Modelling and Computation of Electric and Magnetic Fields – Examples of Questions

Arturo Popoli

January 27, 2025

The following questions are **examples** of possible questions assigned for the exam. All theoretical topics covered during the course are part of the exam syllabus.

To achieve full marks, **the answer must include** all the main concepts covered in the lesson regarding the given topic, including calculations, considerations, and explanatory diagrams/graphs.

1: Discuss the following topology concepts

(a) Definitions of metric space, connected metric space and domain

(b) Simply-connected domains in 2D and 3D.

(c) Reduction methods for multiply-connected domains in 2D and 3D

2: Vector operators:

(a) Define and discuss the properties of the gradient, divergence, and curl operators

(b) Discuss the link between the divergence theorem and solenoidal fields

(c) Discuss the link between Stokes theorem and conservative fields

3: Gauge freedom

(a) Discuss the concept of *gauge freedom* for conservative and solenoidal fields

(b) Discuss the general problem of computing conservative and solenoidal fields, in relation to the use of scalar and vector potentials

4: Clebsh Theorem

(a) Provide a statement of Clebsh's theorem

(b) Check that the theorem works, by decomposing a generic vector field $\boldsymbol{U}$ into a conservative and a solenoidal part.

5: Basics of EM

(a) Discuss Lorentz force, providing examples

(b) Provide a definition of current density and volume charge density

6: Polarization and Magnetization

(a) Discuss the physical effects of electric polarization and magnetization

(b) Show the expression of Maxwell's equations can be extended to account for the two above phoenomena

7: Discuss Maxwell's equations in matter and material constitutive laws

8: Discuss Poynting's theorem for linear and isotropic materials

9: Discuss the uniqueness theorem for general electromagnetic problems

10: Green's identities

(a) Discuss the first and second Green's identities

11: Harmonic functions

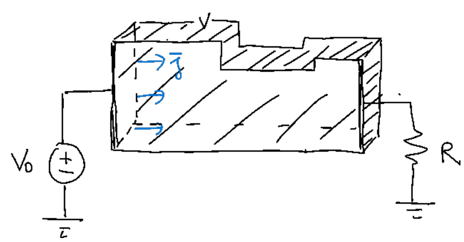
(a) Define a harmonic function and provide examples of harmonic functions

(b) Discuss the mean value theorem for harmonic functions and its three corollaries

12: Exercise: uniform conductive plate

(a) Derive an appropriate formulation to find the electric potential distribution in the geometry below, assuming a uniform electrical conductivity $\sigma = \sigma_0$.

(b) Discuss the values of the α and β coefficients in $\alpha\varphi + \beta\frac{\partial\varphi}{\partial n} = \gamma$ in relation to the uniqueness of the solution.



13: Prove that the following classes of problems have a unique solution

(a) Poisson problems w/ Dirichlet BCs

(b) Poisson problems w/ Dirichlet and Neumann BCs

(c) Poisson problems w/ Dirichlet and Robin BCs ($\alpha\varphi + \beta\frac{\partial\varphi}{\partial n} = \gamma$)

14: Computer arithmetics

(a) Discuss the main features of fixed-point and floating-point representations, and derive estimates for their respective relative and absolute errors

(b) Discuss briefly the main characteristics of the IEEE 754 standard

15: Numerical differentiation

Starting from the theoretical definition of a first-order derivative:

(a) Use Taylor series to derive and discuss expressions for forward/backward/centered finite differences

(b) Sketch a visual representation of the three approaches

(c) Derive a centered expression for a second-order derivative

. . . and provide a visual interpretation of the three approaches

16: Piecewise Linear Interpolation

(a) Discuss the concept of piecewise linear interpolation in 1D, also justifying the need for piecewise linear interpolation based on the drawbacks of polynomial interpolation

(b) Show how can the concept be extended to triangles in 2D?

17: Numerical integration

Discuss:

(a) Rectangles method

(b) Trapezoid method

(c) Gauss integration

    (a) Definition of quadrature rules

    (b) Extension to arbitrary intervals

18: Finite Difference Method (FDM) - Application to a 1D Poisson problem

Topics to be discussed:

(a) Mathematical formulation (use Dirchlet BCs on one side of the domain, and Neumann BCs on the other one)

(b) Discretization $\longrightarrow$ linear system assembly

19: Finite Difference Method (FDM) - Application to a 2D Poisson problem

Topics to be discussed:

(a) Mathematical formulation (use Dirchlet BCs on one side of the domain, and Neumann BCs on the other one)

(b) Discretization $\longrightarrow$ linear system assembly

20: Finite Element Method (FEM): application to a Poisson problem in 1D

Topics to be discussed:

(a) Piecewise interpolation on triangles

21: Finite Element Method (FEM): application to a Poisson problem in 2D

Topics to be discussed:

(a) Piecewise interpolation on triangles

(b) Strong formulation $\longrightarrow$ weighted residuals

(c) Galerkin's approach $\longrightarrow$ weak formulation

(d) Derivation of nodal equations

(e) Linear system assembly