UH.EDU | 2006 | BC EXAM 1
SOLUTIONS TO F.R. # 3,4,6
SHUBLEKA

(3)

$$\text{I}: \begin{cases} x = t+1 \\ y = t^2-1 \end{cases} \qquad \text{II}: \begin{cases} x = 6t+3 \\ y = -2t \end{cases}$$

a):

$$\text{I}: \quad V(4) = \langle x'(t), y'(t) \rangle \Big|_{t=4} \qquad \text{II}: V(4) = \langle 6, -2 \rangle$$

$$= \langle 1, 2t \rangle \Big|_{t=4}$$

$$= \langle 1, 8 \rangle$$

b)

$$\text{Distance} = \int_{t=1}^{t=2} \sqrt{(x'(t))^2 + (y'(t))^2}\, dt$$

$$= \int_1^2 \sqrt{36+4}\, dt = \int_1^2 \sqrt{40}\, dt.$$

c) Collision

$$\begin{cases} x_1(t) = x_2(t) \\ y_1(t) = y_2(t) \end{cases} \iff \begin{cases} t+1 = 6t+3 \\ t^2-1 = -2t \end{cases}$$

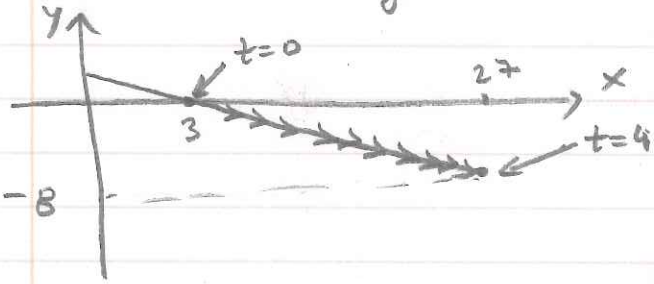
$$\iff \begin{cases} 5t = -2 \\ t^2+2t-1 = 0 \end{cases} \iff \begin{cases} t = -\frac{2}{5} \\ \frac{4}{25} - \frac{4}{5} - 1 \overset{?}{=} 0 \end{cases}$$

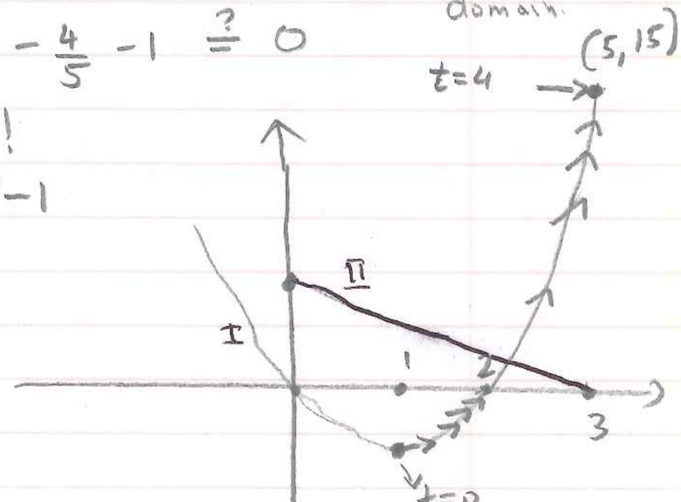
$\longleftarrow$ not in the domain.

No solution, hence no collision!

d) I: $t = x-1 \longrightarrow y = (x-1)^2-1$

II: $x = -3y+3$





④ $f(x) = x \cdot e^x$

a) $f(x) = x \cdot e^x = x\left[1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots + \frac{x^n}{n!} + \ldots\right]$

$= x + x^2 + \frac{x^3}{2!} + \frac{x^4}{3!} + \ldots + \frac{x^{n+1}}{n!} + \ldots = \sum_{n=0}^{\infty} \frac{x^{n+1}}{n!}$

b) $f(x) = x + \frac{x^2}{1!} + \frac{x^3}{2!} + \frac{x^4}{3!} + \ldots + \frac{x^{n+1}}{n!} + \ldots$

$f'(x) = 1 + 2x + 3\frac{x^2}{2!} + 4\frac{x^3}{3!} + \ldots + (n+1)\frac{x^n}{n!} + \ldots$

$= 1 + [x + x] + \left[\frac{2x^2}{2!} + \frac{x^2}{2!}\right] + \left[\frac{3x^3}{3!} + \frac{x^3}{3!}\right] + \ldots + \left[n\frac{x^n}{n!} + \frac{x^n}{n!}\right] + \ldots$

$= \left[1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots + \frac{x^n}{n!} + \ldots\right] + \left[x + \frac{2x^2}{2!} + \frac{3x^3}{3!} + \ldots\right.$

$\left. + \frac{n \cdot x^n}{n!} + \ldots\right] = e^x + x\left[1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots + \frac{x^{n-1}}{(n-1)!} + \ldots\right]$

$= e^x + x \cdot e^x$, as wanted.

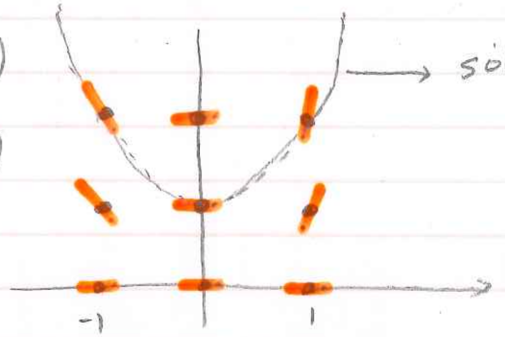
c) $\int_0^1 x \cdot e^x \, dx = \int_0^1 x + x^2 + \frac{x^3}{2!} + \frac{x^4}{3!} + \ldots + \frac{x^{n+1}}{n!} + \ldots \, dx$

$= \left(\frac{x^2}{2} + \frac{x^3}{3} + \frac{x^4}{4 \cdot 2!} + \frac{x^5}{5 \cdot 3!} + \ldots + \frac{x^{n+2}}{(n+2)\, n!} + \ldots\right)\Big|_0^1$

(6) $\frac{dy}{dx} = 2xy$

a)

b)



solution through (0, 1)

c) Euler's Method: $\frac{dy}{dx} = 2xy \quad y(0) = 2 \quad \Delta x = 0.2$

x	y	dy/dx
0	2	0
0.2	2	0.8
0.4	2.16	1.728
0.6	$\boxed{2.5056}$	

Computations

$y = 2$

$y - 2 = 0.8(x - 0.2) \rightarrow y(0.4)$

$y - 2.16 = 1.728(x - 0.4)$

$\hookrightarrow y = 2.16 + 1.728 \cdot 0.2$

$y(0.6) =$

$f(0.6) \approx 2.5056.$

d) $\frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d}{dx}(2xy) = 2\left(1 \cdot y + x \cdot \frac{dy}{dx}\right)$

$= 2y + 2x \cdot [2xy] = 2y\,[1 + 2x^2]$