This exam consists of two sections of 25 (no calculator) and 15 (calculator) multiple choice problems. An Answer Key is included at the end.

SECTION I: 25 PROBLEMS | NO CALCULATOR

1. $\int_0^1 \left(6x^2 - 8x\right) dx$

(A) -2
(B) 2
(C) 12
(D) 0
(E) 6

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2. Find $F'(x)$ if $F(x) = x\sqrt{6x-7}$.

(A) $\frac{x+2}{2\sqrt{6x-7}}$

(B) $\frac{2x\sqrt{(6x-7)^3}+(6x+7)}{3}$

(C) $\frac{9x-7}{\sqrt{6x-7}}$

(D) $\frac{3}{\sqrt{6x-7}}$

(E) $\frac{\sqrt{6x-7}}{2x}$

3. If $\int_c^d f(x)dx = 3c + 2d$, then $\int_c^d (f(x)-4)dx =$

(A) $\frac{3}{2}c^2 + d^2 + 4d - 4c$ (B) $6d - c$ (C) $7c - 2d$ (D) $-c - 2d$ (E) None of these

4. If $f(x) = -2x^4 - x + \frac{1}{x}$, then $f'(-1) =$

(A) -10 (B) 7 (C) -7 (D) 6 (E) 8

5. The graph of $y = x^4 - 6x^3 - 24x^2 + 2x$ is concave down for:

(A) $x > 0$
(B) $1 < x < 4$
(C) $-4 < x < -1$
(D) $x < 0$
(E) $-1 < x < 4$

6.

$$\int \left(e^{\frac{x}{3}} + 4 \right) dx =$$

(A) $\frac{1}{3}e^{\frac{x}{3}} + C$ (B) $3e^{\frac{x}{3}} + C$ (C) $3e^{\frac{x}{3}} + 4x + C$ (D) $e^{\frac{x}{3}} + 4x + C$ (E) $e^{\frac{x}{3}} + \frac{1}{3} + C$

