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Experiment 3

Fourier Series

$$F(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\frac{n\pi x}{l} + \sum_{n=1}^{\infty} b_n \sin\frac{n\pi x}{l}$$

$a_0, a_n, b_n \rightarrow$ Fourier Coefficien

$$c \leq x \leq c+2l$$

$$a_0 = \frac{1}{l}\int_c^{c+2l} f(x)\,dx$$

$$a_n = \frac{1}{l}\int_c^{c+2l} f(x)\cos\left(\frac{n\pi x}{l}\right)dx$$

$$b_n = \frac{1}{l}\int_c^{c+2l} f(x)\cos\left(\frac{n\pi x}{l}\right)dx$$

The fourier series of a periodic function $f(x)$ of period $2l$ defined on the interval $(c, c+2l)$ is given by $F(x)$ as mentioned above can be calculated by formulas.

In the fourier series, the term $(a_1 \cos x + b_1 \sin x)$ is known as the fundamental harmonic. $(a_2 \cos 2x + b_2 \sin 2x)$ is the second harmonic and so on.

Matlab Syntax used :-

Syms var1 var2 → creates symbolic variables

disp (x) → displays the content of x without printing the variable name.

int (exp, var, a, b) → evaluates the definite integration of exp with respect to var from a to b.

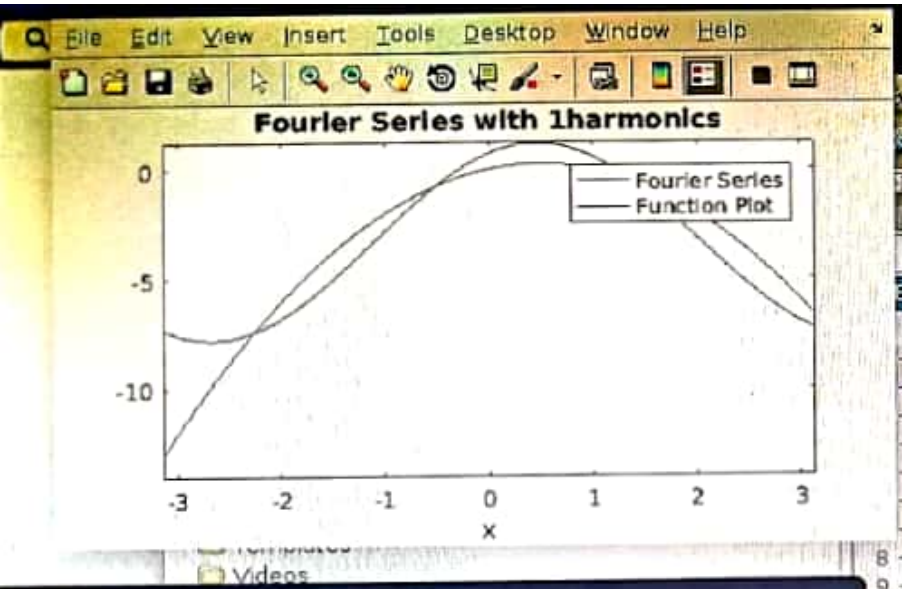
ezplot (fun, [xmin, xmax]) → Plots fun over the domain [xmin, xmax].

