_/_/_

First order process:

A first order system is one whose output y is modeled by a first order differential eq^n.

$$a_1 \frac{dy}{dt} + a_0 y = b f(t)$$

case 1: $a_0 \neq 0$

$$\frac{a_1}{a_0} \frac{dy}{dt} + y = \frac{b_0}{a_0} f$$

$$\tau_p \frac{dy}{dt} + y = k_p f$$

$$\tau_p = \frac{a_1}{a_0} = \text{time constant}$$

$$k_p = \frac{b}{a_0} = \text{static gain}$$

$$= \frac{y}{f} = \frac{\Delta\text{output}}{\Delta\text{input}} \ \{\text{at steady state}\}$$

$$\tau_p s \mathcal{L}\{y(t)\} + \mathcal{L}\{y(t)\} = k_p \bar{F}(s)$$

$$\tau_p s \bar{y}(s) + \bar{y}(s) = k_p \bar{F}(s)$$

$$G(s) = \frac{\bar{y}(s)}{\bar{F}(s)} = \frac{k_p}{\tau_p s + 1}$$ --- TF of 1st-order.

case 2: $a_0 = 0$

$$a_1 \frac{dy}{dt} = b f$$

$$\frac{dy}{dt} = \frac{b}{a_1} f = k_p' f$$

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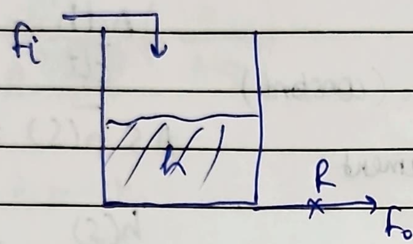
$$G(s) = \frac{\bar{y}(s)}{\bar{F}(s)} = \frac{K_p'}{s}$$ -- pure integrator

Ex



$$A\frac{dh}{dt} = F_i - F_o$$

$$A\frac{dh'}{dt} + \frac{h'}{R} = F_i'$$

$$F_o = \frac{h}{R}$$

$$A\bar{h}'(s) + \frac{\bar{h}'(s)}{R} = \bar{F}_i'(s)$$

$$\cancel{h'(s)(A}$$

$$AR\,\bar{h}'(s) + h'(s) = R\,F_i'(s)$$

$\tau_p = AR$ {unit s}

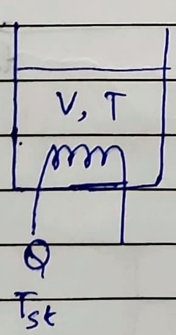
$K_p = R$ {unit depends on the system}

$$\frac{\bar{h}'(s)}{\bar{F}_i(s)} = \frac{R}{AR+1} = \frac{K_p}{\tau+1}$$

$\tau = A \times R$

(storage cap) $\times$ Resistance

Ex



Energy balance:

$$\frac{d}{dt}(\rho V C_p (T - T_{ref})) = Q = UA(T_{st} - T_{ref})$$

$$\frac{d}{dt}(\rho V c_p (T - T_{ref})) = UA'(T_{st} - T_{ref})$$

$$\frac{\rho V c_p}{UA'}\frac{dT'}{dt} + T' = T_{st}'$$

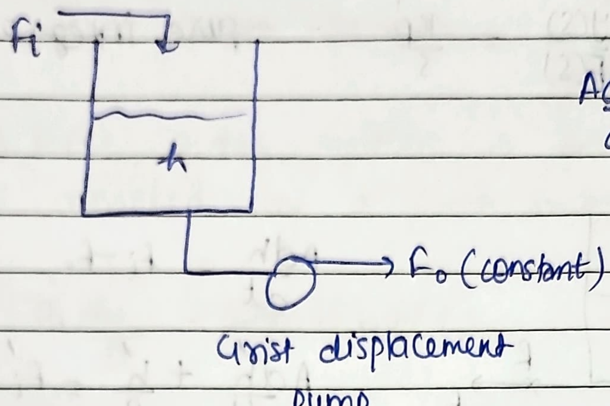
(storage cap)

$\tau_p = \rho V c_p \times \frac{1}{UA'}$ Resistance

$K_p = 1$

$$\boxed{G(s) = \frac{K_p}{\tau_p + 1}}$$

Ex 3



$$A\frac{dh}{dt} = F_i - F_o$$

$$A\frac{dh'}{dt} = F_i'$$

$$A S \bar{h}(S) = F_i(S)$$

$$\frac{h(S)}{F_i(S)} = \frac{1}{AS} = \frac{K_p'}{S}$$

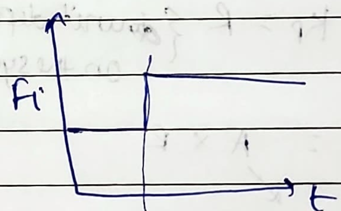
dynamic response

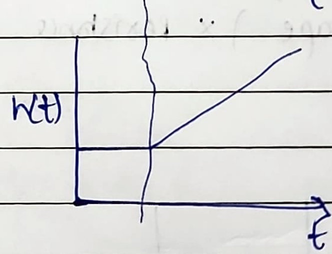
~~$T_p A$~~

$$\bar{h}'(S) = \frac{K_p'}{S} F_i'(S)$$

introducing unit step change in F_i

~~h(S)~~ $h(t) = K_p' t$

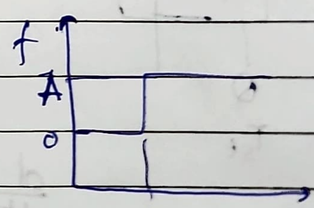




Ramp function

for first order system

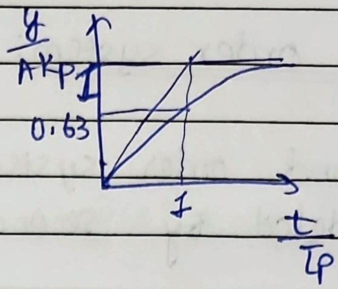
$$\bar{y}(S) = \frac{K_p}{T_p S + 1} \bar{f}(S)$$



$$\bar{y}(S) = \frac{K_p}{T_p S + 1} \times \frac{A}{S} \Rightarrow \frac{K_p}{S} - \frac{K_p T_p}{T_p S + 1}$$

$$y(t) = \mathcal{L}^{-1}\left\{ \frac{\frac{K_p}{T_p}}{S + \frac{1}{T_p}} \times \frac{1}{S} \right\} \Rightarrow \boxed{A K_p \left(1 - e^{-\frac{t}{T_p}}\right)}$$

$\frac{y}{AK_p}$ Vs $\frac{t}{\tau_p}$



$$\lim_{t \to \infty} \left(\frac{y}{AK_p} \right) = 1$$

Remarks

1) Final value = 1

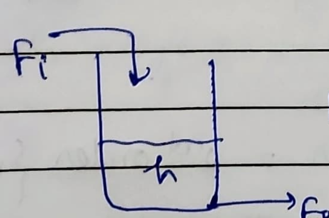
② $\left[\frac{d(y/AK_p)}{d\left(\frac{t}{\tau_p}\right)} \right]_{t=0} = 1$

If the intial rate of change of $y(t)$ were to be maintained the response will reach it's final value in one time constant

③

$\frac{t}{\tau_p}$	1	2	3	4
$\frac{y}{AK_p}$	0.632	0.86	0.9502	0.9816

④ self regulating system



$h_{ss1} = 10m$

$F_i \uparrow$, $h \uparrow$ F_o

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2^{nd} order system:

Second order system is one whose output y is modeled by second order diff^nal eq^n.

