

# The Hydrogen Atom

These are lecture notes by Apoorv Potnis of the lecture ‘Das Wasserstoffatom’ or ‘The Hydrogen Atom’, given by **Prof. Frederic Paul Schuller**, as the eighteenth lecture in the course ‘Theoretische Physik 2: Theoretische Quantenmechanik’ in 2014/15 at the Friedrich-Alexander-Universität Erlangen-Nürnberg. While the original lecture is in German, these notes are in English and have been prepared using YouTube’s automatic subtitle translation tool. The lecture is available at [https://www.youtube.com/watch?v=mcM4S3IMMvI&list=PLP05pgr\\_frzTeqa\\_thbltYjyw8F9ehw7v&index=18](https://www.youtube.com/watch?v=mcM4S3IMMvI&list=PLP05pgr_frzTeqa_thbltYjyw8F9ehw7v&index=18) and at <https://www.fau.tv/clip/id/4511>.

The source code, updates and corrections to this document can be found on this GitHub repository: [https://github.com/apoorvpotnis/schuller\\_hydrogen](https://github.com/apoorvpotnis/schuller_hydrogen). The source code, along with some other files, is embedded in this PDF. Comments and corrections can be mailed at [apoorvpotnis@gmail.com](mailto:apoorvpotnis@gmail.com) or opened as an issue in the GitHub repository. This PDF was compiled on February 2, 2025.

In these notes, We expect that the reader is familiar the previous lecture, ‘Quantenmechanisches Zweikörperproblem’ or ‘The Quantum Mechanical Two-body Problem’. The notes of this lecture can be found here: [https://github.com/apoorvpotnis/schuller\\_two-body\\_problem](https://github.com/apoorvpotnis/schuller_two-body_problem). We additionally assume that the reader is already familiar with all of the material covered in Schuller’s lectures in his English quantum mechanics series [1, 2]. We shall make frequent use of concepts and results from these lectures without mentioning.

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# 1 Hydrogen-like systems

Hydrogen-like systems are systems consisting of a central positive nuclear charge and a single electron. Let the central nuclear charge be of magnitude  $+Z$ , the mass of the nucleus be  $m_N$  and the mass of the electron be  $m_e$ . The electron has unit negative charge in our units. The reduced mass  $\mu$  of the system is then

$$\mu = \frac{m_N m_e}{m_N + m_e}.$$

## References

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