HEAT TRANSFER

[CH21204]

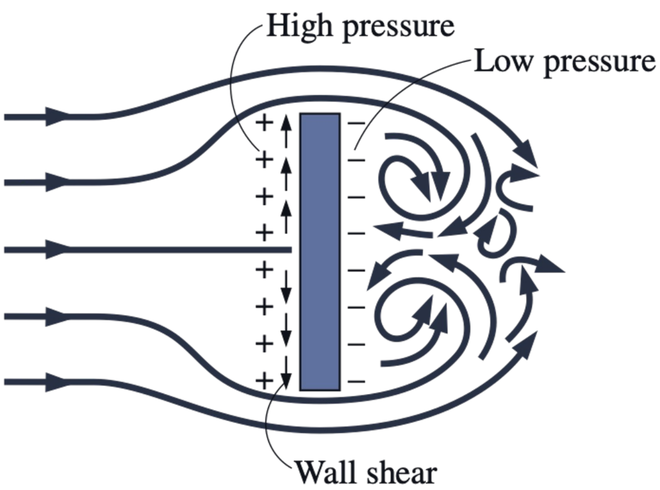
February 03, 2023

Friction & Pressure Drag

- The force a flowing fluid exerts on a body in the flow direction is called **drag.**
- The components of the pressure and wall shear forces in the normal direction to flow tend to move the body in that direction, and their sum is called **lift.**

Drag coefficient:

$$C_D = \frac{F_D}{\frac{1}{2}\rho \mathcal{V}^2 A}$$



- The part of drag that is due directly to wall shear stress is called the **skin friction drag (or just friction drag)** since it is caused by frictional effects.
- The part that is due directly to pressure P is called the **pressure drag** (also called the **form drag** because of its strong dependence on the form or shape of the body).

$$C_D = C_{D,\text{ friction}} + C_{D,\text{ pressure}}$$

$$C_{D,\text{ pressure}} = 0$$

$$C_D = C_{D,\text{ friction}} = C_f$$

$$F_{D,\text{ pressure}} = 0$$

$$F_D = F_{D,\text{ friction}} = F_f = C_f A \frac{\rho \mathcal{V}^2}{2}$$

For **parallel flow over a flat plate**, the **pressure drag is zero**, and thus the drag coefficient is equal to the friction coefficient and the drag force is equal to the friction force.

The **pressure drag** is proportional to the difference between the pressures acting on the front and back of the immersed body, and the frontal area. Therefore, the pressure drag is usually dominant for **blunt bodies**, negligible for streamlined bodies such as airfoils, and zero for thin flat plates parallel to the flow.

$$\mathrm{Nu}_x = f_1(x^*, \mathrm{Re}_x, \mathrm{Pr}) \qquad \text{and} \qquad \mathrm{Nu} = f_2(\mathrm{Re}_L, \mathrm{Pr})$$

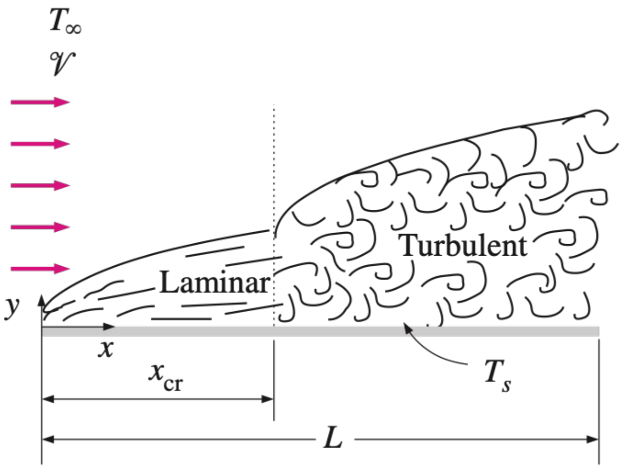
$$\mathrm{Nu} = C\,\mathrm{Re}_L^m \mathrm{Pr}^n$$

$$T_f = \frac{T_s + T_\infty}{2}$$

$$C_D = \frac{1}{L}\int_0^L C_{D,x}\,dx$$

$$h = \frac{1}{L}\int_0^L h_x\,dx$$

$$\dot{Q} = hA_s(T_s - T_\infty)$$



Laminar: $\delta_{v,x} = \dfrac{5x}{Re_x^{1/2}}$ and $C_{f,x} = \dfrac{0.664}{Re_x^{1/2}}$, $Re_x < 5 \times 10^5$

