# Fitzhugh - Nagumo Equation

## Group 6

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# Fitzhugh-Nagumo Equation

- The Fitzhugh-Nagumo equations are a model system for elecrical activity in a neuron, an excitable system. A neuron can be stimulated with an input,with a voltage input. After the stimulus, the neuron is 'excited'.
- When a neuron is excited, physiological processes in the cell will cause the neuron to recover from the excitation.

# Fitzhugh-Nagumo Equation

- The FitzHugh-Nagumo model is a simplified version of the Hodgkin-Huxley model which models in a detailed manner activation and deactivation dynamics of a spiking neuron.
- Neuronal signals are short electrical pulses which can be seen as spikes as seen in the plot of membrane potential vs time.

# Fitzhugh-Nagumo Equation

- The FitzHughNagumo model is
  $u_t = u_{xx} + u(1-u)(a-u) \qquad x \in (0, L) \subset R, t > 0$
  where u is membrane potential and a is the input threshold voltage.
- In our problem we have taken the input threshold voltage as 0.5V(experimental result)[1].

# QUESTION

$$u_t = u_{xx} + u(u - a)(1 - u), a = 0.5$$

$$IC : u(x, 0) = sin(\pi x), 0 < x < 1$$

$$BC : u(0, t) = 0\ , u(1, t) = 0$$

# Discretisation using BTCS

$$\frac{U_j^n - U_j^{n-1}}{\Delta t} = \frac{U_{j-1}^n - 2U_j^n + U_{j+1}^n}{h^2} - (U_j^n)(U_j^n - a)(1 - U_j^n)$$

$$U_j^n[1 + 2\lambda - \Delta t(U_j^n - a)(1 - U_j^n)] - \lambda U_{j-1}^n - \lambda U_{j+1}^n - U_j^{n-1} = 0$$

$$(Taking \frac{\Delta t}{h^2} = \lambda)$$

# Discretisation using BTCS

**Continued**

$$j = 1, (U(0,t) = 0)$$

$$f_1(U) = U_1^n[1 + 2\lambda - \Delta t(U_1^n - a)(1 - U_1^n)] - \lambda U_2^n - U_1^{n-1} = 0$$

$$j = 2, 3, ...N - 2$$

$$f_j(U) = U_j^n[1 + 2\lambda - \Delta t(U_j^n - a)(1 - U_j^n)] - \lambda U_{j-1}^n - \lambda U_{j+1}^n - U_j^{n-1} = 0$$

# Discretisation using BTCS

**Continued**

$$j = N-1, (U(1,t) = 0$$

$$f_{N-1}(U) =$$
$$U_{N-1}^{n}[1 + 2\lambda - \Delta t(U_{N-1}^{n} - a)(1 - U_{N-1}^{n})] - \lambda U_{N-2}^{n} - U_{N-1}^{n} = 0$$

# Discretisation using BTCS

**Continued**

After applying Newton's Method

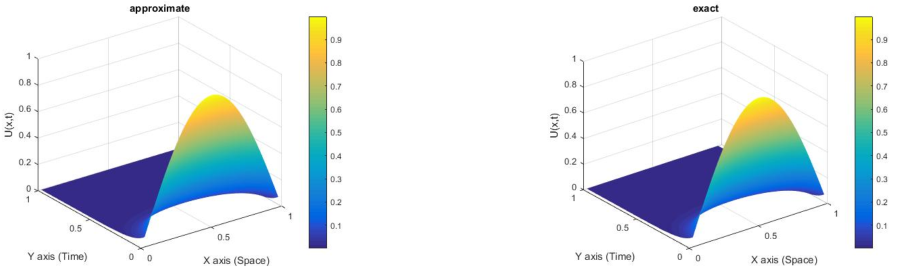
$$U_{n+1} = Un - J(F(U))_n^{-1} * F(U_n)$$

Continued

$$J = \begin{bmatrix} x_{11} & -\lambda & 0 & \dots & 0 & 0 \\ -\lambda & x_{22} & -\lambda & \dots & 0 & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots \\ \dots & \dots & \dots & \dots & \dots & \dots \\ \dots & \dots & \dots & -\lambda & x_{(N-2)(N-2)} & -\lambda \\ 0 & 0 & \dots & 0 & -\lambda & x_{(N-1)(N-1)} \end{bmatrix}$$ where J is a tridiagonal matrix

$x_{ii} = (1 + 2\lambda + a\Delta t) - 2(a+1)\Delta t U_i^n + 3\Delta t((U_i^n)^2)$
i=1,2,....N-1

# Figure 1




Space-Time graph for t $\in (0, 1)$ *and* $x \in (0, 1)$

# Figure 2

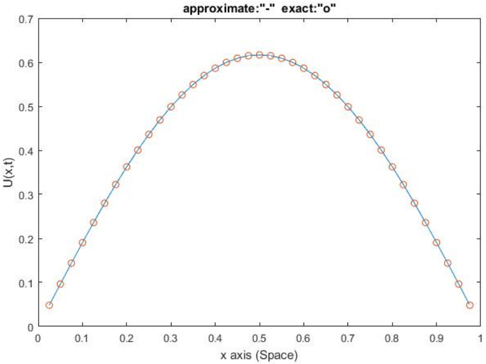



Figure: Approximate and exact at t=0.05

# Figure 3

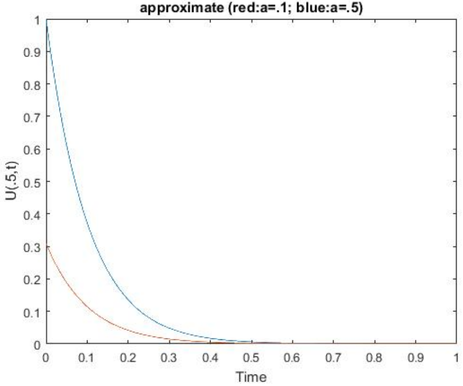



Figure: Membrane Potential at x=0.1 and x=0.5

# Figure 4

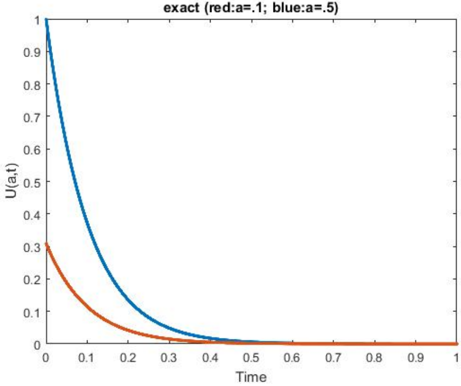



Figure: Membrane Potential at x=0.1 and x=0.5

# References


- [1] The Finite Difference Methods for Fitz Hugh-Nagumo Equation Saad A. Manaa1, Fadhil H. Easif2, Aveen S. Faris3
- [2]Handbook of nonlinear partial differential equation,Andrei D.Polyanin and Valentin F.Zaitsev