Braids and the Jones polynomial

Thesis presentation

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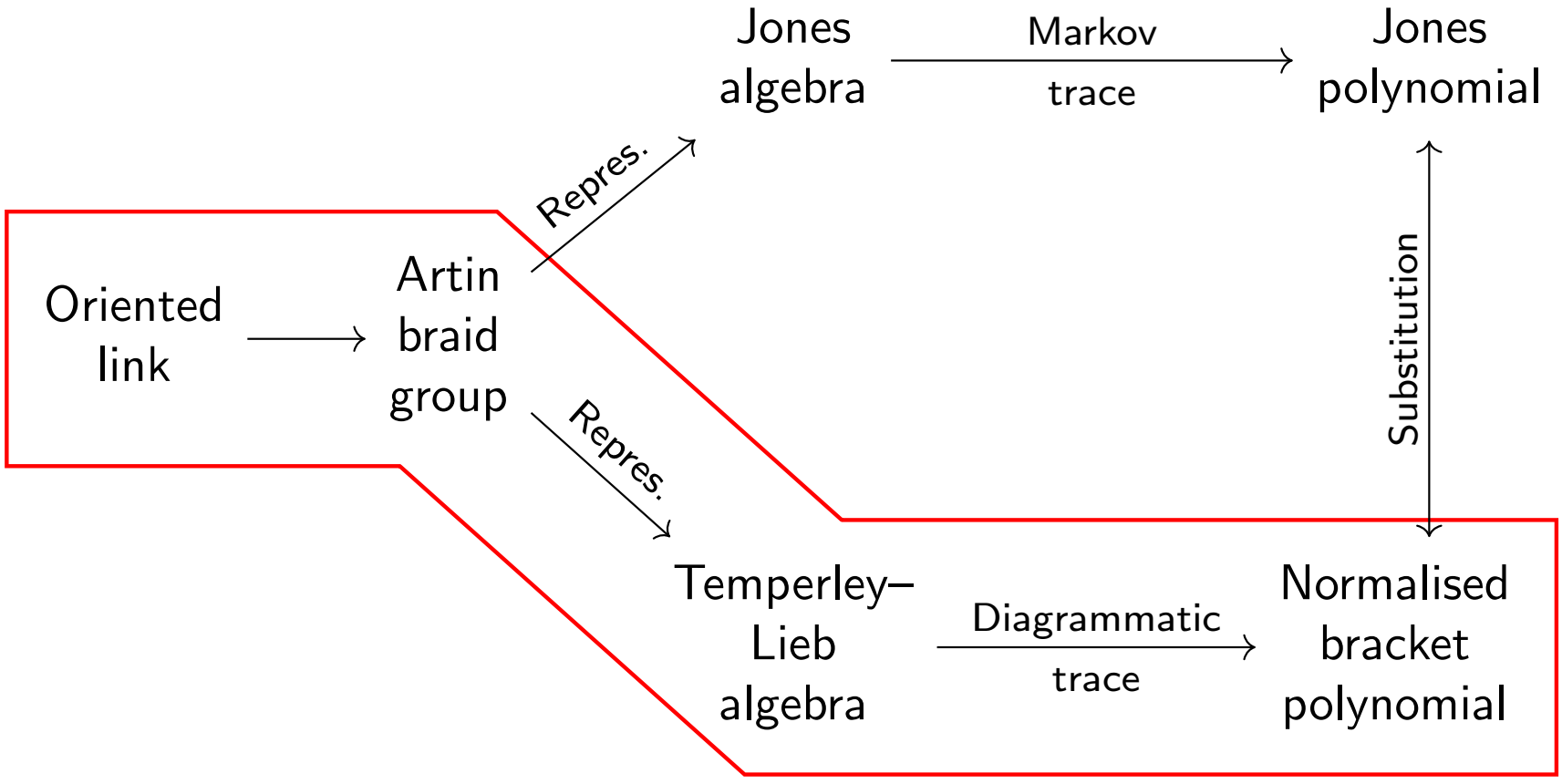
Section 1

Outline


Jones algebra
Markov trace
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Repres.
Oriented link
Artin braid group
Repres.
Substitution
Temperley–Lieb algebra
Diagrammatic trace
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Braids

