AMT - CI 2

(1) Analyse the stability of the following system of equations -

$$\frac{dx}{dt} = x - y - x^3 - xy^2$$

$$\frac{dy}{dt} = x + y - x^2y - y^3$$

Sol^n

$$\frac{dx}{dt} = x - y - x^3 - xy^2 = f_1(x,y)$$

$$\frac{dy}{dt} = x + y - x^2y - y^3 = f_2(x,y)$$

$\Rightarrow$ multiply f_1 by y & f_2 by x

$$\cancel{xy} - y^2 - \cancel{x^3y} - \cancel{xy^3}$$

$$-\quad x^2 \underset{-}{+} \cancel{xy} \underset{+}{-} \cancel{x^3y} \underset{+}{-} \cancel{xy^3}$$

$$-x^2 - y^2 = 0$$

$$x^2 + y^2 = 0$$

$x = 0,\ y = 0 \Rightarrow$ unique S.S.

Calculating Jacobian matrix. 2

$$\begin{vmatrix} \frac{\partial f_1}{\partial x} & \frac{\partial f_1}{\partial y} \\ \frac{\partial f_2}{\partial x} & \frac{\partial f_2}{\partial y} \end{vmatrix}$$

$$\begin{vmatrix} 1 - 3x^2 - y^2 & -1 - 2xy \\ 1 - 2xy & 1 - x^2 - 3y^2 \end{vmatrix}$$

$x = 0 \ , \ y = 0$

$$\begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}$$

Jacobian matrix

$$\begin{vmatrix} 1 - \lambda & -1 \\ 1 & 1 - \lambda \end{vmatrix}$$

$$(1-\lambda)^2 + 1 = 0.$$

$$\lambda^2 - 2\lambda + 2 = 0$$

$$\lambda = \frac{2 \pm \sqrt{4-8}}{2} = 1 \pm i$$

unstable focus.

(if real part is present, unstable syst of eq^n)

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$$\frac{dx}{dt} = y = f_1(x,y)$$

$$\frac{dy}{dt} = -K \sin x = f_2(x,y)$$

$\because$ $\sin y = y$ (given)

$$\frac{dy}{dt} = -K\cancel{y}x = f_2(x,y)$$

$x = 0$
$y = 0$ unique s.s.

$$\begin{vmatrix} \frac{\partial f_1}{\partial x} & \frac{\partial f_1}{\partial y} \\ \frac{\partial f_2}{\partial x} & \frac{\partial f_2}{\partial y} \end{vmatrix} = \begin{vmatrix} 0 & 1 \\ -K & 0 \end{vmatrix}$$

$$\begin{vmatrix} -\lambda & 1 \\ -K & -\lambda \end{vmatrix} = (-\lambda)^2 - (-K) = 0$$

$$\lambda^2 + K = 0$$

$$\lambda = \pm\sqrt{K}$$

$$\frac{dx}{dy} = \frac{y}{-Kx}$$

$$-K\,x\,dx = y\,dy$$

$$\boxed{Kx^2 + y^2 = C}$$ limit cycle.