Turbulent: $\delta_{v,x} = \dfrac{0.382x}{Re_x^{1/5}}$ and $C_{f,x} = \dfrac{0.0592}{Re_x^{1/5}}$, $5 \times 10^5 \le Re_x \le 10^7$

$$C_f = \frac{1}{L}\int_0^L C_{f,x}\,dx = \frac{1}{L}\int_0^L \frac{0.664}{Re_x^{1/2}}\,dx = \frac{0.664}{L}\int_0^L \left(\frac{\mathscr{V}x}{\nu}\right)^{-1/2} dx = \frac{0.664}{L}\left(\frac{\mathscr{V}}{\nu}\right)^{-1/2} \left.\frac{x^{1/2}}{\frac{1}{2}}\right|_0^L = \frac{2\times 0.664}{L}\left(\frac{\mathscr{V}L}{\nu}\right)^{-1/2} = \frac{1.328}{Re_L^{1/2}}$$

Laminar: $C_f = \dfrac{1.328}{Re_L^{1/2}}$ $Re_L < 5 \times 10^5$

Turbulent: $C_f = \dfrac{0.074}{Re_L^{1/5}}$ $5 \times 10^5 \le Re_L \le 10^7$

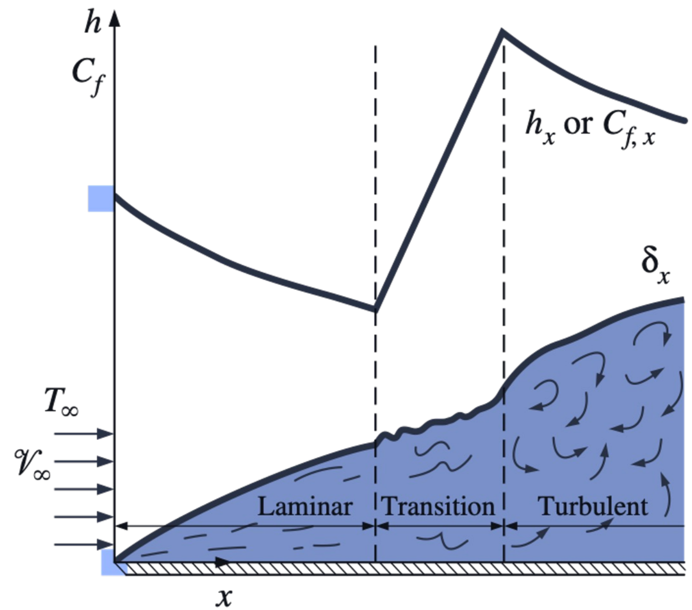
$$C_f = \frac{1}{L}\left(\int_0^{x_{cr}} C_{f,x\ \text{laminar}}\,dx + \int_{x_{cr}}^{L} C_{f,x,\ \text{turbulent}}\,dx\right)$$

$$C_f = \frac{0.074}{Re_L^{1/5}} - \frac{1742}{Re_L} \qquad 5 \times 10^5 \le Re_L \le 10^7$$

smooth, free-stream turbulent free

Laminar: $\mathrm{Nu}_x = \frac{h_x x}{k} = 0.332\,\mathrm{Re}_x^{0.5}\,\mathrm{Pr}^{1/3}$ $\quad \mathrm{Pr} > 0.60$

Turbulent: $\mathrm{Nu}_x = \frac{h_x x}{k} = 0.0296\,\mathrm{Re}_x^{0.8}\,\mathrm{Pr}^{1/3}$ $\quad 0.6 \le \mathrm{Pr} \le 60$, $\quad 5 \times 10^5 \le \mathrm{Re}_x \le 10^7$



Laminar: $\mathrm{Nu} = \frac{hL}{k} = 0.664\,\mathrm{Re}_L^{0.5}\,\mathrm{Pr}^{1/3}$ $\quad \mathrm{Re}_L < 5 \times 10^5$

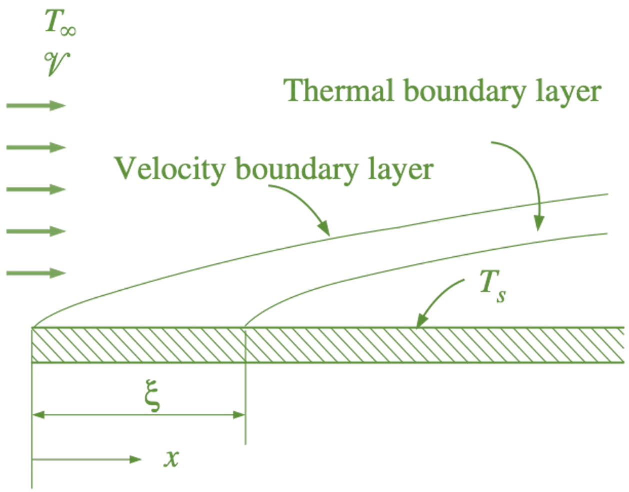
Turbulent: $\mathrm{Nu} = \frac{hL}{k} = 0.037\,\mathrm{Re}_L^{0.8}\,\mathrm{Pr}^{1/3}$ $\quad 0.6 \le \mathrm{Pr} \le 60$, $\quad 5 \times 10^5 \le \mathrm{Re}_L \le 10^7$