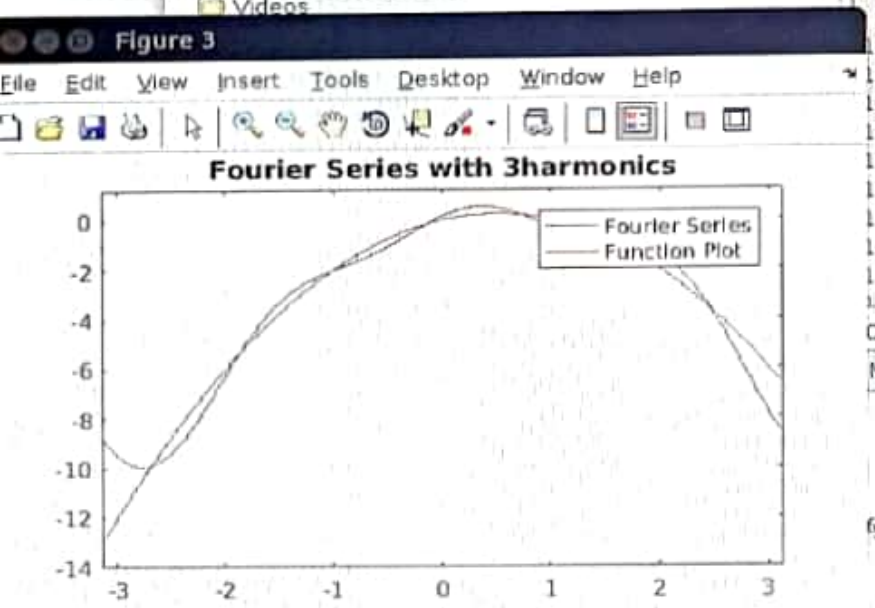
9 Find the fourier series of $f(x) = x - x^2$ in the range $-\pi < x < \pi$ upto 3 harmonics.

