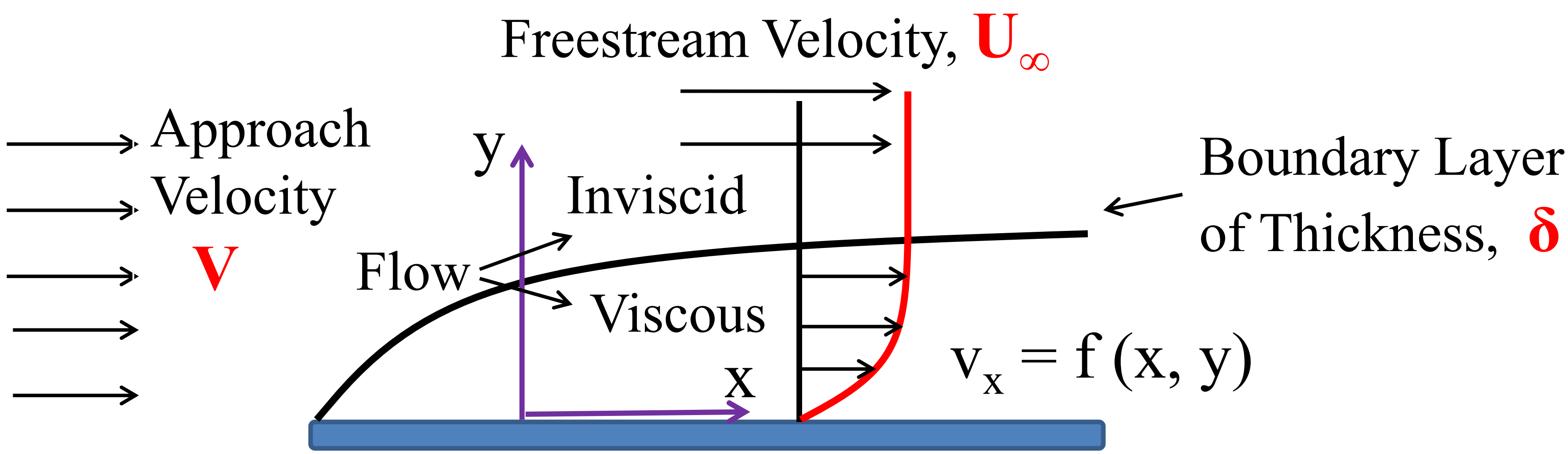
External Incompressible Viscous Flow – Boundary Layer



Flow over a flat plate

$v_x = f(x, y)$ Viscous 2D flow inside BL

$v_x = U_\infty$ Inviscid flow outside BL

$v_x = 0.99U_\infty$
at $y = \delta$

δ - boundary layer thickness

Macroscopic Balance

MACROSCOPIC BALANCE

Fox & McDonald

FOR A CV

$$\left.\frac{dN}{dt}\right|_{SYST} = \frac{\partial}{\partial t}\int_{CV} \eta \rho \, d\forall + \int_{CS} \eta \rho \, \vec{V} \cdot d\vec{A} \quad -(1)$$

$N =$ EXTENSIVE PROPERTY

$\eta = \frac{N}{M}$, INTENSIVE PROPERTY

$\frac{dN}{dt}$ = TOTAL RATE OF CHANGE OF AN ARBITRARY EXTENSIVE PROP. OF THE SYSTEM

$\frac{\partial}{\partial t}\int_{CV} \eta \rho \, d\forall$ = TIME RATE OF CHANGE OF THE EXTENSIVE PROP. 'N' WITHIN CV

$\int_{C.S} \eta \rho \, \vec{V} \cdot d\vec{A}$ = NET RATE OF EFFLUX OF THE EXTENSIVE PROPERTY 'N' THROUGH THE CONTROL SURFACES

Macroscopic Balance

$$\left.\frac{dN}{dt}\right|_{SYST} = \frac{\partial}{\partial t}\int_{CV} \eta \rho \, d\forall + \int_{CS} \eta \rho \, \vec{V}\cdot\vec{dA}$$

MASS $N = M$, $\eta = \frac{N}{M} = 1$

$$\boxed{0 = \frac{\partial}{\partial t}\int_{CV} \rho \, d\forall + \int_{CS} \rho \, \vec{V}\cdot\vec{dA}}$$

CONSERVATION OF MASS

CONTINUITY EQ^{N}.

STEADY STATE INCOMP. $\rho \int_{CS} \vec{V}\cdot\vec{dA} = 0 \Rightarrow V_1 A_1 + V_2 A_2 + \dots = 0$

$$\sum V_i A_i = 0$$

MOMENTUM $\eta = \vec{V}\frac{N}{M} = \vec{V}$ $N = \vec{P}$ $\frac{d\vec{P}}{dt}$

$$\vec{F} = \frac{\partial}{\partial t}\int_{CV} \vec{V} \rho \, d\forall + \int_{CS} \vec{V} \rho \, d\vec{A}$$

Macroscopic Balance – contd.

MOMENTUM

$N = \vec{P}$ $\quad\quad$ $P \equiv$ MOMENTUM

$$\boxed{\vec{F} = \frac{\partial}{\partial t}\int_{CV} \vec{V} \rho \, d\forall + \int_{CS} \vec{V} \rho \, \vec{V} \cdot \vec{dA}}$$

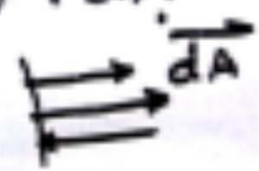
$$\boxed{\vec{F}_S + \vec{F}_B = \frac{\partial}{\partial t}\int_{CV} \vec{V} \rho \, d\forall + \int_{CS} \vec{V} \rho \, \vec{V} \cdot \vec{dA}}$$

MOMENTUM EQ^{N} ✓

→ i) ALL VELOCITIES ARE MEASURED RELATIVE TO THE CV

→ ii) MASS IN ⟶ −ve
MASS OUT ⟶ +ve.

