Indian Institute of Technology Kharagpur

Date of Examination: Session: Duration 3 hrs Full Marks 50
Subject Number: CH30012 Subject: Transport Phenomena Department: Chemical Engineering
Graph paper required: **YES.**
Specific Instructions: Assume and clearly write any assumption and data that you feel are missing.

1. A plane surface, 25 cm wide, has its temperature maintained at 80 C. Atmospheric air, at 25C; flows parallel to the surface with a velocity of 2.8 m/s. Determine the following for a 1-m long plate:
   (i) The total drag force exerted on the plate by the air flow (in mN).
   (ii) The total heat transfer rate from the plate to the air stream (in W).
   For Air at the average temperature the properties can be taken as: $\rho = 1.087$ Kg/m^3, kinematic viscosity = 1.807×10^{-5} m^2/s, $C_p = 1.008$ kJ/(Kg . K), Pr = 0.702, k = 2.816 W/(m . K) **5 + 5 = 10**

2. (a) A crude approximation for the x component of velocity in a laminar boundary layer is a linear variation from $v_x = 0$ at the surface to the freestream velocity, U, at the boundary layer edge ($y = \delta$). The equation for the profile is given below (where C is a constant). Evaluate the maximum value of the ratio v_y / v_x at a location x = 0.5 m and δ = 5 mm.

$$v_x = C\,U\,\frac{y}{x^{1/2}}$$ **3**

   (b) Starting with the governing equation for momentum boundary layer evaluate the value of $\partial^3 u / \partial y^3$ at y = 0 for an incompressible laminar boundary layer on a flat plate with zero-pressure gradient. **2**
   (c) A flat plate, sides a, b in length, is towed through a fluid so that the boundary layer is entirely laminar. Find the ratio of towing speeds so that the drag force remains constant regardless of whether a or b is in the flow direction. U_a is the freestream velocity if side a is in the flow direction and U_b if b is in the flow direction. **5** **3+2+5=10**

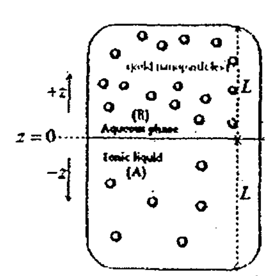
3. A new spray-painting system is being evaluated for the auto industry. The paint is delivered via an atomizer that produces 10 mm particles and propels them toward the surface at a velocity of 10 m/s. The particles are a dilute suspension of pigment agents in a solvent and for modelling purposes can be assumed to be pure solvent. To form a good coating the particles must arrive at the surface with 75% by volume of the solvent remaining. The freestream concentration of the solvent is effectively zero and the solvent has a specific gravity of 0.85 and a molecular weight of 100. The whole system operates at atmospheric pressure and at the temperature of deposition the vapor pressure of the solvent is 250 mm Hg. The diffusivity of the solvent in air was 1×10^{-9} m^2/s, the Schmidt number for the solvent is 500 and R = 8314J/(Kg mol . K). For the specified conditions the relevant mass transfer correlation can be approximated as

$$\overline{Sh_D} = 2 + \left(0.4\,\mathrm{Re}_d^{1/2} + 0.06\,\mathrm{Re}_d^{2/3}\right) Sc^{0.4}$$

   How far away from the surface should the painting nozzle be located? **10**

4. Air passes through a naphthalene tube, that has an inside diameter of 2.5 cm, flowing at a bulk velocity of 15 m/s. The air is at 283 K and an average pressure of 101300 Pa. Assuming that the change in pressure along the tube is negligible and that the naphthalene surface is at 283 K, determine i) the concentration profile of naphthalene as a function of the axial position in the tube, assuming the bulk velocity to be a constant and ii) the length of tube (approx. in m) that is necessary to produce a naphthalene concentration in the exiting gas stream of 4.75×10^{-4} mol/m^3. At 283 K, naphthalene has a vapor pressure of 3 Pa and a diffusivity in air of 5.4×10^{-6} m^2/s. Use $C_f = 0.0058$, $\bar{R}$=8.314 J/(mol•K). **5+5=10**

