Osborne's Lectures on Symmetries and Quantum Mechanics

May 21, 2024

Preface

These are lecture notes by Apoorv Potnis of the lecture series ‘Symmetries and Quantum Mechanics’, given by **Prof. Tobias J. Osborne** in 2023 at the Leibniz Universität Hannover. Prof. Osborne discusses the basics of the representation theory of groups in the context of quantum mechanics in this short lecture series. The video lecture series is available at https://youtube.com/playlist?list=PLDfPUNusx1ErdQhrdAzincNJKgTQahsX_&feature=shared.

The source code, updates and corrections to this document can be found on this GitHub repository: https://github.com/apoorvpotnis/osborne_symmetries. The source code is embedded in this PDF. Comments and corrections can be mailed at apoorvpotnis@gmail.com.

Contents

Chapter 1

Basics, Wigner's theorem and linear representation of groups

1.1 Prerequisites and references

We assume that the reader has a working knowledge of linear algebra and is familiar with basic ideas of quantum mechanics. Mainly, one needs the knowledge of the postulates of quantum mechanics, which can be learnt from Prof. Osborne's video lecture on YouTube titled 'Quantum mechanics essentials: Everything you need for quantum computation' [1].

The main reference book this lecture course series is based on is the famous book of Serre.

> Jean-Pierre Serre. *Linear Representations of Finite Groups.* Graduate Texts in Mathematics 42. Springer-Verlag, New York Inc., 1977. ISBN: 978-1-4684-9460-0. Translated from the French by Leonard L. Scott.

The first part of this book is based on lectures given by the eminent mathematician Jean-Pierre Serre to a group of quantum chemists. One may look at the first quantum field theory volume of Weinberg, but this is too advanced for present purposes and one would have great difficulties reading it.

> Steven Weinberg. *The Quantum Theory of Fields: Foundations.* Vol. 1. Cambridge University Press, Cambridge, 2005. ISBN: 978-0-521-55001-7.

1.2 Postulates of quantum mechanics

We briefly state the postulates of quantum mechanics.

1. A Hilbert space corresponds to every quantum mechanical system.
2. The states of quantum mechanical systems are represented by density matrices.
3. The measurements or detectors are represented by positive operators.
4. Börn rule.
5. Schrodinger's equation.
6. Tensor product for composite systems.

We shall focus on the fifth postulate, namely the Schrödinger equation. We argue that we don't actually need it. It can be derived from deeper principles.

1.3 Symmetries

A symmetry on a quantum system is a physical operation that can be performed or can occur.

Bibliography

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[4] Philip Bowers. *Lectures on Quantum Mechanics*. Cambridge University Press, Cambridge, 2020. ISBN: 978-1-108-42976-4.

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[6] Frederic Schuller, Simon Rea, and Richie Dadhley. *Lectures on the Geometric Anatomy of Theoretical Physics*. Lecture notes in .pdf format. Lecturer: Prof. Frederic Paul Schuller. 2017. URL: https://drive.google.com/file/d/1nchF1fRGSY3R3rP1QmjUg7fe28tAS428/view.

[7] Frederic Schuller. *Lectures on the Geometric Anatomy of Theoretical Physics*. Video lectures on YouTube. 2016. URL: https://www.youtube.com/playlist?list=PLPH7f_7ZlzxTi6kS4vCmv4ZKm9u8g5yic.

[8] Valter Moretti (https://physics.stackexchange.com/users/35354/valter-moretti). *Answer to the question 'Angular momentum Lie algebra for infinite-dimensional Hilbert spaces'*. Physics Stack Exchange (https://physics.stackexchange.com). (version: 2024-05-17 21:03:48Z). URL: https://physics.stackexchange.com/a/814882/81224.