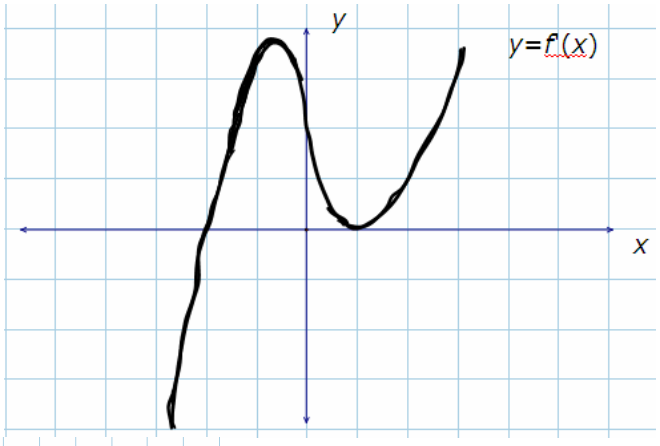
CALCULUS BC AP CHAPTER 1-3 TEST

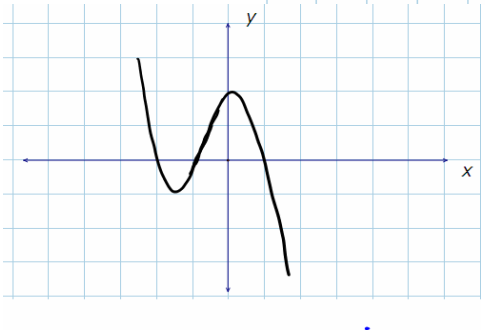
Don't write on the test materials. Put all answers on a separate sheet of paper.

Numbers 1-5: Calculator, 15 minutes. Choose the letter that best completes the statement or answers the question.

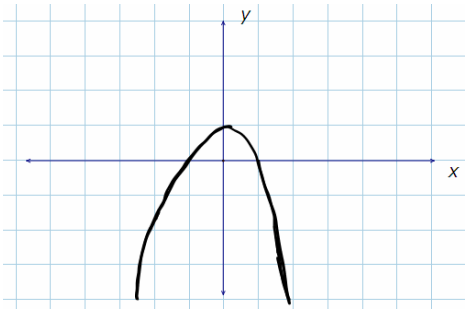
1. The graph of the derivative of f is shown below. Which of the following could be the graph of f ?



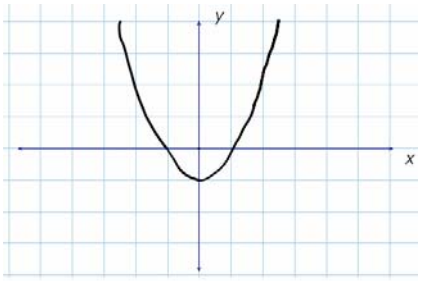
(a)



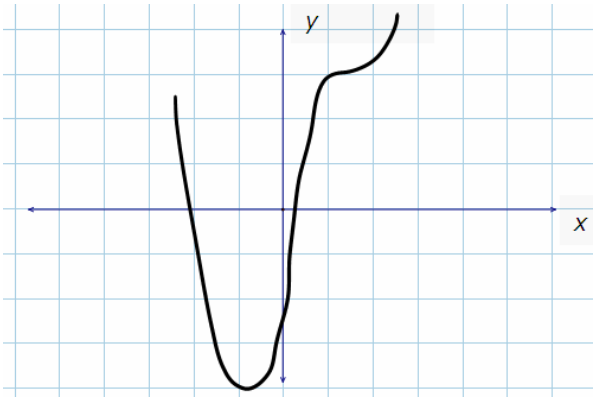
(b)



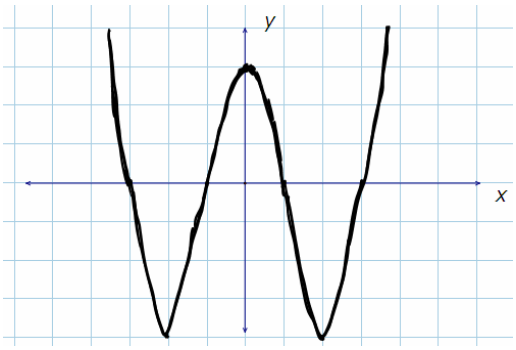
(c)



(d)



(e)



2. A missile rises vertically from a point on the ground 75,000 feet from a radar station. If the missile is rising at the rate of 16,500 feet per minute at the instant when it is 38,000 feet high, what is the rate of change, in radians per minute, of the missile's angle of elevation from the radar station at this instant?

(a) 0.175 (b) 0.219 (c) 0.227 (d) 0.469 (e) 0.507

3.Two cars start at the same place and same time. One car travels west at a constant speed of 50 miles per hour and the second car travels south at a constant speed of 60 miles per hour. Approximately how fast, in miles per hour, is the distance between them changing one-half hour later?

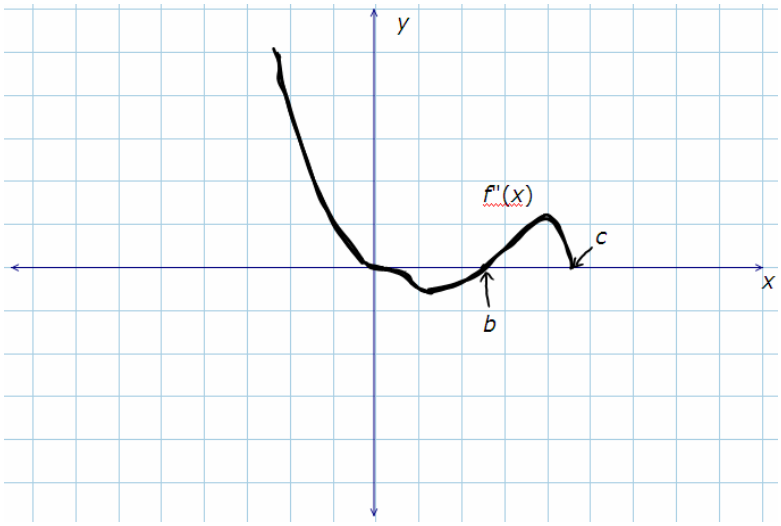
(a) 72 (b) 74 (c) 76 (d) 78 (e) 80

4. A company manufactures x calculators weekly that can be sold for $75-0.01x$ dollars each, at a cost of $1850+28x-x^2+0.001x^3$ dollars for manufacturing x calculators. The number of calculators the company should manufacture weekly to maximize its weekly profit is _____.

(a) 611 (b) 652 (c) 683 (d) 749 (e) 754

5.The figure below shows the graph of $f''(x)$, the second derivative of $f(x)$. The function $f(x)$ is continuous for all x.

Which of the following statements about f are true?



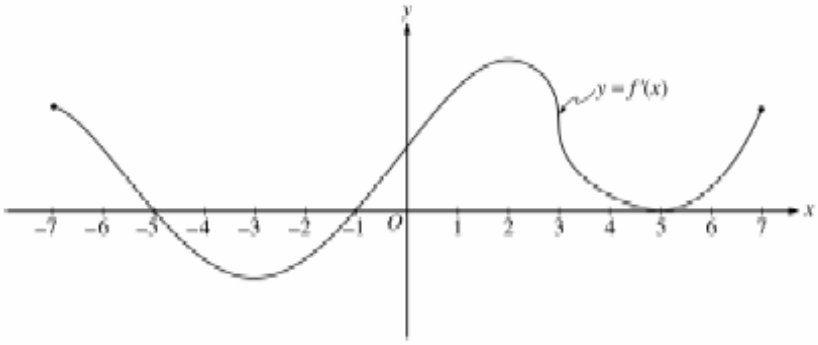
I. f is concave up for $x<0$ and $b<x<c$.

II. f has a relative minimum in the open interval $b<x<c$.

III. f has points of inflection at $x=0$ and $x=b$.

(a) I only (b) II only (c) III only (d) I and III only (e) I, II, and III

6.: Calculator, 15 minutes.



The figure above shows the graph of f', the derivative of the function f, for $-7 \le x \le 7$. The graph of f' has horizontal tangent lines at $x=-3$, $x=2$, and $x=5$, and a vertical tangent line at $x=3$.

(a) Find all values of x, for $-7<x<7$, at which f attains a relative minimum. Justify your answer.

(b) Find all values of x, for $-7<x<7$, at which f attains a relative maximum. Justify your answer.

(c) Find all values of x, for $-7<x<7$, at which $f''(x)<0$.

(d) At what value of x, for $-7 \le x \le 7$, does f attain its absolute maximum? Justify your answer.

Numbers 7-13: NO calculator, 15 minutes. Choose the letter that best completes the statement or answers the question.

7.If $y^2 - 2xy = 21$, then $\frac{dy}{dx}$ at the point (2, -3) is _____.

(a) $-\frac{6}{5}$ (b) $-\frac{3}{5}$ (c) $-\frac{2}{5}$ (d) $\frac{3}{8}$ (e) $\frac{3}{5}$

8.What is the area of the largest rectangle with lower base on the x-axis and upper vertices on the curve $y = 12 - x^2$?

(a) 8 (b) 12 (c) 16 (d) 32 (e) 48

9.If $y = \left(2x^2 + 1\right)^4$, then $\frac{dy}{dx} =$ _____.

(a) $16x^3$ (b) $4\left(2x^2 + 1\right)^3$ (c) $4x\left(2x^2 + 1\right)^3$ (d) $16\left(2x^2 + 1\right)^3$ (e) $16x\left(2x^2 + 1\right)^3$

10.If $\sin\left(xy\right) = x^2$, then $\frac{dy}{dx} =$ _____.

(a) $2x\sec\left(xy\right)$ (b) $\frac{\sec\left(xy\right)}{x^2}$ (c) $2x\sec\left(xy\right) - y$ (d) $\frac{2x\sec\left(xy\right)}{y}$ (e) $\frac{2x\sec\left(xy\right) - y}{x}$

11.If $\lim_{x\to 2}\frac{f\left(x\right)}{x-2} = f'\left(2\right) = 0$, which of the following must be true?

