Bernoulli Equation

$$\frac{p}{\rho} + \frac{V^2}{2} + g\,Z = Const.$$

Relates pressure changes to velocity and elevation changes along a streamline

Restriction to the use of Bernoulli's equation

i) Steady flow
ii) No friction
iii) Flow along a streamline
iv) Incompressible flow

$Case\ \ iv)\quad \Delta P, L, Q \quad known \quad D\ \ unknown$

How to evaluate the smallest pipe size

- Assume D, find Re, ε/D and f
- Calculate head loss (Eq. B & C)
- Solve Eq. A to find ΔP
- If calculated ΔP is large, choose larger D
- If calculated ΔP is small choose smaller D
- Choose commercially available pipes

$$\left(\frac{p_1}{\rho}+\alpha_1\frac{\overline{V_1^2}}{2}+g\,Z_1\right)=\left(\frac{p_2}{\rho}+\alpha_2\frac{\overline{V_2^2}}{2}+g\,Z_2\right)+h_{LT} \qquad (A)$$

$$h_L = f\,\frac{L}{D}\frac{\overline{V^2}}{2},\quad major\ head\ loss, \qquad (B)$$

$$f = {}^{64}\!/_{\mathrm{Re}} - Lamninar \quad OR\ Moody\,diagram - Turbulent$$

$$h_{LM} = K\frac{\overline{V^2}}{2} \qquad (C1) \qquad\qquad h_{LM} = f\,\frac{L_e}{D}\frac{\overline{V^2}}{2} \qquad (C2)$$

$K = Loss\,coefficient \qquad\qquad L_e\ Equiv.\ length\,of\ straight\ pipe$

Solution of Pipe Flow Problems

$$\left(\frac{p_1}{\rho}+\alpha_1\frac{\overline{V_1^2}}{2}+g\,Z_1\right)=\left(\frac{p_2}{\rho}+\alpha_2\frac{\overline{V_2^2}}{2}+g\,Z_2\right)+h_{LT}$$

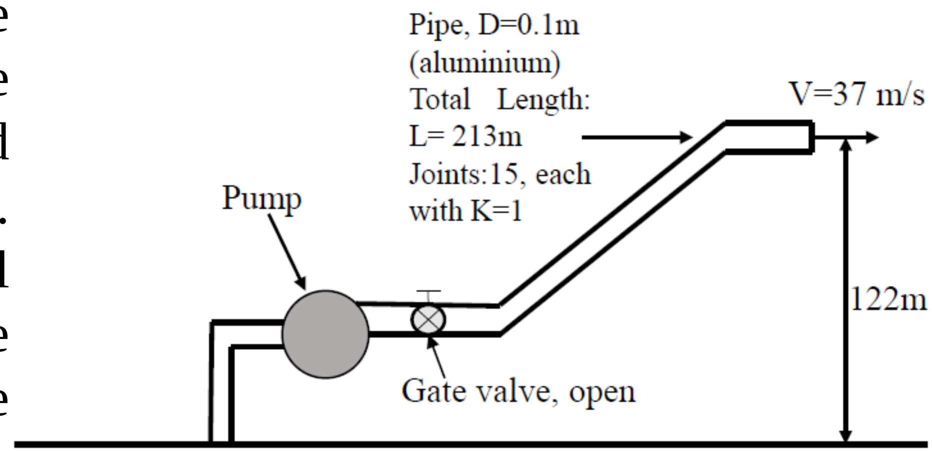
$$h_L = f\,\frac{L}{D}\frac{\overline{V^2}}{2}, \quad major\ head\ loss, \quad h_{LM} = K\frac{\overline{V^2}}{2} \qquad h_{LM} = f\,\frac{L_e}{D}\frac{\overline{V^2}}{2}$$

Head at 1 + Pump Head = Head at 2 + Losses

$$\dot{W}_{in} = \dot{m}\left[\left(\frac{p_2}{\rho}+\alpha_2\frac{\overline{V_2^2}}{2}+g\,Z_2\right)+h_{LT}-\left(\frac{p_1}{\rho}+\alpha_1\frac{\overline{V_1^2}}{2}+g\,Z_1\right)\right]$$

$$Pump\ Head = \frac{\dot{W}_{in}}{\dot{m}}\ \left(in\ \ \frac{m^2}{s^2}\right), \quad Power = \rho Q \times Pump\ Head\,,(W)$$

Cooling water is pumped from a reservoir using the pipe system as shown in the figure. The flow rate through the system is 0.378 m^3/s and the water must leave the nozzle (at the end of the pipe) with a velocity of 37 m/s. The purpose of the nozzle (a small attachment) is to increase the discharge velocity at the end of the pipe significantly from that in the pipe. Note that there are 15 joints in the system including the nozzle each with a loss coefficient K equal to 1.



Calculate i) the minimum pressure needed at the pump outlet and ii) estimate the required power input if the pump efficiency of 70 percent. The pressures at the reservoir and the outlet of the nozzle are atmospheric. The value of the roughness of the pipe is 0.0015 mm. The value of K_{ENTRY} is 0.78, L_e/D values for the gate valve is 8, L_e/D for the 90° bend is 30 and that for the 45° bend is 16. The properties of water are: density = 10^3 kg/^{3}, kinematic viscocity = 1.17 x 10 $^{-6}$ m^2/s.

$$Q = 0.378\ \frac{m^3}{s}\ , \eta_{pump} = 0.7, \vartheta_{water} = 1.7 * 10^{-6}\ \frac{m^2}{s}$$

It is to be noted that for calculating the losses (major and minor) the value of the velocity in the pipes is to be considered, not the velocity out of the nozzle

B` equation between reservoir (1) and nozzle outlet (2)

