 162=c_1+c_2$$

$55=3.084c_1+0.324c_2$ $\quad c_1=0.91$ $\quad c_2=161.09$

$\theta=0.91e^{mx}+161.09e^{-mx}$

$$q\int_0^L hP\theta\,dx=hP\frac{1}{m}[0.91e^{mx}-161.09e^{-mx}]_0^L=\sqrt{hPkA}[0.91e^{mx}-161.09e^{-mx}]_0^{0.3}$$

$$=[(17)\pi(0.0125)(386)\pi(0.0125)^2]^{1/2}\times[0.91e^{mx}-161.09e^{-mx}]_0^{0.3}$$

$$=122.7\text{ W}$$

4.

$k=204\ \frac{\text{W}}{\text{m}\cdot{}^\circ\text{C}}$ $\quad L_c=L+\frac{d}{4}=12+\frac{2}{4}=12.5$ $\quad T_0=250°\text{C}$ $\quad T_\infty=15°\text{C}$

$h=12\ \frac{\text{W}}{\text{m}^2\cdot{}^\circ\text{C}}$ $\quad A=\frac{\pi d^2}{4}$ $\quad P=\pi d$

$$m=\sqrt{\frac{hP}{kA}}=\left[\frac{(12)\pi(0.02)(4)}{(204)\pi(0.02)^2}\right]^{1/2}=3.43$$

$mL_c=(3.43)(0.125)=0.429$ $\qquad q=\sqrt{hPkA}\,\theta_0\tanh(mL_c)$

$$q=\left[(12)\pi(0.02)(204)\pi\frac{(0.02)^2}{4}\right]^{1/2}(250-15)\tanh(0.429)=20.89\text{ W}$$

5.

$$q=\sqrt{hPkA}\,\theta_0=\left[\frac{(20)\pi(0.0005)(372)\pi(0.0005)^2}{4}\right]^{1/2}(120-20)=0.152\text{ W}$$

6.

$r_1=1.0$ cm $\quad L=5$ mm $\quad t=2.5$ mm $\quad h=25$ $\quad T_0=260°\text{C}$

$T_\infty=93°\text{C}$ $\quad k=43$ $\quad L_c=5+1.25=6.25$ mm $\quad r_{2c}=1.625$ cm

$\frac{r_{2c}}{r_1}=1.625$ $\quad A_m=(0.0025)(0.00625)=1.56\times10^{-5}\text{ m}^2$

$$L_c^{3/2}\left(\frac{h}{kA_m}\right)^{1/2}=0.00625^{3/2}\left[\frac{25}{(43)(1.56\times10^{-5})}\right]^{1/2}=0.095 \qquad \eta_f=97\%$$

$$q=(0.97)(25)(2)\pi(0.01625^2-0.01^2)(260-93)=4.17\text{ W}$$

7.

$d = 1.5$ mm $\quad k = 19 \quad L = 12$ mm $\quad T_0 = 45°C \quad T_\infty = 20°C$

$h = 500$

Use insulated tip solution

$$L_c = L + \frac{d}{4} = 12 + 0.375 = 12.375 \text{ mm}$$

$$m = \left(\frac{hP}{kA}\right)^{1/2} = \left[\frac{(500)\pi(0.0015)}{(19)\pi(0.0015)^2(4)}\right]^{1/2} = 264.9$$

$$mL_c = (0.012375)(264.9) = 3.278$$

$$q = \sqrt{hPkA}\,\theta_0 \tanh(mL)$$

$$= \left[(500)\pi(0.0015)(19)\pi(0.0015)\left(\frac{2}{4}\right)\right]^{1/2}(45 - 20)\tanh(3.278) = 0.177 \text{ W}$$

1D heat transfer through fins

(8)

$D = 1m$

$K = 4\ W/mK$

$h = 10\ W/m^2K$

effectiveness = ?

$$\varepsilon = \sqrt{\frac{KP}{hA_c}}$$

$P = \pi D$

$A_c = \frac{\pi D^2}{4}$

$$= \sqrt{\frac{4 \times \cancel{3.14} \times 1 \times 4}{10 \times \cancel{3.14} \times 1 \times 1}}$$

$\varepsilon = 1.265$

effectiveness of fin = 1.265

(9) $D = 10mm$

$L = 300 mm$

$K = 500 W/mK$

$h = 250 W/m^2K$

effectiveness = ?

$$\mathcal{E} = \sqrt{\frac{KP}{hA_c}} \tanh mL$$

$$m = \sqrt{\frac{hP}{KA_c}} = \sqrt{\frac{h\pi D 4}{K\pi D^2}} = \sqrt{\frac{4h}{KD}} = \sqrt{\frac{4\times 250}{500\times 10\times 10^{-3}}} = 14.14$$

$$\therefore \tanh mL = \tanh(14.14\times 300\times 10^{-3}) = \tanh(4.242) = 0.999$$

$$\mathcal{E} = \sqrt{\frac{K\pi D 4}{h\pi D^2}} \tanh mL = \sqrt{\frac{4K}{hD}} \tanh mL = \sqrt{\frac{4\times 500}{250\times 10\times 10^{-3}}} \times 0.999$$

$$\mathcal{E} = 28.255$$

$$\text{effectiveness}(\varepsilon) = \frac{Q \text{ with fin}}{Q \text{ w/o fin}}$$

$$\text{efficiency}(\eta) = \frac{\text{Actual heat transfer through fin}}{\text{max possible heat transfer through fin}}$$

$$= \frac{Q_{actual}}{Q_{max}}$$

$$\therefore \; \varepsilon(Q_{w/o\ fin}) = Q \text{ with fin}$$

$$\therefore \; \varepsilon(Q_{w/o\ fin}) = Q_{max}(\eta)$$

$$\varepsilon = \eta \frac{Q_{max}}{Q_{fin}}$$

$$\varepsilon = \eta \frac{h A_s \theta_b}{h A_c \theta_b}$$

$$\boxed{\varepsilon = \eta \frac{A_{surface}}{A_{crosssectional}}}$$

$$\therefore \; = 0.5 \frac{\pi D L}{\pi D^2/4}$$

$$= 0.5 \frac{4L}{D}$$

$$= \frac{0.5 \times 4 \times 5}{1}$$

$$\varepsilon = \underline{\underline{10}}$$