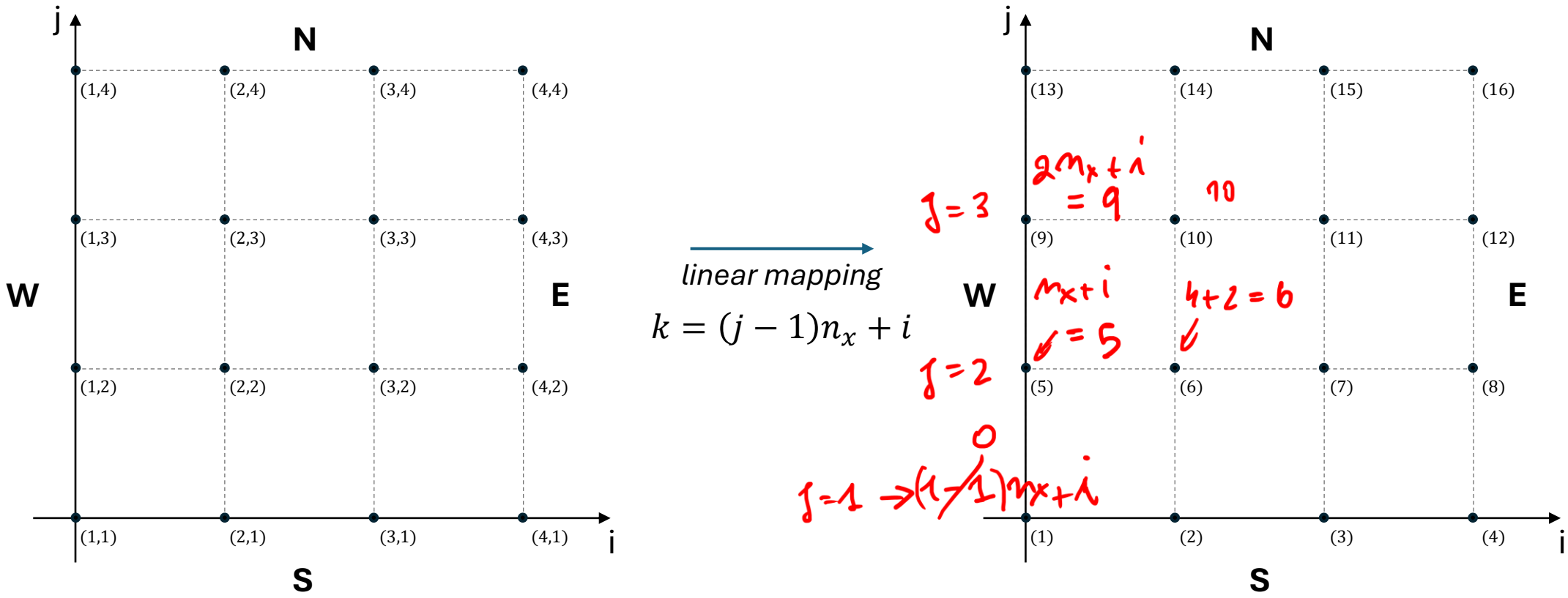
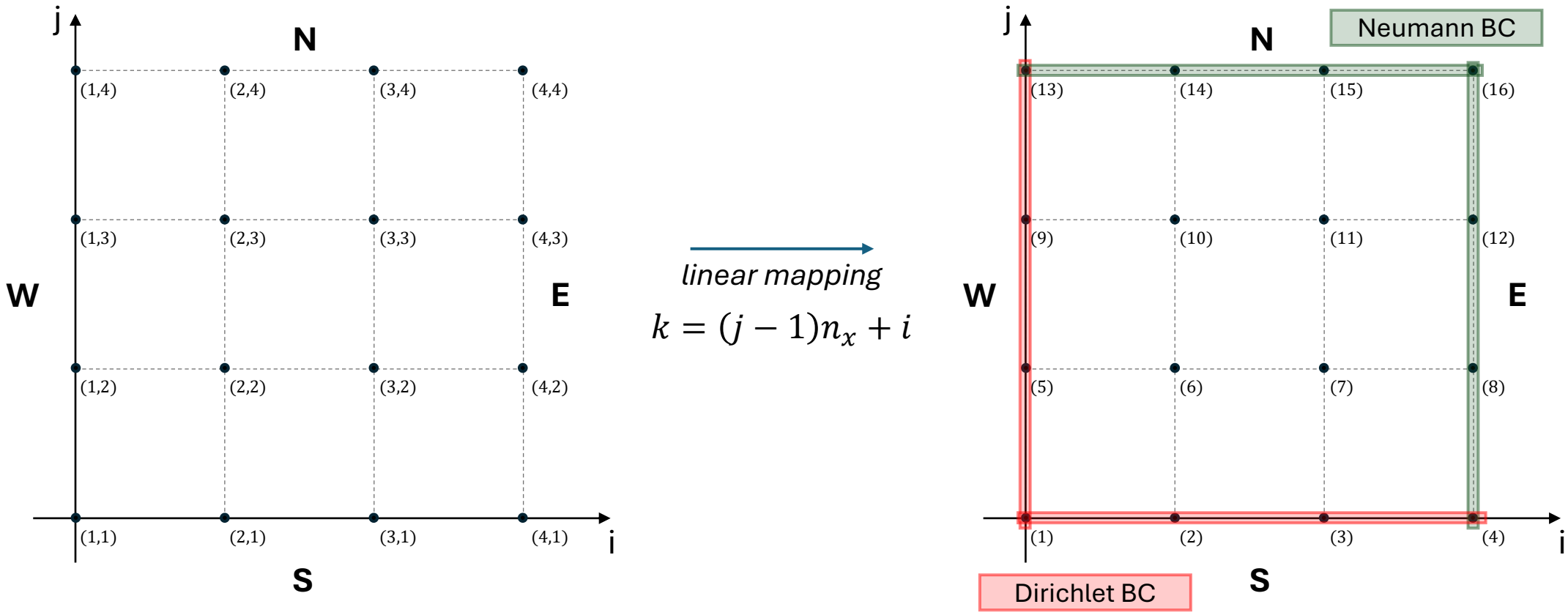
$n_x = 4$; $n_y = 4$



