QuB

A Resource Aware Functional Programming Language

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Hard problems in programming

Naming variables

Introduction and Motivation

Hard problems in programming

Resource management

in evolving production code

Resources: Files, database connections, entity with a shared state

Resource Management: File Handling

- Modified File Handling API in Haskell

$$
\begin{aligned}
\texttt{openFile} &:: \texttt{FilePath} &&\to \texttt{IO FileHandle}\\
\texttt{closeFile} &:: \texttt{FileHandle} &&\to \texttt{IO}\ ()\\
\texttt{readLine} &:: \texttt{FileHandle} &&\to \texttt{IO}\ (\texttt{String}, \texttt{FileHandle})\\
\texttt{writeFile} &:: \texttt{String} &&\to \texttt{FileHandle}\\
& &&\to \texttt{IO}\ ((), \texttt{FileHandle})\\
\\
\texttt{upper} &:: \texttt{String} &&\to \texttt{String}
\end{aligned}
$$

Resource Management: File Handling

- File Handling in Haskell

```
do f  ← openFile "sample.txt"
   (s, f)  ← readLine f
   let c = upper s
   ((), f) ← writeLine f c
          .
          .
          .
   ()  ← closeFile f
```

- File Handling in Haskell Gone Wrong (Part I)

```
do f      ← openFile "sample.txt"
   (s, f)  ← readLine f
   let c = upper s
   ((), f) ← writeLine f c
          .
          .
          .
   ()     ← closeFile f
          .
          .
          .
   ()     ← closeFile f
   return c
```

Resource Management: File Handling

- File Handling in Haskell Gone Wrong (Part I)

```
do f ← openFile "sample.txt"
   (s, f) ← readLine f
   let c = upper s
   ((), f) ← writeLine f c
        .
        .
        .
  () ← closeFile f
        .
        .
        .
  () ← closeFile f
   return c
```

- File is closed twice: Run time crash

Resource Management: File Handling

- File Handling in Haskell Gone Wrong (Part II)

```
do f  ← openFile "sample.txt"
   (s, f)  ← readLine f
   let c = upper s
   ((), f) ← writeLine f c
       .
       .
       .
   return c
```

Resource Management: File Handling

- File Handling in Haskell Gone Wrong (Part II)

```
do f      ← openFile "sample.txt"
   (s, f)  ← readLine f
   let c = upper s
   ((), f) ← writeLine f c
       .
       .
       .
   return c /*File not closed!!*/
```

- File not closed: Memory leak

Resource Management: Exception Handling

- `MonadError`[5] in Haskell

```
class Monad m ⇒ MonadError e m | m → e where
    throwError :: e → m a
    catchError :: m a → (e → m a) → m a
```

- `throwError` starts exception processing
- `catchError` exception handler

Resource Management: Exception Handling

- Using `MonadError` in Haskell

```
do f ← openFile "sample.txt"
   ((s, f)  ← readLine f
   let c = upper s
   ()  ← closeFile f
   return $ Right c)
      `catchError` (\_ →
            return $ Left "Error in reading file")
```

- Exception may cause memory leak

Well typed programs do not go wrong.

— R. Milner

Well typed programs do not go wrong.

— R. Milner

~~Lights~~Types will guide you home

— Coldplay

Contributions

- Design and implement QuB type system
  - Resources as first class citizens
  - Program objects are restricted or unrestricted
  - Functions that share resources with their arguments or are separate.
- Formalizing and proving important properties of QuB
- QuB is logic of ***BI*** with steroids
  - Environments as graphs
- Working examples

Background Work: Simply Typed Lambda Calculus (STLC)

$$\lambda x.M \begin{cases} \text{Abstract over computation} \\ \text{Define functions} \end{cases}$$

$$MN \begin{cases} \text{Do the computation} \\ \text{Use functions} \end{cases}$$

Background Work: Simply Typed Lambda Calculus (STLC)

$$\lambda x.M \begin{cases} \text{Abstract over computation} \\ \text{Define functions} \end{cases}$$

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$$\frac{\Gamma_x, x : \tau \vdash M : \tau'}{\Gamma \vdash \lambda x.M : \tau \to \tau'} [\to \mathrm{I}] \qquad \frac{\Gamma \vdash M : \tau \to \tau' \qquad \Gamma \vdash N : \tau}{\Gamma \vdash MN : \tau'} [\to \mathrm{E}]$$

Background Work: Simply Typed Lambda Calculus (STLC)

$$\lambda x.M \begin{cases} \text{Abstract over computation} \\ \text{Define functions} \end{cases}$$

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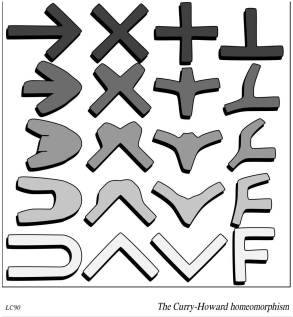
Hinley-Milner (***HM***) type system ensures sane programs

Background Work: Curry-Howard Correspondence

- Types are Propostions
- Programs are Proofs

HM type system ≡ Second Order Propositional Logic



Source: `http://lucacardelli.name/Artifacts/Drawings/CurryHoward/CurryHoward.pdf`

Background Work: Second Order Propositional Logic

$\boxed{\text{Language}}$

Propostions & connectives $\quad A, B, C ::= x \mid A \supset B \mid \forall x.B \mid \ldots$

Context $\quad \Gamma, \Delta ::= \epsilon \mid \Gamma, A$

$\boxed{\text{Logic Rules}}$

$$\frac{}{A \vdash A}\,[\text{Ax}]$$

$$\frac{\Gamma \vdash B \qquad x \notin \Gamma}{\forall x.B}\,[\forall\text{I}] \qquad \frac{\Gamma \vdash \forall x.B \qquad \Gamma \vdash A}{B[x/A]}\,[\forall\text{E}]$$

$$\frac{\Gamma, A \vdash B}{\Gamma \vdash A \supset B}\,[\supset\text{I}] \qquad \frac{\Gamma \vdash A \supset B \qquad \Gamma \vdash A}{\Gamma \vdash B}\,[\supset\text{E}]$$