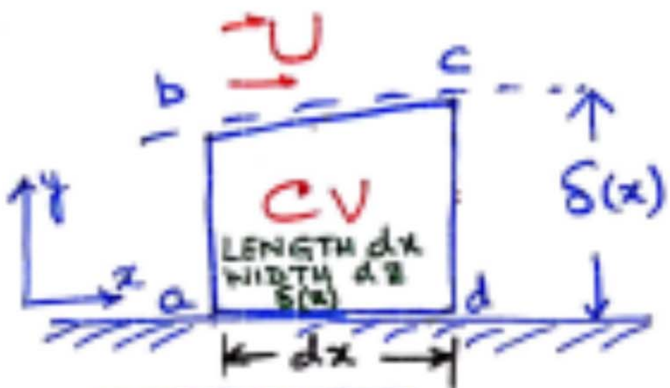
$\vec{V} \cdot \vec{dA}$



Momentum Integral Equation

MOMENTUM INTEGRAL EQUATION

CONT. EQ^N $\quad 0 = \frac{\partial}{\partial t}\int_{CV} \rho \, d\forall + \int_{CS} \rho \, \vec{V}\cdot\vec{dA}$



STEADY, 2D FLOW.

$$\int_{CS} \rho \, \vec{V}\cdot\vec{dA} = 0.$$

$$\dot{m}_{ab} + \dot{m}_{bc} + \dot{m}_{cd} + \dot{m}_{ad} = 0.$$

$$\dot{m}_{ad} \approx 0.$$

$$\boxed{\dot{m}_{bc} = -\dot{m}_{ab} - \dot{m}_{cd}.}$$

$$\int_A \rho \, \vec{V}\cdot\vec{dA}$$

$\boxed{\delta \quad f^n(x)}$

SURF ab LOCATED AT x

$$\dot{m}_{ab} = -\left\{\int_0^\delta \rho \, v_x \, dy\right\} dz. \checkmark$$

Momentum Integral Equation Contd.

$\frac{cd}{x+dx}$ $\dot{m}_{x+dx} = \dot{m}_x + \frac{\partial \dot{m}}{\partial x}\Big|_x dx.$ TAYLOR SER. EXPANSION.

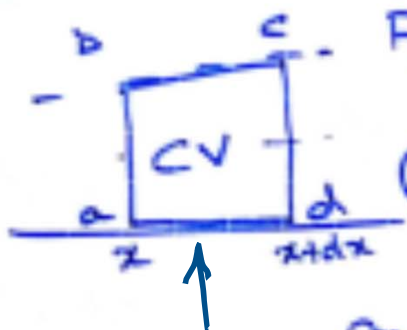
$$\dot{m}_{cd} = \left\{ \int_0^\delta \rho v_x \, dy + \frac{\partial}{\partial x}\left[\int_0^\delta \rho v_x \, dy\right] dx \right\} dz.$$

$$\boxed{\dot{m}_{bc} = -\left\{ \frac{\partial}{\partial x}\left[\int_0^\delta \rho v_x \, dy\right] dx \right\} dz}$$

Momentum Integral Equation Contd.

x COMP OF THE M^2 EQN

$$F_{Sx} + \underbrace{\cancel{F_{Bx}}}_{=0} = \underbrace{\cancel{\frac{\partial}{\partial t}}}_{\substack{=0\\ SS}}\int_{CV} v_x \rho \, d\forall + \int_{CS} v_x \rho \, \vec{V}\cdot d\vec{A}$$



$$F_{Sx} = mf_{ab} + mf_{bc} + mf_{cd}$$

$mf_{ad} = 0$ Since $v_x = 0$ AT 'ad'

ab $\quad mf_{ab} = -\left\{\int_0^{\delta} v_x \rho v_x \, dy\right\} dz$

$$mf_{cd} = mf_{ab} + \frac{\partial}{\partial x}(mf_{ab})\, dx$$

$$mf_{cd} = \left\{\int_0^{\delta} v_x \rho v_x \, dy + \frac{\partial}{\partial x}\left[\int_0^{\delta} v_x \rho v_x \, dy\right] dx\right\} dz.$$

Momentum Integral Equation Contd.

$$mf_{bc} = U \dot{m}_{bc}.$$

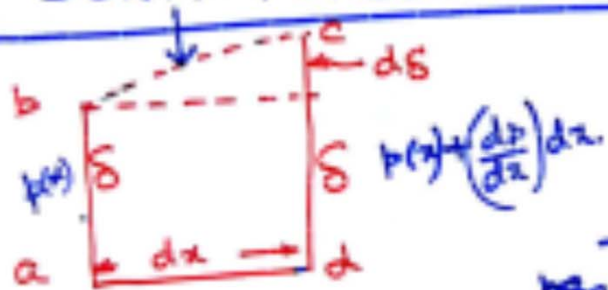
mf_{ab}

mf_{cd}

$$mf_{bc} = -U \left\{ \frac{\partial}{\partial x}\left[\int_0^\delta \rho v_x \, dy \right] dx \right\} dz.$$

$$\rightarrow \int_{cs} v_x \rho \vec{V} \cdot d\vec{A} = \left\{ \frac{\partial}{\partial x}\left[\int_0^\delta v_x \rho v_x \, dy \right] dx - U \frac{\partial}{\partial x}\left[\int_0^\delta \rho v_x \, dy \right] dx \right\} dz.$$

SURF. FORCES ON CV IN THE x DIRECTN.



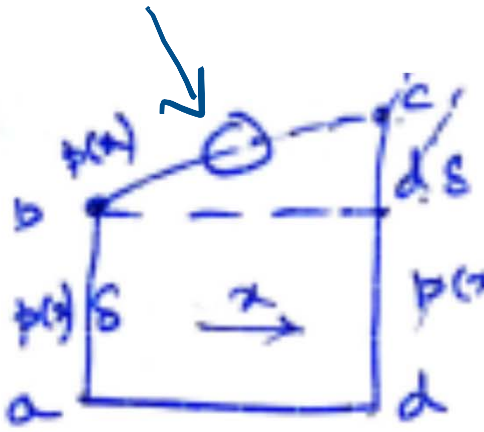
F_{S_x} → PRESSURE

F_{S_x} → SHEAR. → $\propto$ VEL. GRAD

ab: $F_{ab} = p\,\delta\, dz$

~~bc~~cd: $F_{cd} = \left(p + \frac{dp}{dx}\Big|_x dx \right)(\delta + d\delta)\, dz$

Momentum Integral Equation Contd.