$[K]\{\varphi\} = \{RHS\}$ $\quad RHS, \varphi : [16 \times 1]$


j
N
(1,4)
(2,4)
(3,4)
(4,4)
(1,3)
(2,3)
(3,3)
(4,3)
W
E
(1,2)
(2,2)
(3,2)
(4,2)
(1,1)
(2,1)
(3,1)
(4,1)
i
S
linear mapping
$k = (j-1)n_x + i$
j
N
Neumann BC
(13)
(14)
(15)
(16)
(9)
(10)
(11)
(12)
W
E
(5)
(6)
(7)
(8)
(1)
(2)
(3)
(4)
i
S
Dirichlet BC

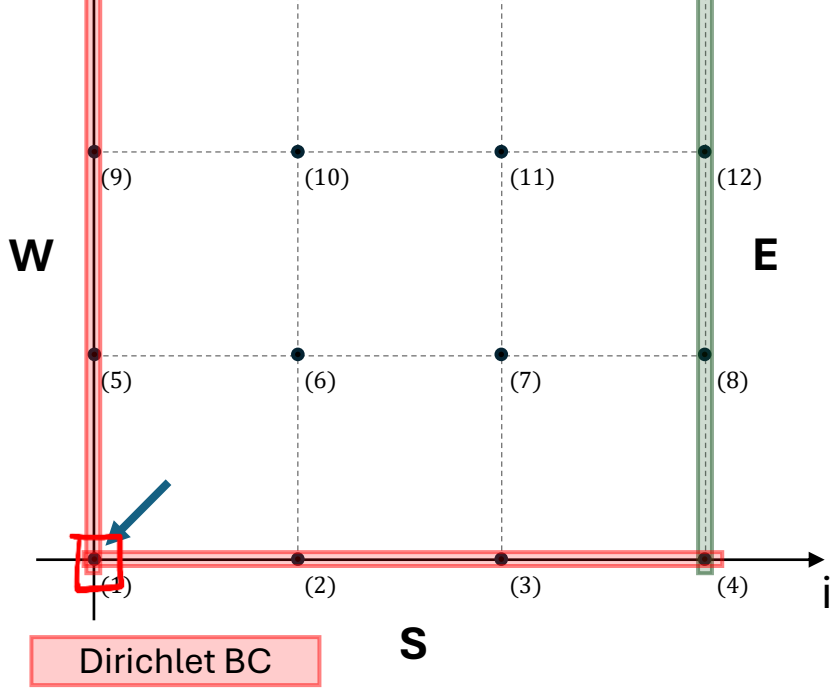
$\boldsymbol{k = 1 \quad (i = 1, j = 1)}$

S-W corner

$(D_W | D_S) \rightarrow$ mean

Nodal equation

$$\varphi_1 = \frac{1}{2}(\varphi_S + \varphi_W)$$



	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
1	1															
2																
3																
4																
5																
6																
7																
8																
9																

$$\begin{bmatrix} \varphi_1 \\ \varphi_2 \\ \varphi_3 \\ \varphi_4 \\ \varphi_5 \\ \varphi_6 \\ \varphi_7 \\ \varphi_8 \\ \varphi_9 \end{bmatrix} = \begin{bmatrix} 0.5(\varphi_S + \varphi_W) \\ \\ \\ \\ \\ \\ \\ \\ \end{bmatrix}$$

