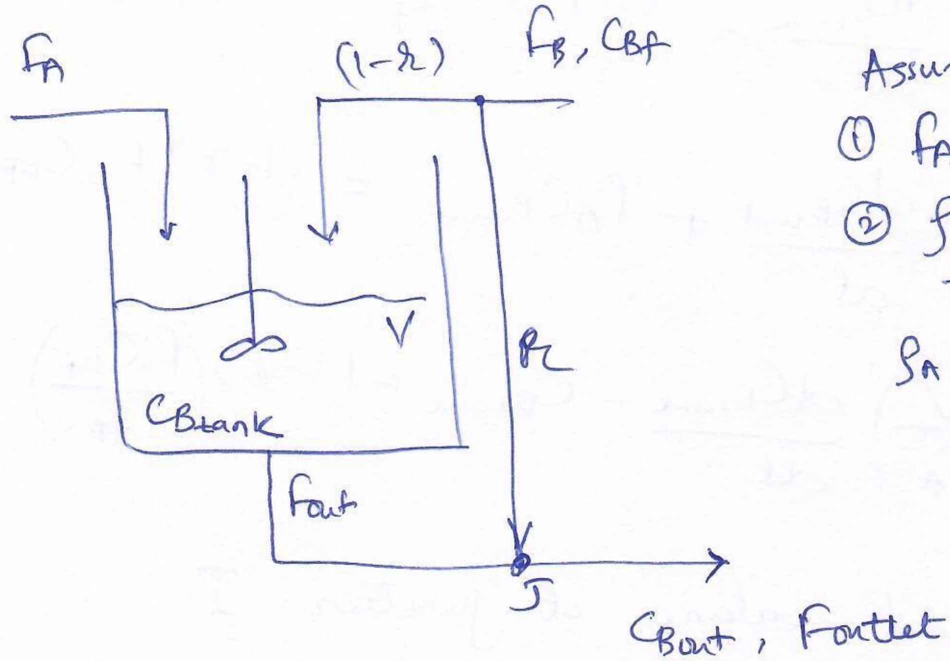
Week 8

Analysis of (p,q) order system.



Assume

① $f_A >>> f_B$

② ρ const. throughout the system

$\rho_A \approx \rho_B \approx \rho_{tank} \approx \rho_{out}$

overall mass balance over the tank:

$$\rho \frac{dV}{dt} = f_A \rho + (1-r)\rho f_B - F_{out}\rho$$

$f_A >> f_B$; V constt

$$\Rightarrow f_A = f_{out} \quad \text{——— ①}$$

overall mass balance @ junction J:

$$\rho \frac{dV_j}{dt} = f_{out}\rho + r f_B \rho - \rho f_{outlet}$$

$$f_{out} = f_A$$

$$\frac{dV_j}{dt} = 0$$

$$\Rightarrow F_A = f_{out} = f_{outlet} \quad \text{——— ②}$$

Component balance over the tank for B

$$V \frac{dC_{B\,tank}}{dt} = (1-\gamma) f_B C_{Bf} - f_A C_{B\,tank}$$

$$V \frac{dC_{B\,tank}}{dt} + f_A C_{B\,tank} = (1-\gamma) F_B C_{BF}$$

$$\left(\frac{V}{F_A}\right) \frac{dC_{B\,tank}}{dt} + C_{B\,tank} = (1-\gamma)\left(\frac{f_B C_{Bf}}{f_A}\right) \quad \text{—} ①$$

component balance at junction : J

$$f_A C_{B\,tank} + \gamma f_B C_{Bf} = f_A C_{B\,out}$$

$$C_{B\,out} = C_{B\,tank} + \gamma \left(\frac{f_B C_{Bf}}{f_A}\right) \quad \text{—} ②$$

