Problem 1

a)

$$R'(2) \approx \frac{R(3)-R(1)}{3-1} = \frac{950-1190}{2} = \frac{-240}{2} = -120\, liters/hr^2$$

b)

$$\int_0^8 R(t)\,dt \approx 1*1340 + 2*1190 + 3*950 + 2*740 = 8050\, liters$$

The rate removal function is decreasing on the given interval; therefore the left-hand Riemann sum is an overestimate.

c)

$$V(8)-V(0) = \int_0^8 W(t)-R(t)\,dt = \int_0^8 W(t)\,dt - \int_0^8 R(t)\,dt$$

$$V(8) = V(0) + \int_0^8 W(t)\,dt - \int_0^8 R(t)\,dt \approx 50{,}000 + \int_0^8 W(t)\,dt - 8050 =$$

$$V(8) \approx 50{,}000 - 7836.195324552195 - 8050 \approx 49786 \text{ liters}$$

d) A graph of $W(t)$ is shown below. Since both rate functions are continuous, then their difference is continuous as well.

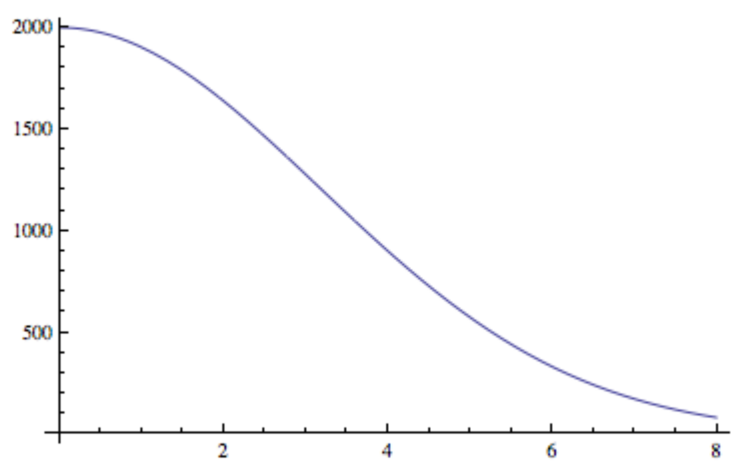
$W(0) - R(0) < 0$

$W(1) - R(1) > 0$

$\rightarrow W(t) - R(t) = 0$ at least once, by IVT.

$\rightarrow W(t) = R(t)$

Therefore, the two rates must be equal at least once for some t value on the interval [0, 8].



Problem 2

a)

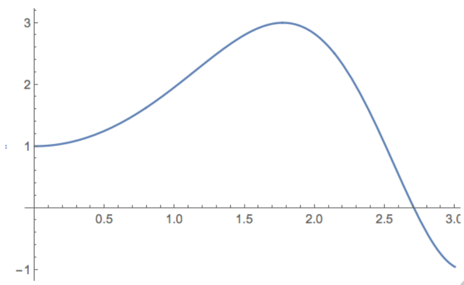
$$v(4) = 1 + 2\sin\left(\frac{4^2}{2}\right) = 1 + 2\sin 8 = 2.97872 > 0$$

$$a(4) = 2t\cos\left(\frac{t^2}{2}\right)_{t=4} = 8\cos(8) = -1.164 < 0$$

Velocity and acceleration have opposite signs, therefore the particle is slowing down

at this time.

b)
The graph of velocity changes from sign once in the given interval, at $t \approx 2.707$.



c)

$$x(4) - x(0) = \int_0^4 v(t)\,dt \to x(0) = x(4) - \int_0^4 v(t)\,dt \approx -3.815$$

d)

$$\int_0^3 |v(t)|\,dt = 5.301$$

Problem 3

a)
$g'(x) = f(x)$
At $x = 10$, the derivative of $g(x)$ does not change sign (it stays negative), therefore $g(10)$ is neither a relative maximum nor a relative minimum.

b)
At $x = 4$, the first derivative of $g(x)$ does not change sign (it stays positive), and its second derivative ($g''(x) = f'(x)$) changes from positive to negative; therefore $g(x)$ has an inflection point at $x = 4$.

c)
Critical Numbers:
$g'(x) = f(x) = 0$ or undefined $\rightarrow x = -2, 2, 6, 10$
$g(-2) = -8$
$g(2) = 0$
$g(6) = 8$
$g(10) = 0$

Endpoints:
$g(-4) = -4$
$g(12) = -4$

By the Closed Interval Method, the absolute maximum and minimum are -8 and 8, respectively. They occur at points $(-2, -8)$ and $(6, 8)$.

d)
$g(x) \le 0 \rightarrow 10 \le x \le 12$ *and* $-4 \le x \le 2$
Net areas are zero or negative for these x – values, by FTC.

