# EXPERIMENT 7

## System of First order Linear Differential Equation

A system of n linear first order differential equations in n unknowns ($n \times n$ system of linear equations) has the general form:-

$$(1) \begin{cases} x_1' = a_{11} x_1 + a_{12} x_2 + \dots a_{1n} x_n + g_1(t) \\ x_2' = a_{21} x_1 + a_{22} x_2 + \dots a_{2n} x_n + g_2(t) \\ \vdots \\ x_n' = a_{n1} x_1 + a_{n2} x_2 + \dots a_{nn} x_n + g_n(t) \end{cases}$$

Where $a_{ij}$ are arbitary constants and $g_i$ are arbitary functions of $t$. Eq$^n$ are homogeneous if all $g_i$ are zero.

(1) is often written in shorthand as

$$X' = AX + G$$

Where $X' = [x_i']_{n \times 1}$ $A = [a_{ij}]_{n \times n}$

$X = [x_i]_{n \times 1}$ & $G = [g_i(t)]_{n \times 1}$

If the matrix $A$ has 2 eigen values (distinct) $\lambda_1$ & $\lambda_2$, & their respective eigen vectors are $X_1$ & $X_2$, then the system $X' = AX$ ~~can be written~~

classmate
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has a general solution,

$$X = C_1 X_1 e^{\lambda_1 t} + C_2 X_2 e^{\lambda_2 t}$$

## System of Second Order Linear Differential ~~Derivatives~~ Equations

consider the system of the form

$$(2) \begin{cases} x_1'' = a_{11} x_1 + a_{12} x_2 + \dots a_{1n} x_n \\ x_2'' = a_{21} x_1 + a_{22} x_2 + \dots a_{2n} x_n \\ \vdots \\ x_n'' = a_{n1} x_1 + a_{n2} x_2 + \dots a_{nn} x_n \end{cases}$$

where $a_{ij}$ are arbitary constants

then the solution of (2), $X'' = AX$ is

$$X = PY$$

where $Y$ is the solution of $Y'' = DY$,

$P$ is the modal matrix of $X$ & $D$ is its diagonal matrix

1) Solve
$$x_1' = x_1 + 2x_2$$
$$x_2' = 0.5x_1 + x_2$$

$$x_1(0) = 16, \quad x_2(0) = -2$$

2) The Governing equations of a certain vibrating system are

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$$x_1'' = 2x_1 + x_2$$

$$x_2'' = 9x_1 + 2x_2$$

Solve the system by Matrix method

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20/12/19

Editor - /home/matlab/diff.m

laplaceode.m | untitled2.m | diff.m

```
clc
clear all
syms t C1 C2
A=input('Enter A: ');
[P,D]=eig(A);
L1=D(1);L2=D(4);
y1=C1*exp(L1*t); y2=C2*exp(L2*t);
Y=[y1;y2];
X=P*Y;
Cond=input('Enter the initial conditions [t0,x10,x20]:');
t0=Cond(1); x10=Cond(2); x20=Cond(3);
eq1=subs(X(1)-x10,t0); eq2=subs(X(2)-x20,t0);
[C1,C2]=solve(eq1,eq2);
X=subs(X)
```

Command Window

```
Enter A: [1 2;0.5 1]
Enter the initial conditions [t0,x10,x20]:[0 16 -2]

X =

 6*exp(2*t) + 10
  3*exp(2*t) - 5

>>
```

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**Editor - /home/matlab/diff1.m**

laplaceode.m | untitled2.m | diff.m | diff1.m | +

```
clc
clear all
A=input('Enter A: ');
[P,D]=eig(A);
Sol1=dsolve(['D2y=',num2str(D(1)),'*y']);
Sol2=dsolve(['D2y=',num2str(D(4)),'*y']);
X=P*[Sol1;Sol2];
disp('x1='); disp(X(1))
disp('x2='); disp(X(2))
```

**Command Window**

```
Warning: Function diff has the same name as a MATLAB builtin. We suggest you rename the function to a
Enter A: [2 1; 9 2]
x1=
(10^(1/2)*(C1*exp(5^(1/2)*t) + C2*exp(-5^(1/2)*t)))/10 - (10^(1/2)*(C3*cos(t) + C4*sin(t)))/10

x2=
(3*10^(1/2)*(C3*cos(t) + C4*sin(t)))/10 + (3*10^(1/2)*(C1*exp(5^(1/2)*t) + C2*exp(-5^(1/2)*t)))/10

>>
```

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## Series solution of ordinary Differential Equations

$$P_0 \frac{d^2y}{dx^2} + P_1 \frac{dy}{dx} + P_2 y = 0 \quad \text{——} \quad (1)$$

Where $P$'s are polynomials in $x$ & $P_0 \neq 0$ at $x=0$.

1) Assume it's solution to be of the form
$y = a_0 + a_1 x + a_2 x^2 + a_3 x^3 + \ldots a_n x^n$ —— (2)

2) Calculate $\frac{dy}{dx}$ & $\frac{d^2y}{dx^2}$ from (2) & ~~substit~~ determine the values of $a_2, a_3, a_4 \ldots a_n$ in terms of $a_0, a_1$.

3) Substituting the values of $a_2, a_3, a_4 \ldots$ in (2), we get the desired series solution having $a_0$ & $a_1$ as its' arbitary constants.

N. Ban
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Q Solve in series, the eq. $\frac{d^2y}{dx^2} + y = 0$

→ $y = A(1 - x^2/2 + \ldots) + B(x - x^3/6 + \ldots)$

Q Solve $y'' + xy = 0$

→ $y = A(1 - x^3/6 + \ldots) + B(x - x^4/12 + \ldots)$

Editor - /home/matlab/sode.m

sode.m

```matlab
clc
clear
syms x a0 a1 a2 a3
a = [a0 a1 a2 a3];
y = sum(a.*(x).^[0:3]);
dy = diff(y);
d2y = diff(dy);
gde = collect(d2y+y,x);
cof=coeffs(gde,x);
A2=solve(cof(1),a2);
A3=solve(cof(2),a3);
y=subs(y,{a2,a3},{A2,A3});
y=coeffs(y,[a1 a0]);
disp('Solution is')
disp(['y=A(',char(y(1)),'+ ...)+B(',char(y(2)),'+ ...)'])
```

Command Window

```
Solution is
y=A(1 - x^2/2+ ...)+B(x - x^3/6+ ...)
fx >>
```

sode.m

```matlab
clc
clear
syms x a0 a1 a2 a3 a4
a = [a0 a1 a2 a3 a4];
y = sum(a.*(x).^[0:4]);
dy = diff(y);
d2y = diff(dy);
gde = collect(d2y+y*x,x);
cof=coeffs(gde,x);
A2=solve(cof(1),a2);
A3=solve(cof(2),a3);
A4=solve(cof(3),a4);
y=subs(y,{a2,a3,a4},{A2,A3,A4});
y=coeffs(y,[a1,a0]);
disp('Solution is')
```

Command Window

```
Solution is
y=A(1 - x^3/6+ ...)+B(x - x^4/12+ ...)
>>
```