7. $\frac{d}{dx}\sin^2\left(x^3\right) =$

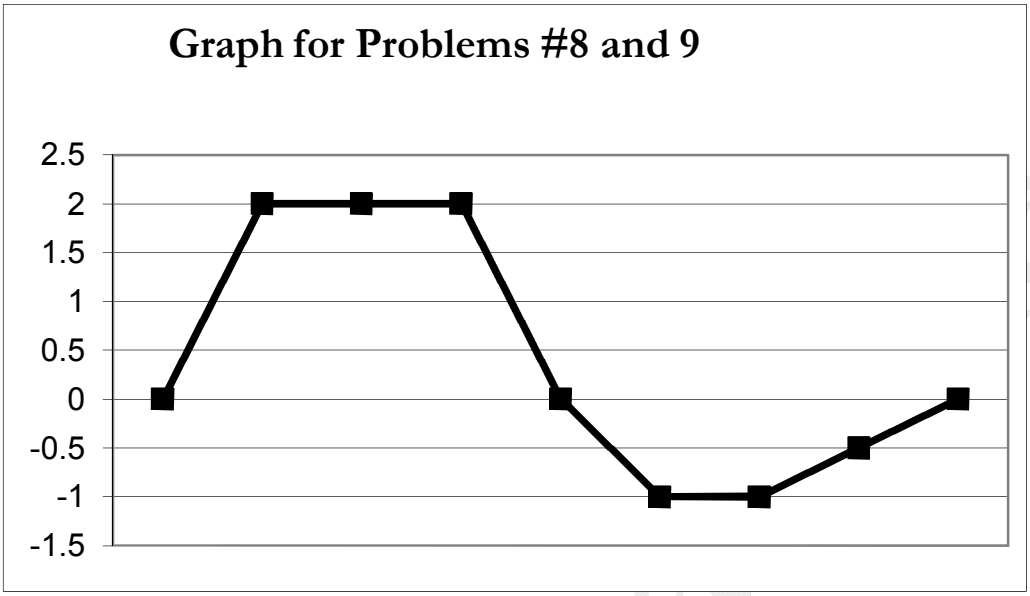
(A) $6x^2 \sin\left(x^3\right)$

(B) $\cos^2\left(x^3\right)$

(C) $6x^2 \sin\left(x^3\right)\cos\left(x^3\right)$

(D) $-6x^2 \sin\left(x^3\right)\cos\left(x^3\right)$

(E) $6\sin\left(x^3\right)\cos\left(x^3\right)$



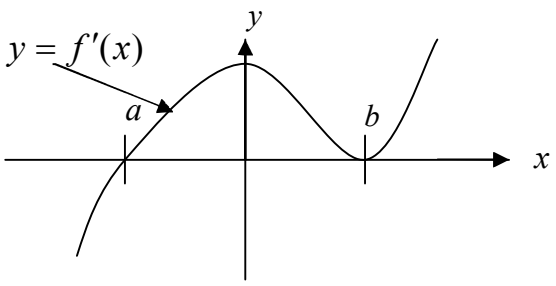
A bug begins to crawl along a vertical wire at time $t = 0$. The velocity v of the bug at time t, for $0 \le t \le 8$, is given by the function whose graph is shown above. Each point represents one unit of time.

8. At which value of t does the bug change direction?
(A) 1 (B) 3 (C) 4 (D) 6 (E) 8

9. What is the total distance the bug traveled from $t = 0$ to $t = 8$?
(A) 5 (B) 2.5 (C)7.5 (D) 8.5 (E) 9.25

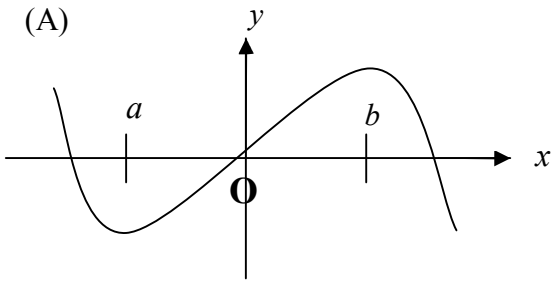
10. An equation of the line tangent to the graph of $y = \sin(4x)$ at $x = \frac{\pi}{2}$ is:

(A) $y = -4x + 2\pi$ (B) $y = x - \frac{\pi}{2}$
(C) $y = 4x - 2\pi$ (D) $y - 1 = 4(x - 2\pi)$ (E) None of these.

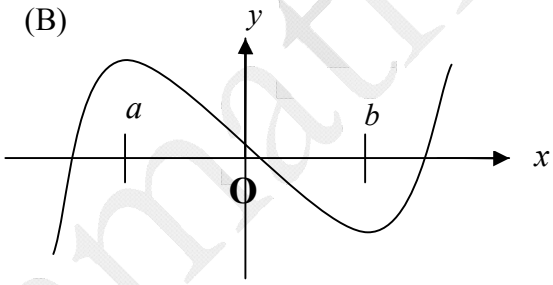


11. The graph of the **derivative** of f is shown in the figure above. Which of the following could be the graph of f ?

(A)



(B)



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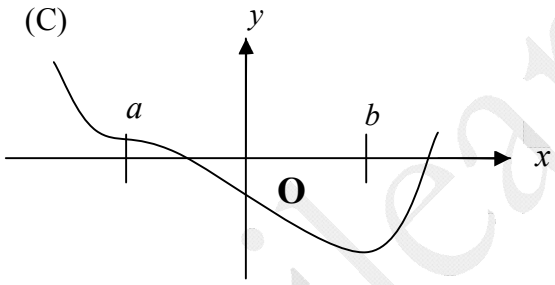

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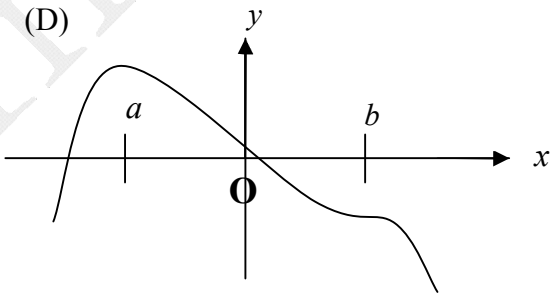

