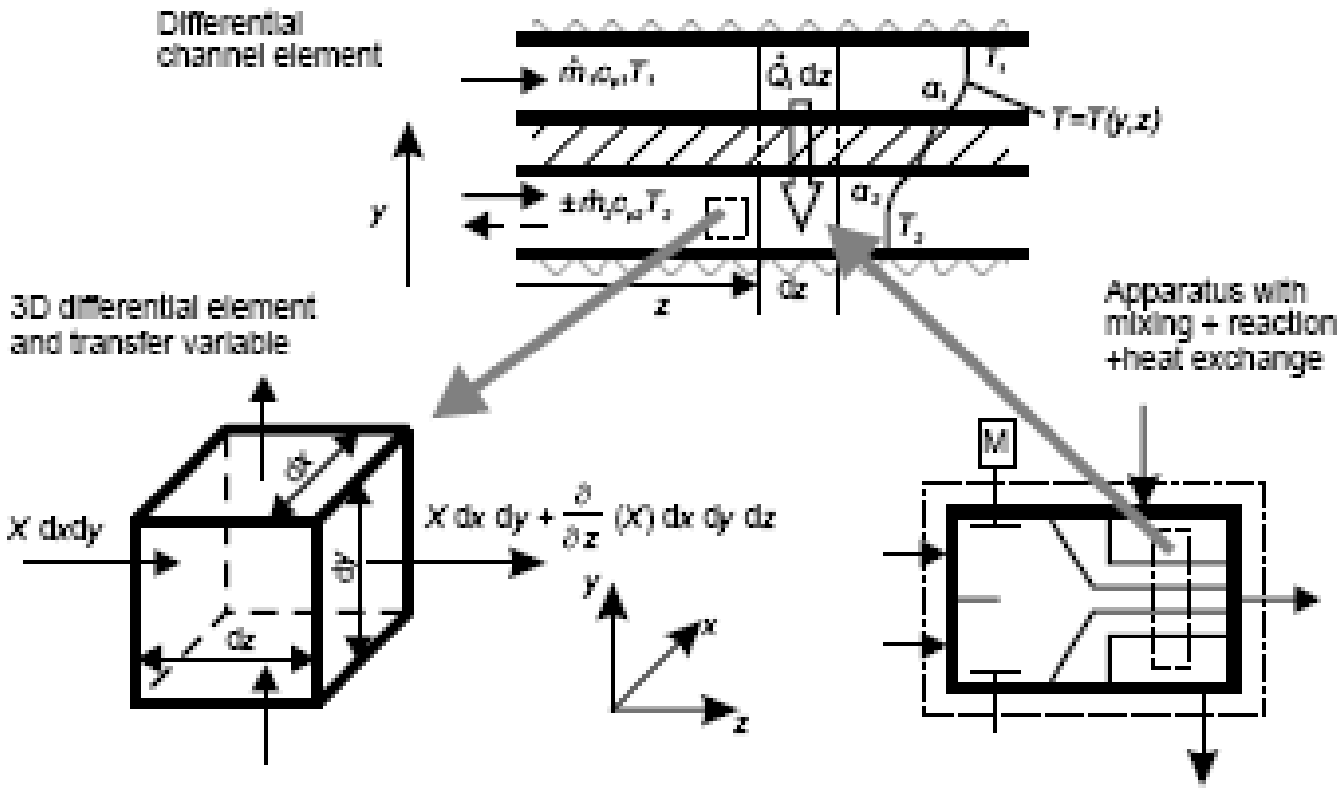
Fundamentals, Balances, and Transport Processes

Conservation and balance equations of mass, species, momentum, energy, as well as the definition of the entropy and its application.

The conservation laws of mass (continuity equations) and energy (first law of thermodynamics) can be described as

$$\begin{bmatrix}\text{System change}\\ \text{with time}\end{bmatrix} = \begin{bmatrix}\text{Incoming}\\ \text{Flow}\end{bmatrix} - \begin{bmatrix}\text{Outgoing}\\ \text{Flow}\end{bmatrix} + \begin{bmatrix}\text{Source or}\\ \text{Sink}\end{bmatrix}$$

Macroscopic balance equations



Overview of the various balancing volumes in process engineering : 3D differential element for general calculations, 1D differential element and complete equipment (active mixer) for process balances.

X stands for mass, species, momentum or energy.

The general balance equation with temporal and spatial derivatives of a system and a differential element with the volume V is written as (with X as the general balanced value):

$$V.\frac{\partial X}{\partial t} = w.\left[X\ dy\ dx - \left(X\ dy\ dx + \frac{\partial X}{\partial z} dz\ dy\ dx \right) \right]$$

In a steady process, the temporal derivative vanishes, $\partial/\partial t = 0$.

In systems with high velocities, the convective transport is dominant compared to conductive and diffusive fluxes.

For systems with dominant chemical reactions, only a change of substance needs to be considered.

Within the systems since no mass is generated or destroyed, hence, no sink or source appears in the balance equation.

For time dependent mass flow rates,

$$\frac{\partial \dot{m}}{\partial t} = \dot{m}_{in} - \dot{m}_{out}$$

The momentum of a moving fluid can be expressed as the product of the mass and the flow velocity.

The integral of the momentum over the volume results in a net force of the fluid on the volume boundary or on the equipment.

In general, the momentum balance of a device can be written as

$$\sum J_{in} = \sum J_{out} + \sum J_{loss}$$

The momentum loss can be interpreted as the viscous momentum loss, which is expressed as pressure loss along the channel or device flow.

For fluid flow through an arbitrary channel:

$$\frac{\partial(mw)}{\partial t} = \left(\dot{mw}\right)_{in} - \left(\dot{mw}\right)_{out} + \left(pA_i\right)_{in} - \left(pA_i\right)_{out} + mg + F_z$$

Starting point for Navier-Stokes equation.

Additional forces in microfluidic applications result from surface effects

Similar to the mass, the energy itself is conserved according to the first law of thermodynamics for open systems.

$$\sum \dot{E}_{in} = \sum \dot{E}_{out} + \sum \dot{E}_{diss}$$

The energy dissipation takes into account that energy conversion from one form into another is accompanied by natural losses.

These losses are characterized by the entropy generation during a process according to the second law of thermodynamics.

Energy equation

$$\rho c_p \frac{dT}{dt} = \frac{dp}{dt} + \varepsilon - div\ (k\ grad\ T)$$

k = thermal conductivity, (W/m K)

The dissipation function ε is the friction loss per volume and time unit.

The solution of this equation gives the temperature distribution.

Isobaric process with no dissipation – usual form of energy equation

$$\rho c_p \frac{dT}{dt} = - div\ (k\ grad\ T)$$

ENERGY EQUATION (in all coordinate systems)

$$\rho \hat{C}_p \left(\frac{\partial T}{\partial t} + v_x \frac{\partial T}{\partial x} + v_y \frac{\partial T}{\partial y} + v_z \frac{\partial T}{\partial z} \right) = k \left[\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} \right] + \mu \phi_v + \dot{Q}$$

$$\rho \hat{C}_p \left(\frac{\partial T}{\partial t} + v_r \frac{\partial T}{\partial r} + \frac{v_\theta}{r} \frac{\partial T}{\partial \theta} + v_z \frac{\partial T}{\partial z} \right) = k \left[\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial T}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 T}{\partial \theta^2} + \frac{\partial^2 T}{\partial z^2} \right] + \mu \phi_v + \dot{Q}$$

$$\rho \hat{C}_p \left(\frac{\partial T}{\partial t} + v_r \frac{\partial T}{\partial r} + \frac{v_\theta}{r} \frac{\partial T}{\partial \theta} + \frac{v_\phi}{r \sin\theta} \frac{\partial T}{\partial \phi} \right) = k \left[\begin{array}{l} \frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial T}{\partial r} \right) + \frac{1}{r^2 \sin\theta} \frac{\partial}{\partial \theta} \left(\sin\theta \frac{\partial T}{\partial \theta} \right) \\ + \frac{1}{r^2 \sin^2\theta} \frac{\partial^2 T}{\partial \phi^2} \end{array} \right] + \mu \phi_v + \dot{Q}$$

Dividing the transferred heat by the temperature, a new state variable, the entropy *s (state function)*, is derived for further characterization of the states and processes.

$$ds \geq \frac{dq}{T} \quad equal\ for\ rev.\ proc$$

The dissipation function ε is the friction loss per volume and time unit.

