MAXWELL'S EQNS. in MATTER

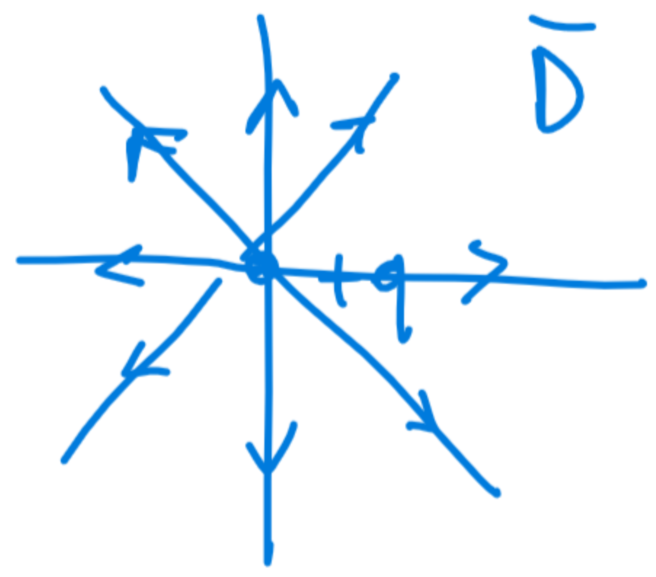
L.F.

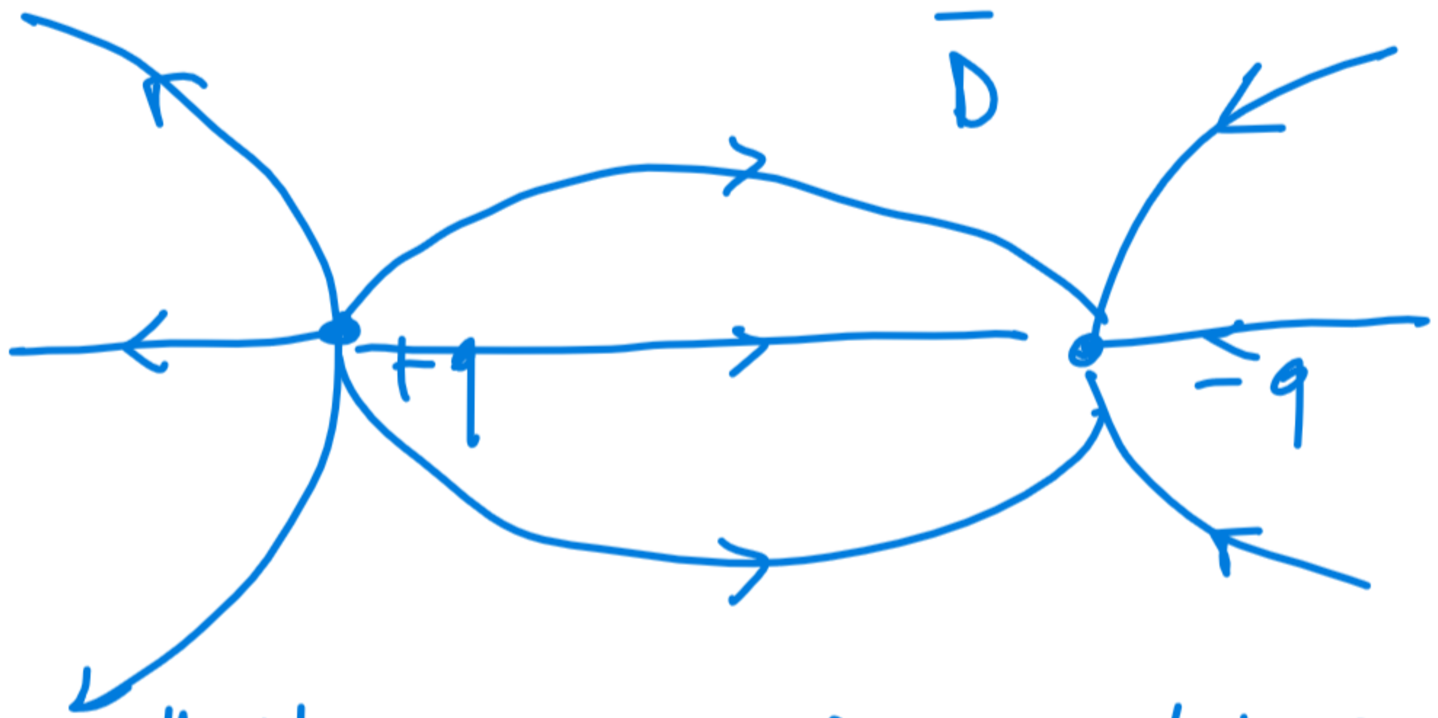
$$\nabla \cdot \bar{D} = \rho$$

$$\int_V \bar{D}\, dV = \int_V \rho\, dV \longrightarrow$$

I.F.

