

## Aus: Static Indeterminacy

$$\text{General Formula} \rightarrow \delta I = \frac{(\alpha_m + \alpha) - (\beta j + r)}{(3m + \alpha) - (3j + r)} \quad \alpha = 3 \\ \beta = 3$$

Here,  $\alpha \rightarrow$  total no. of reaction forces.

$m \rightarrow$  members

$j \rightarrow$  joints

$r \rightarrow$  internal releases.

$$\text{By observation} - \alpha - 3 - r = \delta I$$

$$\text{So, } \delta I = \frac{(3m + \alpha) - (3j + r)}{6+5-9} \quad m=2 \quad j=3 \\ = 11-9 = 2 \quad \alpha = 3+1+1 = 5 \quad r=0$$

and CHECK:

$$\delta I = 5 - 3 - 0 \\ = 2$$

So, static indeterminacy of beam = 2

## Kinematic Indeterminacy

$$\text{General Formula} - (\beta j + r) - (\alpha_m + \alpha) = KI$$

Here,  $\beta = 3$

$\alpha = 0$

$$\text{So, } KI = 3j + r - \alpha$$

By observation -

Total degree of freedom (DOF)  
available at each joint.

$$KI = 3(3) + 0 - 5 = 4$$

CHECK :-

$$KI = \text{total DOF} = 4 = (2+2)$$

Aus 2. Static Indeterminacy

By formula  $\rightarrow$

$$\begin{aligned} SI &= 3m + n - (3j + r) \\ SI &= 6 + 6 - (9 + 1) \\ &= 2 \end{aligned}$$

$$\left\{ \begin{array}{l} m = 2 \\ j = 3 \\ n = 6 \\ r = 1 \end{array} \right.$$

CHECK :-

$$SI = n - 3 - r = 6 - 3 - 1 = 2$$

So, static Indeterminacy of Beam = 2

Kinematic Indeterminacy

$$\begin{aligned} \text{Formula} \rightarrow KI &= 3j + r - n \\ &= 9 + 1 - 6 \\ &= 4 \end{aligned}$$

CHECK :-  $KI = \text{DOF at joints}$

= 4 [i.e. in x, y and two BM's at two beams joined at B].

So, Kinematic Indeterminacy of Beam = 4

### Aus. static indeterminacy

$$m = 2$$

$$\begin{aligned} SI &= r - 3 - \gamma \\ &= 4 - 3 - 2 \\ &= -1 < 0 \end{aligned}$$

$$\gamma = 2$$

$$r = 4$$

$$j = 3$$

structure unstable.

$$\begin{aligned} \text{By formula :- } SI &= 3m + r - (3j + \gamma) \\ &= 6 + 4 - (3(3) + 2) \\ &= 10 - 11 \\ &= -1 < 0 \end{aligned}$$

So, structure is unstable in nature.

### Kinematic Indeterminacy

$$\begin{aligned} KI &= \text{total dof at joints} \\ &= (2+2+1) + 2 \\ &= 7 \end{aligned} \quad \left. \begin{array}{l} \text{at B} \\ \{ \begin{array}{l} 2-\text{axial} \\ 2-\text{BM} \\ 1-\text{vertical} \end{array} \} \end{array} \right.$$

$$\begin{aligned} \text{Formula - } KI &= 3j + r - m \\ &= 4 + 2 - 4 \\ &= 11 - 4 \\ &= 7 \end{aligned}$$

So, Kinematic Indeterminacy of structure = 7

Ans 4: Static Indeterminacy

By Formula -

$$SI = 3m + r - (3j + r)$$

$$SI = 15 + 5 - (18 + 2)$$

$$= 20 - 20$$

$$= 0$$

$$\left\{ \begin{array}{l} m = 5 \quad r = 2 \\ r = 5 \\ j = 6 \end{array} \right\}$$

By observation -

$$SI = r - 3 - r = 5 - 3 - 2 = 0$$

Statically determinate Beam.

Kinematic Indeterminacy.

By Formula -

$$KI = 3j + r - r$$

$$KI = 18 + 2 - 5$$

CHECK -  $KI = 15$

$KI = \text{Total DOF}$

$$KI = 1 + 4 + 4 + 2 + 2 + 2$$

$$KI = 15$$

So, Kinematic Indeterminacy = 5.