Entropy always increases [$ds > 0$] by dissipation and irreversible processes, such as pressure loss or concentration homogenization by mixing.

The entropy production is a major indication of the efficiency of a process.

Various control strategies are used for energy savings and entropy minimization.

For continuous flow systems, high pressure losses and unnecessary throttling should be avoided.

In mixing, high solution concentrations as well as high dilutions lead to high separation effort.

Heat transfer devices with high temperature gradients induce high entropy production.

Elementary transport processes and their description

Elementary transport processes and their description

The entire change of a state variable is described by the transport processes of conduction in the immobile phase (solids or resting fluids), convection in the fluid phase (gases and liquids), and by the generation or depletion in the control volume.

$$\begin{bmatrix} Total\ Flow \\ Density \end{bmatrix} = \begin{bmatrix} Conduction\ over \\ the\ boundary \end{bmatrix} + \begin{bmatrix} Convection\ \ over \\ the\ boundary \end{bmatrix} + \begin{bmatrix} Source \\ or \\ \text{Sink} \end{bmatrix}$$

Conductive transport is driven by a parameter gradient; convective transport is always accompanied by a volume flow rate with a certain mean velocity.

Transfer coefficients: D, $\nu = \mu/\rho$, $\alpha = k/\rho c_p$ (m^2/s)

The dynamic behavior of viscous fluids is governed by Re = $wd\rho/\mu$

Species transfer in convective flow, both Re and Sc (=$\mu/\rho D$)

Sc indicates the ratio between the momentum and species transfer.

For Sc = O(1), as with gases, momentum transfer and species transfer are in the same order-of-magnitude.

Concentration gradients behave similar to velocity gradients.

For high Sc such as for liquids, concentration gradients continue much longer than velocity gradients, important for mixing issues.

Convective heat transfer – both Re and Pr

Low Pr numbers - oils or organic liquids.

The Pr numbers of air or water have the order of O(1), permitting a heat transfer as fast as momentum transfer.

Coupled heat and mass or species transfer, such as convective condensation of aerosol droplets, are described by thermal diffusivity and diffusion coefficient. Both coefficients are combined to **Lewis number Le (=Sc/Pr) = α/D, ($\alpha=k/\rho c_p$)**

A high Le number allows droplet generation from vapor cooling, while for low Le numbers, vapor will directly condense at the wall.

Modeling, Calculation Methods, and Simulation

1. Physical variables and dimensional analysis
2. Similarity and scaling laws
3. Order-of-magnitude analysis
4. Lumped element modeling
5. Numerical simulation and analytical modeling

Momentum Transfer

In single-phase flow, the fluid motion in microchannels is determined by wall friction, viscous forces and inertial forces.

Continuity Equation $$\frac{D\rho}{Dt} = \frac{\partial \rho}{\partial t} + div(\rho \vec{w})$$

Equation of motion $$\frac{\partial (mw)}{\partial t} = \left(\dot{m}w\right)_{in} - \left(\dot{m}w\right)_{out} + \left(pA_i\right)_{in} - \left(pA_i\right)_{out} + mg + F_z$$

With continuity equation, the above equation can be simplified for all space co-ordinates as ($\vec{k}$ is the sum of external forces)

$$\rho \frac{D\vec{w}}{Dt} = \rho\left(\frac{\partial}{\partial t} + \vec{w} \cdot div\right)\vec{w} = -grad\ p + \mu \nabla^2 \vec{w} + \vec{k}$$

The 2nd term on LHS is also called the inertial term and describes convection. It is the only nonlinear term in the Navier-Stokes eqn.

The energy equation for fluid dynamics

The total energy conservation can be expressed as the sum of Mechanical and thermal parts.

Mechanical Energy

$$\frac{\rho}{2}\frac{D\vec{w}}{Dt} = \rho\vec{w}\cdot\vec{g} + \frac{\partial\rho}{\partial t} - \frac{Dp}{Dt} - \Phi$$

Thermal Energy

$$\rho\frac{Dh}{Dt} = -div\,\vec{q} + \frac{Dp}{Dt} + \Phi$$

The dissipation Φ of mechanical energy is accompanied by entropy production

Relevant boundary conditions

• no wall slip, i.e. zero normal and parallel velocity at the wall (if no suction or blowing occurs, and for gases with Kn < 0.01),

• no temperature jump at the wall, i.e. the temperature and/or temperature gradients can be described at the wall.

For constant density, non-viscous flow in all three directions, the momentum balance can be written as

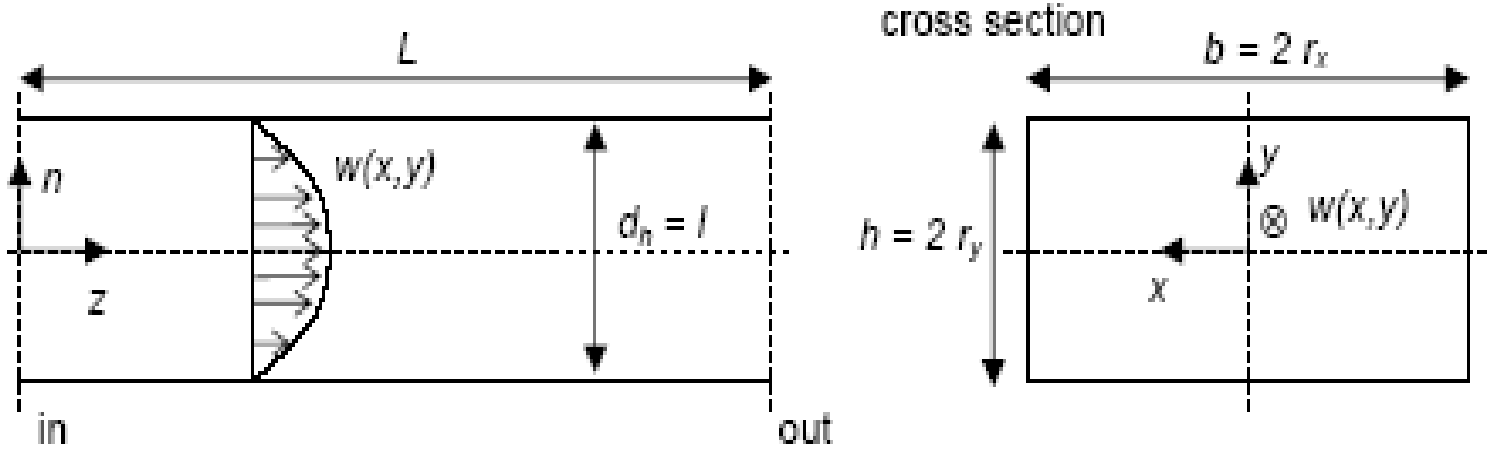
$$\frac{\partial \vec{w}}{\partial t} = -(\vec{w}\, div)\vec{w} - \frac{grad\ p}{\rho} + \vec{g}$$

Euler equation

Additional forces in microchannel flow may occur with surface effects in multiphase flow, such as bubbles and droplets.

