QuB

A Resource Aware Functional Programming Language

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Hard problems in programming

Naming variables

Introduction and Motivation

Hard problems in programming

Resource management

in evolving production code

Resources: Files, database connections, entity with a shared state

Resource Management: File Handling

- Modified File Handling API in Haskell

$$\begin{aligned}
&\texttt{openFile} :: \texttt{FilePath} \to \texttt{IO FileHandle}\\
&\texttt{closeFile} :: \texttt{FileHandle} \to \texttt{IO}\ ()\\
&\texttt{readLine} :: \texttt{FileHandle} \to \texttt{IO}\ (\texttt{String}, \texttt{FileHandle})\\
&\texttt{writeFile} :: \texttt{String} \to \texttt{FileHandle}\\
&\qquad\qquad \to \texttt{IO}\ ((), \texttt{FileHandle})\\
&\texttt{upper} :: \texttt{String} \to \texttt{String}
\end{aligned}$$

Resource Management: File Handling

- File Handling in Haskell

```
do f      ← openFile "sample.txt"
   (s, f)   ← readLine f
   let c = upper s
   ((), f) ← writeLine f c
         .
         .
         .
   ()    ← closeFile f
```

Resource Management: File Handling

- File Handling in Haskell Gone Wrong (Part I)

```
do f      ← openFile "sample.txt"
   (s, f)  ← readLine f
   let c = upper s
   ((), f) ← writeLine f c
        .
        .
        .
   ()  ← closeFile f
        .
        .
        .
   ()  ← closeFile f
   return c
```

Resource Management: File Handling

- File Handling in Haskell Gone Wrong (Part I)

```
do f  ← openFile "sample.txt"
   (s, f)  ← readLine f
   let c = upper s
   ((), f) ← writeLine f c
         .
         .
         .
  ()  ← closeFile f
         .
         .
         .
  ()  ← closeFile f
   return c
```

- File is closed twice: Run time crash

Resource Management: File Handling

- File Handling in Haskell Gone Wrong (Part II)

```
do f  ← openFile "sample.txt"
   (s, f)  ← readLine f
   let c = upper s
   ((), f) ← writeLine f c
       .
       .
       .
   return c
```

Resource Management: File Handling

- File Handling in Haskell Gone Wrong (Part II)

```
do f  ← openFile "sample.txt"
   (s, f)  ← readLine f
   let c = upper s
   ((), f) ← writeLine f c
       .
       .
       .
   return c /*File not closed!!*/
```

- File not closed: Memory leak

Resource Management: Exception Handling

- `MonadError`[4] in Haskell

```
class Monad m ⇒ MonadError e m | m → e where
    throwError :: e → m a
    catchError :: m a → (e → m a) → m a
```

- `throwError` starts exception processing
- `catchError` exception handler

Resource Management: Exception Handling

- Using `MonadError` in Haskell

```
do f ← openFile "sample.txt"
   ((s, f)  ← readLine f
   let c = upper s
   ()  ← closeFile f
   return $ Right c)
       `catchError` (\_ →
               return $ Left "Error in reading file")
```

- Exception may cause memory leak

Well typed programs do not go wrong.

— R. Milner

Well typed programs do not go wrong.

— R. Milner

~~Lights~~Types will guide you home

— Coldplay

Contributions

- Design and implement QuB type system
  - Resources as first class citizens
  - Program objects are restricted or unrestricted
  - Functions that share resources with their arguments or are separate.
- Formalizing and proving important properties of QuB
- QuB is logic of ***BI*** with steroids
  - Environments as graphs
- Working examples

Background Work: Simply Typed Lambda Calculus (STLC)

$$\lambda x.M \begin{cases} \text{Abstract over computation} \\ \text{Define functions} \end{cases}$$

$$MN \begin{cases} \text{Do the computation} \\ \text{Use functions} \end{cases}$$

Background Work: Simply Typed Lambda Calculus (STLC)

$$\lambda x.M \begin{cases} \text{Abstract over computation} \\ \text{Define functions} \end{cases}$$

$$MN \begin{cases} \text{Do the computation} \\ \text{Use functions} \end{cases}$$

$$\frac{\Gamma_x, x : \tau \vdash M : \tau'}{\Gamma \vdash \lambda x.M : \tau \to \tau'}\,[\to \mathrm{I}] \qquad \frac{\Gamma \vdash M : \tau \to \tau' \qquad \Gamma \vdash N : \tau}{\Gamma \vdash MN : \tau'}\,[\to \mathrm{E}]$$

Background Work: Simply Typed Lambda Calculus (STLC)

$$\lambda x.M \begin{cases} \text{Abstract over computation} \\ \text{Define functions} \end{cases}$$

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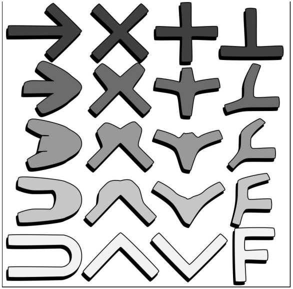
Hinley-Milner (***HM***) type system ensures sane programs