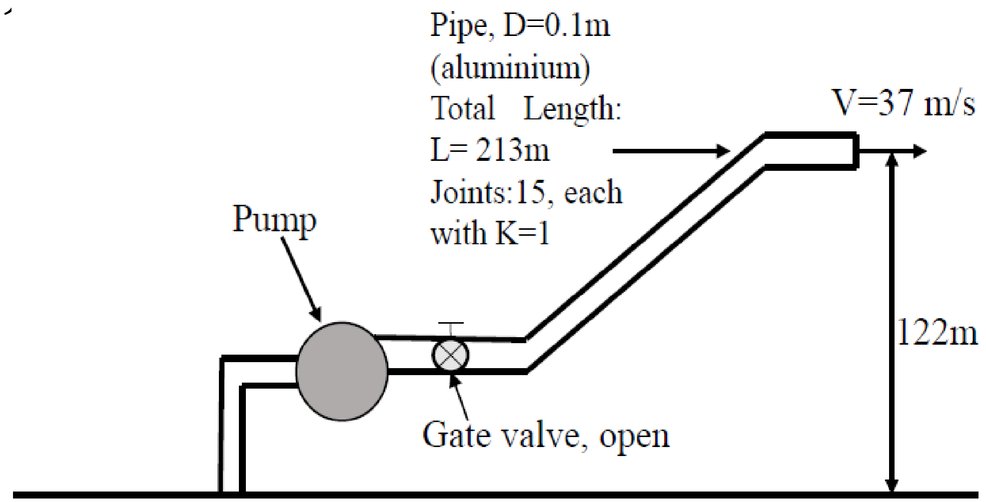
$$\left(\frac{P_1}{\rho} + \alpha\frac{v_1^2}{2} + gz_1\right) - \left(\frac{P_2}{\rho} + \alpha\frac{v_2^2}{2} + gz_2\right) + \Delta h_{pump} = h_{LT}$$

Velocity through the pipe

$$h_{LT} = h_L + h_{LM}$$

$$h_L = f\frac{L}{D}\frac{v^2}{2} \qquad h_{LM} = \frac{v^2}{2}\left(\sum K + \sum f\left(\frac{L_e}{D}\right)\right)$$

Velocity through the pipe



$$V_1 = 0\,, \alpha_1 = \alpha_2 = 1, \mathrm{P}_1 = \mathrm{P}_2 = \mathrm{P}_{\mathrm{atm}}$$

$$\Delta h_p = gz_2 + \frac{v_2^2}{2} + f\frac{L}{D}\frac{v^2}{2} + \frac{v^2}{2}\left[K_{entry} + f\left(\frac{L_e}{D}\right)_{90} + 2f\left(\frac{L_e}{D}\right)_{45^0} + 15K\right]$$

$$v = \frac{Q}{A} = \frac{Q * 4}{\pi D^2} = \frac{0.378\,\frac{m^3}{s} * 4}{\pi * 0.1^2\ m^2} = 48.1\ \frac{m}{s}$$

$$Re = \frac{Dv}{\vartheta} = 4.11 * 10^6, \epsilon = 0.0015\ mm, \frac{\epsilon}{D} = \frac{0.0015 * 10^{-3}}{0.1} = 1.5 * 10^{-5}$$

From Moody diagram, f=0.01: $K_{entry} = 0.78$, K=1, $\left.\frac{L_e}{D}\right|_{gate} = 8$, $\left.\frac{L_e}{D}\right|_{90^0} = 30$, $\left.\frac{L_e}{D}\right|_{45^0} = 16$

$$\Delta h_{pump} = 9.8\,\frac{m}{s^2} * 122\,m + 0.5\,(37)^2\,\frac{m^2}{s^2} + 0.01 * \frac{213}{0.1} * \frac{48.1^2}{2}$$

$$+\frac{48.1^2}{2}[0.78 + 0.01 * 30 + 2 * 0.01 * 16 + 15 * 1]\frac{m^2}{s^2}$$

$$\Delta h_{pump} = 1195.6 + 684.5 + 24634 + \frac{48.1^2}{2}(16.4)\frac{m^2}{s^2} = 45485\,\frac{m^2}{s^2}$$

The theoretical power input to the pump $\dot{w}_P = \dot{m}\Delta \mathrm{h_p}, \eta = \frac{\dot{w}_{Ther}}{\dot{w}_{actual}}$

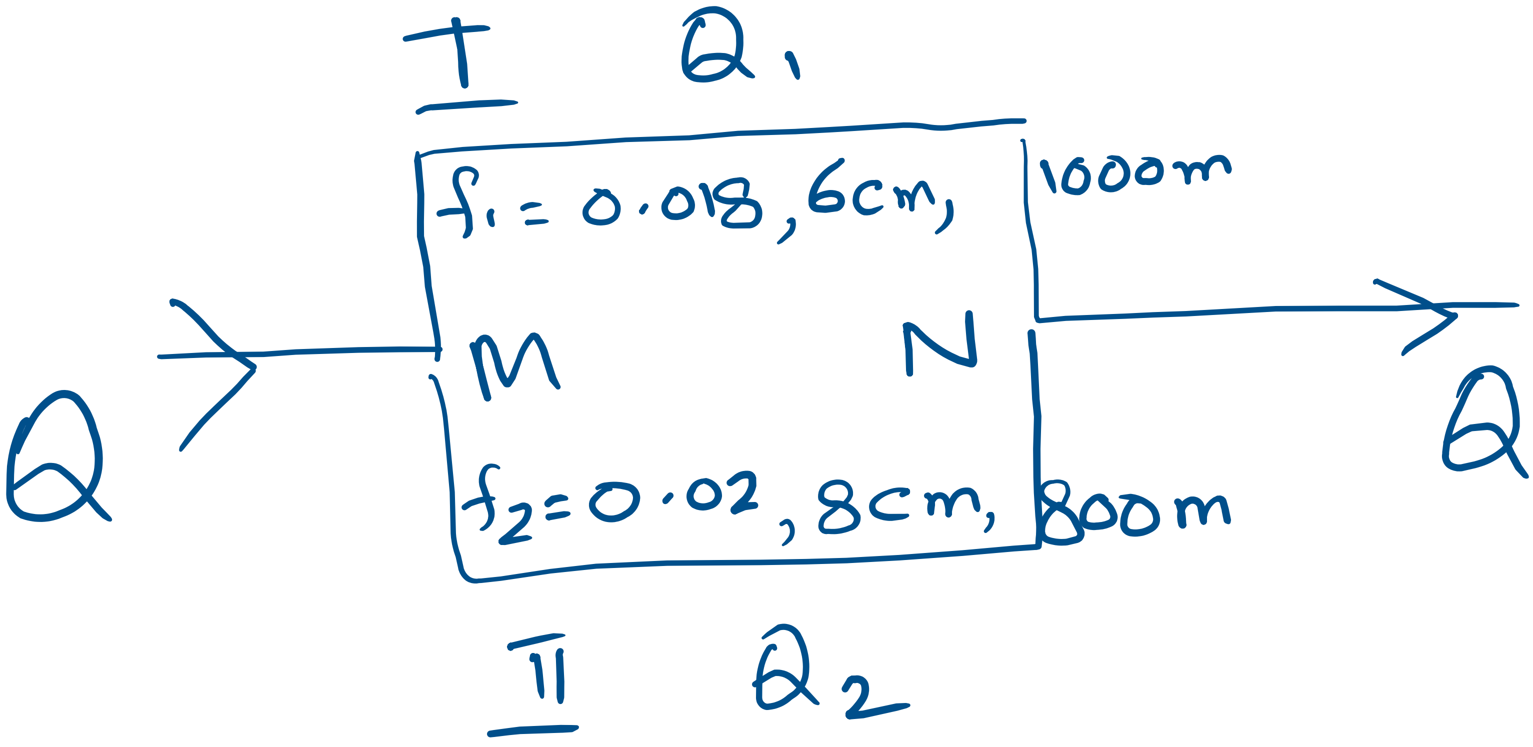
$$\dot{w}_{actual} = \frac{\dot{m}\Delta \mathrm{h_p}}{\eta} = \frac{Q\rho\Delta \mathrm{h_p}}{\eta} = \frac{0.378\ \frac{\mathrm{m}^3}{\mathrm{s}} * 10^3\ \frac{kg}{m^3} * 45485\,\frac{\mathrm{m}^2}{s^2}}{0.7}$$

