Braids and the Jones polynomial

Thesis presentation

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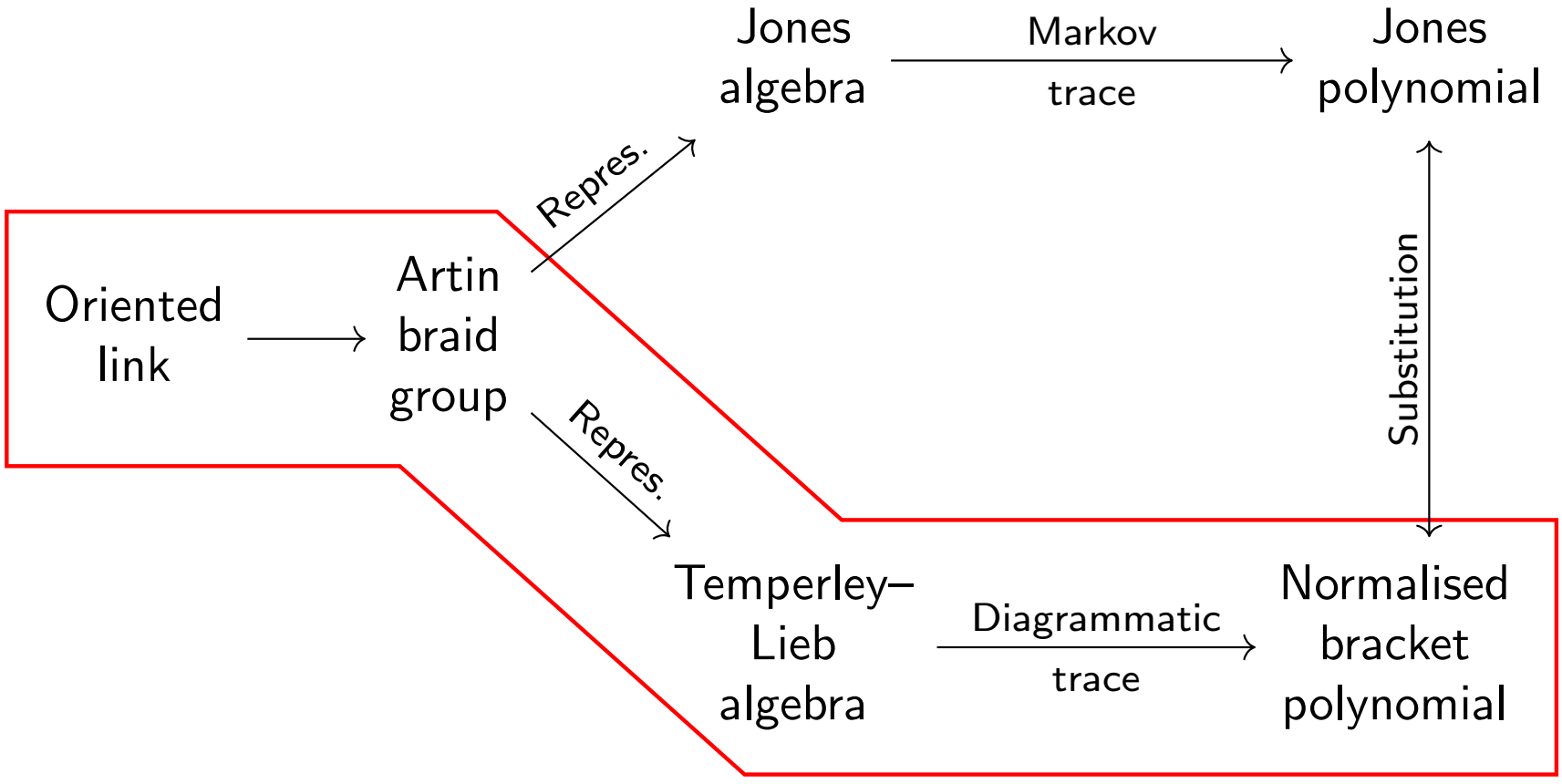
Outline