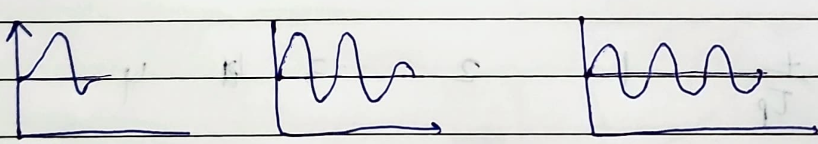
$$a_2 \frac{d^2y}{dt^2} + a_1 \frac{dy}{dt} + a_0 y = b f(t)$$

$$\frac{a_2}{a_0} \frac{d^2y}{dt^2} + \frac{a_1}{a_0} \frac{dy}{dt} + y = \frac{b}{a_0} f(t) \qquad a_0 \neq 0$$

$$\tau^2 \frac{d^2y}{dt^2} + 2\zeta\tau \frac{dy}{dt} + y = K_p f(t)$$

$$\tau^2 = \frac{a_2}{a_0}, \quad 2\zeta\tau = \frac{a_1}{a_0}, \quad K_p = \frac{b}{a_0}$$

τ = natural period
ζ = damping factor
K_p = static gain



ζ more ossicalation ↓ ζ less ossicilation more   zero dampin $\zeta = 0$

~~damping~~ ζ = It is a measure of the amount of damping. The degree of ossicilation

2^{nd} order system

1) 1^{st}-order system + 1^{st} order = 2^{nd} order {multicapacity process}

ii) 1^{st} order + controller = 2^{nd} order

iii)

$$G(s) = \frac{\bar{y}(s)}{\bar{f}(s)}$$

$$\tau^2 s^2 \bar{y}(s) + 2\zeta\tau s \bar{y}(s) + \bar{y}(s) = K_p \bar{f}(s)$$

$$\frac{\bar{y}(s)}{\bar{f}(s)} = \frac{K_p}{\tau^2 s^2 + 2\zeta\tau s + 1}$$

pole: $\tau^2 s^2 + 2\zeta\tau s + 1 = 0$

$$\Rightarrow \frac{-2\zeta\tau \pm \sqrt{4\zeta^2\tau^2 - 4\tau^2}}{2\tau^2}$$

$$\Rightarrow \frac{\cancel{2\tau}\left(-\zeta \pm \sqrt{\zeta^2 - 1}\right)}{\cancel{2}\tau^{\cancel{2}}}$$

$$\Rightarrow \frac{-\zeta \pm \sqrt{\zeta^2 - 1}}{\tau}$$

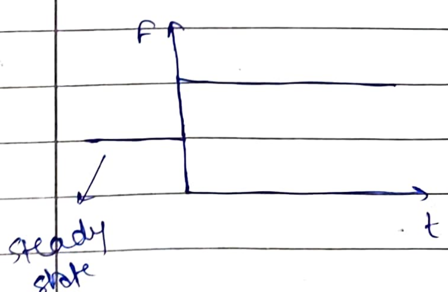
case I $\zeta > 1$ two distinct real poles
overdamped system

case 2 $\zeta = 1$ one pole
critically damped

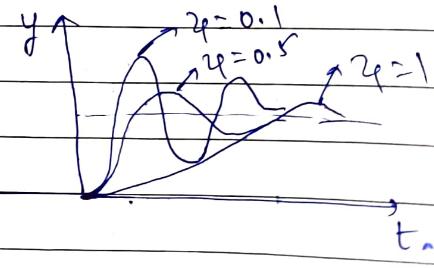
case 3 $0 < \zeta < 1$ conjugate poles
underdamped

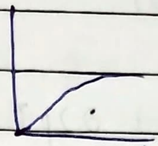
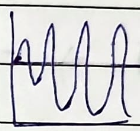
$$P_1 = -\frac{\zeta}{\tau} + i\frac{\sqrt{1-\zeta^2}}{\tau}$$

$$P_2 = -\frac{\zeta}{\tau} - i\frac{\sqrt{1-\zeta^2}}{\tau}$$



$\Rightarrow$



ζ	nature	plot
$\zeta \geq 1$	overdamped, stable or non-oscilatory	
$0 < \zeta < 1$	underdamped, stable, oscilatory	
$\zeta = 0$	undamped, marginal stable osscilarion	
$-1 < \zeta < 0$	unstable, ossilation with ↑ing Amplutide	
$\zeta = -1$		

Response against input change:

$$G_p(s) = \frac{K_p}{\tau^2 s^2 + 2\zeta\tau s + 1} = \frac{\bar{y}(s)}{\bar{f}(s)}$$

$$\bar{y}(s) = \frac{K_p}{\tau^2 s^2 + 2\zeta\tau s + 1}\,\bar{f}(s)$$

$$\bar{y}(s) = \frac{K_p}{\tau^2 s^2 + 2\zeta\tau s + 1}\left(\frac{A}{s}\right) \qquad F(t) = A, \quad \bar{f}(s) = \frac{A}{s}$$

take laplace inverse, $y(t) = ?$

overdamped $y(t) = K_p A \left[1 - e^{-\zeta t/\tau} \left\{ \cosh \frac{\sqrt{\zeta^2-1}}{\tau} t + \frac{\zeta}{\sqrt{\zeta^2-1}} \sinh \frac{\sqrt{\zeta^2-1}}{\tau} t \right\} \right]$

critically damped $y(t) = K_p A \left[1 - \left(1 + \frac{t}{\tau}\right) e^{-t/\tau} \right]$

underdamped

$$y(t) = K_p A\left[1 - e^{-\frac{\zeta t}{\tau}}\left\{\cos\frac{\sqrt{1-\zeta^2}}{\tau}t + \frac{\zeta}{\sqrt{1-\zeta^2}}\sin\frac{\sqrt{1-\zeta^2}}{\tau}t\right\}\right]$$

$$= K_p A\left[1 - \frac{1}{\sqrt{1-\zeta^2}}e^{-\frac{\zeta t}{\tau}}\sin(\omega t+\phi)\right]$$

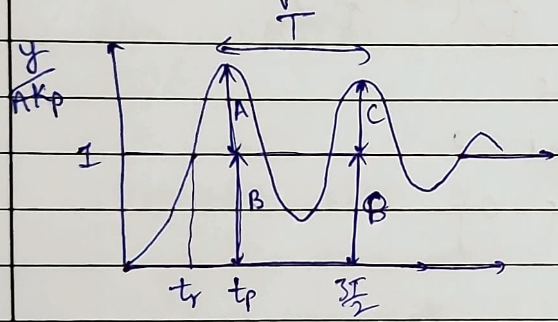
ω = radian frequency rad/time

$= \frac{\sqrt{1-\zeta^2}}{\tau}$, $\omega T = 2\pi$

$\phi = \tan^{-1}\left(\frac{\sqrt{1-\zeta^2}}{\zeta}\right)$

= phase angle

for underdamped:



t_r = rise time
{time required for y to reach the steady state for the first time}

$t_r = \frac{T}{4}$, $t_p = \frac{T}{2}$ {Approximate values}

put $\frac{y}{AK_p} = 1$ in $y(t)$, to get t_r

$t = t_r = \frac{1}{\omega}(\pi - \phi)$

t_p = peak time
time required for the y to reach the max^m value for first time.

