

Solve neatly on separate paper.

Part I

Evaluate the following limits:

$$(a) \lim_{x \rightarrow (\pi/2)^-} (\sec x - \tan x)$$

$$(d) \lim_{x \rightarrow 0^+} \left(\frac{1}{x^3} - \frac{1}{x^2} \right)$$

$$(b) \lim_{x \rightarrow 0^+} x^x$$

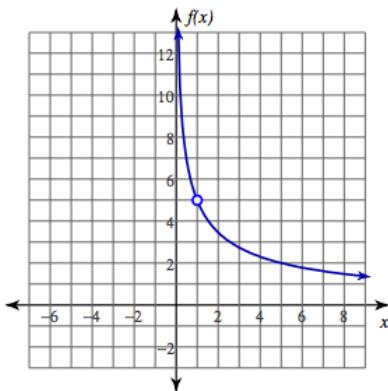
$$(e) \lim_{x \rightarrow \infty} x \tan(1/x)$$

$$(c) \lim_{x \rightarrow \infty} \frac{x}{\sqrt{x^2 + 1}}$$

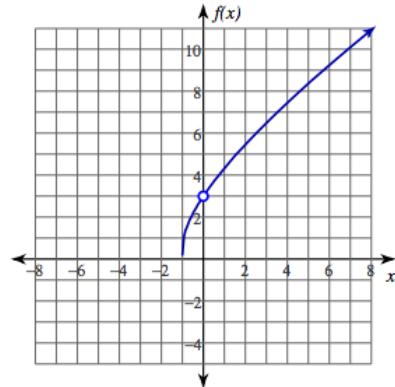
$$(f) \lim_{x \rightarrow 0} \frac{x^2 \cos(\frac{1}{x})}{x}$$

Part II

$$1) \lim_{x \rightarrow 1} \frac{5 \ln x}{x - 1}$$



$$2) \lim_{x \rightarrow 0} \frac{3x}{\ln(x+1)}$$



$$3) \lim_{x \rightarrow 0^+} 5x^2 \ln x$$

$$4) \lim_{x \rightarrow \infty} 4x \cdot e^{-x}$$

$$5) \lim_{x \rightarrow \frac{\pi}{2}} (3\sec x - 3\tan x)$$

$$6) \lim_{x \rightarrow \infty} \left(\frac{x^2}{x-1} - \frac{x^2}{x+1} \right)$$

7) $\lim_{x \rightarrow 0^+} 5 \cdot (\tan x)^{\sin x}$

8) $\lim_{x \rightarrow 0^+} 3x^x$

Evaluate each limit. Use L'Hôpital's Rule if it can be applied. If it cannot be applied, evaluate using another method and write a * next to your answer.

9) $\lim_{x \rightarrow 0} \frac{e^x - e^{-x}}{x}$

10) $\lim_{x \rightarrow 0^+} \frac{e^x + e^{-x}}{\sin(2x)}$

Part III

Use L'Hôpital's rule, if applicable, to find the limit.

1) $\lim_{x \rightarrow 5} \frac{x^2 - 9x + 20}{x - 5}$

2) $\lim_{x \rightarrow \infty} \frac{\ln(x)}{e^{2x}}$

3) $\lim_{y \rightarrow 0^+} \frac{\ln(7y^2 + 15y)}{\ln y}$

4) $\lim_{x \rightarrow 1} \frac{x - x^5}{\ln x}$

5) $\lim_{x \rightarrow \infty} \frac{(\ln x)^3}{x}$

6) $\lim_{x \rightarrow \infty} \frac{8x}{\ln(e^x + 1)}$

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Key/Solutions**Part I**

(a) $\lim_{x \rightarrow (\frac{\pi}{2})^-} (\sec x - \tan x) = \lim_{x \rightarrow (\frac{\pi}{2})^-} \left(\frac{1}{\cos(x)} - \frac{\sin(x)}{\cos(x)} \right) = \lim_{x \rightarrow (\frac{\pi}{2})^-} \frac{1-\sin(x)}{\cos(x)} = \lim_{x \rightarrow (\frac{\pi}{2})^-} \frac{-\cos(x)}{-\sin(x)} = \frac{0}{1} = 0$

(b) $\lim_{x \rightarrow 0^+} x^x = \lim_{x \rightarrow 0^+} e^{x \ln x} = \exp(\lim_{x \rightarrow 0^+} \ln x) = \exp(\lim_{x \rightarrow 0^+} \frac{1/x}{-1/x^2}) = \exp(\lim_{x \rightarrow 0^+} -x) = e^0 = 1$

(c) $\lim_{x \rightarrow \infty} \frac{x}{\sqrt{x^2 + 1}} = \lim_{x \rightarrow \infty} \frac{1}{\frac{1}{x}\sqrt{x^2 + 1}} = \lim_{x \rightarrow \infty} \frac{1}{\sqrt{1 + \frac{1}{x^2}}} = \frac{1}{\sqrt{1+0}} = 1$

(d) $\lim_{x \rightarrow 0^+} \left(\frac{1}{x^3} - \frac{1}{x^2} \right) = \lim_{x \rightarrow 0^+} \left(\frac{1-x}{x^3} \right) = \infty$

(e) $\lim_{x \rightarrow \infty} x \tan(1/x) = \lim_{x \rightarrow \infty} \frac{\tan(1/x)}{1/x} = \lim_{x \rightarrow \infty} \frac{\sec^2(1/x) \cdot \frac{1}{x^2}}{-1/x^2} = \lim_{x \rightarrow \infty} \sec^2(1/x) = 1$

(f) This is a great example of when we cannot use l'Hospital's theorem. By the squeeze theorem the top goes to 0 in the limit, so we could try l'Hospital's theorem:

$$\lim_{x \rightarrow 0} \frac{x^2 \cos(\frac{1}{x})}{x} = \lim_{x \rightarrow 0} \frac{\sin(\frac{1}{x}) + 2x \cos(\frac{1}{x})}{1} = \text{DNE}$$

Since the limit of the quotient of derivatives does not exist, one may be tempted to say that the original limit does not exist, but l'Hospital's theorem *does not say this*. Instead, let's revisit the original limit: $\lim_{x \rightarrow 0} \frac{x^2 \cos(\frac{1}{x})}{x} = \lim_{x \rightarrow 0} x \cos(\frac{1}{x})$. Instead we can just apply the squeeze theorem to this term to get that the limit is 0.

Part II

- 1) 5
- 2) 3
- 3) 0
- 4) 0
- 5) 0
- 6) 2
- 7) 5
- 8) 3
- 9) 2
- 10) infinity, *

Part III

- 1) 1
- 2) 0
- 3) 1
- 4) -4
- 5) 0
- 6) 8

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Source:

https://www.google.com/search?q=L'Hopitals+Rule+worksheet+filetype%3Apdf&oq=L'Hopitals+Rule+worksheet+filetype%3Apdf&aqs=chrome..69i57.9238j0j4&soureceid=chrome&espv=210&es_sm=91&ie=UTF-8

I: http://math.berkeley.edu/~haken/MATH1AFA11/WS10_sol.pdf

II: <http://www.kutasoftware.com/FreeWorksheets/CalcWorksheets/04%20-%20LHopitals%20Rule.pdf>

III:

<http://homepages.ius.edu/MEHRINGE/M120/Worksheets/Worksheet%20A3%20-%20L'Hopital's%20Rule.pdf>