5. Phase transfer of gold nanoparticles from aqueous phase to ionic liquid (IL) phase is greatly desired, as the ionic liquid prevents the solvent vaporization significantly as compared to aqueous phase, which in turn prevents agglomeration of gold nanoparticles. One such recent discovery is use water-immiscible ionic liquid 1-butyl-3-methylimidazolium hexafluorophosphate for phase transfer of gold nanoparticles from aqueous phase (**B**) to ionic liquid (phase A), as shown in figure on the right side. The initial concentration of gold nanoparticles are $C_{A0}(= 0.05\ mol/m^3)$ and $C_{B0}(= 0.5\ mol/m^3)$ in phase **A** and **B** respectively. Consider that the interface of two phases is always located at $z = 0$. $C_A(z,t)$



and $C_B(z,t)$ represent the gold nanoparticle concentration as a function of space and time in phase A and B respectively. The diffusivities of gold nanoparticle in phase A and B are D_A ($=10^{-10}$ m²/s) and D_B ($=2\times10^{-10}$ m²/s) respectively. L = 10mm.

Make the following assumptions:

- Consider unidirectional mass diffusion (only) in $\pm z$-direction. Fick's law of diffusion is applicable for both phases.
- Interface always remains at chemical equilibrium,
  i.e., $C_B(z=0,t>0) = kC_A(z=0,t>0)$ (k =2).
- Interface always remains in quasi-steady state, which implies that net diffusive mass-flux at interface is equal to zero.
- Ignore gravity driven settling.
- Don't mix up initial condition with any boundary condition (In other words, kindly don't try to validate interfacial equilibrium condition at t =0).

a) Report the concentration values of gold nanoparticles in aqueous phase (C_B) at t = 10s for following z = 0.057, 0.115, 0.285, 1.132, 3.0, 5.66 mm. Similarly, report the concentration values of gold nanoparticles in ionic phase (C_A) at t = 10s for following z = −0.057, −0.115, −0.285, −1.132, −3.0, −5.66 mm. Use these concentration values to draw the concentration profile of gold nanoparticles over z at t = 10s (You are free to avoid any derivation, however with proper justification/logic). **(6)**

b) Calculate the mass transfer flux of gold nanoparticle at z = $-0.1mm$ at t = 10s. **(4)**

EQUATION OF CONTINUITY (Cartesian, cylindrical and spherical coordinates)

$$\frac{\partial\rho}{\partial t}+\frac{\partial}{\partial x}(\rho v_x)+\frac{\partial}{\partial y}(\rho v_y)+\frac{\partial}{\partial z}(\rho v_z)=0 \qquad \frac{\partial\rho}{\partial t}+\frac{1}{r}\frac{\partial}{\partial r}(\rho r v_r)+\frac{1}{r}\frac{\partial}{\partial\theta}(\rho v_\theta)+\frac{\partial}{\partial z}(\rho v_z)=0$$

$$\frac{\partial\rho}{\partial t}+\frac{1}{r^2}\frac{\partial}{\partial r}(\rho r^2 v_r)+\frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}(\rho v_\theta\sin\theta)+\frac{1}{r\sin\theta}\frac{\partial}{\partial\phi}(\rho v_\phi)=0$$

EQUATION OF MOTION (Cartesian and Cylindrical coordinates)

$$\rho\left(\frac{\partial v_z}{\partial t}+v_x\frac{\partial v_z}{\partial x}+v_y\frac{\partial v_z}{\partial y}+v_z\frac{\partial v_z}{\partial z}\right)=-\frac{\partial p}{\partial z}+\mu\left(\frac{\partial^2 v_z}{\partial x^2}+\frac{\partial^2 v_z}{\partial y^2}+\frac{\partial^2 v_z}{\partial z^2}\right)+\rho g_z$$

$$\rho\left(\frac{\partial v_r}{\partial t}+v_r\frac{\partial v_r}{\partial r}+\frac{v_\theta}{r}\frac{\partial v_r}{\partial\theta}+v_z\frac{\partial v_r}{\partial z}-\frac{v_\theta^2}{r}\right)=-\frac{\partial p}{\partial r}+\mu\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(r v_r)\right)+\frac{1}{r^2}\frac{\partial^2 v_r}{\partial\theta^2}-\frac{\partial^2 v_r}{\partial z^2}-\frac{2}{r^2}\frac{\partial v_\theta}{\partial\theta}\right]+\rho g_r$$

$$\rho\left(\frac{\partial v_z}{\partial t}+v_r\frac{\partial v_z}{\partial r}+\frac{v_\theta}{r}\frac{\partial v_z}{\partial\theta}+v_z\frac{\partial v_z}{\partial z}\right)=-\frac{\partial p}{\partial z}+\mu\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial v_z}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2 v_z}{\partial\theta^2}+\frac{\partial^2 v_z}{\partial z^2}\right]+\rho g_z$$