$\frac{d\left(\frac{y}{AK_p}\right)}{dt} = 0$ to get t_p

$A+B = \left(\frac{y}{AK_p}\right)_{t=t_p}$ | $\frac{A}{B}$ = overshoot = $\exp\left(\frac{-\pi\zeta}{\sqrt{1-\zeta^2}}\right)$

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$$C + B = \left(\frac{y}{A K_p}\right)_{t = \frac{3T}{2}}$$

$$\frac{C}{A} = \text{decay Ration} = (\text{overshoot})^2 = \exp\left(\frac{-2\pi \zeta}{\sqrt{1-\zeta^2}}\right)$$

$$G(s) = \frac{K_p}{\tau^2 s^2 + 2\zeta\tau s + 1}$$

$$\omega = \frac{\sqrt{1-\zeta^2}}{\tau} \ \frac{\text{rad}}{\text{time}} , \quad \omega T = 2\pi$$

$\zeta = 0$ undamped {sustained oscillation}

$$G(s) = \frac{K_p / \tau^2}{\left(s - \frac{i}{\tau}\right)\left(s + \frac{i}{\tau}\right)}$$

Poles : conjugate
real part = 0

$\omega = \frac{1}{\tau}$ {as $\zeta = 0$}

Q $\frac{\bar{y}(s)}{\bar{f}(s)} = \frac{4}{s^2 + 1.6s + 4}$

(1) ~~Kp~~, ~~τ~~ find ζ

(1) $$\frac{\bar{y}(s)}{\bar{f}(s)} = \frac{1}{\frac{s^2}{4} + 0.4s + 1}$$

$$\tau^2 = \frac{1}{4} \Rightarrow \tau = \frac{1}{2}$$

$$2\zeta\tau = 0.4 \Rightarrow \zeta = 0.4 \ \{\text{underdamped}\}$$

(ii) ultimate value of step change $\bar{f} = \frac{10}{s}$

find ultimate respone and overshoot

$$\bar{y}(s) = \frac{1}{\frac{s^2}{4} + 0.4s + 1}\left(\frac{10}{s}\right)$$

$$y(t) = 10\left[1 - \frac{1}{\sqrt{1-0.16}}\, e^{-0.8t} \sin(\omega t + \phi)\right]$$

$$\omega = \frac{\sqrt{1-\zeta^2}}{\tau} = 2\sqrt{1-0.16}\ , \quad \phi = \tan^{-1}\left(\frac{\sqrt{1-0.16}}{0.4}\right)$$

$$\text{overshoot} = \exp\left(\frac{-\pi}{2\sqrt{1-0.16}}\right) = 0.254$$

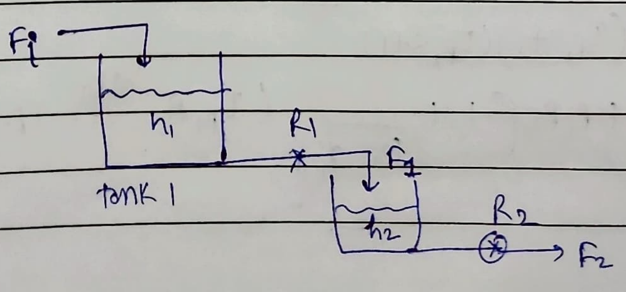
ultimate value $\lim_{s \to 0} s\bar{y}(s) = \lim_{t \to \infty} y(t) = \underline{\underline{10}}$

(iii) calculate rise time

$$t_r = \frac{1}{\omega}(\pi - \phi)$$

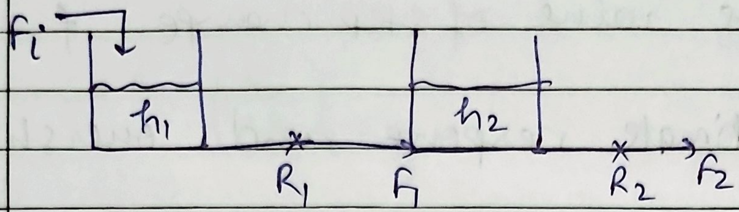
$$= \frac{1}{1.833}(\pi - 1.159) = 1.081$$

Multicapacity processes :



Non-interacting system

$\tau_{p_1}\tau_{p_2}s^2 + (\tau_{p_1}+\tau_{p_2})s + 1$



Interacting system

$$\bar{f} \rightarrow \boxed{G_1} \xrightarrow{\bar{y}_1} \boxed{G_2} \rightarrow \bar{y}_2$$

1st system : $\tau_{p_1}\frac{dy_1}{dt} + y_1 = k_{p_1} f$

$$G_1 = \frac{\bar{y}_1(s)}{\bar{f}(s)} = \frac{k_{p_1}}{\tau_{p_1}s+1}$$

2nd system : $\tau_{p_2}\frac{dy_2}{dt} + y_2 = k_{p_2} y_1$

$$G_2 = \frac{\bar{y}_2}{\bar{y}_1} = \frac{k_{p_2}}{\tau_{p_2}s+1}$$

$$G_{overall} = \frac{\bar{y}_2}{\bar{f}} = G_1 G_2 = \frac{k_{p_1}k_{p_2}}{(\tau_{p_1}s+1)(\tau_{p_2}s+1)}$$

Remarks ① 1st order + 1st order = 2nd order

② $s = \frac{1}{\tau_{p_2}}, \frac{-1}{\tau_{p_1}}$ {poles} → overdamped

③ for unit step change in f

$$\bar{y}_2 = \frac{k_{p_1}k_{p_2}}{(\tau_{p_1}s+1)(\tau_{p_2}s+1)}\frac{1}{s}$$

$$y(t) = k_{p_1 p_2}\left[1 + \frac{1}{\tau_{p_2}-\tau_{p_1}}\left(\tau_{p_1}e^{-t/\tau_{p_1}} - \tau_{p_2}e^{-t/\tau_{p_2}}\right)\right.$$

_/_/_

(4) N , $G_o = G_{P_1} G_{P_2} \text{———} G_{P_N}$:

$$G_o = \frac{K_{P_1} K_{P_2} \text{— —} K_{P_N}}{(\tau_{P_1}S+1)(\tau_{P_2}S+1)(\tau_{P_3}S+1) \text{—} (\tau_{P_n}S+1)}$$

Tank 1 :

$$A\frac{dh_1}{dt} = F_i - F_1 \qquad F_1 = \frac{h_1}{R_1}$$

$$A\frac{dh_1'}{dt} + \frac{h_1'}{R_1} = F_i'$$

$$A_1R_1\frac{dh_1'}{dt} + h_1' = Rf_1'$$

$$A_1R_1\,\bar{h}_1(S) \times S + \bar{h}_1(S) = R\bar{f}_1(S)$$

$$\bar{G}_1 = \frac{\bar{h}_1}{\bar{f}_1} = \frac{R_1}{A_1R_1S+1} \qquad \tau_{P_1} = A_1R_1 \;,\; K_{P_1} = R_1$$

for tank 2 :

$$A_2\frac{dh_2'}{dt} = f_1' - \frac{h_2'}{R_2}$$

$$A_2R_2\frac{dh_2'}{dt} + h_2' = R_2f_1'$$

$$G_2 = \frac{\bar{h}_2'}{\bar{f}_1'} = \frac{R_2}{A_2R_2S+1} \qquad \tau_{P_2} = A_2R_2 \;,\; K_{P_2} = R_2$$

overall transfer f^n :

$$\cancel{G_2/G_1} = \cancel{\frac{R_1R_2}{(A_1R_1S+1)(A_2R_2S+1)}} \qquad \cancel{\frac{\bar{h}_2}{\bar{h}_1}} = \cancel{\frac{R_2\,(A_1R_1S+1)}{(A_2R_2S+1)\,R_1}}$$