Let $C_{B\,tank} - C_{B\,tank,ss} = x$

$C_{B\,out} - C_{B\,out\,ss} = y$

$$\frac{f_B C_{Bf}}{f_A} - \left(\frac{f_B C_{Bf}}{f_A}\right)_{ss} = u$$

eq^n ① can be rewritten as

$$\frac{V}{F_A}\frac{dx}{dt} + x = (1-\gamma) u \quad \text{—} ③$$

eq^n ② can be rewritten as

$$y = x + \gamma u \quad \text{—} ④$$

$$\left(\frac{V}{f_A}s+1\right)\bar{x}(s) = (1-r)\bar{u}(s)$$ taking laplace of eq^n 3

$$\frac{\bar{x}(s)}{\bar{u}(s)} = \frac{1-r}{\left(\frac{V}{f_A}\right)s+1} \quad \text{——— (5)}$$

$$\bar{y}(s) = \bar{x}(s) + r\,\bar{u}(s)$$

$$\bar{y}(s) = \frac{(1-r)\bar{u}(s)}{\left(\frac{V}{f_A}\right)s+1} + r\,\bar{u}(s)$$

$$\frac{\bar{y}(s)}{\bar{u}(s)} = \frac{\left(\frac{Vr}{f_A}\right)s+1}{\left(\frac{V}{f_A}\right)s+1} \quad \text{——— (6)}$$

Let $\frac{V}{f_A} = \tau$ $\quad \frac{Vr}{f_A} = \xi$

$$\frac{\bar{y}(s)}{\bar{u}(s)} = \frac{\xi s+1}{\tau s+1}$$

Comment on dynamics of system !!

$\frac{\bar{x}(s)}{\bar{u}(s)}$ is Ist order system ; $\frac{\bar{y}(s)}{\bar{u}(s)}$ is (1,1) order.

(p, q) order

p → denominator degree
q → numerator degree

$$g_1(s) = \frac{K}{\tau s+1} \Rightarrow y(t) = AK(1-e^{-t/\tau})$$

when ~~[illegible]~~ u(t) is step function of mag. A

(I[st] order)

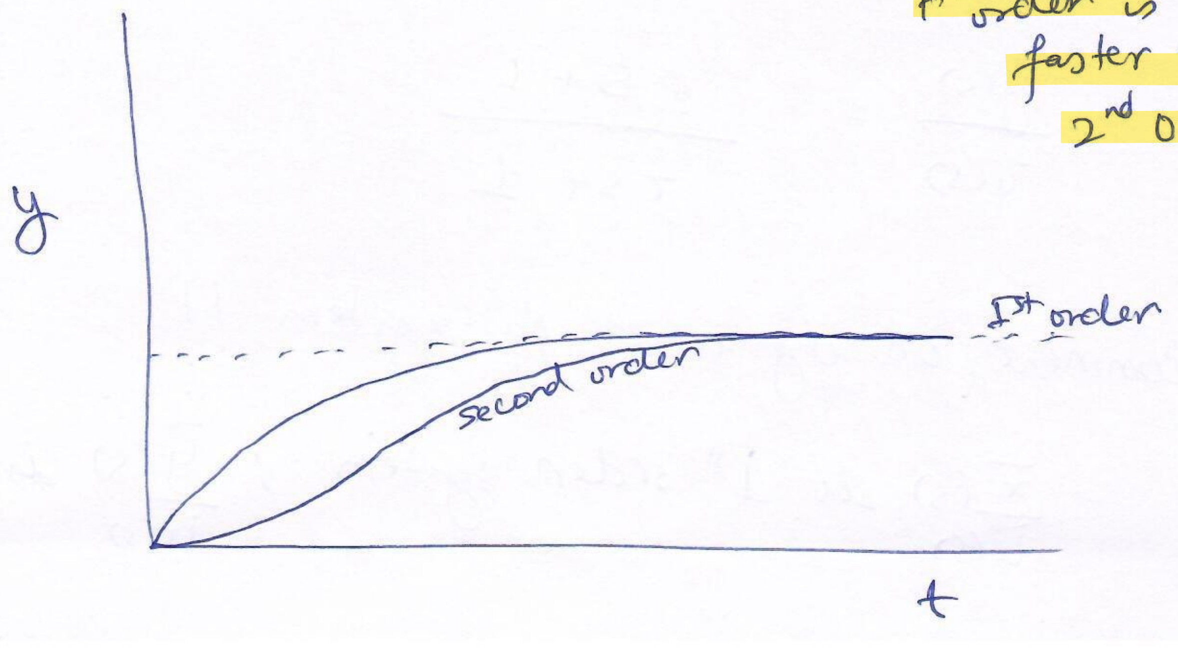
$$g_2(s) = \frac{K_1 k_2}{(\tau_1 s+1)(\tau_2 s+1)}$$

$$y(t) = Ak_1K_2\left[1 - \frac{\tau_1}{\tau_1-\tau_2}e^{-t/\tau_1} - \frac{\tau_2}{\tau_2-\tau_1}e^{-t/\tau_2}\right]$$

when subject to step function of mag. A.

