General problem formulation

Let's consider a rod loosing heat to the surroundings. One end of the rod is kept at 373 K, while the other end is kept at 298 K. The problem can be formulated in one dimension as:

$$\frac{\partial T}{\partial t} = k\frac{\partial^2 T}{\partial x^2} - h(T - T_{ext}) \tag{1}$$

where T_{ext} is the temperature of the surrounding air.

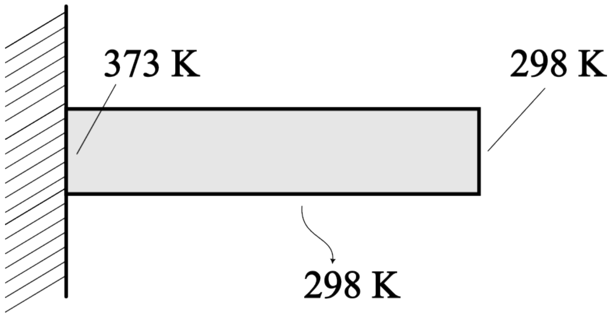

Figure 1: Heat flow problem

1D steady-state heat diffusion

Let's first consider the steady-state formulation of the problem, where Eq. 1 simplifies to:

$$0 = k\frac{\partial^2 T}{\partial x^2} - h(T - T_{ext}). \tag{2}$$

Isolating the spatial derivative, one gets:

$$\frac{\partial^2 T}{\partial x^2} = \frac{h}{k}(T - T_{ext}). \tag{3}$$

The LHS of Eq. 3 can be discretized by means of a central difference scheme (CDS), as:

$$\frac{T_{k-1} - 2T_k + T_{k+1}}{\Delta x^2} = \frac{h}{k}(T_k - T_{ext}). \tag{4}$$

Defining $\alpha = -\frac{h\Delta x^2}{k}$, one gets:

$$T_{k-1} - 2T_k + T_{k+1} = \alpha(T_k - T_{ext}). \tag{5}$$

Eq. 5 must be rearranged to bring the unknown T_k to the LHS:

$$T_{k-1} - (2+\alpha)T_k + T_{k+1} = -\alpha T_{ext}. \tag{6}$$

Finally, Eq. 6 can be assembled into a tri-diagonal linear system.

1D transient heat diffusion

See Video

1 2D transient heat diffusion (FDTD)

See Slides