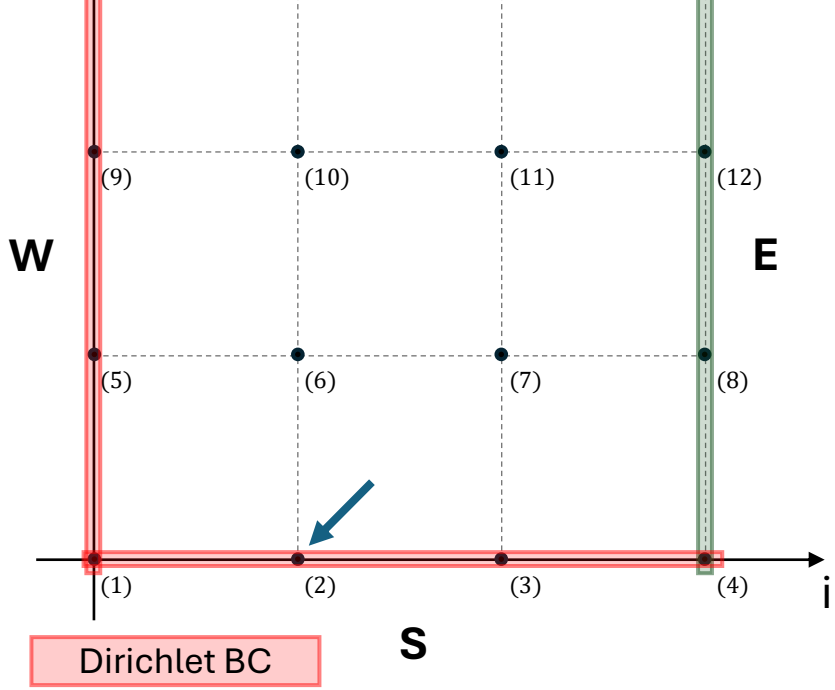
$\boldsymbol{k = 2}$ $\boldsymbol{(i = 2, j = 1)}$

S edge

D_S

Nodal equation

$\varphi_2 = \varphi_S$



	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16				
1	1																φ_1			$0.5(\varphi_S + \varphi_W)$
2		1															φ_2			φ_S
3																	φ_3			
4																	φ_4			
5																	φ_5			
6																	φ_6			
7																	φ_7			
8																	φ_8	=		
9																	φ_9			

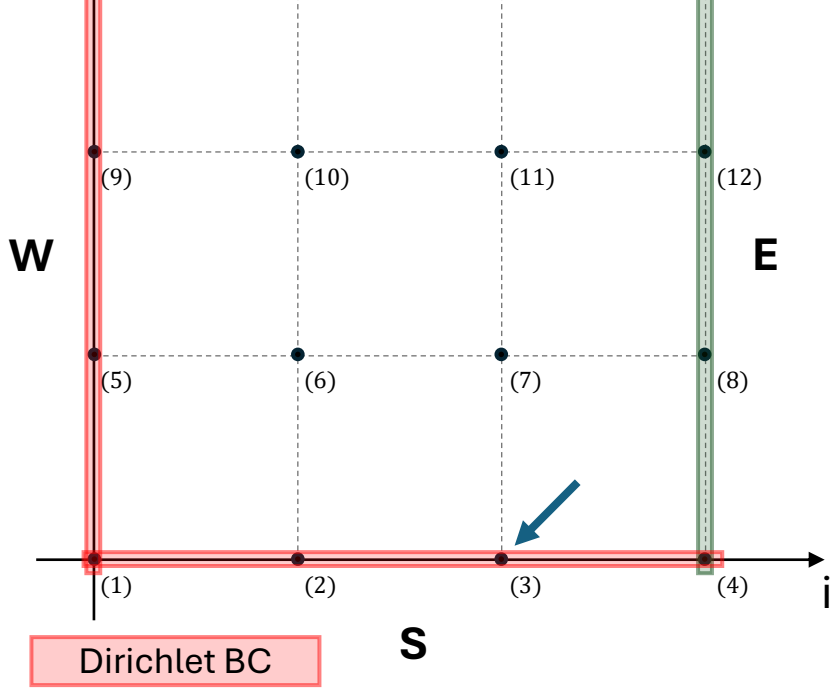
$\boldsymbol{k = 3 \quad (i = 3, j = 1)}$