$$h = \frac{1}{L}\left(\int_0^{x_{cr}} h_{x,\text{ laminar}}\, dx + \int_{x_{cr}}^{L} h_{x,\text{ trubulent}}\, dx\right)$$

$$\text{Nu} = \frac{hL}{k} = (0.037\ \text{Re}_L^{0.8} - 871)\text{Pr}^{1/3} \qquad \begin{array}{l} 0.6 \le \text{Pr} \le 60 \\ 5 \times 10^5 \le \text{Re}_L \le 10^7 \end{array}$$

$$\text{Nu}_x = 0.565(\text{Re}_x\ \text{Pr})^{1/2} \qquad \text{Pr} < 0.05$$

Flat Plate with Unheated Starting Length



Laminar:
$$\mathrm{Nu}_x = \frac{\mathrm{Nu}_{x\,(\text{for } \xi=0)}}{[1-(\xi/x)^{3/4}]^{1/3}} = \frac{0.332\,\mathrm{Re}_x^{0.5}\,\mathrm{Pr}^{1/3}}{[1-(\xi/x)^{3/4}]^{1/3}}$$

Turbulent:
$$\mathrm{Nu}_x = \frac{\mathrm{Nu}_{x\,(\text{for } \xi=0)}}{[1-(\xi/x)^{9/10}]^{1/9}} = \frac{0.0296\,\mathrm{Re}_x^{0.8}\,\mathrm{Pr}^{1/3}}{[1-(\xi/x)^{9/10}]^{1/9}}$$

Laminar:
$$h = \frac{2[1-(\xi/x)^{3/4}]}{1-\xi/L}\,h_{x=L}$$

Turbulent:
$$h = \frac{5[1-(\xi/x)^{9/10}]}{4(1-\xi/L)}\,h_{x=L}$$

Uniform Heat Flux

Laminar: $$\mathrm{Nu}_x = 0.453\,\mathrm{Re}_x^{0.5}\,\mathrm{Pr}^{1/3}$$

Turbulent: $$\mathrm{Nu}_x = 0.0308\,\mathrm{Re}_x^{0.8}\,\mathrm{Pr}^{1/3}$$

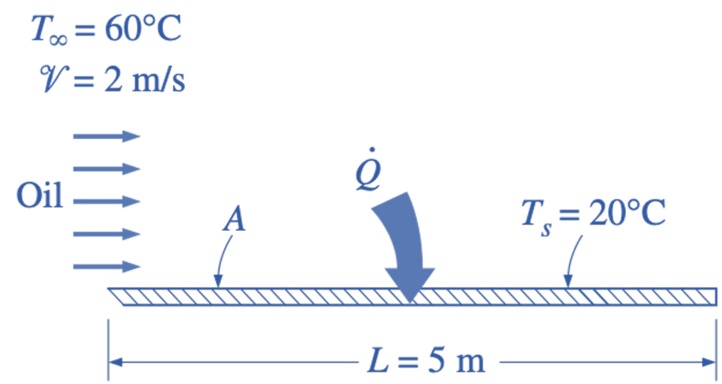
isothermal plate case

Laminar: $$\mathrm{Nu}_x = \frac{h_x x}{k} = 0.332\,\mathrm{Re}_x^{0.5}\,\mathrm{Pr}^{1/3} \qquad \mathrm{Pr} > 0.60$$

Turbulent: $$\mathrm{Nu}_x = \frac{h_x x}{k} = 0.0296\,\mathrm{Re}_x^{0.8}\,\mathrm{Pr}^{1/3} \qquad \begin{array}{l} 0.6 \le \mathrm{Pr} \le 60 \\ 5 \times 10^5 \le \mathrm{Re}_x \le 10^7 \end{array}$$

$$\dot{Q} = \dot{q}_s A_s \qquad \dot{q}_s = h_x[T_s(x) - T_\infty] \qquad T_s(x) = T_\infty + \frac{\dot{q}_s}{h_x}$$

Engine oil at 60°C flows over the upper surface of a 5-m-long flat plate whose temperature is 20°C with a velocity of 2 m/s. Determine the total drag force and the rate of heat transfer per unit width of the entire plate.



