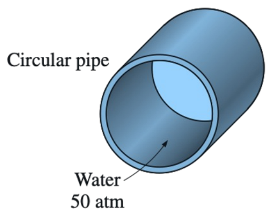
HEAT TRANSFER

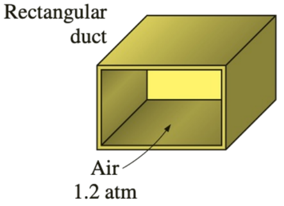
[CH21204]

March 02, 2023

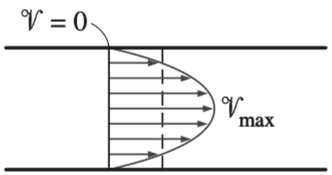
Internal forced convection

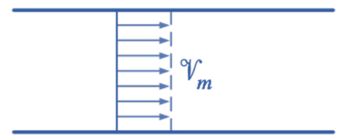
- Pipes with a circular cross section can withstand large pressure differences between the inside and the outside without undergoing any distortion
- Noncircular pipes are usually used in applications such as the heating and cooling systems of buildings where the pressure difference is relatively small and the manufacturing and installation costs are lower
- For a fixed surface area, the circular tube gives the most heat transfer for the least pressure drop, which explains the overwhelming popularity of circular tubes in heat transfer equipment
- pipe/duct/tube/conduit





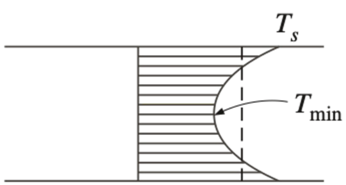
Mean Velocity & Mean Temperature





$$\dot{m} = \rho \mathcal{V}_m A_c = \int_{A_c} \rho \mathcal{V}(r, x) dA_c$$

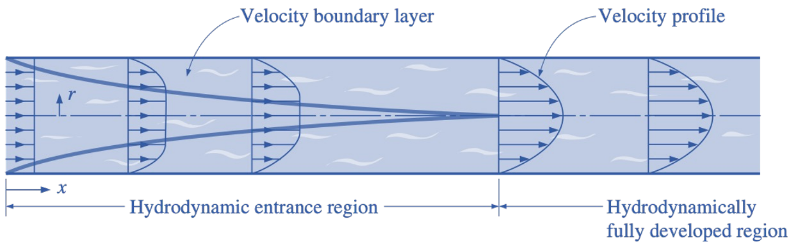
$$\mathcal{V}_m = \frac{\int_{A_c} \rho \mathcal{V}(r, x) dA_c}{\rho A_c} = \frac{\int_0^R \rho \mathcal{V}(r, x) 2\pi r dr}{\rho \pi R^2} = \frac{2}{R^2} \int_0^R \mathcal{V}(r, x) r dr$$



$$\dot{E}_{\text{fluid}} = \dot{m} C_p T_m = \int_{\dot{m}} C_p T \delta\dot{m} = \int_{A_c} \rho C_p T \mathcal{V} dA_c$$

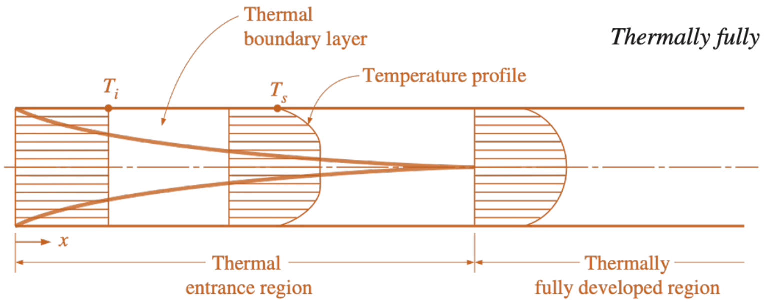
$$T_m = \frac{\int_{\dot{m}} C_p T \delta\dot{m}}{\dot{m} C_p} = \frac{\int_0^R C_p T (\rho \mathcal{V} 2\pi r dr)}{\rho \mathcal{V}_m (\pi R^2) C_p} = \frac{2}{\mathcal{V}_m R^2} \int_0^R T(r, x)\, \mathcal{V}(r, x)\, r dr$$