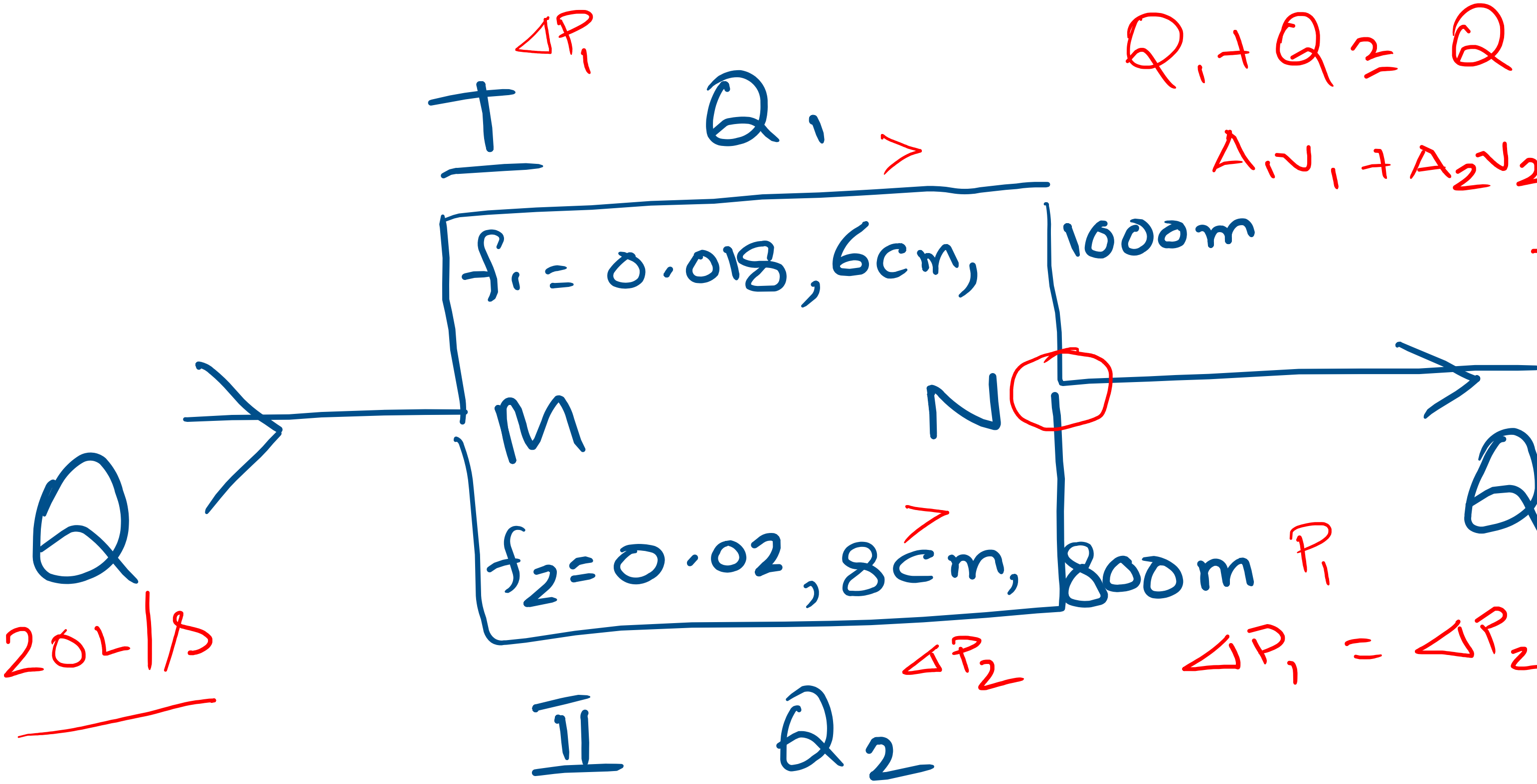
$$w_{actual} = 2.45 * 10^7\ \mathrm{W}$$

The discharge Pressure from the pump is obtained using B` equation between 1 and 3 (just at the exit of the pump, neglecting losses in the inlet section, any elevation changes and kinetic energy at 3) as

$$\mathrm{P_3 - P_1 = \rho\Delta h_{pump}} = 10^3\ \frac{\mathrm{kg}}{\mathrm{m}^3} *\ 45485\,\frac{\mathrm{m}^2}{\mathrm{s}^2} = 4.5 * 10^7\ \mathrm{Pa}$$

A pipe of 6 cm in diameter, 1000 m long and with f = 0.018 is connected in parallel between two points M and N with another pipe 8 cm in diameter, 800m long and having f = 0.02. A total flow of 20 L/s enters the parallel pipes through the division at M to rejoin at N. Estimate the division of flow in the two pipes.





