Braids and the Jones polynomial

Thesis presentation

Apoorv Potnis

IISERB

April 2023

Table of Contents

Section 1

Outline

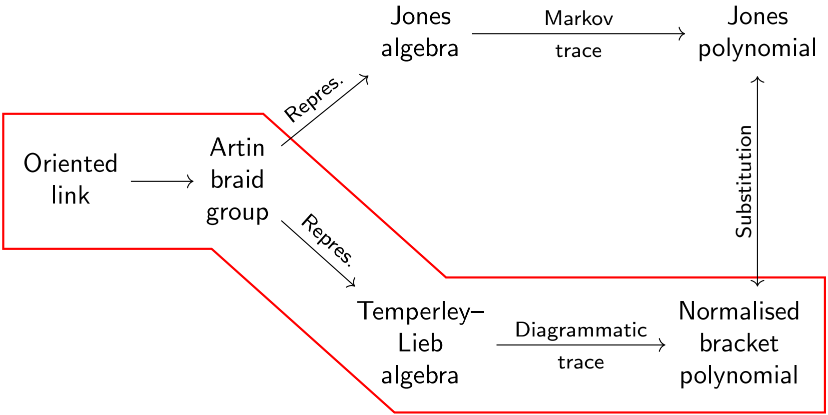

Oriented link
Artin braid group
Repres.
Jones algebra
Markov trace
Jones polynomial
Substitution
Repres.
Temperley–Lieb algebra
Diagrammatic trace
Normalised bracket polynomial