Response of pure 2[nd] order system will never cross response of pure 1[st] order system for any value of τ.

1[st] order is faster than 2[nd] order



zeros of numerator = zeros
zeros of denominator = poles.

Addition of pole makes response sluggish.

$$g_3 = \frac{K(\xi s + 1)}{\tau s + 1} = \text{lead lag system}$$

τ = lag constt

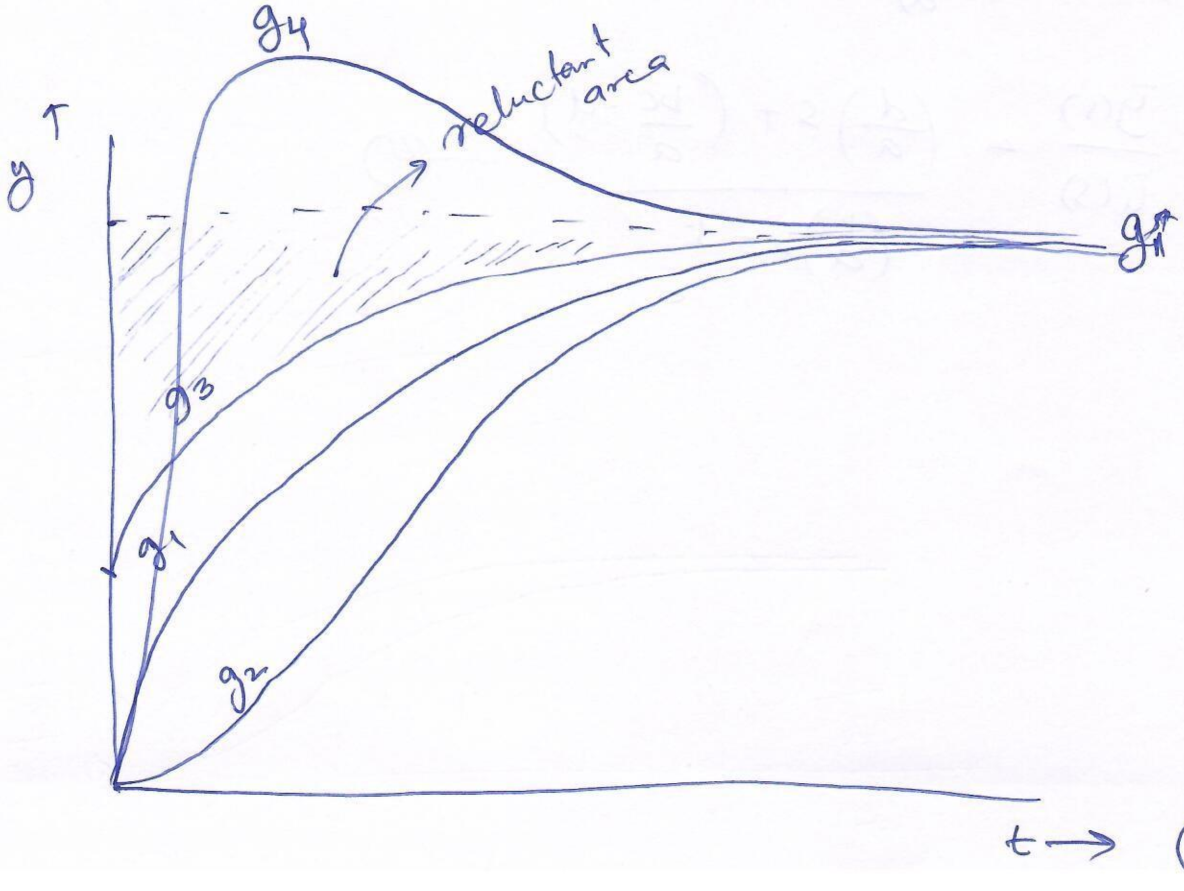
ξ = lead constt.