INTEGRAL EQUATIONS

$$\frac{dN}{dt}\Big|_{system}=\frac{\partial}{\partial t}\int_{CV}\eta\rho d\forall+\int_{CS}\eta\rho\vec{V}.\vec{dA} \qquad \delta^*=\int_0^\delta\left(1-\frac{v_x}{U}\right)dy, \qquad \theta=\int_0^\delta\frac{v_x}{U}\left(1-\frac{v_x}{U}\right)dy$$

$$\frac{\tau_w}{\rho}=\frac{d}{dx}(U^2\theta)+\delta^* U\frac{dU}{dx} \qquad \delta=\frac{5.0x}{\sqrt{Re_x}}\ (laminar\ flow) \quad \delta=\frac{0.37x}{(Re_x)^{\frac{1}{5}}}\ (turbulent\ flow) \quad \frac{\overline{v_z}}{U}=\left(\frac{y}{R}\right)^{\frac{1}{7}}$$

Laminar Flow: $C_f=\frac{0.664}{\sqrt{Re_x}}$ $C_D=\frac{1.328}{\sqrt{Re_L}}$ Turbulent Flow: $C_f=\frac{0.0594}{(Re_x)^{\frac{1}{5}}}$ $C_D=\frac{0.0742}{(Re_L)^{\frac{1}{5}}}$

STOKES LAW $F=3\pi\mu Vd$

For Mixed Flow, $C_{D_{Turb}}=\frac{0.074}{(Re_L)^{\frac{1}{5}}}-\frac{1740}{Re_L}$, $10^5<Re<10^7$, $C_{D_{Turb}}=\frac{0.455}{(\log Re_L)^{2.58}}-\frac{1610}{Re_L}$, $Re>10^7$

ENERGY EQUATION (in all coordinate systems)

$$\rho\hat{C}_p\left(\frac{\partial T}{\partial t}+v_x\frac{\partial T}{\partial x}+v_y\frac{\partial T}{\partial y}+v_z\frac{\partial T}{\partial z}\right)=k\left[\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}+\frac{\partial^2 T}{\partial z^2}\right]+\mu\phi_v+\dot{Q}$$

$$\rho\hat{C}_p\left(\frac{\partial T}{\partial t}+v_r\frac{\partial T}{\partial r}+\frac{v_\theta}{r}\frac{\partial T}{\partial\theta}+v_z\frac{\partial T}{\partial z}\right)=k\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2 T}{\partial\theta^2}+\frac{\partial^2 T}{\partial z^2}\right]+\mu\phi_v+\dot{Q}$$

$$\rho\hat{C}_p\left(\frac{\partial T}{\partial t}+v_r\frac{\partial T}{\partial r}+\frac{v_\theta}{r}\frac{\partial T}{\partial\theta}+\frac{v_\phi}{r\sin\theta}\frac{\partial T}{\partial\phi}\right)=k\left[\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial T}{\partial r}\right)+\frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial T}{\partial\theta}\right)+\frac{1}{r^2\sin^2\theta}\frac{\partial^2 T}{\partial\phi^2}\right]+\mu\phi_v+\dot{Q}$$

$Sc = \frac{\mu}{\rho D_{AB}}$ $Sh = \frac{h_m l}{D_{AB}}$ $St = \frac{Nu}{Re \cdot Pr}$ $Fo = \frac{\alpha t}{l^2}$ $Bi = hl/k_s$ $Le = \frac{\alpha}{D_{AB}}$ $C_f = \frac{\tau_w}{\frac{1}{2}\rho v^2}$ $C_D = \frac{F_D/A_P}{\frac{1}{2}\rho v^2}$