Jones algebra
Markov trace
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Repres.
Oriented link
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Geometric definition of braids

Three dimensional representation

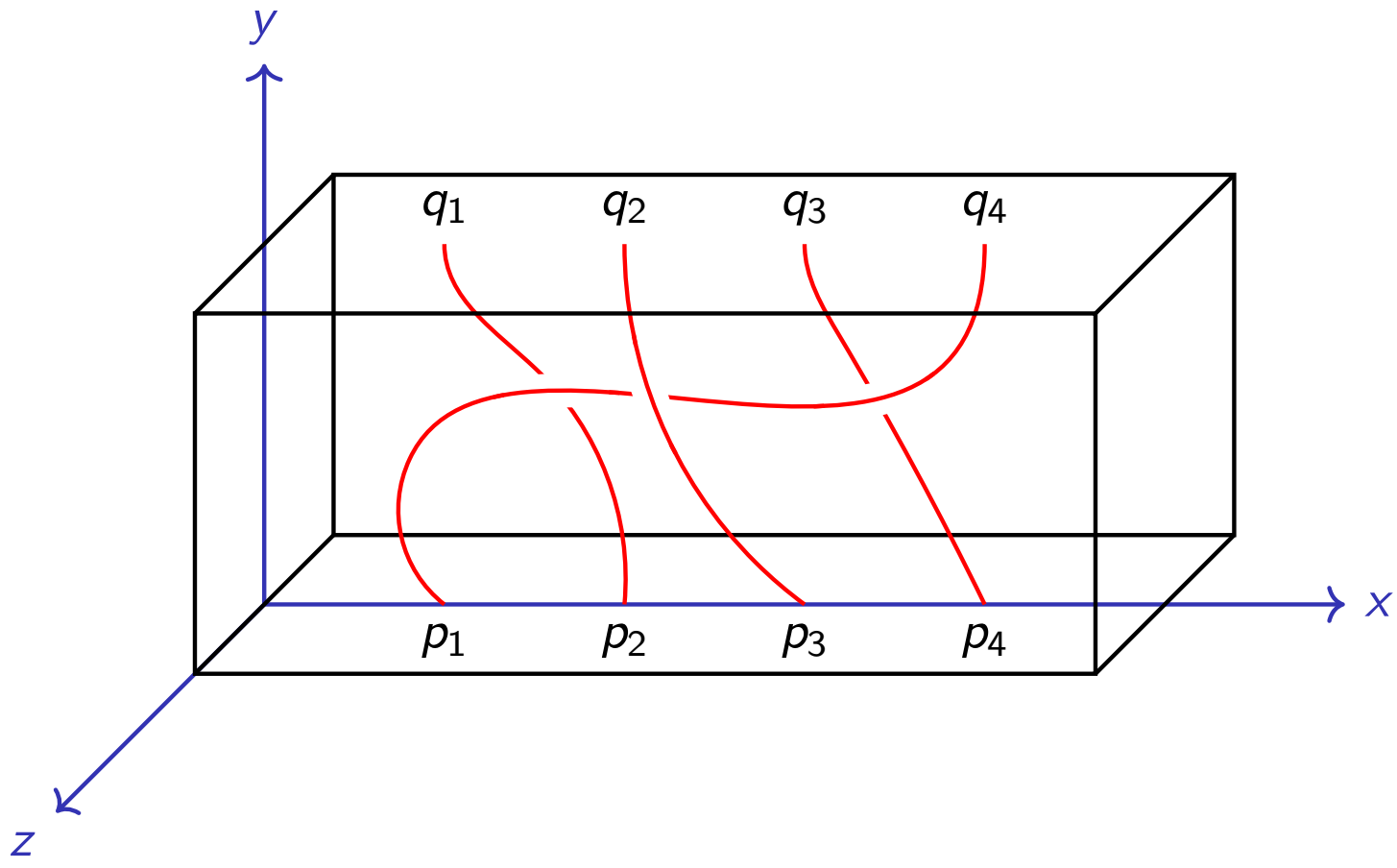


Figure: Three dimensional geometric representation of a braid

Two dimensional representation

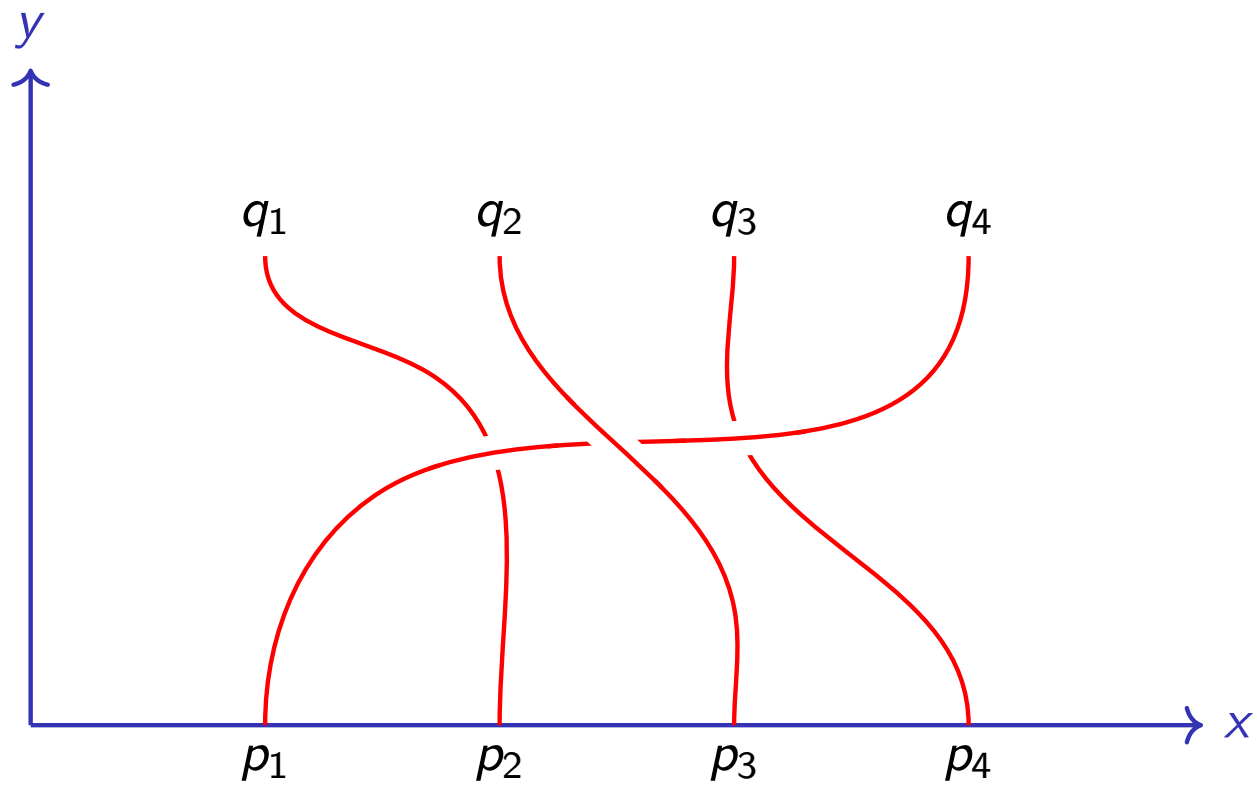


Figure: A projection of the braid

