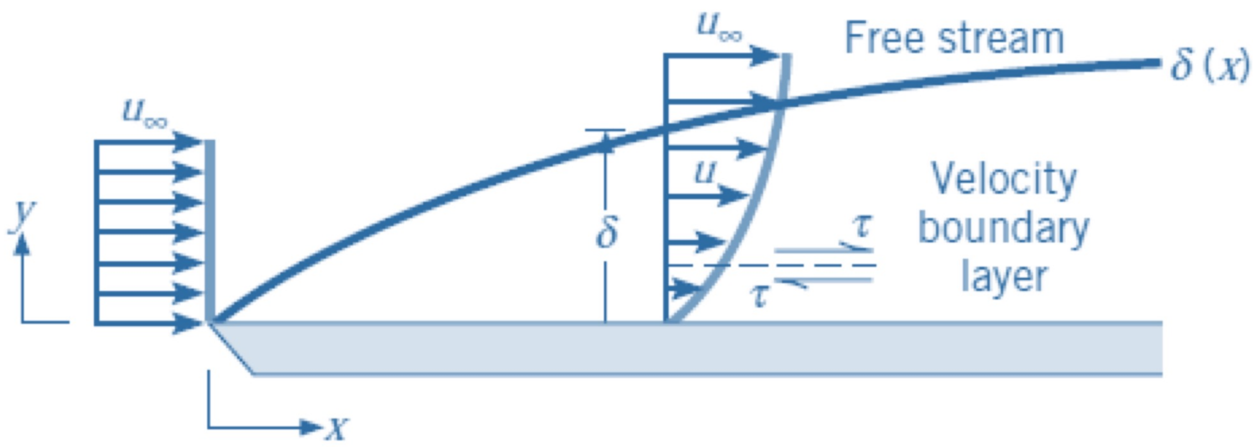
HEAT TRANSFER

[CH21204]

