SECTION II

Part A: 45 minutes, 3 problems
(A graphing calculator is required for some problems or parts of problems.)

During the timed portion for Part A, you may work only on the problems in Part A.

On Part A, you are permitted to use your calculator to solve an equation, find the derivative of a function at a point, or calculate the value of a definite integral. However, you must clearly indicate the setup of your problem, namely the equation, function, or integral you are using. If you use other built-in features or programs, you must show the mathematical steps necessary to produce your results.

Part B: 45 minutes, 3 problems
(No calculator is allowed for these problems.)

During the timed portion for Part B, you may keep Part A, and continue to work on the problems in Part A <u>without the use of any calculator</u>.

GENERAL INSTRUCTIONS FOR SECTION II PART A AND PART B
For each part of Section II, you may wish to look over the problems before starting to work on them, since it is not expected that everyone will be able to complete all parts of all problems. All problems are given equal weight, but the parts of a particular problem are not necessarily given equal weight.

- YOU SHOULD WRITE ALL WORK FOR EACH PART OF EACH PROBLEM WITH A PENCIL OR PEN IN THE SPACE PROVIDED FOR THAT PART IN THE PINK EXAM BOOKLET. Be sure to write clearly and legibly. If you make an error, you may save time by crossing it out rather than trying to erase it. Erased or crossed-out work will not be graded.
- Show all your work. Clearly label any functions, graphs, tables, or other objects that you use. You will be graded on the correctness and completeness of your methods as well as your answers. Answers without supporting work may not receive credit.
- Justifications require that you give mathematical (noncalculator) reasons.
- Your work must be expressed in standard mathematical notation rather than calculator syntax. For example, $\int_1^5 x^2 dx$ may not be written as fnInt (X^2, X, 1, 5).
- Unless otherwise specified, answers (numeric or algebraic) need not be simplified.
- If you use decimal approximations in calculations, you will be graded on accuracy. Unless otherwise specified, your final answers should be accurate to three places after the decimal point.

- Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

A graphing calculator is required for some problems or parts of the problems.

1. Water is poured into a tank in the shape of a right cone, standing on its vertex. The height of the cone is 50 feet and its radius is 15 feet. The water level in the tank is increasing at a constant rate of 0.5 feet per second.

a) Find an expression for the volume of the water (in ft^3) in the tank in terms of its height.

b) How fast (in ft^3/sec) is the water being poured at the instant the depth of the water is 2 feet?

c) How fast is the area of the surface of the water increasing at the instant the depth of the water is 2 feet?

2)
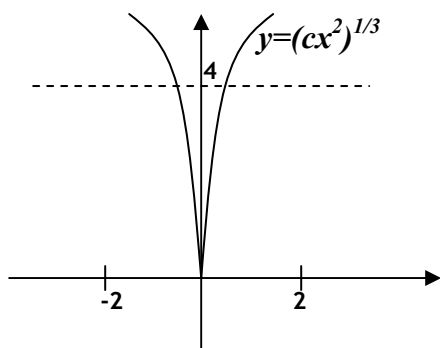
The graph of $y = (cx^2)^{1/3}$ is given above.

a) Find the value of c so that the average value of y on the interval [-2,2] is 1.

b) Using the value of c determined in (a), set up and evaluate the volume of the solid generated by revolving the region enclosed by the curve $y = (cx^2)^{1/3}$ and the line $y = 4$, about the line $y = 4$.

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3. Two particles move in the *xy* - plane. For time $t \geq 0$, the position of particle *I* is given by $x = t - 1$ and $y = t^2 - 1$, and the position of particle *II* is given by $x = 4t + 2$ and $y = -2t$.

a) Find the velocity vector for each particle at time $t = 3$.

b) Give the integral expression (but do not evaluate) for the distance traveled by particle *II* from time $t = 1$ to $t = 3$.

c) Do the particles collide? If yes, at what time? Justify your answer.

d) Sketch the paths of both particles from time $t = 0$ to $t = 3$. Indicate the direction of each particle along its path.

No Calculator is allowed for these problems.

4. Let f be the function $f(x) = xe^{2x}$.

a) Find a power series representation of $f(x)$.

b) Use term-by-term differentiation to show that $f'(x) = e^{2x} + 2xe^{2x}$.

c) Write down (but do not evaluate) $\int_0^1 xe^{2x}dx$ as a series by integrating the power

series in part(a).

5. Let S be the region enclosed by the graphs of $y = 3\ln(x)$ and $y = \dfrac{x-1}{2}$, and the lines $x = 1$ and $x = 2$.

a) Find the area of S.

b) State (but do not evaluate) an integral expression, in terms of a single variable, for the volume of the solid generated when S is revolved about the *x* - axis.

c) State (but do not evaluate) an integral expression, in terms of a single variable, for the volume of the solid generated when S is revolved about the line $x = -1$.

6. Consider the differential equation $\dfrac{dy}{dx} = 4xy$

a) On the axes provided, sketch a slope field for the given differential equation at the 9 points indicated.
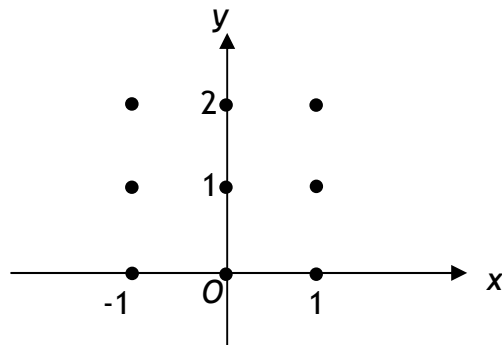
b) Sketch the solution curve through the point (0,1).

c) Let $y = f(x)$ be the particular solution to the given differential equation with the initial condition $f(0) = 1$. Use Euler's Method, starting at $x=0$ with 3 steps of equal size, to approximate $f(0.6)$. Show your work clearly.

d) Find $\dfrac{d^2 y}{dx^2}$ in terms of x and y.

END OF EXAM