

Capital Budgeting

- Capital cost of allowance

Fundamentals of Capital Budgeting

- UCC and CAA for year t , and $CCA_t = UCC_t \times d$

$$UCC_t = \begin{cases} \text{CapEx}/2 & \text{for } t = 1 \\ \text{CapEx} \times (1 - d/2) \times (1 - d)^{t-2} & \text{for } t \geq 2 \end{cases}$$

- Free Cash Flow

$$FCF_t = (\text{Revenues}_t - \text{Costs}_t) \times (1 - \tau_t) - \text{CapEx}_t - \Delta \text{NWC}_t + \tau_c \times CCA_t$$

- PV of CCA tax shields

$$PV_{\text{CCA tax shields}} = \frac{\text{CapEx} \times d \times \tau_c}{r + d} \times \left[\frac{1 + r/2}{1 + r} \right]$$

- PV of lost CCA tax shields

$$PV_{\text{lost CCA tax shield}} = \frac{\min(\text{Sale Price}, \text{CapEx}) \times d \times \tau_c}{r + d} \times \frac{1}{(1 + r)^t}$$

Estimating the Cost of Capital

- CAPM

$$r_i = r_f + \beta_i \times \underbrace{(E[R_{Mkt}] - r_f)}_{\text{market risk premium}}$$

- expected return of the bond is

$$\begin{aligned} r_D &= (1 - p)y + p(y - L) = y - pL \\ &= \text{Yield to Maturity} - \text{Prob}(\text{default}) \times \text{Expected Loss Rate} \end{aligned}$$

- the relationship between different ratios

$$\frac{D}{E} = x \implies \frac{E}{V} = \frac{1}{x + 1} \implies \frac{D}{V} = 1 - \frac{E}{V}$$

- asset or unlevered cost of capital, where D is the net debt (also known as pretax WACC)

$$r_U = \frac{E}{E + D}r_E + \frac{D}{E + D}r_D$$

- asset or unlevered beta

$$\beta_U = \frac{E}{E + D}\beta_E + \frac{D}{E + D}\beta_D$$

- weighted average cost of capital with firm's corporate tax rate τ_C

$$r_{wacc} = \frac{E}{E + D}r_E + \frac{D}{E + D}r_D(1 - \tau_C)$$

Capital Structure of a Perfect Market

- cost of capital of levered equity

$$r_E = r_U + \frac{D}{E}(r_U - r_D)$$

- In a setting of perfect capital markets, there are no taxes, so the firm's WACC and unlevered cost of capital coincide

$$r_{wacc} = r_U = r_A$$

- a firm's unlevered or asset beta is the weighed average of its equity and debt beta

$$\beta_U = \frac{E}{E + D}\beta_E + \frac{D}{E + D}\beta_D$$

- a firm's equity beta

$$\beta_E = \beta_U + \frac{D}{E}(\beta_U - \beta_D)$$

Debt and Taxes

- Interest Tax Shield in the all-equity case the total amount available for all investors is $(\text{EBIT} - \text{Interest})(1 - \tau_c)$, but in the levered case it is

(Note that the interest payment is $r_D \times D$, where D is the debt issued)

$$\begin{aligned}
 & \underbrace{(\text{EBIT} - \text{Interest})(1 - \tau_c)}_{\text{Available to shareholders}} + \underbrace{\text{Interest}}_{\text{Paid to debtholders}} \\
 = & \underbrace{\text{EBIT}(1 - \tau_c)}_{\text{Cash flows to investors without leverage}} + \underbrace{\text{Interest} \times \tau_c}_{\text{Interest tax shield}}
 \end{aligned}$$

- the total value of the levered firm exceeds the value of the firm without leverage due to the PV of the tax savings from debt with corporate taxes

$$V^L = V^U + \text{PV}(\text{Interest tax shield})$$

- PV of the Interest tax shield given permanent debt

$$\begin{aligned}
 \text{PV}(\text{Interest tax shield}) &= \text{PV}(\tau_c \times \text{Future interest payments}) \\
 &= \tau_c \times \text{PV}(\text{Future interest payments}) \\
 &= \tau_c \times D
 \end{aligned}$$

- effective after-tax cost of capital $r_D(1 - \tau_c)$, the after-tax cash flow received by shareholders of a levered firm is

$$(\text{EBIT} - r_D D)(1 - \tau_c)(1 - \tau_e)$$

which can be rearranged

- the weighted average cost of capital given after-tax interest rate

$$\begin{aligned}
 r_{wacc} &= \frac{E}{E + D} r_E + \frac{D}{E + D} r_D (1 - \tau_c) \\
 &= r_U - \frac{D}{E + D} r_D \tau_c \\
 &= \underbrace{\frac{E}{E + D} r_E + \frac{D}{E + D} r_D}_{\text{Pretax WACC}} - \underbrace{\frac{D}{E + D} r_D \tau_c}_{\text{Decrease from interest tax shield}}
 \end{aligned}$$