S edge

D_S

Nodal equation

$\varphi_3 = \varphi_S$



	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
1	1															
2		1														
3			1													
4																
5																
6																
7																
8																
9																

$$\begin{bmatrix} \varphi_1 \\ \varphi_2 \\ \varphi_3 \\ \varphi_4 \\ \varphi_5 \\ \varphi_6 \\ \varphi_7 \\ \varphi_8 \\ \varphi_9 \end{bmatrix} = \begin{bmatrix} 0.5(\varphi_S + \varphi_W) \\ \varphi_S \\ \varphi_S \\ \\ \\ \\ \\ \\ \end{bmatrix}$$

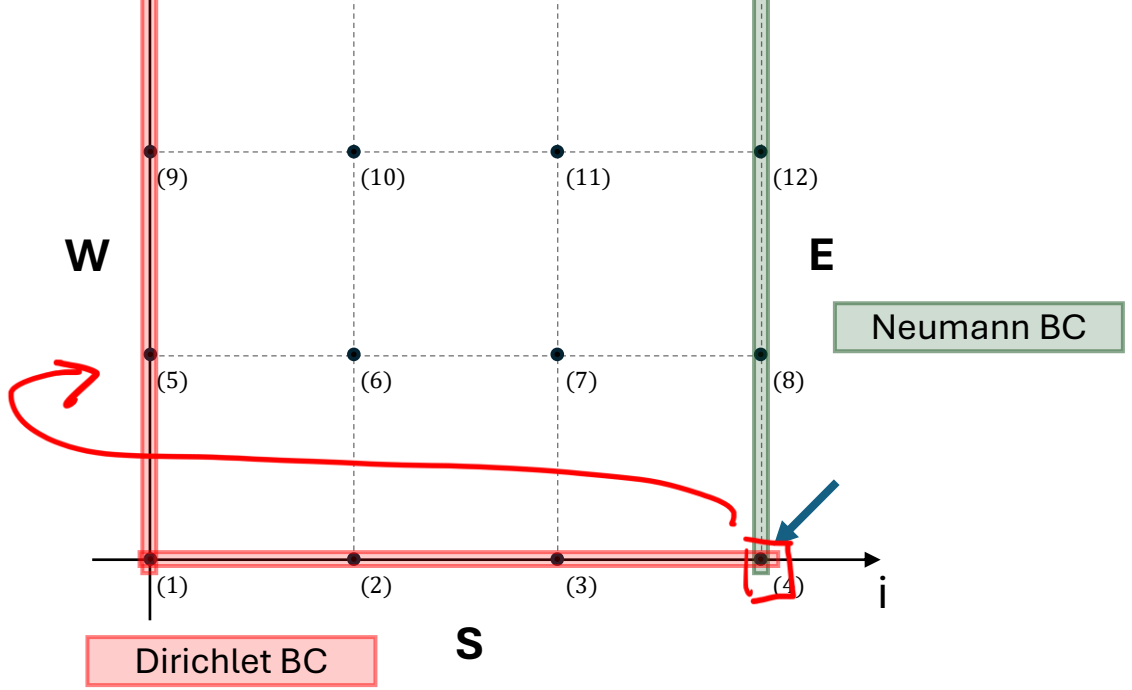
$k = 4 \quad (i = 4, j = 1)$

S-E corner

$$(D_S|N_E) \rightarrow D_S$$

Nodal equation

$$\varphi_4 = \varphi_S$$



	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
1	1															
2		1														
3			1													
4				1												
5																
6																
7																
8																
9																

$$\begin{bmatrix} \varphi_1 \\ \varphi_2 \\ \varphi_3 \\ \varphi_4 \\ \varphi_5 \\ \varphi_6 \\ \varphi_7 \\ \varphi_8 \\ \varphi_9 \end{bmatrix} = \begin{bmatrix} 0.5(\varphi_S + \varphi_W) \\ \varphi_S \\ \varphi_S \\ \varphi_S \\ \\ \\ \\ \\ \\ \end{bmatrix}$$

