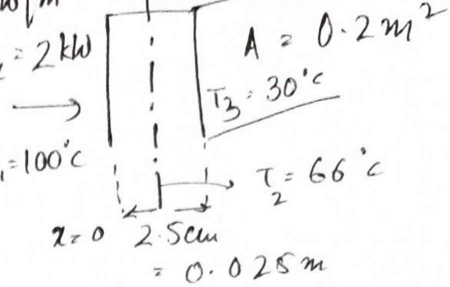
(a)

$Q_x = -KA \frac{dT}{dx}$ $\qquad q_x = \frac{2}{0.2} = 10\ kW/m^2 = 10\times10^3\ W/m^2$ $\qquad Q_x = 2\ kW$



$q_x\, dx = -k\, dT \qquad k = k_0(1+\beta T)$

$q_x \int_{x=0}^{0.025} dx = -k_0 \int_{100}^{30} (1+\beta T)\, dT$

$q_x(0.025) = -k_0 \left[\left(T + \beta \frac{T^2}{2}\right)\right]_{100}^{30}$

$= -k_0 \left[\cancel{100}\,(T_3 - T_1) + \frac{\beta}{2}(T_3^2 - T_1^2)\right] = -k_0\left[-70 + \frac{\beta}{2}(30^2 - 100^2)\right]$

$q_x(0.025) = 70k_0 + \frac{\cancel{70}\,k_0\beta}{2}(100^2 - 30^2)$

$10^3 \times 0.025\, q_x = 70\,k_0 + \cancel{318500}\ 4550\ k_0\beta \longrightarrow$ (1)

$q_x(0.025) = -k_0\left[(66-100) + \frac{\beta}{2}(66^2 - 100^2)\right]$ ...... (2)

$\frac{q_x(0.025)}{2} = 34\,k_0 + 2822\,k_0\beta$

$10^3 \times 0.0125\, q_x = 68\,k_0 + 5644\,k_0\beta$

from (1) and (2) $\quad 70\,k_0 + 4550\,k_0\beta = 68\,k_0 + 5644\,k_0\beta$

$\cancel{140\,k_0} +$

$2\,k_0 = 1094\,k_0\beta \qquad \beta = \frac{2}{1094} = 1.828\times10^{-3}$

$10^3 \times 0.025\,(10) = 70\,k_0 + \underbrace{4550(1.828\times10^{-3})}_{8.3174}\,k_0 = 78.3174\,k_0$

$k_0 = 3.192\ \frac{W}{m\,°C}$ $\qquad \boxed{\therefore\ k = 3.192\,(1 + 1.828\times10^{-3}\,T)}$

(b) S. S. 1d, No ht generation, isotropic, No radiation

(c) $q_x = k_0(1+\beta T)\frac{dT}{dx}$ $\qquad$ from 1d S.S. ht. tr. in Cartesian coordinates for variable k $\quad \frac{d^2(kT)}{dx^2} = 0 \qquad \cancel{\frac{d}{dx}\left(\frac{dT}{dx}\right) = 0}$

or $\cancel{\frac{dT}{dx} = \frac{q_x}{k_0(1+\beta T)}}$ $\qquad \frac{d}{dx}\left[\frac{d(kT)}{dx}\right] = 0$ or $\frac{d}{dx}\left[\frac{d}{dx}\left\{k_0(1+\beta T)\frac{dT}{dx}\right\}\right] = 0$

$k_0 \cancel{\frac{d}{dx}}(1+\beta T)\left[\frac{d^2T}{dx^2} + k_0\beta\left(\frac{dT}{dx}\right)^2 = 0\right.$ or $k_0 \frac{d^2T}{dx^2}(1+\beta T) = -k_0\beta\left(\frac{dT}{dx}\right)^2$

$\frac{d^2T}{dx^2} = -\frac{\beta}{1+\beta T}\left(\frac{dT}{dx}\right)^2$

$\frac{k}{3}$

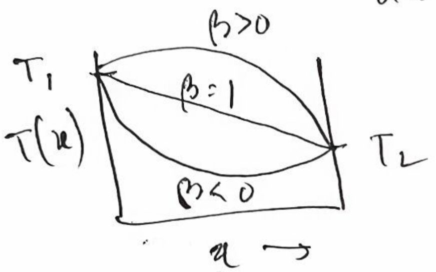
Contd.