Multiplication of braids

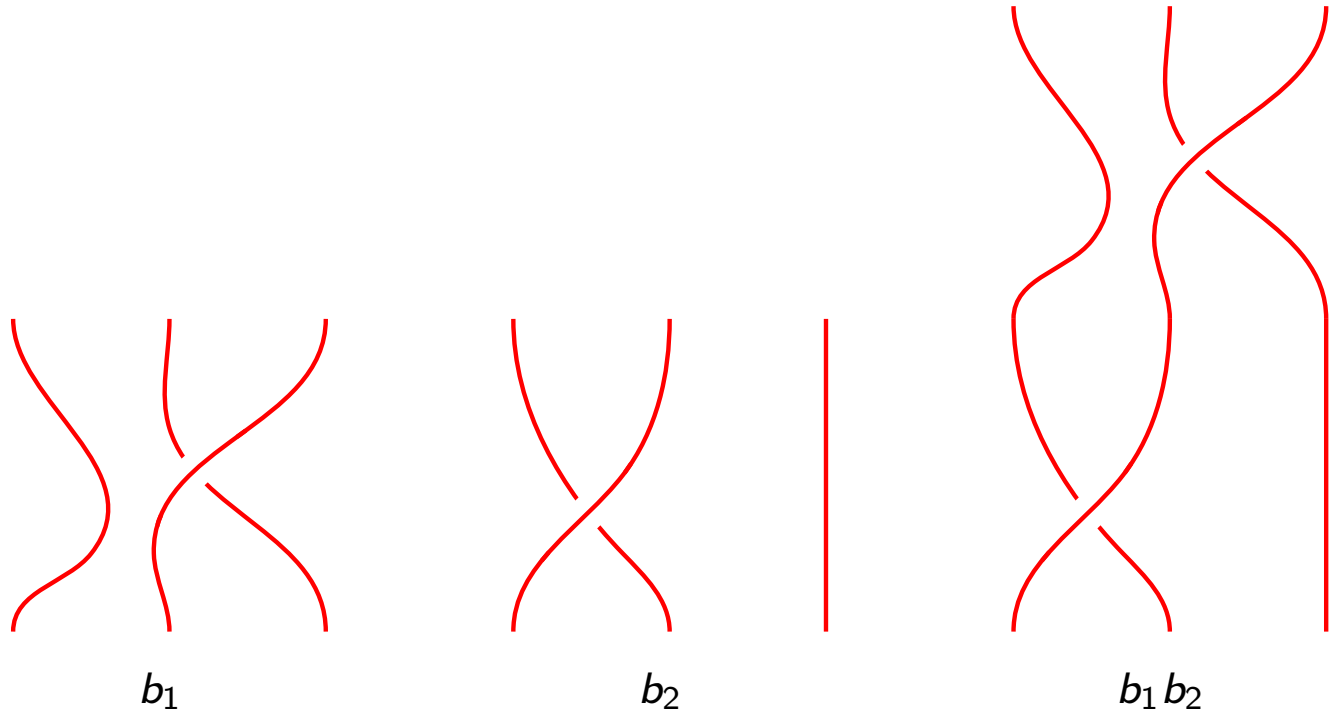


Figure: Multiplication of two braids

The identity braid $\mathbb{1}_n$

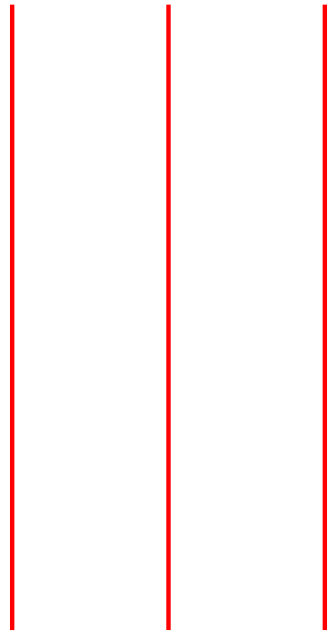


