numerical method order - p

$$err = M h^p \quad , \quad h = \Delta x, \Delta y$$

if h_1, h_2

$$err_1 = M h_1^p$$

$$err_2 = M h_2^p$$

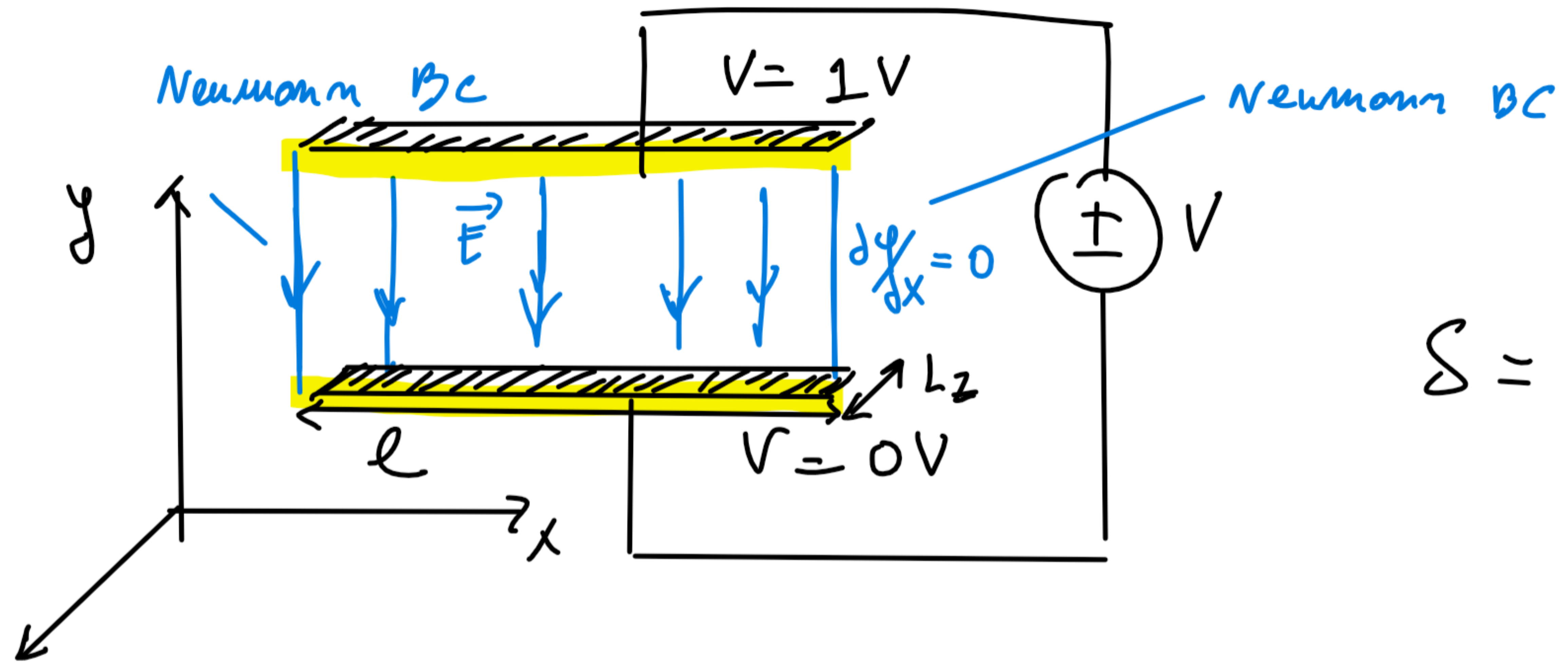
$$\frac{err_1}{err_2} = \left(h_1 / h_2 \right)^p$$

$$\ln\left(\frac{err_1}{err_2}\right) = \ln\left[\left(\frac{h_1}{h_2}\right)^p\right] = p \ln\left(h_1 / h_2 \right)$$

$$p = \frac{\ln\left(\frac{err_1}{err_2}\right)}{\ln\left(h_1 / h_2 \right)}$$ $\Rightarrow$ compute order of convergence method