Background Work: Curry-Howard Correspondence

- Types are Propostions
- Programs are Proofs

HM type system ≡ Second Order Intuitionistic Propositional Logic



LC'90 The Curry-Howard homeomorphism

Source: `http://lucacardelli.name/Artifacts/Drawings/CurryHoward/CurryHoward.pdf`

Background Work: Second Order Intuitionistic Propositional Logic

$\boxed{\text{Language}}$

$$\text{Propostions \& connectives} \quad A, B, C ::= x \mid A \supset B \mid \forall x.B \mid \ldots$$
$$\text{Context} \quad \Gamma, \Delta ::= \epsilon \mid \Gamma, A$$

$\boxed{\text{Logic Rules}}$

$$\frac{}{A \vdash A}\,[\mathrm{Ax}]$$

$$\frac{\Gamma \vdash B \qquad x \notin \Gamma}{\forall x.B}\,[\forall \mathrm{I}] \qquad \frac{\Gamma \vdash \forall x.B \qquad \Gamma \vdash A}{B[x/A]}\,[\forall \mathrm{E}]$$

$$\frac{\Gamma, A \vdash B}{\Gamma \vdash A \supset B}\,[\supset \mathrm{I}] \qquad \frac{\Gamma \vdash A \supset B \qquad \Gamma \vdash A}{\Gamma \vdash B}\,[\supset \mathrm{E}]$$

Background Work: Second Order Intuitionistic Propositional Logic

Propostions are truth values not resources

$\boxed{\text{Language}}$

$$\text{Propostions \& connectives} \quad A, B, C ::= x \mid A \supset B \mid \forall x.B \mid \ldots$$
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Background Work: Substructural Logic

- Structural rules implicit in intuistionistic propositional logics

$$\frac{\Gamma \vdash B}{\Gamma, A \vdash B}\,[\text{WKN}] \qquad \frac{\Gamma, A, A \vdash B}{\Gamma, A \vdash B}\,[\text{CTR}] \qquad \frac{\Gamma, \Delta \vdash B}{\Delta, \Gamma \vdash B}\,[\text{EXCH}]$$

Background Work: Substructural Logic

- Structural rules implicit in intuistionistic propositional logics

$$\frac{\Gamma \vdash B}{\Gamma, A \vdash B}\ [\text{WKN}] \qquad \frac{\Gamma, A, A \vdash B}{\Gamma, A \vdash B}\ [\text{CTR}] \qquad \frac{\Gamma, \Delta \vdash B}{\Delta, \Gamma \vdash B}\ [\text{EXCH}]$$

- Control the use of [WKN] and [CTR]

Propostions now behave like resources

Background Work: Substructural Logic

System	Who	Restrictions
Linear Logic[1]	Girard	[WKN] [CTRN]
Lambek Logic[3]	Lambek	[EXCH]
Logic of Bunched Implications[6]	O'Hearn and Pym	[WKN] [CTRN]
⋮	⋮	⋮

Background Work: Qualified Types

$$P \mid \Gamma \vdash M : \sigma$$

"Type of M is σ
when predicates in P are satisfied
and Γ specifies the free variables in M"[2]

General framework, incorporates predicates into type language

Background Work: Quill

Quill[5]: Qualified types + linear logic

Predicates:

- `Un` τ If τ does not have resources or can be copied or dropped easily.
- `Fun` τ If τ is a function type
- $\tau \geq \tau'$ If τ less restricting than τ'

Background Work: Quill

Quill[5]: Qualified types + linear logic

Qualifying Types:

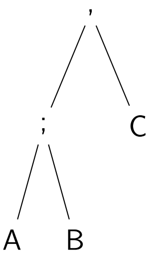
- Unrestricted Types: `Un Int`, `Un` Bool
- Restricted or Linear Types: FileHandle
- Function Types: `Fun` (`Int` → `Int`), `Fun` (`String` → `String`)

Background Work: Logic of Bunched Implications (*BI*)

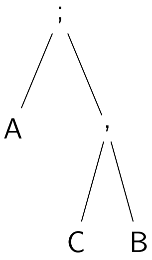
- Contexts are lists or sets

$$\Gamma, A, B$$

- In logic of ***BI***, contexts are trees and called are bunches
- Two connective used to combine bunches: $A; B$ or A, B



$\Gamma = (A; B), C$



$\Gamma = A; (B, C)$

Background Work: Logic of *BI*

Controling structural rules based on context

- Contraction

$$\begin{array}{cc} A; B \vdash A & A; B \vdash B \\ A, B \nvdash A & A, B \nvdash B \end{array}$$

- Weakening

$$\begin{array}{cc} A; B \vdash A; B; B & A; B \vdash A; A; B \\ A, B \nvdash A, B, B & A, B \nvdash A, A, B \end{array}$$

Interpretation:

- Propostions connected with , are separate
- Propostions connected with ; are in sharing