Propostions are truth values not resources

$\boxed{\text{Language}}$

Propostions & connectives $\quad A, B, C ::= x \mid A \supset B \mid \forall x.B \mid \ldots$

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$$\frac{\Gamma, A \vdash B}{\Gamma \vdash A \supset B}\,[\supset \mathrm{I}] \qquad \frac{\Gamma \vdash A \supset B \qquad \Gamma \vdash A}{\Gamma \vdash B}\,[\supset \mathrm{E}]$$

- Two implicit rules in Propositional Calculus

$$\frac{\Gamma \vdash B}{\Gamma, A \vdash B}\ [\mathrm{WKN}] \qquad\qquad \frac{\Gamma, A, A \vdash B}{\Gamma, A \vdash B}\ [\mathrm{CTR}]$$

Background Work: Substructural Logic

- Two implicit rules in Propositional Calculus

$$\frac{\Gamma \vdash B}{\Gamma, A \vdash B}\,[\mathrm{WKN}] \qquad\qquad \frac{\Gamma, A, A \vdash B}{\Gamma, A \vdash B}\,[\mathrm{CTR}]$$

- Control the use of [WKN] and [CTR]

Certain contexts cannot undergo weakening or contraction[3, 10]

$$\text{Context} \quad \Gamma, \Delta ::= \epsilon \mid \Gamma, \langle A \rangle \mid \Gamma, [A]$$

$$\Gamma, \langle A \rangle \nvdash \Gamma$$
$$\Gamma, \langle A \rangle \nvdash \Gamma, \langle A \rangle, \langle A \rangle$$
$$\Gamma, [A] \vdash \Gamma$$
$$\Gamma, [A] \vdash \Gamma, [A], [A]$$

Language

$$\text{Context}\Gamma, \Delta ::= \epsilon \mid \Gamma, \langle A \rangle \mid \Gamma, [A]$$

$$\text{Propostions} A, B, C ::= X \mid !A \mid A \multimap B \mid A \,\&\, B \mid A \otimes B \mid A \oplus B$$

Structural Rules

$$\frac{}{[A] \vdash A}\,[\mathrm{ID}_{[]}] \qquad \frac{}{\langle A \rangle \vdash A}\,[\mathrm{ID}_{\langle\rangle}]$$

$$\frac{\Gamma, \Delta \vdash A}{\Delta, \Gamma \vdash A}\,[\mathrm{EXCH}] \quad \frac{\Gamma, [A], [A] \vdash B}{\Gamma, [A] \vdash B}\,[\mathrm{CTRN}] \quad \frac{\Gamma \vdash B}{\Gamma, [A] \vdash B}\,[\mathrm{WKN}]$$

$$\frac{[\Gamma] \vdash A}{[\Gamma] \vdash !A}\,[!\mathrm{I}] \qquad \frac{\Gamma \vdash !A \qquad \Delta, [A] \vdash B}{\Gamma, \Delta \vdash B}\,[!\mathrm{E}]$$

$\boxed{\text{Connective Rules}}$

$$\frac{\Gamma, A \vdash B}{\Gamma \vdash A \multimap B}\,[\multimap I] \qquad\qquad \frac{\Gamma \vdash A \multimap B \qquad \Delta \vdash A}{\Gamma, \Delta \vdash B}\,[\multimap E]$$

Connective Rules

$$\frac{\Gamma, A \vdash B}{\Gamma \vdash A \multimap B}\,[\multimap I] \qquad \frac{\Gamma \vdash A \multimap B \qquad \Delta \vdash A}{\Gamma, \Delta \vdash B}\,[\multimap E]$$

$$\frac{\Gamma \vdash A \qquad \Gamma \vdash B}{\Gamma \vdash A \,\&\, B}\,[\& I] \qquad \frac{\Gamma \vdash A \,\&\, B}{\Gamma \vdash A}\,[\& E_1] \qquad \frac{\Gamma \vdash A \,\&\, B}{\Gamma \vdash B}\,[\& E_2]$$

Connective Rules

$$\frac{\Gamma, A \vdash B}{\Gamma \vdash A \multimap B}\,[\multimap I] \qquad \frac{\Gamma \vdash A \multimap B \qquad \Delta \vdash A}{\Gamma, \Delta \vdash B}\,[\multimap E]$$

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$$\frac{\Gamma \vdash A \qquad \Delta \vdash B}{\Gamma, \Delta \vdash A \otimes B}\,[\otimes I] \qquad \frac{\Gamma \vdash A \otimes B \qquad \Delta, A, B \vdash C}{\Gamma, \Delta \vdash C}\,[\otimes E]$$

Connective Rules

$$\frac{\Gamma, A \vdash B}{\Gamma \vdash A \multimap B}\ [\multimap I] \qquad \frac{\Gamma \vdash A \multimap B \qquad \Delta \vdash A}{\Gamma, \Delta \vdash B}\ [\multimap E]$$

$$\frac{\Gamma \vdash A \qquad \Gamma \vdash B}{\Gamma \vdash A \,\&\, B}\ [\& I] \qquad \frac{\Gamma \vdash A \,\&\, B}{\Gamma \vdash A}\ [\& E_1] \qquad \frac{\Gamma \vdash A \,\&\, B}{\Gamma \vdash B}\ [\& E_2]$$

$$\frac{\Gamma \vdash A \qquad \Delta \vdash B}{\Gamma, \Delta \vdash A \otimes B}\ [\otimes I] \qquad \frac{\Gamma \vdash A \otimes B \qquad \Delta, A, B \vdash C}{\Gamma, \Delta \vdash C}\ [\otimes E]$$

$$\frac{\Gamma \vdash A}{\Gamma \vdash A \oplus B}\ [\oplus I_1] \qquad \frac{\Delta \vdash B}{\Delta \vdash A \oplus B}\ [\oplus I_2]$$

$$\frac{\Gamma \vdash A \oplus B \qquad \Delta, A \vdash C \qquad \Delta, B \vdash C}{\Gamma, \Delta \vdash C}\ [\oplus E]$$

