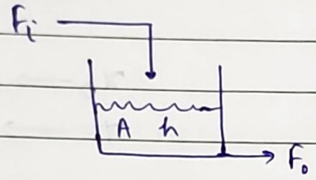
variables

- Input
  - Manipulated variable (MV) / control variable
  - load variable / disturbance
    - Measured load variable
    - unmeasured load variable
- output
  - Measured
    - primary / controlled variable (CV)
    - secondary meast.
  - unmeasured

Ex 1



control obj : $h = h_{\text{safe point}}$

CV	MV
h	F_i

→ Manipulated variable is selected such that it has direct and fast effect on controlled variable

→ Choose the controlled and manuplated variable such that there is min. time lag.

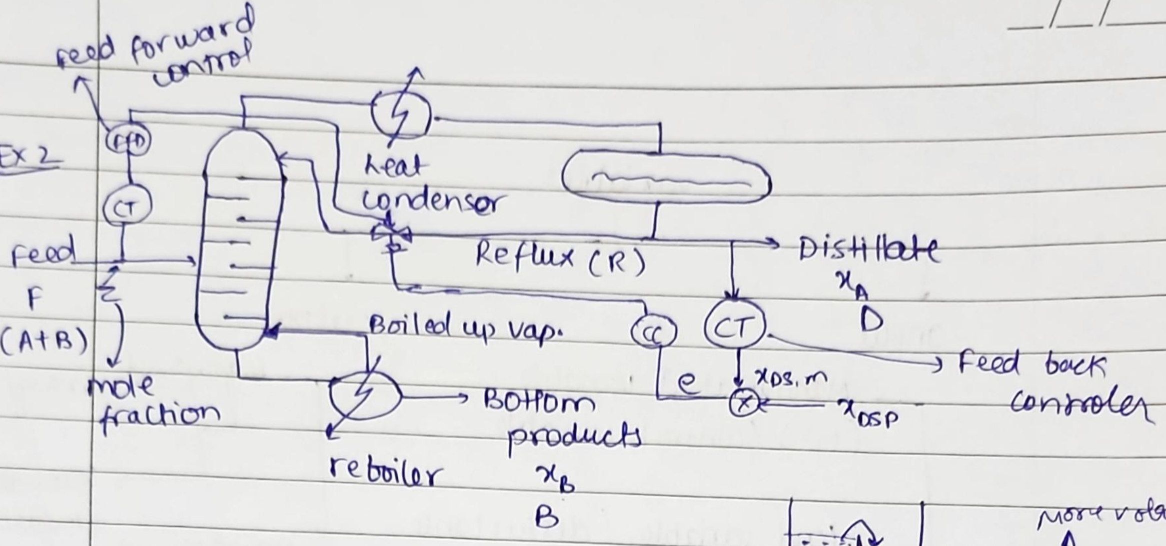
→ There should not be any instability problem.

Input : F_i
output : F_o, h

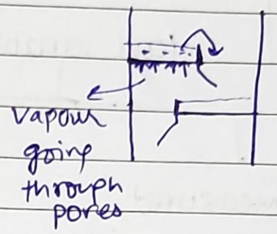
or

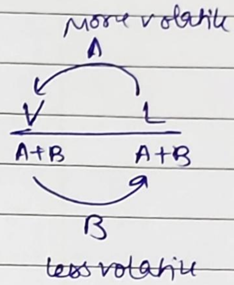
Input : F_i, F_o
output : h

Ex 2



$$e = x_{DSP} - x_{D,m}$$





During startup

Distilate flow rate $D = 0$

total reflux

CV	MV
x_D	R

At steady state

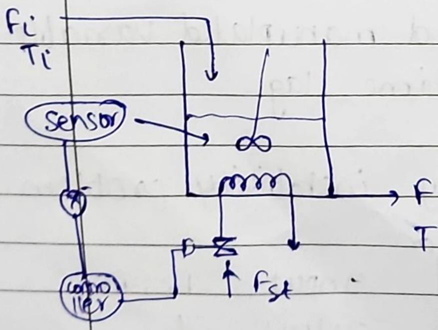
$D \neq 0$

$x_D \rightarrow$ Primary

$T_i \rightarrow$ secondary

→ measured temp is used to infer composition that's why it is called inferential controller

Hardware elements



control obj : $T = T_{sp}$

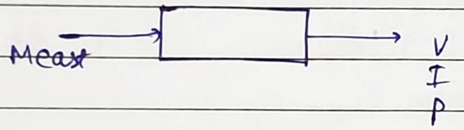
① The process : physical and chemical

② sensor

variable	equipment
T	Thermocouple, RTD
P	manometer, PG
F	orifice, ventury, rotameter
h	DPC
C	GC

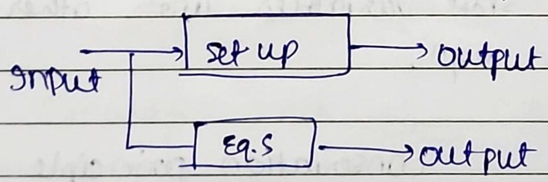
CV - - - FBC
Load variable - - - FFC
secondary - - - - IC

③ Transducer



④ Transmission line
⑤ controller → first Recives measurment signal then calculate manupulated variable.
⑥ Final control element:
⑦ Recording device

Process Modeling:



It is the mathematical represetation of a process intended to promote understanding of a model.