$Q_1 + Q_2 = Q$

$A_1V_1 + A_2V_2 = Q$ — (1)

$\Delta P_1 = \Delta P_2$

P_N SAME

$\Delta P_1 = \Delta P_2$

MAJOR LOSSES BET^N M & N

$$\Rightarrow \frac{f_1 L_1 V_1^2}{2D_1} = \frac{f_2 L_2 V_2^2}{2D_2} \quad -2$$

$Q_1 = 0.0063\ m^3/s$

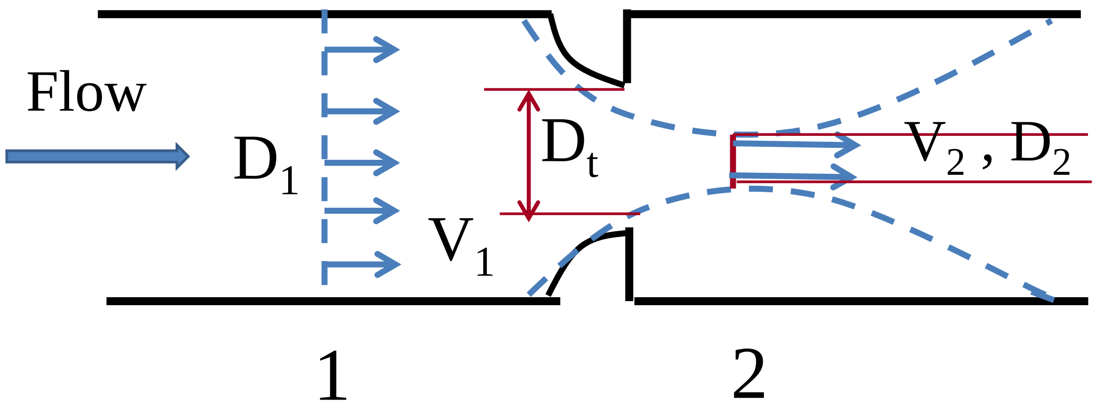
$Q_2 = 0.00137\ m^3/s$

Flow Measurement

Direct Measurement

Restriction Flow meters for Internal Flow

Change in velocity leads to a change in pressure



Internal flow through a generalized nozzle

D_2 = vena contracta

Theoretical flow rates can be obtained using continuity and Bernoulli equations between sections 1 and 2. However, actual flow rates are obtained using empirical factors.

$$\frac{p_1}{\rho} + \frac{V_1^2}{2} + g\,Z_1 = \frac{p_2}{\rho} + \frac{V_2^2}{2} + g\,Z_2$$

Assumptions:
(1) Steady flow.
(2) Incompressible flow.
(3) Flow along a streamline.
(4) No friction.
(5) Uniform velocity at 1 and 2
(6) Pressure is uniform at 1 and 2.
(7) $z_1 = z_2$

$$p_1 - p_2 = \frac{\rho}{2}\left(V_2^2 - V_1^2\right) = \frac{\rho V_2^2}{2}\left[1 - \left(\frac{A_2}{A_1}\right)^2\right]$$

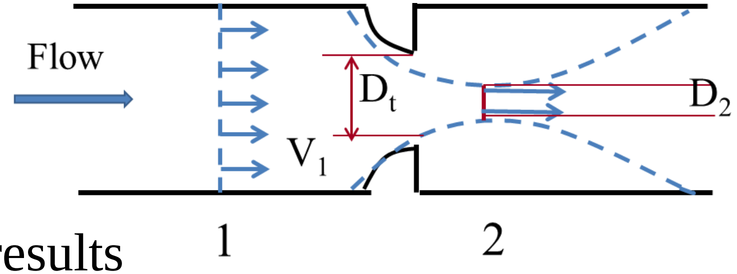
$$V_2 = \sqrt{\frac{2\left(p_1 - p_2\right)}{\rho\left[1 - \left(\frac{A_2}{A_1}\right)^2\right]}} \qquad \dot{m}_{Theoretical} = \rho V_2 A_2$$

$$\dot{m}_{theoretical} \propto \sqrt{\Delta P}$$

$$\dot{m}_{Theoretical} = \frac{A_2}{\sqrt{1 - \left(\frac{A_2}{A_1}\right)^2}}\sqrt{2\rho\left(p_1 - p_2\right)}$$

Limitations

- Actual flow area at 2 is unknown
- Velocities are uniform only at very high Re
- Frictional effects could be important
- Locations of pressure taps can influence the results



Empirical Discharge Coefficients

$$C \equiv \frac{Actual\ mass\ \ flow\ rate}{Theoretical\ mass\ \ flow\ rate}$$

$$With\ \beta = \left({D_t}/{D_1} \right) \qquad \left({A_t}/{A_1} \right)^2 = \left({D_t}/{D_1} \right)^4 = \beta^4$$

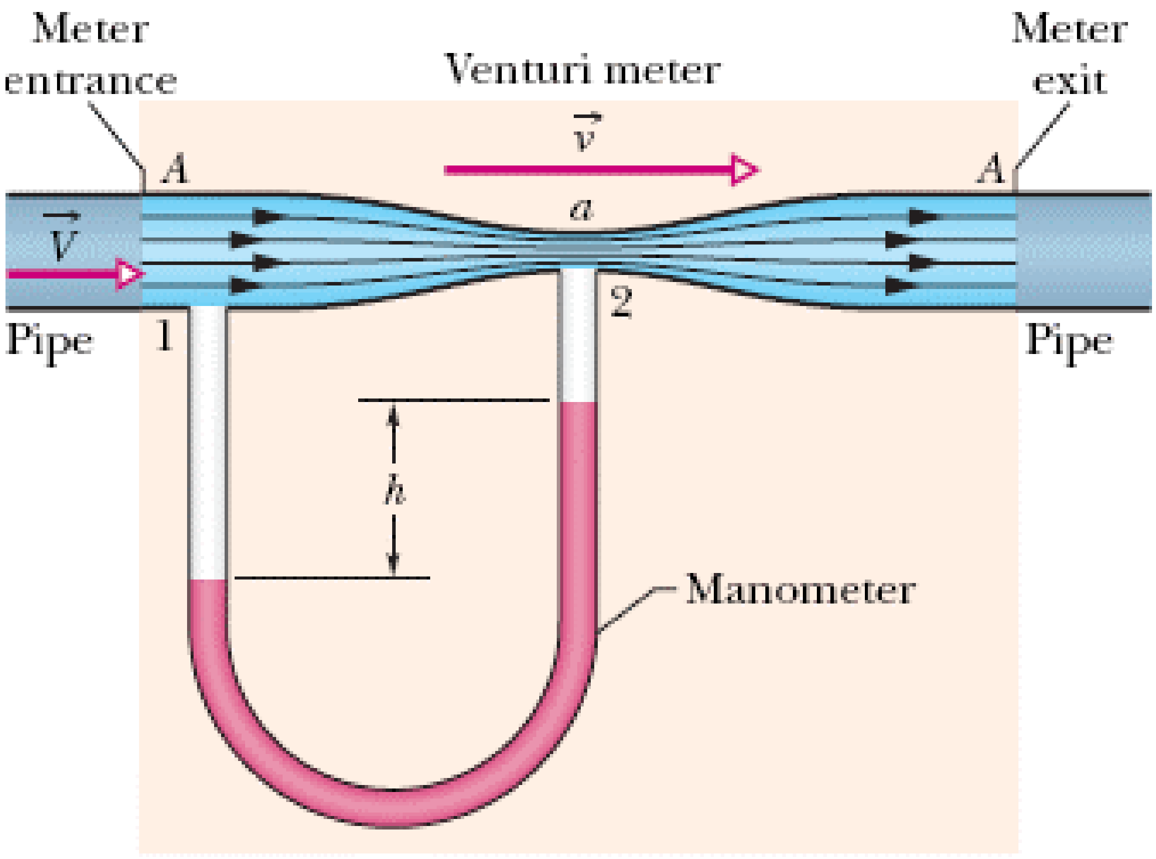
$$\dot{m}_{Actual} = \frac{C\, A_t}{\sqrt{1-\beta^4}} \sqrt{2\,\rho\left(p_1 - p_2\right)} \qquad \frac{1}{\sqrt{1-\beta^4}}\ is\ the\ velocity\ of\ approach\ factor$$