$$y(t) = KA\left[1 - \left(1 - \frac{\xi}{\tau}\right)e^{-t/\tau}\right]$$

$\frac{\xi}{\tau}$ = decides intercept value

If we add zero, reluctant area des in comparison to reluctant area of 1st order system

Addition of zero makes a response faster



$$g_4 = \frac{K(\xi s+1)}{(\tau_1 s+1)(\tau_2 s+1)} \qquad \text{order } (2,1)$$

$$y(t) = KA\left[1 - \left(\frac{\tau_1-\xi}{\tau_1-\tau_2}\right)e^{-t/\tau_1} - \left(\frac{\tau_2-\xi}{\tau_2-\tau_1}\right)e^{-t/\tau_2}\right]$$

MIMO system

~~for distillation column~~

for SISO

$$\frac{dx}{dt} = ax+bu \quad \text{—— (1)}$$

$$y = cx+du \quad \text{—— (2)}$$

$$\frac{\bar{x}(s)}{\bar{u}(s)} = \frac{b/a}{\left(\frac{1}{a}\right)s-1} \quad \text{—— (3)}$$

$$\frac{\bar{y}(s)}{\bar{u}(s)} = \frac{\left(\frac{d}{a}\right)s + \left(\frac{bc}{a}-d\right)}{\left(\frac{1}{a}\right)s-1} \quad \text{—— (4)}$$

Two I/p Two o/p

$$\frac{dx_1}{dt} = a_{11}x_1 + a_{12}x_2 + b_{11}u_1 + b_{12}u_2 \quad \text{--- (1)}$$

$$\frac{dx_2}{dt} = a_{21}x_1 + a_{22}x_2 + b_{21}u_1 + b_{22}u_2 \quad \text{--- (2)}$$

$$y_1 = c_{11}x_1 + c_{12}x_2 + d_{11}u_1 + d_{12}u_2 \quad \text{--- (3)}$$

$$y_2 = c_{21}x_1 + c_{22}x_2 + d_{21}u_1 + d_{22}u_2 \quad \text{--- (4)}$$

laplace of eqⁿ (1) & (2)

$$(s-a_{11})\bar{x}_1(s) - a_{12}\bar{x}_2(s) = b_{11}\bar{u}_1(s) + b_{12}\bar{u}_2(s)$$

$$-a_{21}\bar{x}_1(s) + (s-a_{22})\bar{x}_2(s) = b_{21}\bar{u}_1(s) + b_{22}\bar{u}_2(s)$$

$$\begin{bmatrix} s-a_{11} & -a_{12} \\ -a_{21} & s-a_{22} \end{bmatrix} \begin{bmatrix} \bar{x}_1(s) \\ \bar{x}_2(s) \end{bmatrix} = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix} \begin{bmatrix} \bar{u}_1(s) \\ \bar{u}_2(s) \end{bmatrix}$$

$$\left\{ s\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \right\} \begin{bmatrix} \bar{x}_1(s) \\ \bar{x}_2(s) \end{bmatrix} = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix} \begin{bmatrix} \bar{u}_1(s) \\ \bar{u}_2(s) \end{bmatrix}$$

$$(\underline{\underline{sI}} - \underline{\underline{A}})\underline{\bar{X}}(s) = \underline{\underline{B}}\,\underline{\bar{U}}(s)$$

$$\underline{\bar{X}}(s) = [\underline{\underline{sI}} - \underline{\underline{A}}]^{-1}\underline{\underline{B}}\,\underline{\bar{U}}(s) \quad \text{--- (5)}$$

laplace for eq^n ③ & ④.

$$\bar{y}_1(s) = c_{11}\bar{x}_1(s) + c_{12}\bar{x}_2(s) + d_{11}\bar{u}_1(s) + d_{12}\bar{u}_2(s)$$

$$\bar{y}_2(s) = c_{21}\bar{x}_1(s) + c_{22}\bar{x}_2(s) + d_{21}\bar{u}_1(s) + d_{22}\bar{u}_2(s)$$

$$\begin{bmatrix} \bar{y}_1(s) \\ \bar{y}_2(s) \end{bmatrix} = \begin{bmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{bmatrix} \begin{bmatrix} \bar{x}_1(s) \\ \bar{x}_2(s) \end{bmatrix} + \begin{bmatrix} d_{11} & d_{12} \\ d_{21} & d_{22} \end{bmatrix} \begin{bmatrix} \bar{u}_1(s) \\ \bar{u}_2(s) \end{bmatrix}$$

