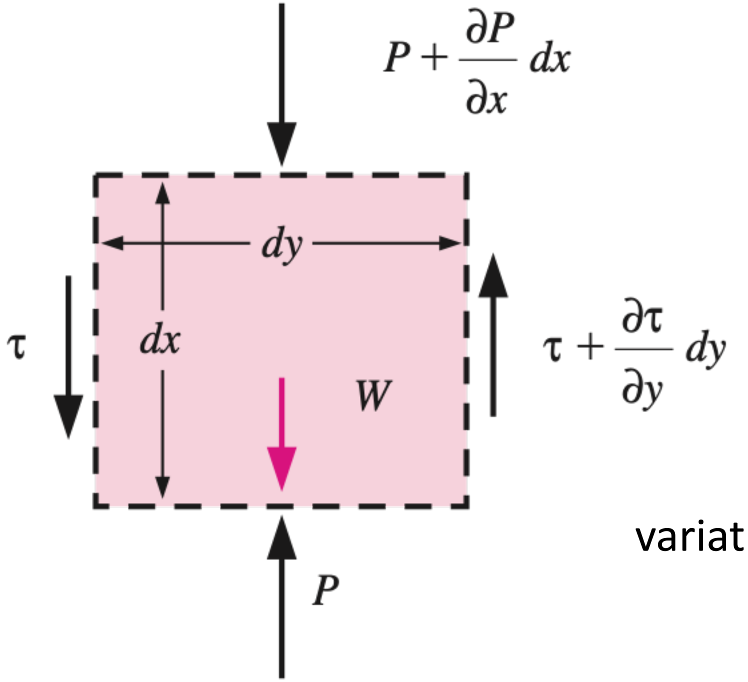
HEAT TRANSFER

[CH21204]

March 31, 2023



in the boundary layer

$$v \ll u$$

$$\partial v/\partial x \approx \partial v/\partial y \approx 0$$

$$\partial P/\partial y = 0$$

variation of pressure in the direction normal to the surface is negligible

$$P = P(x) = P_\infty(x)$$

$$\partial P/\partial x = \partial P_\infty/\partial x = -\rho_\infty g$$

$$\rho\left(u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right) = \mu\frac{\partial^2 u}{\partial y^2} + (\rho_\infty - \rho)g$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \nu\frac{\partial^2 u}{\partial y^2} + g\beta(T - T_\infty)$$

$$x^* = \frac{x}{L_c} \qquad y^* = \frac{y}{L_c} \qquad u^* = \frac{u}{\mathcal{V}} \qquad v^* = \frac{v}{\mathcal{V}} \qquad \text{and} \qquad T^* = \frac{T - T_\infty}{T_s - T_\infty}$$

$$u^* \frac{\partial u^*}{\partial x^*} + v^* \frac{\partial u^*}{\partial y^*} = \left[\frac{g\beta(T_s - T_\infty)L_c^3}{\nu^2}\right] \frac{T^*}{\mathrm{Re}_L^2} + \frac{1}{\mathrm{Re}_L} \frac{\partial^2 u^*}{\partial y^{*2}}$$

Grashof number Gr_L,

$$\mathrm{Gr}_L = \frac{g\beta(T_s - T_\infty)L_c^3}{\nu^2}$$

g = gravitational acceleration, m/s^2

β = coefficient of volume expansion, 1/K ($\beta = 1/T$ for ideal gases)

