

Free Response Set #1

Let $f(x) = \begin{cases} x^2 - ax^2 & \text{if } x < 2 \\ 4 - 2x^2 & \text{if } x \geq 2 \end{cases}$.

- (a) Find $\lim_{x \rightarrow 2^-} f(x)$.
- (b) Find $\lim_{x \rightarrow 2^+} f(x)$.
- (c) Find all values of a that make f continuous at 2. Justify your answer.

Free Response Set #2

Let $f(x) = 2x - x^2$.

- (a) Find $f(4)$
- (b) Find $f(4+h)$
- (c) Find $\frac{f(4+h) - f(4)}{h}$
- (d) Find the instantaneous rate of change
of f at $x=4$.

Free Response Set #3

Let $f(x) = x^4 - 4x^2$.

- (a) Find all the points where f has horizontal tangents.
- (b) Find an equation of the tangent line at $x = 1$.
- (c) Find and equation of the normal line at $x = 1$.

Free Response Set #4

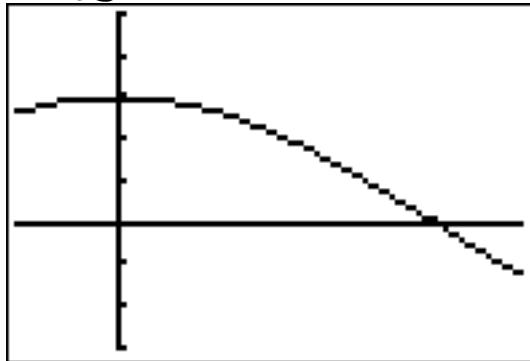
A particle moves along the y -axis with position given by

$$s(t) = -\frac{1}{2} \cos t + \frac{5}{2} \text{ for } t \geq 0.$$

- (a) In which direction (up or down) is the particle moving at $t = 1.5$? Why?
- (b) Find the acceleration of the particle at $t = 1.5$. Is the velocity of the particle increasing or decreasing at $t = 1.5$? Why or why not?
- (c) Find the displacement of the particle during the first two seconds.

Free Response Set #5

Let f be the function given by $f(x) = 3\cos x$. As shown below, the graph of f crosses the y -axis at point P and the x -axis at point Q .



- (a) Write an equation for the line passing through points P and Q .
- (b) Write an equation for the line tangent to the graph of f at point Q . Show the analysis that leads to your equation.
- (c) Find the x -coordinate of the point on the graph of f , between points P and Q , at which the line tangent to the graph of f is parallel to line PQ .

Free Response Set #6 – NO CALCULATOR

Consider the curve given by $xy^2 - x^3y = 6$.

(a) Show that $\frac{dy}{dx} = \frac{3x^2y - y^2}{2xy - x^3}$.

- (b) Find all point on the curve whose x -coordinate is 1, and write and equation for the tangent line at each of these points.
- (c) Find the x -coordinate of each point on the curve where the tangent line is vertical.

Free Response Set #7 – CALCULATOR USE OK

A particle moves along the x -axis so that its velocity at

time t is given by $v(t) = -t + 1 \sin\left(\frac{t^2}{2}\right)$.

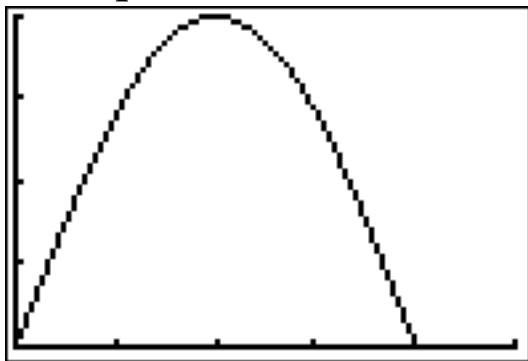
At time $t = 0$, the particle is at position $x = 1$.

- (a) Find the acceleration of the particle at time $t = 2$. Is the speed of the particle increasing at $t = 2$? Why or why not?

(b) Find all times t in the open interval $0 < t < 3$ when the particle changes direction. Justify your answer.

Free Response Set #8 – CALCULATOR USE OK

Let f be the function given by $f(x) = 4 \sin x$. As shown, the graph of f passes through the point $M\left(\frac{\pi}{2}, 4\right)$ and crosses the x -axis at point $N(\pi, 0)$.



- (a) Write an equation for the line passing through points M and N .

