

Apuntes de electromagnetismo: relación entre E y B

Ecuaciones de Maxwell:

$$\text{I} \quad \nabla \cdot \mathbf{E} = \frac{\rho}{\epsilon_0} \quad \text{Ley de Gauss en electricidad}$$

$$\text{II} \quad \nabla \cdot \mathbf{E} + \frac{\partial B}{\partial t} = 0 \quad \text{Ley de Inducción de Faraday}$$

$$\text{III} \quad \nabla \cdot \mathbf{B} = 0 \quad \text{Ley de Gauss en magnetismo}$$

$$\text{IV} \quad \nabla \cdot \mathbf{B} - \frac{1}{C^2} \cdot \frac{\partial B}{\partial t} = \mu_0 \quad \text{Ley de Ampère-Maxwell}$$

Operador diferencial ∇ : $\nabla = \frac{\partial}{\partial x} \mathbf{i} + \frac{\partial}{\partial y} \mathbf{j} + \frac{\partial}{\partial z} \mathbf{k}$

Gradiente: $\nabla \Phi = \frac{\partial \Phi}{\partial x} \mathbf{i} + \frac{\partial \Phi}{\partial y} \mathbf{j} + \frac{\partial \Phi}{\partial z} \mathbf{k}$

Divergencia: $\nabla \cdot \mathbf{V} \Rightarrow \left(\frac{\partial}{\partial x} \mathbf{i} + \frac{\partial}{\partial y} \mathbf{j} + \frac{\partial}{\partial z} \mathbf{k} \right) (V_1 \mathbf{i} + V_2 \mathbf{j} + V_3 \mathbf{k}) \Rightarrow$

$$\nabla \cdot \mathbf{V} = \frac{\partial V_1}{\partial x} \mathbf{i} + \frac{\partial V_2}{\partial y} \mathbf{j} + \frac{\partial V_3}{\partial z} \mathbf{k}$$

Rotacional: $\nabla \wedge \mathbf{V} \Rightarrow \nabla \wedge \mathbf{V} = \left(\frac{\partial}{\partial x} \mathbf{i} + \frac{\partial}{\partial y} \mathbf{j} + \frac{\partial}{\partial z} \mathbf{k} \right) \wedge (V_1 \mathbf{i} + V_2 \mathbf{j} + V_3 \mathbf{k}) \Rightarrow$

$$\nabla \wedge \mathbf{V} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ V_1 & V_2 & V_3 \end{vmatrix} \quad \text{Como: } \bar{E} = E_0 \cos(kx - \omega t), \text{ y } \bar{B} = B_0 \cos(kx - \omega t) \Rightarrow$$

$$\frac{dE}{dx} = -k \cdot E \sin(kx - \omega t), \text{ y } \frac{dB}{dt} = -\omega \cdot B \sin(kx - \omega t). \text{ Por la 2ª Ley de Maxwell:}$$

$$\nabla \wedge \mathbf{E} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ E_1 & E_y & E_z \end{vmatrix} \Rightarrow \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 0 & E_y & 0 \end{vmatrix} = \mathbf{i} \cdot \begin{vmatrix} \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ E_y & 0 \end{vmatrix} - \mathbf{j} \cdot \begin{vmatrix} \frac{\partial}{\partial x} & \frac{\partial}{\partial z} \\ 0 & 0 \end{vmatrix} +$$

$$+\mathbf{k} \cdot \begin{vmatrix} \frac{\partial}{\partial x} & \frac{\partial}{\partial y} \\ 0 & E_y \end{vmatrix} = \mathbf{i} \cdot 0 - \mathbf{j} \cdot 0 + \mathbf{k} \cdot \frac{\partial E_y}{\partial x} \Rightarrow \nabla \wedge \mathbf{E} = \frac{\partial E}{\partial x} \mathbf{k}; \text{ y } \frac{\partial E}{\partial x} = -\frac{\partial B}{\partial t} \Rightarrow$$

$$k \cdot E \sin(kx - \omega t) = \omega \cdot B \sin(kx - \omega t) \Rightarrow \mathbf{K} \cdot \mathbf{E} = \omega \cdot \mathbf{B} \Rightarrow \frac{E}{B} = \frac{2\pi f}{2\pi} \Rightarrow \frac{E}{B} = \frac{2\pi f}{\lambda}$$

$$\frac{E}{B} = \lambda \cdot f = c \Rightarrow \mathbf{E} = c \cdot \mathbf{B}$$