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Model
- → Theoretical — based on conservation Principale
  - → Volume
  - → Temp
- → Empirical

$C_p = a + bT + cT^2$

u	y
input	output

Theoretical model :

Adv : ① better insights
② Extrapolation is easy

disadvantages : ① complex
② time consuming

State variable :

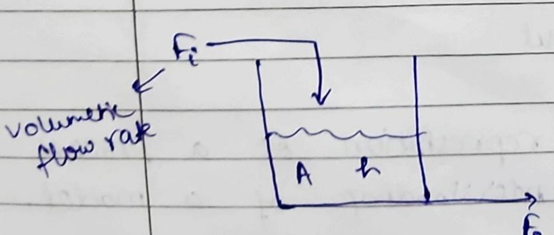
→ natural state

→ mass energy momentum

$\underbrace{h \quad T}_{\text{state variables}}$

present in accumulation term

The eqn which is developed by the application of the conservation Principle or fundamental quanties to relate the state variables with other variables is called state eqn.



conservation principle

Rate of accumulation = Rate of input − Rate of output
+ Rate of generation
− Rate of depletion

$$\frac{d(\rho A h)}{dt} = F_i \rho - F_o \rho$$

$$A\frac{dh}{dt} = F_i - F_o \quad \longrightarrow \text{ state eq}^n$$

h = state variable

Degrees of freedom (f)

F = No of independent variables − No. of eq^{n}s

case 1 :	$F=0$	$V=E$	exactly specified	ideal
case 2 :	$F>0$	$V>E$	underspecified	common
case 3 :	$F<0$	$V<E$	overspecified	uncommon

If $F>0$ or $V-E>0$

~~measat~~ measure some variables such that $V-E=0$. or Degree of freedom $=0$

controller eq^n

$V=3$ F_i, h, f_o

$E=1$

so, $f=2$

CV	MV.
h	F_o

measure : F_i , DOF $=1$

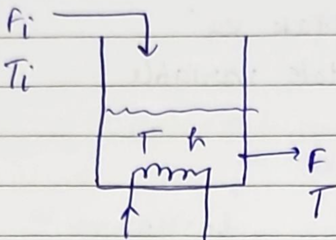
Eq^n : $F_o = F_{os} + K_c(h_{sp} - h)$

: propotional controller.

Simulated model

1. understanding process dynamics
2. Training
3. Identify CV−MV pairs tunning parameter
4. optimization

Heating tank



Q = energy per unit time $\qquad Q = F_{st}\lambda$

Assumption

1. No heat loss
2. perfect mixing
3. ρ, C_p

Mass balance:

$$A\frac{dh}{dt} = F_i - F_o$$

Energy balance:

$$\frac{d}{dt}\left[Ah\rho C_p (T - T_{ref})\right] = F_i \rho C_p (T_i - T_{ref}) - F \rho C_p (T - T_{ref}) + Q$$

Degree of freedom = 6

variable (F_i, F, T_i, T, h, Q)

eq^n: DOF = 6 − 2 = 4

Now measurement

Measure (F_i, T_i)

Now Degree of freedom $\Rightarrow 4-2 \Rightarrow 2$

Now we will use two eqⁿs for height and temperature.

$$\left.\begin{aligned} h &= h_{sp} \\ T &= T_{sp} \end{aligned}\right\} \text{control objective}$$

$$\cancel{c_p\, dT = F_i - F_o + Q}$$

$$\cancel{= F_i(\Delta T) - F_i C(\Delta T) + Q}$$

$$\cancel{\boxed{c_p\, dT = F_1 C(\Delta T) - F_o C + Q}}$$

$$\rho c_p \frac{d}{dt}[V(T)] = \rho c_p (F_i (T_i) - F T) + Q$$

$$T\frac{dV}{dt} + V\frac{dT}{dt} = (F_i T_i - F T) + \frac{Q}{\rho c_p}$$

$$T(\cancel{F_i} - F) + V\frac{dT}{dt} = (F_i - \cancel{F})T + \frac{Q}{\rho c_p}$$

$$V\frac{dT}{dt} = \frac{Q}{\rho c_p}$$

$$\frac{dT}{dt} = \frac{F_i(T_i - T)}{V} + \frac{Q}{V\rho c_p} \quad -(2)$$

Eqs:

CV	MV
T	Q
h	F

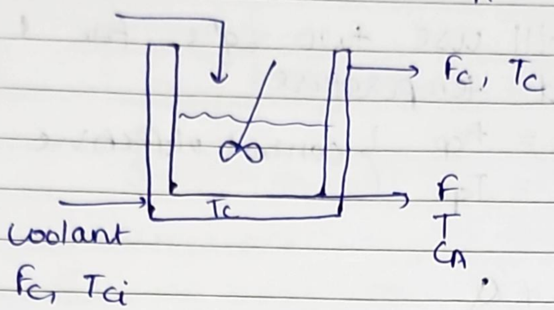
$$Q = Q_s + K_{cQ}(T_{sp} - T)$$

$$F = F_s + K_{cF}(h_{sp} - h)$$

Continuous stirred tank reactor (CSTR)

F_i, T_i, C_{Ai}

$A \longrightarrow pdt$



Assumptions:

1. No heat loss
2. perfect mixing
3. constant parameters (ρ,
4. Exothermic Rx^n

Model

Total mass balance Eq^n:

Rate of accumulation = Rate of input - Rate of output

$$\frac{d}{dt}(\rho V) = F_i\rho - F\rho$$

$$\frac{dV}{dt} = F_i - F \Rightarrow A\frac{dh}{dt} = F_i - F$$

comp A:

Rate of accumulation = Rate of in − Rate of out + Rate of generation

$$\frac{d}{dt}(AhC_A) = F_iC_A - FC_A - (-r_A)V$$

$-r_A = -\frac{dC_A}{dt} = -\frac{1}{V}\frac{dN_A}{dt}$

$(-r_A)V = -\frac{dN_A}{dt}\left(\frac{mol}{time}\right)$

$$\frac{dh}{dt}C_A + h\frac{dC_A}{dt} = \frac{F_iC_{Ai} - FC_A + r_AV}{A}$$

$-r_A$ is rate of disappearance of component A.

