Polynomial Division: Problems and Solutions

APMA Faculty
University of Virginia

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Instructions

Solve the following polynomial division problems. The solutions are provided after each problem.

Problems and Solutions

Problem 1

Divide $2x^3 + 3x^2 + 4x + 5$ by $x + 1$.

Solution:

$$\frac{2x^3 + 3x^2 + 4x + 5}{x + 1} = 2x^2 + x + 3 \quad \text{with remainder } 2$$

$$2x^3 + 3x^2 + 4x + 5 = (x + 1)(2x^2 + x + 3) + 2$$

Problem 2

Divide $x^4 - 2x^3 + x^2 - 3x + 2$ by $x - 1$.

Solution:

$$\frac{x^4 - 2x^3 + x^2 - 3x + 2}{x - 1} = x^3 - x^2 - 3 \quad \text{with remainder } -1$$

$$x^4 - 2x^3 + x^2 - 3x + 2 = (x - 1)(x^3 - x^2 - 3) - 1$$

Problem 3

Divide $6x^3 + 11x^2 - 7x - 6$ by $2x + 3$.

Solution:

$$\frac{6x^3 + 11x^2 - 7x - 6}{2x + 3} = 3x^2 + x - 5 \quad \text{with remainder } 9$$

$$6x^3 + 11x^2 - 7x - 6 = (2x + 3)(3x^2 + x - 5) + 9$$

Problem 4

Divide $x^5 - 4x^4 + 3x^3 - x + 1$ by $x^2 + x - 1$.

Solution:

$$\frac{x^5 - 4x^4 + 3x^3 - x + 1}{x^2 + x - 1} = x^3 - 5x^2 + 9x - 14 \quad \text{with remainder } 22x - 13$$

$$x^5 - 4x^4 + 3x^3 - x + 1 = (x^2 + x - 1)(x^3 - 5x^2 + 9x - 14) + (22x - 13)$$

Problem 5

Divide $4x^4 - 3x^3 + 2x^2 - x + 1$ by $x^2 + 2x + 1$.

Solution:

$$\frac{4x^4 - 3x^3 + 2x^2 - x + 1}{x^2 + 2x + 1} = 4x^2 - 11x + 20 \quad \text{with remainder } -30x - 19$$

$$4x^4 - 3x^3 + 2x^2 - x + 1 = (x^2 + 2x + 1)(4x^2 - 11x + 20) + (-30x - 19)$$

Problem 6

Divide $5x^3 - 6x^2 + 4x - 8$ by $x - 2$.

Solution:

$$\frac{5x^3 - 6x^2 + 4x - 8}{x - 2} = 5x^2 + 4x + 12 \quad \text{with remainder } 16$$

$$5x^3 - 6x^2 + 4x - 8 = (x - 2)(5x^2 + 4x + 12) + 16$$

Problem 7

Divide $3x^4 + 5x^3 - 2x^2 + 7x + 4$ by $x^2 - x + 2$.

Solution:

$$\frac{3x^4 + 5x^3 - 2x^2 + 7x + 4}{x^2 - x + 2} = 3x^2 + 8x \quad \text{with remainder } -9x + 4$$

$$3x^4 + 5x^3 - 2x^2 + 7x + 4 = (x^2 - x + 2)(3x^2 + 8x) + (-9x + 4)$$