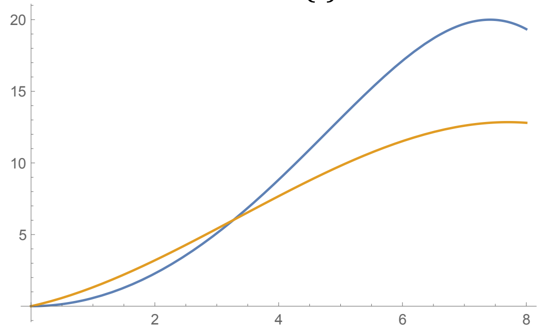
Problem 1

The curve in blue is R(t) and the curve in yellow/orange is D(t).



a)

$$\int_0^8 R(t)\,dt = \int_0^8 20\sin\left(\frac{t^2}{35}\right)dt = 76.570 \quad ft^3$$

b)

$R(3) - D(3) = -0.314 < 0$. Therefore, the amount of water in the pipe is decreasing at t = 3 hours. Note from the graph that removal rate curve is above the inflow rate curve when t equals 3 hours.

c)

We need to find the critical number, where the two rate curves intersect. Using a graphing calculator, we get: t = 3.27166.

$$t = 0 \parallel V(0) = 30$$

$$t = 8 \parallel V(8) = 30 + \int_0^8 R(t) - D(t)\,dt = 30 + 18.544 = 48.544$$

$$t^* = 3.27166 \parallel V(t^*) = 30 + \int_0^{t^*} R(t) - D(t)\,dt = 30 - 2.03544 = 27.965$$

By the Closed Interval Method, the minimum volume occurs is 27.965 cubic feet. The corresponding time value is 3.272 hours.

d)

$$30 + \int_0^w R(t) - D(t)\,dt = 50$$

OR

$$\int_0^w R(t) - D(t)\,dt = 20$$

Problem 2

a)

$$x(2)-x(1)=\int_1^2 x'(t)\,dt$$

$$\rightarrow x(2)=x(1)+\int_1^2 \cos(t^2)\,dt=2.55694\approx 2.557$$

b)

$$\frac{dy/dt}{dx/dt}=2$$

$$\rightarrow \frac{e^{0.5t}}{\cos(t^2)}=2$$

$$\rightarrow t=0.840164472\approx 0.840$$

At time t = 0.840, the object is at a point where the slope of the tangent to the curve is 2.

c)

$$\sqrt{\left(\frac{dx}{dt}\right)^2+\left(\frac{dy}{dt}\right)^2}=3\rightarrow\left(\frac{dx}{dt}\right)^2+\left(\frac{dy}{dt}\right)^2=9$$

$$\left(\cos(t^2)\right)^2+\left(e^{0.5t}\right)^2=9$$

$$\cos^2(t^2)+e^t=9$$

$$\rightarrow t=2.19589515\approx 2.196$$

At t=2.196, the speed of the particle is 3.

d)

$$\int_0^1\sqrt{\left(\frac{dx}{dt}\right)^2+\left(\frac{dy}{dt}\right)^2}\,dt=\int_0^1\sqrt{\left(\cos(t^2)\right)^2+\left(e^{0.5t}\right)^2}\,dt=1.59461\approx 1.595$$

Problem 3

a)

$$v'(16) \approx \frac{v(20)-v(12)}{20-12} = \frac{240-200}{8} = 5m/\min^2$$

b)

$\int_0^{40} |v(t)|dt$ is the total distance traveled during the time interval [0, 40].

$$\int_0^{40} |v(t)|dt \approx R_4 = (12-0)*200+(20-12)*240+(24-20)*220+(40-24)*150$$
$$R_4 = 12*200+8*240+4*220+16*150 = 2400+1920+880+2400 = 7600\,meters.$$

c)

$$v_b(t) = t^3-6t^2+300$$
$$a(t) = 3t^2-12t$$
$$a(5) = 75-60 = 15m/\min^2$$

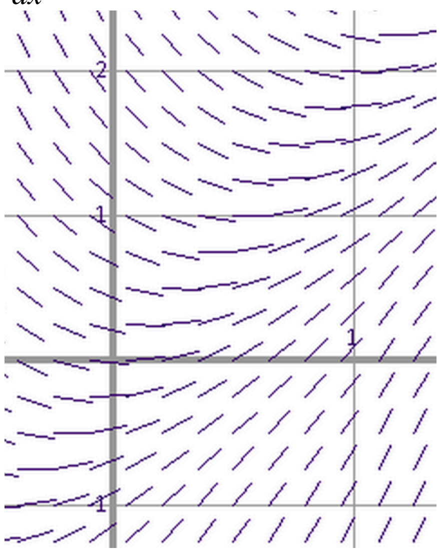
d)

$$v_{avg[0,10]} = \frac{1}{10-0}\int_0^{10} v(t)\,dt = \frac{1}{10}\int_0^{10} t^3-6t^2+300\,dt = \frac{1}{10}\left(\frac{t^4}{4}-2t^3+300t\right)_0^{10} = 350\,m/\min$$

