Ans 1:-

$j = 1$ $\quad$ $i = 1$

$j = 2$ $\quad$ $i = \cancel{1} + 2$

$j = 3$ $\quad$ $i = 1 + 2 + 3$

m - level

for (i)

$\therefore \; 1 + 2 + 3 + \cdots + < n$

$\therefore \; 1 + 2 + 3 + m < n$

$$\therefore \; \frac{m(m+1)}{2} < n$$

$$m \approx \sqrt{n}$$

By Summation Method.

$$\sum_{i=1}^{m} 1 \Rightarrow 1 + + + \cdots + \sqrt{n} \text{ times}$$

$$\underline{T(n) = \sqrt{n}}$$

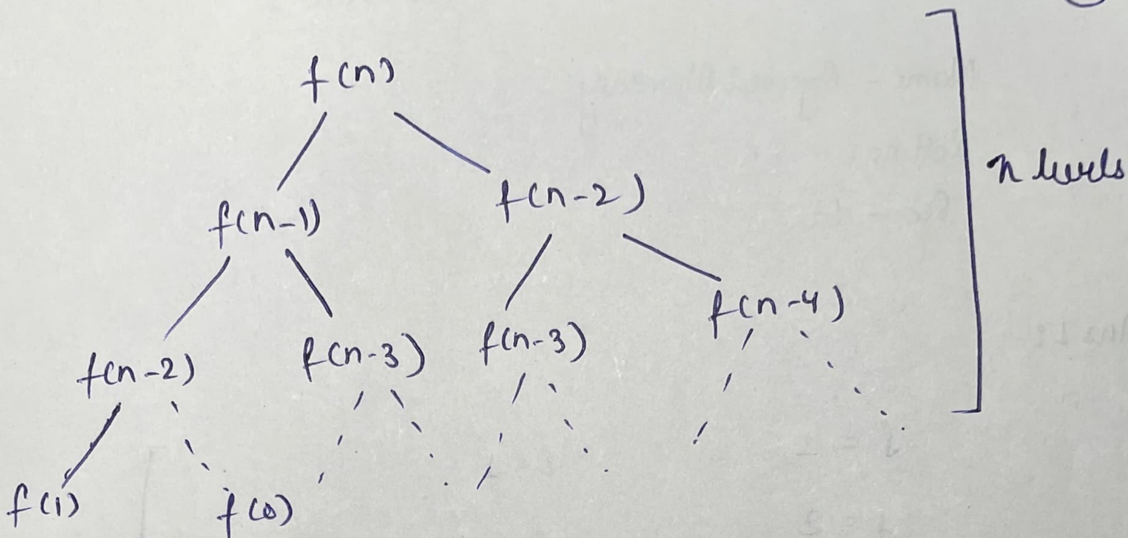
Aus 2:-

for fibonacci series

$f(n) = f(n-1) + f(n-2)$ $\quad$ $f(0) = 0$

$f(1) = 1$

By forming a tree



∴ At every funct. call we get 2 func calls

∴ for n levels

we have = 2 × 2 .... n times

∴ $\boxed{T(n) = 2^n}$

MAXIMUM SPACE

Considering Recursive

Stack :-

no. of calls max. = n

for each call SC (O(1))

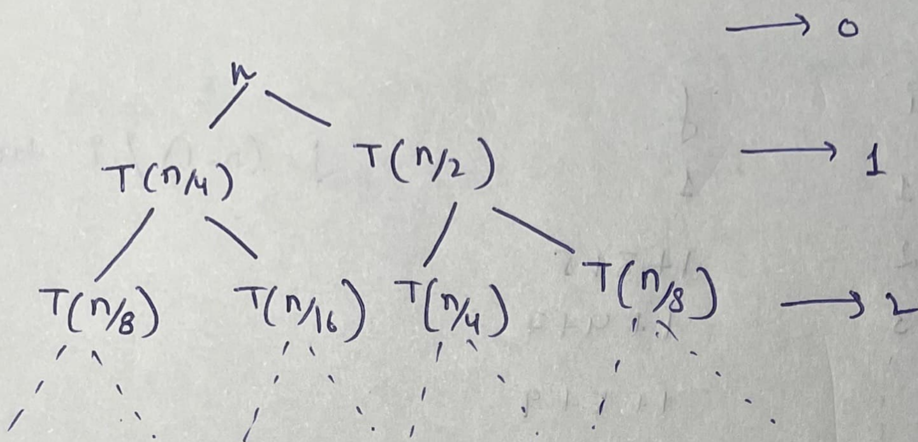
$\boxed{\therefore\ T(n) = O(n)}$

without considering Recursive stack :

each call we have TC O(1)

