AFM 372 Formulas for Exams Spring 2020

Capital Budgeting

- Capital cost of allowance

Fundamentals of Capital Budgeting

- UCC and CAA for year t, and $\text{CCA}_t = \text{UCC}_t \times d$

$$\text{UCC}_t = \begin{cases} \text{CapEx}/2 & \text{for } t = 1 \\ \text{CapEx} \times (1 - d/2) \times (1 - d)^{t-2} & \text{for } t \geq 1 \end{cases}$$

- Free Cash Flow

$$\begin{aligned} \text{FCF}_t =& (\text{Revenues}_t - \text{Costs}_t) \times (1 - \tau_t) - \text{CapEx}_t - \Delta\text{NWC}_t \\ &+ \tau_c \times \text{CCA}_t \end{aligned}$$

- PV of CCA tax shields

$$\text{PV}_{\text{CCA tax shields}} = \frac{\text{CapEx} \times d \times \tau_c}{r + d} \times \left[\frac{1 + r/2}{1 + r}\right]$$

- PV of lost CCA tax shields

$$\text{PV}_{\text{lost CCA tax shield}} = \frac{\min(\text{Sale Price}, \text{CapEx}) \times d \times \tau_c}{r + d} \times \frac{1}{(1 + r)^t}$$

Estimating the Cost of Capital

- CAPM

$$r_i = r_f + \beta_i \times \underbrace{(\mathrm{E}[R_{Mkt}] - r_f)}_{\text{market risk premium}}$$

- expected return of the bond is

$$\begin{aligned} r_D &= (1-p)y + p(y-L) = y - pL \\ &= \text{Yield to Maturity} - \text{Prob(default)} \times \text{Expected Loss Rate} \end{aligned}$$

- the relationship between different ratios

$$\frac{D}{E} = x \implies \frac{E}{V} = \frac{1}{x+1} \implies \frac{D}{V} = 1 - \frac{E}{V}$$

- asset or unlevered cost of capital, where D is the net debt (also known as pretax WACC)

$$r_U = \frac{E}{E+D} r_E + \frac{D}{E+D} r_D$$

- asset or unlevered beta

$$\beta_U = \frac{E}{E+D} \beta_E + \frac{D}{E+D} \beta_E$$

- weighted average cost of capital with firm's corporate tax rate τ_C

$$r_{wacc} = \frac{E}{E+D} r_E + \frac{D}{E+D} r_D (1 - \tau_C)$$

Capital Structure of a Perfect Market

- cost of capital of levered equity

$$r_E = r_U + \frac{D}{E}(r_U - r_D)$$

- In a setting of perfect capital markets, there are no taxes, so the firm's WACC and unlevered cost of capital conincide

$$r_{wacc} = r_U = r_A$$

- a firm's unlevered or asset beta is the weighed average of its equity and debt beta

$$\beta_U = \frac{E}{E+D} \beta_E + \frac{D}{E+D} \beta_D$$

- a firm's equity beta

$$\beta_E = \beta_U + \frac{D}{E}(\beta_U - \beta_D)$$

Debt and Taxes

- Interest Tax Shield in the all-equity case the total amount available for all investors is (EBIT − Interest)$(1-\tau_c)$, but in the levered case it is

  (Note that the interest payment is $r_D \times D$, where D is the debt issued)

$$\begin{aligned} &\underbrace{(\text{EBIT} - \text{Interest})(1-\tau_c)}_{\text{Available to shareholders}} + \underbrace{\text{Interest}}_{\text{Paid to debtholders}} \\ = &\underbrace{\text{EBIT}(1-\tau_c)}_{\substack{\text{Cash flows to investors}\\ \text{without leverage}}} + \underbrace{\text{Interest} \times \tau_c}_{\text{Interest tax shield}} \end{aligned}$$

- the total value of the levered firm exceeds the value of the firm without leverage due to the PV of the tax savings from debt with corporate taxes

$$V^L = V^U + \text{PV(Interest tax shield)}$$

- PV of the Interest tax shield given permanent debt

$$\begin{aligned} \text{PV(Interst tax shield)} &= \text{PV}(\tau_c \times \text{Future interest payments}) \\ &= \tau_c \times \text{PV(Future interest payments)} \\ &= \tau_c \times D \end{aligned}$$

