Instructions

Simplify the following expressions. Show all steps clearly, and express your answers in their simplest forms.

Problems

Problem 1: Polynomial Expression

Simplify the polynomial expression:

$$x^3 + 2x^2 - 4x - 8$$

Factor the polynomial completely.

Problem 2: Rational Expression

Simplify the following rational expression:

$$\frac{3x^2 - 12}{x^2 - 4x + 4}$$

Problem 3: Logarithmic Expression

Simplify the logarithmic expression:

$$\log\left(\frac{100}{x}\right) - \log(10)$$

Problem 4: Trigonometric Identity

Simplify the trigonometric expression using identities:

$$\sin^2(x) - \cos^2(x)$$

Problem 5: Exponential Expression

Simplify the exponential expression:

$$(2^x) \cdot (4^{x+1})$$

Problem 6: Polynomial Division

Perform the division and simplify:

$$\frac{x^4 - 16}{x^2 - 4}$$

Problem 7: Rational Expression with a Complex Fraction

Simplify the complex rational expression:

$$\frac{\frac{2}{x} + \frac{3}{y}}{\frac{1}{x} - \frac{1}{y}}$$

Problem 8: Logarithmic Expression Using Change of Base

Simplify the expression using the change of base formula:

$$\log_3(81)$$

Problem 9: Trigonometric Function Using Double Angle Identity

Simplify the trigonometric expression:

$$2\sin(x)\cos(x)$$

Problem 10: Exponential Expression Using Laws of Exponents

Simplify the expression:

$$\frac{3^{2x} \cdot 9^x}{27^x}$$