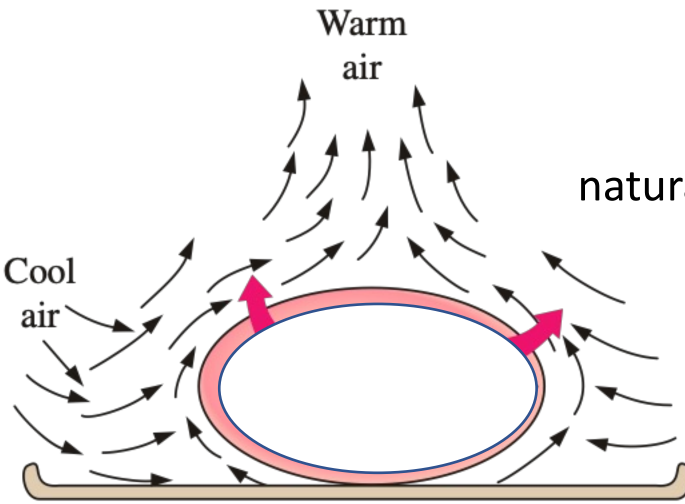
HEAT TRANSFER

[CH21204]

March 23, 2023

NATURAL CONVECTION

FREE CONVECTION



natural convection current

natural convection heat transfer

$$F_{\text{buoyancy}} = \rho_{\text{fluid}}\, g V_{\text{body}}$$

$$\begin{aligned} F_{\text{net}} &= W - F_{\text{buoyancy}} \\ &= \rho_{\text{body}}\, g V_{\text{body}} - \rho_{\text{fluid}}\, g V_{\text{body}} \\ &= (\rho_{\text{body}} - \rho_{\text{fluid}})\, g V_{\text{body}} \end{aligned}$$

- Net force is proportional to the difference in the densities of the fluid and the body immersed in it.
- In heat transfer, the primary variable is temperature, and it is desirable to express the net buoyancy force.
- Expressing the density difference in terms of a temperature difference, which requires a knowledge of a property that represents the variation of the density of a fluid with temperature at constant pressure.
- **Volume expansion coefficient**

$$\beta = \frac{1}{v}\left(\frac{\partial v}{\partial T}\right)_P = -\frac{1}{\rho}\left(\frac{\partial \rho}{\partial T}\right)_P \qquad (1/\mathrm{K})$$

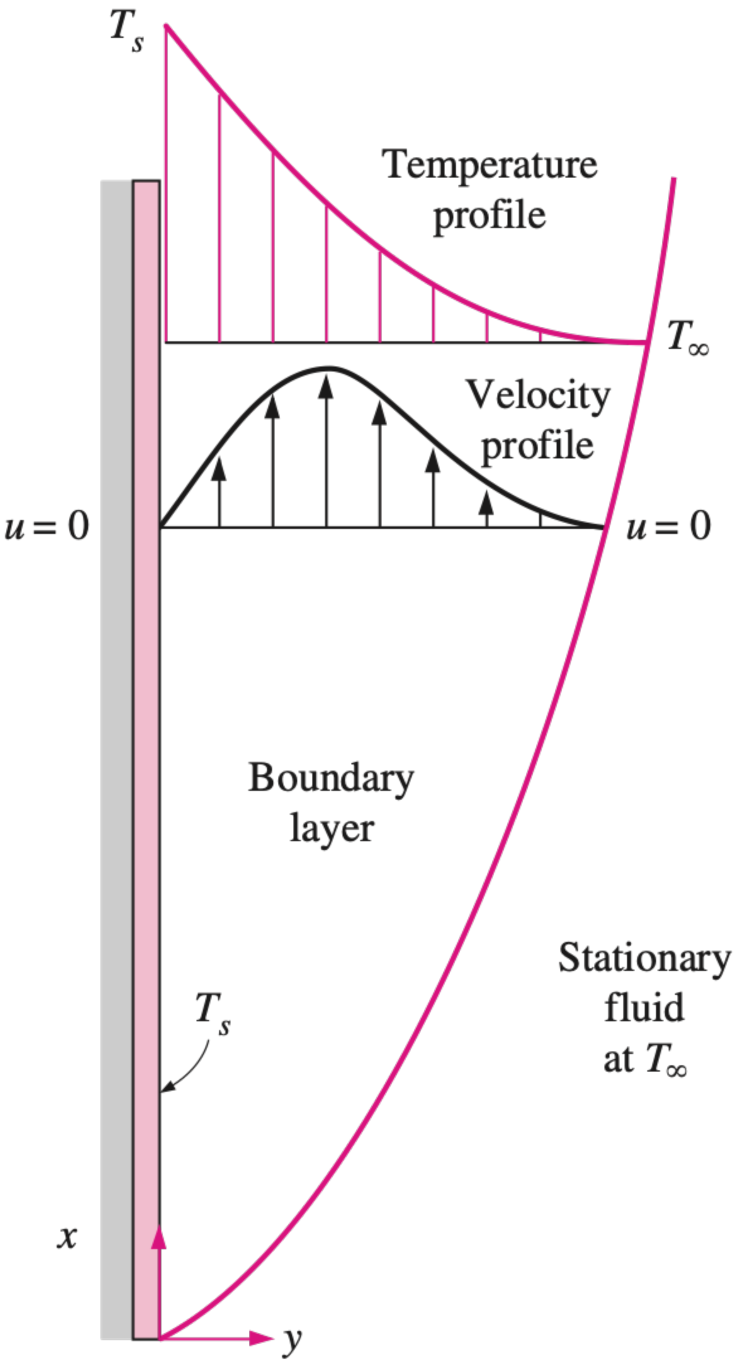
$$\beta \approx -\frac{1}{\rho}\frac{\Delta\rho}{\Delta T} = -\frac{1}{\rho}\frac{\rho_\infty - \rho}{T_\infty - T} \qquad (\text{at constant } P)$$

