# Response to reviewer 01

* 1)There seems to be a huge disconnect between the actual objectives of the game and the proposed optimization model. I do not have first-hand familiarity with the game, but based on the description in the paper, it is my understanding that winning the game entails a lot more than maximizing the sum of scores and depends on the choices of other players. Also, according to the authors, there are eight players, which invalidates the assumption that weakening an opponent's team is the same as strengthening yours. I assume you have the flexibility of deciding which of the eight opponents to weaken. So, it appears that the model ignores some of the most important strategic and interactive (game-theoretic) aspects of the game and is a very unrealistic simplification of the actual situation, which makes the whole study pointless, in my opinion.

>2) The proof of NP-completeness is trivial and does not represent a significant contribution. (Somehow, the authors managed to stretch this into three theorems, but it should really be one proposition with a 3-line proof.) Since the result is put in the title, this is supposed to be the main contribution of the paper. Even if the proof was nontrivial, the statement in the title is quite a bit of a stretch (due to the first point above).

– it is not obvious to us that shortening of the proof size will significantly improve the readability of the article due to the large number of task-specific terms such as alliances, bonuses, hero powers.

>3) It is not clear what does one gain by reducing the considered problem to maximum edge weight clique problem. Given that the problem is NP-complete, one can potentially find reductions to any other NP-complete problem. The question is, why is this useful? It does not appear that the proposed reduction leads to any useful insights in this case.

– We believe that the reduction is valuable by itself, and maybe helpful to someone else with a similar problem.

>4) The way the computational results are presented only proves the point that this study is pointless, because no qualitative analysis of the results, such as, how IP-based approach helps winning the game in practice, is given. This is not surprising, because, as pointed out above, there is a huge disconnect between the proposed model and the actual game.

– we have added a paragraph into the introduction section where we mention that the real game is more complex, and in the present paper we consider one possible simplification of the real game.

>5) The paper is very poorly written, both in terms of logical structure and the language. Even the IP model looks shady, with what the authors call "constraints for the input data" -- this should not be a part of the formulation.

­– In the present revision we improved the language of the paper.

# Response to reviewer 02

* 1. The presentation of the paper should be improved in the sense of a language quality (grammar, articles, terminology use, math symbols and etc.). For example, I would write the introduction in the following edition:

In this paper, we demonstrate how the optimal team choice problem in the popular computer game Dota Underlords can be reduced to the linear integer programming problem. We propose a model and solve it for real data. We also prove that this problem is NP-hard and show that it can be polynomially reduced to the maximum edge-weighted clique problem.

–We changed an abstract according to your suggestion. Thanks for your correction.

* Note, that the terms for problems are «linear integer programming» and «maximum edge-weighted clique problem». You consider the optimization problems and claim that they are NP-complete. It is awkward, as the class of NP-complete problems is constituted by decision problems. The NP-hardness is the proper term, and it is better to say that a problem is NP-hard than it belongs to the NP-hard class. Please, use the term «polynomial-time reduction» (and derivatives from it) instead of «reduction».
* We have pass through the paper and aligned the terms according your suggestion. Thanks

> P. 2, L. 32: the computational complexity;

P. 2, L. 34-35: the decision problem, NP-complete;

P. 2, L. 36: is polynomially;

P. 2, L. 37: the NP-complete computational class;

P. 2, L. 39: the minimum;

P. 2, L. 46: the popular;

P. 3, L. 3: the NP-complete class;

P. 3, L. 4: is organized, In Section 2;

P. 3, L. 5: a formulation;

P. 3, L. 6: the linear integer programming problem, Section 3, In Section 4;

P.3, L. 9-10: a real data is presented in Section 5, in Section 6;

and etc.

* In the present revision we tried to improve the English of the paper. Maybe it is still not perfect, but better than was. Thanks for your corrections.
* The title page should be updated, for example, the addresses and phones must be added. The corresponding author must be emphasized
* We added addresses and phone for bouth authors. The corresponding author was emphasized.
* 3. I believe that the problem of finding the maximum dense subgraph of k vertices should be formulated before Theorem 1. Please, give a reference, where NP-hardness of this problem has been proved. I also believe that the theorem should be formulated as a result about NP-hardness of the Dota Underlords problem
* We have added the following statement before theoreme 1: "The problem of finding a set of k vertices with the most edges in the subgraph induced by this set is called the maximum dense subgraph of k vertices problem. Obviously this problem is NP-hard by reduction from clique. "
* 4. Everywhere, mention that the Dota Underlords problem is NP-hard and withdraw the subsections 4.2 and 4.3. Everywhere, name the problem either as the Dota Underlords problem or Dota Underlords or DU and etc. The same is true for the MWEC and the maximum edge-weighted clique problem. Put the cardinality symbols in the definition of the density.
* Fixed. Thanks.
* 5. I believe that Theorems 4 and 5 should be omitted. Only the general result should be presented.
* Theorems 4, 5, 6 have been merged into one theorem.
* 6. I believe that the conclusions section should re-written by omitting the hopes of the authors, discussions of their contribution to the theory of NP-completeness and etc. They should focus on description of their results and perspectives of the research
* The conclusion section has been rewriten

# Response to reviewer 03

* 1. Please revise the language in the paper and fix the mistakes and typos. To give an example, the first paragraph may be rewritten like below.   
  People love to play games. Many games and puzzles that people play are interesting due to their complexity: you need to be smart enough to solve it. In many cases, such complexity can be expressed as computation complexity depending on the input size. For example, it has been shown [1] that the chess game belongs to the EXPTIME complexity class; the decision problem of a player in a legendary video game called Tetris" is NP-hard [2]. It was shown that the puzzle “Sokoban” is polynomially solvable [3].

– In the present revision we tried to improve the English of the paper. Maybe it is still not perfect, but better than was. Thanks for your corrections.

* 2. A little more introduction into the selection process of heroes by players is needed. Can players pick the same heroes? Can a player see a current selection of the others and change his/her mind on the go? It seems strange that the ultimate selection does account for other players' choices.

– We have added a paragraph into the introduction where we clarify these questions.

* 3. "Also during the game, you can strengthen your heroes by upgrading them to higher levels or by purchasing in-game items. In this work, these aspects will not be taken into account." It looks like the most interesting case to analyse the dynamics of the game. This is where the real complexity comes into play. Otherwise, the most difficult instance is to select 3-5 heroes out of a 100 and it does not constitute a difficult problem.

– Dynamic of the game is hard to model and analyse. In the present revision of the paper, we have mentioned in the “introduction” section that the real game has a complex dynamic, and we would like to avoid such aspect of the game.

* 4. As mentioned earlier, the assumptions are too simplistic. They are enough to carry out the complexity analysis since if the simplified problem is NP-complete then the original one is as well. But the solution to what team is the most efficient should incorporate realistic assumptions.

– With the paper we provide a python code that we used to obtain solution. The solution was found based on our assumption about heroes’ powers and bonuses of alliances. The assumptions maybe not accurate, however any one can use our code with their own values in order to get more accurate results.