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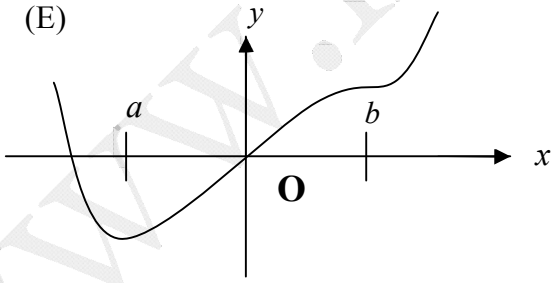

12. At what point on the graph of $y = -3x^2$ is the tangent line perpendicular to the line defined by $5x - 2y = 7$?

(A) $\left(-\frac{5}{12}, -\frac{75}{144}\right)$ (B) $\left(\frac{1}{15}, -\frac{3}{225}\right)$

(C) (5, 3) (D) $\left(\frac{5}{12}, -\frac{75}{144}\right)$ (E) None of these

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13. Let f be a function defined for all real numbers x. If $f'(x) = \frac{9-x^2}{x-3}$, then f is decreasing on the interval

(A) $(-3, \infty)$ (B) $(-\infty, -3)$ (C) $(-3, \infty), x \neq 3$ (D) $(-\infty, \infty), x \neq 3$ (E) $(3, \infty)$

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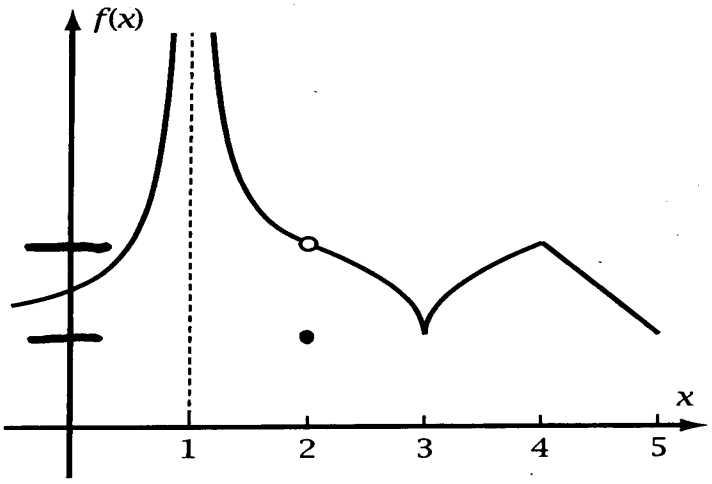
14. Let f be a differentiable function such that $f(4) = 1$ and $f'(4) = 4$. Using the tangent line to the graph of f at $x = 4$, find an approximation to a zero of f. The approximation is:

(A) 1.5 (B) 2.9 (C) 3.8 (D) 4.9 (E) 5.1



15. The graph of the function f is shown in the figure above. Which of the following statements about f is true?

(A) $\lim_{x\to 2} f(x) = f(4)$

(B) $f(2) = f(4)$

(C) $f(2) = \lim_{x\to 4} f(x)$

(D) $\lim_{x\to 2} f(x)$ does not exist

(E) $f'(3) = 1$

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16. Find the area of the region enclosed by the graphs of $y = \sqrt{x} + 2, x = 0$, and the line $y = 4$ in the first quadrant.

(A) $\frac{56}{3}$ (B) $\frac{8}{3}$ (C) $\frac{40}{3}$ (D) $-\frac{8}{3}$ (E) 4

17. If $2x^2 - y^2 = 4$, what is the value of $\frac{d^2y}{dx^2}$ at the point (2,2)?

(A) -1 (B) 1 (C) $\frac{1}{4}$ (D) $\frac{-1}{4}$ (E) Does not exist.