Rearrange this, we can also get

$$r_U = r_{wacc} + \frac{D}{E + D} r_D \tau_c$$

- the effective tax advantage of debt is

$$\tau^* = 1 - \frac{(1 - \tau_c)(1 - \tau_e)}{(1 - \tau_i)}$$

- the value of the firm with leverage becomes with personal taxes

$$V^L = V^U + \tau^* D$$

Financial Distress, Managerial Incentives, and Information

- the total value of a levered firm equals the value of the firm without leverage plus the PV of the tax savings from debt, less the PV of financial distress costs

$$V^L = V^U + \text{PV}(\text{Interest tax shield}) - \text{PV}(\text{Financial distress costs})$$

- Equity holders will benefit from the new investment I if its Profitability Index (NPV/I) exceeds

$$\frac{\text{NPV}}{I} > \frac{\beta_D D}{\beta_E E}$$

- **Agency Costs** are costs arising from conflicts of interest between the firm's stakeholders
- **Debt Overhang** Firms in financial distress may not want to finance new positive NPV projects since much of the gains may go to the debt holders
- The tradeoff theory may be extended to include agency costs

$$\begin{aligned} V^L = V^U &+ \text{PV}(\text{Interest Tax Shield}) - \text{PV}(\text{Financial Distress Costs}) \\ &- \text{PV}(\text{Agency costs of debt}) + \text{PV}(\text{agency benefit of debt}) \end{aligned}$$

Payout Policy

- In a perfect capital market, when a dividend is paid, the share price **drops** by the amount of dividend when the stock begins to trade ex-dividend

$$P_{cum} = \text{Current Dividend} + \text{PV}(\text{Future dividends})$$

- In perfect capital markets, an open market share repurchase has no effect on the stock price, and the stock price is the same as the cum-dividend price if a dividend were paid instead
- the effective dividend tax rate is (τ_d^* is the additional tax paid per dollar of after-tax capital gains income if it is received as a dividend instead)

$$\tau_d^* = \frac{\tau_d - \tau_g}{1 - \tau_g}$$

- the **ex-dividend date price** (aka with-dividend, just before ex-dividend date) can also be written as $\Delta P = P_{cum} - P_{ex} = \text{Div} \times \left[\frac{1 - \tau_d}{1 - \tau_g} \right] = \text{Div} \times (1 - \tau_d^*)$ because

$$(P_{cum} - P_{ex})(1 - \tau_g) = \text{Div}(1 - \tau_d)$$

- the effective tax disadvantage of retaining cash, given personal tax rate on interest income τ_i , corporate tax rate τ_c , and personal tax rate on capital gains τ_g

$$\tau_{\text{retain}}^* = \left[1 - \frac{(1 - \tau_c)(1 - \tau_g)}{(1 - \tau_i)} \right]$$

- A firm's enterprise value is

$$V = E + D - C$$

where C is excess cash

Capital Budgeting and Valuation with Leverage

- Denote the current value of a levered investment by $V_0^L = E + D$, given expected returns on debt and equity of r_D and r_E respectively, the expected value today of the investment one period in the future is

$$E(1 + r_E) + D(1 + r_D)$$

- Current value of a levered investment if r_{wacc} is constant

$$V_0^L = \frac{FCF_1}{1 + r_{wacc}} + \frac{FCF_2}{(1 + r_{wacc})^2} + \frac{FCF_3}{(1 + r_{wacc})^3} + \dots$$

- Continuation value

$$V_t^L = \frac{FCF_{t+1} + \overbrace{V_{t+1}^L}^{\text{Value of FCF in year } t+2 \text{ and beyond}}}{1 + r_{wacc}}$$

- Debt capacity at date t as the amount of debt needed at that time D_t in order to maintain the target D/V ratio d

$$D_t = d \times V_t^L$$

- APV

$$V^L = APV = V^U + PV(\text{Interest Tax Shield}) = E + D$$

- Interest tax shield

$$\text{Interest Paid in Year } t = r_D \times D_{t-1}$$

- Free Cash Flow to Equity, where $D_t - D_{t-1}$ is the net borrowing

$$FCFE_t = FCF_t - \underbrace{r_D D_{t-1}(1 - \tau_c)}_{\text{After tax interest payment}} + (D_t - D_{t-1})$$