Subsection 1

Geometric definition

Three dimensional representation

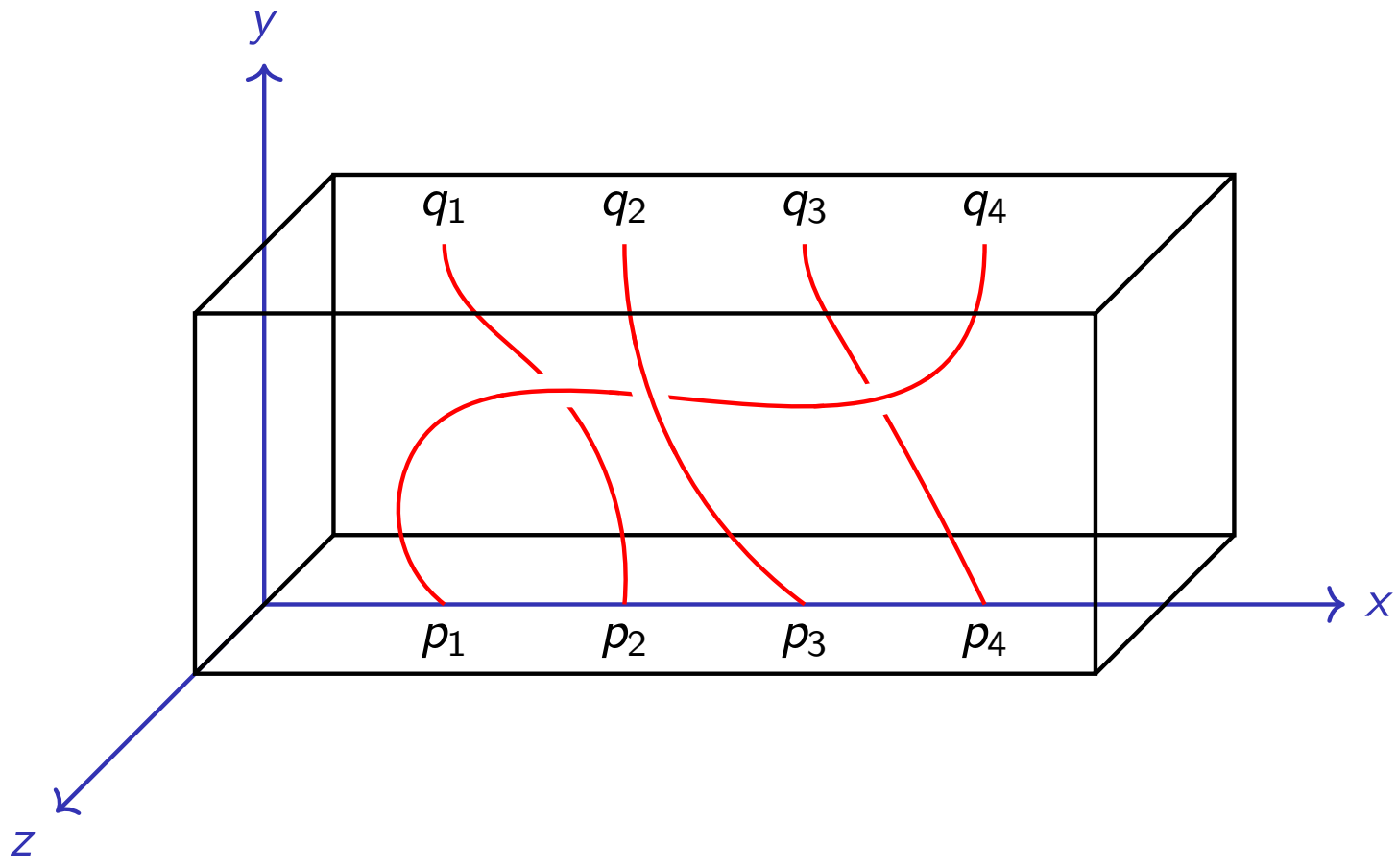


Figure: Three dimensional geometric representation of a braid