Background Work: Logic of *BI*

(Absense of) Structural rules and logical connecitves:

- Meaning of conjunction

$$A, B \vdash A \otimes B \qquad\qquad A; B \vdash A \,\&\, B$$

- Meaning of implication

$$\frac{\Gamma, A \vdash B}{\Gamma \vdash A \mathrel{-\!\!*} B}\,[\mathrel{-\!\!*}\mathrm{I}] \qquad\qquad \frac{\Gamma; A \vdash B}{\Gamma \vdash A \twoheadrightarrow B}\,[\twoheadrightarrow\mathrm{I}]$$

Coffee Shop

1 cup coffee costs \$2

$\otimes$

$\vdash$

Background work: Logic of ***BI***

Coffee Shop

1 cup coffee costs \$2

$\otimes$

$\vdash$

$\&$

$\nvdash$

QuB

- Quill: Qualified types + linear logic
- QuB: Qualified types + logic of bunched implications

QuB: Types and Predicates

$$
\begin{array}{rl}
\text{Types} & \tau, \upsilon, \phi ::= t \mid \iota \mid \tau \to \tau \\
\text{where} & \to \in \{ \overset{!}{\multimap}, \multimap, \overset{!}{\twoheadrightarrow}, \twoheadrightarrow \} \\
\text{Predicates} & \pi, \omega ::= \mathtt{Un}\ \tau \mid \mathtt{SeFun}\ \tau \mid \mathtt{ShFun}\ \tau \mid \tau \geq \tau' \\
\text{Qualified Types} & \rho ::= \tau \mid \pi \Rightarrow \rho \\
\text{Type schemes} & \sigma ::= \rho \mid \forall t.\sigma
\end{array}
$$

- $\mathtt{SeFun}\ \tau$: τ is a function that is separate from its argument
- $\mathtt{ShFun}\ \tau$: τ is a function that is in sharing with its argument
- $\mathtt{Un}\ \tau$: τ does not have resources or they can be copied/dropped easily

$$
\begin{aligned}
\text{Types} \quad & \tau, \upsilon, \phi ::= t \mid \iota \mid \tau \to \tau \\
& \quad \text{where} \qquad \to \in \{\overset{!}{\mathrel{-\!\!*}}, \mathrel{-\!\!*}, \overset{!}{\twoheadrightarrow}, \twoheadrightarrow\} \\
\text{Predicates} \quad & \pi, \omega ::= \texttt{Un}\ \tau \mid \texttt{SeFun}\ \tau \mid \texttt{ShFun}\ \tau \mid \tau \geq \tau' \\
\text{Qualified Types} \quad & \rho ::= \tau \mid \pi \Rightarrow \rho \\
\text{Type schemes} \quad & \sigma ::= \rho \mid \forall t.\sigma
\end{aligned}
$$

- $\mathrel{-\!\!*}$: Function type that is separate with its argument
- $\twoheadrightarrow$: Function type that is in sharing with its argument
- $\overset{!}{\mathrel{-\!\!*}}$, $\overset{!}{\twoheadrightarrow}$: unrestriced versions of $\mathrel{-\!\!*}$ and $\twoheadrightarrow$

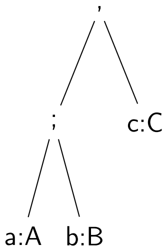
QuB: Language

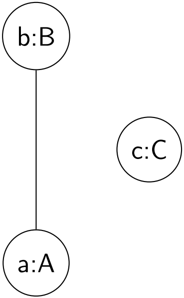
Term Variables	$x, y, z \in \text{Var}$
Expressions	$M, N ::= x \mid \lambda^{\multimap\!\!\ast} x.M \mid \lambda^{\twoheadrightarrow} x.M \mid MN \mid \texttt{let}\ x = M\ \texttt{in}\ N$

- $\lambda^{\multimap\!\!\ast} x.M$: Introduces $\multimap\!\!\ast$ function
- $\lambda^{\twoheadrightarrow} x.N$: Introduces $\twoheadrightarrow$ function

QuB: Typing Environmnet

- Logic of ***BI***: Contexts are trees
- QuB: Contexts are flattened sharing graphs





- Sharing relation Ψ

$$\forall_{x \in \mathtt{dom}(\Psi)}\ x \in \Psi(x) \qquad \text{(reflexive)}$$

$$\forall_{x,y \in \mathtt{dom}(\Psi)}\ \text{if } y \in \Psi(x) \text{ then } x \in \Psi(y) \qquad \text{(symmetric)}$$

$$\forall_{x,y,z \in \mathtt{dom}(\Psi)}\ \text{if } y \in \Psi(x) \text{ and } z \in \Psi(y)\ \not\Rightarrow z \in \Psi(x) \qquad \text{(non-transitive)}$$

