Ans 1

$T(n) = aT(n/b) + f(n^2)$

$a \geq 1,\ b \geq 1$

On comparing

$a = 3,\ b = 2,\ f(n) = n^2$

Now,

$c = \log_b a = \log_2 3 = 1.584$

$n^2 = n^{1.584} < n^2$

$\therefore f(n) > n^c$

$\therefore T(n) = \Theta(n^2)$

Ans 2

$a \geq 1,\ b > 1$

$a = 4,\ b = 2,\ f(n) = n^2$

$c = \log_2 4 = 2$

$n^c = n^2 = f(n) = n^2$

$\therefore T(n) = \Theta(n^2 \log_2 n)$

Ans 3

$a = 1,\ b = 2$

$f(n) = 2^n$

$c = \log_b a = \log_2 c = 0$

$n^c = n^0 = 1$

$f(n) > n^c$

$T(n) = \Theta(2^n)$

Ans 4:-

$a = 2^n$

$b = 2$

$f(n) = n^2$

$c = \log_b a = \log_2 2^n = n$

$n^c \Rightarrow n^n$

$f(n) = n^c$

$T(n) = \Theta(n^2 \log_2 n)$

Ans 5:-

$a = 16, \ b = 4$

$f(n) = n$

$c = \log_4 16 = \log_4 (4)^2 = 2 \log_4 4 = 2$

$n^c \Rightarrow n^2$

$f(n) < n^c$

$\therefore T(n) = \Theta(n^2)$

Ans 6:-

$a = 2, \ b = 2$

$f(n) = n \log n$

$c = \log_2 2 = 1$

$n^c = n^1 = n$

$n \log n > n$

$f(n) > n^c$

$T(n) = \Theta(n \log n)$

Ans 7:-

$a=2, b=2, f(n) = n/\log n$

$c = \log_2 2 = 1$

$n^c = n^1 = n$

$\because \frac{n}{\log n} < n$

$\therefore f(n) < n^c$

$\therefore T(n) = \Theta(n)$

Ans 8:-

$a=2, b=4, f(n) = n^{0.51}$

$c = \log_b a = \log_4 2 = \frac{1}{2} = 0.5$

$n^c = n^{0.5}$

$n^{0.5} < n^{0.51}$

$f(n) > n^c$

$\therefore T(n) = \Theta(n^{0.51})$

Ans 9:-

$a = 0.5, b = 2$

$a \geq 1$ but here a is 0.5

so we cannot apply Masters Theorem

Ans 10:-

$a = 16$, $b = 4$, $f(n) = n!$

$\therefore c = \log_b a = \log_4 16 = 2$

$n^c = n^2$

As $n! > n^2$

$\therefore T(n) = \Theta(n!)$

Ans 11:-

$a = 4$, $b = 2$, $f(n) = \log(n)$

$c = \log_b a = \log_2 4 = 2$

$n^c = n^2$

$f(n) = \log n$

$\log n < n^2$

$f(n) < n^c$

$T(n) = \Theta(n^c)$

$= \Theta(n^2)$

Ans 12:-

$a = \sqrt{n}$

$b = 2$

$c = \log_b a = \log_2 \sqrt{n} = \frac{1}{2} \log_2 n$

$\therefore \frac{1}{2} \log_2 n < \log(n)$

$\therefore f(n) > n^c$

$T(n) = \Theta(f(n))$

$= \Theta(\log(n))$

Ans 13 :-

$a = 3, \ b = 2, \ f(n) = n$

$c = \log_b a = \log_2 3 = 1.5849$

$n^c = n^{1.5849}$

$n < n^{1.5849}$

$f(n) < n^c$

$T(n) = \Theta(n^{1.5849})$

Ans 14 :-

$a = 3, \ b = 3$

$c = \log_b a = \log_3 3 = 1$

$n^c = n^1 = n$

As $sqrt(n) < n$

$f(n) < n^c$

$T(n) = \Theta(n)$

Ans 15 :-

$a = 4, \ b = 2$

$c = \log_b a = \log_2 4 = 2$

$n^c = n^2$

$n < n^2$ (for any constant)

$f(n) < n^c$

$f(n) = \Theta(n^2)$

Ans 16 :-

$a = 3, b = 4, f(n) = n\log n$

$c = \log_b a = \log_4 3 = 0.792$

$n^c = n^{0.792}$

$n^{0.792} < n\log n$

$T(n) = \Theta(n \log n)$

Ans 17 :-

$a = 3, b = 3$

$c = \log_b a = \log_3 3 = 1$

$f(n) = n/2$

$\therefore n^c = n^1 = n$

As $n/2 < n$

$f(n) < n^c$

$\therefore T(n) = \Theta(n)$

Ans 18 :-

$a = 6, b = 3$

$c = \log_b a = \log_3 6 = 1.6309$

$n^c = n^{1.6309}$

As $n^{1.6309} < n^2 \log n$

$\therefore T(n) = \Theta(n^2 \log n)$

Ans 19 :-

$a = 4,\ b = 2,\ f(n) = \frac{n}{\log n}$

$c = \log_b a = \log_2 4 = 2$

$n^c = n^2$

$\frac{n}{\log n} < n^2$

$T(n) = \Theta(n^2)$

Ans 20 :-

$a = 64,\ b = 8$

$c = \log_b a = \log_8 64 = \log_8 8^2 = 2$

$c = 2$

$n^c = n^2$

$n^2 \log n > n^2$

$T(n) = \Theta(n^2 \log n)$

Ans 21 :-

$a = 7,\ b = 3,\ f(n) = n^2$

$c = \log_b a = \log_3 7 = 1.7712$

$n^c = n^{1.7712}$

$n^{1.7712} < n^2$

$T(n) = \Theta(n^2)$

Ans 22 :-

$a = 1 , b = 2$

$c = \log_b a = \log_2 1 = 0$

$n^c = n^0 = 1$

$n(2 - \cos x) > n^c$

$T(n) = \Theta(n(2 - \cos x))$