$\rho = 876\ \text{kg/m}^3$ $\qquad$ $\text{Pr} = 2870$

$k = 0.144\ \text{W/m}\cdot°\text{C}$ $\qquad$ $\nu = 242 \times 10^{-6}\ \text{m}^2/\text{s}$

$$\text{Re}_L = \frac{\mathcal{V}L}{\nu} = \frac{(2\ \text{m/s})(5\ \text{m})}{0.242 \times 10^{-5}\ \text{m}^2/\text{s}} = 4.13 \times 10^4$$

$$C_f = 1.328\ \text{Re}_L^{-0.5} = 1.328 \times (4.13 \times 10^3)^{-0.5} = 0.0207$$

$$F_D = C_f A_s \frac{\rho \mathcal{V}^2}{2} = 0.0207 \times (5 \times 1\ \text{m}^2)\frac{(876\ \text{kg/m}^3)(2\ \text{m/s})^2}{2}\left(\frac{1\ \text{N}}{1\ \text{kg}\cdot\text{m/s}^2}\right)$$

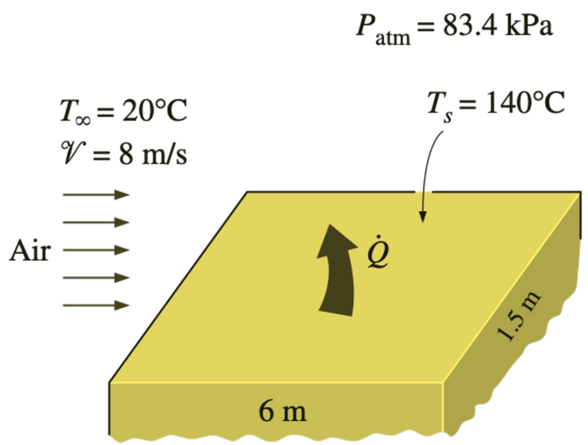
$$= \mathbf{181\ N}$$

$$\mathrm{Nu} = \frac{hL}{k} = 0.664\ \mathrm{Re}_L^{0.5}\ \mathrm{Pr}^{1/3} = 0.664 \times (4.13 \times 10^4)^{0.5} \times 2870^{1/3} = 1918$$

$$h = \frac{k}{L}\mathrm{Nu} = \frac{0.144\ \mathrm{W/m \cdot {}^\circ C}}{5\ \mathrm{m}}(1918) = 55.2\ \mathrm{W/m^2 \cdot {}^\circ C}$$

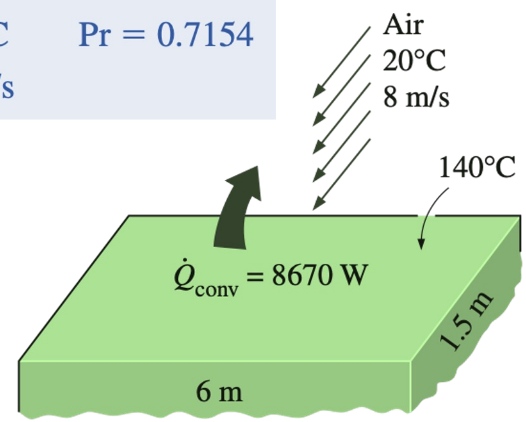
$$\dot{Q} = hA_s(T_\infty - T_s) = (55.2\ \mathrm{W/m^2 \cdot {}^\circ C})(5 \times 1\ \mathrm{m^2})(60 - 20){}^\circ\mathrm{C} = \mathbf{11{,}040\ W}$$

The local atmospheric pressure in a place at an elevation of 1610 m is 83.4 kPa. Air at this pressure and 20°C flows with a velocity of 8 m/s over a 1.5 m X 6 m flat plate whose temperature is 140°C. Determine the rate of heat transfer from the plate if the air flows parallel to the (a) 6-m-long side and (b) the 1.5-m side.