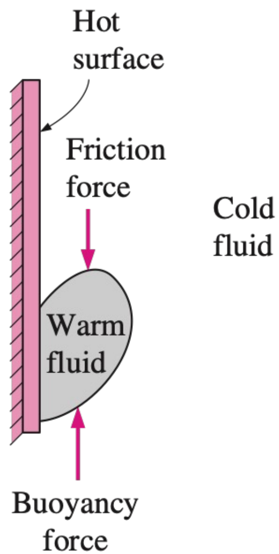
T_s = temperature of the surface, °C

T_∞ = temperature of the fluid sufficiently far from the surface, °C

L_c = characteristic length of the geometry, m

ν = kinematic viscosity of the fluid, m^2/s

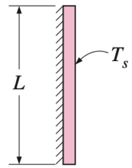
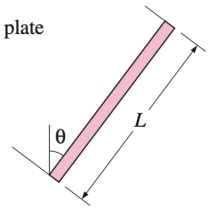
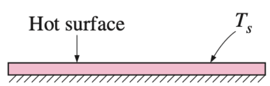
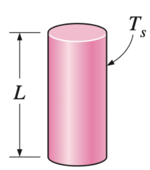
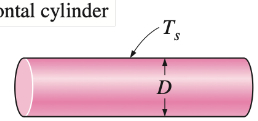
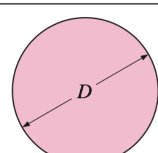
Flow regime in natural convection is governed by the dimensionless **Grashof number**, which represents the ratio of the buoyancy force to the viscous force acting on the fluid



$$\mathrm{Nu} = \frac{hL_c}{k} = C(\mathrm{Gr}_L \, \mathrm{Pr})^n = C \, \mathrm{Ra}_L^n$$

n is usually $\frac{1}{4}$ for laminar flow and $\frac{1}{3}$ for turbulent flow.

$$\mathrm{Ra}_L = \mathrm{Gr}_L \, \mathrm{Pr} = \frac{g\beta(T_s - T_\infty)L_c^3}{\nu^2} \mathrm{Pr}$$

Geometry	Characteristic length L_c	Range of Ra	Nu
Vertical plate 	L	10^4–10^9 10^9–10^{13} Entire range	$\mathrm{Nu} = 0.59\mathrm{Ra}_L^{1/4}$ $\mathrm{Nu} = 0.1\mathrm{Ra}_L^{1/3}$ $\mathrm{Nu} = \left\{0.825 + \dfrac{0.387\mathrm{Ra}_L^{1/6}}{[1 + (0.492/\mathrm{Pr})^{9/16}]^{8/27}}\right\}^2$ (complex but more accurate)
Inclined plate 	L		Use vertical plate equations for the upper surface of a cold plate and the lower surface of a hot plate Replace g by $g\cos\theta$ for $\mathrm{Ra} < 10^9$
Horiontal plate (Surface area A and perimeter p) (*a*) Upper surface of a hot plate (or lower surface of a cold plate)  (*b*) Lower surface of a hot plate (or upper surface of a cold plate) 	A_s/p	10^4–10^7 10^7–10^{11} 10^5–10^{11}	$\mathrm{Nu} = 0.54\mathrm{Ra}_L^{1/4}$ $\mathrm{Nu} = 0.15\mathrm{Ra}_L^{1/3}$ $\mathrm{Nu} = 0.27\mathrm{Ra}_L^{1/4}$
Vertical cylinder 	L		A vertical cylinder can be treated as a vertical plate when $D \geq \dfrac{35L}{\mathrm{Gr}_L^{1/4}}$
Horizontal cylinder 	D	$\mathrm{Ra}_D \leq 10^{12}$	$\mathrm{Nu} = \left\{0.6 + \dfrac{0.387\mathrm{Ra}_D^{1/6}}{[1 + (0.559/\mathrm{Pr})^{9/16}]^{8/27}}\right\}^2$
Sphere 	D	$\mathrm{Ra}_D \leq 10^{11}$ ($\mathrm{Pr} \geq 0.7$)	$\mathrm{Nu} = 2 + \dfrac{0.589\mathrm{Ra}_D^{1/4}}{[1 + (0.469/\mathrm{Pr})^{9/16}]^{4/9}}$

$$\dot{Q}_{\text{conv}} = hA_s(T_s - T_\infty) \qquad (\text{W})$$

Constant surface heat flux

$$\text{Nu} = \frac{hL}{k} = \frac{\dot{q}_s L}{k(T_{L/2} - T_\infty)}$$

A 10-m-long section of a 6-cm-diameter horizontal hot water pipe passes through a large room whose temperature is 22°C. If the temperature of the outer surface of the pipe is 65°C, determine the rate of heat loss from the pipe by natural convection.