Background Work: Linear Logic

- Propositions act like Resources

$$
\begin{array}{rl}
\text{“A consumed to give B”} & A, A \multimap B \vdash B \\
\text{“Both A and B”} & A \otimes B \\
\text{“Choice between A and B”} & A \,\&\, B \\
\text{Cannot create new copies} & A \nvdash A \otimes A \\
\text{Fall back to intuistionstic logic} & !A \multimap B \equiv A \supset B
\end{array}
$$

Background Work: Linear Logic

- Propositions act like Resources

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\text{“A consumed to give B”} & A, A \multimap B \vdash B \\
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\end{array}
$$

- Curry-Howard Corresondence: Type systems based on Linear Logic
- Active Area of research: L^3[1], F°[6], Linear Haskell[2], Quill[7]
- Resources are first class values

Background Work: Linear Logic

Downside: Asymmetry between implication and conjunction
Multiplicative Fragment:

$$A \otimes B \vdash C \text{ iff } A \vdash B \multimap C$$

Additive Fragment:

$$!A \,\&\, !B \vdash C \text{ iff } !A \vdash !B \multimap C$$

Modality is necessary to avoid over-restriction

Background Work: Qualified Types

General framework to incorporate predicates into type language

$$P \mid \Gamma \vdash M : \sigma$$

"Type of M is σ when predicates in P are satisfied and free variables in M are specified in Γ"[4]

HM system with qualified Types:

$$\frac{x : \sigma \in \Gamma}{P \mid \Gamma \vdash x : \sigma}\;[\text{VAR}] \qquad \frac{P \mid \Gamma \vdash M : \sigma \qquad Q \mid \Gamma_x, x : \sigma \vdash N : \tau}{P, Q \mid \Gamma \vdash (\texttt{let}\ x = M\ \texttt{in}\ N) : \tau}\;[\text{LET}]$$

$$\frac{P \mid \Gamma \vdash M : \sigma \qquad t \notin \texttt{fvs}(\Gamma) \cup \texttt{fvs}(P)}{P \mid \Gamma \vdash M : \forall t.\sigma}\;[\forall\ \text{I}] \qquad \frac{P \mid \Gamma \vdash M : \forall t.\sigma}{P \mid \Gamma \vdash M : [\tau/t]\sigma}\;[\forall\ \text{E}]$$

$$\frac{P \mid \Gamma_x, x : \tau \vdash M : \tau'}{P \mid \Gamma \vdash \lambda x.M : \tau \to \tau'}\;[\to \text{I}] \qquad \frac{P \mid \Gamma \vdash M : \tau \to \tau' \qquad P \mid \Gamma \vdash N : \tau}{P \mid \Gamma \vdash MN : \tau'}\;[\to \text{E}]$$

$$\frac{P, \pi \mid \Gamma \vdash M : \rho}{P \mid \Gamma \vdash M : \pi \Rightarrow \rho}\;[\Rightarrow\text{I}] \qquad \frac{P \mid \Gamma \vdash M : \pi \Rightarrow \rho \qquad P \Rightarrow \pi}{P \mid \Gamma \vdash M : \rho}\;[\Rightarrow\text{E}]$$

$\pi \Rightarrow \rho$: π qualifies ρ

$P \Rightarrow \pi$: P entails π

Background Work: Quill

Quill[7]: Language with qualified types for linear types and first class polymorphism

Three Predicates:

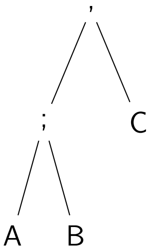
- $\mathtt{Un}\ \tau$ If τ does not have resources or can be copied or dropped easily.
- $\mathtt{Fun}\ \tau$ If τ is a function type
- $\tau \geq \tau'$ If τ less restricting than τ'

Background Work: Quill

- Unrestricted Types: `Un Int`, `Un` Bool
- Restricted or Linear Types: FileHandle
- Function Types: `Fun (Int → Int)`, `Fun (String → String)`

Background Work: Logic of Bunched Implications (***BI***)

- Contexts as trees, called bunches[9]
- Two connective used to combine contexts: $A; B$ or A, B



Background Work: Logic of ***BI***

- Context connective guides choice of implication

$$\frac{\Gamma, A \vdash B}{\Gamma \vdash A \mathrel{-\!\!*} B}\,[\mathrel{-\!\!*}\mathrm{I}] \qquad\qquad \frac{\Gamma; A \vdash B}{\Gamma \vdash A \twoheadrightarrow B}\,[\twoheadrightarrow\mathrm{I}]$$

- Contraction

$$\begin{array}{cc} A; B \vdash A & A; B \vdash B \\ A, B \nvdash A & A, B \nvdash B \end{array}$$

- Weakening

$$\begin{array}{cc} A; B \vdash A; B; B & A; B \vdash A; A; B \\ A, B \nvdash A, B, B & A, B \nvdash A, A, B \end{array}$$

Background Work: Logic of *BI*

Curry-Howard interpretation of logic of ***BI*** [8]

Language

$$
\begin{array}{rl}
\text{Context} & \Gamma, \Delta ::= \{\}_m \mid \{\}_a \mid x : \tau \mid \Gamma, \Delta \mid \Gamma; \Delta \\
\text{Types} & \tau, \upsilon ::= t \mid \iota \mid \tau \twoheadrightarrow \tau \mid \tau \mathrel{-\!\!*} \tau \\
\text{Expressions} & M, N ::= x \mid \lambda x.M \mid \alpha x.M \mid MN
\end{array}
$$