Table of Contents

Section 2

Braids

Subsection 1

Geometric definition

Three dimensional representation

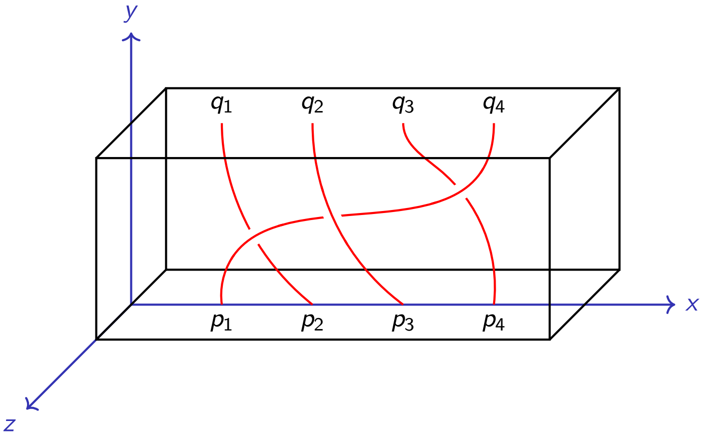


Figure: Three dimensional geometric representation of a braid

Two dimensional representation

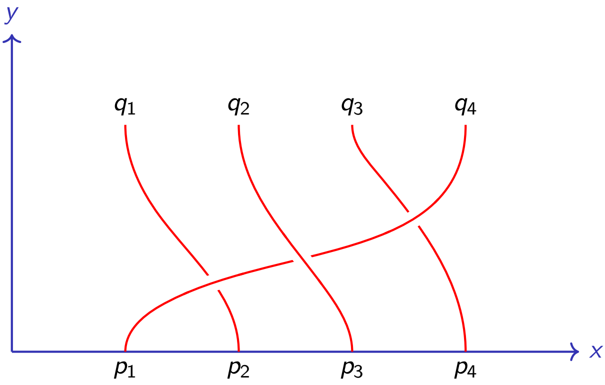


Figure: A projection of the braid

Multiplication of braids

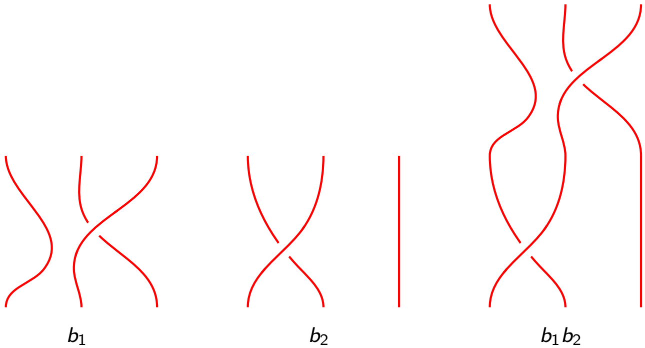


Figure: Multiplication of two braids

The identity braid $\mathbb{1}_n$

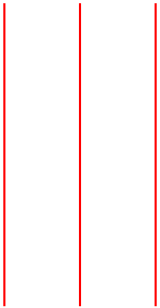
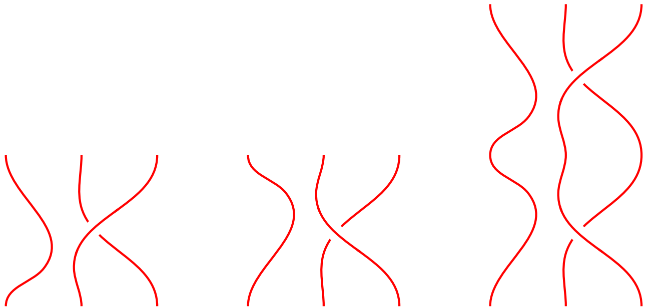


Figure: The identity $\mathbb{1}_3$

Inverse of braids



b $\qquad$ b^{-1} $\qquad$ $bb^{-1} = \mathbb{1}_3$

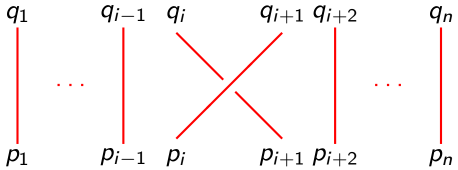
Figure: Inverse of a braid

Thus, braids form a group, known as the Artin braid group B_n.

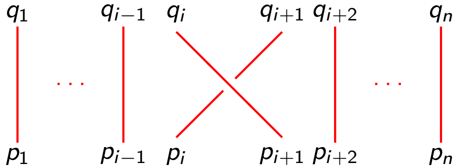
Subsection 2

Generators and relations

Generators of the braid group



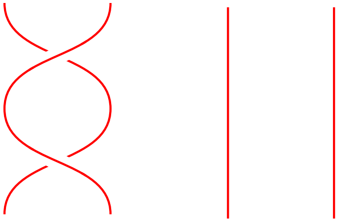
σ_i



σ_i^{-1}

Figure: Generators σ_i and σ_i^{-1}

Type II move: $\sigma_i \sigma_i^{-1} = \mathbb{1}_n$



$\sigma_i \sigma_i^{-1}$ $\mathbb{1}_n$

Figure: A type II move illustrating $\sigma_i \sigma_i^{-1} = \mathbb{1}_n$

Type III move: $\sigma_i\sigma_{i+1}\sigma_i = \sigma_{i+1}\sigma_i\sigma_{i+1}$

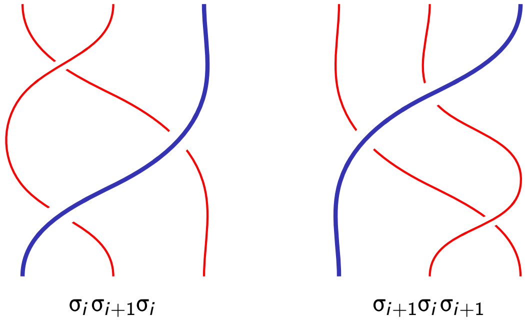


Figure: A type III move illustrating $\sigma_i\sigma_{i+1}\sigma_i = \sigma_{i+1}\sigma_i\sigma_{i+1}$

Sliding of crossings: $\sigma_i \sigma_j = \sigma_j \sigma_i$

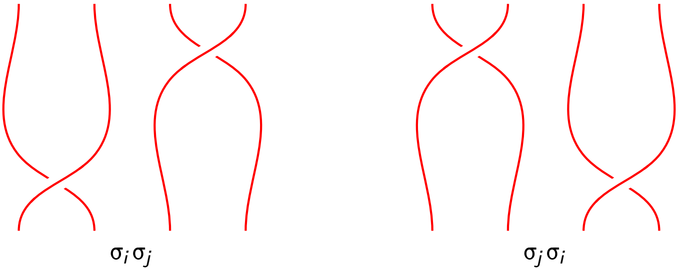


Figure: Sliding of crossings illustrating $\sigma_i \sigma_j = \sigma_j \sigma_i$

Subsection 3

Algebraic definition

Presentation of the braid group

The Artin braid group B_n admits the following presentation on the generators σ_i, for $1 \leq i \leq n-1$.