$$k = 0.02688 \text{ W/m.°C}$$

$$\upsilon = 1.735 \times 10^{-5} \text{ m}^2\text{/s}$$

$$\text{Pr} = 0.7245$$

$$\beta = \frac{1}{T_f} = \frac{1}{(43.5 + 273)\text{K}} = 0.00316 \text{ K}^{-1}$$

$$Ra = \frac{g\beta(T_s - T_\infty)D^3}{\upsilon^2}\text{Pr} = \frac{(9.81 \text{ m/s}^2)(0.00316 \text{ K}^{-1})(65 - 22 \text{ K})(0.06 \text{ m})^3}{(1.735 \times 10^{-5} \text{ m}^2\text{/s})^2}(0.7245) = 692{,}805$$

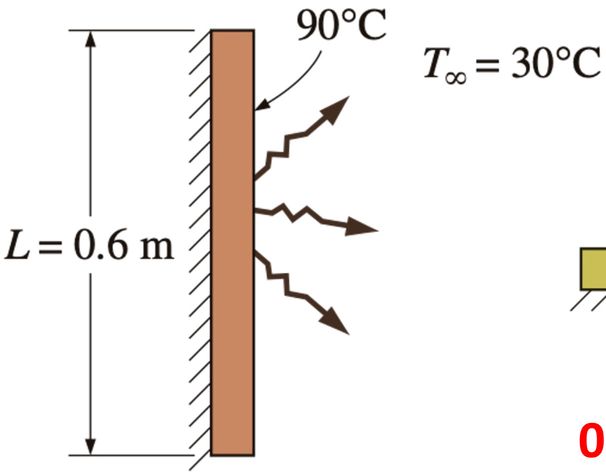
$$Nu = \left\{0.6 + \frac{0.387 Ra^{1/6}}{\left[1 + (0.559/\Pr)^{9/16}\right]^{8/27}}\right\}^2 = \left\{0.6 + \frac{0.387(692{,}805)^{1/6}}{\left[1 + (0.559/0.7245)^{9/16}\right]^{8/27}}\right\}^2 = 13.15$$

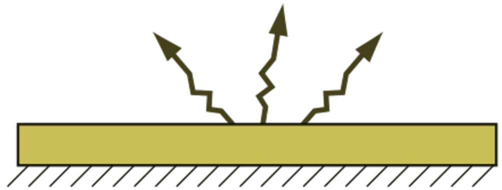
$$h = \frac{k}{D} Nu = \frac{0.02688 \text{ W/m.°C}}{0.06 \text{ m}}(13.15) = 5.893 \text{ W/m}^2\text{.°C}$$

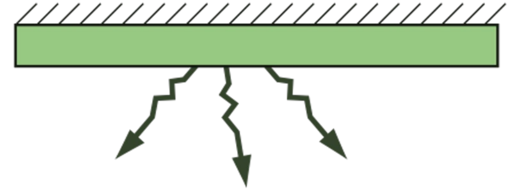
$$A_s = \pi DL = \pi(0.06 \text{ m})(10 \text{ m}) = 1.885 \text{ m}^2$$

$$\dot{Q} = hA_s(T_s - T_\infty) = (5.893 \text{ W/m}^2\text{.°C})(1.885 \text{ m}^2)(65 - 22)\text{°C} = \mathbf{477.6 \text{ W}}$$

$$\dot{Q}_{rad} = \varepsilon A_s \sigma(T_s^{\ 4} - T_{surr}^{\ 4}) = (0.8)(1.885 \text{ m}^2)(5.67\times10^{-8} \text{ W/m}^2\text{.K}^4)\left[(65+273 \text{ K})^4 - (22+273 \text{ K})^4\right] = \mathbf{468.4 \text{ W}}$$