$$\bar{\underline{y}}(s) = \underline{\underline{C}}\,\bar{\underline{x}}(s) + \underline{\underline{D}}\,\bar{\underline{u}}(s)$$

$$\bar{\underline{y}}(s) = \underline{\underline{C}}[s\underline{\underline{I}} - \underline{\underline{A}}]^{-1}\underline{\underline{B}}\,\bar{\underline{u}}(s) + \underline{\underline{D}}\,\bar{\underline{u}}(s)$$

$$\bar{\underline{y}}(s) = \underbrace{\left[\underline{\underline{C}}[s\underline{\underline{I}} - \underline{\underline{A}}]^{-1}\underline{\underline{B}} + \underline{\underline{D}}\right]}_{\text{Transfer function matrix for TITO system.}}\bar{\underline{u}}(s)$$

$$\underline{\underline{G}}(s) = \underline{\underline{C}}(s\underline{\underline{I}} - \underline{\underline{A}})^{-1}\underline{\underline{B}} + \underline{\underline{D}}$$

for M input, P output, you will get P*M matrix

State space model — $\frac{dx}{dt} = ax + bu$

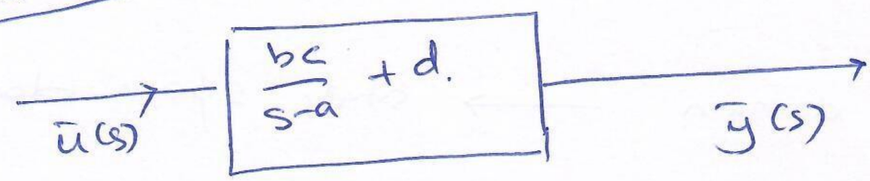
$y = cx + du$

Inverse laplace ↑ ↓ LAPLACE

Transform domain model — $\frac{\bar{x}(s)}{\bar{u}(s)} = \frac{b}{s-a}$

$\frac{\bar{y}(s)}{\bar{u}(s)} = \frac{bc}{s-a} + d$

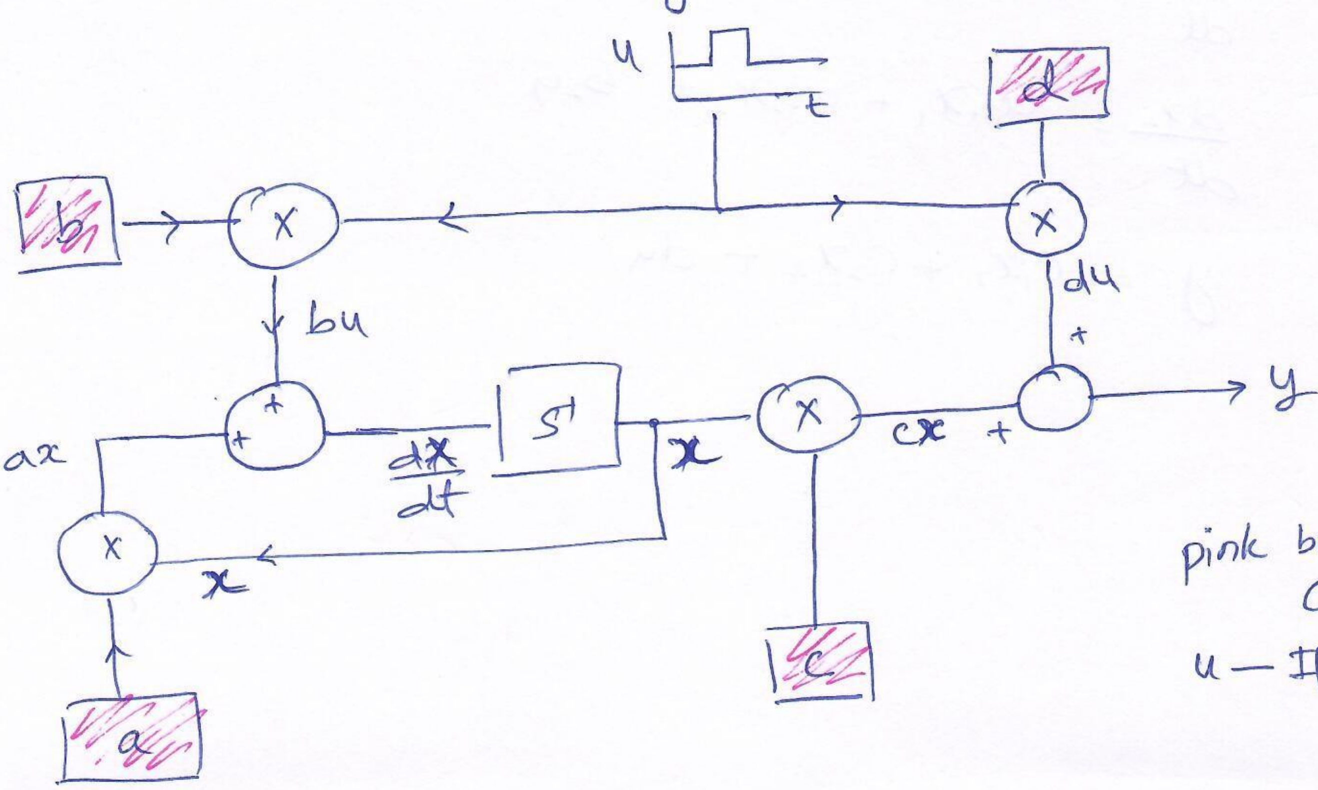
BLOCK DIAGRAM FOR TRANSFORM DOMAIN



STATE SPACE DOMAIN

$\frac{dx}{dt} = ax + bu$

$y = cx + du$



pink box - Const Value

u — I/p signal

$$\bar{y}(s) = \frac{ds + (bc-ad)}{s-a} \bar{u}(s)$$

$$s\bar{y}(s) - a\bar{y}(s) = d\, s\bar{u}(s) + (bc-ad)\,\bar{u}(s)$$

Inverse laplace

$$\frac{dy}{dt} - ay = d\left(\frac{du}{dt}\right) + (bc-ad)\,u$$

Transform domain $\longrightarrow$ state space domain

Q. Draw block diagram for following in both steady state and Transform domain -

$$\frac{dx_1}{dt} = a_{11}x_1 + a_{12}x_2 + b_1 u$$

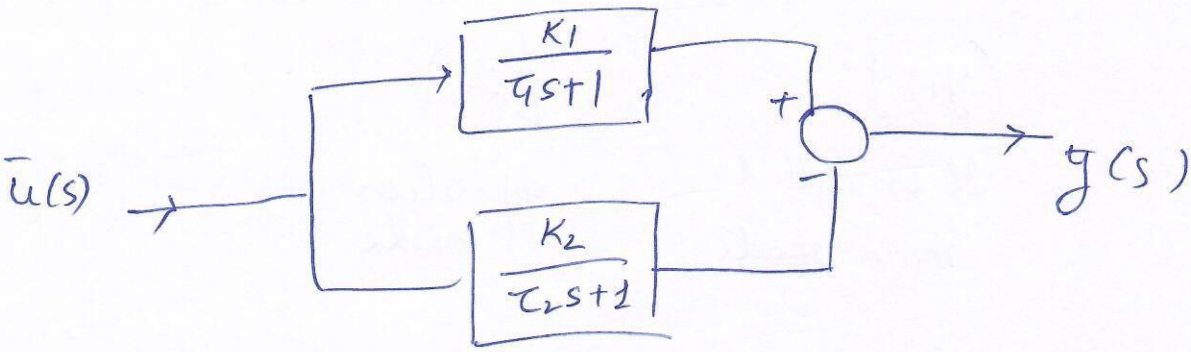
$$\frac{dx_2}{dt} = a_{21}x_1 + a_{22}x_2 + b_2 u$$