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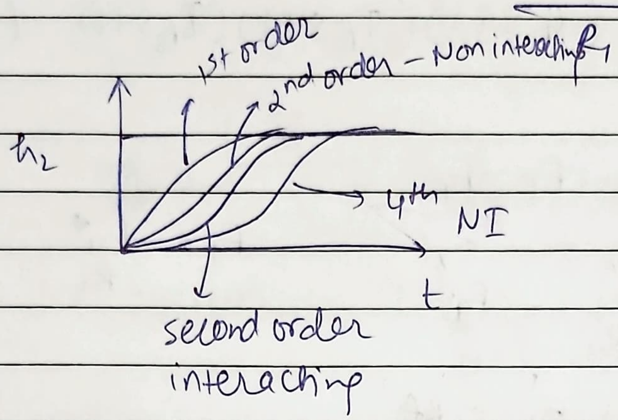
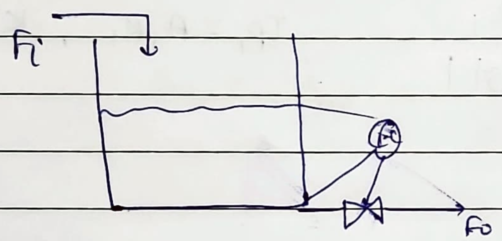
$$\frac{\bar{h}_2}{\bar{F}_i} = \frac{\bar{h}_2}{\bar{F}_1} \times \frac{\bar{F}_1}{\bar{F}_i} = \frac{\bar{h}_2}{\bar{F}_1} \cdot \frac{\bar{h}_1}{\bar{F}_1} R_1$$

$G_1 \quad G_2$



Process + controller :



CV	MV
h	f_o

$$\frac{A\,dh'}{dt} = F_i' - F_o'$$

$h' = h - h_s = h - h_{sp}$

$F_i' = F_i - F_{is}$

$F_o' = F_o - F_s$

controller :

$$F_o = F_{os} + K_c h' + \frac{K_c}{\tau_2}\int h'dt \quad - \text{PI controller}$$

$$F_o' = K_c h' + \frac{K_c}{\tau_i}\int h'dt$$

case 1 : $h' = 0$, $h = h_s$ $F_o = F_s$ { steady state }

case 2 : $h' > 0$, $h > h_s$ $F_o \uparrow$

_/_/_

$$A\frac{dh'}{dt} = \bar{F_i} - f_o'$$

$$A\frac{dh'}{dt} + k_c h' + \frac{k_c}{\tau_i}\int h\,dt = \bar{F_i}'$$

$$As\bar{h}(s) + k_c h'(s) - \frac{k_c}{\tau_i}\left(\frac{\bar{h}(s)}{s}\right) = \bar{F_i}(s)$$

$$G(s) = \frac{\bar{h}(s)}{\bar{F_i}(s)} = \frac{k_p s}{\tau^2 s^2 + 2\zeta\tau s + 1}$$

$$\tau = \sqrt{\frac{A\tau_i}{k_c}}, \quad \zeta = \frac{1}{2}\sqrt{\frac{k_c\tau_i}{A}}, \quad k_p = \frac{\tau_i}{k_c}$$

Remarks:

1) 1^{st} order + PI controller = 2^{nd} order

2) $\sqrt{\frac{k_c\tau_i}{A}} < 2$ $\quad \zeta < 1$ underdamped

$= 2$ $\quad \zeta = 1$ critical

> 2 $\quad \zeta > 1$ overdamped

N^{th} order-system:

Non interacting

$$\bar{f} \longrightarrow \boxed{G_1} - \boxed{G_2} - \boxed{G_3} \longrightarrow \;—\; \boxed{G_N} \longrightarrow \bar{y}_N$$

(1) $\bar{G}_o(s) = G_1 G_2 \ldots G_N$

(ii) S-shaped

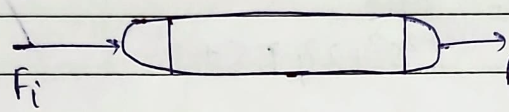
(3) No. of capacities ↑ delay ↑

__/__/__

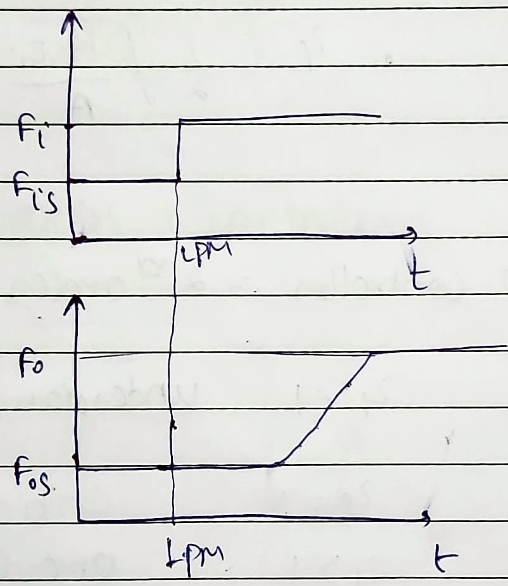
Interacting :

~~$G_0(s) = G_1 G_2 \ldots G_n$~~
(1) No generalization of G_0

processes with dead time :



$F_i = F_{i_s} = F_{o_s}$ at $t = 0$



$f(t) \rightarrow \boxed{\text{Process}} \xrightarrow{y(t)} \boxed{\text{Dead time}} \rightarrow y(t - t_d)$

t_d = dead time

Dead time : $\dfrac{L[y(t-t_d)]}{L[(y(t))]} = e^{-t_d s}$

process $= \dfrac{L[y(t)]}{L[f(t)]} = \dfrac{K_p}{\tau_p s + 1}$

$$\frac{L[y(t - t_d)]}{L[f(t)]} = \frac{L[y(t-t_d)]}{L[y(t)]} \times \frac{L(y(t))}{L[f(t)]}$$

$$= \frac{K_P}{\tau_P s + 1} e^{-t_d s} \quad \left(\text{First order plus dead time (FOPDT)} \right)$$

For second order
similarly

$$\frac{L[y(t-t_d)]}{L[f(t)]} = \frac{K_P}{\tau^2 s^2 + 2\zeta\tau_s + 1} e^{-t_d s} \quad \text{SOPDT}$$

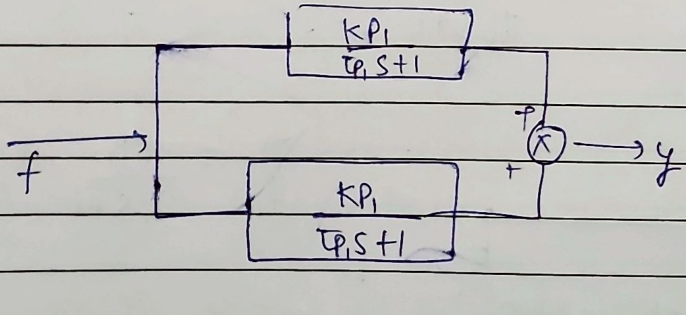
$$e^{-t_d s} = \frac{1 - \frac{t_d}{2} s}{1 + \frac{t_d}{2} s} \quad \text{Pade Approximation (1st order)}$$

$$= \frac{t_d^2 s^2 - 6 t_d s + 12}{t_d^2 s^2 + 6 t_d s + 12} \quad \text{(2nd order)}$$

