Hand-written problems on Fourier series

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Q1. Find the Fourier series of the function

$$f(x) = x \quad 0 < x < 2\pi, \quad f(x+2\pi) = f(x) \ \forall x$$

Ans: $$f(x) = \pi - \sum_{n=1}^{\infty} \frac{2}{n} \sin nx$$

Q2. Obtain the Fourier series expansion of $f(x)$ with period 2 (i.e. $f(x+2) = f(x)$) defined as

$$f(x) = \begin{array}{ll} 1 & \text{if } 0 \le x < 1 \\ 2 & \text{if } 1 \le x < 2 \end{array}$$

Ans: $$f(x) = \frac{3}{2} - \frac{2}{\pi}\left[\frac{\sin \pi x}{1} + \frac{\sin 3\pi x}{3} + \frac{\sin 5\pi x}{5} \cdots\right]$$

Q3. Prove that the f^n. $f(x) = x$ can be expanded

(i) in a series of cosines in $0 < x < \pi$ as

$$x = \frac{\pi}{2} - \frac{4}{\pi}\left[\frac{\cos x}{1^2} + \frac{\cos 3x}{3^2} + \frac{\cos 5x}{5^2} + \cdots\right]$$

Hence deduce that $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \cdots = \frac{\pi^2}{8}$

(ii) in a series of sines in $0 < x < \pi$ as

$$x = 2\left[\frac{\sin x}{1} - \frac{\sin 2x}{2} + \frac{\sin 3x}{3} - \cdots\right]$$

Hence deduce that $1 - \frac{1}{3} + \frac{1}{5} - \cdots = \frac{\pi}{4}$

Q4. Find the half range cosine series of the f^n. $f(x) = x^2$ in $0 < x < \pi$ and hence find the sum of $1 - \frac{1}{2^2} + \frac{1}{3^2} - \cdots$.

Ans: $f(x) = \frac{\pi^2}{3} - 4\left[\frac{\cos x}{1^2} - \frac{\cos 2x}{2^2} + \frac{\cos 3x}{3^2} - \cdots\right]$. Sum $= \frac{\pi^2}{12}$

Q5. Expand $f(x) = \cos x$ $0 < x < \pi$ in a half range sine series.

Ans: $$f(x) = \frac{8}{\pi}\left[\frac{\sin 2x}{3} + \frac{2\sin 4x}{15} + \frac{3\sin 6x}{35} + \cdots\right]$$

***** The End *****