

# Polynomial Division: Problems and Solutions

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## Instructions

Solve the following polynomial division problems. The solutions are provided after each problem.

## Problems and Solutions

### Problem 1

Divide  $2x^3 + 3x^2 + 4x + 5$  by  $x + 1$ .

**Solution:**

$$\frac{2x^3 + 3x^2 + 4x + 5}{x + 1} = 2x^2 + x + 3 \quad \text{with remainder } 2$$
$$2x^3 + 3x^2 + 4x + 5 = (x + 1)(2x^2 + x + 3) + 2$$

### Problem 2

Divide  $x^4 - 2x^3 + x^2 - 3x + 2$  by  $x - 1$ .

**Solution:**

$$\frac{x^4 - 2x^3 + x^2 - 3x + 2}{x - 1} = x^3 - x^2 + 2x - 1 \quad \text{with remainder } 1$$
$$x^4 - 2x^3 + x^2 - 3x + 2 = (x - 1)(x^3 - x^2 + 2x - 1) + 1$$

### Problem 3

Divide  $6x^3 + 11x^2 - 7x - 6$  by  $2x + 3$ .

**Solution:**

$$\frac{6x^3 + 11x^2 - 7x - 6}{2x + 3} = 3x^2 + x - 5 \quad \text{with remainder } 9$$
$$6x^3 + 11x^2 - 7x - 6 = (2x + 3)(3x^2 + x - 5) + 9$$

### Problem 4

Divide  $x^5 - 4x^4 + 3x^3 - x + 1$  by  $x^2 + x - 1$ .

**Solution:**

$$\frac{x^5 - 4x^4 + 3x^3 - x + 1}{x^2 + x - 1} = x^3 - 5x^2 + 8x - 8 \quad \text{with remainder } 7x - 7$$
$$x^5 - 4x^4 + 3x^3 - x + 1 = (x^2 + x - 1)(x^3 - 5x^2 + 8x - 8) + (7x - 7)$$

### Problem 5

Divide  $4x^4 - 3x^3 + 2x^2 - x + 1$  by  $x^2 + 2x + 1$ .

**Solution:**

$$\frac{4x^4 - 3x^3 + 2x^2 - x + 1}{x^2 + 2x + 1} = 4x^2 - 11x + 15 \quad \text{with remainder } -16x + 16$$

$$4x^4 - 3x^3 + 2x^2 - x + 1 = (x^2 + 2x + 1)(4x^2 - 11x + 15) + (-16x + 16)$$

### Problem 6

Divide  $5x^3 - 6x^2 + 4x - 8$  by  $x - 2$ .

**Solution:**

$$\frac{5x^3 - 6x^2 + 4x - 8}{x - 2} = 5x^2 + 4x + 12 \quad \text{with remainder } 16$$

$$5x^3 - 6x^2 + 4x - 8 = (x - 2)(5x^2 + 4x + 12) + 16$$

### Problem 7

Divide  $3x^4 + 5x^3 - 2x^2 + 7x + 4$  by  $x^2 - x + 2$ .

**Solution:**

$$\frac{3x^4 + 5x^3 - 2x^2 + 7x + 4}{x^2 - x + 2} = 3x^2 + 8x + 6 \quad \text{with remainder } -5x - 8$$

$$3x^4 + 5x^3 - 2x^2 + 7x + 4 = (x^2 - x + 2)(3x^2 + 8x + 6) + (-5x - 8)$$