Typing rules

$$
\frac{x : \tau \in \Gamma}{\Gamma \vdash x : \tau}\,[\text{VAR}] \qquad \frac{\Gamma; \Gamma \vdash M : \tau}{\Gamma \vdash M : \tau}\,[\text{CTR}] \qquad \frac{\Gamma \vdash M : \tau}{\Gamma; \Delta \vdash M : \tau}\,[\text{WKN}]
$$

$$
\frac{\Gamma_x, x : \tau \vdash M : \tau'}{\Gamma \vdash \lambda x.M : \tau \mathrel{-\!\!*} \tau'}\,[\mathrel{-\!\!*}\text{I}] \qquad \frac{\Gamma \vdash M : \tau \mathrel{-\!\!*} \tau' \qquad \Delta \vdash N : \tau}{\Gamma, \Delta \vdash MN : \tau'}\,[\mathrel{-\!\!*}\text{E}]
$$

$$
\frac{\Gamma_x; x : \tau \vdash M : \tau'}{\Gamma \vdash \alpha x.M : \tau \twoheadrightarrow \tau'}\,[\twoheadrightarrow\text{I}] \qquad \frac{\Gamma \vdash M : \tau \twoheadrightarrow \tau' \qquad \Delta \vdash N : \tau}{\Gamma; \Delta \vdash MN : \tau'}\,[\twoheadrightarrow\text{E}]
$$

QuB

- Quill: Qualified types for type system based on linear logic
- QuB: Qualified types for type system based on logic of ***BI***

QuB: Types and Predicates

$$
\begin{array}{rl}
\text{Types} & \tau, \upsilon, \phi ::= t \mid \iota \mid \tau \to \tau \\
& \quad \text{where} \qquad \to \in \{\overset{!}{\multimap}, \multimap, \overset{!}{\twoheadrightarrow}, \twoheadrightarrow\} \\
\text{Predicates} & \pi, \omega ::= \mathtt{Un}\ \tau \mid \mathtt{SeFun}\ \tau \mid \mathtt{ShFun}\ \tau \mid \tau \geq \tau' \\
\text{Qualified Types} & \rho ::= \tau \mid \pi \Rightarrow \rho \\
\text{Type schemes} & \sigma ::= \rho \mid \forall t.\sigma
\end{array}
$$

- $\mathtt{SeFun}\ \tau$: τ is a function that is separate from its argument
- $\mathtt{ShFun}\ \tau$: τ is a function that is in sharing with its argument
- $\mathtt{Un}\ \tau$: τ does not have resources or they can be copied/dropped easily

QuB: Types and Predicates

$$
\begin{array}{rl}
\text{Types} & \tau, \upsilon, \phi ::= t \mid \iota \mid \tau \to \tau \\
 & \quad \text{where} \quad \to \in \{\overset{!}{-\!\!\ast}, -\!\!\ast, \overset{!}{\twoheadrightarrow}, \twoheadrightarrow\} \\
\text{Predicates} & \pi, \omega ::= \mathtt{Un}\ \tau \mid \mathtt{SeFun}\ \tau \mid \mathtt{ShFun}\ \tau \mid \tau \geq \tau' \\
\text{Qualified Types} & \rho ::= \tau \mid \pi \Rightarrow \rho \\
\text{Type schemes} & \sigma ::= \rho \mid \forall t.\sigma
\end{array}
$$

- $-\!\!\ast$: Function type that is separate with its argument
- $\twoheadrightarrow$: Function type that is in sharing with its argument
- $\overset{!}{-\!\!\ast}$, $\overset{!}{\twoheadrightarrow}$: unrestriced versions of $-\!\!\ast$ and $\twoheadrightarrow$

QuB: Language

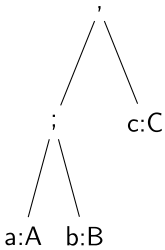
Term Variables $\quad x, y, z \in \mathsf{Var}$

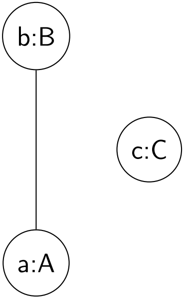
Expressions $\quad M, N ::= x \mid \lambda^{\rightarrow\!\!\!\!\!*} x.M \mid \lambda^{\twoheadrightarrow} x.M \mid MN \mid \texttt{let}\ x = M\ \texttt{in}\ N$

- $\lambda^{\rightarrow\!\!\!\!\!*} x.M$: Introduces $\rightarrow\!\!\!\!\!*$ function
- $\lambda^{\twoheadrightarrow} x.N$: Introduces $\twoheadrightarrow$ function

QuB: Typing Environmnet

- Logic of ***BI***: Contexts are trees
- QuB: Contexts are flattened sharing graphs





- Sharing relation Ψ

$$\forall_{x \in \mathtt{dom}(\Psi)}\; x \in \Psi(x) \qquad \text{(reflexive)}$$

$$\forall_{x,y \in \mathtt{dom}(\Psi)}\; \text{if } y \in \Psi(x) \text{ then } x \in \Psi(y) \qquad \text{(symmetric)}$$

$$\forall_{x,y,z \in \mathtt{dom}(\Psi)}\; \text{if } y \in \Psi(x) \text{ and } z \in \Psi(y) \;\not\Rightarrow\; z \in \Psi(x) \qquad \text{(non-transitive)}$$

QuB: Typing Environment

"x of type σ is in sharing with $\vec{y}$"

$$(x, \sigma, \vec{y}) \in \Gamma$$

$$\text{Typing Context} \quad \Gamma, \Delta ::= \epsilon \mid \Gamma, x^{\vec{y}} : \sigma$$

$$\begin{aligned}
\texttt{Vars}(\Gamma, x^{\vec{y}} : \tau) &= \texttt{Vars}(\Gamma) \cup \{x\} \\
\texttt{Shared}(\Gamma, x^{\vec{y}} : \tau) &= \texttt{Shared}(\Gamma) \cup \{\vec{y}\} \\
\texttt{Used}(\Gamma) &= \texttt{Vars}(\Gamma) \cup \texttt{Shared}(\Gamma)
\end{aligned}$$

