Assignment 7

Advanced Mathematical Techniques in Chemical Engineering (CH 61015)

Full Marks: 30

Note: You can keep the final solution in the form of definite integrals

1. Temperature distribution in a homogeneous solid sphere in non-dimensional form is given as

$$\frac{\partial T}{\partial t} = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial T}{\partial r}\right)$$

At $t = 0$, $T = k(1-r)$, where, k is a constant. At r=1 for all t, T=0. Obtain the temperature distribution.

2. Find the steady state temperature distribution in a semi-circular plate of radius a insulated on both faces. The governing equation is

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 T}{\partial \theta^2} = 0$$

At r=a, T=T_0 for any θ. At $\theta = 0$ *and* π, T=0. This means the temperature is maintained zero on boundary diameter.

3. Solve $\frac{\partial u}{\partial t} = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u}{\partial r}\right)$ at $t = 0, u = r$; at $r = 1, u = 0$

4. Solve the above problem subject to at $t = 0, u = u_0$; at $r = 1, \frac{\partial u}{\partial r} + 2u = 0$

5. Solve $\frac{\partial u}{\partial t} = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 u}{\partial \theta^2}$. At $t = 0, u = 1$; at $r = 1, u = 0$

6. Solve $\frac{\partial u}{\partial t} = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial u}{\partial r}\right)$. At $r = 1$, $\frac{\partial u}{\partial r} + 3u = 0$.