Ans 5: Static Indeterminacy

$$\{m = 5, r = 3, r = 6, j = 6\}$$

By Formula -

~~$$SI = 3m + r - (3j + r)$$~~

$$SI = 15 + 6 - (18 + 3)$$

$$= 21 - 21$$

$$= 0$$

$$\text{CHECK : } SI = n - 3 - 3 \\ = 6 - 6 = 0$$

statically determinate beam

### Kinematic Indeterminacy

Formula -

$$KI = 3j + r - n \\ = 18 + 3 - 6 \\ = 15$$

$$\text{CHECK : } KI = \text{DOF} \\ = 4 \times 3 + 2 + 1 \\ = 15$$

Kinematic Indeterminacy = 15 for this beam.

### Ans 6. Static Indeterminacy

By formula -

$$SI = 3m + n - (3j + r) \\ = 3(3) + 6 - (12 + 0) \\ = 9 + 6 - 12 \\ = 3$$

$$\left\{ \begin{array}{l} m=3 \\ n=6 \\ j=4 \\ r=0 \end{array} \right\}$$

$$\text{CHECK : } SI = n - 3 - r \\ = 3$$

Static Indeterminacy = 3.

## Kinematic Indeterminacy.

By formula -

$$KI = 3j + r - n$$

$$KI = 12 + 0 - 6$$

$$= 6$$

CHECK -

$$KI = \text{total DOF}$$

$$= 1+1+2+2$$

$$= 6$$

Kinematic Indeterminacy = 6.

## SOLUTION FRAMES

Free (2-D)

static Indeterminacy -

$$\text{General formula} - SI = (3m + r) - (3j + r)$$

$$SI = 3m + r - (3j + r)$$

Here,  $m$  = no. of members

$r$  = no. of reactions

$j$  = joints

$r$  = total no. of releases.

Kinematic Indeterminacy -

→ Inextensible member

$$\text{General Formula} \rightarrow KI = -(3m + r) + (3j + r)$$

$$KI = 3j + r - m$$

By formula -

$$\text{Ans 1: } SI = 3m + r - (3j + r) \quad m = 3$$

$$SI = 9 + 4 - (12 + 2) \quad r = 4$$

$$= 13 - 14$$

$$= -1 < 0$$

$$j = 4$$

$$r = 2(n-1) = 2(2-1) = 2$$

↓  
total members meeting at I.H.

by free method →

$$SI = (\text{no. of cuts} \times 3) - 2 - 1 - 1$$

$$= 1 \times 3 - 4$$

$$= -1 < 0$$

structure / frame is unstable.

$$KI = 3j + r - m$$

$$= 12 + 2 - 4$$

$$= 10$$

Kinematic Indeterminacy = 10

Ans 2-

$$\begin{aligned}
 m &= 4, r = 2, x = 2 \times 2 = 4, j = 5 \\
 \delta I &= \alpha m + x - (3j + r) \\
 &= 3m + x - (3j + r) \\
 &= 12 + 4 - (15 + 2) \\
 &= 16 - 17 \\
 &= -1 < 0
 \end{aligned}$$

By tree method →

$$\begin{aligned}
 \delta I &= (\text{no. of cuts} \times 3) - 2 - 2 \\
 &= 3 - 4 \\
 &= -1 < 0
 \end{aligned}$$

Ans. unstable frame.

$$\begin{aligned}
 KI &= (3j + r) - x \\
 &= 3j + r - x \\
 &= 15 + 2 - 4 \\
 &= 13
 \end{aligned}$$

Ans 3-  $m = 2, j = 3, r = 1, x = 5$

$$\begin{aligned}
 \delta I &= 3m + x - (3j + r) \\
 &= 6 + 5 - (9 + 1) \\
 &= 11 - 10 \\
 &= \pm 1 \quad (\text{statically Indeterminate})
 \end{aligned}$$

By tree method —

$$\begin{aligned}
 \delta I &= 3 \times 1 - 1 - 1 \\
 &= 1 \\
 KI &= (3j + r) - x \\
 &= 9 + 1 - 5 \\
 &= 5
 \end{aligned}$$

Ans4.  $m = 5, n = 8, j = 6, r = 0$

$$\begin{aligned} SI &= 3m + n - (pj + r) \\ &= 3m + n - (3j + r) \\ &= 15 + 8 - 18 \\ &= 5 \quad (\text{Statically Indeterminate}) \end{aligned}$$

