INDIAN INSTITUTE OF TECHNOLOGY KHARAGPUR

Mid-Spring Semester Examination 2022-23

Date of Examination: 22/02/2023. **_ Session: (FN/AN) AN.** **_Duration: 2 hrs.** **Full Marks: 30**

Subject No.: CH21204 **Subject: Heat Transfer**

Department/~~Center/School~~: Chemical Engineering _

Specific charts, graph paper, log book etc., required ______________________

Special Instructions (if any): ______________________

1. Assume a fluid is flowing over an isothermally flat plate. If the free-stream velocity of the fluid is doubled (flow is still laminar), then estimate the change in the drag force on the plate and the rate of heat transfer between the fluid and the plate.

(3+3 = 6)

Solution:-

For the laminar flow of a fluid over a flat plate maintained at a constant temperature the drag force is given by:-

$$F_{D1} = C_f A_s \frac{\rho V_\infty^2}{2} \quad \text{where} \quad C_f = \frac{1.328}{\sqrt{Re}}$$

Therefore,

$$F_{D1} = \frac{1.328}{\sqrt{Re}} A_s \frac{\rho V_\infty^2}{2}$$

Substituting Reynold's number relation, we get,

$$F_{D1} = \frac{1.328}{\sqrt{\frac{V_\infty L}{\nu}}} A_s \frac{\rho V_\infty^2}{2} = 0.664 (V_\infty)^{3/2} A_s \frac{\nu^{0.5}}{L^{0.5}}$$

When the free stream velocity of fluid is doubled, the new value of the drag force on the plate becomes

$$F_{D2} = \frac{1.328}{\sqrt{\frac{(2V_\infty) L}{\nu}}} A_s \frac{\rho (2V_\infty)^2}{2} = 0.664 (2V_\infty)^{3/2} A_s \frac{\nu^{0.5}}{L^{0.5}}$$

The ratio of drag forces corresponding to V_∞ and $2V_\infty$ is,

$$\frac{F_{D2}}{F_{D1}} = \frac{(2V_\infty)^{3/2}}{(V_\infty)^{3/2}} = 2^{3/2}$$

For change in rate of heat transfer between the fluid and the plate,

$$\dot{Q}_1 = h A_s (T_s - T_\infty) = \left(\frac{k}{L} Nu\right) A_s (T_s - T_\infty)$$

$$= \frac{k}{L} 0.664 \sqrt{\frac{V_\infty L}{\nu}} Pr^{1/3} A_s (T_s - T_\infty)$$

$$= 0.664 \sqrt{V_\infty} \frac{k}{\sqrt{L\nu}} Pr^{1/3} A_s (T_s - T_\infty)$$

when the free stream velocity of the fluid is doubled, the new value of the heat transfer rate between the fluid and the plate becomes

$$\dot{Q}_2 = 0.664\sqrt{(2V_\infty)}\ \frac{K}{\sqrt{L\gamma}}\ P_r^{1/3} A_s (T_s - T_\infty)$$

Then the ratio is,

$$\frac{\dot{Q}_2}{\dot{Q}_1} = \sqrt{\frac{2V_\infty}{V_\infty}} = \sqrt{2}$$

2. Air at 20 °C is flowing at 15 m/s over an isothermally heated plate (0.5 m length X 0.5 m width, k = 0.0292 W/m K), maintained at 110 °C. What are the average heat transfer coefficient and the total amount of transferred heat? What are *h*, δ_t, and δ at the trailing edge? Consider, Pr = 0.7, and ν = 0.0000195 m²/s of air at 65 °C.

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We evaluate the properties at $T_f = \frac{110+20}{2} = 65^\circ$

Then, $Pr = 0.7$

$$Re_L = \frac{u_\infty L}{\nu} = \frac{15 \times 0.5}{0.0000195} = \cancel{384600}\ 384615$$

$$Nu_x = 0.332\, Re_x^{1/2}\, Pr^{1/3} \quad ; \text{for } Pr \geq 0.6$$

$$\overline{Nu}_L = 0.664\, Re_L^{1/2}\, Pr^{1/3} = \frac{\bar{h} L}{k} \quad ; \text{for } Pr \geq 0.6,\ T_w = \text{constant}$$

$$\therefore \overline{Nu}_L = 0.664 \times Re_L^{1/2} \times Pr^{1/3}$$

$$= 0.664 \times (384615)^{1/2} \times (0.7)^{1/3}$$

$$= 365.67$$

$$\therefore \bar{h} = 365.67 \frac{k}{L} = \frac{365.67 \times (0.0292)}{0.5}$$

$$\therefore \bar{h} = 21.355$$

$$h(\text{trailing edge}) = \tfrac{1}{2}\bar{h} = \frac{21.355}{2} = 10.67\ \text{W/m}^2\text{-K}$$

$$\text{Total heat flux} = Q = \bar{h} A \Delta t$$

$$= 21.355 \times (0.5)^2 \times (110 - 20)$$

$$= 480.5\ \text{W}$$

$$\delta(x = L) = \frac{4.92\, L}{\sqrt{Re_L}} = 3.97 \times 10^{-3}\ \text{m} = 3.97\ \text{mm}$$

$$\delta_t = \frac{\delta}{\sqrt[3]{Pr}} = \frac{3.97 \times 10^{-3}}{\sqrt[3]{0.7}} = 4.47 \times 10^{-3} = 4.47\ \text{mm}$$

3. In case of laminar flow over an isothermal flat plate, the local Nusselt number (Nu) for the entire range of Prandtl number (Pr) is given by:

$$\mathrm{Nu}_x = \frac{0.339\,\mathrm{Re}_x^{1/2}\,\mathrm{Pr}^{1/3}}{\left[1 + (0.0468/\mathrm{Pr})^{2/3}\right]^{1/4}}$$

Derive the expression for the average Nu for a laminar boundary layer over that plate for the identical condition.

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$$\mathrm{Nu}_x = \mathrm{A}\,\mathrm{Re}_x^{1/2}$$

where $\mathrm{A} = \frac{0.339\ \mathrm{Pr}^{1/3}}{[1+(0.0468/\mathrm{Pr})^{2/3}]^{1/4}}$

Furthe, we know that $q_x = \frac{k\,\Delta T}{x}\mathrm{Nu}_x$.

Now, integrating q_x over the length of the flat plate, we get

$$\overline{q}_L = \frac{k\,\mathrm{A}\,\Delta T}{L}\int_0^L \sqrt{\frac{u_\infty x}{\nu}}\,\frac{1}{x}\,dx$$

$$\Rightarrow \overline{q}_L = \frac{2\,k\,\mathrm{A}\,\Delta T}{L}\sqrt{\frac{u_\infty L}{\nu}}$$

$$\Rightarrow \overline{q}_L = \frac{2\,k\,\mathrm{A}\,\Delta T}{L}\mathrm{Re}_L^{1/2}$$

Therefore, the average Nusselt number:

$$\overline{\mathrm{Nu}}_L = \frac{\overline{q}_L\,L}{k\,\Delta T}$$

$$\Rightarrow \overline{\mathrm{Nu}}_L = 2\,\mathrm{A}\,\mathrm{Re}_L^{1/2}$$

$$\Rightarrow \overline{\mathrm{Nu}}_L = \frac{0.678\,\mathrm{Pr}^{1/3}\,\mathrm{Re}_L^{1/2}}{[1 + (0.0468/\mathrm{Pr})^{2/3}]^{1/4}}$$