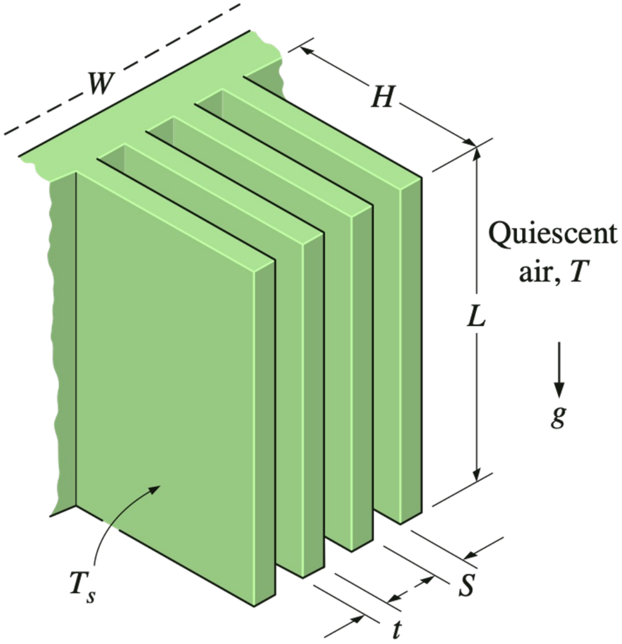


0.6-m X 0.6-m thin square plate

$$k = 0.02808 \text{ W/m} \cdot {}^\circ\text{C}$$

$$\nu = 1.896 \times 10^{-5} \text{ m}^2\text{/s}$$

$$\text{Pr} = 0.7202$$



T_s = constant

$$\mathrm{Ra}_S = \frac{g\beta(T_s - T_\infty)S^3}{\nu^2}\mathrm{Pr}$$

$$\mathrm{Ra}_L = \frac{g\beta(T_s - T_\infty)L^3}{\nu^2}\mathrm{Pr} = \mathrm{Ra}_S\frac{L^3}{S^3}$$

$$\mathrm{Nu} = \frac{hS}{k} = \left[\frac{576}{(\mathrm{Ra}_S S/L)^2} + \frac{2.873}{(\mathrm{Ra}_S S/L)^{0.5}}\right]^{-0.5}$$

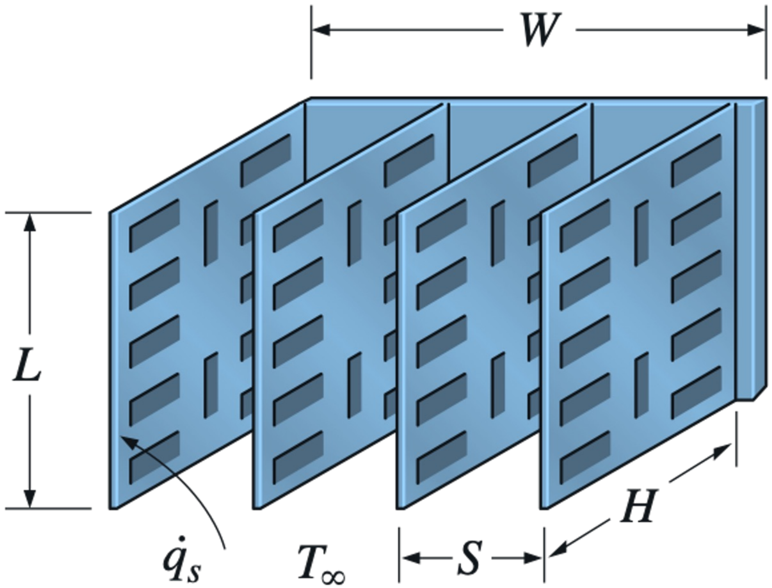
$$S_{\mathrm{opt}} = 2.714\left(\frac{S^3 L}{\mathrm{Ra}_S}\right)^{0.25} = 2.714\frac{L}{\mathrm{Ra}_L^{0.25}}$$

$$\mathrm{Nu} = \frac{hS_{\mathrm{opt}}}{k} = 1.307$$

$$T_{\mathrm{ave}} = (T_s + T_\infty)/2$$

$$\dot{Q} = h(2nLH)(T_s - T_\infty)$$

$$n = W/(S + t) \approx W/S$$



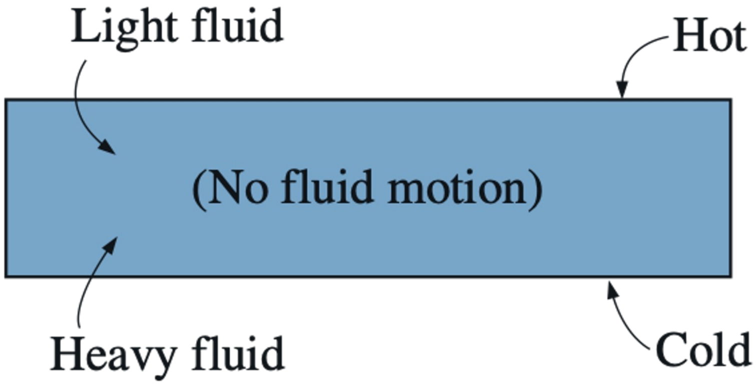
$$\mathrm{Ra}_S^* = \frac{g\beta \dot{q}_s S^4}{k\nu^2}\,\mathrm{Pr}$$