Figure: The identity $\mathbb{1}_3$

Inverse of braids

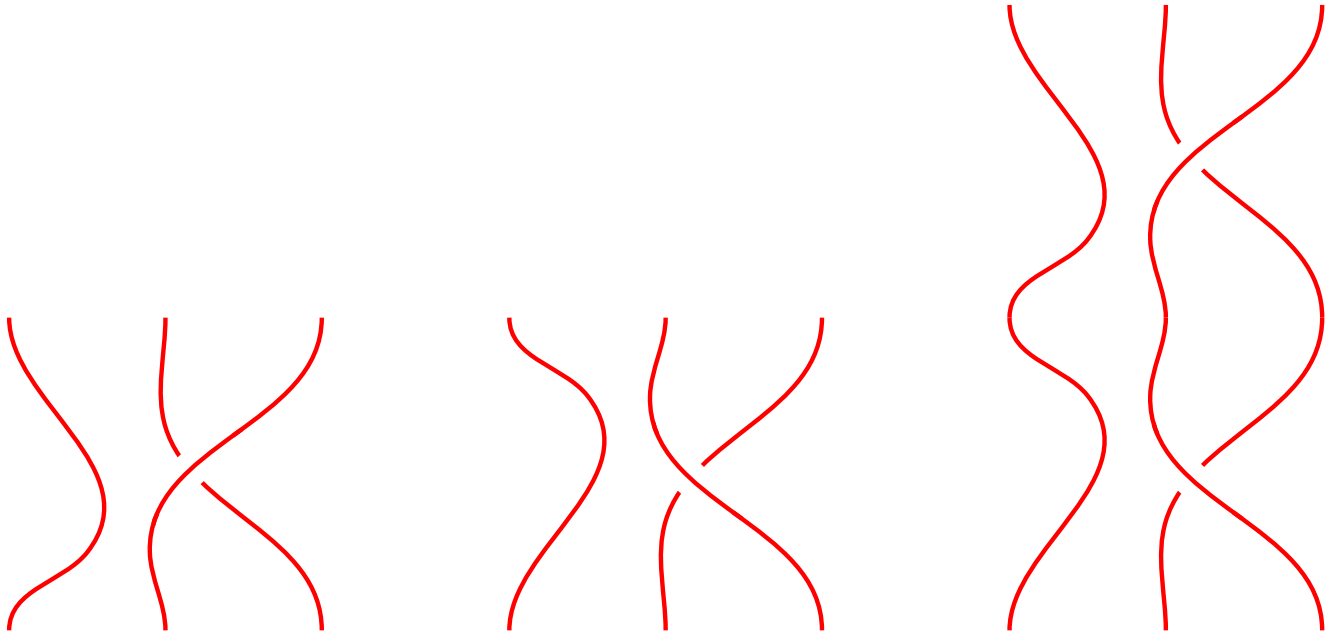


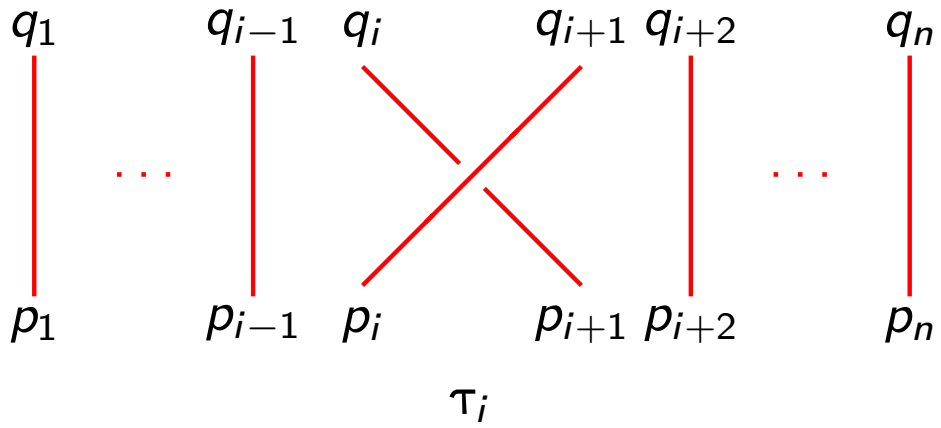
Figure: Inverse of a braid

Thus, braids form a group, known as the Artin braid group.