QuB: Typing Environment

"x of type σ is in sharing with $\vec{y}$"

$$(x, \sigma, \vec{y}) \in \Gamma$$

Typing Context $\quad \Gamma, \Delta ::= \epsilon \mid \Gamma, x^{\vec{y}} : \sigma$

$$\begin{aligned}
\texttt{Vars}(\Gamma, x^{\vec{y}} : \tau) &= \texttt{Vars}(\Gamma) \cup \{x\} \\
\texttt{Shared}(\Gamma, x^{\vec{y}} : \tau) &= \texttt{Shared}(\Gamma) \cup \{\vec{y}\} \\
\texttt{Used}(\Gamma) &= \texttt{Vars}(\Gamma) \cup \texttt{Shared}(\Gamma)
\end{aligned}$$

$$(\Gamma, x^{\vec{y}} : \tau)^{[a \mapsto \vec{b}]} = \begin{cases} a \notin \vec{y} & (\Gamma^{[a \mapsto \vec{b}]}, x^{\vec{y}} : \tau) \\ a \in \vec{y} & (\Gamma^{[a \mapsto \vec{b}]}, x^{(\vec{y} \setminus a) \cup \vec{b}} : \tau) \end{cases}$$

$$\Gamma^{[\vec{a} \mapsto \vec{b}]} = (\ldots((\Gamma^{[a_1 \mapsto \vec{b}]})^{[a_2 \mapsto \vec{b}]})^{\cdots})^{[a_n \mapsto \vec{b}]}$$

$$\begin{aligned}
\Gamma \circledast \Gamma' &= \Gamma \sqcup \Gamma' && \texttt{if } \texttt{Vars}(\Gamma) \# \texttt{Used}(\Gamma') \wedge \texttt{Vars}(\Gamma') \# \texttt{Used}(\Gamma) \\
\Gamma \oplus \Gamma' &= \Gamma \sqcup \Gamma' && \texttt{if } \texttt{Used}(\Gamma) = \texttt{Used}(\Gamma')
\end{aligned}$$

QuB: Typing Rules

$\boxed{\text{Structural Rules}}$

$$\frac{}{P \mid x^{\vec{y}} : \sigma \vdash x : \sigma} \text{[ID]}$$

$$\frac{P \mid \Gamma \circledast \Delta \circledast \Delta \vdash M : \sigma \qquad P \vdash \Delta \ \mathbf{un}}{P \mid \Gamma \circledast \Delta \vdash M : \sigma} \text{[CTR-UN]} \qquad \frac{P \mid \Gamma \oplus \Delta \oplus \Delta \vdash M : \sigma}{P \mid \Gamma \oplus \Delta \vdash M : \sigma} \text{[CTR-SH]}$$

$$\frac{P \mid \Gamma \vdash M : \sigma \qquad P \vdash \Delta \ \mathbf{un}}{P \mid \Gamma \circledast \Delta \vdash M : \sigma} \text{[WKN-UN]} \qquad \frac{P \mid \Gamma \vdash M : \sigma}{P \mid \Gamma \oplus \Delta \vdash M : \sigma} \text{[WKN-SH]}$$

QuB: Typing Rules

Connective Rules

$$\frac{P \mid \Gamma \vdash M : \sigma \qquad P' \mid \Gamma'_x, x : \sigma \vdash N : \tau}{P \cup P' \mid \Gamma \sqcup \Gamma' \vdash (\texttt{let}\ x = M\ \texttt{in}\ N) : \tau}\ [\text{LET}]$$

$$\frac{P \mid \Gamma \vdash M : \sigma \qquad t \notin \texttt{fvs}(\Gamma) \cup \texttt{fvs}(P)}{P \mid \Gamma \vdash M : \forall t.\sigma}\ [\forall\ \text{I}] \qquad \frac{P \mid \Gamma \vdash M : \forall t.\sigma}{P \mid \Gamma \vdash M : [\tau \backslash t]\sigma}\ [\forall\ \text{E}]$$

$$\frac{P, \pi \mid \Gamma \vdash M : \rho}{P \mid \Gamma \vdash M : \pi \Rightarrow \rho}\ [\Rightarrow\ \text{I}] \qquad \frac{P \mid \Gamma \vdash M : \pi \Rightarrow \rho \qquad P \vdash \pi}{P \mid \Gamma \vdash M : \rho}\ [\Rightarrow\ \text{E}]$$

$$\frac{\begin{array}{c} P \Rightarrow \texttt{ShFun}\ \phi \qquad P \vdash \Gamma \geq \phi \\ P \mid \Gamma^{[\varnothing \mapsto \{x\}]}, x^{\mathsf{Vars}(\Gamma)} : \tau \vdash M : \tau' \end{array}}{P \mid \Gamma \vdash \lambda^{\twoheadrightarrow} x.M : \phi\tau\tau'}\ [\twoheadrightarrow\ \text{I}] \qquad \frac{\begin{array}{c} P \Rightarrow \texttt{ShFun}\ \phi \\ P \mid \Gamma \vdash M : \phi\tau\tau' \qquad P \mid \Gamma' \vdash N : \tau \end{array}}{P \mid \Gamma \oplus \Gamma' \vdash MN : \tau'}\ [\twoheadrightarrow\ \text{E}]$$