$p(x) + \left(\frac{dp}{dx}\right) dx.$

Pr on BC

$$\text{Av. Pr} = p(x) + \frac{1}{2} \frac{dp}{dx}\Big| dx$$

<u>AREA</u> $\quad dS\,dz$

$$F_{bc} = \left[p(x) + \frac{1}{2} \frac{dp}{dx}\Big| dx \right] dS\,dz.$$

<u>ad</u>

$$F_{ad} = -\tau_w \, dx\, dz$$

τ_w = WALL SHEAR STRESS

Momentum Integral Equation Contd.

MI EQN

$$\boxed{\frac{\tau_w}{\rho} = \frac{d}{dx}\left(U^2\theta\right) + \delta^* U \frac{dU}{dx}.}$$

$$\theta = \int_0^\delta \frac{v_x}{U}\left(1-\frac{v_x}{U}\right)dy \qquad \delta^* = \int_0^\delta \left(1-\frac{v_x}{U}\right)dy$$

i) ODE

ii) WALL SHEAR STRESS < TURB. ✓ / LAM. ✓

Use of Momentum Integral Equation

USE OF MI EQ^{N} — ZERO PR. GRAD FLOW.

FLOW OVER A FLAT PLATE

$$\frac{\tau_w}{\rho} = \frac{d}{dx}(U^2\theta) + \delta^* U \cancel{\frac{dU}{dx}}$$

$= 0.$

U IS A CONST.

$$\frac{\tau_w}{\rho} = U^2 \frac{d}{dx}\theta$$

$$\frac{\tau_w}{\rho} = U^2 \frac{d\theta}{dx}.$$

$$\tau_w = \rho U^2 \frac{d}{dx}\int_0^\delta \frac{v_x}{U}\left(1-\frac{v_x}{U}\right)dy$$

Use of Momentum Integral Equation

$$\tau_w = \rho U^2 \frac{d\delta}{dx}\int_0^1 \frac{v_x}{U}\left(1-\frac{v_x}{U}\right) d\eta \qquad \frac{y}{\delta} = \eta$$

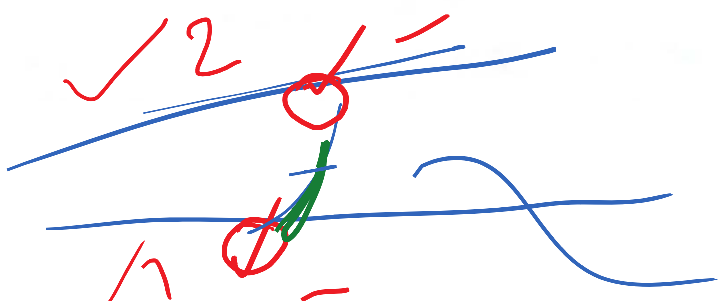
$$\tau_w = \rho U^2 \frac{d\delta}{dx}\left[\int_0^1 \frac{v_x}{U}\left(1-\frac{v_x}{U}\right) d\eta\right] \qquad dy = \delta\, d\eta$$

$$\boxed{\tau_w = \rho U^2 \frac{d\delta}{dx}\cdot \beta} \qquad \beta = \text{CONST.}$$

$$\boxed{\frac{v_x}{U} = f(\eta)}$$

$$\frac{v_x}{U} = a + b\eta + c\eta^2$$

AT	$y = 0$	$v_x = 0.$	$\eta = 0 \quad \frac{v_x}{U} = 0$
AT	$y = \delta$	$v_x = U$	$\eta = 1 \quad \frac{v_x}{U} = 1$
AT	$y = \delta$	$\frac{\partial v_x}{\partial y} = 0.$	$\eta = 1 \quad \frac{d(v_x/U)}{d\eta} = 0$



$$\frac{v_x}{U} =$$

Use of Momentum Integral Equation

$$\frac{v_x}{U} = 2\eta - \eta^2 \qquad \eta = y/\delta.$$

$$\frac{\tau_w}{\rho} = U^2 \frac{d}{dx}\theta =$$

$$\tau_w = \rho U^2 \frac{d\delta}{dx}\left[\int_0^1 \frac{v_x}{U}\left(1-\frac{v_x}{U}\right)d\eta\right]$$

NEWTONIAN FLUID.

$$\tau_w = \mu \left.\frac{\partial v_x}{\partial y}\right|_{y=0} = \frac{2\mu U}{\delta} = \rho U^2 \frac{d\delta}{dx}\left[\int_0^1 (2\eta-\eta^2)(1-2\eta+\eta^2)\,d\eta\right]$$

$$\frac{2\mu}{\delta \rho U} = \frac{2}{15}\frac{d\delta}{dx}$$

Use of Momentum Integral Equation

$$\frac{\delta^2}{2} = \frac{15\mu}{\rho U} x + C.$$

AT $x = 0$ $\delta = 0 \Rightarrow C = 0.$

$$\boxed{\frac{\delta}{x} = \frac{5.48}{\sqrt{Re_x}}}$$

BLASIUS SOL^N $\frac{\delta}{x} = \frac{5.0}{\sqrt{Re_x}}$

$$c_f = \frac{\tau_w}{\frac{1}{2}\rho U^2} = \frac{2\mu(U/\delta)}{\frac{1}{2}\rho U^2} = \frac{4\mu}{\rho U \delta}.$$

$$c_f = \frac{0.73}{\sqrt{Re_x}} \qquad c_f = \frac{0.664}{\sqrt{Re_x}}$$

A flat plate is installed in a water tunnel as a splitter The plate is 0.3m long and 1m wide. The freestream speed is 2 m/s Laminar boundary layer forms on both sides of the plate. The boundary layer velocity profile is approximated by

$$\checkmark \quad \frac{v_x}{U} = 2\left(\frac{y}{\delta}\right) - \left(\frac{y}{\delta}\right)^2$$

