Indian Institute of Technology Kharagpur
Department of Chemical Engineering
End Semester Examination, Autumn 24-25
Sub: **Advanced Fluid Dynamics**
Subject Number: CH61011

22 Nov 2024
Time: 3 Hrs
FM: 50
No. of Students: 116

Instructions: Answer all questions. Closed book, closed notes examination. All symbols carry their usual meaning. Assume missing data suitably. Follow the five step problem solving methodology. Data required are included in the question paper.

1. (7 marks) Consider a solid sphere of radius R rotating around a fixed axis at an angular velocity ω in a large volume of fluid which is otherwise at rest. It is desired to determine the velocity field for this case where Re is vanishingly small, i.e. creeping flow assumption is valid.
   (a) (1 mark) Sketch the system and show the spherical coordinate system with respect to the axis of rotation.
   (b) (2 marks) Analyse the system and provide simplifying assumptions for all velocity components and their dependence.
   (c) (1 mark) Simplify the EOM.
   (d) (3 marks) Obtain v_ϕ by assuming $v_\phi = f(r)\sin\theta$. Justify this assumption.

2. (10 marks) Consider creeping flow around a stationery sphere as considered in the class. The two non-zero velocity components for this flow is given by:

$$v_r = v_\infty\left(1 - \frac{3}{2}\left(\frac{R}{r}\right) + \frac{1}{2}\left(\frac{R}{r}\right)^3\right)\cos\theta; \quad v_\theta = -v_\infty\left(1 - \frac{3}{4}\left(\frac{R}{r}\right) - \frac{1}{4}\left(\frac{R}{r}\right)^3\right)\sin\theta$$

   (a) (6 marks) Using these velocities, obtain the expressions for:

$$\frac{\partial p}{\partial r} - \rho g_r \quad \text{and} \quad \frac{1}{r}\frac{\partial p}{\partial \theta} - \rho g_\theta$$

   (b) (4 marks) Using these expressions, show that:

$$\frac{\partial \mathscr{P}}{\partial r} = 3\left(\frac{\mu v_\infty}{R^2}\right)\left(\frac{R}{r}\right)^3\cos\theta; \quad \frac{\partial \mathscr{P}}{\partial \theta} = \frac{3}{2}\left(\frac{\mu v_\infty}{R}\right)\left(\frac{R}{r}\right)^2\sin\theta$$

3. Explain the following concepts using suitable schematics as applies:
   (a) (2 marks) Kolmogorov's length scale
   (b) (2 marks) Reynold's decomposition
   (c) (2 marks) Turbulent shear stress
   (d) (2 marks) Prandtl mixing length

4. The oxygen intake by humans and its distribution to the blood stream is regulated by lungs. The lung fluid has a surfactant like biomolecule that regulates the adsorption of oxygen from the air up taken through nostrils. In an medico-engineering attempt to understand this highly complex process, a prototype analysis was performed by using

both mathematical correlation as well as common (very basic Newtonian mechanics) physical understandings. The aim is to understand the nature of stresses and deformations occurring on the biomolecule surfactant-air interface (surface). The following initial assumptions were made; (i) the surfactant is a monolayer; (ii) during deformations the surfactant layer acts as purely elastic. Based on this premise answer the following questions.

(a) (3 marks) starting from purely mathematical considerations define the deformation tensor that is responsible for stretching of the surfactant monolayer.

(b) (12 marks) Based on the above definition of the deformation tensor by using a clear diagram (in which the shape of monolayer can be assumed to be of a simple 2D geometry) elaborate how the coordinate transformation is achieved for obtaining the deformation tensor. (diagram with all details-7 marks, details of coordinate transformation-5 marks).

(c) (10 marks) Based on the answers of part (a)&part(b), reach a clear mathematical expression for the deformation tensor based on velocity and displacement assigned in 2D to the monolayer surfactant (Hint: The velocity profile should be representing deformations WITHIN the monolayer not the velocity OF THE monolayer)

Spherical coordinates (r, θ, ϕ):

$$\rho\left(\frac{\partial v_r}{\partial t} + v_r\frac{\partial v_r}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_r}{\partial \theta} + \frac{v_\phi}{r\sin\theta}\frac{\partial v_r}{\partial \phi} + \frac{v_\theta^2 + v_\phi^2}{r}\right) = -\frac{\partial p}{\partial r}$$
$$+ \mu\left[\frac{1}{r^2}\frac{\partial^2}{\partial r^2}(r^2 v_r) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial v_r}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 v_r}{\partial\phi^2}\right] + \rho g_r$$

$$\rho\left(\frac{\partial v_\theta}{\partial t} + v_r\frac{\partial v_\theta}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_\theta}{\partial \theta} + \frac{v_\phi}{r\sin\theta}\frac{\partial v_\theta}{\partial \phi} + \frac{v_r v_\theta - v_\phi^2\cot\theta}{r}\right) = -\frac{1}{r}\frac{\partial p}{\partial \theta}$$
$$+ \mu\left[\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial v_\theta}{\partial r}\right) + \frac{1}{r^2}\frac{\partial}{\partial\theta}\left(\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}(v_\theta\sin\theta)\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 v_\theta}{\partial\phi^2} + \frac{2}{r^2}\frac{\partial v_r}{\partial\theta} - \frac{2\cot\theta}{r^2\sin\theta}\frac{\partial v_\phi}{\partial\phi}\right] + \rho g_\theta$$

$$\rho\left(\frac{\partial v_\phi}{\partial t} + v_r\frac{\partial v_\phi}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_\phi}{\partial \theta} + \frac{v_\phi}{r\sin\theta}\frac{\partial v_\phi}{\partial \phi} + \frac{v_\phi v_r + v_\theta v_\phi\cot\theta}{r}\right) = -\frac{1}{r\sin\theta}\frac{\partial p}{\partial \phi}$$
$$+ \mu\left[\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial v_\phi}{\partial r}\right) + \frac{1}{r^2}\frac{\partial}{\partial\theta}\left(\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}(v_\phi\sin\theta)\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 v_\phi}{\partial\phi^2} + \frac{2}{r^2\sin\theta}\frac{\partial v_r}{\partial\phi} + \frac{2\cot\theta}{r^2\sin\theta}\frac{\partial v_\theta}{\partial\phi}\right] + \rho g_\phi$$

End of Question Paper