I. f(2) = 0
II. f(x) is continuous at x = 2
III. f(x) has a horizontal tangent line at x = 2

(a) I only (b) II only (c) I and II only (d) II and III only (e) I, II, and III

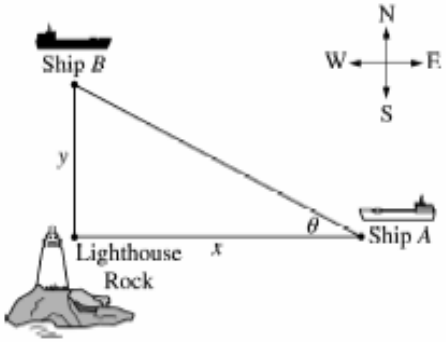
12.If the graph of $y = x^3 + ax^2 + bx - 8$ has a point of inflection at (2, 0), what is the value of b?

(a) 0 (b) 4 (c) 8 (d) 12 (e) can't be determined

13. What is the 20th derivative of $y = \sin\left(2x\right)$?

(a) $-2^{20}\sin\left(2x\right)$ (b) $2^{20}\sin\left(2x\right)$ (c) $-2^{19}\cos\left(2x\right)$ (d) $2^{20}\cos\left(2x\right)$ (e) $2^{21}\cos\left(2x\right)$

14.: NO calculator, 15 minutes.



Ship A is traveling due west toward Lighthouse Rock at a speed of 15 kilometers per hour (km/hr). Ship B is traveling due north away from Lighthouse Rock at a speed of 10 km/hr. Let x be the distance between Ship A and Lighthouse Rock at time t, and let y be the distance between Ship B and Lighthouse Rock at time t, as shown in the figure above.

(a) Find the distance, in kilometers, between Ship A and Ship B when $x = 4$ km and $y = 3$ km.

(b) Find the rate of change, in km/hr, of the distance between the two ships when $x = 4$ km and $y = 3$ km.

(c) Let θ be the angle shown in the figure. Find the rate of change of θ, in radians per hour, when $x = 4$ km and $y = 3$ km.

1. D III-17
2. A VI-35
3. D IV-36
4. C VI-33
5. D VI-38
6. 00AB3
7. E VI-11
8. D VI-16
9. E V-1
10. E V-8
11. E VI-19
12. D V-23
13. B V-12
14. 02BAB6

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test review probs.:

CALCULUS BC AP DIFFERENTIATION QUIZ

Don't write on the test materials. Put all answers on a separate sheet of paper.

Numbers 1-5: Calculator, 15 minutes. Choose the letter that best completes the statement or answers the question.

1. Suppose that f is a polynomial function. Which of the following statements is true?

I. If f is increasing at $x=a$, then $f'(a)>0$. II. If $f'(a)>0$, then f is increasing at $x=a$.

III. If f is decreasing at $x=a$, then $f'(a)<0$.

(a) I only (b) II only (c) I and II only (d) I and III only (e) I, II, and III

2. Let f and g be functions that are differentiable for all real numbers x with $g(x)=\frac{f(x)}{x}$. If $y=2x-3$ is an equation of the line tangent to the graph of f at $x=1$, what is the equation of the line tangent to the graph of g at $x=1$?

(a) $y=3x-4$ (b) $y=x-2$ (c) $y=2x+3$ (d) $y=3x-2$ (e) $y=2x-3$

3. The tangent line to the graph of $y=\sin x$ at the point $\left(\frac{2\pi}{3},\frac{\sqrt{3}}{2}\right)$ crosses the sine graph at the point where $x=$ _____.

(a) -0.781 (b) 4.712 (c) 5.388 (d) 5.760 (e) 6.283

4. A particle moves along the *x*-axis so that its position at any time $t>0$ is given by $x(t)=t^3+22t+3-6\cos(\pi t)$. For what value of t is the velocity negative?

(a) $t=\frac{1}{2}$ (b) $t=1$ (c) $t=\frac{3}{2}$ (d) $t=2$ (e) The velocity is never negative

5. The line $y=mx+b$ with $b\geq 2$ is tangent to the graph of $f(x)=-2(x-2)^2+2$ at a point in the first quadrant. What are all possible values of b?

(a) $b=2$ only (b) $2\leq b<10$ (c) $2\leq b<12$ (d) $2\leq b<14$ (e) $2\leq b<20$

Numbers 6-12: NO calculator, 15 minutes. Choose the letter that best completes the statement or answers the question.

6. If $f(x)=\dfrac{\sin^2 x}{1-\cos x}$, then $f'(x)=$ _____.

(a) $\cos x$ (b) $\sin x$ (c) $-\sin x$ (d) $-\cos x$ (e) $2\cos x$

7. If the line $y=4x+3$ is tangent to the curve $y=x^2+c$, then $c=$ _____.

(a) 2 (b) 4 (c) 7 (d) 11 (e) 15

8. A particle moves along the *x*-axis so that at any time t its position is given by $x(t)=\dfrac{1}{2}\sin t+\cos(2t)$. What is the acceleration of the particle at $t=\dfrac{\pi}{2}$?

(a) 0 (b) $\frac{1}{2}$ (c) $\frac{3}{2}$ (d) $\frac{5}{2}$ (e) $\frac{7}{2}$

9. $\lim\limits_{h\to 0}\dfrac{2(x+h)^5-5(x+h)^3-2x^5+5x^3}{h}=$ _____.

(a) 0 (b) $10x^3-15x$ (c) $10x^4+15x^2$ (d) $10x^4-15x^2$ (e) $-10x^4+15x^2$

10. Let $f(x)$ be a continuous and differentiable function on the interval $0\leq x\leq 1$, and let $g(x)=f(3x)$. The table below gives values of $f'(x)$, the derivative of $f(x)$. What is the value of $g'(0.1)$?

X	0.1	0.2	0.3	0.4	0.5	0.6
f'(x)	1.01	1.041	1.096	1.179	1.298	1.486

(a) 1.010 (b) 1.096 (c) 1.486 (d) 3.030 (e) 3.288

11. If $y=(2x^2+1)^4$, then $\dfrac{dy}{dx}=$ _____.

(a) $16x^3$ (b) $4(2x^2+1)^3$ (c) $4x(2x^2+1)^3$ (d) $16(2x^2+1)^3$ (e) $16x(2x^2+1)^3$

12. If $f(x)=3+|x-2|$, then $f'(2)=$ _____.

(a) 3 (b) 1 (c) -1 (d) 2 (e) nonexistent

All AB D&S probs.

1. B	VI-45	x^3 always increasing, but deriv. not positive at x=0
2. A	VI-39	
3. C	VI-32	
4. E	V-44	
5. C	III-45	
6. C	VI-23	
7. C	VI-7	
8. E	VI-5	
9. D	V-21	
10. E	V-19	
11. E	V-1	
12. E	IV-4	

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quiz review probs.:

CALCULUS BC AP CHAPTER 4 QUIZ

Don't write on the test materials. Put all answers on a separate sheet of paper.

Numbers 1-5: Calculator, 15 minutes. Choose the letter that best completes the statement or answers the question.

1. If $\int_0^{1000} 8^x\,dx - \int_a^{1000} 8^x\,dx = 10.40$, then $a =$ _____.

(a) 1.4 (b) 1.5 (c) 1.6 (d) 1.7 (e) 1.8

2. $\dfrac{d}{dx}\int_0^{2x}\left(e^t + 2t\right)dt =$ _____.

(a) $e^{2x} + 4x$ (b) $e^{2x} + 4x - 1$ (c) $e^{2x} + 4x^2 - 1$ (d) $2e^{2x} + 4x$ (e) $2e^{2x} + 8x$

3. The average value of the function $f(x) = e^{-x}\sin x$ on the closed interval $[1, \pi]$ is _____.

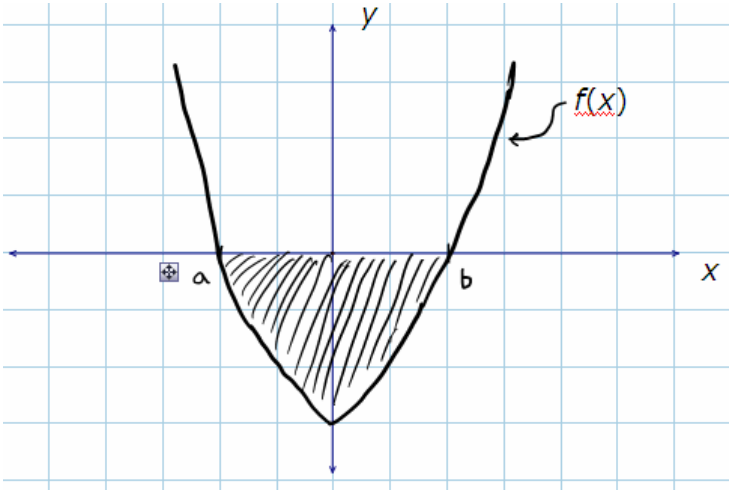
(a) 0.129 (b) 0.145 (c) 0.155 (d) 0.276 (e) 0.310

4. In the interval $0 \le x \le 5$ the graphs of $y = \cos 2x$ and $y = \sin 3x$ intersect four times. Let A, B, C, and D be the *x*-coordinates of these points so that 0 < A < B < C < D < 5. Which of the definite integrals below represents the largest number?

(a) $\int_0^A (\cos 2x - \sin 3x)\,dx$ (b) $\int_A^B (\sin 3x - \cos 2x)\,dx$ (c) $\int_B^C (\sin 3x - \cos 2x)\,dx$

(d) $\int_C^D (\cos 2x - \sin 3x)\,dx$ (e) $\int_C^D (\sin 3x - \cos 2x)\,dx$

5. If f is a continuous function shown in the figure below, then the area of the shaded region is _____.



(a) $\int_a^b f(x)\,dx$ (b) $\int_b^a f(x)\,dx$ (c) $\int_b^{-a} f(x)\,dx$ (d) $\int_{-a}^b f(x)\,dx$ (e) $\int_{-a}^{-b} f(x)\,dx$

Numbers 6-12: NO calculator, 15 minutes. Choose the letter that best completes the statement or answers the question.