Two dimensional representation

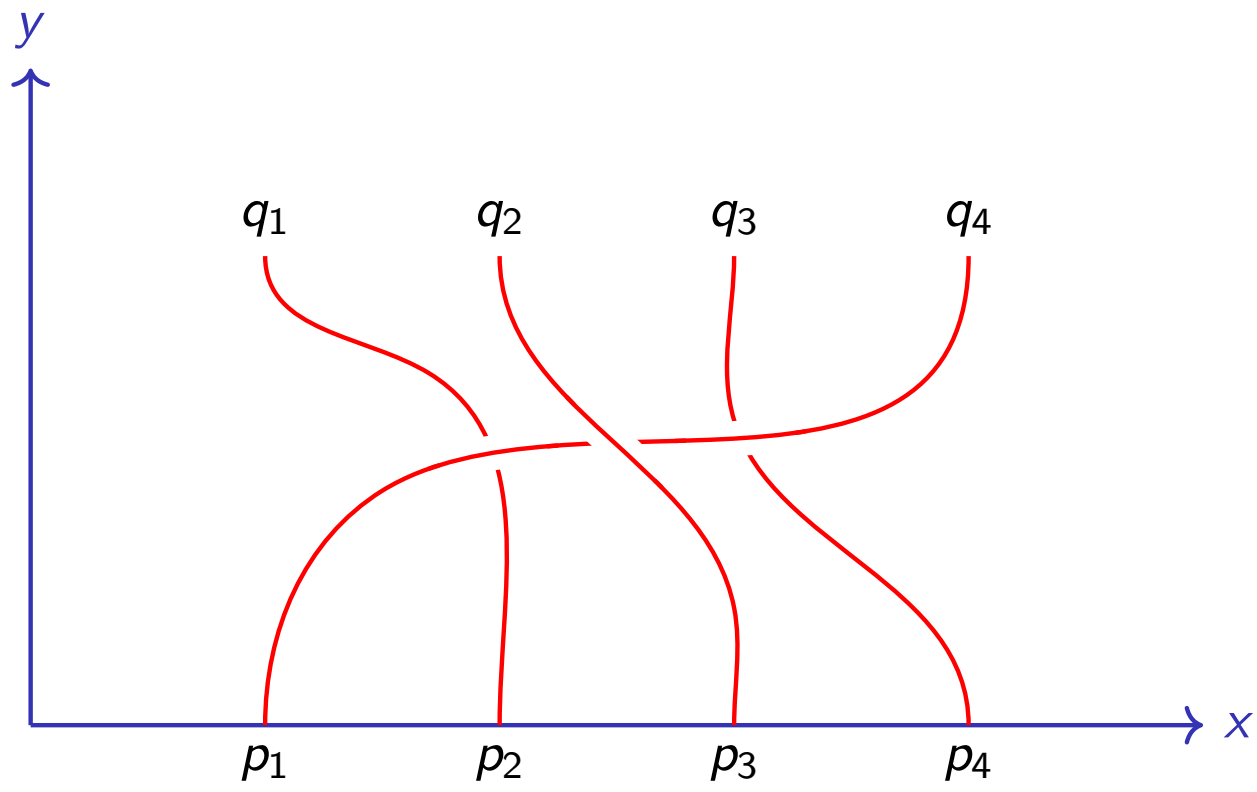


Figure: A projection of the braid

Multiplication of braids

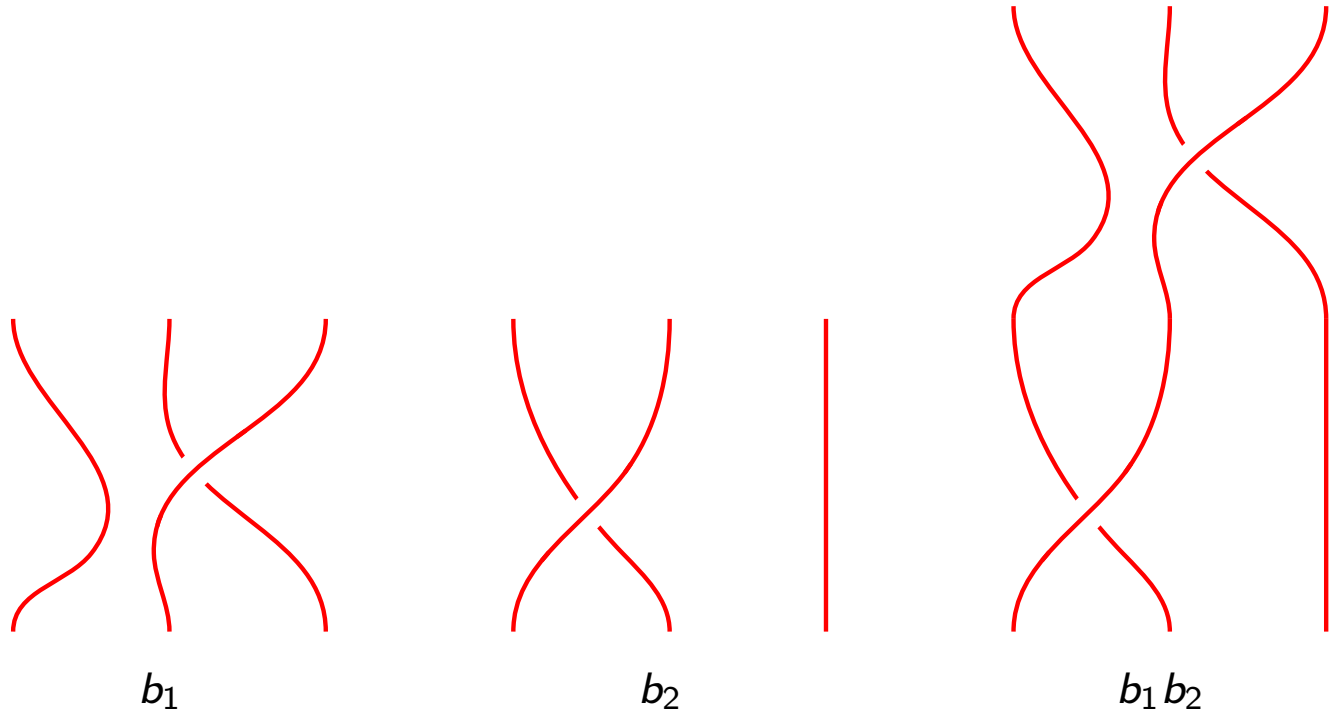