Free method →

$$\begin{aligned} SI &= \text{no. of cuts} \times 3 - 2 - 1 \times 2 \\ &= 3 \times 3 - 4 \\ &= 9 - 4 \\ &= 5 \end{aligned}$$

$$\begin{aligned} KI &= (3j + r) - n \\ &= 18 - 8 \\ &= 10 \end{aligned}$$

Ans5.  $m = 4, j = 4, n = 3, r = 0$

$$\begin{aligned} SI &= 3m + n - (3j + r) \\ &= 12 + 3 - 12 \\ &= 3 \quad (\text{Statically Indeterminate}) \end{aligned}$$

Free method —

$$\begin{aligned} SI &= 3 \times \text{no. of cuts} - 2 - 1 \\ &= 3 \times 2 - 2 - 1 \\ &= 6 - 3 \\ &= 3 \end{aligned}$$

Loop method —

$$II = \text{Internal Indeterminacy} = 3 \times \text{no. of loops} - KI$$

$$= 3$$

$$EI = \text{External Indeterminacy} = n - 3 \\ = 3 - 3 \\ = 0$$

$$SI = II + EI = 3 + 0 = 3$$

$$KI = 3j + r - n \\ = 12 + 0 - 3 \\ = 9$$

Ans 6.  $m = 6, \alpha = 3, j = 3 \times 2 = 6, n = 4, r = 0$

$$SI = 3m + \alpha - (3j + r) \\ = 18 + 4 - (18 + 0) \\ = 4 \quad (\text{Statically Indeterminate})$$

Tree Method —

$$SI = 3 \times \text{no. of cuts} - 1 \times 2 \\ = 3 \times 2 - 2 \\ = 4$$

Loop Method —

$$II = 3 \times \text{no. of loops} - r \\ = 3 \times 1 - 0 \\ = 3$$

$$EI = n - 3 \\ = 4 - 3 = 1$$

$$SI = 3 + 1 = 4 \\ KI = 3j + r - n = 18 + 0 - 4 = 14$$

Ans 7.  $m = 5, j = 6, r = 4, n = 2$

$$SI = 3m + n - (3j + r)$$

$$= 15 + 4 - (18 + 2)$$

$$= 19 - 20$$

$$= -1 < 0$$

Tree Method -

$$SI = 3 \times \text{no. of cuts} - 1 \times 2 - 1 \times 2$$

$$= 3 \times 1 - 4$$

$$= 3 - 4 = -1 < 0$$

Unstable frame.

$$KI = 3j + r - n$$

$$= 18 + 2 - 4$$

$$= 16,$$

Ans 8.  $m = 7, n = 6, r = 1, j = 7$

$$SI = 3m + n - (3j + r)$$

$$= 21 + 6 - (21 + 1)$$

$$= 5$$

Tree Method -

$$SI = \text{no. of cuts} \times 3 - 1$$

$$= 2 \times 3 - 1$$

$$= 5 \quad (\text{Statically Indeterminate})$$

Loop Method -

$$II = \text{no. of loops} \times 3 - r$$

$$= 1 \times 3 - 1$$

$$= 2$$

$$EI = \gamma I - 3 = 6 - 3 = 3.$$

$$SI = EI + II$$

$$= 3 + 2 = 5,$$

$$KI = 3j + r - \gamma I$$

$$= 2I + 1 - 6$$

$$= 22 - 6$$

$$= 16,,$$

Ans 9.

$$m = 6, j = 6+1 = 7, r = 1, \gamma = 3$$

$$SI = 3m + r - (3j + r)$$

$$= 18 + 3 - (2I + 1)$$

$$= 21 - 22$$

$$= -1 < 0$$

True Method -

$$SI = \text{no of cuts} \times 3 - 1 - 3 \times 2 - 1 - 2$$

$$= 3 \times 3 - 1 - 6 - 3$$

$$= 9 - 10$$

$$= -1 < 0$$

unstable structure

$$KI = 3j + r - \gamma I$$

$$= 2I + 1 - 3$$

$$= 19,,$$

Ans. 10.