$$\frac{\begin{array}{c} P \Rightarrow \texttt{SeFun}\ \phi \qquad P \vdash \Gamma \geq \phi \\ P \mid \Gamma, x^{\varnothing} : \tau \vdash M : \tau' \end{array}}{P \mid \Gamma \vdash \lambda^{-\!\!*} x.M : \phi\tau\tau'}\ [-\!\!*\ \text{I}] \qquad \frac{\begin{array}{c} P \Rightarrow \texttt{SeFun}\ \phi \\ P \mid \Gamma \vdash M : \phi\tau\tau' \qquad P \mid \Gamma' \vdash N : \tau \end{array}}{P \mid \Gamma \circledast \Gamma' \vdash MN : \tau'}\ [-\!\!*\ \text{E}]$$

QuB: Typing Terms

$$
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\dfrac{}{\varnothing \mid y^{\varnothing} : \tau' \vdash y : \tau'}\,[\mathrm{ID}]
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\dfrac{}{\varnothing \mid x^{\varnothing} : \tau \vdash x : \tau}\,[\mathrm{ID}]
\qquad
\dfrac{}{\varnothing \mid f^{\varnothing} : \tau \mathrel{-\!\!*} \tau' \mathrel{-\!\!*} \upsilon \vdash f : \tau \mathrel{-\!\!*} \tau' \mathrel{-\!\!*} \upsilon}\,[\mathrm{ID}]
}{\varnothing \mid x^{\varnothing} : \tau \circledast f^{\varnothing} : \tau \mathrel{-\!\!*} \tau' \mathrel{-\!\!*} \upsilon \vdash fx : (\tau' \mathrel{-\!\!*} \upsilon)}\,[\mathrel{-\!\!*}\mathrm{E}]
}{\varnothing \mid y^{\varnothing} : \tau' \circledast x^{\varnothing} : \tau \circledast f^{\varnothing} : \tau \mathrel{-\!\!*} \tau' \mathrel{-\!\!*} \upsilon \vdash fxy : \upsilon}\,[\mathrel{-\!\!*}\mathrm{E}]
}{\varnothing \mid x^{\varnothing} : \tau \circledast y^{\varnothing} : \tau' \circledast f^{\varnothing} : \tau \mathrel{-\!\!*} \tau' \mathrel{-\!\!*} \upsilon \vdash fxy : \upsilon}\,[\mathit{EXCH}]
}{\varnothing \mid x^{\varnothing} : \tau \circledast y^{\varnothing} : \tau' \vdash \lambda^{-\!*} f.fxy : (\tau \mathrel{-\!\!*} \tau' \mathrel{-\!\!*} \upsilon) \mathrel{-\!\!*} \upsilon}\,[\mathrel{-\!\!*}\ \mathrm{I}]
}{\varnothing \mid x^{\varnothing} : \tau \vdash \lambda^{-\!*} y.\lambda^{-\!*} f.fxy : \tau' \mathrel{-\!\!*} (\tau \mathrel{-\!\!*} \tau' \mathrel{-\!\!*} \upsilon) \mathrel{-\!\!*} \upsilon}\,[\mathrel{-\!\!*}\ \mathrm{I}]
}{\varnothing \mid I \vdash \lambda^{-\!*} x.\lambda^{-\!*} y.\lambda^{-\!*} f.fxy : \tau \mathrel{-\!\!*} \tau' \mathrel{-\!\!*} (\tau \mathrel{-\!\!*} \tau' \mathrel{-\!\!*} \upsilon) \mathrel{-\!\!*} \upsilon}\,[\equiv]
$$

