Week-9

We have eq –

$$\int_{-\infty}^{\infty} \psi^* \psi \, dx = 1.$$

$\psi \in \mathbb{R}$ $\therefore \psi^* = \psi$

$$\Rightarrow \int_{-\infty}^{\infty} |\psi|^2 \, dx = 1$$

$$\therefore -\frac{1}{2(1)} \int_{-\infty}^{\infty} -2A_1^2 \left(\frac{4}{\pi}\right)^{1/2} x^2 e^{-x^2} dx$$

$$= -\int_{-\infty}^{\infty} A_1^2 \left(\frac{1}{\pi}\right)^{1/2} x^2 (-2) e^{-x^2} dx$$

$$= -A_1^2 \int_{-\infty}^{\infty} x \left(\frac{1}{\pi}\right)^{1/2} -2x e^{-x^2} dx$$

$$= -A_1^2 \left(\frac{1}{\pi}\right)^{1/2} \int_{-\infty}^{\infty} x \, (-2x e^{-x^2}) dx \quad \{\text{Integration by parts}\}$$

$$= -A_1^2 \left(\frac{1}{\pi}\right)^{1/2} \left\{ \left[e^{-x^2} \right]_{-\infty}^{\infty} - \int_{-\infty}^{\infty} e^{-x^2} dx \right\}$$

$$= -A_1^2 \left(\frac{1}{\pi}\right)^{1/2} \left\{ -\sqrt{\pi} \right\}$$

$$= A_1^2 \left(\frac{1}{\pi}\right)^{1/2} \times \sqrt{\pi}$$

$$= A_1^2 (1)$$

LHS = RHS

$$\Rightarrow A_1^2 = 1$$

$$\therefore A_1 = 1$$

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$$\because x^2 e^{-ax^2}$$

$$= -\frac{1}{2a} x \left(-2ax e^{-ax^2}\right)$$

$$= -\frac{1}{2a} x \left(\frac{d}{dx}\left\{x^2 e^{-ax^2}\right\}\right)$$

$$\int_{-\infty}^{\infty} e^{-ax^2} dx = \sqrt{\frac{\pi}{a}}$$

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