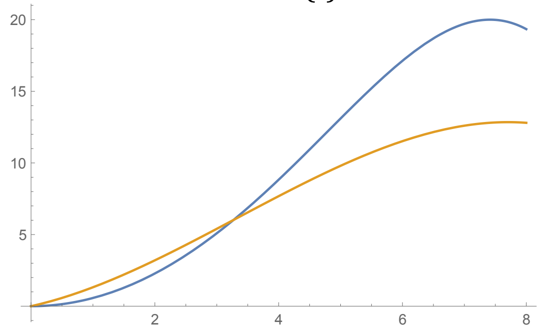
Problem 1

The curve in blue is R(t) and the curve in yellow/orange is D(t).



a)

$$\int_0^8 R(t)\,dt = \int_0^8 20\sin\left(\frac{t^2}{35}\right)dt = 76.570 \quad ft^3$$

b)

$R(3)-D(3)=-0.314<0$. Therefore, the amount of water in the pipe is decreasing at t = 3 hours. Note from the graph that removal rate curve is above the inflow rate curve when t equals 3 hours.

c)

We need to find the critical number, where the two rate curves intersect. Using a graphing calculator, we get: t = 3.27166.

$$t=0 \,\|\, V(0)=30$$

$$t=8 \,\|\, V(8)=30+\int_0^8 R(t)-D(t)\,dt = 30+18.544=48.544$$

$$t^*=3.27166 \,\|\, V(t^*)=30+\int_0^{t^*} R(t)-D(t)\,dt = 30-2.03544=27.965$$

By the Closed Interval Method, the minimum volume occurs is 27.965 cubic feet. The corresponding time value is 3.272 hours.

d)

$$30+\int_0^w R(t)-D(t)\,dt = 50$$

OR

$$\int_0^w R(t)-D(t)\,dt = 20$$

Problem 2

a)

$Area = \int_0^2 |f(x) - g(x)| dx = 2.00435 \approx 2.004$

Or we can compute the areas separately (this takes a bit longer).

Intersection Point:

$f(x) = g(x) \to x = 1.03283 = m$

$Area = \int_0^m g(x) - f(x)\, dx + \int_m^2 f(x) - g(x)\, dx =$

$= \int_0^m \left(x^4 - 6.5x^2 + 6x + 2\right) - \left(1 + x + e^{x^2-2x}\right) dx + \int_m^2 \left(1 + x + e^{x^2-2x}\right) - \left(x^4 - 6.5x^2 + 6x + 2\right) dx \approx 2.004$

b)

Intersection Point:

$f(x) = g(x) \to x = 1.03283 = m$

$A(x) = \left(f(x) - g(x)\right)^2$

$Volume = \int_m^2 \left(\left(1 + x + e^{x^2-2x}\right) - \left(x^4 - 6.5x^2 + 6x + 2\right)\right)^2 dx = 1.28316 \approx 1.283$

c)

$h(x) = f(x) - g(x) = \left(1 + x + e^{x^2-2x}\right) - \left(x^4 - 6.5x^2 + 6x + 2\right)$

$h'(1.8) \approx -3.8112$ (Graphing Calculator)

Problem 3

a)

$$v'(16) \approx \frac{v(20)-v(12)}{20-12} = \frac{240-200}{8} = 5m/\min^2$$

b)

$\int_0^{40} |v(t)|dt$ is the total distance traveled during the time interval [0, 40].

$$\int_0^{40} |v(t)|dt \approx R_4 = (12-0)*200 + (20-12)*240 + (24-20)*220 + (40-24)*150$$
$$R_4 = 12*200 + 8*240 + 4*220 + 16*150 = 2400 + 1920 + 880 + 2400 = 7600\, meters.$$

c)

$$v_b(t) = t^3 - 6t^2 + 300$$
$$a(t) = 3t^2 - 12t$$
$$a(5) = 75 - 60 = 15m/\min^2$$