QuB: Typing Terms

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\dfrac{}{\varnothing \mid y^{\boldsymbol{xf}} : \tau' \vdash y : \tau'}\,[\text{ID}]
\qquad
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\dfrac{}{\varnothing \mid x^{\boldsymbol{fy}} : \tau \vdash x : \tau}\,[\text{ID}]
\qquad
\dfrac{}{\varnothing \mid \boldsymbol{f}^{\boldsymbol{xy}} : \tau \twoheadrightarrow \tau' \twoheadrightarrow \upsilon \vdash \boldsymbol{f} : \tau \twoheadrightarrow \tau' \twoheadrightarrow \upsilon}\,[\text{ID}]
}{\varnothing \mid x^{\boldsymbol{xy}} : \tau \oplus \boldsymbol{f}^{\boldsymbol{xy}} : \tau \twoheadrightarrow \tau' \twoheadrightarrow \upsilon \vdash fx : (\tau' \twoheadrightarrow \upsilon)}\,[\mathrel{-\!\!*}\ E]
}{\varnothing \mid y^{\boldsymbol{xf}} : \tau' \oplus x^{\boldsymbol{yf}} : \tau \oplus f^{\boldsymbol{xy}} : \tau \twoheadrightarrow \tau' \twoheadrightarrow \upsilon \vdash fxy : \upsilon}\,[\twoheadrightarrow E]
}{\varnothing \mid x^{\boldsymbol{yf}} : \tau \oplus y^{\boldsymbol{xf}} : \tau' \oplus f^{\boldsymbol{xy}} : \tau \twoheadrightarrow \tau' \twoheadrightarrow \upsilon \vdash fxy : \upsilon}\,[EXCH]
}{\varnothing \mid x^{\boldsymbol{y}} : \tau \oplus y^{\boldsymbol{x}} : \tau' \vdash \lambda^{\twoheadrightarrow} f.fxy : (\tau \twoheadrightarrow \tau' \twoheadrightarrow \upsilon) \twoheadrightarrow \upsilon}\,[\twoheadrightarrow I]
}{\varnothing \mid x^{\varnothing} : \tau \vdash \lambda^{\twoheadrightarrow} y.\lambda^{\twoheadrightarrow} f.fxy : \tau' \twoheadrightarrow (\tau \twoheadrightarrow \tau' \twoheadrightarrow \upsilon) \twoheadrightarrow \upsilon}\,[\twoheadrightarrow I]
}{\varnothing \mid \boldsymbol{I} \vdash \lambda^{\mathrel{-\!\!*}} x.\lambda^{\twoheadrightarrow} y.\lambda^{\twoheadrightarrow} f.fxy : \tau \mathrel{-\!\!*} \tau' \twoheadrightarrow (\tau \twoheadrightarrow \tau' \twoheadrightarrow \upsilon) \twoheadrightarrow \upsilon}\,[\mathrel{-\!\!*}\ \text{I}]
$$

QuB: Syntax Directed Typing Rules

Term structure $\leftrightarrow$ Typing Rule

QuB: Syntax Directed Typing Rules

$$\frac{P \vdash \Gamma_{\vec{y}}\ \mathtt{un} \qquad (P \Rightarrow \tau) \sqsubseteq \sigma}{P \mid \Gamma, x^{\vec{y}} : \sigma \vdash^{s} x : \tau}\ [\mathrm{VAR}^{s}]$$

$$\frac{\begin{array}{c} Q \mid (\Gamma'_x \oplus \Gamma''_x) \circledast \Delta \vdash^{s} M : \upsilon \qquad P \vdash \Delta\ \mathtt{un} \\ P \mid (\Gamma_x, x^{\varnothing} : \sigma) \oplus \Gamma''_x \circledast \Delta \vdash^{s} N : \tau \qquad \sigma = \mathtt{Gen}(\{\Gamma' \oplus \Gamma''_x \circledast \Delta\}, Q \Rightarrow \upsilon) \end{array}}{P, Q \mid (\Gamma \circledast \Gamma') \oplus \Gamma'' \circledast \Delta \vdash^{s} (\mathtt{let}\ x = M\ \mathtt{in}\ N) : \tau}\ [\mathrm{Let}^{s}]$$

$$[\mathrm{VAR}^{s}] \equiv [\mathrm{ID}] + [\forall\mathrm{E}] + [\Rightarrow\mathrm{E}]$$

$$[\mathrm{LET}^{s}] \equiv [\mathrm{LET}] + [\forall\mathrm{I}] + [\Rightarrow\mathrm{I}]$$

QuB: Syntax Directed Typing Rules

$$\frac{P \Rightarrow \mathtt{SeFun}\ \phi \qquad P \vdash \Gamma \geq \phi \qquad P \mid \Gamma \circledast x^{\varnothing} : \tau \vdash^{s} M : \upsilon}{P \mid \Gamma \vdash^{s} \lambda^{\multimap} x.M : \phi\tau\upsilon}\ [\multimap \mathsf{I}^{s}]$$

$$\frac{P \Rightarrow \mathtt{ShFun}\ \phi \qquad P \vdash \Gamma \geq \phi \qquad P \mid \Gamma^{[\varnothing \mapsto \{x\}]} \oplus x^{\mathtt{Vars}(\Gamma)} : \tau \vdash^{s} M : \upsilon}{P \mid \Gamma \vdash^{s} \lambda^{\twoheadrightarrow} x.M : \phi\tau\upsilon}\ [\twoheadrightarrow \mathsf{I}^{s}]$$

$$\frac{P \mid \Gamma \circledast \Delta \vdash^{s} M : \phi\upsilon\tau \qquad P \mid \Gamma' \circledast \Delta \vdash^{s} N : \upsilon \qquad P \vdash \Delta\ \mathtt{un} \qquad (\Gamma \mathbin{\tilde{\oplus}} \Gamma' \wedge (P \Rightarrow \mathtt{ShFun}\ \phi)) \vee (\Gamma \mathbin{\tilde{\circledast}} \Gamma' \wedge (P \Rightarrow \mathtt{SeFun}\ \phi))}{P \mid \Gamma \sqcup \Gamma' \circledast \Delta \vdash^{s} MN : \tau}\ [\mathsf{App}^{s}]$$