FOPDT $= \dfrac{K_P}{\tau_P s + 1} \times \left(\dfrac{1 - \frac{t_d}{2} s}{1 + \frac{t_d}{2} s} \right)$

SOPDT $= \dfrac{K_P}{\tau^2 s^2 + 2\zeta\tau s + 1} \times \left(\dfrac{1 - \frac{t_d}{2} s}{1 + \frac{t_d}{2} s} \right)$

Process with inverse Response:



$q(s) = \frac{Q(s)}{P(s)}$

$Q(s) = 0$ zero

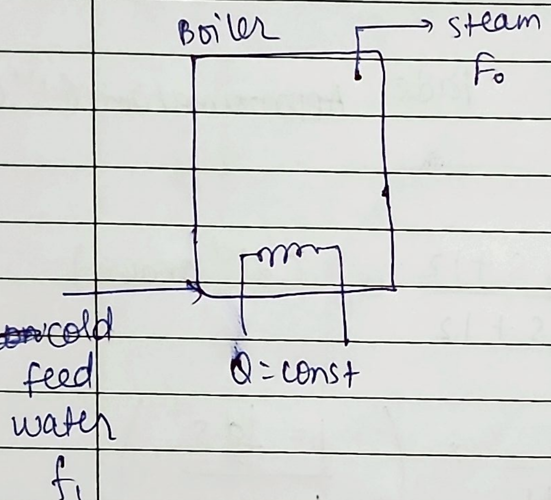
$P(s) = 0$ poles

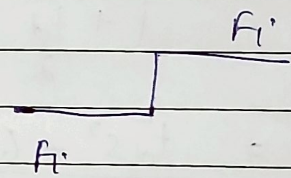
$$\frac{y}{f} = \frac{K_{P_1}}{\tau_{P_1} s + 1} + \frac{K_{P_2}}{\tau_{P_2} s + 1} = \frac{(K_{P_1}\tau_{P_2} + K_{P_2}\tau_{P_1}) + (K_{P_1} + K_{P_2})}{(\tau_{P_1} s + 1)(\tau_{P_2} s + 1)}$$

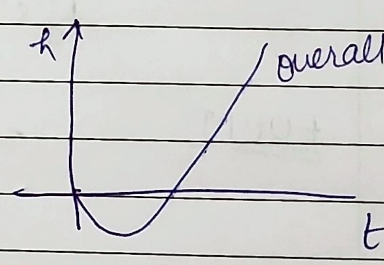
$$\tau = \frac{K_{P_1}\tau_{P_2} + K_{P_2}\tau_{P_2}}{K_{P_1} + K_{P_2}} \qquad \frac{(\tau s+1)(K_{P_1}+K_{P_2})}{(\tau_{P_1}s+1)(\tau_{P_2}s+1)} = \frac{Q(s)}{P(s)}$$

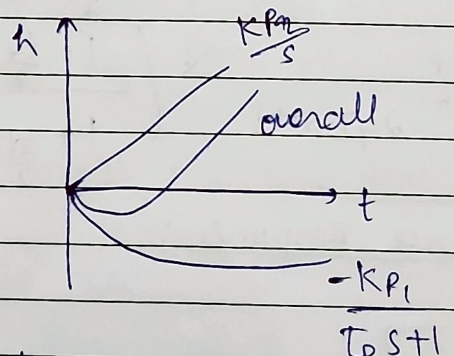
$$Q(s) = 0 \qquad s = -\frac{1}{\tau} \qquad \tau < 0 \Rightarrow s > 0$$

$$\tau < 0 \Rightarrow -\frac{K_{P_2}}{K_{P_1}} > \frac{\tau_{P_2}}{\tau_{P_1}} > 0$$

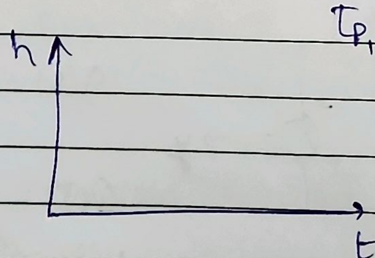






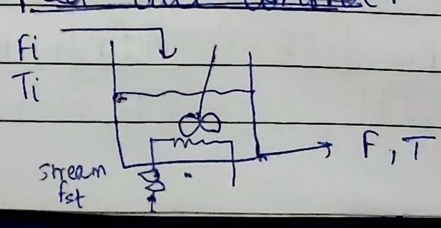


$$\frac{h}{f_1} = \frac{K_{P_2}}{s} - \frac{K_{P_1}}{\tau_{P_1}s+1}$$



$$\frac{h}{f_i} = \frac{K_{P_1}}{\tau_1 s+1} + \frac{K_{P_2}}{\tau_2 s+1}$$

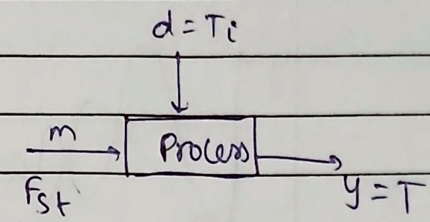
Feed back control!



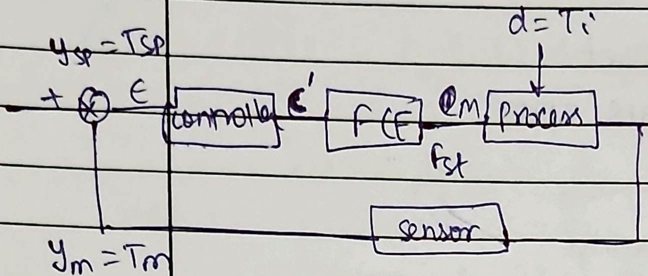
$T = T_{sp}$

___/___/___

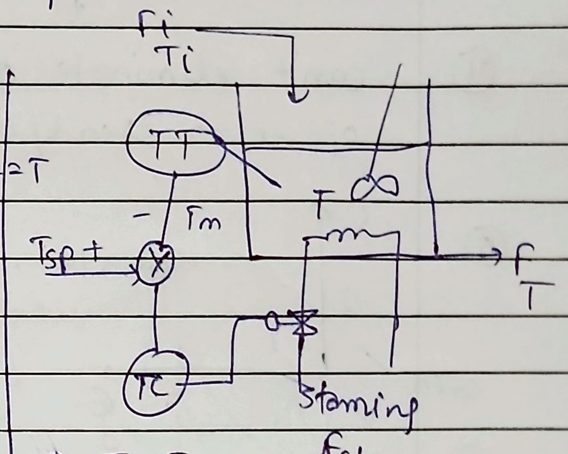
CV	MV	LV
T	F_{st}	T_i



Assume ($F_i = F$ = fixed)



Block diagram



There are three indivisual $\epsilon = T_{sp} - T_m$
control action

1) proportional action ? (a) P-only controller
2) Integral action ? PI controller
3) Derivative action ? (b) → ID controller.

P-only controller ($c'(t) \propto \epsilon(t)$)

$c' = c - c_s$

$$c'(t) = K_c \epsilon$$

$$c(t) = c_s + K_c \epsilon$$

K_c = propotional gain and its tuning parameter.

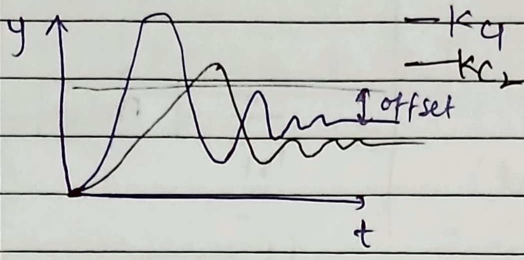
c_s = bias signal {known}

propotional Band $= \frac{100}{K_c}$

<u>Remarks</u> ① $K_c = \frac{c'}{\epsilon}$ = gain

② $G_c(s) = \frac{c'(s)}{\epsilon(s)} = K_c$

③ with the entries of K_c value the controller becomes more sensitive to error signals.

