

Analytical Result

$$\frac{dT(t)}{dt} = -\alpha A_s [T(t) - T_f]$$

given,

$$\text{and } T_f = 10^\circ\text{C}$$

$$\alpha = 0.001$$

A_s = surface area of the asteroid.

$$d = 2\text{ km}$$

$$\therefore r = 1\text{ km}$$

$$\text{Surface area of the sphere} = 4\pi (1\text{ km})^2$$

$$A_s = 4\pi \text{ km}^2$$

$$\frac{dT(t)}{dt} = -\alpha A_s [T(t) - T_f]$$

\therefore Analytical Result

$$T(t) = e^{-\alpha A_s t} [1340] + T_f$$

$$\int \frac{dT(t)}{[T(t) - T_f]} = -\int \alpha A_s dt$$

$$\Rightarrow \ln [T(t) - T_f] = -\alpha A_s t + C$$

$$\Rightarrow T(t) - T_f = e^{-\alpha A_s t + C}$$

$$\Rightarrow T(t) - T_f = C' e^{-\alpha A_s t}$$

$$\text{at } T(0) = 1350^\circ\text{C}$$

$$\Rightarrow 1350 - 10 = C' e^{-0}$$

$$\Rightarrow \boxed{C' = 1340^\circ\text{C}}$$