CMO

Sheet 3 — E0230

Assignment (Due: 31 October 2021)

Instructions

- Use Python for coding
- Please name the code files for question1 and question2 as question1.py and question2.py respectively
- Prepare a report in pdf format. In this report, provide your answers to the questions and instruction to run code
- Compress your folder containing code and report to a **zip file**. (Please do not use .rar format)
- Rename .zip extension to .pdf extension and upload via Microsoft Teams Form
- Late submissions will be penalised

1. *Conjugate Gradient.* Implement the conjugate gradient algorithm to solve the system of linear equations $Ax = b$, where (i, j)-th entry in $A \in R^{(n\times n)}$ is $\frac{1}{i+j-1}$ and all the coordinates in $b \in R^n$ are 1. Start the algorithm from zero vector. For $n = 5, 8, 12, 20$ report the number of iterations required to reduce the error $||Ax - b||_2$ below 10^{-6}. For $n = 20$, plot the error (log scale) vs iteration (10 points).

2. *Quasi Newton.* Consider the function $f(x) = \frac{1}{2}x^T Qx$ where Q is given by

$$\begin{bmatrix} a+10 & 0 & 0 & 0 & 0 & 0 \\ 0 & a+8 & 0 & 0 & 0 & 0 \\ 0 & 0 & a+6 & 0 & 0 & 0 \\ 0 & 0 & 0 & a+4 & 0 & 0 \\ 0 & 0 & 0 & 0 & a+2 & 0 \\ 0 & 0 & 0 & 0 & 0 & a \end{bmatrix}$$

   where a = <your SR number>% 100 (% is the remainder operator). For example if last 5 digits of your SR number is 12345, then $a = 45$. Starting at $x_0 = [10, 10, 10, 10, 10, 10]$, minimize the function $f(x)$ iteratively using the following methods.

   (a) (5 points) Implement steepest descent

   (b) (5 points) Implement DFP with exact line search

   (c) (5 points) Implement DFP with backtracking line search. Backtracking is a form of inexact line search in which a step size is determined at each step which satisfies the Armio-Goldstein condition. Given constants $\alpha, \beta \in (0, 1)$, at each step of the algorithm, if the current point is $x \in \mathbb{R}^d$, the direction of line search is chosen as $u = -\nabla f(x)$, and for determining the step size, an initial step size $t = 1$ is chosen and is repeatedly updated as $t \leftarrow \beta t$ until $f(x + tu) \leq f(x) + \alpha t \nabla f(x)^T u$ and then x is updated as $x \leftarrow x + tu$. Once the update distance $||tu||_2$ for the point x becomes less than ϵ during any epoch, the algorithm is stopped.Apply backtracking line search algorithm with $\alpha = 0.5$, $\beta = 0.5$ and $\epsilon = 10^{-7}$.

   Prepare table as follows, with function value in each iteration:

Iterations	Steepest Descent	DFP with exact line search	DFP with backtracking line search
1			
2			
..			
6			

Table 1: Function value, $f(x)$