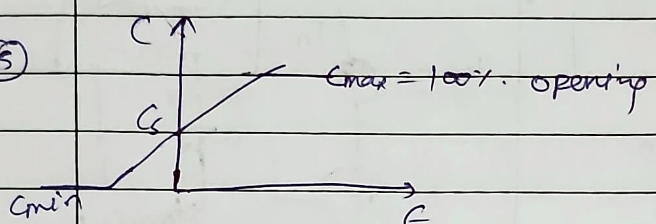


$K_{c1} > K_{c2}$

(4) can't eliminate offset

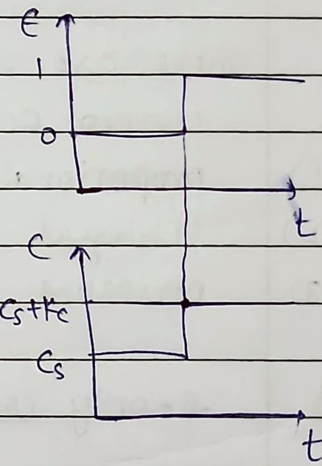
offset → steady state error

(5)



(6) $\bar{C}'(s) = K_c \bar{\epsilon}(s)$ $\quad \bar{\epsilon}(s) = \frac{1}{s}$

$C'(t) = K_c$ $\quad$ {inverse}



(7) If K_c is very very large then p-only controller behaves as on-off controller (Ex - switch)

PI - controller:

$$C(t) = C_s + \underbrace{K\epsilon(t)}_{P} + \underbrace{\frac{K_c}{T_i}\int \epsilon(t)\,dt}_{\text{integral part}}$$

T_i = integral time constant (unit = time) or reset time

$\frac{1}{T_i}$ = reset rate

$$C(S) = K\,\epsilon(S) + \frac{K_c}{T_i}\,\frac{\epsilon(S)}{S}$$

Remarks (1) $\frac{C(S)}{\epsilon(S)} = K_c + \frac{K_c}{T_i S}$

(2) As long as the error is present, PI controller keeps changing its output to make error zero.

_/_/_

It is saying that offset = zero. It eleminates offset.

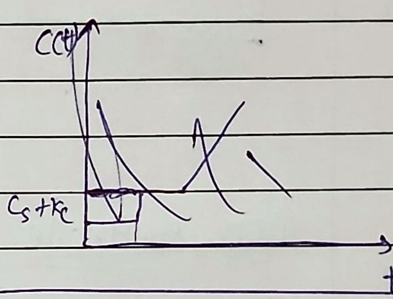
(3) 1st order + PI = 2nd order
system response become sluggish

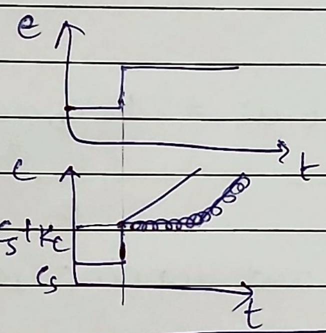
(4) $\bar{C}'(s) = K_c \bar{\epsilon}(s) + \frac{K_c \bar{\epsilon}(s)}{\tau_i s}$ $\bar{\epsilon}(s) = \frac{1}{s}$

$\bar{C}'(s) = \frac{K_c}{s} + \frac{K_c}{\tau_i s^2}$

$C'(t) = K_c + \frac{K_c t}{\tau_i}$

$C(t) = C_s + K_c + \frac{K_c}{\tau_i} t$





Now $t = \tau_i$

$C(t) = C_s + K_c + \frac{K_c \tau_i}{\tau_i}$

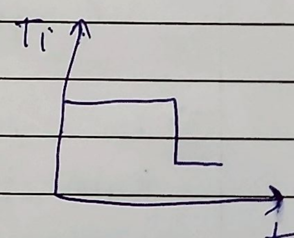
Reset time (τ_i) is the time needed by the controller to reapet its initial propotinal action in the output.

Heating tank:

CV = $T_s = 25°C$

LV = $T_{is} = 40°C$

MV : F_{sts} = 50 % open (value)



Now T_s $25° \longrightarrow 20°C$ $T \downarrow$

$F_{st} \uparrow$ (60%)

P only

	F_{st}	= F_{sts}	+ $K\epsilon$
t = 0	50	50	0
t = t	60	50	10

$K_c \epsilon = 10$, $\epsilon \neq 0$

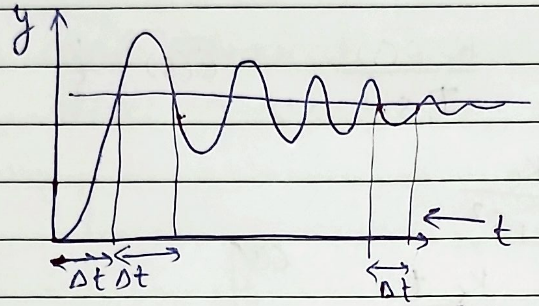
$$f_{st} = f_{sts} + K_c\epsilon + \frac{K_c}{\tau_I}\int \epsilon\, dt$$

t=0	50	50	0	
t=t	60	50	0	10

$$\int_0^t \epsilon\, dt = \sum_{i=1}^{n} \epsilon(i\cdot\Delta t)\Delta t$$

$$= \Delta t(\epsilon\Delta t + 2\epsilon\Delta t \ldots n\epsilon\Delta t)$$

$t = n\Delta t$.



time	ϵ	$\Sigma\epsilon$
S.S → 0	0	0
Δt	5	5
$2\Delta t$	3	8
⋮		
t	0	10

PID controller:

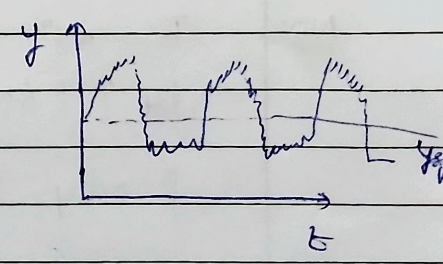
$$C = C_s + \underbrace{K_c\epsilon}_{P} + \underbrace{\frac{K_c}{\tau_i}\int \epsilon . dt}_{I} + \underbrace{K_c\tau_D\frac{d\epsilon}{dt}}_{D}$$

τ_D = derivative time and tunning parameter.