6. If $\int_2^8 f(x)\,dx = -10$ and $\int_2^4 f(x)\,dx = 6$, then $\int_8^4 f(x)\,dx =$ _____.

(a) -16 (b) -6 (c) -4 (d) 4 (e) 16

7. A particle moves along a straight line so that its velocity is given by $v(t) = t^2$. How far does the particle travel between $t = 1$ and $t = 3$?

(a) $\frac{1}{3}$ (b) 8 (c) $\frac{26}{3}$ (d) 26 (e) 27

8. The average value of $\sqrt{3x}$ on the closed interval $[0,9]$ is _____.

(a) $\frac{2\sqrt{3}}{3}$ (b) $2\sqrt{3}$ (c) 6 (d) $6\sqrt{3}$ (e) $18\sqrt{3}$

9. For what value of k, $k > 0$ does $\int_0^k (4kx - 5k)\,dx = k^2$?

(a) 1 (b) 2 (c) 3 (d) 4 (e) 5

10. $\int_{-3}^{3} |x+2|\,dx =$ _____.

(a) 0 (b) 8 (c) 13 (d) 17 (e) 21

11.If $\int_0^4 (x^2 - 6x + 9)\,dx$ is approximated by 4 inscribed rectangles (lower Riemann sum) of equal width on the x-axis, then the approximation is _____.

(a) 14 (b) 10 (c) 6 (d) 5 (e) 4

12. Let f be a differentiable function for all x. Which of the following must be true?

I. $\frac{d}{dx}\int_0^3 f(x)\,dx = f(x)$

II. $\int_3^x f'(x)\,dx = f(x)$

III. $\frac{d}{dx}\int_3^x f(x)\,dx = f(x)$

(a) II only (b) III only (c) I and II only (d) II and III only (e) I, II, and III

1. B VI-44
2. E VI-41
3. A V-37
4. D II-39
5. B VI-34
6. E V-22
7. C III-5
8. B VI-12
9. C III-13
10. C IV-9
11. D V-11
12. B V-7

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quiz review probs.:

CALCULUS BC AP CHAPTER 4 TEST

Don't write on the test materials. Put all answers on a separate sheet of paper.

Numbers 1-5: Calculator, 15 minutes. Choose the letter that best completes the statement or answers the question.

1. Let f be a continuous function such that $\int_2^3 f(2x)\,dx = 8$. What is the value of $\int_4^6 f(x)\,dx$?

(a) 4 (b) 8 (c) 12 (d) 16 (e) 32

2. Let function $f(x)$, its first derivative, and its second derivative all be positive on a closed interval $[a,b]$. The interval $[a,b]$ is partitioned into n equal length sub-intervals and these are used to compute an Upper Sum, U, a Lower Sum, L, and the Trapezoidal Rule approximation, T. If $I = \int_a^b f(x)\,dx$, which statement below is true?

(a) L < U < T < I (b) L < U < I < T (c) L < T < I < U (d) L < I < T < U (e) L < I < U < T

3.Some values of a continuous function are given in the table below. The Trapezoidal Rule approximation for $\int_0^{10} f(x)\,dx$ is _____.

x	0	1	2	3	4	5	6	7	8	9	10
f(x)	20	19.5	18	15.5	12	7.5	2	-4.5	-12	-20.5	-30

(a) 30.825 (b) 32.500 (c) 33.325 (d) 33.333 (e) 35.825

4. For the function whose values are given in the table below, $\int_0^6 f(x)\,dx$ is approximated by a Midpoint Riemann Sum using the three equally spaced intervals. The approximation is _____.

x	0	1	2	3	4	5	6
f(x)	0	0.25	0.48	0.68	0.84	0.95	1

(a) 2.64 (b) 3.64 (c) 3.72 (d) 3.76 (e) 4.64

5. The present price of a new car is $14,500. The price of a new car is changing at a rate of $120+180\sqrt{t}$ dollars per year. How much will a new car cost 5 years from now?

(a) $15,020 (b) $15,300 (c) $16,440 (d) $18,120 (e) $22,600

6.: Calculator, 15 minutes.

A particle moves along the x-axis so that its velocity v at time t, for $0 \le t \le 5$, is given by $v(t) = \ln(t^2 - 3t + 3)$. The particle is at position $x = 8$ at time $t = 0$.

(a) Find the acceleration of the particle at time $t = 4$.

(b) Find all times t in the open interval $0 < t < 5$ at which the particle changes direction. During which time intervals, for $0 \le t \le 5$, does the particle travel to the left?

(c) Find the position of the particle at time $t = 2$.

(d) Find the average speed of the particle over the interval $0 \le t \le 2$.

Numbers 7-13: NO calculator, 15 minutes. Choose the letter that best completes the statement or answers the question.

7. $\int_0^3 \frac{x}{\sqrt{x^2+16}} =$ _____.

(a) 1 (b) 2 (c) 3 (d) 4 (e) 5

8. $\int x\sqrt{x+1}\,dx =$ _____.

(a) $\frac{1}{3}x^2(x+1)^{3/2} + C$ (b) $\sqrt{x+1} + \frac{x}{2\sqrt{x+1}} + C$ (c) $\frac{1}{2\sqrt{x+1}} + C$

(d) $\frac{2}{5}(x+1)^{5/2} - \frac{2}{3}(x+1)^{3/2} + C$ (e) $\frac{3}{2}(x+1)^{3/2} - \frac{5}{2}(x+1)^{5/2} + C$

9. $\int (3x+5)^2\,dx =$ _____.

(a) $\frac{1}{3}(3x+5)^3+C$ (b) $2(3x+5)+C$ (c) $6(3x+5)+C$

(d) $\frac{1}{9}(3x+5)^3+C$ (e) $\frac{1}{9}(3x+5)+C$

10.If the substitution $u=25-x^2$ is made, the integral $\int_0^3 x\sqrt{25-x^2}\,dx =$ _____.

(a) $\frac{1}{2}\int_0^3 \sqrt{u}\,du$ (b) $\frac{1}{2}\int_{25}^{16} \sqrt{u}\,du$ (c) $-\frac{1}{2}\int_0^3 \sqrt{u}\,du$ (d) $\frac{1}{2}\int_{16}^{25} \sqrt{u}\,du$ (e) $2\int_{16}^{25} \sqrt{u}\,du$

11.If f and g are continuously differentiable functions for all real numbers, which of the following definite integrals is equal to $f(g(4))-f(g(2))$?

(a) $\int_2^4 f'(g(x))\,dx$ (b) $\int_2^4 f(g(x))f'(x)\,dx$ (c) $\int_2^4 f(g(x))g'(x)\,dx$

(d) $\int_2^4 f'(g(x))g'(x)\,dx$ (e) $\int_2^4 f(g'(x))g'(x)\,dx$

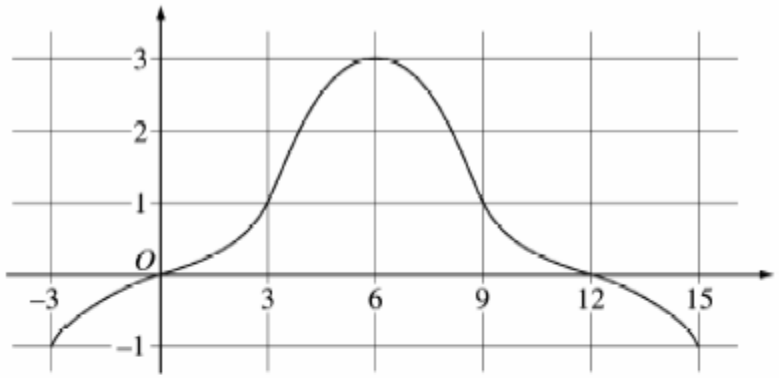
12. If the definite integral $\int_a^b f(x)\,dx$ represents the area of the region bounded by $y=f(x)$, the *x*-axis, and the lines $x = a$ and $x = b$, which of the following must be true?

(a) a > b and $f(x) > 0$ (b) a > b and $f(x) < 0$ (c) a < b and $f(x) > 0$ (d) a < b and $f(x) < 0$ (e) none of the above

13. The velocity of a particle moving along the *y*-axis is given by $v(t)=8-2t$ for $t \geq 0$. The particle moves upward until it reaches the origin and then moves downward. The position of the particle at any time t is given by _____.

(a) $-t^2+8t-16$ (b) $-t^2+8t+16$ (c) $2t^2-8t-16$ (d) $8t-2t^2$ (e) $8t-t^2$

14.: NO calculator, 15 minutes.



Graph of f

The graph of a differentiable function f on the closed interval $[-3, 15]$ is shown in the figure above. The graph of f has a horizontal tangent line at $x = 6$. Let $g(x) = 5 + \int_6^x f(t)\,dt$ for $-3 \leq x \leq 15$.

(a) Find $g(6)$, $g'(6)$, and $g''(6)$.

(b) On what intervals is g decreasing? Justify your answer.

(c) On what intervals is the graph of g concave down? Justify your answer.

(d) Find a trapezoidal approximation of $\int_{-3}^{15} f(t)\,dt$ using six subintervals of length $\Delta t = 3$.

1. D VI-29
2. D IV-33
3. B I-32
4. D II-41
5. C V-39
6. 05BAB3
7. A VI-8
8. D (V-2)
9. D IV-5
10. D IV-22
11. D II-14
12. E V-13
13. A II-15
14. 02BBC4

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test review probs.:

CALCULUS BC AP CHAPTER 5A (5.1-5.5) TEST

Don't write on the test materials. Put all answers on a separate sheet of paper.

Numbers 1-5: Calculator, 15 minutes. Choose the letter that best completes the statement or answers the question.