$$y = c_1 x_1 + c_2 x_2 + du$$

In

Response of Inverse response system

Given $g(s) = \dfrac{K_1}{(\tau_1 s+1)} - \dfrac{K_2}{(\tau_2 s+1)}$



I/p - o/p Block diagram of $g(s)$

Let $\bar{u}$ be step I/p of magnitude A

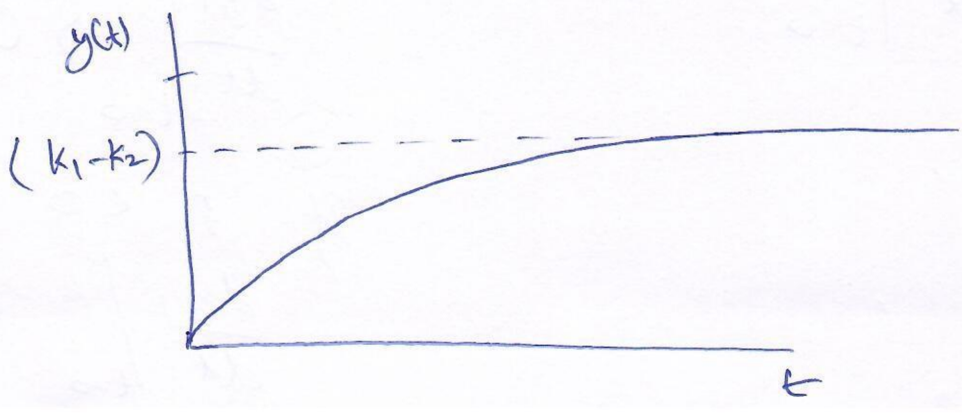
$$\bar{y}(s) = \frac{A K_1}{s(\tau_1 s+1)} - \frac{A K_2}{s(\tau_2 s+1)}$$

Inverse laplace

$$y(t) = A K_1 \left(1 - e^{-t/\tau_1}\right) - A K_2 \left(1 - e^{-t/\tau_2}\right)$$

γ_1

dynamical behaviour of system :-



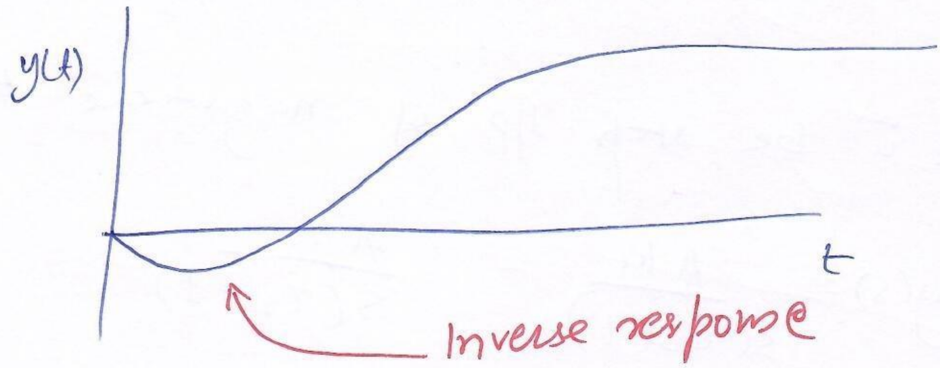
$$\lim_{t \to \infty} y(t) = A(K_1 - K_2) \quad \Leftarrow \text{Steady state value.}$$

$$g(s) = \frac{k_1}{\tau_1 s + 1} - \frac{k_2}{\tau_2 s + 1}$$

(y_1) It is called main mode

(y_2) opposition mode

In some cases, we get response like this



condition for inverse response = ??

$$\left.\frac{dy_1}{dt}\right|_{t=0} = \frac{AK}{\tau} \quad ; \text{ if } A > 0$$

$$\left.\frac{dy_1}{dt}\right|_{t=0} > 0$$

If $A < 0$

$$\left.\frac{dy_1}{dt}\right|_{t=0} < 0$$

$$\left.\frac{dy}{dt}\right|_{t=0} = A\left[\frac{K_1}{\tau_1} - \frac{K_2}{\tau_2}\right] > 0 \quad \text{if } \frac{K_1}{\tau_1} > \frac{K_2}{\tau_2}$$

$$< 0 \quad \text{if } \frac{K_2}{\tau_2} > \frac{K_1}{\tau_1}$$

Condition of inverse response

$\tau_1 >> \tau_2$

If system of order (2,1) such that

$$\frac{K_1\tau_2 - K_2\tau_1}{K_1 - K_2} < 0$$

Will exhibit ~~inverse~~ inverse response.