Obtain an expression for δ. Determine the total viscous drag force on the plate assuming the pressure drag is negligible ($\gamma = 1 \times 10^{-6}$ m²/s)

$$\checkmark \quad \tau_w = \rho U^2 \frac{d\delta}{dx} \int_0^1 \frac{u}{U}\left(1 - \frac{u}{U}\right) d\eta$$

$$\checkmark \quad \frac{\delta}{x} = \frac{5.48}{\sqrt{Re_x}} \ ?$$

$$F_D = 2\int_0^L \tau_w \, b \, dx$$

$$= 2\int_0^L \frac{2\mu U}{\delta} \, b \, dx$$

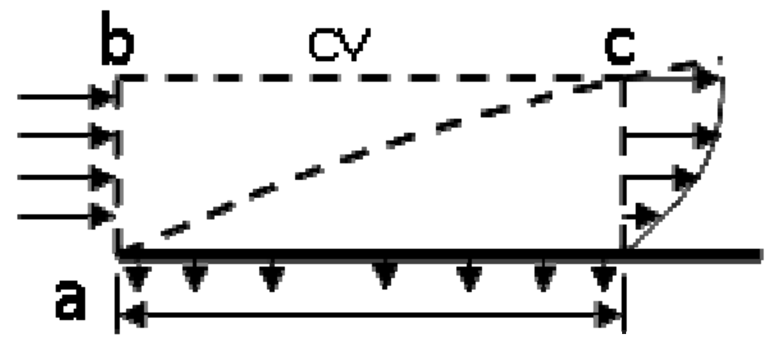
$$= \frac{8}{5.48} \, b \mu U \sqrt{\frac{\rho U L}{\mu}}$$

$$F_D = 2.26 \text{ N}$$

Consider the steady flow of water past a porous plate with a constant suction velocity of 0.2 mm/s (i.e., V = - 0.2j mm/s). A thin boundary layer grows over the flat plate and the velocity profile at section cd is

$$\frac{u}{U_\infty} = \frac{3}{2}\frac{y}{\delta} - 2\left(\frac{y}{\delta}\right)^{1.5}$$

where U_∞ is the velocity of approach at section ab and is equal to 3 m/s. Find the mass flow rate across section bc. Given: width of the plate = 1.5m, length = 2m, δ_{cd} = 1.5 mm



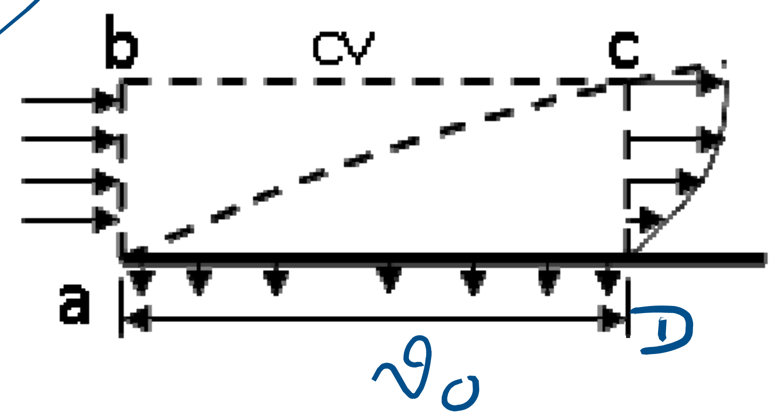
$$\frac{u}{U_\infty} = \frac{3}{2}\frac{y}{\delta} - 2\left(\frac{y}{\delta}\right)^{1.5}$$

$\dot{m}_{ab} = -\rho \delta w U_\infty$

$\dot{m}_{bc} = ?$

$\dot{m}_{cD} = +\int_0^\delta u\, w\, dy\, \rho$

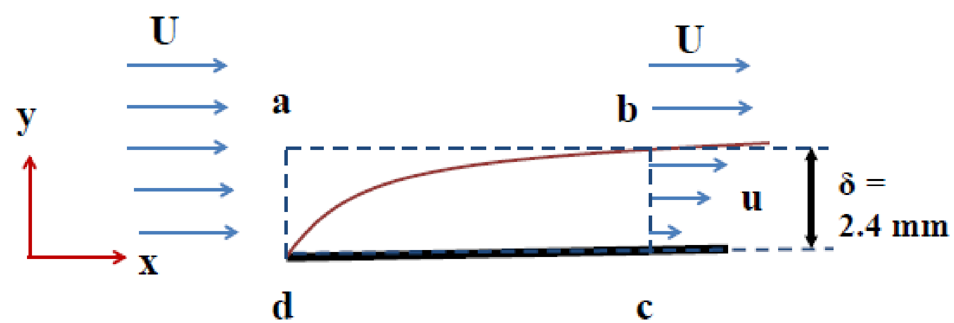
$\dot{m}_{Da} = +\rho v_0 w L$



$0 = \dot{m}_{ab} + \dot{m}_{bc} + \dot{m}_{cd} + \dot{m}_{da}$

$\underline{\dot{m}_{bc}} = 1.042 \text{ kg/s}$

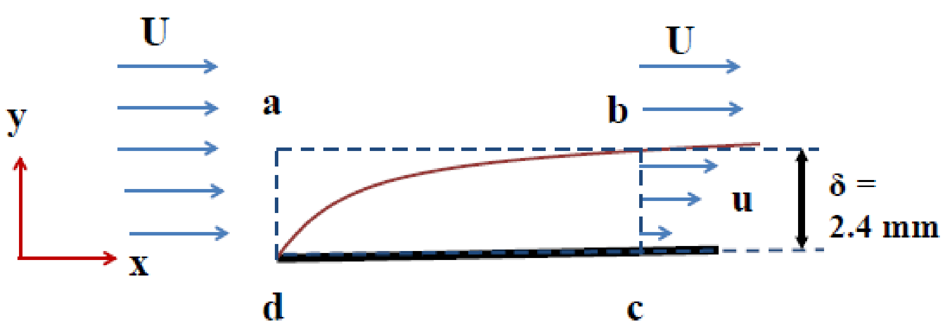
Air at standard conditions is flowing over a thin flat plate which is 1 m long and 0.3 m wide. The flow is uniform at the leading edge of the plate. The velocity profile in the boundary layer is assumed to be linear and U = 30 m/s. Assume that the flow conditions are independent of z and treat the flow as two dimensional. Using the control volume abcd (dc = 1m), compute the mass flow rate across surface ab. The boundary layer thickness at the end of the plate (point c in the figure) is $\delta = 2.4$mm. Determine the magnitude and direction of the x-component of the force required to hold the plate stationary (ρ air = 1.23 kg/m^3).