Remarks ① $G_c(s) = \frac{\bar{C}(s)}{\bar{E}(s)} = K_c\left(1 + \frac{1}{\tau_i s} + \tau_D s\right)$

② ϵ = const $\quad \frac{d\epsilon}{dt} = 0$

③ For a noisy Response, with almost zero error derivative term lead to agressive control action. although it is not needed



$$c'(t) = K_c\tau_D\frac{d\epsilon}{dt}$$

$$\bar{C}(s) = K_c\tau_D\, s\, \bar{E}(s) \qquad \bar{E}(s) = \frac{1}{s}$$

$$\bar{C}(s) = K_c\tau_D$$

$$c(t) = \frac{K_c\tau_D}{s} \Rightarrow K_c\tau_D\delta(t)$$

Remark

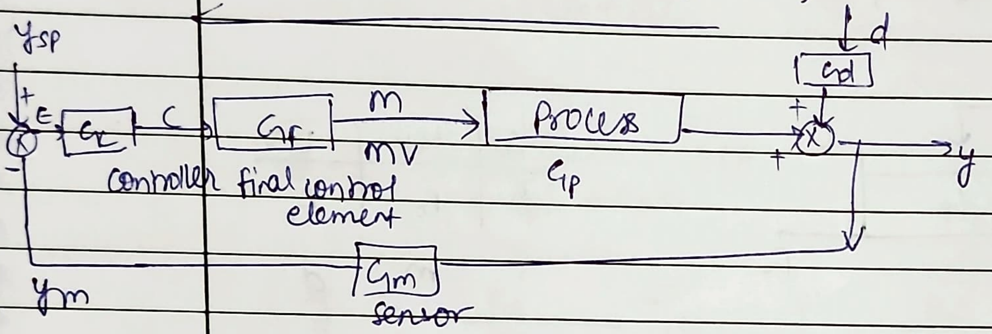
(9) $$G(s) = K_c\left[1 + \frac{1}{\tau_i s} + \frac{\tau_D s F(s)}{(\tau_F s+1)}\right]$$

Filter

$$G(s) = \tau_D s = \frac{Q(s)}{P(s)}$$ order of Q > order of P

$$F(s) = \frac{1}{(\tau_F s+1)^n}$$

If $n=1$ {Lead-lag system}



{open-loop f^n}

process : $\bar{y} = G_p \bar{m} + G_d \bar{d}$ — (1)

sensor : $\bar{y}_m = G_m \bar{y}$

controller : $\bar{c} = G_c \bar{\epsilon} = G_c(\bar{y}_{sp} - \bar{y}_m)$

FCE : $\bar{m} = G_f \bar{c} = G_f G_c(\bar{y}_{sp} - \bar{y}_m)$
$= G_f G_c(y_{sp} - G_m y)$

Now put m in eq^n (1)

$$y = G_p G_f G_c y_{sp} - G_p G_f G_c G_m y + G_d \times d$$

Remember ← $$\bar{y} = \left(\frac{G_p G_f G_c}{G_p G_f G_c G_m + 1}\right)\bar{y}_{sp} + \left(\frac{G_d \times \bar{d}}{G_p G_f G_c G_m + 1}\right)$$

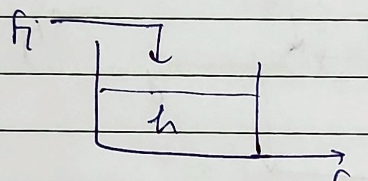
(closed loop transform fun.)

$$G_p = \frac{K_p}{\tau_p s + 1}$$ — open loop transfer function

$$\bar{y} = \frac{G_c G_p G_f}{1 + G_p G_c G_f G_m} \bar{y}_{sp}$$ — servo ($\bar{d} = 0$)

$$\bar{y} = \frac{G_d}{1 + G_c G_f G_p G_m} \bar{d}$$ — Regulatory ($\bar{y}_{sp} = 0$)

#



CV	MV	LV
h	F_o	F_i
	m	d

$$A \frac{dh'}{dt} = F_i' - F_o'$$

$$\bar{h}'(s) = \frac{1}{As} F_i'(s) - \frac{1}{As} F_o'(s)$$

$$\bar{h}'(s) = G_p \bar{m} + G_d \bar{d}$$

$$G_p = \frac{-1}{As}, \quad G_d = \frac{1}{As}$$

Effect of P-action:

$$\bar{y} = \frac{G_c G_p G_f}{1 + G_c G_p G_f G_m} \bar{y}_{sp} + \frac{G_d}{1 + G_c G_p G_f G_m} \bar{d}$$ — closed loop transfer function

Process: $\tau_p \frac{dy}{dt} + y = K_p m + K_d d$

$$\bar{y}(s) = \frac{K_p \bar{m}}{\tau_p s + 1} + \frac{K_d \bar{d}}{\tau_p s + 1}$$

$$= G_p \bar{m} + G_d \bar{d}$$

_/_/_

$$G_p = \frac{k_p}{\tau_p s + 1} \quad , \quad G_d = \frac{k_d}{\tau_p s + 1}$$

controller: $G_c = K_c$ $\quad G_f = G_m = 1$ {for simplicity}

$$\bar{y} = \frac{k_c\left(\frac{k_p}{\tau_p s+1}\right)}{1+\left(\frac{k_p k_c}{(\tau_p s+1)^2}\right)} \bar{y}_{sp} + \frac{\left(\frac{k_d}{\tau_p s+1}\right)}{1+\frac{k_p k_c}{\tau_p s+1}} \bar{d}$$

$$\bar{y} = \frac{K_c k_p}{1+\tau_p s + k_p k_c} \bar{y}_{sp} + \frac{k_d}{1+\tau_p s+1} \bar{d}$$

$$\bar{y} = \frac{k_p'}{\tau_p' s+1} \bar{y}_{sp} + \frac{k_d'}{\tau_p' s+1} \quad \text{--- CLTF}$$

$$\left|\begin{matrix} k_p' \\ \tau_p' s \\ k_d' \end{matrix}\right| = \left|\begin{matrix} k_c k_p \\ 1+\tau_p s \\ k_d \end{matrix}\right| \qquad k_p' = \frac{k_c k_p}{1+k_p k_c}$$

$$\tau_p' = \frac{\tau_p}{1+k_p k_c}$$

$$k_d' = \frac{k_d}{1+k_p k_c}$$

$$k_p' = \frac{k_p}{\frac{1}{k_c}+k_p}$$

Direct action: As the input signal to the controller increases, the output signal should increase in case of direct action.

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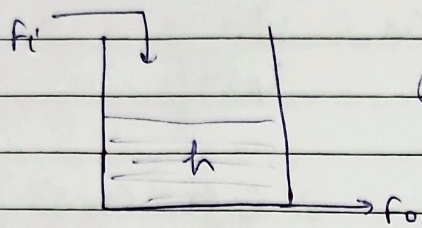


① h↑ then f_o↑ {direct action}

② $f_o = f_{os} + \underset{-ve}{\overline{k_c}}\ (\underset{+ve}{\overline{h_{sp} - h}})$

CV	MV
h	f_o

{gain is −ve for direct action}

* Reverse action:

CV	MV
h	fi

1) h↑ then fi↓

2) +ve ~~gain~~ gain {k_c}

CV	MV	k_c	k_p	$k_c k_p$
h	f_o	−ve	−ve	+ve
h	f_i	+ve	+ve	+ve

Remarks ① $k_p > k_p'$, $k_d > k_d'$

② $\tau_p > \tau_p'$

④ {1st order + P-only = 1st order only}

③ Offset = new set point − ultimate value of the Response

$$\bar{y} = \frac{k_p'}{\tau_p' s + 1}\bar{y}_{sp} + \frac{k_d'}{\tau_p' s + 1}\bar{d}$$