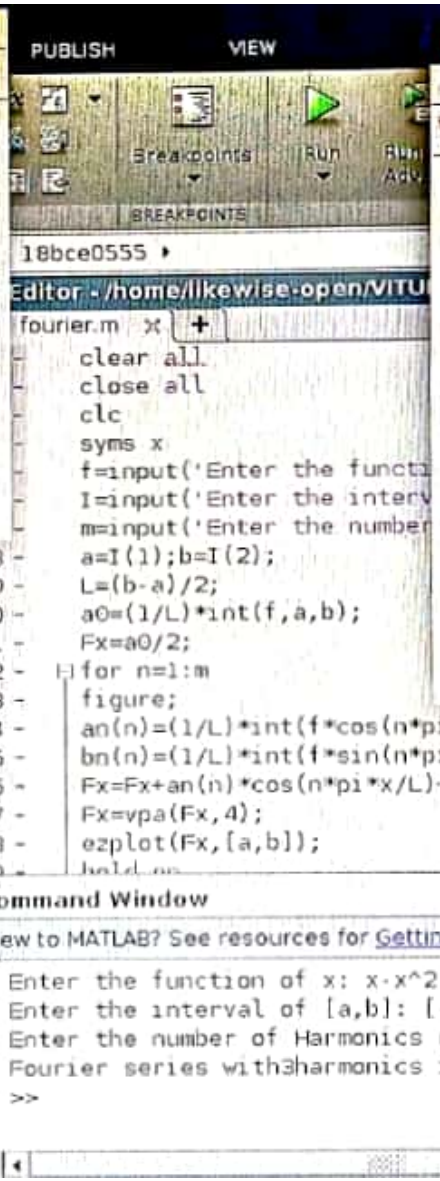
6/12/19

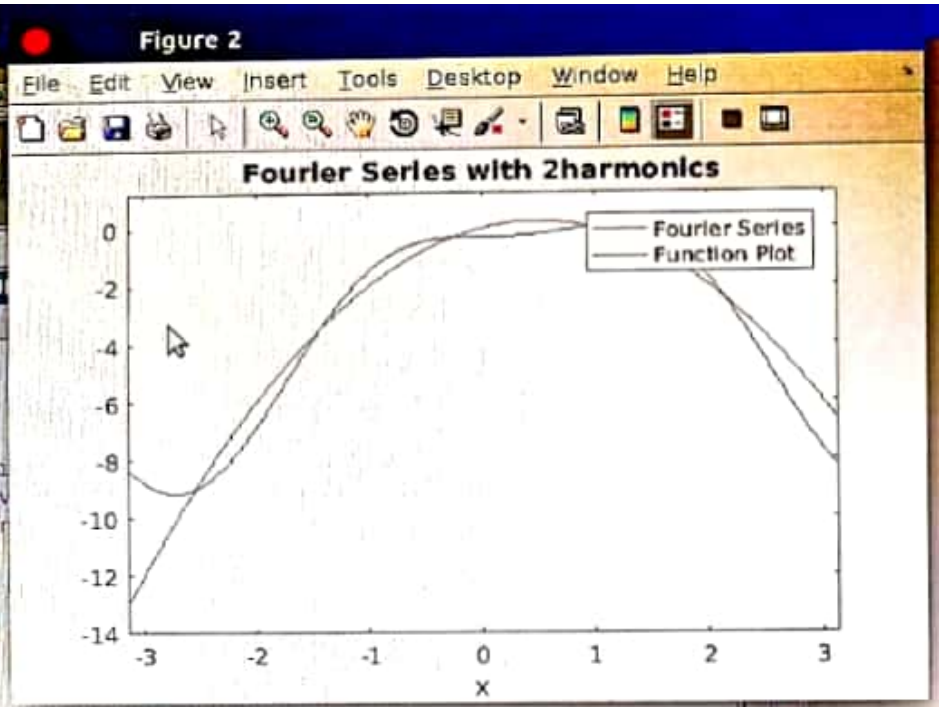
ourier.m* × | untitled2* × | +

```
clear all
close all
clc
syms x
f=input('Enter the function of x: ');
I=input('Enter the interval of [a,b]: ');
m=input('Enter the number of Harmonics required: ');
a=I(1);b=I(2);
L=(b-a)/2;
a0=(1/L)*int(f,a,b);
Fx=a0/2;
for n=1:m
    figure;
    an(n)=(1/L)*int(f*cos(n*pi*x/L),a,b);
    bn(n)=(1/L)*int(f*sin(n*pi*x/L),a,b);
    Fx=Fx+an(n)*cos(n*pi*x/L)+bn(n)*sin(n*pi*x/L);
    Fx=vpa(Fx,4);
    ezplot(Fx,[a,b]);
    hold on
    hold on
    ezplot(f,[a,b]);
    title(['Fourier Series with ',num2str(n+±
    ),'harmonics']);
    legend('Fourier Series', 'Function Plot');
    hold off
end
disp(strcat('Fourier series with', num2str(n),'harmonics is:',char(Fx)))
```









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Experiment 4

Harmonic Analysis

In practice, the function is often not given by a formula, but a table of corresponding values.

Since the mean value of the function $y = f(x)$ is given over the interval (a, b) by $\frac{1}{b-a}\int_a^b f(x)\,dx$, the fourier series in the interval $(a, a+2l)$ can be written as,

$$f(x) = \frac{a_0}{2} + a_1\cos\theta + b_1\sin\theta + a_2\cos\theta + b_2\sin\theta \ldots$$

where $\theta = \frac{\pi x}{l}$

The term $(a_1\cos\theta + b_1\sin\theta)$ is called the Fundamental or First harmonic, $(a_2\cos\theta + b_2\sin\theta)$ is the second harmonic and so on.

Fourier Coefficients :-

$$a_0 = 2\times\frac{1}{2l}\int_a^{a+2l} f(x)\,dx$$

$= 2 \times$ Mean of $f(x)$ in $(a, a+2l)$

$$a_n = 2\times\frac{1}{2l}\int_a^{a+2l} f(x)\cos\left(\frac{n\pi x}{l}\right)dx$$

$= 2 \times$ Mean of $f(x)\cos\frac{n\pi x}{l}$ in $(a, a+2l)$

$$b_n = \frac{2\times 1}{2L}\int_a^{a+2L} f(x)\sin\left(\frac{n\pi x}{L}\right)dx$$

$= 2 \times$ Mean of $f(x)\sin\left(\frac{n\pi x}{L}\right)$ in $(a, a+2L)$

Given m data points of the function $y = f(x)$ of period

$$a_0 = \frac{2\times\sum f(x)}{m} \qquad a_n = \frac{2\sum f(x)\cos\theta}{m}$$

$$b_n = \frac{2\times\sum f(x)\sin n\theta}{m}$$

Matlab syntax used :-

Syms var1 var2 → Creates symbolic variables var1 & var2.

