E0 230
Computational Methods in Optimization
Assignment 1

August 2021

Instructions:

- This is an assignment, and **all work submitted must be your own**!
- Attempt all questions
- You have one week to submit your answers
- All code must be submitted.
- All long answers **must be submitted in a single PDF.**
- For algorithmic questions, a teams form with slots for the answers will be uploaded 24 hours before the submission deadline. If you are asked to provide a number, enter it into the teams form. You answer should be correct to 3 decimal places unless stated otherwise.
- Both your code and your PDF must be submitted in a single zip file, which should be called **student_name_cmo21assn1.zip**.
- Choose the files required for your setup from the concerned directory in the zip file
- For the numericals, you need to call the executables we have created from your script.
- If using python, you can choose use the following command: (Please copy the below carefully and do debugging checks to ensure you're reading the output correctly)
```
someVar = subprocess.run(["filename", "args"],
          stdout=subprocess.PIPE).stdout.decode("utf-8")
```
- In case using linux or mac ensure that you add chmod permissions for executing the file code.
- For mac use the following instructions:
  - Run `chmod 777 <executable name>`
  - You may need to do: `Preferences > Security and Privacy > General` and allow the executable (app) to run (`click "allow"`).
- For MATLAB, use the following instructions:
  - Run `[status,cmdout] = system("filename args")` as described in the problem statement.
  - Convert `cmdout` to floating point and use the output thereof.

1. (5 points) Consider the polynomial

$$p(x, y, z) = x^4y^2 + x^2y^4 + z^6 - 3x^2y^2z^2.$$

Show that $f^* = \inf_{x,y,z} p(x, y, z) = 0$.

2. (15 points) Suppose $f \in C_L^1$, where $L > 0$; that is,

$$\|\nabla f(x) - \nabla f(y)\|_2 \leq L\|x - y\|_2.$$

If $f \in C_L^1$, show that the functions

$$g(x) = \frac{L}{2}x^T x - f(x) \quad \text{and} \quad h(x) = \frac{L}{2}x^T x + f(x)$$

are convex. Then, show that

$$-\frac{L}{2}\|y - x\|^2 \leq f(y) - f(x) - \langle \nabla f(x), y - x\rangle \leq \frac{L}{2}\|y - x\|^2.$$

3. (10 points) You are each given 100 pairs of data points (x_i, y_i), where $x_i \in \mathbb{R}^5$ and $y_i \in \mathbb{R}$. We know that the data is generated by the equation

$$y_i = w^\top x_i + b.$$

Using the provided data, find w that minimizes the least squares error between y_i and $w^\top x_i + b$. Furthermore, for the general case where $x \in \mathbb{R}^n$ and we are given m data points, what is the closed form solution to this problem? Is this solution unique?

Suppose the number of linearly independent data points is less than n - how would you solve this problem, and is the solution unique?

Enter the value of w obtained with precision upto 2 decimal places, and your long form solutions along with any derivations in the PDF.

> Instructions: Call the function through any script (say in python or matlab) as follows:
>
> $\triangleright getDataPoints.exe\ SRNumber$
> $For example:$
> $\triangleright\ getDataPoints.exe$ 10598
>
> You will be returned 100 (x,y) pairs in the format
> $\triangleright y$,[x$_1, x_2, x_3, x_4, x_5$]. For example:
> $\triangleright 1.25, [-1.05, 0.93, 0.34, 0.79, -0.25]$
> $-0.55, [-1.01, 0.63, 0.74, 0.81, -0.45]$
> ...
> $0.78, [-0.45, 0.36, 0.74, 0.95, 0.85]$
>
> You will have to read these 100 lines through some program and output the right w value. Note that the data points for each of you is different.

4. (10 points) You are each given a grey box which takes $x \in \mathbb{R}^2$ and returns $f(x)$. You know that $f(x) = x^T Ax + b^T x$, where $b = [1, 1]^T$ but A is unknown. You may use the grey box to evaluate **only** 1000 points. "Estimate" whether $f(x)$ has a global minimum or not by estimating a single real number and checking it's sign. State what that number is.

   > Instructions: Call the function through any script (say in python or matlab) as follows:
   >
   > $\triangleright$*getFuncValue.exe SRNumber*,[x_1, x_2]
   > For example:
   > $\triangleright$*getFuncValue.exe* 10598,[10,10]
   >
   > You will be returned the value of the function at the point in the format. For example: $\triangleright$ 1.25

5. (10 points) You are each given a black box function which returns $f(x)$ and $\nabla f(x)$. It is not known if $f(x)$ is convex or coercive. Use the following iteration to try and find a minimum:

$$x_{k+1} = x_k - \frac{1}{k+1} \nabla f(x_k).$$

   Question: Starting at $x_0 = [10, 10, 10]$, how many iterations will it take until you reach an ε-approximate point if (a) $\varepsilon = 0.01$, (b) $\varepsilon = 0.001$, and (c) $\varepsilon = 0.0001$? Based on your answers can you say anything about the function? If you are unable to converge, please explain why.

   > Instructions: Call the function through any script (say in python or matlab) as follows:
   >
   > $\triangleright$*getGradient.exe SRNumber*,[x_1, x_2, x_3]
   > For example:
   > $\triangleright$*getGradient.exe* 10598,[10,10,10]
   >
   > You will be returned the value of the function at the point and the gradient in the format
   > $\triangleright$*functionValue*,[$\nabla f(x)_1, \nabla f(x)_2, \nabla f(x)_3$]. For example: $\triangleright$ $1.25, [-1.05, 0.93, 0.34]$