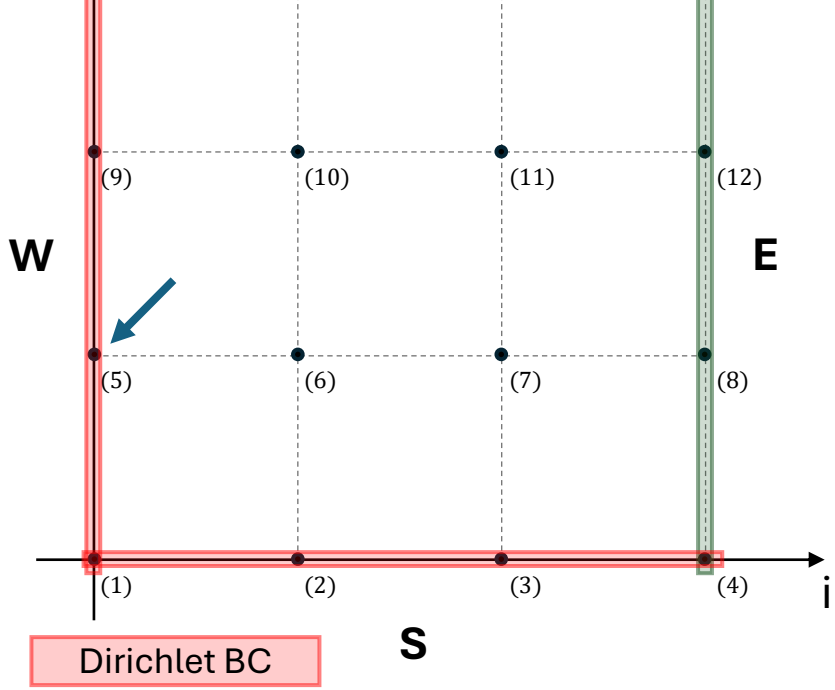
$k = 5 \quad (i = 1, j = 2)$

W edge

D_W

Nodal equation

$\varphi_5 = \varphi_W$



	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16				
1	1																φ_1		$0.5(\varphi_S + \varphi_W)$
2		1															φ_2		φ_S
3			1														φ_3		φ_S
4				1													φ_4		φ_S
5					1												φ_5		φ_W
6																	φ_6		
7																	φ_7		
8																	φ_8	=	
9																	φ_9		

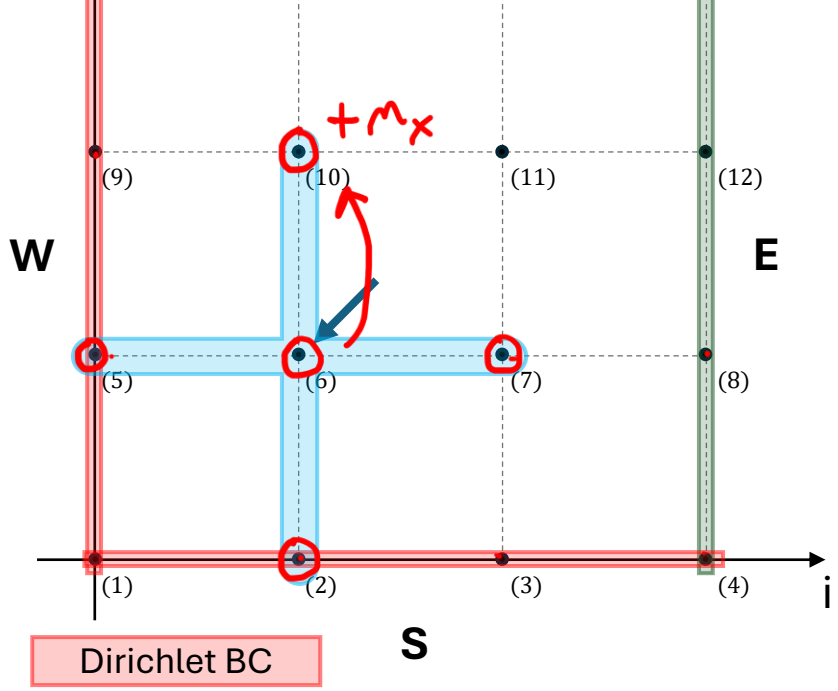
$k = 6 \quad (i = 2, j = 2)$

Internal node

Nodal equation

$$\frac{\varphi_{i+1,j}}{\Delta_x^2} + \frac{\varphi_{i,j+1}}{\Delta_y^2} - 2\left(\frac{1}{\Delta_x^2} + \frac{1}{\Delta_y^2}\right)\varphi_{i,j} + \frac{\varphi_{i-1,j}}{\Delta_x^2} + \frac{\varphi_{i,j-1}}{\Delta_y^2} = t_{i,j}$$

$$\rightarrow \underbrace{\frac{1}{\Delta_x^2}}_{k_x}\varphi_7 + \underbrace{\frac{1}{\Delta_y^2}}_{k_y}\varphi_{10} - \underbrace{2\left(\frac{1}{\Delta_x^2} + \frac{1}{\Delta_y^2}\right)}_{k_c}\varphi_6 + \underbrace{\frac{1}{\Delta_x^2}}_{k_x}\varphi_5 + \underbrace{\frac{1}{\Delta_y^2}}_{k_y}\varphi_2 = t_6$$



	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
1	1															
2		1														
3			1													
4				1												
5					1											
6		k_y			k_x	k_c	k_x			k_y						
7																
8																
9																