1. Let $f(x) = x^3 - 7x^2 + 25x - 39$ and let g be the inverse function of f. What is the value of $g'(0)$?

(a) $-\frac{1}{25}$ (b) $\frac{1}{25}$ (c) $\frac{1}{10}$ (d) 10 (e) 25

2. If $\frac{dy}{dx} = e^x y$ and $y = 3$ when $x = 0$, then $y =$ _____.

(a) $3e^x$ (b) $3e^{e^x}$ (c) $\frac{1}{3}e^{e^x}$ (d) $\frac{3e^{e^x}}{e}$ (e) $\frac{3e^x}{e}$

3. The derivative of f is given by $f'(x) = e^x(-x^3 + 3x) - 3$ for $0 \le x \le 5$. At what value of x is $f(x)$ an absolute minimum?

(a) For no value of x (b) 0 (c) 0.618 (d) 1.623 (e) 5

4. If the derivative of a function f is given by $f'(x) = \sin(x^x)$, then how many critical points does the function f have on the interval $[0.2, 2.6]$?

(a) 0 (b) 1 (c) 2 (d) 3 (e) 4

5. If $f(x) = \left|(x^2 - 12)(x^2 + 4)\right|$, then how many numbers in the interval $-2 \le x \le 3$ satisfy the conclusion of the Mean Value Theorem?

(a) none (b) One (c) Two (d) Three (e) Four

6.: Calculator, 15 minutes.

The number of gallons, $P(t)$, of a pollutant in a lake changes at the rate $P'(t) = 1 - 3e^{-0.2\sqrt{t}}$ gallons per day, where t is measured in days. There are 50 gallons of the pollutant in the lake at time $t = 0$. The lake is considered to be safe when it contains 40 gallons or less of pollutant.

(a) Is the amount of pollutant increasing at time $t = 9$? Why or why not?

(b) For what value of t will the number of gallons of pollutant be at its minimum? Justify your answer.

(c) Is the lake safe when the number of gallons of pollutant is at its minimum? Justify your answer.

(d) An investigator uses the tangent line approximation to $P(t)$ at $t = 0$ as a model for the amount of pollutant in the lake. At what time t does this model predict that the lake becomes safe?

Numbers 7-13: NO calculator, 15 minutes. Choose the letter that best completes the statement or answers the question.

7. If $f(x) = \sqrt{e^{2x} + 1}$, then $f'(0) =$ _____.

(a) $\frac{\sqrt{2}}{4}$ (b) $\sqrt{2}$ (c) $\frac{\sqrt{2}}{2}$ (d) 1 (e) $-\frac{\sqrt{2}}{2}$

8. If $y = \ln(3x + 5)$, then $\frac{d^2y}{dx^2} =$ _____.

(a) $\frac{3}{3x+5}$ (b) $\frac{3}{(3x+5)^2}$ (c) $\frac{9}{(3x+5)^2}$ (d) $\frac{-9}{(3x+5)^2}$ (e) $\frac{-3}{(3x+5)^2}$

9. $\int_e^{e^2} \frac{dx}{x \ln x} =$ _____.

(a) $\ln 2$ (b) $\frac{1}{2}$ (c) 1 (d) 2 (e) e

10. If $y = x(\ln x)^2$, then $\frac{dy}{dx} =$ _____.

(a) $3(\ln x)^2$ (b) $(\ln x)(2x + \ln x)$ (c) $(\ln x)(2 + \ln x)$

(d) $(\ln x)(2 + x\ln x)$ (e) $(\ln x)(1 + \ln x)$

11. If $\frac{dy}{dx} = y\cos x$ and $y = 3$ when $x = 0$, then $y =$ _____.

(a) $e^{\sin x} + 2$ (b) $e^{\sin x} + 3$ (c) $\sin x + 3$ (d) $\sin x + 3e^x$ (e) $3e^{\sin x}$

12. If $\frac{dy}{dx} = \frac{x}{y}$ and $y(3) = 4$, then _____.

(a) $x^2 - y^2 = -7$ (b) $x^2 + y^2 = 7^2$ (c) $x^2 - y^2 = 7^2$

(d) $y^2 - x^2 = 5$ (e) $y^2 - x^2 = 7^2$

13. $\int 2^{3x}\,dx =$ _____.

(a) $\frac{2^{3x}}{\ln 2} + C$ (b) $\frac{2^{3x}}{3\ln 2} + C$ (c) $\frac{2^{3x+1}}{3x+1} + C$ (d) $\frac{2^{3x}}{3} + C$ (e) $(\ln 2)2^{3x} + C$

14.: NO calculator, 15 minutes.

Let f be the function satisfying $f'(x) = x\sqrt{f(x)}$ for all real numbers x, where $f(3) = 25$.

(a) Find $f''(3)$.

(b) Write an expression for $y = f(x)$ by solving the differential equation $\frac{dy}{dx} = x\sqrt{y}$ with the initial condition $f(3) = 25$.

1. C III-40
2. D IV-41
3. E II-36
4. D III-33
5. D I-40
6. 2002B AB/BC2
7. C III-24
8. D VI-9
9. A V-18
10. C I-18
11. E VI-24
12. A V-28
13. B VI-17
14. 2003B AB6

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test review probs.:

CALCULUS BC AP CHAPTER 5B,6 TEST

Don't write on the test materials. Put all answers on a separate sheet of paper.

Numbers 1-5: Calculator, 15 minutes. Choose the letter that best completes the statement or answers the question.

1. The solution of the differential equation $\frac{dy}{dx} = -\frac{x^2}{y}$ contains the point (3, -2). Using Euler's method with $\Delta x = -0.3$ to approximate y when $x = 2.7$ gives $y \approx$ _____.

(a) -2.98 (b) -3.00 (c) -3.08 (d) -3.25 (e) -3.35

2. If $\frac{dy}{dx} = xy - y^2$ and $y(1) = 3$, then $y(2)$ is _____.

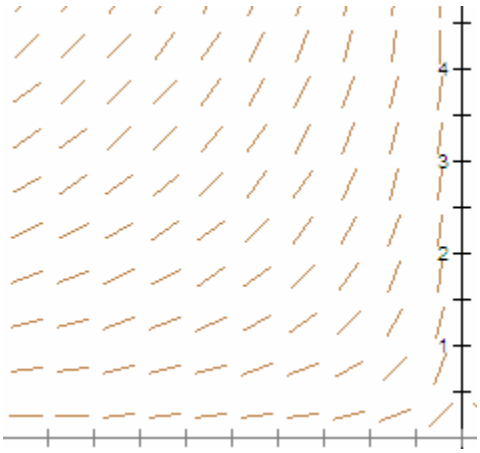
(a) -3 (b) -1 (c) 0 (d) 3 (e) 9

3. A population grows according to the equation $P(t) = 6000 - 5500e^{-0.159t}$ for $t \geq 0$, t is measured in years. This population will approach a limiting value as time goes on. During which year will the population reach <u>half</u> its limiting value?

(a) Second (b) Third (c) Fourth (d) Eighth (e) Twenty-ninth

4. If $e^{xy} = 2$, then at the point $(1, \ln 2)$, $\frac{dy}{dx} =$ _____.

(a) $-\ln 2$ (b) $2\ln 2$ (c) $\ln 2$ (d) $-2e$ (e) $-4\ln 2$



5. The figure above shows the part of a slope field for a differential equation in the second quadrant. Based on the figure, which statement appears to be true?

I. As x approaches zero from the left, y increases without bound.

II. As x decreases without bound, y decreases without bound.

III. For all points in the second quadrant, $\frac{dy}{dx} \geq 0$.

(a) I only (b) II only (c) III only (d) I and II (e) I and III

6.: Calculator, 15 minutes.

A particle moves along the y-axis so that its velocity v at time $t \geq 0$ is given by $v(t) = 1 - \tan^{-1}(e^t)$. At time $t = 0$, the particle is at $y = -1$. (Note: $\tan^{-1} x = \arctan x$)

(a) Find the acceleration of the particle at time $t = 2$.

(b) Is the speed of the particle increasing or decreasing at time $t = 2$? Give a reason for your answer.

(c) Find the time $t \geq 0$ at which the particle reaches its highest point. Justify your answer.

(d) Find the position of the particle at time $t = 2$. Is the particle moving toward the origin or away from the origin at time $t = 2$? Justify your answer.

Numbers 7-13: NO calculator, 15 minutes. Choose the letter that best completes the statement or answers the question.

7. If $y = 5^{(x^3-2)}$, then $\frac{dy}{dx} =$ _____.

(a) $(x^3-2)5^{(x^3-2)}$ (b) $3x^2(\ln 5)5^{(x^3-2)}$ (c) $(3x^2)5^{(x^3-2)}$

(d) $(\ln 5)5^{(x^3-2)}$ (e) $x^3(\ln 5)5^{(x^3-2)}$

8. At each point (x, y) on a certain curve, the slope of the curve is $4xy$. If the curve contains the point $(0,4)$, then its equation is _____.

(a) $y = e^{2x^2} + 4$ (b) $y = e^{2x^2} + 3$ (c) $y = 4e^{2x^2}$ (d) $y^2 = 2x^2 + 4$ (e) $y = 2x^2 + 4$

9. If $f(x) = \arctan\left(\frac{1}{x}\right)$, then $f'(x) =$ _____.

(a) $\frac{-1}{x^2+x}$ (b) $\frac{x}{\sqrt{x^2-1}}$ (c) $\frac{x^2}{x^2+1}$ (d) $\frac{1}{x^2+1}$ (e) $\frac{-1}{x^2+1}$

10. $\int \frac{1}{\sqrt{4-x^2}}dx =$ _____.

(a) $\arcsin\left(\frac{x}{2}\right)+C$ (b) $2\sqrt{4-x^2}+C$ (c) $\arcsin x + C$

(d) $\sqrt{4-x^2}+C$ (e) $\frac{1}{2}\arcsin\left(\frac{x}{2}\right)+C$

11. The rate of decay of a radioactive substance is proportional to the amount of substance present. Four years ago there were 12 grams of substance. Now there are 8 grams. How many grams will there be 8 years from now?

(a) 0 (b) $\frac{8}{3}$ (c) $\frac{32}{9}$ (d) $\frac{81}{16}$ (e) $\frac{16}{3}$

12. The position of a particle on the *x*-axis at time $t, t > 0$ is $\ln t$. The average velocity of the particle for $1 \le t \le e$ is _____.