Subsection 2

Generators and relations

Generators of the braid group



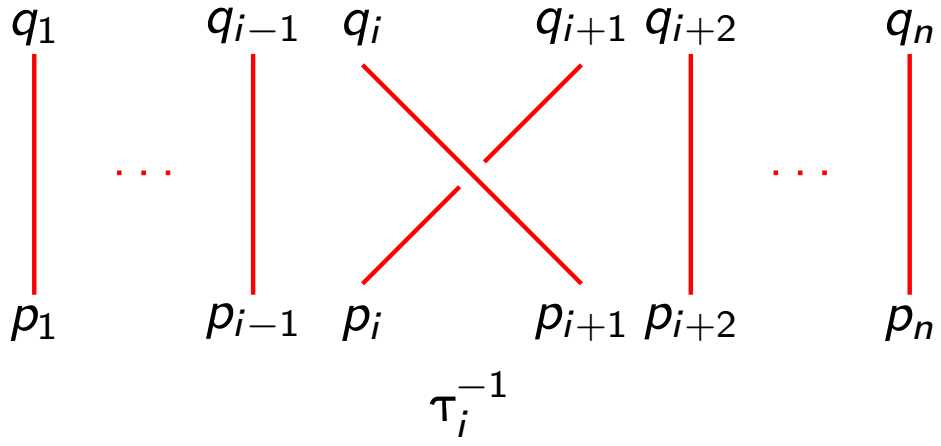


Figure: Generators τ_i and τ_i^{-1}

Type II move: $\tau_i \tau_i^{-1} = \mathbb{I}_n$

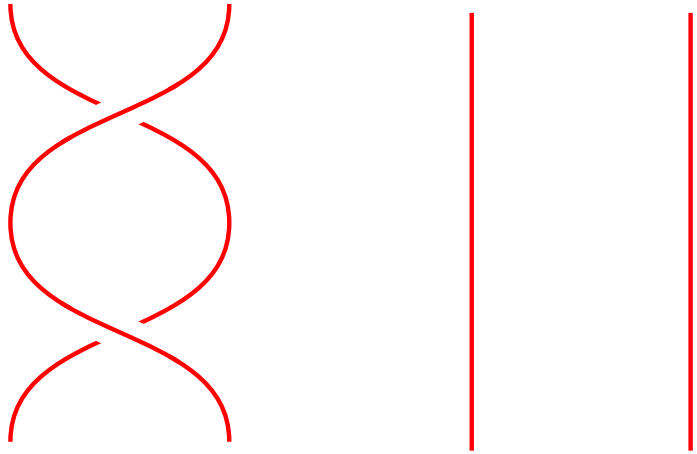
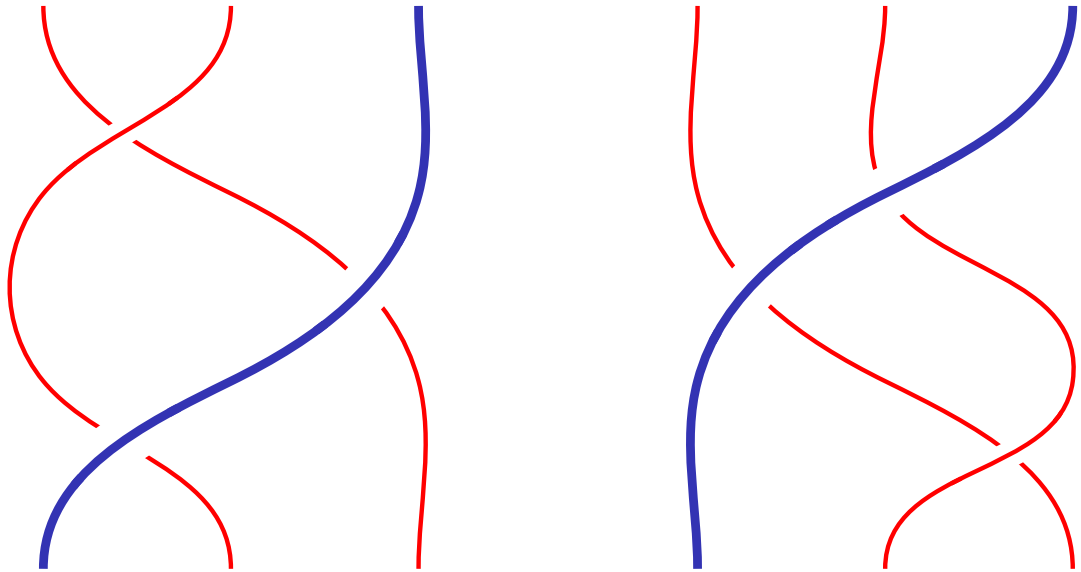