Problem 4

$$\frac{dy}{dx} = \frac{y^2}{x-1}$$

a)

$$\left.\frac{dy}{dx}\right|_{(0,0)} = 0 \,;\, \left.\frac{dy}{dx}\right|_{(0,1)} = -1 \,;\, \left.\frac{dy}{dx}\right|_{(0,2)} = -2 \,;\, \left.\frac{dy}{dx}\right|_{(2,0)} = 0 \,;\, \left.\frac{dy}{dx}\right|_{(2,1)} = 1 \,;\, \left.\frac{dy}{dx}\right|_{(2,2)} = 4$$

b)

$$\left.\frac{dy}{dx}\right|_{(2,3)} = slope = \frac{3^2}{2-1} = 9 \rightarrow y-3 = 9(x-2)$$

$$y = 9x - 15$$

$$f(2.1) \approx 9 * 2.1 - 15 = 3.9$$

c)

$$\frac{dy}{dx} = \frac{y^2}{x-1}$$

$$\frac{1}{y^2}dy = \frac{1}{x-1}dx \rightarrow \int \frac{1}{y^2}dy = \int \frac{1}{x-1}dx$$

$$\frac{-1}{y} = \ln|x-1| + C$$

$$\frac{-1}{3} = \ln 1 + C \rightarrow C = \frac{-1}{3}$$

$$\frac{-1}{y} = \ln|x-1| - \frac{1}{3}$$

$$y = \frac{1}{\frac{1}{3} - \ln|x-1|} = \frac{3}{1 - 3\ln|x-1|}$$

Problem 5

a)

$$r_{avg[0,10]} = \frac{1}{10-0}\int_0^{10} \frac{1}{20}(3+h^2)\,dh = \frac{1}{200}\int_0^{10}(3+h^2)\,dh = \frac{1}{200}\left(\left(3h+\frac{h^3}{3}\right)_{h=10} - \left(3h+\frac{h^3}{3}\right)_{h=0}\right) = \frac{109}{60}$$

b)

$$V = \pi\int_0^{10}\left(\frac{1}{20}(3+h^2)\right)^2 dh = \frac{\pi}{400}\int_0^{10}(3+h^2)^2\,dh = \frac{\pi}{400}\left(\left(9h+2h^3+\frac{h^5}{5}\right)_{h=10} - \left(9h+2h^3+\frac{h^5}{5}\right)_{h=0}\right) =$$

$$= \frac{\pi}{400}\left(90+2000+\frac{100000}{5}\right) = \frac{\pi}{40}\left(9+200+2000\right) = \frac{2209\pi}{40}$$

c)

$$r = \frac{1}{20}(3+h^2)$$

$$\left.\frac{dr}{dt}\right|_{h=3 inches} = \frac{-1}{5}$$

$$\frac{dr}{dt} = \frac{1}{20}2h\frac{dh}{dt} \rightarrow \frac{-1}{5} = \frac{h}{10}\frac{dh}{dt} \rightarrow \frac{dh}{dt} = \frac{-2}{h} = \frac{-2}{3}\text{ inches / second.}$$

Problem 6

a)

$$k'(x) = f'(g(x))\,g'(x)$$

$$slope = k'(3) = f'(g(3))\,g'(3) = f'(6)g'(3) = 5*2 = 10$$

$$k(3) = f(g(3)) = f(6) = 4$$

$$Tangent: y - 4 = 10(x-3) \rightarrow y = 10x - 26$$

b)

$$h'(x) = \frac{f(x)g'(x) - f'(x)g(x)}{(f(x))^2}$$

$$h'(1) = \frac{f(1)g'(1) - f'(1)g(1)}{[f(1)]^2} = \frac{(-6)*8 - 3*2}{(-6)^2} = \frac{-54}{36} = \frac{-3}{2}$$

c)

$$\int_1^3 f''(2x)\,dx = \frac{1}{2}\int_1^3 f''(2x)2\,dx = \frac{1}{2}\int_2^6 f''(u)\,du = \frac{1}{2}\big(f'(6) - f'(2)\big) = \frac{5-(-2)}{2} = \frac{7}{2}$$

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