$$\dot{m}_{Actual} = K\, A_t \sqrt{2\,\rho\left(p_1 - p_2\right)} \qquad K\ is\ termed\ as\ the\ flow\ coefficient$$

Empirical relations for C and K are available as functions of meter bore, pipe diameter, Re

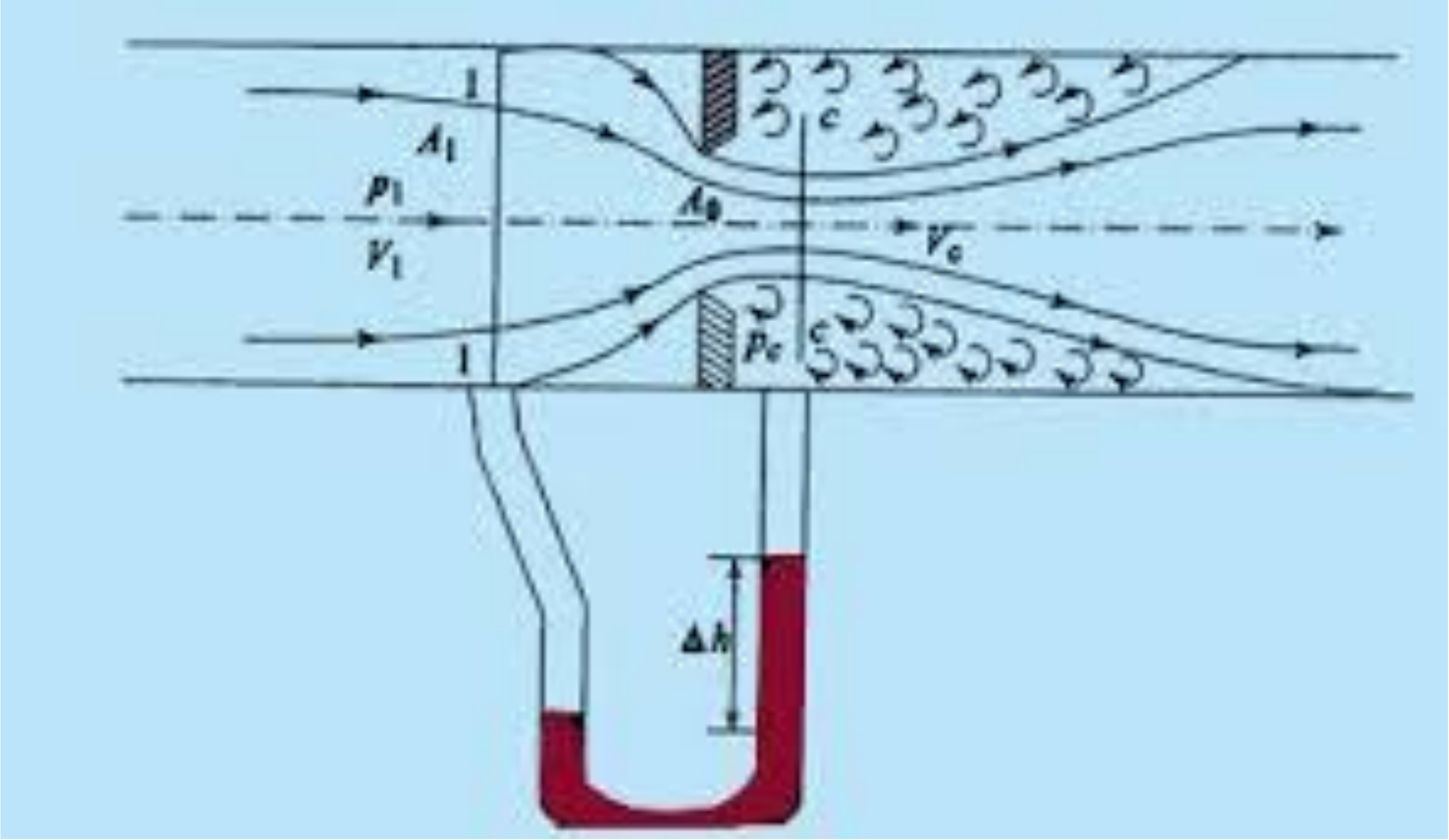
High values of C (closer to 1) are desirable, denoting less head loss

