INDIAN INSTITUTE OF TECHNOLOGY KHARAGPUR

Department of Chemical Engineering

End-semester (Spring) Examination 2022-23

Subject: Process Dynamics and Control (CH61016)

Remarks:

1. This question paper contains two parts: **Part A** and **Part B**. Attempt both parts.
2. Write all the answers of a part together.
3. You may make suitable assumptions but clearly specify and justify them. **No queries will be entertained during exam hours.**
4. Unless otherwise stated, usual notations apply.
5. Time = 3 h; maximum marks = 50; total number of printed pages = 3.

Part A

1. The model for the evolution of dimensionless population of insects, $x(t)$, in a given region is given by the following equation.

$$\frac{dx}{dt} = x(1-x) + \frac{x}{1+x} \tag{1}$$

(a) Analogous to the graphical method to determine the fixed-points of a discrete model, *devise* a graphical method to determine the equilibrium solutions of a continuous autonomous model and determine the equilibrium solutions for this problem for physically realisable conditions. Clearly state the logic behind the graphical method that you have devised. The graph(s) must be plotted on a separate graph-sheet.

(b) For the graphical method that you devised in part (a), state the condition for stability, and identify the equilibrium solutions obtained in part (a) as stable or unstable equilibrium solution on the basis of the graphical method.

(c) The discrete model for Equation (1) is expressed as follows:

$$x_{n+1} = x_n f(x_n, \Delta t) + x_n^2 g(x_n, \Delta t) \tag{2}$$

Determine the expressions for $f(x_n, \Delta t)$ and $g(x_n, \Delta t)$.

(d) Determine the fixed points for this discrete system for the physically realisable conditions. Comment upon the stability of the lower physically realisable fixed point. Show all the calculations/graph(s) in detail.

...2 + 2 + 2 + 2 = 8 marks

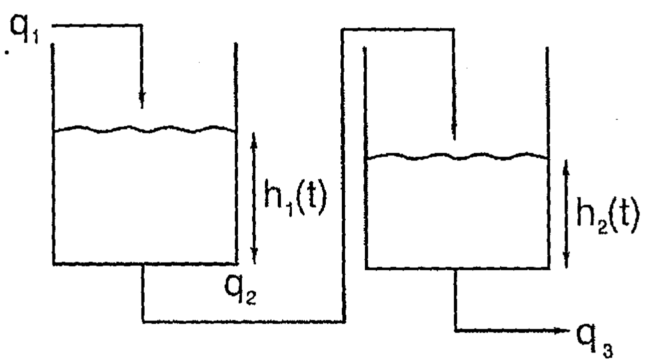
2. Consider a second order non-linear dynamical system modelled by the following equations.

$$\frac{dx_1}{dt} = a^2x_1^2 - b^2x_2^2 - 1 \tag{3}$$
$$\frac{dx_2}{dt} = cx_2 \tag{4}$$

(a) Detemine the condition(s) for the hyperbolicity of the equilibrium solution(s).
(b) Sketch all the relevant projections of the bifurcation diagram for the variable x_1.
You may draw qualitative graph(s) on the answersheet to show the projections of the bifurcation diagram but clearly indicate all the relevant features on the diagram(s) with proper justification.

...3 + 4 = 7 marks

3. Consider a two-tank system, each with unit cross-section, with other details as shown in the figure below. The model equations have been provided alongside.



$$\frac{dh_1}{dt} = q_1 - q_2 \tag{5}$$
$$\frac{dh_2}{dt} = q_2 - q_3 \tag{6}$$
$$q_2 = 0.94h_1 \tag{7}$$
$$q_3 = 0.53h_2 \tag{8}$$

The inlet stream to the system, q_1, is fed via a pump such that $q_1 = f(t)$ while the output from the system, q_3, is used for downstream process operations. Derive an expression for the response of the system in z-domain if the system is subjected to a linear ramp input with no hold element. Assume that all variables are given in deviation variable form.

...10 marks

Part B

4. Consider a process transfer function as given below:

$$y(s) = \frac{-2(-6s+1)}{(10s+1)(3s+1)}u(s) + \frac{3}{5s+1}d(s) \tag{9}$$

where y is output, u is manipulated input and d disturbance. The objective is to control the process using Feedback-Feedforward scheme.
(a) Design the feedback controller using IMC-PID approach with filter time constant is 40% of

dominant time constant.

(b) Design a feedforward controller and make it feasible with proper explanations.

...3 + 2 = 5 marks

5. The transfer function model of the process is given below:

$$y(s) = \frac{0.2}{(2s+1)(4s+1)} m(s) + \frac{0.05}{3s+1} d_1(s) \tag{10}$$

$$m(s) = \frac{1}{s+1} u(s) + d_2(s) \tag{11}$$

(a) Draw a cascade control block diagram.

(b) Find the relationship of ultimate controller gain of master controller (K_{c1u}) with the slave proportional controller (K_{c2}).

...1 + 3 = 4 marks

6. The transfer function model for a particular system is given as:

$$y_1(s) = \frac{4.05e^{-27s}}{50s+1} u_1(s) + \frac{1.2e^{-27s}}{45s+1} u_2(s) \tag{12}$$

$$y_2(s) = \frac{4.06e^{-8s}}{13s+1} u_1(s) + \frac{1.19}{19s+1} u_2(s) \tag{13}$$

(a) Find the RGA for the system and recommend the input-output pairing.

(b) Is the recommended pairing structurally stable?

(c) Design a steady state decoupler for this process using generalized matrix approach.

(d) Design dynamic decoupler for the process using simplified approach and derive the transfer function model of the decoupled process.

...2 + 1 + 2 + 3 = 8 marks

7. Consider the transfer function of a process

$$\frac{y(s)}{u(s)} = \frac{10}{(s+1)(s+2)(s+3)} \tag{14}$$

(a) Derive state space realization of controllable canonical form.

(b) Obtain state feedback controller gain matrix by placing the regulator poles at $[-2 \pm 0.7j, -10]$ using Bass-Gura method.

(c) Design full order observer by placing the poles at [-5,-6,-10].

...2 + 3 + 3 = 8 marks

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