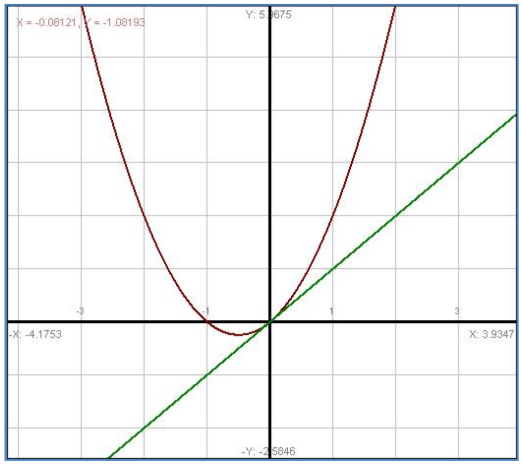
Estimating the Slope of the Tangent Line Numerically

Given a function $f(x)$ and a fixed point $P=(a, f(a))$, approximate the slope of the line tangent to the graph of $f(x)$ at point P.



The Fixed Point to Variable Point Connection

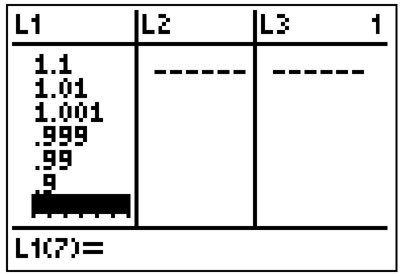
Let $Q_i=(x_i, f(x_i))$ be a variable point along the graph of $f(x)$ *near* P such that $Q_i \neq P$. We numerically estimate the slope of the tangent $y=mx+b$ by observing the slopes of the secant lines connecting Q_i and P as the variable points approach P from both sides.

$$m_{PQ_i} \approx m$$

$$\begin{cases} P=(a, f(a)) \\ Q_i=(x_i, f(x_i)) \end{cases} \Rightarrow m_{PQ_i}=\frac{y_{Q_i}-y_P}{x_{Q_i}-x_P}=\frac{f(x_i)-f(a)}{x_i-a}$$

On the graphing calculator, we create list one (L1) containing the $x-$coordinates of the variable points. For example, we choose numbers to the right of the fixed point: $(a+0.1),(a+0.01),(a+0.001)$, and some to the left $(a-0.001),(a-0.01),(a-0.1)$. Record the numbers in either increasing or decreasing order, so that the two sides "meet" at the center of list where $x=a$ would appear (but it does not!).

For example, suppose that $a = 1$. The list view on the graphing calculator (TI-84) would be:



Move the cursor up to highlight the title of L2, then enter the formula for m_{PQ_i}. In terms of L1, the formula is:

$$m_{PQ_i} = L_2 = \frac{f(L_1) - f(a)}{L_1 - a}$$

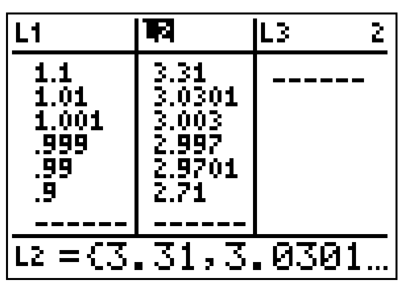
Note that $f(a)$ is constant and can be computed by evaluating the function at $x = a$.

Example:

Let $f(x) = x^3$ and $P = (1,1)$. The formula for L2 is given by:

$$L_2 = \frac{\left((L_1)^3 - 1\right)}{\left(L_1 - 1\right)}$$

Press **Enter** to populate L2 with the slopes of secant lines, as shown below:



Conclusion: We numerically estimate the slope of the tangent to $y = x^3$ at $x = 1$ to be $m = 3$. Simple algebra can be used to compute the equation of the tangent line.

Your turn: Use technology to estimate the slope of the tangent line to the graph of $f(x) = \sin x$ at $x = 0$. How does your answer compare to $\cos 0$?