$k = 0.02953$ W/m · °C Pr = 0.7154

$\nu_{@ 1 \text{ atm}} = 2.097 \times 10^{-5}$ m²/s



$$\nu = \nu_{@ 1 \text{ atm}}/P = (2.097 \times 10^{-5}\ \text{m}^2/\text{s})/0.823 = 2.548 \times 10^{-5}\ \text{m}^2/\text{s}$$

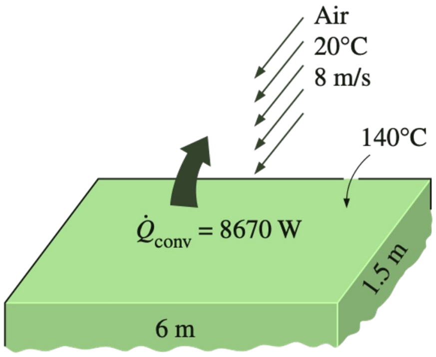
$$\text{Re}_L = \frac{\mathscr{V}L}{\nu} = \frac{(8\ \text{m/s})(6\ \text{m})}{2.548 \times 10^{-5}\ \text{m}^2/\text{s}} = 1.884 \times 10^6$$

$$\begin{aligned} \text{Nu} = \frac{hL}{k} &= (0.037\ \text{Re}_L^{0.8} - 871)\text{Pr}^{1/3} \\ &= [0.037(1.884 \times 10^6)^{0.8} - 871]0.7154^{1/3} \\ &= 2687 \end{aligned}$$

$$h = \frac{k}{L}\,\mathrm{Nu} = \frac{0.02953\ \mathrm{W/m\cdot{}^\circ C}}{6\ \mathrm{m}}(2687) = 13.2\ \mathrm{W/m^2\cdot{}^\circ C}$$

$$A_s = wL = (1.5\ \mathrm{m})(6\ \mathrm{m}) = 9\ \mathrm{m^2}$$

$$\dot{Q} = hA_s(T_s - T_\infty) = (13.2\ \mathrm{W/m^2\cdot{}^\circ C})(9\ \mathrm{m^2})(140 - 20)^\circ\mathrm{C} = \mathbf{1.43 \times 10^4\ W}$$



$$\mathrm{Re}_L = \frac{\mathcal{V}L}{\nu} = \frac{(8\ \mathrm{m/s})(1.5\ \mathrm{m})}{2.548 \times 10^{-5}\ \mathrm{m^2/s}} = 4.71 \times 10^5$$

$$\mathrm{Nu} = \frac{hL}{k} = 0.664\ \mathrm{Re}_L^{0.5}\ \mathrm{Pr}^{1/3} = 0.664 \times (4.71 \times 10^5)^{0.5} \times 0.7154^{1/3} = 408$$

$$h = \frac{k}{L}\,\mathrm{Nu} = \frac{0.02953\ \mathrm{W/m\cdot{}^\circ C}}{1.5\ \mathrm{m}}(408) = 8.03\ \mathrm{W/m^2\cdot{}^\circ C}$$

$$\dot{Q} = hA_s(T_s - T_\infty) = (8.03\ \mathrm{W/m^2\cdot{}^\circ C})(9\ \mathrm{m^2})(140 - 20)^\circ\mathrm{C} = \mathbf{8670\ W}$$

A 15-cm X 15-cm circuit board dissipating 15 W of power uniformly is cooled by air, which approaches the circuit board at 20°C with a velocity of 5 m/s. Disregarding any heat transfer from the back surface of the board, determine the surface temperature of the electronic components (a) at the leading edge and (b) at the end of the board. Assume the flow to be turbulent since the electronic components are expected to act as turbulators.

$$k = 0.0265 \text{ W/m.°C}$$

$$\upsilon = 1.655 \times 10^{-5} \text{ m}^2/\text{s}$$

$$\text{Pr} = 0.7268$$

$$\text{Re}_x = \frac{V_\infty x}{\upsilon} = \frac{(5 \text{ m/s})(0.15 \text{ m})}{1.655 \times 10^{-5} \text{ m}^2/\text{s}} = 4.532 \times 10^4$$

$$Nu_x = \frac{h_x x}{k} = 0.0308 \,\text{Re}_x^{\,0.8} \,\text{Pr}^{1/3} = 0.0308(4.532 \times 10^4)^{0.8}(0.7268)^{1/3} = 147.0$$

$$h_x = \frac{k_x}{x} Nu_x = \frac{0.02625 \text{ W/m.°C}}{0.15 \text{ m}}(147.0) = 25.73 \text{ W/m}^2\text{.°C}$$

$$\dot{q} = h_x(T_s - T_\infty) \longrightarrow T_s = T_\infty + \frac{\dot{q}}{h_x} = 20\text{°C} + \frac{(15 \text{ W})/(0.15 \text{ m})^2}{25.73 \text{ W/m}^2\text{.°C}} = \mathbf{45.9°C}$$