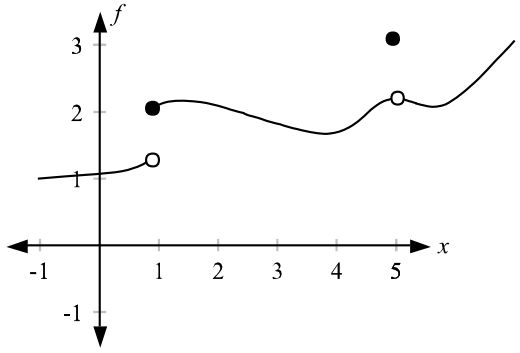
Multiple Choice – Major Theorems & Continuity **Name:**

1. Which of the following statements is true about the figure?



(A) $\lim_{x\to 5} f(x)$ exists

(B) $\lim_{x\to 1} f(x)$ exists

(C) $\lim_{x\to 5} f(x) = f(5)$

(D) $\lim_{x\to 1} f(x) = f(1)$

(E) $\dfrac{f(5)-f(1)}{5-1} = f'(c)$ for all c in (1, 5)

2. Which of the following functions satisfy the hypothesis of the Mean Value Theorem on the interval [0,2]?

I. $f(x) = \sin \pi x + \cos 2x$

II. $f(x) = \sqrt[3]{x-1}$

III. $f(x) = \left|x^2 - 2x\right|$

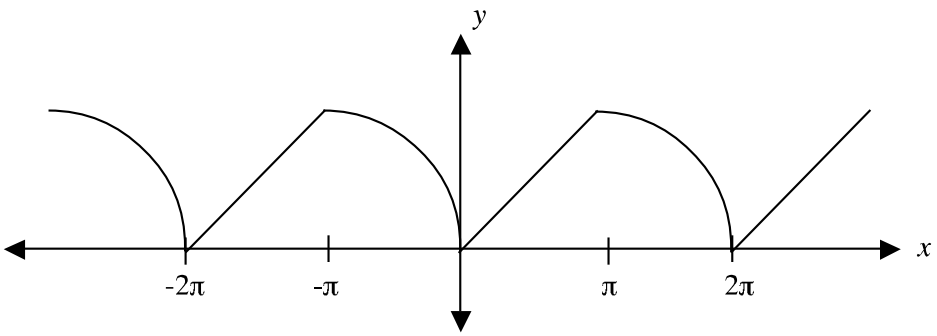
(A) I only (B) II only (C) III only (D) I and II (E) I and III

3. If $\lim_{h\to 0} \dfrac{f(3+h)-f(3)}{h} = 0$, then which of the following must be true?

I. f has derivative at $x = 3$.
II. f is continuous at $x = 3$.
III. f has a critical value at $x = 3$.

(A) I only (B) II only (C) I and II (D) I and III (E) I, II and III

4. The function *f(x)* graphed below is called a sawtooth wave. Which of the following statement about this function is true?



(A) *f(x)* is continuous everywhere.
(B) *f(x)* is differentiable everywhere.
(C) *f(x)* is continuous everywhere but $x = n\pi$.
(D) *f(x)* is an even function.
(E) *f(x)* is a one-to-one function.

5. How many values of *c* satisfy the conclusion of the Mean Value Theorem for $f(x) = x^2 + 1$ on the interval [-1,1]?

(A) 0
(B) 1
(C) 2
(D) 3
(E) 4

Related Rates Multiple Choice

1. A 20-foot ladder leans against the wall of a building. The ladder starts sliding down the wall so that the top of the ladder moves down at the rate of 0.5 ft/sec. How fast is the foot of the ladder moving away from the wall when the foot of the ladder is 12 feet from the wall?

   (A) 0.5 ft/sec (B) $\frac{5}{8}$ ft/sec (C) $\frac{2}{3}$ ft/sec (D) $\frac{4}{3}$ ft/sec (E) $\frac{8}{3}$ ft/sec

2. A spherical balloon is filled with air at 8 $in.^3$/sec. How fast is the diameter of the balloon increasing when the volume of the baloon is 36π $in.^3$? (Volume of a sphere: $V = \frac{4}{3}\pi r^3$.)

   (A) $\frac{4}{9\pi}$ in./sec (B) $\frac{2}{3\pi}$ in./sec (C) $\frac{2}{9\pi}$ in/sec

   (D) $\frac{8}{27\pi}$ in/sec (E) $\frac{2}{27\pi}$ in/sec

3. Sand is falling into a conical pile at the rate of 10 m^3/s such that the height of the pile is always half the diameter of the base of the pile. Find the rate at which the height of the pile is changing when the pile is 5 m high. (Volume of a cone: $V = \frac{1}{3}\pi r^2 h$.)

   (A) $\frac{1}{25\pi}$ m/s (B) $\frac{2}{5\pi}$ m/s (C) $\frac{4}{5\pi}$ m/s

   (D) $\frac{8}{5\pi}$ m/s (E) 250π m/s

4. A sphere is increasing in volume at the rate of 3π cm^3/s. At what rate is its radius changing when the radius is $^1/_2$ cm? (The volume of a sphere is given by $V = \left(\frac{4}{3}\right)\pi r^3$.)

   (A) π cm/s (B) 3 cm/s (C) 2 cm/s (D) 1 cm/s (E) $^1/_2$ cm/s

5. A balloon rises straight up at 10 ft/s. An observer is 40 ft away from the spot where the balloon left the ground. Find the rate of change (in radians per second) of the balloon's angle of elevation when the balloon is 30 ft off the ground.

(A) $\frac{3}{20}$ (B) $\frac{4}{25}$ (C) $\frac{1}{5}$ (D) $\frac{1}{3}$ (E) $\frac{25}{64}$

6. A point moves along the curve $y = x^2 + 1$ such that its x-coordinate is increasing at the rate of 1.5 units per second. At what rate is the point's distance from the origin changing when the point is at (1,2)?

C

(A) $\frac{7\sqrt{5}}{10}$ units/s (B) $\sqrt{5}$ units/s (C) $\frac{3\sqrt{5}}{2}$ units/s

(D) $3\sqrt{5}$ units/s (E) $\frac{15}{2}$ units/s