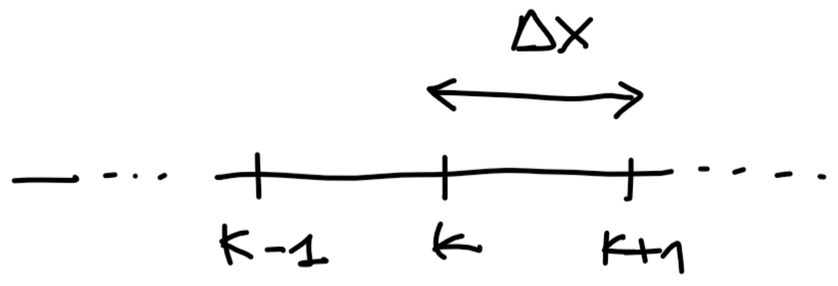
SECOND DERIVATIVE



$$f_{k+1} = f_k + \frac{df(x)}{dx}\bigg|_k \Delta x + \frac{1}{2}\frac{d^2f(x)}{dx^2}\bigg|_k \Delta x^2 + \frac{1}{6}\frac{d^3f(x)}{dx^3}\bigg|_k \Delta x^3 + O(\Delta x^4) \qquad (1)$$

$$f_{k-1} = f_k - \frac{df(x)}{dx}\bigg|_k \Delta x + \frac{1}{2}\frac{d^2f(x)}{dx^2}\bigg|_k \Delta x^2 - \frac{1}{6}\frac{d^3f(x)}{dx^3}\bigg|_k \Delta x^3 + O(\Delta x^4) \qquad (2)$$

(1) + (2)

$$f_{k+1} + f_{k-1} = 2f_k + \frac{d^2f(x)}{dx^2}\bigg|_k \Delta x^2 + O(\Delta x^4)$$

$$\boxed{\frac{d^2f(x)}{dx^2}\bigg|_k = \frac{f_{k+1} - 2f_k + f_{k-1}}{\Delta x^2} + O(\Delta x^2)}$$

Centered finite difference formula for second derivative

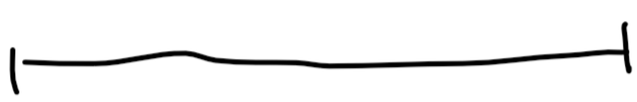
ACCURACY : 2nd order

To Do:

- Develop FORWARD / BACKWARD expressions $O(\Delta x)$ for $\frac{d^2f(x)}{dx^2}$

Hint: $f_{k+1} = f_k + \dots$

$f_{k+2} = f_k + \dots$



$$f(x) = 2x^4 + 3x^3 + x^2 + 0{,}5x - 3$$

$$\frac{df(x)}{dx} = 8x^3 + 9x^2 + 2x + 0{,}5$$

$$\frac{df}{dx}\bigg|_k \quad \begin{cases} \dfrac{f_{k+1} - f_k}{\Delta x} \;;\; \dfrac{f_k - f_{k-1}}{\Delta x} \\ \dfrac{f_{k+1} - f_{k-1}}{2\Delta x} \end{cases}$$

$$\left.\frac{dy}{dx}\right|_k \begin{cases} \frac{f_{k+1} - f_k}{\Delta x} \;;\; \frac{f_k - f_{k-1}}{\Delta x} \\ \frac{f_{k+1} - f_{k-1}}{2\Delta x} \end{cases}$$

define h

$$\frac{f(x+h) - f(x)}{h} \quad \text{FWD}$$

$$\frac{f(x) - f(x-h)}{h} \quad \text{BWD}$$

$$\frac{f(x+h) - f(x-h)}{2h} \quad \text{CD}$$