- effective after-tax cost of capital $r_D(1-\tau_c)$, the after-tax cash flow received by shareholders of a levered firm is

$$(\text{EBIT} - r_D D)(1-\tau_c)(1-\tau_e)$$

  which can be rearranged

- the weighted average cost of capital given after-tax interest rate

$$\begin{aligned} r_{wacc} &= \frac{E}{E+D} r_E + \frac{D}{E+D} r_D (1-\tau_c) \\ &= r_U - \frac{D}{E+D} r_D \tau_c \\ &= \underbrace{\frac{E}{E+D} r_E + \frac{D}{E+D} r_D}_{\text{Pretax WACC}} - \underbrace{\frac{D}{E+D} r_D \tau_c}_{\substack{\text{Decrease from}\\ \text{interest tax shield}}} \end{aligned}$$

  Rearrange this, we can also get

$$r_U = r_{wacc} + \frac{D}{E+D} r_D \tau_c$$

- the effective tax advantage of debt is

$$\tau^* = 1 - \frac{(1-\tau_c)(1-\tau_e)}{(1-\tau_i)}$$

- the value of the firm with leverage becomes with personal taxes

$$V^L = V^U + \tau^* D$$

Financial Distress, Managerial Incentives, and Information

- the total value of a levered firm equals the value of the firm without leverage plus the PV of the tax savings from debt, less the PV of financial distress costs

$$V^L = V^U + \text{PV(Interest tax shield)} - \text{PV(Financial distress costs)}$$

- Equity holders will benefit from the new investment I if its Profitability Index (NPV/I) exceeds

$$\frac{\text{NPV}}{I} > \frac{\beta_D D}{\beta_E E}$$

- **Agency Costs** are costs arising from conflicts of interest between the firm's stakeholders
- **Debt Overhang** Firms in fiancial distress may not want to finance new positive NPV projects since much of the gains may go to the debt holders
- The tradeoff theory may be extended to include agency costs

$$\begin{aligned} V^L = V^U &+ \text{PV(Interest Tax Shield)} - \text{PV(Financial Distress Costs)} \\ &- \text{PV(Agency costs of debt)} + \text{PV(agency benefit of debt)} \end{aligned}$$

Payout Policy

- In a perfect capital market, when a dividend is paid, the share price **drops** by the amount of dividend when the stock begins to trade ex-dividend

$$\text{P}_{cum} = \text{Current Dividend} + \text{PV(Future dividends)}$$

- In perfect capital markets, an open marker share repurchase has no effect on the stock price, and the stock price is the same as the cum-dividend price if a dividend were paid instead
- the effective dividend tax rate is (τ_d^* is the additional tax paid per dollar of after-tax capital gains income if it is received as a dividend instead)

$$\tau_d^* = \frac{\tau_d - \tau_g}{1 - \tau_g}$$

- the **ex-dividend date price** (aka with-dividend, just before ex-dividend date) can also be written as $\Delta P = \text{P}_{cum} - \text{P}_{ex} = \text{Div} \times \left[\frac{1-\tau_d}{1-\tau_g}\right] = \text{Div} \times (1 - \tau_d^*)$ because

$$(\text{P}_{cum} - \text{P}_{ex})(1 - \tau_g) = \text{Div}(1 - \tau_d)$$

- the effective tax disadvantage of retaining cash, given personal tax rate on interest income τ_i, corporate tax rate τ_c, and personal tax rate on capital gains τ_g

$$\tau^*_{\text{retain}} = \left[1 - \frac{(1-\tau_c)(1-\tau_g)}{(1-\tau_i)}\right]$$

- A firm's enterprise value is

$$V = E + D - C$$

  where C is excess cash

Capital Budgeting and Valuation with Leverage

- Denote the current value of a levered investment by $V_0^L = E + D$, given expected returns on debt and equity of r_D and r_E respectively, the expected value today of the investment one period in the future is
$$E(1 + r_E) + D(1 + r_D)$$
- Current value of a levered investment if r_{wacc} is constant
$$V_0^L = \frac{\text{FCF}_1}{1 + r_{wacc}} + \frac{\text{FCF}_2}{(1 + r_{wacc})^2} + \frac{\text{FCF}_3}{(1 + r_{wacc})^3} + \dots$$
- Continuation value
$$V_t^L = \frac{\text{FCF}_{t+1} + \overbrace{V_{t+1}^L}^{\substack{\text{Value of FCF in year} \\ t+2 \text{ and beyond}}}}{1 + r_{wacc}}$$
- Debt capacity at date t as the amount of debt needed at that time D_t in order to maintain the target D/V ratio d
$$D_t = d \times V_t^L$$
- APV
$$V^L = \text{APV} = V^U + \text{PV(Interest Tax Shield)} = E + D$$
- Interest tax shield
$$\text{Interest Paid in Yeat } t = r_D \times D_{t-1}$$
- Free Cash Flow to Equity, where $D_t - D_{t-1}$ is the net borrowing
$$\text{FCFE}_t = \text{FCF}_t - r_D D_{t-1}(1 - \tau_c) + (D_t - D_{t-1})$$
(Note that the PV of FCFE is NPV)
- Project-based WACC, where d is the project's D/V value
$$r_{wacc} = r_U - d\tau_c r_D$$