disp (x) → Displays content of x without variable name

length (x) → Returns length of vector x

plot (fun) → Plots the function whose domain & range is given

Q Compute the first 4 harmonics of the fourier series

x	0	1	2	3	4	5
y	4	8	15	7	6	2

Input :-

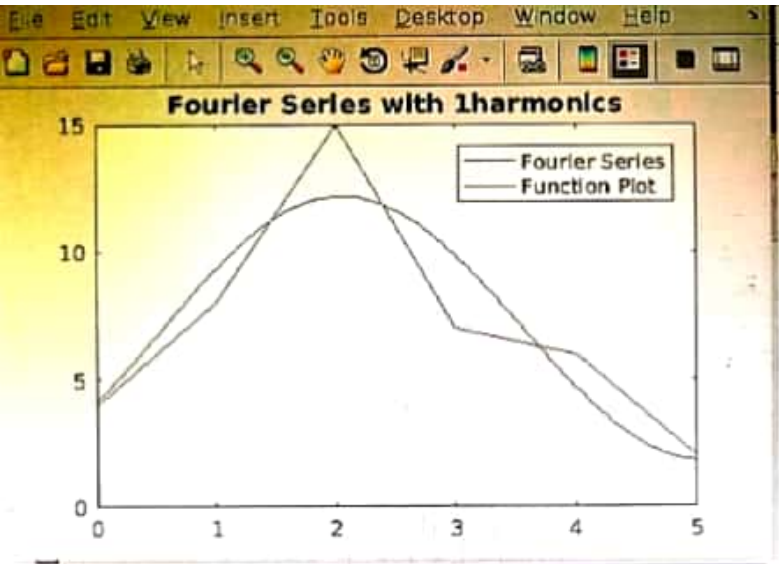
Enter the equally spaced values of x: 0:5

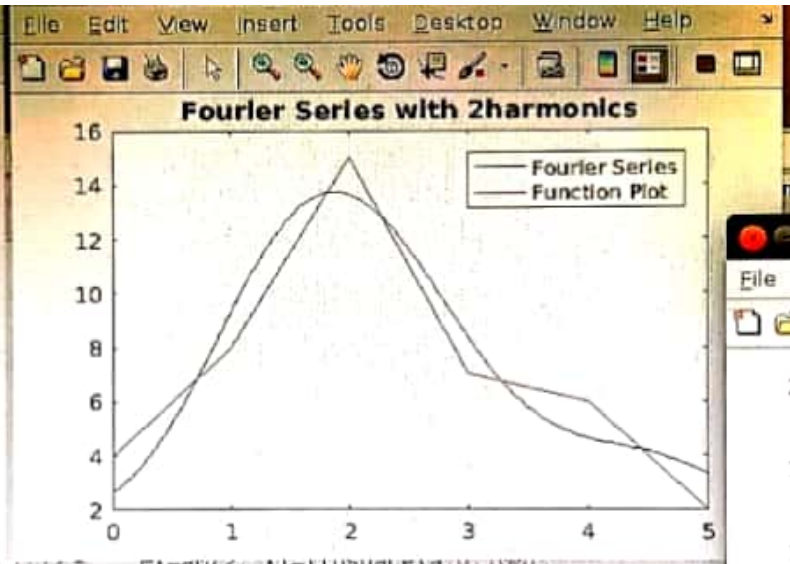
Enter the values of $y = f(x)$: [4, 8 15 7 6 2]