Flow over flat plate

Laminar flow ($0.6 \le \Pr \le 60$, $Re \le 5\times10^5$) $\delta = \frac{5x}{\sqrt{Rex}}$ $C_{fx} = \frac{0.664}{\sqrt{Rex}}$ $C_{fL} = \frac{1.328}{\sqrt{Rex}}$ $Nu_x = 0.332 Re_x^{1/2} \Pr^{1/3}$ $\overline{Nu} = 0.664 Re_x^{1/2} \Pr^{1/3}$

Turbulent flow ($0.5 \le \Pr \le 60$ & $Re \ge 10^7$) $\delta = \frac{0.37x}{Rex^{1/5}}$ $C_{fx} = \frac{0.0577}{Rex^{1/5}}$ $C_D = \frac{0.455}{\log(Re_l)^{2.88}}$

$Nu_x = 0.0296 Re_x^{4/5} \Pr^{1/3}$ $\overline{Nu_l} = 0.037 Re_l^{4/5} \Pr^{1/3}$

Mixed flow

$C_f = \frac{0.072}{Re_l^{1/5}} - \frac{1740}{Re_l}$ $Re \le 10^7$ $C_D = \frac{0.455}{\log(Re_l)^{2.88}} - \frac{1610}{Re_l}$ $Re \ge 10^7$ $\overline{Nu_l} = \left(0.037 Re_l^{\frac{4}{5}} - 871\right) \Pr^{\frac{1}{3}}$ $5\times10^5 \le Re \le 10^7$

Reynold's Analogy $St = \frac{C_{fx}}{2}$, $St = Nu/(Re.\Pr)$; Chilton Coulburn Analogy

$$St \times Pr^{2/3} = \frac{C_{fx}}{2} \quad 0.5 \le Pr \le 50$$

SPECIES BALANCE EQUATIONS

$$\left(\frac{\partial C_A}{\partial t} + v_x\frac{\partial C_A}{\partial x} + v_y\frac{\partial C_A}{\partial y} + v_z\frac{\partial C_A}{\partial z}\right) = D_{AB}\left[\frac{\partial^2 C_A}{\partial x^2} + \frac{\partial^2 C_A}{\partial y^2} + \frac{\partial^2 C_A}{\partial z^2}\right] + R_A$$

$$\left(\frac{\partial C_A}{\partial t} + v_r\frac{\partial C_A}{\partial r} + \frac{v_\theta}{r}\frac{\partial C_A}{\partial \theta} + v_z\frac{\partial C_A}{\partial z}\right) = D_{AB}\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial C_A}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 C_A}{\partial \theta^2} + \frac{\partial^2 C_A}{\partial z^2}\right] + R_A$$

$$\left(\frac{\partial C_A}{\partial t} + v_r\frac{\partial C_A}{\partial r} + \frac{v_\theta}{r}\frac{\partial C_A}{\partial \theta} + \frac{v_\phi}{r\sin\theta}\frac{\partial C_A}{\partial \phi}\right) = D_{AB}\left[\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial C_A}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta\frac{\partial C_A}{\partial \theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 C_A}{\partial \phi^2}\right] + R_A$$

Cauchy Momentum Balance (including turbulent stress contribution) in axial direction, in cylindrical co-ordinate:

$$\rho\left(\frac{\partial \overline{v_z}}{\partial t} + \overline{v_r}\frac{\partial \overline{v_z}}{\partial r} + \frac{\overline{v_\theta}}{r}\frac{\partial \overline{v_z}}{\partial \theta} + \overline{v_z}\frac{\partial \overline{v_z}}{\partial z}\right) = -\frac{\partial p}{\partial z} + \left[\frac{\partial(\tau_{zz} + \tau_{zz}^t)}{\partial z} + \frac{1}{r}\frac{\partial(\tau_{\theta z} + \tau_{\theta z}^t)}{\partial \theta} + \frac{1}{r}\frac{\partial(r(\tau_{rz} + \tau_{rz}^t))}{\partial r}\right]$$

where $\overline{v_z}, \overline{v_\theta}, \overline{v_r}$ are the time averaged mean velocities. $\tau_{zz}/\tau_{\theta z}/\tau_{rz}$ are the time-averaged mean shear stresses, and $\tau_{zz}^t/\tau_{\theta z}^t/\tau_{rz}^t$ are turbulent stress terms. (for unidirectional fully developed axial flow, consider $\tau_{zz}, \tau_{\theta z}, \tau_{zz}^t, \tau_{\theta z}^t = 0$,