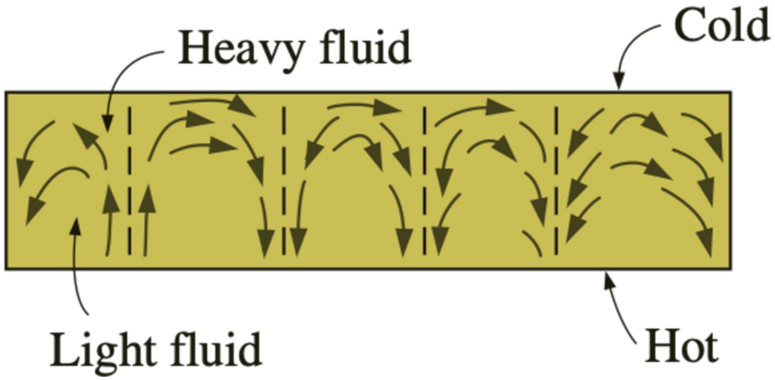
$$\mathrm{Nu}_L = \frac{h_L S}{k} = \left[\frac{48}{\mathrm{Ra}_S^* S/L} + \frac{2.51}{(\mathrm{Ra}_L^* S/L)^{0.4}}\right]^{-0.5}$$

$$S_{\mathrm{opt}} = 2.12\left(\frac{S^4 L}{\mathrm{Ra}_S^*}\right)^{0.2}$$

$$\dot{Q} = \dot{q}_s A_s = \dot{q}_s(2nLH)$$

$$\dot{q}_s = h_L(T_L - T_\infty)$$





Nu = 1 : Pure Conduction

Ra > 1708 : buoyant force overcomes the fluid resistance

Bénard cells : Hexagonal cells

Ra > 3 X 10^5 : Cells break down - Turbulent

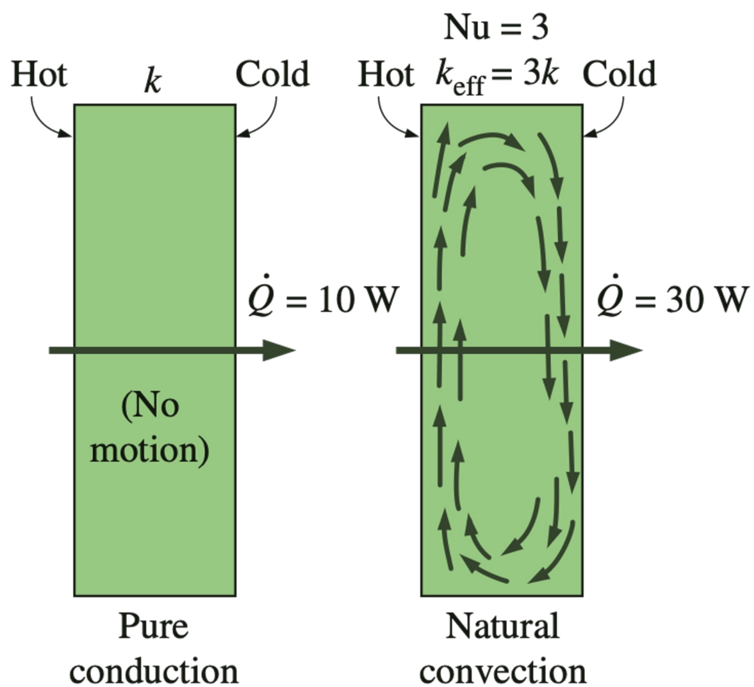
$$Ra_L = \frac{g\beta(T_1 - T_2)L_c^3}{\nu^2} Pr$$