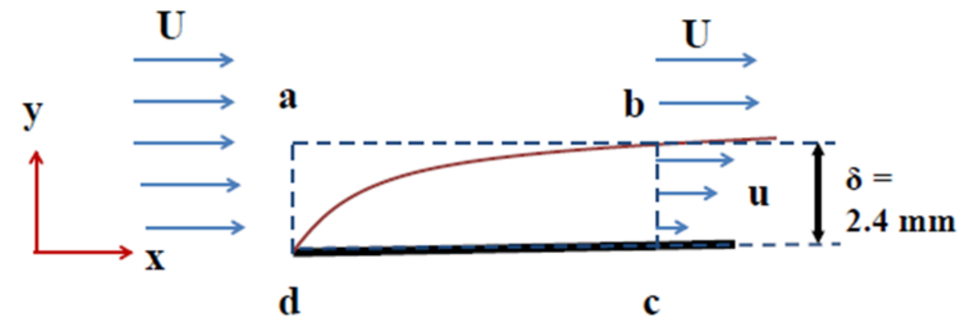


$$\dot{m}_{ab} + \dot{m}_{bc} + \dot{m}_{da} = 0$$

$$0 = \underbrace{\left\{ -\rho U b \delta \right\}}_{\dot{m}_{ad}} + \underbrace{\int_0^{\delta} \rho u b \, dy}_{\dot{m}_{bc}} + \dot{m}_{ab}$$

$$\dot{m}_{ab} = 0.0133 \text{ kg/s}$$





$$R_x = \dot{m}f_{ad} + \dot{m}f_{ab} + \dot{m}f_{bc}$$

CONVEC TIVE M^2

$$= U_{da}\{-\rho U b \delta\} + u_{ab}\,\dot{m}_{ab} + \int_0^{\delta} \rho u^2 b\, dy$$

?

$= U$

$\cancel{R_x = }$ $R_x = -0.133\ N$

$R_x -$

$R_x = -0.133N$

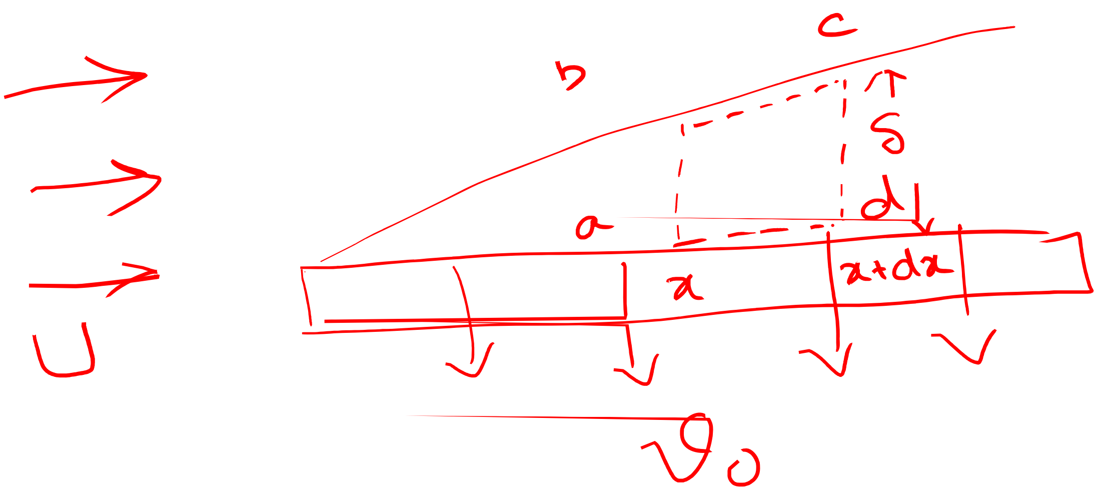
R_x = FORCE **ON** THE CV

FORCE **ON** THE PLATE DUE TO FLOW = $-R_x$

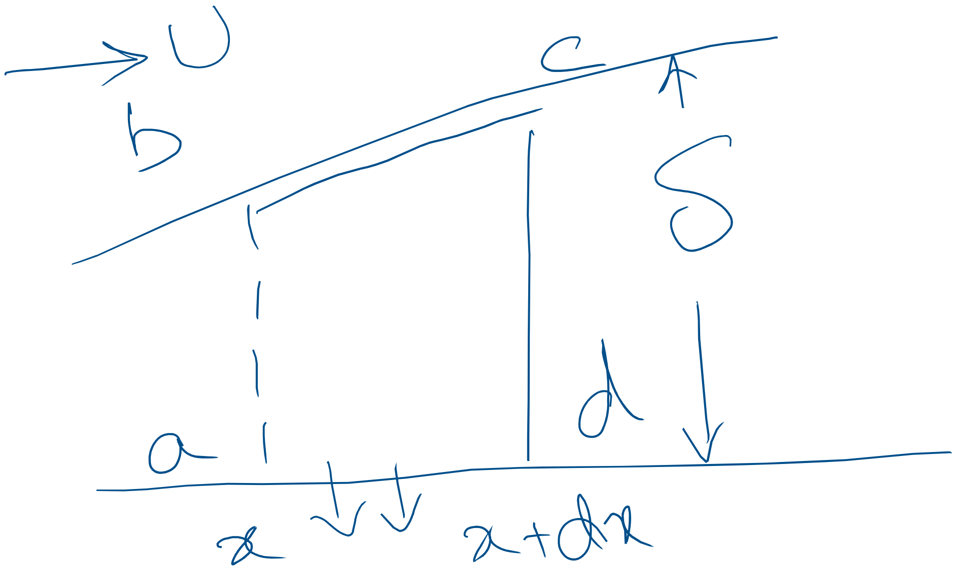
FORCE TO BE APPLIED TO THE PLATE TO KEEP IT FIXED $= R_x$

$= -0.133N$

Consider horizontal, steady, incompressible flow in a boundary layer with uniformly distributed wall suction. The wall suction velocity is constant with $v = - v_o$ at $y = 0$. There is no pressure gradient. Use a differential control volume to express the axial gradient of momentum thickness ($d\theta/dx$) in terms of the wall shear stress (τ_w), v_o, freestream velocity (U) and the density of the fluid (ρ).



$$\frac{d\theta}{dx} = f(\tau_w, v_o, U, \rho)$$



$$\theta = \int_0^{\delta} \frac{u}{U}\left(1 - \frac{u}{U}\right) dy$$

?