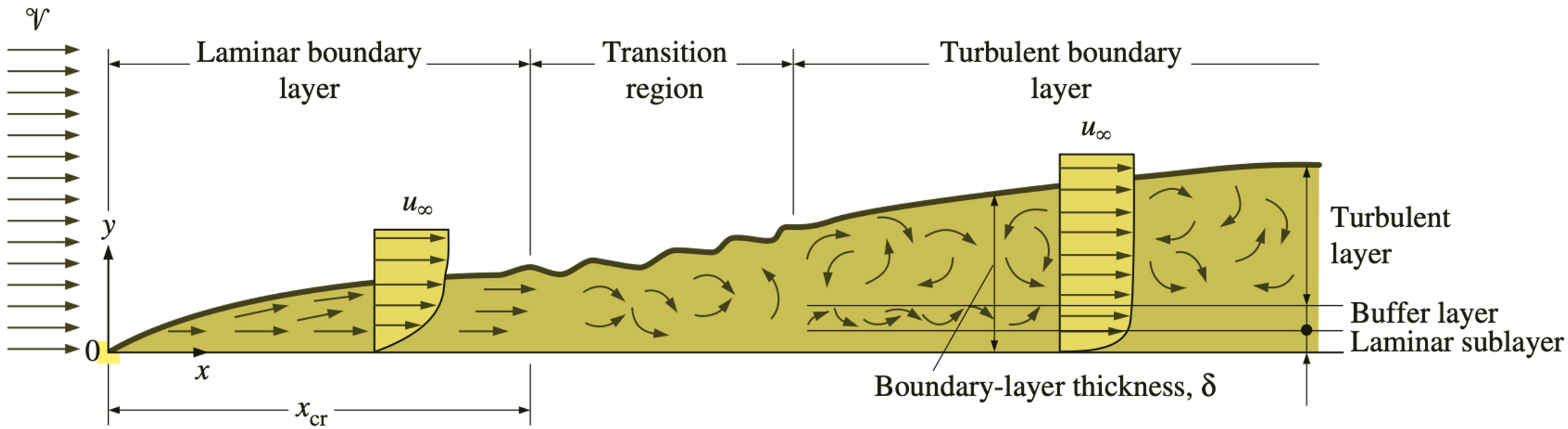
January 12, 2023

Velocity Boundary Layer

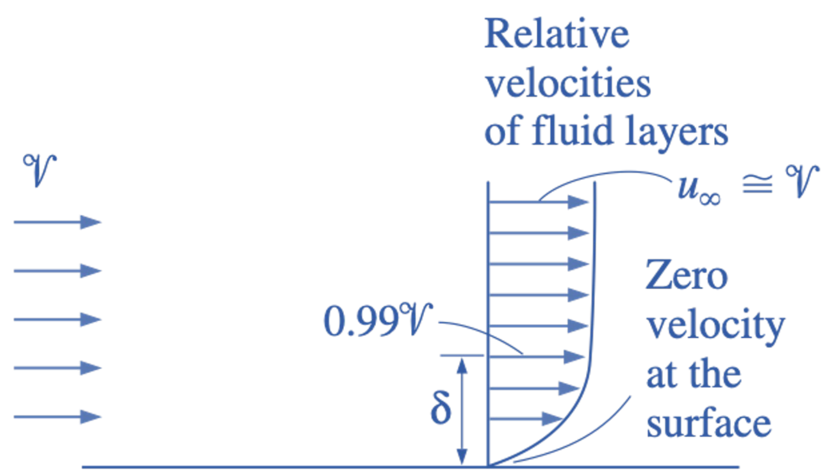


$$C_f \equiv \frac{\tau_s}{\rho u_\infty^2/2}$$

$$\tau_s = \mu \frac{\partial u}{\partial y}\bigg|_{y=0}$$

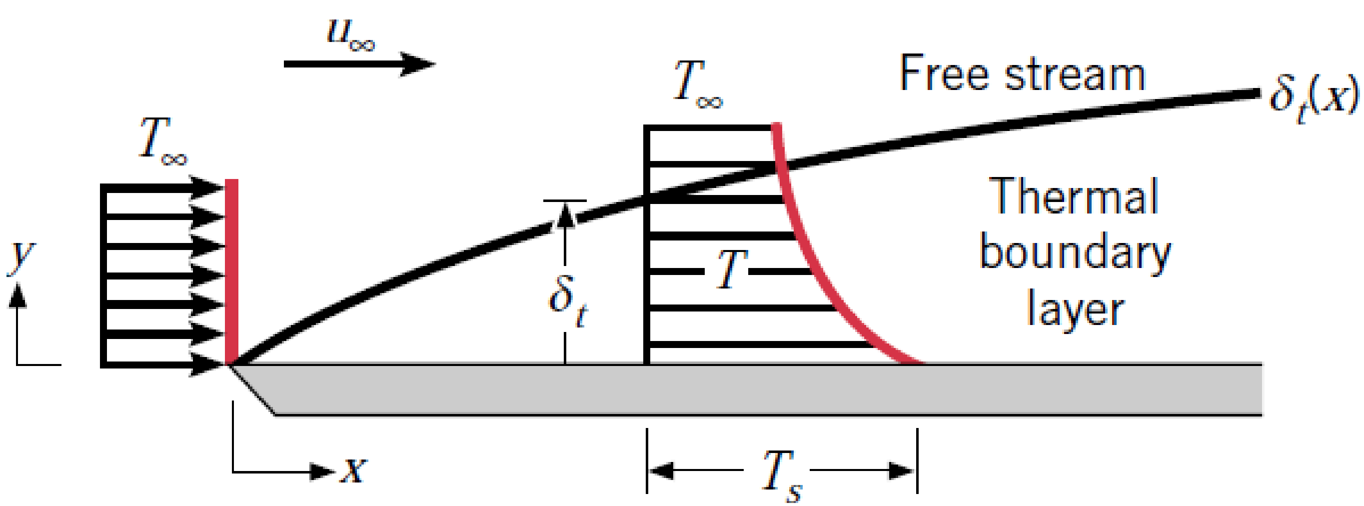


𝒱
Laminar boundary layer
Transition region
Turbulent boundary layer
u∞
u∞
y
0
x
x cr
Turbulent layer
Buffer layer
Laminar sublayer
Boundary-layer thickness, δ