$$\frac{(F_i - \cancel{F})C_A}{A} + h\frac{dC_A}{dt} = \frac{F_iC_{Ai} - \cancel{F}C_A + r_AV}{A}$$

$$h\frac{dC_A}{dt} = \frac{F_i(C_{Ai} - C_A) + r_AV}{Ah}$$

$$\frac{dC_A}{dt} = \frac{F_i(C_{Ai} - C_A)}{V} + r_A \quad \text{— (2)}$$

Energy balance:

$$\frac{d}{dt}[V\rho C_pT] = F_i\rho C_pT_i - F\rho C_pT - Q + (-\Delta H)(-r_A)V$$

$$Q = UA(T - T_c)$$

$$C_p\rho\frac{d(VT)}{dt} = C_p\rho(F_iT_i - FT) - UA(T - T_c) + \Delta H r_AV$$

$$C_p\rho\left[T\frac{dV}{dt} + V\frac{dT}{dt}\right] = C_p\rho(F_iT_i - FT) - UA(T - T_c) + \Delta H r_AV$$

$$T(F_i - \cancel{F}) + V\frac{dT}{dt} = (F_iT_i - \cancel{FT}) - \frac{UA(T - T_c)}{C_p\rho} + \frac{\Delta H r_AV}{C_p\rho}$$

$$\boxed{\frac{dT}{dt} = \frac{F_i(T_i - T)}{V} - \frac{UA(T - T_c)}{C_p\rho V} + \frac{\Delta H r_A\cancel{V}}{C_p\rho\cancel{V}}}$$

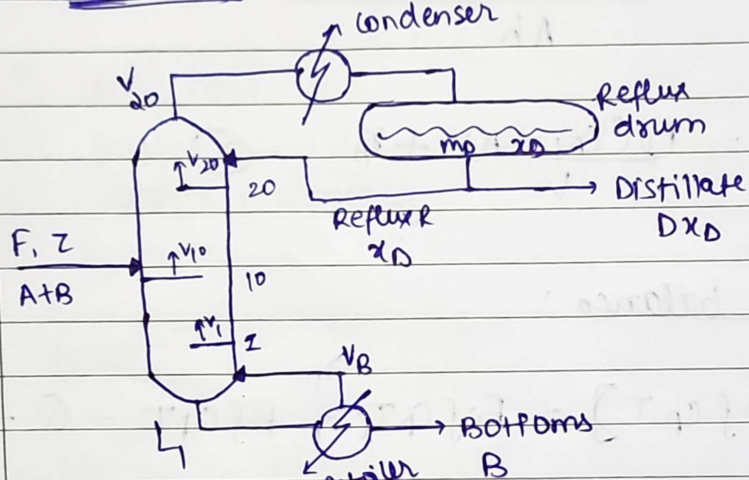
Degree of freedom : 10
$F_i, T_i, C_{Ai}, F, T, C, h, F_c, T_{ci}, T_c$

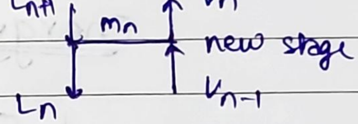
$F = 10 - 3 = 7$

Now measure $F_i, T_i,$ T_{ci}, C_{Ai}
$F = 7 - 4 = 3$

CV	MV
T	F_c
h	F

Distillation :





L = liq. flow rate (mole/min)
V = vap. flow rate - - -
R = Reflux flow rate - - -
D = Distillate " "
B = Bottom - - - - -
x = liq. comp (mole fraction)
y = vap comp.
m = liq hold up (mole)

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Assumption

1) perfect mixing

2) No heat loss

3) No vap holdup

4) molar heats of vapourisation of both comp. A and B are assumed to be identical

5) liq hold up

6) constant Relative volatility

7) ideal trays (100% efficient)

$V_1 = V_2 = \cdots = V_{20} = V_B$

<u>condenser - reflux drum:</u>

mole : $\frac{d}{dt}(m_D)$ {mole/time}

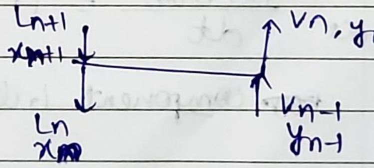
$$\frac{d}{dt}(m_D) = V_{20} - D - R$$

<u>component mole balance:</u>

$$\frac{d}{dt}(m_D x_D) = V_{20} y_{20} - (D+R) x_D$$

$$\frac{dx_D}{dt} = \frac{V_{20}}{m_D}(y_{20} - x_D)$$

<u>For n^{th} tray:</u>



mole :

$$\frac{d}{dt}(m_n) = \cancel{V_{n-1}} + L_{n+1} - (L_n) - \cancel{V_n}$$

component mole balance:

$$\frac{d}{dt}(m_n x_n) = V_{n-1} y_{n-1} + L_{n+1} x_{n+1} - L_n x_n - V_n y_n$$

for 20th tray :

mole balance :

$$\frac{d}{dt}(M_{20}) = V_{19} + R - V_{20} - L_{20}$$

component balance :

$$\frac{d}{dt}(M_{20}\, x_{20}) = V_{19}\, y_{19} + R x_D - V_{20}\, y_{20} - x_{20} L_{20}$$

for 1st tray :

mole balance :

$$\frac{d(m_1)}{dt} = L_2 + V_B - L_1 - V_1$$

component balance :

$$\frac{d}{dt}(m_1 x_1) = V_B y_B + L_2 x_2 - (L_1 x_1 + V_1 y_1)$$

For 10th tray :

mole balance $$\frac{d(m_{10})}{dt} = (F + V_9 + L_{11}) - (V_{10} + L_{10})$$

~~for~~ component balance :

$$\frac{d}{dt}(m_{10}\, x_{10}) = (F z + V_9 y_9 + x_{11} L_{11}) - (x_{10} L_{10} + y_{10} V_{10})$$