$$(\Gamma, x^{\vec{y}} : \tau)^{[a \mapsto \vec{b}]} = \begin{cases} a \notin \vec{y} & (\Gamma^{[a \mapsto \vec{b}]}, x^{\vec{y}} : \tau) \\ a \in \vec{y} & (\Gamma^{[a \mapsto \vec{b}]}, x^{(\vec{y} \setminus a) \cup \vec{b}} : \tau) \end{cases}$$

$$\Gamma^{[\vec{a} \mapsto \vec{b}]} = (\ldots((\Gamma^{[a_1 \mapsto \vec{b}]})^{[a_2 \mapsto \vec{b}]})^{\cdots})^{[a_n \mapsto \vec{b}]}$$

$$\begin{aligned}
\Gamma \circledast \Gamma' &= \Gamma \sqcup \Gamma' \qquad \texttt{if } \texttt{Vars}(\Gamma) \# \texttt{Used}(\Gamma') \wedge \texttt{Vars}(\Gamma') \# \texttt{Used}(\Gamma) \\
\Gamma \oplus \Gamma' &= \Gamma \sqcup \Gamma' \qquad \texttt{if } \texttt{Used}(\Gamma) = \texttt{Used}(\Gamma')
\end{aligned}$$

Structural Rules

$$\frac{}{P \mid x^{\vec{y}} : \sigma \vdash x : \sigma} \text{[ID]}$$

$$\frac{P \mid \Gamma \circledast \Delta \circledast \Delta \vdash M : \sigma \qquad P \vdash \Delta \ \mathbf{un}}{P \mid \Gamma \circledast \Delta \vdash M : \sigma} \text{[CTR-UN]} \qquad \frac{P \mid \Gamma \oplus \Delta \oplus \Delta \vdash M : \sigma}{P \mid \Gamma \oplus \Delta \vdash M : \sigma} \text{[CTR-SH]}$$

$$\frac{P \mid \Gamma \vdash M : \sigma \qquad P \vdash \Delta \ \mathbf{un}}{P \mid \Gamma \circledast \Delta \vdash M : \sigma} \text{[WKN-UN]} \qquad \frac{P \mid \Gamma \vdash M : \sigma}{P \mid \Gamma \oplus \Delta \vdash M : \sigma} \text{[WKN-SH]}$$

Connective Rules

$$\frac{P \mid \Gamma \vdash M : \sigma \qquad P' \mid \Gamma'_x, x : \sigma \vdash N : \tau}{P \cup P' \mid \Gamma \sqcup \Gamma' \vdash (\mathtt{let}\ x = M\ \mathtt{in}\ N) : \tau}\ [\text{LET}]$$

$$\frac{P \mid \Gamma \vdash M : \sigma \qquad t \notin \mathtt{fvs}(\Gamma) \cup \mathtt{fvs}(P)}{P \mid \Gamma \vdash M : \forall t.\sigma}\ [\forall\ \text{I}] \qquad \frac{P \mid \Gamma \vdash M : \forall t.\sigma}{P \mid \Gamma \vdash M : [\tau \backslash t]\sigma}\ [\forall\ \text{E}]$$

$$\frac{P, \pi \mid \Gamma \vdash M : \rho}{P \mid \Gamma \vdash M : \pi \Rightarrow \rho}\ [\Rightarrow\ \text{I}] \qquad \frac{P \mid \Gamma \vdash M : \pi \Rightarrow \rho \qquad P \vdash \pi}{P \mid \Gamma \vdash M : \rho}\ [\Rightarrow\ \text{E}]$$

$$\frac{\begin{array}{c} P \Rightarrow \mathtt{ShFun}\ \phi \qquad P \vdash \Gamma \geq \phi \\ P \mid \Gamma^{[\varnothing \mapsto \{x\}]}, x^{\mathsf{Vars}(\Gamma)} : \tau \vdash M : \tau' \end{array}}{P \mid \Gamma \vdash \lambda^{\twoheadrightarrow} x.M : \phi \tau \tau'}\ [\twoheadrightarrow\ \text{I}] \qquad \frac{\begin{array}{c} P \Rightarrow \mathtt{ShFun}\ \phi \\ P \mid \Gamma \vdash M : \phi \tau \tau' \qquad P \mid \Gamma' \vdash N : \tau \end{array}}{P \mid \Gamma \oplus \Gamma' \vdash MN : \tau'}\ [\twoheadrightarrow\ \text{E}]$$

$$\frac{\begin{array}{c} P \Rightarrow \mathtt{SeFun}\ \phi \qquad P \vdash \Gamma \geq \phi \\ P \mid \Gamma, x^{\varnothing} : \tau \vdash M : \tau' \end{array}}{P \mid \Gamma \vdash \lambda^{-\!\!*} x.M : \phi \tau \tau'}\ [-\!\!*\ \text{I}] \qquad \frac{\begin{array}{c} P \Rightarrow \mathtt{SeFun}\ \phi \\ P \mid \Gamma \vdash M : \phi \tau \tau' \qquad P \mid \Gamma' \vdash N : \tau \end{array}}{P \mid \Gamma \circledast \Gamma' \vdash MN : \tau'}\ [-\!\!*\ \text{E}]$$