$$\oint_S \bar{D} \cdot d\bar{S} = Q$$





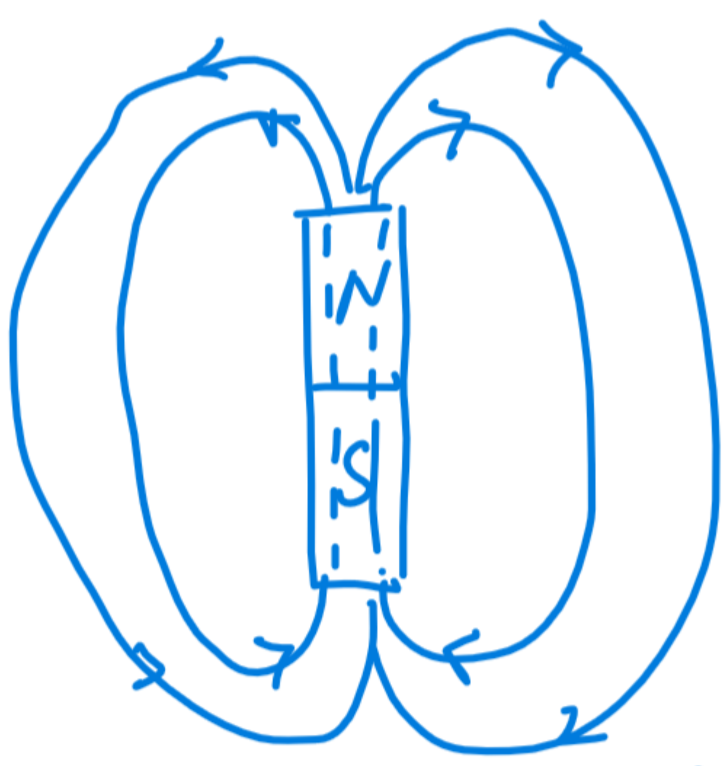
" charges are sources / sinks for $\bar{D}$ field lines "

" Flux of $\bar{D}$ through ANY closed surface S equals the net charge inside S "