$$T_b = (T_{m,\,i} + T_{m,\,e})/2.$$



Hydrodynamically fully developed: $\dfrac{\partial \mathcal{V}(r, x)}{\partial x} = 0 \longrightarrow \mathcal{V} = \mathcal{V}(r)$

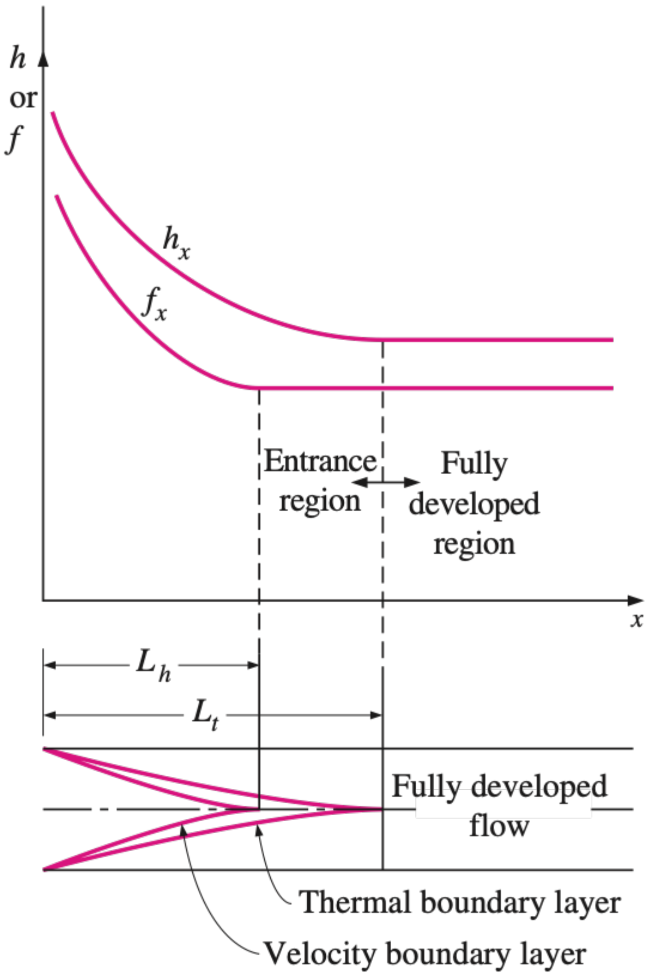
Thermally fully developed: $\dfrac{\partial}{\partial x}\left[\dfrac{T_s(x) - T(r, x)}{T_s(x) - T_m(x)}\right] = 0$



$$\frac{\partial}{\partial r}\left(\frac{T_s - T}{T_s - T_m}\right)\bigg|_{r=R} = \frac{-(\partial T/\partial r)|_{r=R}}{T_s - T_m} \neq f(x)$$

$$\dot{q}_s = h_x(T_s - T_m) = k\,\frac{\partial T}{\partial r}\bigg|_{r=R} \longrightarrow h_x = \frac{k(\partial T/\partial r)|_{r=R}}{T_s - T_m}$$

Both the friction and convection coefficients remain constant in the fully developed region of a tube

