The Hydrogen Atom

These are lecture notes by Apoorv Potnis of the lecture 'Das Wasserstoffatom' or 'The Hydrogen Atom', given by **Prof. Frederic Paul Schuller**, as the eighteenth lecture in the course 'Theoretische Physik 2: Theoretische Quantenmechanik' in 2014/15 at the Friedrich-Alexander-Universität Erlangen-Nürnberg. While the original lecture is in German, these notes are in English and have been prepared using YouTube's automatic subtitle translation tool. The lecture is available at https://www.youtube.com/watch?v=mcM4S3IMMvI&list=PLPO5pgr_frzTeqa_thbltYjyw8F9ehw7v&index=18 and at https://www.fau.tv/clip/id/4511.

The source code, updates and corrections to this document can be found on this GitHub repository: https://github.com/apoorvpotnis/schuller_hydrogen. The source code, along with some other files, is embedded in this PDF. Comments and corrections can be mailed at apoorvpotnis@gmail.com or opened as an issue in the GitHub repository. This PDF was compiled on February 2, 2025.

In these notes, we assume that the reader is already familiar with all of the material covered in Schuller's lectures in his English quantum mechanics series [1, 2]. We shall make frequent use of concepts and results from these lectures without mentioning.

Contents

1 Hydrogen-like systems

Hydrogen-like systems are systems consisting of a central positive nuclear charge and a single electron. Let the central nuclear charge be of magnitude $+Z$, the mass of the nucleus be m_{N} and the mass of the electron be m_{e}. The reduced mass of the system is then $\mu = m_{\mathrm{N}} m_{\mathrm{e}}/m_{\mathrm{N}} + m_{\mathrm{e}}$.

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