Momentum Transport of Single-Phase Flow

Basic equations for long, small channels



Typical setup for a slender microchannel with rectangular cross section.

Length in flow directions is much longer than cross-sectional dimensions

Dimensionless variables are introduced with reference values for slender channels with rectangular cross section (width b, depth h, length L, hydraulic diameter $d_h = l$)

$$z^* = z^* = \frac{z}{l\,\mathrm{Re}};\quad y^* = \frac{y}{l};\quad w^* = \frac{w}{w_r};\quad v^* = \frac{v\,\mathrm{Re}}{w_r};\quad p^* = \frac{p - p_r}{\rho\, w_r^2};$$

$$T^* = \frac{T - T_r}{\Delta T};\quad \eta^* = \frac{\eta}{\eta_r};\quad \lambda^* = \frac{\lambda}{\lambda_r};\quad \Phi^* = \frac{\Phi\, l^2\, \mathrm{Re}^2}{w_r^2}$$

The equations describing a steady and incompressible flow in long channels with variable viscosity and thermal conductivity are

the continuity equation

$$\frac{\partial v^*}{\partial y^*} + \frac{\partial w^*}{\partial z^*} = 0,$$

$$z^* = z^* = \frac{z}{l\,\mathrm{Re}};\quad y^* = \frac{y}{l};\quad w^* = \frac{w}{w_r};\quad v^* = \frac{v\,\mathrm{Re}}{w_r};\quad p^* = \frac{p - p_r}{\rho w_r^2};$$

$$T^* = \frac{T - T_r}{\Delta T};\quad \eta^* = \frac{\eta}{\eta_r};\quad \lambda^* = \frac{\lambda}{\lambda_r};\quad \Phi^* = \frac{\Phi\, l^2\, \mathrm{Re}^2}{w_r^2}$$

the z - momentum equation

$$v^* \frac{\partial w^*}{\partial y^*} + w^* \frac{\partial w^*}{\partial z^*} = -\frac{\partial p}{\partial z^*} + \frac{\partial}{\partial y^*}\left[\eta^* \frac{\partial w^*}{\partial y^*}\right] + \frac{1}{\mathrm{Re}^2}\frac{\partial}{\partial z^*}\left[\eta^* \frac{\partial w^*}{\partial z^*}\right],$$

The y – momentum equation

$$\frac{1}{\mathrm{Re}^2}\left[v^*\frac{\partial v^*}{\partial y^*}+w^*\frac{\partial v^*}{\partial z^*}\right]=-\frac{\partial p}{\partial y^*}+$$

$$+\frac{1}{\mathrm{Re}^2}\left[\frac{\partial}{\partial y^*}\left(\eta^*\frac{\partial v^*}{\partial y^*}\right)+\frac{\partial}{\partial z^*}\eta^*\frac{\partial v^*}{\partial z^*}\right]$$

Thermal energy equation

$$v^*\frac{\partial T^*}{\partial y^*}+w^*\frac{\partial T^*}{\partial z^*}=\frac{1}{\mathrm{Pr}}\frac{\partial}{\partial y^*}\left[\lambda^*\frac{\partial T^*}{\partial y^*}\right]+$$

$$+\frac{1}{\mathrm{PrRe}^2}\frac{\partial}{\partial z^*}\left[\lambda^*\frac{\partial T^*}{\partial z^*}\right]+\frac{\mathrm{Ec}}{\mathrm{Re}^2}\Phi^*$$

$$z^*=z^*=\frac{z}{l\,\mathrm{Re}};\quad y^*=\frac{y}{l};\quad w^*=\frac{w}{w_r};\quad v^*=\frac{v\,\mathrm{Re}}{w_r};$$

$$p^*=\frac{p-p_r}{\rho w_r^2};\quad T^*=\frac{T-T_r}{\Delta T};\quad \eta^*=\frac{\eta}{\eta_r};\quad \lambda^*=\frac{\lambda}{\lambda_r};$$

$$\Phi^*=\frac{\Phi\, l^2\,\mathrm{Re}^2}{w_r^2}$$

Reynold's number = $\rho\, w_r\, l / \eta_r$; Prandtl number = ν / α

Eckert number = $w_r^2/c_p\Delta T$

In flow regimes with high Re numbers, the terms with $1/Re^2$ are very small and can be ignored.

In microflows where the Re number is in the order of O(1), the full Navier-Stokes equations must be considered and the influence of *u* and *v* velocities is not negligible.

When considering the energy equation, the viscous dissipation cannot be neglected in microchannels,

The ratio $Ec/Re^2 = \nu^2/(c_p \Delta T\, d^{\,2})$ (ν = kinematic viscosity) does not incorporate velocity and is proportional to the inverse square of the hydraulic diameter.

For a constant cross section and negligible gravitation forces, the viscous dissipation in a channel element correlates with the pressure loss:

$$\varphi_{12} = \frac{p_1 - p_2}{\rho}$$

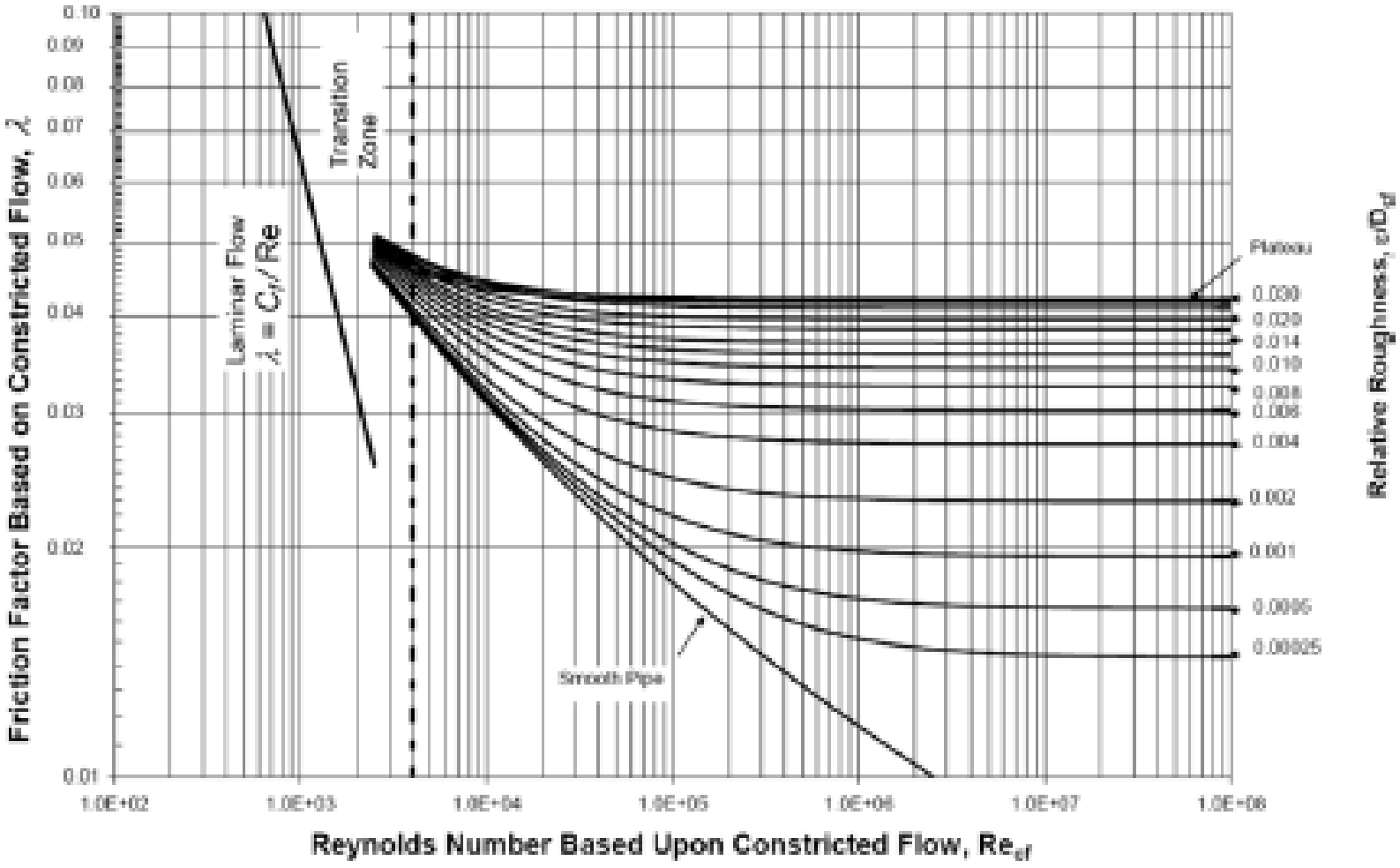
The pressure loss is approximated by the sum of individual losses consisting of fittings, bends, valves and straight pipes of length l_i