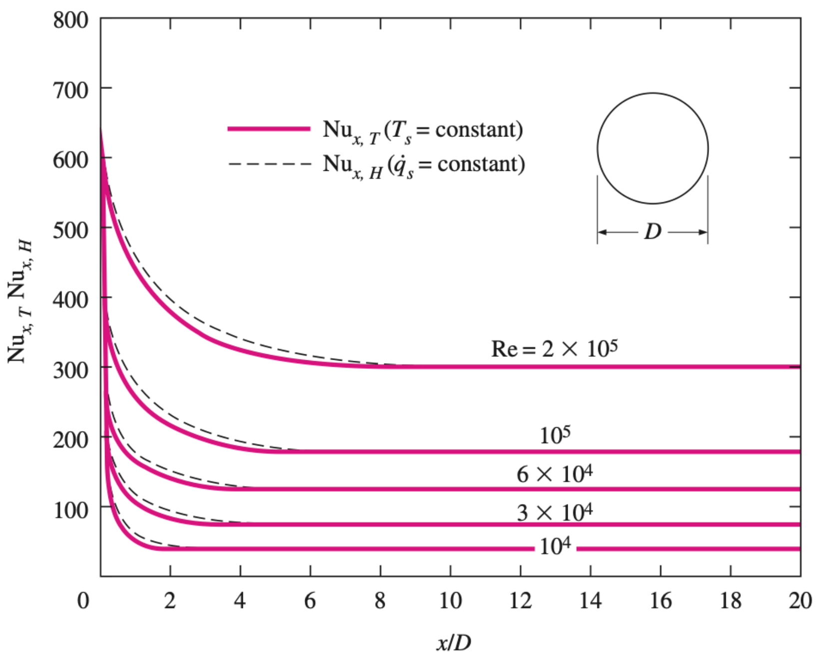


$$L_{h,\ \text{laminar}} \approx 0.05\ \text{Re}\ D$$

$$L_{t,\ \text{laminar}} \approx 0.05\ \text{Re}\ \text{Pr}\ D = \text{Pr}\ L_{h,\ \text{laminar}}$$

$$L_{h,\ \text{turbulent}} = 1.359\ \text{Re}^{1/4}$$

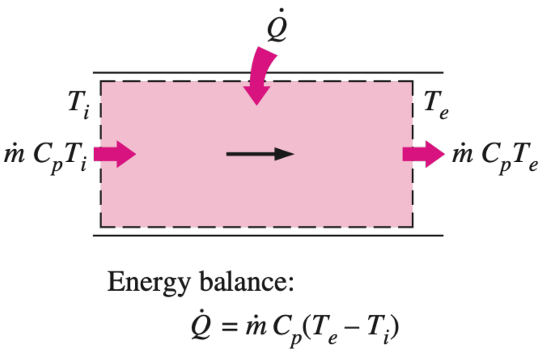
$$L_{h,\ \text{turbulent}} \approx L_{t,\ \text{turbulent}} \approx 10D$$



- The Nusselt numbers and thus the convection heat transfer coefficients are much higher in the entrance region.

- The Nusselt number reaches a constant value at a distance of less than 10 diameters, and thus the flow can be assumed to be fully developed for x > 10D.

- The Nusselt numbers for the uniform surface temperature and uniform surface heat flux conditions are identical in the fully developed regions, and nearly identical in the entrance regions. Therefore, Nusselt number is insensitive to the type of thermal boundary condition, and the turbulent flow correlations can be used for either type of boundary condition.



