**Problem 1 BC**

a)

$$R'(2) \approx \frac{R(3)-R(1)}{3-1} = \frac{950-1190}{2} = \frac{-240}{2} = -120\,liters\,/\,hr^2$$

b)

$$\int_0^8 R(t)\,dt \approx 1*1340 + 2*1190 + 3*950 + 2*740 = 8050\,liters$$

The rate removal function is decreasing on the given interval; therefore the left-hand Riemann sum is an overestimate.

c)

$$V(8)-V(0) = \int_0^8 W(t)-R(t)\,dt = \int_0^8 W(t)\,dt - \int_0^8 R(t)\,dt$$

$$V(8) = V(0) + \int_0^8 W(t)\,dt - \int_0^8 R(t)\,dt \approx 50{,}000 + \int_0^8 W(t)\,dt - 8050 =$$

$$V(8) \approx 50{,}000 - 7836.195324552195 - 8050 \approx 49786 \text{ liters}$$

d) A graph of $W(t)$ is shown below. Since both rate functions are continuous, then their difference is continuous as well.

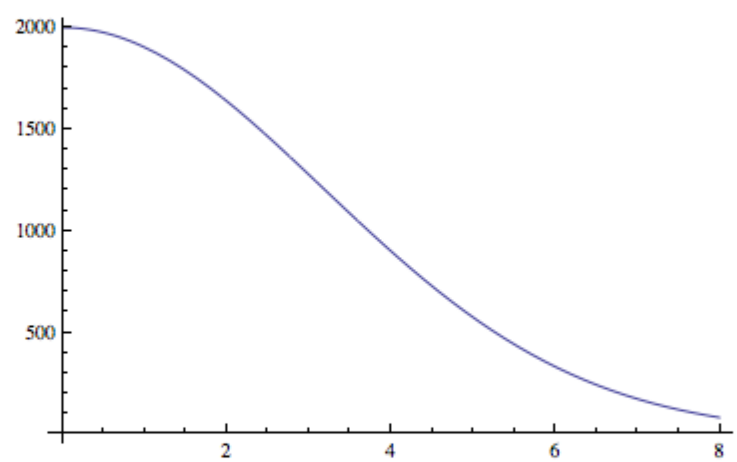
$W(0)-R(0)<0$

$W(1)-R(1)>0$

$\rightarrow W(t)-R(t)=0$ at least once, by IVT.

$\rightarrow W(t)=R(t)$

Therefore, the two rates must be equal at least once for some $t$ value on the interval [0, 8].



**Problem 2 BC**

a)

$$x(3) = x(0) + \int_0^3 x'(t)\,dt = 5 + \int_0^3 t^2 + \sin(3t^2)\,dt \approx 14.377$$

$$y(3) = \frac{-1}{2} = -0.500$$

$$\left(x(3), y(3)\right) \approx \left(14.377, -0.500\right)$$

b)

$$\left.\frac{dy}{dx}\right|_{t=3} = \left.\frac{\frac{dy}{dt}}{\frac{dx}{dt}}\right|_{t=3} = \frac{y'(3)}{x'(3)} = \frac{0.5}{9+\sin(27)} \approx 0.0502191 = 0.050$$

c)

$$speed_{t=3} = \sqrt{\left(x'(3)\right)^2 + \left(y'(3)\right)^2} \approx 9.96892 \approx 9.969$$

d)

$$\text{Distance}_{[0,\,2]} = \int_0^2 \sqrt{\left(x'(t)\right)^2 + \left(y'(t)\right)^2}\,dt = \int_0^1 \sqrt{\left(x'(t)\right)^2 + \left(y'(t)\right)^2}\,dt + \int_1^2 \sqrt{\left(x'(t)\right)^2 + \left(y'(t)\right)^2}\,dt$$

$$= \int_0^1 \sqrt{\left(x'(t)\right)^2 + \left(-2\right)^2}\,dt + \int_1^2 \sqrt{\left(x'(t)\right)^2 + \left(0\right)^2}\,dt = 4.34987 \approx 4.350$$

**Problem 3 BC**

a)
$g'(x) = f(x)$
At $x = 10$, the derivative of $g(x)$ does not change sign (it stays negative), therefore $g(10)$ is neither a relative maximum nor a relative minimum.

b)
At $x = 4$, the first derivative of $g(x)$ does not change sign (it stays positive), and its second derivative ( $g''(x) = f'(x)$ ) changes from positive to negative; therefore $g(x)$ has an inflection point at $x = 4$.

c)
Critical Numbers:
$g'(x) = f(x) = 0$ or undefined $\rightarrow x = -2, 2, 6, 10$

$g(-2) = -8$

$g(2) = 0$

$g(6) = 8$

$g(10) = 0$

Endpoints:
$g(-4) = -4$

$g(12) = -4$

By the Closed Interval Method, the absolute maximum and minimum are $-8$ and $8$, respectively. They occur at points $(-2, -8)$ and $(6, 8)$.

d)
$g(x) \leq 0 \rightarrow 10 \leq x \leq 12$ *and* $-4 \leq x \leq 2$
Net areas are zero or negative for these $x$ – values, by FTC.