$$\Delta P = \sum_i \left(\lambda_i \frac{l_i}{d_{h,i}} + \zeta_i \right) \frac{\rho}{2} w_{ref,i}^2$$

The reference velocity $w_{ref,\,i}$ must be determined for each channel element *i*.

The channel friction factor λ of the straight pipe is determined by the flow regime and the cross section.

The pressure loss coefficient ζ_i is affected by flow internals, such as curves, bends, fittings, and other channel joints.



Flow friction factors for straight channels with a rough surface according to Moody

Channel friction factor coefficients C_f in fully developed flow through straight microchannels with different cross-sections.

Cross section, char. length		$C_f = \lambda \cdot \mathrm{Re}$	$w_{max}/\bar{w}$
circle, D		64	2.000
square, h		56.92	2.0962
rectangular, $h;b$ aspect ratio $\alpha_A = h/b$ see also Eq. 3.67		$96\ [1 - 1.3553\alpha_A + 1.9467\alpha_A^2 - 1.7012\alpha_A^3 + 0.9564\alpha_A^4 - 0.2537\alpha_A^5]$	–
slab, $\alpha \to 0$		96	1.500
hexagon		60	–
60° trapezoid $h/b =$	4.00	55.66	2.181
	2.00	55.22	2.162
	1.00	56.60	2.119
	0.50	62.77	1.969
	0.25	72.20	1.766
KOH trapezoid $h/b =$	1.00	56.15	2.137

Convective Fluid Dynamics in Microchannels

The flow in microchannels is generally regarded as straight laminar flow. This is correct for straight channels with low flow velocity and, therefore, low Re numbers.

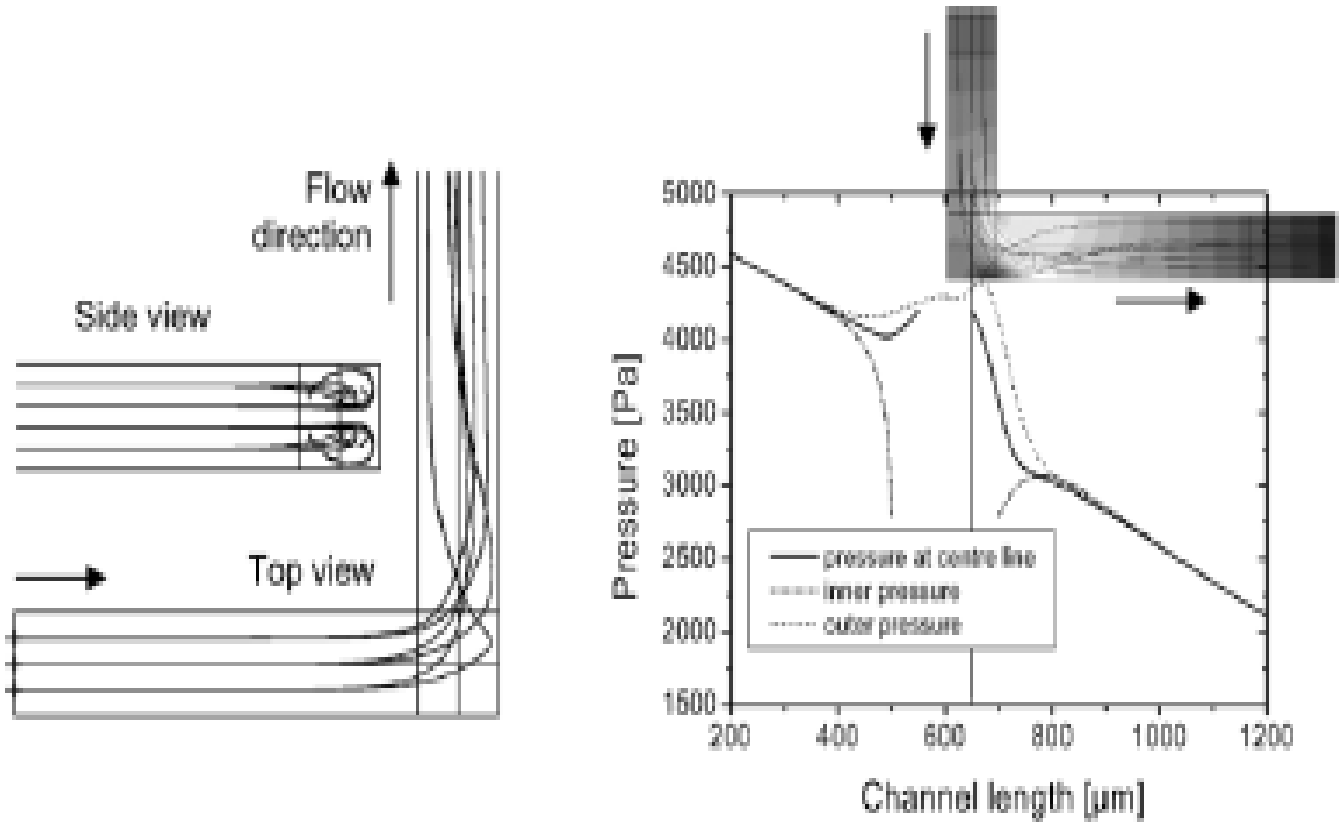
Centrifugal forces in bends push the fluid from the center of the channel, where the bulk fluid flows with high velocity, to the outward side. At the wall, the fluid is forced either upwards or downwards, producing a symmetric, double vortex filling the entire channel.

This flow regime in curved channel elements is often called Dean flow.