$$\rho_\infty - \rho = \rho\beta(T - T_\infty) \qquad (\text{at constant } P)$$

$$\beta_{\text{ideal gas}} = \frac{1}{T} \qquad (1/\text{K})$$

- The buoyancy force is proportional to the density difference, which is proportional to the temperature difference at constant pressure.
- Therefore, the larger the temperature difference between the fluid adjacent to a hot (or cold) surface and the fluid away from it, the larger the buoyancy force and the stronger the natural convection currents, and thus the higher the heat transfer rate.

- **Heat sinks with closely spaced fins are not suitable for natural convection cooling. Why?**

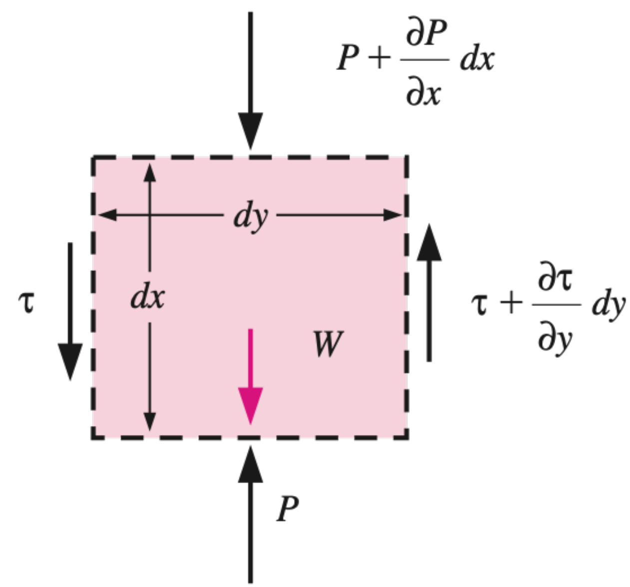


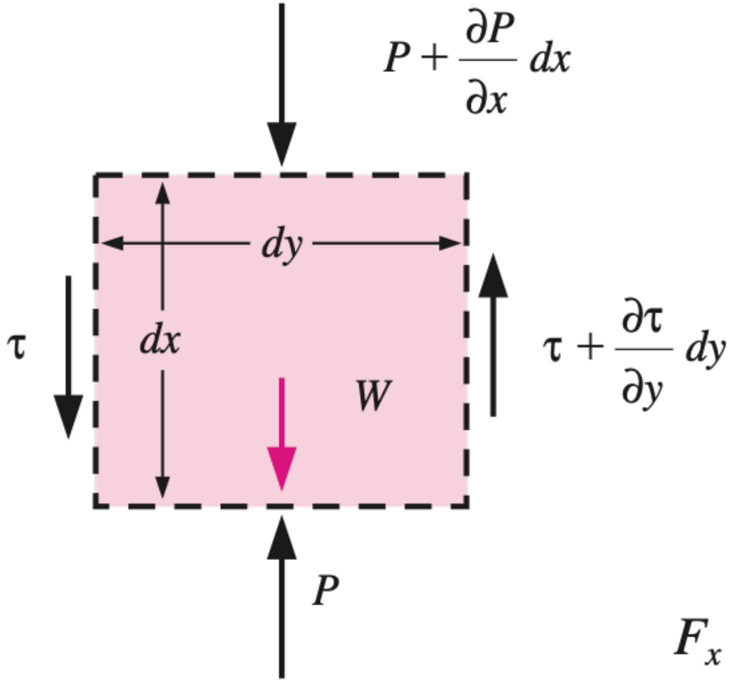
Steady, laminar, and two-dimensional, and the fluid to be Newtonian with constant properties, including density.