- (b) Write an equation for the line tangent to the graph of f at point N . Show the analysis that leads to your equation.

- (c) Find the x -coordinate of the point on the graph of, between points M and N , at which the line tangent to the graph of f is parallel to line MN .

Free Response Set #9 – CALCULATOR USE OK

Let f be the function given by $f(x) = 2xe^{2x}$.

- (a) Find $\lim_{x \rightarrow -\infty} f(x)$ and $\lim_{x \rightarrow \infty} f(x)$.

- (b) Find the absolute minimum value of f . Justify that your answer is an absolute minimum.

- (c) What is the range of f ?

- (d) Consider the family of functions defined by $y = bxe^{bx}$, where b is a nonzero constant. Show that the absolute minimum value of bxe^{bx} is the same for all nonzero values of b .

Free Response Set #10 – NO CALCULATOR

Let h be a function defined for all $x \neq 0$ such that

$h(4) = -3$ and the derivative of h is given by

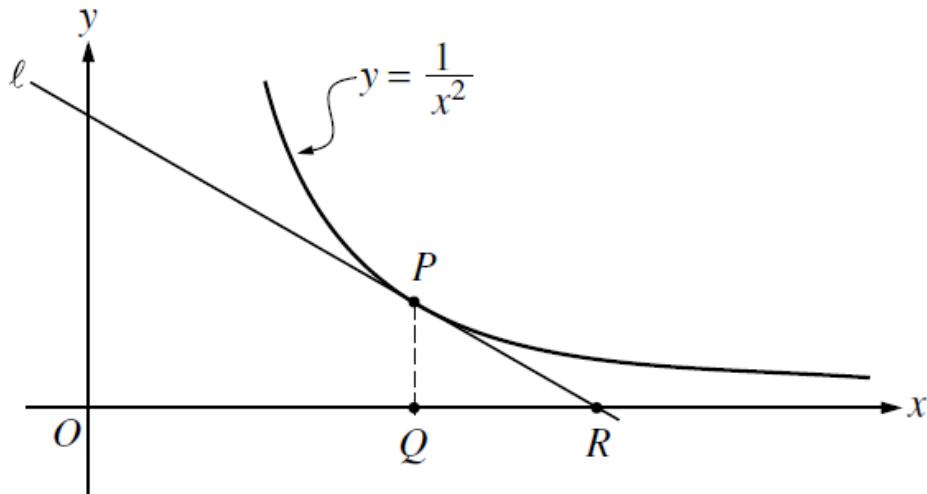
$$h'(x) = \frac{x^2 - 2}{x} \text{ for all } x \neq 0.$$

- (a) Find all values of x for which the graph of h has a horizontal tangent, and determine whether h has a local maximum, a local minimum, or neither at each of these values. Justify your answers.

- (b) On what intervals, if any, is the graph of h concave up? Justify that your answer.

- (c) Write an equation for the line tangent to the graph of h at $x = 4$.

- (d) Does the line tangent to the graph of h at $x = 4$ lie above or below the graph of h for $x > 4$? Why?

Free Response Set #11 – NO CALCULATOR

In the figure above, line ℓ is tangent to the graph of $y = \frac{1}{x^2}$ at point P , with coordinates $(w, \frac{1}{w^2})$, where $w > 0$. Point Q has coordinates $(w, 0)$. Line ℓ crosses the x -axis at point R , with coordinates $(k, 0)$.

(a) Find the value of k when $w = 3$.

(b) For all $w > 0$, find k in terms of w .

- (c) Suppose that w is increasing at the constant rate of 7 units per second. When $w = 5$, what is the rate of change of k with respect to time?
- (d) Suppose that w is increasing at the constant rate of 7 units per second. When $w = 5$, what is the rate of change of the area of $\triangle PQR$ with respect to time? Determine whether the area is increasing or decreasing at this instant.

Free Response Set #12 CALCULATOR USE OK

t (hours)	$R(t)$ (gallons per hour)
0	9.6
3	10.4
6	10.8
9	11.2
12	11.4
15	11.3
18	10.7
21	10.2
24	9.6

The rate at which water flows out of a pipe, in gallons per hour, is given by a differentiable function R of time t . The table above shows the rate as measured every 3 hours for a 24-hour period.

- (a) Use a midpoint Riemann sum with 4 subdivision of equal length to approximate $\int_0^{24} R(t)dt$. Using correct units, explain the meaning of your answer in terms of water flow.

