Computational Assignment 3

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Q7

For a general DFT algorithm with Brute force takes $O(N^2)$ time complexity.

FFT

→ 1^{st} step is to take $N = 2^r$, where $r \in \mathbb{I}^+$

$$X_n = \sum_{k=0}^{N-1} x_k \cdot e^{-\frac{i2\pi kn}{N}} \quad , \left(X_n \xleftrightarrow{FFT} x_n \right)$$

→ Now divide our original sum into $N/2$ odd & $N/2$ even part.

$$X_n = \sum_{k=0}^{\frac{N}{2}-1} x_{2n} e^{-\frac{i2\pi kn}{N/2}} + e^{-\frac{i2\pi n}{N}} \sum_{k=0}^{\frac{N}{2}-1} x_{2n+1} e^{-\frac{i2\pi kn}{N/2}}$$

→ every sum can be recursively divided $\log_2(N)$ times.

N	— N Calculation
$N/2$ $N/2$	— N Calculation
$N/4$ $N/4$ $N/4$ $N/4$	— //
⋮	—
1 1 . . . . 1 1	= //

→ Here we have $\log_2(N)$ steps & N computation on each steps hence we have

$$FFT \sim O(N \log_2 N)$$

→ Here we are not computing single X_n, but each step computes the entire X_n.

Q12

$$g(x) = \exp(-x^2)$$

$$h(x) = \exp(-4x^2)$$

$$g \otimes h(x) = \int_{-\infty}^{\infty} dr \; g(r) \; h(x-r)$$

$$= \int_{-\infty}^{\infty} dr \; e^{-r^2} \; e^{-4(x-r)^2}$$

$$= \int_{-\infty}^{\infty} dr \; \exp\left[-5r^2 - 4x^2 + 8xr\right]$$

$$= \int_{-\infty}^{\infty} dr \; \exp\left[\frac{-1}{5}\left(25r^2 + 16x^2 - 40xr\right) - \frac{4}{5}x^2\right]$$

$$= e^{-\frac{4x^2}{5}} \int_{-\infty}^{\infty} dr \; e^{-\frac{(5r-4x)^2}{5}}$$

$$= \sqrt{\frac{\pi}{5}} \; e^{-\frac{4x^2}{5}}$$