φ_1		$0.5(\varphi_S + \varphi_W)$
φ_2		φ_S
φ_3		φ_S
φ_4		φ_S
φ_5		φ_W
φ_6	=	t_6
φ_7		
φ_8		
φ_9		

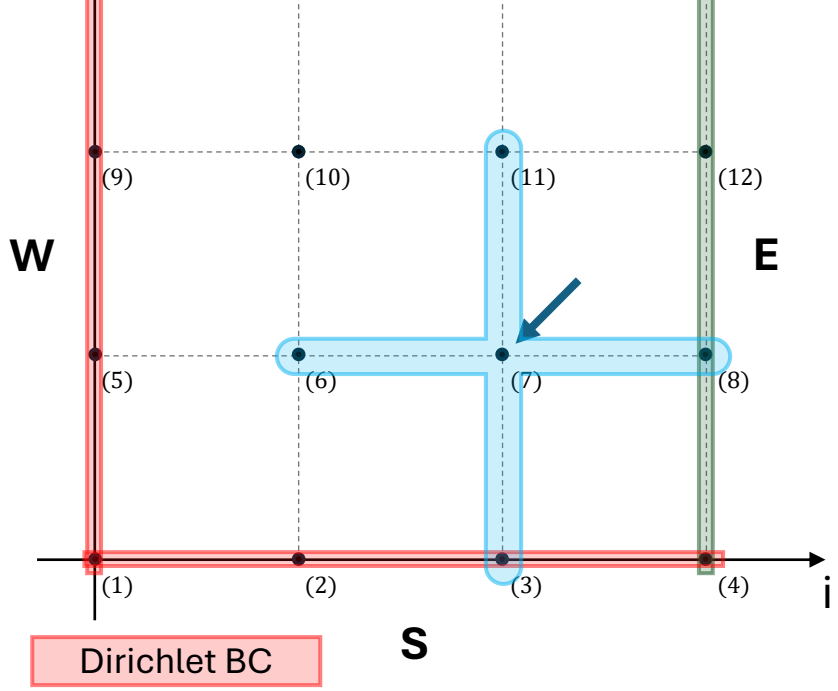
$k = 7 \quad (i = 3, j = 2)$

Internal node

Nodal equation

$$\frac{\varphi_{i+1,j}}{\Delta_x^2} + \frac{\varphi_{i,j+1}}{\Delta_y^2} - 2\left(\frac{1}{\Delta_x^2} + \frac{1}{\Delta_y^2}\right)\varphi_{i,j} + \frac{\varphi_{i-1,j}}{\Delta_x^2} + \frac{\varphi_{i,j-1}}{\Delta_y^2} = t_{i,j}$$

$$\rightarrow \underbrace{\frac{1}{\Delta_x^2}}_{k_x}\varphi_8 + \underbrace{\frac{1}{\Delta_y^2}}_{k_y}\varphi_{11} - \underbrace{2\left(\frac{1}{\Delta_x^2} + \frac{1}{\Delta_y^2}\right)}_{k_c}\varphi_7 + \underbrace{\frac{1}{\Delta_x^2}}_{k_x}\varphi_6 + \underbrace{\frac{1}{\Delta_y^2}}_{k_y}\varphi_3 = t_7$$



	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16				
1	1																φ_1		$0.5(\varphi_S + \varphi_W)$
2		1															φ_2		φ_S
3			1														φ_3		φ_S
4				1													φ_4		φ_S
5					1												φ_5		φ_W
6		k_y			k_x	k_c	k_x			k_y							φ_6		t_6
7			k_y			k_x	k_c	k_x			k_y						φ_7		t_7
8																	φ_8	=	
9																	φ_9		