∴ $\boxed{T(n) = O(1)}$

Ans :-



At level

$0 \to cn^2$

$1 \to \frac{n^2}{4^2} + \frac{n^2}{2^2} = \frac{c5n^2}{16}$

$2 \to \frac{n^2}{8^2} + \frac{n^2}{16^2} + \frac{n^2}{4^2} + \frac{n^2}{8^2} = \left(\frac{5}{16}\right)^2 n^2 c$

$\vdots$

max level $= \frac{n}{2^k} = 1$

$\Rightarrow k = \log_2 n$

$$T(n) = c\left(n^2 + \left(\frac{5}{16}\right)n^2 + \left(\frac{5}{16}\right)^2 n^2 + - + \left(\frac{5}{16}\right)^{\log n} n^2\right)$$

$$T(n) = cn^2\left[1 + \left(\frac{5}{16}\right) + \left(\frac{5}{16}\right)^2 + - + \left(\frac{5}{16}\right)^{\log n}\right]$$

$$T(n) = cn^2 \times 1 \times \left(\frac{1 - \left(\frac{5}{16}\right)^{\log n}}{1 - \left(\frac{5}{16}\right)}\right)$$

$$T(n) = cn^2 \times \frac{11}{5}\left(1 - \left(\frac{5}{16}\right)^{\log n}\right)$$

$$T(n) = O(n^2 c)$$

$$\underline{\underline{O(cn^2)}}$$

Ans 5:

②

for	i	j	$J = (n-1)/i$ times
	1	1	
	2	1+3+5	
	3	1+4+7	
	⋮	1+5+9	
	n		

$$\sum_{i=1}^{n} \frac{(n-1)}{i}$$

$$\therefore T(n) = \frac{(n-1)}{1} + \frac{(n-1)}{2} + \frac{(n-1)}{3} + \cdots + \frac{(n-1)}{n}$$

$$T(n) = n\left[1 + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{n}\right] - 1 \times \left[1 + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{n}\right]$$

$$= n \log n - \log n$$

$$\underline{\underline{T(n) = O(n \log n)}}$$

Ans 6:-

for i

2^1

2^k

2^{k^2}

2^{k^3}

⋮

2^{k^m}

where

$$2^{k^m} < = n$$

$$k^m = \log_2 n$$

$$m = \log_k \log_2 n$$

$$\therefore \sum_{i=1}^{m} 1$$

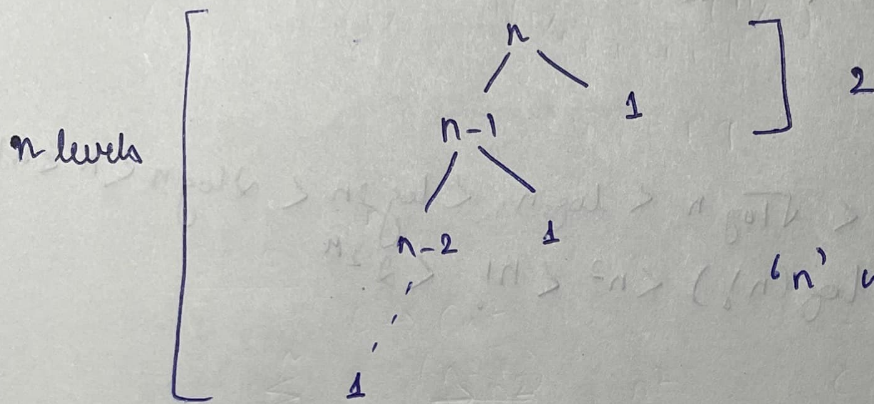
$1 + 1 + 1 + \cdots$ m times

$$T(n) = \underline{\underline{O(\log_k \log n)}}$$

Ans 7 :-

Given Algorithm devides array in 99% and 1 % part

$\therefore T(n) = T(n-1) + O(1)$



'n' work is done at each level

$T(n) = (T(n-1) + T(n-2) + \dots + T(1) + O(1)) \times n$

$= n \times n$

$\therefore \boxed{T(n) = O(n^2)}$

lowest height = 2

highest height = n

$\therefore \boxed{\text{diff} = n-2}$ $\quad n>1$

The given algorithm produces linear result

Ans 8.

(a)

$100 < \log\log n < \log n < (\log n)^2 < \sqrt{n} < n < n\log n < \log(n!) < 2^n < 4^n < 2^{2^n}$

(b)

$1 < \log\log n < \sqrt{\log n} < \log n < \log 2n < 2\log n < n < n\log n < 2n < 4n < \log(n!) < n^2 < n! < 2^{2n}$

(c)

$96 < \log_8 n < \log 2n < 5n < \cancel{n}\ \log_6(n) < n\log_2 n < \log(n!) < 8n^2 < 7n^3 < n! < 8^{2n}$