Newton's law of viscosity: $\tau_{rz} = \eta\frac{\partial \overline{v_z}}{\partial r}$, **STOKES LAW** $F = 3\pi\mu V d$

Prandtl's law of mixing for turbulent flow: $\tau_{rz}^t = \eta_t\frac{\partial \overline{v_z}}{\partial r}$, and $\eta_t = \rho l^2 \left|\frac{\partial \overline{v_z}}{\partial r}\right|$, where η_t is eddy viscosity, l is mixing length.

error function: $erf(z) = \left(2/\sqrt{\pi}\right)\int_0^z \exp(-t^2)dt$, gamma function: $\Gamma(z) = \int_0^\infty t^{z-1}e^{-t}dt$, and $\Gamma(1/2) = \sqrt{\pi}$

complementary error function, $\text{erfc}(y) = 1 - \text{erf}(y)$, $\text{erf}(0)=0$, $\text{erf}(\infty)=1$, $\text{erf}(-\infty) = -1$.

$\int erfc(ay)dy = y(erfc(ay)) - (1/a\sqrt{\pi})\exp(-a^2y^2)$, $\frac{d(erf(y))}{dy} = \frac{2}{\sqrt{\pi}}\exp(-y^2)$, $\text{erf}(-z) = -\text{erf}(z)$

The complementary error function

ξ	erfc (ξ)	ξ	erfc (ξ)	ξ	erfc (ξ)	ξ	erfc (ξ)	ξ	erfc (ξ)	ξ	erfc (ξ)
0.00	1.00000	0.38	0.5910	0.76	0.2825	1.14	0.1069	1.52	0.03159	1.90	0.00721
0.02	0.9774	0.40	0.5716	0.78	0.2700	1.16	0.10090	1.54	0.02941	1.92	0.00662
0.04	0.9549	0.42	0.5525	0.80	0.2579	1.18	0.09516	1.56	0.02737	1.94	0.00608
0.06	0.9324	0.44	0.5338	0.82	0.2462	1.20	0.08969	1.58	0.02545	1.96	0.00557
0.08	0.9099	0.46	0.5153	0.84	0.2349	1.22	0.08447	1.60	0.02365	1.98	0.00511
0.10	0.8875	0.48	0.4973	0.86	0.2239	1.24	0.07950	1.62	0.02196	2.00	0.00468
0.12	0.8652	0.50	0.4795	0.88	0.2133	1.26	0.07476	1.64	0.02038	2.10	0.00298
0.14	0.8431	0.52	0.4621	0.90	0.2031	1.28	0.07027	1.66	0.01890	2.20	0.00186
0.16	0.8210	0.54	0.4451	0.92	0.1932	1.30	0.06599	1.68	0.01751	2.30	0.00114
0.18	0.7991	0.56	0.4284	0.94	0.1837	1.32	0.06194	1.70	0.01612	2.40	0.00069
0.20	0.7773	0.58	0.4121	0.96	0.1746	1.34	0.05809	1.72	0.01500	2.50	0.00041
0.22	0.7557	0.60	0.3961	0.98	0.1658	1.36	0.05444	1.74	0.01387	2.60	0.00024
0.24	0.7343	0.62	0.3806	1.00	0.1573	1.38	0.05098	1.76	0.01281	2.70	0.00013
0.26	0.7131	0.64	0.3654	1.02	0.1492	1.40	0.04772	1.78	0.01183	2.80	0.00008
0.28	0.6921	0.66	0.3506	1.04	0.1413	1.42	0.04462	1.80	0.01091	2.90	0.00004
0.30	0.6714	0.68	0.3362	1.06	0.1339	1.44	0.04170	1.82	0.01006	3.00	0.00002
0.32	0.6509	0.70	0.3222	1.08	0.1267	1.46	0.03895	1.84	0.00926	3.20	0.00001
0.34	0.6306	0.72	0.3086	1.10	0.1198	1.48	0.03635	1.86	0.00853	3.40	0.00000
0.36	0.6107	0.74	0.2953	1.12	0.1132	1.50	0.03390	1.88	0.00784	3.60	0.00000