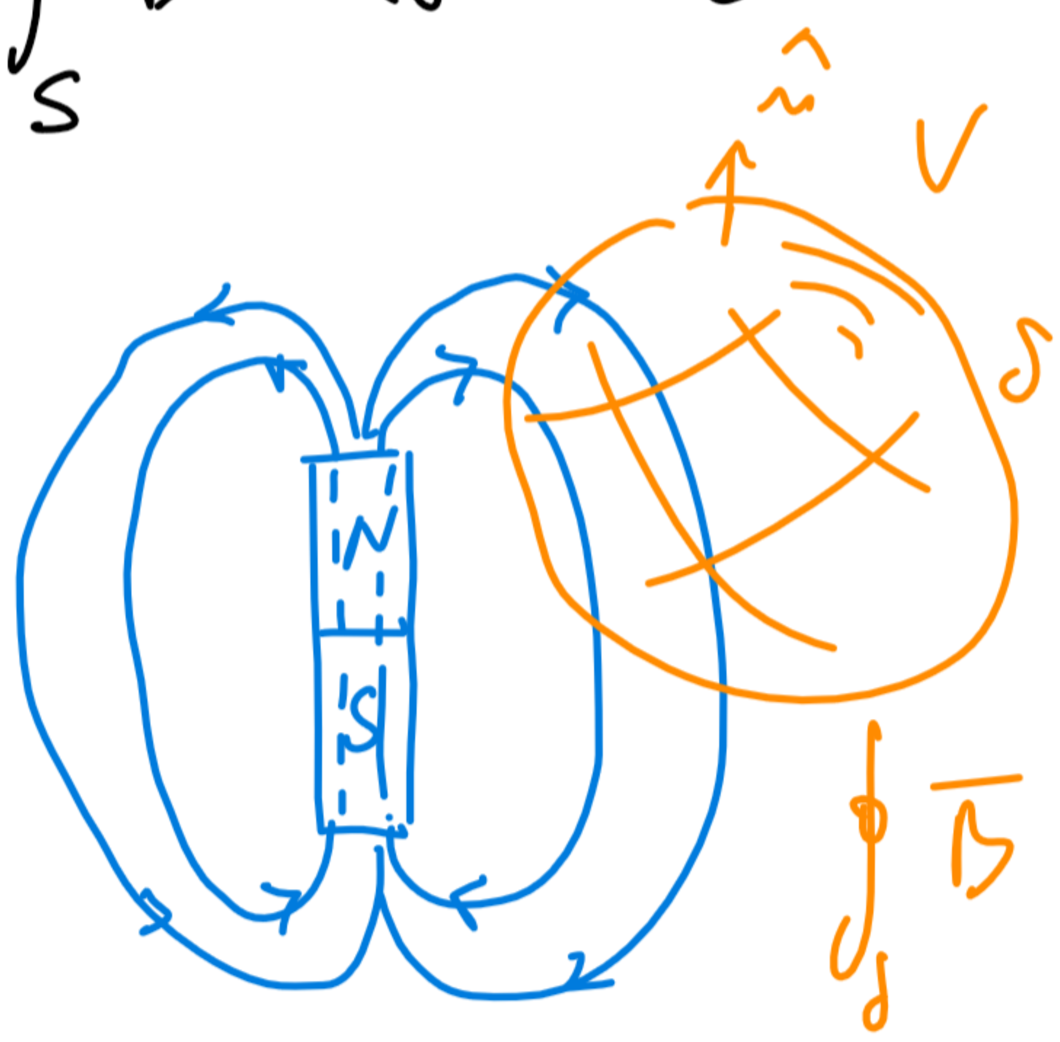
$$\nabla \cdot \bar{B} = 0$$

$$\int_V \nabla \cdot \bar{B}\, dV = 0 \longrightarrow$$

$$\oint_S \bar{B} \cdot d\bar{S} = 0$$



" Field lines of $\bar{B}$ are always closed "



$$\oint_S \bar{B} \cdot d\bar{S} = 0$$

" The flux of $\bar{B}$ through ANY closed surface is ZERO "

$$\nabla\times\bar E = -\partial\bar B/\partial t \xrightarrow[\text{STOKES}]{\int_S \nabla\times\bar E\cdot d\bar S = \int_S -\partial\bar B/\partial t\, dS} \oint_\Gamma \bar E\cdot d\bar l = \int_S -\partial\bar B/\partial t \cdot dS$$

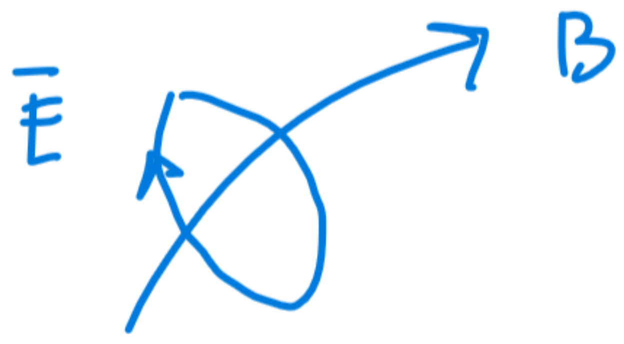
ELECTROMOTIVE FORCE → $\mathcal{E} = -\, d/dt \int_S \bar B\cdot dS$

"Field lines $\bar E$ are CURLED around the field lines of $\bar B$"

FLUX of $\bar B$ LINKED with Γ CURVE BOUNDING S ⇒ $\mathcal{E} = -\, d/dt\; \Phi_{B,\Gamma}$