Dean Number = $Re\ (\ D/\ R_c\ )^{1/2}$

D = diameter of the channel, R_c is the radius of curvature of the bend

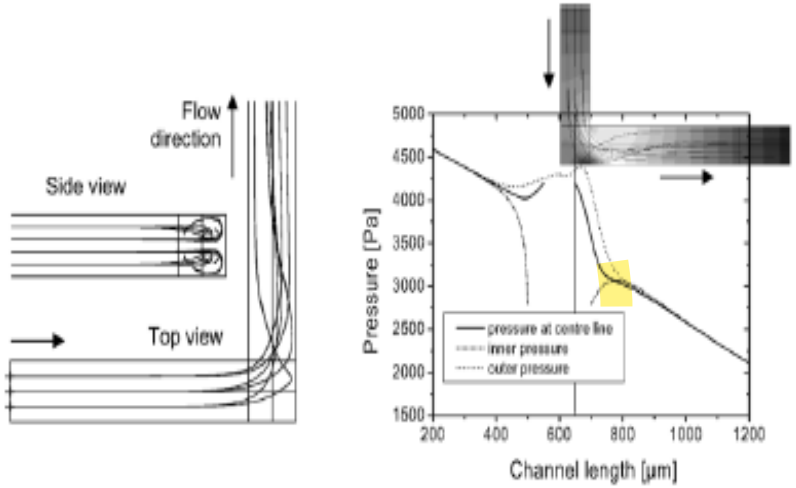
The viscous wall friction acts against the centrifugal force and dampens the vortex flow.



Left: Vortices in a 90° bend (L-mixer) with laminar vortex flow, Re = 99, $\bar{w}$ = 0.85 m/s; Right: Pressure distribution in the 90° bend at the center plane of the channels, 100×100 µm^2, Re = 99.

The investigated flow regimes are laminar with vortex formation. No onset of turbulence was observed in the bends.

In the inlet channel, a uniform pressure distribution in the cross section can be observed.



Due to the curvature of the bend the flow is altered into a new direction. At the outer side of the bend, the pressure is increased, comparable to the stagnation point of an impinging jet.

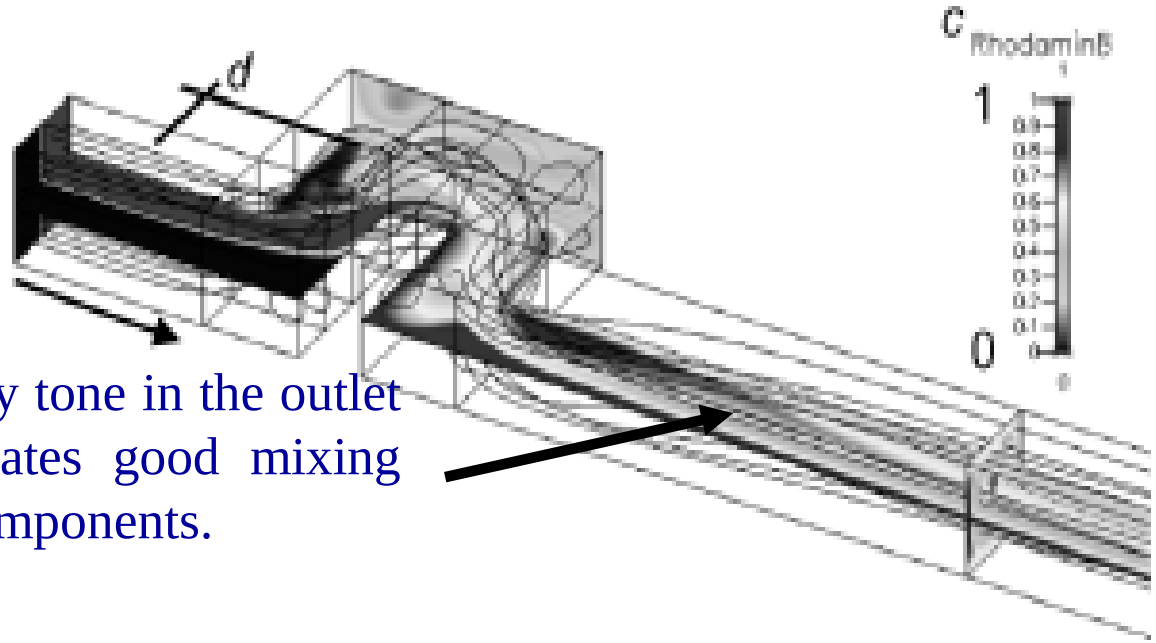
At the inner side of the bend, the pressure decreases directly at or shortly behind the sharp corner, which often results in recirculation, separation flow, or cavitation.

Approx. 100 µ*m* behind the bend, a uniform pressure establishes in the cross section.

At this point, the vortices are already dampened and straight laminar flow is established again, as shown by the streamlines in left side

Mixing Elements

U-shaped 90^{o} bend mixing element



The light gray tone in the outlet section indicates good mixing of the two components.

Streamlines and concentration profiles (Sc = 3700 for aqueous Rhodamin B solution) in a combined 90° bend mixer (U-mixer) with channel dimensions of 300×300 µm, offset d, mass flow rate of 250 g/h, (Re = 270.7)

Heat Transfer and Micro Heat Exchangers

Heat Transfer Fundamentals: The energy balance

For a process device with mass flow rate m, heat flux q over the boundary, technical work W_t, or mechanical power P, and chemical reaction, the energy equation is written as

$$\dot{Q} = \left(\dot{m}(u_2 - u_1) \right) + \left(\dot{m} \left(\frac{p_2}{\rho} - \frac{p_1}{\rho} \right) \right) + \left(\dot{m} g (z_2 - z_1) \right) + \dot{m} \left(\frac{\alpha_2 \overline{V_2^2}}{2} - \frac{\alpha_1 \overline{V_1^2}}{2} \right) + \dot{E}_q + P$$

The temporal energy change in a system consists of

- energy flowing in and out/dissipation,
- pressure,
- gravity,
- KE,
- energy produced inside the system (chemical reactions).
- technical work or mechanical power P.

Heat conduction in small systems

Fourier's law of heat transfer describes the correlation between the steady heat flow and the driving temperature difference.

$$\dot{Q} = -\lambda A(r) \frac{\mathrm{d}T}{\mathrm{d}r}$$

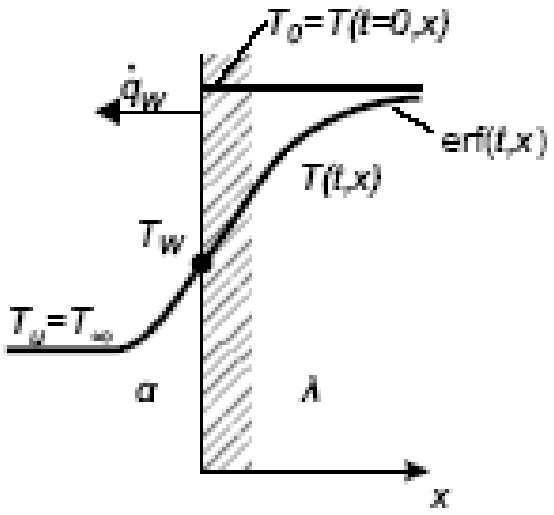
The heat conductivity in microsystems is influenced by the microstructure of the material. Grain boundaries and crystal lattices form additional resistances to heat transfer.

In regular crystals, the heat transfer coefficient is dependent on the crystal orientation, and the Fourier equation of heat transfer must be expanded to the tensor notation

The transient temperature development in a semi-infinite body, is given in one-dimensional form by

$$\frac{\partial T}{\partial t} = a\frac{\partial^2 T}{\partial x^2}; \quad t \geq 0, \; x \geq 0.$$

The introduction of the dimensionless temperature θ leads to



$$\frac{\partial \theta}{\partial t} = a\frac{\partial^2 \theta}{\partial x^2} \quad \text{with} \quad \theta = \frac{T - T_0}{T_W - T_0},$$

describing the temperature development in a solid body with defined wall temperature T_w

Temperature distribution in a semi-infinite body during cooling

With constant wall temperature T_W = const., the dimensionless temperature is determined by the error function

$$\frac{T - T_0}{T_W - T_0} = \mathrm{erf}(x^*) \quad \text{with} \quad x^* = \frac{x}{2\sqrt{at}}$$

The wall heat flux can be calculated from this equation.

