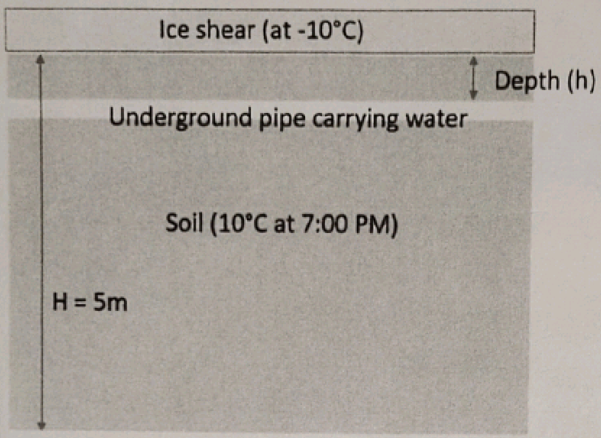
CT>1

45

Time: 40 minutes

$$\frac{T-T_s}{T_o-T_s} = erf\left(\frac{x}{\sqrt{4\alpha t}}\right)$$

In extremely cold places on earth, freezing of water in underground pipe-lines is a major concern due to sub-freezing temperature (below 0°C). Thanks to nature, that soil acts as bad conductor due to its porous architecture. Consider one such scenario as described below (with schematic).



The complementary error function

η	erfc (η)	η	erfc (η)
0.00	1.00000	0.38	0.5910
0.02	0.9774	0.40	0.5716
0.04	0.9549	0.42	0.5525
0.06	0.9324	0.44	0.5338
0.08	0.9099	0.46	0.5153
0.10	0.8875	0.48	0.4973
0.12	0.8652	0.50	0.4795
0.14	0.8431	0.52	0.4621
0.16	0.8210	0.54	0.4451
0.18	0.7991	0.56	0.4284
0.20	0.7773	0.58	0.4121

In harsh winter season, in the Himalayan region, the soil remains covered with ice sheet at temp -10°C for continuous 12 hours (7:00PM to 7:00 AM) every day. You have been assigned a job to find out the minimum appropriate depth for water carrying underground pipe to avoid water freezing inside it over these 12 hours. Assume that the initial temp of soil is 10°C throughout the depth (before 7:00PM). The soil thermal properties are, k= 0.4W/(m.K), thermal diffusivity $(\alpha) = 10^{-6}$ m^2/s. Assume, the **total depth** of soil, H= 5m. Freezing point of ice is 0°C. Assumption: Kindly ignore the pipe diameter. Assume pipe material is very conductive such that pipe material temperature is practically equal to water flowing inside it. (if you want, you can skip derivation).

Marks (5)

A crystal is growing from a supersaturated solution. The solution contains 4 moles/lit of solute and saturated conditions have been measured to be 3.95 moles/lit at the conditions of the test. The crystal is 1 mm in diameter and the fluid flows past the crystal at a velocity of 0.5 m/s. The viscosity of the fluid is 5x10^{-3} Ns/m^2 and its density is 1100 kg/m^3. The solid has a density of 1500 kg/m^3 and its molecular weight is 150. The diffusion coefficient for the solute in the liquid is 4x10^{-10} m^2/s. The Sherwood number for this situation can be expressed by the following correlation:

$$Sh_D = 2 + \left(0.4\,\mathrm{Re}_D^{1/2} + 0.06\,\mathrm{Re}_D^{2/3}\right) Sc^{0.4}$$

How fast could the crystal grow in this solution?

Marks (5)