Problem 4

a)

$$\frac{dy}{dx} = 2x - y$$



b)

$$\frac{d^2y}{dx^2} = 2 - \frac{dy}{dx} = 2 - (2x - y) = 2 - 2x + y$$

In quadrant II, x-values are negative and y-values are positive. Therefore, the second derivative will be always positive. The solution curves will hence be concave upward.

c)

f(2) = 3

@ x=2, the first derivative is: 2*2 – 3 = 1. If a curve has a local extremum at a point, then the derivative must be zero or undefined there. In our case, it is neither zero nor undefined. Therefore, the given point is neither a local maximum nor a local minimum.

d)

$$\frac{dy}{dx} = 2x - y$$

$$y = mx + b \rightarrow \frac{dy}{dx} = m = 2x - y$$

$Also:$

$$\frac{dy}{dx} = 2x - (mx + b) = (2 - m)x - b = 0x + m$$

$$2 - m = 0 \rightarrow m = 2$$

$$m = -b \rightarrow b = -2$$

Problem 5

a) k=3

$$f(x)=\frac{1}{x^2-3x}$$

$$f(4)=\frac{1}{16-12}=\frac{1}{4}$$

$$f'(x)=\frac{-2x+3}{(x^2-3x)^2}\rightarrow f'(4)=\frac{-5}{16}$$

$$L(x)=f(4)+f'(4)(x-4)$$

$$L(x)=\frac{1}{4}-\frac{5}{16}(x-4)$$

b) k=4

$$f(x)=\frac{1}{x^2-4x}\rightarrow f'(x)=\frac{4-2x}{(x^2-4x)^2}\rightarrow f'(2)=\frac{0}{16}=0$$

$$f''(x)=\frac{(x^2-4x)^2(-2)-(4-2x)*2*(x^2-4x)(2x-4)}{(x^2-4x)^4}\rightarrow f''(2)=\frac{(x^2-4x)^2(-2)}{(x^2-4x)^4}\Bigg|_{x=2}<0$$

By the Second Derivative Test, the curve has a local maximum at x=2.

c)

$$f(x)=\frac{1}{x^2-kx}\rightarrow f'(x)=\frac{k-2x}{x^2-kx}$$

$$f'(-5)=\frac{k+10}{25+5k}$$

$$k=-10\ OR\ k=-5$$

Note, however, that when k = - 5, the function isn't defined at x = - 5. Therefore, k = - 10 is the only k value that makes x = -5 a critical number in the domain of f(x).

d)

$$f(x)=\frac{1}{x^2-6x}=\frac{A}{x}+\frac{B}{x-6}\rightarrow 0x+1=A(x-6)+Bx\rightarrow 0x+1=(A+B)x-6A$$

$$A+B=0$$

$$-6A=1\rightarrow A=\frac{-1}{6}\rightarrow B=\frac{1}{6}$$

$$\int f(x)dx=\int \frac{-1}{6x}+\frac{1}{6(x-6)}dx=\frac{-1}{6}\ln|x|+\frac{1}{6}\ln|x-6|+C=\frac{1}{6}\ln\left|\frac{x-6}{x}\right|+C$$

Problem 6

a)

$$\left|\frac{a_{n+1}}{a_n}\right| = \left|\frac{\dfrac{(-3)^{n+1-1}x^{n+1}}{n+1}}{\dfrac{(-3)^{n-1}x^n}{n}}\right| = \left|\frac{(-3)^n x^{n+1}}{n+1}\frac{n}{(-3)^{n-1}x^n}\right| = \left|-3x\frac{n}{n+1}\right| \to |-3x|<1 \to |x|<\frac{1}{3}$$

$$R=\frac{1}{3}$$

b)

$$f(x)=x-\frac{3}{2}x^2+3x^3-\frac{27}{4}x^4+\ldots+\frac{(-3)^{n-1}}{n}x^n+\ldots$$

$$f'(x)=1-3x+9x^2-27x^3+\ldots+(-3)^{n-1}x^{n-1}+\ldots$$

$$f'(x)=\sum_{n=0}^{\infty}(-3x)^n=\frac{a}{1-r}=\frac{1}{1-(-3x)}=\frac{1}{1+3x}$$

Note that in the last step above we recognized the derivative as a geometric series, and we can only write it as a rational function in its interval of convergence, which is the same as that of the original function f(x).

c)

$$e^x=1+\frac{x}{1!}+\frac{x^2}{2!}+\frac{x^3}{3!}+\ldots+\frac{x^n}{n!}+\ldots$$

$$e^x f(x)=\left(1+\frac{x}{1!}+\frac{x^2}{2!}+\frac{x^3}{3!}+\ldots+\frac{x^n}{n!}+\ldots\right)*\left(x-\frac{3}{2}x^2+3x^3-\ldots+\frac{(-3)^{n-1}}{n}x^n\right)$$

$$T_3(x)=x-\frac{3}{2}x^2+x^2+3x^3-\frac{3}{2}x^3+\ldots=x-\frac{1}{2}x^2-\frac{3}{2}x^3+\ldots$$