For a constant α at the wall, the solution is derived with the help of two dimensionless numbers, the Fourier number (a = thermal diffusivity) and the Biot number (α = heat transfer coefficient)

$$Fo = \frac{a\,t}{x^2} \qquad Bi = \frac{\alpha\,x}{\lambda_s}$$

The temperature development during cooling of the body is given by

$$\theta_C = \frac{T - T_\infty}{T_0 - T_\infty} = \operatorname{erf}(x^*) - e^{\mathrm{FoBi}^2 + \mathrm{Bi}} \operatorname{erfc}\left(\sqrt{\mathrm{Fo}}\,\mathrm{Bi} + x^*\right)$$

and for heating $\theta_H = 1 - \theta_C.$

$$Fo = \frac{a\,t}{x^2} \qquad Bi = \frac{\alpha\,x}{\lambda_s}$$

$$t_c = a\,t\left(\frac{\alpha}{\lambda}\right)^2 = Fo.Bi^2$$

The characteristic time for heating or cooling is drawn from the combination of the Fo and Bi numbers and does not depend on the length (semi-infinite body).

Miniaturization will not influence the temperature development and the heat flux for a semi-infinite body.

Convective heat transfer in microchannels

The total heat transfer in microstructured devices consists of heat conduction through the walls and convective heat transfer from the wall into the fluid in microchannels.

For straight laminar flow, the dimensionless heat transfer coefficient, the Nu number, is constant.

For constant wall heat flux, $Nu_q = 4.3$, for constant wall temperature $Nu_T = 3.66$.

In a wide gap or narrow slit, the Nu number is 7.54 (*q* = const.) and 8.24 (*T* = const.) for double-sided heat transfer, and Nu = 4.86 (*q* = const.) and 5.39 (*T* = const.) for single-sided heat transfer.

For smaller channels, the heat transfer coefficient increases due to the constant Nu number.

With decreasing channel dimensions, the transfer area and the mass flow through the channel are also decreased, hence, the transported heat is limited by these conditions.

To maintain a high heat transfer coefficient with a high transport rate, an optimum channel dimension must be found - fabrication process.

At the entrance of a channel or behind channel elements, such as channel junctions, expansions or contractions, the disturbed flow enhances the radial transport in the channel.

This results in increased pressure loss as well as increased heat or mass transfer.

Dimensionless channel length X^*, starting at the entrance,

$$X^* = \frac{L}{d_h}\,\mathrm{Pe} = \frac{L}{d_h}\,\mathrm{Re}\cdot\mathrm{Pr}$$

Pe is the heat transfer Peclet number.

The mean Nu number in the entrance flow Nu_{me} is calculated with the mean Nu number in straight channel flow, Nu_m according to the following correlation

$$Nu_{me} = \frac{Nu_m}{\tanh\left(2.432\,Pr^{1/6}\,X^{*\,1/6}\right)}.$$

This equation is valid for the entire channel length X^* and $Pr > 0.1$

Beyond a certain length, the velocity and temperature profiles do not alter - fully-developed flow

In turbulent flow (for Re > Re_{crit} = 2300 in channel flow) the pressure loss is proportional to the square mean velocity and the heat transfer can be calculated according to Gnielinski

$$Nu = \frac{\xi/8\ (Re - 1\,000)\ Pr}{1 + 12.7\sqrt{\xi/8}\left(Pr^{2/3} - 1\right)} \left(1 + \left(\frac{d_h}{l}\right)^{2/3}\right) K_{Pr}$$

$$\text{with } \xi = (1.8 \log_{10}(Re) - 1.5)^{-2} \text{ and } K_{Pr} = (Pr_{fluid}/Pr_{wall})^{0.11}$$

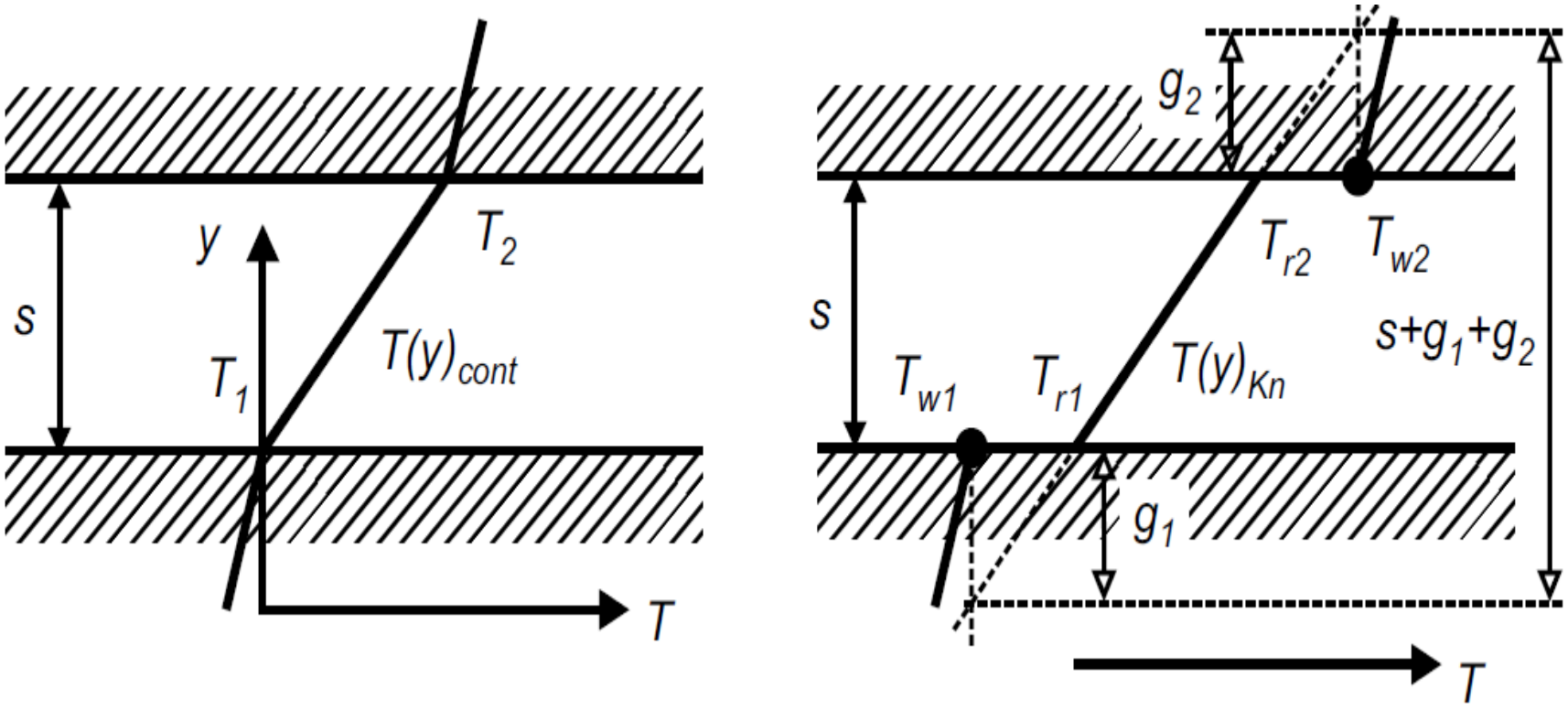
This correlation is valid for 0.5 < Pr < 2000, 2300 < Re < $5 \cdot 10^6$ and 1 < L/d_h < ∞.