Figure: Multiplication of two braids

The identity braid $\mathbb{1}_n$

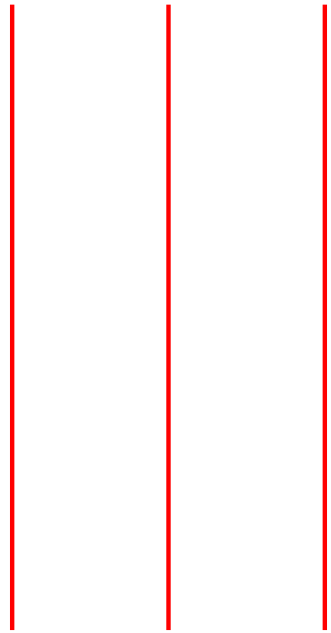


Figure: The identity $\mathbb{1}_3$

Inverse of braids

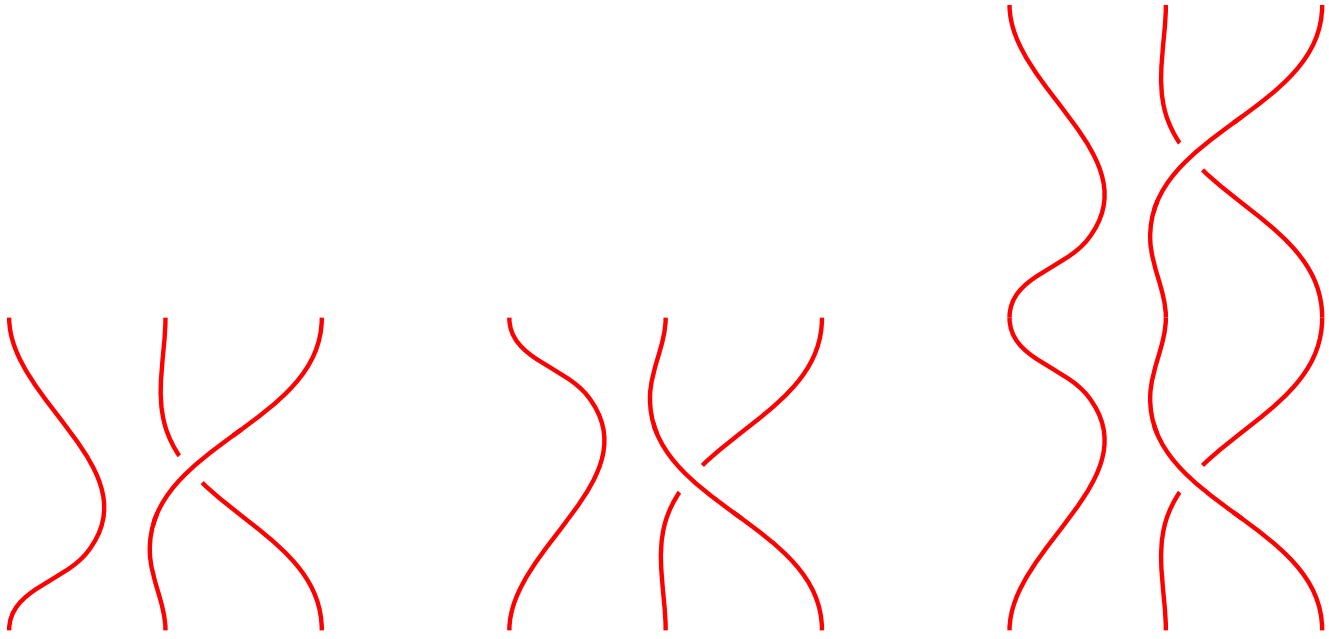


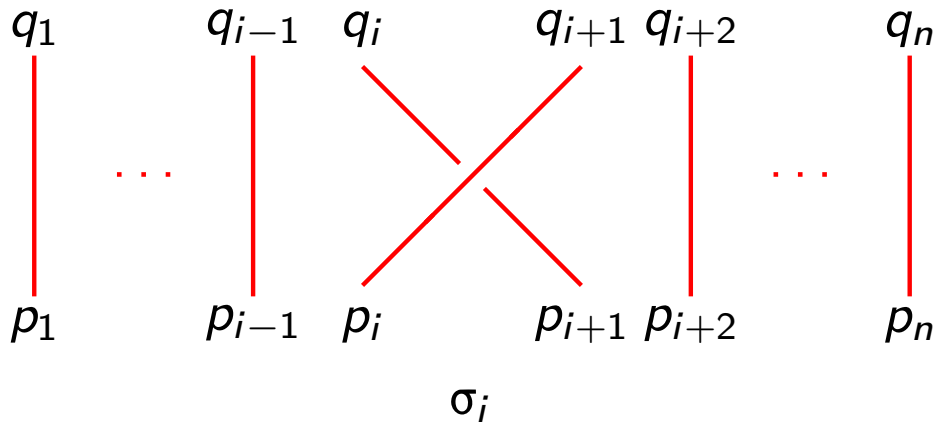
Figure: Inverse of a braid

Thus, braids form a group, known as the Artin braid group B_n.

