Can Perform Polynomial Division

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Introduction

Polynomial division is a process similar to the long division of numbers. It allows us to divide one polynomial (the dividend) by another polynomial (the divisor) to obtain a quotient and a remainder. Polynomial division is particularly useful when simplifying expressions or solving polynomial equations.

Step-by-Step Process

To divide one polynomial by another, follow these steps:

1. **Arrange the Polynomials:** Write the dividend and divisor polynomials in descending order of their degrees.
2. **Divide the Leading Terms:** Divide the leading term (the term with the highest degree) of the dividend by the leading term of the divisor to get the first term of the quotient.
3. **Multiply and Subtract:** Multiply the entire divisor by the term obtained in step 2, and subtract the result from the dividend.
4. **Repeat:** Repeat the process with the new polynomial obtained after subtraction until the degree of the remainder is less than the degree of the divisor.

Example 1: Simple Polynomial Division

Divide $2x^3 + 3x^2 + 4x + 5$ by $x + 1$.

Solution:

$$
\begin{aligned}
\text{Step 1:} \quad & \frac{2x^3 + 3x^2 + 4x + 5}{x + 1} \\
\text{Step 2:} \quad & \frac{2x^3}{x} = 2x^2 \\
\text{Step 3:} \quad & (2x^3 + 3x^2 + 4x + 5) - (2x^2 \cdot (x + 1)) \\
& = 2x^3 + 3x^2 + 4x + 5 - (2x^3 + 2x^2) = x^2 + 4x + 5 \\
\text{Step 4:} \quad & \frac{x^2}{x} = x \\
\text{Step 5:} \quad & (x^2 + 4x + 5) - (x \cdot (x + 1))
\end{aligned}
$$

Example 1: Simple Polynomial Division

Solution Continued:

$$\begin{aligned}
\text{Step 6:} \quad & \frac{3x}{x} = 3 \\
\text{Step 7:} \quad & (3x + 5) - (3 \cdot (x + 1)) \\
& = 3x + 5 - (3x + 3) = 2
\end{aligned}$$

Quotient: $2x^2 + x + 3$
Remainder: 2

Example 2: Division a Higher-Degree Dividend

Divide $x^4 - 3x^3 + 5x^2 - x + 2$ by $x - 1$.

Solution:

Step 1: $\dfrac{x^4 - 3x^3 + 5x^2 - x + 2}{x - 1}$

Step 2: $\dfrac{x^4}{x} = x^3$

Step 3: $(x^4 - 3x^3 + 5x^2 - x + 2) - (x^3 \cdot (x - 1))$

$= x^4 - 3x^3 + 5x^2 - x + 2 - (x^4 - x^3) = -2x^3 + 5x^2 -$

Step 4: $\dfrac{-2x^3}{x} = -2x^2$

Step 5: $(-2x^3 + 5x^2 - x + 2) - (-2x^2 \cdot (x - 1))$

$= -2x^3 + 5x^2 - x + 2 - (-2x^3 + 2x^2) = 7x^2 - x + 2$

Example 2: Division a Higher-Degree Dividend

Solution Continued:

$$\begin{aligned}
\text{Step 6:}\quad & \frac{7x^2}{x} = 7x \\
\text{Step 7:}\quad & (7x^2 - x + 2) - (7x \cdot (x-1)) \\
& = 7x^2 - x + 2 - (7x^2 - 7x) = 6x + 2 \\
\text{Step 8:}\quad & \frac{6x}{x} = 6 \\
\text{Step 9:}\quad & (6x + 2) - (6 \cdot (x-1)) \\
& = 6x + 2 - (6x - 6) = 8
\end{aligned}$$

Quotient: $x^3 - 2x^2 + 7x + 6$
Remainder: 8

Example 3: Division results in a Zero Remainder

Divide $x^2 - 6x + 9$ by $x - 3$.

Solution:

$$\begin{aligned}
\text{Step 1:} \quad & \frac{x^2 - 6x + 9}{x - 3} \\
\text{Step 2:} \quad & \frac{x^2}{x} = x \\
\text{Step 3:} \quad & (x^2 - 6x + 9) - (x \cdot (x - 3)) \\
& = x^2 - 6x + 9 - (x^2 - 3x) = -3x + 9 \\
\text{Step 4:} \quad & \frac{-3x}{x} = -3 \\
\text{Step 5:} \quad & (-3x + 9) - (-3 \cdot (x - 3)) \\
& = -3x + 9 - (-3x + 9) = 0
\end{aligned}$$

Quotient: $x - 3$

Remainder: 0

Conclusion

Polynomial division is a fundamental algebraic technique that is essential for simplifying expressions and solving equations. By following the step-by-step process outlined above, you can successfully divide any polynomial by another, producing a quotient and remainder.