

Name\_\_\_\_\_

Label and present your answers neatly on separate paper.

1. a) Draw the line  $y = 2t + 1$  and use geometry to find the area under this line, above the  $t$ -axis, and between the vertical lines  $t = 1$  and  $t = 3$ .  
b) If  $x > 1$ , let  $A(x)$  be the area of the region that lies under the line  $y = 2t + 1$  between  $t = 1$  and  $t = x$ . Sketch this region and use geometry to find an expression for  $A(x)$ .  
c) Differentiate the area function  $A(x)$ . What do you notice?

2. a) If  $x \geq -1$ , let

$$A(x) = \int_{-1}^x (1+t^2) dt$$

$A(x)$  represents the area of a region. Sketch that region.

- b) Use the result below to find an expression for  $A(x)$ .

$$\int_a^b x^2 dx = \frac{b^3}{3} - \frac{a^3}{3}$$

- c) Find  $A'(x)$ . What do you notice?

- d) If  $x \geq -1$  and  $h$  is a small positive number, then  $A(x+h) - A(x)$  represents the area of a region. Describe and sketch the region.

- e) Draw a rectangle that approximates the region in part d). By comparing the areas of these two regions, show that:

$$\frac{A(x+h) - A(x)}{h} \approx 1 + x^2$$

- f) Use part e) to give an intuitive explanation for the result of part c).

3. a) Draw the graph of the function  $f(x) = \cos(x^2)$  in the viewing rectangle  $[0, 2]$  by  $[-1.25, 1.25]$ .

b) If we define a new function  $g$  by

$$g(x) = \int_0^x \cos(t^2) dt$$

then  $g(x)$  is the area under the graph of  $f$  from 0 to  $x$  [until  $f(x)$  becomes negative, at which point  $g(x)$  becomes a difference of areas]. Use part a) to determine the value of  $x$  at which  $g(x)$  starts to decrease.

- c) Use the definite integral command on your calculator to estimate  $g(0.2), g(0.4), g(0.6), \dots, g(1.8), g(2)$ . Then use these values to sketch a graph of  $g$ .
- d) Use your graph of  $g$  from part c) to sketch the graph of  $g'$  using the interpretation of  $g'(x)$  as the slope of the tangent line. How does the graph of  $g'$  compare with the graph of  $f$ ?

4. Suppose  $f$  is a continuous function on the interval  $[a, b]$  and we define a new function

$$g(x) = \int_a^x f(t) dt$$

Based on your results from problems 1 – 3, conjecture an expression for  $g'(x)$ .