All about triangles

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1 What is a triangle?

A triangle is a polygon with three straight sides. Let's get used to some notation. In Fig. 1, the vertices

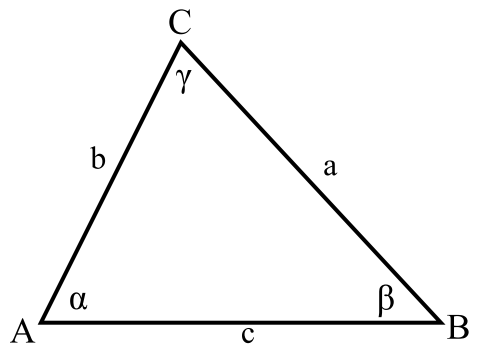


Figure 1: A triangle

of the triangle are denoted by A, B, and C. It is customary to use uppercase Latin alphabet to label vertices of polygons. The sides opposite to these vertices are denoted by a, b, and c respectively. The angles at these vertices are denoted by α, β, and γ. It is customary to use lowercase Greek alphabet to denote angles in polygons. The triangle itself is denoted by $\triangle ABC$.
We also use the following notation:

- AB denotes the line segment joining points A and B and $\overline{AB}$ denotes the length of that segment. Obviously, $\overline{AB} = c$. Similarly, $\overline{BC} = a$ and $\overline{CA} = b$.
- $\angle ABC$ denotes the angle at vertex B formed by line segments AB and BC and is equal to β. Similarly, $\angle BCA = \gamma$ and $\angle CAB = \alpha$.
- The perimeter of a triangle is the sum of the lengths of all its sides. For our $\triangle ABC$, it is $a+b+c$. The semi-perimeter (or half-perimeter) of $\triangle ABC$ is denoted by:

$$s = \frac{a+b+c}{2}$$

An important property of triangles is that the sum of the angles in a triangle is always 180°. This is a provable statement and in mathematics, we call such statements **theorems** and state them like so:

Theorem 1.1 *The sum of the (interior) angles in a triangle is* 180°.

We have specified that the angles are interior angles. This is because a triangle can also have exterior angles, which are formed by extending one of its sides. The sum of the exterior angles of a triangle is always 360°.

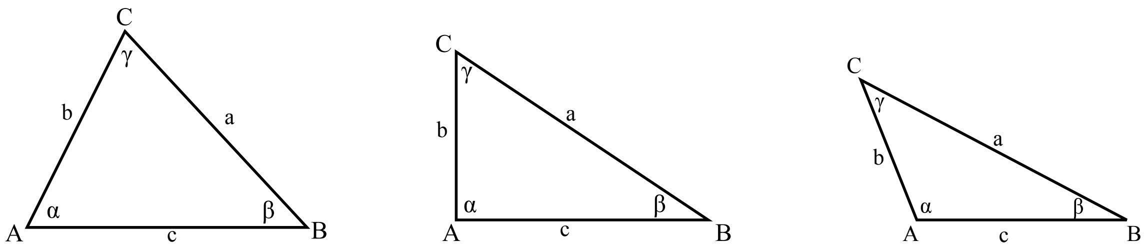
Exercise: First, draw a triangle and measure its angles. Verify this theorem. But all you would have done is verify this theorem for a single triangle. Can you prove that this theorem holds for all triangles? *Hint: Use the fact that the sum of angles on a straight line is* 180°.

2 Types of triangles

Triangles can be classified based on their angles and sides.

2.1 Based on angles

Triangles can be classified based on their angles:



(a) An **acute triangle** has all angles less than 90°.

(b) A **right triangle** has one angle equal to 90°. Here, $\angle BAC = \alpha = 90°$.

(c) An **obtuse triangle** has one angle greater than 90°. Here, $\angle BAC = \alpha > 90°$.

Figure 2: Different types of triangles based on angles

2.2 Based on sides

Triangles can also be classified based on the lengths of their sides. The hatches in the figure below denote the sides that are equal to one another.

(a) An **equilateral triangle** has all its sides equal. Here, $\overline{AB} = \overline{BC} = \overline{CA}$.

(b) An **isosceles triangle** has two angles equal. Here, $\overline{AC} = \overline{BC} \neq \overline{AB}$.

Figure 3: Different types of triangles based on sides

2.3 Exercises

Using Theorem 1.1, let's try to answer the following questions:

1. Can a triangle have two right angles? Why or why not?
2. Can a triangle have two obtuse angles? Why or why not?
3. Can a triangle have two acute angles? Why or why not?

3 More properties of triangles

Here are some relationships between the sides and angles of triangles that are useful to know.

1. The sum of the lengths of any two sides of a triangle is greater than the length of the third side. This is known as the **triangle inequality property**. Therefore, in any $\triangle ABC$:

$$\overline{AB} + \overline{BC} > \overline{AC}, \quad \overline{AC} + \overline{BC} > \overline{AB}, \quad \overline{AB} + \overline{AC} > \overline{BC}$$

2. **Theorem 3.1** *Angles opposite to equal sides of a triangle are equal. This is known as the **isosceles triangle theorem**. In any $\triangle ABC$,*

$$\textit{If } \overline{AB} = \overline{AC}, \textit{ then } \angle ABC = \angle ACB$$

When mathematicians write a statement using *if* like here, they mean that "whenever the condition on the left-hand side is true, the conclusion on the right-hand side is also true". In this case, it means that whenever $\overline{AB} = \overline{AC}$, then $\angle ABC$ must equal $\angle ACB$.

3. **Theorem 3.2** *Angles opposite to a larger side of a triangle are larger than those opposite to a smaller side. In any $\triangle ABC$,*

$$\textit{If } \overline{AB} > \overline{AC}, \textit{ then } \angle ACB > \angle ABC$$

3.1 Exercises

1. Can you prove theorems 3.1 and 3.2 in this list above?
2. Prove that the angles opposite to equal sides of a triangle are equal using the isosceles triangle theorem.
3. Prove that all angles of an equilateral triangle are equal.
4. In Fig. 3b, AB is the largest side. Which is the largest angle?
5. Can a triangle with sides 4 cm, 5 cm, and 6 cm exist? Why or why not?
6. Can a triangle with sides 2 cm, 3 cm, and 6 cm exist? Why or why not?