QuB: Typing Terms

$$
\begin{prooftree}
\AxiomC{}
\RightLabel{[ID]}
\UnaryInfC{$\varnothing \mid y^{\varnothing} : \tau' \vdash y : \tau'$}
\AxiomC{}
\RightLabel{[ID]}
\UnaryInfC{$\varnothing \mid x^{\varnothing} : \tau \vdash x : \tau$}
\AxiomC{}
\RightLabel{[ID]}
\UnaryInfC{$\varnothing \mid f^{\varnothing} : \tau \mathrel{-\!\!*} \tau' \mathrel{-\!\!*} \upsilon \vdash f : \tau \mathrel{-\!\!*} \tau' \mathrel{-\!\!*} \upsilon$}
\RightLabel{[$\mathrel{-\!\!*}$E]}
\BinaryInfC{$\varnothing \mid x^{\varnothing} : \tau \circledast f^{\varnothing} : \tau \mathrel{-\!\!*} \tau' \mathrel{-\!\!*} \upsilon \vdash fx : (\tau' \mathrel{-\!\!*} \upsilon)$}
\RightLabel{[$\mathrel{-\!\!*}$E]}
\BinaryInfC{$\varnothing \mid y^{\varnothing} : \tau' \circledast x^{\varnothing} : \tau \circledast f^{\varnothing} : \tau \mathrel{-\!\!*} \tau' \mathrel{-\!\!*} \upsilon \vdash fxy : \upsilon$}
\RightLabel{[$EXCH$]}
\UnaryInfC{$\varnothing \mid x^{\varnothing} : \tau \circledast y^{\varnothing} : \tau' \circledast f^{\varnothing} : \tau \mathrel{-\!\!*} \tau' \mathrel{-\!\!*} \upsilon \vdash fxy : \upsilon$}
\RightLabel{[$\mathrel{-\!\!*}$ I]}
\UnaryInfC{$\varnothing \mid x^{\varnothing} : \tau \circledast y^{\varnothing} : \tau' \vdash \lambda^{-\!*} f.fxy : (\tau \mathrel{-\!\!*} \tau' \mathrel{-\!\!*} \upsilon) \mathrel{-\!\!*} \upsilon$}
\RightLabel{[$\mathrel{-\!\!*}$ I]}
\UnaryInfC{$\varnothing \mid x^{\varnothing} : \tau \vdash \lambda^{-\!*} y.\lambda^{-\!*} f.fxy : \tau' \mathrel{-\!\!*} (\tau \mathrel{-\!\!*} \tau' \mathrel{-\!\!*} \upsilon) \mathrel{-\!\!*} \upsilon$}
\RightLabel{[$\equiv$]}
\UnaryInfC{$\varnothing \mid I \vdash \lambda^{-\!*} x.\lambda^{-\!*} y.\lambda^{-\!*} f.fxy : \tau \mathrel{-\!\!*} \tau' \mathrel{-\!\!*} (\tau \mathrel{-\!\!*} \tau' \mathrel{-\!\!*} \upsilon) \mathrel{-\!\!*} \upsilon$}
\end{prooftree}
$$

QuB: Typing Terms

$$
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\dfrac{}{\varnothing \mid y^{xf} : \tau' \vdash y : \tau'}\,[ID]
\qquad
\dfrac{
\dfrac{}{\varnothing \mid x^{fy} : \tau \vdash x : \tau}\,[ID]
\quad
\dfrac{}{\varnothing \mid f^{xy} : \tau \twoheadrightarrow \tau' \twoheadrightarrow \upsilon \vdash f : \tau \twoheadrightarrow \tau' \twoheadrightarrow \upsilon}\,[ID]
}{\varnothing \mid x^{xy} : \tau \oplus f^{xy} : \tau \twoheadrightarrow \tau' \twoheadrightarrow \upsilon \vdash fx : (\tau' \twoheadrightarrow \upsilon)}\,[\mathrel{-\!\!*}\, E]
}{\varnothing \mid y^{xf} : \tau' \oplus x^{yf} : \tau \oplus f^{xy} : \tau \twoheadrightarrow \tau' \twoheadrightarrow \upsilon \vdash fxy : \upsilon}\,[\twoheadrightarrow E]
}{\varnothing \mid x^{yf} : \tau \oplus y^{xf} : \tau' \oplus f^{xy} : \tau \twoheadrightarrow \tau' \twoheadrightarrow \upsilon \vdash fxy : \upsilon}\,[EXCH]
}{\varnothing \mid x^{y} : \tau \oplus y^{x} : \tau' \vdash \lambda^{\twoheadrightarrow} f.fxy : (\tau \twoheadrightarrow \tau' \twoheadrightarrow \upsilon) \twoheadrightarrow \upsilon}\,[\twoheadrightarrow I]
}{\varnothing \mid x^{\varnothing} : \tau \vdash \lambda^{\twoheadrightarrow} y.\lambda^{\twoheadrightarrow} f.fxy : \tau' \twoheadrightarrow (\tau \twoheadrightarrow \tau' \twoheadrightarrow \upsilon) \twoheadrightarrow \upsilon}\,[\twoheadrightarrow I]
}{\varnothing \mid I \vdash \lambda^{\mathrel{-\!\!*}} x.\lambda^{\twoheadrightarrow} y.\lambda^{\twoheadrightarrow} f.fxy : \tau \mathrel{-\!\!*} \tau' \twoheadrightarrow (\tau \twoheadrightarrow \tau' \twoheadrightarrow \upsilon) \twoheadrightarrow \upsilon}\,[\mathrel{-\!\!*}\, I]
$$

QuB: Syntax Directed Typing Rules

Term structure $\leftrightarrow$ Typing Rule

QuB: Syntax Directed Typing Rules

$$\frac{P \vdash \Gamma_{\vec{y}}\ \mathtt{un} \qquad (P \Rightarrow \tau) \sqsubseteq \sigma}{P \mid \Gamma, x^{\vec{y}} : \sigma \vdash^{s} x : \tau}\ [\mathsf{VAR}^{s}]$$

$$\frac{\begin{array}{c} Q \mid (\Gamma'_x \oplus \Gamma''_x) \circledast \Delta \vdash^{s} M : \upsilon \qquad P \vdash \Delta\ \mathtt{un} \\ P \mid (\Gamma_x, x^{\varnothing} : \sigma) \oplus \Gamma''_x \circledast \Delta \vdash^{s} N : \tau \qquad \sigma = \mathtt{Gen}(\{\Gamma' \oplus \Gamma''_x \circledast \Delta\}, Q \Rightarrow \upsilon) \end{array}}{P, Q \mid (\Gamma \circledast \Gamma') \oplus \Gamma'' \circledast \Delta \vdash^{s} (\mathtt{let}\ x = M\ \mathtt{in}\ N) : \tau}\ [\mathsf{Let}^{s}]$$

$$[\mathsf{VAR}^{s}] \equiv [\mathsf{ID}] + [\forall \mathsf{E}] + [\Rightarrow \mathsf{E}]$$

$$[\mathsf{LET}^{s}] \equiv [\mathsf{LET}] + [\forall \mathsf{I}] + [\Rightarrow \mathsf{I}]$$