Relative velocities of fluid layers
𝒱
u∞ ≅ 𝒱
Zero velocity at the surface
0.99𝒱
δ

Thermal Boundary Layer



$$[(T_s - T)/(T_s - T_\infty)] = 0.99$$

$$q_s'' = h(T_s - T_\infty)$$

$$h = \frac{-k_f \, \partial T/\partial y|_{y=0}}{T_s - T_\infty}$$

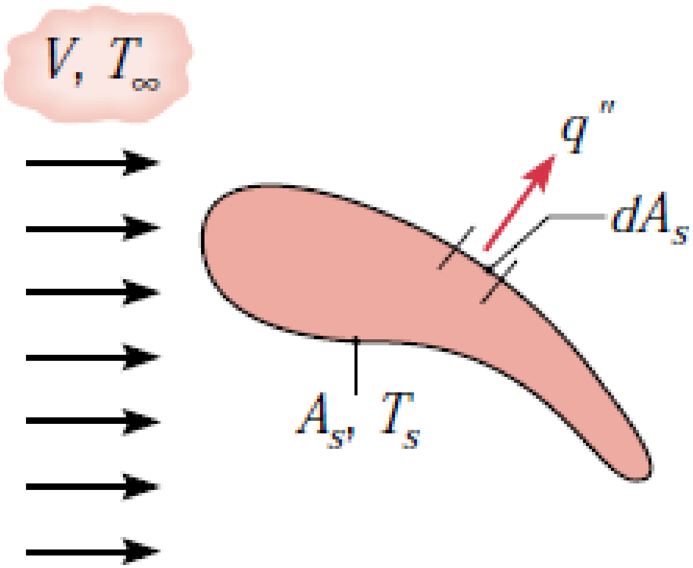
$$\text{Pr} = \frac{\text{Molecular diffusivity of momentum}}{\text{Molecular diffusivity of heat}} = \frac{\nu}{\alpha} = \frac{\mu C_p}{k}$$

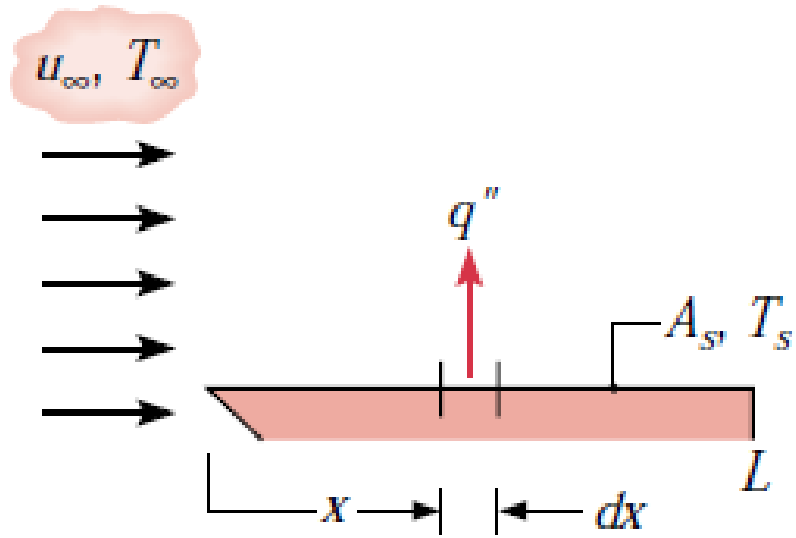
Typical ranges of Prandtl numbers for common fluids

Fluid	Pr
Liquid metals	0.004–0.030
Gases	0.7–1.0
Water	1.7–13.7
Light organic fluids	5–50
Oils	50–100,000
Glycerin	2000–100,000

- Prandtl numbers of gases are about 1, which indicates that both momentum and heat dissipate through the fluid at about the same rate.
- Heat diffuses very quickly in **liquid metals** (Pr << 1) and very slowly in **oils** (Pr >> 1) relative to momentum.
- Consequently the thermal boundary layer is much **thicker for liquid metals** and much **thinner for oils** relative to the velocity boundary layer.