**Problem 4 BC**

$$\frac{dy}{dx} = x^2 - \frac{1}{2}y$$

a)

$$\frac{dy}{dx} = x^2 - \frac{1}{2}y$$

$$\frac{d^2y}{dx^2} = 2x - \frac{1}{2}\frac{dy}{dx} = 2x - \frac{1}{2}\left(x^2 - \frac{1}{2}y\right) = 2x - \frac{x^2}{2} + \frac{y}{4}$$

b)

$$\left.\frac{dy}{dx}\right|_{x=-2,y=8} = 4 - 4 = 0$$

$$\left.\frac{d^2y}{dx^2}\right|_{x=-2,y=8} = -4 - 2 + 2 = -4 < 0$$

By the Second Derivative Test for Local Extrema, the solution function will have a local maximum at $(-2, 8)$.

c)

We use L'Hospital's Rule twice below.

$$g(-1) = 2$$

$$\lim_{x\to-1}\left(\frac{g(x)-2}{3(x+1)^2}\right) = \lim_{x\to-1}\left(\frac{g'(x)}{6(x+1)}\right) = \lim_{x\to-1}\left(\frac{x^2 - \frac{1}{2}y}{6(x+1)}\right) = \lim_{x\to-1}\left(\frac{2x - \frac{1}{2}\frac{dy}{dx}}{6}\right) =$$

$$\lim_{x\to-1}\left(\frac{2x - \frac{x^2}{2} + \frac{y}{4}}{6}\right) = \frac{-2 - \frac{1}{2} + \frac{1}{2}}{6} = \frac{-1}{3}$$

d)

$$h(0) = 2$$

$$\Delta x = 0.5$$

$$y - 2 = \left.\frac{dy}{dx}\right|_{x=0}(x - 0) \to y = 2 + (-1) * 0.5 = 1.5$$

$$y - 1.5 = \left.\frac{dy}{dx}\right|_{x=0.5}(x - 0.5) \to y = 1.5 + (-0.5) * 0.5 = 1.25$$

$$h(1) \approx 1.250$$

**Problem 5 BC**

a)

$$r_{avg[0,10]} = \frac{1}{10-0}\int_0^{10}\frac{1}{20}(3+h^2)\,dh = \frac{1}{200}\int_0^{10}(3+h^2)\,dh = \frac{1}{200}\left(\left(3h+\frac{h^3}{3}\right)_{h=10} - \left(3h+\frac{h^3}{3}\right)_{h=0}\right) = \frac{109}{60}$$

b)

$$V = \pi\int_0^{10}\left(\frac{1}{20}(3+h^2)\right)^2 dh = \frac{\pi}{400}\int_0^{10}(3+h^2)^2\,dh = \frac{\pi}{400}\left(\left(9h+2h^3+\frac{h^5}{5}\right)_{h=10} - \left(9h+2h^3+\frac{h^5}{5}\right)_{h=0}\right) =$$

$$= \frac{\pi}{400}\left(90+2000+\frac{100000}{5}\right) = \frac{\pi}{40}\left(9+200+2000\right) = \frac{2209\pi}{40}$$

c)

$$r = \frac{1}{20}(3+h^2)$$

$$\left.\frac{dr}{dt}\right|_{h=3\,inches} = \frac{-1}{5}$$

$$\frac{dr}{dt} = \frac{1}{20}2h\frac{dh}{dt} \rightarrow \frac{-1}{5} = \frac{h}{10}\frac{dh}{dt} \rightarrow \frac{dh}{dt} = \frac{-2}{h} = \frac{-2}{3}\text{ inches / second.}$$

**Problem 6 BC**

a)

$$f(x)=f(1)+f'(1)(x-1)+\frac{f''(1)}{2!}(x-1)^2+\frac{f'''(1)}{3!}(x-1)^3+\ldots+\frac{f^{(n)}(1)}{n!}(x-1)^n+\ldots$$

$$f(x)=1-\frac{1}{2}(x-1)+\frac{1/4}{2}(x-1)^2-\frac{2/8}{6}(x-1)^3+\ldots+(-1)^n\frac{(n-1)!/2^n}{n!}(x-1)^n+\ldots$$

$$f(x)=1-\frac{1}{2}(x-1)+\frac{1}{8}(x-1)^2-\frac{1}{24}(x-1)^3+\ldots+(-1)^n\frac{1}{n\,2^n}(x-1)^n+\ldots$$

b)

The center is $x=1$; with a radius of 2, we need to test the endpoints at $x=-1$ and $x=3$.

$$f(x)=\sum\frac{(-1)^n}{n2^n}(x-1)^n$$

$$x=-1\rightarrow\sum\frac{(-1)^n}{n2^n}(-1-1)^n=\sum\frac{1}{n2^n}(2)^n=\sum\frac{1}{n}\ \text{(Harmonic Series; Diverges)}$$

$$x=3\rightarrow\sum\frac{(-1)^n}{n2^n}(3-1)^n=\sum\frac{(-1)^n}{n}\quad\text{(Alternating Harmonic Series; Converges)}$$

The interval of convergence is $(-1,3]$.

c)

$$f(x)=1-\frac{1}{2}(x-1)+\frac{1}{8}(x-1)^2-\frac{1}{24}(x-1)^3+\ldots+(-1)^n\frac{1}{n\,2^n}(x-1)^n+\ldots$$

$$f(1.2)\approx1-\frac{1}{2}(1.2-1)+\frac{1}{8}(1.2-1)^2=0.905$$

d)

For Alternating Series, the error bound is the absolute value of the next term:

$$\left|-\frac{1}{24}(1.2-1)^3\right|=\frac{1}{24}\frac{1}{5^3}=\frac{1}{3000}<0.001$$

**Last edited: May 11th, 2016 4:24pm Eastern Time**