(a) 1 (b) $\frac{1}{e}-1$ (c) $\frac{1}{e-1}$ (d) e (e) $e-1$

13. If $f(x) = x^3 - x + 3$ and if c is the only real number such that $f(c) = 0$, then c is between _____.

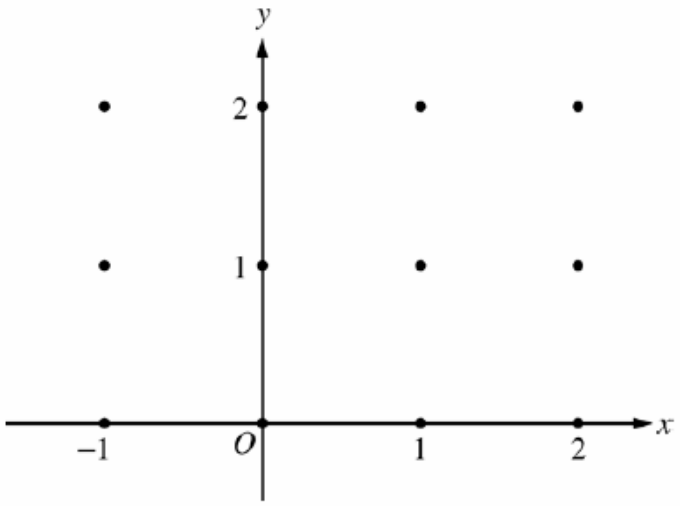
(a) -2 and -1 (b) -1 and 0 (c) 0 and 1 (d) 1 and 2 (e) 2 and 3

14.: NO calculator, 15 minutes.

Note: For part a, copy the axes and points on your paper and put the slope field on that.

Consider the differential equation $\frac{dy}{dx} = \frac{-xy^2}{2}$. Let $y = f(x)$ be the particular solution to this differential equation with the initial condition $f(-1) = 2$.

(a) On the axes provided, sketch a slope field for the given differential equation at the twelve points indicated. **(Note: Use the axes provided in the test booklet.)**



(b) Write an equation for the line tangent to the graph of f at $x = -1$.

(c) Find the solution $y = f(x)$ to the given differential equation with the initial condition $f(-1) = 2$.

1. E IV-45
2. A III-41
3. C I-44
4. A AB:IV-29
5. E VI-45
6. 2004 AB3
7. B III-15
8. C III-20
9. E III-9
10. A I-12
11. C II-23
12. C IV-6
13. A V-3
14. 2005B AB/BC6

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test review probs.:

CALCULUS BC AP CHAPTER 7 QUIZ

Don't write on the test materials. Put all answers on a separate sheet of paper.

Numbers 1-5: Calculator, 15 minutes. Choose the letter that best completes the statement or answers the question.

1. The area of the QI region enclosed by the *y*-axis and the graphs of $y = 3\cos x$ and $y = x$ is _____.

(a) 1.170 (b) 1.1571 (c) 2.078 (d) 3.142 (e) 3.447

2. The area of the region enclosed by the graphs of $y = e^{x^2} - 2$ and $y = \sqrt{4 - x^2}$ is _____.

(a) 2.525 (b) 4.049 (c) 4.328 (d) 5.050 (e) 6.289

3. The region enclosed by the *x*-axis, the curve $y = \arcsin x$, and the line $x = 1$ is rotated about the line $x = 1$. What is the volume of the solid generated?

(a) 0.178 (b) 0.356 (c) 0.571 (d) 1.119 (e) 2.047

4. A region in the first quadrant is enclosed by the coordinate axes and the line $y = k$ and $x = 3k, k > 0$. If the volume of the solid that is generated by rotating the region about the <u>*y*-axis</u> is 72π, then $k =$ _____.

(a) $\sqrt[3]{3}$ (b) 2 (c) 3 (d) $2\sqrt{3}$ (e) 4

5. The base of a solid is the region enclosed by the graph of $y = 3(x-2)^2$ and the coordinate axes. If every cross section perpendicular to the *x*-axis is a square, then the volume of the solid is _____.

(a) 8.0 (b) 19.2 (c) 24.0 (d) 25.6 (e) 57.6

Numbers 6-12: NO calculator, 15 minutes. Choose the letter that best completes the statement or answers the question.

6. Let R be the region in the <u>fourth quadrant</u> enclosed by the *x*-axis and the curve $y = x^2 - 2kx$, where $k > 0$. If the area of the region R is 36, then the value of k is _____.

(a) 2 (b) 3 (c) 4 (d) 6 (e) 9

7. The area of the region between the graph of $y = 3x^2 + 2x$ and the *x*-axis from $x = 1$ to $x = 3$ is _____.

(a) 36 (b) 34 (c) 31 (d) 26 (e) 12

8. The base of a solid is the region enclosed by the ellipse $4x^2 + y^2 = 1$. If all plane cross sections perpendicular to the *x*-axis are semicircles, then its volume is _____.

(a) $\frac{\pi}{3}$ (b) $\frac{\pi}{6}$ (c) $\frac{\pi}{2}$ (d) $\frac{2\pi}{3}$ (e) $\frac{\pi}{4}$

9. A solid has a circular base of radius 3. If every plane cross section perpendicular to the *x*-axis is an equilateral triangle, then its volume is _____.

(a) 36 (b) $12\sqrt{3}$ (c) $18\sqrt{3}$ (d) $24\sqrt{3}$ (e) $36\sqrt{3}$

10. The base of a solid is the region in the first quadrant bounded by the line $x + 2y = 4$ and the coordinate axes. What is the volume of the solid if every cross section perpendicular to the *x*-axis is a semicircle?

(a) $\frac{2\pi}{3}$ (b) $\frac{4\pi}{3}$ (c) $\frac{8\pi}{3}$ (d) $\frac{32\pi}{3}$ (e) $\frac{64\pi}{3}$

11. Which definite integral represents the volume of a sphere with radius 2?

(a) $\pi\int_{-2}^{2}\left(x^2 - 4\right)dx$ (b) $\pi\int_{-2}^{2}\left(x^2 + 4\right)dx$ (c) $2\pi\int_{0}^{2}\left(4 - x^2\right)dx$

(d) $2\pi\int_{-2}^{2}\left(4 - x^2\right)dx$ (e) $\pi\int_{0}^{2}\left(4 - x^2\right)dx$

12. The region R in the first quadrant is enclosed by the lines $x = 0$ and $y = 2$ and the graph of $y = e^x$. The volume of the solid generated when R is revolved about the <u>*x*-axis</u> is given by _____.

(a) $\pi\int_{0}^{2}\left(4 - e^{2x}\right)dx$ (b) $\pi\int_{0}^{\ln 2}\left(2 - e^{x}\right)^2 dx$ (c) $2\pi\int_{0}^{\ln 2} x\left(2 - e^{x}\right)dx$

(d) $\pi\int_{0}^{\ln 2}\left(4 - e^{2x}\right)dx$ (e) $2\pi\int_{0}^{2} x\left(2 - e^{x}\right)dx$

Give after 6.3 if shells?

1. B V-30
2. D I-39
3. D VI-43 (make sure this one does not require shells)
4. B V-40
5. E V-34
6. B IV-10
7. B III-1
8. A VI-22
9. E IV-21
10. A III-26
11. C AB:IV-7
12. D AB:VI-37 (make sure this one does not require shells)

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quiz review probs.:

CALCULUS BC AP CHAPTER 7 TEST

Don't write on the test materials. Put all answers on a separate sheet of paper.

Numbers 1-5: Calculator, 15 minutes. Choose the letter that best completes the statement or answers the question.

1. A square with vertices $(0,0),(2,0),(2,2),(0,2)$ is divided into two regions by the graph of $y=-x^2+2x$. If a point is picked at random from inside the square, what is the probability that the point lies in the region above the parabola?

(a) 0 (b) $\frac{1}{6}$ (c) $\frac{1}{3}$ (d) $\frac{1}{2}$ (e) $\frac{2}{3}$

2. The tangent line to the graph $y=e^{2-x}$ at the point $(1,e)$ intersects both coordinate axes. What is the area of the triangle formed by this tangent line and the coordinate axes?

(a) $2e$ (b) e^2-1 (c) e^2 (d) $2e\sqrt{e}$ (e) $4e$

3. The length of the curve $y=x^3$ from $(0,0)$ to $(1,1)$ is _____.

(a) 1.000 (b) 1.260 (c) 1.414 (d) 1.548 (e) 1.732

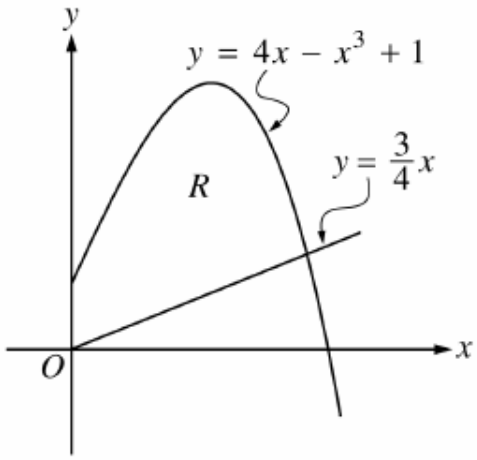
4. The region R is enclosed by the positive coordinate axes and the graph of $y=k(x-2)^2$. When this region is revolved around the y-axis, the solid formed has a volume of 8π cubic units. What is the value of k?

(a) 1 (b) $\frac{4}{3}$ (c) $\frac{3}{\pi}$ (d) 2 (e) 3

5. The region in the first quadrant enclosed by the graphs $y=x$ and $y=2\sin x$ is revolved about the *x*-axis. The volume of the resulting solid figure is _____.

(a) 1.895 (b) 2.126 (c) 5.811 (d) 6.678 (e) 13.355

6.: Calculator, 15 minutes.



Let R be the region in the first quadrant bounded by the y-axis and the graphs of $y = 4x - x^3 + 1$ and $y = \frac{3}{4}x$.

(a) Find the area of R.

(b) Find the volume of the solid generated when R is revolved about the x-axis.