Figure: A type II move illustrating $\tau_i \tau_i^{-1} = \mathbb{I}_n$

Type III move: $\tau_i \tau_{i+1} \tau_i = \tau_{i+1} \tau_i \tau_{i+1}$



$\tau_i \tau_{i+1} \tau_i$ $\tau_{i+1} \tau_i \tau_{i+1}$

Figure: A type III move illustrating $\tau_i \tau_{i+1} \tau_i = \tau_{i+1} \tau_i \tau_{i+1}$

Sliding of crossings: $\tau_i \tau_j = \tau_j \tau_i$

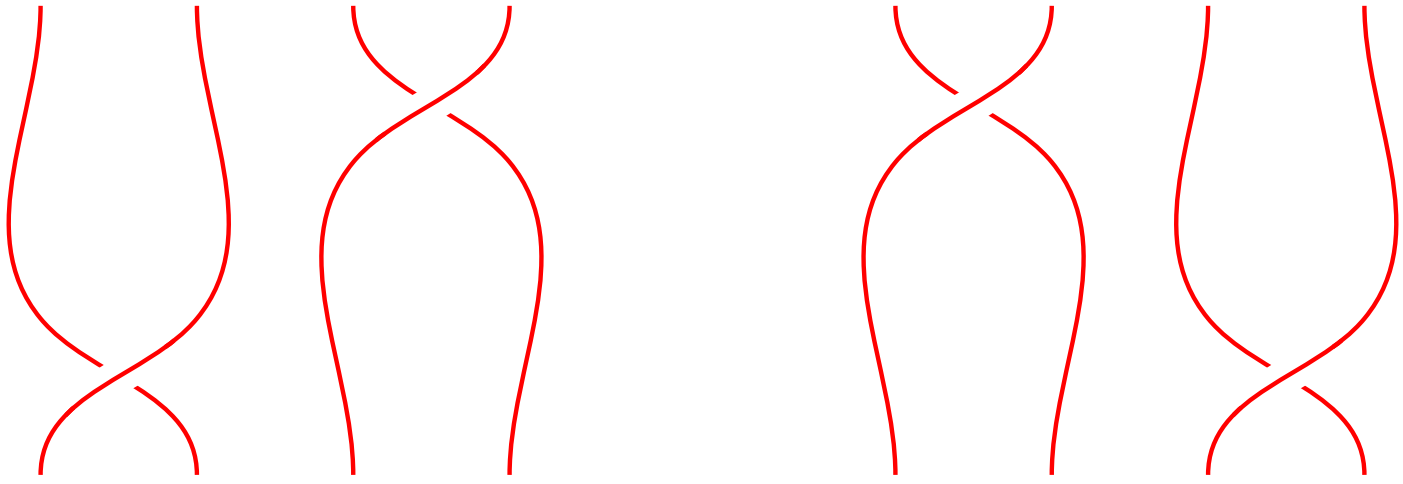


Figure: Sliding of crossings illustrating $\tau_i \tau_j = \tau_j \tau_i$