The Quantum Mechanical Two-body Problem

These are lecture notes by Apoorv Potnis of the lecture 'Quantenmechanisches Zweikörperproblem' or 'The Quantum Mechanical Two-body Problem', given by **Prof. Frederic Paul Schuller**, as the seventeenth lecture in the course 'Theoretische Physik 2: Theoretische Quantenmechanik' in 2014/15 at the Friedrich-Alexander-Universität Erlangen-Nürnberg. While the original lecture is in German, these notes are in English and have been prepared using YouTube's automatic subtitle translation tool. The lecture is available at https://www.youtube.com/watch?v=mcM4S3IMMvI&list=PLPO5pgr_frzTeqa_thbltYjyw8F9ehw7v&index=17 and at https://www.fau.tv/clip/id/44891.

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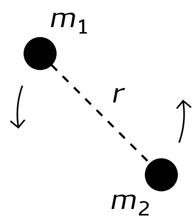
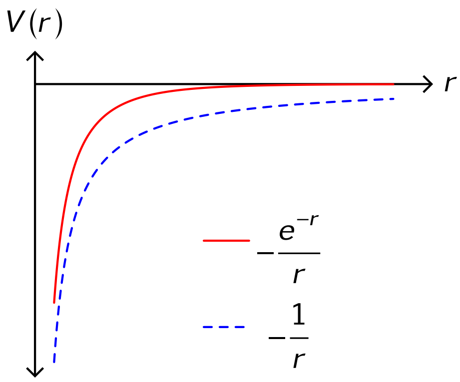
1 Introduction

Consider a classical system system consisting of two interacting particles of masses m_1 and m_2, such that the interaction is completely determined by the



Classical picture



Yukawa and Coulomb potentials

potential $V(r)$, and the potential depends only on the distance r between the particles. If we consider the central force problem, like a planet orbiting the sun, then we know we have bound states corresponding to *all* the negative energy values, and scattering states corresponding to all the positive energy values. It turns out that if consider the system quantum-mechanically, then bound states cannot admit all the negative energy solutions. Only certain energy levels are allowed. This can be seen experimentally from the discrete lines in the spectra of atoms, demonstrating the quantised nature of energy levels. In this lecture, we shall consider the quantum-mechanical case of the two-body problem.

An example of a potential as described above would be the *Yukawa potential*, defined as

$$V_{\text{Yukawa}}(r) := a\frac{\exp(-kmr)}{r},$$

where k, m and a are constants. $a \in \mathbb{R} \setminus \{0\}$, $m \geq 0$. According to quantum field theory, very roughly speaking, interaction between particles takes place via a 'mediating particle'. If the interaction is mediated by a 'scalar field'[1], then the mass associated to the particle of that scalar field is the mass m appearing in the Yukawa potential. If we plot a graph of the Yukawa potential for a massive scalar field, then we see that the magnitude of the potential becomes very close to zero after a certain distance. Thus, these interactions are short-ranged. If instead we have $m = 0$, corresponding to a photon, then we get the familiar long-range Coulomb potential

$$V_{\text{Coulomb}}(r) := a\frac{1}{r}.$$

[1]Whatever that means

We also have the finite wall potential $V_{\text{finite wall}}(r) := a\Theta(r - r_0)$ and the isotropic harmonic oscillator $V_{\text{ihc}}(r) := ar^2$.

References

[1] Frederic Schuller, Simon Rea, and Richie Dadhley. 'Lectures on Quantum Theory'. Lecturer: Prof. Frederic Paul Schuller. 2019. URL: https://docs.wixstatic.com/ugd/6b203f_a94140db21404ae69fd8b367d9fcd360.pdf.

[2] Frederic Schuller. *Lectures on Quantum Theory*. 2015. URL: https://youtube.com/playlist?list=PLPH7f_7ZlzxQVx5jRjbfRGEzWY_upS5K6.

[3] Philip Bowers. *Lectures on Quantum Mechanics*. Cambridge University Press, Cambridge, 2020. ISBN: 978-1-108-42976-4.

[4] Michael Reed and Barry Simon. *Methods of Modern Mathematical Analysis I: Functional Analysis*. Revised and Enlarged editon. Vol. 1. Academic Press, Inc. London, 1980. ISBN: 978-0-080-57048-8.

The source code, updates and corrections to this document can be found on this GitHub repository: https://github.com/apoorvpotnis/schuller_two-body_problem. The source code, along with the .bib file is embedded in this PDF. Comments and corrections can be mailed at apoorvpotnis@gmail.com.