18.

$$\int_0^{\frac{\pi}{3}} \frac{\sin(x)e^{\sec x}}{\cos^2 x}dx =$$

(A) $e-\frac{1}{e}$ (B) $e-1$ (C) 1 (D) $e(e-1)$ (E) $\frac{1-e^2}{e}$

19. If $f(x)=\ln(x^4-1)$, then $f'(x)=$

(A) $\frac{5x}{x^4-1}$ (B) $\frac{1}{x^4-1}$ (C) $\frac{4x^3}{x^4-1}$

(D) $\frac{3x^3}{x^3-1}$ (E) $\frac{(4x)^3}{x^4-1}$

20. Find the average value of $f(x)=\sin x$ on the interval $[-1,4]$.

(A) $\frac{-\cos(4)+\cos(-1)}{5}$ (B) $\frac{\cos(-1)+\cos(4)}{5}$

(C) $\frac{-\cos(4)+\cos(-1)}{5}$ (D) $\frac{\cos(4)-\cos(-1)}{3}$

(E) $\frac{\sin(4)-\sin(-1)}{5}$

21. $\lim_{x\to\pi}\frac{x}{\tan x}=$

(A) ∞ (B) $-\infty$ (C) 1 (D) –1 (E) nonexistent

22. On what interval(s) is $f(x)=e^{x}\left(2x^{2}-x-1\right)$ decreasing?

(A) $(-\infty,-2]\cup[0.5,\infty)$
(B) $(-\infty,-0.5)$
(C) $[-2,0.5]$
(D) $(1,\infty)$
(E) $(-\infty,-2)$

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23. The region enclosed by the $x-$axis, the curve $\sqrt{y}$ = x, and the line $x=2$ is revolved about the $x-$axis. The volume of the resulting solid is:

(A) $\frac{32\pi}{5}$ (B) $\frac{32}{5}$ (C) 16π (D) 16 (E) None of these.

24. The expression $\frac{1}{15}\left(\left(\frac{16}{15}\right)^{2}+\left(\frac{17}{15}\right)^{2}+\left(\frac{18}{15}\right)^{2}\ldots+\left(\frac{30}{15}\right)^{2}\right)$ is a Riemann Sum approximation for:

(A) $\int_{0}^{2}\left(\frac{x}{15}\right)^{2}dx$

(B) $\frac{1}{15}\int_{1}^{2}\left(x^{2}\right)dx$

(C) $\int_{1}^{2}\left(x^{2}\right)dx$

(D) $\frac{1}{15}\int_{0}^{15}\left(x^{2}\right)dx$

(E) None of the above.

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25. $\int x\sin(3x)dx$ = (No longer an AB topic: Integration by Parts)

(A) $-\frac{1}{3}x\cos(3x)+\frac{1}{9}\sin(3x)+C$

(B) $\frac{1}{3}x\cos(3x)+\frac{1}{9}\sin(3x)+C$

(C) $\frac{1}{3}x\cos(3x)-\frac{1}{6}\sin(3x)+C$

(D) $-\frac{1}{6}\sin(3x)+\frac{1}{3}\cos(3x)+C$

(E) $3\sin(3x)$

SECTION II: 15 PROBLEMS | CALCULATOR

76. Find $f'(x)$ if $f(x)=\dfrac{e^{4x}}{5x}$.

(A) $\dfrac{e^{4x}(4x-1)}{5x^2}$ (B) $\dfrac{e^{4x}(1-4x)}{2x^2}$

(C) $\dfrac{e^{4x}(1-x)}{x^2}$ (D) $\dfrac{e^{4x}}{x}$ (E) Other.

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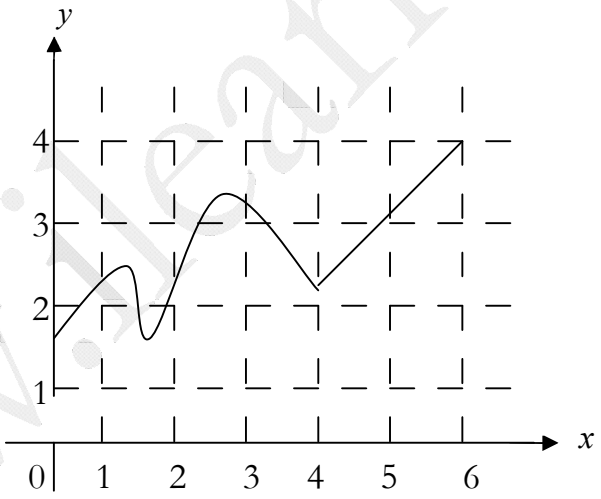
77. The graph of the function $y=x^4-3x^3+5x-3\sin x$ is concave down on the interval(s):

(A) (0,1.316)
(B) $(-\infty,0)\cup(1.316,\infty)$
(C) (0, 1.651)
(D) $(-\infty, .605)\cup(.901, 1.872)$
(E) None of the above.