Local and Average Convection Coefficients





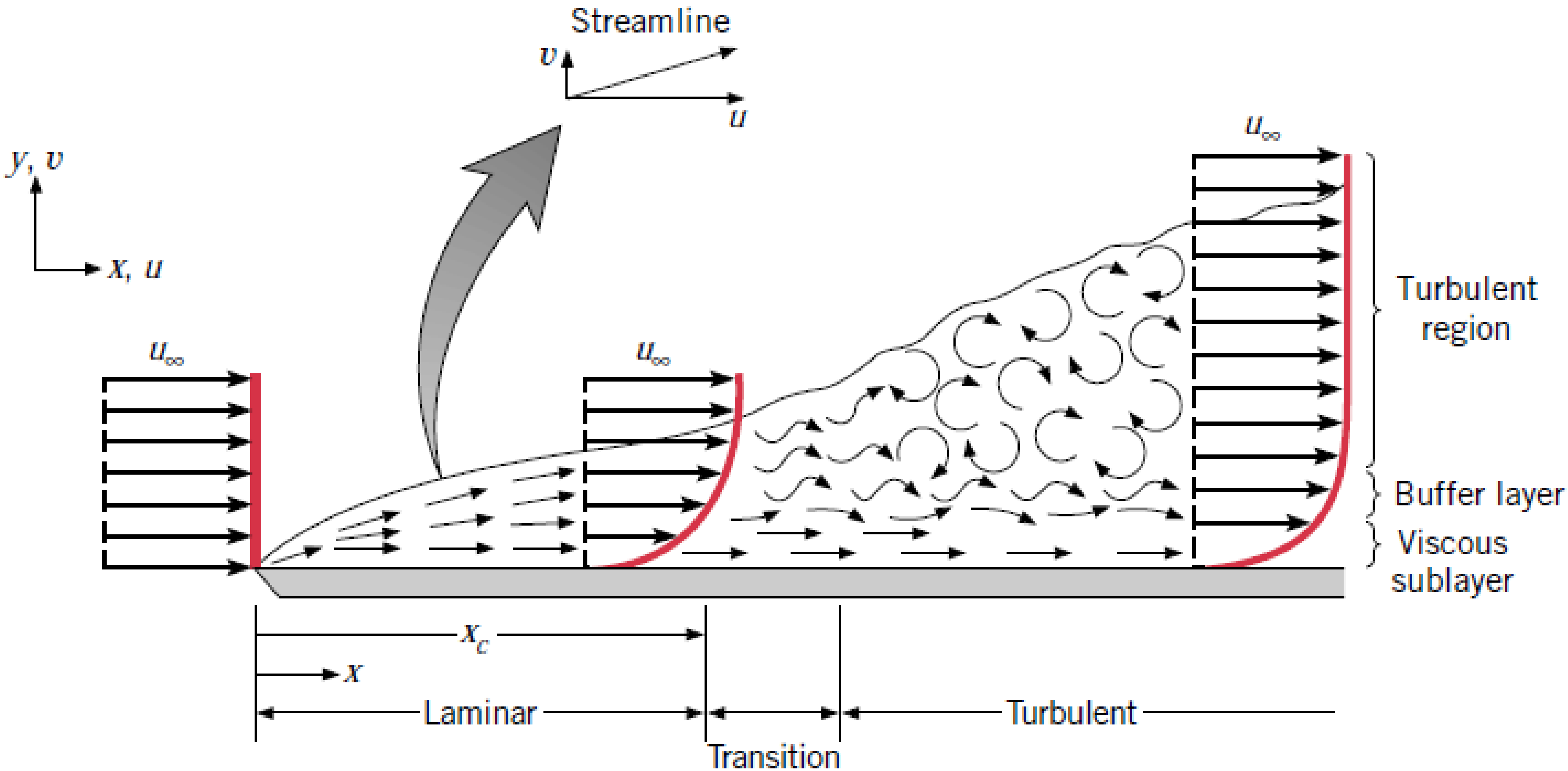
$$q = \int_{A_s} q'' dA_s$$

$$q = (T_s - T_\infty)\int_{A_s} h dA_s$$

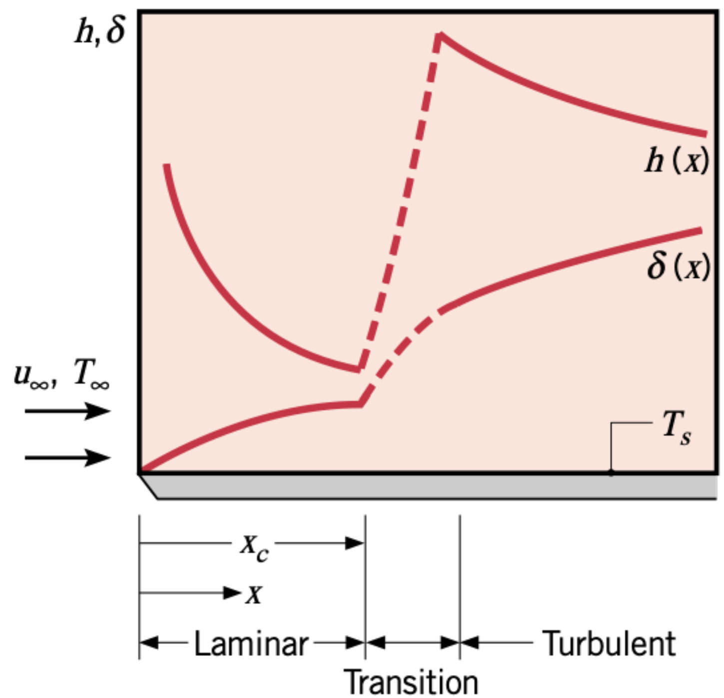
$$q = \bar{h} A_s (T_s - T_\infty)$$

$$\bar{h} = \frac{1}{A_s}\int_{A_s} h dA_s$$

$$\bar{h} = \frac{1}{L}\int_0^L h dx$$



$$Re_{x,c} \equiv \frac{\rho u_\infty x_c}{\mu} = 5 \times 10^5$$



h, δ
$h(x)$
$\delta(x)$
$u_\infty,\ T_\infty$
T_s
x_c
x
Laminar
Transition
Turbulent

Water flows at a velocity $u_\infty = 1$ m/s over a flat plate of length $L = 0.6$ m. Consider two cases, one for which the water temperature is approximately 300 K and the other for an approximate water temperature of 350 K. In the laminar and turbulent regions, experimental measurements show that the local convection coefficients are well described by

$$h_{\text{lam}}(x) = C_{\text{lam}}\, x^{-0.5} \qquad h_{\text{turb}}(x) = C_{\text{turb}}\, x^{-0.2}$$

where x has units of m. At 300 K,

$$C_{\text{lam},300} = 395 \text{ W/m}^{1.5} \cdot \text{K} \qquad C_{\text{turb},300} = 2330 \text{ W/m}^{1.8} \cdot \text{K}$$

while at 350 K,

$$C_{\text{lam},350} = 477 \text{ W/m}^{1.5} \cdot \text{K} \qquad C_{\text{turb},350} = 3600 \text{ W/m}^{1.8} \cdot \text{K}$$

As is evident, the constant, C, depends on the nature of the flow as well as the water temperature because of the thermal dependence of various properties of the fluid.

Determine the average convection coefficient, $\bar{h}$, over the entire plate for the two water temperatures.