Able to simplify expressions

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Introduction

Simplifying mathematical expressions is essential in both calculus and algebra. This document provides a variety of examples demonstrating how to simplify different types of expressions, including polynomial expressions, rational expressions, logarithms, trigonometric identities, derivatives, and integrals.

Simplifying Polynomial Expressions

Example 1: Factoring Polynomials: Simplify the expression by factoring:

$$x^2 - 5x + 6$$

Solution:

$$x^2 - 5x + 6 = (x - 2)(x - 3)$$

Simplified Expression:

$$x^2 - 5x + 6 = (x - 2)(x - 3)$$

Simplifying Polynomial Expressions

Example 2: Expanding a Binomial Expression: Simplify the expression by expanding:

$$(x+2)(x-3)$$

Solution:

$$(x+2)(x-3) = x^2 - 3x + 2x - 6 = x^2 - x - 6$$

Simplified Expression:

$$(x+2)(x-3) = x^2 - x - 6$$

Simplifying Polynomial Expressions

Example 3: Factoring by Grouping: Simplify the expression by factoring:

$$x^3 + 3x^2 + 2x + 6$$

Solution:

$$x^3 + 3x^2 + 2x + 6 = x^2(x + 3) + 2(x + 3) = (x^2 + 2)(x + 3)$$

Simplified Expression:

$$x^3 + 3x^2 + 2x + 6 = (x^2 + 2)(x + 3)$$

Simplifying Rational Expressions

Example 1: Simplifying a Rational Expression: Simplify the following rational expression:

$$\frac{x^2 - 4}{x^2 - 2x}$$

Solution:

$$\frac{x^2 - 4}{x^2 - 2x} = \frac{(x-2)(x+2)}{x(x-2)} = \frac{x+2}{x}, \quad \text{for } x \neq 2$$

Simplified Expression:

$$\frac{x^2 - 4}{x^2 - 2x} = \frac{x+2}{x}$$

Simplifying Rational Expressions

Example 2: Adding Rational Expressions: Simplify the expression:

$$\frac{2}{x} + \frac{3}{x+1}$$

Solution:

$$\frac{2}{x} + \frac{3}{x+1} = \frac{2(x+1)+3x}{x(x+1)} = \frac{2x+2+3x}{x(x+1)} = \frac{5x+2}{x(x+1)}$$

Simplified Expression:

$$\frac{2}{x} + \frac{3}{x+1} = \frac{5x+2}{x(x+1)}$$

Simplifying Rational Expressions

Example 3: Simplifying Complex Rational Expressions: Simplify the following rational expression:

$$\frac{\frac{1}{x}+\frac{1}{y}}{\frac{1}{x}-\frac{1}{y}}$$

Solution:

$$\frac{\frac{1}{x}+\frac{1}{y}}{\frac{1}{x}-\frac{1}{y}}=\frac{\frac{x+y}{xy}}{\frac{x-y}{xy}}=\frac{x+y}{x-y}$$

Simplified Expression:

$$\frac{\frac{1}{x}+\frac{1}{y}}{\frac{1}{x}-\frac{1}{y}}=\frac{x+y}{x-y}$$

Simplifying Rational Expressions

Example 4: Simplifying a Rational Expression with Polynomial Denominators: Simplify the expression:

$$\frac{x^2 - 9}{x^2 - 6x + 9}$$

Solution:

$$\frac{x^2 - 9}{x^2 - 6x + 9} = \frac{(x-3)(x+3)}{(x-3)^2} = \frac{x+3}{x-3}, \quad \text{for } x \neq 3$$

Simplified Expression:

$$\frac{x^2 - 9}{x^2 - 6x + 9} = \frac{x+3}{x-3}$$

Simplifying Logarithmic Expressions

Example 1: Simplifying a Logarithmic Expression Using the Power Rule: Simplify the expression:

$$\log(8x^3)$$

Solution:

$$\log(8x^3) = \log(8) + \log(x^3) = \log(8) + 3\log(x)$$

Simplified Expression:

$$\log(8x^3) = \log(8) + 3\log(x)$$

Simplifying Logarithmic Expressions

Example 2: Simplifying Using the Change of Base Formula: Simplify the expression:

$$\log_2(32)$$

Solution:

$$\log_2(32) = \frac{\log(32)}{\log(2)} = \frac{5}{1} = 5$$

Simplified Expression:

$$\log_2(32) = 5$$

Simplifying Logarithmic Expressions

Example 3: Expanding Logarithms Using the Quotient Rule: Simplify the expression:

$$\log\left(\frac{10x}{5y}\right)$$

Solution:

$$\log\left(\frac{10x}{5y}\right) = \log(10x) - \log(5y) = \log(10) + \log(x) - \log(5) - \log(y)$$

Simplified Expression:

$$\log\left(\frac{10x}{5y}\right) = \log(2) + \log(x) - \log(y)$$

Simplifying Trigonometric Expressions

Example 1: Simplifying Using Trigonometric Identities:
Simplify the expression:

$$1 - \sin^2(x)$$

Solution: Using the Pythagorean identity:

$$1 - \sin^2(x) = \cos^2(x)$$

Simplified Expression:

$$1 - \sin^2(x) = \cos^2(x)$$

Simplifying Trigonometric Expressions

Example 2: Simplifying Using the Sum-to-Product Identity:
Simplify the expression:

$$\sin(x) - \sin(y)$$

Solution: Using the sum-to-product identity:

$$\sin(x) - \sin(y) = 2\cos\left(\frac{x+y}{2}\right)\sin\left(\frac{x-y}{2}\right)$$

Simplified Expression:

$$\sin(x) - \sin(y) = 2\cos\left(\frac{x+y}{2}\right)\sin\left(\frac{x-y}{2}\right)$$

Simplifying Exponential Expressions

Example 1: Simplifying Exponential Expressions Using the Power Rule Simplify the expression:

$$(3^x)^2$$

Solution:

$$(3^x)^2 = 3^{2x}$$

Simplified Expression:

$$(3^x)^2 = 3^{2x}$$

Simplifying Exponential Expressions

Example 2: Simplifying Exponential Expressions with Different Bases Simplify the expression:

$$2^x \cdot 4^x$$

Solution:

$$2^x \cdot 4^x = 2^x \cdot (2^2)^x = 2^x \cdot 2^{2x} = 2^{3x}$$

Simplified Expression:

$$2^x \cdot 4^x = 2^{3x}$$

Conclusion

This document provides a wide range of examples to demonstrate how to simplify various mathematical expressions in algebra and calculus. Each example is carefully explained to help students understand the underlying techniques and apply them to other problems.