(Note that the PV of FCFE is NPV and the discount rate is r_E)

- Project-based WACC, where d is the project's D/V value

$$r_{wacc} = r_U - d\tau_c r_D$$

Capital Budgeting and Valuation with Leverage EXTENSION

- Levered value with a constant interest coverage ratio, where k is the firm's incremental interest payments as a target fraction

$$\begin{aligned} V^L &= V^U + PV(\text{Interest Tax Shield}) \\ &= V^U + \tau_c k \times V^U \\ &= (1 + \tau_c k) V^U \end{aligned}$$

- Leverage value with permanent debt, **affected by discount rate** r_D

$$\begin{aligned} V^L &= V^U + PV(\text{Interest Tax Shield}) \\ &= V^U + \tau_c \times D \end{aligned}$$

Note: since the risk of the tax shield is not the same as the risk of the free cash flows, the unlevered cost of capital r_U is not equal to the pretax WACC in this context

- When a firm finances a project by issuing securities, it incurs **flotation costs** (underwriting costs, legal expenses, accounting fees, etc.) which will reduce project NPV

- Present value of interest expense at year t , (aka Int_t) when the debt is adjusted annually

$$\begin{aligned}\text{PV}(\tau_c \times \text{Int}_t) &= \text{PV}(\tau_c r_D D_{t-1}) \\ &= \frac{\tau_c r_D D_{t-1}}{(1+r_U)^{t-1}(1+r_D)} \\ &= \frac{\tau_c r_D D_{t-1}}{(1+r_U)^t} \times \left(\frac{1+r_U}{1+r_D} \right)\end{aligned}$$

- The project-based WACC when the debt is adjusted annually to a target debt-to-value ratio $d = D/V$

$$r_{wacc} = r_U - d\tau_c r_D \left(\frac{1+r_U}{1+r_D} \right)$$

- Levered value when a firm sets debt annually based on expected future cash flows, where k is the firm's incremental interest payments as a target fraction (i.e. $k = \text{Interest payment}/\text{Cash flow}$)

$$V^L = V^U \left[1 + \tau_c k \left(\frac{1+r_U}{1+r_D} \right) \right]$$

- Leverage and the cost of capital with a fixed debt schedule, where $D^s = D - T^s$

$$r_U = \frac{E}{E+D} r_E + \frac{D^s}{E+D^s} r_D$$

or equivalently

$$r_E = r_U + \frac{D^s}{E} (r_U - r_D)$$

– continuously adjusted debt $T^s = 0 \implies D^s = D$

– this is equivalent to $r_{wacc} + d\tau_c r_D = r_U$

– permanent debt, $T^s = \tau_c D \implies D^s = D(1 - \tau_c)$

- Project WACC with a fixed debt schedule, where $d = D/(D+E)$, and $\phi = T^s/(\tau_c D)$

$$r_{wacc} = r_U - d\tau_c [r_D + \phi(r_U - r_D)]$$

– permanent debt $T^s = \tau_c D$, $\phi = 1$

– continuously adjusted debt, $T^s = 0$, $\phi = 0$

– annually adjusted debt $T^s = \tau_c r_D D/(1+r_D)$, $\phi = r_D/(1+r_D)$

- levered value each year if a firm does not keep a constant debt-equity ratio

$$V_t^L = \frac{\text{FCF}_{t+1} + V_{t+1}^L}{1 + r_{wacc}(t)}$$

- unlevered cost of capital with personal taxes, given $r_D^* = r_D \times \left[\frac{(1-\tau_i)}{(1-\tau_e)} \right]$

$$r_U = \frac{E}{E+D^s} r_E + \frac{D^s}{E+D^s} r_D^*$$

- Interest tax shield given the effective tax advantage of debt τ^* and r_D^* , where

$$\tau^* = 1 - \frac{(1-\tau_c)(1-\tau_e)}{(1-\tau_i)}, \text{ and the interest tax shield in year } t \text{ is}$$

$$\tau^* r_D^* D_{t-1}$$

and generalize for r_{wacc} , where the debt permanent measure $\phi = T^s/(\tau^* D)$

$$r_{wacc} = r_U - d\tau^* [r_D^* + \phi(r_U - r_D^*)]$$

Raising Equity Capital

- Given pre-money evaluation and the amount invested

$$\text{Post-money Valuation} = \text{Pre-money Valuation} + \text{Amount invested}$$

Percentage Ownership

$$\text{Percentage Ownership} = \text{Amount Invested} / \text{Post-money valuation}$$