The Quantum Mechanical Two-body Problem

These are lecture notes by Apoorv Potnis of the lecture 'Quantenmechanisches Zweikörperproblem' or 'The Quantum Mechanical Two-body Problem', given by **Prof. Frederic Paul Schuller**, as the seventeenth lecture in the course 'Theoretische Physik 2: Theoretische Quantenmechanik' in 2014/15 at the Friedrich-Alexander-Universität Erlangen-Nürnberg. While the original lecture is in German, these notes are in English and have been prepared using YouTube's automatic subtitle translation tool. The lecture is available at https://www.youtube.com/watch?v=mcM4S3IMMvI&list=PLPO5pgr_frzTeqa_thbltYjyw8F9ehw7v&index=17 and at https://www.fau.tv/clip/id/44891.

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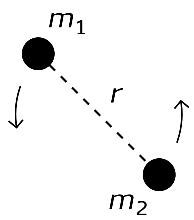
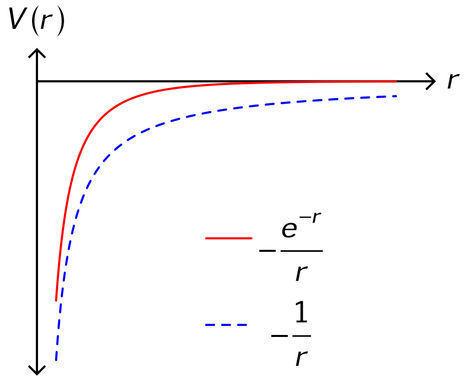
1 Introduction

In this lecture, we shall consider a quantum-mechanical system consisting of two interacting particles of masses m_1 and m_2, such that the interaction



Classical picture



is completely determined by the potential $V(r)$, and the potential depends only on the distance r between the particles. An example of such a potential would be the *Yukawa potential*, defined as

$$V_{\text{Yukawa}}(r) := a\frac{\exp(-kmr)}{r},$$

where k, m and a are constants. $a \in \mathbb{R} \setminus \{0\}$, $m \geq 0$. According to quantum field theory, very roughly speaking, interaction between particles takes place via a 'mediating particle'. If the interaction is mediated by a 'scalar field'[1], then the mass associated to the particle of that scalar field is the mass m appearing in the Yukawa potential. If we plot a graph of the Yukawa potential for a massive scalar field, then we see that the magnitude of the potential becomes very close to zero after a certain distance. Thus, these interactions are short-ranged. If instead we have $m = 0$, corresponding to a photon, then we get the familiar long-range Coulomb potential

$$V_{\text{Coulomb}}(r) := a\frac{1}{r}.$$

We also have the finite wall potential $V_{\text{finite wall}}(r) := a\Theta(r - r_0)$ and the isotropic harmonic oscillator $V_{\text{ihc}}(r) := ar^2$.

References

[1] Frederic Schuller, Simon Rea, and Richie Dadhley. 'Lectures on Quantum Theory'. Lecturer: Prof. Frederic Paul Schuller. 2019. URL: https://docs.wixstatic.com/ugd/6b203f_a94140db21404ae69fd8b367d9fcd360.pdf.

[1] Whatever that means

[2] Frederic Schuller. *Lectures on Quantum Theory*. 2015. URL: https://youtube.com/playlist?list=PLPH7f_7ZlzxQVx5jRjbfRGEzWY_upS5K6.

[3] Philip Bowers. *Lectures on Quantum Mechanics*. Cambridge University Press, Cambridge, 2020. ISBN: 978-1-108-42976-4.

[4] Michael Reed and Barry Simon. *Methods of Modern Mathematical Analysis I: Functional Analysis*. Revised and Enlarged editon. Vol. 1. Academic Press, Inc. London, 1980. ISBN: 978-0-080-57048-8.

The source code, updates and corrections to this document can be found on this GitHub repository: https://github.com/apoorvpotnis/schuller_two-body_problem. The source code, along with the .bib file is embedded in this PDF. Comments and corrections can be mailed at apoorvpotnis@gmail.com.