(c) Write an expression involving one or more integrals that gives the perimeter of R. Do not evaluate.

Numbers 7-13: NO calculator, 15 minutes. Choose the letter that best completes the statement or answers the question.

7. If f is a differentiable function such that the slope of the graph of f at each point $(x, f(x))$ is $\sqrt{x^2-2x}$, then the length of f between $(0, f(0))$ and $(2, f(2))$ is _____.

(a) $\frac{1}{2}$ (b) $\frac{2}{3}$ (c) $\frac{3}{4}$ (d) 1 (e) 2

8. Which of the following definite integrals gives the length of the graph $y = e^{\left(e^x\right)}$ between $x = 0$ and $x = 1$?

(a) $\int_0^1 \sqrt{1+e^{2\left(x+e^x\right)}}\,dx$ (b) $\int_0^1 \sqrt{1+e^{\left(x+e^x\right)}}\,dx$ (c) $\int_0^1 \sqrt{1+e^{4x}}\,dx$

(d) $\int_0^1 \sqrt{1+e^{2e^x}}\,dx$ (e) $\int_0^1 \sqrt{e^{\left(e^x\right)}+e^{\left(x+e^x\right)}}\,dx$

9. If the length of a curve $y = f\left(x\right)$ from $x = a$ to $x = b$ is given by $L = \int_a^b \sqrt{e^{2x}+2e^x+2}\,dx$, then $f\left(x\right)$ may be _____.

(a) $2e^{2x}+2e^x$ (b) $\frac{1}{2}e^{2x}+2e^x+2x$ (c) e^x-x+3 (d) e^x+1 (e) e^x+x-2

10. The base of a solid is the region enclosed by the ellipse $4x^2+y^2=1$. If all plane cross sections perpendicular to the *x*-axis are semicircles, then its volume is _____.

(a) $\frac{\pi}{3}$ (b) $\frac{\pi}{6}$ (c) $\frac{\pi}{2}$ (d) $\frac{2\pi}{3}$ (e) $\frac{\pi}{4}$

11. Let $R\left(t\right)$ represent the rate at which water is leaking out of tank, where t is measured in hours. Which of the following expressions represents the total amount of water that leaks out in the first three hours?

(a) $R\left(3\right)-R\left(0\right)$ (b) $\dfrac{R\left(3\right)-R\left(0\right)}{3-0}$ (c) $\int_0^3 R\left(t\right)dt$ (d) $\int_0^3 R'\left(t\right)dt$ (e) $\frac{1}{3}\int_0^3 R\left(t\right)dt$

12. The local linear approximation of a function f will always be greater than or equal to the function's value if, for all x in an interval containing the point of tangency, _____.

(a) $f'<0$ (b) $f'>0$ (c) $f''<0$ (d) $f''>0$ (e) $f'=f''=0$

13. Let $f\left(x\right)=\begin{cases}1+e^{-x}, & 0\le x\le 5\\ 1+e^{x-10}, & 5\le x\le 10\end{cases}$. Which of the following statements are true?

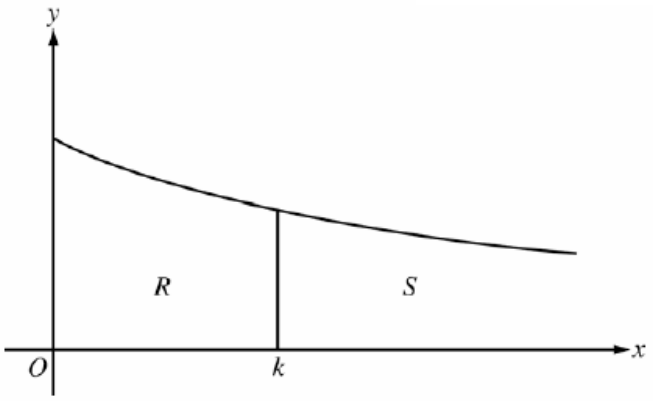
I. $f\left(x\right)$ is continuous for all values of x in the interval $\left[0,10\right]$.

II. $f'\left(x\right)$, the derivative of $f\left(x\right)$, is continuous for all values of x in the interval $\left[0,10\right]$.

III. The graph of $f\left(x\right)$ is concave upwards for all values of x in the interval $\left[0,10\right]$.

(a) I only (b) II only (c) III only (d) I and III only (e) I, II, and III

14.: NO calculator, 15 minutes.



Consider the graph of the function f given by $f(x) = \frac{1}{x+2}$ for $x \geq 0$, as shown in the figure above. Let R be the region bounded by the graph of f, the x- and y-axes, and the vertical line $x = k$, where $k \geq 0$.

(a) Find the area of R in terms of k.

(b) Find the volume of the solid generated when R is revolved about the x-axis in terms of k.

(c) Let S be the unbounded region in the first quadrant to the right of the vertical line $x = k$ and below the graph of f, as shown in the figure above. Find all values of k such that the volume of the solid generated when S is revolved about the x-axis is equal to the volume of the solid found in part (b).

1. E IV-37
2. A II-45
3. D IV-34
4. E IV-42
5. D II-44
6. 2002B BC3
7. D V-17
8. A III-22
9. E I-23
10. A AB:VI-22
11. C [I-25]
12. C [III-28]
13. D [IV-20]
14. 2005B BC6

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test review probs.:

CALCULUS BC AP CHAPTER 8 TEST

Don't write on the test materials. Put all answers on a separate sheet of paper.

Numbers 1-5: Calculator, 15 minutes. Choose the letter that best completes the statement or answers the question.

1. $\lim_{h\to 0}\frac{1}{h}\int_0^h \frac{\sin^2 t}{t^2}\,dt =$ _____.

(a) 0 (b) $\frac{1}{2}$ (c) 1 (d) 2 (e) ∞

2. If $\int x\sec^2 x\,dx = f(x)+\ln|\cos x|+C$, then $f(x) =$ _____.

(a) $\tan x$ (b) $\frac{1}{2}x^2$ (c) $x\tan x$ (d) $x^2\tan x$ (e) $\tan^2 x$

3. $\int \frac{dx}{x^2-9} =$ _____.

(a) $\frac{1}{3}\ln\left|\frac{x+3}{x-3}\right|+C$ (b) $\ln|x^2-9|+C$ (c) $\frac{1}{6}\ln|(x-3)(x+3)|+C$

(d) $\frac{1}{6}\ln\left|\frac{(x-3)}{(x+3)}\right|+C$ (e) $\frac{1}{6}\ln\left|\frac{(x+3)}{(x-3)}\right|+C$

4. $\int x\sin x\,dx =$ _____.

(a) $-\frac{1}{2}x^2\cos x+C$ (b) $-x\cos x+C$ (c) $x\cos x-\sin x+C$

(d) $-x\cos x+\sin x+C$ (e) $-x\cos x-\sin x+C$

5. If $\frac{dy}{dt}=\frac{2y}{t(t+2)}$ for $t>0$ and $y=1$ when $t=1$, then when $t=2$, $y =$ _____.

(a) 0 (b) $\frac{1}{2}$ (c) $\frac{2}{3}$ (d) 1 (e) $\frac{3}{2}$

6.: Calculator, 15 minutes.

Let f be the function given by $f(x) = 2xe^{2x}$.

(a) Find $\lim_{x\to-\infty} f(x)$ and $\lim_{x\to\infty} f(x)$.

(b) Find the absolute minimum value of f. Justify that your answer is an absolute minimum.

(c) What is the range of f?

(d) Consider the family of functions defined by $y = bxe^{bx}$, where b is a nonzero constant. Show that the absolute minimum value of bxe^{bx} is the same for all nonzero values of b.

Numbers 7-13: NO calculator, 15 minutes. Choose the letter that best completes the statement or answers the question.

7. $\int_0^2 xe^x dx =$ _____.

(a) $1-e^2$ (b) e^2-1 (c) e^2+1 (d) e^4-e^2+1 (e) e^4+e^2-1

8. $\int_0^8 \frac{1}{\sqrt[3]{8-x}}\,dx =$ _____.

(a) -6 (b) 2 (c) 6 (d) 12 (e) nonexistent

9. $\int_1^\infty x^{-\frac{5}{4}}\,dx =$ _____.

(a) $\frac{5}{4}$ (b) $\frac{1}{4}$ (c) 4 (d) -4 (e) nonexistent

10. Which of the following improper integrals converges?

I. $\int_0^\infty e^{-x}dx$ II. $\int_0^1 \frac{1}{x^2}dx$ III. $\int_0^1 \frac{1}{\sqrt{x}}dx$

(a) I only (b) III only (c) I and II (d) II and III (e) I and III

11. $\int_{-1}^{1} \frac{dx}{x^2+5x+6} =$ _____.

(a) $\ln\frac{3}{2}$ (b) $\ln\frac{1}{4}$ (c) $\ln\frac{2}{3}$ (d) $\ln 6$ (e) $\ln 12$

12. $\lim_{x\to 2} \frac{2^{\frac{x}{2}}-2}{2^x-4} =$ _____.

(a) 0 (b) $\frac{1}{4}$ (c) $\frac{1}{2}$ (d) $\ln 2$ (e) nonexistent

13. $\int_0^2 \ln x\,dx =$ _____.

(a) Undefined (b) $2\ln 2-2$ (c) $2\ln 2-1$ (d) $2\ln 2$ (e) $-\infty$

14.: NO calculator, 15 minutes.

Let g be the function given by $g(x) = \frac{1}{\sqrt{x}}$.

(a) Find the average value of g on the closed interval $[1, 4]$.

(b) Let S be the solid generated when the region bounded by the graph of $y = g(x)$, the vertical lines $x = 1$ and $x = 4$, and the x-axis is revolved about the x-axis. Find the volume of S.

(c) For the solid S, given in part (b), find the average value of the areas of the cross sections perpendicular to the x-axis.

(d) The average value of a function f on the unbounded interval $[a, \infty)$ is defined to be $\lim_{b\to\infty}\left[\frac{\int_a^b f(x)\,dx}{b-a}\right]$. Show that the improper integral $\int_4^\infty g(x)\,dx$ is divergent, but the average value of g on the interval $[4, \infty)$ is finite.