Boussinesq approximation

$$\rho - \rho_\infty$$

density difference between the inside and the outside of the boundary layer





$$\tau = \mu(\partial u/\partial y)$$

$$\delta m \cdot a_x = F_x$$

$$\delta m = \rho(dx \cdot dy \cdot 1)$$

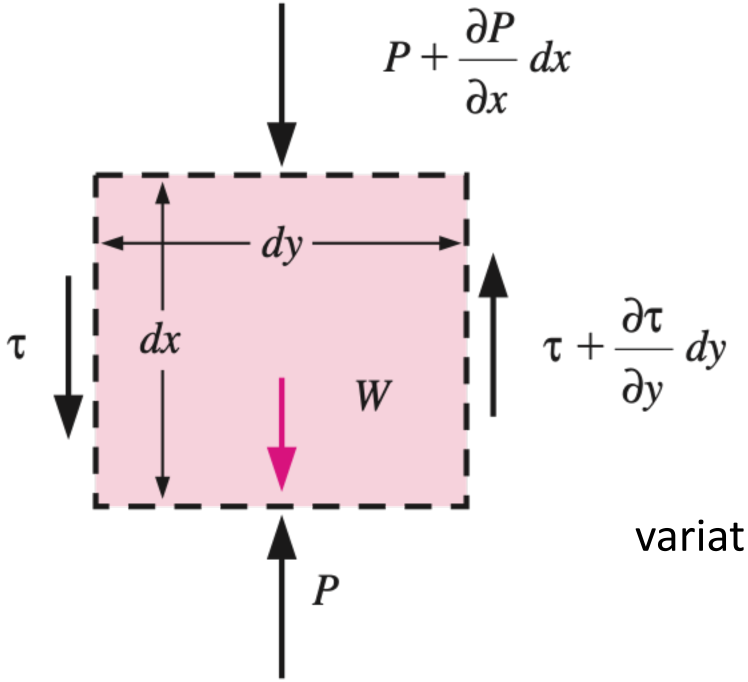
$$a_x = \frac{du}{dt} = \frac{\partial u}{\partial x}\frac{dx}{dt} + \frac{\partial u}{\partial y}\frac{dy}{dt} = u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}$$

$$\begin{aligned} F_x &= \left(\frac{\partial \tau}{\partial y}\,dy\right)(dx \cdot 1) - \left(\frac{\partial P}{\partial x}\,dx\right)(dy \cdot 1) - \rho g(dx \cdot dy \cdot 1) \\ &= \left(\mu\frac{\partial^2 u}{\partial y^2} - \frac{\partial P}{\partial x} - \rho g\right)(dx \cdot dy \cdot 1) \end{aligned}$$

$$\rho\left(u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right) = \mu\frac{\partial^2 u}{\partial y^2} - \frac{\partial P}{\partial x} - \rho g$$

$$\frac{\partial P_\infty}{\partial x} = -\rho_\infty g$$

relation for the variation of hydrostatic pressure in a quiescent fluid with height



in the boundary layer

$$v \ll u$$

$$\partial v/\partial x \approx \partial v/\partial y \approx 0.$$

$$\partial P/\partial y = 0.$$

variation of pressure in the direction normal to the surface is negligible

$$P = P(x) = P_\infty(x)$$

$$\partial P/\partial x = \partial P_\infty/\partial x = -\rho_\infty g.$$

$$\rho\left(u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right) = \mu\frac{\partial^2 u}{\partial y^2} + (\rho_\infty - \rho)g$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \nu\frac{\partial^2 u}{\partial y^2} + g\beta(T - T_\infty)$$

$$x^* = \frac{x}{L_c} \qquad y^* = \frac{y}{L_c} \qquad u^* = \frac{u}{\mathcal{V}} \qquad v^* = \frac{v}{\mathcal{V}} \qquad \text{and} \qquad T^* = \frac{T - T_\infty}{T_s - T_\infty}$$

$$u^* \frac{\partial u^*}{\partial x^*} + v^* \frac{\partial u^*}{\partial y^*} = \left[\frac{g\beta(T_s - T_\infty)L_c^3}{\nu^2}\right] \frac{T^*}{\mathrm{Re}_L^2} + \frac{1}{\mathrm{Re}_L} \frac{\partial^2 u^*}{\partial y^{*2}}$$

Grashof number Gr_L,

$$\mathrm{Gr}_L = \frac{g\beta(T_s - T_\infty)L_c^3}{\nu^2}$$

g = gravitational acceleration, m/s^2
β = coefficient of volume expansion, 1/K ($\beta = 1/T$ for ideal gases)
T_s = temperature of the surface, °C
T_∞ = temperature of the fluid sufficiently far from the surface, °C
L_c = characteristic length of the geometry, m
ν = kinematic viscosity of the fluid, m^2/s