Turbulent flow does not often occur in microchannels, however, the manifolds or inlet and outlet headers may produce turbulent conditions.

Surface roughness effect on heat transfer is less significant than on momentum transfer

Rarefied gases with slip boundary conditions

Temperature gradient in a gap



Left: linear development for dense gases (Kn = λ / L) < 0.01); Right: temperature jump in rarefied gases (Kn > 0.01).

Accommodation Coefficient, β

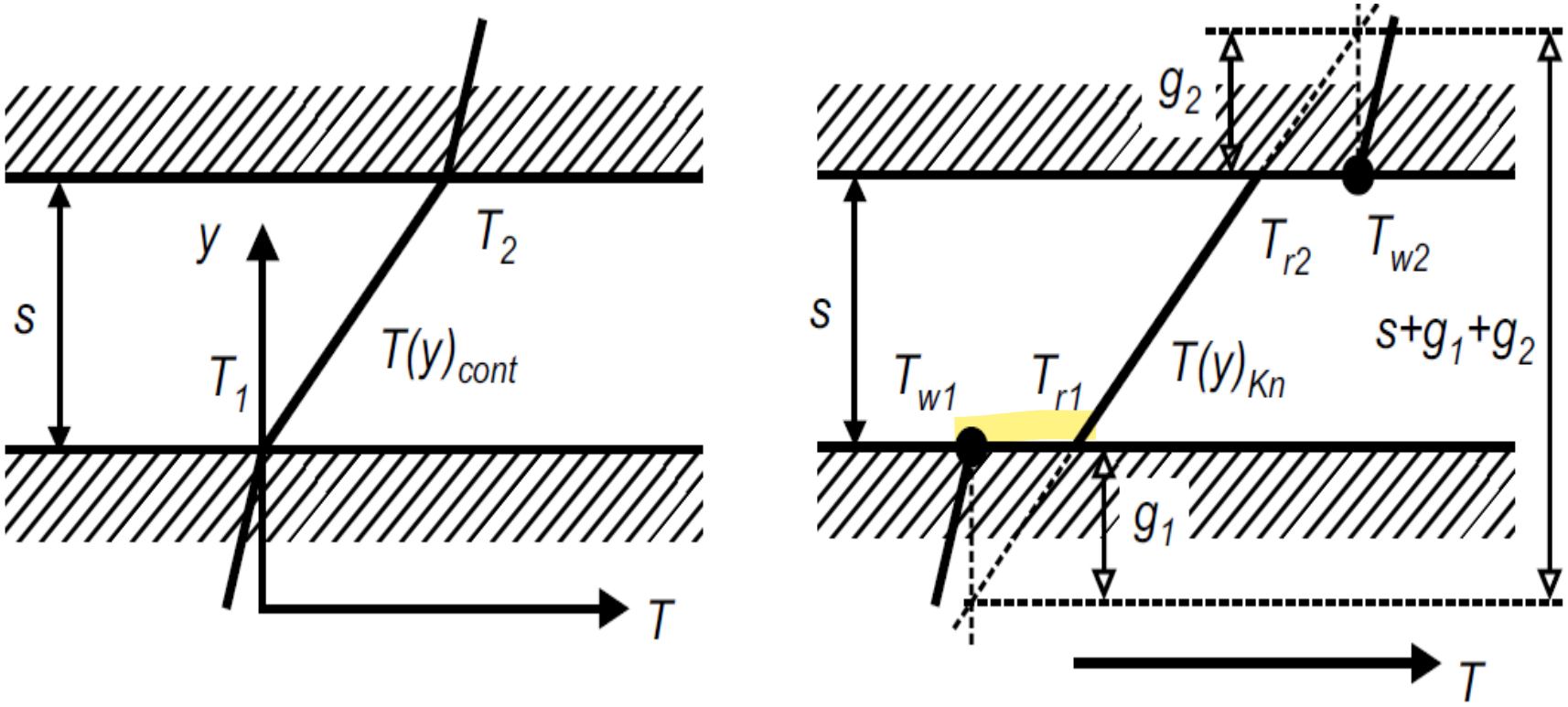
When a particle impacts with a surface, energy is transferred in the form of heat and stress, which leads to two main types of accommodation coefficients - thermal and transverse momentum.

The thermal accommodation coefficient is the fraction of heat transferred between the surface and the molecule. "The ratio of the average energy actually transferred between a surface and impinging gas molecules scattered by the surface, to the average energy which would theoretically be transferred if the impinging molecules reached complete thermal equilibrium with the surface.

Transverse Momentum Accommodation Coefficients (TMAC)

The TMAC is the fraction of the momentum normal to the wall that is transferred to the wall in terms of stress. This stress is commonly known as pressure. By creating a pressure on the wall some of the vertical momentum is lost.

Temperature gradient in a gap



left: linear development for dense gases (Kn < 0.01); right: temperature jump in rarefied gases (Kn > 0.01).

The temperature jump coefficient *g* can be regarded as an additional distance of the gap.

For 0.1 < Kn < 10, monatomic gases, and complete accommodation, the temperature gradient is only a function of the Kn number.

For rarefied gas flow (0.01 < Kn < 0.1), the boundary condition of the gas velocity at the wall is described by, (slip length ζ,)

$$w\,(x=0)=\zeta\left(\frac{\partial w}{\partial x}\right)_{y=0}$$

The slip length ζ, can be calculated with the accommodation coefficient β and the mean free path Λ of the molecules. β describes the efficiency of the momentum and energy transfer from molecules to the wall and vice versa.

$$\zeta\approx\frac{2-\beta}{\beta}\,\Lambda$$

β = 2, from Kinetic theory for continuum regime

Experimental data for β can be found in the literature.

The temperature jump at the wall is described in a similar way with the temperature jump coefficient g

$$T\,(x=0)=T_r=T_W+g\left(\frac{\partial\,T}{\partial x}\right)_{x=0}$$

The temperature jump coefficient g can be determined via kinetic theory from the thermal accommodation coefficient γ, a material parameter f, and the mean free path Λ

$$g = \frac{2-\gamma}{\gamma}\,\frac{15}{8}\,f\,\Lambda$$

The material parameter f is calculated from the linearized Boltzmann transport equation and is given by

$$f = \frac{16}{15}\,\frac{\lambda}{\eta\,c_v}\,\frac{1}{\kappa+1} = \frac{16}{15}\,\frac{1}{\mathrm{Pr}}\,\frac{\kappa}{\kappa+1}$$

λ = thermal conductivity, η = viscosity, κ = isentropic exponent = c_p/c_v

Microfluidic Networks for Heat Exchange

Transport processes in micro process engineering are governed by two different mechanisms: the conductive and the convective transfer of a species or energy.

The fluid flow in microchannels is often regarded to be laminar with dominant conductive transport.

On structured surfaces and in bent and curved flow, secondary transversal flow components are introduced into the straight laminar flow, which enhance the transport processes.

Thus, two strategies enhance the overall transport in pressure-driven flow in passive devices: the implementation of small diameter channels with a short diffusion length and the creation of secondary flow structures perpendicular to the main flow direction.

The typically low flow rates in single microstructured elements can be enlarged by internal and external numbering-up or equal-up of the desired effects.