$\Rightarrow A - \lim_{s \to 0} \bar{y}(s) \times s$

$\Rightarrow A - A k_p'$

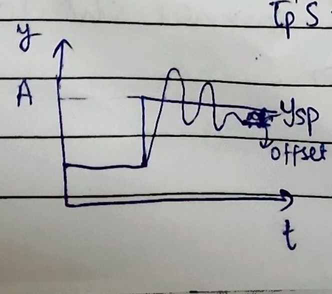
$\Rightarrow \boxed{\frac{A}{1 + k_p k_c}} \neq 0$

servo:

$$\bar{y} = \frac{k_p'}{\tau_p' s + 1}\bar{y}_{sp}$$

↓

$$\bar{y} = \frac{k_p'}{\tau_p' s + 1}\left(\frac{A}{s}\right)$$



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Regulatory case:

$$\bar{y} = \frac{K_d'}{\tau_p' s+1}\,\bar{d}' \qquad d = \frac{A}{s}$$

$$\bar{y} = \frac{K_d'}{\tau_p' s+1}\left(\frac{A}{s}\right)$$

Offset = ~~new~~ set point − ultimate value of Response

$$= \cancel{A} - \lim_{s\to 0} s\,\bar{y}(s)$$

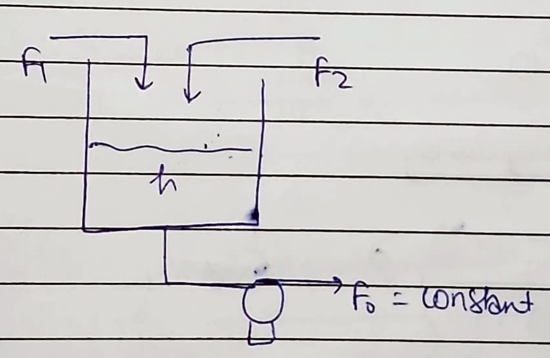
0

for regulatory case $= (\cancel{A} - K_d' \cancel{\otimes} A)$

$$= \cancel{A} - \frac{A' K_d}{1+K_pK_c} \Rightarrow$$

$\boxed{\text{as } y_{sp} = 0}$
in regulatory

Ex



CV	MV	LV
h	F_1	F_2

$$A\frac{dh}{dt} = F_1 + F_2 - F_0$$

$$A\frac{dh'}{dt} = F_1' + F_2'$$

$$A\,s\bar{h}'(s) = \bar{F}_1'(s) + \bar{F}_2'(s)$$

$$\bar{h}(S) = \frac{1}{As}\bar{F}_1(s) + \frac{\bar{F}_2(s)}{As}$$

$$\bar{h}(s) = G_p\,\bar{m} + G_d\,\bar{d} \qquad G_p = \frac{1}{As},\; G_d = \frac{1}{As}$$

$G_p = G_d = \frac{1}{As}$, $G_c = K_c$, $G_f = G_m = 1$

~~Offset~~

$$\bar{y} = \frac{1}{\frac{A}{K_c}s+1}\bar{y}_{sp} + \left(\frac{\frac{1}{K_c}}{\frac{A}{K_c}s+1}\right)\bar{d}$$

Offset for servo :

$$\bar{y} = \frac{1}{\frac{A}{K_c}s+1}\bar{y}_{sp}$$

$$y_{sp} = \frac{A}{s}$$

$$\text{Offset} = A - \lim_{s\to 0}\left(\frac{A}{\frac{A}{K_c}s+1}\right)$$

$$= 0$$

Offset for regulatory :

$$\bar{y} = \left(\frac{\frac{1}{K_c}}{\frac{A}{K_c}s+1}\right)\bar{d}$$

$$d = \frac{A}{s}$$

$$\text{offset} = 0 - \lim_{s\to 0}\left(\frac{\frac{1}{K_c}}{\frac{A}{K_c}s+1}\right)A$$

$$= -\frac{A}{K_c}$$

2nd order

$$G_p = \frac{k_p}{\tau^2 s^2 + 2\zeta\tau s + 1}$$

$$G_c = k_c \quad , \quad G_f = G_m = 1$$

$$y = \frac{\left(\frac{k_p}{\tau^2 s^2 + 2\zeta\tau s + 1}\right) k_c}{1 + \frac{k_c k_p}{\tau^2 s^2 + 2\zeta\tau s + 1}} \bar{y}_{sp} + (\quad)\bar{d}$$

$$y = \frac{k_p k_c}{\tau^2 s^2 + 2\zeta\tau s + 1 + k_p k_c} \bar{y}_{sp} + (\quad)\bar{d}$$

$$\text{offset} = A - \lim_{s \to 0} \frac{A k_p k_c}{1 + k_p k_c} \quad \left\{ \bar{y}_{sp} = \frac{A}{s} \right\}$$

$$= \frac{A}{1 + k_p k_c}$$

Effect of Integral action:

$$G_p = \frac{k_p}{\tau_p s + 1} \quad , \quad G_c = \frac{k_c}{\tau_i s} \quad , \quad G_m = G_f = 1$$

servo case:

$$\bar{y} = \frac{\left(\frac{k_p}{\tau_p s + 1}\right)\left(\frac{k_c}{\tau_i s}\right)}{1 + \left(\frac{k_p}{\tau_p s + 1}\right)\left(\frac{k_c}{\tau_i s}\right)}$$

$$\bar{y} = \frac{k_p k_c}{(\tau_p s + 1)(\tau_i s) + k_p k_c} \bar{y}_{sp}$$

$$y = \frac{k_p k_c}{\tau_i \tau_p s^2 + \tau_i s + k_p k_c} \bar{y}_{sp}$$

$$\bar{y} = \frac{1}{\frac{\tau_i \tau_p s^2}{K_p K_c} + \frac{\tau_i}{K_p K_c} s + 1} \bar{y}_{sp}$$

$$\bar{y} = \frac{K_p}{\tau^2 s^2 + 2\zeta \tau s + 1} \bar{y}_{sp}$$

$$\boxed{\tau = \sqrt{\frac{\tau_i \tau_p}{K_p K_c}}} \quad , \quad 2\zeta \tau s = \frac{\tau_i s}{K_p K_c}$$

$$\zeta = \frac{\tau_i}{2 K_p K_c} \frac{\sqrt{K_p K_c}}{\sqrt{\tau_i \cdot \tau_p}}$$

$$\boxed{K_p = 1} \quad , \quad \boxed{\zeta = \frac{1}{2} \sqrt{\frac{\tau_i}{\tau_p K_p K_c}}}$$

Remark ① 1st order + Integral action = 2nd order
② system response becomes sluggish

$$\text{Offset} = A - \lim_{s \to 0} \left(\frac{K_p A}{1} \right)$$

$$= A - A$$

$$= 0 \qquad \{ \text{As } K_p = 1 \}$$

Effect of Derivative action:

$$G_p = \frac{K_p}{\tau_p s + 1} \quad , \quad G_c = K_c \tau_D s \, , \quad G_m = G_f = 1$$

$$\bar{y} = \frac{\left(\frac{K_p}{\tau_p s + 1}\right)\left(K_c \tau_D s\right)}{1 + \frac{K_p K_c \tau_D s}{\tau_p s + 1}} \bar{y}_{sp}$$

__/__/____

$$\bar{y} = \frac{K_p K_c \tau_D s}{\tau_p s + 1 + K_p K_c \tau_D s} \bar{y}_{sp}$$

$$\bar{y} = \frac{K_p K_c \tau_D s}{\underbrace{(\tau_p + K_p K_c \tau_D)}_{\tau_p'} s + 1} \bar{y}_{sp}$$

Remark ① 1st order + Derivative = 1st order

② $\tau_p' > \tau_p$ {Response become slower}
{robustness}

③ offset = $A - \lim_{s \to 0} s \bar{y}_{sp}$

$= \underline{\underline{A}}$

offset is not eliminated.

P → speeds up the response but can't eliminate offset

PI → (i) eliminate offset
(ii) slow, $K_c \uparrow$, $\tau_I \downarrow$
(iii) unstable

PID : ~~obset~~ offset removed ✓
Robust ✓