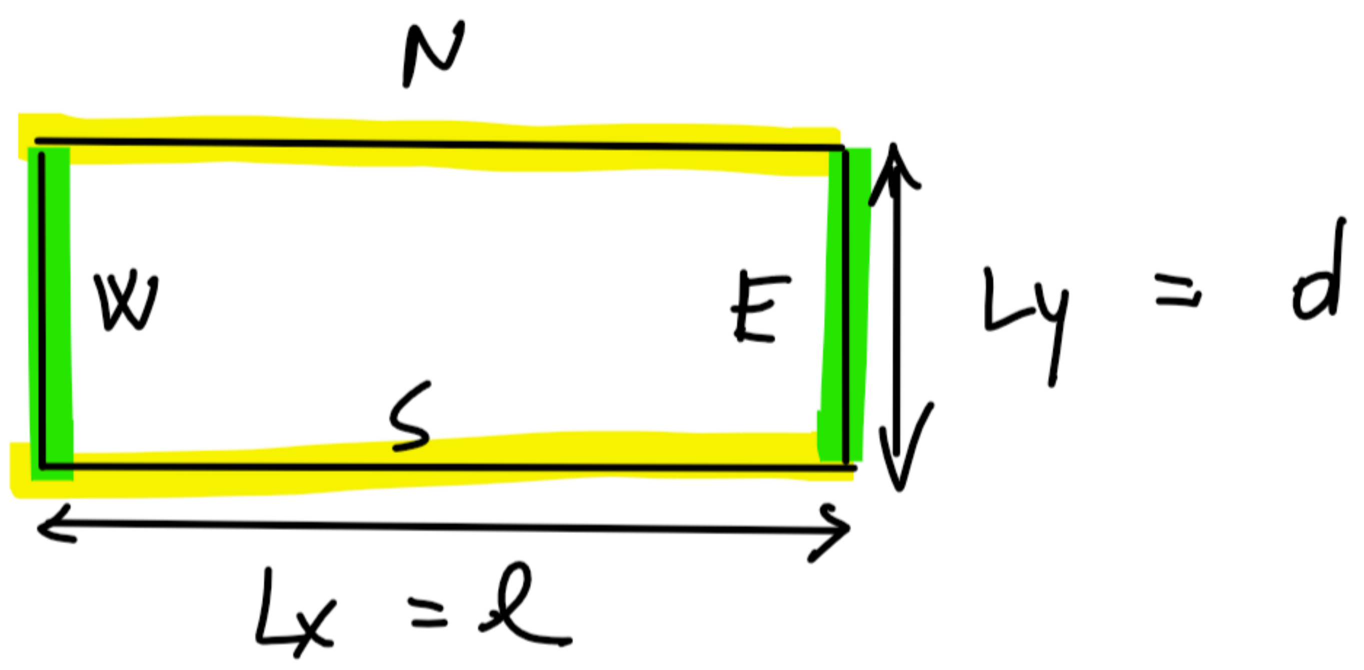
→ $\boxed{\text{TRY TO VERIFY this with the code !}}$

Homework



$\boxed{L_z = 1}$

$S = \ell \cdot L_z$



OPTION 1: $S = \ell \cdot L_z \quad [m^2]$

SAME RESULT

$$W = \int \frac{1}{2} |E^2| \, dV$$

dV: dx dy dz (dz: 1)

OPTION 2: $S = \ell$

$$W = \int \frac{1}{2} |E^2| \, dS$$

dS: dx dy

to compute electric field $E = -\nabla\varphi$

x-component:

$$E_{x_{i,j}} = - \frac{\varphi_{i+1,j} - \varphi_{i-1,j}}{2\Delta x}$$

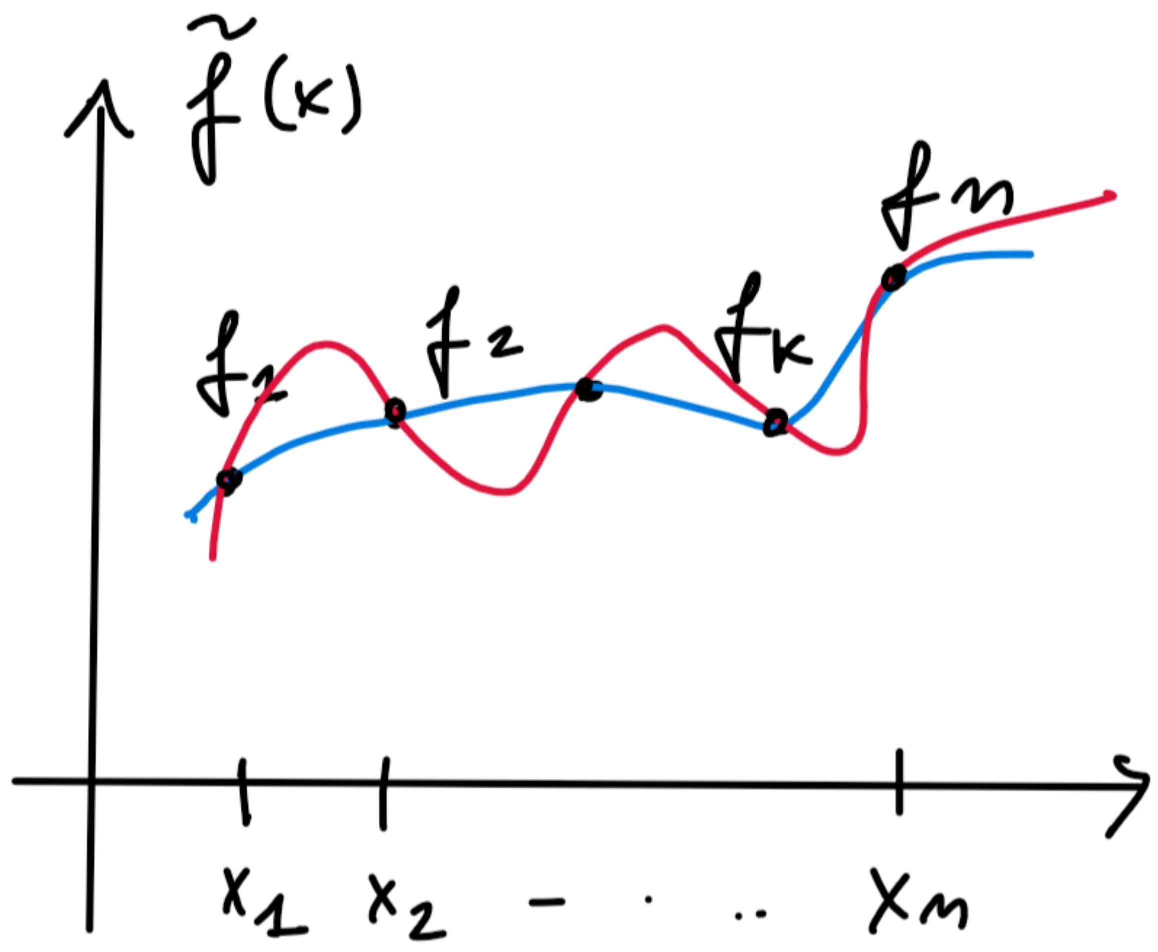
$$E_{y_{i,j}} = - \frac{\varphi_{i,j+1} - \varphi_{i,j-1}}{2\Delta y}$$