CONT.

$$0 = \dot{m}_{ab} + \dot{m}_{cd} + \dot{m}_{bc} + \dot{m}_{da}$$

$$\dot{m}_{cd} = \dot{m}_{ab} + \frac{\partial}{\partial x}\left(\dot{m}_{ab}\right)\Delta x$$

$$\dot{m}_{ab} = -\int_0^{\delta} \rho\, u\, w\, dy$$

$$\dot{m}_{cD} = \text{TAYLOR SER.}$$

$$\dot{m}_{ad} = +\rho\, v_0\, w\, L$$

$$\dot{m}_{bc} = ?$$

$$\dot{m}_{bc} = \checkmark \quad \text{Exbc}$$

m^2 EQN

$$\dot{m}f_{ab} = -\int_0^\delta \rho u^2 w \, dy$$

$$\dot{m}f_{cd} = \text{TAYLOR SER } \dot{m}_{ab}$$

$$\dot{m}f_{bc} = \dot{m}_{bc} \times U$$

$$mf_{ad} = \tau_w \, w \, dx$$

Turbulent Flow

TURBULENT FLOW

FULLY DEV STEADY FLOW

$$0 = -\frac{\partial P}{\partial z} + \frac{1}{r}\frac{\partial}{\partial r}(r\tau_{rz}).$$

Z COMP. OF EQ^N OF MOTION

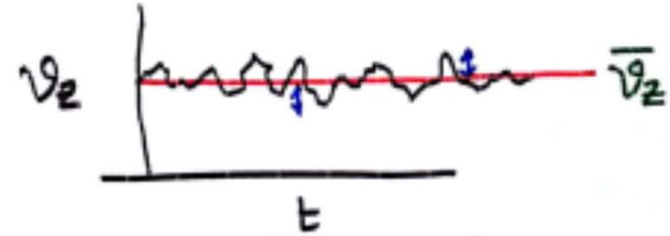
$$\tau_{rz} = \frac{r}{2}\frac{\partial P}{\partial z} + \boxed{C}$$

AT $r=0$ $\tau_{rz}=0 \Rightarrow C=0$

$$\tau_{rz} = \frac{r}{2}\frac{\partial P}{\partial z}$$

$$\tau_W = -\tau_{rz}\Big|_{r=R} = -\frac{R}{2}\frac{\partial P}{\partial z}$$

VALID BOTH FOR LAM. & TURB FLOW ✓



$$v_z = \bar{v}_z + v_z'.$$

v_z: INSTANTANEOUS VEL.

$\bar{v}_z$: TIME SMOOTHED VEL.

v_z': FLUCTUATING COMPONENT OF VEL.

$$\overline{v_z'} = 0 \qquad \overline{(v_z')^2} \neq 0$$

$$\frac{\sqrt{\overline{(v_z')^2}}}{\langle \bar{v}_z \rangle} = \text{MEASURE OF TURBULENCE}$$

$\tau_w = \,?$

$\Downarrow$

$$\boxed{\frac{d\theta}{dx} = \frac{\tau_w}{\rho U^2} - \frac{v_0}{U}}$$

Turbulent Flow

TURBULENT FLOW

FULLY DEV STEADY FLOW

$$0 = -\frac{\partial P}{\partial z} + \frac{1}{r}\frac{\partial}{\partial r}(r\tau_{rz}).$$

Z COMP. OF EQ^N OF MOTION

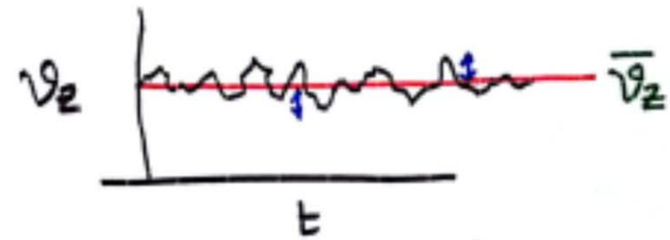
$$\tau_{rz} = \frac{r}{2}\frac{\partial P}{\partial z} + \boxed{C}$$

AT $r=0$ $\tau_{rz}=0 \Rightarrow C=0$

$$\tau_{rz} = \frac{r}{2}\frac{\partial P}{\partial z}$$

$$\tau_W = -\tau_{rz}\Big|_{r=R} = -\frac{R}{2}\frac{\partial P}{\partial z}$$

VALID BOTH FOR LAM. & TURB FLOW



$$v_z = \bar{v}_z + v_z'$$

INSTANTANEOUS VEL. / TIME SMOOTHED VEL / FLUCTUATING COMPONENT OF VEL.

$\overline{v_z'} = 0$ $\overline{(v_z')^2} \neq 0$

$$\frac{\sqrt{\overline{(v_z')^2}}}{\langle \bar{v}_z \rangle} = \text{MEASURE OF TURBULENCE}$$

Turbulent Flow Contd

Using in NS equation

x COMP.

$$\frac{\partial}{\partial t}\rho(\bar{v}_x + \cancel{v'_x}) = -\frac{\partial}{\partial x}(\bar{p} + \cancel{p'}) - \left[\frac{\partial}{\partial x}\left(\rho(\bar{v}_x + v'_x)(\bar{v}_x + v'_x)\right) + \frac{\partial}{\partial y}\rho(\bar{v}_y + v'_y)(\bar{v}_x + v'_x) + \frac{\partial}{\partial z}(\bar{v}_z + v'_z)(\bar{v}_x + v'_x)\rho\right] + \mu\nabla^2(\bar{v}_x + v'_x) + \rho g_x.$$

$\overline{v'_x v'_y} \neq 0$

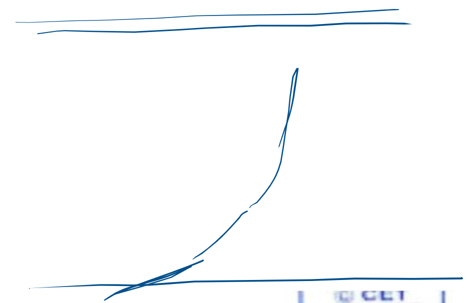
$\overline{v'_x} = 0 \quad \overline{v'_y} = 0$

TAKE TIME AVERAGE

$$\frac{\partial}{\partial t}\rho\bar{v}_x = -\frac{\partial \bar{p}}{\partial x} - \left(\frac{\partial}{\partial x}\rho\bar{v}_x\bar{v}_x + \frac{\partial}{\partial y}\rho\bar{v}_y\bar{v}_x + \frac{\partial}{\partial z}\rho\bar{v}_z\bar{v}_x\right) + \mu\nabla^2\bar{v}_x + \rho g_x - \left(\frac{\partial}{\partial x}\rho\overline{v'_x v'_x} + \frac{\partial}{\partial y}\rho\overline{v'_x v'_y} + \frac{\partial}{\partial z}\rho\overline{v'_x v'_z}\right)$$