- (b) Is there some time t , $0 < t < 24$, such that $R'(t) = 0$? Justify your answer.

Free Response Set #13 Calculator Use OK

Distance x (cm)	0	1	5	6	8
Temperature $T(x)$ $^{\circ}\text{C}$	100	93	70	62	55

A metal wire of length 8 centimeters (cm) is heated at one end. The table above gives selected values of the temperature $T(x)$, in degrees Celsius $^{\circ}\text{C}$, of the wire x cm from the heated end. The function T is decreasing and twice differentiable.

- (a) Estimate $T'(7)$. Show the work that leads to your answer. Indicate units of measure. Using correct units, explain the meaning of your answer in terms of water flow.
- (b) Write an integral expression in terms of $T(x)$ for the average temperature of the wire.
- (c) Find $\int_0^8 T'(x)dx$, and indicate units of measure.
Explain the meaning of $\int_0^8 T'(x)dx$ in terms of the temperature of the wire.

- (d) Are the data in the table consistent with the assertion that $T''(x) > 0$ for every x in the interval $0 < x < 8$? Explain your answer.

Free Response Set #14

The temperature outside a house during a 24-hour period is given by $F(t) = 80 - 10 \cos\left(\frac{\pi t}{12}\right)$, $0 \leq t \leq 24$, where $F(t)$ is measured in degree Fahrenheit and t is measure in hours.

- (a) Find the average temperature, to the nearest degree Fahrenheit, between $t = 6$ and $t = 14$?

 - (b) An air conditioner cooled the house whenever the outside temperature was at or above 78 degrees Fahrenheit. For what values of t was the air conditioner cooling the house?

 - (c) The cost of cooling the house accumulates at the rate of \$0.05 per hour for each degree the

outside temperature exceeds 78 degrees Fahrenheit. What was the total cost, to the nearest cent, to cool the house for this 24-hour period?

Free Response Set #15 Calculator Use OK

A particle moves along the y -axis so that its velocity v at time $t \geq 0$ is given by $v(t) = 1 - \tan^{-1} e^t$. At time $t = 0$, the particle is at $y = -1$. (Note: $\tan^{-1} x = \arctan x$).

- (a) Find the acceleration of the particle at time $t = 2$.

(b) Is the speed of the particle increasing or decreasing at time $t = 2$? Give a reason for your answer

(c) Find the position of the particle at time $t = 2$. Is the particle moving toward the origin or away from the origin at time $t = 2$? Justify your answer

Free Response Set #16

Consider the differential equation $\frac{dy}{dx} = \frac{3x^2}{e^{2y}}$.

- (a) Find a solution $y = f(x)$ to the differential equation satisfying $f(0) = \frac{1}{2}$.
- (b) Find the domain and range of the function f found in part (a).

Free Response Set #17

A graphing calculator is required for some problems or parts of problems.

- There is no snow on Janet's driveway when snow begins to fall at midnight. From midnight to 9 A.M., snow accumulates on the driveway at a rate modeled by $f(t) = 7te^{\cos t}$ cubic feet per hour, where t is measured in hours since midnight. Janet starts removing snow at 6 A.M. ($t = 6$). The rate $g(t)$, in cubic feet per hour, at which Janet removes snow from the driveway at time t hours after midnight is modeled by

$$g(t) = \begin{cases} 0 & \text{for } 0 \leq t < 6 \\ 125 & \text{for } 6 \leq t < 7 \\ 108 & \text{for } 7 \leq t \leq 9. \end{cases}$$

- How many cubic feet of snow have accumulated on the driveway by 6 A.M.?
- Find the rate of change of the volume of snow on the driveway at 8 A.M.
- Let $h(t)$ represent the total amount of snow, in cubic feet, that Janet has removed from the driveway at time t hours after midnight. Express h as a piecewise-defined function with domain $0 \leq t \leq 9$.
- How many cubic feet of snow are on the driveway at 9 A.M.?

Free Response Set #18

A graphing calculator is required for some problems or parts of problems.

Let R be the region in the first and second quadrants bounded above by the

graph of $y = \frac{20}{1+x^2}$ and below by the horizontal line $y=2$.

a. Find the area of R

b. The region R is the base of a solid. For this solid, the cross sections

perpendicular to the x -axis are semicircles. Find the volume of this solid.