78.



The graph of f is shown in the figure above. If $\int_0^4 f(x)dx = 6.775$ and $F'(x)=f(x)$, , then $F(6)-F(0)=$

(A) 0.775 (B) 12.775 (C) 4.775 (D) 2.775 (E) 11.775

79. Let F be a function such that

$$\lim_{h\to 0}\frac{F(5+h)-F(5)}{h}=2.$$

Which of the following *must* be true?

I. F is continuous at $x=5$
II. F is differentiable at $x=5$
III. The derivative of F is continuous at $x=2$

(A) I only
(B) II only
(C) I and II only
(D) I and III only
(E) II and III only

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80. Let f be the function given by $f(x)=4xe^{6x}$. For what value of x is the slope of the line tangent to the graph of f at $(x, f(x))$ equal to 14?

(A) 0 (B) 0.119 (C) –0.167 (D) 0.181 (E) 0.306

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81. A sidewalk and a road cross at right angles. An observer stands on the sidewalk 40 meters south of the crossing and watches an eastbound car traveling at 50 meters per second. At how many meter per second is the car moving away from the observer 4 seconds after it passes through the intersection?

(A) 50.00

(B) 49.02

(C) 51.02

(D) 49.22

(E) 64.03

82. If $y = -\frac{3}{2}x + 18$, what is the minimum value of the product xy?

(A) 0 (B) 6 (C) 9 (D) 36 (E) 54

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83. What is the area of the region in the first quadrant enclosed by the graphs of $y = \sin x, y = x - \frac{\pi}{2}$, and the $x-$axis?

(A) 2.611 (B) 0.599 (C) 1.395 (D) 0.438 (E) 0.385

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84. The base of a solid S is the region enclosed by the graph of $y = \sqrt{\ln x}$, the line $x = e$, and the $x-$axis. If the cross sections of S perpendicular to the $x-$axis are rectangles whose height is twice as big as their base, the volume of S is

(A) $\frac{1}{2}$ (B) $\frac{2}{3}$ (C) 1 (D) 2 (E) $\frac{1}{3}\left(e^3 - 1\right)$

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85. If the derivative of f is given by $f'(x) = 6x^3 + 7x^2 - 8x$, at what $x-$value does f have a relative maximum?

(A) –1.877
(B) –0.7103
(C) 0.877
(D) 0.7103
(E) 0

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86. Let $f(x) = \frac{1}{x^2}$. If the rate of change of f at $x = c$ is one-fourth its rate of change at $x = -3$, then c =

(A) $\frac{-3\sqrt[3]{16}}{4}$ (B) –108 (C) $\frac{-27}{4}$ (D) $-3\sqrt[3]{4}$ (E) –12

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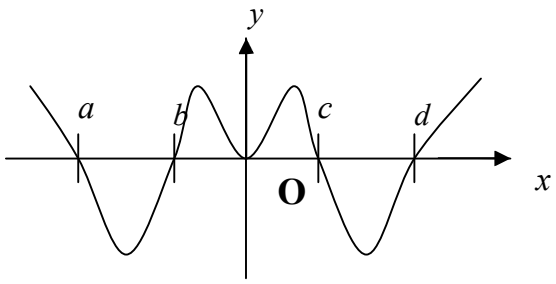
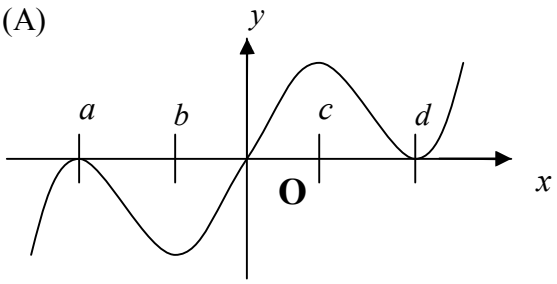
87. At time $t \geq 0$ the acceleration of a particle moving on the x-axis is $a(t) = t - \cos t$. At $t = 0$, the velocity of the particle is -1. For what value of t will the velocity of the particle be zero?