Subsection 2

Generators and relations

Generators of the braid group



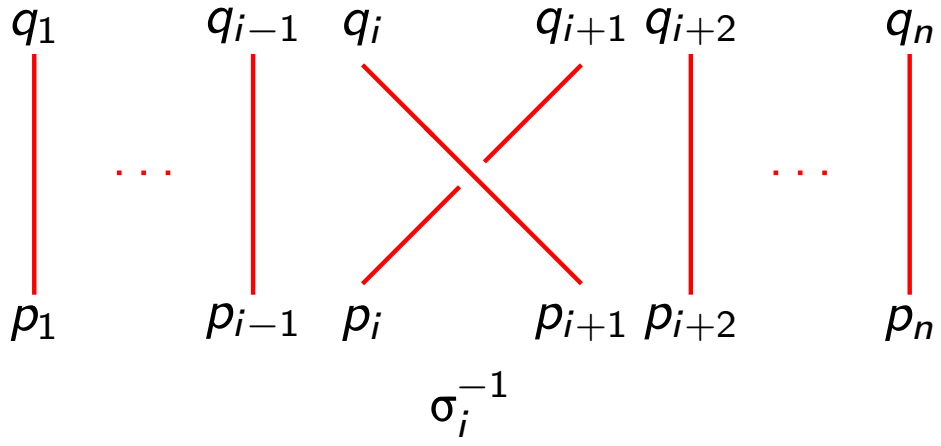


Figure: Generators σ_i and σ_i^{-1}

Type II move: $\sigma_i \sigma_i^{-1} = \mathbb{1}_n$

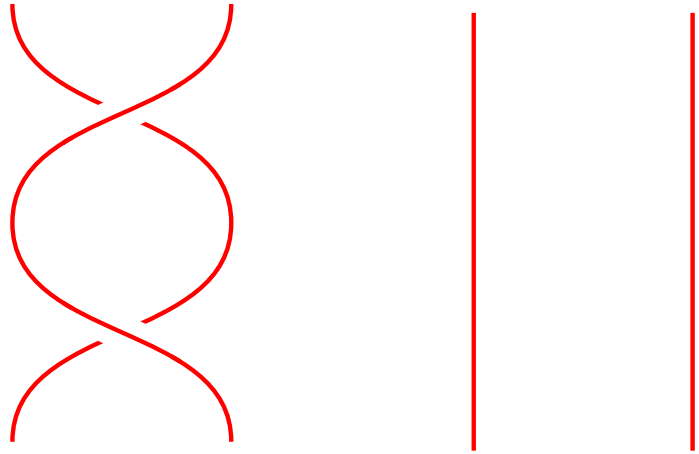
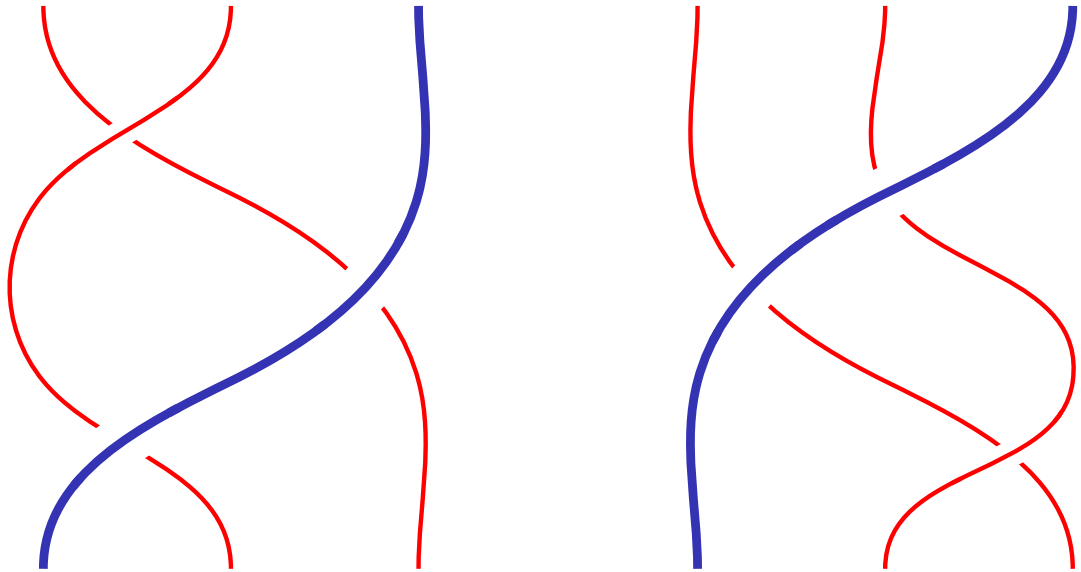


Figure: A type II move illustrating $\sigma_i \sigma_i^{-1} = \mathbb{1}_n$

Type III move: $\sigma_i\sigma_{i+1}\sigma_i = \sigma_{i+1}\sigma_i\sigma_{i+1}$



$\sigma_i\sigma_{i+1}\sigma_i$ $\sigma_{i+1}\sigma_i\sigma_{i+1}$

Figure: A type III move illustrating $\sigma_i\sigma_{i+1}\sigma_i = \sigma_{i+1}\sigma_i\sigma_{i+1}$

