SOME TOPICS IN KNOT THEORY

A REPORT

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ABSTRACT

We shall study some selected topics in knot theory from the book *Knots and Physics*, 4th ed. by Louis Kauffman [1]. The basics of knot theory have been studied from the book *Knots and Links* by Peter Cromwell [2].

A knot is an embedding of the circle in three-dimensional space. We discuss the state model for bracket polynomial, Jones polynomial and some of its generalisations. One can apply these knot invariants to show that various knots are not ambient isotopic, which is the notion we shall use to distinguish knots. We shall see a resolution of some of the old conjectures in knot theory using the bracket polynomial. Vaughn Jones found the polynomial named after him while he was studying towers of von Neumann algebras. We shall follow a different route following Kauffman to understand the relation amongst braids, links, Temperley–Lieb algebras, which arose in statistical physics, and the Jones algebras.

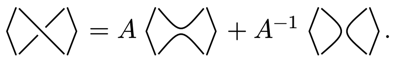
Contents

Chapter 1

The Jones polynomial and its Generalisations

In the previous chapters, we defined Kauffman's bracket polynomial using the state model and then normalised it to get a link invariant. Instead of using the state model, one can define the bracket polynomial in an axiomatic way as well. One defines the polynomial as a function on link diagrams which satisfies certain properties. We would need to check the well-definedness and invariance of such a function under the Reidemeister moves. We thus define the bracket polynomial in the following way.

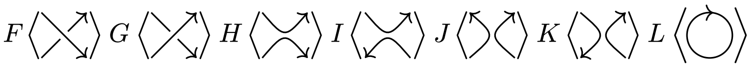
Definition 1 (Kauffman's bracket polynomial)**.** Let K an unoriented link diagram. Then the bracket $\langle K \rangle \in \mathbb{Z}[A, A^{-1}]$ is defined by the rules:

1. $\left\langle \bigcirc \right\rangle = 1.$
2. $\left\langle \bigcirc \cup K \right\rangle = (-A^2 - A^{-2})\langle K \rangle.$
3. 

This definition is consistent with the state model. $\langle K \rangle$ is invariant under the type II and type III moves. In order to gain type I invariance, we normalise it by adding an orientation and multiplying $\langle K \rangle$ with $(-A^3)^{-w(K)}$.

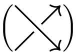
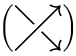
1.0.1 Jones polynomial

We now give an axiomatic definition of the Jones polynomial.



Definition 2 (Jones polynomial)**.** Let K be an oriented link diagram. Then the Jones polynomial $\mathrm{V}(K) \in \mathbb{Z}\left[\sqrt{t}, \dfrac{1}{\sqrt{t}}\right]$ is defined by the rules:

1. $\mathrm{V}\left(\bigcirc\right) = 1.$

2. $\frac{1}{t}\,\mathrm{V}\Big($  $\Big) - t\,\mathrm{V}\Big($  $\Big) = \left(\sqrt{t} - \frac{1}{\sqrt{t}}\right).$

Bibliography

[1] Louis H. Kauffman. *Knots and physics*. 4th ed. Knots and Everything 53. Singapore: World Scientific, 2013. ISBN: 978-981-4383-00-4.

[2] Peter R. Cromwell. *Knots and links*. Cambridge, UK: Cambridge University Press, 2004. ISBN: 0-521-83947-5.