Capital Budgeting and Valuation with Leverage EXTENSION

- Levered value with a constant interest coverage ratio, where k is the firm's incremental interest payments as a target fraction
$$\begin{aligned} V^L &= V^U + \text{PV(Interest Tax Shield)} \\ &= V^U + \tau_c k \times V^U \\ &= (1 + \tau_c k)V^U \end{aligned}$$
- Leverage value with permanent debt, **affected by discount rate** r_D
$$\begin{aligned} V^L &= V^U + \text{PV(Interest Tax Shield)} \\ &= V^U + \tau_c \times D \end{aligned}$$
Note: since the risk of the tax shield is not the same as the risk of the free cash flows, the unlevered cost of capital r_U is not equal to the pretax WACC in this context
- When a firm finances a project by issuing securities, it incurs **flotation costs** (underwriting costs, legal expenses, accounting fees, etc.) which will reduce project NPV

- Present value of interest expense at year t, (aka Int_t) when the debt is adjusted annually

$$\begin{aligned}\text{PV}(\tau_c \times \text{Int}_t) &= \text{PV}(\tau_c r_D D_{t-1}) \\ &= \frac{\tau_c r_D D_{t-1}}{(1+r_U)^{t-1}(1+r_D)} \\ &= \frac{\tau_c r_D D_{t-1}}{(1+r_U)^t} \times \left(\frac{1+r_U}{1+r_D}\right)\end{aligned}$$

- The project-based WACC when the debt is adjusted annually to a target debt-to-value ratio $d = D/V$

$$r_{wacc} = r_U - d\tau_c r_D \left(\frac{1+r_U}{1+r_D}\right)$$

- Levered value when a firm sets debt annually based on expected future cash flows, where k is the firm's incremental interest payments as a target fraction (i.e. $k = \text{Interest payment/Cash flow}$)

$$V^L = V^U \left[1 + \tau_c k \left(\frac{1+r_U}{1+r_D}\right)\right]$$

- Leverage and the cost of capital with a fixed debt schedule, where $D^s = D - T^s$

$$r_U = \frac{E}{E+D} r_E + \frac{D^s}{E+D^s} r_D$$

  or equivalently

$$r_E = r_U + \frac{D^s}{E}(r_U - r_D)$$

  - continuously adjusted debt $T^s = 0 \implies D^s = D$
    this is equivalent to $r_{wacc} + d\tau_c r_D = r_U$
  - permanent debt, $T^s = \tau_c D \implies D^s = D(1-\tau_c)$

- Project WACC with a fixed debt schedule, where $d = D/(D+E)$, and $\phi = T^s/(\tau_c D)$

$$r_{wacc} = r_U - d\tau_c[r_D + \phi(r_U - r_D)]$$

  - permanent debt $T^s = \tau_c D$, $\phi = 1$
  - continuously adjusted debt, $T^s = 0$, $\phi = 0$
  - annually adjusted debt $T^s = \tau_c r_D D/(1+r_D)$, $\phi = r_D(1+r_D)$

- levered value each year if a firm does not keep a constant debt-equity ratio

$$V_t^L = \frac{\text{FCF}_{t+1} + V_{t+1}^L}{1 + r_{wacc}(t)}$$

- unlevered cost of capital with personal taxes, given $r_D^* = r_D \times \left[\frac{(1-\tau_i)}{(1-\tau_e)}\right]$

$$r_U = \frac{E}{E+D^s} r_E + \frac{D^s}{E+D^s} r_D^*$$

- Interest tax shield given the effective tax advantage of debt τ^* and r_D^*, where $\tau^* = 1 - \frac{(1-\tau_c)(1-\tau_e)}{(1-\tau_i)}$, and the interest tax shield in year t is

$$\tau^* r_D^* D_{t-1}$$

  and generalize for r_{wacc}, where the debt permanent measure $\phi = T^s/(\tau^* D)$

$$r_{wacc} = r_U - d\tau^*[r_D^* + \phi(r_U - r_d^*)]$$