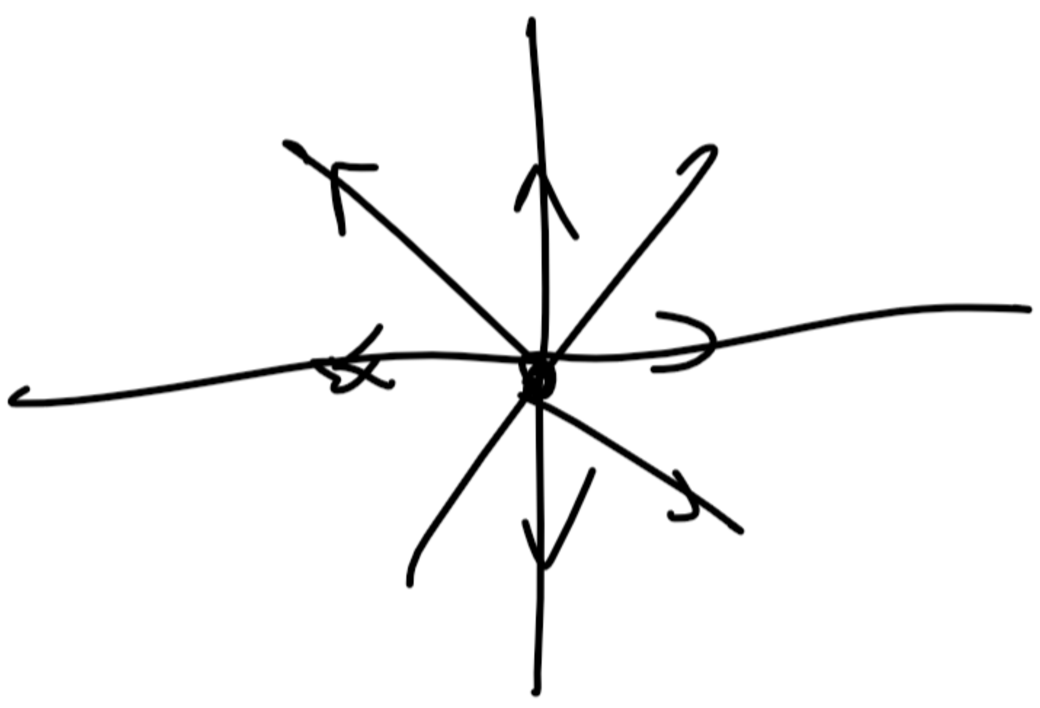
"Time-variations of the linked magnetic flux with a curve Γ produce a circulation of $\bar E$ on Γ"

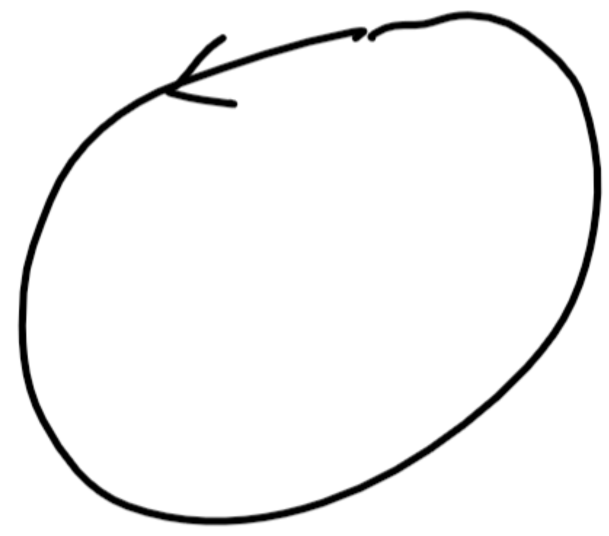
↳ if Γ is conductor, $\mathcal{E}$ will produce a current i which will "oppose" the flux that generated the $\mathcal{E}$



↑ ($\bar E$-field due to charges)

