

## Increasing/Decreasing Test

If  $f'(x) > 0$  in  $(a, b)$ , then  $f(x)$  is increasing in  $(a, b)$ .

If  $f'(x) < 0$  in  $(a, b)$ , then  $f(x)$  is decreasing in  $(a, b)$ .

## First Derivative Test

Suppose  $x = c$  is a critical number of  $f(x)$ .

- I. If  $f'(x)$  changes sign from  $+$  to  $-$  at  $x = a$ , then  $f(a)$  is a local maximum.
- II. If  $f'(x)$  changes sign from  $-$  to  $+$  at  $x = a$ , then  $f(a)$  is a local minimum.
- III. If  $f'(x)$  does not change sign at  $x = a$ , then  $f(a)$  is not a local extremum.

## Concavity Test

If  $f''(x) > 0$  in  $(a, b)$ , then  $f(x)$  is concave up in  $(a, b)$ .

If  $f''(x) < 0$  in  $(a, b)$ , then  $f(x)$  is concave down in  $(a, b)$ .

## Second Derivative Test

Suppose  $f(x)$  is twice differentiable near a critical number  $x = c$ .

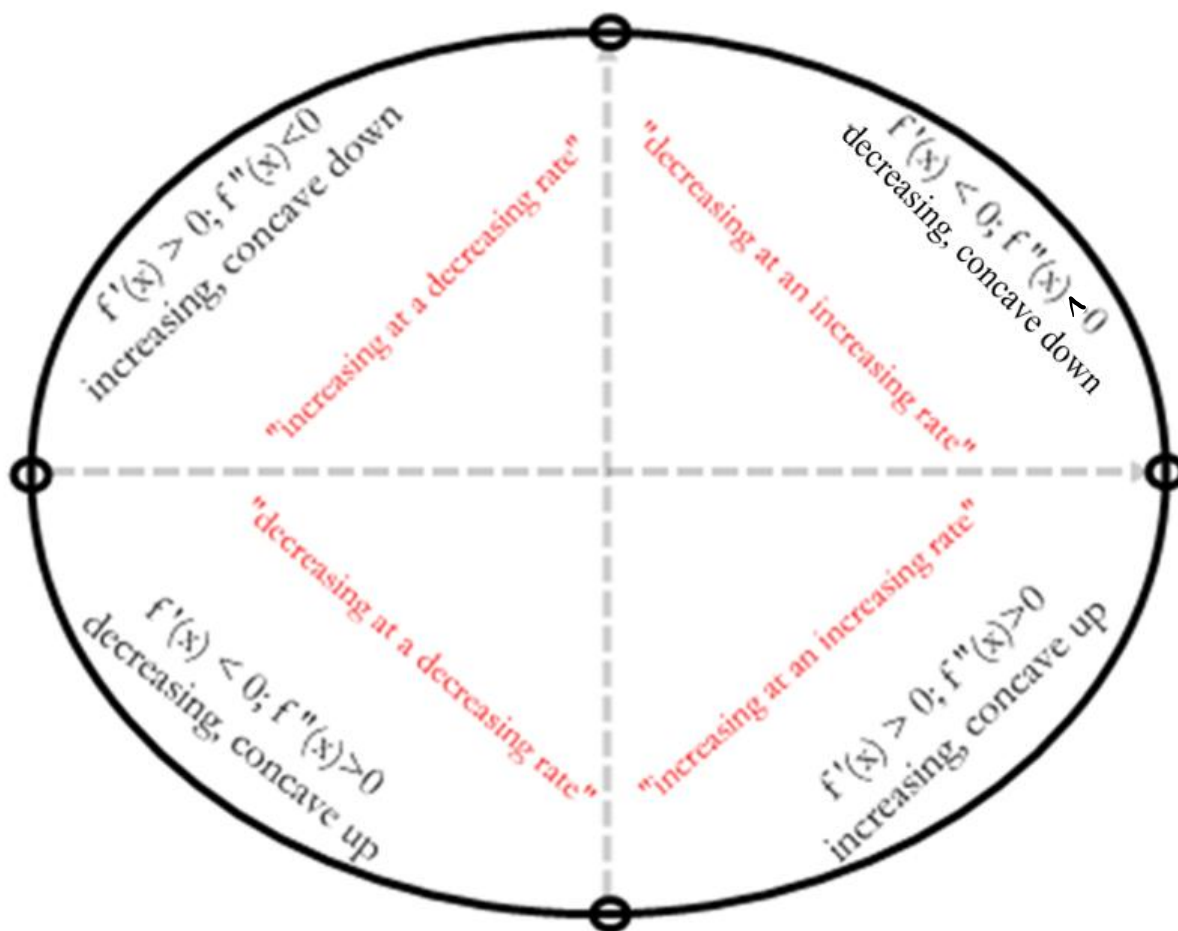
- I. If  $f'(c) = 0$  and  $f''(c) > 0$ , then  $f(c)$  is a local minimum.
- II. If  $f'(c) = 0$  and  $f''(c) < 0$ , then  $f(c)$  is a local maximum.
- IV. If  $f'(c) = f''(c) = 0$ , then the second derivative test is inconclusive.

## Inflection Point

$f(x)$  has an inflection point at  $x = c$  if and only if it satisfies the following conditions:

- I.  $f(x)$  is continuous at  $x = c$
- II.  $f''(x)$  changes sign at  $x = c$  (concavity changes)
- III.  $f'(x)$  does not change sign at  $x = c$ .

**Summary: What do  $f'$  and  $f''$  say about  $f$ ?**



**Sketching Guidelines: Discussing a Function.**

- Find the domain, the  $x$ -intercepts and the  $y$ -intercepts. If possible, comment on range.
- Find asymptotes: vertical, horizontal, slant.
- Find any discontinuities
- Describe the end behavior: what happens to  $y$ -values as  $x$  approaches infinity or negative infinity?
- If possible, factor  $f(x)$  completely, and then study its sign using a table.
- Find  $f'(x)$ , and then factor it completely to determine the critical numbers of  $f(x)$ .

- Study the sign of  $f'(x)$  to determine the increasing/decreasing intervals for the original function  $f(x)$ .
- Find  $f''(x)$ , and then factor it completely to determine the critical numbers of  $f'(x)$ .
- Study the sign of  $f''(x)$  to determine concavity for the original function  $f(x)$ .
- Combine all the information (increasing/decreasing, concavity, intercepts, sign of  $f(x)$  etc. to draw a sketch of the original function; verify your sketch using a graphing device.