(4) $\dfrac{d^2T}{dx^2} = \dfrac{-\beta}{1+\beta T}\left(\dfrac{dT}{dx}\right)^2 \Rightarrow$ For $\beta > 0$, $\dfrac{d^2T}{dx^2} < 0$

(Maxima at $\dfrac{dT}{dx} = 0$)

For $\beta < 0$, $\dfrac{d^2T}{dx^2} > 0$ (Minima at $\dfrac{dT}{dx} = 0$)

For $\beta = 0$ $\quad \dfrac{d^2T}{dx^2} = 0 \quad \dfrac{dT}{dx} = C_1 \quad \boxed{T = C_1 x + C_2}$ linear



(5) $D = 4$ cm $\quad R = 2$ cm $= 2 \times 10^{-2}$ m $= 0.02$ m

$\dot{q}_{gen}$ S.S. sphere

$h = 15$ W/m^2 °C , $k = 20 \dfrac{W}{m\,°C}$

$T_\infty = 30$ °C

1.0 MW/m^3 $= 10^6 \dfrac{W}{m^3}$

For uniform ht generation, S.S 1d ht conduction equ in a sphere - $\dfrac{1}{r^2}\dfrac{d^2(rT)}{dr^2} + \dfrac{\dot{q}}{k} = 0 \Rightarrow$

$\dfrac{d}{dr^2}(rT) = -\dfrac{\dot{q}\,r^2}{k}$

$\dfrac{d}{dr}\left[\dfrac{d}{dr}(rT)\right] = -\dfrac{\dot{q}\,r^2}{k} \Rightarrow \dfrac{d}{dr}(rT) = -\dfrac{\dot{q}}{k}\dfrac{r^3}{3} + C_1$

$rT = -\dfrac{\dot{q}}{k}\dfrac{r^4}{12} + C_1 r + C_2 \qquad T = -\dfrac{\dot{q}}{k}\dfrac{r^3}{12} + C_1 + \dfrac{C_2}{r}$

$T(r) = -\dfrac{\dot{q}}{12k}(r^3) + C_1 + \dfrac{C_2}{r} \qquad \dfrac{dT}{dr} = -\dfrac{\dot{q}}{12k}(3r^2) + C_2 \ln r$

At $r = 0$, $\dfrac{dT}{dr} = 0 \Rightarrow C_2 = 0 \quad \therefore \dfrac{dT}{dr} = -\dfrac{\dot{q}}{4k} r^2$

At $r = R$, $-kA\dfrac{dT}{dr} = hA(T_w - T_\infty) \qquad T(r) = -\dfrac{\dot{q}}{12k} r^3 + C_1$

$= \dot{q}\left(\dfrac{4}{3}\pi R^3\right)$

Same result from Ht generated = ht lost to surr. $\quad -k\,4\pi R^2\left(\dfrac{dT}{dr}\right) = \cancel{-k\,4\pi R^2\left(-\dot{q}\dfrac{R^3}{4k}\right)}$

$\dot{q}\left(\dfrac{4}{3}\pi R^3\right) = \cancel{h(4\pi R^2)(}$ $\Rightarrow C_2 = 0$

$\cancel{\dfrac{\dot{k}}{3} = -k}$

(5) contd

At $r = R$, $T = T_w$

Or $T_w = -\frac{\dot{q}}{12k} R^3 + C_1$

$C_1 = T_w + \frac{\dot{q}}{12k} R^3$

$C_1 = 474.44 + \frac{10^6}{12 \times 20} \times (0.02)^3$

$= 474.473$

$\dot{q}\left(\frac{4}{3}\pi R^3\right) = hA\left(T_w - T_\infty\right)$

$\dot{q}\left(\frac{4}{3}\pi R^3\right) = h \, 4\pi R^2 \left(T_w - T_\infty\right)$