$\bar E$ is ≠ from ELECTROSTATIC FIELD





CLEBSH THEOREM

$$\bar U = \nabla f + \nabla\times\bar F$$

⇑ $\bar E_{CHARGE}$ ⇑ $\bar E_{FNL}$

↑ $\nabla^2 f = t$ ↑ $\nabla^2 \bar F = -\bar T$

$\bar E$ {

Electrostatic field $\bar E_{CHARGE}$ →
- Field lines OPEN
- is conservative

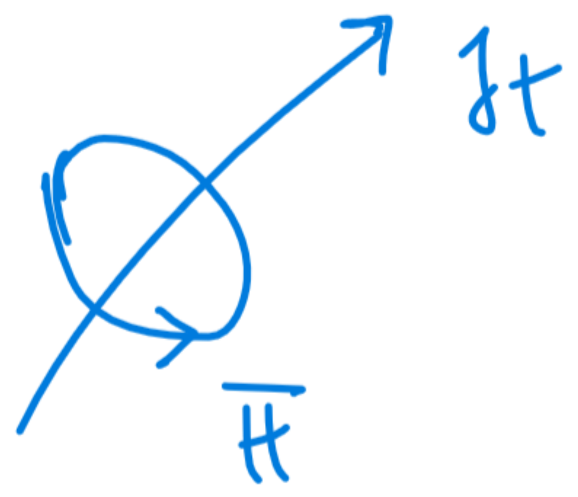
$$\oint_\Gamma \bar E_{CHARGE} = 0$$

F-N-L electric field $\bar E_{FNL}$ →
- CLOSED field lines
- is not conservative

$$\oint_\Gamma \bar E_{FNL} = \mathcal{E}$$

$$\nabla \times \vec{H} = \underbrace{\bar{J} + \partial \bar{D}/\partial t}_{\bar{J}_t} \longrightarrow \oint_{\Gamma} \bar{H} \cdot d\bar{\ell} = \int_{S} \left(\bar{J} + \partial \bar{D}/\partial t\right) \cdot d\bar{S}$$

"Field lines of $\bar{H}$ are CURLED around the ones of $\bar{J}_t$"



magnetomotive force

conduction current

DISPLACEM. CURRENT

$$MMF = \underbrace{i_C + i_D}_{i_T}$$

TOTAL CURRENT ENCLOSED BY Γ

LINKED FLUX of $\bar{J}_t$

"The linked flux of the total current density $\bar{J}_t$ produces a circulation of $\bar{H}$ around the closed path Γ (MMF)"

(charge conservation equation)

Current continuity equation

CONDUCTION CURRENT

$$\frac{\partial \rho}{\partial t} + \nabla \cdot \bar{J} = 0$$

$$\nabla \cdot \bar{J} = -\partial \rho/\partial t \xrightarrow[\text{V DOES NOT CHANGE OVER time}]{\text{GAUSS}} \int_V \nabla \cdot \bar{J}\, dV = \int_V -\partial \rho/\partial t\, dV$$

$$\oint_S \bar{J} \cdot d\bar{S} = -d/dt \int \rho\, dV$$

$$i = -\frac{dQ}{dt}$$

current leaving a closed surface corresponds to a DECREASE of the enclosed net charge

$$\Rightarrow \text{if } dQ/dt = 0 \text{ for a } \underbrace{\text{closed surface } V}_{\text{system}} \Rightarrow i = 0 \Rightarrow \text{ISOLATED SYSTEM}$$

MATERIAL CONSTITUTIVE RELATIONS

+ M.E. in matter
continuity eq

for linear and isotropic materials

$$\begin{cases} \bar{D} = \varepsilon_0 \varepsilon_r \bar{E} \\ \bar{B} = \mu_0 \mu_r \bar{H} \\ \bar{J} = \sigma(\bar{E} + \bar{E}_i) \end{cases}$$

$\rightarrow$ LOCAL OHM'S LAW

$$\bar{J} = \sigma \vec{E}$$

$\Uparrow$ Electrical Conductivity [S/m]

$\Uparrow$ ELECTROMOTIVE FIELDS // IMPRESSED FIELDS

(Electrical fields produced by non-electrical sources)

$$\oint_\Gamma \bar{E} \cdot d\bar{l} = 0 \qquad \oint \bar{E}_i \cdot d\bar{l} = \mathcal{E}$$

Poynting theorem

- ENERGY conservation principle for EM fields
- Hp: Linear and Isotropic materials

$$\begin{cases} \nabla \times \bar{E} = -\partial \bar{B}/\partial t \rightarrow \bar{H} \cdot \nabla \times \bar{E} = \bar{H} \cdot (-\partial \bar{B}/\partial t) & (1) \\ \nabla \times \bar{H} = \bar{J} + \partial \bar{D}/\partial t \Rightarrow \bar{E} \cdot \nabla \times \bar{H} = \bar{E} \cdot (\bar{J} + \partial \bar{D}/\partial t) & (2) \\ \bar{D} = \varepsilon \bar{E}, \quad \varepsilon = \varepsilon_0 \varepsilon_r \\ \bar{B} = \mu \bar{H}, \quad \mu = \mu_0 \mu_r \\ \bar{J} = \sigma(\bar{E} + \bar{E}_i) \end{cases}$$

