Can use Laws of Exponents

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Introduction and Functions

A function is such that any input in the domain results in exactly one output in the range. Also exponents are mathematical operations or functions that represent repeated multiplication of the same factor. The laws of exponents simplify expressions involving exponents.

For example:

$f(x) = 2^x$

$f(x) = 5^x$

$f(x) = e^x$

$f(x) = (1/2)^x$

Product of Powers Rule

When multiplying two exponents with the same base, add the exponents. if a and b are positive numbers and m and n are any real numbers, then:

$$\text{Formula:} \quad a^m \cdot a^n = a^{m+n}$$

$$\text{Example:} \quad 2^3 \cdot 2^4 = 2^{3+4} = 2^7 = 128$$

Quotient of Powers Rule

When dividing two exponents with the same base, subtract the exponents. if a and b are positive numbers and m and n are any real numbers, then:

$$\text{Formula:} \quad \frac{a^m}{a^n} = a^{m-n}$$

$$\text{Example:} \quad \frac{10^5}{10^2} = 10^{5-2} = 10^3 = 1000$$

Power of a Power Rule

When raising an exponent to another exponent, multiply the exponents. if a and b are positive numbers and m and n are any real numbers, then:

$$\text{Formula:} \quad (a^m)^n = a^{m \cdot n}$$

$$\text{Example:} \quad (3^2)^3 = 3^{2 \cdot 3} = 3^6 = 729$$

Power of a Product Rule

When raising a product to an exponent, raise each factor to the exponent. if a and b are positive numbers and m and n are any real numbers, then:

$$\begin{aligned} \text{Formula:} \quad & (ab)^n = a^n \cdot b^n \\ \text{Example:} \quad & (2 \cdot 5)^3 = 2^3 \cdot 5^3 = 8 \cdot 125 = 1000 \end{aligned}$$

Power of a Quotient Rule

When raising a quotient to an exponent, raise both the numerator and the denominator to the exponent. if a and b are positive numbers and m and n are any real numbers, then:

$$\text{Formula:} \quad \left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$$

$$\text{Example:} \quad \left(\frac{4}{2}\right)^3 = \frac{4^3}{2^3} = \frac{64}{8} = 8$$

Zero Exponent Rule

Any nonzero number raised to the power of zero equals one.

$$\text{Formula:} \quad a^0 = 1 \quad (\text{where } a \neq 0)$$
$$\text{Example:} \quad 5^0 = 1$$

Negative Exponent Rule

A negative exponent represents the reciprocal of the base raised to the absolute value of the exponent. if a and b are positive numbers and m and n are any real numbers, then:

$$\text{Formula:} \quad a^{-n} = \frac{1}{a^n}$$

$$\text{Example:} \quad 2^{-3} = \frac{1}{2^3} = \frac{1}{8}$$

Examples of Use

*Simplifying Logarithmic Expressions Given: $\log_2(32) + \log_2(4)$

Using the Product Rule: $\log_2(32 \cdot 4) = \log_2(128) = 7$

*Solving Logarithmic Equations Given: $\log_5(x) + \log_5(25) = 3$

Simplify using the Product Rule: $\log_5(25x) = 3$

Rewrite as an exponential equation: $5^3 = 25x$

Solve for x: $125 = 25x \rightarrow x = 5$

These laws are essential for solving logarithmic equations and simplifying expressions in various fields of science and engineering.

The Natural Exponential Function

The natural exponential function is one of the most important functions in mathematics. It is defined as $f(x) = e^x$, where e is the base of the natural logarithm, approximately equal to 2.71828. The function e^x is unique in that it is its own derivative, making it extremely useful in various fields such as calculus, physics, and engineering.

*Definition: The natural exponential function is defined as:

$$f(x) = e^x$$

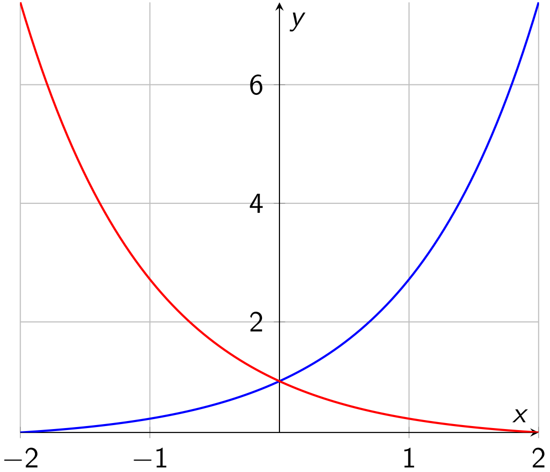
where $e \approx 2.71828$ is the base of the natural logarithm.

Properties of the Natural Exponential Function

- The function e^x is always positive.
- The derivative of e^x is e^x.
- The function e^x passes through the point (0,1), i.e., $e^0 = 1$.
- As $x \to \infty$, $e^x \to \infty$ and as $x \to -\infty$, $e^x \to 0$.

Plot of e^x

Below is the plot of the natural exponential function and $f(x) = e^x$ and $f(x) = e^{-x}$:



Properties of the Plot

- The curve passes through the point $(0, 1)$ because $e^0 = 1$.
- As x increases, the function grows exponentially.
- As x decreases, the function approaches zero but never reaches it.