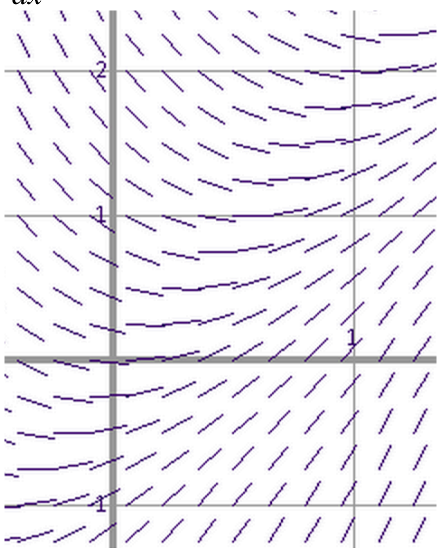
d)

$$v_{avg[0,10]} = \frac{1}{10-0}\int_0^{10} v(t)\,dt = \frac{1}{10}\int_0^{10} t^3 - 6t^2 + 300\,dt = \frac{1}{10}\left(\frac{t^4}{4} - 2t^3 + 300t\right)_0^{10} = 350\,m/\min$$

Problem 4

a)

$$\frac{dy}{dx} = 2x - y$$



b)

$$\frac{d^2y}{dx^2} = 2 - \frac{dy}{dx} = 2 - (2x - y) = 2 - 2x + y$$

In quadrant II, x-values are negative and y-values are positive. Therefore, the second derivative will be always positive. The solution curves will hence be concave upward.

c)

f(2) = 3

@ x=2, the first derivative is: 2*2 – 3 = 1. If a curve has a local extremum at a point, then the derivative must be zero or undefined there. In our case, it is neither zero nor undefined. Therefore, the given point is neither a local maximum nor a local minimum.

d)

$$\frac{dy}{dx} = 2x - y$$

$$y = mx + b \rightarrow \frac{dy}{dx} = m = 2x - y$$

$Also:$

$$\frac{dy}{dx} = 2x - (mx + b) = (2 - m)x - b = 0x + m$$

$$2 - m = 0 \rightarrow m = 2$$

$$m = -b \rightarrow b = -2$$

Problem 5

a)
f has a relative maximum whenever f' changes sign from positive to negative. This occurs, according to the graph, at $x = -2$.

b) f is decreasing whenever f' is negative. f is concave down whenever f'' is negative. In other words, f is concave down whenever the slope of f' is negative or whenever f' is decreasing. These two conditions are met on the interval $(-2,-1)$ and $(1,3)$.

c) f has a point inflection whenever change f''s sign, f' stays the same, and f itself is continuous. There are three inflection points, at $x = -1,1,3$.

d)
$f(x) = f(1) + \int_1^x f'(t)\,dt = 3 + \int_1^x f'(t)\,dt$

$\int_1^4 f'(x)\,dx = f(4) - f(1) \rightarrow f(4) = f(1) + \int_1^4 f'(x)\,dx = 3 - 12 = -9$

$\int_{-2}^1 f'(x)\,dx = f(1) - f(-2) \rightarrow f(-2) = f(1) - \int_{-2}^1 f'(x)\,dx = 3 - (-9) = 12$

Problem 6

a)

$$slope = \frac{dy}{dx}_{(-1,1)} = \frac{y}{3y^2 - x}_{(-1,1)} = \frac{1}{4}$$

$$L(x) = f(-1) + f'(-1)(x+1) = 1 + \frac{1}{4}(x+1)$$

b)

$$\frac{dy}{dx} = undefined$$

$$3y^2 - x = 0 \rightarrow x = 3y^2$$

$$y^3 - (3y^2)y = 2 \rightarrow -2y^3 = 2 \rightarrow y = -1 \rightarrow x = 3$$

$$(3,-1)$$

c)

$$\frac{dy}{dx}_{(-1,1)} = \frac{1}{4}$$

$$\frac{d^2y}{dx^2} = \frac{\left(3y^2 - x\right)\frac{dy}{dx} - y\left(6y\frac{dy}{dx} - 1\right)}{\left(3y^2 - x\right)^2}_{(-1,1)} = \frac{4 * \frac{1}{4} - 1 * \left(\frac{6}{4} - 1\right)}{16} = \frac{1 - \frac{1}{2}}{16} = \frac{1}{32}$$