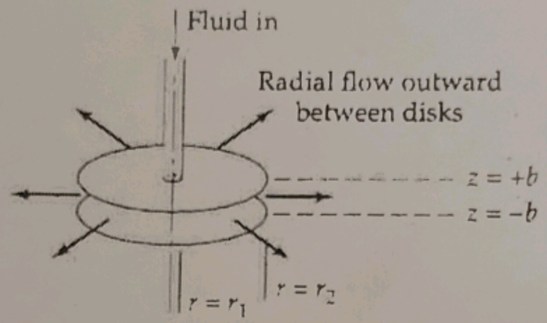
NR311 - (45)

Indian Institute of Technology Kharagpur
Department of Chemical Engineering
Mid Semester Examination, Autumn 24-25
Sub: **Advanced Fluid Dynamics**
Subject Number: CH61011

23 Sept 2024 AN
Time: 2 Hrs
FM: 30
No. of Students: 116

Instructions: Answer all questions. Closed book, closed notes examination. All symbols carry their usual meaning. Assume missing data suitably. Follow the five step problem solving methodology. Data required are included in the question paper.

1. (8 marks) Consider the system shown below where a fluid enters through a tube and flows radially through the gap between two disks.



(a) Simplify the equation of continuity and motion for this system assuming steady laminar incompressible Newtonian flow. Consider only the region $r_1 < r < r_2$ and flow that is radially directed. If you use any additional assumption, that has to be specified explicitly and JUSTIFIED. You MUST write down the equations on the answer script and connect cancellation of each term with appropriate assumption.

(b) Show that the EOM can be written as:

$$-\rho\frac{\phi^2}{r^3} = -\frac{dp}{dr} + \mu\frac{1}{r}\frac{d^2\phi}{dz^2}$$

where $\phi = rv_r$ is independent of z

$$\frac{\partial\rho}{\partial t} + \frac{1}{r}\frac{\partial}{\partial r}(r\rho u_r) + \frac{1}{r}\frac{\partial(\rho u_\theta)}{\partial\theta} + \frac{\partial(\rho u_z)}{\partial z} = 0$$

$$\rho\left(\frac{\partial v_r}{\partial t} + v_r\frac{\partial v_r}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_r}{\partial\theta} + v_z\frac{\partial v_r}{\partial z} - \frac{v_\theta^2}{r}\right) = -\frac{\partial p}{\partial r} + \mu\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(rv_r)\right) + \frac{1}{r^2}\frac{\partial^2 v_r}{\partial\theta^2} + \frac{\partial^2 v_r}{\partial z^2} - \frac{2}{r^2}\frac{\partial v_\theta}{\partial\theta}\right] + \rho g_r$$

$$\rho\left(\frac{\partial v_\theta}{\partial t} + v_r\frac{\partial v_\theta}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_\theta}{\partial\theta} + v_z\frac{\partial v_\theta}{\partial z} + \frac{v_r v_\theta}{r}\right) = -\frac{1}{r}\frac{\partial p}{\partial\theta} + \mu\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(rv_\theta)\right) + \frac{1}{r^2}\frac{\partial^2 v_\theta}{\partial\theta^2} + \frac{\partial^2 v_\theta}{\partial z^2} + \frac{2}{r^2}\frac{\partial v_r}{\partial\theta}\right] + \rho g_\theta$$

$$\rho\left(\frac{\partial v_z}{\partial t} + v_r\frac{\partial v_z}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_z}{\partial\theta} + v_z\frac{\partial v_z}{\partial z}\right) = -\frac{\partial p}{\partial z} + \mu\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial v_z}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 v_z}{\partial\theta^2} + \frac{\partial^2 v_z}{\partial z^2}\right] + \rho g_z$$

2. (4 marks) Using summation notation prove that:

$$[\mathbf{v}\times[\nabla\times\mathbf{v}]] = \frac{1}{2}\nabla(\mathbf{v}\cdot\mathbf{v}) - \mathbf{v}\cdot\nabla\mathbf{v}$$

You may find the following relations useful:

$$\sum_i \epsilon_{ijk}\epsilon_{mnk} = \delta_{im}\delta_{jn} - \delta_{in}\delta_{jm}$$

3. (3 marks) For each of the equations given below, identify/name the equation and mention the restrictions that applies to it:
   (a) $\frac{\partial}{\partial t}(\rho \mathbf{v}) = -\nabla \cdot \rho \mathbf{v}\mathbf{v} - \nabla p - \nabla \cdot \tau + \rho \mathbf{g}$
   (b) $\frac{D\rho}{Dt} = -\rho(\nabla \cdot \mathbf{v})$
   (c) $\rho \frac{D\mathbf{v}}{Dt} = -\nabla p - \nabla \cdot \tau + \rho \mathbf{g}$
   (d) $\rho \frac{D\mathbf{v}}{Dt} = -\nabla \mathscr{P} + \mu \nabla^2 \mathbf{v}$
   (e) $\rho \frac{D\mathbf{v}}{Dt} = -\nabla p + \rho \mathbf{g}$

4. In a paint industry, a new coating has to be developed which will sustain significant stresses applied on its surface during its use. The R&D engineering team has identified the new coating to be a 'complex fluid'. The force per unit area was measured on three mutually perpendicular test surfaces at a point P (which can be considered as the origin if one construct a coordinate system at the surface to solve the problem). The results of the measurements are shown below Based on the above, answer the following questions.

Direction of vector normal to the surface	Measured force/area (Pa)
$\hat{x}$	$\hat{x}$
$\hat{y}$	$3\hat{y} - \hat{x}$
$\hat{z}$	$-\hat{y} - 3\hat{x}$

   (a) (5 marks) What is the magnitude of the stress vector acting on surface whose normal is $\hat{n} = \frac{1}{\sqrt{2}}(\hat{x} + \hat{y})$
   (b) (10 marks) Determine the invariants and the magnitudes and directions of the principal stresses for the stress tensor.

End of Question Paper