AS-level-Y12-LogBaseChange

September 30, 2021

Reminder:
$log\,(x^n) = n \cdot log\,(x)$

True in any base.

Statements:
$(A) : x = a^y$
$(B) : x = b^{v \cdot y} = (b^y)^v$

Developing:
$(A\&B) \implies a = b^v \implies v = log_b a$

$(A) \implies y = log_a x$
$(B) \implies b^y = x^{\frac{1}{v}} \implies y = log_b x^{\frac{1}{v}} = \frac{1}{v} log_b x = \frac{log_b x}{log_b a}$

Conclusion:
$log_a x = \frac{log_b x}{log_b a}$

$log_b x = \frac{log_a x}{log_a b}$

$log_a x = \frac{log_b x}{log_b a} = \frac{log_a x}{log_a b \cdot log_b a}$

$log_a b = \frac{1}{log_b a}$

This is called a base change.