$$\dot{Q} = hA_s(T_1 - T_2) = k\mathrm{Nu}A_s \frac{T_1 - T_2}{L_c}$$

$$\dot{Q}_{\mathrm{cond}} = kA_s \frac{T_1 - T_2}{L_c}$$

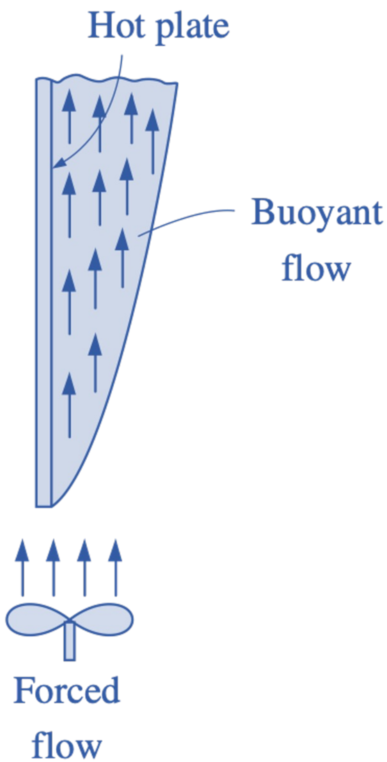
The fluid in an enclosure behaves like a fluid whose thermal conductivity is **kNu** as a result of convection currents.

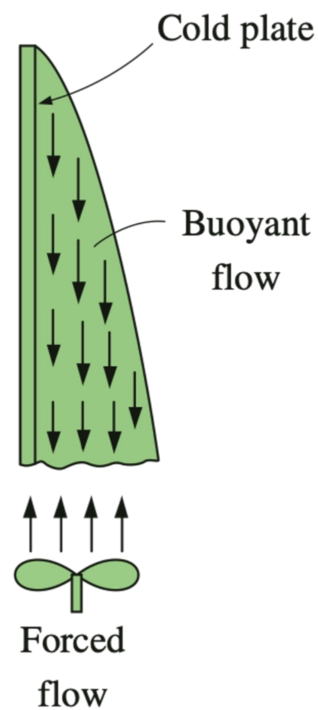
$$k_{\mathrm{eff}} = k\mathrm{Nu}$$

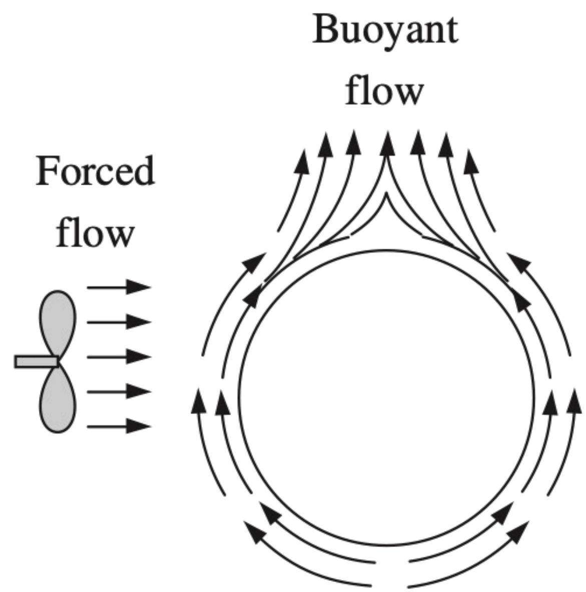


$$\dot{Q}_{\text{total}} = \dot{Q}_{\text{conv}} + \dot{Q}_{\text{rad}}$$

$$\dot{Q}_{\text{rad}} = \varepsilon\sigma A_s(T_s^4 - T_{\text{surr}}^4)$$







Gr/Re^2

$\mathrm{Gr}/\mathrm{Re}^2 < 0.1$ **natural convection is negligible**

$\mathrm{Gr}/\mathrm{Re}^2 > 10$ **forced convection is negligible**

$0.1 < \mathrm{Gr}/\mathrm{Re}^2 < 10$

$$\mathrm{Nu}_{\text{combined}} = (\mathrm{Nu}^n_{\text{forced}} \pm \mathrm{Nu}^n_{\text{natural}})^{1/n}$$