Venturimeter



C = 0.98 – 0.995, Low head loss, high cost, excellent recovery of pressure

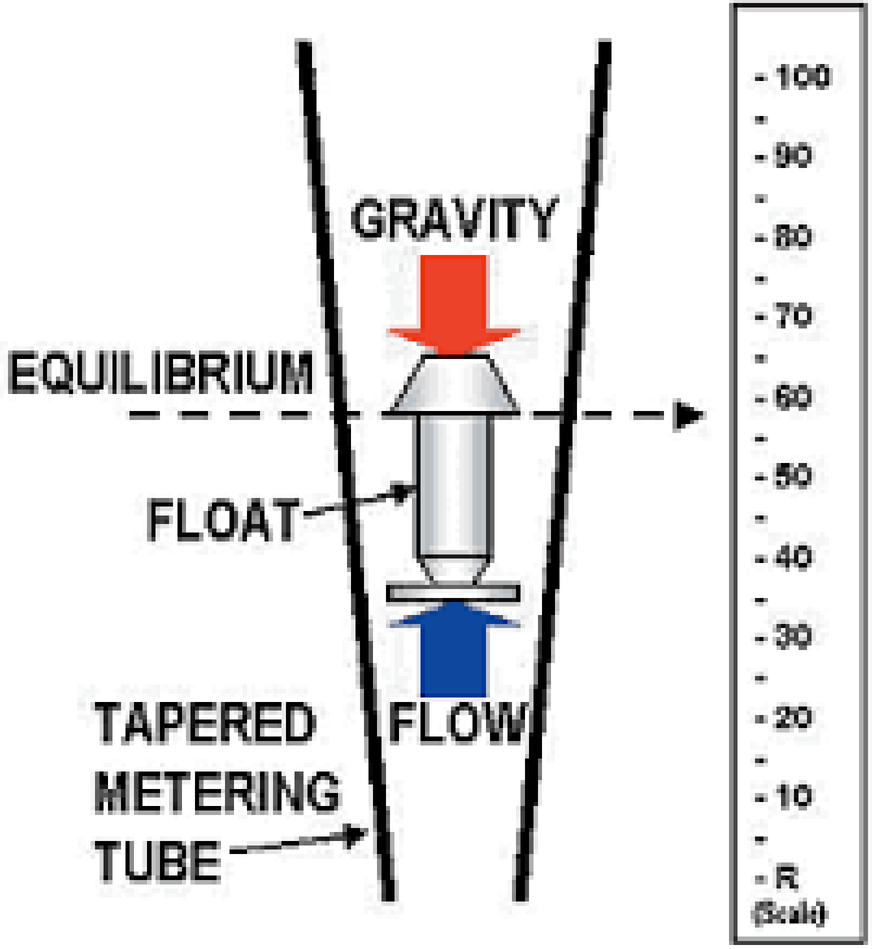
Orificemeter



High head loss, low cost, suspended matters may start to build up

Linear Flow meter (Output directly proportional to flow rate

Rotameter



Variable area flow meter

Summation of

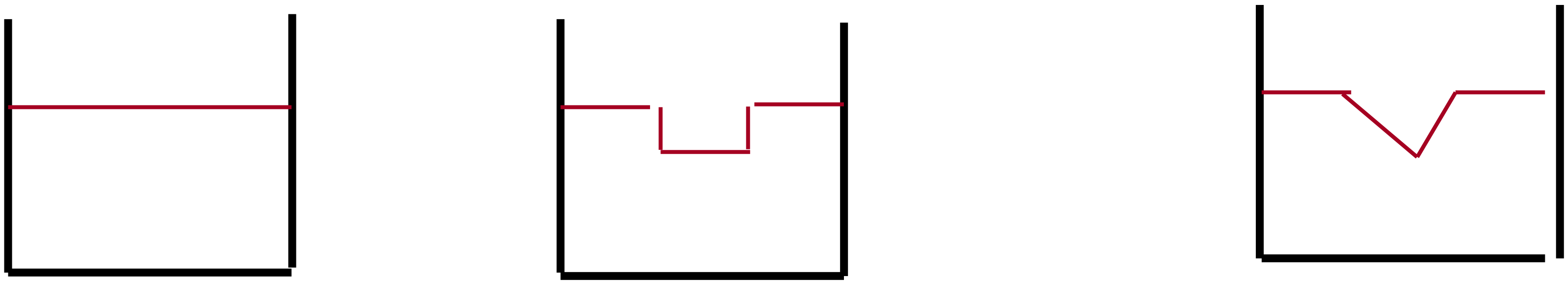
drag force ↑

weight ↓

buoyancy ↑ = 0

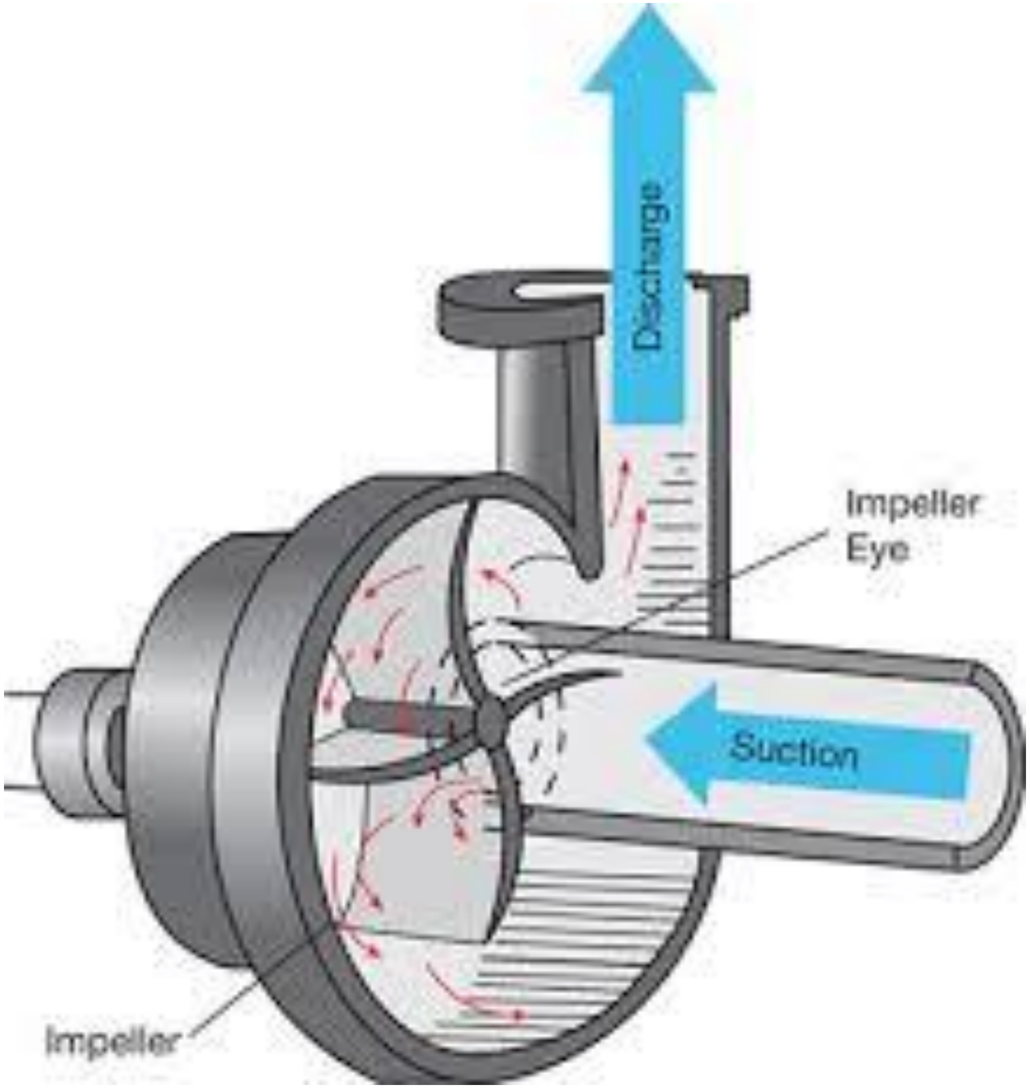
Calibrated for common fluids and flow rate regimes