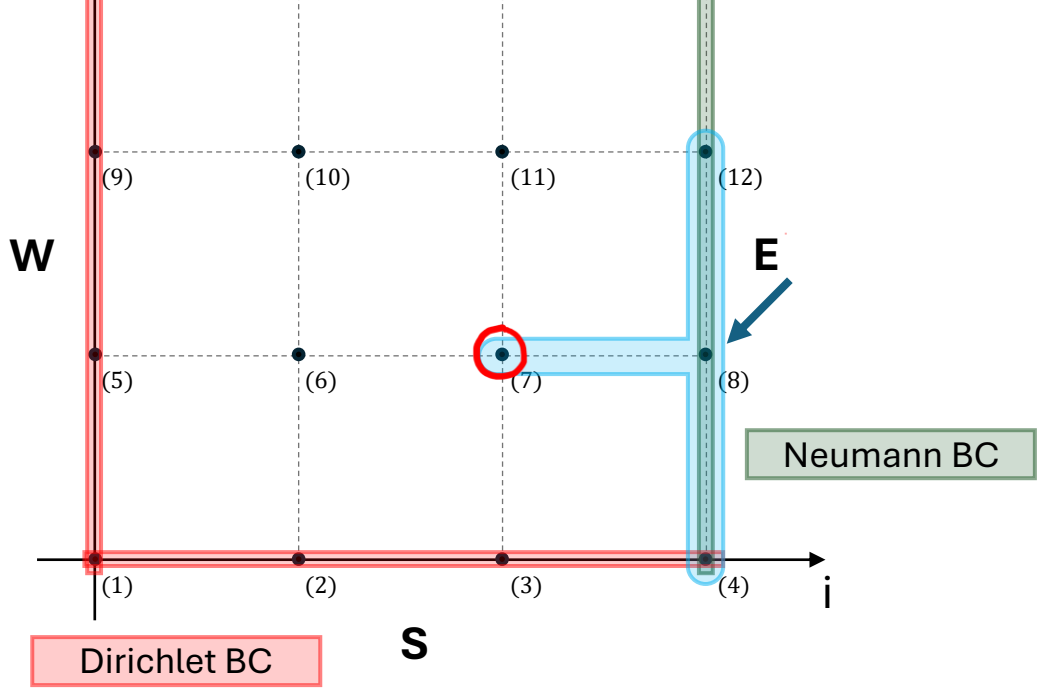
$k = 8 \quad (i = 4, j = 2)$

Boundary node – Neumann BC

Nodal equation

$$\frac{\varphi_{i,j+1}}{\Delta_y^2} - 2\left(\frac{1}{\Delta_x^2} + \frac{1}{\Delta_y^2}\right)\varphi_{i,j} + 2\frac{\varphi_{i-1,j}}{\Delta_x^2} + \frac{\varphi_{i,j-1}}{\Delta_y^2} = t_{i,j} - \frac{2\varphi'_E}{\Delta_x}$$

$$\rightarrow \underbrace{\frac{1}{\Delta_y^2}}_{k_y}\varphi_{12} - \underbrace{2\left(\frac{1}{\Delta_x^2} + \frac{1}{\Delta_y^2}\right)}_{k_c}\varphi_8 + \underbrace{\frac{2}{\Delta_x^2}}_{2k_x}\varphi_7 + \underbrace{\frac{1}{\Delta_y^2}}_{k_y}\varphi_4 = t_8 - \frac{2\varphi'_E}{\Delta_x}$$



	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16				
1	1																φ_1		$0.5(\varphi_S + \varphi_W)$
2		1															φ_2		φ_S
3			1														φ_3		φ_S
4				1													φ_4		φ_S
5					1												φ_5		φ_W
6		k_y			k_x	k_c	k_x			k_y							φ_6		t_6
7			k_y			k_x	k_c	k_x			k_y						φ_7		t_7
8				k_y			$2k_x$	k_c				k_y					φ_8	=	$t_8 - \frac{2\varphi'_E}{\Delta_x}$
9																	φ_9		