General Formula -

$$SI = \alpha m + n - (\beta j + r) \quad i$$

$$\alpha = 6, \beta = 6$$

$$SI = 6m + n - (6j + r)$$

General Formula - (assuming axially invertable)

$$KI = \cancel{(\beta j + r)} - (\alpha m + n) \quad \alpha = 0$$

$$KI = \cancel{\beta j} + r - n$$

So,

$$SI = 6m + n - (6j + r)$$

$$m = 18$$

$$= 18 \times 6 + 16 - (6 \times 14 + 6)$$

$$j = 14$$

$$= 108 + 16 - (84 + 6)$$

$$r = 3 \times 2 = 6$$

$$= 124 - 90$$

$$n = 6 \times 2 + 1 + 3$$

$$= 34$$

$$= 12 + 4 = 16$$

True Method -

$$SI = 6 \times \text{no. of cuts} - 3 \times \text{no. of hinges} - 5 \times \text{no. of rollers} \\ - 3 \times \text{no. of hinge.}$$

$$= 6 \times 8 - 3 \times 2 - 5 \times 1 - 3 \times 1$$

$$= 48 - 6 - 5 - 3$$

$$= 48 - 14$$

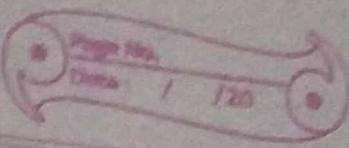
$$= 34$$

Loop Method -

$$\cancel{II} \quad II = \text{no. of loops} \times 6 - r$$

$$= 4 \times 6 - 6$$

$$= 12 - 6 = 6$$



$$EI = \pi - 6$$

$$= 10$$

$$\begin{aligned} KI &= 6j + r - \pi \\ &= 6 \times 14 + 6 - 16 \\ &= 84 + 6 - 16 \\ &= 90 - 16 \\ &= 74 \end{aligned}$$

SOLUTION TRUSSES

trusses

For frames, KI is not defined because if members are considered inextensible, then there is no need to calculate KI because for trusses only axial forces are present at each member.

Static Indeterminacy. (SI)

$$\text{General formula.} \rightarrow (\alpha m + \alpha) - (\beta j + r)$$

$$\alpha = 1, \beta = 2$$

$$SI = m + \alpha - (2j + r)$$

$\alpha$  - reactions

$m$  - total members

$j$  - joints

$r$  - releases = 0 (because all joints are smooth pin joints)

$$SI = m + \alpha - 2j$$

$$\underline{\text{Ans 1.}} \quad m = 6, j = 4, \alpha = 3$$

$$SI = 6 + 3 - 8 \\ = 1$$

$$\left\{ \begin{array}{l} EI = \alpha - 3 = 3 - 3 = 0 \\ II = m - (2j - 3) \\ = 1 \end{array} \right.$$

Statically Indeterminate,

$$\underline{\text{Ans 2.}} \quad m = 9, j = 6, \alpha = 3$$

$$SI = m + \alpha - 2j \\ = 9 + 3 - 12 \\ = 0$$

Statically determinate.

Ans 3.  $m = 9, j = 6, x = 3$

$$SI = m + x - 2j$$

$$SI = 0$$

Statically Indeterminate.

Ans 4.  $m = 16, x = 3, j = 8$

$$SI = m + x - 2j$$

$$= 16 + 3 - 16$$

$$= 3$$

Statically indeterminate.

$$II = m - (2j - 3)$$

$$= 16 - (16 - 3)$$

$$= 3$$

$$II = 3 \therefore$$

$$EI = 3 - 3 = 0$$

Ans 5.  $m = 15, j = 9, x = 3$

$$SI = m + x - 2j$$

$$= 15 + 3 - 18$$

$$= 0$$

Statically determinate

Ans 6.  $m = 9, j = 6, x = 3$

$$SI = m + x - 2j$$

$$= 9 + 3 - 12$$

$$= 0$$

Statically determinate

$$\text{Ans 7: } m = 22, j = 13, x = 2 \times 2 = 4$$

$$SI = m + x - 2j$$

$$= 22 + 4 - 26$$

$$SF = 0$$

$$II = m - (2j - 3)$$

$$= 22 - (26 - 3)$$

$$= -1$$

$$EI = 1 \quad ST = EI + II$$

$$= 0.$$

statically determinate structure.

$$\text{Ans 8: } m = 16, j = 9, x = 3$$

$$SI = m + x - 2j$$

$$= 16 + 3 - 18$$

$$= 1$$

$$II = m - 2j + 3$$

$$= 16 - 18 + 3$$

$$= 1$$

$$EI = 0$$