1. C IV-11
2. C I-14
3. D III-21
4. D V-6
5. E III-44
6. 98BC 2
7. C VI-2
8. C V-5
9. C III-10
10. E II-12
11. A VI-10
12. B II-11
13. B I-4
14. 04B BC5

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test review probs.:

CALCULUS BC AP CHAPTER 9A TEST

Don't write on the test materials. Put all answers on a separate sheet of paper.

Numbers 1-5: Calculator, 15 minutes. Choose the letter that best completes the statement or answers the question.

1. $\sum_{k=0}^{\infty}\left(\sin\left(\frac{\pi}{6}\right)\right)^k =$ _____.

(a) 1 (b) 2 (c) $\frac{1}{1-\frac{\sqrt{3}}{2}}$ (d) $\frac{\frac{\sqrt{3}}{2}}{1-\frac{\sqrt{3}}{2}}$ (e) does not converge

2. For which pair of functions $f(x)$ and $g(x)$ below, will the $\lim_{x\to\infty}\frac{f(x)}{g(x)}=0$?

(a) $f(x)=e^x, g(x)=x^2$ (b) $f(x)=e^x, g(x)=\ln x$ (c) $f(x)=\ln x, g(x)=e^x$

(d) $f(x)=x, g(x)=\ln x$ (e) $f(x)=3^x, g(x)=2^x$

3. Of the choices given, which value is NOT in the domain of the function $f(x)=(\cos x)^x$?

(a) 1 (b) $\frac{\pi}{2}$ (c) $\frac{4\pi}{3}$ (d) 4 (e) 2π

4. If $y=7$ is a horizontal asymptote of a rational function f, then which of the following must be true?

(a) $\lim_{x\to 7} f(x)=\infty$ (b) $\lim_{x\to\infty} f(x)=7$ (c) $\lim_{x\to 0} f(x)=7$

(d) $\lim_{x\to 7} f(x)=0$ (e) $\lim_{x\to-\infty} f(x)=-7$

5. If $f(x)>0$ is continuous and $g(x)=\int_0^x\sqrt{(f(t))^2-1}\,dt$, what is the length of the graph of $g(x)$ from $x=a$ to $x=b$?

(a) $\int_a^b f(x)\,dx$ (b) $\int_a^b g(x)\,dx$ (c) $\int_a^b\sqrt{(f(x))^2+1}\,dx$

(d) $\int_a^b\sqrt{g(x)+1}\,dx$ (e) $\int_a^b\sqrt{(g(x))^2+1}\,dx$

Numbers 6-12: NO calculator, 15 minutes. Choose the letter that best completes the statement or answers the question.

6. Let a_n, b_n, and c_n be sequences of positive numbers such that for all positive integers n, $a_n\le b_n\le c_n$. If $\sum_{n=1}^{\infty} b_n$ converges, then which of the following statements must be true?

I. $\sum_{n=1}^{\infty} a_n$ converges II. $\sum_{n=1}^{\infty} c_n$ converges III. $\sum_{n=1}^{\infty}(a_n+b_n)$ converges

(a) I only (b) II only (c) III only (d) I and III only (e) I, II, and III

7. Which of the following series are convergent?

I. $\sum_{n=1}^{\infty}(-1)^{n+1}$ II. $\sum_{n=1}^{\infty}(-1)^{n+1}n$ III. $\sum_{n=1}^{\infty}\left(\frac{1+n}{n}\right)^n$

(a) none (b) II only (c) III only (d) I and II only (e) I and III only

8. Which of the following series are convergent?

I. $1+\frac{1}{2\sqrt{2}}+\frac{1}{3\sqrt{3}}+\ldots+\frac{1}{n\sqrt{n}}+\ldots$ II. $\frac{1}{1\cdot 2}+\frac{1}{2\cdot 3}+\frac{1}{3\cdot 4}+\ldots+\frac{1}{n\cdot(n+1)}+\ldots$ III. $1+\frac{1}{\ln 2}+\frac{1}{\ln 3}+\ldots+\frac{1}{\ln n}+\ldots$

(a) I only (b) II only (c) I and II only (d) II and III only (e) I, II, and III

9. If $s_n = \left(\frac{(8-n)^{200}}{8^{n+2}}\right)\left(\frac{8^n}{(3-n^2)^{100}}\right)$, to what number does the sequence $\{s_n\}$ converge?

(a) $-\frac{1}{8}$ (b) $-\frac{1}{64}$ (c) 0 (d) $\frac{1}{64}$ (e) $\frac{1}{8}$

10. What is the sum of the series $\frac{3}{2}-\frac{3}{8}+\frac{3}{32}-\frac{3}{128}+\ldots$?

(a) $\frac{6}{7}$ (b) $\frac{9}{8}$ (c) $\frac{6}{5}$ (d) $\frac{15}{8}$ (e) 2

11. $\sum_{n=1}^{\infty}\left(\frac{1}{2}\right)^{2n} = ?$

(a) $\frac{1}{3}$ (b) $\frac{1}{2}$ (c) 1 (d) 2 (e) ∞

12. $\sum_{n=0}^{\infty}\frac{(-1)^n(\pi)^{2n}}{(2n)!} = ?$

(a) 1 (b) -1 (c) π (d) $\frac{\pi}{2}$ (e) e^{π}

1. B I-36
2. C [I-33]
3. C [II-30]
4. B [II-38]
5. A [VI-39]
6. D IV-19
7. A III-27
8. C II-24
9. D IV-23
10. C I-8
11. A II-19
12. B V-27

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test review probs.:

CALCULUS BC AP CHAPTER 9B TEST

Don't write on the test materials. Put all answers on a separate sheet of paper.

Numbers 1-5: Calculator, 15 minutes. Choose the letter that best completes the statement or answers the question.

1. If $\sum_{n=1}^{\infty}|a_n|$ converges, then which of the following is true?

I. $\sum_{n=1}^{\infty}a_n$ converges II. $\sum_{n=1}^{\infty}a_n$ is absolutely convergent III. $\sum_{n=1}^{\infty}-a_n$ converges

(a) I only (b) II only (c) III only (d) I and III only (e) 1, II, and III

2. Let E be the error when the Taylor polynomial $T(x)=x-\frac{x^3}{3!}$ is used to approximate $f(x)=\sin x$ at $x=0.5$. Which of the following is true?

(a) $|E|<0.0001$ (b) $0.0001<|E|<0.0003$ (c) $0.0003<|E|<0.005$

(d) $0.005<|E|<0.007$ (e) $0.07<|E|$

3. Let $f(x)$ be a function whose Taylor series converges for all x. If $\left|f^{(n)}(x)\right|<1$ where $f^{(n)}(x)$ is the *n*-th derivative of $f(x)$, what is the minimum number of terms of the Taylor series centered at $x=1$ necessary to approximate $f(1.2)$ with an error less than 0.000001?

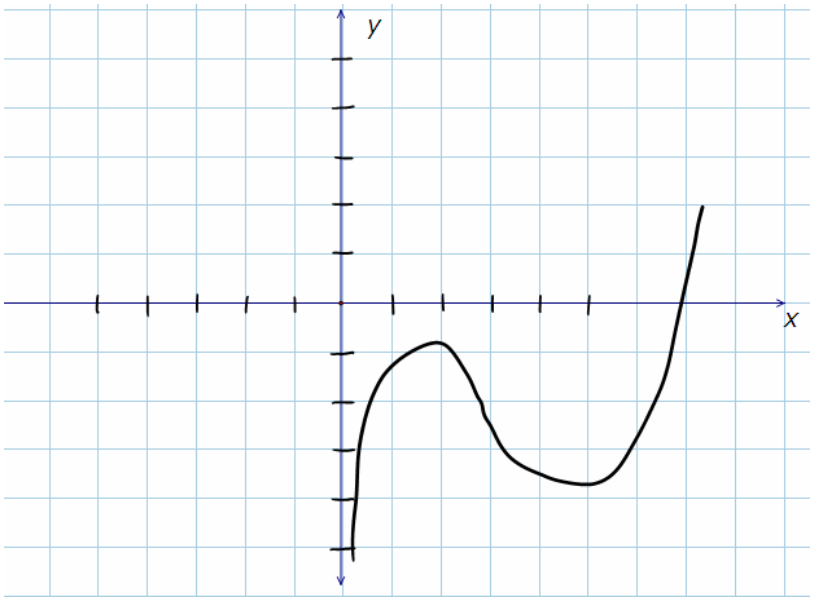
(a) 3 (b) 4 (c) 4 (d) 6 (e) 10

4. The Taylor series expansion of $\frac{1}{x}$ about the point $a=2$ is _____.

(a) $\sum_{n=0}^{\infty}\frac{(-1)^n(x-2)^n}{2^{n+1}}$ (b) $\sum_{n=0}^{\infty}\frac{(-1)^{n+1}(x-2)^n}{2^{n+1}}$ (c) $\sum_{n=0}^{\infty}\frac{(x-2)^n}{2^{n+1}}$

(d) $\sum_{n=0}^{\infty}\frac{(x-2)^n}{2^n}$ (e) $\sum_{n=0}^{\infty}\frac{(-1)^n(x-2)^n}{2^n}$

5. The graph below shows a function f with a relative maximum at $x=2$ and a relative minimum at $x=5$. The approximation of $f(x)$ near $x=5$ by the second-degree Taylor polynomial centered about $x=5$ is given by $a+b(x-5)+c(x-5)^2$. Which of the following is true about a, b, and c?



(a) $a<0, b=0, c>0$ (b) $a>0, b=0, c<0$ (c) $a<0, b<0, c<0$

(d) $a<0, b>0, c>0$ (e) $a>0, b=0, c>0$

6.: Calculator, 15 minutes.

Let f be a function having derivatives of all orders for all real numbers. The third-degree Taylor polynomial for f about $x = 2$ is given by

$$T(x) = 7 - 9(x - 2)^2 - 3(x - 2)^3.$$

(a) Find $f(2)$ and $f''(2)$.

(b) Is there enough information given to determine whether f has a critical point at $x = 2$?

If not, explain why not.

