

Simplifying Various Functions: Problems and Solutions

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Instructions

Simplify the following expressions. Solutions are provided after each problem.

Problems and Solutions

Problem 1: Polynomial Expression

Simplify the polynomial expression:

$$x^3 + 2x^2 - 4x - 8$$

Solution:

$$x^3 + 2x^2 - 4x - 8 = (x^2 - 4)(x + 2) = (x - 2)(x + 2)(x + 2)$$

Simplified Expression:

$$x^3 + 2x^2 - 4x - 8 = (x - 2)(x + 2)^2$$

Problem 2: Rational Expression

Simplify the following rational expression:

$$\frac{3x^2 - 12}{x^2 - 4x + 4}$$

Solution:

$$\frac{3x^2 - 12}{x^2 - 4x + 4} = \frac{3(x^2 - 4)}{(x - 2)^2} = \frac{3(x - 2)(x + 2)}{(x - 2)^2} = \frac{3(x + 2)}{x - 2}, \quad \text{for } x \neq 2$$

Simplified Expression:

$$\frac{3x^2 - 12}{x^2 - 4x + 4} = \frac{3(x + 2)}{x - 2}$$

Problem 3: Logarithmic Expression

Simplify the logarithmic expression:

$$\log\left(\frac{100}{x}\right) - \log(10)$$

Solution:

$$\log\left(\frac{100}{x}\right) - \log(10) = \log\left(\frac{100}{x \cdot 10}\right) = \log\left(\frac{10}{x}\right)$$

Simplified Expression:

$$\log\left(\frac{100}{x}\right) - \log(10) = \log\left(\frac{10}{x}\right)$$

Problem 4: Trigonometric Identity

Simplify the trigonometric expression using identities:

$$\sin^2(x) - \cos^2(x)$$

Solution: Using the identity $\sin^2(x) - \cos^2(x) = -\cos(2x)$:

$$\sin^2(x) - \cos^2(x) = -\cos(2x)$$

Simplified Expression:

$$\sin^2(x) - \cos^2(x) = -\cos(2x)$$

Problem 5: Exponential Expression

Simplify the exponential expression:

$$(2^x) \cdot (4^{x+1})$$

Solution:

$$(2^x) \cdot (4^{x+1}) = 2^x \cdot (2^2)^{x+1} = 2^x \cdot 2^{2x+2} = 2^{3x+2}$$

Simplified Expression:

$$(2^x) \cdot (4^{x+1}) = 2^{3x+2}$$

Problem 6: Polynomial Division

Perform the division and simplify:

$$\frac{x^4 - 16}{x^2 - 4}$$

Solution:

$$\frac{x^4 - 16}{x^2 - 4} = \frac{(x^2 - 4)(x^2 + 4)}{x^2 - 4} = x^2 + 4$$

Simplified Expression:

$$\frac{x^4 - 16}{x^2 - 4} = x^2 + 4, \quad \text{for } x \neq \pm 2$$

Problem 7: Rational Expression with a Complex Fraction

Simplify the complex rational expression:

$$\frac{\frac{2}{x} + \frac{3}{y}}{\frac{1}{x} - \frac{1}{y}}$$

Solution:

$$\frac{\frac{2}{x} + \frac{3}{y}}{\frac{1}{x} - \frac{1}{y}} = \frac{\frac{2y+3x}{xy}}{\frac{y-x}{xy}} = \frac{2y+3x}{y-x}$$

Simplified Expression:

$$\frac{\frac{2}{x} + \frac{3}{y}}{\frac{1}{x} - \frac{1}{y}} = \frac{2y+3x}{y-x}$$

Problem 8: Logarithmic Expression Using Change of Base

Simplify the expression using the change of base formula:

$$\log_3(81)$$

Solution:

$$\log_3(81) = \log_3(3^4) = 4 \log_3(3) = 4$$

Simplified Expression:

$$\log_3(81) = 4$$

Problem 9: Trigonometric Function Using Double Angle Identity

Simplify the trigonometric expression:

$$2 \sin(x) \cos(x)$$

Solution: Using the double angle identity:

$$2 \sin(x) \cos(x) = \sin(2x)$$

Simplified Expression:

$$2 \sin(x) \cos(x) = \sin(2x)$$

Problem 10: Exponential Expression Using Laws of Exponents

Simplify the expression:

$$\frac{3^{2x} \cdot 9^x}{27^x}$$

Solution:

$$\frac{3^{2x} \cdot 9^x}{27^x} = \frac{3^{2x} \cdot (3^2)^x}{(3^3)^x} = \frac{3^{2x} \cdot 3^{2x}}{3^{3x}} = \frac{3^{4x}}{3^{3x}} = 3^x$$

Simplified Expression:

$$\frac{3^{2x} \cdot 9^x}{27^x} = 3^x$$