Sliding of crossings: $\sigma_i\sigma_j = \sigma_j\sigma_i$

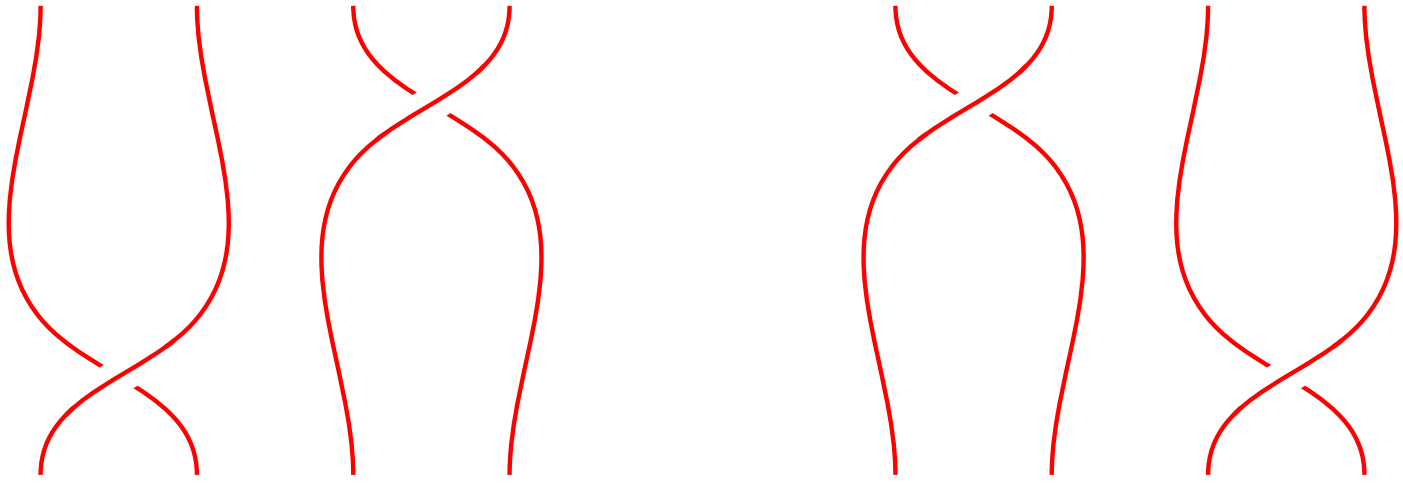


Figure: Sliding of crossings illustrating $\sigma_i\sigma_j = \sigma_j\sigma_i$

Subsection 3

Algebraic definition

Presentation of the braid group

The Artin braid group B_n admits the following presentation on the generators σ_i, for $1 \leq i \leq n-1$.

$$B_n = \left\langle \sigma_1, \ldots, \sigma_{n-1} \;\middle|\; \begin{array}{rcll} \sigma_i \sigma_i^{-1} & = & \mathbb{1}_n & \\ \sigma_i \sigma_{i+1} \sigma_i & = & \sigma_{i+1} \sigma_i \sigma_{i+1} & \text{if } i+1 \leq n-1 \\ \sigma_i \sigma_j & = & \sigma_j \sigma_i & \text{if } |i-j| \geq 2 \end{array} \right\rangle$$