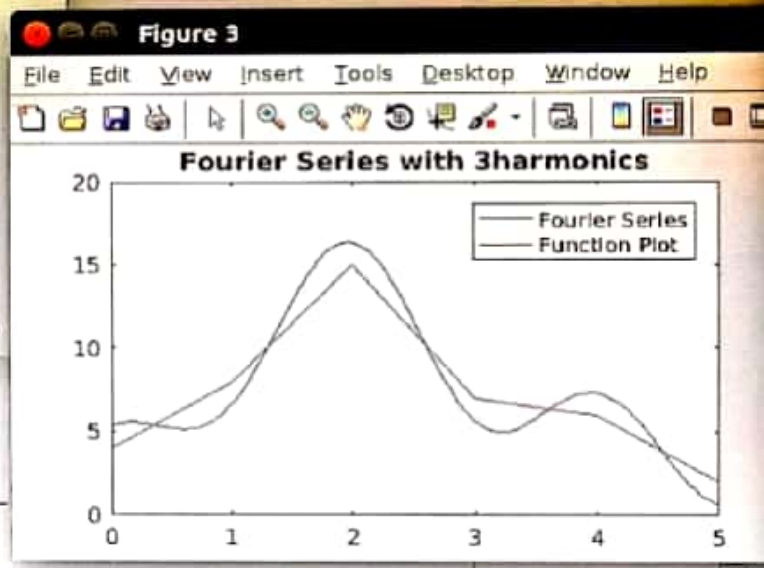
Enter the no. of harmonics : 4

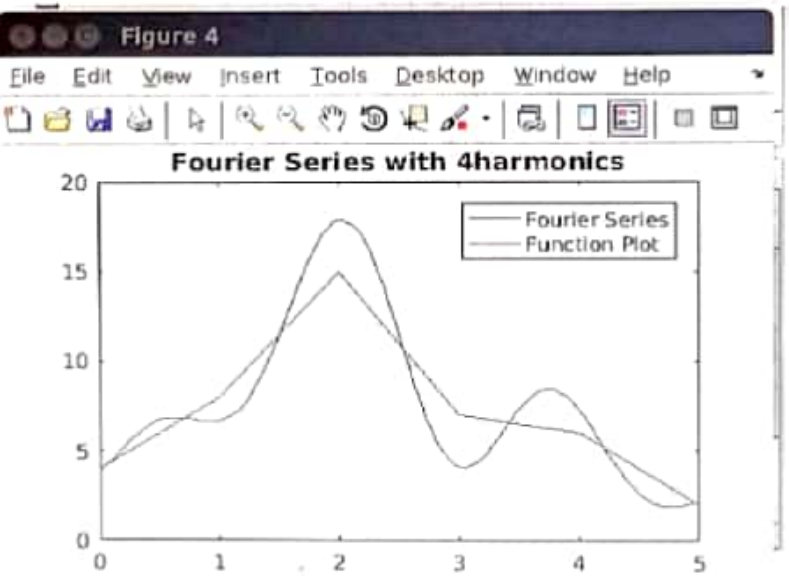
fourier.m | harmonics.m | +

```
clear all
clc
syms t
x=input('Enter the equally spaced values of x: ');
y=input('Enter the values of y=f(x): ');
m=input('Enter the number of harmonics required: ');
n=length(x);a=x(1);b=x(n);
h=x(2)-x(1);
L=(b-a+h)/2;
theta=pi*x/L;
a0=(2/n)*sum(y);
Fx=a0/2; x1=linspace(a,b,100);
for i=1:m
    figure
    an=(2/n)*sum(y.*cos(i*theta));
    bn=(2/n)*sum(y.*sin(i*theta));
    Fx=Fx+an*cos(i*pi*t/L)+bn*sin(i*pi*t/L) ;
    Fx=vpa(Fx,4);
    Fx1=subs(Fx,t,x1);
    plot(x1,Fx1);
    hold on
    plot(x,y);
    title(['Fourier Series with ',num2str( i ),'harmonics'])
    legend('Fourier Series', 'Function Plot')
    hold off;
end
disp(strcat('Fourier series with', num2str(i),'harmonics is:',char(Fx)));
```










```
13 -  for i=1:m
14 -      figure
15 -      an=(2/n)*sum(y.*cos(i*theta));
16 -      bn=(2/n)*sum(y.*sin(i*theta));
17 -      Fx=Fx+an*cos(i*pi*t/L)+bn*sin(i*pi*t/L
18 -      Fx=vpa(Fx,4);
19 -      Fx1=subs(Fx,t,x1);
20 -      plot(x1,Fx1);
```

Command Window

New to MATLAB? See resources for Getting Started.

```
Enter the equally spaced values of x: 0:5
Enter the values of y=f(x): [4 8 15 7 6 2]
Enter the number of harmonics required: 4
Fourier series with4harmonics is:4.33*sin(1.047*t) - 1.5*cos(2.094*t) - 1.5*cos(4.189*t) - 2.833*cos(
fx >>
```