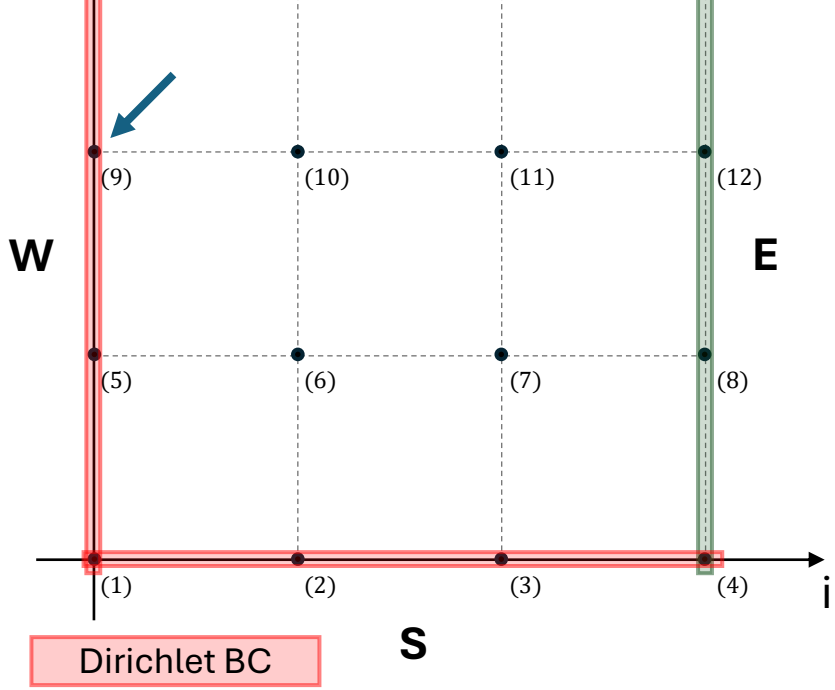
$k = 9 \quad (i = 1, j = 3)$

W edge

D_W

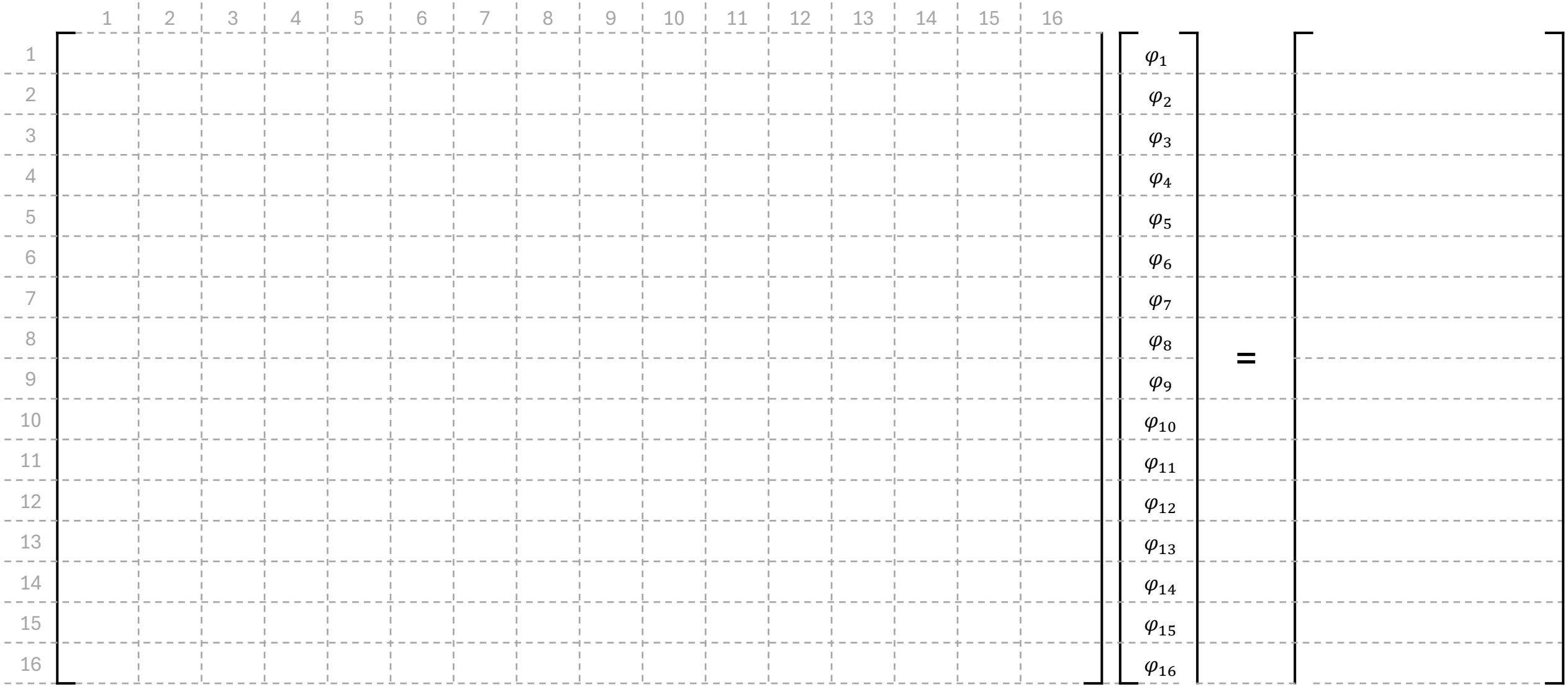
Nodal equation

$\varphi_9 = \varphi_W$



SPARSE : many zero elements

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16				
1	1																φ_1			$0.5(\varphi_S + \varphi_W)$
2		1															φ_2			φ_S
3			1														φ_3			φ_S
4				1													φ_4			φ_S
5					1												φ_5			φ_W
6		k_y			k_x	k_c	k_x			k_y							φ_6			t_6
7			k_y			k_x	k_c	k_x			k_y						φ_7			t_7
8				k_y			$2k_x$	k_c				k_y					φ_8			$t_8 - \frac{2\varphi'_E}{\Delta_x}$
9									1								φ_9			φ_W


1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16
φ_1 φ_2 φ_3 φ_4 φ_5 φ_6 φ_7 φ_8 φ_9 φ_{10} φ_{11} φ_{12} φ_{13} φ_{14} φ_{15} φ_{16}
=