$\frac{\dot{q}}{3h} R = T_w - T_\infty$

$T_w = \frac{\dot{q} R}{3h} + T_\infty$

$= \frac{10^6 \times 0.02}{3 \times 15} + 30$

$= 474.44\ °C$

$\therefore T(r) = -\frac{\dot{q}}{12K} r^3 + 474.44 + \frac{\dot{q}}{12K} R^3$

$= \frac{\dot{q}}{12K}\left(R^3 - r^3\right) + 474.44$

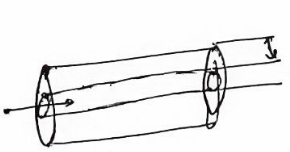
$T(r) = \frac{10^6}{12 \times 20}\left(R^3 - r^3\right) + 474.44 = 4166.67\left(R^3 - r^3\right) + 474.44$

$= 4166.67 R^3 \left[1 - \left(\frac{r}{R}\right)^3\right] + 474.44$

$T(r) = 474.44 + 0.833 R^3 \left[1 - \left(\frac{r}{R}\right)^3\right]$

$\therefore$ At $r = 0$ $T_{centre} = 474.44\ °C$.

(6)



2 mm

25 mm = 2.5 cm = 0.025 m = D_i

$D_o = 25 + 4 = 29$ mm $= 0.029$ m

$K = 20 \frac{W}{m\,°C}$

$h_i = 100 \frac{W}{m^2\,°C}$, $h_o = 12 \frac{W}{m^2\,°C}$,

For unit pipe length

$R_i = \frac{1}{h_i A_i} = \frac{1}{100(\pi \times 0.025)} = \cancel{7.854}\ 0.1273 \frac{°C\,m}{W}$

$R_o = \frac{1}{h_o A_o} = \frac{1}{12(\pi \times 0.029)} = 0.9147 \frac{°C\,m}{W}$

$R_t = \frac{\ln(r_o/r_i)}{2\pi k L}$, $\frac{\ln(29/25)}{2\pi \times 20} = 1.181 \times 10^{-3} \frac{°C\,m}{W}$

$R_{ov} = 1.0432 \frac{°C\,m}{W}$

$Q = \frac{\Delta T_{ov}}{R_{ov}} = U_o A_o (\Delta T_{ov})$

$U_o = \frac{1}{R_{ov} A_o}$

$= 10.52 \frac{W}{m^2\,°C}$

$\boxed{\text{Main determining factor} = h_o}$

profile - parabolic

6 contd.

Temp profile for 1d SS ht conduction $\frac{d^2 T}{dr^2} + \frac{1}{r}\frac{dT}{dr} = 0$

$$\frac{d}{dr}\left(r\frac{dT}{dr}\right) = 0$$

$$r\frac{dT}{dr} = C_1 \Rightarrow \frac{dT}{dr} = \frac{C_1}{r}$$

$$T(r) = C_1 \ln r + C_2$$

At $r = R_1, \quad T = T_{w1}$

$r = R_2 \quad T = T_{w2}$

$$T_{w1} = C_1 \ln R_1 + C_2$$

$$T_{w2} = C_1 \ln R_2 + C_2$$

$$T_{w1} - T_{w2} = C_1 \ln \frac{R_1}{R_2} \Rightarrow \dot{q} = \frac{T_{w1} - T_{w2}}{\ln (R_1/R_2)}$$

$$\frac{T(r)}{\cancel{T_{w1}}} = \frac{T_{w1} - T_{w2}}{\ln (R_1/R_2)} \ln r/r_2 + T_{w2} \Rightarrow \text{logarithmic}$$