The velocity profile in a laminar boundary layer flow at zero pressure gradient is to be approximated by the linear expression, $u/U = a + b\eta$, where $\eta = y/\delta$. Use the momentum integral equation with this profile to obtain ab expression for the ratio, δ/x and the skin friction coefficient, C_f, in terms of the Reynold's number.

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$$\tau_W = \rho U^2 \frac{d}{dx}\int_0^\delta \frac{v_x}{U}\left(1-\frac{v_x}{U}\right)dy$$

Given $\frac{u}{U} = a + b\eta$. $\eta = y/\delta$.

at $\eta = 0$ $(y=0)$ $u = 0 \Rightarrow a = 0$.

at $\eta = 1$ $(y=\delta)$ $u = U \Rightarrow b = 1$.

$\therefore$ $\boxed{\frac{u}{U} = \frac{y}{\delta} = \eta.}$ 1

If this step is not done −3

The MI eqn for flat plate, incompressible flow, zero pre grad.

$$T_w = \rho U^2 \frac{d\delta}{dx}\int_0^1 \frac{u}{U}\left(1-\frac{u}{U}\right)d(y/\delta)$$

$$\boxed{T_w = \rho U^2 \beta \frac{d\delta}{dx}} - ②$$

β is the value of the definite integral.

$$\int_0^1 \frac{u}{U}\left(1-\frac{u}{U}\right)d\eta = \int_0^1 \eta(1-\eta)d\eta = \left[\frac{1}{2}\eta^2 - \frac{1}{3}\eta^3\right]_0^1 = \frac{1}{6} = \beta.$$ 1

$$T_w = \mu \frac{\partial u}{\partial y}\bigg|_{y=0} = \frac{\mu U}{\delta}\frac{\partial(u/U)}{\partial(y/\delta)}\bigg|_{y/\delta=0} = \frac{\mu U}{\delta}\frac{d\left(\frac{u}{U}\right)}{d\eta}\bigg|_{\eta=0} = \frac{\mu U}{\delta}.$$ 2

From ②

$$\frac{\mu U}{\delta} = \rho U^2 \frac{d\delta}{dx}\left(\frac{1}{6}\right).$$

$$\frac{\delta^2}{2} = \frac{6\mu}{\rho U}x + c.$$ 2

BC. $\delta = 0$ at $x = 0 \Rightarrow c = 0. \Rightarrow \frac{\delta}{x} = \sqrt{\frac{12\mu}{\rho U x}}.$

$$\boxed{\frac{\delta}{x} = \frac{3.46}{\sqrt{Re_x}}}$$ 2

$$C_f = T_w / \tfrac{1}{2}\rho U^2 = \frac{\mu U/\delta}{\frac{1}{2}\rho U^2} = \frac{2\mu}{\rho U\delta} = \frac{2\mu}{\rho U x}\cdot\frac{x}{\delta} \Rightarrow \boxed{C_f = \frac{0.577}{\sqrt{Re_x}}}$$