QuB: Syntax Directed Typing Rules

$$\frac{P \Rightarrow \texttt{SeFun}\ \phi \qquad P \vdash \Gamma \geq \phi \qquad P \mid \Gamma \circledast x^{\varnothing} : \tau \vdash^{s} M : \upsilon}{P \mid \Gamma \vdash^{s} \lambda^{\multimap} x.M : \phi\tau\upsilon}\ [\multimap \mathsf{I}^{s}] \qquad \frac{P \Rightarrow \texttt{ShFun}\ \phi \qquad P \vdash \Gamma \geq \phi \qquad P \mid \Gamma^{[\varnothing \mapsto \{x\}]} \oplus x^{\texttt{Vars}(\Gamma)} : \tau \vdash^{s} M : \upsilon}{P \mid \Gamma \vdash^{s} \lambda^{\twoheadrightarrow} x.M : \phi\tau\upsilon}\ [\twoheadrightarrow \mathsf{I}^{s}]$$

$$\frac{P \mid \Gamma \circledast \Delta \vdash^{s} M : \phi\upsilon\tau \qquad P \mid \Gamma' \circledast \Delta \vdash^{s} N : \upsilon \qquad P \vdash \Delta\ \texttt{un} \qquad (\Gamma \mathbin{\tilde{\oplus}} \Gamma' \wedge (P \Rightarrow \texttt{ShFun}\ \phi)) \vee (\Gamma \mathbin{\tilde{\circledast}} \Gamma' \wedge (P \Rightarrow \texttt{SeFun}\ \phi))}{P \mid \Gamma \sqcup \Gamma' \circledast \Delta \vdash^{s} MN : \tau}\ [\mathsf{App}^{s}]$$

$\boxed{\text{Context Operations}}$

$$\Gamma \mathbin{\tilde{\circledast}} \Delta = \texttt{Used}(\Gamma) \# \texttt{Vars}(\Delta) \wedge \texttt{Used}(\Delta) \# \texttt{Vars}(\Gamma)$$

$$\Gamma \mathbin{\tilde{\oplus}} \Delta = \texttt{Used}(\Gamma) = \texttt{Used}(\Delta)$$

$$[\twoheadrightarrow \mathsf{I}] \equiv [\twoheadrightarrow \mathsf{I}] \qquad [\multimap \mathsf{I}] \equiv [\multimap \mathsf{I}]$$

$$[\mathsf{APP}^{s}] \equiv [\twoheadrightarrow \mathsf{E}] + [\multimap \mathsf{E}]$$

QuB: Soundess and Completeness of $\vdash^s$

Theorem (Soundness of $\vdash^s$)

If $P \mid \Gamma \vdash^s M : \tau$ *then* $P \mid \Gamma \vdash M : \tau$

Theorem (Completeness of $\vdash^s$)

If $P \mid \Gamma \vdash M : \sigma$ *then* $\exists Q, \tau$ *such that* $Q \mid \Gamma \vdash^s M : \tau$ *and* $(P \mid \sigma) \sqsubseteq \mathit{Gen}(\Gamma, Q \Rightarrow \tau)$

$\boxed{\text{Original Type System}} \equiv \boxed{\text{Syntax Directed Typing Rules}}$

$\boxed{\text{Proof in Original Type System}} \equiv \boxed{\text{Proof in Syntax Directed Typing Rules}}$

QuB: Algorithm $\mathcal{M}$

$$\boxed{\mathcal{M}(S,\Psi,\Gamma \vdash M:\tau) = P, S', \Sigma, \Psi'}$$

$$\begin{aligned}
\mathcal{M}(S,\Psi,\Gamma \vdash x:\tau) &= ([\vec{u}/\vec{t}]P), S'\circ S, \{x\}, \Psi \\
\text{where} \quad (x : \forall\vec{t}.P \Rightarrow \upsilon) &\in S\Gamma \\
S' &= \mathcal{U}([\vec{u}/\vec{t}]\upsilon, S\tau)
\end{aligned}$$

$$\begin{aligned}
&\mathcal{M}(S,\Psi,\Gamma \vdash \lambda^{\twoheadrightarrow}x.M:\tau) = \{P \cup Q\}, S', \Sigma\backslash x, \Psi'' \\
&\quad \text{where} \quad P;S';\Sigma;\Psi' = \mathcal{M}(\mathcal{U}(\tau, u_1 u_2 u_3)\circ S, \Psi, \Gamma, x:u_2 \vdash M:u_3) \\
&\qquad\qquad \Psi'' = \{\forall_{y\in\mathtt{dom}(\Psi')}.\Psi'(y)+x\} \cup \{(x,\{y \mid y \in \mathtt{dom}(\Gamma)\})\} \\
&\qquad\qquad Q = \{\mathtt{ShFun}\ u_1\} \cup \mathtt{Leq}(u_1, \Gamma|_\Sigma) \cup \mathtt{Weaken}(x, u_2, \Sigma, \Psi'')
\end{aligned}$$

$$\begin{aligned}
&\mathcal{M}(S,\Psi,\Gamma \vdash \lambda^{\multimap}x.M:\tau) = \{P \cup Q\}, S', \Sigma\backslash x, \Psi'' \\
&\quad \text{where} \quad P;S';\Sigma;\Psi' = \mathcal{M}(\mathcal{U}(\tau, u_1 u_2 u_3)\circ S, X;\Gamma, x:u_2 \vdash M:u_3) \\
&\qquad\qquad \Psi'' = \Psi' \cup \{(x,\{x\})\} \\
&\qquad\qquad Q = \{\mathtt{SeFun}\ u_1\} \cup \mathtt{Leq}(u_1, \Gamma\,|_\Sigma) \cup \mathtt{Weaken}(x, u_2, \Sigma, \Psi'')
\end{aligned}$$