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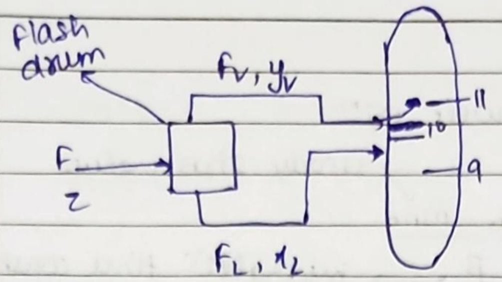

$$V_1 = V_2 - - - V_{10} = V_B$$
$$V_{11} = V_{12} - - - V_{20} = V_B + F_V$$

For Reboiler :

mole balance

$$\frac{d(m_R)}{dt} = L_1 - V_B - B$$

component balance

$$\frac{d}{dt}(m_R x_R) = L_1 x_1 - B x_B - V_B y_B$$

Simulation :

① $m \longrightarrow$ Total mole balance
② $x \longrightarrow$ component mole balance
③ $y \longrightarrow$ α - based eq^n

constant Relative volatility $\longrightarrow \alpha_{ij} = \frac{k_i}{k_j}$ K is vap-~~to~~liq ~~cst~~ equb^n coff

$$k_i = \frac{y_i}{x_i}$$

$$\alpha_{ij} = \frac{y_i/x_i}{y_j/x_j} = \frac{(y_i/x_i)}{\frac{(1-y_i)}{(1-x_i)}}$$

$$y_i = \frac{\alpha_{ij} x_i}{1+(\alpha_{ij}-1)x_i}$$

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④ liquid flow rate

$L \rightarrow$ Francis-weir eq^n

$$L_n = L_{n_0} + \frac{m_n - m_{n_0}}{\beta}$$

L_{n_0}, m_{n_0} → steady state values

β → hydraulic time constant
typically adopted (in b/w 3 to 6 sec)

⑤ vapour flow rate → V_B { Q_R - reboiler duty
$= m_S \lambda_S$
$= V_B \lambda_S$

<u>Distillation</u> : Degree of freedom:

$F = V - E$, No. of trays $= N$

No. of eq^{n}s	origin
$N+1$	$y = \frac{\alpha x}{1+(\alpha-1)x}$
N	$L = L_0 + \frac{m - m_0}{\beta}$
$N+2$	$\frac{dm}{dt}$
$N+2$	$\frac{dx}{dt}$

No. of variables	Type
$N+1$	y
N	L
$N+2$	M
$N+2$	x
6	F, Z, D, R, B, V_B
$4N+11$	

$F = 6$

measure : F, z $\qquad f = 6 - 2 = 4$

CV	MV
x_D	R
x_B	V_B
M_D	D
M_B	B

Laplace transform:

$$\mathcal{L}[f(t)] = F(s) = \int_0^\infty f(t)\, e^{-st}\, dt$$

$f(1)$	$1/s$
t^n	$\frac{n!}{s^{n+1}}$
$e^{-at} \sinh wt$	$\frac{w}{(s+a)^2 - w^2}$

$$\mathcal{L}\left[\int_0^t f(t)\, dt\right] = \frac{1}{s} \bar{f}(s)$$

$$\mathcal{L}\left[\frac{df}{dt}\right] = s f(s) - F(0)$$

$$\mathcal{L}\left[\frac{d^n f}{dt^n}\right] = s^n F(s) - s^{n-1} F(0) - s^{n-2} F'(0) \ldots - f^{n-1}(0)$$

$$= s^n F(s)$$

f = deviation variable $= f_t - f_{ss}$

$f(t = 0) = 0$

Final value Theorem

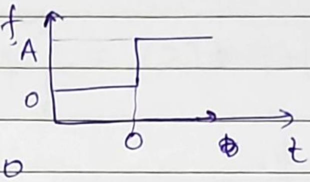
$$\lim_{t\to\infty} f(t) = \lim_{s\to 0} sF(s)$$

initial value theorem

$$\lim_{t\to 0} f(t) = \lim_{s\to\infty} sF(s)$$

Input / Forcing functions:

① step input

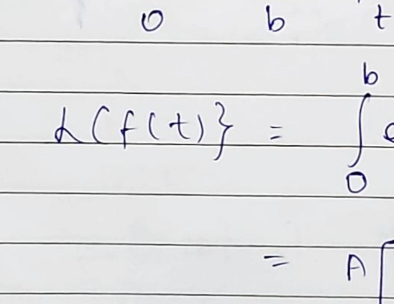


$$f(t) = \begin{cases} 0 & t<0 \\ A & t>0 \end{cases}$$

$$F(s) = \frac{A}{s}$$

unit step change $F(s) = \frac{1}{s}$

② pulse function



$$\mathcal{L}\{f(t)\} = \int_0^b e^{-st} A\,dt + \int_b^\infty 0$$

$$= A\left[\frac{e^{-(s)t}}{-(s)}\right]_0^b$$

$$= \frac{A}{s}\left[\frac{1}{s} - \frac{e^{sb}}{s}\right) = \frac{A}{s}\left(1 - e^{-sb}\right]$$

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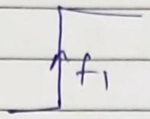
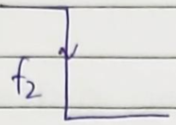


$$f_1 = \begin{cases} 0 & t<0 \\ A & t>0 \end{cases}$$

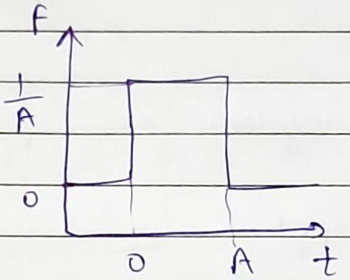


$$f_2 = \begin{cases} 0 & t<b \\ A & t>b \end{cases}$$

$$= f_1(t-b)$$

$$F(t) = f_1(t) - f_2(t-b)$$

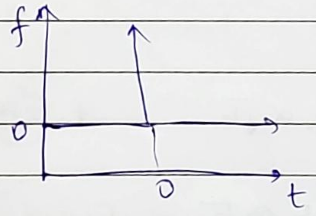
(3) unit pulse f^n:



$$F(s) = \frac{1-e^{AS}}{AS} = \int_0^A \frac{e^{-st}}{A}\,dt = \delta_A$$

(4) unit impulse :