$\boxed{\text{Context Operations}}$

$$\Gamma \mathbin{\tilde{\circledast}} \Delta = \mathtt{Used}(\Gamma) \mathbin{\#} \mathtt{Vars}(\Delta) \wedge \mathtt{Used}(\Delta) \mathbin{\#} \mathtt{Vars}(\Gamma)$$

$$\Gamma \mathbin{\tilde{\oplus}} \Delta = \mathtt{Used}(\Gamma) = \mathtt{Used}(\Delta)$$

$$[\twoheadrightarrow \mathsf{I}] \equiv [\twoheadrightarrow \mathsf{I}] \qquad [\multimap \mathsf{I}] \equiv [\multimap \mathsf{I}]$$

$$[\mathsf{APP}^{s}] \equiv [\twoheadrightarrow \mathsf{E}] + [\multimap \mathsf{E}]$$

QuB: Soundess and Completeness of $\vdash^s$

Theorem (Soundness of $\vdash^s$)

If $P \mid \Gamma \vdash^s M : \tau$ *then* $P \mid \Gamma \vdash M : \tau$

Theorem (Completeness of $\vdash^s$)

If $P \mid \Gamma \vdash M : \sigma$ *then* $\exists Q, \tau$ *such that* $Q \mid \Gamma \vdash^s M : \tau$ *and* $(P \mid \sigma) \sqsubseteq \mathit{Gen}(\Gamma, Q \Rightarrow \tau)$

$\boxed{\text{Original Type System}} \equiv \boxed{\text{Syntax Directed Typing Rules}}$

$\boxed{\text{Proof in Original Type System}} \equiv \boxed{\text{Proof in Syntax Directed Typing Rules}}$

$$\boxed{\mathcal{M}(S, \Psi, \Gamma \vdash M : \tau) = P, S', \Sigma, \Psi'}$$

$$\begin{aligned}
& \mathcal{M}(S, \Psi, \Gamma \vdash x : \tau) = ([\vec{u}/\vec{t}]P), S' \circ S, \{x\}, \Psi \\
\text{where} \quad & (x : \forall \vec{t}.P \Rightarrow \upsilon) \in S\Gamma \\
& S' = \mathcal{U}([\vec{u}/\vec{t}]\upsilon, S\tau)
\end{aligned}$$

$$\begin{aligned}
& \mathcal{M}(S, \Psi, \Gamma \vdash \lambda^{\twoheadrightarrow} x.M : \tau) = \{P \cup Q\}, S', \Sigma \backslash x, \Psi'' \\
\text{where} \quad & P; S'; \Sigma; \Psi' = \mathcal{M}(\mathcal{U}(\tau, u_1 u_2 u_3) \circ S, \Psi, \Gamma, x : u_2 \vdash M : u_3) \\
& \Psi'' = \{\forall_{y \in \mathtt{dom}(\Psi')}.\Psi'(y) + x\} \cup \{(x, \{y \mid y \in \mathtt{dom}(\Gamma)\})\} \\
& Q = \{\mathtt{ShFun}\ u_1\} \cup \mathtt{Leq}(u_1, \Gamma|_\Sigma) \cup \mathtt{Weaken}(x, u_2, \Sigma, \Psi'')
\end{aligned}$$

$$\begin{aligned}
& \mathcal{M}(S, \Psi, \Gamma \vdash \lambda^{\rightarrowtail} x.M : \tau) = \{P \cup Q\}, S', \Sigma \backslash x, \Psi'' \\
\text{where} \quad & P; S'; \Sigma; \Psi' = \mathcal{M}(\mathcal{U}(\tau, u_1 u_2 u_3) \circ S, X; \Gamma, x : u_2 \vdash M : u_3) \\
& \Psi'' = \Psi' \cup \{(x, \{x\})\} \\
& Q = \{\mathtt{SeFun}\ u_1\} \cup \mathtt{Leq}(u_1, \Gamma\ |_\Sigma) \cup \mathtt{Weaken}(x, u_2, \Sigma, \Psi'')
\end{aligned}$$

QuB: Algorithm $\mathcal{M}$

$$\boxed{\mathcal{M}(S, \Psi, \Gamma \vdash M : \tau) = P, S', \Sigma, \Psi'}$$

$$
\begin{aligned}
\mathcal{M}(S, \Psi, \Gamma \vdash MN : \tau) &= \{P \cup P' \cup Q\}, R', \Sigma \cup \Sigma', \Psi'' \\
\text{where} \quad P; R; \Sigma; \Psi' &= \mathcal{M}(S, \Psi, \Gamma \vdash M : u_1 u_2 \tau) \\
P'; R'; \Sigma'; \Psi'' &= \mathcal{M}(SR, \Psi', S\Gamma \vdash N : u_2) \\
&\text{if } \mathcal{C}(\Gamma, \Psi'', \Sigma) = \mathcal{C}(\Gamma, \Psi'', \Sigma') \\
&\quad \text{then } Q = \{\mathtt{ShFun}\ u_1\} \\
&\quad \text{else if } (\Sigma \# \mathcal{C}(R\Gamma, \Psi'', \Sigma') \text{ and } \Sigma' \# \mathcal{C}(R\Gamma, \Psi'', \Sigma)) \\
&\qquad \text{then } Q = \{\mathtt{SeFun}\ u_1\}
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{M}(S, \Psi, \Gamma \vdash \mathtt{let}\ x = M\ \mathtt{in}\ N : \tau) &= (P \cup Q), R', \Sigma \cup \{\Sigma' \backslash x\}, \Psi'' \\
\text{where} \quad P; R; \Sigma; \Psi' &= \mathcal{M}(S, \Psi, \Gamma \vdash M : u_1) \\
\sigma &= \mathtt{GenI}(R\Gamma; R(P \Rightarrow u_1)) \\
P'; R'; \Sigma'; \Psi'' &= \mathcal{M}(R, \Psi', \Gamma, x : \sigma \vdash N : \tau) \\
Q &= \mathtt{Un}(\Gamma|_{\Sigma \cap \Sigma'}) \cup \mathtt{Weaken}(x, \sigma, \Sigma', \Psi'')
\end{aligned}
$$

QuB: Soundess of Algorithm $\mathcal{M}$

Theorem (Soundness of $\mathcal{M}$)

if $\mathcal{M}(S, \Psi, \Gamma \vdash M : \tau) = P, S', \Sigma, \Psi'$ *then* $S'P \mid S'(\Gamma|_{\Sigma}) \vdash M : S'\tau$

$$\boxed{\text{Algorithm M}} \rightarrow \boxed{\text{Type system}}$$

Examples: Basic Structures

- Multiplicative Product

$$\tau \otimes \tau' = \tau \twoheadrightarrow \tau' \twoheadrightarrow (\tau \twoheadrightarrow \tau' \twoheadrightarrow \upsilon) \twoheadrightarrow \upsilon$$
$$(,) = \lambda^{\twoheadrightarrow} x.\lambda^{\twoheadrightarrow} y.\lambda^{\twoheadrightarrow} f.fxy$$

- Additive Product

$$\tau \,\&\, \tau' = \tau \twoheadrightarrow \tau' \twoheadrightarrow (\tau \twoheadrightarrow \tau' \twoheadrightarrow \upsilon) \twoheadrightarrow \upsilon$$
$$(;) = \lambda^{\twoheadrightarrow} x.\lambda^{\twoheadrightarrow} y.\lambda^{\twoheadrightarrow} f.fxy$$

- Sums

$$\tau \oplus \tau' = (\tau \to \upsilon) \to (\tau' \to \upsilon) \to \upsilon$$
$$\texttt{case } c \texttt{ of } \{f; g\} = \lambda^{\twoheadrightarrow} c.\lambda^{\twoheadrightarrow} f.\lambda^{\twoheadrightarrow} g.cfg$$

$$\texttt{inl} : \tau \twoheadrightarrow (\tau \oplus \tau') \qquad \texttt{inr} : \tau' \twoheadrightarrow (\tau \oplus \tau')$$
$$\texttt{inl} = \lambda^{\twoheadrightarrow} x.\lambda^{\twoheadrightarrow} f.\lambda^{\twoheadrightarrow} g.fx \qquad \texttt{inr} = \lambda^{\twoheadrightarrow} y.\lambda^{\twoheadrightarrow} f.\lambda^{\twoheadrightarrow} g.gy$$

QuB: Extension

- User defined types and type classes

QuB: Extension

- User defined types and type classes
- Kind System with type constructors

$$
\begin{array}{rl}
\text{Type Variables} & t, u \in \text{Type Variables} \\
\text{Kinds} & \kappa ::= \star \mid \kappa' \to \kappa \\
\text{Types} & \tau^{\kappa} ::= t^{\kappa} \mid T^{\kappa} \mid \tau^{\kappa' \to \kappa} \tau^{\kappa'} \\
\text{Type Constructors} & T^{\kappa} \in \mathcal{T}^{\kappa} \text{ where } \quad \{\otimes, \&, \oplus, \overset{!}{\twoheadrightarrow}, \twoheadrightarrow, \overset{!}{\twoheadrightarrow}, \twoheadrightarrow\} \subseteq \mathcal{T}^{\star \to \star \to} \\
\text{Predicates} & \pi, \omega ::= \mathtt{Un}\ \tau \mid \mathtt{SeFun}\ \tau \mid \mathtt{ShFun}\ \tau \mid \tau \geq \tau' \\
\text{Qualified Types} & \rho ::= \tau^{\star} \mid \pi \Rightarrow \rho \\
\text{Type schemes} & \sigma ::= \rho \mid \forall t.\sigma
\end{array}
$$

Conclusion and Future Work

What next?

Thank You!

Q & A

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