The internal numbering-up is limited by the available space and the uniform distribution of the fluids. A relative maldistribution of the liquids of approx. 5 % will lead to a corresponding decrease in the heat transfer efficiency.

Status-quo of microfluidic networks for device cooling

Goal: to provide high thermal performance with low pressure loss.

Cooling of electronic equipment is often accomplished in long, straight channels, which produces relatively low heat transfer coefficients.

With a cost effective manufacturing and proper system integration, a heat dissipation rate of up to 10 MW/m^2 (1 kW/cm^2) appears possible for single phase liquid flow.

The setup of different branching levels to spread fluid over an area and collect it again can be managed with the **Constructal Theory of Bejan**

Constructal theory of Bejan

The constructal design approach begins with the smallest elements on zero level and connects these with those on the next higher level.

This approach is inverse to the fractal description of branched systems, where an element is repeatedly miniaturized till almost infinitely small structures.

In nature, systems have a finite smallest size, and therefore, follow the constructal approach.

Optimum size of channel elements and the corresponding area covered

Murray's law

The Physiological Principle of Minimum Work: The Vascular System and the Cost of Blood Volume, 1926, PNAS

In every segment of vessel, flow is achieved with the least possible biological work.

Two energy terms contribute to the cost of maintaining blood flow: (a) the energy required to overcome viscous drag in a fluid obeying Poiseuille's law, and (b) the energy metabolically required to maintain the volume of blood and vessel tissue involved in the flow.

The larger the radius, the smaller is the power, Pf, required for flow, but the larger is the power, Pm, required for metabolic maintenance of the blood and vessel wall tissue.

The vessel can be neither too large nor too small if the total power, Pt = Pf + Pm, is to be minimized.

Murray's law

The optimum flow distribution and cross sections of the channels on different branching levels are influenced by - Murray's law, applicable, for example, in the branching of blood vessels or plant capillaries.

If the sum of the inner radii to the power of three on each branching level is constant, the channel network will need minimal power consumption or exhibit minimal pressure loss for a given flow rate.

For channels with circular cross section, the diameter of the highest and largest level element d_n to the power of 3 is equal to the sum of the diameters d_z of the next level elements to the power of 3.

$$d_n^3 = \sum_i d_{z,i}^3$$

The level with the largest elements has the notation “n” due to n branching levels of the system. The zero level of the system is the smallest level following the notation of the constructal design method.

For a symmetrical bifurcation, the above equation gives the following correlation for the diameters on various branching levels z

$$d_n^3 = 2^z d_z^3 \quad or \quad d_z / d_n = 2^{-z/3}$$

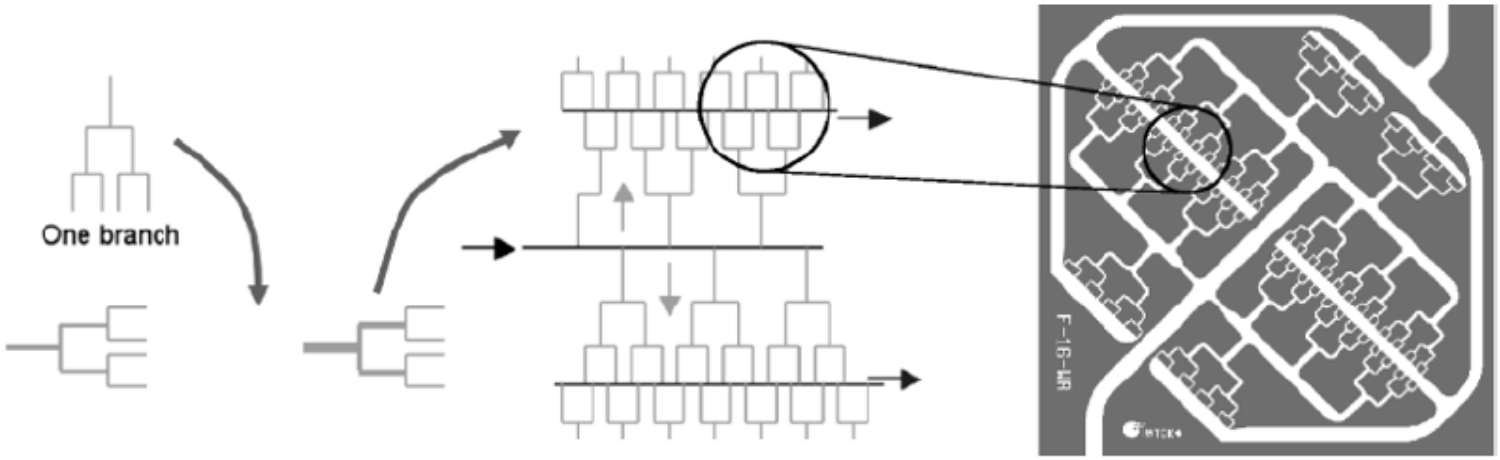
This correlation serves for an equal wall shear stress in the channels on each branching level z

Since the original derivation of Murray's law, it has been noted that the application of other optimization principles (not just that of minimum work) result in the final expression: minimizing the total mass of the network, minimizing volume for a constant pressure drop and flow rate, keeping the shear stress constant in all channels, or minimizing flow resistance for a constant volume.

Combined channel elements

Low pressure loss and a low mean driving temperature difference ΔT are essential for an optimum operation and suitable heat exchanger performance.

Both effects lead to appropriate geometrical optimization of dendritic channel networks with minimal entropy generation, presented under the concept of "Bejan's"constructal theory"



Setup of a two level heat exchanger network from branched channels (left to right)

Figure shows the combination of the branched elements from zero level to the desired covered area. The channel dimensions and the covered area are determined by the actual situation

The channel cross sections are designed according to Murray's law - the relation between the channel cross sections in different branched and connected levels.

The optimization of a channel network to a minimal pressure loss shows that the cube of the diameters of a parent channel should equal the sum of the cubes of the daughter channel diameters

This law can be derived from laminar flow in a branched circular tube system, but is also present in biological systems, such as plants and mammals.

To maintain a homogeneous fluid flow and temperature distribution, the emphasis must be focused on proper fluid distribution.

Micro Heat Exchanger Devices

The balance equations for cooling and heating, as well as the transfer correlation, are valid independently from the length scale.

Two approaches: LMTD and ε – NTU method

Typical overall heat transfer coefficients of microstructured heat exchangers range from 2.6 kW/m^2 K for gas/liquid flow to 26 kW/m^2 K for liquid/liquid flow.

Conventional plate heat exchangers exhibit very good heat transfer characteristics, ranging from 0.2 to 2.5 kW/m^2 K, under optimum conditions up to 5 kW/m^2 K for gas/liquid and liquid/liquid flow, respectively.

Design issues for exchange equipment

While microstructured heat exchangers deliver high transfer rates, they also have drawbacks, application limits, and factors to be considered during design and operation.

- Axial heat conduction in the relatively thick walls
- Equal distribution on a large number of channels
- Fouling or blocking of single passages or complete parts

SUMMARY

Microstructured equipment and related transport processes promise successful application in various fields, where high transfer rates, intelligent incorporation of microstructures in macro devices, and new process routes are needed.

High gradients and high specific surface in devices with various construction materials lead to fast equilibrium state.

The characteristic dimensions of microstructured internals are in the range of boundary layers, where high gradients enforce the transfer processes.

Cascading of effects and integration of various elements enhance and guide the entire process. Opportunities in the area of process intensification/new technologies.

Microfluidic network for flow, heat and mass transfer

Example