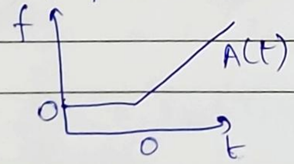
$A \to 0$
in unit pulse



dirac function
$\delta(t)$

$$\mathcal{L}\left[\lim_{A\to 0} \delta_A(t)\right] = \mathcal{L}[\delta(t)] = 1$$

(5) Ramp function



$$f(t) = \begin{cases} 0 & t<0 \\ At & t>0 \end{cases}$$

$$F(s) = \frac{A}{s^2}$$

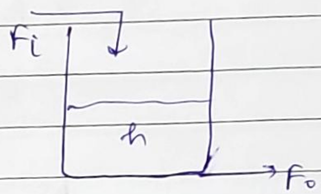
Nonlinear system:

$$\frac{dx}{dt} = F(x) \quad \text{--- nonlinear system} \quad (1)$$

$$f(x) = f(x_s) + \left(\frac{dF}{dx}\right)_{x=x_s} \frac{(x-x_s)}{1!} + \left(\frac{d^2f}{dx^2}\right)_{x=x_s} \frac{(x-x_s)^2}{2!} \; \text{---}$$

$$\frac{dx}{dt} = f(x) = f(x_s) + \left(\frac{dF}{dx}\right)_{x=x_s}(x-x_s) \quad \text{--- linear form} \quad (II)$$

Ex Liq tank system



$$A\frac{dh}{dt} = f_i - F_o$$

Case 1 $F_o \propto h$ $F_o = \alpha h$

$$A\frac{dh}{dt} + \alpha h = f_i \quad \text{--- linear model}$$

Case II

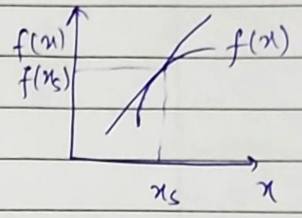
$F_o \propto \sqrt{h}$ $f_o = \beta\sqrt{h}$

$$A\frac{dh}{dt} + \beta\sqrt{h} = f_i \quad \text{--- Non linear}$$

$$\beta\sqrt{h} = \beta\sqrt{h_s} + \frac{\beta}{2\sqrt{h_s}}(h - h_s)$$

$$A\frac{dh}{dt} + \beta\sqrt{h_s} + \frac{\beta}{2\sqrt{h_s}}(h-h_s) = f_i \quad (III)$$

$$\frac{A\,dh}{dt} + \frac{\beta\, h}{2\sqrt{h_s}} = F_i - \frac{\beta\sqrt{h_s}}{2}$$



$$\frac{dx_s}{dt} = F(x_s) \quad - (3) \quad \{\text{AT steady state}\}$$

Now eq^n (2) - (3)

$$\frac{d(x-x_s)}{dt} = \left(\frac{df}{dx}\right)_{x=x_s} (x-x_s)$$

$x - x_s$ = deviation variable

$$\boxed{\frac{dx'}{dt} = \left(\frac{df}{dx}\right)_{x=x_s} x'}$$

$$\frac{A\,dh_s}{dt} + \beta\sqrt{h_s} = F_{i_s} \quad \{\text{At steady state}\} \quad - (IV)$$

$$A\frac{d(h-h_s)}{dt} + \beta(\sqrt{h} - \sqrt{h_s}) = F_i - F_{i_s}$$

$$A\frac{dh'}{dt} + \beta(\sqrt{h} - \sqrt{h_s}) = F_i'$$

eq^n (III) - (IV)

$$\frac{A\,dh'}{dt} + \frac{\beta(h')}{2\sqrt{h_s}} = F_i'$$

<u>Multivariable system:</u>

$$\frac{dx_1}{dt} = F_1(x_1, x_2) \quad (I)$$

$$\frac{dx_2}{dt} = F_2(x_1, x_2) \quad (II)$$

nonlinear system

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$$f_1(x_1, x_2) = f_1(x_{1s}, x_{2s}) + \left(\frac{\partial f_1}{\partial x_1}\right)_{x_{1s}, x_{2s}} (x - x_{1s}) + \left(\frac{\partial f_1}{\partial x_2}\right)_{x_{1s}, x_{2s}} (x_2 - x_{2s})$$

$$\underbrace{+ \left(\frac{\partial^2 f_1}{\partial x_1^2}\right)_{x_{1s}, x_{2s}} \frac{(x_1 - x_{1s})^2}{2!} + \left(\frac{\partial^2 f_1}{\partial x_2^2}\right)_{x_s} \frac{(x_2 - x_{2s})}{2!}}_{\text{neglet}}$$

$$\frac{dx_1}{dt} = f_1(x_1, x_2) = f_1(x_{1s}, x_{2s}) + \left(\frac{\partial f_1}{\partial x_1}\right)_{x_{1s}, x_{2s}} (x_1 - x_{1s}) + \left(\frac{\partial f_1}{\partial x_2}\right)_{x_{1s}, x_{2s}} (x_2 - x_{2s}) \quad -③$$

$$\frac{dx_s}{dt} = f_1(x_{1s}, x_{2s}) \quad -④$$

$$\frac{dx_1'}{dt} = \left(\frac{\partial f_1}{\partial x_1}\right)_{x_s} x_1' + \left(\frac{\partial f_1}{\partial x_2}\right)_{x_s} x_2'$$

$$\frac{dx_2'}{dt} = \left(\frac{\partial f_2}{\partial x_1}\right)_{x_s} x_1' + \left(\frac{\partial f_2}{\partial x_2}\right)_{x_s} x_2'$$