$\downarrow$ (1) − (2)

$$\overbrace{\bar{H} \cdot \nabla \times \bar{E} - \bar{E} \cdot \nabla \times \bar{H}}^{\mathrm{I}} = \overbrace{-\bar{E} \cdot \bar{J}}^{\mathrm{II}} \overbrace{- \bar{E} \cdot \partial \bar{D}/\partial t}^{\mathrm{III}} \overbrace{- \bar{H}\, \partial \bar{B}/\partial t}^{\mathrm{IV}}$$

I: $\nabla \cdot (\bar{A} \times \bar{B}) = \bar{B} \cdot (\nabla \times \bar{A}) - \bar{A} \cdot (\nabla \times \bar{B})$

$\hookrightarrow \nabla \cdot (\bar{E} \times \bar{H})$

$$\overbrace{\bar H\cdot\nabla\times\bar E - \bar E\cdot\nabla\times\bar H}^{\mathrm{I}} = \overbrace{-\bar E\cdot\bar J}^{\mathrm{II}} - \overbrace{\bar E\cdot\partial\bar D/\partial t}^{\mathrm{III}} - \overbrace{\bar H\,\partial\bar B/\partial t}^{\mathrm{IV}}$$

$$\mathrm{II}:\quad \bar E\cdot\bar J = \left(\bar J/\sigma - \bar E_i\right)\cdot\bar J = \underbrace{J^2/\sigma}_{W/m^3} - \underbrace{\bar E_i\cdot\bar J}_{W/m^3}$$

$$\bar J = \sigma(\bar E + \bar E_i)$$

$$\Rightarrow \bar E = \bar J/\sigma - \bar E_i$$

HP: ε is constant over time

$$\mathrm{III}:\quad \underset{\bar D=\varepsilon\bar E}{\bar E\cdot\partial\bar D/\partial t =} \bar E\cdot\frac{\partial\varepsilon\bar E}{\partial t} = \varepsilon\,\underbrace{\bar E\cdot\partial\bar E/\partial t}_{\frac12\frac{\partial E^2}{\partial t}} = \frac12\varepsilon\,\partial E^2/\partial t$$

$[\frac12\varepsilon E^2] = [J/m^3]$ ELECTRIC ENERGY DENSITY

$[d/dt\,\frac12\varepsilon E^2] = [W/m^3]$ ELECTRIC POWER DENSITY

$$\mathrm{IV}:\quad \underset{\bar B=\mu\bar H}{\bar H\cdot\partial\bar B/\partial t =} \frac1\mu\,\bar B\cdot\underbrace{\frac{\partial\bar B}{\partial t}}_{\frac12\,\partial B^2/\partial t} = \frac1{2\mu}\,\partial B^2/\partial t$$

$$\Rightarrow \bar H = \bar B/\mu$$

$$\nabla\cdot(\bar E\times\bar H) = -J^2/\sigma + \bar E_i\cdot\bar J - \frac12\varepsilon\frac{\partial E^2}{\partial t} - \frac1{2\mu}\,\partial B^2/\partial t$$

POWER DENSITIES

$$\bar E_i\cdot\bar J = J^2/\sigma + \underbrace{\frac12\varepsilon\,\partial E^2/\partial t + \frac1{2\mu}\,\partial B^2/\partial t}_{} + \nabla\cdot(\bar E\times\bar H)$$

V is not changing over time

$$\int_V \bar E_i\cdot\bar J\,dV = \int_V J^2/\sigma\,dV + d/dt\int_V \frac12\varepsilon E^2\,dV + d/dt\int_V \frac1{2\mu}B^2\,dV + \int_V \nabla\cdot(\bar E\times\bar H)\,dV$$

$$\frac{J}{s\,m^3} = \frac{W}{m^3}$$

$\boxed{\bar E\times\bar H:\ \text{POYNTING VECTOR}}$

GAUSS: $\oint_S \bar E\times\bar H\cdot d\bar S \Rightarrow$ POYNTING VECTOR FLUX