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Section 3

Closure

Subsection 1

Closure of a braid

Closure of a braid $\overline{b}$

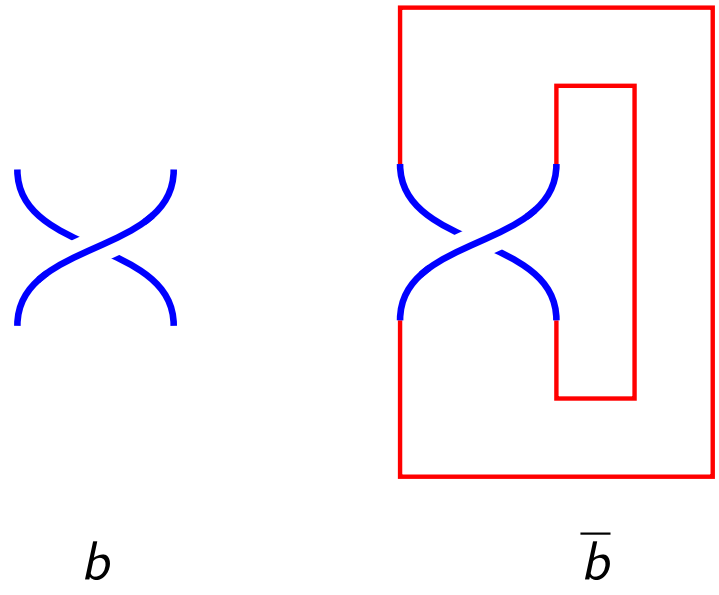

Figure: Closure of a braid

Braids and links

Every closure of a braid is a link.

Theorem (Alexander)

Every link is ambient isotopic to a closure of a braid.

Subsection 2

Equivalence of closures of braids

Conjugation

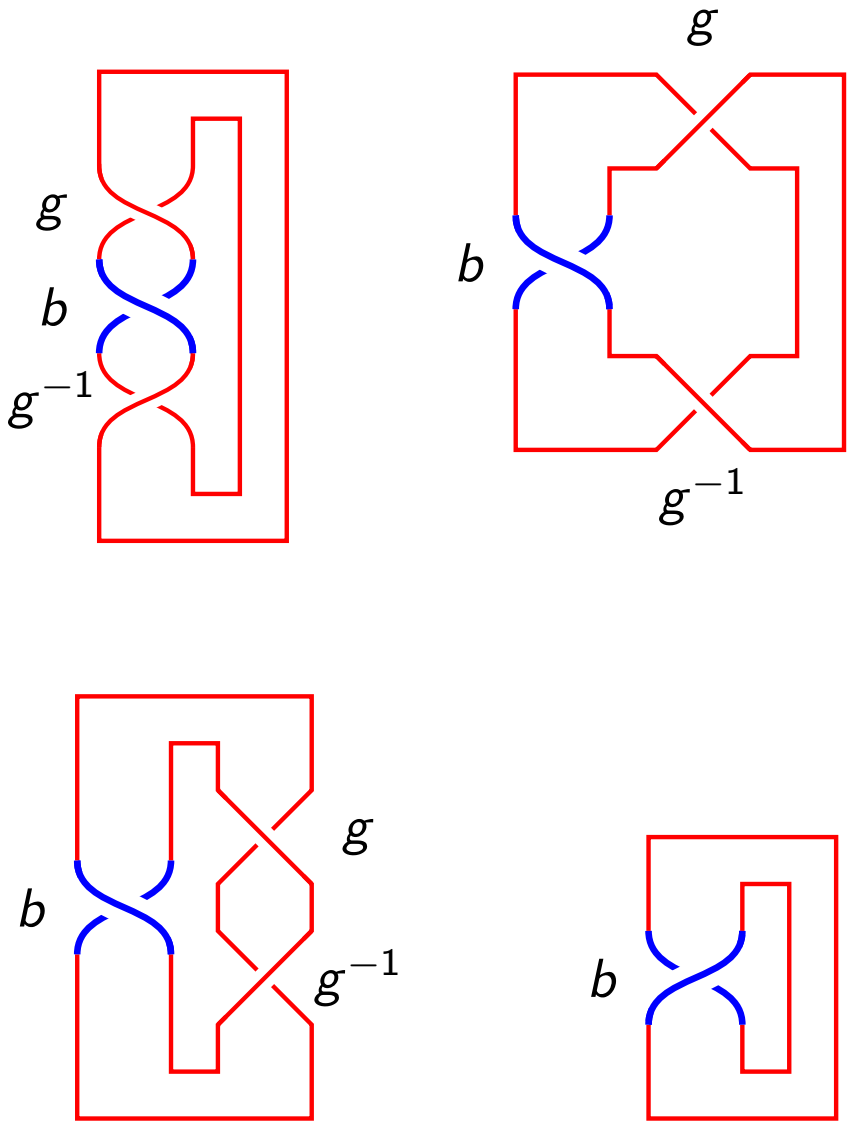

Figure: Conjugation process illustrating the link equivalence of $\overline{gbg^{-1}}$ and $\overline{b}$.