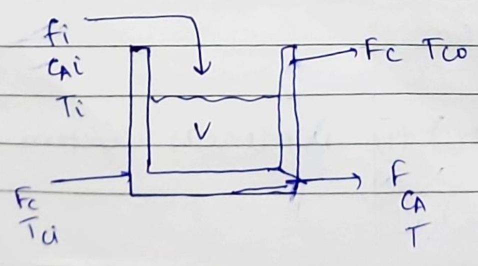
$$\frac{dx_1'}{dt} = a_{11} x_1' + a_{12} x_2'$$

$$a_{ij} = \left(\frac{\partial f_i}{\partial x_j}\right)_{x_s}$$

$$\frac{dx_2'}{dt} = a_{21} x_1' + a_{22} x_2'$$

$$\begin{bmatrix} \dot{x}_1' \\ \dot{x}_2' \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} x_1' \\ x_2' \end{bmatrix}$$

<u>Ex</u> <u>CSTR</u>



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$$A\frac{dh}{dt} = F_i - F \quad (1)$$

$$\frac{dC_A}{dt} = \frac{F_i(C_{Ai} - C_A)}{V} + r_A \quad (2)$$

$$\frac{dC_A}{dt} = \frac{1}{\tau}(C_{Ai} - C_A) + r_A \quad (2) \quad \left\{ \frac{1}{\tau} = \frac{F_i}{V} \right\}$$

residance time

$$\frac{dC_A}{dt} = \frac{1}{\tau}(C_{Ai} - C_A) - k_0 C_A e^{-E/RT} \quad (2)$$

nonlinear terms

$$\frac{dT}{dt} = \frac{1}{\tau}(T_i - T) - \frac{Q}{V\rho C_p} - \frac{\Delta H\, k_0 C_A e^{-E/RT}}{\rho C_p} \quad (3)$$

$$C_A e^{-E/RT} = C_{As} e^{-E/RT_s} + e^{-E/RT_s}(C_A - C_{As}) + \frac{E\, C_{As} e^{-E/RT_s}}{RT_s^2}(T - T_s)$$

$$\frac{dC_A'}{dt} = \frac{1}{\tau}(C_{Ai}' - C_A') - k_0\left(C_{As} e^{-E/RT_s} + e^{-E/RT_s}(C_A - C_{As})' + \frac{E\, C_{As} e^{-E/RT_s}}{RT_s^2}(T - T_s)'\right)$$

$$\frac{dT}{dt} = \frac{1}{\tau}(T_i - T) - \frac{Q}{V\rho C_p} - \frac{\Delta H\, k_0}{\rho C_p}\left[C_A e^{-E/RT_s} + e^{-E/RT_s}(C_A - C_{As}) + \frac{E\, C_{As} e^{-E/RT_s}}{RT_s^2}(T - T_s)\right]$$

Transfer function:

input $f(t)$ → [Process] → output $y(t)$ input $\bar{f}(s)$ → [$G(s)$] → output $\bar{y}(s)$

single input single output

linear process

$$G(s) \equiv TFn = \frac{\bar{y}(s)}{\bar{F}(s)}$$

$$a_n\frac{d^n y}{dt^n} + a_{n-1}\frac{d^{n-1}y}{dt^{n-1}} \text{ ------ } a_1\frac{dy}{dt} + a_0 y = bf(t)$$

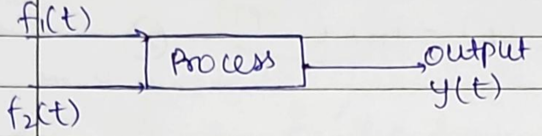
y, f = deviation variable

SS at $t=0$

$$a_n\{s^n\bar{y}(s)\} + a_{n-1}s^{n-1}\bar{y}(s) \cdots a_1 s\bar{y}(s) + a_0\bar{y}(s) = b\bar{f}(s)$$

$$\frac{\bar{y}(s)}{\bar{f}(s)} = \frac{b}{a_n s^n + a_{n-1}s^{n-1} \cdots a_1 s + a_0}$$

Multiinput single output:

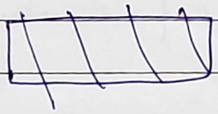


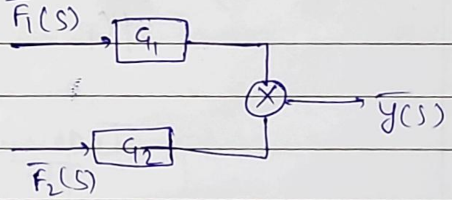
$$a_n\frac{d^n y}{dt^n} + a_{n-1}\frac{d^{n-1} y}{dt^{n-1}} \cdots a_1\frac{dy}{dt} + a_0 y = b_1 f_1(t) + b_2 f_2(t)$$

$$a_n s^n\bar{y}(s) + a_{n-1}s^{n-1}\bar{y}(s) \cdots a_1 s\bar{y}(s) + a_0\bar{y}(s) = b_1\bar{f}_1(s) + b_2\bar{f}_2(s)$$