REYNOLD'S STRESSES

Turbulent Flow Contd



Universal velocity profile

VISCOUS SUB LAYER

$$v_z^+ = \frac{\overline{v_z}}{v_*} = \frac{y v_*}{\nu} = y^+ \quad \boxed{0 \le y^+ \le 5}$$

$$v_* = \sqrt{\frac{\tau_w}{\rho}} = \text{FRICTION VEL}$$

TRANSITION REGION

$$\frac{\overline{v_z}}{v_*} = v_z^+ = 2.5 \ln \frac{y v_*}{\nu} + 5.0, \quad \boxed{5 < y^+ < 26}$$

TURB. CORE

$$v_z^+ = \frac{1}{0.36} \ln y^+ + 3.8, \quad \boxed{y^+ \ge 26}$$

Turbulent Flow Contd

Universal velocity profile

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POWER LAW EQ^N EMPIRICAL

$$\frac{\bar{v}_z}{U} = \left(\frac{y}{R}\right)^{1/7}$$

CENTRELINE VEL

DIST. FROM THE PIPE WALL

RAD.

$\boxed{Re \sim 10^4 - 10^5}$

$1/7$ th POWER LAW PROFILE

$$\frac{\bar{V}}{U} = \frac{2n^2}{(n+1)(2n+1)} \qquad n \sim 1/7 \qquad \frac{\bar{V}}{U} = 0.8 \quad \Big| \quad 0.5$$

$$\frac{y}{R} < 0.04 \longrightarrow$$ INFINITE VEL. GRAD AT THE WALL

DOES NOT GIVE ZERO SLOPE AT THE CENTRE LINE

$$\frac{\tau_w}{\rho} = U^2 \frac{d\delta}{dx} \int_0^1 \frac{u}{U}\left(1 - \frac{u}{U}\right) d\eta$$

Turbulent Boundary Layers

BOUNDARY LAYERS IN TURBULENT FLOW

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$$\frac{v_x}{U} = \left(\frac{y}{\delta}\right)^{1/7} = \eta^{1/7}$$

MI EQN

$$\frac{\tau_w}{\rho} = U^2 \frac{d\theta}{dx}$$

$$\tau_w = -\frac{R}{2}\frac{\partial p}{\partial z}.$$

$$\frac{p_1 - p_2}{\rho} \equiv \frac{\Delta P}{\rho} = h \quad \left(\begin{array}{l} h = \text{HEAD LOSS} \\ \text{FOR A HORIZONTAL} \\ \text{PIPE} \end{array}\right)$$

$$h = f \frac{L}{D} \frac{\bar{v}^2}{2} \quad (\bar{v} = \text{AV. FLOW VEL.})$$

$$\tau_w = -\frac{R}{2}\frac{\partial p}{\partial z} = +\frac{R}{2}\frac{\Delta P}{L} = \frac{R}{2L}\rho h.$$

$$\tau_w = \frac{R}{2L}\rho f \frac{L}{D}\frac{\bar{v}^2}{2} = \frac{1}{8}\rho f \bar{v}^2$$

BLASIUS CORRELATION

$$f = \frac{0.3164}{Re^{0.25}}$$

$$\tau_w = 0.03325\,\rho \bar{v}^2 \left(\frac{\nu}{R\bar{v}}\right)^{0.25}$$

USING 1/7th POWER LAW

$$\boxed{\tau_w = 0.0225\,\rho U^2 \left(\frac{\nu}{U\delta}\right)^{1/4}}$$

Turbulent Boundary Layers

MI eqn

$$0.0225\left(\frac{\nu}{U\delta}\right)^{1/4} = \frac{d\delta}{dx}\int_0^1 \eta^{1/7}\left(1-\eta^{1/7}\right)d\eta = \frac{7}{72}\frac{d\delta}{dx}$$

$$\rightarrow \frac{4}{5}\delta^{5/4} = 0.23\left(\frac{\nu}{U}\right)^{1/4} x + c$$

BC $\boxed{x = 0 \quad \delta = 0} \Rightarrow c = 0.$

$$\boxed{\delta = 0.37/(Re_x)^{1/5}}$$

$$\boxed{c_f = \frac{\tau_w}{\frac{1}{2}\rho U^2} = 0.045\left(\frac{\nu}{U\delta}\right)^{1/4} = \frac{0.0577}{(Re_x)^{1/5}}}$$

$$5\times10^5 < Re_x < 10^7$$

Transition from laminar to turbulent boundary layer flow actually occurs over a finite length of surface, during which the velocity profile and wall shear stress adjust from laminar to turbulent forms. A useful approximation during transition is that the momentum thickness of the boundary layer remains constant. Assuming constant momentum thickness, evaluate the ratio of $\delta_{turbulent}$ to $\delta_{laminar}$ for transition from a parabolic laminar velocity profile to a 1/7 - power turbulent velocity profile.

✓ $$\frac{u}{U} = \frac{2\eta - \eta^2}{1} \qquad \eta \equiv y/\delta$$

$$\frac{\theta}{\delta} = \int_0^1 \frac{u}{U}\left(1 - \frac{u}{U}\right) d\eta$$

$$\frac{\theta}{\delta_{lam}} = \int_0^1 (2\eta - \eta^2)(1 - 2\eta + \eta^2)\, d\eta = \frac{2}{15}$$

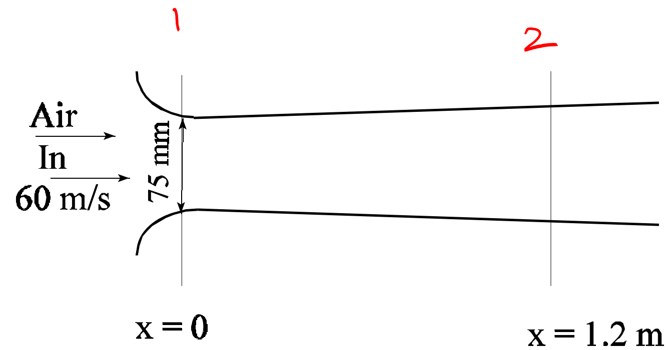
$$\frac{\theta}{\delta_{turb}} = \int_0^1 \eta^{1/7}\left(1-\eta^{1/7}\right)d\eta \qquad \frac{u}{U} = \eta^{1/7}$$

$$= 7/72$$

θ CONST.