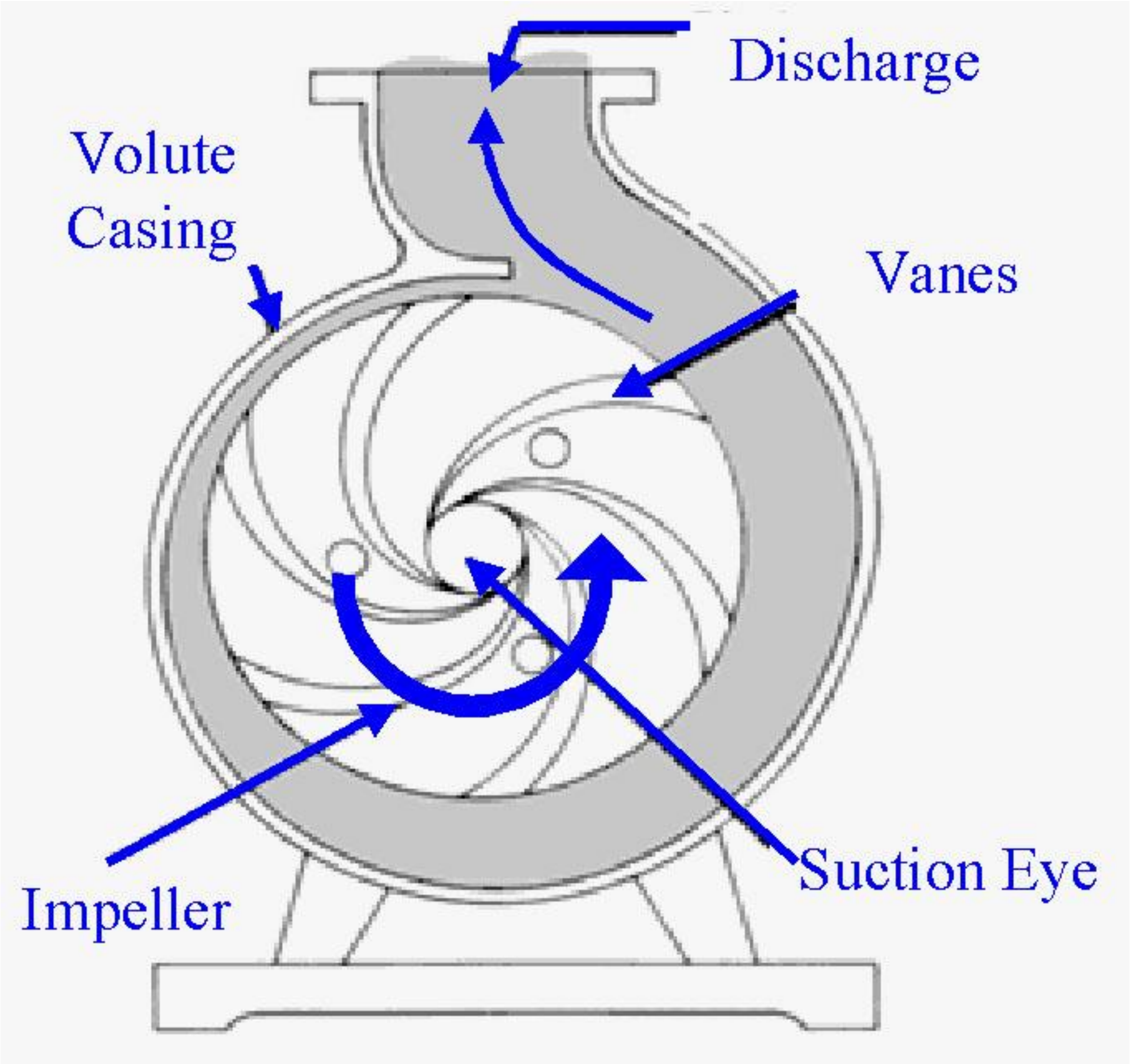
Measurement of Open Channel Flows – by Weirs



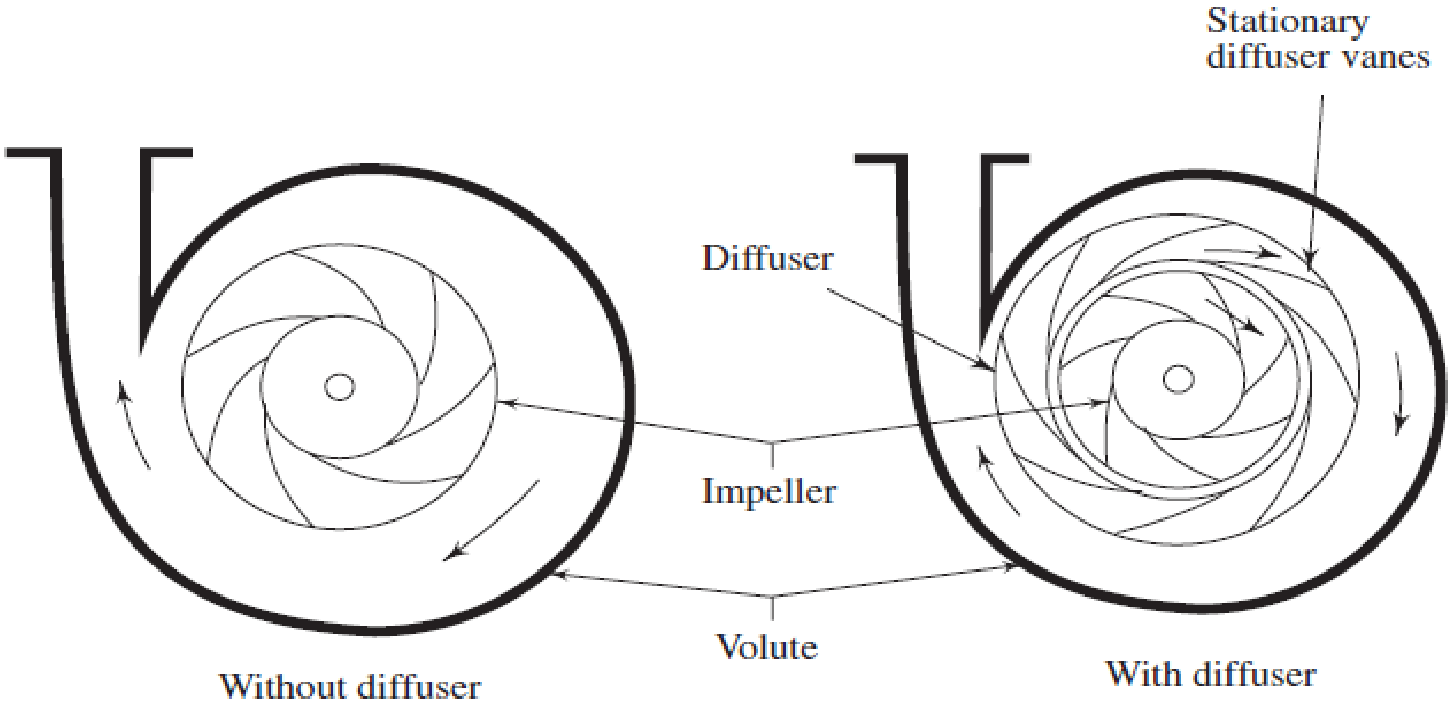
Empirical discharge coefficients and correlations
for flow over the weir are available

