

## Firm Theory with Differential Calculus



Function	Description	
$x$	Quantity	Quantity supplied or demanded
$p(x)$	Price Function	Price at which $x$ units are demanded
$R(x) = xp(x)$	Revenue	The product of price and quantity
$F$	Fixed Cost	Cost that does not vary with output
$V(x)$	Variable Cost	Cost that varies with the output level
$C(x) = F + V(x)$	Total Cost	Direct sum of fixed and variable costs
$A(x) = \frac{C(x)}{x}$	Average Cost	Total Cost divided by quantity
$R'(x) = MR(x)$	Marginal Revenue	The rate of revenue with respect to output
$C'(x) = MC(x)$	Marginal Cost	The rate of total cost with respect to output
$\Pi(x) = R(x) - C(x)$	Profit Function	The difference between revenue and cost.

Interpretations:

$R'(100)$  = the revenue brought in by the 101<sup>st</sup> unit.

$R'(100)$  = the cost of producing the 101<sup>st</sup> unit

$F$  = the  $y$ -intercept of the total cost function

Two types of firm behavior are discussed here: average-cost-minimization and profit-maximization.

I. Minimizing Average Cost

Objective Function:  $A(x) = \frac{C(x)}{x}$

$$\Rightarrow A'(x) = \frac{x C'(x) - C(x)}{x^2} \rightarrow x = 0 \vee x C'(x) - C(x) = 0$$

$$\Rightarrow x C'(x) - C(x) = 0$$

$$\Rightarrow C'(x) = \frac{C(x)}{x}$$

$$\Rightarrow C'(x) = A(x)$$

$$\Rightarrow MC = AC$$

Remark: One must justify a minimum average cost using one of the derivative tests.

II. Maximizing Profit

Objective Function:  $\Pi(x) = R(x) - C(x)$

$$\Rightarrow \Pi'(x) = R'(x) - C'(x) = 0$$

$$\Rightarrow R'(x) = C'(x)$$

$$\Rightarrow MR = MC$$

Remark: One must justify a maximum profit using one of the derivative tests.