Lab Assignment 10

May 27, 2021

Newton's Law of Cooling states that the rate of heat loss of a body is directly proportional to the difference in the temperatures between the body and its surroundings. Mathematically, it's expressed using the first order differential equation as given below.

$$\frac{dT(t)}{dt} = -\alpha A_s \left[T(t) - T_f\right] \tag{1}$$

where

$T(t)$ is the temperature of the body at time t,
α is the proportionality constant,
A_s is the surface area of the body, and
T_f is the final temperature of the body.

Question:
A large asteroid of size 2 km, had hit the earth in the South Pacific Ocean at a speed of around 20 km/s 2.5 million years ago. The surface temperature of the asteroid was, by some estimates, around 1350 °C at the time of impact (i.e. t= 0 s) and the temperature of the water was 10 °C. This was the Eltanin event. Assuming that the temperature of the water did not change appreciably during the impact, the asteroid will cool down, and become equal to the temperature of surrounding water. Write a program to solve this first order differential equation using RK4 method. You can easily solve the differential equation to obtain the analytical solution.

Your code should have the following components.

- Firstly, write two user defined functions, one for the differential equation, and second, analytical solution.
- Write another function that solves the differential equation using RK4 for a given value of time t, and return the value of temperature T

  The function should be of the form ($t \rightarrow$ time, $T \rightarrow$ temperature)

  ```
  double RK4( double t, double T, double h );
  ```

  Use this function in `main()`, and store the results in a file, along with the analytical result. Your calculations should stop only when the temperature of the object estimated by you is within 0.01% of the final temperature.
- Make a graph of the data stored in the file and also print on the terminal, the time the asteroid took to cool to the temperature of the surrounding water.

For the purpose of calculations, you may model the asteroid as a spherical body of diameter 2 km, $\alpha = 0.001$, and use a step size of $h = 10.0$. (**Note:** Perform calculations in given units only i.e. do not convert km to m and °C to °K.)