$$\frac{\delta_t}{\delta_{lam}} = 144/105$$

A uniform flow of standard air ($\mu/\rho = 1.5\text{x}10^{-5}\ \text{m}^2/\text{s}$) at 60 m/s enters a plane wall diffuser with negligible boundary layer thickness at the inlet. The inlet width is 75 mm. The diffuser walls diverge slightly to accommodate the boundary layer growth so that the pressure gradient is negligible. Flat plate boundary-layer behavior may be assumed for each plate. Explain why the Bernoulli equation is applicable to this flow. Estimate the diffuser width 1.2m downstream from the entrance for this condition.



$$\cancel{\frac{p_1}{\rho}} + \frac{V_1^2}{2} = \cancel{\frac{p_2}{\rho}} + \frac{V_2^2}{2}$$

$$\Rightarrow V_1 = V_2$$

$$\rho_1 v_1 A_1 = \rho v_2 A_{2\,eff} \quad ?$$

$$A_{2\,eff} = (w_2 - 2\delta_2^*)\, b$$

b

depth

$v_1 = v_2$

$$(w_2 - 2\delta_2^*)\, b = w_1\, b$$

$$\frac{\delta}{L} = \frac{0.37}{Re_L^{1/5}}$$

$\delta_2^* = ?$ TURB / LAM.

$Re = 4.8 \times 10^6 \rightarrow$ TURB

REL. FOR δ_{turb} at 1.2m

$\delta_2 = 0.02$ m $\leftarrow$

$$\delta^* = \int_0^{\delta} \left(1 - \frac{u}{U}\right) dy$$

$$\uparrow \text{TURB} = \eta^{1/7}, \quad \eta \equiv y/\delta$$

$$\delta^* = 2.56\,\text{mm}$$

$$W_2 = \cancel{W +} W_1 + 2\delta_2^*$$

$$= 80.1\,\text{mm}$$

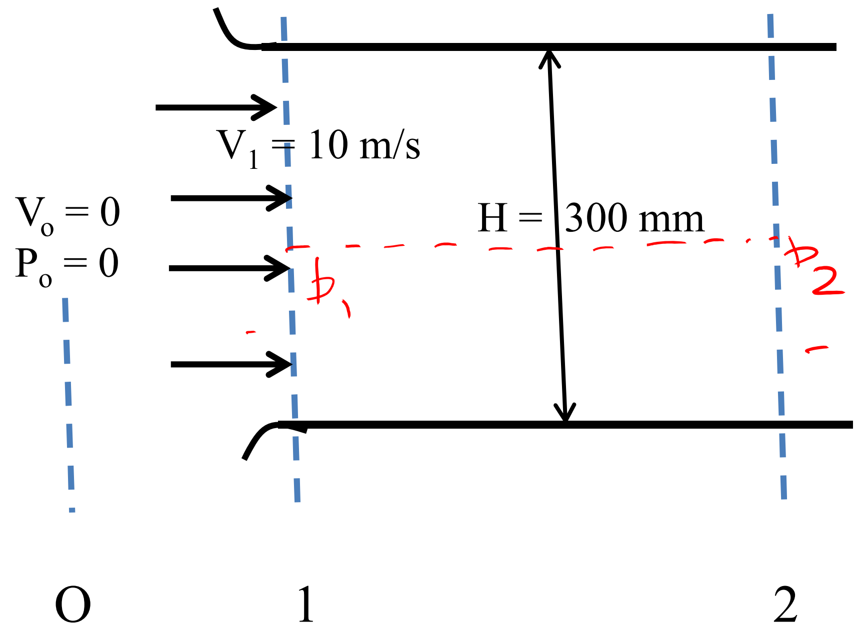
Flow of air (= 123 kg/m3) develops in a flat horizontal duct following a well rounded entrance section. The duct height is H = 300 mm. Turbulent boundary layer grows on the duct walls, but the flow is not yet fully developed. Assume that the velocity profile in each boundary layer is given by $u/U = (y/\delta)^{1/7}$. The inlet flow is uniform with V = 10 m/s at section 1. At section 2, the boundary layer thickness on each wall of the channel is δ_2 = 100 mm.

(a) Show that for this flow, $\delta^* = \delta/8$.

(b) Evaluate the static gage pressure at section 2

(c) Find the average wall shear stress between the entrance and section 2 at L = 5 m.

The region outside the entrance (location O in the figure) an be taken to be still air with pressure equal to the atmospheric pressure



$$\delta^* = \delta/8$$

$$\delta^* = \delta \int_0^1 \left(1 - \frac{u}{U}\right) d\eta = \delta \int_0^1 \left(1 - \eta^{1/7}\right) d\eta$$

$$\frac{\delta^*}{\delta} = \frac{1}{8}$$

$$V_1 A_1 = V_1 W H = V_2 A_2$$

$$V_1 A_1 = V_2 W \left(H - 2\delta_2^*\right)$$

$$V_2 = 10.9 \text{ m/s}$$

$$\frac{p_0}{\rho} + \cancel{\frac{v_0^2}{2}} = \frac{p}{\rho} + \frac{v^2}{2}$$

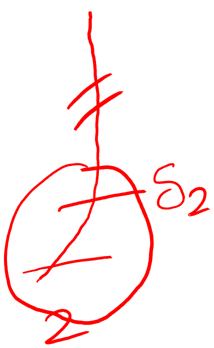
$$p_{1g} = p_1 - p_0 = -\frac{1}{2}\rho v_1^2 = -6105 \text{ Pa}$$

$$p_{2g} \rightarrow -73 \text{ Pa}.$$

Apply M^2 eq^n

$$F_{sx} = \int_{CS} u \rho \vec{V} \cdot d\vec{A}$$

$$(p_1 - p_2) w \frac{H}{2} - \tau w L = V_1 \left\{ -\rho V_1 \frac{H}{2} w \right\}$$



$$+ v_2 \left\{ \rho v_2 \left(\frac{H}{2} - \delta_2 \right) w \right\}$$

$$+ \int_0^{\delta_2} u \rho u w \, dy = (A)$$

$$A = \rho V_2^2 \delta_2 w \int_0^1 \eta^{2/7}\, d\eta$$

1/7th Power Law

$$A = \rho V_2^2 \frac{7}{9} \delta_2 w$$

$$\tau = 0.3\ N/m^2$$