If so, determine whether $f(2)$ is a relative maximum, a relative minimum, or neither, and justify your answer.

(c) Use $T(x)$ to find an approximation for $f(0)$. Is there enough information given to determine whether f has a critical point at $x = 0$?

If not, explain why not.

If so, determine whether $f(0)$ is a relative maximum, a relative minimum, or neither, and justify your answer.

(d) The fourth derivative of f satisfies the inequality $\left|f^{(4)}(x)\right| \le 6$ for all x in the closed interval $[0, 2]$. Use the Lagrange error bound on the approximation to $f(0)$ found in part (c) to explain why $f(0)$ is negative.

Numbers 7-13: NO calculator, 15 minutes. Choose the letter that best completes the statement or answers the question.

7. What are the values of x for which the series $x-\frac{x^2}{2}+\frac{x^3}{3}-\frac{x^4}{4}+\ldots$ converges?

(a) $-1\le x\le 1$ (b) $-1\le x<1$ (c) $-1<x\le 1$ (d) $-1<x<1$ (e) All real x

8. For all x if $f(x)=\sum_{n=0}^{\infty}\frac{(-1)^{n+1}x^{2n+1}}{(2n+1)!}$, then $f'(x)=$ _____.

(a) $\sum_{n=0}^{\infty}\frac{(-1)^{n+1}x^{2n}}{(2n+1)!}$ (b) $\sum_{n=0}^{\infty}\frac{(-1)^{n}x^{2n}}{(2n)!}$ (c) $\sum_{n=0}^{\infty}\frac{(-1)^{n+1}x^{2n}}{(2n+2)!}$

(d) $\sum_{n=0}^{\infty}\frac{(-1)^{n+1}x^{2n}}{(2n)!}$ (e) $\sum_{n=0}^{\infty}\frac{(-1)^{n}x^{2n}}{(2n+1)!}$

9. Let $f(x)$ be the function defined by the power series $f(x)=\sum_{n=0}^{\infty}2x^n$. If $g'(x)=f(x)$ and $g(0)=2$, then $g(x)=$____.

(a) $2+\sum_{n=1}^{\infty}2x^n$ (b) $2\sum_{n=1}^{\infty}nx^{n-1}$ (c) $2+\sum_{n=1}^{\infty}\frac{2x^n}{n}$ (d) $\sum_{n=0}^{\infty}\frac{2}{n+1}x^{n+1}$ (e) $4+\sum_{n=1}^{\infty}\frac{2x^n}{n}$

10. What are all the values of x for which $\sum_{n=2}^{\infty}\frac{(-1)^n}{\ln n}x^n$ converges?

(a) $-e<x\le e$ (b) $-1\le x<1$ (c) $-e\le x<e$ (d) $-1<x\le 1$ (e) $-1\le x\le 1$

11. $T_n(x) = b_0 - b_1(x-a) + \frac{b_2(x-a)^2}{2!} - \frac{b_3(x-a)^3}{3!} + \ldots + (-1)^n \frac{b_n(x-a)^n}{n!}$ is the Taylor polynomial for a function $f(x)$ centered at $x = a$. If the numbers b_n are all positive, then which statement is true for all values of x in the interval of convergence?

I. If n is odd then $T_n(x) \le f(x)$ II. If n is even then $T_n(x) \ge f(x)$ III. If n is odd then $T_n(x) \ge f(x)$

(a) I only (b) II only (c) III only (d) I and II (e) II and III

12. The Maclaurin series for $\frac{\sin(x^2)}{x^2}$ is _____.

(a) $\sum_{k=0}^{\infty} \frac{(-1)^k x^{2k+1}}{(2k+1)!}$ (b) $\sum_{k=0}^{\infty} \frac{(-1)^k x^{2k}}{(2k+1)!}$ (c) $\sum_{k=0}^{\infty} \frac{(-1)^k x^{2k+1}}{(2k)!}$

(d) $\frac{1}{x} + \sum_{k=1}^{\infty} \frac{(-1)^k x^{2k-1}}{(2k-1)!}$ (e) $\sum_{k=0}^{\infty} \frac{(-1)^k x^{4k}}{(2k+1)!}$

13. The first three nonzero terms in the Maclaurin series about $x = 0$ of xe^{-x} are _____.

(a) $x - x^2 - \frac{x^3}{2!}$ (b) $x + x^2 + \frac{x^3}{2!}$ (c) $-x + x^2 - \frac{x^3}{2!}$

(d) $x - x^2 + \frac{x^3}{2!}$ (e) $1 - x + \frac{x^2}{2!}$

14.: NO calculator, 15 minutes.

The function f has a Taylor series about $x = 2$ that converges to $f(x)$ for all x in the interval of convergence. The nth derivative of f at $x = 2$ is given by $f^{(n)}(2) = \frac{(n+1)!}{3^n}$ for $n \ge 1$, and $f(2) = 1$.

(a) Write the first four terms and the general term of the Taylor series for f about $x = 2$.

(b) Find the radius of convergence for the Taylor series for f about $x = 2$. Show the work that leads to your answer.

(c) Let g be a function satisfying $g(2) = 3$ and $g'(x) = f(x)$ for all x. Write the first four terms and the general term of the Taylor series for g about $x = 2$.

(d) Does the Taylor series for g as defined in part (c) converge at $x = -2$? Give a reason for your answer.

1. E V-33
2. B III-38
3. C VI-36
4. A VI-26
5. A IV-44
6. 2004B BC2
7. C V-26
8. D II-25
9. C VI-21
10. D I-11
11. D VI-18
12. E I-22
13. D II-17
14. 2003B BC6

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test review probs.:

CALCULUS BC AP CHAPTER 10 QUIZ

Don't write on the test materials. Put all answers on a separate sheet of paper.

Numbers 1-5: Calculator, 15 minutes. Choose the letter that best completes the statement or answers the question.

1. Which of the following gives the area of the region enclosed by the graph of the polar curve $r = 1 + \cos\theta$?

(a) $\int_0^{\pi} \left(1 + \cos^2\theta\right) d\theta$ (b) $\int_0^{\pi} \left(1 + \cos\theta\right)^2 d\theta$ (c) $\int_0^{2\pi} \left(1 + \cos\theta\right) d\theta$

(d) $\int_0^{2\pi} \left(1 + \cos\theta\right)^2 d\theta$ (e) $\frac{1}{2}\int_0^{2\pi} \left(1 + \cos^2\theta\right) d\theta$

2. The area of the region enclosed by the polar curve $r = 2\left(\sin\theta + \cos\theta\right)$ is _____.

(a) 1 (b) 2 (c) π (d) 2π (e) 4π

3. The Cartesian equation of the polar curve $r = \cos\theta$ is _____.

(a) $x^2 + y^2 = x$ (b) $x^2 + y^2 = y$ (c) $x^2 + y^2 = x + y$ (d) $\left(x + y\right)^2 = y$ (e) $\left(x + y\right)^2 = x$

4. The region in the first quadrant enclosed by the *x*-axis, the line $x = \pi$, and the curve $y = \cos\left(\cos x\right)$ is rotated about the *x*-axis. What is the volume of the solid generated?

(a) 1.921 (b) 3.782 (c) 6.040 (d) 8.130 (e) 23.781

5. The table below gives values of a differentiable function f. What is the approximate value of $f'(4)$?

x	3.998	3.999	4	4.001	4.002
f(x)	1.15315	1.15548	1.15782	1.16016	1.1625

(a) 0.00234 (b) 0.289 (c) 0.427 (d) 2.340 (e) cannot be determined

Numbers 6-12: NO calculator, 15 minutes. Choose the letter that best completes the statement or answers the question.

6. A curve is given parametrically by the equations $x = 3t - t^3$ and $y = 3t^2$. The length of the arc from $t = 0$ to $t = 2$ is _____.

(a) 6 (b) 8 (c) 10 (d) 12 (e) 14

7. If $x = t - t^2$ and $y = \sqrt{2t + 5}$, then $\frac{dy}{dx}$ at $t = 2$ is _____.

(a) $-\frac{1}{9}$ (b) -1 (c) $\frac{3}{2}$ (d) -9 (e) $-\frac{1}{18}$

8. If $x = 2t^2$ and $y = t^3$, then $\frac{d^2y}{dx^2}$ at $t = 3$ is _____.

(a) $\frac{1}{16}$ (b) $\frac{9}{2}$ (c) $\frac{3}{4}$ (d) $\frac{1}{4}$ (e) $\frac{9}{4}$

9. A curve in the *xy*-plane is defined parametrically by the equations $x = t^2 + t$ and $y = t^2 - t$. For what value of t is the tangent line to the curve horizontal?

(a) -1 (b) $-\frac{1}{2}$ (c) 0 (d) $\frac{1}{2}$ (e) 1

10. A curve is given parametrically by the equation $x = 3 - 4\sin t$ and $y = 4 + 3\cos t$ for $0 \le t < 2\pi$. What are all points $\left(x, y\right)$ at which the curve has a vertical tangent?

(a) (-1, 4) only (b) (3, 7) only (c) (-1, 4) and (7, 4)
(d) (3, 7) and (3, 1) (e) (4, -1) and (4, 7)

11. Which of the following is an equation of the line tangent to the curve with parametric equations $x = 3e^{-t}, y = 6e^{t}$ at the point where $t = 0$.

(a) $2x + y - 12 = 0$ (b) $-2x + y - 12 = 0$ (c) $2x + y - 6 = 0$ (d) $-2x + y - 6 = 0$ (e) $2x + y = 0$

12. The rectangular equation of the curve given parametrically by $x = 1 + e^{-t}$ and $y = 1 + e^{t}$ is _____.

(a) $y = 2 - x$ (b) $y = x$ (c) $y = \frac{1}{x}$ (d) $y = \frac{x}{x - 1}$ (e) $y = \frac{x}{1 - x}$

1. B I-34
2. D III-36
3. A III-29
4. C [III-30]
5. D [II-37]
6. E IV-8
7. A III-18
8. A IV-4
9. D II-7
10. C IV-3
11. A I-15
12. D IV-25

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quiz review probs.: