QuB
A Resource Aware Functional Programming Language

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Hard problems in programming

Naming variables

Hard problems in programming

Resource management
in evolving production code

Resources: Files, database connections, entity with a shared state

Resource Management: File Handling

- Modified File Handling API in Haskell

  ```
  openFile :: FilePath → IO FileHandle

  closeFile :: FileHandle → IO ()

  readLine :: FileHandle → IO (String, FileHandle)

  writeFile :: String    → FileHandle
                         → IO ((), FileHandle)


  upper    :: String     → String
  ```

Resource Management: File Handling

- File Handling in Haskell

```
do f  ← openFile "sample.txt"
   (s, f)  ← readLine f
   let c = upper s
   ((), f) ← writeLine f c
           .
           .
           .
   () ← closeFile f
```

Resource Management: File Handling

- File Handling in Haskell Gone Wrong (Part I)

```
do f  ← openFile "sample.txt"
   (s, f)  ← readLine f
   let c = upper s
   ((),  f) ← writeLine f c
       .
       .
       .
   () ← closeFile f
       .
       .
       .
   () ← closeFile f
   return c
```

Resource Management: File Handling

- File Handling in Haskell Gone Wrong (Part I)

```
do f  ← openFile "sample.txt"
   (s, f)  ← readLine f
   let c = upper s
   (() , f) ← writeLine f c
            .
            .
            .
    () ← closeFile f
            .
            .
            .
    () ← closeFile f
    return c
```

- File is closed twice: Run time crash

Resource Management: File Handling

- File Handling in Haskell Gone Wrong (Part II)

```
do f ← openFile "sample.txt"
   (s, f) ← readLine f
   let c = upper s
   ((), f) ← writeLine f c
        .
        .
        .
   return c
```

Resource Management: File Handling

- File Handling in Haskell Gone Wrong (Part II)

```
do f  ← openFile "sample.txt"
   (s, f)  ← readLine f
   let c = upper s
   ((),  f) ← writeLine f c
        .
        .
        .
   return c /*File not closed!!*/
```

- File not closed: Memory leak

Resource Management: Exception Handling

- `MonadError[5]` in Haskell

  ```
  class Monad m ⇒ MonadError e m | m → e where
      throwError :: e → m a
      catchError :: m a → (e → m a) → m a
  ```

- `throwError` starts exception processing

- `catchError` exception handler

Resource Management: Exception Handling

- Using `MonadError` in Haskell

```
do f ← openFile "sample.txt"
   ((s, f) ← readLine f
   let c = upper s
   () ← closeFile f
   return $ Right c)
       `catchError` (\_ →
               return $ Left "Error in reading file")
```

- Exception may cause memory leak

Well typed programs do not go wrong.
— R. Milner

Well typed programs do not go wrong.

— R. Milner

~~Lights~~ Types will guide you home

— Coldplay

Contributions

- Design and implement QuB type system

  - Resources as first class citizens
  - Program objects are restricted or unrestricted
  - Functions that share resources with their arguments or are separate.

- Formalizing and proving important properties of QuB
- QuB is logic of *BI* with steroids
  - Environments as graphs
- Working examples

$$\lambda x.M \begin{cases} \text{Abstract over computation} \\ \text{Define functions} \end{cases}$$

$$MN \begin{cases} \text{Do the computation} \\ \text{Use functions} \end{cases}$$

$$\lambda x.M \begin{cases} \text{Abstract over computation} \\ \text{Define functions} \end{cases}$$

$$MN \begin{cases} \text{Do the computation} \\ \text{Use functions} \end{cases}$$

$$\frac{\Gamma_x, x : \tau \vdash M : \tau'}{\Gamma \vdash \lambda x.M : \tau \to \tau'} \, [\to \text{I}] \qquad \frac{\Gamma \vdash M : \tau \to \tau' \qquad \Gamma \vdash N : \tau}{\Gamma \vdash MN : \tau'} \, [\to \text{E}]$$

$$\lambda x.M \begin{cases} \text{Abstract over computation} \\ \text{Define functions} \end{cases}$$

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$$\frac{\Gamma_x, x : \tau \vdash M : \tau'}{\Gamma \vdash \lambda x.M : \tau \to \tau'} \ [\to \text{I}] \qquad \frac{\Gamma \vdash M : \tau \to \tau' \qquad \Gamma \vdash N : \tau}{\Gamma \vdash MN : \tau'} \ [\to \text{E}]$$

Hinley-Milner ($\mathbf{HM}$) type system ensures sane programs

- Types are Propostions

- Programs are Proofs

  HM type system ≡ Second Order Propositional Logic

Background Work: Second Order Propositional Logic

Language:

Propostions & connectives $\quad A, B, C ::= x \mid A \supset B \mid \forall x.B \mid ...$

Context $\quad \Gamma, \Delta ::= \epsilon \mid \Gamma, A$

Logic Rules:

$$\frac{}{A \vdash A} \ [\text{Ax}]$$

$$\frac{\Gamma \vdash B \quad x \notin \Gamma}{\forall x.B} \ [\forall \text{I}] \qquad\qquad \frac{\Gamma \vdash \forall x.B \quad \Gamma \vdash A}{B[x/A]} \ [\forall \text{E}]$$

$$\frac{\Gamma, A \vdash B}{\Gamma \vdash A \supset B} \ [\supset \text{I}] \qquad\qquad \frac{\Gamma \vdash A \supset B \quad \Gamma \vdash A}{\Gamma \vdash B} \ [\supset \text{E}]$$

- Two implicit rules in Propositional Calculus

$$\frac{\Gamma \vdash B}{\Gamma, A \vdash B} \ [\text{WKN}] \qquad\qquad \frac{\Gamma, A, A \vdash B}{\Gamma, A \vdash B} \ [\text{CTR}]$$

- Two implicit rules in Propositional Calculus

$$\frac{\Gamma \vdash B}{\Gamma, A \vdash B} \text{ [WKN]} \qquad \qquad \frac{\Gamma, A, A \vdash B}{\Gamma, A \vdash B} \text{ [CTR]}$$

- Control the use of [WKN] and [CTR]

Background Work: Linear Logic

Certain contexts cannot undergo weakening or contraction[3, 10]

$$\text{Context} \quad \Gamma, \Delta ::= \epsilon \mid \Gamma, \langle A \rangle \mid \Gamma, [A]$$

$$\Gamma, \langle A \rangle \nvdash \Gamma$$
$$\Gamma, \langle A \rangle \nvdash \Gamma, \langle A \rangle, \langle A \rangle$$
$$\Gamma, [A] \vdash \Gamma$$
$$\Gamma, [A] \vdash \Gamma, [A], [A]$$

Background Work: Linear Logic

$$\boxed{\text{Language}}$$

$$\text{Context } \Gamma, \Delta ::= \epsilon \mid \Gamma, \langle A \rangle \mid \Gamma, [A]$$

$$\text{Propostions } A, B, C ::= X \mid !A \mid A \multimap B \mid A \,\&\, B \mid A \otimes B \mid A \oplus B$$

$$\boxed{\text{Structural Rules}}$$

$$\frac{}{[A] \vdash A} \, [\text{ID}_{[]}] \qquad \frac{}{\langle A \rangle \vdash A} \, [\text{ID}_{\langle\rangle}]$$

$$\frac{\Gamma, \Delta \vdash A}{\Delta, \Gamma \vdash A} \, [\text{EXCH}] \quad \frac{\Gamma, [A], [A] \vdash B}{\Gamma, [A] \vdash B} \, [\text{CTRN}] \, \frac{\Gamma \vdash B}{\Gamma, [A] \vdash B} \, [\text{WKN}]$$

$$\frac{[\Gamma] \vdash A}{[\Gamma] \vdash !A} \, [!\text{I}] \qquad \frac{\Gamma \vdash !A \qquad \Delta, [A] \vdash B}{\Gamma, \Delta \vdash B} \, [!\text{E}]$$

Background Work: Linear Logic

$$\boxed{\text{Connective Rules}}$$

$$\frac{\Gamma, A \vdash B}{\Gamma \vdash A \multimap B} \; [\multimap I] \qquad\qquad \frac{\Gamma \vdash A \multimap B \quad \Delta \vdash A}{\Gamma, \Delta \vdash B} \; [\multimap E]$$

Background Work: Linear Logic

Connective Rules

$$\frac{\Gamma, A \vdash B}{\Gamma \vdash A \multimap B} \; [\multimap I] \qquad\qquad \frac{\Gamma \vdash A \multimap B \quad \Delta \vdash A}{\Gamma, \Delta \vdash B} \; [\multimap E]$$

$$\frac{\Gamma \vdash A \quad \Gamma \vdash B}{\Gamma \vdash A \,\&\, B} \; [\&I] \qquad \frac{\Gamma \vdash A \,\&\, B}{\Gamma \vdash A} \; [\&E_1] \qquad \frac{\Gamma \vdash A \,\&\, B}{\Gamma \vdash B} \; [\&E_2]$$

Background Work: Linear Logic

$$\frac{\Gamma, A \vdash B}{\Gamma \vdash A \multimap B} \, [\multimap I] \qquad\qquad \frac{\Gamma \vdash A \multimap B \qquad \Delta \vdash A}{\Gamma, \Delta \vdash B} \, [\multimap E]$$

$$\frac{\Gamma \vdash A \qquad \Gamma \vdash B}{\Gamma \vdash A \,\&\, B} \, [\&I] \qquad \frac{\Gamma \vdash A \,\&\, B}{\Gamma \vdash A} \, [\&E_1] \qquad \frac{\Gamma \vdash A \,\&\, B}{\Gamma \vdash B} \, [\&E_2]$$

$$\frac{\Gamma \vdash A \qquad \Delta \vdash B}{\Gamma, \Delta \vdash A \otimes B} \, [\otimes I] \qquad \frac{\Gamma \vdash A \otimes B \qquad \Delta, A, B \vdash C}{\Gamma, \Delta \vdash C} \, [\otimes E]$$

Background Work: Linear Logic

$$\boxed{\text{Connective Rules}}$$

$$\frac{\Gamma, A \vdash B}{\Gamma \vdash A \multimap B} \; [\multimap I] \qquad\qquad \frac{\Gamma \vdash A \multimap B \qquad \Delta \vdash A}{\Gamma, \Delta \vdash B} \; [\multimap E]$$

$$\frac{\Gamma \vdash A \qquad \Gamma \vdash B}{\Gamma \vdash A \, \& \, B} \; [\& I] \qquad \frac{\Gamma \vdash A \, \& \, B}{\Gamma \vdash A} \; [\& E_1] \qquad \frac{\Gamma \vdash A \, \& \, B}{\Gamma \vdash B} \; [\& E_2]$$

$$\frac{\Gamma \vdash A \qquad \Delta \vdash B}{\Gamma, \Delta \vdash A \otimes B} \; [\otimes I] \qquad \frac{\Gamma \vdash A \otimes B \qquad \Delta, A, B \vdash C}{\Gamma, \Delta \vdash C} \; [\otimes E]$$

$$\frac{\Gamma \vdash A}{\Gamma \vdash A \oplus B} \; [\oplus I_1] \qquad\qquad \frac{\Delta \vdash B}{\Delta \vdash A \oplus B} \; [\oplus I_2]$$

$$\frac{\Gamma \vdash A \oplus B \qquad \Delta, A \vdash C \qquad \Delta, B \vdash C}{\Gamma, \Delta \vdash C} \; [\oplus E]$$

Background Work: Linear Logic

- Propositions act like Resources

$$\text{"A consumed to give B"} \quad A, A \multimap B \vdash B$$
$$\text{"Both A and B"} \quad A \otimes B$$
$$\text{"Choice between A and B"} \quad A \mathbin{\&} B$$
$$\text{Cannot create new copies} \quad A \nvdash A \otimes A$$
$$\text{Fall back to intuistionstic logic} \quad !A \multimap B \equiv A \supset B$$

- Propositions act like Resources

$$\text{"A consumed to give B"} \quad A, A \multimap B \vdash B$$
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$$\text{Cannot create new copies} \quad A \nvdash A \otimes A$$
$$\text{Fall back to intuistionstic logic} \quad !A \multimap B \equiv A \supset B$$

- Curry-Howard Corresondence: Type systems based on Linear Logic
- Active Area of research: L^3[1], F°[6], Linear Haskell[2], Quill[7]
- Resources are first class values

Background Work: Linear Logic

Downside: Asymmetry between implication and conjunction
Multiplicative Fragment:

$$A \otimes B \vdash C \text{ iff } A \vdash B \multimap C$$

Additive Fragment:

$$!A \,\&\, !B \vdash C \text{ iff } !A \vdash !B \multimap C$$

Modality is necessary to avoid over-restriction

General framework to incorporate predicates into type language

$$P \mid \Gamma \vdash M : \sigma$$

"Type of M is σ when predicates in P are satisfied and free variables in M are specified in Γ"[4]

HM system with qualified Types:

$$\frac{x : \sigma \in \Gamma}{P \mid \Gamma \vdash x : \sigma} \text{ [VAR]} \qquad \frac{P \mid \Gamma \vdash M : \sigma \qquad Q \mid \Gamma_x, x : \sigma \vdash N : \tau}{P, Q \mid \Gamma \vdash (\texttt{let } x = M \texttt{ in } N) : \tau} \text{ [LET]}$$

$$\frac{P \mid \Gamma \vdash M : \sigma \qquad t \notin \texttt{fvs}(\Gamma) \cup \texttt{fvs}(P)}{P \mid \Gamma \vdash M : \forall t.\sigma} \text{ [}\forall\text{ I]} \qquad \frac{P \mid \Gamma \vdash M : \forall t.\sigma}{P \mid \Gamma \vdash M : [\tau/t]\sigma} \text{ [}\forall\text{ E]}$$

$$\frac{P \mid \Gamma_x, x : \tau \vdash M : \tau'}{P \mid \Gamma \vdash \lambda x.M : \tau \to \tau'} \text{ [}\to\text{ I]} \qquad \frac{P \mid \Gamma \vdash M : \tau \to \tau' \qquad P \mid \Gamma \vdash N : \tau}{P \mid \Gamma \vdash MN : \tau'} \text{ [}\to\text{ E]}$$

$$\frac{P, \pi \mid \Gamma \vdash M : \rho}{P \mid \Gamma \vdash M : \pi \Rightarrow \rho} \text{ [}\Rightarrow\text{I]} \qquad \frac{P \mid \Gamma \vdash M : \pi \Rightarrow \rho \qquad P \Rightarrow \pi}{P \mid \Gamma \vdash M : \rho} \text{ [}\Rightarrow\text{E]}$$

$\pi \Rightarrow \rho$: π qualifies ρ $\qquad\qquad$ $P \Rightarrow \pi$: P entails π

Quill[7]: Language with qualified types for linear types and first class polymorphism
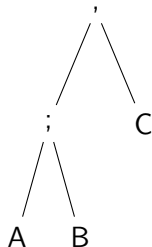
Three Predicates:

- Un τ If τ does not have resources or can be copied or dropped easily.

- Fun τ If τ is a function type

- $\tau \geq \tau'$ If τ less restricting than τ'

- Unrestricted Types: `Un Int`, `Un Bool`

- Restricted or Linear Types: `FileHandle`

- Function Types: `Fun (Int → Int)`, `Fun (String → String)`

- Contexts as trees, called bunches[9]
- Two connective used to combine contexts: $A; B$ or A, B

Background Work: Logic of *BI*

- Context connective guides choice of implication

$$\frac{\Gamma, A \vdash B}{\Gamma \vdash A \ast B} \, [\ast I] \qquad\qquad \frac{\Gamma; A \vdash B}{\Gamma \vdash A \twoheadrightarrow B} \, [\twoheadrightarrow I]$$

- Contraction

$$A; B \vdash A \qquad A; B \vdash B$$
$$A, B \nvdash A \qquad A, B \nvdash B$$

- Weakening

$$A; B \vdash A; B; B \qquad A; B \vdash A; A; B$$
$$A, B \nvdash A, B, B \qquad A, B \nvdash A, A, B$$

Background Work: $\alpha\lambda$-calculus

Curry-Howard interpretation of logic of **BI**[8]

$$\boxed{\text{Language}}$$

$$
\begin{aligned}
\text{Context} \quad & \Gamma, \Delta ::= \{\}_m \mid \{\}_a \mid x : \tau \mid \Gamma, \Delta \mid \Gamma; \Delta \\
\text{Types} \quad & \tau, \upsilon ::= t \mid \iota \mid \tau \twoheadrightarrow \tau \mid \tau \ast \tau \\
\text{Expressions} \quad & M, N ::= x \mid \lambda x.M \mid \alpha x.M \mid MN
\end{aligned}
$$

$$\boxed{\text{Typing rules}}$$

$$\frac{x : \tau \in \Gamma}{\Gamma \vdash x : \tau} \ [\text{VAR}] \qquad \frac{\Gamma; \Gamma \vdash M : \tau}{\Gamma \vdash M : \tau} \ [\text{CTR}] \ \frac{\Gamma \vdash M : \tau}{\Gamma; \Delta \vdash M : \tau} \ [\text{WKN}]$$

$$\frac{\Gamma_x, x : \tau \vdash M : \tau'}{\Gamma \vdash \lambda x.M : \tau \ast \tau'} \ [\ast\text{I}] \qquad \frac{\Gamma \vdash M : \tau \ast \tau' \quad \Delta \vdash N : \tau}{\Gamma, \Delta \vdash MN : \tau'} \ [\ast\text{E}]$$

$$\frac{\Gamma_x; x : \tau \vdash M : \tau'}{\Gamma \vdash \alpha x.M : \tau \twoheadrightarrow \tau'} \ [\twoheadrightarrow\text{I}] \qquad \frac{\Gamma \vdash M : \tau \twoheadrightarrow \tau' \quad \Delta \vdash N : \tau}{\Gamma; \Delta \vdash MN : \tau'} \ [\twoheadrightarrow\text{E}]$$

- Quill: Qualified types for type system based on linear logic
- QuB: Qualified types for type system based on logic of *BI*

$$\text{Types} \quad \tau, \upsilon, \phi ::= t \mid \iota \mid \tau \to \tau$$

$$\text{where} \quad \to \in \left\{ \overset{\shortmid}{\twoheadrightarrow}, \rightarrowtail, \overset{\shortmid}{\twoheadrightarrow}, \twoheadrightarrow \right\}$$

$$\text{Predicates} \quad \pi, \omega ::= \text{Un } \tau \mid \text{SeFun } \tau \mid \text{ShFun } \tau \mid \tau \geq \tau'$$

$$\text{Qualified Types} \quad \rho ::= \tau \mid \pi \Rightarrow \rho$$

$$\text{Type schemes} \quad \sigma ::= \rho \mid \forall t.\sigma$$

- SeFun τ: τ is a function that is separate from its argument

- ShFun τ: τ is a function that is in sharing with its argument

- Un τ: τ does not have resources or they can be copied/dropped easily

$$\text{Types} \quad \tau, \upsilon, \phi ::= t \mid \iota \mid \tau \to \tau$$

$$\text{where} \quad \to \in \left\{ \overset{|}{\twoheadrightarrow}, \twoheadrightarrow, \overset{|}{\twoheadrightarrow}, \twoheadrightarrow \right\}$$

$$\text{Predicates} \quad \pi, \omega ::= \text{Un } \tau \mid \text{SeFun } \tau \mid \text{ShFun } \tau \mid \tau \geq \tau'$$

$$\text{Qualified Types} \quad \rho ::= \tau \mid \pi \Rightarrow \rho$$

$$\text{Type schemes} \quad \sigma ::= \rho \mid \forall t.\sigma$$

- $\twoheadrightarrow$: Function type that is separate with its argument

- $\twoheadrightarrow$: Function type that is in sharing with its argument

- $\overset{|}{\twoheadrightarrow}$, $\overset{|}{\twoheadrightarrow}$: unrestriced versions of $\twoheadrightarrow$ and $\twoheadrightarrow$

Term Variables $x, y, z \in \mathrm{Var}$

Expressions $M, N ::= x \mid \lambda^{\twoheadrightarrow^*} x.M \mid \lambda^{\twoheadrightarrow} x.M \mid MN \mid \mathtt{let}\ x = M\ \mathtt{in}\ N$

- $\lambda^{\twoheadrightarrow^*} x.M$: Introduces $\twoheadrightarrow^*$ function
- $\lambda^{\twoheadrightarrow} x.N$: Introduces $\twoheadrightarrow$ function

QuB: Typing Environmnet

- Logic of *BI*: Contexts are trees

- QuB: Contexts are flattened sharing graphs
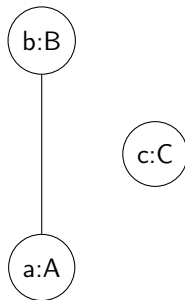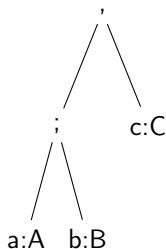
- Sharing relation Ψ

$$\forall_{x \in \mathrm{dom}(\Psi)} \ x \in \Psi(x) \qquad \text{(reflexive)}$$

$$\forall_{x,y \in \mathrm{dom}(\Psi)} \ \text{if } y \in \Psi(x) \text{ then } x \in \Psi(y) \qquad \text{(symmetric)}$$

$$\forall_{x,y,z \in \mathrm{dom}(\Psi)} \ \text{if } y \in \Psi(x) \text{ and } z \in \Psi(y) \ \not\Rightarrow z \in \Psi(x) \quad \text{(non-transitive)}$$

QuB: Typing Environment

"x of type σ is in sharing with $\vec{y}$"

$$(x, \sigma, \vec{y}) \in \Gamma$$

Typing Context $\quad \Gamma, \Delta ::= \epsilon \mid \Gamma, x^{\vec{y}} : \sigma$

$$\mathtt{Vars}(\Gamma, x^{\vec{y}} : \tau) = \mathtt{Vars}(\Gamma) \cup \{x\}$$

$$\mathtt{Shared}(\Gamma, x^{\vec{y}} : \tau) = \mathtt{Shared}(\Gamma) \cup \{\vec{y}\}$$

$$\mathtt{Used}(\Gamma) = \mathtt{Vars}(\Gamma) \cup \mathtt{Shared}(\Gamma)$$

$$(\Gamma, x^{\vec{y}} : \tau)^{[a \mapsto \vec{b}]} = \begin{cases} a \notin \vec{y} & (\Gamma^{[a \mapsto \vec{b}]}, x^{\vec{y}} : \tau) \\ a \in \vec{y} & (\Gamma^{[a \mapsto \vec{b}]}, x^{(\vec{y} \backslash a) \cup \vec{b}} : \tau) \end{cases}$$

$$\Gamma^{[\vec{a} \mapsto \vec{b}]} = (\ldots((\Gamma^{[a_1 \mapsto \vec{b}]})^{[a_2 \mapsto \vec{b}]})^{\cdots})^{[a_n \mapsto \vec{b}]}$$

$\Gamma \otimes \Gamma' = \Gamma \sqcup \Gamma' \qquad$ if $\mathtt{Vars}(\Gamma) \# \mathtt{Used}(\Gamma') \wedge \mathtt{Vars}(\Gamma') \# \mathtt{Used}(\Gamma)$

$\Gamma \oplus \Gamma' = \Gamma \sqcup \Gamma' \qquad$ if $\mathtt{Used}(\Gamma) = \mathtt{Used}(\Gamma')$

QuB: Typing Rules

$$\boxed{\text{Structural Rules}}$$

$$\frac{}{P \mid x^{\check{y}} : \sigma \vdash x : \sigma} \text{ [ID]}$$

$$\frac{P \mid \Gamma \otimes \Delta \otimes \Delta \vdash M : \sigma \qquad P \vdash \Delta \text{ un}}{P \mid \Gamma \otimes \Delta \vdash M : \sigma} \text{ [CTR-UN]} \qquad \frac{P \mid \Gamma \oplus \Delta \oplus \Delta \vdash M : \sigma}{P \mid \Gamma \oplus \Delta \vdash M : \sigma} \text{ [CTR-SH]}$$

$$\frac{P \mid \Gamma \vdash M : \sigma \qquad P \vdash \Delta \text{ un}}{P \mid \Gamma \otimes \Delta \vdash M : \sigma} \text{ [WKN-UN]} \qquad \frac{P \mid \Gamma \vdash M : \sigma}{P \mid \Gamma \oplus \Delta \vdash M : \sigma} \text{ [WKN-SH]}$$

QuB: Typing Rules

$$\boxed{\text{Connective Rules}}$$

$$\frac{P \mid \Gamma \vdash M : \sigma \qquad P' \mid \Gamma'_x, x : \sigma \vdash N : \tau}{P \cup P' \mid \Gamma \sqcup \Gamma' \vdash (\texttt{let } x = M \texttt{ in } N) : \tau} \ [\textsf{LET}]$$

$$\frac{P \mid \Gamma \vdash M : \sigma \qquad t \notin \texttt{fvs}(\Gamma) \cup \texttt{fvs}(P)}{P \mid \Gamma \vdash M : \forall t.\sigma} \ [\forall \ \textsf{I}] \qquad \frac{P \mid \Gamma \vdash M : \forall t.\sigma}{P \mid \Gamma \vdash M : [\tau \backslash t]\sigma} \ [\forall \ \textsf{E}]$$

$$\frac{P, \pi \mid \Gamma \vdash M : \rho}{P \mid \Gamma \vdash M : \pi \Rightarrow \rho} \ [\Rightarrow \textsf{I}] \qquad \frac{P \mid \Gamma \vdash M : \pi \Rightarrow \rho \qquad P \vdash \pi}{P \mid \Gamma \vdash M : \rho} \ [\Rightarrow \textsf{E}]$$

$$\frac{P \Rightarrow \texttt{ShFun } \phi \qquad P \vdash \Gamma \geq \phi}{P \mid \Gamma^{[\varnothing \mapsto \{x\}]}, x^{\textsf{Vars}(\Gamma)} : \tau \vdash M : \tau'}{P \mid \Gamma \vdash \lambda^{\twoheadrightarrow} x.M : \phi\tau\tau'} \ [\twoheadrightarrow \textsf{I}] \quad \frac{P \Rightarrow \texttt{ShFun } \phi}{P \mid \Gamma \vdash M : \phi\tau\tau' \qquad P \mid \Gamma' \vdash N : \tau}{P \mid \Gamma \oplus \Gamma' \vdash MN : \tau'} \ [\twoheadrightarrow \textsf{E}]$$

$$\frac{P \Rightarrow \texttt{SeFun } \phi \qquad P \vdash \Gamma \geq \phi}{P \mid \Gamma, x^{\varnothing} : \tau \vdash M : \tau'}{P \mid \Gamma \vdash \lambda^{\twoheadrightarrow *} x.M : \phi\tau\tau'} \ [\twoheadrightarrow* \textsf{I}] \quad \frac{P \Rightarrow \texttt{SeFun } \phi}{P \mid \Gamma \vdash M : \phi\tau\tau' \qquad P \mid \Gamma' \vdash N : \tau}{P \mid \Gamma \otimes \Gamma' \vdash MN : \tau'} \ [\twoheadrightarrow* \textsf{E}]$$

QuB: Typing Terms

$$\dfrac{}{\varnothing \mid x^{\varnothing} : \tau \vdash x : \tau} \text{ [ID]}$$

$$\dfrac{}{\varnothing \mid f^{\varnothing} : \tau \multimap \tau' \multimap \upsilon \vdash f : \tau \multimap \tau' \multimap \upsilon} \text{ [ID]}$$

$$\dfrac{}{\varnothing \mid y^{\varnothing} : \tau' \vdash y : \tau'} \text{ [ID]}$$

$$\dfrac{\varnothing \mid x^{\varnothing} : \tau \oplus f^{\varnothing} : \tau \multimap \tau' \multimap \upsilon \vdash fx : (\tau' \multimap \upsilon)}{} \text{ [$\multimap$E]}$$

$$\dfrac{\varnothing \mid y^{\varnothing} : \tau' \oplus x^{\varnothing} : \tau \oplus f^{\varnothing} : \tau \multimap \tau' \multimap \upsilon \vdash fxy : \upsilon}{} \text{ [$\multimap$E]}$$

$$\dfrac{\varnothing \mid x^{\varnothing} : \tau \oplus y^{\varnothing} : \tau' \oplus f^{\varnothing} : \tau \multimap \tau' \multimap \upsilon \vdash fxy : \upsilon}{} \text{ [EXCH]}$$

$$\dfrac{\varnothing \mid x^{\varnothing} : \tau \oplus y^{\varnothing} : \tau' \vdash \lambda^{\multimap} f . fxy : (\tau \multimap \tau' \multimap \upsilon) \multimap \upsilon}{} \text{ [$\multimap$ I]}$$

$$\dfrac{\varnothing \mid x^{\varnothing} : \tau \vdash \lambda^{\multimap} y . \lambda^{\multimap} f . fxy : \tau' \multimap (\tau \multimap \tau' \multimap \upsilon) \multimap \upsilon}{} \text{ [$\multimap$ I]}$$

$$\dfrac{\varnothing \mid I \vdash \lambda^{\multimap} x . \lambda^{\multimap} y . \lambda^{\multimap} f . fxy : \tau \multimap \tau' \multimap (\tau \multimap \tau' \multimap \upsilon) \multimap \upsilon}{} \text{ [$\equiv$]}$$

$$\frac{}{\varnothing \mid y^{xf} : \tau' \vdash y : \tau'} \text{ [ID]}$$

$$\frac{\dfrac{}{\varnothing \mid x^{fy} : \tau \vdash x : \tau} \text{ [ID]} \quad \dfrac{}{\varnothing \mid f^{xy} : \tau \twoheadrightarrow \tau' \twoheadrightarrow \upsilon \vdash f : \tau \twoheadrightarrow \tau' \twoheadrightarrow \upsilon} \text{ [ID]}}{\varnothing \mid x^{xy} : \tau \oplus f^{xy} : \tau \twoheadrightarrow \tau' \twoheadrightarrow \upsilon \vdash fx : (\tau' \twoheadrightarrow \upsilon)} \text{ [$*$ E]}$$

$$\frac{\varnothing \mid y^{xf} : \tau' \oplus x^{yf} : \tau \oplus f^{xy} : \tau \twoheadrightarrow \tau' \twoheadrightarrow \upsilon \vdash fxy : \upsilon}{} \text{ [$\twoheadrightarrow$ E]}$$

$$\frac{\varnothing \mid x^{yf} : \tau \oplus y^{xf} : \tau' \oplus f^{xy} : \tau \twoheadrightarrow \tau' \twoheadrightarrow \upsilon \vdash fxy : \upsilon}{} \text{ [EXCH]}$$

$$\frac{\varnothing \mid x^{y} : \tau \oplus y^{x} : \tau' \vdash \lambda^{\twoheadrightarrow} f.fxy : (\tau \twoheadrightarrow \tau' \twoheadrightarrow \upsilon) \twoheadrightarrow \upsilon}{} \text{ [$\twoheadrightarrow$ I]}$$

$$\frac{\varnothing \mid x^{\varnothing} : \tau \vdash \lambda^{\twoheadrightarrow} y.\lambda^{\twoheadrightarrow} f.fxy : \tau' \twoheadrightarrow (\tau \twoheadrightarrow \tau' \twoheadrightarrow \upsilon) \twoheadrightarrow \upsilon}{} \text{ [$\twoheadrightarrow$ I]}$$

$$\frac{\varnothing \mid I \vdash \lambda^{*} x.\lambda^{\twoheadrightarrow} y.\lambda^{\twoheadrightarrow} f.fxy : \tau * \tau' \twoheadrightarrow (\tau \twoheadrightarrow \tau' \twoheadrightarrow \upsilon) \twoheadrightarrow \upsilon}{} \text{ [$*$ I]}$$

Term structure $\leftrightarrow$ Typing Rule

QuB: Syntax Directed Typing Rules

$$\frac{P \vdash \Gamma_{\bar{y}} \text{ un} \qquad (P \Rightarrow \tau) \sqsubseteq \sigma}{P \mid \Gamma, x^{\bar{y}} : \sigma \vdash^{s} x : \tau} \text{ [VAR}^{s}\text{]}$$

$$\frac{Q \mid (\Gamma_{x}' \oplus \Gamma_{x}'') \circledast \Delta \vdash^{s} M : \upsilon \qquad P \vdash \Delta \text{ un}}{P \mid (\Gamma_{x}, x^{\varnothing} : \sigma) \oplus \Gamma_{x}'' \circledast \Delta \vdash^{s} N : \tau \qquad \sigma = \text{Gen}(\{\Gamma' \oplus \Gamma_{x}'' \circledast \Delta\}, Q \Rightarrow \upsilon)}{P, Q \mid (\Gamma \circledast \Gamma') \oplus \Gamma'' \circledast \Delta \vdash^{s} (\text{let } x = M \text{ in } N) : \tau} \text{ [Let}^{s}\text{]}$$

$$[\text{VAR}^{s}] \equiv [\text{ID}] + [\forall \text{E}] + [\Rightarrow \text{E}]$$

$$[\text{LET}^{s}] \equiv [\text{LET}] + [\forall \text{I}] + [\Rightarrow \text{I}]$$

QuB: Syntax Directed Typing Rules

$$\frac{\begin{array}{cc} P \Rightarrow \mathtt{SeFun}\ \phi & P \vdash \Gamma \geq \phi \\ P \mid \Gamma \circledast x^{\varnothing} : \tau \vdash^s M : \upsilon \end{array}}{P \mid \Gamma \vdash^s \lambda^{\twoheadleftarrow}x.M : \phi\tau\upsilon}\ [\twoheadleftarrow\mathsf{I}^s] \qquad \frac{\begin{array}{cc} P \Rightarrow \mathtt{ShFun}\ \phi & P \vdash \Gamma \geq \phi \\ P \mid \Gamma^{[\varnothing \mapsto \{x\}]} \oplus x^{\mathtt{Vars}(\Gamma)} : \tau \vdash^s M : \upsilon \end{array}}{P \mid \Gamma \vdash^s \lambda^{\twoheadrightarrow}x.M : \phi\tau\upsilon}\ [\twoheadrightarrow\mathsf{I}^s]$$

$$\frac{\begin{array}{ccc} P \mid \Gamma \circledast \Delta \vdash^s M : \phi\upsilon\tau & P \mid \Gamma' \circledast \Delta \vdash^s N : \upsilon & P \vdash \Delta\ \mathtt{un} \\ (\Gamma \stackrel{\sim}{\oplus} \Gamma' \wedge (P \Rightarrow \mathtt{ShFun}\ \phi)) \vee (\Gamma \stackrel{\sim}{\circledast} \Gamma' \wedge (P \Rightarrow \mathtt{SeFun}\ \phi)) \end{array}}{P \mid \Gamma \sqcup \Gamma' \circledast \Delta \vdash^s MN : \tau}\ [\mathsf{App}^s]$$

$$\boxed{\text{Context Operations}}$$

$$\Gamma \stackrel{\sim}{\circledast} \Delta = \mathtt{Used}(\Gamma)\ \#\ \mathtt{Vars}(\Delta) \wedge \mathtt{Used}(\Delta)\ \#\ \mathtt{Vars}(\Gamma)$$
$$\Gamma \stackrel{\sim}{\oplus} \Delta = \mathtt{Used}(\Gamma) = \mathtt{Used}(\Delta)$$

$$[\twoheadrightarrow\mathsf{I}] \equiv [\twoheadrightarrow\mathsf{I}] \qquad [\twoheadleftarrow\mathsf{I}] \equiv [\twoheadleftarrow\mathsf{I}]$$
$$[\mathsf{APP}^s] \equiv [\twoheadrightarrow\mathsf{E}] + [\twoheadleftarrow\mathsf{E}]$$

Theorem (Soundness of $\vdash^s$)

If $P \mid \Gamma \vdash^s M : \tau$ then $P \mid \Gamma \vdash M : \tau$

Theorem (Completeness of $\vdash^s$)

If $P \mid \Gamma \vdash M : \sigma$ then $\exists Q, \tau$ such that $Q \mid \Gamma \vdash^s M : \tau$ and $(P \mid \sigma) \sqsubseteq \mathcal{Gen}(\Gamma, Q \Rightarrow \tau)$

| Original Type System | $\equiv$ | Syntax Directed Typing Rules |

| Proof in Original Type System | $\equiv$ | Proof in Syntax Directed Typing Rules |

QuB: Algorithm $\mathcal{M}$

$$\boxed{\mathcal{M}(S, \Psi, \Gamma \vdash M : \tau) = P, S', \Sigma, \Psi'}$$

$$\mathcal{M}(S, \Psi, \Gamma \vdash x : \tau) = ([\vec{u}/\vec{t}]P), S' \circ S, \{x\}, \Psi$$

where $\quad (x : \forall \vec{t}.P \Rightarrow \upsilon) \in S\Gamma$

$$S' = \mathcal{U}([\vec{u}/\vec{t}]\upsilon, S\tau)$$

$$\mathcal{M}(S, \Psi, \Gamma \vdash \lambda^{\twoheadrightarrow}x.M : \tau) = \{P \cup Q\}, S', \Sigma \backslash x, \Psi''$$

where $\quad P; S'; \Sigma; \Psi' = \mathcal{M}(\mathcal{U}(\tau, u_1 u_2 u_3) \circ S, \Psi, \Gamma, x : u_2 \vdash M : u_3)$

$$\Psi'' = \{\forall_{y \in \text{dom}(\Psi')}.\Psi'(y) + x\} \cup \{(x, \{y \mid y \in \text{dom}(\Gamma)\})\}$$

$$Q = \{\text{ShFun } u_1\} \cup \text{Leq}(u_1, \Gamma|_\Sigma) \cup \text{Weaken}(x, u_2, \Sigma, \Psi'')$$

$$\mathcal{M}(S, \Psi, \Gamma \vdash \lambda^{\dashv*}x.M : \tau) = \{P \cup Q\}, S', \Sigma \backslash x, \Psi''$$

where $\quad P; S'; \Sigma; \Psi' = \mathcal{M}(\mathcal{U}(\tau, u_1 u_2 u_3) \circ S, X; \Gamma, x : u_2 \vdash M : u_3)$

$$\Psi'' = \Psi' \cup \{(x, \{x\})\}$$

$$Q = \{\text{SeFun } u_1\} \cup \text{Leq}(u_1, \Gamma \mid_\Sigma) \cup \text{Weaken}(x, u_2, \Sigma, \Psi'')$$

$$\boxed{\mathcal{M}(S, \Psi, \Gamma \vdash M : \tau) = P, S', \Sigma, \Psi'}$$

$$\mathcal{M}(S, \Psi, \Gamma \vdash MN : \tau) = \{P \cup P' \cup Q\}, R', \Sigma \cup \Sigma', \Psi''$$

$$\text{where} \quad P; R; \Sigma; \Psi' = \mathcal{M}(S, \Psi, \Gamma \vdash M : u_1 u_2 \tau)$$

$$P'; R'; \Sigma'; \Psi'' = \mathcal{M}(SR, \Psi', S\Gamma \vdash N : u_2)$$

$$\text{if } \mathcal{C}(\Gamma, \Psi'', \Sigma) = \mathcal{C}(\Gamma, \Psi'', \Sigma')$$

$$\text{then } Q = \{\text{ShFun } u_1\}$$

$$\text{else if } (\Sigma \# \mathcal{C}(R\Gamma, \Psi'', \Sigma') \text{ and } \Sigma' \# \mathcal{C}(R\Gamma, \Psi'', \Sigma))$$

$$\text{then } Q = \{\text{SeFun } u_1\}$$

$$\mathcal{M}(S, \Psi, \Gamma \vdash \text{let } x = M \text{ in } N : \tau) = (P \cup Q), R', \Sigma \cup \{\Sigma' \backslash x\}, \Psi''$$

$$\text{where} \quad P; R; \Sigma; \Psi' = \mathcal{M}(S, \Psi, \Gamma \vdash M : u_1)$$

$$\sigma = \text{GenI}(R\Gamma; R(P \Rightarrow u_1))$$

$$P'; R'; \Sigma'; \Psi'' = \mathcal{M}(R, \Psi', \Gamma, x : \sigma \vdash N : \tau)$$

$$Q = \text{Un}(\Gamma|_{\Sigma \cap \Sigma'}) \cup \text{Weaken}(x, \sigma, \Sigma', \Psi'')$$

Theorem (Soundness of $\mathcal{M}$)

if $\mathcal{M}(S, \Psi, \Gamma \vdash M : \tau) = P, S', \Sigma, \Psi'$ then $S'P \mid S'(\Gamma|_\Sigma) \vdash M : S'\tau$

$$\boxed{\text{Algorithm M}} \rightarrow \boxed{\text{Type system}}$$

Examples: Basic Structures

- Multiplicative Product

$$\tau \otimes \tau' = \tau \twoheadrightarrow \tau' \twoheadrightarrow (\tau \twoheadrightarrow \tau' \twoheadrightarrow \upsilon) \twoheadrightarrow \upsilon$$
$$(,) = \lambda^{\twoheadrightarrow}x.\lambda^{\twoheadrightarrow}y.\lambda^{\twoheadrightarrow}f.fxy$$

- Additive Product

$$\tau \mathbin{\&} \tau' = \tau \twoheadrightarrow \tau' \twoheadrightarrow (\tau \twoheadrightarrow \tau' \twoheadrightarrow \upsilon) \twoheadrightarrow \upsilon$$
$$(;) = \lambda^{\twoheadrightarrow}x.\lambda^{\twoheadrightarrow}y.\lambda^{\twoheadrightarrow}f.fxy$$

- Sums

$$\tau \oplus \tau' = (\tau \to \upsilon) \to (\tau' \to \upsilon) \to \upsilon$$
$$\texttt{case } c \texttt{ of } \{f;g\} = \lambda^{\twoheadrightarrow}c.\lambda^{\twoheadrightarrow}f.\lambda^{\twoheadrightarrow}g.cfg$$

$$\texttt{inl} : \tau \twoheadrightarrow (\tau \oplus \tau') \qquad\qquad \texttt{inr} : \tau' \twoheadrightarrow (\tau \oplus \tau')$$
$$\texttt{inl} = \lambda^{\twoheadrightarrow}x.\lambda^{\twoheadrightarrow}f.\lambda^{\twoheadrightarrow}g.fx \qquad \texttt{inr} = \lambda^{\twoheadrightarrow}y.\lambda^{\twoheadrightarrow}f.\lambda^{\twoheadrightarrow}g.gy$$

- User defined types and type classes

QuB: Extension

- User defined types and type classes

- Kind System with type constructors

$$\text{Type Variables} \quad t, u \in \text{Type Variables}$$

$$\text{Kinds} \quad \kappa ::= \star \mid \kappa' \to \kappa$$

$$\text{Types} \quad \tau^\kappa ::= t^\kappa \mid T^\kappa \mid \tau^{\kappa' \to \kappa} \tau^{\kappa'}$$

$$\text{Type Constructors} \quad T^\kappa \in \mathcal{T}^\kappa \text{ where } \quad \{\otimes, \&, \oplus, \stackrel{\cdot}{\ast}, \ast\!\!\ast, \twoheadrightarrow, \twoheadrightarrow\} \subseteq \mathcal{T}^{\star \to \star \to \star \to}$$

$$\text{Predicates} \quad \pi, \omega ::= \text{Un } \tau \mid \text{SeFun } \tau \mid \text{ShFun } \tau \mid \tau \geq \tau'$$

$$\text{Qualified Types} \quad \rho ::= \tau^\star \mid \pi \Rightarrow \rho$$

$$\text{Type schemes} \quad \sigma ::= \rho \mid \forall t.\sigma$$

Conclusion and Future Work

What next?

Thank You!

Q & A

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