(A) 0.511
(B) 0.637
(C) 0.775
(D) 1.962
(E) None of the above.



88. Let $f(x) = \int_a^x h(t)\,dt$, where h has the graph shown above. Which of the following could be the graph of f?

(A)



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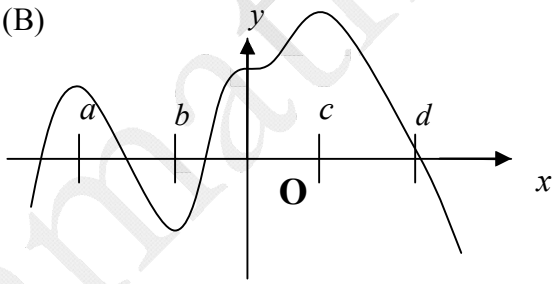

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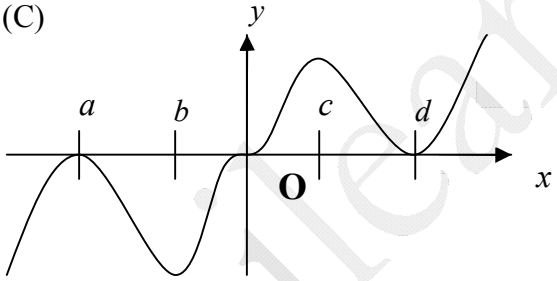

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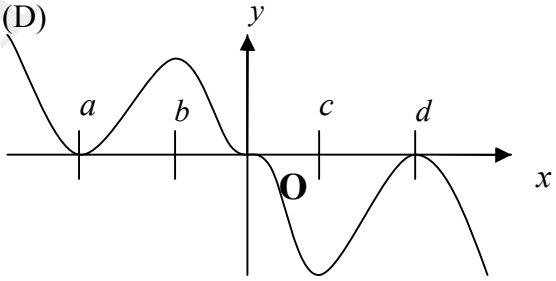
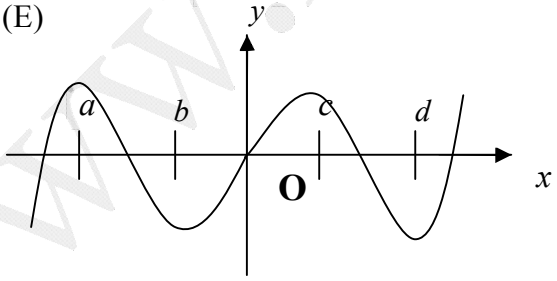


(E)



Minutes	0	1	2	3	4	5	6	7	8	9	10
Miles per Hour	50	56	60	65	69	72	73	75	78	81	82

89. The speedometer reading $v(t)$ on a car was observed at 1 – minute intervals and recorded in the chart above. Use the Trapezoidal Rule to estimate the distance traveled by the car on the interval [0, 10]. Round your answer to one decimal place.

(A) 12.7
(B) 695
(C) 11.6
(D) 6.3
(E) None of the above.

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90. Which of the following are anti-derivatives of $f(x) = \sin^2 x$?

I. $F(x) = \frac{1}{2} + \frac{1}{2}\cos(2x)$

II. $F(x) = \ln|\sin x| + C$

III. $F(x) = \frac{1}{2}x - \frac{1}{4}\sin(2x) + C$

IV. $F(x) = \sec x$

(A) I & III only (B) IV only (C) II & IV only (D) III only (E) II only

Question	Answer	Question	Answer
1	A	**21**	E
2	C	**22**	C
3	C	**23**	A
4	D	**24**	C
5	E	**25**	A
6	C	**76**	A
7	C	**77**	A
8	C	**78**	B
9	D	**79**	C
10	C	**80**	B
11	E	**81**	B
12	B	**82**	D
13	C	**83**	B
14	C	**84**	D
15	A	**85**	E
16	B	**86**	D
17	A	**87**	C
18	D	**88**	C
19	C	**89**	C
20	A	**90**	D

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