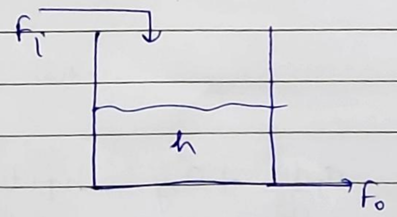
$$\frac{\bar{y}(s)}{\cancel{f(s)}} = \frac{b_1(\bar{f}_1(s)) + b_2\bar{f}_2(s)}{a_n s^n + a_{n-1}s^{n-1} \cdots a_1 s + a_0}$$

$$\bar{y}(s) = G_1\bar{F}_1(s) + G_2\bar{F}_2(s)$$





Ex liquid tank



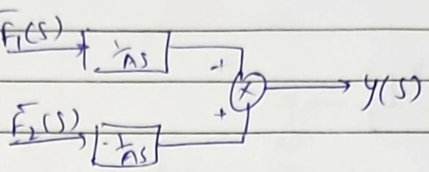
$$A\frac{dh}{dt} = F_i - F_o$$

$$A(s)\bar{h}(s) = \bar{F}_i(s) - \bar{F}_o(s)$$

$$\bar{h}(s) = \frac{\bar{F}_i(s) - \bar{F}_o(s)}{As}$$

$$h(S) = G_1 \bar{F}_1(S) + G_2 \bar{F}_2(S)$$

$$G_1 = \frac{1}{As}, \quad G_2 = \frac{-1}{As}$$



Multiinput multioutput:

$$\frac{dy_1}{dt} = a_{11} y_1 + a_{12} y_2 + b_{11} f_1 + b_{12} f_2$$

$$\frac{dy_2}{dt} = a_{22} y_1 + a_{22} y_2 + b_{21} f_1 + b_{22} f_2$$

$$\bar{y}_1(s) = \frac{(s-a_{22}) b_{11} + a_{12} b_{21}}{P(s)} \bar{F}_1(s) + \frac{(s-a_{22}) b_{12} + a_{12} b_{22}}{P(s)} \bar{F}_2(s)$$

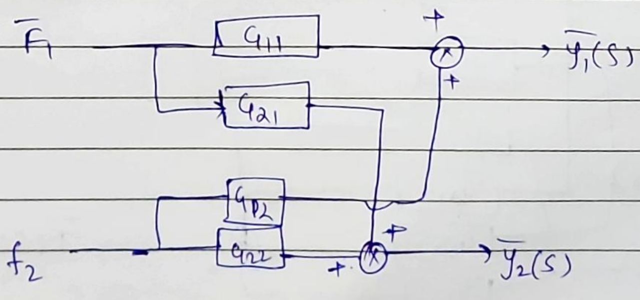
$$\bar{y}_2(s) = \frac{(s-a_{11}) b_{21} + a_{21} b_{11}}{P(s)} \bar{F}_1(s) + \frac{(s-a_{11}) b_{22} + a_{21} b_{12}}{P(s)} \bar{F}_2(s)$$

$$P(s) = s^2 - (a_{11} + a_{12}) s - (a_{12} a_{21} - a_{11} a_{22})$$

$$\bar{y}_1(s) = G_{11} \bar{F}_1(s) + G_{12} \bar{F}_2(s)$$

$$y_2(s) = G_{21} \bar{F}_1(s) + G_{22} \bar{F}_2(s)$$

i = output
j = input



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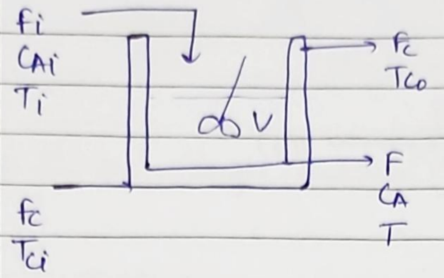
CSTR : MIMO



$$\frac{dC_A'}{dt} + \left[\frac{1}{\tau} + k_0 e^{-E/RT_0}\right] C_A' + \left[\frac{k_0 E}{RT_0^2} e^{-E/RT_0} C_{A0}\right] T' = \frac{1}{\tau} C_{Ai}'$$

$$\frac{dT'}{dt} + \left[\frac{1}{t} - \frac{S k_0 E}{RT_0^2} e^{-E/RT_0} C_{A0} + \frac{U A_t}{V \rho C_p}\right] T' - \left[S k_0 e^{-E/RT_0}\right] C_A' = \frac{1}{\tau} T_i' + \frac{U A_t}{V \rho C_p} T_c'$$

$$S = \frac{(-\Delta H)}{\rho C_p} \quad , \quad Q = U A_t (T - T_c)$$

Heat transfer area

$$\frac{dC_A'}{dt} + a_{11} C_A' + a_{12} T' = b_1 C_{Ai}'$$

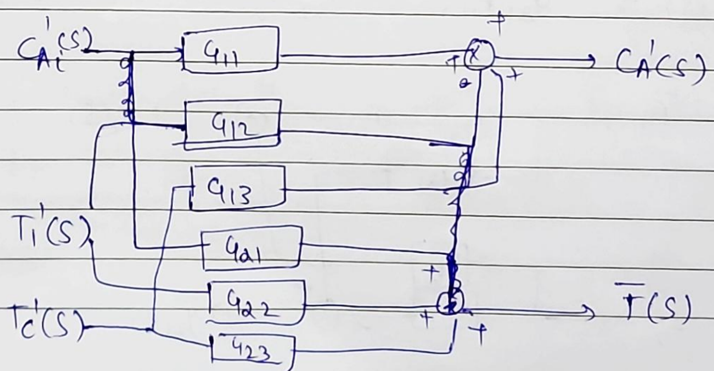
$T'(0) = 0$

$$\frac{dT}{dt} + a_{21} C_A' + a_{22} T' = b_1 T_i' + b_2 T_c'$$

$C_A'(0) = 0$

$$C_A'(s) = G_{11} \overline{C_{Ai}}'(s) + G_{12} T_i'(s) + G_{13} \overline{T_c}'(s)$$

$$\overline{T}(s) = G_{21} \overline{C_A}'(s) + G_{22} T_i'(s) + G_{23} T_c'(s)$$