$$\dot{Q} = \dot{m} C_p (T_e - T_i)$$

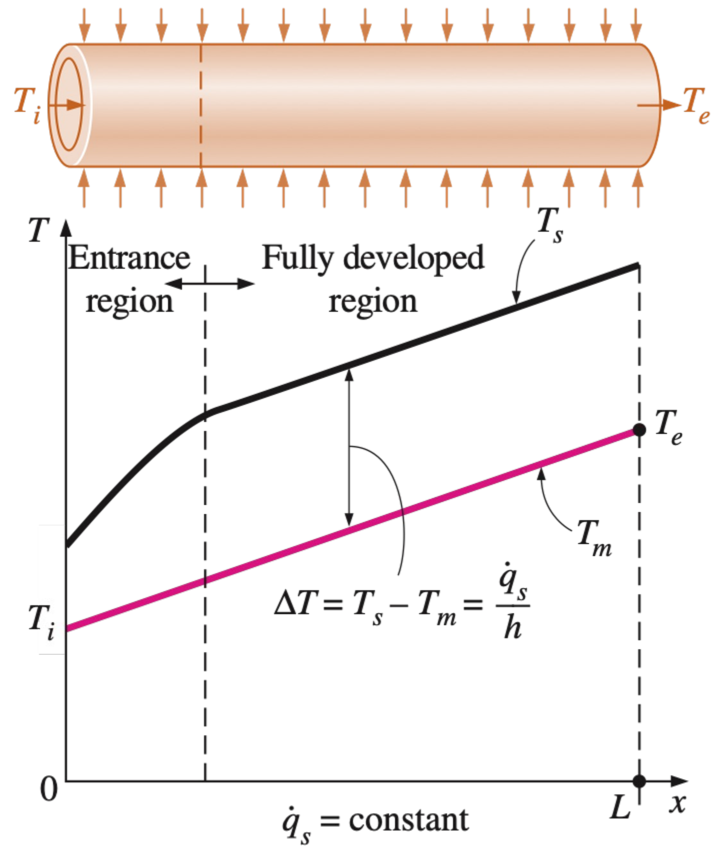
$$\dot{q}_s = h_x (T_s - T_m)$$

Constant Surface Heat Flux:

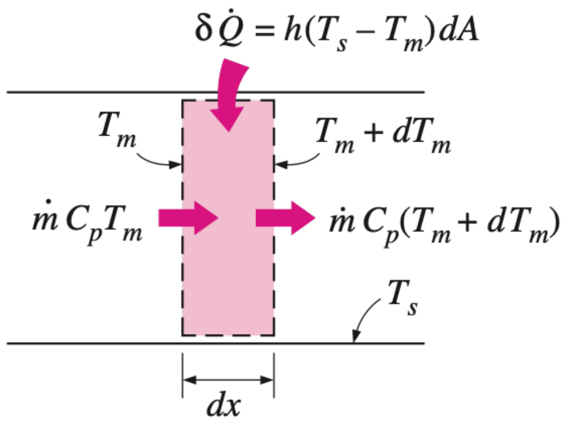
$$\dot{Q} = \dot{q}_s A_s = \dot{m} C_p (T_e - T_i)$$

$$T_e = T_i + \frac{\dot{q}_s A_s}{\dot{m} C_p}$$

$$\dot{q}_s = h(T_s - T_m) \longrightarrow T_s = T_m + \frac{\dot{q}_s}{h}$$



T_i
T_e
T
Entrance region
Fully developed region
T_s
T_e
T_m
T_i
$\Delta T = T_s - T_m = \frac{\dot{q}_s}{h}$
0
L
x
$\dot{q}_s$ = constant



$$\dot{m}C_p\,dT_m = \dot{q}_s(p\,dx) \longrightarrow \frac{dT_m}{dx} = \frac{\dot{q}_s p}{\dot{m}C_p} = \text{constant}$$

$$\frac{dT_m}{dx} = \frac{dT_s}{dx}$$

$$\frac{\partial}{\partial x}\left(\frac{T_s - T}{T_s - T_m}\right) = 0 \longrightarrow \frac{1}{T_s - T_m}\left(\frac{\partial T_s}{\partial x} - \frac{\partial T}{\partial x}\right) = 0 \longrightarrow \frac{\partial T}{\partial x} = \frac{dT_s}{dx}$$

$$\frac{\partial T}{\partial x} = \frac{dT_s}{dx} = \frac{dT_m}{dx} = \frac{\dot{q}_s p}{\dot{m}C_p} = \text{constant}$$

In fully developed flow in a tube subjected to constant surface heat flux, the temperature gradient is independent of x and thus the shape of the temperature profile does not change along the tube.