A crystal is growing from a supersaturated solution. The solution contains 4 moles/lit of solute and saturated conditions have been measured to be 3.95 moles/lit at the conditions of the test. The crystal is 1 mm in diameter and the fluid flows past the crystal at a velocity of 0.5 m/s. The viscosity of the fluid is $5x10^{-3}$ Ns/m^2 and its density is 1100 kg/m^3. The solid has a density of 1500 kg/m^3 and its molecular weight is 150. The diffusion coefficient for the solute in the liquid is $4x10^{-10}$ m^2/s. The Sherwood number for this situation can be expressed by the following correlation:

$$Sh_D = 2+\left(0.4\,\mathrm{Re}_D^{1/2}+0.06\,\mathrm{Re}_D^{2/3}\right)Sc^{0.4}$$

How fast could the crystal grow in this solution? **Marks (5)**

We need to calculate the rate of mass addition to the crystal at the specific time we are interested in. Thus, we need a mass balance with the driving force for mass transfer being the difference in concentration of solute between the fluid and the saturated state at the crystal surface.

Basic Equation

In - Out + Gen = Accumulation

$$\left(\dot{M}_{IN}\right)-0+0=\frac{dM}{dt}$$

$$\overline{k_c}A_s\left(c_{A\infty}-C_{AS}\right)=\frac{d}{dt}\left(C_{AC}V\right)$$

Where $\overline{k_c}$ is the effective convective mass transfer coefficient.

Assuming a spherical catalyst, and substituting the volume and the surface area,

$$\overline{k_c}\left(4\pi r^2\right)\left(c_{A\infty}-C_{AS}\right)=C_{AC}\frac{d}{dt}\left(\frac{4}{3}\pi r^3\right)$$

$$\overline{k_c}\frac{\left(c_{A\infty}-C_{AS}\right)}{C_{AC}}=\frac{dr}{dt}$$

The next step is to obtain $\overline{k_c}$ using the relation $Sh_D = 2+\left(0.4\,\mathrm{Re}_D^{1/2}+0.06\,\mathrm{Re}_D^{2/3}\right)Sc^{0.4}$

The Reynold's number for the crystal is

$$\mathrm{Re}_D=\frac{V_\infty d}{\gamma}=\frac{0.5\times 0.001}{\frac{5\times 10^{-3}}{1100}}=110$$

$$Sc=\frac{\gamma}{D_{AB}}=\frac{\frac{5\times 10^{-3}}{1100}}{4\times 10^{-10}}=11360$$

$$Sh_D=2+\left(0.4\left(110\right)^{0.5}+0.06\left(110\right)^{2/3}\right)\left(11360\right)^{0.4}=235.5$$

$$\overline{k_c}=\frac{Sh_D D_{AB}}{d}=\frac{235.5\times 4\times 10^{-10}}{0.001}=9.42\times 10^{-5}\,\frac{m}{s}$$

The growth rate, dr/dt can be calculated now-

$$\frac{dr}{dt}=\overline{k_c}\frac{\left(c_{A\infty}-C_{AS}\right)}{C_{AC}}=9.42\times 10^{-5}\left(\frac{4\,{}^{kmol}\!/\!{}_{m^3}-3.95\,{}^{kmol}\!/\!{}_{m^3}}{\frac{1500\,{}^{kg}\!/\!{}_{m^3}}{150\,{}^{kg}\!/\!{}_{kmol}}}\right)=0.47\,\frac{\mu m}{s}$$