$$|E_{i,j}| = \sqrt{E_{x_{i,j}}^2 + E_{y_{i,j}}^2}$$

Interpolation

Def: process of constructing an approximate function $\tilde{f}(x)$ that ESTIMATES values of $f(x)$ given a set of known values of $f(x)$

$$[\, x_1, f(x_1) \,;\ x_2, f(x_2) \,;\ \cdots\ x_n, f(x_n) \,] \Rightarrow \tilde{f}(x)$$



$f_1 = f(x_1)$

$f_2 = f(x_2)$

$\vdots$

$f_K = f(x_K)$

$\vdots$

$f_n = f(x_n)$

$\tilde{f}_1(x)$

$\tilde{f}_2(x)$

$\Downarrow$

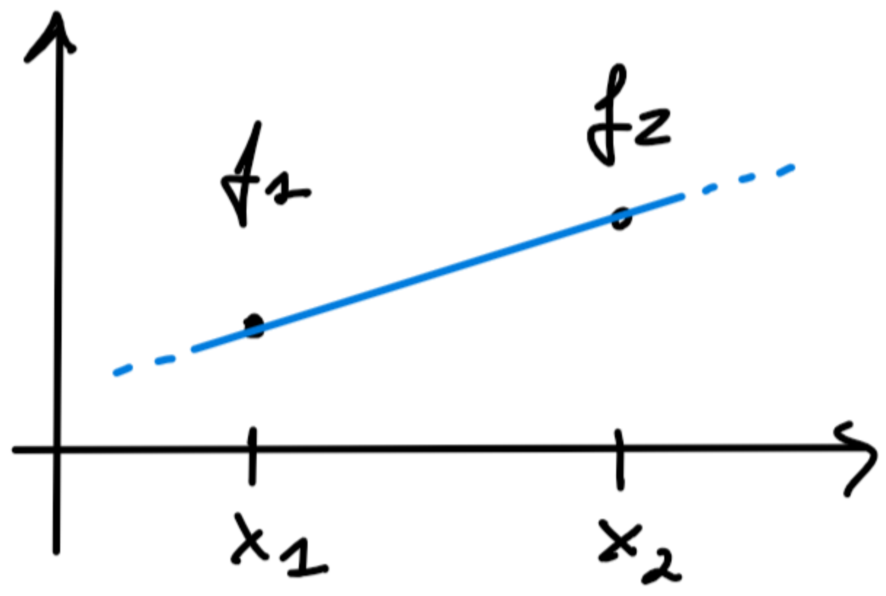
$\tilde{f}(x)$ is not UNIQUE

∞ $\tilde{f}(x)$ passing from $f_1, f_2, \dots f_n$

Polynomial Interpolation

$$\tilde{f}(x) = a_0 + a_1 x + a_2 x^2 + \cdots + a_k x^k + \cdots + a_{n-1} x^{n-1}$$

polynomial of order: $n-1$; n - data points



2 data points

$\downarrow$

straight line $\Rightarrow$ polynomial order 1

$\tilde{f}(x) = a_0 + a_1 x$