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zeros and poles

$$\frac{\bar{y}(s)}{\bar{f}(s)} = G(s) = \frac{Q(s)}{P(s)}$$

$Q(s) = 0$ and find s ← zeros

$P(s) = 0$ and find s ← poles

$$G(s) = \frac{\frac{1}{\tau}}{s+a}$$ No zeros, pole $= -a$

$$G(s) = \frac{s-1}{s^2-3s+2}$$ zeros $= 1$, poles $= 1, 2$

$$a_n \frac{d^n y}{dt^n} + a_{n-1} \frac{d^{n-1} y}{dt^{n-1}} \cdots a_1 \frac{dy}{dt} + a_0 y = b_m \frac{d^m f}{dt^m} + b_{m-1} \frac{d^{m-1} f}{dt^{m-1}} \cdots b_1 \frac{df}{dt} + b_0 f$$

$$\frac{\bar{y}(s)}{\bar{F}(s)} = G(s) = \left(\frac{b_m}{a_m}\right)\left(\frac{(s-z_1)(s-z_2) \cdots (s-z_m)}{(s-p_1)(s-p_2) \cdots (s-p_n)}\right)$$

$n > m$ strictly proper

$n = m$ proper

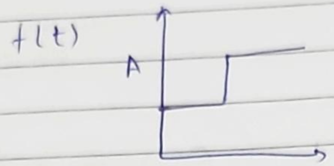
$n < m$ improper @ Not physically realizable

$n < m$ $n = 0$, $m = 1$

$$a_0 y = b_1 \frac{df}{dt} + b_0 f$$

$$a_0 \bar{y}(s) = b_1 s \bar{f}(s) + b_0 \bar{f}(s)$$

$$\bar{y}(s) = \frac{b_1}{a_0} s \bar{f}(s) + \frac{b_0}{a_0} \bar{f}(s)$$



$$f(t) = A$$

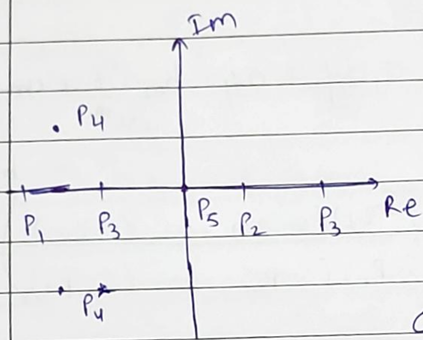
$$\bar{f}(s) = \frac{A}{s}$$

$$\bar{y}(s) = \frac{b_1}{a_0} A + \frac{b_0}{a_0} \frac{A}{s}$$

inverse of laplace transform

$$y(t) = \frac{b_1}{a_0} A\, \delta(t) + \frac{b_0}{a_0} A$$

Qualitative analysis:



$$G(s) = \frac{Q(s)}{P(s)}$$

$Q(s)$ → plays role in process dynamics

$P(s)$ → plays role in stability of a process

$$G(s) = \frac{Q(s)}{(s-P_1)(s-P_2)(s-P_3)^m(s-P_4)(s-P_4^*)(s-P_5)}$$

$$G(s) = \frac{C_1}{s-P_1} + \frac{C_2}{s-P_2} + \left\{ \frac{C_{31}}{s-P_3} + \frac{C_{32}}{(s-P_3)^2} \cdots \frac{C_{3m}}{(s-P_3)^m} \right\} + \frac{C_4}{s-P_4} + \frac{C_4^*}{s-P_4^*} + \frac{C_5}{s-P_5}$$

case: 1 Two distinct Real poles

$$G(s) = \frac{C_1}{s-P_1} + \frac{C_2}{s-P_2}$$

$$\mathcal{L}^{-1}(G(s)) = C_1 e^{P_1 t} + C_2 e^{P_2 t}$$

$P_1 < 0$ $\quad e^{P_1 t} \to 0$ as $t \to \infty$ $\quad$ stable

$P_2 > 0$ $\quad e^{P_2 t} \to \infty$ as $t \to \infty$ $\quad$ unstable

$P_1 < 0$ and $P_2 < 0$ {stable}

$P_1 > 0$ and $P_2 > 0$ {unstable}

case ② **Multiple real roots:**

$$G(S) = \frac{C_{31}}{S-P_3} + \frac{C_{32}}{(S-P_3)^2} \text{ —— } \frac{C_{3m}}{(S-P_3)^m}$$

$$\mathcal{L}^{-1}(G(S)) = \left[C_{31} + \frac{C_{32}t}{1!} + \frac{C_{33}}{2!}t^2 \text{ —— } \frac{C_{3m}t^{m-1}}{(m-1)!} \right] e^{P_3 t}$$

$P_3 < 0$ $\quad e^{P_3 t} \to 0$ as $t \to \infty$ $\quad$ stable

$P_3 > 0$ $\quad e^{P_3 t} \to \infty$ as $t \to \infty$ $\quad$ unstable

case ③ **conjugate poles**

$$G(S) = \frac{C_4}{S-P_4} + \frac{C_4^*}{S-P_4^*}$$

$P_4 = \alpha + i\beta$

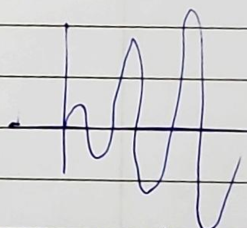
$P_4^* = \alpha - i\beta$

$$\mathcal{L}^{-1}(G(S)) = e^{\alpha t} \sin(\beta t + \phi)$$

$\alpha > 0$ $\quad e^{\alpha t} \to \infty$ as $t \to \infty$ $\quad$ unstable

$\alpha < 0$ $\quad e^{\alpha t} \to 0$ as $t \to \infty$ $\quad$ stable.

$\alpha = 0$ $\quad$ sinosodal wave as $t \to \infty$



case ④ $G(S) = \frac{C_5}{S-P_5} = \frac{C_5}{S}$ as $P_5 = 0$ {pole at origin}

$$\mathcal{L}^{-1}\{G(S)\} = C_5$$