$$B_n = \left\langle \ \sigma_1, \ldots, \sigma_{n-1} \ \middle| \ \begin{array}{rcll} \sigma_i \sigma_i^{-1} & = & \mathbb{1}_n & \\ \sigma_i \sigma_{i+1} \sigma_i & = & \sigma_{i+1} \sigma_i \sigma_{i+1} & \text{if } i+1 \leq n-1 \\ \sigma_i \sigma_j & = & \sigma_j \sigma_i & \text{if } |i-j| \geq 2 \end{array} \ \right\rangle$$

Table of Contents

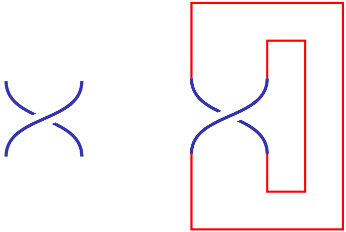
Section 3

Closure

Subsection 1

Closure of a braid

Closure of a braid $\overline{b}$



b $\overline{b}$

Figure: Closure of a braid

Braids and links

Every closure of a braid is a link.

Theorem (Alexander)

Every link is ambient isotopic to a closure of a braid.

Subsection 2

Equivalence of closures of braids

Conjugation

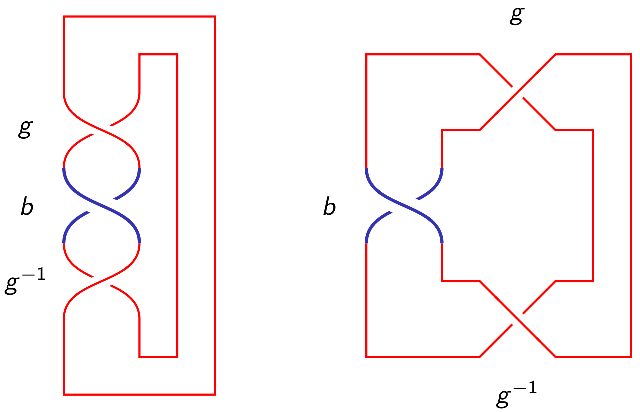


Figure: Conjugation process illustrating the link equivalence of $\overline{gbg^{-1}}$ and $\overline{b}$ (part 1)

Conjugation

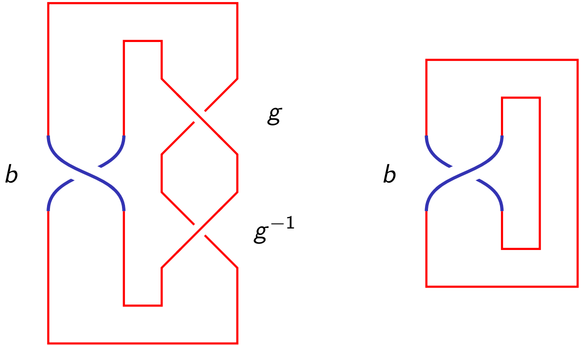


Figure: Conjugation process illustrating the link equivalence of $\overline{gbg^{-1}}$ and $\overline{b}$ (part 2)

Markov theorem

Theorem (Markov)

Two braids whose closures are ambient isotopic to each other are related by a finite sequence of the following operations.

1. Braid equivalences, i.e. equivalences resulting due to the braid relations.
2. Conjugation.
3. Markov moves.

Markov move

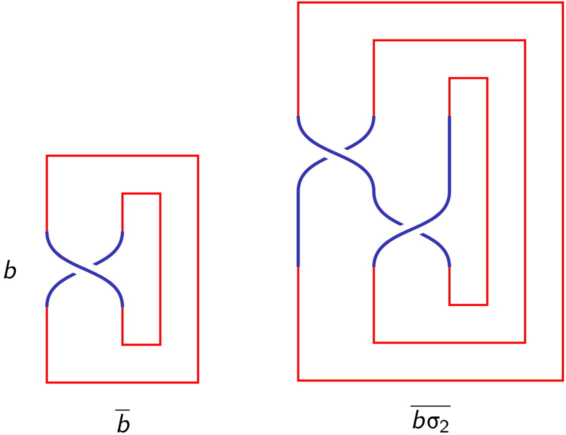


Figure: Markov move with $b = \sigma_1^{-1}$.

Table of Contents

Section 4

Orientation

Orientation