Centrifugal Pump




Discharge
Volute
Casing
Vanes
Impeller
Suction Eye

Centrifugal pumps and Characteristic Curves



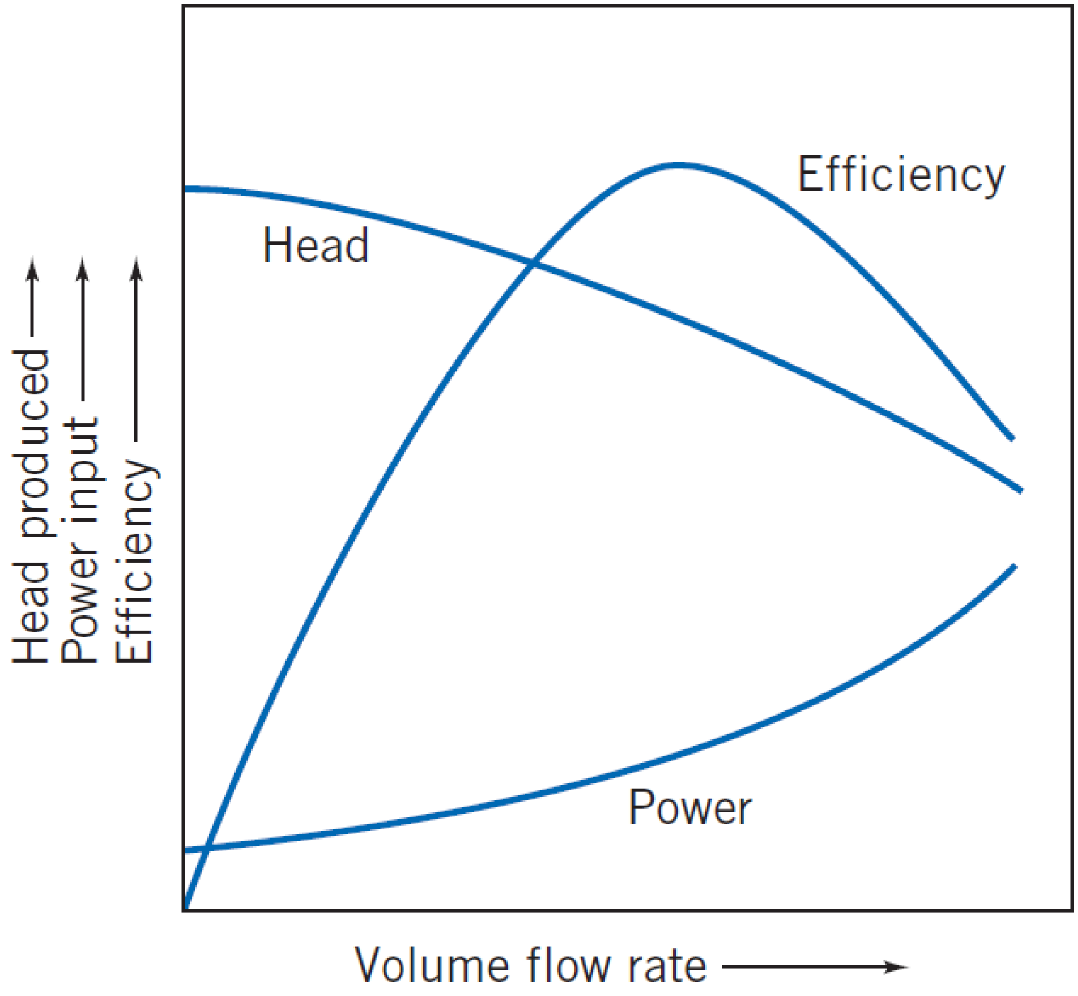
The detailed flow pattern within a pump changes with volume flow rate and speed; these changes affect the pump's performance.

Performance parameters of interest include the pressure rise (or head developed), the power input required, and the machine efficiency measured under specific operating conditions.

The independent variables are volume flow rate, angular speed, impeller diameter, and fluid properties. Dependent variables are the several performance quantities of interest.

Efficiency is defined as the ratio of power delivered to the fluid divided by input power, $\eta = P/P_{in}$. For incompressible flow, the energy equation reduces to $P = \rho Q h$ (when "head" h is expressed as energy per unit mass) or to $P = \rho g Q H$ (when head H is expressed as energy per unit weight).

Typical characteristics curves for centrifugal pumps tested at constant speed (experimental)



Cavitation and Net Positive Suction Head

Cavitation can occur in any machine handling liquid whenever the local static pressure falls below the vapor pressure of the liquid. When this occurs, the liquid can locally flash to vapor, forming a vapor cavity.

The flow may become unsteady. The entire flow may oscillate and the machine starts to vibrate. As cavitation commences, it reduces the performance of a pump or turbine rapidly.

Thus cavitation must be avoided to maintain stable and efficient operation and to reduce erosion damage or surface pitting.

Cavitation can be avoided if the pressure everywhere in the machine is kept above the vapor pressure of the operating liquid.

Net positive suction head (NPSH) is defined as the difference between the absolute stagnation pressure in the flow at the pump suction and the liquid vapor pressure, expressed as head of flowing liquid.

Hence the NPSH is a measure of the difference between the maximum possible pressure in the given flow and the pressure at which the liquid will start flashing over to a vapor; the larger the NPSH, the less likely cavitation is to occur.

The net positive suction head required (NPSHR) by a specific pump to suppress cavitation varies with the liquid pumped, and with the liquid temperature and pump condition.

Reciprocating Pump