$$\boxed{\mathcal{M}(S, \Psi, \Gamma \vdash M : \tau) = P, S', \Sigma, \Psi'}$$

$$\begin{aligned}
&\mathcal{M}(S, \Psi, \Gamma \vdash MN : \tau) = \{P \cup P' \cup Q\}, R', \Sigma \cup \Sigma', \Psi'' \\
&\text{where} \qquad P; R; \Sigma; \Psi' = \mathcal{M}(S, \Psi, \Gamma \vdash M : u_1 u_2 \tau) \\
&\qquad P'; R'; \Sigma'; \Psi'' = \mathcal{M}(SR, \Psi', S\Gamma \vdash N : u_2) \\
&\qquad \text{if } \mathcal{C}(\Gamma, \Psi'', \Sigma) = \mathcal{C}(\Gamma, \Psi'', \Sigma') \\
&\qquad\qquad \text{then } Q = \{\texttt{ShFun}\ u_1\} \\
&\qquad\qquad \text{else if } (\Sigma \# \mathcal{C}(R\Gamma, \Psi'', \Sigma') \text{ and } \Sigma' \# \mathcal{C}(R\Gamma, \Psi'', \Sigma)) \\
&\qquad\qquad\qquad \text{then } Q = \{\texttt{SeFun}\ u_1\}
\end{aligned}$$

$$\begin{aligned}
&\mathcal{M}(S, \Psi, \Gamma \vdash \texttt{let}\ x = M\ \texttt{in}\ N : \tau) = (P \cup Q), R', \Sigma \cup \{\Sigma' \backslash x\}, \Psi'' \\
&\text{where} \qquad P; R; \Sigma; \Psi' = \mathcal{M}(S, \Psi, \Gamma \vdash M : u_1) \\
&\qquad \sigma = \texttt{GenI}(R\Gamma; R(P \Rightarrow u_1)) \\
&\qquad P'; R'; \Sigma'; \Psi'' = \mathcal{M}(R, \Psi', \Gamma, x : \sigma \vdash N : \tau) \\
&\qquad Q = \texttt{Un}(\Gamma|_{\Sigma \cap \Sigma'}) \cup \texttt{Weaken}(x, \sigma, \Sigma', \Psi'')
\end{aligned}$$

QuB: Soundess of Algorithm $\mathcal{M}$

Theorem (Soundness of $\mathcal{M}$)

if $\mathcal{M}(S, \Psi, \Gamma \vdash M : \tau) = P, S', \Sigma, \Psi'$ *then* $S'P \mid S'(\Gamma|_{\Sigma}) \vdash M : S'\tau$

$$\boxed{\text{Algorithm M}} \rightarrow \boxed{\text{Type system}}$$

Examples: Basic Structures

- Multiplicative Product

$$\tau \otimes \tau' = \tau \twoheadrightarrow^{*} \tau' \twoheadrightarrow^{*} (\tau \twoheadrightarrow^{*} \tau' \twoheadrightarrow^{*} \upsilon) \twoheadrightarrow^{*} \upsilon$$

$$(,) = \lambda^{\twoheadrightarrow^{*}} x.\lambda^{\twoheadrightarrow^{*}} y.\lambda^{\twoheadrightarrow^{*}} f.fxy$$

- Additive Product

$$\tau \,\&\, \tau' = \tau \twoheadrightarrow^{*} \tau' \twoheadrightarrow (\tau \twoheadrightarrow^{*} \tau' \twoheadrightarrow \upsilon) \twoheadrightarrow \upsilon$$

$$(;) = \lambda^{\twoheadrightarrow^{*}} x.\lambda^{\twoheadrightarrow} y.\lambda^{\twoheadrightarrow} f.fxy$$

- Sums

$$\tau \oplus \tau' = (\tau \to \upsilon) \to (\tau' \to \upsilon) \to \upsilon$$

$$\texttt{case}\ c\ \texttt{of}\ \{f; g\} = \lambda^{\twoheadrightarrow^{*}} c.\lambda^{\twoheadrightarrow} f.\lambda^{\twoheadrightarrow} g.cfg$$

$$\texttt{inl} : \tau \twoheadrightarrow^{*} (\tau \oplus \tau') \qquad \texttt{inr} : \tau' \twoheadrightarrow^{*} (\tau \oplus \tau')$$

$$\texttt{inl} = \lambda^{\twoheadrightarrow^{*}} x.\lambda^{\twoheadrightarrow} f.\lambda^{\twoheadrightarrow} g.fx \qquad \texttt{inr} = \lambda^{\twoheadrightarrow^{*}} y.\lambda^{\twoheadrightarrow} f.\lambda^{\twoheadrightarrow} g.gy$$

QuB: Extension

- User defined types and type classes

- User defined types and type classes
- Kind System with type constructors

$$
\begin{aligned}
\text{Type Variables} &\quad t, u \in \text{Type Variables} \\
\text{Kinds} &\quad \kappa ::= \star \mid \kappa' \to \kappa \\
\text{Types} &\quad \tau^{\kappa} ::= t^{\kappa} \mid T^{\kappa} \mid \tau^{\kappa' \to \kappa} \tau^{\kappa'} \\
\text{Type Constructors} &\quad T^{\kappa} \in \mathcal{T}^{\kappa} \text{ where } \quad \{\otimes, \&, \oplus, \overset{!}{\multimap}, \multimap, \overset{!}{\twoheadrightarrow}, \twoheadrightarrow\} \subseteq \mathcal{T}^{\star \to \star \to} \\
\text{Predicates} &\quad \pi, \omega ::= \mathtt{Un}\ \tau \mid \mathtt{SeFun}\ \tau \mid \mathtt{ShFun}\ \tau \mid \tau \geq \tau' \\
\text{Qualified Types} &\quad \rho ::= \tau^{\star} \mid \pi \Rightarrow \rho \\
\text{Type schemes} &\quad \sigma ::= \rho \mid \forall t.\sigma
\end{aligned}
$$

What next?

Thank You!

Q & A

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